Computer algebra independent integration tests

4-Trig-functions/4.1-Sine/4.1.2.1-a+b-sin-^m-c+d-sin-^n

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May 18, 2020 Compiled on May 18, 2020 at 6:44am

Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [837]. This is test number [73].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (837)	% 0. (0)
Mathematica	% 97.97 (820)	% 2.03 (17)
Maple	% 75.87 (635)	% 24.13 (202)
Maxima	% 25.81 (216)	% 74.19 (621)
Fricas	% 61.17 (512)	% 38.83 (325)
Sympy	% 14.7 (123)	% 85.3 (714)
Giac	% 38.59 (323)	% 61.41 (514)

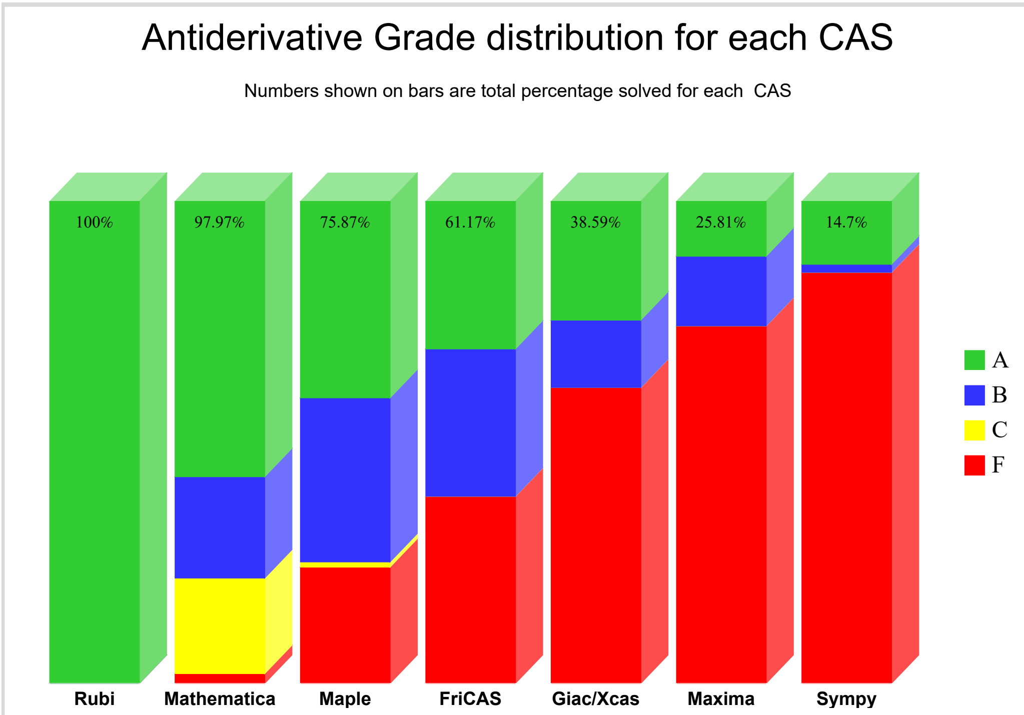
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

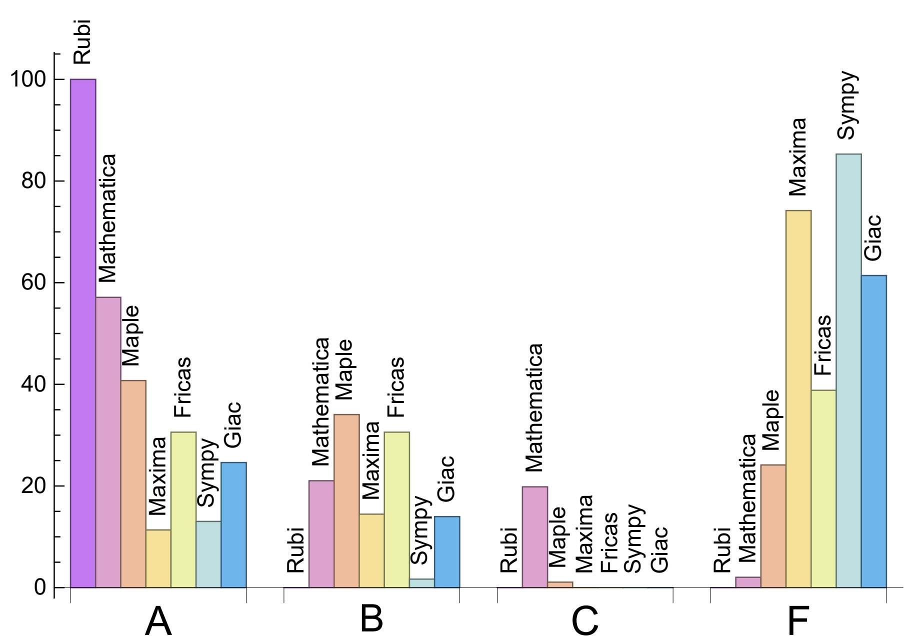
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	100.	0.	0.	0.
Mathematica	57.11	21.03	19.83	2.03
Maple	40.74	34.05	1.08	24.13
Maxima	11.35	14.46	0.	74.19
Fricas	30.59	30.59	0.	38.83
Sympy	13.02	1.67	0.	85.3
Giac	24.61	13.98	0.	61.41

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.37	170.11	0.99	122.	1.
Mathematica	3.43	1685.59	6.49	173.5	1.44
Maple	1.72	37291.8	56.75	193.	1.76
Maxima	1.64	417.28	4.28	238.	3.02
Fricas	2.13	1122.66	7.71	541.	5.45
Sympy	16.02	516.32	5.67	314.	3.11
Giac	1.75	354.95	2.82	205.	2.07

1.4 list of integrals that has no closed form antiderivative

{221, 226, 801, 805, 806, 807, 808, 809, 810, 811, 812, 813, 831}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {665, 666, 667}

Mathematica {97, 102, 103, 104, 109, 115, 116, 117, 118, 119, 120, 122, 123, 124, 125, 126, 127, 128, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 141, 142, 143, 145, 146, 148, 165, 174, 211, 216, 217, 218, 224, 225, 404, 406, 407, 408, 409, 410, 415, 416, 422, 423, 424, 482, 483, 484, 485, 486, 487, 488, 542, 563, 571, 579, 580, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 624, 627, 629, 630, 631, 632, 635, 638, 640, 641, 642, 643, 645, 646, 648, 652, 653, 654, 658, 665, 666, 667, 668, 669, 670, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 787, 788, 789, 790, 792, 793, 794, 795, 796, 797, 798, 799, 800, 803, 804, 814, 816, 819, 822, 828, 829, 830, 835, 836, 837}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
```

```
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in-

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

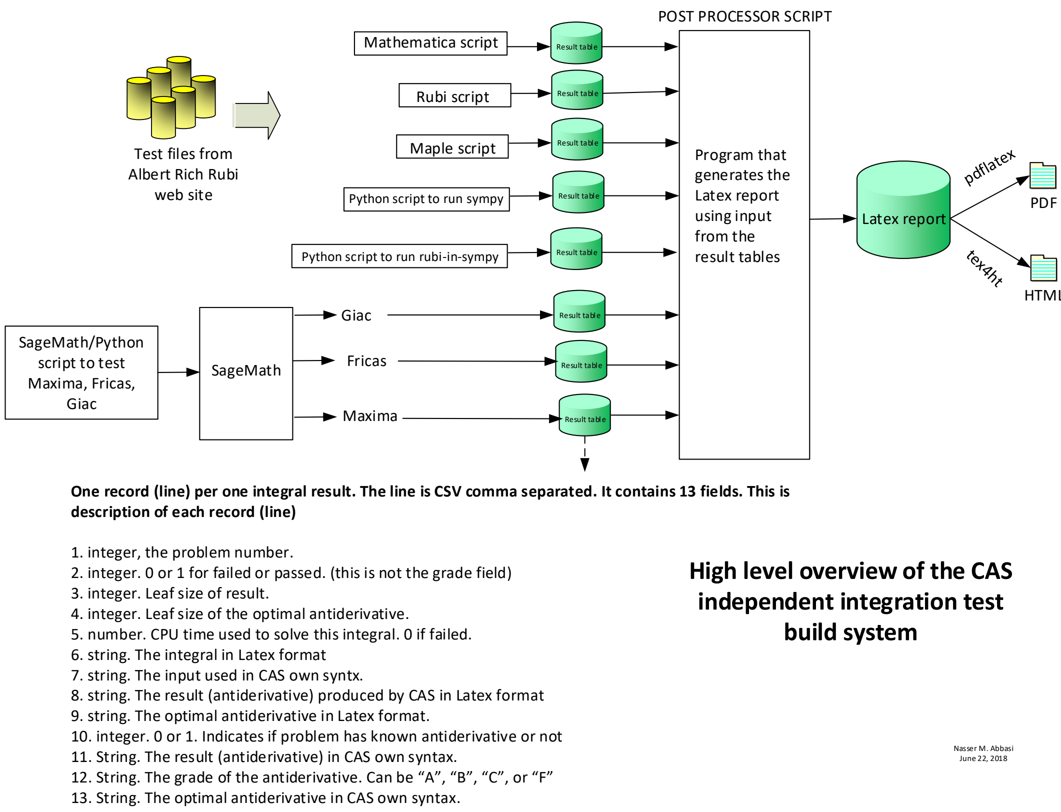
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 5, 10, 12, 13, 14, 15, 16, 21, 22, 23, 24, 25, 26, 27, 32, 33, 34, 35, 44, 45, 46, 47, 48, 51, 52, 53, 54, 55, 56, 57, 59, 91, 92, 93, 94, 95, 104, 105, 106, 107, 110, 111, 112, 113, 144, 147, 148, 149, 150, 151, 152, 154, 155, 156, 157, 158, 159, 160, 164, 166, 167, 168, 169, 170, 171, 172, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 207, 208, 209, 212, 213, 214, 215, 219, 220, 221, 224, 225, 226, 227, 228, 229, 230, 233, 234, 235, 236, 237, 238, 239, 240, 242, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 257, 258, 259, 260, 261, 262, 265, 266, 267, 268, 269, 270, 272, 274, 275, 276, 277, 278, 279, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 294, 295, 296, 297, 299, 300, 307, 309, 317, 318, 319, 320, 324, 325, 326, 327, 332, 333, 334, 335, 340, 341, 342, 343, 348, 349, 350, 351, 352, 353, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 381, 382, 383, 384, 386, 387, 388, 389, 390, 391, 393, 394, 395, 396, 397, 398, 401, 402, 403, 411, 412, 413, 419, 420, 425, 426, 427, 428, 429, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 458, 459, 460, 462, 463, 465, 466, 467, 468, 469, 474, 475, 476, 477, 478, 480, 481, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 529, 530, 531, 532, 535, 536, 537, 538, 539, 542, 568, 569, 570, 571, 572, 573, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 599, 604, 610, 621, 622, 623, 624, 625, 626, 627, 632, 636, 640, 643, 645, 646, 647, 648, 649, 651, 654, 658, 662, 663, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 717, 718, 719, 720, 721, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 747, 748, 768, 769, 771, 777, 778, 785, 786, 791, 801, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 820, 821, 823, 824, 825, 826, 827, 831, 832, 833, 834 }

B grade: { 4, 6, 7, 8, 9, 11, 17, 18, 19, 20, 28, 29, 30, 31, 36, 37, 38, 39, 40, 41, 42, 43, 49, 50, 58, 60, 118, 119, 122, 123, 126, 127, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 153, 161, 162, 163, 165, 173, 174, 216, 217, 218, 231, 232, 241, 243, 255, 256, 263, 264, 271, 273, 280, 281, 282, 293, 298, 301, 308, 310, 328, 336, 345, 346, 347, 354, 365, 378, 379, 380, 392, 399, 400, 414, 417, 418, 421, 456, 457, 461, 464, 471, 472, 473, 479, 525, 533, 534, 540, 541, 574, 575, 591, 592, 593, 596, 597, 598, 601, 602, 603, 605, 606, 607, 608, 611, 612, 613, 614, 615, 616, 617, 618, 619, 628, 629, 633, 634, 635, 637, 638, 639, 644, 650, 665, 666, 667, 668, 669, 670, 692, 715, 716, 722, 765, 766, 770, 772, 773, 775, 776, 779, 780, 781, 782, 783, 787, 788, 789, 790, 792, 793, 794, 795, 796, 797, 798, 799, 800, 819, 828, 829, 830, 835, 836, 837 }

C grade: { 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 97, 98, 99, 100, 101, 102, 103, 109, 115, 116, 117, 120, 121, 124, 125, 128, 129, 141, 142, 143, 145, 146, 206, 210, 211, 302, 303, 304, 305, 306, 311, 312, 313, 314, 315, 316, 321, 322, 323, 329, 330, 331, 337, 338, 339, 344, 385, 404, 406, 407, 408, 409, 410, 415, 416, 422, 423, 424, 430, 431, 432, 433, 470, 482, 483, 484, 485, 486, 487, 488, 526, 527, 528, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 588, 589, 590, 594, 595, 600, 609, 620, 630, 631, 641, 642, 652, 653, 714, 745, 746, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 767, 774, 784, 816, 822 }

F grade: { 96, 108, 114, 140, 222, 223, 405, 655, 656, 657, 659, 660, 661, 664, 802, 817, 818}

2.1.3 Maple

A grade: { 1, 2, 5, 6, 7, 8, 9, 10, 11, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 38, 39, 40, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 88, 90, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 188, 189, 190, 191, 192, 205, 206, 207, 208, 209, 210, 221, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 239, 240, 241, 242, 244, 245, 246, 247, 250, 252, 254, 255, 258, 259, 260, 261, 263, 264, 266, 267, 268, 269, 270, 271, 272, 274, 276, 277, 278, 279, 280, 281, 283, 284, 285, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 342, 343, 345, 348, 349, 350, 351, 355, 358, 359, 360, 367, 369, 370, 371, 372, 381, 382, 387, 392, 393, 394, 395, 402, 403, 425, 426, 427, 428, 429, 431, 434, 435, 436, 437, 438, 446, 447, 456, 457, 458, 459, 465, 466, 467, 468, 474, 475, 476, 477, 478, 480, 481, 485, 486, 493, 506, 507, 513, 519, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 536, 537, 538, 539, 540, 543, 544, 545, 546, 547, 553, 671, 672, 673, 674, 675, 678, 679, 680, 681, 686, 687, 688, 689, 695, 697, 701, 702, 703, 710, 726, 746, 747, 748, 755, 763, 801, 805, 806, 807, 808, 809, 810, 811, 812, 813, 831 }

B grade: { 3, 4, 12, 37, 41, 42, 79, 80, 81, 82, 83, 84, 85, 86, 87, 89, 176, 187, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 212, 238, 243, 248, 249, 251, 253, 256, 257, 262, 273, 282, 287, 340, 341, 344, 346, 347, 352, 353, 354, 356, 357, 361, 362, 363, 364, 365, 366, 368, 373, 374, 375, 376, 377, 378, 379, 380, 383, 384, 385, 386, 388, 389, 390, 391, 396, 397, 398, 399, 400, 401, 430, 432, 433, 439, 440, 441, 442, 443, 444, 445, 448, 449, 450, 451, 452, 453, 454, 455, 460, 461, 462, 463, 464, 469, 470, 471, 472, 473, 479, 482, 483, 484, 487, 488, 489, 490, 491, 492, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 508, 509, 510, 511, 512, 514, 515, 516, 517, 518, 520, 521, 534, 535, 541, 542, 548, 549, 550, 551, 552, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 567, 568, 569, 570, 575, 576, 577, 578, 584, 585, 586, 587, 590, 591, 592, 593, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 676, 677, 682, 683, 684, 685, 690, 691, 692, 693, 694, 696, 698, 699, 700, 704, 705, 706, 707, 708, 709, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 749, 750, 751, 752, 753, 754, 756, 757, 758, 759, 760, 761, 762, 764, 769, 770, 771, 774, 775, 776, 777, 778, 780, 781, 783, 784, 786, 787, 788, 789, 790, 791, 792, 793, 794, 796, 797, 798, 799, 800 }

C grade: { 211, 765, 766, 767, 768, 772, 773, 779, 785 }

F grade: { 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 213, 214, 215, 216, 217, 218, 219, 220, 222, 223, 224, 225, 265, 275, 286, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 564, 565, 566, 571, 572, 573, 574, 579, 580, 581, 582, 583, 588, 589, 594, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 782, 795, 802, 803, 804, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 832, 833, 834, 835, 836, 837 }

2.1.4 Maxima

A grade: { 1, 2, 6, 7, 8, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 221, 226, 227, 228, 229, 230, 236, 237, 239, 240, 247, 250, 252, 265, 275, 286, 344, 353, 364, 377, 385, 391, 398, 411, 425, 426, 427, 428, 429, 434, 435, 436, 437, 438, 445, 446, 447, 457, 622, 633, 650, 671, 672, 673, 674, 678, 679, 680, 681, 686, 687, 688, 689, 697, 801, 805, 808, 809, 810, 811, 812, 813, 831

B grade: { 3, 4, 5, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 85, 231, 232, 233, 234, 235, 238, 241, 242, 243, 244, 245, 246, 248, 249, 251, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 266, 267, 268, 269, 270, 271, 272, 273, 274, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 287, 288, 289, 317, 318, 319, 320, 324, 325, 326, 327, 328, 332, 333, 334, 335, 336, 412, 413, 444, 453, 454, 455, 456, 461, 462, 463, 464, 465, 466, 470, 471, 472, 473, 474, 475, 476, 480, 481, 568, 569, 570, 576, 577, 578, 585, 586, 587 }

C grade: { }

F grade: { 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 222, 223, 224, 225, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 321, 322, 323, 329, 330, 331, 337, 338, 339, 340, 341, 342, 343, 345, 346, 347, 348, 349, 350, 351, 352, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 378, 379, 380, 381, 382, 383, 384, 386, 387, 388, 389, 390, 392, 393, 394, 395, 396, 397, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 430, 431, 432, 433, 439, 440, 441, 442, 443, 448, 449, 450, 451, 452, 458, 459, 460, 467, 468, 469, 477, 478, 479, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 571, 572, 573, 574, 575, 579, 580, 581, 582, 583, 584, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 675, 676, 677, 682, 683, 684, 685, 690, 691, 692, 693, 694, 695, 696, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 802, 803, 804, 806, 807, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 832, 833, 834, 835, 836, 837 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 12, 16, 21, 22, 32, 33, 34, 35, 37, 41, 42, 43, 44, 45, 46, 47, 52, 53, 54, 55, 61, 64, 87, 88, 89, 90, 149, 150, 151, 152, 157, 158, 159, 160, 161, 164, 166, 167, 168, 169, 170, 171, 173, 174, 175, 176, 177, 178, 179, 180, 181, 185, 186, 188, 189, 221, 226, 227, 228, 229, 230, 231, 236, 237, 238, 239, 240, 241, 247, 248, 249, 250, 251, 252, 253, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 274, 275, 276, 277, 278, 279, 284, 285, 286, 287, 288, 289, 290, 291, 292, 298, 317, 318, 319, 320, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 342, 343, 345, 346, 348, 349, 350, 351, 355, 356, 357, 358, 359, 360, 366, 367, 368, 369, 370, 371, 372, 380, 381, 382, 386, 387, 388, 392, 393, 394, 395, 400, 401, 402, 403, 411, 412, 413, 419, 420, 421, 425, 426, 427, 428, 429, 430, 431, 434, 435, 436, 437, 438, 439, 440, 444, 445, 446, 447, 448, 449, 453, 456, 457, 465, 466, 475, 476, 522, 523, 524, 526, 529, 530, 531, 532, 536, 537, 538, 539, 546, 591, 622, 628, 633, 639, 644, 650, 671, 672, 673, 674, 675, 676, 678, 679, 680, 681, 682, 686, 687, 688, 689, 690, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 709, 710, 801, 805, 806, 807, 808, 809, 810, 811, 812, 813, 831 }

B grade: { 8, 9, 10, 11, 13, 14, 15, 17, 18, 19, 20, 23, 24, 25, 26, 27, 28, 29, 30, 31, 36, 38, 39, 40, 48, 49, 50, 51, 56, 57, 58, 59, 60, 62, 63, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 153, 154, 155, 156, 162, 163, 165, 172, 182, 183, 184, 187, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 232, 233, 234, 235, 242, 243, 244, 245, 246, 254, 255, 256, 257, 258, 259, 260, 271, 272, 273, 280, 281, 282, 283, 293, 294, 295, 296, 297, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 321, 322, 340, 341, 347, 354, 361, 365, 373, 378, 379, 399, 432, 433, 441, 442, 443, 450, 451, 452, 454, 455, 458, 459, 460, 461, 462, 463, 464, 467, 468, 469, 470, 471, 472, 473, 474, 477, 478, 479, 480, 481, 525, 527, 528, 533, 534, 535, 540, 541, 542, 543, 544, 545, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 677, 683, 684, 685, 691, 692, 693, 706, 707, 708, 714, 715, 716, 717, 718, 719 }

C grade: { }

F grade: { 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 222, 223, 224, 225, 344, 352, 353, 362, 363, 364, 374, 375, 376, 377, 383, 384, 385, 389, 390, 391, 396, 397, 398, 404, 405, 406, 407, 408, 409, 410, 414, 415, 416, 417, 418, 422, 423, 424, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 623, 624, 625, 626, 627, 629, 630, 631, 632, 634, 635, 636, 637, 638, 640, 641, 642, 643, 645, 646, 647, 648, 649, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 704, 705, 711, 712, 713, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 802, 803, 804, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 832, 833, 834, 835, 836, 837 }

2.1.6 Sympy

A grade: { 1, 2, 7, 149, 150, 151, 152, 157, 158, 159, 160, 166, 167, 168, 169, 175, 179, 180, 221, 226, 227, 228, 229, 230, 231, 232, 233, 234, 236, 237, 238, 239, 240, 241, 242, 243, 247, 248, 249, 250, 251, 252, 253, 254, 261, 262, 263, 264, 265, 266, 267, 271, 272, 273, 274, 275, 282, 283, 284, 286, 425, 426, 427, 428, 429, 430, 434, 435, 436, 437, 438, 444, 445, 446, 447, 453, 454, 455, 456, 457, 463, 464, 465, 466, 474, 475, 476, 671, 672, 673, 674, 675, 678, 679, 680, 681, 686, 687, 688, 689, 694, 695, 697, 701, 702, 810, 811, 812, 831 }

B grade: { 3, 4, 5, 6, 13, 14, 15, 16, 25, 26, 27, 153, 480, 481 }

C grade: { }

F grade: { 8, 9, 10, 11, 12, 17, 18, 19, 20, 21, 22, 23, 24, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 154, 155, 156, 161, 162, 163, 164, 165, 170, 171, 172, 173, 174, 176, 177, 178, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 222, 223, 224, 225, 235, 244, 245, 246, 255, 256, 257, 258, 259, 260, 268, 269, 270, 276, 277, 278, 279, 280, 281, 285, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 431, 432, 433, 439, 440, 441, 442, 443, 448, 449, 450, 451, 452, 458, 459, 460, 461, 462, 467, 468, 469, 470, 471, 472, 473, 477, 478, 479, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 676, 677, 682, 683, 684, 685, 690, 691, 692, 693, 696, 698, 699, 700, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 832, 833, 834, 835, 836, 837 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 149, 150, 151, 152, 153, 157, 158, 159, 160, 161, 166, 167, 168, 169, 170, 172, 173, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 194, 195, 196, 199, 200, 221, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 288, 289, 425, 426, 427, 428, 429, 430, 431, 434, 435, 436, 437, 438, 439, 440, 444, 445, 446, 447, 448, 453, 454, 456, 457, 458, 462, 463, 464, 465, 466, 467, 468, 474, 475, 476, 478, 480, 481, 671, 672, 673, 674, 675, 676, 678, 679, 680, 681, 682, 686, 687, 688, 689, 690, 694, 695, 697, 699, 700, 701, 702, 704, 706, 709, 710, 711, 801, 805, 806, 807, 808, 810, 811, 812, 813, 831 }

B grade: { 9, 37, 38, 39, 40, 41, 42, 43, 48, 49, 50, 51, 56, 57, 58, 59, 60, 61, 62, 63, 66, 67, 69, 70, 72, 73, 74, 82, 154, 155, 156, 162, 163, 164, 165, 171, 174, 193, 197, 198, 201, 202, 256, 257, 258, 287, 294, 295, 302, 311, 317, 318, 319, 320, 321, 322, 324, 325, 326, 327, 328, 329, 331, 332, 333, 334, 335, 336, 337, 432, 433, 441, 442, 443, 449, 450, 451, 452, 455, 459, 460, 461, 469, 470, 471, 472, 473, 477, 479, 543, 544, 545, 551, 552, 553, 558, 677, 683, 684, 685, 691, 692, 693, 696, 698, 705, 707, 708, 713, 714, 715, 716, 717, 718, 719, 720, 721 }

C grade: { }

F grade: { 32, 33, 34, 35, 36, 44, 45, 46, 47, 52, 53, 54, 55, 64, 65, 68, 71, 75, 76, 77, 78, 79, 80, 81, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 222, 223, 224, 225, 290, 291, 292, 293, 296, 297, 298, 299, 300, 301, 303, 304, 305, 306, 307, 308, 309, 310, 312, 313, 314, 315, 316, 323, 330, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 546, 547, 548, 549, 550, 554, 555, 556, 557, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 703, 712, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 802, 803, 804, 809, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 832, 833, 834, 835, 836, 837 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	105	96	128	205	221	127
normalized size	1	1.	1.03	0.94	1.25	2.01	2.17	1.25
time (sec)	N/A	0.102	0.429	0.031	1.729	1.816	2.24	2.085

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	115	143	193	248	379	151
normalized size	1	1.	0.89	1.11	1.5	1.92	2.94	1.17
time (sec)	N/A	0.145	0.516	0.034	1.801	1.759	4.459	2.105

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	101	121	243	211	1300	90
normalized size	1	1.	1.91	2.28	4.58	3.98	24.53	1.7
time (sec)	N/A	0.069	0.116	0.026	2.569	1.754	37.374	2.219

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	87	100	173	166	665	76
normalized size	1	1.	2.07	2.38	4.12	3.95	15.83	1.81
time (sec)	N/A	0.049	0.079	0.026	2.564	1.709	3.265	1.804

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	48	40	105	122	250	59
normalized size	1	1.	1.78	1.48	3.89	4.52	9.26	2.19
time (sec)	N/A	0.066	0.061	0.024	2.578	1.722	1.457	1.893

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	42	25	43	93	39	26
normalized size	1	1.	2.47	1.47	2.53	5.47	2.29	1.53
time (sec)	N/A	0.029	0.037	0.022	2.368	1.579	0.632	2.361

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	29	14	22	68	10	18
normalized size	1	1.	2.42	1.17	1.83	5.67	0.83	1.5
time (sec)	N/A	0.01	0.025	0.017	1.738	1.598	0.261	1.661

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	74	24	42	204	0	32
normalized size	1	1.	3.7	1.2	2.1	10.2	0.	1.6
time (sec)	N/A	0.038	0.05	0.029	1.656	1.911	0.	2.236

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	63	45	92	301	0	72
normalized size	1	1.	2.42	1.73	3.54	11.58	0.	2.77
time (sec)	N/A	0.06	0.153	0.033	1.674	1.73	0.	1.792

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	83	67	131	433	0	99
normalized size	1	1.	1.98	1.6	3.12	10.31	0.	2.36
time (sec)	N/A	0.066	0.308	0.037	1.739	1.766	0.	2.123

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	113	89	162	537	0	130
normalized size	1	1.	2.05	1.62	2.95	9.76	0.	2.36
time (sec)	N/A	0.07	0.717	0.04	1.786	1.834	0.	2.462

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	100	126	267	297	0	97
normalized size	1	1.	1.52	1.91	4.05	4.5	0.	1.47
time (sec)	N/A	0.121	0.24	0.042	2.489	1.397	0.	2.384

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	84	66	194	259	848	69
normalized size	1	1.	1.79	1.4	4.13	5.51	18.04	1.47
time (sec)	N/A	0.143	0.225	0.04	2.497	1.459	105.718	1.349

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	69	51	122	215	369	47
normalized size	1	1.	1.97	1.46	3.49	6.14	10.54	1.34
time (sec)	N/A	0.073	0.127	0.037	2.689	1.456	7.368	1.64

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	29	27	84	163	178	28
normalized size	1	1.	0.88	0.82	2.55	4.94	5.39	0.85
time (sec)	N/A	0.031	0.042	0.032	1.079	1.39	3.419	1.887

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	31	35	100	161	87	39
normalized size	1	1.	0.94	1.06	3.03	4.88	2.64	1.18
time (sec)	N/A	0.021	0.028	0.027	1.698	1.337	1.461	1.726

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	129	50	120	367	0	54
normalized size	1	1.	3.39	1.32	3.16	9.66	0.	1.42
time (sec)	N/A	0.088	0.136	0.049	1.647	1.483	0.	1.768

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	166	71	170	512	0	93
normalized size	1	1.	3.69	1.58	3.78	11.38	0.	2.07
time (sec)	N/A	0.135	0.346	0.053	1.636	1.438	0.	2.046

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	203	92	209	666	0	126
normalized size	1	1.	3.17	1.44	3.27	10.41	0.	1.97
time (sec)	N/A	0.145	0.598	0.061	1.843	1.554	0.	2.132

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	71	238	115	240	819	0	154
normalized size	1	1.09	3.66	1.77	3.69	12.6	0.	2.37
time (sec)	N/A	0.15	3.182	0.062	1.489	1.489	0.	2.03

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	191	174	413	479	0	134
normalized size	1	1.	1.89	1.72	4.09	4.74	0.	1.33
time (sec)	N/A	0.226	0.102	0.052	2.167	1.559	0.	2.086

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	170	152	340	432	0	119
normalized size	1	1.	1.89	1.69	3.78	4.8	0.	1.32
time (sec)	N/A	0.208	0.076	0.046	2.747	1.412	0.	1.638

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	140	79	267	379	0	90
normalized size	1	1.	1.97	1.11	3.76	5.34	0.	1.27
time (sec)	N/A	0.221	0.077	0.046	2.086	1.396	0.	1.32

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	112	77	194	340	0	69
normalized size	1	1.	1.9	1.31	3.29	5.76	0.	1.17
time (sec)	N/A	0.157	0.177	0.043	2.555	1.426	0.	1.24

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	47	37	140	250	212	39
normalized size	1	1.	0.94	0.74	2.8	5.	4.24	0.78
time (sec)	N/A	0.076	0.064	0.039	1.759	1.408	111.999	1.228

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	41	45	157	246	418	50
normalized size	1	1.	0.82	0.9	3.14	4.92	8.36	1.
time (sec)	N/A	0.046	0.04	0.035	1.627	1.386	5.199	1.321

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	45	57	173	252	348	61
normalized size	1	1.	0.9	1.14	3.46	5.04	6.96	1.22
time (sec)	N/A	0.035	0.058	0.034	1.627	1.398	3.402	1.233

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	160	76	193	536	0	76
normalized size	1	1.	2.76	1.31	3.33	9.24	0.	1.31
time (sec)	N/A	0.161	0.069	0.055	2.053	1.385	0.	1.282

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	206	97	243	691	0	115
normalized size	1	1.	3.17	1.49	3.74	10.63	0.	1.77
time (sec)	N/A	0.229	0.14	0.06	1.899	1.521	0.	1.279

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	247	119	282	861	0	147
normalized size	1	1.	2.87	1.38	3.28	10.01	0.	1.71
time (sec)	N/A	0.235	0.407	0.068	1.831	1.557	0.	1.462

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	299	141	313	1026	0	173
normalized size	1	1.	2.9	1.37	3.04	9.96	0.	1.68
time (sec)	N/A	0.245	0.853	0.071	1.75	1.514	0.	1.313

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	165	83	0	373	0	0
normalized size	1	1.	1.04	0.53	0.	2.36	0.	0.
time (sec)	N/A	0.23	0.499	0.503	0.	1.429	0.	0.

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	141	73	0	300	0	0
normalized size	1	1.	1.16	0.6	0.	2.46	0.	0.
time (sec)	N/A	0.169	0.292	0.539	0.	1.376	0.	0.

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	117	63	0	246	0	0
normalized size	1	1.	1.36	0.73	0.	2.86	0.	0.
time (sec)	N/A	0.111	0.182	0.44	0.	1.395	0.	0.

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	81	51	0	190	0	0
normalized size	1	1.	1.45	0.91	0.	3.39	0.	0.
time (sec)	N/A	0.045	0.118	0.377	0.	1.446	0.	0.

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	65	43	0	136	0	0
normalized size	1	1.	2.5	1.65	0.	5.23	0.	0.
time (sec)	N/A	0.013	0.033	0.332	0.	1.442	0.	0.

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	94	68	0	590	0	243
normalized size	1	1.	2.54	1.84	0.	15.95	0.	6.57
time (sec)	N/A	0.055	0.098	0.399	0.	1.596	0.	1.868

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	178	104	0	695	0	475
normalized size	1	1.	2.78	1.62	0.	10.86	0.	7.42
time (sec)	N/A	0.103	0.676	0.579	0.	1.462	0.	2.434

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	249	132	0	853	0	721
normalized size	1	1.	2.44	1.29	0.	8.36	0.	7.07
time (sec)	N/A	0.156	0.699	0.589	0.	1.453	0.	2.544

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	285	158	0	968	0	829
normalized size	1	1.	2.07	1.14	0.	7.01	0.	6.01
time (sec)	N/A	0.213	1.284	0.58	0.	1.618	0.	2.448

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	97	67	0	591	0	248
normalized size	1	1.	2.55	1.76	0.	15.55	0.	6.53
time (sec)	N/A	0.051	0.101	0.404	0.	1.506	0.	2.569

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	96	70	0	587	0	269
normalized size	1	1.	2.46	1.79	0.	15.05	0.	6.9
time (sec)	N/A	0.05	0.077	0.382	0.	1.494	0.	2.132

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	95	69	0	589	0	258
normalized size	1	1.	2.38	1.72	0.	14.72	0.	6.45
time (sec)	N/A	0.053	0.085	0.447	0.	1.527	0.	2.133

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	165	85	0	405	0	0
normalized size	1	1.	1.02	0.52	0.	2.5	0.	0.
time (sec)	N/A	0.241	0.53	0.463	0.	1.393	0.	0.

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	141	75	0	338	0	0
normalized size	1	1.	1.22	0.65	0.	2.91	0.	0.
time (sec)	N/A	0.135	0.352	0.609	0.	1.516	0.	0.

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	115	63	0	259	0	0
normalized size	1	1.	1.34	0.73	0.	3.01	0.	0.
time (sec)	N/A	0.062	0.159	0.437	0.	1.523	0.	0.

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	89	53	0	204	0	0
normalized size	1	1.	1.51	0.9	0.	3.46	0.	0.
time (sec)	N/A	0.028	0.132	0.371	0.	1.438	0.	0.

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	118	84	0	649	0	350
normalized size	1	1.	1.79	1.27	0.	9.83	0.	5.3
time (sec)	N/A	0.098	0.154	0.57	0.	1.462	0.	2.415

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	180	103	0	709	0	485
normalized size	1	1.	2.73	1.56	0.	10.74	0.	7.35
time (sec)	N/A	0.11	0.646	0.628	0.	1.467	0.	2.328

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	250	126	0	878	0	734
normalized size	1	1.	2.36	1.19	0.	8.28	0.	6.92
time (sec)	N/A	0.166	0.603	0.656	0.	1.543	0.	2.015

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	286	144	0	1002	0	844
normalized size	1	1.	1.99	1.	0.	6.96	0.	5.86
time (sec)	N/A	0.231	0.928	0.744	0.	1.53	0.	3.184

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	189	95	0	508	0	0
normalized size	1	1.	0.93	0.47	0.	2.5	0.	0.
time (sec)	N/A	0.352	1.269	0.628	0.	1.538	0.	0.

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	165	85	0	437	0	0
normalized size	1	1.	1.13	0.58	0.	2.99	0.	0.
time (sec)	N/A	0.155	1.017	0.622	0.	1.38	0.	0.

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	141	75	0	355	0	0
normalized size	1	1.	1.22	0.65	0.	3.06	0.	0.
time (sec)	N/A	0.083	0.614	0.506	0.	1.383	0.	0.

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	117	65	0	292	0	0
normalized size	1	1.	1.31	0.73	0.	3.28	0.	0.
time (sec)	N/A	0.046	0.309	0.506	0.	1.404	0.	0.

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	143	103	0	733	0	429
normalized size	1	1.	1.46	1.05	0.	7.48	0.	4.38
time (sec)	N/A	0.192	0.372	0.615	0.	1.446	0.	2.721

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	182	123	0	790	0	567
normalized size	1	1.	1.94	1.31	0.	8.4	0.	6.03
time (sec)	N/A	0.193	0.801	0.703	0.	1.493	0.	2.78

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	252	126	0	911	0	740
normalized size	1	1.	2.38	1.19	0.	8.59	0.	6.98
time (sec)	N/A	0.218	0.75	0.623	0.	1.534	0.	2.601

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	288	144	0	1040	0	852
normalized size	1	1.	2.	1.	0.	7.22	0.	5.92
time (sec)	N/A	0.275	1.159	0.732	0.	1.912	0.	2.643

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	182	370	162	0	1238	0	1102
normalized size	1	1.	2.03	0.89	0.	6.8	0.	6.05
time (sec)	N/A	0.335	1.602	0.727	0.	1.918	0.	3.286

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	150	130	0	648	0	410
normalized size	1	1.	1.08	0.94	0.	4.66	0.	2.95
time (sec)	N/A	0.232	0.233	0.589	0.	1.906	0.	3.416

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	105	96	0	585	0	332
normalized size	1	1.	1.	0.91	0.	5.57	0.	3.16
time (sec)	N/A	0.119	0.218	0.566	0.	1.735	0.	3.321

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	98	94	0	533	0	246
normalized size	1	1.	1.36	1.31	0.	7.4	0.	3.42
time (sec)	N/A	0.048	0.106	0.547	0.	1.769	0.	2.89

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	73	75	0	463	0	0
normalized size	1	1.	1.55	1.6	0.	9.85	0.	0.
time (sec)	N/A	0.02	0.046	0.336	0.	1.704	0.	0.

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	128	96	0	795	0	0
normalized size	1	1.	1.52	1.14	0.	9.46	0.	0.
time (sec)	N/A	0.114	0.099	0.506	0.	1.904	0.	0.

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	168	133	0	1119	0	628
normalized size	1	1.	1.54	1.22	0.	10.27	0.	5.76
time (sec)	N/A	0.204	1.229	0.526	0.	1.944	0.	3.011

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	307	162	0	1328	0	830
normalized size	1	1.	2.1	1.11	0.	9.1	0.	5.68
time (sec)	N/A	0.343	3.465	0.619	0.	1.925	0.	2.858

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	178	183	0	834	0	0
normalized size	1	1.	0.97	1.	0.	4.56	0.	0.
time (sec)	N/A	0.382	0.453	0.533	0.	1.828	0.	0.

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	156	183	0	783	0	552
normalized size	1	1.	1.08	1.26	0.	5.4	0.	3.81
time (sec)	N/A	0.253	0.268	0.589	0.	1.884	0.	2.99

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	134	143	0	725	0	486
normalized size	1	1.	1.28	1.36	0.	6.9	0.	4.63
time (sec)	N/A	0.128	0.271	0.549	0.	1.768	0.	2.304

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	108	123	0	668	0	0
normalized size	1	1.	1.4	1.6	0.	8.68	0.	0.
time (sec)	N/A	0.057	0.195	0.438	0.	1.809	0.	0.

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	108	125	0	666	0	396
normalized size	1	1.	1.4	1.62	0.	8.65	0.	5.14
time (sec)	N/A	0.039	0.154	0.386	0.	1.87	0.	2.73

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	223	173	0	1215	0	556
normalized size	1	1.	1.96	1.52	0.	10.66	0.	4.88
time (sec)	N/A	0.213	0.2	0.551	0.	1.971	0.	2.091

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	449	219	0	1434	0	684
normalized size	1	1.	3.12	1.52	0.	9.96	0.	4.75
time (sec)	N/A	0.358	0.627	0.599	0.	1.91	0.	2.61

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	186	620	299	0	1651	0	0
normalized size	1	1.	3.33	1.61	0.	8.88	0.	0.
time (sec)	N/A	0.486	4.707	0.652	0.	1.952	0.	0.

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	221	221	323	0	1041	0	0
normalized size	1	1.	1.	1.46	0.	4.71	0.	0.
time (sec)	N/A	0.521	0.566	0.783	0.	1.878	0.	0.

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	197	269	0	976	0	0
normalized size	1	1.	1.08	1.47	0.	5.33	0.	0.
time (sec)	N/A	0.385	0.482	0.787	0.	1.889	0.	0.

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	173	233	0	910	0	0
normalized size	1	1.	1.19	1.61	0.	6.28	0.	0.
time (sec)	N/A	0.268	0.321	0.742	0.	1.796	0.	0.

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	196	193	0	855	0	0
normalized size	1	1.	1.83	1.8	0.	7.99	0.	0.
time (sec)	N/A	0.128	0.206	0.658	0.	1.793	0.	0.

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	196	193	0	848	0	0
normalized size	1	1.	1.83	1.8	0.	7.93	0.	0.
time (sec)	N/A	0.074	0.183	0.561	0.	1.817	0.	0.

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	196	195	0	851	0	0
normalized size	1	1.	1.83	1.82	0.	7.95	0.	0.
time (sec)	N/A	0.058	0.159	0.768	0.	1.729	0.	0.

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	296	262	0	1458	0	776
normalized size	1	1.	2.06	1.82	0.	10.12	0.	5.39
time (sec)	N/A	0.325	0.276	0.696	0.	1.98	0.	3.172

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	509	356	0	1683	0	0
normalized size	1	1.	2.93	2.05	0.	9.67	0.	0.
time (sec)	N/A	0.507	0.588	0.839	0.	2.029	0.	0.

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	224	680	404	0	1909	0	0
normalized size	1	1.	3.04	1.8	0.	8.52	0.	0.
time (sec)	N/A	0.66	1.074	0.853	0.	2.078	0.	0.

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	164	320	284	934	0	0
normalized size	1	1.	4.43	8.65	7.68	25.24	0.	0.
time (sec)	N/A	0.056	0.514	0.141	2.09	2.574	0.	0.

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	119	271	0	941	0	0
normalized size	1	1.	3.13	7.13	0.	24.76	0.	0.
time (sec)	N/A	0.07	0.468	0.112	0.	2.631	0.	0.

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	123	52	0	107	0	0
normalized size	1	1.	7.24	3.06	0.	6.29	0.	0.
time (sec)	N/A	0.039	2.44	0.086	0.	1.689	0.	0.

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	125	54	0	539	0	0
normalized size	1	1.	2.98	1.29	0.	12.83	0.	0.
time (sec)	N/A	0.057	0.09	0.089	0.	2.005	0.	0.

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	125	52	0	104	0	0
normalized size	1	1.	4.03	1.68	0.	3.35	0.	0.
time (sec)	N/A	0.046	2.402	0.098	0.	1.674	0.	0.

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	128	53	0	543	0	0
normalized size	1	1.	3.05	1.26	0.	12.93	0.	0.
time (sec)	N/A	0.064	0.102	0.093	0.	2.096	0.	0.

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	121	0	0	0	0	0
normalized size	1	1.	0.66	0.	0.	0.	0.	0.
time (sec)	N/A	0.213	0.404	0.219	0.	0.	0.	0.

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	160	0	0	0	0	0
normalized size	1	1.	0.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.278	0.696	0.193	0.	0.	0.	0.

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	151	0	0	0	0	0
normalized size	1	1.	1.2	0.	0.	0.	0.	0.
time (sec)	N/A	0.145	0.439	0.197	0.	0.	0.	0.

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	138	0	0	0	0	0
normalized size	1	1.	1.44	0.	0.	0.	0.	0.
time (sec)	N/A	0.068	0.231	0.075	0.	0.	0.	0.

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	124	0	0	0	0	0
normalized size	1	1.	1.88	0.	0.	0.	0.	0.
time (sec)	N/A	0.031	0.203	0.003	0.	0.	0.	0.

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	77	77	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.111	2.724	0.096	0.	0.	0.	0.

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	77	77	143	0	0	0	0	0
normalized size	1	1.	1.86	0.	0.	0.	0.	0.
time (sec)	N/A	0.131	14.615	0.098	0.	0.	0.	0.

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	373	0	0	0	0	0
normalized size	1	1.	2.3	0.	0.	0.	0.	0.
time (sec)	N/A	0.279	2.575	0.193	0.	0.	0.	0.

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	363	0	0	0	0	0
normalized size	1	1.	2.86	0.	0.	0.	0.	0.
time (sec)	N/A	0.143	2.563	0.201	0.	0.	0.	0.

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	351	0	0	0	0	0
normalized size	1	1.	3.62	0.	0.	0.	0.	0.
time (sec)	N/A	0.067	1.924	0.075	0.	0.	0.	0.

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	341	0	0	0	0	0
normalized size	1	1.	5.09	0.	0.	0.	0.	0.
time (sec)	N/A	0.03	1.649	0.005	0.	0.	0.	0.

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	78	78	2791	0	0	0	0	0
normalized size	1	1.	35.78	0.	0.	0.	0.	0.
time (sec)	N/A	0.114	9.516	0.096	0.	0.	0.	0.

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	78	78	2800	0	0	0	0	0
normalized size	1	1.	35.9	0.	0.	0.	0.	0.
time (sec)	N/A	0.13	10.537	0.102	0.	0.	0.	0.

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	161	161	110	0	0	0	0	0
normalized size	1	1.	0.68	0.	0.	0.	0.	0.
time (sec)	N/A	0.253	0.458	0.286	0.	0.	0.	0.

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	95	0	0	0	0	0
normalized size	1	1.	0.75	0.	0.	0.	0.	0.
time (sec)	N/A	0.132	0.297	0.162	0.	0.	0.	0.

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	84	0	0	0	0	0
normalized size	1	1.	0.9	0.	0.	0.	0.	0.
time (sec)	N/A	0.063	0.148	0.132	0.	0.	0.	0.

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	70	0	0	0	0	0
normalized size	1	1.	1.06	0.	0.	0.	0.	0.
time (sec)	N/A	0.031	0.099	0.006	0.	0.	0.	0.

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	77	77	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.106	3.439	0.096	0.	0.	0.	0.

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	77	77	184	0	0	0	0	0
normalized size	1	1.	2.39	0.	0.	0.	0.	0.
time (sec)	N/A	0.119	8.749	0.105	0.	0.	0.	0.

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	116	0	0	0	0	0
normalized size	1	1.	0.72	0.	0.	0.	0.	0.
time (sec)	N/A	0.27	0.483	0.284	0.	0.	0.	0.

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	108	0	0	0	0	0
normalized size	1	1.	0.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.136	0.327	0.162	0.	0.	0.	0.

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	130	0	0	0	0	0
normalized size	1	1.	1.31	0.	0.	0.	0.	0.
time (sec)	N/A	0.079	0.295	0.082	0.	0.	0.	0.

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	130	0	0	0	0	0
normalized size	1	1.	1.88	0.	0.	0.	0.	0.
time (sec)	N/A	0.031	0.197	0.005	0.	0.	0.	0.

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	80	80	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.117	10.167	0.099	0.	0.	0.	0.

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	80	80	230	0	0	0	0	0
normalized size	1	1.	2.88	0.	0.	0.	0.	0.
time (sec)	N/A	0.137	14.028	0.105	0.	0.	0.	0.

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	96	96	5109	0	0	0	0	0
normalized size	1	1.	53.22	0.	0.	0.	0.	0.
time (sec)	N/A	0.111	23.221	0.118	0.	0.	0.	0.

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	43	43	186	0	0	0	0	0
normalized size	1	1.	4.33	0.	0.	0.	0.	0.
time (sec)	N/A	0.048	0.408	0.088	0.	0.	0.	0.

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	58	58	225	0	0	0	0	0
normalized size	1	1.	3.88	0.	0.	0.	0.	0.
time (sec)	N/A	0.068	1.537	0.088	0.	0.	0.	0.

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	60	60	263	0	0	0	0	0
normalized size	1	1.	4.38	0.	0.	0.	0.	0.
time (sec)	N/A	0.072	3.652	0.088	0.	0.	0.	0.

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	106	106	5111	0	0	0	0	0
normalized size	1	1.	48.22	0.	0.	0.	0.	0.
time (sec)	N/A	0.14	6.331	0.125	0.	0.	0.	0.

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	264	0	0	0	0	0
normalized size	1	1.	5.74	0.	0.	0.	0.	0.
time (sec)	N/A	0.062	4.085	0.138	0.	0.	0.	0.

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	60	60	234	0	0	0	0	0
normalized size	1	1.	3.9	0.	0.	0.	0.	0.
time (sec)	N/A	0.124	1.313	0.124	0.	0.	0.	0.

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	65	65	274	0	0	0	0	0
normalized size	1	1.	4.22	0.	0.	0.	0.	0.
time (sec)	N/A	0.136	2.225	0.123	0.	0.	0.	0.

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	130	130	5129	0	0	0	0	0
normalized size	1	1.	39.45	0.	0.	0.	0.	0.
time (sec)	N/A	0.141	6.302	0.116	0.	0.	0.	0.

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	72	72	215	0	0	0	0	0
normalized size	1	1.	2.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.056	0.306	0.109	0.	0.	0.	0.

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	78	78	227	0	0	0	0	0
normalized size	1	1.	2.91	0.	0.	0.	0.	0.
time (sec)	N/A	0.118	0.405	0.109	0.	0.	0.	0.

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	80	80	265	0	0	0	0	0
normalized size	1	1.	3.31	0.	0.	0.	0.	0.
time (sec)	N/A	0.135	0.857	0.109	0.	0.	0.	0.

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	131	131	5131	0	0	0	0	0
normalized size	1	1.	39.17	0.	0.	0.	0.	0.
time (sec)	N/A	0.17	6.321	0.145	0.	0.	0.	0.

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	266	0	0	0	0	0
normalized size	1	1.	4.03	0.	0.	0.	0.	0.
time (sec)	N/A	0.075	0.409	0.161	0.	0.	0.	0.

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	80	80	242	0	0	0	0	0
normalized size	1	1.	3.02	0.	0.	0.	0.	0.
time (sec)	N/A	0.18	0.697	0.148	0.	0.	0.	0.

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	85	85	276	0	0	0	0	0
normalized size	1	1.	3.25	0.	0.	0.	0.	0.
time (sec)	N/A	0.202	1.024	0.142	0.	0.	0.	0.

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	71	71	2805	0	0	0	0	0
normalized size	1	1.	39.51	0.	0.	0.	0.	0.
time (sec)	N/A	0.06	14.804	0.547	0.	0.	0.	0.

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	68	68	300	0	0	0	0	0
normalized size	1	1.	4.41	0.	0.	0.	0.	0.
time (sec)	N/A	0.061	2.253	0.62	0.	0.	0.	0.

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	91	91	2813	0	0	0	0	0
normalized size	1	1.	30.91	0.	0.	0.	0.	0.
time (sec)	N/A	0.1	6.21	0.607	0.	0.	0.	0.

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	90	90	300	0	0	0	0	0
normalized size	1	1.	3.33	0.	0.	0.	0.	0.
time (sec)	N/A	0.11	0.712	0.597	0.	0.	0.	0.

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	87	87	2807	0	0	0	0	0
normalized size	1	1.	32.26	0.	0.	0.	0.	0.
time (sec)	N/A	0.098	6.301	0.636	0.	0.	0.	0.

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	85	85	301	0	0	0	0	0
normalized size	1	1.	3.54	0.	0.	0.	0.	0.
time (sec)	N/A	0.11	0.604	0.675	0.	0.	0.	0.

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	107	107	2815	0	0	0	0	0
normalized size	1	1.	26.31	0.	0.	0.	0.	0.
time (sec)	N/A	0.15	6.227	0.658	0.	0.	0.	0.

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	107	107	301	0	0	0	0	0
normalized size	1	1.	2.81	0.	0.	0.	0.	0.
time (sec)	N/A	0.16	0.38	0.655	0.	0.	0.	0.

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	294	294	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.51	180.089	1.428	0.	0.	0.	0.

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	215	215	60244	0	0	0	0	0
normalized size	1	1.	280.2	0.	0.	0.	0.	0.
time (sec)	N/A	0.294	127.957	1.111	0.	0.	0.	0.

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	156	156	28439	0	0	0	0	0
normalized size	1	1.	182.3	0.	0.	0.	0.	0.
time (sec)	N/A	0.143	54.176	0.907	0.	0.	0.	0.

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	109	109	178	0	0	0	0	0
normalized size	1	1.	1.63	0.	0.	0.	0.	0.
time (sec)	N/A	0.064	0.444	0.875	0.	0.	0.	0.

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	90	0	0	0	0	0
normalized size	1	1.	1.22	0.	0.	0.	0.	0.
time (sec)	N/A	0.03	0.155	0.003	0.	0.	0.	0.

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	85	85	2560	0	0	0	0	0
normalized size	1	1.	30.12	0.	0.	0.	0.	0.
time (sec)	N/A	0.111	15.841	0.736	0.	0.	0.	0.

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	85	85	4206	0	0	0	0	0
normalized size	1	1.	49.48	0.	0.	0.	0.	0.
time (sec)	N/A	0.115	21.28	0.355	0.	0.	0.	0.

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	88	0	0	0	0	0
normalized size	1	1.	1.52	0.	0.	0.	0.	0.
time (sec)	N/A	0.014	0.121	0.26	0.	0.	0.	0.

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	57	57	90	0	0	0	0	0
normalized size	1	1.	1.58	0.	0.	0.	0.	0.
time (sec)	N/A	0.018	0.105	0.283	0.	0.	0.	0.

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	76	60	77	157	144	89
normalized size	1	1.	0.99	0.78	1.	2.04	1.87	1.16
time (sec)	N/A	0.058	0.164	0.014	1.663	1.616	1.748	1.68

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	60	49	65	120	92	68
normalized size	1	1.	1.09	0.89	1.18	2.18	1.67	1.24
time (sec)	N/A	0.045	0.058	0.014	1.622	1.59	0.919	1.688

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	35	39	49	86	66	46
normalized size	1	1.	0.9	1.	1.26	2.21	1.69	1.18
time (sec)	N/A	0.014	0.09	0.013	1.739	1.606	0.36	1.682

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	27	17	22	38	19	23
normalized size	1	1.	1.69	1.06	1.38	2.38	1.19	1.44
time (sec)	N/A	0.008	0.006	0.009	1.648	1.548	0.237	1.641

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	43	32	39	111	51	36
normalized size	1	1.	2.53	1.88	2.29	6.53	3.	2.12
time (sec)	N/A	0.022	0.02	0.027	1.749	1.76	7.324	2.152

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	52	35	54	180	0	84
normalized size	1	1.	2.	1.35	2.08	6.92	0.	3.23
time (sec)	N/A	0.037	0.024	0.029	1.767	1.597	0.	2.182

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	91	54	81	251	0	124
normalized size	1	1.	1.9	1.12	1.69	5.23	0.	2.58
time (sec)	N/A	0.047	0.028	0.089	1.615	1.745	0.	2.056

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	115	74	99	344	0	165
normalized size	1	1.	1.8	1.16	1.55	5.38	0.	2.58
time (sec)	N/A	0.051	0.027	0.084	1.726	2.017	0.	2.335

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	91	95	127	232	221	173
normalized size	1	1.	0.81	0.85	1.13	2.07	1.97	1.54
time (sec)	N/A	0.104	0.341	0.023	1.858	1.912	3.371	1.568

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	117	89	113	201	211	116
normalized size	1	1.	1.16	0.88	1.12	1.99	2.09	1.15
time (sec)	N/A	0.089	0.153	0.023	1.712	1.868	2.106	2.119

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	59	64	84	139	107	103
normalized size	1	1.	0.83	0.9	1.18	1.96	1.51	1.45
time (sec)	N/A	0.049	0.21	0.019	1.607	1.975	1.036	2.03

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	46	51	62	109	78	61
normalized size	1	1.	0.92	1.02	1.24	2.18	1.56	1.22
time (sec)	N/A	0.015	0.098	0.017	1.84	1.886	0.46	1.956

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	76	52	59	147	0	70
normalized size	1	1.	2.17	1.49	1.69	4.2	0.	2.
time (sec)	N/A	0.059	0.022	0.044	1.649	2.072	0.	1.801

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	76	52	70	209	0	107
normalized size	1	1.	2.24	1.53	2.06	6.15	0.	3.15
time (sec)	N/A	0.066	0.22	0.042	1.656	1.996	0.	1.758

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	133	82	120	316	0	169
normalized size	1	1.	2.25	1.39	2.03	5.36	0.	2.86
time (sec)	N/A	0.076	0.458	0.052	1.973	1.926	0.	1.664

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	132	93	120	387	0	224
normalized size	1	1.	1.61	1.13	1.46	4.72	0.	2.73
time (sec)	N/A	0.087	0.039	0.052	1.991	1.893	0.	1.851

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	110	110	255	146	198	558	0	311
normalized size	1	1.	2.32	1.33	1.8	5.07	0.	2.83
time (sec)	N/A	0.094	0.041	0.111	1.997	1.662	0.	1.825

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	193	147	145	196	340	393	243
normalized size	1	1.13	0.86	0.85	1.15	1.99	2.3	1.42
time (sec)	N/A	0.209	0.7	0.025	2.555	1.74	6.077	2.215

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	180	117	124	163	285	284	174
normalized size	1	1.12	0.73	0.78	1.02	1.78	1.78	1.09
time (sec)	N/A	0.216	0.635	0.027	1.737	1.673	3.529	1.715

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	100	104	131	217	233	157
normalized size	1	1.	0.83	0.86	1.08	1.79	1.93	1.3
time (sec)	N/A	0.114	0.338	0.022	1.423	1.61	2.012	1.6

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	71	76	100	169	128	101
normalized size	1	1.	0.79	0.84	1.11	1.88	1.42	1.12
time (sec)	N/A	0.066	0.157	0.022	1.962	1.749	1.324	1.711

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	81	92	96	208	0	154
normalized size	1	1.	1.09	1.24	1.3	2.81	0.	2.08
time (sec)	N/A	0.115	0.16	0.056	1.722	1.852	0.	2.253

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	87	72	92	266	0	197
normalized size	1	1.	1.28	1.06	1.35	3.91	0.	2.9
time (sec)	N/A	0.12	0.517	0.052	1.65	1.736	0.	2.33

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	152	99	138	383	0	192
normalized size	1	1.	1.92	1.25	1.75	4.85	0.	2.43
time (sec)	N/A	0.133	0.642	0.063	2.37	1.728	0.	2.019

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	525	122	159	471	0	271
normalized size	1	1.	4.82	1.12	1.46	4.32	0.	2.49
time (sec)	N/A	0.181	6.183	0.063	2.641	1.721	0.	2.111

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	134	134	322	166	219	589	0	363
normalized size	1	1.	2.4	1.24	1.63	4.4	0.	2.71
time (sec)	N/A	0.205	6.171	0.067	1.656	1.796	0.	1.907

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	106	116	153	244	240	151
normalized size	1	1.	0.77	0.85	1.12	1.78	1.75	1.1
time (sec)	N/A	0.146	0.359	0.023	2.406	1.78	2.111	1.557

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	98	213	0	728	0	201
normalized size	1	1.	0.89	1.94	0.	6.62	0.	1.83
time (sec)	N/A	0.278	0.256	0.039	0.	1.86	0.	1.567

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	78	142	0	630	0	151
normalized size	1	1.	0.95	1.73	0.	7.68	0.	1.84
time (sec)	N/A	0.164	0.108	0.036	0.	1.784	0.	1.83

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	56	72	0	504	0	104
normalized size	1	1.	0.92	1.18	0.	8.26	0.	1.7
time (sec)	N/A	0.104	0.089	0.033	0.	1.8	0.	1.879

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	47	54	0	423	202	78
normalized size	1	1.	0.94	1.08	0.	8.46	4.04	1.56
time (sec)	N/A	0.055	0.04	0.029	0.	1.809	99.124	1.859

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	40	39	0	344	114	65
normalized size	1	1.	1.	0.98	0.	8.6	2.85	1.62
time (sec)	N/A	0.031	0.022	0.023	0.	1.687	8.713	1.861

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	62	53	0	582	0	85
normalized size	1	1.	1.17	1.	0.	10.98	0.	1.6
time (sec)	N/A	0.066	0.058	0.041	0.	2.096	0.	1.297

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	91	77	0	752	0	132
normalized size	1	1.	1.47	1.24	0.	12.13	0.	2.13
time (sec)	N/A	0.111	0.239	0.043	0.	2.187	0.	1.24

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	144	112	0	1131	0	190
normalized size	1	1.	1.71	1.33	0.	13.46	0.	2.26
time (sec)	N/A	0.271	0.474	0.05	0.	2.954	0.	1.247

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	125	162	0	1361	0	262
normalized size	1	1.	1.12	1.45	0.	12.15	0.	2.34
time (sec)	N/A	0.433	1.549	0.051	0.	3.053	0.	1.506

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	115	266	0	1277	0	248
normalized size	1	1.	0.68	1.57	0.	7.56	0.	1.47
time (sec)	N/A	0.383	0.529	0.051	0.	2.012	0.	1.769

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	94	196	0	1071	0	275
normalized size	1	1.	0.76	1.58	0.	8.64	0.	2.22
time (sec)	N/A	0.218	0.391	0.05	0.	1.943	0.	1.864

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	83	170	0	896	0	167
normalized size	1	1.	0.95	1.95	0.	10.3	0.	1.92
time (sec)	N/A	0.125	0.214	0.045	0.	1.851	0.	1.938

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	67	99	0	612	0	122
normalized size	1	1.	1.02	1.5	0.	9.27	0.	1.85
time (sec)	N/A	0.063	0.105	0.039	0.	1.725	0.	1.846

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	66	98	0	614	0	128
normalized size	1	1.	1.02	1.51	0.	9.45	0.	1.97
time (sec)	N/A	0.051	0.096	0.038	0.	1.813	0.	1.662

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	99	174	0	1184	0	181
normalized size	1	1.	1.06	1.87	0.	12.73	0.	1.95
time (sec)	N/A	0.18	0.239	0.062	0.	4.096	0.	1.602

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	127	201	0	1775	0	316
normalized size	1	1.	1.03	1.63	0.	14.43	0.	2.57
time (sec)	N/A	0.335	0.676	0.064	0.	4.128	0.	1.704

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	171	236	0	2635	0	290
normalized size	1	1.	1.02	1.4	0.	15.68	0.	1.73
time (sec)	N/A	0.574	0.864	0.075	0.	6.921	0.	1.604

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	243	243	164	712	0	2384	0	697
normalized size	1	1.	0.67	2.93	0.	9.81	0.	2.87
time (sec)	N/A	0.661	0.917	0.066	0.	2.445	0.	1.708

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	144	634	0	2013	0	346
normalized size	1	1.	0.8	3.54	0.	11.25	0.	1.93
time (sec)	N/A	0.411	0.757	0.062	0.	2.281	0.	1.982

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	136	612	0	1742	0	316
normalized size	1	1.	0.94	4.25	0.	12.1	0.	2.19
time (sec)	N/A	0.245	0.514	0.057	0.	2.161	0.	1.648

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	94	265	0	1138	0	246
normalized size	1	1.	0.8	2.25	0.	9.64	0.	2.08
time (sec)	N/A	0.144	0.352	0.051	0.	2.107	0.	1.806

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	94	221	0	1072	0	255
normalized size	1	1.	0.91	2.15	0.	10.41	0.	2.48
time (sec)	N/A	0.103	0.278	0.049	0.	2.201	0.	1.694

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	93	300	0	1135	0	290
normalized size	1	1.	0.91	2.94	0.	11.13	0.	2.84
time (sec)	N/A	0.095	0.182	0.052	0.	2.126	0.	1.589

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	140	614	0	2233	0	332
normalized size	1	1.	0.97	4.23	0.	15.4	0.	2.29
time (sec)	N/A	0.37	0.795	0.075	0.	8.209	0.	1.625

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	174	641	0	3114	0	378
normalized size	1	1.	0.93	3.43	0.	16.65	0.	2.02
time (sec)	N/A	0.639	1.248	0.085	0.	8.02	0.	1.625

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	241	241	220	686	0	4528	0	694
normalized size	1	1.	0.91	2.85	0.	18.79	0.	2.88
time (sec)	N/A	0.873	1.889	0.088	0.	15.143	0.	1.644

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	182	157	1733	0	2101	0	689
normalized size	1	1.	0.86	9.52	0.	11.54	0.	3.79
time (sec)	N/A	0.225	1.065	0.076	0.	2.035	0.	1.238

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	143	460	0	0	0	0
normalized size	1	1.	0.83	2.67	0.	0.	0.	0.
time (sec)	N/A	0.17	3.156	0.918	0.	0.	0.	0.

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	61	239	0	0	0	0
normalized size	1	1.	0.98	3.85	0.	0.	0.	0.
time (sec)	N/A	0.037	0.074	0.882	0.	0.	0.	0.

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	89	169	0	0	0	0
normalized size	1	1.	0.7	1.32	0.	0.	0.	0.
time (sec)	N/A	0.235	15.415	0.832	0.	0.	0.	0.

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	213	312	456	0	0	0	0
normalized size	1	1.	1.46	2.14	0.	0.	0.	0.
time (sec)	N/A	0.481	8.888	1.047	0.	0.	0.	0.

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	94	202	0	0	0	0
normalized size	1	1.	0.71	1.53	0.	0.	0.	0.
time (sec)	N/A	0.107	2.4	0.819	0.	0.	0.	0.

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	61	126	0	0	0	0
normalized size	1	1.	0.98	2.03	0.	0.	0.	0.
time (sec)	N/A	0.037	0.056	0.695	0.	0.	0.	0.

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	62	135	0	0	0	0
normalized size	1	1.	0.98	2.14	0.	0.	0.	0.
time (sec)	N/A	0.13	0.077	0.589	0.	0.	0.	0.

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	222	222	315	412	0	0	0	0
normalized size	1	1.	1.42	1.86	0.	0.	0.	0.
time (sec)	N/A	0.495	10.025	1.823	0.	0.	0.	0.

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	371	371	10847	9567	0	0	0	0
normalized size	1	1.	29.24	25.79	0.	0.	0.	0.
time (sec)	N/A	0.563	26.756	0.568	0.	0.	0.	0.

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	172	310	0	0	0	0
normalized size	1	1.	1.58	2.84	0.	0.	0.	0.
time (sec)	N/A	0.068	3.268	0.207	0.	0.	0.	0.

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	270	270	199	0	0	0	0	0
normalized size	1	1.	0.74	0.	0.	0.	0.	0.
time (sec)	N/A	0.38	0.771	2.731	0.	0.	0.	0.

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	144	0	0	0	0	0
normalized size	1	1.	0.74	0.	0.	0.	0.	0.
time (sec)	N/A	0.135	0.305	3.307	0.	0.	0.	0.

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	111	0	0	0	0	0
normalized size	1	1.	0.8	0.	0.	0.	0.	0.
time (sec)	N/A	0.072	0.156	1.076	0.	0.	0.	0.

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	195	195	1593	0	0	0	0	0
normalized size	1	1.	8.17	0.	0.	0.	0.	0.
time (sec)	N/A	0.248	17.553	0.399	0.	0.	0.	0.

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	306	306	1856	0	0	0	0	0
normalized size	1	1.	6.07	0.	0.	0.	0.	0.
time (sec)	N/A	0.42	18.892	0.66	0.	0.	0.	0.

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	406	406	2388	0	0	0	0	0
normalized size	1	1.	5.88	0.	0.	0.	0.	0.
time (sec)	N/A	0.549	18.591	0.835	0.	0.	0.	0.

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	188	0	0	0	0	0
normalized size	1	1.	1.32	0.	0.	0.	0.	0.
time (sec)	N/A	0.123	0.311	5.474	0.	0.	0.	0.

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	73	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.074	0.105	0.183	0.	0.	0.	0.

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.04	2.272	0.602	0.	0.	0.	0.

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	351	351	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.486	4.271	0.282	0.	0.	0.	0.

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	274	274	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.298	6.787	0.222	0.	0.	0.	0.

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	220	220	193	0	0	0	0	0
normalized size	1	1.	0.88	0.	0.	0.	0.	0.
time (sec)	N/A	0.188	0.47	0.145	0.	0.	0.	0.

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	104	104	120	0	0	0	0	0
normalized size	1	1.	1.15	0.	0.	0.	0.	0.
time (sec)	N/A	0.065	0.241	0.006	0.	0.	0.	0.

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.031	2.403	0.824	0.	0.	0.	0.

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	64	149	197	196	314	135
normalized size	1	1.	0.55	1.28	1.7	1.69	2.71	1.16
time (sec)	N/A	0.162	0.548	0.021	1.206	1.641	4.438	1.715

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	54	89	116	154	196	109
normalized size	1	1.	0.65	1.07	1.4	1.86	2.36	1.31
time (sec)	N/A	0.113	0.371	0.017	1.213	1.505	2.187	1.903

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	42	77	104	111	133	84
normalized size	1	1.	0.81	1.48	2.	2.13	2.56	1.62
time (sec)	N/A	0.065	0.287	0.015	1.122	1.332	0.7	1.918

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	25	40	50	66	70	31
normalized size	1	1.	0.86	1.38	1.72	2.28	2.41	1.07
time (sec)	N/A	0.018	0.024	0.013	1.19	1.509	0.406	1.998

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	83	43	111	159	88	50
normalized size	1	1.	2.52	1.3	3.36	4.82	2.67	1.52
time (sec)	N/A	0.048	0.182	0.057	2.238	1.63	1.916	1.672

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	74	56	293	250	158	53
normalized size	1	1.	2.47	1.87	9.77	8.33	5.27	1.77
time (sec)	N/A	0.073	0.27	0.067	1.247	1.257	6.834	1.937

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	96	86	525	382	573	113
normalized size	1	1.	1.6	1.43	8.75	6.37	9.55	1.88
time (sec)	N/A	0.117	0.328	0.076	1.258	1.301	15.001	1.634

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	109	116	757	517	1061	154
normalized size	1	1.	1.18	1.26	8.23	5.62	11.53	1.67
time (sec)	N/A	0.17	0.466	0.086	1.255	1.407	48.928	1.736

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	124	146	990	653	0	194
normalized size	1	1.	0.98	1.16	7.86	5.18	0.	1.54
time (sec)	N/A	0.22	0.585	0.096	1.268	1.383	0.	1.68

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	89	255	346	246	629	208
normalized size	1	1.	0.59	1.68	2.28	1.62	4.14	1.37
time (sec)	N/A	0.197	1.124	0.022	1.188	1.451	19.739	1.654

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	79	211	282	207	530	180
normalized size	1	1.	0.67	1.79	2.39	1.75	4.49	1.53
time (sec)	N/A	0.145	0.746	0.022	1.144	1.448	10.094	1.666

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	69	159	213	163	340	151
normalized size	1	1.	0.81	1.87	2.51	1.92	4.	1.78
time (sec)	N/A	0.097	1.543	0.016	1.152	1.452	5.081	1.873

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	39	88	109	122	206	70
normalized size	1	1.	0.61	1.38	1.7	1.91	3.22	1.09
time (sec)	N/A	0.069	0.048	0.014	1.117	1.429	2.322	1.979

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	43	78	104	112	133	84
normalized size	1	1.	0.83	1.5	2.	2.15	2.56	1.62
time (sec)	N/A	0.077	0.339	0.014	1.129	1.238	1.547	1.911

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	130	73	294	238	456	139
normalized size	1	1.	2.28	1.28	5.16	4.18	8.	2.44
time (sec)	N/A	0.141	0.371	0.073	1.701	1.325	7.495	2.062

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	121	71	491	362	486	81
normalized size	1	1.	1.68	0.99	6.82	5.03	6.75	1.12
time (sec)	N/A	0.137	0.613	0.075	1.778	1.35	16.866	2.009

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	81	88	752	398	364	81
normalized size	1	1.	2.38	2.59	22.12	11.71	10.71	2.38
time (sec)	N/A	0.094	0.402	0.088	1.269	1.282	49.375	2.171

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	117	118	1102	537	0	173
normalized size	1	1.	1.75	1.76	16.45	8.01	0.	2.58
time (sec)	N/A	0.136	0.601	0.092	1.272	1.335	0.	2.122

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	121	148	1449	684	0	219
normalized size	1	1.	1.23	1.51	14.79	6.98	0.	2.23
time (sec)	N/A	0.182	0.536	0.104	1.404	1.314	0.	2.203

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	133	178	1798	832	0	265
normalized size	1	1.	1.01	1.35	13.62	6.3	0.	2.01
time (sec)	N/A	0.233	0.673	0.107	1.456	1.394	0.	2.179

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	109	297	406	297	838	265
normalized size	1	1.	0.61	1.65	2.26	1.65	4.66	1.47
time (sec)	N/A	0.206	2.143	0.023	1.225	1.477	47.784	2.07

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	89	276	381	247	740	208
normalized size	1	1.	0.61	1.9	2.63	1.7	5.1	1.43
time (sec)	N/A	0.157	1.23	0.02	1.211	1.356	29.937	1.695

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	89	255	346	207	631	208
normalized size	1	1.	0.79	2.28	3.09	1.85	5.63	1.86
time (sec)	N/A	0.108	1.055	0.016	1.192	1.388	18.636	1.725

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	49	140	178	163	398	99
normalized size	1	1.	0.54	1.54	1.96	1.79	4.37	1.09
time (sec)	N/A	0.079	0.051	0.013	1.225	1.402	10.191	1.775

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	69	160	213	165	340	151
normalized size	1	1.	0.81	1.88	2.51	1.94	4.	1.78
time (sec)	N/A	0.092	1.533	0.015	1.6	1.393	5.522	2.482

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	54	89	116	155	196	109
normalized size	1	1.	0.66	1.09	1.41	1.89	2.39	1.33
time (sec)	N/A	0.098	0.386	0.017	1.292	1.461	3.192	2.442

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	153	181	585	312	1170	158
normalized size	1	1.	1.63	1.93	6.22	3.32	12.45	1.68
time (sec)	N/A	0.183	0.507	0.08	2.127	1.409	17.9	2.294

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	149	121	802	435	1282	136
normalized size	1	1.	1.62	1.32	8.72	4.73	13.93	1.48
time (sec)	N/A	0.186	0.961	0.086	1.803	1.305	49.115	1.759

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	249	143	1060	551	0	150
normalized size	1	1.	2.35	1.35	10.	5.2	0.	1.42
time (sec)	N/A	0.189	0.455	0.092	2.663	1.353	0.	2.355

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	93	118	1411	528	0	104
normalized size	1	1.	2.74	3.47	41.5	15.53	0.	3.06
time (sec)	N/A	0.09	0.776	0.099	1.547	1.365	0.	2.049

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	135	148	1875	675	0	219
normalized size	1	1.	1.96	2.14	27.17	9.78	0.	3.17
time (sec)	N/A	0.139	0.721	0.109	2.191	1.39	0.	2.081

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	145	178	2341	832	0	265
normalized size	1	1.	1.44	1.76	23.18	8.24	0.	2.62
time (sec)	N/A	0.182	0.925	0.119	1.896	1.345	0.	2.22

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	157	208	2805	976	0	311
normalized size	1	1.	1.19	1.58	21.25	7.39	0.	2.36
time (sec)	N/A	0.23	1.865	0.129	2.047	1.469	0.	2.299

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	209	238	3270	1138	0	356
normalized size	1	1.	1.26	1.43	19.7	6.86	0.	2.14
time (sec)	N/A	0.287	2.113	0.139	1.948	1.442	0.	2.202

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	175	219	972	390	2106	182
normalized size	1	1.	1.48	1.86	8.24	3.31	17.85	1.54
time (sec)	N/A	0.196	1.381	0.092	2.302	1.319	42.412	2.277

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	155	181	572	313	1170	158
normalized size	1	1.	1.68	1.97	6.22	3.4	12.72	1.72
time (sec)	N/A	0.176	0.508	0.078	2.028	1.395	16.818	2.073

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	129	73	284	238	456	135
normalized size	1	1.	2.3	1.3	5.07	4.25	8.14	2.41
time (sec)	N/A	0.136	0.367	0.071	2.148	1.365	7.789	1.933

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	79	43	104	159	90	50
normalized size	1	1.	2.47	1.34	3.25	4.97	2.81	1.56
time (sec)	N/A	0.044	0.18	0.056	1.884	1.32	3.36	2.051

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	0	22	47	49	23
normalized size	1	1.	1.	0.	1.38	2.94	3.06	1.44
time (sec)	N/A	0.067	0.012	180.	1.751	1.277	2.815	2.032

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	87	73	192	142	328	104
normalized size	1	1.	1.64	1.38	3.62	2.68	6.19	1.96
time (sec)	N/A	0.11	0.426	0.049	1.412	1.242	8.539	2.044

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	111	103	285	209	738	142
normalized size	1	1.	1.31	1.21	3.35	2.46	8.68	1.67
time (sec)	N/A	0.158	0.669	0.063	1.465	1.307	17.941	2.019

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	131	133	431	279	0	180
normalized size	1	1.	1.11	1.13	3.65	2.36	0.	1.53
time (sec)	N/A	0.207	0.741	0.059	1.569	1.331	0.	2.18

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	276	267	1760	581	0	274
normalized size	1	1.	1.86	1.8	11.89	3.93	0.	1.85
time (sec)	N/A	0.24	0.726	0.108	3.305	1.442	0.	2.151

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	243	229	1219	513	0	205
normalized size	1	1.	1.8	1.7	9.03	3.8	0.	1.52
time (sec)	N/A	0.224	0.504	0.093	2.357	1.439	0.	2.222

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	210	121	797	433	1282	136
normalized size	1	1.	2.33	1.34	8.86	4.81	14.24	1.51
time (sec)	N/A	0.175	0.361	0.089	3.916	1.379	51.751	2.164

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	119	71	487	362	486	78
normalized size	1	1.	1.7	1.01	6.96	5.17	6.94	1.11
time (sec)	N/A	0.132	0.586	0.078	2.155	1.396	12.476	2.139

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	70	56	290	251	158	53
normalized size	1	1.	2.41	1.93	10.	8.66	5.45	1.83
time (sec)	N/A	0.067	0.264	0.069	1.779	1.297	6.883	2.048

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	87	73	192	142	328	104
normalized size	1	1.	1.67	1.4	3.69	2.73	6.31	2.
time (sec)	N/A	0.107	0.462	0.046	1.559	1.37	9.14	1.989

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	29	0	38	92	286	41
normalized size	1	1.	0.76	0.	1.	2.42	7.53	1.08
time (sec)	N/A	0.063	0.054	180.	1.436	1.209	13.275	2.239

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	131	133	452	211	0	180
normalized size	1	1.	1.72	1.75	5.95	2.78	0.	2.37
time (sec)	N/A	0.113	0.822	0.059	1.379	1.297	0.	2.025

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	151	163	576	278	0	217
normalized size	1	1.	1.36	1.47	5.19	2.5	0.	1.95
time (sec)	N/A	0.159	0.934	0.066	1.615	1.382	0.	2.181

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	193	193	701	351	0	255
normalized size	1	1.	1.34	1.34	4.87	2.44	0.	1.77
time (sec)	N/A	0.215	1.131	0.067	1.74	1.383	0.	2.04

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	303	277	2020	707	0	251
normalized size	1	1.	1.88	1.72	12.55	4.39	0.	1.56
time (sec)	N/A	0.276	0.849	0.108	3.688	1.387	0.	2.139

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	270	145	1480	633	0	182
normalized size	1	1.	2.18	1.17	11.94	5.1	0.	1.47
time (sec)	N/A	0.221	0.601	0.099	1.962	1.371	0.	2.058

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	239	143	1054	551	0	150
normalized size	1	1.	2.32	1.39	10.23	5.35	0.	1.46
time (sec)	N/A	0.179	0.427	0.092	2.066	1.338	0.	1.893

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	81	88	748	400	362	81
normalized size	1	1.	2.45	2.67	22.67	12.12	10.97	2.45
time (sec)	N/A	0.086	0.389	0.083	1.198	1.286	49.998	1.892

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	92	86	522	381	573	113
normalized size	1	1.	1.59	1.48	9.	6.57	9.88	1.95
time (sec)	N/A	0.107	0.33	0.074	1.208	1.282	19.125	2.049

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	111	101	285	208	738	142
normalized size	1	1.	1.34	1.22	3.43	2.51	8.89	1.71
time (sec)	N/A	0.148	0.588	0.06	1.137	1.377	17.937	2.118

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	131	133	452	211	0	180
normalized size	1	1.	1.75	1.77	6.03	2.81	0.	2.4
time (sec)	N/A	0.11	0.744	0.055	1.102	1.54	0.	2.093

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	41	0	54	119	687	58
normalized size	1	1.	0.69	0.	0.92	2.02	11.64	0.98
time (sec)	N/A	0.073	0.128	180.	1.138	1.565	61.744	2.015

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	193	193	701	263	0	255
normalized size	1	1.	1.99	1.99	7.23	2.71	0.	2.63
time (sec)	N/A	0.12	1.071	0.069	1.299	1.667	0.	2.271

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	213	223	824	333	0	293
normalized size	1	1.	1.63	1.7	6.29	2.54	0.	2.24
time (sec)	N/A	0.171	1.321	0.081	1.284	1.616	0.	2.024

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	233	253	949	412	0	331
normalized size	1	1.	1.4	1.51	5.68	2.47	0.	1.98
time (sec)	N/A	0.217	1.565	0.076	1.351	1.649	0.	2.103

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	104	79	0	466	0	0
normalized size	1	1.	0.76	0.58	0.	3.4	0.	0.
time (sec)	N/A	0.295	0.841	0.573	0.	1.159	0.	0.

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	94	69	0	385	0	0
normalized size	1	1.	0.91	0.67	0.	3.74	0.	0.
time (sec)	N/A	0.222	0.532	0.52	0.	1.098	0.	0.

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	82	59	0	284	0	0
normalized size	1	1.	1.19	0.86	0.	4.12	0.	0.
time (sec)	N/A	0.157	0.336	0.506	0.	1.074	0.	0.

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	71	47	0	203	0	0
normalized size	1	1.	2.09	1.38	0.	5.97	0.	0.
time (sec)	N/A	0.092	0.118	0.375	0.	1.043	0.	0.

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	135	94	0	544	0	267
normalized size	1	1.	1.75	1.22	0.	7.06	0.	3.47
time (sec)	N/A	0.138	0.586	0.576	0.	1.073	0.	2.394

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	107	120	0	687	0	428
normalized size	1	1.	1.41	1.58	0.	9.04	0.	5.63
time (sec)	N/A	0.141	0.623	0.425	0.	1.067	0.	2.384

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	176	189	0	878	0	0
normalized size	1	1.	1.56	1.67	0.	7.77	0.	0.
time (sec)	N/A	0.163	0.912	0.714	0.	1.143	0.	0.

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	189	243	0	1073	0	0
normalized size	1	1.	1.3	1.68	0.	7.4	0.	0.
time (sec)	N/A	0.193	1.16	0.719	0.	1.191	0.	0.

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	1105	81	0	576	0	0
normalized size	1	1.	7.62	0.56	0.	3.97	0.	0.
time (sec)	N/A	0.327	6.433	0.632	0.	1.099	0.	0.

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	96	71	0	486	0	0
normalized size	1	1.	0.88	0.65	0.	4.46	0.	0.
time (sec)	N/A	0.26	5.381	0.478	0.	1.077	0.	0.

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	84	61	0	374	0	0
normalized size	1	1.	1.15	0.84	0.	5.12	0.	0.
time (sec)	N/A	0.195	1.363	0.67	0.	1.023	0.	0.

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	73	49	0	281	0	0
normalized size	1	1.	2.03	1.36	0.	7.81	0.	0.
time (sec)	N/A	0.125	0.236	0.333	0.	1.03	0.	0.

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	130	112	0	630	0	375
normalized size	1	1.	1.13	0.97	0.	5.48	0.	3.26
time (sec)	N/A	0.244	0.443	0.674	0.	1.092	0.	3.508

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	149	145	0	771	0	0
normalized size	1	1.	1.3	1.26	0.	6.7	0.	0.
time (sec)	N/A	0.244	0.651	0.622	0.	1.147	0.	0.

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	163	191	0	915	0	0
normalized size	1	1.	1.34	1.57	0.	7.5	0.	0.
time (sec)	N/A	0.24	0.961	0.735	0.	1.132	0.	0.

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	307	245	0	1118	0	0
normalized size	1	1.	1.97	1.57	0.	7.17	0.	0.
time (sec)	N/A	0.273	0.972	0.802	0.	1.154	0.	0.

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	190	190	371	299	0	1327	0	0
normalized size	1	1.	1.95	1.57	0.	6.98	0.	0.
time (sec)	N/A	0.303	1.472	0.812	0.	1.206	0.	0.

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	112	81	0	656	0	0
normalized size	1	1.	0.77	0.56	0.	4.52	0.	0.
time (sec)	N/A	0.33	6.185	0.545	0.	1.099	0.	0.

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	1105	71	0	567	0	0
normalized size	1	1.	10.14	0.65	0.	5.2	0.	0.
time (sec)	N/A	0.268	6.465	0.525	0.	1.093	0.	0.

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	84	61	0	451	0	0
normalized size	1	1.	1.15	0.84	0.	6.18	0.	0.
time (sec)	N/A	0.203	2.98	0.536	0.	1.044	0.	0.

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	73	49	0	342	0	0
normalized size	1	1.	2.03	1.36	0.	9.5	0.	0.
time (sec)	N/A	0.127	0.374	0.383	0.	1.051	0.	0.

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	156	129	0	707	0	466
normalized size	1	1.	1.03	0.85	0.	4.68	0.	3.09
time (sec)	N/A	0.316	0.724	0.767	0.	1.154	0.	2.526

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	173	189	0	844	0	0
normalized size	1	1.	1.15	1.26	0.	5.63	0.	0.
time (sec)	N/A	0.32	0.799	0.599	0.	1.121	0.	0.

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	187	239	0	984	0	0
normalized size	1	1.	1.19	1.52	0.	6.27	0.	0.
time (sec)	N/A	0.325	0.999	0.744	0.	1.193	0.	0.

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	307	245	0	1121	0	0
normalized size	1	1.	1.96	1.56	0.	7.14	0.	0.
time (sec)	N/A	0.323	1.569	0.645	0.	1.126	0.	0.

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	191	371	299	0	1338	0	0
normalized size	1	1.	1.94	1.57	0.	7.01	0.	0.
time (sec)	N/A	0.353	2.575	0.991	0.	1.235	0.	0.

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	225	435	353	0	1553	0	0
normalized size	1	1.	1.93	1.57	0.	6.9	0.	0.
time (sec)	N/A	0.388	4.147	0.837	0.	1.259	0.	0.

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	112	69	321	173	0	575
normalized size	1	1.	0.85	0.52	2.43	1.31	0.	4.36
time (sec)	N/A	0.345	2.124	0.479	1.895	1.033	0.	1.789

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	102	59	259	139	0	486
normalized size	1	1.	1.04	0.6	2.64	1.42	0.	4.96
time (sec)	N/A	0.269	0.705	0.403	1.795	1.07	0.	1.676

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	88	49	197	97	0	358
normalized size	1	1.	1.47	0.82	3.28	1.62	0.	5.97
time (sec)	N/A	0.204	0.297	0.454	2.431	1.002	0.	1.579

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	39	103	66	0	262
normalized size	1	1.	1.	1.34	3.55	2.28	0.	9.03
time (sec)	N/A	0.128	0.096	0.408	2.244	1.008	0.	1.503

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	97	85	0	425	0	408
normalized size	1	1.	1.17	1.02	0.	5.12	0.	4.92
time (sec)	N/A	0.159	0.314	0.625	0.	1.098	0.	1.62

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	125	134	0	563	0	628
normalized size	1	1.	1.07	1.15	0.	4.81	0.	5.37
time (sec)	N/A	0.192	0.603	0.638	0.	1.134	0.	1.993

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	162	210	0	663	0	0
normalized size	1	1.	1.04	1.35	0.	4.25	0.	0.
time (sec)	N/A	0.267	0.779	0.703	0.	1.132	0.	0.

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	124	91	513	261	0	887
normalized size	1	1.	0.7	0.52	2.91	1.48	0.	5.04
time (sec)	N/A	0.42	3.071	0.638	1.737	1.068	0.	2.098

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	112	79	451	223	0	802
normalized size	1	1.	0.82	0.58	3.32	1.64	0.	5.9
time (sec)	N/A	0.332	1.182	0.581	1.861	1.066	0.	2.676

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	104	71	389	190	0	603
normalized size	1	1.	1.04	0.71	3.89	1.9	0.	6.03
time (sec)	N/A	0.264	0.78	0.601	1.775	1.074	0.	2.498

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	92	61	323	147	0	579
normalized size	1	1.	1.35	0.9	4.75	2.16	0.	8.51
time (sec)	N/A	0.196	0.352	0.582	1.88	1.035	0.	1.799

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	73	49	201	117	0	578
normalized size	1	1.	2.03	1.36	5.58	3.25	0.	16.06
time (sec)	N/A	0.136	0.119	0.404	1.567	1.058	0.	1.515

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	109	109	0	563	0	664
normalized size	1	1.	0.88	0.88	0.	4.54	0.	5.35
time (sec)	N/A	0.224	0.482	0.681	0.	1.151	0.	1.702

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	164	157	0	512	0	0
normalized size	1	1.	1.06	1.01	0.	3.3	0.	0.
time (sec)	N/A	0.25	0.786	0.569	0.	1.149	0.	0.

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	156	233	0	643	0	1083
normalized size	1	1.	0.81	1.21	0.	3.35	0.	5.64
time (sec)	N/A	0.331	1.136	0.728	0.	1.25	0.	3.086

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	124	91	637	301	0	1084
normalized size	1	1.	0.71	0.52	3.66	1.73	0.	6.23
time (sec)	N/A	0.403	3.014	0.761	1.823	1.144	0.	2.341

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	114	81	575	262	0	975
normalized size	1	1.	0.85	0.6	4.29	1.96	0.	7.28
time (sec)	N/A	0.33	1.243	0.622	1.835	1.117	0.	2.177

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	104	71	513	230	0	878
normalized size	1	1.	1.	0.68	4.93	2.21	0.	8.44
time (sec)	N/A	0.269	0.841	0.504	1.828	1.121	0.	1.944

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	92	61	447	186	0	803
normalized size	1	1.	1.26	0.84	6.12	2.55	0.	11.
time (sec)	N/A	0.196	0.376	0.651	1.839	1.056	0.	1.86

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	73	49	294	153	0	875
normalized size	1	1.	2.03	1.36	8.17	4.25	0.	24.31
time (sec)	N/A	0.122	0.142	0.425	1.748	1.078	0.	1.709

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	189	122	0	663	0	891
normalized size	1	1.	1.18	0.76	0.	4.14	0.	5.57
time (sec)	N/A	0.294	0.598	0.618	0.	1.163	0.	1.78

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	191	174	170	0	645	0	0
normalized size	1	1.	0.91	0.89	0.	3.38	0.	0.
time (sec)	N/A	0.328	1.231	0.603	0.	1.216	0.	0.

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	228	228	443	246	0	578	0	0
normalized size	1	1.	1.94	1.08	0.	2.54	0.	0.
time (sec)	N/A	0.407	1.506	0.767	0.	1.201	0.	0.

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	83	103	0	232	0	0
normalized size	1	1.	1.93	2.4	0.	5.4	0.	0.
time (sec)	N/A	0.083	0.396	0.211	0.	1.131	0.	0.

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	74	78	0	200	0	0
normalized size	1	1.	1.72	1.81	0.	4.65	0.	0.
time (sec)	N/A	0.082	0.286	0.179	0.	1.109	0.	0.

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	60	61	0	155	0	0
normalized size	1	1.	1.4	1.42	0.	3.6	0.	0.
time (sec)	N/A	0.081	0.212	0.176	0.	1.096	0.	0.

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	39	44	0	111	0	0
normalized size	1	1.	0.95	1.07	0.	2.71	0.	0.
time (sec)	N/A	0.073	0.087	0.178	0.	1.027	0.	0.

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	119	106	85	0	0	0
normalized size	1	1.	2.29	2.04	1.63	0.	0.	0.
time (sec)	N/A	0.101	0.92	0.161	1.823	0.	0.	0.

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	84	68	0	146	0	0
normalized size	1	1.	2.1	1.7	0.	3.65	0.	0.
time (sec)	N/A	0.082	0.208	0.171	0.	1.05	0.	0.

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	87	96	0	188	0	0
normalized size	1	1.	2.02	2.23	0.	4.37	0.	0.
time (sec)	N/A	0.085	0.207	0.178	0.	1.096	0.	0.

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	87	120	0	224	0	0
normalized size	1	1.	2.02	2.79	0.	5.21	0.	0.
time (sec)	N/A	0.084	0.274	0.18	0.	1.167	0.	0.

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	146	106	0	248	0	0
normalized size	1	1.	1.64	1.19	0.	2.79	0.	0.
time (sec)	N/A	0.188	1.022	0.168	0.	1.469	0.	0.

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	137	90	0	215	0	0
normalized size	1	1.	1.54	1.01	0.	2.42	0.	0.
time (sec)	N/A	0.179	0.632	0.148	0.	1.424	0.	0.

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	70	55	0	155	0	0
normalized size	1	1.	0.79	0.62	0.	1.74	0.	0.
time (sec)	N/A	0.178	0.429	0.141	0.	1.448	0.	0.

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	60	63	0	157	0	0
normalized size	1	1.	1.4	1.47	0.	3.65	0.	0.
time (sec)	N/A	0.082	0.203	0.158	0.	1.355	0.	0.

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	113	252	0	0	0	0
normalized size	1	1.	1.18	2.62	0.	0.	0.	0.
time (sec)	N/A	0.192	0.318	0.17	0.	0.	0.	0.

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	153	377	185	0	0	0
normalized size	1	1.	1.58	3.89	1.91	0.	0.	0.
time (sec)	N/A	0.195	0.454	0.14	1.721	0.	0.	0.

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	99	90	0	203	0	0
normalized size	1	1.	2.36	2.14	0.	4.83	0.	0.
time (sec)	N/A	0.091	0.469	0.132	0.	1.066	0.	0.

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	106	141	0	255	0	0
normalized size	1	1.	1.2	1.6	0.	2.9	0.	0.
time (sec)	N/A	0.184	0.576	0.141	0.	1.108	0.	0.

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	106	169	0	288	0	0
normalized size	1	1.	1.15	1.84	0.	3.13	0.	0.
time (sec)	N/A	0.177	1.053	0.15	0.	1.132	0.	0.

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	106	196	0	331	0	0
normalized size	1	1.	1.15	2.13	0.	3.6	0.	0.
time (sec)	N/A	0.176	1.504	0.163	0.	1.196	0.	0.

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	156	116	0	265	0	0
normalized size	1	1.	1.16	0.87	0.	1.98	0.	0.
time (sec)	N/A	0.27	1.376	0.165	0.	1.209	0.	0.

Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	77	67	0	207	0	0
normalized size	1	1.	0.57	0.5	0.	1.54	0.	0.
time (sec)	N/A	0.271	0.538	0.145	0.	1.093	0.	0.

Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	133	90	0	216	0	0
normalized size	1	1.	1.49	1.01	0.	2.43	0.	0.
time (sec)	N/A	0.172	0.664	0.141	0.	1.088	0.	0.

Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	72	80	0	201	0	0
normalized size	1	1.	1.67	1.86	0.	4.67	0.	0.
time (sec)	N/A	0.08	0.266	0.183	0.	1.056	0.	0.

Problem 362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	127	315	0	0	0	0
normalized size	1	1.	0.9	2.23	0.	0.	0.	0.
time (sec)	N/A	0.279	0.577	0.184	0.	0.	0.	0.

Problem 363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	169	439	0	0	0	0
normalized size	1	1.	1.17	3.05	0.	0.	0.	0.
time (sec)	N/A	0.29	0.806	0.154	0.	0.	0.	0.

Problem 364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	190	556	248	0	0	0
normalized size	1	1.	1.29	3.78	1.69	0.	0.	0.
time (sec)	N/A	0.293	1.153	0.151	2.495	0.	0.	0.

Problem 365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	110	136	0	265	0	0
normalized size	1	1.	2.62	3.24	0.	6.31	0.	0.
time (sec)	N/A	0.092	0.983	0.142	0.	1.097	0.	0.

Problem 366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	118	199	0	328	0	0
normalized size	1	1.	1.34	2.26	0.	3.73	0.	0.
time (sec)	N/A	0.193	2.189	0.155	0.	1.121	0.	0.

Problem 367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	118	226	0	369	0	0
normalized size	1	1.	0.89	1.7	0.	2.77	0.	0.
time (sec)	N/A	0.283	3.354	0.197	0.	1.155	0.	0.

Problem 368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	118	252	0	409	0	0
normalized size	1	1.	0.84	1.8	0.	2.92	0.	0.
time (sec)	N/A	0.273	4.741	0.187	0.	1.194	0.	0.

Problem 369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	127	143	0	306	0	0
normalized size	1	1.	0.71	0.8	0.	1.71	0.	0.
time (sec)	N/A	0.366	5.623	0.203	0.	1.21	0.	0.

Problem 370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	87	77	0	244	0	0
normalized size	1	1.	0.49	0.43	0.	1.36	0.	0.
time (sec)	N/A	0.366	1.012	0.156	0.	1.183	0.	0.

Problem 371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	107	116	0	266	0	0
normalized size	1	1.	0.8	0.87	0.	1.99	0.	0.
time (sec)	N/A	0.264	1.225	0.167	0.	1.162	0.	0.

Problem 372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	93	106	0	250	0	0
normalized size	1	1.	1.04	1.19	0.	2.81	0.	0.
time (sec)	N/A	0.17	0.963	0.161	0.	1.119	0.	0.

Problem 373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	82	103	0	231	0	0
normalized size	1	1.	1.91	2.4	0.	5.37	0.	0.
time (sec)	N/A	0.082	0.341	0.17	0.	1.11	0.	0.

Problem 374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	150	367	0	0	0	0
normalized size	1	1.	0.82	1.99	0.	0.	0.	0.
time (sec)	N/A	0.376	1.024	0.197	0.	0.	0.	0.

Problem 375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	179	491	0	0	0	0
normalized size	1	1.	0.93	2.56	0.	0.	0.	0.
time (sec)	N/A	0.396	1.598	0.176	0.	0.	0.	0.

Problem 376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	207	618	0	0	0	0
normalized size	1	1.	1.06	3.17	0.	0.	0.	0.
time (sec)	N/A	0.402	1.904	0.173	0.	0.	0.	0.

Problem 377	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	232	748	454	0	0	0
normalized size	1	1.	1.2	3.88	2.35	0.	0.	0.
time (sec)	N/A	0.408	2.394	0.168	1.802	0.	0.	0.

Problem 378	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	115	154	0	309	0	0
normalized size	1	1.	2.74	3.67	0.	7.36	0.	0.
time (sec)	N/A	0.093	4.422	0.139	0.	1.13	0.	0.

Problem 379	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	331	247	0	402	0	0
normalized size	1	1.	3.76	2.81	0.	4.57	0.	0.
time (sec)	N/A	0.186	6.6	0.165	0.	1.211	0.	0.

Problem 380	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	335	276	0	446	0	0
normalized size	1	1.	2.52	2.08	0.	3.35	0.	0.
time (sec)	N/A	0.294	6.64	0.179	0.	1.223	0.	0.

Problem 381	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	333	302	0	481	0	0
normalized size	1	1.	1.87	1.7	0.	2.7	0.	0.
time (sec)	N/A	0.38	6.689	0.207	0.	1.274	0.	0.

Problem 382	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	128	328	0	516	0	0
normalized size	1	1.	0.68	1.74	0.	2.74	0.	0.
time (sec)	N/A	0.384	6.024	0.228	0.	1.345	0.	0.

Problem 383	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	136	320	0	0	0	0
normalized size	1	1.	0.98	2.3	0.	0.	0.	0.
time (sec)	N/A	0.281	0.561	0.198	0.	0.	0.	0.

Problem 384	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	119	261	0	0	0	0
normalized size	1	1.	1.28	2.81	0.	0.	0.	0.
time (sec)	N/A	0.187	0.293	0.174	0.	0.	0.	0.

Problem 385	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	118	106	86	0	0	0
normalized size	1	1.	2.41	2.16	1.76	0.	0.	0.
time (sec)	N/A	0.1	0.976	0.127	1.818	0.	0.	0.

Problem 386	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	89	92	0	393	0	0
normalized size	1	1.	1.93	2.	0.	8.54	0.	0.
time (sec)	N/A	0.086	0.243	0.164	0.	1.219	0.	0.

Problem 387	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	161	165	0	803	0	0
normalized size	1	1.	1.69	1.74	0.	8.45	0.	0.
time (sec)	N/A	0.175	0.409	0.177	0.	1.299	0.	0.

Problem 388	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	224	252	0	990	0	0
normalized size	1	1.	1.6	1.8	0.	7.07	0.	0.
time (sec)	N/A	0.269	0.617	0.188	0.	1.353	0.	0.

Problem 389	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	191	162	501	0	0	0	0
normalized size	1	1.	0.85	2.62	0.	0.	0.	0.
time (sec)	N/A	0.377	1.711	0.174	0.	0.	0.	0.

Problem 390	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	153	446	0	0	0	0
normalized size	1	1.	1.07	3.12	0.	0.	0.	0.
time (sec)	N/A	0.288	0.781	0.157	0.	0.	0.	0.

Problem 391	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	134	388	184	0	0	0
normalized size	1	1.	1.38	4.	1.9	0.	0.	0.
time (sec)	N/A	0.197	0.454	0.141	1.779	0.	0.	0.

Problem 392	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	85	69	0	146	0	0
normalized size	1	1.	2.07	1.68	0.	3.56	0.	0.
time (sec)	N/A	0.085	0.193	0.161	0.	1.039	0.	0.

Problem 393	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	148	167	0	803	0	0
normalized size	1	1.	1.56	1.76	0.	8.45	0.	0.
time (sec)	N/A	0.176	0.398	0.178	0.	1.326	0.	0.

Problem 394	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	170	117	0	676	0	0
normalized size	1	1.	1.19	0.82	0.	4.73	0.	0.
time (sec)	N/A	0.278	0.608	0.15	0.	1.295	0.	0.

Problem 395	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	191	287	227	0	950	0	0
normalized size	1	1.	1.5	1.19	0.	4.97	0.	0.
time (sec)	N/A	0.374	0.773	0.167	0.	1.382	0.	0.

Problem 396	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	237	202	685	0	0	0	0
normalized size	1	1.	0.85	2.89	0.	0.	0.	0.
time (sec)	N/A	0.493	5.144	0.196	0.	0.	0.	0.

Problem 397	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	187	633	0	0	0	0
normalized size	1	1.	0.97	3.28	0.	0.	0.	0.
time (sec)	N/A	0.387	2.023	0.168	0.	0.	0.	0.

Problem 398	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	172	566	247	0	0	0
normalized size	1	1.	1.2	3.96	1.73	0.	0.	0.
time (sec)	N/A	0.303	1.181	0.148	1.815	0.	0.	0.

Problem 399	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	86	96	0	203	0	0
normalized size	1	1.	2.05	2.29	0.	4.83	0.	0.
time (sec)	N/A	0.092	0.467	0.132	0.	1.39	0.	0.

Problem 400	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	87	92	0	186	0	0
normalized size	1	1.	2.02	2.14	0.	4.33	0.	0.
time (sec)	N/A	0.085	0.214	0.164	0.	1.143	0.	0.

Problem 401	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	211	252	0	990	0	0
normalized size	1	1.	1.51	1.8	0.	7.07	0.	0.
time (sec)	N/A	0.271	0.628	0.181	0.	1.329	0.	0.

Problem 402	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	287	227	0	950	0	0
normalized size	1	1.	1.53	1.21	0.	5.05	0.	0.
time (sec)	N/A	0.383	0.745	0.17	0.	1.356	0.	0.

Problem 403	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	236	236	237	134	0	745	0	0
normalized size	1	1.	1.	0.57	0.	3.16	0.	0.
time (sec)	N/A	0.479	0.999	0.171	0.	1.324	0.	0.

Problem 404	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	110	110	365	0	0	0	0	0
normalized size	1	1.	3.32	0.	0.	0.	0.	0.
time (sec)	N/A	0.155	3.079	0.95	0.	0.	0.	0.

Problem 405	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	86	86	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.143	180.047	2.334	0.	0.	0.	0.

Problem 406	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	86	86	88512	0	0	0	0	0
normalized size	1	1.	1029.21	0.	0.	0.	0.	0.
time (sec)	N/A	0.14	155.36	2.758	0.	0.	0.	0.

Problem 407	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	84	84	261	0	0	0	0	0
normalized size	1	1.	3.11	0.	0.	0.	0.	0.
time (sec)	N/A	0.113	1.652	0.977	0.	0.	0.	0.

Problem 408	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	76	76	3844	0	0	0	0	0
normalized size	1	1.	50.58	0.	0.	0.	0.	0.
time (sec)	N/A	0.133	15.828	0.181	0.	0.	0.	0.

Problem 409	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	86	86	5391	0	0	0	0	0
normalized size	1	1.	62.69	0.	0.	0.	0.	0.
time (sec)	N/A	0.132	20.801	0.312	0.	0.	0.	0.

Problem 410	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	86	86	7184	0	0	0	0	0
normalized size	1	1.	83.53	0.	0.	0.	0.	0.
time (sec)	N/A	0.133	23.397	0.368	0.	0.	0.	0.

Problem 411	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	149	0	392	630	0	0
normalized size	1	1.	0.93	0.	2.45	3.94	0.	0.
time (sec)	N/A	0.252	2.547	4.983	1.987	1.158	0.	0.

Problem 412	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	110	0	261	359	0	0
normalized size	1	1.	1.1	0.	2.61	3.59	0.	0.
time (sec)	N/A	0.149	0.498	0.148	1.872	1.142	0.	0.

Problem 413	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	85	0	157	205	0	0
normalized size	1	1.	1.85	0.	3.41	4.46	0.	0.
time (sec)	N/A	0.068	0.172	0.149	1.815	1.103	0.	0.

Problem 414	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	157	0	0	0	0	0
normalized size	1	1.	2.31	0.	0.	0.	0.	0.
time (sec)	N/A	0.134	1.437	0.148	0.	0.	0.	0.

Problem 415	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	74	74	3006	0	0	0	0	0
normalized size	1	1.	40.62	0.	0.	0.	0.	0.
time (sec)	N/A	0.157	17.343	0.148	0.	0.	0.	0.

Problem 416	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	74	74	5136	0	0	0	0	0
normalized size	1	1.	69.41	0.	0.	0.	0.	0.
time (sec)	N/A	0.16	22.069	0.144	0.	0.	0.	0.

Problem 417	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	157	0	0	0	0	0
normalized size	1	1.	2.31	0.	0.	0.	0.	0.
time (sec)	N/A	0.129	0.492	0.006	0.	0.	0.	0.

Problem 418	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	157	0	0	0	0	0
normalized size	1	1.	2.31	0.	0.	0.	0.	0.
time (sec)	N/A	0.138	0.427	0.175	0.	0.	0.	0.

Problem 419	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	174	0	0	251	0	0
normalized size	1	1.	1.06	0.	0.	1.53	0.	0.
time (sec)	N/A	0.223	8.606	0.342	0.	1.089	0.	0.

Problem 420	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	136	0	0	178	0	0
normalized size	1	1.	1.35	0.	0.	1.76	0.	0.
time (sec)	N/A	0.135	3.127	0.298	0.	1.145	0.	0.

Problem 421	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	107	0	0	108	0	0
normalized size	1	1.	2.33	0.	0.	2.35	0.	0.
time (sec)	N/A	0.067	1.549	0.263	0.	1.098	0.	0.

Problem 422	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	112	112	388	0	0	0	0	0
normalized size	1	1.	3.46	0.	0.	0.	0.	0.
time (sec)	N/A	0.158	2.939	0.463	0.	0.	0.	0.

Problem 423	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	114	114	602	0	0	0	0	0
normalized size	1	1.	5.28	0.	0.	0.	0.	0.
time (sec)	N/A	0.186	7.555	0.256	0.	0.	0.	0.

Problem 424	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	114	114	1201	0	0	0	0	0
normalized size	1	1.	10.54	0.	0.	0.	0.	0.
time (sec)	N/A	0.182	12.74	0.289	0.	0.	0.	0.

Problem 425	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	227	207	259	338	481	580	367
normalized size	1	1.	0.91	1.14	1.49	2.12	2.56	1.62
time (sec)	N/A	0.282	1.391	0.046	1.056	1.188	4.014	1.293

Problem 426	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	124	182	236	340	386	258
normalized size	1	1.	0.77	1.12	1.46	2.1	2.38	1.59
time (sec)	N/A	0.189	0.805	0.039	1.173	1.134	1.818	1.239

Problem 427	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	89	115	151	215	199	158
normalized size	1	1.	0.9	1.16	1.53	2.17	2.01	1.6
time (sec)	N/A	0.093	0.399	0.033	0.987	1.093	0.803	1.277

Problem 428	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	45	59	77	120	94	74
normalized size	1	1.	0.94	1.23	1.6	2.5	1.96	1.54
time (sec)	N/A	0.024	0.113	0.029	1.101	1.069	0.331	1.251

Problem 429	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	27	17	22	38	19	23
normalized size	1	1.	1.69	1.06	1.38	2.38	1.19	1.44
time (sec)	N/A	0.008	0.006	0.007	1.125	1.024	0.159	1.293

Problem 430	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	182	119	0	520	269	116
normalized size	1	1.	2.89	1.89	0.	8.25	4.27	1.84
time (sec)	N/A	0.091	0.321	0.083	0.	1.183	162.134	1.325

Problem 431	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	220	147	0	790	0	174
normalized size	1	1.	2.65	1.77	0.	9.52	0.	2.1
time (sec)	N/A	0.092	0.557	0.102	0.	1.428	0.	1.301

Problem 432	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	242	1104	0	1729	0	518
normalized size	1	1.	1.81	8.24	0.	12.9	0.	3.87
time (sec)	N/A	0.184	1.144	0.122	0.	1.719	0.	1.406

Problem 433	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	428	3104	0	2877	0	1091
normalized size	1	1.	2.23	16.17	0.	14.98	0.	5.68
time (sec)	N/A	0.335	2.693	0.145	0.	2.287	0.	1.44

Problem 434	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	318	318	262	462	609	662	1136	618
normalized size	1	1.	0.82	1.45	1.92	2.08	3.57	1.94
time (sec)	N/A	0.463	1.386	0.056	1.222	1.893	8.37	1.397

Problem 435	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	233	233	204	329	429	474	729	437
normalized size	1	1.	0.88	1.41	1.84	2.03	3.13	1.88
time (sec)	N/A	0.309	0.91	0.046	1.179	1.792	4.253	1.39

Problem 436	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	148	219	285	324	459	281
normalized size	1	1.	0.95	1.4	1.83	2.08	2.94	1.8
time (sec)	N/A	0.202	0.525	0.039	1.293	1.688	1.886	1.341

Problem 437	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	106	117	154	192	199	147
normalized size	1	1.	1.13	1.24	1.64	2.04	2.12	1.56
time (sec)	N/A	0.062	0.332	0.033	1.134	1.595	0.8	1.39

Problem 438	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	34	52	63	97	78	54
normalized size	1	1.	0.76	1.16	1.4	2.16	1.73	1.2
time (sec)	N/A	0.015	0.188	0.023	1.108	1.582	0.296	1.327

Problem 439	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	130	228	0	668	0	184
normalized size	1	1.	1.41	2.48	0.	7.26	0.	2.
time (sec)	N/A	0.202	0.406	0.095	0.	1.725	0.	1.411

Problem 440	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	115	139	389	0	1054	0	277
normalized size	1	1.03	1.24	3.47	0.	9.41	0.	2.47
time (sec)	N/A	0.182	0.459	0.112	0.	1.837	0.	1.387

Problem 441	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	140	799	0	1449	0	470
normalized size	1	1.	1.01	5.79	0.	10.5	0.	3.41
time (sec)	N/A	0.182	0.664	0.132	0.	1.849	0.	1.452

Problem 442	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	207	196	2425	0	2874	0	1054
normalized size	1	1.	0.95	11.71	0.	13.88	0.	5.09
time (sec)	N/A	0.315	2.439	0.155	0.	2.435	0.	1.764

Problem 443	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	286	286	269	6466	0	4629	0	2099
normalized size	1	1.	0.94	22.61	0.	16.19	0.	7.34
time (sec)	N/A	0.507	3.901	0.192	0.	3.121	0.	1.617

Problem 444	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	326	233	481	633	590	1176	504
normalized size	1	1.52	1.08	2.24	2.94	2.74	5.47	2.34
time (sec)	N/A	0.545	1.414	0.051	1.199	2.145	8.794	1.417

Problem 445	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	189	177	319	416	413	702	339
normalized size	1	1.15	1.08	1.95	2.54	2.52	4.28	2.07
time (sec)	N/A	0.26	0.73	0.045	1.165	2.032	4.031	1.313

Problem 446	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	117	120	178	231	252	371	186
normalized size	1	1.06	1.09	1.62	2.1	2.29	3.37	1.69
time (sec)	N/A	0.097	0.51	0.039	1.174	1.997	1.691	1.373

Problem 447	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	44	74	97	134	121	78
normalized size	1	1.	0.7	1.17	1.54	2.13	1.92	1.24
time (sec)	N/A	0.05	0.343	0.029	1.119	1.916	0.625	1.307

Problem 448	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	162	480	0	909	0	323
normalized size	1	1.	1.13	3.36	0.	6.36	0.	2.26
time (sec)	N/A	0.389	0.65	0.101	0.	2.215	0.	1.329

Problem 449	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	162	600	0	1389	0	533
normalized size	1	1.	1.01	3.73	0.	8.63	0.	3.31
time (sec)	N/A	0.381	0.678	0.131	0.	2.004	0.	1.408

Problem 450	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	196	1400	0	2256	0	705
normalized size	1	1.	1.05	7.49	0.	12.06	0.	3.77
time (sec)	N/A	0.477	0.986	0.149	0.	2.125	0.	1.488

Problem 451	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	207	178	1924	0	2369	0	900
normalized size	1	1.	0.86	9.29	0.	11.44	0.	4.35
time (sec)	N/A	0.475	2.458	0.17	0.	2.147	0.	1.487

Problem 452	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	289	289	240	5149	0	4343	0	1806
normalized size	1	1.	0.83	17.82	0.	15.03	0.	6.25
time (sec)	N/A	0.696	3.181	0.233	0.	2.625	0.	1.621

Problem 453	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	234	673	979	799	8344	413
normalized size	1	1.	1.24	3.56	5.18	4.23	44.15	2.19
time (sec)	N/A	0.227	0.411	0.069	1.813	1.688	25.654	1.366

Problem 454	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	192	364	574	537	3499	232
normalized size	1	1.	1.59	3.01	4.74	4.44	28.92	1.92
time (sec)	N/A	0.125	0.562	0.068	1.745	1.649	9.642	1.361

Problem 455	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	122	140	282	321	877	193
normalized size	1	1.	1.97	2.26	4.55	5.18	14.15	3.11
time (sec)	N/A	0.137	0.449	0.059	1.71	1.59	4.124	1.195

Problem 456	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	79	65	105	166	109	54
normalized size	1	1.	2.26	1.86	3.	4.74	3.11	1.54
time (sec)	N/A	0.047	0.158	0.036	1.803	1.538	1.817	1.4

Problem 457	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	48	22	36	108	27	30
normalized size	1	1.	2.09	0.96	1.57	4.7	1.17	1.3
time (sec)	N/A	0.012	0.042	0.023	1.07	1.499	0.868	1.417

Problem 458	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	115	87	0	1089	0	135
normalized size	1	1.	1.29	0.98	0.	12.24	0.	1.52
time (sec)	N/A	0.13	0.471	0.073	0.	1.701	0.	1.544

Problem 459	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	162	273	0	2422	0	726
normalized size	1	1.	1.08	1.82	0.	16.15	0.	4.84
time (sec)	N/A	0.178	0.638	0.104	0.	1.921	0.	1.62

Problem 460	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	213	232	1224	0	5072	0	651
normalized size	1	1.	1.09	5.75	0.	23.81	0.	3.06
time (sec)	N/A	0.323	1.344	0.115	0.	2.38	0.	1.475

Problem 461	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	260	260	837	982	1771	1314	0	1319
normalized size	1	1.	3.22	3.78	6.81	5.05	0.	5.07
time (sec)	N/A	0.495	1.797	0.102	1.97	1.815	0.	1.532

Problem 462	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	378	618	1226	1000	0	456
normalized size	1	1.	1.94	3.17	6.29	5.13	0.	2.34
time (sec)	N/A	0.362	1.934	0.087	1.8	1.678	0.	1.327

Problem 463	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	212	340	798	699	3471	282
normalized size	1	1.	1.77	2.83	6.65	5.82	28.92	2.35
time (sec)	N/A	0.367	0.368	0.076	1.732	1.629	29.065	1.287

Problem 464	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	172	213	486	454	853	178
normalized size	1	1.	2.02	2.51	5.72	5.34	10.04	2.09
time (sec)	N/A	0.142	0.284	0.059	1.707	1.625	9.749	1.267

Problem 465	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	43	70	289	288	309	92
normalized size	1	1.	0.66	1.08	4.45	4.43	4.75	1.42
time (sec)	N/A	0.052	0.057	0.051	1.17	1.524	4.329	1.256

Problem 466	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	54	53	158	239	146	68
normalized size	1	1.	0.98	0.96	2.87	4.35	2.65	1.24
time (sec)	N/A	0.028	0.1	0.039	1.183	1.456	1.995	1.287

Problem 467	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	204	175	0	2129	0	263
normalized size	1	1.	1.56	1.34	0.	16.25	0.	2.01
time (sec)	N/A	0.282	0.363	0.093	0.	1.829	0.	1.347

Problem 468	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	221	267	361	0	4787	0	432
normalized size	1	1.	1.21	1.63	0.	21.66	0.	1.95
time (sec)	N/A	0.417	1.376	0.104	0.	2.28	0.	1.361

Problem 469	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	294	294	338	1313	0	7625	0	829
normalized size	1	1.	1.15	4.47	0.	25.94	0.	2.82
time (sec)	N/A	0.615	1.183	0.127	0.	2.813	0.	1.42

Problem 470	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	354	354	560	1340	2691	1949	0	1048
normalized size	1	1.	1.58	3.79	7.6	5.51	0.	2.96
time (sec)	N/A	0.79	2.962	0.104	2.142	1.909	0.	1.322

Problem 471	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	278	278	992	924	2030	1548	0	761
normalized size	1	1.	3.57	3.32	7.3	5.57	0.	2.74
time (sec)	N/A	0.614	8.027	0.101	2.564	1.84	0.	1.336

Problem 472	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	683	593	1486	1154	0	533
normalized size	1	1.	3.5	3.04	7.62	5.92	0.	2.73
time (sec)	N/A	0.613	1.425	0.092	2.239	1.759	0.	1.237

Problem 473	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	408	438	1058	824	0	378
normalized size	1	1.	2.87	3.08	7.45	5.8	0.	2.66
time (sec)	N/A	0.333	5.593	0.075	2.658	1.648	0.	1.324

Problem 474	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	84	139	747	570	1248	244
normalized size	1	1.	0.67	1.11	5.98	4.56	9.98	1.95
time (sec)	N/A	0.182	0.114	0.062	1.562	1.569	29.641	1.265

Problem 475	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	63	114	522	466	915	176
normalized size	1	1.	0.62	1.12	5.12	4.57	8.97	1.73
time (sec)	N/A	0.075	0.07	0.056	1.684	1.52	10.486	1.324

Problem 476	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	76	85	274	369	558	105
normalized size	1	1.	0.92	1.02	3.3	4.45	6.72	1.27
time (sec)	N/A	0.047	0.123	0.045	1.758	1.497	4.776	1.232

Problem 477	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	186	301	325	0	3800	0	491
normalized size	1	1.	1.62	1.75	0.	20.43	0.	2.64
time (sec)	N/A	0.522	0.713	0.095	0.	2.075	0.	1.402

Problem 478	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	298	298	361	511	0	6917	0	698
normalized size	1	1.	1.21	1.71	0.	23.21	0.	2.34
time (sec)	N/A	0.73	2.483	0.118	0.	2.637	0.	1.431

Problem 479	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	378	378	914	1462	0	11534	0	1072
normalized size	1	1.	2.42	3.87	0.	30.51	0.	2.84
time (sec)	N/A	0.962	6.266	0.141	0.	3.519	0.	1.57

Problem 480	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	55	115	417	428	818	151
normalized size	1	1.	0.73	1.53	5.56	5.71	10.91	2.01
time (sec)	N/A	0.057	0.051	0.035	1.85	1.553	18.973	1.344

Problem 481	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	54	115	417	429	818	151
normalized size	1	1.	0.67	1.42	5.15	5.3	10.1	1.86
time (sec)	N/A	0.066	0.065	0.036	1.53	1.539	19.06	1.325

Problem 482	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	290	290	3531	1315	0	0	0	0
normalized size	1	1.	12.18	4.53	0.	0.	0.	0.
time (sec)	N/A	0.443	6.726	1.273	0.	0.	0.	0.

Problem 483	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	231	231	2625	1034	0	0	0	0
normalized size	1	1.	11.36	4.48	0.	0.	0.	0.
time (sec)	N/A	0.317	6.368	0.927	0.	0.	0.	0.

Problem 484	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	179	179	1736	657	0	0	0	0
normalized size	1	1.	9.7	3.67	0.	0.	0.	0.
time (sec)	N/A	0.198	6.284	0.967	0.	0.	0.	0.

Problem 485	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	138	138	880	203	0	0	0	0
normalized size	1	1.	6.38	1.47	0.	0.	0.	0.
time (sec)	N/A	0.122	6.247	0.8	0.	0.	0.	0.

Problem 486	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	169	169	938	246	0	0	0	0
normalized size	1	1.	5.55	1.46	0.	0.	0.	0.
time (sec)	N/A	0.206	6.392	0.955	0.	0.	0.	0.

Problem 487	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	237	237	1870	884	0	0	0	0
normalized size	1	1.	7.89	3.73	0.	0.	0.	0.
time (sec)	N/A	0.336	6.693	3.656	0.	0.	0.	0.

Problem 488	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	318	318	2815	1046	0	0	0	0
normalized size	1	1.	8.85	3.29	0.	0.	0.	0.
time (sec)	N/A	0.507	7.067	5.059	0.	0.	0.	0.

Problem 489	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	378	378	322	1614	0	0	0	0
normalized size	1	1.	0.85	4.27	0.	0.	0.	0.
time (sec)	N/A	0.67	1.978	1.148	0.	0.	0.	0.

Problem 490	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	298	298	262	1316	0	0	0	0
normalized size	1	1.	0.88	4.42	0.	0.	0.	0.
time (sec)	N/A	0.479	2.104	1.149	0.	0.	0.	0.

Problem 491	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	239	239	244	1035	0	0	0	0
normalized size	1	1.	1.02	4.33	0.	0.	0.	0.
time (sec)	N/A	0.336	1.429	1.029	0.	0.	0.	0.

Problem 492	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	193	758	0	0	0	0
normalized size	1	1.	1.02	4.01	0.	0.	0.	0.
time (sec)	N/A	0.246	1.09	0.998	0.	0.	0.	0.

Problem 493	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	175	463	0	0	0	0
normalized size	1	1.	0.93	2.45	0.	0.	0.	0.
time (sec)	N/A	0.239	0.92	0.826	0.	0.	0.	0.

Problem 494	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	247	247	207	1221	0	0	0	0
normalized size	1	1.	0.84	4.94	0.	0.	0.	0.
time (sec)	N/A	0.369	1.762	1.007	0.	0.	0.	0.

Problem 495	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	320	320	283	1436	0	0	0	0
normalized size	1	1.	0.88	4.49	0.	0.	0.	0.
time (sec)	N/A	0.578	2.027	5.667	0.	0.	0.	0.

Problem 496	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	467	467	377	1926	0	0	0	0
normalized size	1	1.	0.81	4.12	0.	0.	0.	0.
time (sec)	N/A	1.032	1.899	1.256	0.	0.	0.	0.

Problem 497	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	390	390	318	1613	0	0	0	0
normalized size	1	1.	0.82	4.14	0.	0.	0.	0.
time (sec)	N/A	0.775	2.303	1.2	0.	0.	0.	0.

Problem 498	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	318	318	266	1316	0	0	0	0
normalized size	1	1.	0.84	4.14	0.	0.	0.	0.
time (sec)	N/A	0.58	2.741	1.246	0.	0.	0.	0.

Problem 499	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	258	258	246	1035	0	0	0	0
normalized size	1	1.	0.95	4.01	0.	0.	0.	0.
time (sec)	N/A	0.476	1.63	1.255	0.	0.	0.	0.

Problem 500	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	270	270	234	1031	0	0	0	0
normalized size	1	1.	0.87	3.82	0.	0.	0.	0.
time (sec)	N/A	0.481	1.433	1.208	0.	0.	0.	0.

Problem 501	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	280	280	232	1257	0	0	0	0
normalized size	1	1.	0.83	4.49	0.	0.	0.	0.
time (sec)	N/A	0.577	1.526	4.573	0.	0.	0.	0.

Problem 502	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	336	336	298	1589	0	0	0	0
normalized size	1	1.	0.89	4.73	0.	0.	0.	0.
time (sec)	N/A	0.72	2.107	6.027	0.	0.	0.	0.

Problem 503	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	419	419	351	2079	0	0	0	0
normalized size	1	1.	0.84	4.96	0.	0.	0.	0.
time (sec)	N/A	0.928	3.563	8.848	0.	0.	0.	0.

Problem 504	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	246	246	298	1372	0	0	0	0
normalized size	1	1.	1.21	5.58	0.	0.	0.	0.
time (sec)	N/A	0.381	1.365	1.304	0.	0.	0.	0.

Problem 505	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	186	223	925	0	0	0	0
normalized size	1	1.	1.2	4.97	0.	0.	0.	0.
time (sec)	N/A	0.257	1.564	1.28	0.	0.	0.	0.

Problem 506	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	201	382	0	0	0	0
normalized size	1	1.	1.18	2.25	0.	0.	0.	0.
time (sec)	N/A	0.207	1.099	1.27	0.	0.	0.	0.

Problem 507	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	210	443	0	0	0	0
normalized size	1	1.	1.16	2.45	0.	0.	0.	0.
time (sec)	N/A	0.214	1.095	3.408	0.	0.	0.	0.

Problem 508	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	244	244	264	925	0	0	0	0
normalized size	1	1.	1.08	3.79	0.	0.	0.	0.
time (sec)	N/A	0.327	1.997	1.509	0.	0.	0.	0.

Problem 509	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	333	333	367	1291	0	0	0	0
normalized size	1	1.	1.1	3.88	0.	0.	0.	0.
time (sec)	N/A	0.488	4.23	5.365	0.	0.	0.	0.

Problem 510	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	256	256	310	1372	0	0	0	0
normalized size	1	1.	1.21	5.36	0.	0.	0.	0.
time (sec)	N/A	0.553	2.63	4.717	0.	0.	0.	0.

Problem 511	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	237	283	1049	0	0	0	0
normalized size	1	1.	1.19	4.43	0.	0.	0.	0.
time (sec)	N/A	0.538	2.791	4.859	0.	0.	0.	0.

Problem 512	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	233	233	256	906	0	0	0	0
normalized size	1	1.	1.1	3.89	0.	0.	0.	0.
time (sec)	N/A	0.41	2.561	4.282	0.	0.	0.	0.

Problem 513	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	257	257	290	507	0	0	0	0
normalized size	1	1.	1.13	1.97	0.	0.	0.	0.
time (sec)	N/A	0.441	2.462	3.622	0.	0.	0.	0.

Problem 514	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	326	326	405	1299	0	0	0	0
normalized size	1	1.	1.24	3.98	0.	0.	0.	0.
time (sec)	N/A	0.642	4.777	5.844	0.	0.	0.	0.

Problem 515	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	405	405	674	1758	0	0	0	0
normalized size	1	1.	1.66	4.34	0.	0.	0.	0.
time (sec)	N/A	0.834	6.62	7.269	0.	0.	0.	0.

Problem 516	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	322	322	385	1615	0	0	0	0
normalized size	1	1.	1.2	5.02	0.	0.	0.	0.
time (sec)	N/A	0.833	5.927	6.812	0.	0.	0.	0.

Problem 517	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	323	323	631	1462	0	0	0	0
normalized size	1	1.	1.95	4.53	0.	0.	0.	0.
time (sec)	N/A	0.877	6.245	6.334	0.	0.	0.	0.

Problem 518	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	334	334	449	1056	0	0	0	0
normalized size	1	1.	1.34	3.16	0.	0.	0.	0.
time (sec)	N/A	0.768	5.96	5.708	0.	0.	0.	0.

Problem 519	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	344	344	638	593	0	0	0	0
normalized size	1	1.	1.85	1.72	0.	0.	0.	0.
time (sec)	N/A	0.769	6.336	4.102	0.	0.	0.	0.

Problem 520	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	423	423	745	1851	0	0	0	0
normalized size	1	1.	1.76	4.38	0.	0.	0.	0.
time (sec)	N/A	1.025	6.545	7.014	0.	0.	0.	0.

Problem 521	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	518	518	828	2311	0	0	0	0
normalized size	1	1.	1.6	4.46	0.	0.	0.	0.
time (sec)	N/A	1.215	6.835	9.667	0.	0.	0.	0.

Problem 522	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	146	141	0	576	0	0
normalized size	1	1.	0.91	0.88	0.	3.58	0.	0.
time (sec)	N/A	0.279	0.521	0.647	0.	1.963	0.	0.

Problem 523	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	111	92	0	390	0	0
normalized size	1	1.	0.99	0.82	0.	3.48	0.	0.
time (sec)	N/A	0.169	0.287	0.648	0.	1.876	0.	0.

Problem 524	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	82	58	0	225	0	0
normalized size	1	1.	1.32	0.94	0.	3.63	0.	0.
time (sec)	N/A	0.055	0.123	0.614	0.	1.836	0.	0.

Problem 525	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	65	43	0	136	0	0
normalized size	1	1.	2.5	1.65	0.	5.23	0.	0.
time (sec)	N/A	0.014	0.033	0.467	0.	1.775	0.	0.

Problem 526	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	657	80	0	1061	0	0
normalized size	1	1.	10.77	1.31	0.	17.39	0.	0.
time (sec)	N/A	0.115	5.173	0.654	0.	2.507	0.	0.

Problem 527	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	871	155	0	1874	0	0
normalized size	1	1.	8.3	1.48	0.	17.85	0.	0.
time (sec)	N/A	0.186	5.831	1.078	0.	2.614	0.	0.

Problem 528	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	920	254	0	2890	0	0
normalized size	1	1.	5.97	1.65	0.	18.77	0.	0.
time (sec)	N/A	0.269	7.246	1.	0.	3.139	0.	0.

Problem 529	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	231	203	195	0	872	0	0
normalized size	1	1.	0.88	0.84	0.	3.77	0.	0.
time (sec)	N/A	0.384	1.678	0.596	0.	1.734	0.	0.

Problem 530	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	136	130	0	590	0	0
normalized size	1	1.	0.87	0.83	0.	3.76	0.	0.
time (sec)	N/A	0.229	0.875	0.668	0.	1.622	0.	0.

Problem 531	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	101	77	0	355	0	0
normalized size	1	1.	1.	0.76	0.	3.51	0.	0.
time (sec)	N/A	0.084	0.411	0.665	0.	1.565	0.	0.

Problem 532	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	89	53	0	204	0	0
normalized size	1	1.	1.51	0.9	0.	3.46	0.	0.
time (sec)	N/A	0.029	0.142	0.46	0.	1.52	0.	0.

Problem 533	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	233	137	0	1539	0	0
normalized size	1	1.	2.38	1.4	0.	15.7	0.	0.
time (sec)	N/A	0.2	2.201	0.934	0.	2.34	0.	0.

Problem 534	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	268	233	0	2241	0	0
normalized size	1	1.	2.25	1.96	0.	18.83	0.	0.
time (sec)	N/A	0.193	2.354	1.197	0.	2.606	0.	0.

Problem 535	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	313	429	0	3536	0	0
normalized size	1	1.	1.75	2.4	0.	19.75	0.	0.
time (sec)	N/A	0.29	3.96	1.422	0.	3.165	0.	0.

Problem 536	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	328	328	246	249	0	1227	0	0
normalized size	1	1.	0.75	0.76	0.	3.74	0.	0.
time (sec)	N/A	0.656	6.354	0.668	0.	1.815	0.	0.

Problem 537	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	202	180	168	0	822	0	0
normalized size	1	1.	0.89	0.83	0.	4.07	0.	0.
time (sec)	N/A	0.274	3.354	0.638	0.	1.673	0.	0.

Problem 538	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	119	99	0	518	0	0
normalized size	1	1.	0.86	0.72	0.	3.75	0.	0.
time (sec)	N/A	0.108	1.507	0.743	0.	1.648	0.	0.

Problem 539	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	117	65	0	292	0	0
normalized size	1	1.	1.31	0.73	0.	3.28	0.	0.
time (sec)	N/A	0.049	0.319	0.592	0.	1.529	0.	0.

Problem 540	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	330	228	0	1987	0	0
normalized size	1	1.	2.32	1.61	0.	13.99	0.	0.
time (sec)	N/A	0.411	3.595	0.804	0.	2.522	0.	0.

Problem 541	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	350	392	0	2916	0	0
normalized size	1	1.	2.11	2.36	0.	17.57	0.	0.
time (sec)	N/A	0.389	4.113	1.224	0.	2.706	0.	0.

Problem 542	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	B	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	194	194	379	567	0	4378	0	0
normalized size	1	1.	1.95	2.92	0.	22.57	0.	0.
time (sec)	N/A	0.443	4.723	1.523	0.	3.47	0.	0.

Problem 543	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	155	285	0	965	0	1029
normalized size	1	1.	0.87	1.6	0.	5.42	0.	5.78
time (sec)	N/A	0.44	0.599	0.859	0.	1.726	0.	2.963

Problem 544	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	125	185	0	741	0	583
normalized size	1	1.	1.02	1.5	0.	6.02	0.	4.74
time (sec)	N/A	0.201	0.385	0.839	0.	1.69	0.	2.336

Problem 545	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	106	128	0	578	0	297
normalized size	1	1.	1.34	1.62	0.	7.32	0.	3.76
time (sec)	N/A	0.07	0.213	0.741	0.	1.647	0.	2.182

Problem 546	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	73	75	0	463	0	0
normalized size	1	1.	1.55	1.6	0.	9.85	0.	0.
time (sec)	N/A	0.023	0.048	0.421	0.	1.608	0.	0.

Problem 547	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	215	131	0	1701	0	0
normalized size	1	1.	1.75	1.07	0.	13.83	0.	0.
time (sec)	N/A	0.214	1.73	0.947	0.	2.552	0.	0.

Problem 548	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	324	453	0	3519	0	0
normalized size	1	1.	1.85	2.59	0.	20.11	0.	0.
time (sec)	N/A	0.425	3.384	1.22	0.	4.343	0.	0.

Problem 549	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	247	247	414	1066	0	6543	0	0
normalized size	1	1.	1.68	4.32	0.	26.49	0.	0.
time (sec)	N/A	0.732	4.961	1.689	0.	7.329	0.	0.

Problem 550	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	328	490	0	1211	0	0
normalized size	1	1.	1.71	2.55	0.	6.31	0.	0.
time (sec)	N/A	0.463	0.542	0.769	0.	1.763	0.	0.

Problem 551	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	239	316	0	953	0	911
normalized size	1	1.	1.73	2.29	0.	6.91	0.	6.6
time (sec)	N/A	0.216	0.332	0.78	0.	1.691	0.	3.347

Problem 552	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	150	176	0	755	0	599
normalized size	1	1.	1.72	2.02	0.	8.68	0.	6.89
time (sec)	N/A	0.073	0.198	0.54	0.	1.698	0.	2.935

Problem 553	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	108	125	0	666	0	414
normalized size	1	1.	1.4	1.62	0.	8.65	0.	5.38
time (sec)	N/A	0.041	0.178	0.609	0.	1.628	0.	2.676

Problem 554	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	385	338	0	3133	0	0
normalized size	1	1.	2.35	2.06	0.	19.1	0.	0.
time (sec)	N/A	0.417	1.783	0.836	0.	3.962	0.	0.

Problem 555	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	243	243	491	978	0	5644	0	0
normalized size	1	1.	2.02	4.02	0.	23.23	0.	0.
time (sec)	N/A	0.743	4.544	1.296	0.	7.392	0.	0.

Problem 556	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	318	318	570	2222	0	9231	0	0
normalized size	1	1.	1.79	6.99	0.	29.03	0.	0.
time (sec)	N/A	1.108	6.267	1.677	0.	13.286	0.	0.

Problem 557	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	400	688	0	1497	0	0
normalized size	1	1.	2.06	3.55	0.	7.72	0.	0.
time (sec)	N/A	0.469	0.768	1.017	0.	1.807	0.	0.

Problem 558	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	252	379	0	1234	0	1540
normalized size	1	1.	1.71	2.58	0.	8.39	0.	10.48
time (sec)	N/A	0.23	0.554	0.929	0.	1.875	0.	4.75

Problem 559	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	227	279	0	1015	0	0
normalized size	1	1.	1.8	2.21	0.	8.06	0.	0.
time (sec)	N/A	0.098	0.353	1.065	0.	2.009	0.	0.

Problem 560	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	196	195	0	851	0	0
normalized size	1	1.	1.83	1.82	0.	7.95	0.	0.
time (sec)	N/A	0.059	0.155	0.7	0.	1.889	0.	0.

Problem 561	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	218	218	501	732	0	4721	0	0
normalized size	1	1.	2.3	3.36	0.	21.66	0.	0.
time (sec)	N/A	0.741	3.235	1.442	0.	6.606	0.	0.

Problem 562	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	313	313	570	1972	0	8342	0	0
normalized size	1	1.	1.82	6.3	0.	26.65	0.	0.
time (sec)	N/A	1.1	5.764	1.894	0.	12.723	0.	0.

Problem 563	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	B	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	400	400	958	3535	0	13377	0	0
normalized size	1	1.	2.4	8.84	0.	33.44	0.	0.
time (sec)	N/A	1.515	9.301	2.497	0.	24.137	0.	0.

Problem 564	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	391	0	0	3033	0	0
normalized size	1	1.	1.93	0.	0.	14.94	0.	0.
time (sec)	N/A	0.415	3.772	180.	0.	5.829	0.	0.

Problem 565	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	365	0	0	2600	0	0
normalized size	1	1.	2.34	0.	0.	16.67	0.	0.
time (sec)	N/A	0.29	2.073	180.	0.	5.218	0.	0.

Problem 566	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	350	0	0	2311	0	0
normalized size	1	1.	3.33	0.	0.	22.01	0.	0.
time (sec)	N/A	0.182	1.299	180.	0.	5.2	0.	0.

Problem 567	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	305	2707	0	1825	0	0
normalized size	1	1.	5.	44.38	0.	29.92	0.	0.
time (sec)	N/A	0.093	1.239	0.378	0.	5.657	0.	0.

Problem 568	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	84	99	242	323	0	0
normalized size	1	1.	1.87	2.2	5.38	7.18	0.	0.
time (sec)	N/A	0.092	0.196	0.205	1.831	2.283	0.	0.

Problem 569	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	100	222	459	703	0	0
normalized size	1	1.	1.05	2.34	4.83	7.4	0.	0.
time (sec)	N/A	0.192	0.274	0.215	1.953	2.367	0.	0.

Problem 570	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	128	430	734	1264	0	0
normalized size	1	1.	0.9	3.03	5.17	8.9	0.	0.
time (sec)	N/A	0.295	0.393	0.244	2.278	2.604	0.	0.

Problem 571	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	285	285	318	0	0	3841	0	0
normalized size	1	1.	1.12	0.	0.	13.48	0.	0.
time (sec)	N/A	0.567	1.373	180.	0.	14.063	0.	0.

Problem 572	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	228	228	281	0	0	3214	0	0
normalized size	1	1.	1.23	0.	0.	14.1	0.	0.
time (sec)	N/A	0.435	0.834	180.	0.	9.254	0.	0.

Problem 573	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	247	0	0	2716	0	0
normalized size	1	1.	1.44	0.	0.	15.88	0.	0.
time (sec)	N/A	0.311	0.62	180.	0.	7.059	0.	0.

Problem 574	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	301	0	0	2390	0	0
normalized size	1	1.	2.71	0.	0.	21.53	0.	0.
time (sec)	N/A	0.209	0.6	0.29	0.	6.798	0.	0.

Problem 575	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	377	6627	0	3043	0	0
normalized size	1	1.	3.22	56.64	0.	26.01	0.	0.
time (sec)	N/A	0.214	7.274	0.428	0.	7.015	0.	0.

Problem 576	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	104	345	414	752	0	0
normalized size	1	1.	0.9	3.	3.6	6.54	0.	0.
time (sec)	N/A	0.215	0.587	0.208	1.736	2.69	0.	0.

Problem 577	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	140	625	682	1369	0	0
normalized size	1	1.	0.81	3.63	3.97	7.96	0.	0.
time (sec)	N/A	0.324	0.889	0.234	2.087	3.324	0.	0.

Problem 578	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	229	193	979	1013	2163	0	0
normalized size	1	1.	0.84	4.28	4.42	9.45	0.	0.
time (sec)	N/A	0.448	1.534	0.319	2.418	3.653	0.	0.

Problem 579	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	377	377	395	0	0	4905	0	0
normalized size	1	1.	1.05	0.	0.	13.01	0.	0.
time (sec)	N/A	0.855	2.964	180.	0.	21.	0.	0.

Problem 580	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	312	312	327	0	0	4084	0	0
normalized size	1	1.	1.05	0.	0.	13.09	0.	0.
time (sec)	N/A	0.726	1.761	180.	0.	14.768	0.	0.

Problem 581	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	241	241	285	0	0	3374	0	0
normalized size	1	1.	1.18	0.	0.	14.	0.	0.
time (sec)	N/A	0.576	0.921	180.	0.	9.829	0.	0.

Problem 582	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	256	0	0	2873	0	0
normalized size	1	1.	1.44	0.	0.	16.14	0.	0.
time (sec)	N/A	0.433	0.875	0.237	0.	6.698	0.	0.

Problem 583	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	263	0	0	3758	0	0
normalized size	1	1.	1.46	0.	0.	20.88	0.	0.
time (sec)	N/A	0.438	1.009	0.252	0.	5.575	0.	0.

Problem 584	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	261	16223	0	5053	0	0
normalized size	1	1.	1.43	88.65	0.	27.61	0.	0.
time (sec)	N/A	0.444	7.886	0.435	0.	5.436	0.	0.

Problem 585	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	152	793	626	1467	0	0
normalized size	1	1.	0.8	4.2	3.31	7.76	0.	0.
time (sec)	N/A	0.485	2.052	0.262	2.048	2.083	0.	0.

Problem 586	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	254	254	216	1223	949	2342	0	0
normalized size	1	1.	0.85	4.81	3.74	9.22	0.	0.
time (sec)	N/A	0.625	4.201	0.374	2.255	2.371	0.	0.

Problem 587	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	317	317	616	1730	1328	3441	0	0
normalized size	1	1.	1.94	5.46	4.19	10.85	0.	0.
time (sec)	N/A	0.776	6.535	0.395	2.369	2.945	0.	0.

Problem 588	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	249	249	1893	0	0	7116	0	0
normalized size	1	1.	7.6	0.	0.	28.58	0.	0.
time (sec)	N/A	0.929	17.357	180.	0.	6.919	0.	0.

Problem 589	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	188	188	1639	0	0	6151	0	0
normalized size	1	1.	8.72	0.	0.	32.72	0.	0.
time (sec)	N/A	0.598	16.924	0.328	0.	6.355	0.	0.

Problem 590	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	141	141	1251	3359	0	4674	0	0
normalized size	1	1.	8.87	23.82	0.	33.15	0.	0.
time (sec)	N/A	0.296	15.016	0.273	0.	4.083	0.	0.

Problem 591	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	79	79	283	191	0	1199	0	0
normalized size	1	1.	3.58	2.42	0.	15.18	0.	0.
time (sec)	N/A	0.107	4.073	0.139	0.	2.596	0.	0.

Problem 592	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	131	131	306	879	0	2430	0	0
normalized size	1	1.	2.34	6.71	0.	18.55	0.	0.
time (sec)	N/A	0.243	6.354	0.224	0.	3.161	0.	0.

Problem 593	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	191	191	387	2575	0	4224	0	0
normalized size	1	1.	2.03	13.48	0.	22.12	0.	0.
time (sec)	N/A	0.476	6.191	0.278	0.	5.064	0.	0.

Problem 594	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	251	251	1844	0	0	8276	0	0
normalized size	1	1.	7.35	0.	0.	32.97	0.	0.
time (sec)	N/A	0.903	17.168	0.362	0.	5.992	0.	0.

Problem 595	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	194	194	1625	6675	0	7039	0	0
normalized size	1	1.	8.38	34.41	0.	36.28	0.	0.
time (sec)	N/A	0.564	16.956	0.299	0.	4.289	0.	0.

Problem 596	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	126	126	372	1373	0	2199	0	0
normalized size	1	1.	2.95	10.9	0.	17.45	0.	0.
time (sec)	N/A	0.221	5.245	0.229	0.	2.446	0.	0.

Problem 597	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	135	135	381	1268	0	2431	0	0
normalized size	1	1.	2.82	9.39	0.	18.01	0.	0.
time (sec)	N/A	0.237	5.535	0.224	0.	2.813	0.	0.

Problem 598	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	197	197	401	2246	0	4350	0	0
normalized size	1	1.	2.04	11.4	0.	22.08	0.	0.
time (sec)	N/A	0.494	5.36	0.273	0.	4.239	0.	0.

Problem 599	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	271	271	478	5040	0	6974	0	0
normalized size	1	1.	1.76	18.6	0.	25.73	0.	0.
time (sec)	N/A	0.855	8.992	0.286	0.	5.96	0.	0.

Problem 600	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	260	260	1845	10738	0	9397	0	0
normalized size	1	1.	7.1	41.3	0.	36.14	0.	0.
time (sec)	N/A	0.864	17.159	0.398	0.	5.101	0.	0.

Problem 601	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	184	184	396	3050	0	3136	0	0
normalized size	1	1.	2.15	16.58	0.	17.04	0.	0.
time (sec)	N/A	0.538	7.188	0.23	0.	2.781	0.	0.

Problem 602	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	191	191	412	3050	0	3490	0	0
normalized size	1	1.	2.16	15.97	0.	18.27	0.	0.
time (sec)	N/A	0.488	6.802	0.234	0.	3.203	0.	0.

Problem 603	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	201	201	411	2805	0	3838	0	0
normalized size	1	1.	2.04	13.96	0.	19.09	0.	0.
time (sec)	N/A	0.494	6.557	0.273	0.	4.549	0.	0.

Problem 604	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	270	270	462	4262	0	6607	0	0
normalized size	1	1.	1.71	15.79	0.	24.47	0.	0.
time (sec)	N/A	0.861	8.394	0.263	0.	6.621	0.	0.

Problem 605	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	355	355	717	8041	0	10797	0	0
normalized size	1	1.	2.02	22.65	0.	30.41	0.	0.
time (sec)	N/A	1.264	9.853	0.279	0.	15.593	0.	0.

Problem 606	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	129	129	373	0	0	0	0	0
normalized size	1	1.	2.89	0.	0.	0.	0.	0.
time (sec)	N/A	0.168	1.411	1.124	0.	0.	0.	0.

Problem 607	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	320	320	3599	0	0	0	0	0
normalized size	1	1.	11.25	0.	0.	0.	0.	0.
time (sec)	N/A	0.663	56.073	2.727	0.	0.	0.	0.

Problem 608	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	193	193	1774	0	0	0	0	0
normalized size	1	1.	9.19	0.	0.	0.	0.	0.
time (sec)	N/A	0.269	103.011	2.927	0.	0.	0.	0.

Problem 609	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	117	117	275	0	0	0	0	0
normalized size	1	1.	2.35	0.	0.	0.	0.	0.
time (sec)	N/A	0.08	1.833	0.976	0.	0.	0.	0.

Problem 610	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	90	0	0	0	0	0
normalized size	1	1.	1.22	0.	0.	0.	0.	0.
time (sec)	N/A	0.03	0.141	0.005	0.	0.	0.	0.

Problem 611	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	100	100	363	0	0	0	0	0
normalized size	1	1.	3.63	0.	0.	0.	0.	0.
time (sec)	N/A	0.128	1.176	0.819	0.	0.	0.	0.

Problem 612	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	100	100	363	0	0	0	0	0
normalized size	1	1.	3.63	0.	0.	0.	0.	0.
time (sec)	N/A	0.126	1.306	0.744	0.	0.	0.	0.

Problem 613	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	100	100	363	0	0	0	0	0
normalized size	1	1.	3.63	0.	0.	0.	0.	0.
time (sec)	N/A	0.126	1.43	0.942	0.	0.	0.	0.

Problem 614	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	138	138	365	0	0	0	0	0
normalized size	1	1.	2.64	0.	0.	0.	0.	0.
time (sec)	N/A	0.187	1.862	0.178	0.	0.	0.	0.

Problem 615	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	136	136	365	0	0	0	0	0
normalized size	1	1.	2.68	0.	0.	0.	0.	0.
time (sec)	N/A	0.184	1.298	0.175	0.	0.	0.	0.

Problem 616	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	131	131	365	0	0	0	0	0
normalized size	1	1.	2.79	0.	0.	0.	0.	0.
time (sec)	N/A	0.165	1.096	0.168	0.	0.	0.	0.

Problem 617	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	131	131	373	0	0	0	0	0
normalized size	1	1.	2.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.164	1.162	0.171	0.	0.	0.	0.

Problem 618	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	138	138	373	0	0	0	0	0
normalized size	1	1.	2.7	0.	0.	0.	0.	0.
time (sec)	N/A	0.182	1.384	0.169	0.	0.	0.	0.

Problem 619	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	138	138	373	0	0	0	0	0
normalized size	1	1.	2.7	0.	0.	0.	0.	0.
time (sec)	N/A	0.18	1.557	0.175	0.	0.	0.	0.

Problem 620	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	62	111	238	0	0	0	0	0
normalized size	1	1.79	3.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.106	1.42	0.241	0.	0.	0.	0.

Problem 621	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	64	122	88	0	0	0	0	0
normalized size	1	1.91	1.38	0.	0.	0.	0.	0.
time (sec)	N/A	0.116	0.473	0.257	0.	0.	0.	0.

Problem 622	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	45	0	47	132	0	0
normalized size	1	1.	1.61	0.	1.68	4.71	0.	0.
time (sec)	N/A	0.02	0.037	0.22	1.798	1.008	0.	0.

Problem 623	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	131	0	0	0	0	0
normalized size	1	1.	1.07	0.	0.	0.	0.	0.
time (sec)	N/A	0.115	0.53	0.25	0.	0.	0.	0.

Problem 624	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	106	106	167	0	0	0	0	0
normalized size	1	1.	1.58	0.	0.	0.	0.	0.
time (sec)	N/A	0.101	0.58	0.247	0.	0.	0.	0.

Problem 625	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	95	0	0	0	0	0
normalized size	1	1.	1.46	0.	0.	0.	0.	0.
time (sec)	N/A	0.021	0.146	0.297	0.	0.	0.	0.

Problem 626	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	182	0	0	0	0	0
normalized size	1	1.	1.94	0.	0.	0.	0.	0.
time (sec)	N/A	0.092	1.066	0.258	0.	0.	0.	0.

Problem 627	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	114	114	177	0	0	0	0	0
normalized size	1	1.	1.55	0.	0.	0.	0.	0.
time (sec)	N/A	0.099	0.884	0.256	0.	0.	0.	0.

Problem 628	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	97	0	0	105	0	0
normalized size	1	1.	2.26	0.	0.	2.44	0.	0.
time (sec)	N/A	0.054	0.523	0.262	0.	1.063	0.	0.

Problem 629	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	83	113	176	0	0	0	0	0
normalized size	1	1.36	2.12	0.	0.	0.	0.	0.
time (sec)	N/A	0.096	0.882	0.262	0.	0.	0.	0.

Problem 630	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	78	111	246	0	0	0	0	0
normalized size	1	1.42	3.15	0.	0.	0.	0.	0.
time (sec)	N/A	0.093	1.763	0.243	0.	0.	0.	0.

Problem 631	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	81	115	240	0	0	0	0	0
normalized size	1	1.42	2.96	0.	0.	0.	0.	0.
time (sec)	N/A	0.106	0.569	0.246	0.	0.	0.	0.

Problem 632	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	83	118	90	0	0	0	0	0
normalized size	1	1.42	1.08	0.	0.	0.	0.	0.
time (sec)	N/A	0.113	0.455	0.251	0.	0.	0.	0.

Problem 633	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	104	0	51	119	0	0
normalized size	1	1.	2.67	0.	1.31	3.05	0.	0.
time (sec)	N/A	0.017	5.035	0.254	1.758	1.014	0.	0.

Problem 634	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	118	179	0	0	0	0	0
normalized size	1	1.42	2.16	0.	0.	0.	0.	0.
time (sec)	N/A	0.113	0.552	0.237	0.	0.	0.	0.

Problem 635	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	81	117	166	0	0	0	0	0
normalized size	1	1.44	2.05	0.	0.	0.	0.	0.
time (sec)	N/A	0.107	0.651	0.25	0.	0.	0.	0.

Problem 636	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	97	0	0	0	0	0
normalized size	1	1.	1.2	0.	0.	0.	0.	0.
time (sec)	N/A	0.041	0.155	0.313	0.	0.	0.	0.

Problem 637	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	118	184	0	0	0	0	0
normalized size	1	1.64	2.56	0.	0.	0.	0.	0.
time (sec)	N/A	0.109	0.778	0.254	0.	0.	0.	0.

Problem 638	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	77	116	179	0	0	0	0	0
normalized size	1	1.51	2.32	0.	0.	0.	0.	0.
time (sec)	N/A	0.105	0.624	0.24	0.	0.	0.	0.

Problem 639	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	99	0	0	108	0	0
normalized size	1	1.	2.2	0.	0.	2.4	0.	0.
time (sec)	N/A	0.062	0.601	0.254	0.	1.304	0.	0.

Problem 640	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	115	115	178	0	0	0	0	0
normalized size	1	1.	1.55	0.	0.	0.	0.	0.
time (sec)	N/A	0.106	0.655	0.263	0.	0.	0.	0.

Problem 641	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	113	113	248	0	0	0	0	0
normalized size	1	1.	2.19	0.	0.	0.	0.	0.
time (sec)	N/A	0.102	0.553	0.261	0.	0.	0.	0.

Problem 642	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	72	113	247	0	0	0	0	0
normalized size	1	1.57	3.43	0.	0.	0.	0.	0.
time (sec)	N/A	0.098	1.529	0.228	0.	0.	0.	0.

Problem 643	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	77	116	154	0	0	0	0	0
normalized size	1	1.51	2.	0.	0.	0.	0.	0.
time (sec)	N/A	0.105	0.613	0.251	0.	0.	0.	0.

Problem 644	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	110	0	0	107	0	0
normalized size	1	1.	2.44	0.	0.	2.38	0.	0.
time (sec)	N/A	0.063	0.669	0.261	0.	1.5	0.	0.

Problem 645	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	117	117	155	0	0	0	0	0
normalized size	1	1.	1.32	0.	0.	0.	0.	0.
time (sec)	N/A	0.101	0.626	0.232	0.	0.	0.	0.

Problem 646	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	116	116	155	0	0	0	0	0
normalized size	1	1.	1.34	0.	0.	0.	0.	0.
time (sec)	N/A	0.097	0.725	0.241	0.	0.	0.	0.

Problem 647	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	97	0	0	0	0	0
normalized size	1	1.	1.2	0.	0.	0.	0.	0.
time (sec)	N/A	0.047	0.157	0.332	0.	0.	0.	0.

Problem 648	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	119	119	131	0	0	0	0	0
normalized size	1	1.	1.1	0.	0.	0.	0.	0.
time (sec)	N/A	0.102	0.909	0.26	0.	0.	0.	0.

Problem 649	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	186	0	0	0	0	0
normalized size	1	1.	1.56	0.	0.	0.	0.	0.
time (sec)	N/A	0.114	1.137	0.251	0.	0.	0.	0.

Problem 650	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	106	0	61	119	0	0
normalized size	1	1.	2.72	0.	1.56	3.05	0.	0.
time (sec)	N/A	0.016	0.5	0.247	1.776	1.547	0.	0.

Problem 651	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	187	0	0	0	0	0
normalized size	1	1.	1.6	0.	0.	0.	0.	0.
time (sec)	N/A	0.1	1.174	0.266	0.	0.	0.	0.

Problem 652	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	115	115	241	0	0	0	0	0
normalized size	1	1.	2.1	0.	0.	0.	0.	0.
time (sec)	N/A	0.098	1.561	0.263	0.	0.	0.	0.

Problem 653	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	116	116	194	0	0	0	0	0
normalized size	1	1.	1.67	0.	0.	0.	0.	0.
time (sec)	N/A	0.189	1.507	0.24	0.	0.	0.	0.

Problem 654	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	129	129	187	0	0	0	0	0
normalized size	1	1.	1.45	0.	0.	0.	0.	0.
time (sec)	N/A	0.149	1.381	0.25	0.	0.	0.	0.

Problem 655	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	107	107	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.123	35.626	0.381	0.	0.	0.	0.

Problem 656	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	107	107	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.107	16.618	0.357	0.	0.	0.	0.

Problem 657	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	105	105	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.081	5.627	0.222	0.	0.	0.	0.

Problem 658	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	104	104	120	0	0	0	0	0
normalized size	1	1.	1.15	0.	0.	0.	0.	0.
time (sec)	N/A	0.064	0.278	0.382	0.	0.	0.	0.

Problem 659	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	107	107	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.107	2.614	0.395	0.	0.	0.	0.

Problem 660	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	109	109	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.106	6.056	0.379	0.	0.	0.	0.

Problem 661	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	109	109	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.108	12.821	0.572	0.	0.	0.	0.

Problem 662	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	257	257	190	0	0	0	0	0
normalized size	1	1.	0.74	0.	0.	0.	0.	0.
time (sec)	N/A	0.481	32.538	0.179	0.	0.	0.	0.

Problem 663	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	133	0	0	0	0	0
normalized size	1	1.	0.83	0.	0.	0.	0.	0.
time (sec)	N/A	0.223	7.491	0.161	0.	0.	0.	0.

Problem 664	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	85	85	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.099	4.51	0.177	0.	0.	0.	0.

Problem 665	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	99	129	236	0	0	0	0	0
normalized size	1	1.3	2.38	0.	0.	0.	0.	0.
time (sec)	N/A	0.169	2.908	0.16	0.	0.	0.	0.

Problem 666	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	104	130	319	0	0	0	0	0
normalized size	1	1.25	3.07	0.	0.	0.	0.	0.
time (sec)	N/A	0.161	4.69	0.161	0.	0.	0.	0.

Problem 667	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	104	137	414	0	0	0	0	0
normalized size	1	1.32	3.98	0.	0.	0.	0.	0.
time (sec)	N/A	0.178	9.466	0.158	0.	0.	0.	0.

Problem 668	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	107	107	1736	0	0	0	0	0
normalized size	1	1.	16.22	0.	0.	0.	0.	0.
time (sec)	N/A	0.091	6.374	0.147	0.	0.	0.	0.

Problem 669	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	107	107	886	0	0	0	0	0
normalized size	1	1.	8.28	0.	0.	0.	0.	0.
time (sec)	N/A	0.087	6.269	0.331	0.	0.	0.	0.

Problem 670	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	112	112	942	0	0	0	0	0
normalized size	1	1.	8.41	0.	0.	0.	0.	0.
time (sec)	N/A	0.096	6.402	0.151	0.	0.	0.	0.

Problem 671	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	143	182	236	340	386	205
normalized size	1	1.	0.84	1.06	1.38	1.99	2.26	1.2
time (sec)	N/A	0.211	0.67	0.029	1.113	1.655	1.759	1.292

Problem 672	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	90	115	151	215	199	130
normalized size	1	1.	0.85	1.08	1.42	2.03	1.88	1.23
time (sec)	N/A	0.101	0.297	0.026	1.147	1.626	0.791	1.347

Problem 673	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	52	59	77	120	94	65
normalized size	1	1.	0.98	1.11	1.45	2.26	1.77	1.23
time (sec)	N/A	0.022	0.091	0.022	1.033	1.597	0.338	1.152

Problem 674	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	27	17	22	38	19	23
normalized size	1	1.	1.69	1.06	1.38	2.38	1.19	1.44
time (sec)	N/A	0.008	0.006	0.009	1.209	1.553	0.139	1.243

Problem 675	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	67	119	0	547	502	116
normalized size	1	1.	1.03	1.83	0.	8.42	7.72	1.78
time (sec)	N/A	0.088	0.13	0.043	0.	1.72	151.013	1.326

Problem 676	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	96	309	0	871	0	213
normalized size	1	1.	0.98	3.15	0.	8.89	0.	2.17
time (sec)	N/A	0.096	0.289	0.073	0.	1.743	0.	1.228

Problem 677	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	157	1291	0	1724	0	578
normalized size	1	1.	0.96	7.87	0.	10.51	0.	3.52
time (sec)	N/A	0.198	0.603	0.085	0.	2.002	0.	1.445

Problem 678	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	314	314	249	325	424	566	729	370
normalized size	1	1.	0.79	1.04	1.35	1.8	2.32	1.18
time (sec)	N/A	0.546	1.373	0.036	1.339	1.749	4.187	1.425

Problem 679	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	217	160	216	281	371	459	238
normalized size	1	1.	0.74	1.	1.29	1.71	2.12	1.1
time (sec)	N/A	0.281	0.762	0.03	1.179	1.707	1.807	1.24

Problem 680	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	90	115	151	215	199	130
normalized size	1	1.	0.84	1.07	1.41	2.01	1.86	1.21
time (sec)	N/A	0.094	0.295	0.026	1.123	1.592	0.786	1.339

Problem 681	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	46	51	62	109	78	61
normalized size	1	1.	0.92	1.02	1.24	2.18	1.56	1.22
time (sec)	N/A	0.016	0.097	0.018	1.014	1.54	0.309	1.243

Problem 682	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	89	226	0	798	0	181
normalized size	1	1.	0.96	2.43	0.	8.58	0.	1.95
time (sec)	N/A	0.182	0.212	0.062	0.	1.771	0.	1.425

Problem 683	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	134	556	0	1434	0	336
normalized size	1	1.	1.04	4.31	0.	11.12	0.	2.6
time (sec)	N/A	0.231	0.526	0.082	0.	1.99	0.	1.389

Problem 684	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	196	202	1923	0	2167	0	822
normalized size	1	1.	1.03	9.81	0.	11.06	0.	4.19
time (sec)	N/A	0.279	0.922	0.088	0.	2.191	0.	1.348

Problem 685	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	305	305	346	4818	0	3687	0	1777
normalized size	1	1.	1.13	15.8	0.	12.09	0.	5.83
time (sec)	N/A	0.563	1.39	0.105	0.	2.534	0.	1.497

Problem 686	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	400	493	552	489	644	834	1217	562
normalized size	1	1.23	1.38	1.22	1.61	2.08	3.04	1.4
time (sec)	N/A	0.948	1.156	0.041	1.113	1.924	8.552	1.67

Problem 687	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	315	315	246	325	424	568	729	370
normalized size	1	1.	0.78	1.03	1.35	1.8	2.31	1.17
time (sec)	N/A	0.46	1.648	0.036	1.06	1.802	4.129	1.382

Problem 688	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	142	182	236	340	386	205
normalized size	1	1.	0.83	1.06	1.38	1.99	2.26	1.2
time (sec)	N/A	0.197	0.645	0.029	1.084	1.722	1.734	1.395

Problem 689	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	71	76	100	169	128	101
normalized size	1	1.	0.79	0.84	1.11	1.88	1.42	1.12
time (sec)	N/A	0.069	0.17	0.023	1.038	1.569	0.655	1.438

Problem 690	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	137	506	0	1208	0	340
normalized size	1	1.	0.88	3.24	0.	7.74	0.	2.18
time (sec)	N/A	0.378	0.361	0.075	0.	1.875	0.	1.57

Problem 691	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	208	152	842	0	2221	0	790
normalized size	1	1.	0.73	4.05	0.	10.68	0.	3.8
time (sec)	N/A	0.495	1.06	0.096	0.	2.281	0.	1.401

Problem 692	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	255	521	2785	0	3468	0	1200
normalized size	1	1.	2.04	10.92	0.	13.6	0.	4.71
time (sec)	N/A	0.631	2.322	0.108	0.	2.512	0.	1.455

Problem 693	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	325	325	345	6128	0	4540	0	2264
normalized size	1	1.	1.06	18.86	0.	13.97	0.	6.97
time (sec)	N/A	0.725	5.156	0.125	0.	3.105	0.	1.487

Problem 694	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	52	99	0	377	87	99
normalized size	1	1.	0.96	1.83	0.	6.98	1.61	1.83
time (sec)	N/A	0.082	0.075	0.037	0.	1.577	126.164	1.27

Problem 695	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	0	9	3	8
normalized size	1	1.	1.	1.17	0.	1.5	0.5	1.33
time (sec)	N/A	0.001	0.	0.006	0.	1.117	0.332	1.289

Problem 696	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	34	0	32	0	43
normalized size	1	1.	1.	2.83	0.	2.67	0.	3.58
time (sec)	N/A	0.029	0.03	0.04	0.	1.364	0.	1.25

Problem 697	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	28	24	49	88	42	69
normalized size	1	1.	0.82	0.71	1.44	2.59	1.24	2.03
time (sec)	N/A	0.033	0.028	0.036	1.703	1.33	1.089	1.301

Problem 698	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	235	235	203	948	0	1594	0	628
normalized size	1	1.	0.86	4.03	0.	6.78	0.	2.67
time (sec)	N/A	0.648	0.568	0.084	0.	1.744	0.	1.372

Problem 699	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	138	506	0	1153	0	340
normalized size	1	1.	0.88	3.24	0.	7.39	0.	2.18
time (sec)	N/A	0.363	0.336	0.076	0.	1.598	0.	1.376

Problem 700	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	90	226	0	771	0	181
normalized size	1	1.	0.97	2.43	0.	8.29	0.	1.95
time (sec)	N/A	0.167	0.159	0.065	0.	1.599	0.	1.359

Problem 701	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	67	119	0	543	502	116
normalized size	1	1.	1.03	1.83	0.	8.35	7.72	1.78
time (sec)	N/A	0.072	0.098	0.042	0.	1.421	142.884	1.348

Problem 702	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	47	47	0	428	162	84
normalized size	1	1.	1.	1.	0.	9.11	3.45	1.79
time (sec)	N/A	0.034	0.031	0.033	0.	1.37	7.72	1.279

Problem 703	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	104	116	0	2238	0	0
normalized size	1	1.	0.89	0.99	0.	19.13	0.	0.
time (sec)	N/A	0.153	0.166	0.09	0.	10.395	0.	0.

Problem 704	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	165	514	0	0	0	416
normalized size	1	1.	0.89	2.78	0.	0.	0.	2.25
time (sec)	N/A	0.468	0.884	0.121	0.	0.	0.	1.284

Problem 705	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	284	284	263	2644	0	0	0	1061
normalized size	1	1.	0.93	9.31	0.	0.	0.	3.74
time (sec)	N/A	1.08	2.139	0.136	0.	0.	0.	1.347

Problem 706	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	306	306	199	1303	0	2984	0	698
normalized size	1	1.	0.65	4.26	0.	9.75	0.	2.28
time (sec)	N/A	0.941	2.015	0.109	0.	2.967	0.	1.524

Problem 707	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	151	842	0	2078	0	782
normalized size	1	1.	0.74	4.11	0.	10.14	0.	3.81
time (sec)	N/A	0.459	1.1	0.101	0.	2.547	0.	1.508

Problem 708	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	133	556	0	1401	0	336
normalized size	1	1.	1.03	4.31	0.	10.86	0.	2.6
time (sec)	N/A	0.218	0.55	0.079	0.	2.271	0.	1.391

Problem 709	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	96	309	0	871	0	212
normalized size	1	1.	0.99	3.19	0.	8.98	0.	2.19
time (sec)	N/A	0.092	0.304	0.072	0.	2.069	0.	1.432

Problem 710	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	82	155	0	752	0	171
normalized size	1	1.	0.99	1.87	0.	9.06	0.	2.06
time (sec)	N/A	0.06	0.183	0.056	0.	1.773	0.	1.371

Problem 711	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	178	514	0	0	0	410
normalized size	1	1.	0.98	2.84	0.	0.	0.	2.27
time (sec)	N/A	0.438	0.852	0.119	0.	0.	0.	1.429

Problem 712	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	290	290	227	886	0	0	0	0
normalized size	1	1.	0.78	3.06	0.	0.	0.	0.
time (sec)	N/A	1.197	2.844	0.138	0.	0.	0.	0.

Problem 713	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	458	458	346	4023	0	0	0	1497
normalized size	1	1.	0.76	8.78	0.	0.	0.	3.27
time (sec)	N/A	2.439	6.441	0.174	0.	0.	0.	1.461

Problem 714	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	534	534	341	4767	0	6529	0	4269
normalized size	1	1.	0.64	8.93	0.	12.23	0.	7.99
time (sec)	N/A	2.155	3.806	0.139	0.	4.28	0.	1.527

Problem 715	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	318	318	894	3683	0	4743	0	1565
normalized size	1	1.	2.81	11.58	0.	14.92	0.	4.92
time (sec)	N/A	0.972	4.242	0.123	0.	3.494	0.	1.381

Problem 716	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	248	248	524	2785	0	3313	0	1197
normalized size	1	1.	2.11	11.23	0.	13.36	0.	4.83
time (sec)	N/A	0.806	2.373	0.109	0.	2.173	0.	1.347

Problem 717	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	196	204	1923	0	2137	0	822
normalized size	1	1.	1.04	9.81	0.	10.9	0.	4.19
time (sec)	N/A	0.285	0.922	0.096	0.	1.75	0.	1.39

Problem 718	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	157	1291	0	1696	0	579
normalized size	1	1.	0.97	7.97	0.	10.47	0.	3.57
time (sec)	N/A	0.174	0.62	0.087	0.	1.84	0.	1.375

Problem 719	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	114	705	0	1346	0	383
normalized size	1	1.	0.87	5.38	0.	10.27	0.	2.92
time (sec)	N/A	0.114	0.383	0.075	0.	1.653	0.	1.256

Problem 720	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-2)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	285	285	275	2644	0	0	0	1064
normalized size	1	1.	0.96	9.28	0.	0.	0.	3.73
time (sec)	N/A	1.037	2.231	0.136	0.	0.	0.	1.561

Problem 721	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	454	454	346	3241	0	0	0	1500
normalized size	1	1.	0.76	7.14	0.	0.	0.	3.3
time (sec)	N/A	2.439	5.318	0.186	0.	0.	0.	1.501

Problem 722	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	669	669	1815	7348	0	0	0	0
normalized size	1	1.	2.71	10.98	0.	0.	0.	0.
time (sec)	N/A	3.309	8.379	0.234	0.	0.	0.	0.

Problem 723	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	298	298	275	1839	0	0	0	0
normalized size	1	1.	0.92	6.17	0.	0.	0.	0.
time (sec)	N/A	0.481	1.078	1.23	0.	0.	0.	0.

Problem 724	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	235	235	218	1449	0	0	0	0
normalized size	1	1.	0.93	6.17	0.	0.	0.	0.
time (sec)	N/A	0.359	0.728	1.217	0.	0.	0.	0.

Problem 725	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	152	862	0	0	0	0
normalized size	1	1.	0.84	4.76	0.	0.	0.	0.
time (sec)	N/A	0.213	0.63	1.213	0.	0.	0.	0.

Problem 726	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	101	243	0	0	0	0
normalized size	1	1.	0.72	1.74	0.	0.	0.	0.
time (sec)	N/A	0.124	2.56	1.069	0.	0.	0.	0.

Problem 727	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	159	567	0	0	0	0
normalized size	1	1.	0.82	2.91	0.	0.	0.	0.
time (sec)	N/A	0.216	0.574	2.57	0.	0.	0.	0.

Problem 728	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	285	285	199	887	0	0	0	0
normalized size	1	1.	0.7	3.11	0.	0.	0.	0.
time (sec)	N/A	0.395	1.483	4.556	0.	0.	0.	0.

Problem 729	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	369	369	297	1049	0	0	0	0
normalized size	1	1.	0.8	2.84	0.	0.	0.	0.
time (sec)	N/A	0.526	2.978	6.319	0.	0.	0.	0.

Problem 730	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	451	451	382	2112	0	0	0	0
normalized size	1	1.	0.85	4.68	0.	0.	0.	0.
time (sec)	N/A	0.945	1.748	5.909	0.	0.	0.	0.

Problem 731	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	347	347	292	1575	0	0	0	0
normalized size	1	1.	0.84	4.54	0.	0.	0.	0.
time (sec)	N/A	0.634	1.176	4.46	0.	0.	0.	0.

Problem 732	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	254	254	214	1100	0	0	0	0
normalized size	1	1.	0.84	4.33	0.	0.	0.	0.
time (sec)	N/A	0.411	0.895	3.353	0.	0.	0.	0.

Problem 733	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	173	695	0	0	0	0
normalized size	1	1.	0.85	3.42	0.	0.	0.	0.
time (sec)	N/A	0.285	0.906	2.608	0.	0.	0.	0.

Problem 734	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	228	228	172	888	0	0	0	0
normalized size	1	1.	0.75	3.89	0.	0.	0.	0.
time (sec)	N/A	0.308	0.866	3.56	0.	0.	0.	0.

Problem 735	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	329	329	302	1043	0	0	0	0
normalized size	1	1.	0.92	3.17	0.	0.	0.	0.
time (sec)	N/A	0.499	2.406	4.746	0.	0.	0.	0.

Problem 736	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	460	460	424	1450	0	0	0	0
normalized size	1	1.	0.92	3.15	0.	0.	0.	0.
time (sec)	N/A	0.863	4.966	7.268	0.	0.	0.	0.

Problem 737	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	642	642	545	2728	0	0	0	0
normalized size	1	1.	0.85	4.25	0.	0.	0.	0.
time (sec)	N/A	1.401	2.624	7.577	0.	0.	0.	0.

Problem 738	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	496	496	410	2112	0	0	0	0
normalized size	1	1.	0.83	4.26	0.	0.	0.	0.
time (sec)	N/A	1.026	2.368	5.964	0.	0.	0.	0.

Problem 739	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	375	375	306	1561	0	0	0	0
normalized size	1	1.	0.82	4.16	0.	0.	0.	0.
time (sec)	N/A	0.678	1.41	4.738	0.	0.	0.	0.

Problem 740	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	302	302	219	1085	0	0	0	0
normalized size	1	1.	0.73	3.59	0.	0.	0.	0.
time (sec)	N/A	0.479	1.183	3.21	0.	0.	0.	0.

Problem 741	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	361	361	311	1398	0	0	0	0
normalized size	1	1.	0.86	3.87	0.	0.	0.	0.
time (sec)	N/A	0.622	2.058	4.096	0.	0.	0.	0.

Problem 742	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	391	391	357	1379	0	0	0	0
normalized size	1	1.	0.91	3.53	0.	0.	0.	0.
time (sec)	N/A	0.748	3.636	5.682	0.	0.	0.	0.

Problem 743	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	532	532	584	1621	0	0	0	0
normalized size	1	1.	1.1	3.05	0.	0.	0.	0.
time (sec)	N/A	1.109	5.307	8.145	0.	0.	0.	0.

Problem 744	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	716	716	1127	2111	0	0	0	0
normalized size	1	1.	1.57	2.95	0.	0.	0.	0.
time (sec)	N/A	1.436	6.941	12.731	0.	0.	0.	0.

Problem 745	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	296	296	606	1190	0	0	0	0
normalized size	1	1.	2.05	4.02	0.	0.	0.	0.
time (sec)	N/A	1.088	5.881	3.692	0.	0.	0.	0.

Problem 746	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	229	242	391	0	0	0	0
normalized size	1	1.	1.06	1.71	0.	0.	0.	0.
time (sec)	N/A	0.491	3.921	1.343	0.	0.	0.	0.

Problem 747	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	114	181	0	0	0	0
normalized size	1	1.	0.75	1.18	0.	0.	0.	0.
time (sec)	N/A	0.329	2.819	1.151	0.	0.	0.	0.

Problem 748	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	74	151	0	0	0	0
normalized size	1	1.	0.99	2.01	0.	0.	0.	0.
time (sec)	N/A	0.214	0.114	0.948	0.	0.	0.	0.

Problem 749	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	220	220	617	610	0	0	0	0
normalized size	1	1.	2.8	2.77	0.	0.	0.	0.
time (sec)	N/A	0.63	6.954	3.242	0.	0.	0.	0.

Problem 750	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	399	399	1079	1072	0	0	0	0
normalized size	1	1.	2.7	2.69	0.	0.	0.	0.
time (sec)	N/A	1.653	7.177	5.758	0.	0.	0.	0.

Problem 751	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	534	534	1109	1886	0	0	0	0
normalized size	1	1.	2.08	3.53	0.	0.	0.	0.
time (sec)	N/A	2.04	8.109	6.068	0.	0.	0.	0.

Problem 752	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	390	390	986	1363	0	0	0	0
normalized size	1	1.	2.53	3.49	0.	0.	0.	0.
time (sec)	N/A	1.261	7.994	5.217	0.	0.	0.	0.

Problem 753	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	351	351	891	1027	0	0	0	0
normalized size	1	1.	2.54	2.93	0.	0.	0.	0.
time (sec)	N/A	1.037	7.208	4.618	0.	0.	0.	0.

Problem 754	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	307	307	846	872	0	0	0	0
normalized size	1	1.	2.76	2.84	0.	0.	0.	0.
time (sec)	N/A	0.867	7.191	4.293	0.	0.	0.	0.

Problem 755	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	325	325	871	690	0	0	0	0
normalized size	1	1.	2.68	2.12	0.	0.	0.	0.
time (sec)	N/A	0.968	7.25	3.542	0.	0.	0.	0.

Problem 756	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	449	449	1057	1266	0	0	0	0
normalized size	1	1.	2.35	2.82	0.	0.	0.	0.
time (sec)	N/A	1.627	7.638	5.793	0.	0.	0.	0.

Problem 757	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	661	661	1319	1731	0	0	0	0
normalized size	1	1.	2.	2.62	0.	0.	0.	0.
time (sec)	N/A	2.799	8.618	10.434	0.	0.	0.	0.

Problem 758	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	816	816	1526	2775	0	0	0	0
normalized size	1	1.	1.87	3.4	0.	0.	0.	0.
time (sec)	N/A	3.179	9.059	11.506	0.	0.	0.	0.

Problem 759	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	605	605	1323	2237	0	0	0	0
normalized size	1	1.	2.19	3.7	0.	0.	0.	0.
time (sec)	N/A	2.201	8.319	9.81	0.	0.	0.	0.

Problem 760	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	549	549	1149	1888	0	0	0	0
normalized size	1	1.	2.09	3.44	0.	0.	0.	0.
time (sec)	N/A	2.076	8.	8.871	0.	0.	0.	0.

Problem 761	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	472	472	1001	1718	0	0	0	0
normalized size	1	1.	2.12	3.64	0.	0.	0.	0.
time (sec)	N/A	1.809	7.516	8.563	0.	0.	0.	0.

Problem 762	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	487	487	1038	1525	0	0	0	0
normalized size	1	1.	2.13	3.13	0.	0.	0.	0.
time (sec)	N/A	1.605	7.836	8.669	0.	0.	0.	0.

Problem 763	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	503	503	1069	867	0	0	0	0
normalized size	1	1.	2.13	1.72	0.	0.	0.	0.
time (sec)	N/A	1.675	7.694	4.98	0.	0.	0.	0.

Problem 764	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	682	682	1318	2099	0	0	0	0
normalized size	1	1.	1.93	3.08	0.	0.	0.	0.
time (sec)	N/A	2.768	8.703	11.483	0.	0.	0.	0.

Problem 765	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	888	888	1948	404333	0	0	0	0
normalized size	1	1.	2.19	455.33	0.	0.	0.	0.
time (sec)	N/A	3.386	7.123	9.008	0.	0.	0.	0.

Problem 766	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	784	784	1849	277165	0	0	0	0
normalized size	1	1.	2.36	353.53	0.	0.	0.	0.
time (sec)	N/A	3.516	9.484	4.23	0.	0.	0.	0.

Problem 767	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	628	628	228392	146613	0	0	0	0
normalized size	1	1.	363.68	233.46	0.	0.	0.	0.
time (sec)	N/A	1.327	31.683	1.348	0.	0.	0.	0.

Problem 768	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	195	248028	0	0	0	0
normalized size	1	1.	0.98	1252.67	0.	0.	0.	0.
time (sec)	N/A	0.114	0.219	4.787	0.	0.	0.	0.

Problem 769	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	409	409	263	46599	0	0	0	0
normalized size	1	1.	0.64	113.93	0.	0.	0.	0.
time (sec)	N/A	0.494	7.344	0.766	0.	0.	0.	0.

Problem 770	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	489	489	2037	197178	0	0	0	0
normalized size	1	1.	4.17	403.23	0.	0.	0.	0.
time (sec)	N/A	0.865	6.394	2.937	0.	0.	0.	0.

Problem 771	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1080	1080	2061	577146	0	0	0	0
normalized size	1	1.	1.91	534.39	0.	0.	0.	0.
time (sec)	N/A	5.238	7.475	20.865	0.	0.	0.	0.

Problem 772	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	870	870	1922	409584	0	0	0	0
normalized size	1	1.	2.21	470.79	0.	0.	0.	0.
time (sec)	N/A	3.359	6.291	8.178	0.	0.	0.	0.

Problem 773	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	F	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	740	740	1849	278382	0	0	0	0
normalized size	1	1.	2.5	376.19	0.	0.	0.	0.
time (sec)	N/A	2.294	9.45	3.464	0.	0.	0.	0.

Problem 774	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	644	644	222963	529273	0	0	0	0
normalized size	1	1.	346.22	821.85	0.	0.	0.	0.
time (sec)	N/A	1.538	32.695	8.722	0.	0.	0.	0.

Problem 775	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	600	600	1866	2626517	0	0	0	0
normalized size	1	1.	3.11	4377.53	0.	0.	0.	0.
time (sec)	N/A	0.934	9.357	123.669	0.	0.	0.	0.

Problem 776	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	497	497	1982	189727	0	0	0	0
normalized size	1	1.	3.99	381.74	0.	0.	0.	0.
time (sec)	N/A	0.954	6.322	3.327	0.	0.	0.	0.

Problem 777	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1295	1295	2246	752449	0	0	0	0
normalized size	1	1.	1.73	581.04	0.	0.	0.	0.
time (sec)	N/A	7.844	8.429	38.718	0.	0.	0.	0.

Problem 778	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1071	1071	2061	577718	0	0	0	0
normalized size	1	1.	1.92	539.42	0.	0.	0.	0.
time (sec)	N/A	4.995	7.397	17.068	0.	0.	0.	0.

Problem 779	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	894	894	1949	409146	0	0	0	0
normalized size	1	1.	2.18	457.66	0.	0.	0.	0.
time (sec)	N/A	3.269	7.045	10.796	0.	0.	0.	0.

Problem 780	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	745	745	1864	730173	0	0	0	0
normalized size	1	1.	2.5	980.1	0.	0.	0.	0.
time (sec)	N/A	2.189	10.184	18.651	0.	0.	0.	0.

Problem 781	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	780	780	1976	3434386	0	0	0	0
normalized size	1	1.	2.53	4403.06	0.	0.	0.	0.
time (sec)	N/A	2.468	6.801	39.216	0.	0.	0.	0.

Problem 782	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	737	737	2139	0	0	0	0	0
normalized size	1	1.	2.9	0.	0.	0.	0.	0.
time (sec)	N/A	1.781	6.923	180.	0.	0.	0.	0.

Problem 783	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	772	772	1864	731123	0	0	0	0
normalized size	1	1.	2.41	947.05	0.	0.	0.	0.
time (sec)	N/A	2.303	10.301	13.859	0.	0.	0.	0.

Problem 784	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	644	644	222963	544147	0	0	0	0
normalized size	1	1.	346.22	844.95	0.	0.	0.	0.
time (sec)	N/A	1.58	32.536	7.095	0.	0.	0.	0.

Problem 785	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	195	248962	0	0	0	0
normalized size	1	1.	0.98	1257.38	0.	0.	0.	0.
time (sec)	N/A	0.11	0.265	4.023	0.	0.	0.	0.

Problem 786	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	189	1228	0	0	0	0
normalized size	1	1.	0.98	6.4	0.	0.	0.	0.
time (sec)	N/A	0.121	0.218	0.516	0.	0.	0.	0.

Problem 787	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	405	405	90261	41715	0	0	0	0
normalized size	1	1.	222.87	103.	0.	0.	0.	0.
time (sec)	N/A	0.459	32.45	0.846	0.	0.	0.	0.

Problem 788	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	521	521	2072	219162	0	0	0	0
normalized size	1	1.	3.98	420.66	0.	0.	0.	0.
time (sec)	N/A	1.019	6.441	3.897	0.	0.	0.	0.

Problem 789	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	822	822	1975	3901706	0	0	0	0
normalized size	1	1.	2.4	4746.6	0.	0.	0.	0.
time (sec)	N/A	2.652	6.791	51.96	0.	0.	0.	0.

Problem 790	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	600	600	1866	2946560	0	0	0	0
normalized size	1	1.	3.11	4910.93	0.	0.	0.	0.
time (sec)	N/A	0.928	9.392	134.514	0.	0.	0.	0.

Problem 791	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	409	409	226	46962	0	0	0	0
normalized size	1	1.	0.55	114.82	0.	0.	0.	0.
time (sec)	N/A	0.455	4.166	0.763	0.	0.	0.	0.

Problem 792	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	405	405	90261	40572	0	0	0	0
normalized size	1	1.	222.87	100.18	0.	0.	0.	0.
time (sec)	N/A	0.451	32.763	0.822	0.	0.	0.	0.

Problem 793	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	495	495	2052	119239	0	0	0	0
normalized size	1	1.	4.15	240.89	0.	0.	0.	0.
time (sec)	N/A	0.908	6.735	1.815	0.	0.	0.	0.

Problem 794	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	681	681	2320	415876	0	0	0	0
normalized size	1	1.	3.41	610.68	0.	0.	0.	0.
time (sec)	N/A	2.68	7.393	8.03	0.	0.	0.	0.

Problem 795	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	736	736	2142	0	0	0	0	0
normalized size	1	1.	2.91	0.	0.	0.	0.	0.
time (sec)	N/A	1.843	7.016	180.	0.	0.	0.	0.

Problem 796	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	497	497	1982	195453	0	0	0	0
normalized size	1	1.	3.99	393.27	0.	0.	0.	0.
time (sec)	N/A	1.001	6.34	3.203	0.	0.	0.	0.

Problem 797	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	489	489	2037	212512	0	0	0	0
normalized size	1	1.	4.17	434.58	0.	0.	0.	0.
time (sec)	N/A	0.86	6.412	2.955	0.	0.	0.	0.

Problem 798	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	516	516	2072	243193	0	0	0	0
normalized size	1	1.	4.02	471.3	0.	0.	0.	0.
time (sec)	N/A	0.994	6.432	4.226	0.	0.	0.	0.

Problem 799	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	688	688	2322	439275	0	0	0	0
normalized size	1	1.	3.38	638.48	0.	0.	0.	0.
time (sec)	N/A	1.847	7.378	8.531	0.	0.	0.	0.

Problem 800	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	941	941	2639	1123207	0	0	0	0
normalized size	1	1.	2.8	1193.63	0.	0.	0.	0.
time (sec)	N/A	4.472	8.602	18.731	0.	0.	0.	0.

Problem 801	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.05	3.009	1.065	0.	0.	0.	0.

Problem 802	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	311	311	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.439	17.935	0.369	0.	0.	0.	0.

Problem 803	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	229	229	200	0	0	0	0	0
normalized size	1	1.	0.87	0.	0.	0.	0.	0.
time (sec)	N/A	0.203	0.57	0.202	0.	0.	0.	0.

Problem 804	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	104	104	120	0	0	0	0	0
normalized size	1	1.	1.15	0.	0.	0.	0.	0.
time (sec)	N/A	0.066	0.254	0.393	0.	0.	0.	0.

Problem 805	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.057	2.586	0.913	0.	0.	0.	0.

Problem 806	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.056	4.586	0.632	0.	0.	0.	0.

Problem 807	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.055	13.133	0.967	0.	0.	0.	0.

Problem 808	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.078	34.107	0.171	0.	0.	0.	0.

Problem 809	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	F(-2)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.076	13.637	0.163	0.	0.	0.	0.

Problem 810	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.069	0.494	0.157	0.	0.	0.	0.

Problem 811	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.069	3.155	0.156	0.	0.	0.	0.

Problem 812	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.076	4.575	0.155	0.	0.	0.	0.

Problem 813	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.078	9.853	0.155	0.	0.	0.	0.

Problem 814	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	272	272	493	0	0	0	0	0
normalized size	1	1.	1.81	0.	0.	0.	0.	0.
time (sec)	N/A	0.459	11.561	2.757	0.	0.	0.	0.

Problem 815	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	342	0	0	0	0	0
normalized size	1	1.	1.68	0.	0.	0.	0.	0.
time (sec)	N/A	0.257	6.111	3.602	0.	0.	0.	0.

Problem 816	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	149	149	280	0	0	0	0	0
normalized size	1	1.	1.88	0.	0.	0.	0.	0.
time (sec)	N/A	0.15	1.644	1.033	0.	0.	0.	0.

Problem 817	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	171	171	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.238	2.801	0.483	0.	0.	0.	0.

Problem 818	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	231	231	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.442	4.642	0.776	0.	0.	0.	0.

Problem 819	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	113	113	2967	0	0	0	0	0
normalized size	1	1.	26.26	0.	0.	0.	0.	0.
time (sec)	N/A	0.232	15.132	0.41	0.	0.	0.	0.

Problem 820	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	299	299	297	0	0	0	0	0
normalized size	1	1.	0.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.495	1.397	0.44	0.	0.	0.	0.

Problem 821	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	222	222	222	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.249	0.676	0.427	0.	0.	0.	0.

Problem 822	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	163	163	270	0	0	0	0	0
normalized size	1	1.	1.66	0.	0.	0.	0.	0.
time (sec)	N/A	0.119	1.534	0.19	0.	0.	0.	0.

Problem 823	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	157	0	0	0	0	0
normalized size	1	1.	0.83	0.	0.	0.	0.	0.
time (sec)	N/A	0.23	0.299	0.155	0.	0.	0.	0.

Problem 824	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	288	288	195	0	0	0	0	0
normalized size	1	1.	0.68	0.	0.	0.	0.	0.
time (sec)	N/A	0.487	2.867	0.349	0.	0.	0.	0.

Problem 825	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	298	298	167	0	0	0	0	0
normalized size	1	1.	0.56	0.	0.	0.	0.	0.
time (sec)	N/A	0.566	0.544	2.937	0.	0.	0.	0.

Problem 826	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	213	135	0	0	0	0	0
normalized size	1	1.	0.63	0.	0.	0.	0.	0.
time (sec)	N/A	0.269	0.375	3.652	0.	0.	0.	0.

Problem 827	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	105	0	0	0	0	0
normalized size	1	1.	0.7	0.	0.	0.	0.	0.
time (sec)	N/A	0.148	0.246	1.145	0.	0.	0.	0.

Problem 828	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	204	204	1668	0	0	0	0	0
normalized size	1	1.	8.18	0.	0.	0.	0.	0.
time (sec)	N/A	0.399	16.924	0.493	0.	0.	0.	0.

Problem 829	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	321	321	1938	0	0	0	0	0
normalized size	1	1.	6.04	0.	0.	0.	0.	0.
time (sec)	N/A	0.544	19.073	0.859	0.	0.	0.	0.

Problem 830	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	432	432	2496	0	0	0	0	0
normalized size	1	1.	5.78	0.	0.	0.	0.	0.
time (sec)	N/A	0.703	20.328	1.06	0.	0.	0.	0.

Problem 831	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	55	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.112	2.502	0.214	0.	0.	0.	0.

Problem 832	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	323	303	230	0	0	0	0	0
normalized size	1	0.94	0.71	0.	0.	0.	0.	0.
time (sec)	N/A	0.548	1.078	0.451	0.	0.	0.	0.

Problem 833	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	221	152	0	0	0	0	0
normalized size	1	0.96	0.66	0.	0.	0.	0.	0.
time (sec)	N/A	0.233	0.331	0.382	0.	0.	0.	0.

Problem 834	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	129	0	0	0	0	0
normalized size	1	1.	0.79	0.	0.	0.	0.	0.
time (sec)	N/A	0.114	0.248	0.165	0.	0.	0.	0.

Problem 835	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	204	204	1811	0	0	0	0	0
normalized size	1	1.	8.88	0.	0.	0.	0.	0.
time (sec)	N/A	0.338	18.176	0.144	0.	0.	0.	0.

Problem 836	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	322	322	2036	0	0	0	0	0
normalized size	1	1.	6.32	0.	0.	0.	0.	0.
time (sec)	N/A	0.508	19.004	0.394	0.	0.	0.	0.

Problem 837	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	428	428	2660	0	0	0	0	0
normalized size	1	1.	6.21	0.	0.	0.	0.	0.
time (sec)	N/A	0.652	20.207	0.436	0.	0.	0.	0.

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [198] had the largest ratio of [0.75]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	9	4	1.	21	0.19
2	A	13	4	1.	21	0.19
3	A	6	5	1.	13	0.385
4	A	2	2	1.	13	0.154
5	A	4	4	1.	13	0.308
6	A	2	2	1.	11	0.182
7	A	1	1	1.	8	0.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
8	A	3	3	1.	11	0.273
9	A	5	5	1.	13	0.385
10	A	6	6	1.	13	0.462
11	A	6	5	1.	13	0.385
12	A	3	3	1.	13	0.231
13	A	6	6	1.	13	0.462
14	A	3	3	1.	13	0.231
15	A	2	2	1.	11	0.182
16	A	2	2	1.	8	0.25
17	A	4	4	1.	11	0.364
18	A	6	6	1.	13	0.462
19	A	7	7	1.	13	0.538
20	A	7	6	1.09	13	0.462
21	A	8	6	1.	13	0.462
22	A	4	3	1.	13	0.231
23	A	7	7	1.	13	0.538
24	A	5	5	1.	13	0.385
25	A	3	3	1.	13	0.231
26	A	3	3	1.	11	0.273
27	A	3	2	1.	8	0.25
28	A	5	4	1.	11	0.364
29	A	7	6	1.	13	0.462
30	A	8	7	1.	13	0.538
31	A	8	6	1.	13	0.462
32	A	5	4	1.	23	0.174
33	A	4	4	1.	23	0.174
34	A	3	3	1.	23	0.13
35	A	2	2	1.	21	0.095
36	A	1	1	1.	14	0.071
37	A	2	2	1.	21	0.095
38	A	3	3	1.	23	0.13
39	A	4	3	1.	23	0.13
40	A	5	3	1.	23	0.13
41	A	2	2	1.	22	0.091
42	A	2	2	1.	23	0.087
43	A	2	2	1.	24	0.083
44	A	6	6	1.	23	0.261
45	A	4	4	1.	23	0.174
46	A	3	3	1.	21	0.143
47	A	2	2	1.	14	0.143
48	A	4	4	1.	21	0.19
49	A	4	4	1.	23	0.174
50	A	5	5	1.	23	0.217
51	A	6	5	1.	23	0.217

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
52	A	6	6	1.	23	0.261
53	A	5	4	1.	23	0.174
54	A	4	3	1.	21	0.143
55	A	3	2	1.	14	0.143
56	A	4	4	1.	21	0.19
57	A	4	4	1.	23	0.174
58	A	4	4	1.	23	0.174
59	A	5	5	1.	23	0.217
60	A	6	5	1.	23	0.217
61	A	6	6	1.	23	0.261
62	A	4	4	1.	23	0.174
63	A	3	3	1.	21	0.143
64	A	2	2	1.	14	0.143
65	A	5	4	1.	21	0.19
66	A	6	5	1.	23	0.217
67	A	7	6	1.	23	0.261
68	A	7	7	1.	23	0.304
69	A	6	6	1.	23	0.261
70	A	4	4	1.	23	0.174
71	A	3	3	1.	21	0.143
72	A	3	3	1.	14	0.214
73	A	6	5	1.	21	0.238
74	A	7	6	1.	23	0.261
75	A	8	6	1.	23	0.261
76	A	8	8	1.	23	0.348
77	A	7	7	1.	23	0.304
78	A	6	6	1.	23	0.261
79	A	4	4	1.	23	0.174
80	A	4	4	1.	21	0.19
81	A	4	3	1.	14	0.214
82	A	7	6	1.	21	0.286
83	A	8	7	1.	23	0.304
84	A	9	7	1.	23	0.304
85	A	2	2	1.	25	0.08
86	A	2	2	1.	28	0.071
87	A	2	2	1.	15	0.133
88	A	2	2	1.	17	0.118
89	A	2	2	1.	17	0.118
90	A	2	2	1.	18	0.111
91	A	5	4	1.	23	0.174
92	A	6	6	1.	23	0.261
93	A	4	4	1.	23	0.174
94	A	3	3	1.	21	0.143
95	A	2	2	1.	14	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
96	A	4	4	1.	21	0.19
97	A	4	4	1.	23	0.174
98	A	6	6	1.	23	0.261
99	A	4	4	1.	23	0.174
100	A	3	3	1.	21	0.143
101	A	2	2	1.	14	0.143
102	A	4	4	1.	21	0.19
103	A	4	4	1.	23	0.174
104	A	6	6	1.	23	0.261
105	A	4	4	1.	23	0.174
106	A	3	3	1.	21	0.143
107	A	2	2	1.	14	0.143
108	A	4	4	1.	21	0.19
109	A	4	4	1.	23	0.174
110	A	6	6	1.	23	0.261
111	A	4	4	1.	23	0.174
112	A	3	3	1.	21	0.143
113	A	2	2	1.	14	0.143
114	A	4	4	1.	21	0.19
115	A	4	4	1.	23	0.174
116	A	4	4	1.	21	0.19
117	A	2	2	1.	21	0.095
118	A	3	3	1.	21	0.143
119	A	3	3	1.	21	0.143
120	A	4	4	1.	23	0.174
121	A	2	2	1.	23	0.087
122	A	4	4	1.	23	0.174
123	A	4	4	1.	23	0.174
124	A	4	4	1.	23	0.174
125	A	2	2	1.	23	0.087
126	A	4	4	1.	23	0.174
127	A	4	4	1.	23	0.174
128	A	5	5	1.	25	0.2
129	A	3	3	1.	25	0.12
130	A	5	5	1.	25	0.2
131	A	5	5	1.	25	0.2
132	A	2	2	1.	19	0.105
133	A	2	2	1.	23	0.087
134	A	3	3	1.	21	0.143
135	A	3	3	1.	23	0.13
136	A	3	3	1.	21	0.143
137	A	3	3	1.	24	0.125
138	A	4	4	1.	23	0.174
139	A	4	4	1.	24	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
140	A	7	7	1.	21	0.333
141	A	6	6	1.	21	0.286
142	A	4	4	1.	21	0.19
143	A	3	3	1.	19	0.158
144	A	2	2	1.	12	0.167
145	A	4	4	1.	19	0.21
146	A	4	4	1.	21	0.19
147	A	1	1	1.	10	0.1
148	A	1	1	1.	12	0.083
149	A	6	4	1.	19	0.21
150	A	5	4	1.	19	0.21
151	A	1	1	1.	17	0.059
152	A	2	1	1.	10	0.1
153	A	2	2	1.	17	0.118
154	A	4	4	1.	19	0.21
155	A	5	5	1.	19	0.263
156	A	5	4	1.	19	0.21
157	A	7	5	1.	21	0.238
158	A	6	5	1.	21	0.238
159	A	2	2	1.	19	0.105
160	A	1	1	1.	12	0.083
161	A	3	3	1.	19	0.158
162	A	4	4	1.	21	0.19
163	A	5	5	1.	21	0.238
164	A	6	6	1.	21	0.286
165	A	6	5	1.	21	0.238
166	A	8	6	1.13	21	0.286
167	A	4	3	1.12	21	0.143
168	A	3	2	1.	19	0.105
169	A	2	2	1.	12	0.167
170	A	4	4	1.	19	0.21
171	A	4	4	1.	21	0.19
172	A	4	4	1.	21	0.19
173	A	6	6	1.	21	0.286
174	A	7	7	1.	21	0.333
175	A	3	3	1.	12	0.25
176	A	7	7	1.	13	0.538
177	A	6	6	1.	13	0.462
178	A	6	6	1.	13	0.462
179	A	4	4	1.	11	0.364
180	A	3	3	1.	8	0.375
181	A	5	5	1.	11	0.454
182	A	7	7	1.	13	0.538
183	A	7	7	1.	13	0.538

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
184	A	8	7	1.	13	0.538
185	A	7	7	1.	13	0.538
186	A	6	6	1.	13	0.462
187	A	5	5	1.	13	0.385
188	A	5	5	1.	11	0.454
189	A	5	5	1.	8	0.625
190	A	6	6	1.	11	0.546
191	A	7	7	1.	13	0.538
192	A	8	7	1.	13	0.538
193	A	8	8	1.	13	0.615
194	A	7	7	1.	13	0.538
195	A	6	6	1.	13	0.462
196	A	6	6	1.	13	0.462
197	A	6	5	1.	11	0.454
198	A	6	6	1.	8	0.75
199	A	7	7	1.	11	0.636
200	A	8	7	1.	13	0.538
201	A	9	7	1.	13	0.538
202	A	7	6	1.	12	0.5
203	A	6	6	1.	21	0.286
204	A	2	2	1.	14	0.143
205	A	5	5	1.	21	0.238
206	A	9	9	1.	23	0.391
207	A	5	5	1.	21	0.238
208	A	2	2	1.	14	0.143
209	A	2	2	1.	21	0.095
210	A	9	9	1.	23	0.391
211	A	7	7	1.	25	0.28
212	A	1	1	1.	25	0.04
213	A	5	4	1.	23	0.174
214	A	4	3	1.	23	0.13
215	A	3	2	1.	21	0.095
216	A	5	3	1.	23	0.13
217	A	10	4	1.	23	0.174
218	A	13	4	1.	23	0.174
219	A	3	3	1.	36	0.083
220	A	3	3	1.	23	0.13
221	A	0	0	0.	0	0.
222	A	9	6	1.	21	0.286
223	A	8	5	1.	21	0.238
224	A	7	4	1.	19	0.21
225	A	3	3	1.	12	0.25
226	A	0	0	0.	0	0.
227	A	6	5	1.	24	0.208

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
228	A	5	5	1.	24	0.208
229	A	4	4	1.	24	0.167
230	A	1	1	1.	22	0.045
231	A	2	2	1.	24	0.083
232	A	2	2	1.	24	0.083
233	A	3	3	1.	24	0.125
234	A	4	3	1.	24	0.125
235	A	5	3	1.	24	0.125
236	A	7	5	1.	26	0.192
237	A	6	5	1.	26	0.192
238	A	5	4	1.	26	0.154
239	A	4	3	1.	26	0.115
240	A	4	4	1.	24	0.167
241	A	4	4	1.	26	0.154
242	A	4	3	1.	26	0.115
243	A	2	2	1.	26	0.077
244	A	3	3	1.	26	0.115
245	A	4	3	1.	26	0.115
246	A	5	3	1.	26	0.115
247	A	8	5	1.	26	0.192
248	A	7	5	1.	26	0.192
249	A	6	4	1.	26	0.154
250	A	5	3	1.	26	0.115
251	A	5	4	1.	26	0.154
252	A	5	5	1.	24	0.208
253	A	5	5	1.	26	0.192
254	A	5	4	1.	26	0.154
255	A	5	3	1.	26	0.115
256	A	2	2	1.	26	0.077
257	A	3	3	1.	26	0.115
258	A	4	3	1.	26	0.115
259	A	5	3	1.	26	0.115
260	A	6	3	1.	26	0.115
261	A	6	5	1.	26	0.192
262	A	5	5	1.	26	0.192
263	A	4	4	1.	26	0.154
264	A	2	2	1.	24	0.083
265	A	3	3	1.	26	0.115
266	A	4	4	1.	26	0.154
267	A	5	4	1.	26	0.154
268	A	6	4	1.	26	0.154
269	A	7	5	1.	26	0.192
270	A	6	5	1.	26	0.192
271	A	5	4	1.	26	0.154

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
272	A	4	3	1.	26	0.115
273	A	2	2	1.	24	0.083
274	A	4	4	1.	26	0.154
275	A	3	2	1.	26	0.077
276	A	4	3	1.	26	0.115
277	A	5	3	1.	26	0.115
278	A	6	3	1.	26	0.115
279	A	7	5	1.	26	0.192
280	A	6	4	1.	26	0.154
281	A	5	3	1.	26	0.115
282	A	2	2	1.	26	0.077
283	A	3	3	1.	24	0.125
284	A	5	4	1.	26	0.154
285	A	4	3	1.	26	0.115
286	A	3	2	1.	26	0.077
287	A	4	3	1.	26	0.115
288	A	5	3	1.	26	0.115
289	A	6	3	1.	26	0.115
290	A	5	3	1.	26	0.115
291	A	4	3	1.	26	0.115
292	A	3	3	1.	26	0.115
293	A	2	2	1.	26	0.077
294	A	4	4	1.	26	0.154
295	A	4	4	1.	26	0.154
296	A	5	5	1.	26	0.192
297	A	6	5	1.	26	0.192
298	A	5	3	1.	28	0.107
299	A	4	3	1.	28	0.107
300	A	3	3	1.	28	0.107
301	A	2	2	1.	28	0.071
302	A	5	4	1.	28	0.143
303	A	5	5	1.	28	0.179
304	A	5	4	1.	28	0.143
305	A	6	5	1.	28	0.179
306	A	7	5	1.	28	0.179
307	A	5	3	1.	28	0.107
308	A	4	3	1.	28	0.107
309	A	3	3	1.	28	0.107
310	A	2	2	1.	28	0.071
311	A	6	4	1.	28	0.143
312	A	6	5	1.	28	0.179
313	A	6	5	1.	28	0.179
314	A	6	4	1.	28	0.143
315	A	7	5	1.	28	0.179

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
316	A	8	5	1.	28	0.179
317	A	5	3	1.	28	0.107
318	A	4	3	1.	28	0.107
319	A	3	3	1.	28	0.107
320	A	2	2	1.	28	0.071
321	A	4	4	1.	28	0.143
322	A	5	5	1.	28	0.179
323	A	6	6	1.	28	0.214
324	A	6	3	1.	28	0.107
325	A	5	3	1.	28	0.107
326	A	4	3	1.	28	0.107
327	A	3	3	1.	28	0.107
328	A	2	2	1.	28	0.071
329	A	5	4	1.	28	0.143
330	A	6	6	1.	28	0.214
331	A	7	6	1.	28	0.214
332	A	6	3	1.	28	0.107
333	A	5	3	1.	28	0.107
334	A	4	3	1.	28	0.107
335	A	3	3	1.	28	0.107
336	A	2	2	1.	28	0.071
337	A	6	4	1.	28	0.143
338	A	7	6	1.	28	0.214
339	A	8	7	1.	28	0.25
340	A	1	1	1.	30	0.033
341	A	1	1	1.	30	0.033
342	A	1	1	1.	30	0.033
343	A	1	1	1.	30	0.033
344	A	3	3	1.	30	0.1
345	A	1	1	1.	30	0.033
346	A	1	1	1.	30	0.033
347	A	1	1	1.	30	0.033
348	A	2	2	1.	30	0.067
349	A	2	2	1.	30	0.067
350	A	2	2	1.	30	0.067
351	A	1	1	1.	30	0.033
352	A	4	4	1.	30	0.133
353	A	4	4	1.	30	0.133
354	A	1	1	1.	30	0.033
355	A	2	2	1.	30	0.067
356	A	2	2	1.	30	0.067
357	A	2	2	1.	30	0.067
358	A	3	2	1.	30	0.067
359	A	3	2	1.	30	0.067

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
360	A	2	2	1.	30	0.067
361	A	1	1	1.	30	0.033
362	A	5	4	1.	30	0.133
363	A	5	5	1.	30	0.167
364	A	5	4	1.	30	0.133
365	A	1	1	1.	30	0.033
366	A	2	2	1.	30	0.067
367	A	3	2	1.	30	0.067
368	A	3	2	1.	30	0.067
369	A	4	2	1.	30	0.067
370	A	4	2	1.	30	0.067
371	A	3	2	1.	30	0.067
372	A	2	2	1.	30	0.067
373	A	1	1	1.	30	0.033
374	A	6	4	1.	30	0.133
375	A	6	5	1.	30	0.167
376	A	6	5	1.	30	0.167
377	A	6	4	1.	30	0.133
378	A	1	1	1.	30	0.033
379	A	2	2	1.	30	0.067
380	A	3	2	1.	30	0.067
381	A	4	2	1.	30	0.067
382	A	4	2	1.	30	0.067
383	A	5	4	1.	30	0.133
384	A	4	4	1.	30	0.133
385	A	3	3	1.	30	0.1
386	A	2	2	1.	30	0.067
387	A	3	3	1.	30	0.1
388	A	4	3	1.	30	0.1
389	A	6	5	1.	30	0.167
390	A	5	5	1.	30	0.167
391	A	4	4	1.	30	0.133
392	A	1	1	1.	30	0.033
393	A	3	3	1.	30	0.1
394	A	4	3	1.	30	0.1
395	A	5	3	1.	30	0.1
396	A	7	5	1.	30	0.167
397	A	6	5	1.	30	0.167
398	A	5	4	1.	30	0.133
399	A	1	1	1.	30	0.033
400	A	1	1	1.	30	0.033
401	A	4	3	1.	30	0.1
402	A	5	3	1.	30	0.1
403	A	6	3	1.	30	0.1

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
404	A	4	4	1.	26	0.154
405	A	4	4	1.	26	0.154
406	A	4	4	1.	26	0.154
407	A	4	4	1.	24	0.167
408	A	4	4	1.	26	0.154
409	A	4	4	1.	26	0.154
410	A	4	4	1.	26	0.154
411	A	3	2	1.	28	0.071
412	A	2	2	1.	28	0.071
413	A	1	1	1.	28	0.036
414	A	3	3	1.	28	0.107
415	A	3	3	1.	28	0.107
416	A	3	3	1.	28	0.107
417	A	3	3	1.	28	0.107
418	A	3	3	1.	28	0.107
419	A	3	2	1.	30	0.067
420	A	2	2	1.	30	0.067
421	A	1	1	1.	30	0.033
422	A	4	4	1.	28	0.143
423	A	4	4	1.	30	0.133
424	A	4	4	1.	30	0.133
425	A	4	2	1.	23	0.087
426	A	3	2	1.	23	0.087
427	A	2	2	1.	23	0.087
428	A	1	1	1.	21	0.048
429	A	2	1	1.	10	0.1
430	A	4	4	1.	23	0.174
431	A	5	5	1.	23	0.217
432	A	6	5	1.	23	0.217
433	A	7	5	1.	23	0.217
434	A	5	3	1.	25	0.12
435	A	4	3	1.	25	0.12
436	A	3	3	1.	25	0.12
437	A	2	2	1.	23	0.087
438	A	1	1	1.	12	0.083
439	A	5	5	1.	25	0.2
440	A	5	5	1.03	25	0.2
441	A	6	6	1.	25	0.24
442	A	7	6	1.	25	0.24
443	A	8	6	1.	25	0.24
444	A	6	5	1.52	25	0.2
445	A	9	7	1.15	25	0.28
446	A	8	6	1.06	23	0.261
447	A	7	5	1.	12	0.417

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
448	A	7	7	1.	25	0.28
449	A	7	7	1.	25	0.28
450	A	7	7	1.	25	0.28
451	A	8	8	1.	25	0.32
452	A	9	8	1.	25	0.32
453	A	3	3	1.	25	0.12
454	A	2	2	1.	25	0.08
455	A	3	3	1.	25	0.12
456	A	2	2	1.	23	0.087
457	A	1	1	1.	12	0.083
458	A	5	5	1.	25	0.2
459	A	6	6	1.	25	0.24
460	A	7	6	1.	25	0.24
461	A	4	4	1.	25	0.16
462	A	3	3	1.	25	0.12
463	A	5	5	1.	25	0.2
464	A	3	3	1.	25	0.12
465	A	2	2	1.	23	0.087
466	A	2	2	1.	12	0.167
467	A	6	6	1.	25	0.24
468	A	7	7	1.	25	0.28
469	A	8	7	1.	25	0.28
470	A	5	4	1.	25	0.16
471	A	4	3	1.	25	0.12
472	A	6	6	1.	25	0.24
473	A	5	5	1.	25	0.2
474	A	3	3	1.	25	0.12
475	A	3	3	1.	23	0.13
476	A	3	2	1.	12	0.167
477	A	7	6	1.	25	0.24
478	A	8	7	1.	25	0.28
479	A	9	7	1.	25	0.28
480	A	4	3	1.	13	0.231
481	A	4	3	1.	15	0.2
482	A	8	6	1.	25	0.24
483	A	7	6	1.	25	0.24
484	A	6	6	1.	25	0.24
485	A	5	5	1.	25	0.2
486	A	6	6	1.	25	0.24
487	A	7	6	1.	25	0.24
488	A	8	6	1.	25	0.24
489	A	9	7	1.	27	0.259
490	A	8	7	1.	27	0.259
491	A	7	7	1.	27	0.259

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
492	A	6	6	1.	27	0.222
493	A	6	6	1.	27	0.222
494	A	7	7	1.	27	0.259
495	A	8	7	1.	27	0.259
496	A	11	9	1.	27	0.333
497	A	10	9	1.	27	0.333
498	A	9	9	1.	27	0.333
499	A	8	8	1.	27	0.296
500	A	8	8	1.	27	0.296
501	A	8	8	1.	27	0.296
502	A	9	9	1.	27	0.333
503	A	10	9	1.	27	0.333
504	A	7	7	1.	27	0.259
505	A	6	6	1.	27	0.222
506	A	6	6	1.	27	0.222
507	A	6	6	1.	27	0.222
508	A	7	7	1.	27	0.259
509	A	8	7	1.	27	0.259
510	A	7	7	1.	27	0.259
511	A	7	7	1.	27	0.259
512	A	7	7	1.	27	0.259
513	A	7	7	1.	27	0.259
514	A	8	8	1.	27	0.296
515	A	9	8	1.	27	0.296
516	A	8	8	1.	27	0.296
517	A	8	7	1.	27	0.259
518	A	8	7	1.	27	0.259
519	A	8	7	1.	27	0.259
520	A	9	8	1.	27	0.296
521	A	10	8	1.	27	0.296
522	A	4	4	1.	27	0.148
523	A	3	3	1.	27	0.111
524	A	2	2	1.	25	0.08
525	A	1	1	1.	14	0.071
526	A	2	2	1.	27	0.074
527	A	3	3	1.	27	0.111
528	A	4	3	1.	27	0.111
529	A	6	6	1.	27	0.222
530	A	4	4	1.	27	0.148
531	A	3	3	1.	25	0.12
532	A	2	2	1.	14	0.143
533	A	4	4	1.	27	0.148
534	A	4	4	1.	27	0.148
535	A	5	5	1.	27	0.185

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
536	A	6	6	1.	27	0.222
537	A	5	4	1.	27	0.148
538	A	4	3	1.	25	0.12
539	A	3	2	1.	14	0.143
540	A	4	4	1.	27	0.148
541	A	4	4	1.	27	0.148
542	A	4	4	1.	27	0.148
543	A	6	6	1.	27	0.222
544	A	4	4	1.	27	0.148
545	A	3	3	1.	25	0.12
546	A	2	2	1.	14	0.143
547	A	5	5	1.	27	0.185
548	A	6	6	1.	27	0.222
549	A	7	7	1.	27	0.259
550	A	6	6	1.	27	0.222
551	A	4	4	1.	27	0.148
552	A	3	3	1.	25	0.12
553	A	3	3	1.	14	0.214
554	A	6	6	1.	27	0.222
555	A	7	7	1.	27	0.259
556	A	8	7	1.	27	0.259
557	A	6	6	1.	27	0.222
558	A	4	4	1.	27	0.148
559	A	4	4	1.	25	0.16
560	A	4	3	1.	14	0.214
561	A	7	7	1.	27	0.259
562	A	8	8	1.	27	0.296
563	A	9	8	1.	27	0.296
564	A	5	3	1.	29	0.103
565	A	4	3	1.	29	0.103
566	A	3	3	1.	29	0.103
567	A	2	2	1.	29	0.069
568	A	1	1	1.	29	0.034
569	A	2	2	1.	29	0.069
570	A	3	2	1.	29	0.069
571	A	7	5	1.	29	0.172
572	A	6	5	1.	29	0.172
573	A	5	5	1.	29	0.172
574	A	4	4	1.	29	0.138
575	A	4	4	1.	29	0.138
576	A	3	3	1.	29	0.103
577	A	4	4	1.	29	0.138
578	A	5	4	1.	29	0.138
579	A	7	5	1.	29	0.172

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
580	A	6	5	1.	29	0.172
581	A	5	5	1.	29	0.172
582	A	4	4	1.	29	0.138
583	A	4	4	1.	29	0.138
584	A	4	4	1.	29	0.138
585	A	3	3	1.	29	0.103
586	A	4	4	1.	29	0.138
587	A	5	4	1.	29	0.138
588	A	7	7	1.	29	0.241
589	A	6	6	1.	29	0.207
590	A	5	5	1.	29	0.172
591	A	2	2	1.	29	0.069
592	A	4	4	1.	29	0.138
593	A	5	5	1.	29	0.172
594	A	7	7	1.	29	0.241
595	A	6	6	1.	29	0.207
596	A	4	4	1.	29	0.138
597	A	4	4	1.	29	0.138
598	A	5	5	1.	29	0.172
599	A	6	5	1.	29	0.172
600	A	7	7	1.	29	0.241
601	A	5	5	1.	29	0.172
602	A	5	5	1.	29	0.172
603	A	5	5	1.	29	0.172
604	A	6	6	1.	29	0.207
605	A	7	6	1.	29	0.207
606	A	4	4	1.	25	0.16
607	A	6	6	1.	25	0.24
608	A	4	4	1.	25	0.16
609	A	3	3	1.	23	0.13
610	A	2	2	1.	12	0.167
611	A	3	3	1.	25	0.12
612	A	3	3	1.	25	0.12
613	A	3	3	1.	25	0.12
614	A	4	4	1.	27	0.148
615	A	4	4	1.	27	0.148
616	A	4	4	1.	27	0.148
617	A	4	4	1.	27	0.148
618	A	4	4	1.	27	0.148
619	A	4	4	1.	27	0.148
620	A	2	2	1.79	27	0.074
621	A	2	2	1.91	27	0.074
622	A	2	2	1.	27	0.074
623	A	2	2	1.	27	0.074

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
624	A	2	2	1.	25	0.08
625	A	2	2	1.	18	0.111
626	A	2	2	1.	27	0.074
627	A	2	2	1.	27	0.074
628	A	1	1	1.	27	0.037
629	A	2	2	1.36	27	0.074
630	A	2	2	1.42	27	0.074
631	A	2	2	1.42	29	0.069
632	A	2	2	1.42	29	0.069
633	A	2	2	1.	29	0.069
634	A	2	2	1.42	29	0.069
635	A	2	2	1.44	27	0.074
636	A	3	3	1.	20	0.15
637	A	2	2	1.64	29	0.069
638	A	2	2	1.51	29	0.069
639	A	1	1	1.	29	0.034
640	A	2	2	1.	29	0.069
641	A	2	2	1.	29	0.069
642	A	2	2	1.57	29	0.069
643	A	2	2	1.51	29	0.069
644	A	1	1	1.	29	0.034
645	A	2	2	1.	29	0.069
646	A	2	2	1.	27	0.074
647	A	3	3	1.	20	0.15
648	A	2	2	1.	29	0.069
649	A	2	2	1.	29	0.069
650	A	2	2	1.	29	0.069
651	A	2	2	1.	29	0.069
652	A	2	2	1.	29	0.069
653	A	4	4	1.	27	0.148
654	A	2	2	1.	29	0.069
655	A	3	3	1.	25	0.12
656	A	3	3	1.	25	0.12
657	A	3	3	1.	23	0.13
658	A	3	3	1.	12	0.25
659	A	3	3	1.	25	0.12
660	A	3	3	1.	25	0.12
661	A	3	3	1.	25	0.12
662	A	5	5	1.	27	0.185
663	A	5	5	1.	27	0.185
664	A	3	3	1.	27	0.111
665	A	3	3	1.3	27	0.111
666	A	3	3	1.25	27	0.111
667	A	3	3	1.32	27	0.111

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
668	A	3	3	1.	25	0.12
669	A	3	3	1.	25	0.12
670	A	3	3	1.	25	0.12
671	A	3	2	1.	23	0.087
672	A	2	2	1.	23	0.087
673	A	1	1	1.	21	0.048
674	A	2	1	1.	10	0.1
675	A	4	4	1.	23	0.174
676	A	5	5	1.	23	0.217
677	A	6	5	1.	23	0.217
678	A	4	3	1.	25	0.12
679	A	3	3	1.	25	0.12
680	A	2	2	1.	23	0.087
681	A	1	1	1.	12	0.083
682	A	5	5	1.	25	0.2
683	A	5	5	1.	25	0.2
684	A	6	6	1.	25	0.24
685	A	7	6	1.	25	0.24
686	A	5	4	1.23	25	0.16
687	A	4	3	1.	25	0.12
688	A	3	2	1.	23	0.087
689	A	2	2	1.	12	0.167
690	A	6	6	1.	25	0.24
691	A	6	6	1.	25	0.24
692	A	6	6	1.	25	0.24
693	A	7	7	1.	25	0.28
694	A	4	4	1.	20	0.2
695	A	2	2	1.	20	0.1
696	A	2	2	1.	15	0.133
697	A	2	2	1.	13	0.154
698	A	7	7	1.	25	0.28
699	A	6	6	1.	25	0.24
700	A	5	5	1.	25	0.2
701	A	4	4	1.	23	0.174
702	A	3	3	1.	12	0.25
703	A	7	4	1.	25	0.16
704	A	8	5	1.	25	0.2
705	A	9	6	1.	25	0.24
706	A	7	7	1.	25	0.28
707	A	6	6	1.	25	0.24
708	A	5	5	1.	25	0.2
709	A	5	5	1.	23	0.217
710	A	5	5	1.	12	0.417
711	A	8	5	1.	25	0.2

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
712	A	9	6	1.	25	0.24
713	A	10	6	1.	25	0.24
714	A	8	8	1.	25	0.32
715	A	7	7	1.	25	0.28
716	A	6	6	1.	25	0.24
717	A	6	6	1.	25	0.24
718	A	6	5	1.	23	0.217
719	A	6	6	1.	12	0.5
720	A	9	6	1.	25	0.24
721	A	10	6	1.	25	0.24
722	A	11	6	1.	25	0.24
723	A	8	6	1.	25	0.24
724	A	7	6	1.	25	0.24
725	A	6	6	1.	25	0.24
726	A	5	5	1.	25	0.2
727	A	6	6	1.	25	0.24
728	A	7	6	1.	25	0.24
729	A	8	6	1.	25	0.24
730	A	9	7	1.	27	0.259
731	A	8	7	1.	27	0.259
732	A	7	7	1.	27	0.259
733	A	6	6	1.	27	0.222
734	A	6	6	1.	27	0.222
735	A	7	7	1.	27	0.259
736	A	8	7	1.	27	0.259
737	A	10	8	1.	27	0.296
738	A	9	8	1.	27	0.296
739	A	8	8	1.	27	0.296
740	A	7	7	1.	27	0.259
741	A	7	7	1.	27	0.259
742	A	7	7	1.	27	0.259
743	A	8	8	1.	27	0.296
744	A	9	8	1.	27	0.296
745	A	9	9	1.	27	0.333
746	A	8	8	1.	27	0.296
747	A	5	5	1.	27	0.185
748	A	2	2	1.	27	0.074
749	A	7	7	1.	27	0.259
750	A	10	10	1.	27	0.37
751	A	10	10	1.	27	0.37
752	A	9	9	1.	27	0.333
753	A	9	9	1.	27	0.333
754	A	9	9	1.	27	0.333
755	A	9	9	1.	27	0.333

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
756	A	10	10	1.	27	0.37
757	A	11	10	1.	27	0.37
758	A	11	11	1.	27	0.407
759	A	10	10	1.	27	0.37
760	A	10	10	1.	27	0.37
761	A	10	10	1.	27	0.37
762	A	10	10	1.	27	0.37
763	A	10	10	1.	27	0.37
764	A	11	10	1.	27	0.37
765	A	8	8	1.	29	0.276
766	A	8	8	1.	29	0.276
767	A	7	7	1.	29	0.241
768	A	1	1	1.	29	0.034
769	A	3	3	1.	29	0.103
770	A	4	4	1.	29	0.138
771	A	9	8	1.	29	0.276
772	A	8	8	1.	29	0.276
773	A	7	7	1.	29	0.241
774	A	6	6	1.	29	0.207
775	A	5	5	1.	29	0.172
776	A	4	4	1.	29	0.138
777	A	10	8	1.	29	0.276
778	A	9	8	1.	29	0.276
779	A	8	8	1.	29	0.276
780	A	7	7	1.	29	0.241
781	A	7	7	1.	29	0.241
782	A	6	6	1.	29	0.207
783	A	7	7	1.	29	0.241
784	A	6	6	1.	29	0.207
785	A	1	1	1.	29	0.034
786	A	1	1	1.	29	0.034
787	A	3	3	1.	29	0.103
788	A	4	4	1.	29	0.138
789	A	7	7	1.	29	0.241
790	A	5	5	1.	29	0.172
791	A	3	3	1.	29	0.103
792	A	3	3	1.	29	0.103
793	A	4	4	1.	29	0.138
794	A	5	5	1.	29	0.172
795	A	6	6	1.	29	0.207
796	A	4	4	1.	29	0.138
797	A	4	4	1.	29	0.138
798	A	4	4	1.	29	0.138
799	A	5	5	1.	29	0.172

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
800	A	6	5	1.	29	0.172
801	A	0	0	0.	0	0.
802	A	8	5	1.	25	0.2
803	A	7	4	1.	23	0.174
804	A	3	3	1.	12	0.25
805	A	0	0	0.	0	0.
806	A	0	0	0.	0	0.
807	A	0	0	0.	0	0.
808	A	0	0	0.	0	0.
809	A	0	0	0.	0	0.
810	A	0	0	0.	0	0.
811	A	0	0	0.	0	0.
812	A	0	0	0.	0	0.
813	A	0	0	0.	0	0.
814	A	8	6	1.	23	0.261
815	A	7	5	1.	23	0.217
816	A	6	4	1.	21	0.19
817	A	7	5	1.	23	0.217
818	A	8	6	1.	23	0.261
819	A	5	5	1.	27	0.185
820	A	7	6	1.	27	0.222
821	A	5	4	1.	27	0.148
822	A	4	3	1.	25	0.12
823	A	5	4	1.	27	0.148
824	A	6	5	1.	27	0.185
825	A	8	6	1.	23	0.261
826	A	7	5	1.	23	0.217
827	A	6	4	1.	21	0.19
828	A	7	5	1.	23	0.217
829	A	10	5	1.	23	0.217
830	A	12	5	1.	23	0.217
831	A	0	0	0.	0	0.
832	A	6	5	0.94	27	0.185
833	A	5	4	0.96	27	0.148
834	A	4	3	1.	25	0.12
835	A	6	4	1.	27	0.148
836	A	11	5	1.	27	0.185
837	A	14	5	1.	27	0.185

Chapter 3

Listing of integrals

3.1 $\int \sin^3(e+fx)(a+a\sin(e+fx))^2\,dx$

Optimal. Leaf size=102

$$-\frac{a^2\cos^5(e+fx)}{5f}+\frac{a^2\cos^3(e+fx)}{f}-\frac{2a^2\cos(e+fx)}{f}-\frac{a^2\sin^3(e+fx)\cos(e+fx)}{2f}-\frac{3a^2\sin(e+fx)\cos(e+fx)}{4f}+$$

```
[Out] (3*a^2*x)/4 - (2*a^2*Cos[e + f*x])/f + (a^2*Cos[e + f*x]^3)/f - (a^2*Cos[e
+ f*x]^5)/(5*f) - (3*a^2*Cos[e + f*x]*Sin[e + f*x])/(4*f) - (a^2*Cos[e + f*
x]*Sin[e + f*x]^3)/(2*f)
```

Rubi [A] time = 0.101512, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2757, 2633, 2635, 8}

$$-\frac{a^2\cos^5(e+fx)}{5f}+\frac{a^2\cos^3(e+fx)}{f}-\frac{2a^2\cos(e+fx)}{f}-\frac{a^2\sin^3(e+fx)\cos(e+fx)}{2f}-\frac{3a^2\sin(e+fx)\cos(e+fx)}{4f}+$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^3*(a + a*Sin[e + f*x])^2,x]
```

```
[Out] (3*a^2*x)/4 - (2*a^2*Cos[e + f*x])/f + (a^2*Cos[e + f*x]^3)/f - (a^2*Cos[e
+ f*x]^5)/(5*f) - (3*a^2*Cos[e + f*x]*Sin[e + f*x])/(4*f) - (a^2*Cos[e + f*
x]*Sin[e + f*x]^3)/(2*f)
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \sin^3(e+fx)(a+a\sin(e+fx))^2\,dx = \int \left(a^2\sin^3(e+fx)+2a^2\sin^4(e+fx)+a^2\sin^5(e+fx)\right)dx$$

$$= a^2\int \sin^3(e+fx)\,dx + a^2\int \sin^5(e+fx)\,dx + \left(2a^2\right)\int \sin^4(e+fx)\,dx$$

$$= -\frac{a^2\cos(e+fx)\sin^3(e+fx)}{2f} + \frac{1}{2}\left(3a^2\right)\int \sin^2(e+fx)\,dx - \frac{a^2\,\mathrm{Subst}\left(\int\left(1-x^2\right.\right.}{}$$

$$= -\frac{2a^2\cos(e+fx)}{f} + \frac{a^2\cos^3(e+fx)}{f} - \frac{a^2\cos^5(e+fx)}{5f} - \frac{3a^2\cos(e+fx)\sin(e+}{4f}$$

$$= \frac{3a^2x}{4} - \frac{2a^2\cos(e+fx)}{f} + \frac{a^2\cos^3(e+fx)}{f} - \frac{a^2\cos^5(e+fx)}{5f} - \frac{3a^2\cos(e+fx)\,\mathrm{s}}{4f}$$

Mathematica [A] time = 0.428919, size = 105, normalized size = 1.03

$$-\frac{a^2\cos(e+fx)\left(30\sin^{-1}\left(\frac{\sqrt{1-\sin(e+fx)}}{\sqrt{2}}\right)+\left(4\sin^4(e+fx)+10\sin^3(e+fx)+12\sin^2(e+fx)+15\sin(e+fx)+24\right)\sqrt{\mathrm{co}}\right.}{20f\sqrt{\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^3*(a + a*Sin[e + f*x])^2,x]
```

```
[Out] -(a^2*Cos[e + f*x]*(30*ArcSin[Sqrt[1 - Sin[e + f*x]]/Sqrt[2]] + Sqrt[Cos[e
+ f*x]^2]*(24 + 15*Sin[e + f*x] + 12*Sin[e + f*x]^2 + 10*Sin[e + f*x]^3 + 4
*Sin[e + f*x]^4)))/(20*f*Sqrt[Cos[e + f*x]^2])
```

Maple [A] time = 0.031, size = 96, normalized size = 0.9

$$\frac{1}{f}\left(-\frac{a^2\cos\left(fx+e\right)}{5}\left(\frac{8}{3}+\left(\sin\left(fx+e\right)\right)^4+\frac{4\left(\sin\left(fx+e\right)\right)^2}{3}\right)+2a^2\left(-1/4\left(\left(\sin\left(fx+e\right)\right)^3+3/2\sin\left(fx+e\right)\right)\cos\left(fx\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3*(a+a*sin(f*x+e))^2,x)
```

```
[Out] 1/f*(-1/5*a^2*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)+2*a^2*(-1/4*(s
in(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-1/3*a^2*(2+sin(f*x+e)
^2)*cos(f*x+e))
```

Maxima [A] time = 1.72938, size = 128, normalized size = 1.25

$$-\frac{16\left(3\cos\left(fx+e\right)^{5}-10\cos\left(fx+e\right)^{3}+15\cos\left(fx+e\right)\right)a^{2}-80\left(\cos\left(fx+e\right)^{3}-3\cos\left(fx+e\right)\right)a^{2}-15\left(12fx+1\right.}{240f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+a*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] -1/240*(16*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*a^2 - 8
0*(cos(f*x + e)^3 - 3*cos(f*x + e))*a^2 - 15*(12*f*x + 12*e + sin(4*f*x + 4
*e) - 8*sin(2*f*x + 2*e))*a^2)/f
```

Fricas [A] time = 1.8163, size = 205, normalized size = 2.01

$$-\frac{4a^{2}\cos\left(fx+e\right)^{5}-20a^{2}\cos\left(fx+e\right)^{3}-15a^{2}fx+40a^{2}\cos\left(fx+e\right)-5\left(2a^{2}\cos\left(fx+e\right)^{3}-5a^{2}\cos\left(fx+e\right)\right)\sin}{20f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -1/20*(4*a^2*cos(f*x + e)^5 - 20*a^2*cos(f*x + e)^3 - 15*a^2*f*x + 40*a^2*c
os(f*x + e) - 5*(2*a^2*cos(f*x + e)^3 - 5*a^2*cos(f*x + e))*sin(f*x + e))/f
```

Sympy [A] time = 2.24003, size = 221, normalized size = 2.17

$$\begin{cases}\frac{3a^{2}x\sin^{4}\left(e+fx\right)}{4}+\frac{3a^{2}x\sin^{2}\left(e+fx\right)\cos^{2}\left(e+fx\right)}{2}+\frac{3a^{2}x\cos^{4}\left(e+fx\right)}{4}-\frac{a^{2}\sin^{4}\left(e+fx\right)\cos\left(e+fx\right)}{f}-\frac{5a^{2}\sin^{3}\left(e+fx\right)\cos\left(e+fx\right)}{4f}-\frac{4a^{2}\sin^{2}\left(e+fx\right)}{3f} \\ x\left(a\sin\left(e\right)+a\right)^{2}\sin^{3}\left(e\right)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3*(a+a*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((3*a**2*x*sin(e + f*x)**4/4 + 3*a**2*x*sin(e + f*x)**2*cos(e + f*
x)**2/2 + 3*a**2*x*cos(e + f*x)**4/4 - a**2*sin(e + f*x)**4*cos(e + f*x)/f
- 5*a**2*sin(e + f*x)**3*cos(e + f*x)/(4*f) - 4*a**2*sin(e + f*x)**2*cos(e
+ f*x)**3/(3*f) - a**2*sin(e + f*x)**2*cos(e + f*x)/f - 3*a**2*sin(e + f*x)
*cos(e + f*x)**3/(4*f) - 8*a**2*cos(e + f*x)**5/(15*f) - 2*a**2*cos(e + f*x
)**3/(3*f), Ne(f, 0)), (x*(a*sin(e) + a)**2*sin(e)**3, True))
```

Giac [A] time = 2.08549, size = 127, normalized size = 1.25

$$\frac{3}{4}a^{2}x-\frac{a^{2}\cos\left(5fx+5e\right)}{80f}+\frac{3a^{2}\cos\left(3fx+3e\right)}{16f}-\frac{11a^{2}\cos\left(fx+e\right)}{8f}+\frac{a^{2}\sin\left(4fx+4e\right)}{16f}-\frac{a^{2}\sin\left(2fx+2e\right)}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 3/4*a^2*x - 1/80*a^2*cos(5*f*x + 5*e)/f + 3/16*a^2*cos(3*f*x + 3*e)/f - 11/
8*a^2*cos(f*x + e)/f + 1/16*a^2*sin(4*f*x + 4*e)/f - 1/2*a^2*sin(2*f*x + 2*
e)/f
```

3.2 $\int \sin^3(e+fx)(a+a\sin(e+fx))^3\,dx$

Optimal. Leaf size=129

$$-\frac{3a^3\cos^5(e+fx)}{5f}+\frac{7a^3\cos^3(e+fx)}{3f}-\frac{4a^3\cos(e+fx)}{f}-\frac{a^3\sin^5(e+fx)\cos(e+fx)}{6f}-\frac{23a^3\sin^3(e+fx)\cos(e+f\ldots}{24f}$$

```
[Out] (23*a^3*x)/16 - (4*a^3*Cos[e + f*x])/f + (7*a^3*Cos[e + f*x]^3)/(3*f) - (3*
a^3*Cos[e + f*x]^5)/(5*f) - (23*a^3*Cos[e + f*x]*Sin[e + f*x])/(16*f) - (23
*a^3*Cos[e + f*x]*Sin[e + f*x]^3)/(24*f) - (a^3*Cos[e + f*x]*Sin[e + f*x]^5
)/(6*f)
```

Rubi [A] time = 0.144826, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2757, 2633, 2635, 8}

$$-\frac{3a^3\cos^5(e+fx)}{5f}+\frac{7a^3\cos^3(e+fx)}{3f}-\frac{4a^3\cos(e+fx)}{f}-\frac{a^3\sin^5(e+fx)\cos(e+fx)}{6f}-\frac{23a^3\sin^3(e+fx)\cos(e+f\ldots}{24f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^3*(a + a*Sin[e + f*x])^3,x]
```

```
[Out] (23*a^3*x)/16 - (4*a^3*Cos[e + f*x])/f + (7*a^3*Cos[e + f*x]^3)/(3*f) - (3*
a^3*Cos[e + f*x]^5)/(5*f) - (23*a^3*Cos[e + f*x]*Sin[e + f*x])/(16*f) - (23
*a^3*Cos[e + f*x]*Sin[e + f*x]^3)/(24*f) - (a^3*Cos[e + f*x]*Sin[e + f*x]^5
)/(6*f)
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \sin^3(e+fx)(a+a\sin(e+fx))^3\,dx = \int \left(a^3\sin^3(e+fx)+3a^3\sin^4(e+fx)+3a^3\sin^5(e+fx)+a^3\sin^6(e+fx)\right)dx$$
$$= a^3\int \sin^3(e+fx)\,dx + a^3\int \sin^6(e+fx)\,dx + \left(3a^3\right)\int \sin^4(e+fx)\,dx + \left(3a^3\right)$$
$$= -\frac{3a^3\cos(e+fx)\sin^3(e+fx)}{4f} - \frac{a^3\cos(e+fx)\sin^5(e+fx)}{6f} + \frac{1}{6}\left(5a^3\right)\int \sin^4(e$$
$$= -\frac{4a^3\cos(e+fx)}{f} + \frac{7a^3\cos^3(e+fx)}{3f} - \frac{3a^3\cos^5(e+fx)}{5f} - \frac{9a^3\cos(e+fx)\sin(e}{8f}$$
$$= \frac{9a^3x}{8} - \frac{4a^3\cos(e+fx)}{f} + \frac{7a^3\cos^3(e+fx)}{3f} - \frac{3a^3\cos^5(e+fx)}{5f} - \frac{23a^3\cos(e+f}{16}$$
$$= \frac{23a^3x}{16} - \frac{4a^3\cos(e+fx)}{f} + \frac{7a^3\cos^3(e+fx)}{3f} - \frac{3a^3\cos^5(e+fx)}{5f} - \frac{23a^3\cos(e+}{1}$$

Mathematica [A] time = 0.516368, size = 115, normalized size = 0.89

$$-\frac{a^3\cos(e+fx)\left(690\sin^{-1}\left(\frac{\sqrt{1-\sin(e+fx)}}{\sqrt{2}}\right)+\left(40\sin^5(e+fx)+144\sin^4(e+fx)+230\sin^3(e+fx)+272\sin^2(e+fx)+3\right.}{240f\sqrt{\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^3*(a + a*Sin[e + f*x])^3,x]
```

```
[Out] -(a^3*Cos[e + f*x]*(690*ArcSin[Sqrt[1 - Sin[e + f*x]]/Sqrt[2]] + Sqrt[Cos[e
 + f*x]^2]*(544 + 345*Sin[e + f*x] + 272*Sin[e + f*x]^2 + 230*Sin[e + f*x]^
3 + 144*Sin[e + f*x]^4 + 40*Sin[e + f*x]^5)))/(240*f*Sqrt[Cos[e + f*x]^2])
```

Maple [A] time = 0.034, size = 143, normalized size = 1.1

$$\frac{1}{f}\left(a^3\left(-\frac{\cos(fx+e)}{6}\left(\left(\sin(fx+e)\right)^5+\frac{5\left(\sin(fx+e)\right)^3}{4}+\frac{15\sin(fx+e)}{8}\right)+\frac{5fx}{16}+\frac{5e}{16}\right)-\frac{3a^3\cos(fx+e)}{5}\left(\frac{8}{3}+(\mathrm{si}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3*(a+a*sin(f*x+e))^3,x)
```

```
[Out] 1/f*(a^3*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+e)+5
/16*f*x+5/16*e)-3/5*a^3*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)+3*a^
3*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-1/3*a^3*(2+
sin(f*x+e)^2)*cos(f*x+e))
```

Maxima [A] time = 1.8007, size = 193, normalized size = 1.5

$$-\frac{192\left(3\cos(fx+e)^5-10\cos(fx+e)^3+15\cos(fx+e)\right)a^3-320\left(\cos(fx+e)^3-3\cos(fx+e)\right)a^3-5\left(4\sin(2fx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+a*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] -1/960*(192*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*a^3 -
320*(cos(f*x + e)^3 - 3*cos(f*x + e))*a^3 - 5*(4*sin(2*f*x + 2*e)^3 + 60*f*
x + 60*e + 9*sin(4*f*x + 4*e) - 48*sin(2*f*x + 2*e))*a^3 - 90*(12*f*x + 12*
e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*a^3)/f
```

Fricas [A] time = 1.75889, size = 248, normalized size = 1.92

$$-\frac{144\,a^3\cos\left(fx+e\right)^5 - 560\,a^3\cos\left(fx+e\right)^3 - 345\,a^3fx + 960\,a^3\cos\left(fx+e\right) + 5\left(8\,a^3\cos\left(fx+e\right)^5 - 62\,a^3\cos\left(fx\right.\right.}{240\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+a*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -1/240*(144*a^3*cos(f*x + e)^5 - 560*a^3*cos(f*x + e)^3 - 345*a^3*f*x + 960
*a^3*cos(f*x + e) + 5*(8*a^3*cos(f*x + e)^5 - 62*a^3*cos(f*x + e)^3 + 123*a
^3*cos(f*x + e))*sin(f*x + e))/f
```

Sympy [A] time = 4.45884, size = 379, normalized size = 2.94

$$\begin{cases} \frac{5a^3x\sin^6(e+fx)}{16} + \frac{15a^3x\sin^4(e+fx)\cos^2(e+fx)}{16} + \frac{9a^3x\sin^4(e+fx)}{8} + \frac{15a^3x\sin^2(e+fx)\cos^4(e+fx)}{16} + \frac{9a^3x\sin^2(e+fx)\cos^2(e+fx)}{4} + \frac{5a^3x\cos}{} \\ x\left(a\sin(e)+a\right)^3\sin^3(e) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3*(a+a*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((5*a**3*x*sin(e + f*x)**6/16 + 15*a**3*x*sin(e + f*x)**4*cos(e +
f*x)**2/16 + 9*a**3*x*sin(e + f*x)**4/8 + 15*a**3*x*sin(e + f*x)**2*cos(e +
 f*x)**4/16 + 9*a**3*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 5*a**3*x*cos(e +
 f*x)**6/16 + 9*a**3*x*cos(e + f*x)**4/8 - 11*a**3*sin(e + f*x)**5*cos(e +
f*x)/(16*f) - 3*a**3*sin(e + f*x)**4*cos(e + f*x)/f - 5*a**3*sin(e + f*x)**
3*cos(e + f*x)**3/(6*f) - 15*a**3*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 4*a*
*3*sin(e + f*x)**2*cos(e + f*x)**3/f - a**3*sin(e + f*x)**2*cos(e + f*x)/f
- 5*a**3*sin(e + f*x)*cos(e + f*x)**5/(16*f) - 9*a**3*sin(e + f*x)*cos(e +
f*x)**3/(8*f) - 8*a**3*cos(e + f*x)**5/(5*f) - 2*a**3*cos(e + f*x)**3/(3*f)
, Ne(f, 0)), (x*(a*sin(e) + a)**3*sin(e)**3, True))
```

Giac [A] time = 2.10458, size = 151, normalized size = 1.17

$$\frac{23}{16}\,a^3x - \frac{3\,a^3\cos\left(5\,fx+5\,e\right)}{80\,f} + \frac{19\,a^3\cos\left(3\,fx+3\,e\right)}{48\,f} - \frac{21\,a^3\cos\left(fx+e\right)}{8\,f} - \frac{a^3\sin\left(6\,fx+6\,e\right)}{192\,f} + \frac{9\,a^3\sin\left(4\,fx+4\,e\right.}{64\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+a*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 23/16*a^3*x - 3/80*a^3*cos(5*f*x + 5*e)/f + 19/48*a^3*cos(3*f*x + 3*e)/f -
21/8*a^3*cos(f*x + e)/f - 1/192*a^3*sin(6*f*x + 6*e)/f + 9/64*a^3*sin(4*f*x
 + 4*e)/f - 63/64*a^3*sin(2*f*x + 2*e)/f
```

3.3 $\int \frac{\sin^4(x)}{a+a\sin(x)}\,dx$

Optimal. Leaf size=53

$$-\frac{3x}{2a}+\frac{4\cos^3(x)}{3a}-\frac{4\cos(x)}{a}+\frac{\sin^3(x)\cos(x)}{a\sin(x)+a}+\frac{3\sin(x)\cos(x)}{2a}$$

```
[Out] (-3*x)/(2*a) - (4*Cos[x])/a + (4*Cos[x]^3)/(3*a) + (3*Cos[x]*Sin[x])/(2*a)
+ (Cos[x]*Sin[x]^3)/(a + a*Sin[x])
```

Rubi [A] time = 0.069486, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2767, 2748, 2635, 8, 2633}

$$-\frac{3x}{2a}+\frac{4\cos^3(x)}{3a}-\frac{4\cos(x)}{a}+\frac{\sin^3(x)\cos(x)}{a\sin(x)+a}+\frac{3\sin(x)\cos(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^4/(a + a*Sin[x]),x]
```

```
[Out] (-3*x)/(2*a) - (4*Cos[x])/a + (4*Cos[x]^3)/(3*a) + (3*Cos[x]*Sin[x])/(2*a)
+ (Cos[x]*Sin[x]^3)/(a + a*Sin[x])
```

Rule 2767

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e + f
*x])^(n - 1))/(a*f*(a + b*Sin[e + f*x])), x] - Dist[d/(a*b), Int[(c + d*Sin
[e + f*x])^(n - 2)*Simp[b*d*(n - 1) - a*c*n + (b*c*(n - 1) - a*d*n)*Sin[e +
 f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && E
qQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && (IntegerQ[2*n] || EqQ
[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
```

```
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^4(x)}{a+a\sin(x)}\,dx &= \frac{\cos(x)\sin^3(x)}{a+a\sin(x)}-\frac{\int \sin^2(x)(3a-4a\sin(x))\,dx}{a^2}\\ &= \frac{\cos(x)\sin^3(x)}{a+a\sin(x)}-\frac{3\int \sin^2(x)\,dx}{a}+\frac{4\int \sin^3(x)\,dx}{a}\\ &= \frac{3\cos(x)\sin(x)}{2a}+\frac{\cos(x)\sin^3(x)}{a+a\sin(x)}-\frac{3\int 1\,dx}{2a}-\frac{4\,\mathrm{Subst}\left(\int \left(1-x^2\right)\,dx,x,\cos(x)\right)}{a}\\ &= -\frac{3x}{2a}-\frac{4\cos(x)}{a}+\frac{4\cos^3(x)}{3a}+\frac{3\cos(x)\sin(x)}{2a}+\frac{\cos(x)\sin^3(x)}{a+a\sin(x)}\end{aligned}$$

Mathematica [A] time = 0.115781, size = 101, normalized size = 1.91

$$\frac{\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)\left(-36x\sin\left(\frac{x}{2}\right)+69\sin\left(\frac{x}{2}\right)-18\sin\left(\frac{3x}{2}\right)+2\sin\left(\frac{5x}{2}\right)+\sin\left(\frac{7x}{2}\right)-3(12x+7)\cos\left(\frac{x}{2}\right)-18\cos\left(\frac{3x}{2}\right)\right.}{24a(\sin(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^4/(a + a*Sin[x]),x]
```

```
[Out] ((Cos[x/2] + Sin[x/2])*(-3*(7 + 12*x)*Cos[x/2] - 18*Cos[(3*x)/2] - 2*Cos[(5
*x)/2] + Cos[(7*x)/2] + 69*Sin[x/2] - 36*x*Sin[x/2] - 18*Sin[(3*x)/2] + 2*S
in[(5*x)/2] + Sin[(7*x)/2]))/(24*a*(1 + Sin[x]))
```

Maple [B] time = 0.026, size = 121, normalized size = 2.3

$$-\frac{1}{a}\left(\tan\left(\frac{x}{2}\right)\right)^5\left(\left(\tan\left(\frac{x}{2}\right)\right)^2+1\right)^{-3}-2\frac{(\tan(x/2))^4}{a\left((\tan(x/2))^2+1\right)^3}-8\frac{(\tan(x/2))^2}{a\left((\tan(x/2))^2+1\right)^3}+\frac{1}{a}\tan\left(\frac{x}{2}\right)\left(\left(\tan\left(\frac{x}{2}\right)\right)^2+1\right)^{-3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^4/(a+a*sin(x)),x)
```

```
[Out] -1/a/(tan(1/2*x)^2+1)^3*tan(1/2*x)^5-2/a/(tan(1/2*x)^2+1)^3*tan(1/2*x)^4-8/
a/(tan(1/2*x)^2+1)^3*tan(1/2*x)^2+1/a/(tan(1/2*x)^2+1)^3*tan(1/2*x)-10/3/a/
(tan(1/2*x)^2+1)^3-3/a*arctan(tan(1/2*x))-2/a/(tan(1/2*x)+1)
```

Maxima [B] time = 2.56916, size = 243, normalized size = 4.58

$$-\frac{\frac{7\sin(x)}{\cos(x)+1}+\frac{39\sin(x)^2}{(\cos(x)+1)^2}+\frac{24\sin(x)^3}{(\cos(x)+1)^3}+\frac{24\sin(x)^4}{(\cos(x)+1)^4}+\frac{9\sin(x)^5}{(\cos(x)+1)^5}+\frac{9\sin(x)^6}{(\cos(x)+1)^6}+16}{3\left(a+\frac{a\sin(x)}{\cos(x)+1}+\frac{3a\sin(x)^2}{(\cos(x)+1)^2}+\frac{3a\sin(x)^3}{(\cos(x)+1)^3}+\frac{3a\sin(x)^4}{(\cos(x)+1)^4}+\frac{3a\sin(x)^5}{(\cos(x)+1)^5}+\frac{a\sin(x)^6}{(\cos(x)+1)^6}+\frac{a\sin(x)^7}{(\cos(x)+1)^7}\right)}-\frac{3\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^4/(a+a*sin(x)),x, algorithm="maxima")
```

```
[Out] -1/3*(7*sin(x)/(cos(x) + 1) + 39*sin(x)^2/(cos(x) + 1)^2 + 24*sin(x)^3/(cos
(x) + 1)^3 + 24*sin(x)^4/(cos(x) + 1)^4 + 9*sin(x)^5/(cos(x) + 1)^5 + 9*sin
(x)^6/(cos(x) + 1)^6 + 16)/(a + a*sin(x)/(cos(x) + 1) + 3*a*sin(x)^2/(cos(x
) + 1)^2 + 3*a*sin(x)^3/(cos(x) + 1)^3 + 3*a*sin(x)^4/(cos(x) + 1)^4 + 3*a*
sin(x)^5/(cos(x) + 1)^5 + a*sin(x)^6/(cos(x) + 1)^6 + a*sin(x)^7/(cos(x) +
1)^7) - 3*arctan(sin(x)/(cos(x) + 1))/a
```

Fricas [A] time = 1.75444, size = 211, normalized size = 3.98

$$\frac{2\cos(x)^4 - \cos(x)^3 - 3(3x+5)\cos(x) - 12\cos(x)^2 + \left(2\cos(x)^3 + 3\cos(x)^2 - 9x - 9\cos(x) + 6\right)\sin(x) - 9x - 6}{6\left(a\cos(x) + a\sin(x) + a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^4/(a+a*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*cos(x)^4 - cos(x)^3 - 3*(3*x + 5)*cos(x) - 12*cos(x)^2 + (2*cos(x)^3
 + 3*cos(x)^2 - 9*x - 9*cos(x) + 6)*sin(x) - 9*x - 6)/(a*cos(x) + a*sin(x)
+ a)
```

Sympy [B] time = 37.374, size = 1300, normalized size = 24.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**4/(a+a*sin(x)),x)
```

```
[Out] -18*x*tan(x/2)**7/(12*a*tan(x/2)**7 + 12*a*tan(x/2)**6 + 36*a*tan(x/2)**5 +
 36*a*tan(x/2)**4 + 36*a*tan(x/2)**3 + 36*a*tan(x/2)**2 + 12*a*tan(x/2) + 1
2*a) - 18*x*tan(x/2)**6/(12*a*tan(x/2)**7 + 12*a*tan(x/2)**6 + 36*a*tan(x/2
)**5 + 36*a*tan(x/2)**4 + 36*a*tan(x/2)**3 + 36*a*tan(x/2)**2 + 12*a*tan(x/
2) + 12*a) - 54*x*tan(x/2)**5/(12*a*tan(x/2)**7 + 12*a*tan(x/2)**6 + 36*a*t
an(x/2)**5 + 36*a*tan(x/2)**4 + 36*a*tan(x/2)**3 + 36*a*tan(x/2)**2 + 12*a*
tan(x/2) + 12*a) - 54*x*tan(x/2)**4/(12*a*tan(x/2)**7 + 12*a*tan(x/2)**6 +
36*a*tan(x/2)**5 + 36*a*tan(x/2)**4 + 36*a*tan(x/2)**3 + 36*a*tan(x/2)**2 +
 12*a*tan(x/2) + 12*a) - 54*x*tan(x/2)**3/(12*a*tan(x/2)**7 + 12*a*tan(x/2)
**6 + 36*a*tan(x/2)**5 + 36*a*tan(x/2)**4 + 36*a*tan(x/2)**3 + 36*a*tan(x/2
)**2 + 12*a*tan(x/2) + 12*a) - 54*x*tan(x/2)**2/(12*a*tan(x/2)**7 + 12*a*ta
n(x/2)**6 + 36*a*tan(x/2)**5 + 36*a*tan(x/2)**4 + 36*a*tan(x/2)**3 + 36*a*t
an(x/2)**2 + 12*a*tan(x/2) + 12*a) - 18*x*tan(x/2)/(12*a*tan(x/2)**7 + 12*a
*tan(x/2)**6 + 36*a*tan(x/2)**5 + 36*a*tan(x/2)**4 + 36*a*tan(x/2)**3 + 36*
a*tan(x/2)**2 + 12*a*tan(x/2) + 12*a) - 18*x/(12*a*tan(x/2)**7 + 12*a*tan(x
/2)**6 + 36*a*tan(x/2)**5 + 36*a*tan(x/2)**4 + 36*a*tan(x/2)**3 + 36*a*tan(
x/2)**2 + 12*a*tan(x/2) + 12*a) + 55*tan(x/2)**7/(12*a*tan(x/2)**7 + 12*a*t
an(x/2)**6 + 36*a*tan(x/2)**5 + 36*a*tan(x/2)**4 + 36*a*tan(x/2)**3 + 36*a*
tan(x/2)**2 + 12*a*tan(x/2) + 12*a) + 19*tan(x/2)**6/(12*a*tan(x/2)**7 + 12
*a*tan(x/2)**6 + 36*a*tan(x/2)**5 + 36*a*tan(x/2)**4 + 36*a*tan(x/2)**3 + 3
6*a*tan(x/2)**2 + 12*a*tan(x/2) + 12*a) + 129*tan(x/2)**5/(12*a*tan(x/2)**7
 + 12*a*tan(x/2)**6 + 36*a*tan(x/2)**5 + 36*a*tan(x/2)**4 + 36*a*tan(x/2)**
3 + 36*a*tan(x/2)**2 + 12*a*tan(x/2) + 12*a) + 69*tan(x/2)**4/(12*a*tan(x/2
)**7 + 12*a*tan(x/2)**6 + 36*a*tan(x/2)**5 + 36*a*tan(x/2)**4 + 36*a*tan(x/
2)**3 + 36*a*tan(x/2)**2 + 12*a*tan(x/2) + 12*a) + 69*tan(x/2)**3/(12*a*tan
(x/2)**7 + 12*a*tan(x/2)**6 + 36*a*tan(x/2)**5 + 36*a*tan(x/2)**4 + 36*a*ta
```

```
n(x/2)**3 + 36*a*tan(x/2)**2 + 12*a*tan(x/2) + 12*a) + 9*tan(x/2)**2/(12*a*
tan(x/2)**7 + 12*a*tan(x/2)**6 + 36*a*tan(x/2)**5 + 36*a*tan(x/2)**4 + 36*a
*tan(x/2)**3 + 36*a*tan(x/2)**2 + 12*a*tan(x/2) + 12*a) + 27*tan(x/2)/(12*a
*tan(x/2)**7 + 12*a*tan(x/2)**6 + 36*a*tan(x/2)**5 + 36*a*tan(x/2)**4 + 36*
a*tan(x/2)**3 + 36*a*tan(x/2)**2 + 12*a*tan(x/2) + 12*a) - 9/(12*a*tan(x/2)
**7 + 12*a*tan(x/2)**6 + 36*a*tan(x/2)**5 + 36*a*tan(x/2)**4 + 36*a*tan(x/2
)**3 + 36*a*tan(x/2)**2 + 12*a*tan(x/2) + 12*a)
```

Giac [A] time = 2.21867, size = 90, normalized size = 1.7

$$-\frac{3x}{2a}-\frac{2}{a\left(\tan\left(\frac{1}{2}x\right)+1\right)}-\frac{3\tan\left(\frac{1}{2}x\right)^{5}+6\tan\left(\frac{1}{2}x\right)^{4}+24\tan\left(\frac{1}{2}x\right)^{2}-3\tan\left(\frac{1}{2}x\right)+10}{3\left(\tan\left(\frac{1}{2}x\right)^{2}+1\right)^{3}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^4/(a+a*sin(x)),x, algorithm="giac")
```

```
[Out] -3/2*x/a - 2/(a*(tan(1/2*x) + 1)) - 1/3*(3*tan(1/2*x)^5 + 6*tan(1/2*x)^4 +
24*tan(1/2*x)^2 - 3*tan(1/2*x) + 10)/((tan(1/2*x)^2 + 1)^3*a)
```

3.4 $\int \frac{\sin^3(x)}{a+a\sin(x)}\,dx$

Optimal. Leaf size=42

$$\frac{3x}{2a}+\frac{2\cos(x)}{a}+\frac{\sin^2(x)\cos(x)}{a\sin(x)+a}-\frac{3\sin(x)\cos(x)}{2a}$$

```
[Out] (3*x)/(2*a) + (2*Cos[x])/a - (3*Cos[x]*Sin[x])/(2*a) + (Cos[x]*Sin[x]^2)/(a
 + a*Sin[x])
```

Rubi [A] time = 0.048874, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2767, 2734}

$$\frac{3x}{2a}+\frac{2\cos(x)}{a}+\frac{\sin^2(x)\cos(x)}{a\sin(x)+a}-\frac{3\sin(x)\cos(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^3/(a + a*Sin[x]),x]
```


```
[Out] (3*x)/(2*a) + (2*Cos[x])/a - (3*Cos[x]*Sin[x])/(2*a) + (Cos[x]*Sin[x]^2)/(a
 + a*Sin[x])
```


Rule 2767

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e + f
*x])^(n - 1))/(a*f*(a + b*Sin[e + f*x])), x] - Dist[d/(a*b), Int[(c + d*Sin
[e + f*x])^(n - 2)*Simp[b*d*(n - 1) - a*c*n + (b*c*(n - 1) - a*d*n)*Sin[e +
 f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && E
qQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && (IntegerQ[2*n] || EqQ
[c, 0])
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^3(x)}{a+a\sin(x)}\,dx &= \frac{\cos(x)\sin^2(x)}{a+a\sin(x)}-\frac{\int \sin(x)(2a-3a\sin(x))\,dx}{a^2}\\ &= \frac{3x}{2a}+\frac{2\cos(x)}{a}-\frac{3\cos(x)\sin(x)}{2a}+\frac{\cos(x)\sin^2(x)}{a+a\sin(x)}\end{aligned}$$

Mathematica [B] time = 0.0792168, size = 87, normalized size = 2.07

$$\frac{\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)\left(12x\sin\left(\frac{x}{2}\right)-20\sin\left(\frac{x}{2}\right)+3\sin\left(\frac{3x}{2}\right)-\sin\left(\frac{5x}{2}\right)+4(3x+1)\cos\left(\frac{x}{2}\right)+3\cos\left(\frac{3x}{2}\right)+\cos\left(\frac{5x}{2}\right)\right)}{8a(\sin(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^3/(a + a*Sin[x]),x]
```

```
[Out] ((Cos[x/2] + Sin[x/2])*(4*(1 + 3*x)*Cos[x/2] + 3*Cos[(3*x)/2] + Cos[(5*x)/2
] - 20*Sin[x/2] + 12*x*Sin[x/2] + 3*Sin[(3*x)/2] - Sin[(5*x)/2]))/(8*a*(1 +
 Sin[x]))
```

Maple [B] time = 0.026, size = 100, normalized size = 2.4

$$\frac{1}{a}\left(\tan\left(\frac{x}{2}\right)\right)^{3}\left(\left(\tan\left(\frac{x}{2}\right)\right)^{2}+1\right)^{-2}+2\,\frac{(\tan(x/2))^{2}}{a\left((\tan(x/2))^{2}+1\right)^{2}}-\frac{1}{a}\tan\left(\frac{x}{2}\right)\left(\left(\tan\left(\frac{x}{2}\right)\right)^{2}+1\right)^{-2}+2\,\frac{1}{a\left((\tan(x/2))^{2}+1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3/(a+a*sin(x)),x)
```

```
[Out] 1/a/(tan(1/2*x)^2+1)^2*tan(1/2*x)^3+2/a/(tan(1/2*x)^2+1)^2*tan(1/2*x)^2-1/a
/(tan(1/2*x)^2+1)^2*tan(1/2*x)+2/a/(tan(1/2*x)^2+1)^2+3/a*arctan(tan(1/2*x)
)+2/a/(tan(1/2*x)+1)
```

Maxima [B] time = 2.5637, size = 173, normalized size = 4.12

$$\frac{\frac{\sin(x)}{\cos(x)+1}+\frac{5\,\sin(x)^{2}}{(\cos(x)+1)^{2}}+\frac{3\,\sin(x)^{3}}{(\cos(x)+1)^{3}}+\frac{3\,\sin(x)^{4}}{(\cos(x)+1)^{4}}+4}{a+\frac{a\sin(x)}{\cos(x)+1}+\frac{2\,a\sin(x)^{2}}{(\cos(x)+1)^{2}}+\frac{2\,a\sin(x)^{3}}{(\cos(x)+1)^{3}}+\frac{a\sin(x)^{4}}{(\cos(x)+1)^{4}}+\frac{a\sin(x)^{5}}{(\cos(x)+1)^{5}}}+\frac{3\,\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(a+a*sin(x)),x, algorithm="maxima")
```

```
[Out] (sin(x)/(cos(x) + 1) + 5*sin(x)^2/(cos(x) + 1)^2 + 3*sin(x)^3/(cos(x) + 1)^
3 + 3*sin(x)^4/(cos(x) + 1)^4 + 4)/(a + a*sin(x)/(cos(x) + 1) + 2*a*sin(x)^
2/(cos(x) + 1)^2 + 2*a*sin(x)^3/(cos(x) + 1)^3 + a*sin(x)^4/(cos(x) + 1)^4
+ a*sin(x)^5/(cos(x) + 1)^5) + 3*arctan(sin(x)/(cos(x) + 1))/a
```

Fricas [A] time = 1.70878, size = 166, normalized size = 3.95

$$\frac{\cos(x)^{3}+3\,(x+1)\cos(x)+2\,\cos(x)^{2}-\left(\cos(x)^{2}-3\,x-\cos(x)+2\right)\sin(x)+3\,x+2}{2\,(a\cos(x)+a\sin(x)+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(a+a*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(cos(x)^3 + 3*(x + 1)*cos(x) + 2*cos(x)^2 - (cos(x)^2 - 3*x - cos(x) +
2)*sin(x) + 3*x + 2)/(a*cos(x) + a*sin(x) + a)
```

Sympy [B] time = 3.26524, size = 665, normalized size = 15.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**3/(a+a*sin(x)),x)
```

```
[Out] 3*x*tan(x/2)**5/(2*a*tan(x/2)**5 + 2*a*tan(x/2)**4 + 4*a*tan(x/2)**3 + 4*a*
tan(x/2)**2 + 2*a*tan(x/2) + 2*a) + 3*x*tan(x/2)**4/(2*a*tan(x/2)**5 + 2*a*
tan(x/2)**4 + 4*a*tan(x/2)**3 + 4*a*tan(x/2)**2 + 2*a*tan(x/2) + 2*a) + 6*x
*tan(x/2)**3/(2*a*tan(x/2)**5 + 2*a*tan(x/2)**4 + 4*a*tan(x/2)**3 + 4*a*tan
(x/2)**2 + 2*a*tan(x/2) + 2*a) + 6*x*tan(x/2)**2/(2*a*tan(x/2)**5 + 2*a*tan
(x/2)**4 + 4*a*tan(x/2)**3 + 4*a*tan(x/2)**2 + 2*a*tan(x/2) + 2*a) + 3*x*ta
n(x/2)/(2*a*tan(x/2)**5 + 2*a*tan(x/2)**4 + 4*a*tan(x/2)**3 + 4*a*tan(x/2)*
*2 + 2*a*tan(x/2) + 2*a) + 3*x/(2*a*tan(x/2)**5 + 2*a*tan(x/2)**4 + 4*a*tan
(x/2)**3 + 4*a*tan(x/2)**2 + 2*a*tan(x/2) + 2*a) - 6*tan(x/2)**5/(2*a*tan(x
/2)**5 + 2*a*tan(x/2)**4 + 4*a*tan(x/2)**3 + 4*a*tan(x/2)**2 + 2*a*tan(x/2)
 + 2*a) - 6*tan(x/2)**3/(2*a*tan(x/2)**5 + 2*a*tan(x/2)**4 + 4*a*tan(x/2)**
3 + 4*a*tan(x/2)**2 + 2*a*tan(x/2) + 2*a) - 2*tan(x/2)**2/(2*a*tan(x/2)**5
+ 2*a*tan(x/2)**4 + 4*a*tan(x/2)**3 + 4*a*tan(x/2)**2 + 2*a*tan(x/2) + 2*a)
 - 4*tan(x/2)/(2*a*tan(x/2)**5 + 2*a*tan(x/2)**4 + 4*a*tan(x/2)**3 + 4*a*ta
n(x/2)**2 + 2*a*tan(x/2) + 2*a) + 2/(2*a*tan(x/2)**5 + 2*a*tan(x/2)**4 + 4*
a*tan(x/2)**3 + 4*a*tan(x/2)**2 + 2*a*tan(x/2) + 2*a)
```

Giac [A] time = 1.80424, size = 76, normalized size = 1.81

$$\frac{3\,x}{2\,a}+\frac{\tan\left(\frac{1}{2}\,x\right)^{3}+2\,\tan\left(\frac{1}{2}\,x\right)^{2}-\tan\left(\frac{1}{2}\,x\right)+2}{\left(\tan\left(\frac{1}{2}\,x\right)^{2}+1\right)^{2}a}+\frac{2}{a\left(\tan\left(\frac{1}{2}\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(a+a*sin(x)),x, algorithm="giac")
```

```
[Out] 3/2*x/a + (tan(1/2*x)^3 + 2*tan(1/2*x)^2 - tan(1/2*x) + 2)/((tan(1/2*x)^2 +
 1)^2*a) + 2/(a*(tan(1/2*x) + 1))
```

3.5 $\int \frac{\sin^2(x)}{a+a\sin(x)}\,dx$

Optimal. Leaf size=27

$$-\frac{x}{a}-\frac{\cos(x)}{a}-\frac{\cos(x)}{a(\sin(x)+1)}$$

```
[Out] -(x/a) - Cos[x]/a - Cos[x]/(a*(1 + Sin[x]))
```

Rubi [A] time = 0.0664069, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2746, 12, 2735, 2648}

$$-\frac{x}{a}-\frac{\cos(x)}{a}-\frac{\cos(x)}{a(\sin(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^2/(a + a*Sin[x]),x]
```

```
[Out] -(x/a) - Cos[x]/a - Cos[x]/(a*(1 + Sin[x]))
```

Rule 2746

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2/((c_.) + (d_.)*sin[(e_.) + (f
_.)*(x_)]), x_Symbol] :> -Simp[(b^2*Cos[e + f*x])/(d*f), x] + Dist[1/d, Int
[Simp[a^2*d - b*(b*c - 2*a*d)*Sin[e + f*x], x]/(c + d*Sin[e + f*x]), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{\sin^2(x)}{a+a\sin(x)}\,dx = -\frac{\cos(x)}{a} - \frac{\int \frac{a\sin(x)}{a+a\sin(x)}\,dx}{a}$$
$$= -\frac{\cos(x)}{a} - \int \frac{\sin(x)}{a+a\sin(x)}\,dx$$
$$= -\frac{x}{a} - \frac{\cos(x)}{a} + \int \frac{1}{a+a\sin(x)}\,dx$$
$$= -\frac{x}{a} - \frac{\cos(x)}{a} - \frac{\cos(x)}{a+a\sin(x)}$$

Mathematica [A] time = 0.0606443, size = 48, normalized size = 1.78

$$-\frac{\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)\left(\cos\left(\frac{x}{2}\right)(x+\cos(x))+\sin\left(\frac{x}{2}\right)(x+\cos(x)-2)\right)}{a(\sin(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^2/(a + a*Sin[x]),x]
```

```
[Out] -(((Cos[x/2] + Sin[x/2])*(Cos[x/2]*(x + Cos[x]) + (-2 + x + Cos[x])*Sin[x/2
]))/(a*(1 + Sin[x])))
```

Maple [A] time = 0.024, size = 40, normalized size = 1.5

$$-2\,\frac{1}{a\left((\tan(x/2))^2+1\right)}-2\,\frac{\arctan(\tan(x/2))}{a}-2\,\frac{1}{a(\tan(x/2)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2/(a+a*sin(x)),x)
```

```
[Out] -2/a/(tan(1/2*x)^2+1)-2/a*arctan(tan(1/2*x))-2/a/(tan(1/2*x)+1)
```

Maxima [B] time = 2.5776, size = 105, normalized size = 3.89

$$-\frac{2\left(\frac{\sin(x)}{\cos(x)+1}+\frac{\sin(x)^2}{(\cos(x)+1)^2}+2\right)}{a+\frac{a\sin(x)}{\cos(x)+1}+\frac{a\sin(x)^2}{(\cos(x)+1)^2}+\frac{a\sin(x)^3}{(\cos(x)+1)^3}}-\frac{2\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a+a*sin(x)),x, algorithm="maxima")
```

```
[Out] -2*(sin(x)/(cos(x) + 1) + sin(x)^2/(cos(x) + 1)^2 + 2)/(a + a*sin(x)/(cos(x
) + 1) + a*sin(x)^2/(cos(x) + 1)^2 + a*sin(x)^3/(cos(x) + 1)^3) - 2*arctan(
sin(x)/(cos(x) + 1))/a
```

Fricas [A] time = 1.72242, size = 122, normalized size = 4.52

$$-\frac{(x+2)\cos(x)+\cos(x)^2+(x+\cos(x)-1)\sin(x)+x+1}{a\cos(x)+a\sin(x)+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a+a*sin(x)),x, algorithm="fricas")
```

```
[Out] -((x + 2)*cos(x) + cos(x)^2 + (x + cos(x) - 1)*sin(x) + x + 1)/(a*cos(x) +
a*sin(x) + a)
```

Sympy [B] time = 1.45736, size = 250, normalized size = 9.26

$$-\frac{x\tan^3\left(\frac{x}{2}\right)}{a\tan^3\left(\frac{x}{2}\right)+a\tan^2\left(\frac{x}{2}\right)+a\tan\left(\frac{x}{2}\right)+a}-\frac{x\tan^2\left(\frac{x}{2}\right)}{a\tan^3\left(\frac{x}{2}\right)+a\tan^2\left(\frac{x}{2}\right)+a\tan\left(\frac{x}{2}\right)+a}-\frac{x\tan\left(\frac{x}{2}\right)}{a\tan^3\left(\frac{x}{2}\right)+a\tan^2\left(\frac{x}{2}\right)+a\tan}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**2/(a+a*sin(x)),x)
```

```
[Out] -x*tan(x/2)**3/(a*tan(x/2)**3 + a*tan(x/2)**2 + a*tan(x/2) + a) - x*tan(x/2
)**2/(a*tan(x/2)**3 + a*tan(x/2)**2 + a*tan(x/2) + a) - x*tan(x/2)/(a*tan(x
/2)**3 + a*tan(x/2)**2 + a*tan(x/2) + a) - x/(a*tan(x/2)**3 + a*tan(x/2)**2
 + a*tan(x/2) + a) + 3*tan(x/2)**3/(a*tan(x/2)**3 + a*tan(x/2)**2 + a*tan(x
/2) + a) + tan(x/2)**2/(a*tan(x/2)**3 + a*tan(x/2)**2 + a*tan(x/2) + a) + t
an(x/2)/(a*tan(x/2)**3 + a*tan(x/2)**2 + a*tan(x/2) + a) - 1/(a*tan(x/2)**3
 + a*tan(x/2)**2 + a*tan(x/2) + a)
```

Giac [A] time = 1.89314, size = 59, normalized size = 2.19

$$-\frac{x}{a}-\frac{2\left(\tan\left(\frac{1}{2}x\right)^2+\tan\left(\frac{1}{2}x\right)+2\right)}{\left(\tan\left(\frac{1}{2}x\right)^3+\tan\left(\frac{1}{2}x\right)^2+\tan\left(\frac{1}{2}x\right)+1\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a+a*sin(x)),x, algorithm="giac")
```

```
[Out] -x/a - 2*(tan(1/2*x)^2 + tan(1/2*x) + 2)/((tan(1/2*x)^3 + tan(1/2*x)^2 + ta
n(1/2*x) + 1)*a)
```

3.6 $\int \frac{\sin(x)}{a+a\sin(x)}\,dx$

Optimal. Leaf size=17

$$\frac{x}{a}+\frac{\cos(x)}{a\sin(x)+a}$$

```
[Out] x/a + Cos[x]/(a + a*Sin[x])
```

Rubi [A] time = 0.0285508, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2735, 2648}

$$\frac{x}{a}+\frac{\cos(x)}{a\sin(x)+a}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]/(a + a*Sin[x]),x]
```

```
[Out] x/a + Cos[x]/(a + a*Sin[x])
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(x)}{a+a\sin(x)}\,dx &= \frac{x}{a}-\int \frac{1}{a+a\sin(x)}\,dx\\ &= \frac{x}{a}+\frac{\cos(x)}{a+a\sin(x)}\end{aligned}$$

Mathematica [B] time = 0.0366239, size = 42, normalized size = 2.47

$$\frac{\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)\left((x-2)\sin\left(\frac{x}{2}\right)+x\cos\left(\frac{x}{2}\right)\right)}{a(\sin(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/(a + a*Sin[x]),x]
```

```
[Out] ((Cos[x/2] + Sin[x/2])*(x*Cos[x/2] + (-2 + x)*Sin[x/2]))/(a*(1 + Sin[x]))
```

Maple [A] time = 0.022, size = 25, normalized size = 1.5

$$2\,\frac{\arctan\left(\tan\left(x/2\right)\right)}{a}+2\,\frac{1}{a\left(\tan\left(x/2\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(a+a*sin(x)),x)
```

```
[Out] 2/a*arctan(tan(1/2*x))+2/a/(tan(1/2*x)+1)
```

Maxima [A] time = 2.36792, size = 43, normalized size = 2.53

$$\frac{2\,\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a}+\frac{2}{a+\frac{a\sin(x)}{\cos(x)+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+a*sin(x)),x, algorithm="maxima")
```

```
[Out] 2*arctan(sin(x)/(cos(x) + 1))/a + 2/(a + a*sin(x)/(cos(x) + 1))
```

Fricas [A] time = 1.57898, size = 93, normalized size = 5.47

$$\frac{(x+1)\cos\left(x\right)+(x-1)\sin\left(x\right)+x+1}{a\cos\left(x\right)+a\sin\left(x\right)+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+a*sin(x)),x, algorithm="fricas")
```

```
[Out] ((x + 1)*cos(x) + (x - 1)*sin(x) + x + 1)/(a*cos(x) + a*sin(x) + a)
```

Sympy [B] time = 0.631777, size = 39, normalized size = 2.29

$$\frac{x\tan\left(\frac{x}{2}\right)}{a\tan\left(\frac{x}{2}\right)+a}+\frac{x}{a\tan\left(\frac{x}{2}\right)+a}-\frac{2\tan\left(\frac{x}{2}\right)}{a\tan\left(\frac{x}{2}\right)+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+a*sin(x)),x)
```

```
[Out] x*tan(x/2)/(a*tan(x/2) + a) + x/(a*tan(x/2) + a) - 2*tan(x/2)/(a*tan(x/2) +
 a)
```

Giac [A] time = 2.36122, size = 26, normalized size = 1.53

$$\frac{x}{a} + \frac{2}{a\left(\tan\left(\frac{1}{2}x\right) + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+a*sin(x)),x, algorithm="giac")
```

```
[Out] x/a + 2/(a*(tan(1/2*x) + 1))
```

3.7 $\int \frac{1}{a+a\sin(x)}\,dx$

Optimal. Leaf size=12

$$-\frac{\cos(x)}{a\sin(x)+a}$$

```
[Out] -(Cos[x]/(a + a*Sin[x]))
```

Rubi [A] time = 0.0102863, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2648}

$$-\frac{\cos(x)}{a\sin(x)+a}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[x])^(-1),x]
```

```
[Out] -(Cos[x]/(a + a*Sin[x]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{1}{a+a\sin(x)}\,dx = -\frac{\cos(x)}{a+a\sin(x)}$$

Mathematica [B] time = 0.0254837, size = 29, normalized size = 2.42

$$\frac{2\sin\left(\frac{x}{2}\right)\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)}{a\sin(x)+a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[x])^(-1),x]
```

```
[Out] (2*Sin[x/2]*(Cos[x/2] + Sin[x/2]))/(a + a*Sin[x])
```

Maple [A] time = 0.017, size = 14, normalized size = 1.2

$$-2\,\frac{1}{a\left(\tan(x/2)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(x)),x)
```

```
[Out] -2/a/(tan(1/2*x)+1)
```

Maxima [A] time = 1.73839, size = 22, normalized size = 1.83

$$-\frac{2}{a + \frac{a\sin(x)}{\cos(x)+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(x)),x, algorithm="maxima")
```

```
[Out] -2/(a + a*sin(x)/(cos(x) + 1))
```

Fricas [A] time = 1.59791, size = 68, normalized size = 5.67

$$-\frac{\cos(x) - \sin(x) + 1}{a\cos(x) + a\sin(x) + a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(x)),x, algorithm="fricas")
```

```
[Out] -(cos(x) - sin(x) + 1)/(a*cos(x) + a*sin(x) + a)
```

Sympy [A] time = 0.261398, size = 10, normalized size = 0.83

$$-\frac{2}{a\tan\left(\frac{x}{2}\right) + a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(x)),x)
```

```
[Out] -2/(a*tan(x/2) + a)
```

Giac [A] time = 1.66146, size = 18, normalized size = 1.5

$$-\frac{2}{a\left(\tan\left(\frac{1}{2}x\right) + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(x)),x, algorithm="giac")
```

```
[Out] -2/(a*(tan(1/2*x) + 1))
```

3.8 $\int \frac{\csc(x)}{a+a\sin(x)}\,dx$

Optimal. Leaf size=20

$$\frac{\cos(x)}{a\sin(x)+a}-\frac{\tanh^{-1}(\cos(x))}{a}$$

```
[Out] -(ArcTanh[Cos[x]]/a) + Cos[x]/(a + a*Sin[x])
```

Rubi [A] time = 0.0379892, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2747, 3770, 2648}

$$\frac{\cos(x)}{a\sin(x)+a}-\frac{\tanh^{-1}(\cos(x))}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]/(a + a*Sin[x]),x]
```

```
[Out] -(ArcTanh[Cos[x]]/a) + Cos[x]/(a + a*Sin[x])
```

Rule 2747

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]),
x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc(x)}{a+a\sin(x)}\,dx &= \frac{\int \csc(x)\,dx}{a}-\int \frac{1}{a+a\sin(x)}\,dx\\ &= -\frac{\tanh^{-1}(\cos(x))}{a}+\frac{\cos(x)}{a+a\sin(x)}\end{aligned}$$

Mathematica [B] time = 0.0501187, size = 74, normalized size = 3.7

$$-\frac{\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)\left(\cos\left(\frac{x}{2}\right)\left(\log\left(\cos\left(\frac{x}{2}\right)\right)-\log\left(\sin\left(\frac{x}{2}\right)\right)\right)+\sin\left(\frac{x}{2}\right)\left(-\log\left(\sin\left(\frac{x}{2}\right)\right)+\log\left(\cos\left(\frac{x}{2}\right)\right)+2\right)\right)}{a(\sin(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]/(a + a*Sin[x]),x]
```

```
[Out] -(((Cos[x/2] + Sin[x/2])*(Cos[x/2]*(Log[Cos[x/2]] - Log[Sin[x/2]]) + (2 + L
og[Cos[x/2]] - Log[Sin[x/2]])*Sin[x/2]))/(a*(1 + Sin[x])))
```

Maple [A] time = 0.029, size = 24, normalized size = 1.2

$$2\,\frac{1}{a\left(\tan\left(x/2\right)+1\right)}+\frac{1}{a}\ln\left(\tan\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)/(a+a*sin(x)),x)
```

```
[Out] 2/a/(tan(1/2*x)+1)+1/a*ln(tan(1/2*x))
```

Maxima [A] time = 1.65596, size = 42, normalized size = 2.1

$$\frac{\log\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a}+\frac{2}{a+\frac{a\sin(x)}{\cos(x)+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a+a*sin(x)),x, algorithm="maxima")
```

```
[Out] log(sin(x)/(cos(x) + 1))/a + 2/(a + a*sin(x)/(cos(x) + 1))
```

Fricas [B] time = 1.91135, size = 204, normalized size = 10.2

$$-\frac{\left(\cos\left(x\right)+\sin\left(x\right)+1\right)\log\left(\frac{1}{2}\,\cos\left(x\right)+\frac{1}{2}\right)-\left(\cos\left(x\right)+\sin\left(x\right)+1\right)\log\left(-\frac{1}{2}\,\cos\left(x\right)+\frac{1}{2}\right)-2\,\cos\left(x\right)+2\,\sin\left(x\right)-2}{2\left(a\cos\left(x\right)+a\sin\left(x\right)+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a+a*sin(x)),x, algorithm="fricas")
```

```
[Out] -1/2*((cos(x) + sin(x) + 1)*log(1/2*cos(x) + 1/2) - (cos(x) + sin(x) + 1)*l
og(-1/2*cos(x) + 1/2) - 2*cos(x) + 2*sin(x) - 2)/(a*cos(x) + a*sin(x) + a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{\csc\left(x\right)}{\sin\left(x\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a+a*sin(x)),x)
```

```
[Out] Integral(csc(x)/(sin(x) + 1), x)/a
```

Giac [A] time = 2.23577, size = 32, normalized size = 1.6

$$\frac{\log\left(\left|\tan\left(\frac{1}{2}x\right)\right|\right)}{a}+\frac{2}{a\left(\tan\left(\frac{1}{2}x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a+a*sin(x)),x, algorithm="giac")
```

```
[Out] log(abs(tan(1/2*x)))/a + 2/(a*(tan(1/2*x) + 1))
```

3.9 $\int \frac{\csc^2(x)}{a+a\sin(x)}\,dx$

Optimal. Leaf size=26

$$-\frac{2\cot(x)}{a}+\frac{\tanh^{-1}(\cos(x))}{a}+\frac{\cot(x)}{a\sin(x)+a}$$

```
[Out] ArcTanh[Cos[x]]/a - (2*Cot[x])/a + Cot[x]/(a + a*Sin[x])
```

Rubi [A] time = 0.0602264, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2768, 2748, 3767, 8, 3770}

$$-\frac{2\cot(x)}{a}+\frac{\tanh^{-1}(\cos(x))}{a}+\frac{\cot(x)}{a\sin(x)+a}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^2/(a + a*Sin[x]),x]
```

```
[Out] ArcTanh[Cos[x]]/a - (2*Cot[x])/a + Cot[x]/(a + a*Sin[x])
```

Rule 2768

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n
+ 1))/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x])), x] + Dist[d/(a*(b*c - a*d)),
Int[(c + d*Sin[e + f*x])^n*(a*n - b*(n + 1)*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\csc^2(x)}{a+a\sin(x)}\,dx = \frac{\cot(x)}{a+a\sin(x)} - \frac{\int \csc^2(x)(-2a+a\sin(x))\,dx}{a^2}$$
$$= \frac{\cot(x)}{a+a\sin(x)} - \frac{\int \csc(x)\,dx}{a} + \frac{2\int \csc^2(x)\,dx}{a}$$
$$= \frac{\tanh^{-1}(\cos(x))}{a} + \frac{\cot(x)}{a+a\sin(x)} - \frac{2\,\mathrm{Subst}(\int 1\,dx, x, \cot(x))}{a}$$
$$= \frac{\tanh^{-1}(\cos(x))}{a} - \frac{2\cot(x)}{a} + \frac{\cot(x)}{a+a\sin(x)}$$

Mathematica [B] time = 0.152978, size = 63, normalized size = 2.42

$$\frac{\tan\left(\frac{x}{2}\right) - \cot\left(\frac{x}{2}\right) - 2\log\left(\sin\left(\frac{x}{2}\right)\right) + 2\log\left(\cos\left(\frac{x}{2}\right)\right) + \frac{4\sin\left(\frac{x}{2}\right)}{\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)}}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^2/(a + a*Sin[x]),x]
```

```
[Out] (-Cot[x/2] + 2*Log[Cos[x/2]] - 2*Log[Sin[x/2]] + (4*Sin[x/2])/(Cos[x/2] + S
in[x/2]) + Tan[x/2])/(2*a)
```

Maple [A] time = 0.033, size = 45, normalized size = 1.7

$$\frac{1}{2a}\tan\left(\frac{x}{2}\right) - 2\frac{1}{a\left(\tan\left(x/2\right)+1\right)} - \frac{1}{2a}\left(\tan\left(\frac{x}{2}\right)\right)^{-1} - \frac{1}{a}\ln\left(\tan\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^2/(a+a*sin(x)),x)
```

```
[Out] 1/2/a*tan(1/2*x)-2/a/(tan(1/2*x)+1)-1/2/a/tan(1/2*x)-1/a*ln(tan(1/2*x))
```

Maxima [B] time = 1.67394, size = 92, normalized size = 3.54

$$-\frac{\frac{5\sin(x)}{\cos(x)+1}+1}{2\left(\frac{a\sin(x)}{\cos(x)+1}+\frac{a\sin(x)^2}{(\cos(x)+1)^2}\right)} - \frac{\log\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a} + \frac{\sin(x)}{2a(\cos(x)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a+a*sin(x)),x, algorithm="maxima")
```

```
[Out] -1/2*(5*sin(x)/(cos(x) + 1) + 1)/(a*sin(x)/(cos(x) + 1) + a*sin(x)^2/(cos(x
) + 1)^2) - log(sin(x)/(cos(x) + 1))/a + 1/2*sin(x)/(a*(cos(x) + 1))
```

Fricas [B] time = 1.72996, size = 301, normalized size = 11.58

$$\frac{4\cos(x)^2 + \left(\cos(x)^2 - (\cos(x)+1)\sin(x) - 1\right)\log\left(\frac{1}{2}\cos(x) + \frac{1}{2}\right) - \left(\cos(x)^2 - (\cos(x)+1)\sin(x) - 1\right)\log\left(-\frac{1}{2}\cos(x\right.}{2\left(a\cos(x)^2 - (a\cos(x)+a)\sin(x) - a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a+a*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(4*cos(x)^2 + (cos(x)^2 - (cos(x) + 1)*sin(x) - 1)*log(1/2*cos(x) + 1/2
) - (cos(x)^2 - (cos(x) + 1)*sin(x) - 1)*log(-1/2*cos(x) + 1/2) + 2*(2*cos(
x) + 1)*sin(x) + 2*cos(x) - 2)/(a*cos(x)^2 - (a*cos(x) + a)*sin(x) - a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\csc^2(x)}{\sin(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**2/(a+a*sin(x)),x)
```

```
[Out] Integral(csc(x)**2/(sin(x) + 1), x)/a
```

Giac [B] time = 1.79242, size = 72, normalized size = 2.77

$$-\frac{\log\left(\left|\tan\left(\frac{1}{2}x\right)\right|\right)}{a} + \frac{\tan\left(\frac{1}{2}x\right)}{2a} + \frac{\tan\left(\frac{1}{2}x\right)^2 - 4\tan\left(\frac{1}{2}x\right) - 1}{2\left(\tan\left(\frac{1}{2}x\right)^2 + \tan\left(\frac{1}{2}x\right)\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a+a*sin(x)),x, algorithm="giac")
```

```
[Out] -log(abs(tan(1/2*x)))/a + 1/2*tan(1/2*x)/a + 1/2*(tan(1/2*x)^2 - 4*tan(1/2*
x) - 1)/((tan(1/2*x)^2 + tan(1/2*x))*a)
```

3.10 $\int \frac{\csc^3(x)}{a+a\sin(x)}\,dx$

Optimal. Leaf size=42

$$\frac{2\cot(x)}{a}-\frac{3\tanh^{-1}(\cos(x))}{2a}-\frac{3\cot(x)\csc(x)}{2a}+\frac{\cot(x)\csc(x)}{a\sin(x)+a}$$

```
[Out] (-3*ArcTanh[Cos[x]])/(2*a) + (2*Cot[x])/a - (3*Cot[x]*Csc[x])/(2*a) + (Cot[
x]*Csc[x])/(a + a*Sin[x])
```

Rubi [A] time = 0.0663447, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2768, 2748, 3768, 3770, 3767, 8}

$$\frac{2\cot(x)}{a}-\frac{3\tanh^{-1}(\cos(x))}{2a}-\frac{3\cot(x)\csc(x)}{2a}+\frac{\cot(x)\csc(x)}{a\sin(x)+a}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^3/(a + a*Sin[x]),x]
```

```
[Out] (-3*ArcTanh[Cos[x]])/(2*a) + (2*Cot[x])/a - (3*Cot[x]*Csc[x])/(2*a) + (Cot[
x]*Csc[x])/(a + a*Sin[x])
```

Rule 2768

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n
+ 1))/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x])), x] + Dist[d/(a*(b*c - a*d)),
Int[(c + d*Sin[e + f*x])^n*(a*n - b*(n + 1)*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^3(x)}{a+a\sin(x)}\,dx &= \frac{\cot(x)\csc(x)}{a+a\sin(x)}-\frac{\int \csc^3(x)(-3a+2a\sin(x))\,dx}{a^2}\\ &= \frac{\cot(x)\csc(x)}{a+a\sin(x)}-\frac{2\int \csc^2(x)\,dx}{a}+\frac{3\int \csc^3(x)\,dx}{a}\\ &= -\frac{3\cot(x)\csc(x)}{2a}+\frac{\cot(x)\csc(x)}{a+a\sin(x)}+\frac{3\int \csc(x)\,dx}{2a}+\frac{2\,\mathrm{Subst}\left(\int 1\,dx,x,\cot(x)\right)}{a}\\ &= -\frac{3\tanh^{-1}(\cos(x))}{2a}+\frac{2\cot(x)}{a}-\frac{3\cot(x)\csc(x)}{2a}+\frac{\cot(x)\csc(x)}{a+a\sin(x)}\end{aligned}$$

Mathematica [A] time = 0.308347, size = 83, normalized size = 1.98

$$\frac{-4\tan\left(\frac{x}{2}\right)+4\cot\left(\frac{x}{2}\right)-\csc^2\left(\frac{x}{2}\right)+\sec^2\left(\frac{x}{2}\right)+12\log\left(\sin\left(\frac{x}{2}\right)\right)-12\log\left(\cos\left(\frac{x}{2}\right)\right)-\frac{16\sin\left(\frac{x}{2}\right)}{\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)}}{8a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^3/(a + a*Sin[x]),x]
```

```
[Out] (4*Cot[x/2] - Csc[x/2]^2 - 12*Log[Cos[x/2]] + 12*Log[Sin[x/2]] + Sec[x/2]^2
 - (16*Sin[x/2])/(Cos[x/2] + Sin[x/2]) - 4*Tan[x/2])/(8*a)
```

Maple [A] time = 0.037, size = 67, normalized size = 1.6

$$\frac{1}{8\,a}\left(\tan\left(\frac{x}{2}\right)\right)^2-\frac{1}{2\,a}\tan\left(\frac{x}{2}\right)+2\,\frac{1}{a\left(\tan(x/2)+1\right)}-\frac{1}{8\,a}\left(\tan\left(\frac{x}{2}\right)\right)^{-2}+\frac{1}{2\,a}\left(\tan\left(\frac{x}{2}\right)\right)^{-1}+\frac{3}{2\,a}\ln\left(\tan\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^3/(a+a*sin(x)),x)
```

```
[Out] 1/8/a*tan(1/2*x)^2-1/2/a*tan(1/2*x)+2/a/(tan(1/2*x)+1)-1/8/a/tan(1/2*x)^2+1
/2/a/tan(1/2*x)+3/2/a*ln(tan(1/2*x))
```

Maxima [B] time = 1.73902, size = 131, normalized size = 3.12

$$-\frac{\frac{4\sin(x)}{\cos(x)+1}-\frac{\sin(x)^2}{(\cos(x)+1)^2}}{8\,a}+\frac{\frac{3\sin(x)}{\cos(x)+1}+\frac{20\sin(x)^2}{(\cos(x)+1)^2}-1}{8\left(\frac{a\sin(x)^2}{(\cos(x)+1)^2}+\frac{a\sin(x)^3}{(\cos(x)+1)^3}\right)}+\frac{3\log\left(\frac{\sin(x)}{\cos(x)+1}\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(a+a*sin(x)),x, algorithm="maxima")
```

```
[Out] -1/8*(4*sin(x)/(cos(x) + 1) - sin(x)^2/(cos(x) + 1)^2)/a + 1/8*(3*sin(x)/(c
os(x) + 1) + 20*sin(x)^2/(cos(x) + 1)^2 - 1)/(a*sin(x)^2/(cos(x) + 1)^2 + a
*sin(x)^3/(cos(x) + 1)^3) + 3/2*log(sin(x)/(cos(x) + 1))/a
```

Fricas [B] time = 1.76625, size = 433, normalized size = 10.31

$$\frac{8\cos(x)^3+6\cos(x)^2-3\left(\cos(x)^3+\cos(x)^2+\left(\cos(x)^2-1\right)\sin(x)-\cos(x)-1\right)\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)+3\left(\cos(x)^3\right.}{4\left(a\cos(x)^3+a\cos(x)^2-a\,\mathrm{co}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(a+a*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/4*(8*cos(x)^3 + 6*cos(x)^2 - 3*(cos(x)^3 + cos(x)^2 + (cos(x)^2 - 1)*sin(
x) - cos(x) - 1)*log(1/2*cos(x) + 1/2) + 3*(cos(x)^3 + cos(x)^2 + (cos(x)^2
 - 1)*sin(x) - cos(x) - 1)*log(-1/2*cos(x) + 1/2) - 2*(4*cos(x)^2 + cos(x)
- 2)*sin(x) - 6*cos(x) - 4)/(a*cos(x)^3 + a*cos(x)^2 - a*cos(x) + (a*cos(x)
^2 - a)*sin(x) - a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\csc^3(x)}{\sin(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**3/(a+a*sin(x)),x)
```

```
[Out] Integral(csc(x)**3/(sin(x) + 1), x)/a
```

Giac [A] time = 2.12281, size = 99, normalized size = 2.36

$$\frac{3\log\left(\left|\tan\left(\frac{1}{2}x\right)\right|\right)}{2a}+\frac{a\tan\left(\frac{1}{2}x\right)^2-4a\tan\left(\frac{1}{2}x\right)}{8a^2}+\frac{2}{a\left(\tan\left(\frac{1}{2}x\right)+1\right)}-\frac{18\tan\left(\frac{1}{2}x\right)^2-4\tan\left(\frac{1}{2}x\right)+1}{8a\tan\left(\frac{1}{2}x\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(a+a*sin(x)),x, algorithm="giac")
```

```
[Out] 3/2*log(abs(tan(1/2*x)))/a + 1/8*(a*tan(1/2*x)^2 - 4*a*tan(1/2*x))/a^2 + 2/
(a*(tan(1/2*x) + 1)) - 1/8*(18*tan(1/2*x)^2 - 4*tan(1/2*x) + 1)/(a*tan(1/2*
x)^2)
```

3.11 $\int \frac{\csc^4(x)}{a+a\sin(x)}\,dx$

Optimal. Leaf size=55

$$-\frac{4\cot^3(x)}{3a}-\frac{4\cot(x)}{a}+\frac{3\tanh^{-1}(\cos(x))}{2a}+\frac{3\cot(x)\csc(x)}{2a}+\frac{\cot(x)\csc^2(x)}{a\sin(x)+a}$$

```
[Out] (3*ArcTanh[Cos[x]])/(2*a) - (4*Cot[x])/a - (4*Cot[x]^3)/(3*a) + (3*Cot[x]*C
sc[x])/(2*a) + (Cot[x]*Csc[x]^2)/(a + a*Sin[x])
```

Rubi [A] time = 0.0696497, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2768, 2748, 3767, 3768, 3770}

$$-\frac{4\cot^3(x)}{3a}-\frac{4\cot(x)}{a}+\frac{3\tanh^{-1}(\cos(x))}{2a}+\frac{3\cot(x)\csc(x)}{2a}+\frac{\cot(x)\csc^2(x)}{a\sin(x)+a}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^4/(a + a*Sin[x]),x]
```

```
[Out] (3*ArcTanh[Cos[x]])/(2*a) - (4*Cot[x])/a - (4*Cot[x]^3)/(3*a) + (3*Cot[x]*C
sc[x])/(2*a) + (Cot[x]*Csc[x]^2)/(a + a*Sin[x])
```

Rule 2768

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n
+ 1))/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x])), x] + Dist[d/(a*(b*c - a*d)),
Int[(c + d*Sin[e + f*x])^n*(a*n - b*(n + 1)*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^4(x)}{a+a\sin(x)}\,dx &= \frac{\cot(x)\csc^2(x)}{a+a\sin(x)}-\frac{\int \csc^4(x)(-4a+3a\sin(x))\,dx}{a^2}\\ &= \frac{\cot(x)\csc^2(x)}{a+a\sin(x)}-\frac{3\int \csc^3(x)\,dx}{a}+\frac{4\int \csc^4(x)\,dx}{a}\\ &= \frac{3\cot(x)\csc(x)}{2a}+\frac{\cot(x)\csc^2(x)}{a+a\sin(x)}-\frac{3\int \csc(x)\,dx}{2a}-\frac{4\,\mathrm{Subst}\left(\int \left(1+x^2\right)dx,x,\cot(x)\right)}{a}\\ &= \frac{3\tanh^{-1}(\cos(x))}{2a}-\frac{4\cot(x)}{a}-\frac{4\cot^3(x)}{3a}+\frac{3\cot(x)\csc(x)}{2a}+\frac{\cot(x)\csc^2(x)}{a+a\sin(x)}\end{aligned}$$

Mathematica [B] time = 0.716829, size = 113, normalized size = 2.05

$$\frac{20\tan\left(\frac{x}{2}\right)-20\cot\left(\frac{x}{2}\right)+3\csc^2\left(\frac{x}{2}\right)-3\sec^2\left(\frac{x}{2}\right)-36\log\left(\sin\left(\frac{x}{2}\right)\right)+36\log\left(\cos\left(\frac{x}{2}\right)\right)+\frac{48\sin\left(\frac{x}{2}\right)}{\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)}+8\sin^4\left(\frac{x}{2}\right)\csc^3}{24a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^4/(a + a*Sin[x]),x]
```

```
[Out] (-20*Cot[x/2] + 3*Csc[x/2]^2 + 36*Log[Cos[x/2]] - 36*Log[Sin[x/2]] - 3*Sec[
x/2]^2 + 8*Csc[x]^3*Sin[x/2]^4 + (48*Sin[x/2])/(Cos[x/2] + Sin[x/2]) - (Csc
[x/2]^4*Sin[x])/2 + 20*Tan[x/2])/(24*a)
```

Maple [A] time = 0.04, size = 89, normalized size = 1.6

$$\frac{1}{24\,a}\left(\tan\left(\frac{x}{2}\right)\right)^3-\frac{1}{8\,a}\left(\tan\left(\frac{x}{2}\right)\right)^2+\frac{7}{8\,a}\tan\left(\frac{x}{2}\right)-2\,\frac{1}{a\left(\tan(x/2)+1\right)}-\frac{1}{24\,a}\left(\tan\left(\frac{x}{2}\right)\right)^{-3}+\frac{1}{8\,a}\left(\tan\left(\frac{x}{2}\right)\right)^{-2}-\frac{7}{8\,a}\left(\mathrm{ta}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^4/(a+a*sin(x)),x)
```

```
[Out] 1/24/a*tan(1/2*x)^3-1/8/a*tan(1/2*x)^2+7/8/a*tan(1/2*x)-2/a/(tan(1/2*x)+1)-
1/24/a/tan(1/2*x)^3+1/8/a/tan(1/2*x)^2-7/8/a/tan(1/2*x)-3/2/a*ln(tan(1/2*x)
)
```

Maxima [B] time = 1.786, size = 162, normalized size = 2.95

$$\frac{\frac{21\sin(x)}{\cos(x)+1}-\frac{3\sin(x)^2}{(\cos(x)+1)^2}+\frac{\sin(x)^3}{(\cos(x)+1)^3}}{24\,a}+\frac{\frac{2\sin(x)}{\cos(x)+1}-\frac{18\sin(x)^2}{(\cos(x)+1)^2}-\frac{69\sin(x)^3}{(\cos(x)+1)^3}-1}{24\left(\frac{a\sin(x)^3}{(\cos(x)+1)^3}+\frac{a\sin(x)^4}{(\cos(x)+1)^4}\right)}-\frac{3\log\left(\frac{\sin(x)}{\cos(x)+1}\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^4/(a+a*sin(x)),x, algorithm="maxima")
```

```
[Out] 1/24*(21*sin(x)/(cos(x) + 1) - 3*sin(x)^2/(cos(x) + 1)^2 + sin(x)^3/(cos(x)
 + 1)^3)/a + 1/24*(2*sin(x)/(cos(x) + 1) - 18*sin(x)^2/(cos(x) + 1)^2 - 69*
```

```
sin(x)^3/(cos(x) + 1)^3 - 1)/(a*sin(x)^3/(cos(x) + 1)^3 + a*sin(x)^4/(cos(x
) + 1)^4) - 3/2*log(sin(x)/(cos(x) + 1))/a
```

Fricas [B] time = 1.8338, size = 537, normalized size = 9.76

$$\frac{32\,\cos\left(x\right)^{4}+14\,\cos\left(x\right)^{3}-48\,\cos\left(x\right)^{2}+9\left(\cos\left(x\right)^{4}-2\,\cos\left(x\right)^{2}-\left(\cos\left(x\right)^{3}+\cos\left(x\right)^{2}-\cos\left(x\right)-1\right)\sin\left(x\right)+1\right)\log\left(\frac{1}{2}\right.}{12\left(a\cos\left(x\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^4/(a+a*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/12*(32*cos(x)^4 + 14*cos(x)^3 - 48*cos(x)^2 + 9*(cos(x)^4 - 2*cos(x)^2 -
(cos(x)^3 + cos(x)^2 - cos(x) - 1)*sin(x) + 1)*log(1/2*cos(x) + 1/2) - 9*(c
os(x)^4 - 2*cos(x)^2 - (cos(x)^3 + cos(x)^2 - cos(x) - 1)*sin(x) + 1)*log(-
1/2*cos(x) + 1/2) + 2*(16*cos(x)^3 + 9*cos(x)^2 - 15*cos(x) - 6)*sin(x) - 1
8*cos(x) + 12)/(a*cos(x)^4 - 2*a*cos(x)^2 - (a*cos(x)^3 + a*cos(x)^2 - a*co
s(x) - a)*sin(x) + a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\csc^{4}\left(x\right)}{\sin\left(x\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**4/(a+a*sin(x)),x)
```

```
[Out] Integral(csc(x)**4/(sin(x) + 1), x)/a
```

Giac [A] time = 2.46184, size = 130, normalized size = 2.36

$$-\frac{3\,\log\left(\left|\tan\left(\frac{1}{2}\,x\right)\right|\right)}{2\,a}+\frac{a^{2}\tan\left(\frac{1}{2}\,x\right)^{3}-3\,a^{2}\tan\left(\frac{1}{2}\,x\right)^{2}+21\,a^{2}\tan\left(\frac{1}{2}\,x\right)}{24\,a^{3}}-\frac{2}{a\left(\tan\left(\frac{1}{2}\,x\right)+1\right)}+\frac{66\,\tan\left(\frac{1}{2}\,x\right)^{3}-21\,\tan\left(\frac{1}{2}\right.}{24\,a\tan\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^4/(a+a*sin(x)),x, algorithm="giac")
```

```
[Out] -3/2*log(abs(tan(1/2*x)))/a + 1/24*(a^2*tan(1/2*x)^3 - 3*a^2*tan(1/2*x)^2 +
 21*a^2*tan(1/2*x))/a^3 - 2/(a*(tan(1/2*x) + 1)) + 1/24*(66*tan(1/2*x)^3 -
21*tan(1/2*x)^2 + 3*tan(1/2*x) - 1)/(a*tan(1/2*x)^3)
```

3.12 $\int \frac{\sin^4(x)}{(a+a\sin(x))^2}\,dx$

Optimal. Leaf size=66

$$\frac{7x}{2a^2}+\frac{16\cos(x)}{3a^2}+\frac{8\sin^2(x)\cos(x)}{3a^2(\sin(x)+1)}-\frac{7\sin(x)\cos(x)}{2a^2}+\frac{\sin^3(x)\cos(x)}{3(a\sin(x)+a)^2}$$

```
[Out] (7*x)/(2*a^2) + (16*Cos[x])/(3*a^2) - (7*Cos[x]*Sin[x])/(2*a^2) + (8*Cos[x]
*Sin[x]^2)/(3*a^2*(1 + Sin[x])) + (Cos[x]*Sin[x]^3)/(3*(a + a*Sin[x])^2)
```

Rubi [A] time = 0.120751, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2765, 2977, 2734}

$$\frac{7x}{2a^2}+\frac{16\cos(x)}{3a^2}+\frac{8\sin^2(x)\cos(x)}{3a^2(\sin(x)+1)}-\frac{7\sin(x)\cos(x)}{2a^2}+\frac{\sin^3(x)\cos(x)}{3(a\sin(x)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^4/(a + a*Sin[x])^2,x]
```

```
[Out] (7*x)/(2*a^2) + (16*Cos[x])/(3*a^2) - (7*Cos[x]*Sin[x])/(2*a^2) + (8*Cos[x]
*Sin[x]^2)/(3*a^2*(1 + Sin[x])) + (Cos[x]*Sin[x]^3)/(3*(a + a*Sin[x])^2)
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{\sin^4(x)}{(a+a\sin(x))^2}\,dx = \frac{\cos(x)\sin^3(x)}{3(a+a\sin(x))^2} - \frac{\int \frac{\sin^2(x)(3a-5a\sin(x))}{a+a\sin(x)}\,dx}{3a^2}$$
$$= \frac{8\cos(x)\sin^2(x)}{3a^2(1+\sin(x))} + \frac{\cos(x)\sin^3(x)}{3(a+a\sin(x))^2} - \frac{\int \sin(x)\left(16a^2-21a^2\sin(x)\right)dx}{3a^4}$$
$$= \frac{7x}{2a^2} + \frac{16\cos(x)}{3a^2} - \frac{7\cos(x)\sin(x)}{2a^2} + \frac{8\cos(x)\sin^2(x)}{3a^2(1+\sin(x))} + \frac{\cos(x)\sin^3(x)}{3(a+a\sin(x))^2}$$

Mathematica [A] time = 0.239894, size = 100, normalized size = 1.52

$$\frac{\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)\left(21(12x-7)\cos\left(\frac{x}{2}\right)+(239-84x)\cos\left(\frac{3x}{2}\right)+3\left(-5\cos\left(\frac{5x}{2}\right)+\cos\left(\frac{7x}{2}\right)+2\sin\left(\frac{x}{2}\right)(56x+(28x+27\right.\right.}{48a^2(\sin(x)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^4/(a + a*Sin[x])^2,x]
```

```
[Out] ((Cos[x/2] + Sin[x/2])*(21*(-7 + 12*x)*Cos[x/2] + (239 - 84*x)*Cos[(3*x)/2]
 + 3*(-5*Cos[(5*x)/2] + Cos[(7*x)/2] + 2*(-50 + 56*x + (27 + 28*x)*Cos[x] +
 6*Cos[2*x] + Cos[3*x])*Sin[x/2])))/(48*a^2*(1 + Sin[x])^2)
```

Maple [B] time = 0.042, size = 126, normalized size = 1.9

$$\frac{1}{a^2}\left(\tan\left(\frac{x}{2}\right)\right)^3\left(\left(\tan\left(\frac{x}{2}\right)\right)^2+1\right)^{-2}+4\,\frac{(\tan(x/2))^2}{a^2\left((\tan(x/2))^2+1\right)^2}-\frac{1}{a^2}\tan\left(\frac{x}{2}\right)\left(\left(\tan\left(\frac{x}{2}\right)\right)^2+1\right)^{-2}+4\,\frac{1}{a^2\left((\tan(x/2))^2+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^4/(a+a*sin(x))^2,x)
```

```
[Out] 1/a^2/(tan(1/2*x)^2+1)^2*tan(1/2*x)^3+4/a^2/(tan(1/2*x)^2+1)^2*tan(1/2*x)^2
-1/a^2/(tan(1/2*x)^2+1)^2*tan(1/2*x)+4/a^2/(tan(1/2*x)^2+1)^2+7/a^2*arctan(
tan(1/2*x))-4/3/a^2/(tan(1/2*x)+1)^3+2/a^2/(tan(1/2*x)+1)^2+6/a^2/(tan(1/2*
x)+1)
```

Maxima [B] time = 2.4889, size = 267, normalized size = 4.05

$$\frac{\frac{75\sin(x)}{\cos(x)+1}+\frac{97\sin(x)^2}{(\cos(x)+1)^2}+\frac{126\sin(x)^3}{(\cos(x)+1)^3}+\frac{98\sin(x)^4}{(\cos(x)+1)^4}+\frac{63\sin(x)^5}{(\cos(x)+1)^5}+\frac{21\sin(x)^6}{(\cos(x)+1)^6}+32}{3\left(a^2+\frac{3a^2\sin(x)}{\cos(x)+1}+\frac{5a^2\sin(x)^2}{(\cos(x)+1)^2}+\frac{7a^2\sin(x)^3}{(\cos(x)+1)^3}+\frac{7a^2\sin(x)^4}{(\cos(x)+1)^4}+\frac{5a^2\sin(x)^5}{(\cos(x)+1)^5}+\frac{3a^2\sin(x)^6}{(\cos(x)+1)^6}+\frac{a^2\sin(x)^7}{(\cos(x)+1)^7}\right)}+\frac{7\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^4/(a+a*sin(x))^2,x, algorithm="maxima")
```

```
[Out] 1/3*(75*sin(x)/(cos(x) + 1) + 97*sin(x)^2/(cos(x) + 1)^2 + 126*sin(x)^3/(co
s(x) + 1)^3 + 98*sin(x)^4/(cos(x) + 1)^4 + 63*sin(x)^5/(cos(x) + 1)^5 + 21*
sin(x)^6/(cos(x) + 1)^6 + 32)/(a^2 + 3*a^2*sin(x)/(cos(x) + 1) + 5*a^2*sin(
x)^2/(cos(x) + 1)^2 + 7*a^2*sin(x)^3/(cos(x) + 1)^3 + 7*a^2*sin(x)^4/(cos(x
```

```
) + 1)^4 + 5*a^2*sin(x)^5/(cos(x) + 1)^5 + 3*a^2*sin(x)^6/(cos(x) + 1)^6 +
a^2*sin(x)^7/(cos(x) + 1)^7) + 7*arctan(sin(x)/(cos(x) + 1))/a^2
```

Fricas [A] time = 1.3966, size = 297, normalized size = 4.5

$$-\frac{3\cos(x)^4-(21x-31)\cos(x)^2-6\cos(x)^3+(21x+38)\cos(x)+\left(3\cos(x)^3+(21x+40)\cos(x)+9\cos(x)^2+4\right.}{6\left(a^2\cos(x)^2-a^2\cos(x)-2a^2-\left(a^2\cos(x)+2a^2\right)\sin(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^4/(a+a*sin(x))^2,x, algorithm="fricas")
```

```
[Out] -1/6*(3*cos(x)^4 - (21*x - 31)*cos(x)^2 - 6*cos(x)^3 + (21*x + 38)*cos(x) +
 (3*cos(x)^3 + (21*x + 40)*cos(x) + 9*cos(x)^2 + 42*x + 2)*sin(x) + 42*x -
2)/(a^2*cos(x)^2 - a^2*cos(x) - 2*a^2 - (a^2*cos(x) + 2*a^2)*sin(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**4/(a+a*sin(x))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 2.38438, size = 97, normalized size = 1.47

$$\frac{7x}{2a^2}+\frac{\tan\left(\frac{1}{2}x\right)^3+4\tan\left(\frac{1}{2}x\right)^2-\tan\left(\frac{1}{2}x\right)+4}{\left(\tan\left(\frac{1}{2}x\right)^2+1\right)^2a^2}+\frac{2\left(9\tan\left(\frac{1}{2}x\right)^2+21\tan\left(\frac{1}{2}x\right)+10\right)}{3a^2\left(\tan\left(\frac{1}{2}x\right)+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^4/(a+a*sin(x))^2,x, algorithm="giac")
```

```
[Out] 7/2*x/a^2 + (tan(1/2*x)^3 + 4*tan(1/2*x)^2 - tan(1/2*x) + 4)/((tan(1/2*x)^2
 + 1)^2*a^2) + 2/3*(9*tan(1/2*x)^2 + 21*tan(1/2*x) + 10)/(a^2*(tan(1/2*x) +
 1)^3)
```

3.13 $\int \frac{\sin^3(x)}{(a+a\sin(x))^2}\,dx$

Optimal. Leaf size=47

$$-\frac{2x}{a^2}-\frac{4\cos(x)}{3a^2}-\frac{2\cos(x)}{a^2(\sin(x)+1)}+\frac{\sin^2(x)\cos(x)}{3(a\sin(x)+a)^2}$$

```
[Out] (-2*x)/a^2 - (4*Cos[x])/(3*a^2) - (2*Cos[x])/(a^2*(1 + Sin[x])) + (Cos[x]*S
in[x]^2)/(3*(a + a*Sin[x])^2)
```

Rubi [A] time = 0.143152, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2765, 2968, 3023, 12, 2735, 2648}

$$-\frac{2x}{a^2}-\frac{4\cos(x)}{3a^2}-\frac{2\cos(x)}{a^2(\sin(x)+1)}+\frac{\sin^2(x)\cos(x)}{3(a\sin(x)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^3/(a + a*Sin[x])^2,x]
```

```
[Out] (-2*x)/a^2 - (4*Cos[x])/(3*a^2) - (2*Cos[x])/(a^2*(1 + Sin[x])) + (Cos[x]*S
in[x]^2)/(3*(a + a*Sin[x])^2)
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sin^3(x)}{(a+a\sin(x))^2}\,dx &= \frac{\cos(x)\sin^2(x)}{3(a+a\sin(x))^2}-\frac{\int \frac{\sin(x)(2a-4a\sin(x))}{a+a\sin(x)}\,dx}{3a^2}\\
&= \frac{\cos(x)\sin^2(x)}{3(a+a\sin(x))^2}-\frac{\int \frac{2a\sin(x)-4a\sin^2(x)}{a+a\sin(x)}\,dx}{3a^2}\\
&= -\frac{4\cos(x)}{3a^2}+\frac{\cos(x)\sin^2(x)}{3(a+a\sin(x))^2}-\frac{\int \frac{6a^2\sin(x)}{a+a\sin(x)}\,dx}{3a^3}\\
&= -\frac{4\cos(x)}{3a^2}+\frac{\cos(x)\sin^2(x)}{3(a+a\sin(x))^2}-\frac{2\int \frac{\sin(x)}{a+a\sin(x)}\,dx}{a}\\
&= -\frac{2x}{a^2}-\frac{4\cos(x)}{3a^2}+\frac{\cos(x)\sin^2(x)}{3(a+a\sin(x))^2}+\frac{2\int \frac{1}{a+a\sin(x)}\,dx}{a}\\
&= -\frac{2x}{a^2}-\frac{4\cos(x)}{3a^2}+\frac{\cos(x)\sin^2(x)}{3(a+a\sin(x))^2}-\frac{2\cos(x)}{a^2+a^2\sin(x)}
\end{aligned}
$$

Mathematica [A] time = 0.224515, size = 84, normalized size = 1.79

$$
-\frac{\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)\left(6(6x-5)\cos\left(\frac{x}{2}\right)+(41-12x)\cos\left(\frac{3x}{2}\right)-3\cos\left(\frac{5x}{2}\right)+6\sin\left(\frac{x}{2}\right)(8x+4(x+1)\cos(x)+\cos(2x)\right.}{12a^2(\sin(x)+1)^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^3/(a + a*Sin[x])^2,x]
```

```
[Out] -((Cos[x/2] + Sin[x/2])*(6*(-5 + 6*x)*Cos[x/2] + (41 - 12*x)*Cos[(3*x)/2] -
 3*Cos[(5*x)/2] + 6*(-9 + 8*x + 4*(1 + x)*Cos[x] + Cos[2*x])*Sin[x/2]))/(12
*a^2*(1 + Sin[x])^2)
```

Maple [A] time = 0.04, size = 66, normalized size = 1.4

$$
-2\,\frac{1}{a^2\left((\tan(x/2))^2+1\right)}-4\,\frac{\arctan(\tan(x/2))}{a^2}+\frac{4}{3\,a^2}\left(\tan\left(\frac{x}{2}\right)+1\right)^{-3}-2\,\frac{1}{a^2\,(\tan(x/2)+1)^2}-4\,\frac{1}{a^2\,(\tan(x/2)+1)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3/(a+a*sin(x))^2,x)
```

```
[Out] -2/a^2/(tan(1/2*x)^2+1)-4/a^2*arctan(tan(1/2*x))+4/3/a^2/(tan(1/2*x)+1)^3-2
/a^2/(tan(1/2*x)+1)^2-4/a^2/(tan(1/2*x)+1)
```

Maxima [B] time = 2.49702, size = 194, normalized size = 4.13

$$-\frac{4\left(\frac{12\sin(x)}{\cos(x)+1}+\frac{11\sin(x)^2}{(\cos(x)+1)^2}+\frac{9\sin(x)^3}{(\cos(x)+1)^3}+\frac{3\sin(x)^4}{(\cos(x)+1)^4}+5\right)}{3\left(a^2+\frac{3a^2\sin(x)}{\cos(x)+1}+\frac{4a^2\sin(x)^2}{(\cos(x)+1)^2}+\frac{4a^2\sin(x)^3}{(\cos(x)+1)^3}+\frac{3a^2\sin(x)^4}{(\cos(x)+1)^4}+\frac{a^2\sin(x)^5}{(\cos(x)+1)^5}\right)}-\frac{4\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(a+a*sin(x))^2,x, algorithm="maxima")
```

```
[Out] -4/3*(12*sin(x)/(cos(x) + 1) + 11*sin(x)^2/(cos(x) + 1)^2 + 9*sin(x)^3/(cos
(x) + 1)^3 + 3*sin(x)^4/(cos(x) + 1)^4 + 5)/(a^2 + 3*a^2*sin(x)/(cos(x) + 1
) + 4*a^2*sin(x)^2/(cos(x) + 1)^2 + 4*a^2*sin(x)^3/(cos(x) + 1)^3 + 3*a^2*s
in(x)^4/(cos(x) + 1)^4 + a^2*sin(x)^5/(cos(x) + 1)^5) - 4*arctan(sin(x)/(co
s(x) + 1))/a^2
```

Fricas [B] time = 1.45906, size = 259, normalized size = 5.51

$$-\frac{(6x-11)\cos(x)^2+3\cos(x)^3-(6x+13)\cos(x)-\left(2(3x+7)\cos(x)+3\cos(x)^2+12x+1\right)\sin(x)-12x+1}{3\left(a^2\cos(x)^2-a^2\cos(x)-2a^2-\left(a^2\cos(x)+2a^2\right)\sin(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(a+a*sin(x))^2,x, algorithm="fricas")
```

```
[Out] -1/3*((6*x - 11)*cos(x)^2 + 3*cos(x)^3 - (6*x + 13)*cos(x) - (2*(3*x + 7)*c
os(x) + 3*cos(x)^2 + 12*x + 1)*sin(x) - 12*x + 1)/(a^2*cos(x)^2 - a^2*cos(x
) - 2*a^2 - (a^2*cos(x) + 2*a^2)*sin(x))
```

Sympy [B] time = 105.718, size = 848, normalized size = 18.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**3/(a+a*sin(x))**2,x)
```

```
[Out] -6*x*tan(x/2)**5/(3*a**2*tan(x/2)**5 + 9*a**2*tan(x/2)**4 + 12*a**2*tan(x/2
)**3 + 12*a**2*tan(x/2)**2 + 9*a**2*tan(x/2) + 3*a**2) - 18*x*tan(x/2)**4/(
3*a**2*tan(x/2)**5 + 9*a**2*tan(x/2)**4 + 12*a**2*tan(x/2)**3 + 12*a**2*tan
(x/2)**2 + 9*a**2*tan(x/2) + 3*a**2) - 24*x*tan(x/2)**3/(3*a**2*tan(x/2)**5
 + 9*a**2*tan(x/2)**4 + 12*a**2*tan(x/2)**3 + 12*a**2*tan(x/2)**2 + 9*a**2*
tan(x/2) + 3*a**2) - 24*x*tan(x/2)**2/(3*a**2*tan(x/2)**5 + 9*a**2*tan(x/2)
**4 + 12*a**2*tan(x/2)**3 + 12*a**2*tan(x/2)**2 + 9*a**2*tan(x/2) + 3*a**2)
 - 18*x*tan(x/2)/(3*a**2*tan(x/2)**5 + 9*a**2*tan(x/2)**4 + 12*a**2*tan(x/2
)**3 + 12*a**2*tan(x/2)**2 + 9*a**2*tan(x/2) + 3*a**2) - 6*x/(3*a**2*tan(x/
2)**5 + 9*a**2*tan(x/2)**4 + 12*a**2*tan(x/2)**3 + 12*a**2*tan(x/2)**2 + 9*
a**2*tan(x/2) + 3*a**2) + 14*tan(x/2)**5/(3*a**2*tan(x/2)**5 + 9*a**2*tan(x
```

```
/2)**4 + 12*a**2*tan(x/2)**3 + 12*a**2*tan(x/2)**2 + 9*a**2*tan(x/2) + 3*a*
*2) + 30*tan(x/2)**4/(3*a**2*tan(x/2)**5 + 9*a**2*tan(x/2)**4 + 12*a**2*tan
(x/2)**3 + 12*a**2*tan(x/2)**2 + 9*a**2*tan(x/2) + 3*a**2) + 20*tan(x/2)**3
/(3*a**2*tan(x/2)**5 + 9*a**2*tan(x/2)**4 + 12*a**2*tan(x/2)**3 + 12*a**2*t
an(x/2)**2 + 9*a**2*tan(x/2) + 3*a**2) + 12*tan(x/2)**2/(3*a**2*tan(x/2)**5
 + 9*a**2*tan(x/2)**4 + 12*a**2*tan(x/2)**3 + 12*a**2*tan(x/2)**2 + 9*a**2*
tan(x/2) + 3*a**2) - 6*tan(x/2)/(3*a**2*tan(x/2)**5 + 9*a**2*tan(x/2)**4 +
12*a**2*tan(x/2)**3 + 12*a**2*tan(x/2)**2 + 9*a**2*tan(x/2) + 3*a**2) - 6/(
3*a**2*tan(x/2)**5 + 9*a**2*tan(x/2)**4 + 12*a**2*tan(x/2)**3 + 12*a**2*tan
(x/2)**2 + 9*a**2*tan(x/2) + 3*a**2)
```

Giac [A] time = 1.34885, size = 69, normalized size = 1.47

$$-\frac{2\,x}{a^2}-\frac{2}{\left(\tan\left(\frac{1}{2}\,x\right)^2+1\right)a^2}-\frac{2\left(6\,\tan\left(\frac{1}{2}\,x\right)^2+15\,\tan\left(\frac{1}{2}\,x\right)+7\right)}{3\,a^2\left(\tan\left(\frac{1}{2}\,x\right)+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(a+a*sin(x))^2,x, algorithm="giac")
```

```
[Out] -2*x/a^2 - 2/((tan(1/2*x)^2 + 1)*a^2) - 2/3*(6*tan(1/2*x)^2 + 15*tan(1/2*x)
 + 7)/(a^2*(tan(1/2*x) + 1)^3)
```

3.14 $\int \frac{\sin^2(x)}{(a+a\sin(x))^2}\,dx$

Optimal. Leaf size=35

$$\frac{x}{a^2}+\frac{5\cos(x)}{3a^2(\sin(x)+1)}-\frac{\cos(x)}{3(a\sin(x)+a)^2}$$

```
[Out] x/a^2 + (5*Cos[x])/(3*a^2*(1 + Sin[x])) - Cos[x]/(3*(a + a*Sin[x])^2)
```

Rubi [A] time = 0.0725763, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2758, 2735, 2648}

$$\frac{x}{a^2}+\frac{5\cos(x)}{3a^2(\sin(x)+1)}-\frac{\cos(x)}{3(a\sin(x)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^2/(a + a*Sin[x])^2,x]
```

```
[Out] x/a^2 + (5*Cos[x])/(3*a^2*(1 + Sin[x])) - Cos[x]/(3*(a + a*Sin[x])^2)
```

Rule 2758

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)),
x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(a*m - b*(2*m
 + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -2^(-1)]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^2(x)}{(a+a\sin(x))^2}\,dx &= -\frac{\cos(x)}{3(a+a\sin(x))^2}+\frac{\int \frac{-2a+3a\sin(x)}{a+a\sin(x)}\,dx}{3a^2}\\ &= \frac{x}{a^2}-\frac{\cos(x)}{3(a+a\sin(x))^2}-\frac{5\int \frac{1}{a+a\sin(x)}\,dx}{3a}\\ &= \frac{x}{a^2}-\frac{\cos(x)}{3(a+a\sin(x))^2}+\frac{5\cos(x)}{3\left(a^2+a^2\sin(x)\right)}\end{aligned}$$

Mathematica [A] time = 0.126773, size = 69, normalized size = 1.97

$$\frac{\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)\left(3(3x-4)\cos\left(\frac{x}{2}\right)+(10-3x)\cos\left(\frac{3x}{2}\right)+6\sin\left(\frac{x}{2}\right)(2x+x\cos(x)-3)\right)}{6a^2(\sin(x)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^2/(a + a*Sin[x])^2,x]
```

```
[Out] ((Cos[x/2] + Sin[x/2])*(3*(-4 + 3*x)*Cos[x/2] + (10 - 3*x)*Cos[(3*x)/2] + 6
*(-3 + 2*x + x*Cos[x])*Sin[x/2]))/(6*a^2*(1 + Sin[x])^2)
```

Maple [A] time = 0.037, size = 51, normalized size = 1.5

$$2\,\frac{\arctan\left(\tan\left(x/2\right)\right)}{a^2}-\frac{4}{3\,a^2}\left(\tan\left(\frac{x}{2}\right)+1\right)^{-3}+2\,\frac{1}{a^2\left(\tan\left(x/2\right)+1\right)^2}+2\,\frac{1}{a^2\left(\tan\left(x/2\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2/(a+a*sin(x))^2,x)
```

```
[Out] 2/a^2*arctan(tan(1/2*x))-4/3/a^2/(tan(1/2*x)+1)^3+2/a^2/(tan(1/2*x)+1)^2+2/
a^2/(tan(1/2*x)+1)
```

Maxima [B] time = 2.68941, size = 122, normalized size = 3.49

$$\frac{2\left(\frac{9\sin(x)}{\cos(x)+1}+\frac{3\sin(x)^2}{(\cos(x)+1)^2}+4\right)}{3\left(a^2+\frac{3\,a^2\sin(x)}{\cos(x)+1}+\frac{3\,a^2\sin(x)^2}{(\cos(x)+1)^2}+\frac{a^2\sin(x)^3}{(\cos(x)+1)^3}\right)}+\frac{2\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a+a*sin(x))^2,x, algorithm="maxima")
```

```
[Out] 2/3*(9*sin(x)/(cos(x) + 1) + 3*sin(x)^2/(cos(x) + 1)^2 + 4)/(a^2 + 3*a^2*si
n(x)/(cos(x) + 1) + 3*a^2*sin(x)^2/(cos(x) + 1)^2 + a^2*sin(x)^3/(cos(x) +
1)^3) + 2*arctan(sin(x)/(cos(x) + 1))/a^2
```

Fricas [B] time = 1.45629, size = 215, normalized size = 6.14

$$\frac{(3\,x-5)\cos\left(x\right)^2-(3\,x+4)\cos\left(x\right)-((3\,x+5)\cos\left(x\right)+6\,x+1)\sin\left(x\right)-6\,x+1}{3\left(a^2\cos\left(x\right)^2-a^2\cos\left(x\right)-2\,a^2-\left(a^2\cos\left(x\right)+2\,a^2\right)\sin\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a+a*sin(x))^2,x, algorithm="fricas")
```

```
[Out] 1/3*((3*x - 5)*cos(x)^2 - (3*x + 4)*cos(x) - ((3*x + 5)*cos(x) + 6*x + 1)*s
in(x) - 6*x + 1)/(a^2*cos(x)^2 - a^2*cos(x) - 2*a^2 - (a^2*cos(x) + 2*a^2)*
```

```
sin(x))
```

Sympy [B] time = 7.36811, size = 369, normalized size = 10.54

$$\frac{15x\tan^3\left(\frac{x}{2}\right)}{15a^2\tan^3\left(\frac{x}{2}\right)+45a^2\tan^2\left(\frac{x}{2}\right)+45a^2\tan\left(\frac{x}{2}\right)+15a^2}+\frac{45x\tan^2\left(\frac{x}{2}\right)}{15a^2\tan^3\left(\frac{x}{2}\right)+45a^2\tan^2\left(\frac{x}{2}\right)+45a^2\tan\left(\frac{x}{2}\right)+15a^2}+\frac{}{15a^2\tan}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**2/(a+a*sin(x))**2,x)
```

```
[Out] 15*x*tan(x/2)**3/(15*a**2*tan(x/2)**3 + 45*a**2*tan(x/2)**2 + 45*a**2*tan(x
/2) + 15*a**2) + 45*x*tan(x/2)**2/(15*a**2*tan(x/2)**3 + 45*a**2*tan(x/2)**
2 + 45*a**2*tan(x/2) + 15*a**2) + 45*x*tan(x/2)/(15*a**2*tan(x/2)**3 + 45*a
**2*tan(x/2)**2 + 45*a**2*tan(x/2) + 15*a**2) + 15*x/(15*a**2*tan(x/2)**3 +
 45*a**2*tan(x/2)**2 + 45*a**2*tan(x/2) + 15*a**2) - 22*tan(x/2)**3/(15*a**
2*tan(x/2)**3 + 45*a**2*tan(x/2)**2 + 45*a**2*tan(x/2) + 15*a**2) - 36*tan(
x/2)**2/(15*a**2*tan(x/2)**3 + 45*a**2*tan(x/2)**2 + 45*a**2*tan(x/2) + 15*
a**2) + 24*tan(x/2)/(15*a**2*tan(x/2)**3 + 45*a**2*tan(x/2)**2 + 45*a**2*ta
n(x/2) + 15*a**2) + 18/(15*a**2*tan(x/2)**3 + 45*a**2*tan(x/2)**2 + 45*a**2
*tan(x/2) + 15*a**2)
```

Giac [A] time = 1.63975, size = 47, normalized size = 1.34

$$\frac{x}{a^2}+\frac{2\left(3\tan\left(\frac{1}{2}x\right)^2+9\tan\left(\frac{1}{2}x\right)+4\right)}{3a^2\left(\tan\left(\frac{1}{2}x\right)+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a+a*sin(x))^2,x, algorithm="giac")
```

```
[Out] x/a^2 + 2/3*(3*tan(1/2*x)^2 + 9*tan(1/2*x) + 4)/(a^2*(tan(1/2*x) + 1)^3)
```

3.15 $\int \frac{\sin(x)}{(a+a\sin(x))^2} dx$

Optimal. Leaf size=33

$$\frac{\cos(x)}{3(a\sin(x)+a)^2} - \frac{2\cos(x)}{3\left(a^2\sin(x)+a^2\right)}$$

```
[Out] Cos[x]/(3*(a + a*Sin[x])^2) - (2*Cos[x])/(3*(a^2 + a^2*Sin[x]))
```

Rubi [A] time = 0.0314165, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2750, 2648}

$$\frac{\cos(x)}{3(a\sin(x)+a)^2} - \frac{2\cos(x)}{3\left(a^2\sin(x)+a^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]/(a + a*Sin[x])^2,x]
```

```
[Out] Cos[x]/(3*(a + a*Sin[x])^2) - (2*Cos[x])/(3*(a^2 + a^2*Sin[x]))
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{\sin(x)}{(a+a\sin(x))^2} dx &= \frac{\cos(x)}{3(a+a\sin(x))^2} + \frac{2\int \frac{1}{a+a\sin(x)} dx}{3a} \\ &= \frac{\cos(x)}{3(a+a\sin(x))^2} - \frac{2\cos(x)}{3\left(a^2+a^2\sin(x)\right)} \end{aligned}$$

Mathematica [A] time = 0.0421223, size = 29, normalized size = 0.88

$$-\frac{-4\sin(x)+\sin(2x)+\cos(x)+\cos(2x)-3}{3a^2(\sin(x)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/(a + a*Sin[x])^2,x]
```

```
[Out] -(-3 + Cos[x] + Cos[2*x] - 4*Sin[x] + Sin[2*x])/(3*a^2*(1 + Sin[x])^2)
```

Maple [A] time = 0.032, size = 27, normalized size = 0.8

$$4\,\frac{1}{a^2}\left(1/3\,(\tan(x/2)+1)^{-3}-1/2\,(\tan(x/2)+1)^{-2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(a+a*sin(x))^2,x)
```

```
[Out] 4/a^2*(1/3/(tan(1/2*x)+1)^3-1/2/(tan(1/2*x)+1)^2)
```

Maxima [B] time = 1.07915, size = 84, normalized size = 2.55

$$-\frac{2\left(\frac{3\sin(x)}{\cos(x)+1}+1\right)}{3\left(a^2+\frac{3\,a^2\sin(x)}{\cos(x)+1}+\frac{3\,a^2\sin(x)^2}{(\cos(x)+1)^2}+\frac{a^2\sin(x)^3}{(\cos(x)+1)^3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+a*sin(x))^2,x, algorithm="maxima")
```

```
[Out] -2/3*(3*sin(x)/(cos(x) + 1) + 1)/(a^2 + 3*a^2*sin(x)/(cos(x) + 1) + 3*a^2*s
in(x)^2/(cos(x) + 1)^2 + a^2*sin(x)^3/(cos(x) + 1)^3)
```

Fricas [B] time = 1.39024, size = 163, normalized size = 4.94

$$\frac{2\cos(x)^2+(2\cos(x)+1)\sin(x)+\cos(x)-1}{3\left(a^2\cos(x)^2-a^2\cos(x)-2\,a^2-\left(a^2\cos(x)+2\,a^2\right)\sin(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+a*sin(x))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(2*cos(x)^2 + (2*cos(x) + 1)*sin(x) + cos(x) - 1)/(a^2*cos(x)^2 - a^2*c
os(x) - 2*a^2 - (a^2*cos(x) + 2*a^2)*sin(x))
```

Sympy [B] time = 3.41913, size = 178, normalized size = 5.39

$$\frac{\tan^3\left(\frac{x}{2}\right)}{6a^2\tan^3\left(\frac{x}{2}\right)+18a^2\tan^2\left(\frac{x}{2}\right)+18a^2\tan\left(\frac{x}{2}\right)+6a^2}+\frac{3\tan^2\left(\frac{x}{2}\right)}{6a^2\tan^3\left(\frac{x}{2}\right)+18a^2\tan^2\left(\frac{x}{2}\right)+18a^2\tan\left(\frac{x}{2}\right)+6a^2}-\frac{}{6a^2\tan^3\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+a*sin(x))**2,x)
```

```
[Out] tan(x/2)**3/(6*a**2*tan(x/2)**3 + 18*a**2*tan(x/2)**2 + 18*a**2*tan(x/2) +
6*a**2) + 3*tan(x/2)**2/(6*a**2*tan(x/2)**3 + 18*a**2*tan(x/2)**2 + 18*a**2
```

```
*tan(x/2) + 6*a**2) - 9*tan(x/2)/(6*a**2*tan(x/2)**3 + 18*a**2*tan(x/2)**2
+ 18*a**2*tan(x/2) + 6*a**2) - 3/(6*a**2*tan(x/2)**3 + 18*a**2*tan(x/2)**2
+ 18*a**2*tan(x/2) + 6*a**2)
```

Giac [A] time = 1.88682, size = 28, normalized size = 0.85

$$-\frac{2\left(3\tan\left(\frac{1}{2}x\right)+1\right)}{3a^2\left(\tan\left(\frac{1}{2}x\right)+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+a*sin(x))^2,x, algorithm="giac")
```

```
[Out] -2/3*(3*tan(1/2*x) + 1)/(a^2*(tan(1/2*x) + 1)^3)
```

3.16 $\int \frac{1}{(a+a\sin(x))^2}\,dx$

Optimal. Leaf size=33

$$-\frac{\cos(x)}{3\left(a^2\sin(x)+a^2\right)}-\frac{\cos(x)}{3(a\sin(x)+a)^2}$$

```
[Out] -Cos[x]/(3*(a + a*Sin[x])^2) - Cos[x]/(3*(a^2 + a^2*Sin[x]))
```

Rubi [A] time = 0.0211535, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2650, 2648}

$$-\frac{\cos(x)}{3\left(a^2\sin(x)+a^2\right)}-\frac{\cos(x)}{3(a\sin(x)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[x])^(-2),x]
```

```
[Out] -Cos[x]/(3*(a + a*Sin[x])^2) - Cos[x]/(3*(a^2 + a^2*Sin[x]))
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+a\sin(x))^2}\,dx &= -\frac{\cos(x)}{3(a+a\sin(x))^2}+\frac{\int \frac{1}{a+a\sin(x)}\,dx}{3a}\\ &= -\frac{\cos(x)}{3(a+a\sin(x))^2}-\frac{\cos(x)}{3\left(a^2+a^2\sin(x)\right)}\end{aligned}$$

Mathematica [A] time = 0.0277993, size = 31, normalized size = 0.94

$$-\frac{-4\sin(x)+\sin(2x)+4\cos(x)+\cos(2x)-3}{6a^2(\sin(x)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[x])^(-2),x]
```

```
[Out] -(-3 + 4*Cos[x] + Cos[2*x] - 4*Sin[x] + Sin[2*x])/(6*a^2*(1 + Sin[x])^2)
```

Maple [A] time = 0.027, size = 35, normalized size = 1.1

$$2\frac{1}{a^2}\left((\tan(x/2)+1)^{-2} - 2/3\,(\tan(x/2)+1)^{-3} - (\tan(x/2)+1)^{-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(x))^2,x)
```

```
[Out] 2/a^2*(1/(tan(1/2*x)+1)^2-2/3/(tan(1/2*x)+1)^3-1/(tan(1/2*x)+1))
```

Maxima [B] time = 1.69752, size = 100, normalized size = 3.03

$$-\frac{2\left(\frac{3\sin(x)}{\cos(x)+1} + \frac{3\sin(x)^2}{(\cos(x)+1)^2} + 2\right)}{3\left(a^2 + \frac{3a^2\sin(x)}{\cos(x)+1} + \frac{3a^2\sin(x)^2}{(\cos(x)+1)^2} + \frac{a^2\sin(x)^3}{(\cos(x)+1)^3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(x))^2,x, algorithm="maxima")
```

```
[Out] -2/3*(3*sin(x)/(cos(x) + 1) + 3*sin(x)^2/(cos(x) + 1)^2 + 2)/(a^2 + 3*a^2*s
in(x)/(cos(x) + 1) + 3*a^2*sin(x)^2/(cos(x) + 1)^2 + a^2*sin(x)^3/(cos(x) +
 1)^3)
```

Fricas [A] time = 1.33675, size = 161, normalized size = 4.88

$$\frac{\cos(x)^2 + (\cos(x)-1)\sin(x) + 2\cos(x) + 1}{3\left(a^2\cos(x)^2 - a^2\cos(x) - 2a^2 - \left(a^2\cos(x) + 2a^2\right)\sin(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(x))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(cos(x)^2 + (cos(x) - 1)*sin(x) + 2*cos(x) + 1)/(a^2*cos(x)^2 - a^2*cos
(x) - 2*a^2 - (a^2*cos(x) + 2*a^2)*sin(x))
```

Sympy [B] time = 1.46067, size = 87, normalized size = 2.64

$$\frac{2\tan^3\left(\frac{x}{2}\right)}{3a^2\tan^3\left(\frac{x}{2}\right) + 9a^2\tan^2\left(\frac{x}{2}\right) + 9a^2\tan\left(\frac{x}{2}\right) + 3a^2} - \frac{2}{3a^2\tan^3\left(\frac{x}{2}\right) + 9a^2\tan^2\left(\frac{x}{2}\right) + 9a^2\tan\left(\frac{x}{2}\right) + 3a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(x))**2,x)
```

```
[Out] 2*tan(x/2)**3/(3*a**2*tan(x/2)**3 + 9*a**2*tan(x/2)**2 + 9*a**2*tan(x/2) +
3*a**2) - 2/(3*a**2*tan(x/2)**3 + 9*a**2*tan(x/2)**2 + 9*a**2*tan(x/2) + 3*
a**2)
```

Giac [A] time = 1.72625, size = 39, normalized size = 1.18

$$-\frac{2\left(3\tan\left(\frac{1}{2}x\right)^2+3\tan\left(\frac{1}{2}x\right)+2\right)}{3a^2\left(\tan\left(\frac{1}{2}x\right)+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(x))^2,x, algorithm="giac")
```

```
[Out] -2/3*(3*tan(1/2*x)^2 + 3*tan(1/2*x) + 2)/(a^2*(tan(1/2*x) + 1)^3)
```

3.17 $\int \frac{\csc(x)}{(a+a\sin(x))^2}\,dx$

Optimal. Leaf size=38

$$\frac{4\cos(x)}{3a^2(\sin(x)+1)}-\frac{\tanh^{-1}(\cos(x))}{a^2}+\frac{\cos(x)}{3(a\sin(x)+a)^2}$$

```
[Out] -(ArcTanh[Cos[x]]/a^2) + (4*Cos[x])/(3*a^2*(1 + Sin[x])) + Cos[x]/(3*(a + a
*Sin[x])^2)
```

Rubi [A] time = 0.0882768, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2766, 2978, 12, 3770}

$$\frac{4\cos(x)}{3a^2(\sin(x)+1)}-\frac{\tanh^{-1}(\cos(x))}{a^2}+\frac{\cos(x)}{3(a\sin(x)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]/(a + a*Sin[x])^2,x]
```

```
[Out] -(ArcTanh[Cos[x]]/a^2) + (4*Cos[x])/(3*a^2*(1 + Sin[x])) + Cos[x]/(3*(a + a
*Sin[x])^2)
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\csc(x)}{(a+a\sin(x))^2}\,dx &= \frac{\cos(x)}{3(a+a\sin(x))^2} + \frac{\int \frac{\csc(x)(3a-a\sin(x))}{a+a\sin(x)}\,dx}{3a^2} \\
&= \frac{4\cos(x)}{3a^2(1+\sin(x))} + \frac{\cos(x)}{3(a+a\sin(x))^2} + \frac{\int 3a^2\csc(x)\,dx}{3a^4} \\
&= \frac{4\cos(x)}{3a^2(1+\sin(x))} + \frac{\cos(x)}{3(a+a\sin(x))^2} + \frac{\int \csc(x)\,dx}{a^2} \\
&= -\frac{\tanh^{-1}(\cos(x))}{a^2} + \frac{4\cos(x)}{3a^2(1+\sin(x))} + \frac{\cos(x)}{3(a+a\sin(x))^2}
\end{aligned}
$$

Mathematica [B] time = 0.135999, size = 129, normalized size = 3.39

$$
\frac{\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)\left(\cos\left(\frac{3x}{2}\right)\left(-3\log\left(\sin\left(\frac{x}{2}\right)\right)+3\log\left(\cos\left(\frac{x}{2}\right)\right)+8\right)+\cos\left(\frac{x}{2}\right)\left(9\log\left(\sin\left(\frac{x}{2}\right)\right)-9\log\left(\cos\left(\frac{x}{2}\right)\right)-6\right)-6}{6a^2(\sin(x)+1)^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]/(a + a*Sin[x])^2,x]
```

```
[Out] ((Cos[x/2] + Sin[x/2])*(Cos[(3*x)/2]*(8 + 3*Log[Cos[x/2]] - 3*Log[Sin[x/2]]
) + Cos[x/2]*(-6 - 9*Log[Cos[x/2]] + 9*Log[Sin[x/2]]) - 6*(3 + 2*Log[Cos[x/
2]] + Cos[x]*(Log[Cos[x/2]] - Log[Sin[x/2]]) - 2*Log[Sin[x/2]])*Sin[x/2]))/
(6*a^2*(1 + Sin[x])^2)
```

Maple [A] time = 0.049, size = 50, normalized size = 1.3

$$
\frac{4}{3a^2}\left(\tan\left(\frac{x}{2}\right)+1\right)^{-3} - 2\frac{1}{a^2\left(\tan(x/2)+1\right)^2} + 4\frac{1}{a^2\left(\tan(x/2)+1\right)} + \frac{1}{a^2}\ln\left(\tan\left(\frac{x}{2}\right)\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)/(a+a*sin(x))^2,x)
```

```
[Out] 4/3/a^2/(tan(1/2*x)+1)^3-2/a^2/(tan(1/2*x)+1)^2+4/a^2/(tan(1/2*x)+1)+1/a^2*
ln(tan(1/2*x))
```

Maxima [B] time = 1.64702, size = 120, normalized size = 3.16

$$
\frac{2\left(\frac{9\sin(x)}{\cos(x)+1}+\frac{6\sin(x)^2}{(\cos(x)+1)^2}+5\right)}{3\left(a^2+\frac{3a^2\sin(x)}{\cos(x)+1}+\frac{3a^2\sin(x)^2}{(\cos(x)+1)^2}+\frac{a^2\sin(x)^3}{(\cos(x)+1)^3}\right)} + \frac{\log\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a^2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a+a*sin(x))^2,x, algorithm="maxima")
```

```
[Out] 2/3*(9*sin(x)/(cos(x) + 1) + 6*sin(x)^2/(cos(x) + 1)^2 + 5)/(a^2 + 3*a^2*si
n(x)/(cos(x) + 1) + 3*a^2*sin(x)^2/(cos(x) + 1)^2 + a^2*sin(x)^3/(cos(x) +
```

```
1)^3) + log(sin(x)/(cos(x) + 1))/a^2
```

Fricas [B] time = 1.48328, size = 367, normalized size = 9.66

$$-\frac{8\cos(x)^2+3\left(\cos(x)^2-(\cos(x)+2)\sin(x)-\cos(x)-2\right)\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)-3\left(\cos(x)^2-(\cos(x)+2)\sin(x)-\right.}{6\left(a^2\cos(x)^2-a^2\cos(x)-2a^2-\left(a^2\cos(x)+2a\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a+a*sin(x))^2,x, algorithm="fricas")
```

```
[Out] -1/6*(8*cos(x)^2 + 3*(cos(x)^2 - (cos(x) + 2)*sin(x) - cos(x) - 2)*log(1/2*
cos(x) + 1/2) - 3*(cos(x)^2 - (cos(x) + 2)*sin(x) - cos(x) - 2)*log(-1/2*co
s(x) + 1/2) + 2*(4*cos(x) - 1)*sin(x) + 10*cos(x) + 2)/(a^2*cos(x)^2 - a^2*
cos(x) - 2*a^2 - (a^2*cos(x) + 2*a^2)*sin(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\csc(x)}{\sin^2(x)+2\sin(x)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a+a*sin(x))**2,x)
```

```
[Out] Integral(csc(x)/(sin(x)**2 + 2*sin(x) + 1), x)/a**2
```

Giac [A] time = 1.76838, size = 54, normalized size = 1.42

$$\frac{\log\left(\left|\tan\left(\frac{1}{2}x\right)\right|\right)}{a^2}+\frac{2\left(6\tan\left(\frac{1}{2}x\right)^2+9\tan\left(\frac{1}{2}x\right)+5\right)}{3a^2\left(\tan\left(\frac{1}{2}x\right)+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a+a*sin(x))^2,x, algorithm="giac")
```

```
[Out] log(abs(tan(1/2*x)))/a^2 + 2/3*(6*tan(1/2*x)^2 + 9*tan(1/2*x) + 5)/(a^2*(ta
n(1/2*x) + 1)^3)
```

3.18 $\int \frac{\csc^2(x)}{(a+a\sin(x))^2}\,dx$

Optimal. Leaf size=45

$$-\frac{10\cot(x)}{3a^2}+\frac{2\tanh^{-1}(\cos(x))}{a^2}+\frac{2\cot(x)}{a^2(\sin(x)+1)}+\frac{\cot(x)}{3(a\sin(x)+a)^2}$$

```
[Out] (2*ArcTanh[Cos[x]])/a^2 - (10*Cot[x])/(3*a^2) + (2*Cot[x])/(a^2*(1 + Sin[x]
)) + Cot[x]/(3*(a + a*Sin[x])^2)
```

Rubi [A] time = 0.134503, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2766, 2978, 2748, 3767, 8, 3770}

$$-\frac{10\cot(x)}{3a^2}+\frac{2\tanh^{-1}(\cos(x))}{a^2}+\frac{2\cot(x)}{a^2(\sin(x)+1)}+\frac{\cot(x)}{3(a\sin(x)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^2/(a + a*Sin[x])^2,x]
```

```
[Out] (2*ArcTanh[Cos[x]])/a^2 - (10*Cot[x])/(3*a^2) + (2*Cot[x])/(a^2*(1 + Sin[x]
)) + Cot[x]/(3*(a + a*Sin[x])^2)
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\csc^2(x)}{(a+a\sin(x))^2}\,dx &= \frac{\cot(x)}{3(a+a\sin(x))^2}+\frac{\int \frac{\csc^2(x)(4a-2a\sin(x))}{a+a\sin(x)}\,dx}{3a^2}\\
&= \frac{2\cot(x)}{a^2(1+\sin(x))}+\frac{\cot(x)}{3(a+a\sin(x))^2}+\frac{\int \csc^2(x)\left(10a^2-6a^2\sin(x)\right)\,dx}{3a^4}\\
&= \frac{2\cot(x)}{a^2(1+\sin(x))}+\frac{\cot(x)}{3(a+a\sin(x))^2}-\frac{2\int \csc(x)\,dx}{a^2}+\frac{10\int \csc^2(x)\,dx}{3a^2}\\
&= \frac{2\tanh^{-1}(\cos(x))}{a^2}+\frac{2\cot(x)}{a^2(1+\sin(x))}+\frac{\cot(x)}{3(a+a\sin(x))^2}-\frac{10\,\mathrm{Subst}(\int 1\,dx,x,\cot(x))}{3a^2}\\
&= \frac{2\tanh^{-1}(\cos(x))}{a^2}-\frac{10\cot(x)}{3a^2}+\frac{2\cot(x)}{a^2(1+\sin(x))}+\frac{\cot(x)}{3(a+a\sin(x))^2}
\end{aligned}
$$

Mathematica [B] time = 0.345518, size = 166, normalized size = 3.69

$$
\frac{\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)\left(4\sin\left(\frac{x}{2}\right)+28\sin\left(\frac{x}{2}\right)\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)^2-2\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)+12\log\left(\cos\left(\frac{x}{2}\right)\right)\left(\sin\left(\frac{x}{2}\right)+c\right.}{6(a\sin(x)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^2/(a + a*Sin[x])^2,x]
```

```
[Out] ((Cos[x/2] + Sin[x/2])*(4*Sin[x/2] - 2*(Cos[x/2] + Sin[x/2]) + 28*Sin[x/2]*
(Cos[x/2] + Sin[x/2])^2 - 3*Cot[x/2]*(Cos[x/2] + Sin[x/2])^3 + 12*Log[Cos[x
/2]]*(Cos[x/2] + Sin[x/2])^3 - 12*Log[Sin[x/2]]*(Cos[x/2] + Sin[x/2])^3 + 3
*(Cos[x/2] + Sin[x/2])^3*Tan[x/2]))/(6*(a + a*Sin[x])^2)
```

Maple [A] time = 0.053, size = 71, normalized size = 1.6

$$
\frac{1}{2a^2}\tan\left(\frac{x}{2}\right)-\frac{4}{3a^2}\left(\tan\left(\frac{x}{2}\right)+1\right)^{-3}+2\frac{1}{a^2\left(\tan\left(x/2\right)+1\right)^2}-6\frac{1}{a^2\left(\tan\left(x/2\right)+1\right)}-\frac{1}{2a^2}\left(\tan\left(\frac{x}{2}\right)\right)^{-1}-2\frac{\ln\left(\tan\left(x/2\right.\right.}{a^2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^2/(a+a*sin(x))^2,x)
```

```
[Out] 1/2/a^2*tan(1/2*x)-4/3/a^2/(tan(1/2*x)+1)^3+2/a^2/(tan(1/2*x)+1)^2-6/a^2/(t
an(1/2*x)+1)-1/2/a^2/tan(1/2*x)-2/a^2*ln(tan(1/2*x))
```

Maxima [B] time = 1.63623, size = 170, normalized size = 3.78

$$-\frac{\frac{41\sin(x)}{\cos(x)+1}+\frac{69\sin(x)^2}{(\cos(x)+1)^2}+\frac{39\sin(x)^3}{(\cos(x)+1)^3}+3}{6\left(\frac{a^2\sin(x)}{\cos(x)+1}+\frac{3a^2\sin(x)^2}{(\cos(x)+1)^2}+\frac{3a^2\sin(x)^3}{(\cos(x)+1)^3}+\frac{a^2\sin(x)^4}{(\cos(x)+1)^4}\right)}-\frac{2\log\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a^2}+\frac{\sin(x)}{2a^2(\cos(x)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a+a*sin(x))^2,x, algorithm="maxima")
```

```
[Out] -1/6*(41*sin(x)/(cos(x) + 1) + 69*sin(x)^2/(cos(x) + 1)^2 + 39*sin(x)^3/(co
s(x) + 1)^3 + 3)/(a^2*sin(x)/(cos(x) + 1) + 3*a^2*sin(x)^2/(cos(x) + 1)^2 +
 3*a^2*sin(x)^3/(cos(x) + 1)^3 + a^2*sin(x)^4/(cos(x) + 1)^4) - 2*log(sin(x
)/(cos(x) + 1))/a^2 + 1/2*sin(x)/(a^2*(cos(x) + 1))
```

Fricas [B] time = 1.43824, size = 512, normalized size = 11.38

$$-\frac{10\cos(x)^3-4\cos(x)^2-3\left(\cos(x)^3+2\cos(x)^2+\left(\cos(x)^2-\cos(x)-2\right)\sin(x)-\cos(x)-2\right)\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)+\cdots}{3\left(a^2\cos(x)^3+2a^2\cos(x)^2-\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a+a*sin(x))^2,x, algorithm="fricas")
```

```
[Out] -1/3*(10*cos(x)^3 - 4*cos(x)^2 - 3*(cos(x)^3 + 2*cos(x)^2 + (cos(x)^2 - cos
(x) - 2)*sin(x) - cos(x) - 2)*log(1/2*cos(x) + 1/2) + 3*(cos(x)^3 + 2*cos(x
)^2 + (cos(x)^2 - cos(x) - 2)*sin(x) - cos(x) - 2)*log(-1/2*cos(x) + 1/2) -
 (10*cos(x)^2 + 14*cos(x) + 1)*sin(x) - 13*cos(x) + 1)/(a^2*cos(x)^3 + 2*a^
2*cos(x)^2 - a^2*cos(x) - 2*a^2 + (a^2*cos(x)^2 - a^2*cos(x) - 2*a^2)*sin(x
))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{\csc^2(x)}{\sin^2(x)+2\sin(x)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**2/(a+a*sin(x))**2,x)
```

```
[Out] Integral(csc(x)**2/(sin(x)**2 + 2*sin(x) + 1), x)/a**2
```

Giac [A] time = 2.04621, size = 93, normalized size = 2.07

$$-\frac{2\log\left(\left|\tan\left(\frac{1}{2}x\right)\right|\right)}{a^2}+\frac{\tan\left(\frac{1}{2}x\right)}{2a^2}+\frac{4\tan\left(\frac{1}{2}x\right)-1}{2a^2\tan\left(\frac{1}{2}x\right)}-\frac{2\left(9\tan\left(\frac{1}{2}x\right)^2+15\tan\left(\frac{1}{2}x\right)+8\right)}{3a^2\left(\tan\left(\frac{1}{2}x\right)+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a+a*sin(x))^2,x, algorithm="giac")
```

```
[Out] -2*log(abs(tan(1/2*x)))/a^2 + 1/2*tan(1/2*x)/a^2 + 1/2*(4*tan(1/2*x) - 1)/(
a^2*tan(1/2*x)) - 2/3*(9*tan(1/2*x)^2 + 15*tan(1/2*x) + 8)/(a^2*(tan(1/2*x)
 + 1)^3)
```

3.19 $\int \frac{\csc^3(x)}{(a+a\sin(x))^2}\,dx$

Optimal. Leaf size=64

$$\frac{16\cot(x)}{3a^2}-\frac{7\tanh^{-1}(\cos(x))}{2a^2}-\frac{7\cot(x)\csc(x)}{2a^2}+\frac{8\cot(x)\csc(x)}{3a^2(\sin(x)+1)}+\frac{\cot(x)\csc(x)}{3(a\sin(x)+a)^2}$$

```
[Out] (-7*ArcTanh[Cos[x]])/(2*a^2) + (16*Cot[x])/(3*a^2) - (7*Cot[x]*Csc[x])/(2*a
^2) + (8*Cot[x]*Csc[x])/(3*a^2*(1 + Sin[x])) + (Cot[x]*Csc[x])/(3*(a + a*Si
n[x])^2)
```

Rubi [A] time = 0.145165, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {2766, 2978, 2748, 3768, 3770, 3767, 8}

$$\frac{16\cot(x)}{3a^2}-\frac{7\tanh^{-1}(\cos(x))}{2a^2}-\frac{7\cot(x)\csc(x)}{2a^2}+\frac{8\cot(x)\csc(x)}{3a^2(\sin(x)+1)}+\frac{\cot(x)\csc(x)}{3(a\sin(x)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^3/(a + a*Sin[x])^2,x]
```


```
[Out] (-7*ArcTanh[Cos[x]])/(2*a^2) + (16*Cot[x])/(3*a^2) - (7*Cot[x]*Csc[x])/(2*a
^2) + (8*Cot[x]*Csc[x])/(3*a^2*(1 + Sin[x])) + (Cot[x]*Csc[x])/(3*(a + a*Si
n[x])^2)
```


Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc^3(x)}{(a+a\sin(x))^2}\,dx &= \frac{\cot(x)\csc(x)}{3(a+a\sin(x))^2}+\frac{\int \frac{\csc^3(x)(5a-3a\sin(x))}{a+a\sin(x)}\,dx}{3a^2}\\
&= \frac{8\cot(x)\csc(x)}{3a^2(1+\sin(x))}+\frac{\cot(x)\csc(x)}{3(a+a\sin(x))^2}+\frac{\int \csc^3(x)\left(21a^2-16a^2\sin(x)\right)dx}{3a^4}\\
&= \frac{8\cot(x)\csc(x)}{3a^2(1+\sin(x))}+\frac{\cot(x)\csc(x)}{3(a+a\sin(x))^2}-\frac{16\int \csc^2(x)\,dx}{3a^2}+\frac{7\int \csc^3(x)\,dx}{a^2}\\
&= -\frac{7\cot(x)\csc(x)}{2a^2}+\frac{8\cot(x)\csc(x)}{3a^2(1+\sin(x))}+\frac{\cot(x)\csc(x)}{3(a+a\sin(x))^2}+\frac{7\int \csc(x)\,dx}{2a^2}+\frac{16\,\mathrm{Subst}(\int 1\,dx,x,c}{3a^2}\\
&= -\frac{7\tanh^{-1}(\cos(x))}{2a^2}+\frac{16\cot(x)}{3a^2}-\frac{7\cot(x)\csc(x)}{2a^2}+\frac{8\cot(x)\csc(x)}{3a^2(1+\sin(x))}+\frac{\cot(x)\csc(x)}{3(a+a\sin(x))^2}
\end{aligned}$$

Mathematica [B] time = 0.597961, size = 203, normalized size = 3.17

$$\frac{\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)\left(-16\sin\left(\frac{x}{2}\right)-160\sin\left(\frac{x}{2}\right)\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)^2+8\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)+3\cos\left(\frac{x}{2}\right)\left(\tan\left(\frac{x}{2}\right)+1\right)^3\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^3/(a + a*Sin[x])^2,x]
```

```
[Out] ((Cos[x/2] + Sin[x/2])*(-16*Sin[x/2] - 3*(1 + Cot[x/2])^3*Sin[x/2] + 8*(Cos
[x/2] + Sin[x/2]) - 160*Sin[x/2]*(Cos[x/2] + Sin[x/2])^2 + 24*Cot[x/2]*(Cos
[x/2] + Sin[x/2])^3 - 84*Log[Cos[x/2]]*(Cos[x/2] + Sin[x/2])^3 + 84*Log[Sin
[x/2]]*(Cos[x/2] + Sin[x/2])^3 - 24*(Cos[x/2] + Sin[x/2])^3*Tan[x/2] + 3*Co
s[x/2]*(1 + Tan[x/2])^3))/(24*a^2*(1 + Sin[x])^2)
```

Maple [A] time = 0.061, size = 92, normalized size = 1.4

$$\frac{1}{8a^2}\left(\tan\left(\frac{x}{2}\right)\right)^2-\frac{1}{a^2}\tan\left(\frac{x}{2}\right)+\frac{4}{3a^2}\left(\tan\left(\frac{x}{2}\right)+1\right)^{-3}-2\frac{1}{a^2\left(\tan(x/2)+1\right)^2}+8\frac{1}{a^2\left(\tan(x/2)+1\right)}-\frac{1}{8a^2}\left(\tan\left(\frac{x}{2}\right)\right)^{-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^3/(a+a*sin(x))^2,x)
```

```
[Out] 1/8/a^2*tan(1/2*x)^2-1/a^2*tan(1/2*x)+4/3/a^2/(tan(1/2*x)+1)^3-2/a^2/(tan(1
/2*x)+1)^2+8/a^2/(tan(1/2*x)+1)-1/8/a^2/tan(1/2*x)^2+1/a^2/tan(1/2*x)+7/2/a
^2*ln(tan(1/2*x))
```

Maxima [B] time = 1.84253, size = 209, normalized size = 3.27

$$\frac{\frac{15\sin(x)}{\cos(x)+1}+\frac{239\sin(x)^2}{(\cos(x)+1)^2}+\frac{405\sin(x)^3}{(\cos(x)+1)^3}+\frac{216\sin(x)^4}{(\cos(x)+1)^4}-3}{24\left(\frac{a^2\sin(x)^2}{(\cos(x)+1)^2}+\frac{3a^2\sin(x)^3}{(\cos(x)+1)^3}+\frac{3a^2\sin(x)^4}{(\cos(x)+1)^4}+\frac{a^2\sin(x)^5}{(\cos(x)+1)^5}\right)}-\frac{\frac{8\sin(x)}{\cos(x)+1}-\frac{\sin(x)^2}{(\cos(x)+1)^2}}{8a^2}+\frac{7\log\left(\frac{\sin(x)}{\cos(x)+1}\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(a+a*sin(x))^2,x, algorithm="maxima")
```

```
[Out] 1/24*(15*sin(x)/(cos(x) + 1) + 239*sin(x)^2/(cos(x) + 1)^2 + 405*sin(x)^3/(
cos(x) + 1)^3 + 216*sin(x)^4/(cos(x) + 1)^4 - 3)/(a^2*sin(x)^2/(cos(x) + 1)
^2 + 3*a^2*sin(x)^3/(cos(x) + 1)^3 + 3*a^2*sin(x)^4/(cos(x) + 1)^4 + a^2*si
n(x)^5/(cos(x) + 1)^5) - 1/8*(8*sin(x)/(cos(x) + 1) - sin(x)^2/(cos(x) + 1)
^2)/a^2 + 7/2*log(sin(x)/(cos(x) + 1))/a^2
```

Fricas [B] time = 1.55372, size = 666, normalized size = 10.41

$$-\frac{64\cos(x)^4+86\cos(x)^3-54\cos(x)^2+21\left(\cos(x)^4-\cos(x)^3-3\cos(x)^2-\left(\cos(x)^3+2\cos(x)^2-\cos(x)-2\right)\sin(x)\right.}{12\left(a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(a+a*sin(x))^2,x, algorithm="fricas")
```

```
[Out] -1/12*(64*cos(x)^4 + 86*cos(x)^3 - 54*cos(x)^2 + 21*(cos(x)^4 - cos(x)^3 -
3*cos(x)^2 - (cos(x)^3 + 2*cos(x)^2 - cos(x) - 2)*sin(x) + cos(x) + 2)*log(
1/2*cos(x) + 1/2) - 21*(cos(x)^4 - cos(x)^3 - 3*cos(x)^2 - (cos(x)^3 + 2*co
s(x)^2 - cos(x) - 2)*sin(x) + cos(x) + 2)*log(-1/2*cos(x) + 1/2) + 2*(32*co
s(x)^3 - 11*cos(x)^2 - 38*cos(x) + 2)*sin(x) - 80*cos(x) - 4)/(a^2*cos(x)^4
 - a^2*cos(x)^3 - 3*a^2*cos(x)^2 + a^2*cos(x) + 2*a^2 - (a^2*cos(x)^3 + 2*a
^2*cos(x)^2 - a^2*cos(x) - 2*a^2)*sin(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\csc^3(x)}{\sin^2(x)+2\sin(x)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**3/(a+a*sin(x))**2,x)
```

```
[Out] Integral(csc(x)**3/(sin(x)**2 + 2*sin(x) + 1), x)/a**2
```

Giac [A] time = 2.1323, size = 126, normalized size = 1.97

$$\frac{7\log\left(\left|\tan\left(\frac{1}{2}x\right)\right|\right)}{2a^2}+\frac{a^2\tan\left(\frac{1}{2}x\right)^2-8a^2\tan\left(\frac{1}{2}x\right)}{8a^4}-\frac{42\tan\left(\frac{1}{2}x\right)^2-8\tan\left(\frac{1}{2}x\right)+1}{8a^2\tan\left(\frac{1}{2}x\right)^2}+\frac{2\left(12\tan\left(\frac{1}{2}x\right)^2+21\tan\left(\right.\right.}{3a^2\left(\tan\left(\frac{1}{2}x\right)+1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(a+a*sin(x))^2,x, algorithm="giac")
```

```
[Out] 7/2*log(abs(tan(1/2*x)))/a^2 + 1/8*(a^2*tan(1/2*x)^2 - 8*a^2*tan(1/2*x))/a^
4 - 1/8*(42*tan(1/2*x)^2 - 8*tan(1/2*x) + 1)/(a^2*tan(1/2*x)^2) + 2/3*(12*t
an(1/2*x)^2 + 21*tan(1/2*x) + 11)/(a^2*(tan(1/2*x) + 1)^3)
```

3.20 $\int \frac{\csc^4(x)}{(a+a\sin(x))^2}\,dx$

Optimal. Leaf size=65

$$-\frac{\cot^3(x)}{3a^2}-\frac{4\cot(x)}{a^2}-\frac{13\cos(x)}{3a^2(\sin(x)+1)}-\frac{\cos(x)}{3a^2(\sin(x)+1)^2}+\frac{5\tanh^{-1}(\cos(x))}{a^2}+\frac{\cot(x)\csc(x)}{a^2}$$

```
[Out] (5*ArcTanh[Cos[x]])/a^2 - (4*Cot[x])/a^2 - Cot[x]^3/(3*a^2) + (Cot[x]*Csc[x
])/a^2 - Cos[x]/(3*a^2*(1 + Sin[x])^2) - (13*Cos[x])/(3*a^2*(1 + Sin[x]))
```

Rubi [A] time = 0.150109, antiderivative size = 71, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2766, 2978, 2748, 3767, 3768, 3770}

$$-\frac{4\cot^3(x)}{a^2}-\frac{12\cot(x)}{a^2}+\frac{5\tanh^{-1}(\cos(x))}{a^2}+\frac{5\cot(x)\csc(x)}{a^2}+\frac{10\cot(x)\csc^2(x)}{3a^2(\sin(x)+1)}+\frac{\cot(x)\csc^2(x)}{3(a\sin(x)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^4/(a + a*Sin[x])^2,x]
```

```
[Out] (5*ArcTanh[Cos[x]])/a^2 - (12*Cot[x])/a^2 - (4*Cot[x]^3)/a^2 + (5*Cot[x]*Cs
c[x])/a^2 + (10*Cot[x]*Csc[x]^2)/(3*a^2*(1 + Sin[x])) + (Cot[x]*Csc[x]^2)/(
3*(a + a*Sin[x])^2)
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^4(x)}{(a+a\sin(x))^2}\,dx &= \frac{\cot(x)\csc^2(x)}{3(a+a\sin(x))^2}+\frac{\int \frac{\csc^4(x)(6a-4a\sin(x))}{a+a\sin(x)}\,dx}{3a^2}\\ &= \frac{10\cot(x)\csc^2(x)}{3a^2(1+\sin(x))}+\frac{\cot(x)\csc^2(x)}{3(a+a\sin(x))^2}+\frac{\int \csc^4(x)\left(36a^2-30a^2\sin(x)\right)\,dx}{3a^4}\\ &= \frac{10\cot(x)\csc^2(x)}{3a^2(1+\sin(x))}+\frac{\cot(x)\csc^2(x)}{3(a+a\sin(x))^2}-\frac{10\int \csc^3(x)\,dx}{a^2}+\frac{12\int \csc^4(x)\,dx}{a^2}\\ &= \frac{5\cot(x)\csc(x)}{a^2}+\frac{10\cot(x)\csc^2(x)}{3a^2(1+\sin(x))}+\frac{\cot(x)\csc^2(x)}{3(a+a\sin(x))^2}-\frac{5\int \csc(x)\,dx}{a^2}-\frac{12\,\mathrm{Subst}\left(\int \left(1+x^2\right)\right.}{a^2}\\ &= \frac{5\tanh^{-1}(\cos(x))}{a^2}-\frac{12\cot(x)}{a^2}-\frac{4\cot^3(x)}{a^2}+\frac{5\cot(x)\csc(x)}{a^2}+\frac{10\cot(x)\csc^2(x)}{3a^2(1+\sin(x))}+\frac{\cot(x)\csc}{3(a+a\sin}\end{aligned}$$

Mathematica [B] time = 3.18151, size = 238, normalized size = 3.66

$$\frac{\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)\left(16\sin\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\left(\tan\left(\frac{x}{2}\right)+1\right)^3+208\sin\left(\frac{x}{2}\right)\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)^2-8\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)-6}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^4/(a + a*Sin[x])^2,x]
```

```
[Out] ((Cos[x/2] + Sin[x/2])*(-(Cos[x/2]*(1 + Cot[x/2])^3) + 16*Sin[x/2] + 6*(1 +
 Cot[x/2])^3*Sin[x/2] - 8*(Cos[x/2] + Sin[x/2]) + 208*Sin[x/2]*(Cos[x/2] +
Sin[x/2])^2 - 44*Cot[x/2]*(Cos[x/2] + Sin[x/2])^3 + 120*Log[Cos[x/2]]*(Cos[
x/2] + Sin[x/2])^3 - 120*Log[Sin[x/2]]*(Cos[x/2] + Sin[x/2])^3 + 44*(Cos[x/
2] + Sin[x/2])^3*Tan[x/2] - 6*Cos[x/2]*(1 + Tan[x/2])^3 + Sin[x/2]*(1 + Tan
[x/2])^3))/(24*a^2*(1 + Sin[x])^2)
```

Maple [A] time = 0.062, size = 115, normalized size = 1.8

$$\frac{1}{24a^2}\left(\tan\left(\frac{x}{2}\right)\right)^3-\frac{1}{4a^2}\left(\tan\left(\frac{x}{2}\right)\right)^2+\frac{15}{8a^2}\tan\left(\frac{x}{2}\right)-\frac{4}{3a^2}\left(\tan\left(\frac{x}{2}\right)+1\right)^{-3}+2\frac{1}{a^2\left(\tan(x/2)+1\right)^2}-10\frac{1}{a^2\left(\tan(x/2)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^4/(a+a*sin(x))^2,x)
```

```
[Out] 1/24/a^2*tan(1/2*x)^3-1/4/a^2*tan(1/2*x)^2+15/8/a^2*tan(1/2*x)-4/3/a^2/(tan
(1/2*x)+1)^3+2/a^2/(tan(1/2*x)+1)^2-10/a^2/(tan(1/2*x)+1)-1/24/a^2/tan(1/2*
x)^3+1/4/a^2/tan(1/2*x)^2-15/8/a^2/tan(1/2*x)-5/a^2*ln(tan(1/2*x))
```

Maxima [B] time = 1.48871, size = 240, normalized size = 3.69

$$\frac{\frac{3\sin(x)}{\cos(x)+1} - \frac{30\sin(x)^2}{(\cos(x)+1)^2} - \frac{342\sin(x)^3}{(\cos(x)+1)^3} - \frac{561\sin(x)^4}{(\cos(x)+1)^4} - \frac{285\sin(x)^5}{(\cos(x)+1)^5} - 1}{24\left(\frac{a^2\sin(x)^3}{(\cos(x)+1)^3} + \frac{3a^2\sin(x)^4}{(\cos(x)+1)^4} + \frac{3a^2\sin(x)^5}{(\cos(x)+1)^5} + \frac{a^2\sin(x)^6}{(\cos(x)+1)^6}\right)} + \frac{\frac{45\sin(x)}{\cos(x)+1} - \frac{6\sin(x)^2}{(\cos(x)+1)^2} + \frac{\sin(x)^3}{(\cos(x)+1)^3}}{24a^2} - \frac{5\log\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^4/(a+a*sin(x))^2,x, algorithm="maxima")
```

```
[Out] 1/24*(3*sin(x)/(cos(x) + 1) - 30*sin(x)^2/(cos(x) + 1)^2 - 342*sin(x)^3/(co
s(x) + 1)^3 - 561*sin(x)^4/(cos(x) + 1)^4 - 285*sin(x)^5/(cos(x) + 1)^5 - 1
)/(a^2*sin(x)^3/(cos(x) + 1)^3 + 3*a^2*sin(x)^4/(cos(x) + 1)^4 + 3*a^2*sin(
x)^5/(cos(x) + 1)^5 + a^2*sin(x)^6/(cos(x) + 1)^6) + 1/24*(45*sin(x)/(cos(x
) + 1) - 6*sin(x)^2/(cos(x) + 1)^2 + sin(x)^3/(cos(x) + 1)^3)/a^2 - 5*log(s
in(x)/(cos(x) + 1))/a^2
```

Fricas [B] time = 1.48931, size = 819, normalized size = 12.6

$$-\frac{48\cos(x)^5 - 18\cos(x)^4 - 108\cos(x)^3 + 22\cos(x)^2 - 15\left(\cos(x)^5 + 2\cos(x)^4 - 2\cos(x)^3 - 4\cos(x)^2 + \left(\cos(x)^4 - \right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^4/(a+a*sin(x))^2,x, algorithm="fricas")
```

```
[Out] -1/6*(48*cos(x)^5 - 18*cos(x)^4 - 108*cos(x)^3 + 22*cos(x)^2 - 15*(cos(x)^5
 + 2*cos(x)^4 - 2*cos(x)^3 - 4*cos(x)^2 + (cos(x)^4 - cos(x)^3 - 3*cos(x)^2
 + cos(x) + 2)*sin(x) + cos(x) + 2)*log(1/2*cos(x) + 1/2) + 15*(cos(x)^5 +
2*cos(x)^4 - 2*cos(x)^3 - 4*cos(x)^2 + (cos(x)^4 - cos(x)^3 - 3*cos(x)^2 +
cos(x) + 2)*sin(x) + cos(x) + 2)*log(-1/2*cos(x) + 1/2) - 2*(24*cos(x)^4 +
33*cos(x)^3 - 21*cos(x)^2 - 32*cos(x) - 1)*sin(x) + 62*cos(x) - 2)/(a^2*cos
(x)^5 + 2*a^2*cos(x)^4 - 2*a^2*cos(x)^3 - 4*a^2*cos(x)^2 + a^2*cos(x) + 2*a
^2 + (a^2*cos(x)^4 - a^2*cos(x)^3 - 3*a^2*cos(x)^2 + a^2*cos(x) + 2*a^2)*si
n(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\csc^4(x)}{\sin^2(x)+2\sin(x)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**4/(a+a*sin(x))**2,x)
```

```
[Out] Integral(csc(x)**4/(sin(x)**2 + 2*sin(x) + 1), x)/a**2
```

Giac [A] time = 2.02992, size = 154, normalized size = 2.37

$$-\frac{5\log\left(\left|\tan\left(\frac{1}{2}x\right)\right|\right)}{a^2}+\frac{110\tan\left(\frac{1}{2}x\right)^6+45\tan\left(\frac{1}{2}x\right)^5-231\tan\left(\frac{1}{2}x\right)^4-232\tan\left(\frac{1}{2}x\right)^3-30\tan\left(\frac{1}{2}x\right)^2+3\tan\left(\frac{1}{2}\right.}{24\left(\tan\left(\frac{1}{2}x\right)^2+\tan\left(\frac{1}{2}x\right)\right)^3a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^4/(a+a*sin(x))^2,x, algorithm="giac")
```

```
[Out] -5*log(abs(tan(1/2*x)))/a^2 + 1/24*(110*tan(1/2*x)^6 + 45*tan(1/2*x)^5 - 23
1*tan(1/2*x)^4 - 232*tan(1/2*x)^3 - 30*tan(1/2*x)^2 + 3*tan(1/2*x) - 1)/((t
an(1/2*x)^2 + tan(1/2*x))^3*a^2) + 1/24*(a^4*tan(1/2*x)^3 - 6*a^4*tan(1/2*x
)^2 + 45*a^4*tan(1/2*x))/a^6
```

3.21 $\int \frac{\sin^6(x)}{(a+a\sin(x))^3}\,dx$

Optimal. Leaf size=101

$$-\frac{23x}{2a^3}+\frac{136\cos^3(x)}{15a^3}-\frac{136\cos(x)}{5a^3}+\frac{23\sin^3(x)\cos(x)}{3\left(a^3\sin(x)+a^3\right)}+\frac{23\sin(x)\cos(x)}{2a^3}+\frac{\sin^5(x)\cos(x)}{5(a\sin(x)+a)^3}+\frac{13\sin^4(x)\cos(x)}{15a(a\sin(x)+a)^2}$$

```
[Out] (-23*x)/(2*a^3) - (136*Cos[x])/(5*a^3) + (136*Cos[x]^3)/(15*a^3) + (23*Cos[
x]*Sin[x])/(2*a^3) + (Cos[x]*Sin[x]^5)/(5*(a + a*Sin[x])^3) + (13*Cos[x]*Si
n[x]^4)/(15*a*(a + a*Sin[x])^2) + (23*Cos[x]*Sin[x]^3)/(3*(a^3 + a^3*Sin[x]
))
```

Rubi [A] time = 0.226027, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2765, 2977, 2748, 2635, 8, 2633}

$$-\frac{23x}{2a^3}+\frac{136\cos^3(x)}{15a^3}-\frac{136\cos(x)}{5a^3}+\frac{23\sin^3(x)\cos(x)}{3\left(a^3\sin(x)+a^3\right)}+\frac{23\sin(x)\cos(x)}{2a^3}+\frac{\sin^5(x)\cos(x)}{5(a\sin(x)+a)^3}+\frac{13\sin^4(x)\cos(x)}{15a(a\sin(x)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^6/(a + a*Sin[x])^3,x]
```

```
[Out] (-23*x)/(2*a^3) - (136*Cos[x])/(5*a^3) + (136*Cos[x]^3)/(15*a^3) + (23*Cos[
x]*Sin[x])/(2*a^3) + (Cos[x]*Sin[x]^5)/(5*(a + a*Sin[x])^3) + (13*Cos[x]*Si
n[x]^4)/(15*a*(a + a*Sin[x])^2) + (23*Cos[x]*Sin[x]^3)/(3*(a^3 + a^3*Sin[x]
))
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^6(x)}{(a+a\sin(x))^3}\,dx &= \frac{\cos(x)\sin^5(x)}{5(a+a\sin(x))^3} - \frac{\int \frac{\sin^4(x)(5a-8a\sin(x))}{(a+a\sin(x))^2}\,dx}{5a^2} \\
&= \frac{\cos(x)\sin^5(x)}{5(a+a\sin(x))^3} + \frac{13\cos(x)\sin^4(x)}{15a(a+a\sin(x))^2} - \frac{\int \frac{\sin^3(x)\left(52a^2-63a^2\sin(x)\right)}{a+a\sin(x)}\,dx}{15a^4} \\
&= \frac{\cos(x)\sin^5(x)}{5(a+a\sin(x))^3} + \frac{13\cos(x)\sin^4(x)}{15a(a+a\sin(x))^2} + \frac{23\cos(x)\sin^3(x)}{3\left(a^3+a^3\sin(x)\right)} - \frac{\int \sin^2(x)\left(345a^3-408a^3\sin(x)\right)}{15a^6} \\
&= \frac{\cos(x)\sin^5(x)}{5(a+a\sin(x))^3} + \frac{13\cos(x)\sin^4(x)}{15a(a+a\sin(x))^2} + \frac{23\cos(x)\sin^3(x)}{3\left(a^3+a^3\sin(x)\right)} - \frac{23\int \sin^2(x)\,dx}{a^3} + \frac{136\int \sin^3(x)\,d}{5a^3} \\
&= \frac{23\cos(x)\sin(x)}{2a^3} + \frac{\cos(x)\sin^5(x)}{5(a+a\sin(x))^3} + \frac{13\cos(x)\sin^4(x)}{15a(a+a\sin(x))^2} + \frac{23\cos(x)\sin^3(x)}{3\left(a^3+a^3\sin(x)\right)} - \frac{23\int 1\,dx}{2a^3} - \frac{13}{} \\
&= -\frac{23x}{2a^3} - \frac{136\cos(x)}{5a^3} + \frac{136\cos^3(x)}{15a^3} + \frac{23\cos(x)\sin(x)}{2a^3} + \frac{\cos(x)\sin^5(x)}{5(a+a\sin(x))^3} + \frac{13\cos(x)\sin^4(x)}{15a(a+a\sin(x))^2}
\end{aligned}$$

Mathematica [A] time = 0.102024, size = 191, normalized size = 1.89

$$\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)\left(24\sin\left(\frac{x}{2}\right)-690x\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)^5-405\cos(x)\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)^5+5\cos(3x)\left(\sin\left(\frac{x}{2}\right)+c\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^6/(a + a*Sin[x])^3,x]
```

```
[Out] ((Cos[x/2] + Sin[x/2])*(24*Sin[x/2] - 12*(Cos[x/2] + Sin[x/2]) - 224*Sin[x/
2]*(Cos[x/2] + Sin[x/2])^2 + 112*(Cos[x/2] + Sin[x/2])^3 + 1576*Sin[x/2]*(C
os[x/2] + Sin[x/2])^4 - 690*x*(Cos[x/2] + Sin[x/2])^5 - 405*Cos[x]*(Cos[x/2
] + Sin[x/2])^5 + 5*Cos[3*x]*(Cos[x/2] + Sin[x/2])^5 + 45*(Cos[x/2] + Sin[x
/2])^5*Sin[2*x]))/(60*(a + a*Sin[x])^3)
```

Maple [A] time = 0.052, size = 174, normalized size = 1.7

$$-3\,\frac{(\tan(x/2))^5}{a^3\left((\tan(x/2))^2+1\right)^3}-12\,\frac{(\tan(x/2))^4}{a^3\left((\tan(x/2))^2+1\right)^3}-28\,\frac{(\tan(x/2))^2}{a^3\left((\tan(x/2))^2+1\right)^3}+3\,\frac{\tan(x/2)}{a^3\left((\tan(x/2))^2+1\right)^3}-\frac{40}{3\,a^3}\left(\left(\tan\left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^6/(a+a*sin(x))^3,x)
```

```
[Out] -3/a^3/(tan(1/2*x)^2+1)^3*tan(1/2*x)^5-12/a^3/(tan(1/2*x)^2+1)^3*tan(1/2*x)
^4-28/a^3/(tan(1/2*x)^2+1)^3*tan(1/2*x)^2+3/a^3/(tan(1/2*x)^2+1)^3*tan(1/2*
x)-40/3/a^3/(tan(1/2*x)^2+1)^3-23/a^3*arctan(tan(1/2*x))-8/5/a^3/(tan(1/2*x
)+1)^5+4/a^3/(tan(1/2*x)+1)^4+8/3/a^3/(tan(1/2*x)+1)^3-8/a^3/(tan(1/2*x)+1)
^2-20/a^3/(tan(1/2*x)+1)
```

Maxima [B] time = 2.16743, size = 413, normalized size = 4.09

$$-\frac{\frac{2375\sin(x)}{\cos(x)+1}+\frac{5347\sin(x)^2}{(\cos(x)+1)^2}+\frac{9230\sin(x)^3}{(\cos(x)+1)^3}+\frac{12622\sin(x)^4}{(\cos(x)+1)^4}+\frac{13340\sin(x)^5}{(\cos(x)+1)^5}+\frac{11684\sin(x)^6}{(\cos(x)+1)^6}+\frac{8050\sin(x)^7}{(\cos(x)+1)^7}+\frac{4370\sin(x)^8}{(\cos(x)+1)^8}+\frac{1725\sin(}{(\cos(x)+1}}{15\left(a^3+\frac{5a^3\sin(x)}{\cos(x)+1}+\frac{13a^3\sin(x)^2}{(\cos(x)+1)^2}+\frac{25a^3\sin(x)^3}{(\cos(x)+1)^3}+\frac{38a^3\sin(x)^4}{(\cos(x)+1)^4}+\frac{46a^3\sin(x)^5}{(\cos(x)+1)^5}+\frac{46a^3\sin(x)^6}{(\cos(x)+1)^6}+\frac{38a^3\sin(x)^7}{(\cos(x)+1)^7}+\frac{25a^3\sin(x)^8}{(\cos(x)+1)^8}+\frac{13a^3\sin(x)^9}{(\cos(x)+1)^9}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^6/(a+a*sin(x))^3,x, algorithm="maxima")
```

```
[Out] -1/15*(2375*sin(x)/(cos(x) + 1) + 5347*sin(x)^2/(cos(x) + 1)^2 + 9230*sin(x
)^3/(cos(x) + 1)^3 + 12622*sin(x)^4/(cos(x) + 1)^4 + 13340*sin(x)^5/(cos(x)
 + 1)^5 + 11684*sin(x)^6/(cos(x) + 1)^6 + 8050*sin(x)^7/(cos(x) + 1)^7 + 43
70*sin(x)^8/(cos(x) + 1)^8 + 1725*sin(x)^9/(cos(x) + 1)^9 + 345*sin(x)^10/(
cos(x) + 1)^10 + 544)/(a^3 + 5*a^3*sin(x)/(cos(x) + 1) + 13*a^3*sin(x)^2/(c
os(x) + 1)^2 + 25*a^3*sin(x)^3/(cos(x) + 1)^3 + 38*a^3*sin(x)^4/(cos(x) + 1
)^4 + 46*a^3*sin(x)^5/(cos(x) + 1)^5 + 46*a^3*sin(x)^6/(cos(x) + 1)^6 + 38*
a^3*sin(x)^7/(cos(x) + 1)^7 + 25*a^3*sin(x)^8/(cos(x) + 1)^8 + 13*a^3*sin(x
)^9/(cos(x) + 1)^9 + 5*a^3*sin(x)^10/(cos(x) + 1)^10 + a^3*sin(x)^11/(cos(x
) + 1)^11) - 23*arctan(sin(x)/(cos(x) + 1))/a^3
```

Fricas [A] time = 1.55864, size = 479, normalized size = 4.74

$$\frac{10\cos(x)^6-15\cos(x)^5-(345\,x+839)\cos(x)^3-140\cos(x)^4-(1035\,x-668)\cos(x)^2+6\,(115\,x+233)\cos(x)+\left(10\right.}{30\left(a^3\cos(x)^3+3\,a^3\cos(x)^2-2\,a^3\cos(x)-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^6/(a+a*sin(x))^3,x, algorithm="fricas")
```

```
[Out] 1/30*(10*cos(x)^6 - 15*cos(x)^5 - (345*x + 839)*cos(x)^3 - 140*cos(x)^4 - (
1035*x - 668)*cos(x)^2 + 6*(115*x + 233)*cos(x) + (10*cos(x)^5 + 25*cos(x)^
4 - (345*x - 724)*cos(x)^2 - 115*cos(x)^3 + 6*(115*x + 232)*cos(x) + 1380*x
 - 6)*sin(x) + 1380*x + 6)/(a^3*cos(x)^3 + 3*a^3*cos(x)^2 - 2*a^3*cos(x) -
4*a^3 + (a^3*cos(x)^2 - 2*a^3*cos(x) - 4*a^3)*sin(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**6/(a+a*sin(x))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 2.08568, size = 134, normalized size = 1.33

$$-\frac{23\,x}{2\,a^3} - \frac{9\,\tan\left(\frac{1}{2}\,x\right)^5 + 36\,\tan\left(\frac{1}{2}\,x\right)^4 + 84\,\tan\left(\frac{1}{2}\,x\right)^2 - 9\,\tan\left(\frac{1}{2}\,x\right) + 40}{3\left(\tan\left(\frac{1}{2}\,x\right)^2 + 1\right)^3 a^3} - \frac{4\left(75\,\tan\left(\frac{1}{2}\,x\right)^4 + 330\,\tan\left(\frac{1}{2}\,x\right)^3 + 530 \cdots\right.}{15\,a^3\left(\tan\left(\frac{1}{2} \cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^6/(a+a*sin(x))^3,x, algorithm="giac")
```

```
[Out] -23/2*x/a^3 - 1/3*(9*tan(1/2*x)^5 + 36*tan(1/2*x)^4 + 84*tan(1/2*x)^2 - 9*t
an(1/2*x) + 40)/((tan(1/2*x)^2 + 1)^3*a^3) - 4/15*(75*tan(1/2*x)^4 + 330*ta
n(1/2*x)^3 + 530*tan(1/2*x)^2 + 355*tan(1/2*x) + 86)/(a^3*(tan(1/2*x) + 1)^
5)
```

3.22 $\int \frac{\sin^5(x)}{(a+a\sin(x))^3}\,dx$

Optimal. Leaf size=90

$$\frac{13x}{2a^3}+\frac{152\cos(x)}{15a^3}+\frac{76\sin^2(x)\cos(x)}{15\left(a^3\sin(x)+a^3\right)}-\frac{13\sin(x)\cos(x)}{2a^3}+\frac{\sin^4(x)\cos(x)}{5(a\sin(x)+a)^3}+\frac{11\sin^3(x)\cos(x)}{15a(a\sin(x)+a)^2}$$

```
[Out] (13*x)/(2*a^3) + (152*Cos[x])/(15*a^3) - (13*Cos[x]*Sin[x])/(2*a^3) + (Cos[
x]*Sin[x]^4)/(5*(a + a*Sin[x])^3) + (11*Cos[x]*Sin[x]^3)/(15*a*(a + a*Sin[x
])^2) + (76*Cos[x]*Sin[x]^2)/(15*(a^3 + a^3*Sin[x]))
```

Rubi [A] time = 0.208, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2765, 2977, 2734}

$$\frac{13x}{2a^3}+\frac{152\cos(x)}{15a^3}+\frac{76\sin^2(x)\cos(x)}{15\left(a^3\sin(x)+a^3\right)}-\frac{13\sin(x)\cos(x)}{2a^3}+\frac{\sin^4(x)\cos(x)}{5(a\sin(x)+a)^3}+\frac{11\sin^3(x)\cos(x)}{15a(a\sin(x)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^5/(a + a*Sin[x])^3,x]
```

```
[Out] (13*x)/(2*a^3) + (152*Cos[x])/(15*a^3) - (13*Cos[x]*Sin[x])/(2*a^3) + (Cos[
x]*Sin[x]^4)/(5*(a + a*Sin[x])^3) + (11*Cos[x]*Sin[x]^3)/(15*a*(a + a*Sin[x
])^2) + (76*Cos[x]*Sin[x]^2)/(15*(a^3 + a^3*Sin[x]))
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{\sin^5(x)}{(a+a\sin(x))^3}\,dx = \frac{\cos(x)\sin^4(x)}{5(a+a\sin(x))^3} - \frac{\int \frac{\sin^3(x)(4a-7a\sin(x))}{(a+a\sin(x))^2}\,dx}{5a^2}$$

$$= \frac{\cos(x)\sin^4(x)}{5(a+a\sin(x))^3} + \frac{11\cos(x)\sin^3(x)}{15a(a+a\sin(x))^2} - \frac{\int \frac{\sin^2(x)\left(33a^2-43a^2\sin(x)\right)}{a+a\sin(x)}\,dx}{15a^4}$$

$$= \frac{\cos(x)\sin^4(x)}{5(a+a\sin(x))^3} + \frac{11\cos(x)\sin^3(x)}{15a(a+a\sin(x))^2} + \frac{76\cos(x)\sin^2(x)}{15\left(a^3+a^3\sin(x)\right)} - \frac{\int \sin(x)\left(152a^3-195a^3\sin(x)\right)}{15a^6}$$

$$= \frac{13x}{2a^3} + \frac{152\cos(x)}{15a^3} - \frac{13\cos(x)\sin(x)}{2a^3} + \frac{\cos(x)\sin^4(x)}{5(a+a\sin(x))^3} + \frac{11\cos(x)\sin^3(x)}{15a(a+a\sin(x))^2} + \frac{76\cos(x)\sin}{15\left(a^3+a^3\sin\right.}$$

Mathematica [A] time = 0.076316, size = 170, normalized size = 1.89

$$\frac{\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)\left(-24\sin\left(\frac{x}{2}\right)+390x\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)^5+180\cos(x)\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)^5-15\sin(2x)\left(\sin\left(\frac{x}{2}\right)+\right.}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^5/(a + a*Sin[x])^3,x]
```

```
[Out] ((Cos[x/2] + Sin[x/2])*(-24*Sin[x/2] + 12*(Cos[x/2] + Sin[x/2]) + 184*Sin[x
/2]*(Cos[x/2] + Sin[x/2])^2 - 92*(Cos[x/2] + Sin[x/2])^3 - 1016*Sin[x/2]*(C
os[x/2] + Sin[x/2])^4 + 390*x*(Cos[x/2] + Sin[x/2])^5 + 180*Cos[x]*(Cos[x/2
] + Sin[x/2])^5 - 15*(Cos[x/2] + Sin[x/2])^5*Sin[2*x]))/(60*(a + a*Sin[x])^
3)
```

Maple [A] time = 0.046, size = 152, normalized size = 1.7

$$\frac{1}{a^3}\left(\tan\left(\frac{x}{2}\right)\right)^3\left(\left(\tan\left(\frac{x}{2}\right)\right)^2+1\right)^{-2}+6\frac{(\tan(x/2))^2}{a^3\left((\tan(x/2))^2+1\right)^2}-\frac{1}{a^3}\tan\left(\frac{x}{2}\right)\left(\left(\tan\left(\frac{x}{2}\right)\right)^2+1\right)^{-2}+6\frac{1}{a^3\left((\tan(x/2))^2+1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^5/(a+a*sin(x))^3,x)
```

```
[Out] 1/a^3/(tan(1/2*x)^2+1)^2*tan(1/2*x)^3+6/a^3/(tan(1/2*x)^2+1)^2*tan(1/2*x)^2
-1/a^3/(tan(1/2*x)^2+1)^2*tan(1/2*x)+6/a^3/(tan(1/2*x)^2+1)^2+13/a^3*arctan
(tan(1/2*x))+8/5/a^3/(tan(1/2*x)+1)^5-4/a^3/(tan(1/2*x)+1)^4-4/3/a^3/(tan(1
/2*x)+1)^3+6/a^3/(tan(1/2*x)+1)^2+12/a^3/(tan(1/2*x)+1)
```

Maxima [B] time = 2.74715, size = 340, normalized size = 3.78

$$\frac{\frac{1325\sin(x)}{\cos(x)+1}+\frac{2673\sin(x)^2}{(\cos(x)+1)^2}+\frac{3805\sin(x)^3}{(\cos(x)+1)^3}+\frac{4329\sin(x)^4}{(\cos(x)+1)^4}+\frac{3575\sin(x)^5}{(\cos(x)+1)^5}+\frac{2275\sin(x)^6}{(\cos(x)+1)^6}+\frac{975\sin(x)^7}{(\cos(x)+1)^7}+\frac{195\sin(x)^8}{(\cos(x)+1)^8}+304}{15\left(a^3+\frac{5a^3\sin(x)}{\cos(x)+1}+\frac{12a^3\sin(x)^2}{(\cos(x)+1)^2}+\frac{20a^3\sin(x)^3}{(\cos(x)+1)^3}+\frac{26a^3\sin(x)^4}{(\cos(x)+1)^4}+\frac{26a^3\sin(x)^5}{(\cos(x)+1)^5}+\frac{20a^3\sin(x)^6}{(\cos(x)+1)^6}+\frac{12a^3\sin(x)^7}{(\cos(x)+1)^7}+\frac{5a^3\sin(x)^8}{(\cos(x)+1)^8}+\frac{a^3\sin(x)^9}{(\cos(x)+1)}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^5/(a+a*sin(x))^3,x, algorithm="maxima")
```

```
[Out] 1/15*(1325*sin(x)/(cos(x) + 1) + 2673*sin(x)^2/(cos(x) + 1)^2 + 3805*sin(x)
^3/(cos(x) + 1)^3 + 4329*sin(x)^4/(cos(x) + 1)^4 + 3575*sin(x)^5/(cos(x) +
1)^5 + 2275*sin(x)^6/(cos(x) + 1)^6 + 975*sin(x)^7/(cos(x) + 1)^7 + 195*sin
(x)^8/(cos(x) + 1)^8 + 304)/(a^3 + 5*a^3*sin(x)/(cos(x) + 1) + 12*a^3*sin(x
)^2/(cos(x) + 1)^2 + 20*a^3*sin(x)^3/(cos(x) + 1)^3 + 26*a^3*sin(x)^4/(cos(
x) + 1)^4 + 26*a^3*sin(x)^5/(cos(x) + 1)^5 + 20*a^3*sin(x)^6/(cos(x) + 1)^6
 + 12*a^3*sin(x)^7/(cos(x) + 1)^7 + 5*a^3*sin(x)^8/(cos(x) + 1)^8 + a^3*sin
(x)^9/(cos(x) + 1)^9) + 13*arctan(sin(x)/(cos(x) + 1))/a^3
```

Fricas [A] time = 1.41183, size = 432, normalized size = 4.8

$$\frac{15 \cos(x)^5 + (195x + 449)\cos(x)^3 + 60 \cos(x)^4 + (585x - 358)\cos(x)^2 - 6(65x + 128)\cos(x) - \left(15 \cos(x)^4 - (195x\right.}{30\left(a^3 \cos(x)^3 + 3a^3 \cos(x)^2 - 2a^3 \cos(x) - 4a^3 + \left(a^3 \cos(x)^2\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^5/(a+a*sin(x))^3,x, algorithm="fricas")
```

```
[Out] 1/30*(15*cos(x)^5 + (195*x + 449)*cos(x)^3 + 60*cos(x)^4 + (585*x - 358)*co
s(x)^2 - 6*(65*x + 128)*cos(x) - (15*cos(x)^4 - (195*x - 404)*cos(x)^2 - 45
*cos(x)^3 + 6*(65*x + 127)*cos(x) + 780*x - 6)*sin(x) - 780*x - 6)/(a^3*cos
(x)^3 + 3*a^3*cos(x)^2 - 2*a^3*cos(x) - 4*a^3 + (a^3*cos(x)^2 - 2*a^3*cos(x
) - 4*a^3)*sin(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**5/(a+a*sin(x))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.63836, size = 119, normalized size = 1.32

$$\frac{13x}{2a^3} + \frac{\tan\left(\frac{1}{2}x\right)^3 + 6\tan\left(\frac{1}{2}x\right)^2 - \tan\left(\frac{1}{2}x\right) + 6}{\left(\tan\left(\frac{1}{2}x\right)^2 + 1\right)^2 a^3} + \frac{2\left(90\tan\left(\frac{1}{2}x\right)^4 + 405\tan\left(\frac{1}{2}x\right)^3 + 665\tan\left(\frac{1}{2}x\right)^2 + 445\tan\left(\frac{1}{2}x\right.\right.}{15a^3\left(\tan\left(\frac{1}{2}x\right) + 1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^5/(a+a*sin(x))^3,x, algorithm="giac")
```

```
[Out] 13/2*x/a^3 + (tan(1/2*x)^3 + 6*tan(1/2*x)^2 - tan(1/2*x) + 6)/((tan(1/2*x)^
2 + 1)^2*a^3) + 2/15*(90*tan(1/2*x)^4 + 405*tan(1/2*x)^3 + 665*tan(1/2*x)^2
 + 445*tan(1/2*x) + 107)/(a^3*(tan(1/2*x) + 1)^5)
```

3.23 $\int \frac{\sin^4(x)}{(a+a\sin(x))^3}\,dx$

Optimal. Leaf size=71

$$-\frac{3x}{a^3}-\frac{9\cos(x)}{5a^3}-\frac{3\cos(x)}{a^3\sin(x)+a^3}+\frac{\sin^3(x)\cos(x)}{5(a\sin(x)+a)^3}+\frac{3\sin^2(x)\cos(x)}{5a(a\sin(x)+a)^2}$$

```
[Out] (-3*x)/a^3 - (9*Cos[x])/(5*a^3) + (Cos[x]*Sin[x]^3)/(5*(a + a*Sin[x])^3) +
(3*Cos[x]*Sin[x]^2)/(5*a*(a + a*Sin[x])^2) - (3*Cos[x])/(a^3 + a^3*Sin[x])
```

Rubi [A] time = 0.221008, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {2765, 2977, 2968, 3023, 12, 2735, 2648}

$$-\frac{3x}{a^3}-\frac{9\cos(x)}{5a^3}-\frac{3\cos(x)}{a^3\sin(x)+a^3}+\frac{\sin^3(x)\cos(x)}{5(a\sin(x)+a)^3}+\frac{3\sin^2(x)\cos(x)}{5a(a\sin(x)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^4/(a + a*Sin[x])^3,x]
```

```
[Out] (-3*x)/a^3 - (9*Cos[x])/(5*a^3) + (Cos[x]*Sin[x]^3)/(5*(a + a*Sin[x])^3) +
(3*Cos[x]*Sin[x]^2)/(5*a*(a + a*Sin[x])^2) - (3*Cos[x])/(a^3 + a^3*Sin[x])
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sin^4(x)}{(a+a\sin(x))^3}\,dx &= \frac{\cos(x)\sin^3(x)}{5(a+a\sin(x))^3}-\frac{\int \frac{\sin^2(x)(3a-6a\sin(x))}{(a+a\sin(x))^2}\,dx}{5a^2}\\
&= \frac{\cos(x)\sin^3(x)}{5(a+a\sin(x))^3}+\frac{3\cos(x)\sin^2(x)}{5a(a+a\sin(x))^2}-\frac{\int \frac{\sin(x)\left(18a^2-27a^2\sin(x)\right)}{a+a\sin(x)}\,dx}{15a^4}\\
&= \frac{\cos(x)\sin^3(x)}{5(a+a\sin(x))^3}+\frac{3\cos(x)\sin^2(x)}{5a(a+a\sin(x))^2}-\frac{\int \frac{18a^2\sin(x)-27a^2\sin^2(x)}{a+a\sin(x)}\,dx}{15a^4}\\
&= -\frac{9\cos(x)}{5a^3}+\frac{\cos(x)\sin^3(x)}{5(a+a\sin(x))^3}+\frac{3\cos(x)\sin^2(x)}{5a(a+a\sin(x))^2}-\frac{\int \frac{45a^3\sin(x)}{a+a\sin(x)}\,dx}{15a^5}\\
&= -\frac{9\cos(x)}{5a^3}+\frac{\cos(x)\sin^3(x)}{5(a+a\sin(x))^3}+\frac{3\cos(x)\sin^2(x)}{5a(a+a\sin(x))^2}-\frac{3\int \frac{\sin(x)}{a+a\sin(x)}\,dx}{a^2}\\
&= -\frac{3x}{a^3}-\frac{9\cos(x)}{5a^3}+\frac{\cos(x)\sin^3(x)}{5(a+a\sin(x))^3}+\frac{3\cos(x)\sin^2(x)}{5a(a+a\sin(x))^2}+\frac{3\int \frac{1}{a+a\sin(x)}\,dx}{a^2}\\
&= -\frac{3x}{a^3}-\frac{9\cos(x)}{5a^3}+\frac{\cos(x)\sin^3(x)}{5(a+a\sin(x))^3}+\frac{3\cos(x)\sin^2(x)}{5a(a+a\sin(x))^2}-\frac{3\cos(x)}{a^3+a^3\sin(x)}
\end{aligned}
$$

Mathematica [A] time = 0.0765933, size = 140, normalized size = 1.97

$$
\frac{\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)\left(\sin\left(\frac{x}{2}\right)-\cos\left(\frac{x}{2}\right)-15x\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)^5-5\cos(x)\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)^5+48\sin\left(\frac{x}{2}\right)\left(\sin\left(\frac{x}{2}\right)+c\right.}{5(a\sin(x)+a)^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^4/(a + a*Sin[x])^3,x]
```

```
[Out] ((Cos[x/2] + Sin[x/2])*(-Cos[x/2] + Sin[x/2] - 12*Sin[x/2]*(Cos[x/2] + Sin[
x/2])^2 + 6*(Cos[x/2] + Sin[x/2])^3 + 48*Sin[x/2]*(Cos[x/2] + Sin[x/2])^4 -
 15*x*(Cos[x/2] + Sin[x/2])^5 - 5*Cos[x]*(Cos[x/2] + Sin[x/2])^5))/(5*(a +
a*Sin[x])^3)
```

Maple [A] time = 0.046, size = 79, normalized size = 1.1

$$-2\frac{1}{a^3\left((\tan(x/2))^2+1\right)}-6\frac{\arctan(\tan(x/2))}{a^3}-\frac{8}{5a^3}\left(\tan\left(\frac{x}{2}\right)+1\right)^{-5}+4\frac{1}{a^3(\tan(x/2)+1)^4}-4\frac{1}{a^3(\tan(x/2)+1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^4/(a+a*sin(x))^3,x)
```

```
[Out] -2/a^3/(tan(1/2*x)^2+1)-6/a^3*arctan(tan(1/2*x))-8/5/a^3/(tan(1/2*x)+1)^5+4
/a^3/(tan(1/2*x)+1)^4-4/a^3/(tan(1/2*x)+1)^2-6/a^3/(tan(1/2*x)+1)
```

Maxima [B] time = 2.08608, size = 267, normalized size = 3.76

$$-\frac{2\left(\frac{105\sin(x)}{\cos(x)+1}+\frac{189\sin(x)^2}{(\cos(x)+1)^2}+\frac{200\sin(x)^3}{(\cos(x)+1)^3}+\frac{160\sin(x)^4}{(\cos(x)+1)^4}+\frac{75\sin(x)^5}{(\cos(x)+1)^5}+\frac{15\sin(x)^6}{(\cos(x)+1)^6}+24\right)}{5\left(a^3+\frac{5a^3\sin(x)}{\cos(x)+1}+\frac{11a^3\sin(x)^2}{(\cos(x)+1)^2}+\frac{15a^3\sin(x)^3}{(\cos(x)+1)^3}+\frac{15a^3\sin(x)^4}{(\cos(x)+1)^4}+\frac{11a^3\sin(x)^5}{(\cos(x)+1)^5}+\frac{5a^3\sin(x)^6}{(\cos(x)+1)^6}+\frac{a^3\sin(x)^7}{(\cos(x)+1)^7}\right)}-\frac{6\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^4/(a+a*sin(x))^3,x, algorithm="maxima")
```

```
[Out] -2/5*(105*sin(x)/(cos(x) + 1) + 189*sin(x)^2/(cos(x) + 1)^2 + 200*sin(x)^3/
(cos(x) + 1)^3 + 160*sin(x)^4/(cos(x) + 1)^4 + 75*sin(x)^5/(cos(x) + 1)^5 +
 15*sin(x)^6/(cos(x) + 1)^6 + 24)/(a^3 + 5*a^3*sin(x)/(cos(x) + 1) + 11*a^3
*sin(x)^2/(cos(x) + 1)^2 + 15*a^3*sin(x)^3/(cos(x) + 1)^3 + 15*a^3*sin(x)^4
/(cos(x) + 1)^4 + 11*a^3*sin(x)^5/(cos(x) + 1)^5 + 5*a^3*sin(x)^6/(cos(x) +
 1)^6 + a^3*sin(x)^7/(cos(x) + 1)^7) - 6*arctan(sin(x)/(cos(x) + 1))/a^3
```

Fricas [B] time = 1.39637, size = 379, normalized size = 5.34

$$-\frac{3(5x+13)\cos(x)^3+5\cos(x)^4+(45x-28)\cos(x)^2-3(10x+21)\cos(x)+\left((15x-34)\cos(x)^2+5\cos(x)^3-2\right.}{5\left(a^3\cos(x)^3+3a^3\cos(x)^2-2a^3\cos(x)-4a^3+\left(a^3\cos(x)^2-2a^3\cos(x)-4\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^4/(a+a*sin(x))^3,x, algorithm="fricas")
```

```
[Out] -1/5*(3*(5*x + 13)*cos(x)^3 + 5*cos(x)^4 + (45*x - 28)*cos(x)^2 - 3*(10*x +
 21)*cos(x) + ((15*x - 34)*cos(x)^2 + 5*cos(x)^3 - 2*(15*x + 31)*cos(x) - 6
0*x + 1)*sin(x) - 60*x - 1)/(a^3*cos(x)^3 + 3*a^3*cos(x)^2 - 2*a^3*cos(x) -
 4*a^3 + (a^3*cos(x)^2 - 2*a^3*cos(x) - 4*a^3)*sin(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**4/(a+a*sin(x))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.31993, size = 90, normalized size = 1.27

$$-\frac{3x}{a^3} - \frac{2}{\left(\tan\left(\frac{1}{2}x\right)^2 + 1\right)a^3} - \frac{2\left(15\tan\left(\frac{1}{2}x\right)^4 + 70\tan\left(\frac{1}{2}x\right)^3 + 120\tan\left(\frac{1}{2}x\right)^2 + 80\tan\left(\frac{1}{2}x\right) + 19\right)}{5a^3\left(\tan\left(\frac{1}{2}x\right) + 1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^4/(a+a*sin(x))^3,x, algorithm="giac")
```

```
[Out] -3*x/a^3 - 2/((tan(1/2*x)^2 + 1)*a^3) - 2/5*(15*tan(1/2*x)^4 + 70*tan(1/2*x
)^3 + 120*tan(1/2*x)^2 + 80*tan(1/2*x) + 19)/(a^3*(tan(1/2*x) + 1)^5)
```

3.24 $\int \frac{\sin^3(x)}{(a+a\sin(x))^3}\,dx$

<u>**Optimal**</u>. Leaf size=59

$$\frac{x}{a^3}+\frac{29\cos(x)}{15\left(a^3\sin(x)+a^3\right)}+\frac{\sin^2(x)\cos(x)}{5(a\sin(x)+a)^3}-\frac{7\cos(x)}{15a(a\sin(x)+a)^2}$$

```
[Out] x/a^3 + (Cos[x]*Sin[x]^2)/(5*(a + a*Sin[x])^3) - (7*Cos[x])/(15*a*(a + a*Si
n[x])^2) + (29*Cos[x])/(15*(a^3 + a^3*Sin[x]))
```

<u>**Rubi**</u> **[A]** time = 0.156979, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2765, 2968, 3019, 2735, 2648}

$$\frac{x}{a^3}+\frac{29\cos(x)}{15\left(a^3\sin(x)+a^3\right)}+\frac{\sin^2(x)\cos(x)}{5(a\sin(x)+a)^3}-\frac{7\cos(x)}{15a(a\sin(x)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^3/(a + a*Sin[x])^3,x]
```

```
[Out] x/a^3 + (Cos[x]*Sin[x]^2)/(5*(a + a*Sin[x])^3) - (7*Cos[x])/(15*a*(a + a*Si
n[x])^2) + (29*Cos[x])/(15*(a^3 + a^3*Sin[x]))
```

<u>Rule 2765</u>

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

<u>Rule 2968</u>

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

<u>Rule 3019</u>

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b - a
*B + b*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[1
/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*
B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

<u>Rule 2735</u>

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
```

```
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sin^3(x)}{(a+a\sin(x))^3}\,dx &= \frac{\cos(x)\sin^2(x)}{5(a+a\sin(x))^3}-\frac{\int \frac{\sin(x)(2a-5a\sin(x))}{(a+a\sin(x))^2}\,dx}{5a^2}\\
&= \frac{\cos(x)\sin^2(x)}{5(a+a\sin(x))^3}-\frac{\int \frac{2a\sin(x)-5a\sin^2(x)}{(a+a\sin(x))^2}\,dx}{5a^2}\\
&= \frac{\cos(x)\sin^2(x)}{5(a+a\sin(x))^3}-\frac{7\cos(x)}{15a(a+a\sin(x))^2}+\frac{\int \frac{-14a^2+15a^2\sin(x)}{a+a\sin(x)}\,dx}{15a^4}\\
&= \frac{x}{a^3}+\frac{\cos(x)\sin^2(x)}{5(a+a\sin(x))^3}-\frac{7\cos(x)}{15a(a+a\sin(x))^2}-\frac{29\int \frac{1}{a+a\sin(x)}\,dx}{15a^2}\\
&= \frac{x}{a^3}+\frac{\cos(x)\sin^2(x)}{5(a+a\sin(x))^3}-\frac{7\cos(x)}{15a(a+a\sin(x))^2}+\frac{29\cos(x)}{15\left(a^3+a^3\sin(x)\right)}
\end{aligned}
$$

Mathematica [A] time = 0.176702, size = 112, normalized size = 1.9

$$\frac{\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)\left(150x\sin\left(\frac{x}{2}\right)-370\sin\left(\frac{x}{2}\right)+75x\sin\left(\frac{3x}{2}\right)-90\sin\left(\frac{3x}{2}\right)-15x\sin\left(\frac{5x}{2}\right)+64\sin\left(\frac{5x}{2}\right)+30(5x-9)\cos\right.}{60a^3(\sin(x)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^3/(a + a*Sin[x])^3,x]
```

```
[Out] ((Cos[x/2] + Sin[x/2])*(30*(-9 + 5*x)*Cos[x/2] + (230 - 75*x)*Cos[(3*x)/2]
- 15*x*Cos[(5*x)/2] - 370*Sin[x/2] + 150*x*Sin[x/2] - 90*Sin[(3*x)/2] + 75*
x*Sin[(3*x)/2] + 64*Sin[(5*x)/2] - 15*x*Sin[(5*x)/2]))/(60*a^3*(1 + Sin[x])
^3)
```

Maple [A] time = 0.043, size = 77, normalized size = 1.3

$$2\,\frac{\arctan\left(\tan\left(x/2\right)\right)}{a^3}-4\,\frac{1}{a^3\left(\tan\left(x/2\right)+1\right)^4}+\frac{8}{5\,a^3}\left(\tan\left(\frac{x}{2}\right)+1\right)^{-5}+\frac{4}{3\,a^3}\left(\tan\left(\frac{x}{2}\right)+1\right)^{-3}+2\,\frac{1}{a^3\left(\tan\left(x/2\right)+1\right)^2}+2\,\frac{}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3/(a+a*sin(x))^3,x)
```

```
[Out] 2/a^3*arctan(tan(1/2*x))-4/a^3/(tan(1/2*x)+1)^4+8/5/a^3/(tan(1/2*x)+1)^5+4/
3/a^3/(tan(1/2*x)+1)^3+2/a^3/(tan(1/2*x)+1)^2+2/a^3/(tan(1/2*x)+1)
```

Maxima [B] time = 2.55501, size = 194, normalized size = 3.29

$$\frac{2\left(\frac{95\,\sin(x)}{\cos(x)+1} + \frac{145\,\sin(x)^2}{(\cos(x)+1)^2} + \frac{75\,\sin(x)^3}{(\cos(x)+1)^3} + \frac{15\,\sin(x)^4}{(\cos(x)+1)^4} + 22\right)}{15\left(a^3 + \frac{5\,a^3\sin(x)}{\cos(x)+1} + \frac{10\,a^3\sin(x)^2}{(\cos(x)+1)^2} + \frac{10\,a^3\sin(x)^3}{(\cos(x)+1)^3} + \frac{5\,a^3\sin(x)^4}{(\cos(x)+1)^4} + \frac{a^3\sin(x)^5}{(\cos(x)+1)^5}\right)} + \frac{2\,\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(a+a*sin(x))^3,x, algorithm="maxima")
```

```
[Out] 2/15*(95*sin(x)/(cos(x) + 1) + 145*sin(x)^2/(cos(x) + 1)^2 + 75*sin(x)^3/(c
os(x) + 1)^3 + 15*sin(x)^4/(cos(x) + 1)^4 + 22)/(a^3 + 5*a^3*sin(x)/(cos(x)
 + 1) + 10*a^3*sin(x)^2/(cos(x) + 1)^2 + 10*a^3*sin(x)^3/(cos(x) + 1)^3 + 5
*a^3*sin(x)^4/(cos(x) + 1)^4 + a^3*sin(x)^5/(cos(x) + 1)^5) + 2*arctan(sin(
x)/(cos(x) + 1))/a^3
```

Fricas [B] time = 1.42613, size = 340, normalized size = 5.76

$$\frac{(15\,x+32)\cos(x)^3 + (45\,x-19)\cos(x)^2 - 6\,(5\,x+9)\cos(x) + \left((15\,x-32)\cos(x)^2 - 3\,(10\,x+17)\cos(x) - 60\,x + 3\right.}{15\left(a^3\cos(x)^3 + 3\,a^3\cos(x)^2 - 2\,a^3\cos(x) - 4\,a^3 + \left(a^3\cos(x)^2 - 2\,a^3\cos(x) - 4\,a^3\right)\sin(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(a+a*sin(x))^3,x, algorithm="fricas")
```

```
[Out] 1/15*((15*x + 32)*cos(x)^3 + (45*x - 19)*cos(x)^2 - 6*(5*x + 9)*cos(x) + ((
15*x - 32)*cos(x)^2 - 3*(10*x + 17)*cos(x) - 60*x + 3)*sin(x) - 60*x - 3)/(
a^3*cos(x)^3 + 3*a^3*cos(x)^2 - 2*a^3*cos(x) - 4*a^3 + (a^3*cos(x)^2 - 2*a^
3*cos(x) - 4*a^3)*sin(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**3/(a+a*sin(x))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.23968, size = 69, normalized size = 1.17

$$\frac{x}{a^3} + \frac{2\left(15\,\tan\left(\frac{1}{2}\,x\right)^4 + 75\,\tan\left(\frac{1}{2}\,x\right)^3 + 145\,\tan\left(\frac{1}{2}\,x\right)^2 + 95\,\tan\left(\frac{1}{2}\,x\right) + 22\right)}{15\,a^3\left(\tan\left(\frac{1}{2}\,x\right) + 1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(a+a*sin(x))^3,x, algorithm="giac")
```

```
[Out] x/a^3 + 2/15*(15*tan(1/2*x)^4 + 75*tan(1/2*x)^3 + 145*tan(1/2*x)^2 + 95*tan
(1/2*x) + 22)/(a^3*(tan(1/2*x) + 1)^5)
```

3.25 $\int \frac{\sin^2(x)}{(a+a\sin(x))^3}\,dx$

Optimal. Leaf size=50

$$-\frac{7\cos(x)}{15\left(a^3\sin(x)+a^3\right)}+\frac{8\cos(x)}{15a(a\sin(x)+a)^2}-\frac{\cos(x)}{5(a\sin(x)+a)^3}$$

```
[Out] -Cos[x]/(5*(a + a*Sin[x])^3) + (8*Cos[x])/(15*a*(a + a*Sin[x])^2) - (7*Cos[
x])/(15*(a^3 + a^3*Sin[x]))
```

Rubi [A] time = 0.0760257, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2758, 2750, 2648}

$$-\frac{7\cos(x)}{15\left(a^3\sin(x)+a^3\right)}+\frac{8\cos(x)}{15a(a\sin(x)+a)^2}-\frac{\cos(x)}{5(a\sin(x)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^2/(a + a*Sin[x])^3,x]
```

```
[Out] -Cos[x]/(5*(a + a*Sin[x])^3) + (8*Cos[x])/(15*a*(a + a*Sin[x])^2) - (7*Cos[
x])/(15*(a^3 + a^3*Sin[x]))
```

Rule 2758

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)),
x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(a*m - b*(2*m
 + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -2^(-1)]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{\sin^2(x)}{(a+a\sin(x))^3}\,dx = -\frac{\cos(x)}{5(a+a\sin(x))^3} + \frac{\int \frac{-3a+5a\sin(x)}{(a+a\sin(x))^2}\,dx}{5a^2}$$
$$= -\frac{\cos(x)}{5(a+a\sin(x))^3} + \frac{8\cos(x)}{15a(a+a\sin(x))^2} + \frac{7\int \frac{1}{a+a\sin(x)}\,dx}{15a^2}$$
$$= -\frac{\cos(x)}{5(a+a\sin(x))^3} + \frac{8\cos(x)}{15a(a+a\sin(x))^2} - \frac{7\cos(x)}{15\left(a^3+a^3\sin(x)\right)}$$

Mathematica [A] time = 0.0638365, size = 47, normalized size = 0.94

$$\frac{105\sin(x) - 12\sin(2x) - 7\sin(3x) - 15\cos(x) - 42\cos(2x) + 7\cos(3x) + 70}{60a^3(\sin(x)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^2/(a + a*Sin[x])^3,x]
```

```
[Out] (70 - 15*Cos[x] - 42*Cos[2*x] + 7*Cos[3*x] + 105*Sin[x] - 12*Sin[2*x] - 7*S
in[3*x])/(60*a^3*(1 + Sin[x])^3)
```

Maple [A] time = 0.039, size = 37, normalized size = 0.7

$$8\,\frac{1}{a^3}\left(-1/5\,\left(\tan(x/2)+1\right)^{-5} + 1/2\,\left(\tan(x/2)+1\right)^{-4} - 1/3\,\left(\tan(x/2)+1\right)^{-3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2/(a+a*sin(x))^3,x)
```

```
[Out] 8/a^3*(-1/5/(tan(1/2*x)+1)^5+1/2/(tan(1/2*x)+1)^4-1/3/(tan(1/2*x)+1)^3)
```

Maxima [B] time = 1.75882, size = 140, normalized size = 2.8

$$-\frac{4\left(\frac{5\,\sin(x)}{\cos(x)+1} + \frac{10\,\sin(x)^2}{(\cos(x)+1)^2} + 1\right)}{15\left(a^3 + \frac{5\,a^3\sin(x)}{\cos(x)+1} + \frac{10\,a^3\sin(x)^2}{(\cos(x)+1)^2} + \frac{10\,a^3\sin(x)^3}{(\cos(x)+1)^3} + \frac{5\,a^3\sin(x)^4}{(\cos(x)+1)^4} + \frac{a^3\sin(x)^5}{(\cos(x)+1)^5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a+a*sin(x))^3,x, algorithm="maxima")
```

```
[Out] -4/15*(5*sin(x)/(cos(x) + 1) + 10*sin(x)^2/(cos(x) + 1)^2 + 1)/(a^3 + 5*a^3
*sin(x)/(cos(x) + 1) + 10*a^3*sin(x)^2/(cos(x) + 1)^2 + 10*a^3*sin(x)^3/(co
s(x) + 1)^3 + 5*a^3*sin(x)^4/(cos(x) + 1)^4 + a^3*sin(x)^5/(cos(x) + 1)^5)
```

Fricas [B] time = 1.40828, size = 250, normalized size = 5.

$$-\frac{7\,\cos(x)^3 + \cos(x)^2 - \left(7\,\cos(x)^2 + 6\,\cos(x) - 3\right)\sin(x) - 9\,\cos(x) - 3}{15\left(a^3\cos(x)^3 + 3\,a^3\cos(x)^2 - 2\,a^3\cos(x) - 4\,a^3 + \left(a^3\cos(x)^2 - 2\,a^3\cos(x) - 4\,a^3\right)\sin(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a+a*sin(x))^3,x, algorithm="fricas")
```

```
[Out] -1/15*(7*cos(x)^3 + cos(x)^2 - (7*cos(x)^2 + 6*cos(x) - 3)*sin(x) - 9*cos(x
) - 3)/(a^3*cos(x)^3 + 3*a^3*cos(x)^2 - 2*a^3*cos(x) - 4*a^3 + (a^3*cos(x)^
2 - 2*a^3*cos(x) - 4*a^3)*sin(x))
```

Sympy [B] time = 111.999, size = 212, normalized size = 4.24

$$\frac{4\tan^5\left(\frac{x}{2}\right)}{15a^3\tan^5\left(\frac{x}{2}\right)+75a^3\tan^4\left(\frac{x}{2}\right)+150a^3\tan^3\left(\frac{x}{2}\right)+150a^3\tan^2\left(\frac{x}{2}\right)+75a^3\tan\left(\frac{x}{2}\right)+15a^3}+\frac{}{15a^3\tan^5\left(\frac{x}{2}\right)+75a^3\tan^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**2/(a+a*sin(x))**3,x)
```

```
[Out] 4*tan(x/2)**5/(15*a**3*tan(x/2)**5 + 75*a**3*tan(x/2)**4 + 150*a**3*tan(x/2
)**3 + 150*a**3*tan(x/2)**2 + 75*a**3*tan(x/2) + 15*a**3) + 20*tan(x/2)**4/
(15*a**3*tan(x/2)**5 + 75*a**3*tan(x/2)**4 + 150*a**3*tan(x/2)**3 + 150*a**
3*tan(x/2)**2 + 75*a**3*tan(x/2) + 15*a**3) + 40*tan(x/2)**3/(15*a**3*tan(x
/2)**5 + 75*a**3*tan(x/2)**4 + 150*a**3*tan(x/2)**3 + 150*a**3*tan(x/2)**2
+ 75*a**3*tan(x/2) + 15*a**3)
```

Giac [A] time = 1.22799, size = 39, normalized size = 0.78

$$-\frac{4\left(10\tan\left(\frac{1}{2}x\right)^2+5\tan\left(\frac{1}{2}x\right)+1\right)}{15a^3\left(\tan\left(\frac{1}{2}x\right)+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a+a*sin(x))^3,x, algorithm="giac")
```

```
[Out] -4/15*(10*tan(1/2*x)^2 + 5*tan(1/2*x) + 1)/(a^3*(tan(1/2*x) + 1)^5)
```

3.26 $\int \frac{\sin(x)}{(a+a\sin(x))^3}\,dx$

Optimal. Leaf size=50

$$-\frac{\cos(x)}{5\left(a^3\sin(x)+a^3\right)}-\frac{\cos(x)}{5a(a\sin(x)+a)^2}+\frac{\cos(x)}{5(a\sin(x)+a)^3}$$

```
[Out] Cos[x]/(5*(a + a*Sin[x])^3) - Cos[x]/(5*a*(a + a*Sin[x])^2) - Cos[x]/(5*(a^
3 + a^3*Sin[x]))
```

Rubi [A] time = 0.0460519, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2750, 2650, 2648}

$$-\frac{\cos(x)}{5\left(a^3\sin(x)+a^3\right)}-\frac{\cos(x)}{5a(a\sin(x)+a)^2}+\frac{\cos(x)}{5(a\sin(x)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]/(a + a*Sin[x])^3,x]
```

```
[Out] Cos[x]/(5*(a + a*Sin[x])^3) - Cos[x]/(5*a*(a + a*Sin[x])^2) - Cos[x]/(5*(a^
3 + a^3*Sin[x]))
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin(x)}{(a+a\sin(x))^3}\,dx &= \frac{\cos(x)}{5(a+a\sin(x))^3}+\frac{3\int \frac{1}{(a+a\sin(x))^2}\,dx}{5a}\\
&= \frac{\cos(x)}{5(a+a\sin(x))^3}-\frac{\cos(x)}{5a(a+a\sin(x))^2}+\frac{\int \frac{1}{a+a\sin(x)}\,dx}{5a^2}\\
&= \frac{\cos(x)}{5(a+a\sin(x))^3}-\frac{\cos(x)}{5a(a+a\sin(x))^2}-\frac{\cos(x)}{5\left(a^3+a^3\sin(x)\right)}
\end{aligned}$$

Mathematica [A] time = 0.0403954, size = 41, normalized size = 0.82

$$\frac{\sin^2\left(\frac{x}{2}\right)(8\sin(x)+\sin(2x)+4\cos(x)-\cos(2x)+7)}{5a^3(\sin(x)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/(a + a*Sin[x])^3,x]
```

```
[Out] (Sin[x/2]^2*(7 + 4*Cos[x] - Cos[2*x] + 8*Sin[x] + Sin[2*x]))/(5*a^3*(1 + Si
n[x])^3)
```

Maple [A] time = 0.035, size = 45, normalized size = 0.9

$$4\frac{1}{a^3}\left((\tan(x/2)+1)^{-3}+2/5\,(\tan(x/2)+1)^{-5}-(\tan(x/2)+1)^{-4}-1/2\,(\tan(x/2)+1)^{-2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(a+a*sin(x))^3,x)
```

```
[Out] 4/a^3*(1/(tan(1/2*x)+1)^3+2/5/(tan(1/2*x)+1)^5-1/(tan(1/2*x)+1)^4-1/2/(tan(
1/2*x)+1)^2)
```

Maxima [B] time = 1.62675, size = 157, normalized size = 3.14

$$-\frac{2\left(\frac{5\sin(x)}{\cos(x)+1}+\frac{5\sin(x)^2}{(\cos(x)+1)^2}+\frac{5\sin(x)^3}{(\cos(x)+1)^3}+1\right)}{5\left(a^3+\frac{5a^3\sin(x)}{\cos(x)+1}+\frac{10a^3\sin(x)^2}{(\cos(x)+1)^2}+\frac{10a^3\sin(x)^3}{(\cos(x)+1)^3}+\frac{5a^3\sin(x)^4}{(\cos(x)+1)^4}+\frac{a^3\sin(x)^5}{(\cos(x)+1)^5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+a*sin(x))^3,x, algorithm="maxima")
```

```
[Out] -2/5*(5*sin(x)/(cos(x) + 1) + 5*sin(x)^2/(cos(x) + 1)^2 + 5*sin(x)^3/(cos(x
) + 1)^3 + 1)/(a^3 + 5*a^3*sin(x)/(cos(x) + 1) + 10*a^3*sin(x)^2/(cos(x) +
1)^2 + 10*a^3*sin(x)^3/(cos(x) + 1)^3 + 5*a^3*sin(x)^4/(cos(x) + 1)^4 + a^3
*sin(x)^5/(cos(x) + 1)^5)
```

Fricas [B] time = 1.38556, size = 246, normalized size = 4.92

$$-\frac{\cos(x)^3-2\cos(x)^2-\left(\cos(x)^2+3\cos(x)+1\right)\sin(x)-2\cos(x)+1}{5\left(a^3\cos(x)^3+3a^3\cos(x)^2-2a^3\cos(x)-4a^3+\left(a^3\cos(x)^2-2a^3\cos(x)-4a^3\right)\sin(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+a*sin(x))^3,x, algorithm="fricas")
```

```
[Out] -1/5*(cos(x)^3 - 2*cos(x)^2 - (cos(x)^2 + 3*cos(x) + 1)*sin(x) - 2*cos(x) +
 1)/(a^3*cos(x)^3 + 3*a^3*cos(x)^2 - 2*a^3*cos(x) - 4*a^3 + (a^3*cos(x)^2 -
```

```
2*a^3*cos(x) - 4*a^3)*sin(x))
```

Sympy [B] time = 5.19911, size = 418, normalized size = 8.36

$$\frac{12\tan^5\left(\frac{x}{2}\right)}{55a^3\tan^5\left(\frac{x}{2}\right)+275a^3\tan^4\left(\frac{x}{2}\right)+550a^3\tan^3\left(\frac{x}{2}\right)+550a^3\tan^2\left(\frac{x}{2}\right)+275a^3\tan\left(\frac{x}{2}\right)+55a^3}+\frac{}{55a^3\tan^5\left(\frac{x}{2}\right)+275a^3\tan}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+a*sin(x))**3,x)
```

```
[Out] 12*tan(x/2)**5/(55*a**3*tan(x/2)**5 + 275*a**3*tan(x/2)**4 + 550*a**3*tan(x
/2)**3 + 550*a**3*tan(x/2)**2 + 275*a**3*tan(x/2) + 55*a**3) + 60*tan(x/2)*
*4/(55*a**3*tan(x/2)**5 + 275*a**3*tan(x/2)**4 + 550*a**3*tan(x/2)**3 + 550
*a**3*tan(x/2)**2 + 275*a**3*tan(x/2) + 55*a**3) + 10*tan(x/2)**3/(55*a**3*
tan(x/2)**5 + 275*a**3*tan(x/2)**4 + 550*a**3*tan(x/2)**3 + 550*a**3*tan(x/
2)**2 + 275*a**3*tan(x/2) + 55*a**3) + 10*tan(x/2)**2/(55*a**3*tan(x/2)**5
+ 275*a**3*tan(x/2)**4 + 550*a**3*tan(x/2)**3 + 550*a**3*tan(x/2)**2 + 275*
a**3*tan(x/2) + 55*a**3) - 50*tan(x/2)/(55*a**3*tan(x/2)**5 + 275*a**3*tan(
x/2)**4 + 550*a**3*tan(x/2)**3 + 550*a**3*tan(x/2)**2 + 275*a**3*tan(x/2) +
 55*a**3) - 10/(55*a**3*tan(x/2)**5 + 275*a**3*tan(x/2)**4 + 550*a**3*tan(x
/2)**3 + 550*a**3*tan(x/2)**2 + 275*a**3*tan(x/2) + 55*a**3)
```

Giac [A] time = 1.32119, size = 50, normalized size = 1.

$$-\frac{2\left(5\tan\left(\frac{1}{2}x\right)^3+5\tan\left(\frac{1}{2}x\right)^2+5\tan\left(\frac{1}{2}x\right)+1\right)}{5a^3\left(\tan\left(\frac{1}{2}x\right)+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+a*sin(x))^3,x, algorithm="giac")
```

```
[Out] -2/5*(5*tan(1/2*x)^3 + 5*tan(1/2*x)^2 + 5*tan(1/2*x) + 1)/(a^3*(tan(1/2*x)
+ 1)^5)
```

3.27 $\int \frac{1}{(a+a\sin(x))^3}\,dx$

Optimal. Leaf size=50

$$-\frac{2\cos(x)}{15\left(a^3\sin(x)+a^3\right)}-\frac{2\cos(x)}{15a(a\sin(x)+a)^2}-\frac{\cos(x)}{5(a\sin(x)+a)^3}$$

```
[Out] -Cos[x]/(5*(a + a*Sin[x])^3) - (2*Cos[x])/(15*a*(a + a*Sin[x])^2) - (2*Cos[
x])/(15*(a^3 + a^3*Sin[x]))
```

Rubi [A] time = 0.0350515, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2650, 2648}

$$-\frac{2\cos(x)}{15\left(a^3\sin(x)+a^3\right)}-\frac{2\cos(x)}{15a(a\sin(x)+a)^2}-\frac{\cos(x)}{5(a\sin(x)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[x])^(-3),x]
```

```
[Out] -Cos[x]/(5*(a + a*Sin[x])^3) - (2*Cos[x])/(15*a*(a + a*Sin[x])^2) - (2*Cos[
x])/(15*(a^3 + a^3*Sin[x]))
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+a\sin(x))^3}\,dx &= -\frac{\cos(x)}{5(a+a\sin(x))^3}+\frac{2\int \frac{1}{(a+a\sin(x))^2}\,dx}{5a}\\ &= -\frac{\cos(x)}{5(a+a\sin(x))^3}-\frac{2\cos(x)}{15a(a+a\sin(x))^2}+\frac{2\int \frac{1}{a+a\sin(x)}\,dx}{15a^2}\\ &= -\frac{\cos(x)}{5(a+a\sin(x))^3}-\frac{2\cos(x)}{15a(a+a\sin(x))^2}-\frac{2\cos(x)}{15\left(a^3+a^3\sin(x)\right)}\end{aligned}$$

Mathematica [A] time = 0.0578181, size = 45, normalized size = 0.9

$$-\frac{-10\sin\left(\frac{x}{2}\right)+\sin\left(\frac{5x}{2}\right)+5\cos\left(\frac{3x}{2}\right)}{15a^3\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[x])^(-3),x]
```

```
[Out] -(5*Cos[(3*x)/2] - 10*Sin[x/2] + Sin[(5*x)/2])/(15*a^3*(Cos[x/2] + Sin[x/2]
)^5)
```

Maple [A] time = 0.034, size = 57, normalized size = 1.1

$$2\frac{1}{a^3}\left(-4/5\ (\tan(x/2)+1)^{-5}+2\ (\tan(x/2)+1)^{-4}+2\ (\tan(x/2)+1)^{-2}-(\tan(x/2)+1)^{-1}-8/3\ (\tan(x/2)+1)^{-3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(x))^3,x)
```

```
[Out] 2/a^3*(-4/5/(tan(1/2*x)+1)^5+2/(tan(1/2*x)+1)^4+2/(tan(1/2*x)+1)^2-1/(tan(1
/2*x)+1)-8/3/(tan(1/2*x)+1)^3)
```

Maxima [B] time = 1.62658, size = 173, normalized size = 3.46

$$-\frac{2\left(\frac{20\sin(x)}{\cos(x)+1}+\frac{40\sin(x)^2}{(\cos(x)+1)^2}+\frac{30\sin(x)^3}{(\cos(x)+1)^3}+\frac{15\sin(x)^4}{(\cos(x)+1)^4}+7\right)}{15\left(a^3+\frac{5a^3\sin(x)}{\cos(x)+1}+\frac{10a^3\sin(x)^2}{(\cos(x)+1)^2}+\frac{10a^3\sin(x)^3}{(\cos(x)+1)^3}+\frac{5a^3\sin(x)^4}{(\cos(x)+1)^4}+\frac{a^3\sin(x)^5}{(\cos(x)+1)^5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(x))^3,x, algorithm="maxima")
```

```
[Out] -2/15*(20*sin(x)/(cos(x) + 1) + 40*sin(x)^2/(cos(x) + 1)^2 + 30*sin(x)^3/(c
os(x) + 1)^3 + 15*sin(x)^4/(cos(x) + 1)^4 + 7)/(a^3 + 5*a^3*sin(x)/(cos(x)
+ 1) + 10*a^3*sin(x)^2/(cos(x) + 1)^2 + 10*a^3*sin(x)^3/(cos(x) + 1)^3 + 5*
a^3*sin(x)^4/(cos(x) + 1)^4 + a^3*sin(x)^5/(cos(x) + 1)^5)
```

Fricas [B] time = 1.39788, size = 252, normalized size = 5.04

$$-\frac{2\cos(x)^3-4\cos(x)^2-\left(2\cos(x)^2+6\cos(x)-3\right)\sin(x)-9\cos(x)-3}{15\left(a^3\cos(x)^3+3a^3\cos(x)^2-2a^3\cos(x)-4a^3+\left(a^3\cos(x)^2-2a^3\cos(x)-4a^3\right)\sin(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(x))^3,x, algorithm="fricas")
```

```
[Out] -1/15*(2*cos(x)^3 - 4*cos(x)^2 - (2*cos(x)^2 + 6*cos(x) - 3)*sin(x) - 9*cos
(x) - 3)/(a^3*cos(x)^3 + 3*a^3*cos(x)^2 - 2*a^3*cos(x) - 4*a^3 + (a^3*cos(x
)^2 - 2*a^3*cos(x) - 4*a^3)*sin(x))
```

Sympy [B] time = 3.40163, size = 348, normalized size = 6.96

$$-\frac{30\tan^4\left(\frac{x}{2}\right)}{15a^3\tan^5\left(\frac{x}{2}\right)+75a^3\tan^4\left(\frac{x}{2}\right)+150a^3\tan^3\left(\frac{x}{2}\right)+150a^3\tan^2\left(\frac{x}{2}\right)+75a^3\tan\left(\frac{x}{2}\right)+15a^3}-\frac{}{15a^3\tan^5\left(\frac{x}{2}\right)+75a^3\tan}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(x))**3,x)
```

```
[Out] -30*tan(x/2)**4/(15*a**3*tan(x/2)**5 + 75*a**3*tan(x/2)**4 + 150*a**3*tan(x
/2)**3 + 150*a**3*tan(x/2)**2 + 75*a**3*tan(x/2) + 15*a**3) - 60*tan(x/2)**
3/(15*a**3*tan(x/2)**5 + 75*a**3*tan(x/2)**4 + 150*a**3*tan(x/2)**3 + 150*a
**3*tan(x/2)**2 + 75*a**3*tan(x/2) + 15*a**3) - 80*tan(x/2)**2/(15*a**3*tan
(x/2)**5 + 75*a**3*tan(x/2)**4 + 150*a**3*tan(x/2)**3 + 150*a**3*tan(x/2)**
2 + 75*a**3*tan(x/2) + 15*a**3) - 40*tan(x/2)/(15*a**3*tan(x/2)**5 + 75*a**
3*tan(x/2)**4 + 150*a**3*tan(x/2)**3 + 150*a**3*tan(x/2)**2 + 75*a**3*tan(x
/2) + 15*a**3) - 14/(15*a**3*tan(x/2)**5 + 75*a**3*tan(x/2)**4 + 150*a**3*t
an(x/2)**3 + 150*a**3*tan(x/2)**2 + 75*a**3*tan(x/2) + 15*a**3)
```

Giac [A] time = 1.23253, size = 61, normalized size = 1.22

$$-\frac{2\left(15\tan\left(\frac{1}{2}x\right)^4+30\tan\left(\frac{1}{2}x\right)^3+40\tan\left(\frac{1}{2}x\right)^2+20\tan\left(\frac{1}{2}x\right)+7\right)}{15a^3\left(\tan\left(\frac{1}{2}x\right)+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(x))^3,x, algorithm="giac")
```

```
[Out] -2/15*(15*tan(1/2*x)^4 + 30*tan(1/2*x)^3 + 40*tan(1/2*x)^2 + 20*tan(1/2*x)
+ 7)/(a^3*(tan(1/2*x) + 1)^5)
```

3.28 $\int \frac{\csc(x)}{(a+a\sin(x))^3}\,dx$

Optimal. Leaf size=58

$$\frac{22\cos(x)}{15\left(a^3\sin(x)+a^3\right)}-\frac{\tanh^{-1}(\cos(x))}{a^3}+\frac{7\cos(x)}{15a(a\sin(x)+a)^2}+\frac{\cos(x)}{5(a\sin(x)+a)^3}$$

```
[Out] -(ArcTanh[Cos[x]]/a^3) + Cos[x]/(5*(a + a*Sin[x])^3) + (7*Cos[x])/(15*a*(a
+ a*Sin[x])^2) + (22*Cos[x])/(15*(a^3 + a^3*Sin[x]))
```

Rubi [A] time = 0.160529, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2766, 2978, 12, 3770}

$$\frac{22\cos(x)}{15\left(a^3\sin(x)+a^3\right)}-\frac{\tanh^{-1}(\cos(x))}{a^3}+\frac{7\cos(x)}{15a(a\sin(x)+a)^2}+\frac{\cos(x)}{5(a\sin(x)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]/(a + a*Sin[x])^3,x]
```

```
[Out] -(ArcTanh[Cos[x]]/a^3) + Cos[x]/(5*(a + a*Sin[x])^3) + (7*Cos[x])/(15*a*(a
+ a*Sin[x])^2) + (22*Cos[x])/(15*(a^3 + a^3*Sin[x]))
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc(x)}{(a+a\sin(x))^3}\,dx &= \frac{\cos(x)}{5(a+a\sin(x))^3}+\frac{\int \frac{\csc(x)(5a-2a\sin(x))}{(a+a\sin(x))^2}\,dx}{5a^2}\\
&= \frac{\cos(x)}{5(a+a\sin(x))^3}+\frac{7\cos(x)}{15a(a+a\sin(x))^2}+\frac{\int \frac{\csc(x)\left(15a^2-7a^2\sin(x)\right)}{a+a\sin(x)}\,dx}{15a^4}\\
&= \frac{\cos(x)}{5(a+a\sin(x))^3}+\frac{7\cos(x)}{15a(a+a\sin(x))^2}+\frac{22\cos(x)}{15\left(a^3+a^3\sin(x)\right)}+\frac{\int 15a^3\csc(x)\,dx}{15a^6}\\
&= \frac{\cos(x)}{5(a+a\sin(x))^3}+\frac{7\cos(x)}{15a(a+a\sin(x))^2}+\frac{22\cos(x)}{15\left(a^3+a^3\sin(x)\right)}+\frac{\int \csc(x)\,dx}{a^3}\\
&= -\frac{\tanh^{-1}(\cos(x))}{a^3}+\frac{\cos(x)}{5(a+a\sin(x))^3}+\frac{7\cos(x)}{15a(a+a\sin(x))^2}+\frac{22\cos(x)}{15\left(a^3+a^3\sin(x)\right)}
\end{aligned}$$

Mathematica [B] time = 0.0689821, size = 160, normalized size = 2.76

$$\frac{\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)\left(-6\sin\left(\frac{x}{2}\right)-44\sin\left(\frac{x}{2}\right)\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)^4+7\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)^3-14\sin\left(\frac{x}{2}\right)\left(\sin\left(\frac{x}{2}\right)+\cos\left(\right.\right.}{15(a\sin(x)+}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]/(a + a*Sin[x])^3,x]
```

```
[Out] ((Cos[x/2] + Sin[x/2])*(-6*Sin[x/2] + 3*(Cos[x/2] + Sin[x/2]) - 14*Sin[x/2]
*(Cos[x/2] + Sin[x/2])^2 + 7*(Cos[x/2] + Sin[x/2])^3 - 44*Sin[x/2]*(Cos[x/2
] + Sin[x/2])^4 - 15*Log[Cos[x/2]]*(Cos[x/2] + Sin[x/2])^5 + 15*Log[Sin[x/2
]]*(Cos[x/2] + Sin[x/2])^5))/(15*(a + a*Sin[x])^3)
```

Maple [A] time = 0.055, size = 76, normalized size = 1.3

$$\frac{8}{5a^3}\left(\tan\left(\frac{x}{2}\right)+1\right)^{-5}-4\frac{1}{a^3(\tan(x/2)+1)^4}+\frac{20}{3a^3}\left(\tan\left(\frac{x}{2}\right)+1\right)^{-3}-6\frac{1}{a^3(\tan(x/2)+1)^2}+6\frac{1}{a^3(\tan(x/2)+1)}+\frac{1}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)/(a+a*sin(x))^3,x)
```

```
[Out] 8/5/a^3/(tan(1/2*x)+1)^5-4/a^3/(tan(1/2*x)+1)^4+20/3/a^3/(tan(1/2*x)+1)^3-6
/a^3/(tan(1/2*x)+1)^2+6/a^3/(tan(1/2*x)+1)+1/a^3*ln(tan(1/2*x))
```

Maxima [B] time = 2.05255, size = 193, normalized size = 3.33

$$\frac{2\left(\frac{115\sin(x)}{\cos(x)+1}+\frac{185\sin(x)^2}{(\cos(x)+1)^2}+\frac{135\sin(x)^3}{(\cos(x)+1)^3}+\frac{45\sin(x)^4}{(\cos(x)+1)^4}+32\right)}{15\left(a^3+\frac{5a^3\sin(x)}{\cos(x)+1}+\frac{10a^3\sin(x)^2}{(\cos(x)+1)^2}+\frac{10a^3\sin(x)^3}{(\cos(x)+1)^3}+\frac{5a^3\sin(x)^4}{(\cos(x)+1)^4}+\frac{a^3\sin(x)^5}{(\cos(x)+1)^5}\right)}+\frac{\log\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a+a*sin(x))^3,x, algorithm="maxima")
```

```
[Out] 2/15*(115*sin(x)/(cos(x) + 1) + 185*sin(x)^2/(cos(x) + 1)^2 + 135*sin(x)^3/
(cos(x) + 1)^3 + 45*sin(x)^4/(cos(x) + 1)^4 + 32)/(a^3 + 5*a^3*sin(x)/(cos(
x) + 1) + 10*a^3*sin(x)^2/(cos(x) + 1)^2 + 10*a^3*sin(x)^3/(cos(x) + 1)^3 +
 5*a^3*sin(x)^4/(cos(x) + 1)^4 + a^3*sin(x)^5/(cos(x) + 1)^5) + log(sin(x)/
(cos(x) + 1))/a^3
```

Fricas [B] time = 1.38515, size = 536, normalized size = 9.24

$$\frac{44\cos(x)^3 - 58\cos(x)^2 - 15\left(\cos(x)^3 + 3\cos(x)^2 + \left(\cos(x)^2 - 2\cos(x) - 4\right)\sin(x) - 2\cos(x) - 4\right)\log\left(\frac{1}{2}\cos(x) + \right.}{30\left(a^3\cos(x)^3 + 3a^3\cos(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a+a*sin(x))^3,x, algorithm="fricas")
```

```
[Out] 1/30*(44*cos(x)^3 - 58*cos(x)^2 - 15*(cos(x)^3 + 3*cos(x)^2 + (cos(x)^2 - 2
*cos(x) - 4)*sin(x) - 2*cos(x) - 4)*log(1/2*cos(x) + 1/2) + 15*(cos(x)^3 +
3*cos(x)^2 + (cos(x)^2 - 2*cos(x) - 4)*sin(x) - 2*cos(x) - 4)*log(-1/2*cos(
x) + 1/2) - 2*(22*cos(x)^2 + 51*cos(x) - 3)*sin(x) - 108*cos(x) - 6)/(a^3*c
os(x)^3 + 3*a^3*cos(x)^2 - 2*a^3*cos(x) - 4*a^3 + (a^3*cos(x)^2 - 2*a^3*cos
(x) - 4*a^3)*sin(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\csc(x)}{\sin^3(x)+3\sin^2(x)+3\sin(x)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a+a*sin(x))**3,x)
```

```
[Out] Integral(csc(x)/(sin(x)**3 + 3*sin(x)**2 + 3*sin(x) + 1), x)/a**3
```

Giac [A] time = 1.28153, size = 76, normalized size = 1.31

$$\frac{\log\left(\left|\tan\left(\frac{1}{2}x\right)\right|\right)}{a^3} + \frac{2\left(45\tan\left(\frac{1}{2}x\right)^4 + 135\tan\left(\frac{1}{2}x\right)^3 + 185\tan\left(\frac{1}{2}x\right)^2 + 115\tan\left(\frac{1}{2}x\right) + 32\right)}{15a^3\left(\tan\left(\frac{1}{2}x\right) + 1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a+a*sin(x))^3,x, algorithm="giac")
```

```
[Out] log(abs(tan(1/2*x)))/a^3 + 2/15*(45*tan(1/2*x)^4 + 135*tan(1/2*x)^3 + 185*t
an(1/2*x)^2 + 115*tan(1/2*x) + 32)/(a^3*(tan(1/2*x) + 1)^5)
```

3.29 $\int \frac{\csc^2(x)}{(a+a\sin(x))^3}\,dx$

Optimal. Leaf size=65

$$-\frac{24\cot(x)}{5a^3}+\frac{3\tanh^{-1}(\cos(x))}{a^3}+\frac{3\cot(x)}{a^3\sin(x)+a^3}+\frac{3\cot(x)}{5a(a\sin(x)+a)^2}+\frac{\cot(x)}{5(a\sin(x)+a)^3}$$

```
[Out] (3*ArcTanh[Cos[x]])/a^3 - (24*Cot[x])/(5*a^3) + Cot[x]/(5*(a + a*Sin[x])^3)
 + (3*Cot[x])/(5*a*(a + a*Sin[x])^2) + (3*Cot[x])/(a^3 + a^3*Sin[x])
```

Rubi [A] time = 0.228711, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2766, 2978, 2748, 3767, 8, 3770}

$$-\frac{24\cot(x)}{5a^3}+\frac{3\tanh^{-1}(\cos(x))}{a^3}+\frac{3\cot(x)}{a^3\sin(x)+a^3}+\frac{3\cot(x)}{5a(a\sin(x)+a)^2}+\frac{\cot(x)}{5(a\sin(x)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^2/(a + a*Sin[x])^3,x]
```

```
[Out] (3*ArcTanh[Cos[x]])/a^3 - (24*Cot[x])/(5*a^3) + Cot[x]/(5*(a + a*Sin[x])^3)
 + (3*Cot[x])/(5*a*(a + a*Sin[x])^2) + (3*Cot[x])/(a^3 + a^3*Sin[x])
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc^2(x)}{(a+a\sin(x))^3}\,dx &= \frac{\cot(x)}{5(a+a\sin(x))^3}+\frac{\int \frac{\csc^2(x)(6a-3a\sin(x))}{(a+a\sin(x))^2}\,dx}{5a^2}\\
&= \frac{\cot(x)}{5(a+a\sin(x))^3}+\frac{3\cot(x)}{5a(a+a\sin(x))^2}+\frac{\int \frac{\csc^2(x)\left(27a^2-18a^2\sin(x)\right)}{a+a\sin(x)}\,dx}{15a^4}\\
&= \frac{\cot(x)}{5(a+a\sin(x))^3}+\frac{3\cot(x)}{5a(a+a\sin(x))^2}+\frac{3\cot(x)}{a^3+a^3\sin(x)}+\frac{\int \csc^2(x)\left(72a^3-45a^3\sin(x)\right)\,dx}{15a^6}\\
&= \frac{\cot(x)}{5(a+a\sin(x))^3}+\frac{3\cot(x)}{5a(a+a\sin(x))^2}+\frac{3\cot(x)}{a^3+a^3\sin(x)}-\frac{3\int \csc(x)\,dx}{a^3}+\frac{24\int \csc^2(x)\,dx}{5a^3}\\
&= \frac{3\tanh^{-1}(\cos(x))}{a^3}+\frac{\cot(x)}{5(a+a\sin(x))^3}+\frac{3\cot(x)}{5a(a+a\sin(x))^2}+\frac{3\cot(x)}{a^3+a^3\sin(x)}-\frac{24\,\mathrm{Subst}(\int 1\,dx,x,c}{5a^3}\\
&= \frac{3\tanh^{-1}(\cos(x))}{a^3}-\frac{24\cot(x)}{5a^3}+\frac{\cot(x)}{5(a+a\sin(x))^3}+\frac{3\cot(x)}{5a(a+a\sin(x))^2}+\frac{3\cot(x)}{a^3+a^3\sin(x)}
\end{aligned}$$

Mathematica [B] time = 0.139807, size = 206, normalized size = 3.17

$$\frac{\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)\left(4\sin\left(\frac{x}{2}\right)+76\sin\left(\frac{x}{2}\right)\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)^4-8\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)^3+16\sin\left(\frac{x}{2}\right)\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)^2}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^2/(a + a*Sin[x])^3,x]
```

```
[Out] ((Cos[x/2] + Sin[x/2])*(4*Sin[x/2] - 2*(Cos[x/2] + Sin[x/2]) + 16*Sin[x/2]*
(Cos[x/2] + Sin[x/2])^2 - 8*(Cos[x/2] + Sin[x/2])^3 + 76*Sin[x/2]*(Cos[x/2]
 + Sin[x/2])^4 - 5*Cot[x/2]*(Cos[x/2] + Sin[x/2])^5 + 30*Log[Cos[x/2]]*(Cos
[x/2] + Sin[x/2])^5 - 30*Log[Sin[x/2]]*(Cos[x/2] + Sin[x/2])^5 + 5*(Cos[x/2
] + Sin[x/2])^5*Tan[x/2]))/(10*(a + a*Sin[x])^3)
```

Maple [A] time = 0.06, size = 97, normalized size = 1.5

$$\frac{1}{2a^3}\tan\left(\frac{x}{2}\right)-\frac{8}{5a^3}\left(\tan\left(\frac{x}{2}\right)+1\right)^{-5}+4\frac{1}{a^3\left(\tan(x/2)+1\right)^4}-8\frac{1}{a^3\left(\tan(x/2)+1\right)^3}+8\frac{1}{a^3\left(\tan(x/2)+1\right)^2}-12\frac{1}{a^3\left(\tan(x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^2/(a+a*sin(x))^3,x)
```

```
[Out] 1/2/a^3*tan(1/2*x)-8/5/a^3/(tan(1/2*x)+1)^5+4/a^3/(tan(1/2*x)+1)^4-8/a^3/(t
an(1/2*x)+1)^3+8/a^3/(tan(1/2*x)+1)^2-12/a^3/(tan(1/2*x)+1)-1/2/a^3/tan(1/2
*x)-3/a^3*ln(tan(1/2*x))
```

Maxima [B] time = 1.89945, size = 243, normalized size = 3.74

$$-\frac{\frac{121\sin(x)}{\cos(x)+1}+\frac{410\sin(x)^2}{(\cos(x)+1)^2}+\frac{610\sin(x)^3}{(\cos(x)+1)^3}+\frac{425\sin(x)^4}{(\cos(x)+1)^4}+\frac{125\sin(x)^5}{(\cos(x)+1)^5}+5}{10\left(\frac{a^3\sin(x)}{\cos(x)+1}+\frac{5a^3\sin(x)^2}{(\cos(x)+1)^2}+\frac{10a^3\sin(x)^3}{(\cos(x)+1)^3}+\frac{10a^3\sin(x)^4}{(\cos(x)+1)^4}+\frac{5a^3\sin(x)^5}{(\cos(x)+1)^5}+\frac{a^3\sin(x)^6}{(\cos(x)+1)^6}\right)}-\frac{3\log\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a^3}+\frac{\sin(x)}{2a^3(\cos(x)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a+a*sin(x))^3,x, algorithm="maxima")
```

```
[Out] -1/10*(121*sin(x)/(cos(x) + 1) + 410*sin(x)^2/(cos(x) + 1)^2 + 610*sin(x)^3
/(cos(x) + 1)^3 + 425*sin(x)^4/(cos(x) + 1)^4 + 125*sin(x)^5/(cos(x) + 1)^5
 + 5)/(a^3*sin(x)/(cos(x) + 1) + 5*a^3*sin(x)^2/(cos(x) + 1)^2 + 10*a^3*sin
(x)^3/(cos(x) + 1)^3 + 10*a^3*sin(x)^4/(cos(x) + 1)^4 + 5*a^3*sin(x)^5/(cos
(x) + 1)^5 + a^3*sin(x)^6/(cos(x) + 1)^6) - 3*log(sin(x)/(cos(x) + 1))/a^3
+ 1/2*sin(x)/(a^3*(cos(x) + 1))
```

Fricas [B] time = 1.5206, size = 691, normalized size = 10.63

$$48\cos(x)^4+114\cos(x)^3-60\cos(x)^2+15\left(\cos(x)^4-2\cos(x)^3-5\cos(x)^2-\left(\cos(x)^3+3\cos(x)^2-2\cos(x)-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a+a*sin(x))^3,x, algorithm="fricas")
```

```
[Out] 1/10*(48*cos(x)^4 + 114*cos(x)^3 - 60*cos(x)^2 + 15*(cos(x)^4 - 2*cos(x)^3
- 5*cos(x)^2 - (cos(x)^3 + 3*cos(x)^2 - 2*cos(x) - 4)*sin(x) + 2*cos(x) + 4
)*log(1/2*cos(x) + 1/2) - 15*(cos(x)^4 - 2*cos(x)^3 - 5*cos(x)^2 - (cos(x)^
3 + 3*cos(x)^2 - 2*cos(x) - 4)*sin(x) + 2*cos(x) + 4)*log(-1/2*cos(x) + 1/2
) + 2*(24*cos(x)^3 - 33*cos(x)^2 - 63*cos(x) - 1)*sin(x) - 124*cos(x) + 2)/
(a^3*cos(x)^4 - 2*a^3*cos(x)^3 - 5*a^3*cos(x)^2 + 2*a^3*cos(x) + 4*a^3 - (a
^3*cos(x)^3 + 3*a^3*cos(x)^2 - 2*a^3*cos(x) - 4*a^3)*sin(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\csc^2(x)}{\sin^3(x)+3\sin^2(x)+3\sin(x)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**2/(a+a*sin(x))**3,x)
```

```
[Out] Integral(csc(x)**2/(sin(x)**3 + 3*sin(x)**2 + 3*sin(x) + 1), x)/a**3
```

Giac [A] time = 1.27868, size = 115, normalized size = 1.77

$$-\frac{3\,\log\left(\left|\tan\left(\frac{1}{2}\,x\right)\right|\right)}{a^3}+\frac{\tan\left(\frac{1}{2}\,x\right)}{2\,a^3}+\frac{6\,\tan\left(\frac{1}{2}\,x\right)-1}{2\,a^3\tan\left(\frac{1}{2}\,x\right)}-\frac{4\left(15\,\tan\left(\frac{1}{2}\,x\right)^4+50\,\tan\left(\frac{1}{2}\,x\right)^3+70\,\tan\left(\frac{1}{2}\,x\right)^2+45\,\tan\left(\frac{1}{2}\,x\right)\right.}{5\,a^3\left(\tan\left(\frac{1}{2}\,x\right)+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a+a*sin(x))^3,x, algorithm="giac")
```

```
[Out] -3*log(abs(tan(1/2*x)))/a^3 + 1/2*tan(1/2*x)/a^3 + 1/2*(6*tan(1/2*x) - 1)/(
a^3*tan(1/2*x)) - 4/5*(15*tan(1/2*x)^4 + 50*tan(1/2*x)^3 + 70*tan(1/2*x)^2
+ 45*tan(1/2*x) + 12)/(a^3*(tan(1/2*x) + 1)^5)
```

3.30 $\int \frac{\csc^3(x)}{(a+a\sin(x))^3}\,dx$

Optimal. Leaf size=86

$$\frac{152\cot(x)}{15a^3}-\frac{13\tanh^{-1}(\cos(x))}{2a^3}-\frac{13\cot(x)\csc(x)}{2a^3}+\frac{76\cot(x)\csc(x)}{15\left(a^3\sin(x)+a^3\right)}+\frac{11\cot(x)\csc(x)}{15a(a\sin(x)+a)^2}+\frac{\cot(x)\csc(x)}{5(a\sin(x)+a)^3}$$

```
[Out] (-13*ArcTanh[Cos[x]])/(2*a^3) + (152*Cot[x])/(15*a^3) - (13*Cot[x]*Csc[x])/
(2*a^3) + (Cot[x]*Csc[x])/(5*(a + a*Sin[x])^3) + (11*Cot[x]*Csc[x])/(15*a*(
a + a*Sin[x])^2) + (76*Cot[x]*Csc[x])/(15*(a^3 + a^3*Sin[x]))
```

Rubi [A] time = 0.23502, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {2766, 2978, 2748, 3768, 3770, 3767, 8}

$$\frac{152\cot(x)}{15a^3}-\frac{13\tanh^{-1}(\cos(x))}{2a^3}-\frac{13\cot(x)\csc(x)}{2a^3}+\frac{76\cot(x)\csc(x)}{15\left(a^3\sin(x)+a^3\right)}+\frac{11\cot(x)\csc(x)}{15a(a\sin(x)+a)^2}+\frac{\cot(x)\csc(x)}{5(a\sin(x)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^3/(a + a*Sin[x])^3,x]
```

```
[Out] (-13*ArcTanh[Cos[x]])/(2*a^3) + (152*Cot[x])/(15*a^3) - (13*Cot[x]*Csc[x])/
(2*a^3) + (Cot[x]*Csc[x])/(5*(a + a*Sin[x])^3) + (11*Cot[x]*Csc[x])/(15*a*(
a + a*Sin[x])^2) + (76*Cot[x]*Csc[x])/(15*(a^3 + a^3*Sin[x]))
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc^3(x)}{(a+a\sin(x))^3}\,dx &= \frac{\cot(x)\csc(x)}{5(a+a\sin(x))^3}+\frac{\int \frac{\csc^3(x)(7a-4a\sin(x))}{(a+a\sin(x))^2}\,dx}{5a^2}\\
&= \frac{\cot(x)\csc(x)}{5(a+a\sin(x))^3}+\frac{11\cot(x)\csc(x)}{15a(a+a\sin(x))^2}+\frac{\int \frac{\csc^3(x)\left(43a^2-33a^2\sin(x)\right)}{a+a\sin(x)}\,dx}{15a^4}\\
&= \frac{\cot(x)\csc(x)}{5(a+a\sin(x))^3}+\frac{11\cot(x)\csc(x)}{15a(a+a\sin(x))^2}+\frac{76\cot(x)\csc(x)}{15\left(a^3+a^3\sin(x)\right)}+\frac{\int \csc^3(x)\left(195a^3-152a^3\sin(x)\right)\,d}{15a^6}\\
&= \frac{\cot(x)\csc(x)}{5(a+a\sin(x))^3}+\frac{11\cot(x)\csc(x)}{15a(a+a\sin(x))^2}+\frac{76\cot(x)\csc(x)}{15\left(a^3+a^3\sin(x)\right)}-\frac{152\int \csc^2(x)\,dx}{15a^3}+\frac{13\int \csc^3(x)\,dx}{a^3}\\
&= -\frac{13\cot(x)\csc(x)}{2a^3}+\frac{\cot(x)\csc(x)}{5(a+a\sin(x))^3}+\frac{11\cot(x)\csc(x)}{15a(a+a\sin(x))^2}+\frac{76\cot(x)\csc(x)}{15\left(a^3+a^3\sin(x)\right)}+\frac{13\int \csc(x)\,dx}{2a^3}\\
&= -\frac{13\tanh^{-1}(\cos(x))}{2a^3}+\frac{152\cot(x)}{15a^3}-\frac{13\cot(x)\csc(x)}{2a^3}+\frac{\cot(x)\csc(x)}{5(a+a\sin(x))^3}+\frac{11\cot(x)\csc(x)}{15a(a+a\sin(x))^2}+\frac{}{1}
\end{aligned}$$

Mathematica [B] time = 0.406562, size = 247, normalized size = 2.87

$$\left(\sin\left(\tfrac{x}{2}\right)+\cos\left(\tfrac{x}{2}\right)\right)\left(-48\sin\left(\tfrac{x}{2}\right)+15\cos^3\left(\tfrac{x}{2}\right)\left(\tan\left(\tfrac{x}{2}\right)+1\right)^5-1712\sin\left(\tfrac{x}{2}\right)\left(\sin\left(\tfrac{x}{2}\right)+\cos\left(\tfrac{x}{2}\right)\right)^4+136\left(\sin\left(\tfrac{x}{2}\right)+\cos\left(\tfrac{x}{2}\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^3/(a + a*Sin[x])^3,x]
```

```
[Out] ((Cos[x/2] + Sin[x/2])*(-48*Sin[x/2] - 15*(1 + Cot[x/2])^5*Sin[x/2]^3 + 24*
(Cos[x/2] + Sin[x/2]) - 272*Sin[x/2]*(Cos[x/2] + Sin[x/2])^2 + 136*(Cos[x/2
] + Sin[x/2])^3 - 1712*Sin[x/2]*(Cos[x/2] + Sin[x/2])^4 + 180*Cot[x/2]*(Cos
[x/2] + Sin[x/2])^5 - 780*Log[Cos[x/2]]*(Cos[x/2] + Sin[x/2])^5 + 780*Log[S
in[x/2]]*(Cos[x/2] + Sin[x/2])^5 - 180*(Cos[x/2] + Sin[x/2])^5*Tan[x/2] + 1
```

```
5*Cos[x/2]^3*(1 + Tan[x/2])^5))/(120*a^3*(1 + Sin[x])^3)
```

Maple [A] time = 0.068, size = 119, normalized size = 1.4

$$\frac{1}{8a^3}\left(\tan\left(\frac{x}{2}\right)\right)^2-\frac{3}{2a^3}\tan\left(\frac{x}{2}\right)+\frac{8}{5a^3}\left(\tan\left(\frac{x}{2}\right)+1\right)^{-5}-4\frac{1}{a^3(\tan(x/2)+1)^4}+\frac{28}{3a^3}\left(\tan\left(\frac{x}{2}\right)+1\right)^{-3}-10\frac{1}{a^3(\tan(x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^3/(a+a*sin(x))^3,x)
```

```
[Out] 1/8/a^3*tan(1/2*x)^2-3/2/a^3*tan(1/2*x)+8/5/a^3/(tan(1/2*x)+1)^5-4/a^3/(tan
(1/2*x)+1)^4+28/3/a^3/(tan(1/2*x)+1)^3-10/a^3/(tan(1/2*x)+1)^2+20/a^3/(tan(
1/2*x)+1)-1/8/a^3/tan(1/2*x)^2+3/2/a^3/tan(1/2*x)+13/2/a^3*ln(tan(1/2*x))
```

Maxima [B] time = 1.83054, size = 282, normalized size = 3.28

$$\frac{\frac{105\sin(x)}{\cos(x)+1}+\frac{2782\sin(x)^2}{(\cos(x)+1)^2}+\frac{9410\sin(x)^3}{(\cos(x)+1)^3}+\frac{13645\sin(x)^4}{(\cos(x)+1)^4}+\frac{9285\sin(x)^5}{(\cos(x)+1)^5}+\frac{2580\sin(x)^6}{(\cos(x)+1)^6}-15}{120\left(\frac{a^3\sin(x)^2}{(\cos(x)+1)^2}+\frac{5a^3\sin(x)^3}{(\cos(x)+1)^3}+\frac{10a^3\sin(x)^4}{(\cos(x)+1)^4}+\frac{10a^3\sin(x)^5}{(\cos(x)+1)^5}+\frac{5a^3\sin(x)^6}{(\cos(x)+1)^6}+\frac{a^3\sin(x)^7}{(\cos(x)+1)^7}\right)}-\frac{\frac{12\sin(x)}{\cos(x)+1}-\frac{\sin(x)^2}{(\cos(x)+1)^2}}{8a^3}+\frac{13\log\left(\frac{\sin}{\cos(}\right.}{2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(a+a*sin(x))^3,x, algorithm="maxima")
```

```
[Out] 1/120*(105*sin(x)/(cos(x) + 1) + 2782*sin(x)^2/(cos(x) + 1)^2 + 9410*sin(x)
^3/(cos(x) + 1)^3 + 13645*sin(x)^4/(cos(x) + 1)^4 + 9285*sin(x)^5/(cos(x) +
 1)^5 + 2580*sin(x)^6/(cos(x) + 1)^6 - 15)/(a^3*sin(x)^2/(cos(x) + 1)^2 + 5
*a^3*sin(x)^3/(cos(x) + 1)^3 + 10*a^3*sin(x)^4/(cos(x) + 1)^4 + 10*a^3*sin(
x)^5/(cos(x) + 1)^5 + 5*a^3*sin(x)^6/(cos(x) + 1)^6 + a^3*sin(x)^7/(cos(x)
+ 1)^7) - 1/8*(12*sin(x)/(cos(x) + 1) - sin(x)^2/(cos(x) + 1)^2)/a^3 + 13/2
*log(sin(x)/(cos(x) + 1))/a^3
```

Fricas [B] time = 1.55687, size = 861, normalized size = 10.01

$$\frac{608\cos(x)^5-826\cos(x)^4-2174\cos(x)^3+784\cos(x)^2-195\left(\cos(x)^5+3\cos(x)^4-3\cos(x)^3-7\cos(x)^2+\left(\cos\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(a+a*sin(x))^3,x, algorithm="fricas")
```

```
[Out] 1/60*(608*cos(x)^5 - 826*cos(x)^4 - 2174*cos(x)^3 + 784*cos(x)^2 - 195*(cos
(x)^5 + 3*cos(x)^4 - 3*cos(x)^3 - 7*cos(x)^2 + (cos(x)^4 - 2*cos(x)^3 - 5*c
os(x)^2 + 2*cos(x) + 4)*sin(x) + 2*cos(x) + 4)*log(1/2*cos(x) + 1/2) + 195*
(cos(x)^5 + 3*cos(x)^4 - 3*cos(x)^3 - 7*cos(x)^2 + (cos(x)^4 - 2*cos(x)^3 -
 5*cos(x)^2 + 2*cos(x) + 4)*sin(x) + 2*cos(x) + 4)*log(-1/2*cos(x) + 1/2) -
 2*(304*cos(x)^4 + 717*cos(x)^3 - 370*cos(x)^2 - 762*cos(x) + 6)*sin(x) + 1
536*cos(x) + 12)/(a^3*cos(x)^5 + 3*a^3*cos(x)^4 - 3*a^3*cos(x)^3 - 7*a^3*co
s(x)^2 + 2*a^3*cos(x) + 4*a^3 + (a^3*cos(x)^4 - 2*a^3*cos(x)^3 - 5*a^3*cos(
```

```
x)^2 + 2*a^3*cos(x) + 4*a^3)*sin(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\csc^3(x)}{\sin^3(x)+3\sin^2(x)+3\sin(x)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**3/(a+a*sin(x))**3,x)
```

```
[Out] Integral(csc(x)**3/(sin(x)**3 + 3*sin(x)**2 + 3*sin(x) + 1), x)/a**3
```

Giac [A] time = 1.46168, size = 147, normalized size = 1.71

$$\frac{13\log\left(\left|\tan\left(\frac{1}{2}x\right)\right|\right)}{2a^3} - \frac{78\tan\left(\frac{1}{2}x\right)^2 - 12\tan\left(\frac{1}{2}x\right) + 1}{8a^3\tan\left(\frac{1}{2}x\right)^2} + \frac{a^3\tan\left(\frac{1}{2}x\right)^2 - 12a^3\tan\left(\frac{1}{2}x\right)}{8a^6} + \frac{2\left(150\tan\left(\frac{1}{2}x\right)^4 + 525\,\mathrm{ta}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(a+a*sin(x))^3,x, algorithm="giac")
```

```
[Out] 13/2*log(abs(tan(1/2*x)))/a^3 - 1/8*(78*tan(1/2*x)^2 - 12*tan(1/2*x) + 1)/(
a^3*tan(1/2*x)^2) + 1/8*(a^3*tan(1/2*x)^2 - 12*a^3*tan(1/2*x))/a^6 + 2/15*(
150*tan(1/2*x)^4 + 525*tan(1/2*x)^3 + 745*tan(1/2*x)^2 + 485*tan(1/2*x) + 1
27)/(a^3*(tan(1/2*x) + 1)^5)
```

3.31 $\int \frac{\csc^4(x)}{(a+a\sin(x))^3}\,dx$

Optimal. Leaf size=103

$$-\frac{136\cot^3(x)}{15a^3}-\frac{136\cot(x)}{5a^3}+\frac{23\tanh^{-1}(\cos(x))}{2a^3}+\frac{23\cot(x)\csc(x)}{2a^3}+\frac{23\cot(x)\csc^2(x)}{3\left(a^3\sin(x)+a^3\right)}+\frac{13\cot(x)\csc^2(x)}{15a(a\sin(x)+a)^2}+\frac{\cot(}{5(a\,\mathrm{s}}$$

```
[Out] (23*ArcTanh[Cos[x]])/(2*a^3) - (136*Cot[x])/(5*a^3) - (136*Cot[x]^3)/(15*a^
3) + (23*Cot[x]*Csc[x])/(2*a^3) + (Cot[x]*Csc[x]^2)/(5*(a + a*Sin[x])^3) +
(13*Cot[x]*Csc[x]^2)/(15*a*(a + a*Sin[x])^2) + (23*Cot[x]*Csc[x]^2)/(3*(a^3
 + a^3*Sin[x]))
```

Rubi [A] time = 0.244845, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2766, 2978, 2748, 3767, 3768, 3770}

$$-\frac{136\cot^3(x)}{15a^3}-\frac{136\cot(x)}{5a^3}+\frac{23\tanh^{-1}(\cos(x))}{2a^3}+\frac{23\cot(x)\csc(x)}{2a^3}+\frac{23\cot(x)\csc^2(x)}{3\left(a^3\sin(x)+a^3\right)}+\frac{13\cot(x)\csc^2(x)}{15a(a\sin(x)+a)^2}+\frac{\cot(}{5(a\,\mathrm{s}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^4/(a + a*Sin[x])^3,x]
```

```
[Out] (23*ArcTanh[Cos[x]])/(2*a^3) - (136*Cot[x])/(5*a^3) - (136*Cot[x]^3)/(15*a^
3) + (23*Cot[x]*Csc[x])/(2*a^3) + (Cot[x]*Csc[x]^2)/(5*(a + a*Sin[x])^3) +
(13*Cot[x]*Csc[x]^2)/(15*a*(a + a*Sin[x])^2) + (23*Cot[x]*Csc[x]^2)/(3*(a^3
 + a^3*Sin[x]))
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\csc^4(x)}{(a+a\sin(x))^3}\,dx = \frac{\cot(x)\csc^2(x)}{5(a+a\sin(x))^3} + \frac{\int \frac{\csc^4(x)(8a-5a\sin(x))}{(a+a\sin(x))^2}\,dx}{5a^2}$$

$$= \frac{\cot(x)\csc^2(x)}{5(a+a\sin(x))^3} + \frac{13\cot(x)\csc^2(x)}{15a(a+a\sin(x))^2} + \frac{\int \frac{\csc^4(x)\left(63a^2-52a^2\sin(x)\right)}{a+a\sin(x)}\,dx}{15a^4}$$

$$= \frac{\cot(x)\csc^2(x)}{5(a+a\sin(x))^3} + \frac{13\cot(x)\csc^2(x)}{15a(a+a\sin(x))^2} + \frac{23\cot(x)\csc^2(x)}{3\left(a^3+a^3\sin(x)\right)} + \frac{\int \csc^4(x)\left(408a^3-345a^3\sin(x)\right)\,dx}{15a^6}$$

$$= \frac{\cot(x)\csc^2(x)}{5(a+a\sin(x))^3} + \frac{13\cot(x)\csc^2(x)}{15a(a+a\sin(x))^2} + \frac{23\cot(x)\csc^2(x)}{3\left(a^3+a^3\sin(x)\right)} - \frac{23\int \csc^3(x)\,dx}{a^3} + \frac{136\int \csc^4(x)\,dx}{5a^3}$$

$$= \frac{23\cot(x)\csc(x)}{2a^3} + \frac{\cot(x)\csc^2(x)}{5(a+a\sin(x))^3} + \frac{13\cot(x)\csc^2(x)}{15a(a+a\sin(x))^2} + \frac{23\cot(x)\csc^2(x)}{3\left(a^3+a^3\sin(x)\right)} - \frac{23\int \csc(x)\,dx}{2a^3} -$$

$$= \frac{23\tanh^{-1}(\cos(x))}{2a^3} - \frac{136\cot(x)}{5a^3} - \frac{136\cot^3(x)}{15a^3} + \frac{23\cot(x)\csc(x)}{2a^3} + \frac{\cot(x)\csc^2(x)}{5(a+a\sin(x))^3} + \frac{13\cot(x}{15a(a+}$$

Mathematica [B] time = 0.852753, size = 299, normalized size = 2.9

$$\frac{\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)\left(48\sin\left(\frac{x}{2}\right)-45\cos^3\left(\frac{x}{2}\right)\left(\tan\left(\frac{x}{2}\right)+1\right)^5+2752\sin\left(\frac{x}{2}\right)\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)^4-176\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^4/(a + a*Sin[x])^3,x]
```

```
[Out] ((Cos[x/2] + Sin[x/2])*(48*Sin[x/2] - 5*Cos[x/2]*(1 + Cot[x/2])^5*Sin[x/2]^
2 + 45*(1 + Cot[x/2])^5*Sin[x/2]^3 - 24*(Cos[x/2] + Sin[x/2]) + 352*Sin[x/2
]*(Cos[x/2] + Sin[x/2])^2 - 176*(Cos[x/2] + Sin[x/2])^3 + 2752*Sin[x/2]*(Co
s[x/2] + Sin[x/2])^4 - 400*Cot[x/2]*(Cos[x/2] + Sin[x/2])^5 + 1380*Log[Cos[
x/2]]*(Cos[x/2] + Sin[x/2])^5 - 1380*Log[Sin[x/2]]*(Cos[x/2] + Sin[x/2])^5
+ 400*(Cos[x/2] + Sin[x/2])^5*Tan[x/2] - 45*Cos[x/2]^3*(1 + Tan[x/2])^5 + 5
```

```
*Cos[x/2]^2*Sin[x/2]*(1 + Tan[x/2])^5))/(120*a^3*(1 + Sin[x])^3)
```

Maple [A] time = 0.071, size = 141, normalized size = 1.4

$$\frac{1}{24a^3}\left(\tan\left(\frac{x}{2}\right)\right)^3-\frac{3}{8a^3}\left(\tan\left(\frac{x}{2}\right)\right)^2+\frac{27}{8a^3}\tan\left(\frac{x}{2}\right)-\frac{8}{5a^3}\left(\tan\left(\frac{x}{2}\right)+1\right)^{-5}+4\frac{1}{a^3(\tan(x/2)+1)^4}-\frac{32}{3a^3}\left(\tan\left(\frac{x}{2}\right)+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^4/(a+a*sin(x))^3,x)
```

```
[Out] 1/24/a^3*tan(1/2*x)^3-3/8/a^3*tan(1/2*x)^2+27/8/a^3*tan(1/2*x)-8/5/a^3/(tan
(1/2*x)+1)^5+4/a^3/(tan(1/2*x)+1)^4-32/3/a^3/(tan(1/2*x)+1)^3+12/a^3/(tan(1
/2*x)+1)^2-30/a^3/(tan(1/2*x)+1)-1/24/a^3/tan(1/2*x)^3+3/8/a^3/tan(1/2*x)^2
-27/8/a^3/tan(1/2*x)-23/2/a^3*ln(tan(1/2*x))
```

Maxima [B] time = 1.74956, size = 313, normalized size = 3.04

$$\frac{\frac{20\sin(x)}{\cos(x)+1}-\frac{230\sin(x)^2}{(\cos(x)+1)^2}-\frac{4777\sin(x)^3}{(\cos(x)+1)^3}-\frac{15785\sin(x)^4}{(\cos(x)+1)^4}-\frac{22390\sin(x)^5}{(\cos(x)+1)^5}-\frac{14940\sin(x)^6}{(\cos(x)+1)^6}-\frac{4005\sin(x)^7}{(\cos(x)+1)^7}-5}{120\left(\frac{a^3\sin(x)^3}{(\cos(x)+1)^3}+\frac{5a^3\sin(x)^4}{(\cos(x)+1)^4}+\frac{10a^3\sin(x)^5}{(\cos(x)+1)^5}+\frac{10a^3\sin(x)^6}{(\cos(x)+1)^6}+\frac{5a^3\sin(x)^7}{(\cos(x)+1)^7}+\frac{a^3\sin(x)^8}{(\cos(x)+1)^8}\right)}+\frac{\frac{81\sin(x)}{\cos(x)+1}-\frac{9\sin(x)^2}{(\cos(x)+1)^2}+\ldots}{24a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^4/(a+a*sin(x))^3,x, algorithm="maxima")
```

```
[Out] 1/120*(20*sin(x)/(cos(x) + 1) - 230*sin(x)^2/(cos(x) + 1)^2 - 4777*sin(x)^3
/(cos(x) + 1)^3 - 15785*sin(x)^4/(cos(x) + 1)^4 - 22390*sin(x)^5/(cos(x) +
1)^5 - 14940*sin(x)^6/(cos(x) + 1)^6 - 4005*sin(x)^7/(cos(x) + 1)^7 - 5)/(a
^3*sin(x)^3/(cos(x) + 1)^3 + 5*a^3*sin(x)^4/(cos(x) + 1)^4 + 10*a^3*sin(x)^
5/(cos(x) + 1)^5 + 10*a^3*sin(x)^6/(cos(x) + 1)^6 + 5*a^3*sin(x)^7/(cos(x)
+ 1)^7 + a^3*sin(x)^8/(cos(x) + 1)^8) + 1/24*(81*sin(x)/(cos(x) + 1) - 9*si
n(x)^2/(cos(x) + 1)^2 + sin(x)^3/(cos(x) + 1)^3)/a^3 - 23/2*log(sin(x)/(cos
(x) + 1))/a^3
```

Fricas [B] time = 1.51376, size = 1026, normalized size = 9.96

$$1088\cos(x)^6+2574\cos(x)^5-2428\cos(x)^4-5338\cos(x)^3+1372\cos(x)^2+345\left(\cos(x)^6-2\cos(x)^5-6\cos(x\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^4/(a+a*sin(x))^3,x, algorithm="fricas")
```

```
[Out] 1/60*(1088*cos(x)^6 + 2574*cos(x)^5 - 2428*cos(x)^4 - 5338*cos(x)^3 + 1372*
cos(x)^2 + 345*(cos(x)^6 - 2*cos(x)^5 - 6*cos(x)^4 + 4*cos(x)^3 + 9*cos(x)^
2 - (cos(x)^5 + 3*cos(x)^4 - 3*cos(x)^3 - 7*cos(x)^2 + 2*cos(x) + 4)*sin(x)
 - 2*cos(x) - 4)*log(1/2*cos(x) + 1/2) - 345*(cos(x)^6 - 2*cos(x)^5 - 6*cos
(x)^4 + 4*cos(x)^3 + 9*cos(x)^2 - (cos(x)^5 + 3*cos(x)^4 - 3*cos(x)^3 - 7*c
os(x)^2 + 2*cos(x) + 4)*sin(x) - 2*cos(x) - 4)*log(-1/2*cos(x) + 1/2) + 2*(
```

```
544*cos(x)^5 - 743*cos(x)^4 - 1957*cos(x)^3 + 712*cos(x)^2 + 1398*cos(x) +
6)*sin(x) + 2784*cos(x) - 12)/(a^3*cos(x)^6 - 2*a^3*cos(x)^5 - 6*a^3*cos(x)
^4 + 4*a^3*cos(x)^3 + 9*a^3*cos(x)^2 - 2*a^3*cos(x) - 4*a^3 - (a^3*cos(x)^5
 + 3*a^3*cos(x)^4 - 3*a^3*cos(x)^3 - 7*a^3*cos(x)^2 + 2*a^3*cos(x) + 4*a^3)
*sin(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\csc^4(x)}{\sin^3(x)+3\sin^2(x)+3\sin(x)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**4/(a+a*sin(x))**3,x)
```

```
[Out] Integral(csc(x)**4/(sin(x)**3 + 3*sin(x)**2 + 3*sin(x) + 1), x)/a**3
```

Giac [A] time = 1.31258, size = 173, normalized size = 1.68

$$-\frac{23\log\left(\left|\tan\left(\frac{1}{2}x\right)\right|\right)}{2a^3}+\frac{506\tan\left(\frac{1}{2}x\right)^3-81\tan\left(\frac{1}{2}x\right)^2+9\tan\left(\frac{1}{2}x\right)-1}{24a^3\tan\left(\frac{1}{2}x\right)^3}-\frac{2\left(225\tan\left(\frac{1}{2}x\right)^4+810\tan\left(\frac{1}{2}x\right)^3+1160\right.}{15a^3\left(\tan\left(\frac{1}{2}x\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^4/(a+a*sin(x))^3,x, algorithm="giac")
```

```
[Out] -23/2*log(abs(tan(1/2*x)))/a^3 + 1/24*(506*tan(1/2*x)^3 - 81*tan(1/2*x)^2 +
 9*tan(1/2*x) - 1)/(a^3*tan(1/2*x)^3) - 2/15*(225*tan(1/2*x)^4 + 810*tan(1/
2*x)^3 + 1160*tan(1/2*x)^2 + 760*tan(1/2*x) + 197)/(a^3*(tan(1/2*x) + 1)^5)
 + 1/24*(a^6*tan(1/2*x)^3 - 9*a^6*tan(1/2*x)^2 + 81*a^6*tan(1/2*x))/a^9
```

3.32 $\int \sin^4(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=158

$$-\frac{2a\sin^4(c+dx)\cos(c+dx)}{9d\sqrt{a\sin(c+dx)+a}}-\frac{16a\sin^3(c+dx)\cos(c+dx)}{63d\sqrt{a\sin(c+dx)+a}}-\frac{32\cos(c+dx)(a\sin(c+dx)+a)^{3/2}}{105ad}+\frac{64\cos(c+dx)\sqrt{}}{31}$$

```
[Out] (-32*a*Cos[c + d*x])/(45*d*Sqrt[a + a*Sin[c + d*x]]) - (16*a*Cos[c + d*x]*S
in[c + d*x]^3)/(63*d*Sqrt[a + a*Sin[c + d*x]]) - (2*a*Cos[c + d*x]*Sin[c +
d*x]^4)/(9*d*Sqrt[a + a*Sin[c + d*x]]) + (64*Cos[c + d*x]*Sqrt[a + a*Sin[c
+ d*x]])/(315*d) - (32*Cos[c + d*x]*(a + a*Sin[c + d*x])^(3/2))/(105*a*d)
```

Rubi [A] time = 0.230115, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2770, 2759, 2751, 2646}

$$-\frac{2a\sin^4(c+dx)\cos(c+dx)}{9d\sqrt{a\sin(c+dx)+a}}-\frac{16a\sin^3(c+dx)\cos(c+dx)}{63d\sqrt{a\sin(c+dx)+a}}-\frac{32\cos(c+dx)(a\sin(c+dx)+a)^{3/2}}{105ad}+\frac{64\cos(c+dx)\sqrt{}}{31}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^4*Sqrt[a + a*Sin[c + d*x]],x]
```


```
[Out] (-32*a*Cos[c + d*x])/(45*d*Sqrt[a + a*Sin[c + d*x]]) - (16*a*Cos[c + d*x]*S
in[c + d*x]^3)/(63*d*Sqrt[a + a*Sin[c + d*x]]) - (2*a*Cos[c + d*x]*Sin[c +
d*x]^4)/(9*d*Sqrt[a + a*Sin[c + d*x]]) + (64*Cos[c + d*x]*Sqrt[a + a*Sin[c
+ d*x]])/(315*d) - (32*Cos[c + d*x]*(a + a*Sin[c + d*x])^(3/2))/(105*a*d)
```


Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2759

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin[e
 + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ
[m, -2^(-1)]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
```

```
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int \sin^4(c+dx)\sqrt{a+a\sin(c+dx)}\,dx = -\frac{2a\cos(c+dx)\sin^4(c+dx)}{9d\sqrt{a+a\sin(c+dx)}}+\frac{8}{9}\int \sin^3(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$$
$$= -\frac{16a\cos(c+dx)\sin^3(c+dx)}{63d\sqrt{a+a\sin(c+dx)}}-\frac{2a\cos(c+dx)\sin^4(c+dx)}{9d\sqrt{a+a\sin(c+dx)}}+\frac{16}{21}\int \sin^2(c+dx)$$
$$= -\frac{16a\cos(c+dx)\sin^3(c+dx)}{63d\sqrt{a+a\sin(c+dx)}}-\frac{2a\cos(c+dx)\sin^4(c+dx)}{9d\sqrt{a+a\sin(c+dx)}}-\frac{32\cos(c+dx)(a+}{105}$$
$$= -\frac{16a\cos(c+dx)\sin^3(c+dx)}{63d\sqrt{a+a\sin(c+dx)}}-\frac{2a\cos(c+dx)\sin^4(c+dx)}{9d\sqrt{a+a\sin(c+dx)}}+\frac{64\cos(c+dx)\sqrt{a}}{315}$$
$$= -\frac{32a\cos(c+dx)}{45d\sqrt{a+a\sin(c+dx)}}-\frac{16a\cos(c+dx)\sin^3(c+dx)}{63d\sqrt{a+a\sin(c+dx)}}-\frac{2a\cos(c+dx)\sin^4(c+}{9d\sqrt{a+a\sin(c+dx}}$$

Mathematica [A] time = 0.49884, size = 165, normalized size = 1.04

$$\frac{\sqrt{a(\sin(c+dx)+1)}\left(1890\sin\left(\frac{1}{2}(c+dx)\right)-420\sin\left(\frac{3}{2}(c+dx)\right)-252\sin\left(\frac{5}{2}(c+dx)\right)+45\sin\left(\frac{7}{2}(c+dx)\right)+35\sin\left(\frac{9}{2}(c\right.\right.}{2520d\left(\sin\left(\frac{1}{2}(c+dx)\right)\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^4*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (Sqrt[a*(1 + Sin[c + d*x])]*(-1890*Cos[(c + d*x)/2] - 420*Cos[(3*(c + d*x))
/2] + 252*Cos[(5*(c + d*x))/2] + 45*Cos[(7*(c + d*x))/2] - 35*Cos[(9*(c + d
*x))/2] + 1890*Sin[(c + d*x)/2] - 420*Sin[(3*(c + d*x))/2] - 252*Sin[(5*(c
+ d*x))/2] + 45*Sin[(7*(c + d*x))/2] + 35*Sin[(9*(c + d*x))/2]))/(2520*d*(C
os[(c + d*x)/2] + Sin[(c + d*x)/2]))
```

Maple [A] time = 0.503, size = 83, normalized size = 0.5

$$\frac{(2+2\sin(dx+c))\,a\,(\sin(dx+c)-1)\left(35\,(\sin(dx+c))^4+40\,(\sin(dx+c))^3+48\,(\sin(dx+c))^2+64\sin(dx+c)+12\right.}{315\,d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^4*(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] 2/315*(1+sin(d*x+c))*a*(sin(d*x+c)-1)*(35*sin(d*x+c)^4+40*sin(d*x+c)^3+48*s
in(d*x+c)^2+64*sin(d*x+c)+128)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\sin(dx+c)+a}\,\sin(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4*(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*sin(d*x + c)^4, x)
```

Fricas [A] time = 1.42856, size = 373, normalized size = 2.36

$$-\frac{2\left(35\cos\left(dx+c\right)^5-5\cos\left(dx+c\right)^4-118\cos\left(dx+c\right)^3+26\cos\left(dx+c\right)^2-\left(35\cos\left(dx+c\right)^4+40\cos\left(dx+c\right)^3-\right.}{315\left(d\cos\left(dx+c\right)+d\sin\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4*(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -2/315*(35*cos(d*x + c)^5 - 5*cos(d*x + c)^4 - 118*cos(d*x + c)^3 + 26*cos(
d*x + c)^2 - (35*cos(d*x + c)^4 + 40*cos(d*x + c)^3 - 78*cos(d*x + c)^2 - 1
04*cos(d*x + c) + 107)*sin(d*x + c) + 211*cos(d*x + c) + 107)*sqrt(a*sin(d*
x + c) + a)/(d*cos(d*x + c) + d*sin(d*x + c) + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a\left(\sin\left(c+dx\right)+1\right)}\sin^4\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**4*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sin(c + d*x) + 1))*sin(c + d*x)**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a\sin\left(dx+c\right)+a}\sin\left(dx+c\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4*(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*sin(d*x + c)^4, x)
```

3.33 $\int \sin^3(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=122

$$-\frac{2a\sin^3(c+dx)\cos(c+dx)}{7d\sqrt{a\sin(c+dx)+a}}-\frac{12\cos(c+dx)(a\sin(c+dx)+a)^{3/2}}{35ad}+\frac{8\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{35d}-\frac{4a\cos(c+dx)}{5d\sqrt{a\sin(c+dx)+a}}$$

```
[Out] (-4*a*Cos[c + d*x])/(5*d*Sqrt[a + a*Sin[c + d*x]]) - (2*a*Cos[c + d*x]*Sin[
c + d*x]^3)/(7*d*Sqrt[a + a*Sin[c + d*x]]) + (8*Cos[c + d*x]*Sqrt[a + a*Sin
[c + d*x]])/(35*d) - (12*Cos[c + d*x]*(a + a*Sin[c + d*x])^(3/2))/(35*a*d)
```

Rubi [A] time = 0.16932, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2770, 2759, 2751, 2646}

$$-\frac{2a\sin^3(c+dx)\cos(c+dx)}{7d\sqrt{a\sin(c+dx)+a}}-\frac{12\cos(c+dx)(a\sin(c+dx)+a)^{3/2}}{35ad}+\frac{8\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{35d}-\frac{4a\cos(c+dx)}{5d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^3*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-4*a*Cos[c + d*x])/(5*d*Sqrt[a + a*Sin[c + d*x]]) - (2*a*Cos[c + d*x]*Sin[
c + d*x]^3)/(7*d*Sqrt[a + a*Sin[c + d*x]]) + (8*Cos[c + d*x]*Sqrt[a + a*Sin
[c + d*x]])/(35*d) - (12*Cos[c + d*x]*(a + a*Sin[c + d*x])^(3/2))/(35*a*d)
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2759

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin[e
 + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ
[m, -2^(-1)]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int \sin^3(c+dx)\sqrt{a+a\sin(c+dx)}\,dx = -\frac{2a\cos(c+dx)\sin^3(c+dx)}{7d\sqrt{a+a\sin(c+dx)}} + \frac{6}{7}\int \sin^2(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$$

$$= -\frac{2a\cos(c+dx)\sin^3(c+dx)}{7d\sqrt{a+a\sin(c+dx)}} - \frac{12\cos(c+dx)(a+a\sin(c+dx))^{3/2}}{35ad} + \frac{12\int\left(\frac{3a}{2}\right.}{}$$

$$= -\frac{2a\cos(c+dx)\sin^3(c+dx)}{7d\sqrt{a+a\sin(c+dx)}} + \frac{8\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{35d} - \frac{12\cos(c+}{}$$

$$= -\frac{4a\cos(c+dx)}{5d\sqrt{a+a\sin(c+dx)}} - \frac{2a\cos(c+dx)\sin^3(c+dx)}{7d\sqrt{a+a\sin(c+dx)}} + \frac{8\cos(c+dx)\sqrt{a+a\,\mathrm{s}}}{35d}$$

Mathematica [A] time = 0.292307, size = 141, normalized size = 1.16

$$\frac{\sqrt{a(\sin(c+dx)+1)}\left(105\sin\left(\frac{1}{2}(c+dx)\right)-35\sin\left(\frac{3}{2}(c+dx)\right)-7\sin\left(\frac{5}{2}(c+dx)\right)+5\sin\left(\frac{7}{2}(c+dx)\right)-105\cos\left(\frac{1}{2}(c+\right.\right.}{140d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^3*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (Sqrt[a*(1 + Sin[c + d*x])]*(-105*Cos[(c + d*x)/2] - 35*Cos[(3*(c + d*x))/2
] + 7*Cos[(5*(c + d*x))/2] + 5*Cos[(7*(c + d*x))/2] + 105*Sin[(c + d*x)/2]
- 35*Sin[(3*(c + d*x))/2] - 7*Sin[(5*(c + d*x))/2] + 5*Sin[(7*(c + d*x))/2]
))/(140*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
```

Maple [A] time = 0.539, size = 73, normalized size = 0.6

$$\frac{(2+2\sin(dx+c))\,a\,(\sin(dx+c)-1)\left(5(\sin(dx+c))^3+6(\sin(dx+c))^2+8\sin(dx+c)+16\right)}{35\,d\cos(dx+c)}\frac{1}{\sqrt{a+a\sin(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3*(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] 2/35*(1+sin(d*x+c))*a*(sin(d*x+c)-1)*(5*sin(d*x+c)^3+6*sin(d*x+c)^2+8*sin(d
*x+c)+16)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\sin(dx+c)+a}\,\sin(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*sin(d*x + c)^3, x)
```

Fricas [A] time = 1.37575, size = 300, normalized size = 2.46

$$\frac{2\left(5\cos\left(dx+c\right)^{4}+6\cos\left(dx+c\right)^{3}-12\cos\left(dx+c\right)^{2}+\left(5\cos\left(dx+c\right)^{3}-\cos\left(dx+c\right)^{2}-13\cos\left(dx+c\right)+9\right)\sin\left(dx+\right.}{35\left(d\cos\left(dx+c\right)+d\sin\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/35*(5*cos(d*x + c)^4 + 6*cos(d*x + c)^3 - 12*cos(d*x + c)^2 + (5*cos(d*x
+ c)^3 - cos(d*x + c)^2 - 13*cos(d*x + c) + 9)*sin(d*x + c) - 22*cos(d*x +
c) - 9)*sqrt(a*sin(d*x + c) + a)/(d*cos(d*x + c) + d*sin(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a\sin\left(dx+c\right)+a}\sin\left(dx+c\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*sin(d*x + c)^3, x)
```

3.34 $\int \sin^2(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=86

$$-\frac{2\cos(c+dx)(a\sin(c+dx)+a)^{3/2}}{5ad}+\frac{4\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{15d}-\frac{14a\cos(c+dx)}{15d\sqrt{a\sin(c+dx)+a}}$$

```
[Out] (-14*a*Cos[c + d*x])/(15*d*Sqrt[a + a*Sin[c + d*x]]) + (4*Cos[c + d*x]*Sqrt
[a + a*Sin[c + d*x]])/(15*d) - (2*Cos[c + d*x]*(a + a*Sin[c + d*x])^(3/2))/
(5*a*d)
```

Rubi [A] time = 0.110794, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {2759, 2751, 2646}

$$-\frac{2\cos(c+dx)(a\sin(c+dx)+a)^{3/2}}{5ad}+\frac{4\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{15d}-\frac{14a\cos(c+dx)}{15d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-14*a*Cos[c + d*x])/(15*d*Sqrt[a + a*Sin[c + d*x]]) + (4*Cos[c + d*x]*Sqrt
[a + a*Sin[c + d*x]])/(15*d) - (2*Cos[c + d*x]*(a + a*Sin[c + d*x])^(3/2))/
(5*a*d)
```

Rule 2759

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin[e
 + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ
[m, -2^(-1)]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int \sin^2(c+dx)\sqrt{a+a\sin(c+dx)}\,dx = -\frac{2\cos(c+dx)(a+a\sin(c+dx))^{3/2}}{5ad} + \frac{2\int\left(\frac{3a}{2}-a\sin(c+dx)\right)\sqrt{a+a\sin(c+dx}}{5a}$$
$$= \frac{4\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{15d} - \frac{2\cos(c+dx)(a+a\sin(c+dx))^{3/2}}{5ad} + \frac{7}{15}\int\sqrt{a}$$
$$= -\frac{14a\cos(c+dx)}{15d\sqrt{a+a\sin(c+dx)}} + \frac{4\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{15d} - \frac{2\cos(c+dx)(a+a}{5ad}$$

Mathematica [A] time = 0.18153, size = 117, normalized size = 1.36

$$-\frac{\sqrt{a(\sin(c+dx)+1)}\left(-30\sin\left(\frac{1}{2}(c+dx)\right)+5\sin\left(\frac{3}{2}(c+dx)\right)+3\sin\left(\frac{5}{2}(c+dx)\right)+30\cos\left(\frac{1}{2}(c+dx)\right)+5\cos\left(\frac{3}{2}(c+dx\right.\right.}{30d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] -(Sqrt[a*(1 + Sin[c + d*x])]*(30*Cos[(c + d*x)/2] + 5*Cos[(3*(c + d*x))/2]
- 3*Cos[(5*(c + d*x))/2] - 30*Sin[(c + d*x)/2] + 5*Sin[(3*(c + d*x))/2] + 3
*Sin[(5*(c + d*x))/2]))/(30*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
```

Maple [A] time = 0.44, size = 63, normalized size = 0.7

$$\frac{(2+2\sin(dx+c))\,a\,(\sin(dx+c)-1)\left(3(\sin(dx+c))^2+4\sin(dx+c)+8\right)}{15\,d\cos(dx+c)}\frac{1}{\sqrt{a+a\sin(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2*(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] 2/15*(1+sin(d*x+c))*a*(sin(d*x+c)-1)*(3*sin(d*x+c)^2+4*sin(d*x+c)+8)/cos(d*
x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\sin(dx+c)+a}\sin(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*sin(d*x + c)^2, x)
```

Fricas [A] time = 1.39477, size = 246, normalized size = 2.86

$$\frac{2\left(3\cos(dx+c)^3-\cos(dx+c)^2-\left(3\cos(dx+c)^2+4\cos(dx+c)-7\right)\sin(dx+c)-11\cos(dx+c)-7\right)\sqrt{a\sin(dx+}}{15\left(d\cos(dx+c)+d\sin(dx+c)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*cos(d*x + c)^3 - cos(d*x + c)^2 - (3*cos(d*x + c)^2 + 4*cos(d*x + c
) - 7)*sin(d*x + c) - 11*cos(d*x + c) - 7)*sqrt(a*sin(d*x + c) + a)/(d*cos(
d*x + c) + d*sin(d*x + c) + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\left(\sin\left(c+dx\right)+1\right)}\sin^{2}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sin(c + d*x) + 1))*sin(c + d*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\sin\left(dx+c\right)+a}\sin\left(dx+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*sin(d*x + c)^2, x)
```

3.35 $\int \sin(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=56

$$-\frac{2\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{3d}-\frac{2a\cos(c+dx)}{3d\sqrt{a\sin(c+dx)+a}}$$

```
[Out] (-2*a*Cos[c + d*x])/(3*d*Sqrt[a + a*Sin[c + d*x]]) - (2*Cos[c + d*x]*Sqrt[a
 + a*Sin[c + d*x]])/(3*d)
```

Rubi [A] time = 0.0451658, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2751, 2646}

$$-\frac{2\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{3d}-\frac{2a\cos(c+dx)}{3d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-2*a*Cos[c + d*x])/(3*d*Sqrt[a + a*Sin[c + d*x]]) - (2*Cos[c + d*x]*Sqrt[a
 + a*Sin[c + d*x]])/(3*d)
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \sin(c+dx)\sqrt{a+a\sin(c+dx)}\,dx &= -\frac{2\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{3d}+\frac{1}{3}\int \sqrt{a+a\sin(c+dx)}\,dx\\ &= -\frac{2a\cos(c+dx)}{3d\sqrt{a+a\sin(c+dx)}}-\frac{2\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{3d}\end{aligned}$$

Mathematica [A] time = 0.117609, size = 81, normalized size = 1.45

$$-\frac{\sqrt{a(\sin(c+dx)+1)}\left(-4\sin^3\left(\frac{1}{2}(c+dx)\right)+3\cos\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{3}{2}(c+dx)\right)\right)}{3d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] -((3*Cos[(c + d*x)/2] + Cos[(3*(c + d*x))/2] - 4*Sin[(c + d*x)/2]^3)*Sqrt[a
*(1 + Sin[c + d*x])])/(3*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
```

Maple [A] time = 0.377, size = 51, normalized size = 0.9

$$\frac{(2+2\sin(dx+c))\,a\,(\sin(dx+c)-1)\,(\sin(dx+c)+2)}{3\,d\cos(dx+c)}\frac{1}{\sqrt{a+a\sin(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)*(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] 2/3*(1+sin(d*x+c))*a*(sin(d*x+c)-1)*(sin(d*x+c)+2)/cos(d*x+c)/(a+a*sin(d*x+
c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a\sin(dx+c)+a}\sin(dx+c)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*sin(d*x + c), x)
```

Fricas [A] time = 1.44562, size = 190, normalized size = 3.39

$$-\frac{2\left(\cos(dx+c)^2+(\cos(dx+c)-1)\sin(dx+c)+2\cos(dx+c)+1\right)\sqrt{a\sin(dx+c)+a}}{3\,(d\cos(dx+c)+d\sin(dx+c)+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*(cos(d*x + c)^2 + (cos(d*x + c) - 1)*sin(d*x + c) + 2*cos(d*x + c) + 1
)*sqrt(a*sin(d*x + c) + a)/(d*cos(d*x + c) + d*sin(d*x + c) + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a(\sin(c+dx)+1)}\sin(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sin(c + d*x) + 1))*sin(c + d*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\sin(dx+c)+a}\sin(dx+c)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*sin(d*x + c), x)
```

3.36 $\int \sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=26

$$-\frac{2a\cos(c+dx)}{d\sqrt{a\sin(c+dx)+a}}$$

```
[Out] (-2*a*Cos[c + d*x])/(d*Sqrt[a + a*Sin[c + d*x]])
```

Rubi [A] time = 0.0131947, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2646}

$$-\frac{2a\cos(c+dx)}{d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-2*a*Cos[c + d*x])/(d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int \sqrt{a+a\sin(c+dx)}\,dx = -\frac{2a\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}}$$

Mathematica [B] time = 0.0325276, size = 65, normalized size = 2.5

$$\frac{2\sqrt{a(\sin(c+dx)+1)}\left(\sin\left(\frac{1}{2}(c+dx)\right)-\cos\left(\frac{1}{2}(c+dx)\right)\right)}{d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (2*(-Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*Sqrt[a*(1 + Sin[c + d*x])])/(d*(C
os[(c + d*x)/2] + Sin[(c + d*x)/2]))
```

Maple [A] time = 0.332, size = 43, normalized size = 1.7

$$2\,\frac{a\,(1+\sin(dx+c))\,(\sin(dx+c)-1)}{\cos(dx+c)\,\sqrt{a+a\sin(dx+c)}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] 2*(1+sin(d*x+c))*a*(sin(d*x+c)-1)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a \sin (dx+c)+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a), x)
```

Fricas [B] time = 1.44243, size = 136, normalized size = 5.23

$$-\frac{2\sqrt{a \sin (dx+c)+a}(\cos (dx+c)-\sin (dx+c)+1)}{d \cos (dx+c)+d \sin (dx+c)+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(a*sin(d*x + c) + a)*(cos(d*x + c) - sin(d*x + c) + 1)/(d*cos(d*x +
c) + d*sin(d*x + c) + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a \sin (c+dx)+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*sin(c + d*x) + a), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a \sin (dx+c)+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a), x)
```

3.37 $\int \csc(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=37

$$-\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{d}$$

```
[Out] (-2*Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/d
```

Rubi [A] time = 0.0548745, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2773, 206}

$$-\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-2*Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/d
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \csc(c+dx)\sqrt{a+a\sin(c+dx)}\,dx &= -\frac{(2a)\,\text{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, \frac{a\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d}\\ &= -\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d}\end{aligned}$$

Mathematica [B] time = 0.098415, size = 94, normalized size = 2.54

$$\frac{\sqrt{a(\sin(c+dx)+1)}\left(\log\left(\sin\left(\frac{1}{2}(c+dx)\right)-\cos\left(\frac{1}{2}(c+dx)\right)+1\right)-\log\left(-\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)+1\right)\right)}{d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] ((-Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + Log[1 - Cos[(c + d*x)/2]
+ Sin[(c + d*x)/2]])*Sqrt[a*(1 + Sin[c + d*x])])/(d*(Cos[(c + d*x)/2] + Sin
[(c + d*x)/2]))
```

Maple [B] time = 0.399, size = 68, normalized size = 1.8

$$-2\,\frac{(1+\sin(dx+c))\,\sqrt{-a\,(\sin(dx+c)-1)}\sqrt{a}}{\cos(dx+c)\,\sqrt{a+a\sin(dx+c)}d}\mathrm{Artanh}\left(\frac{\sqrt{-a\,(\sin(dx+c)-1)}}{\sqrt{a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] -2*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*a^(1/2)*arctanh((-a*(sin(d*x+c)
-1))^(1/2)/a^(1/2))/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\sin(dx+c)+a}\csc(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*csc(d*x + c), x)
```

Fricas [A] time = 1.59607, size = 590, normalized size = 15.95

$$\left[\frac{\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-4\left(\cos(dx+c)^2+(\cos(dx+c)+3)\sin(dx+c)-2\,\cos(dx+c)-3\right)\sqrt{a\sin(dx+c)+a}\sqrt{a}-9\,a\cos(dx+c)+\left(a\cos(dx+c)^2+8\,a\cos(d\right.}{\cos(dx+c)^3+\cos(dx+c)^2+\left(\cos(dx+c)^2-1\right)\sin(dx+c)-\cos(dx+c)-1}\right.}{2\,d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2
 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*cos(d*x + c) - 3)*sqrt(a*sin(d*x + c
) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*cos(d*x + c)^2 + 8*a*cos(d*x + c) -
a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1
)*sin(d*x + c) - cos(d*x + c) - 1))/d, -sqrt(-a)*arctan(1/2*sqrt(a*sin(d*x
+ c) + a)*sqrt(-a)*(sin(d*x + c) - 2)/(a*cos(d*x + c)))/d]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\,(\sin(c+dx)+1)}\csc(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sin(c + d*x) + 1))*csc(c + d*x), x)
```

Giac [B] time = 1.86797, size = 243, normalized size = 6.57

$$\frac{\frac{2\,a\arctan\left(-\frac{\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}}{\sqrt{-a}}\right)\operatorname{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{\sqrt{-a}}-\sqrt{a}\log\left(\left|-\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+\right.\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] (2*a*arctan(-(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2
+ a))/sqrt(-a))*sgn(tan(1/2*d*x + 1/2*c) + 1)/sqrt(-a) - sqrt(a)*log(abs(-s
qrt(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)))*sgn(tan(
1/2*d*x + 1/2*c) + 1) - (2*a*arctan((sqrt(2)*sqrt(a) + sqrt(a))/sqrt(-a)) -
 sqrt(-a)*sqrt(a)*log(sqrt(2)*sqrt(a) + sqrt(a)))*sgn(tan(1/2*d*x + 1/2*c)
+ 1)/sqrt(-a))/d
```

3.38 $\int \csc^2(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=64

$$-\frac{a\cot(c+dx)}{d\sqrt{a\sin(c+dx)+a}}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{d}$$

```
[Out] -((Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/d) - (
a*Cot[c + d*x])/(d*Sqrt[a + a*Sin[c + d*x]])
```

Rubi [A] time = 0.103492, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {2772, 2773, 206}

$$-\frac{a\cot(c+dx)}{d\sqrt{a\sin(c+dx)+a}}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] -((Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/d) - (
a*Cot[c + d*x])/(d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \csc^2(c+dx)\sqrt{a+a\sin(c+dx)}\,dx = -\frac{a\cot(c+dx)}{d\sqrt{a+a\sin(c+dx)}} + \frac{1}{2}\int \csc(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$$
$$= -\frac{a\cot(c+dx)}{d\sqrt{a+a\sin(c+dx)}} - \frac{a\,\mathrm{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, \frac{a\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d}$$
$$= -\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d} - \frac{a\cot(c+dx)}{d\sqrt{a+a\sin(c+dx)}}$$

Mathematica [B] time = 0.675921, size = 178, normalized size = 2.78

$$-\frac{\csc^4\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sin(c+dx)+1)}\left(-2\sin\left(\frac{1}{2}(c+dx)\right)+2\cos\left(\frac{1}{2}(c+dx)\right)+\sin(c+dx)\left(\log\left(-\sin\left(\frac{1}{2}(c+dx)\right)+\right.\right.\right.}{d\left(\cot\left(\frac{1}{2}(c+dx)\right)+1\right)\left(\csc\left(\frac{1}{4}(c+dx)\right)-\sec\left(\frac{1}{4}(c+dx)\right)\right)\left(\csc\left(\frac{1}{4}(c+\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] -((Csc[(c + d*x)/2]^4*Sqrt[a*(1 + Sin[c + d*x])]*(2*Cos[(c + d*x)/2] - 2*Si
n[(c + d*x)/2] + (Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[1 - Co
s[(c + d*x)/2] + Sin[(c + d*x)/2]])*Sin[c + d*x]))/(d*(1 + Cot[(c + d*x)/2]
)*(Csc[(c + d*x)/4] - Sec[(c + d*x)/4])*(Csc[(c + d*x)/4] + Sec[(c + d*x)/4
])))
```

Maple [A] time = 0.579, size = 104, normalized size = 1.6

$$-\frac{1+\sin(dx+c)}{\cos(dx+c)\sin(dx+c)d}\sqrt{-a(\sin(dx+c)-1)}\left(\sqrt{a-a\sin(dx+c)}a^{\frac{3}{2}}+\mathrm{Artanh}\left(\sqrt{a-a\sin(dx+c)}\frac{1}{\sqrt{a}}\right)a^2\sin(d\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] -(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*((a-a*sin(d*x+c))^(1/2)*a^(3/2)+a
rctanh((a-a*sin(d*x+c))^(1/2)/a^(1/2))*a^2*sin(d*x+c))/sin(d*x+c)/a^(3/2)/c
os(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\sin(dx+c)+a}\csc(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*csc(d*x + c)^2, x)
```

Fricas [B] time = 1.46216, size = 695, normalized size = 10.86

$$\frac{\left(\cos\left(dx+c\right)^2-\left(\cos\left(dx+c\right)+1\right)\sin\left(dx+c\right)-1\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-4\left(\cos(dx+c)^2+(\cos(dx+c)+3)\sin(dx+c)-2\,\cos(dx\cdots\right.}{\cos(dx+c)^3+\cos(dx+c)^2+\left(\cos\cdots\right.}\right.}{4\left(d\cos\left(dx+c\right)^2-\left(d\cos\left(\cdots\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*((cos(d*x + c)^2 - (cos(d*x + c) + 1)*sin(d*x + c) - 1)*sqrt(a)*log((a*
cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3
)*sin(d*x + c) - 2*cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a
*cos(d*x + c) + (a*cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)
/(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos
(d*x + c) - 1)) + 4*sqrt(a*sin(d*x + c) + a)*(cos(d*x + c) - sin(d*x + c) +
 1))/(d*cos(d*x + c)^2 - (d*cos(d*x + c) + d)*sin(d*x + c) - d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a\left(\sin\left(c+dx\right)+1\right)}\csc^2\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sin(c + d*x) + 1))*csc(c + d*x)**2, x)
```

Giac [B] time = 2.43371, size = 475, normalized size = 7.42

$$\frac{\frac{2\,a\arctan\left(-\frac{\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}}{\sqrt{-a}}\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{\sqrt{-a}}-\sqrt{a}\log\left(\left|-\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right|\right)\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(2*a*arctan(-(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c
)^2 + a))/sqrt(-a))*sgn(tan(1/2*d*x + 1/2*c) + 1)/sqrt(-a) - sqrt(a)*log(ab
s(-sqrt(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)))*sgn(
tan(1/2*d*x + 1/2*c) + 1) + sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)*sgn(tan(1/2*
d*x + 1/2*c) + 1) + 2*a^(3/2)*sgn(tan(1/2*d*x + 1/2*c) + 1)/((sqrt(a)*tan(1
/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a) - (2*sqrt(2)*a
*arctan((sqrt(2)*sqrt(a) + sqrt(a))/sqrt(-a)) - sqrt(2)*sqrt(-a)*sqrt(a)*lo
g(sqrt(2)*sqrt(a) + sqrt(a)) + 2*a*arctan((sqrt(2)*sqrt(a) + sqrt(a))/sqrt(
-a)) - sqrt(-a)*sqrt(a)*log(sqrt(2)*sqrt(a) + sqrt(a)) + sqrt(2)*sqrt(-a)*s
qrt(a) + 3*sqrt(-a)*sqrt(a))*sgn(tan(1/2*d*x + 1/2*c) + 1)/(sqrt(2)*sqrt(-a
) + sqrt(-a)))/d
```

3.39 $\int \csc^3(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=102

$$-\frac{3a\cot(c+dx)}{4d\sqrt{a\sin(c+dx)+a}}-\frac{3\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{4d}-\frac{a\cot(c+dx)\csc(c+dx)}{2d\sqrt{a\sin(c+dx)+a}}$$

```
[Out] (-3*Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(4*d)
 - (3*a*Cot[c + d*x])/(4*d*Sqrt[a + a*Sin[c + d*x]]) - (a*Cot[c + d*x]*Csc[
c + d*x])/(2*d*Sqrt[a + a*Sin[c + d*x]])
```

Rubi [A] time = 0.156279, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {2772, 2773, 206}

$$-\frac{3a\cot(c+dx)}{4d\sqrt{a\sin(c+dx)+a}}-\frac{3\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{4d}-\frac{a\cot(c+dx)\csc(c+dx)}{2d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-3*Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(4*d)
 - (3*a*Cot[c + d*x])/(4*d*Sqrt[a + a*Sin[c + d*x]]) - (a*Cot[c + d*x]*Csc[
c + d*x])/(2*d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \csc^3(c+dx)\sqrt{a+a\sin(c+dx)}\,dx = -\frac{a\cot(c+dx)\csc(c+dx)}{2d\sqrt{a+a\sin(c+dx)}} + \frac{3}{4}\int \csc^2(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$$
$$= -\frac{3a\cot(c+dx)}{4d\sqrt{a+a\sin(c+dx)}} - \frac{a\cot(c+dx)\csc(c+dx)}{2d\sqrt{a+a\sin(c+dx)}} + \frac{3}{8}\int \csc(c+dx)\sqrt{a+a\sin$$
$$= -\frac{3a\cot(c+dx)}{4d\sqrt{a+a\sin(c+dx)}} - \frac{a\cot(c+dx)\csc(c+dx)}{2d\sqrt{a+a\sin(c+dx)}} - \frac{(3a)\,\text{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, \right.}{4d}$$
$$= -\frac{3\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{4d} - \frac{3a\cot(c+dx)}{4d\sqrt{a+a\sin(c+dx)}} - \frac{a\cot(c+dx)\csc(c+dx)}{2d\sqrt{a+a\sin(c+dx)}}$$

Mathematica [B] time = 0.699265, size = 249, normalized size = 2.44

$$\csc^7\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sin(c+dx)+1)}\left(2\sin\left(\frac{1}{2}(c+dx)\right) - 6\sin\left(\frac{3}{2}(c+dx)\right) - 2\cos\left(\frac{1}{2}(c+dx)\right) - 6\cos\left(\frac{3}{2}(c+dx)\right) + 3\,\text{co}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (Csc[(c + d*x)/2]^7*Sqrt[a*(1 + Sin[c + d*x])]*(-2*Cos[(c + d*x)/2] - 6*Cos
[(3*(c + d*x))/2] - 3*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 3*Cos[
2*(c + d*x)]*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 3*Log[1 - Cos[(
c + d*x)/2] + Sin[(c + d*x)/2]] - 3*Cos[2*(c + d*x)]*Log[1 - Cos[(c + d*x)/
2] + Sin[(c + d*x)/2]] + 2*Sin[(c + d*x)/2] - 6*Sin[(3*(c + d*x))/2]))/(4*d
*(1 + Cot[(c + d*x)/2])*(Csc[(c + d*x)/4]^2 - Sec[(c + d*x)/4]^2)^2)
```

Maple [A] time = 0.589, size = 132, normalized size = 1.3

$$-\frac{1+\sin(dx+c)}{4(\sin(dx+c))^2\cos(dx+c)\,d}\sqrt{-a(\sin(dx+c)-1)}\left(3\sqrt{-a(\sin(dx+c)-1)}a^{3/2}\sin(dx+c) + 3\,\text{Artanh}\left(\frac{\sqrt{-a(\sin(d}}{\sqrt{}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] -1/4*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(3*(-a*(sin(d*x+c)-1))^(1/2)*
a^(3/2)*sin(d*x+c)+3*arctanh((-a*(sin(d*x+c)-1))^(1/2)/a^(1/2))*sin(d*x+c)^
2*a^2+2*(-a*(sin(d*x+c)-1))^(1/2)*a^(3/2))/sin(d*x+c)^2/a^(3/2)/cos(d*x+c)/
(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\sin(dx+c)+a}\csc(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*csc(d*x + c)^3, x)
```

Fricas [B] time = 1.45284, size = 853, normalized size = 8.36

$$\frac{3\left(\cos(dx+c)^3+\cos(dx+c)^2+\left(\cos(dx+c)^2-1\right)\sin(dx+c)-\cos(dx+c)-1\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-4(\cdots}{\cdots}\right.}{16(d\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/16*(3*(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c
) - cos(d*x + c) - 1)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 -
4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*cos(d*x + c) - 3)*s
qrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*cos(d*x + c)^2 + 8*
a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + cos(d*x + c)^2 + (c
os(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) + 4*(3*cos(d*x + c)^2
+ (3*cos(d*x + c) + 1)*sin(d*x + c) + 2*cos(d*x + c) - 1)*sqrt(a*sin(d*x +
c) + a))/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2 - d*cos(d*x + c) + (d*cos(d*x
 + c)^2 - d)*sin(d*x + c) - d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 2.54367, size = 721, normalized size = 7.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*(6*a*arctan(-(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c
)^2 + a))/sqrt(-a))*sgn(tan(1/2*d*x + 1/2*c) + 1)/sqrt(-a) - 3*sqrt(a)*log(
abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)))*sg
n(tan(1/2*d*x + 1/2*c) + 1) + sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)*(sgn(tan(1
/2*d*x + 1/2*c) + 1)*tan(1/2*d*x + 1/2*c) + 2*sgn(tan(1/2*d*x + 1/2*c) + 1)
) - (12*sqrt(2)*a*arctan((sqrt(2)*sqrt(a) + sqrt(a))/sqrt(-a)) - 6*sqrt(2)*
sqrt(-a)*sqrt(a)*log(sqrt(2)*sqrt(a) + sqrt(a)) + 18*a*arctan((sqrt(2)*sqrt
(a) + sqrt(a))/sqrt(-a)) - 9*sqrt(-a)*sqrt(a)*log(sqrt(2)*sqrt(a) + sqrt(a)
) + 2*sqrt(2)*sqrt(-a)*sqrt(a) + 2*sqrt(-a)*sqrt(a))*sgn(tan(1/2*d*x + 1/2*
c) + 1)/(2*sqrt(2)*sqrt(-a) + 3*sqrt(-a)) + 2*((sqrt(a)*tan(1/2*d*x + 1/2*c
) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^3*a*sgn(tan(1/2*d*x + 1/2*c) + 1) +
```

```
 2*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a^
(3/2)*sgn(tan(1/2*d*x + 1/2*c) + 1) + (sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(
a*tan(1/2*d*x + 1/2*c)^2 + a))*a^2*sgn(tan(1/2*d*x + 1/2*c) + 1) - 2*a^(5/2
)*sgn(tan(1/2*d*x + 1/2*c) + 1))/((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*ta
n(1/2*d*x + 1/2*c)^2 + a))^2 - a)^2)/d
```

3.40 $\int \csc^4(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=138

$$-\frac{5a\cot(c+dx)}{8d\sqrt{a\sin(c+dx)+a}}-\frac{5\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{8d}-\frac{a\cot(c+dx)\csc^2(c+dx)}{3d\sqrt{a\sin(c+dx)+a}}-\frac{5a\cot(c+dx)\csc(c+dx)}{12d\sqrt{a\sin(c+dx)+a}}$$

```
[Out] (-5*Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(8*d)
 - (5*a*Cot[c + d*x])/(8*d*Sqrt[a + a*Sin[c + d*x]]) - (5*a*Cot[c + d*x]*Cs
c[c + d*x])/(12*d*Sqrt[a + a*Sin[c + d*x]]) - (a*Cot[c + d*x]*Csc[c + d*x]^
2)/(3*d*Sqrt[a + a*Sin[c + d*x]])
```

Rubi [A] time = 0.212697, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {2772, 2773, 206}

$$-\frac{5a\cot(c+dx)}{8d\sqrt{a\sin(c+dx)+a}}-\frac{5\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{8d}-\frac{a\cot(c+dx)\csc^2(c+dx)}{3d\sqrt{a\sin(c+dx)+a}}-\frac{5a\cot(c+dx)\csc(c+dx)}{12d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4*Sqrt[a + a*Sin[c + d*x]],x]
```


```
[Out] (-5*Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(8*d)
 - (5*a*Cot[c + d*x])/(8*d*Sqrt[a + a*Sin[c + d*x]]) - (5*a*Cot[c + d*x]*Cs
c[c + d*x])/(12*d*Sqrt[a + a*Sin[c + d*x]]) - (a*Cot[c + d*x]*Csc[c + d*x]^
2)/(3*d*Sqrt[a + a*Sin[c + d*x]])
```


Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \csc^4(c+dx)\sqrt{a+a\sin(c+dx)}\,dx = -\frac{a\cot(c+dx)\csc^2(c+dx)}{3d\sqrt{a+a\sin(c+dx)}} + \frac{5}{6}\int \csc^3(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$$
$$= -\frac{5a\cot(c+dx)\csc(c+dx)}{12d\sqrt{a+a\sin(c+dx)}} - \frac{a\cot(c+dx)\csc^2(c+dx)}{3d\sqrt{a+a\sin(c+dx)}} + \frac{5}{8}\int \csc^2(c+dx)\sqrt{a}$$
$$= -\frac{5a\cot(c+dx)}{8d\sqrt{a+a\sin(c+dx)}} - \frac{5a\cot(c+dx)\csc(c+dx)}{12d\sqrt{a+a\sin(c+dx)}} - \frac{a\cot(c+dx)\csc^2(c+dx)}{3d\sqrt{a+a\sin(c+dx)}}$$
$$= -\frac{5a\cot(c+dx)}{8d\sqrt{a+a\sin(c+dx)}} - \frac{5a\cot(c+dx)\csc(c+dx)}{12d\sqrt{a+a\sin(c+dx)}} - \frac{a\cot(c+dx)\csc^2(c+dx)}{3d\sqrt{a+a\sin(c+dx)}}$$
$$= -\frac{5\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{8d} - \frac{5a\cot(c+dx)}{8d\sqrt{a+a\sin(c+dx)}} - \frac{5a\cot(c+dx)\csc(c+d}{12d\sqrt{a+a\sin(c+dx)}}$$

Mathematica [B] time = 1.28357, size = 285, normalized size = 2.07

$$\frac{\csc^{10}\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sin(c+dx)+1)}\left(84\sin\left(\frac{1}{2}(c+dx)\right)-10\sin\left(\frac{3}{2}(c+dx)\right)-30\sin\left(\frac{5}{2}(c+dx)\right)-84\cos\left(\frac{1}{2}(c+dx)\right)-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (Csc[(c + d*x)/2]^10*Sqrt[a*(1 + Sin[c + d*x])]*(-84*Cos[(c + d*x)/2] - 10*
Cos[(3*(c + d*x))/2] + 30*Cos[(5*(c + d*x))/2] + 84*Sin[(c + d*x)/2] - 45*L
og[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[c + d*x] + 45*Log[1 - Cos[(
c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[c + d*x] - 10*Sin[(3*(c + d*x))/2] - 30
*Sin[(5*(c + d*x))/2] + 15*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin
[3*(c + d*x)] - 15*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[3*(c +
d*x)]))/(24*d*(1 + Cot[(c + d*x)/2])*(Csc[(c + d*x)/4]^2 - Sec[(c + d*x)/4]
^2)^3)
```

Maple [A] time = 0.58, size = 158, normalized size = 1.1

$$-\frac{1+\sin(dx+c)}{24(\sin(dx+c))^3\cos(dx+c)d}\sqrt{-a(\sin(dx+c)-1)}\left(15\sqrt{-a(\sin(dx+c)-1)}a^{3/2}(\sin(dx+c))^2+15\,\mathrm{Artanh}\left(\frac{\sqrt{-a}}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4*(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] -1/24*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(15*(-a*(sin(d*x+c)-1))^(1/2
)*a^(3/2)*sin(d*x+c)^2+15*arctanh((-a*(sin(d*x+c)-1))^(1/2)/a^(1/2))*sin(d*
x+c)^3*a^2+10*(-a*(sin(d*x+c)-1))^(1/2)*a^(3/2)*sin(d*x+c)+8*(-a*(sin(d*x+c
)-1))^(1/2)*a^(3/2))/sin(d*x+c)^3/a^(3/2)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)
/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\sin(dx+c)+a}\,\csc(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*csc(d*x + c)^4, x)
```

Fricas [B] time = 1.61838, size = 968, normalized size = 7.01

$$\frac{15\left(\cos\left(dx+c\right)^{4}-2\,\cos\left(dx+c\right)^{2}-\left(\cos\left(dx+c\right)^{3}+\cos\left(dx+c\right)^{2}-\cos\left(dx+c\right)-1\right)\sin\left(dx+c\right)+1\right)\sqrt{a}\log\left(\frac{a\cos(dx}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/96*(15*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 - (cos(d*x + c)^3 + cos(d*x + c
)^2 - cos(d*x + c) - 1)*sin(d*x + c) + 1)*sqrt(a)*log((a*cos(d*x + c)^3 - 7
*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2
*cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a
*cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 +
 cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) +
4*(15*cos(d*x + c)^3 + 5*cos(d*x + c)^2 - (15*cos(d*x + c)^2 + 10*cos(d*x +
 c) - 13)*sin(d*x + c) - 23*cos(d*x + c) - 13)*sqrt(a*sin(d*x + c) + a))/(d
*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 - (d*cos(d*x + c)^3 + d*cos(d*x + c)^2
 - d*cos(d*x + c) - d)*sin(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 2.44771, size = 829, normalized size = 6.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*(30*a*arctan(-(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2
*c)^2 + a))/sqrt(-a))*sgn(tan(1/2*d*x + 1/2*c) + 1)/sqrt(-a) - 15*sqrt(a)*l
og(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)))
*sgn(tan(1/2*d*x + 1/2*c) + 1) + sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)*((2*sgn
(tan(1/2*d*x + 1/2*c) + 1)*tan(1/2*d*x + 1/2*c) + 3*sgn(tan(1/2*d*x + 1/2*c
) + 1))*tan(1/2*d*x + 1/2*c) + 14*sgn(tan(1/2*d*x + 1/2*c) + 1)) - (150*sqr
```

```
t(2)*a*arctan((sqrt(2)*sqrt(a) + sqrt(a))/sqrt(-a)) - 75*sqrt(2)*sqrt(-a)*s
qrt(a)*log(sqrt(2)*sqrt(a) + sqrt(a)) + 210*a*arctan((sqrt(2)*sqrt(a) + sqr
t(a))/sqrt(-a)) - 105*sqrt(-a)*sqrt(a)*log(sqrt(2)*sqrt(a) + sqrt(a)) + 112
*sqrt(2)*sqrt(-a)*sqrt(a) + 162*sqrt(-a)*sqrt(a))*sgn(tan(1/2*d*x + 1/2*c)
+ 1)/(5*sqrt(2)*sqrt(-a) + 7*sqrt(-a)) + 2*(3*(sqrt(a)*tan(1/2*d*x + 1/2*c)
 - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^5*a*sgn(tan(1/2*d*x + 1/2*c) + 1) +
18*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*a^
(3/2)*sgn(tan(1/2*d*x + 1/2*c) + 1) - 24*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sq
rt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a^(5/2)*sgn(tan(1/2*d*x + 1/2*c) + 1) -
 3*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))*a^3*
sgn(tan(1/2*d*x + 1/2*c) + 1) + 14*a^(7/2)*sgn(tan(1/2*d*x + 1/2*c) + 1))/(
(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a)^
3)/d
```

3.41 $\int \csc(c+dx)\sqrt{a-a\sin(c+dx)}\,dx$

Optimal. Leaf size=38

$$-\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a-a\sin(c+dx)}}\right)}{d}$$

```
[Out] (-2*Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a - a*Sin[c + d*x]]])/d
```

Rubi [A] time = 0.0510491, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2773, 206}

$$-\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a-a\sin(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*Sqrt[a - a*Sin[c + d*x]],x]
```

```
[Out] (-2*Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a - a*Sin[c + d*x]]])/d
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \csc(c+dx)\sqrt{a-a\sin(c+dx)}\,dx &= \frac{(2a)\operatorname{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, -\frac{a\cos(c+dx)}{\sqrt{a-a\sin(c+dx)}}\right)}{d}\\ &= -\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a-a\sin(c+dx)}}\right)}{d}\end{aligned}$$

Mathematica [B] time = 0.100621, size = 97, normalized size = 2.55

$$\frac{\sqrt{a-a\sin(c+dx)}\left(\log\left(-\sin\left(\frac{1}{2}(c+dx)\right)-\cos\left(\frac{1}{2}(c+dx)\right)+1\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)+1\right)\right)}{d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]*Sqrt[a - a*Sin[c + d*x]],x]
```

```
[Out] ((Log[1 - Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[1 + Cos[(c + d*x)/2] +
 Sin[(c + d*x)/2]])*Sqrt[a - a*Sin[c + d*x]])/(d*(Cos[(c + d*x)/2] - Sin[(c
 + d*x)/2]))
```

Maple [B] time = 0.404, size = 67, normalized size = 1.8

$$2\,\frac{(\sin(dx+c)-1)\sqrt{a(1+\sin(dx+c))}\sqrt{a}}{\cos(dx+c)\sqrt{a-a\sin(dx+c)}d}\mathrm{Artanh}\left(\frac{\sqrt{a(1+\sin(dx+c))}}{\sqrt{a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*(a-a*sin(d*x+c))^(1/2),x)
```

```
[Out] 2*(sin(d*x+c)-1)*(a*(1+sin(d*x+c)))^(1/2)*a^(1/2)*arctanh((a*(1+sin(d*x+c))
)^(1/2)/a^(1/2))/cos(d*x+c)/(a-a*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{-a\sin(dx+c)+a}\csc(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a-a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a*sin(d*x + c) + a)*csc(d*x + c), x)
```

Fricas [A] time = 1.50561, size = 591, normalized size = 15.55

$$\left[\frac{\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-4\left(\cos(dx+c)^2-(\cos(dx+c)+3)\sin(dx+c)-2\,\cos(dx+c)-3\right)\sqrt{-a\sin(dx+c)+a}\sqrt{a}-9\,a\cos(dx+c)-\left(a\cos(dx+c)^2+8\,a\cos(\right.}{\cos(dx+c)^3+\cos(dx+c)^2-\left(\cos(dx+c)^2-1\right)\sin(dx+c)-\cos(dx+c)-1}\right.}{2\,d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a-a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2
 - (cos(d*x + c) + 3)*sin(d*x + c) - 2*cos(d*x + c) - 3)*sqrt(-a*sin(d*x +
c) + a)*sqrt(a) - 9*a*cos(d*x + c) - (a*cos(d*x + c)^2 + 8*a*cos(d*x + c) -
 a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + cos(d*x + c)^2 - (cos(d*x + c)^2 -
1)*sin(d*x + c) - cos(d*x + c) - 1))/d, sqrt(-a)*arctan(1/2*sqrt(-a*sin(d*x
 + c) + a)*sqrt(-a)*(sin(d*x + c) + 2)/(a*cos(d*x + c)))/d]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{-a(\sin(c+dx)-1)}\csc(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a-a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(-a*(sin(c + d*x) - 1))*csc(c + d*x), x)
```

Giac [B] time = 2.56855, size = 248, normalized size = 6.53

$$-\frac{\frac{2\,a\arctan\left(-\frac{\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}}{\sqrt{-a}}\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}{\sqrt{-a}}+\sqrt{a}\log\left(\left|-\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2\cdots}\right.\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a-a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -(2*a*arctan(-(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2
 + a))/sqrt(-a))*sgn(tan(1/2*d*x + 1/2*c) - 1)/sqrt(-a) + sqrt(a)*log(abs(-
sqrt(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)))*sgn(tan
(1/2*d*x + 1/2*c) - 1) - (2*a*arctan((sqrt(2)*sqrt(a) - sqrt(a))/sqrt(-a))
+ sqrt(-a)*sqrt(a)*log(abs(sqrt(2)*sqrt(a) - sqrt(a))))*sgn(tan(1/2*d*x + 1
/2*c) - 1)/sqrt(-a))/d
```

3.42 $\int \csc(c+dx)\sqrt{-a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=39

$$\frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)-a}}\right)}{d}$$

```
[Out] (2*Sqrt[a]*ArcTan[(Sqrt[a]*Cos[c + d*x])/Sqrt[-a + a*Sin[c + d*x]]])/d
```

Rubi [A] time = 0.049979, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2773, 204}

$$\frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)-a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*Sqrt[-a + a*Sin[c + d*x]],x]
```

```
[Out] (2*Sqrt[a]*ArcTan[(Sqrt[a]*Cos[c + d*x])/Sqrt[-a + a*Sin[c + d*x]]])/d
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \csc(c+dx)\sqrt{-a+a\sin(c+dx)}\,dx &= -\frac{(2a)\,\text{Subst}\left(\int \frac{1}{-a-x^2}\,dx, x, \frac{a\cos(c+dx)}{\sqrt{-a+a\sin(c+dx)}}\right)}{d}\\ &= \frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{-a+a\sin(c+dx)}}\right)}{d}\end{aligned}$$

Mathematica [B] time = 0.0766714, size = 96, normalized size = 2.46

$$\frac{\sqrt{a(\sin(c+dx)-1)}\left(\log\left(-\sin\left(\frac{1}{2}(c+dx)\right)-\cos\left(\frac{1}{2}(c+dx)\right)+1\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)+1\right)\right)}{d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]*Sqrt[-a + a*Sin[c + d*x]],x]
```

```
[Out] ((Log[1 - Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[1 + Cos[(c + d*x)/2] +
 Sin[(c + d*x)/2]])*Sqrt[a*(-1 + Sin[c + d*x])])/(d*(Cos[(c + d*x)/2] - Sin
[(c + d*x)/2]))
```

Maple [B] time = 0.382, size = 70, normalized size = 1.8

$$2\,{\frac {(\sin \left(dx+c \right) -1)\sqrt {-a \left(1+\sin \left(dx+c \right) \right) }\sqrt {a}}{\cos \left(dx+c \right) \sqrt {a\sin \left(dx+c \right) -a}d}\arctan \left({\frac {\sqrt {-a \left(1+\sin \left(dx+c \right) \right) }}{\sqrt {a}}} \right) }$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*(a*sin(d*x+c)-a)^(1/2),x)
```

```
[Out] 2*(sin(d*x+c)-1)*(-a*(1+sin(d*x+c)))^(1/2)*a^(1/2)*arctan((-a*(1+sin(d*x+c)
))^(1/2)/a^(1/2))/cos(d*x+c)/(a*sin(d*x+c)-a)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a \sin \left(d x + c\right) - a} \csc \left(d x + c\right)\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(-a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) - a)*csc(d*x + c), x)
```

Fricas [A] time = 1.49424, size = 587, normalized size = 15.05

$$\left[\frac{\sqrt{-a} \log\left(\frac{a \cos\left(d x + c\right)^{3} - 7 \, a \cos\left(d x + c\right)^{2} + 4 \, {\left(\cos\left(d x + c\right)^{2} - {\left(\cos\left(d x + c\right) + 3\right)} \sin\left(d x + c\right) - 2 \, \cos\left(d x + c\right) - 3\right)} \sqrt{a \sin\left(d x + c\right) - a} \sqrt{-a} - 9 \, a \cos\left(d x + c\right) - {\left(a \cos\left(d x + c\right)^{2} + 8\right.}}{\cos\left(d x + c\right)^{3} + \cos\left(d x + c\right)^{2} - {\left(\cos\left(d x + c\right)^{2} - 1\right)} \sin\left(d x + c\right) - \cos\left(d x + c\right) - 1}\right.}{2 \, d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(-a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(-a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 + 4*(cos(d*x + c)^
2 - (cos(d*x + c) + 3)*sin(d*x + c) - 2*cos(d*x + c) - 3)*sqrt(a*sin(d*x +
c) - a)*sqrt(-a) - 9*a*cos(d*x + c) - (a*cos(d*x + c)^2 + 8*a*cos(d*x + c)
- a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + cos(d*x + c)^2 - (cos(d*x + c)^2 -
 1)*sin(d*x + c) - cos(d*x + c) - 1))/d, -sqrt(a)*arctan(1/2*sqrt(a*sin(d*x
 + c) - a)*(sin(d*x + c) + 2)/(sqrt(a)*cos(d*x + c)))/d]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a \left(\sin{\left (c + d x \right)} - 1\right)} \csc{\left (c + d x \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(-a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sin(c + d*x) - 1))*csc(c + d*x), x)
```

Giac [B] time = 2.13176, size = 269, normalized size = 6.9

$$-\frac{2\sqrt{a}\arctan\left(-\frac{\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a}}{\sqrt{a}}\right)\mathrm{sgn}\left(-\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)-\sqrt{-a}\log\left(\left|-\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(-a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -(2*sqrt(a)*arctan(-(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x +
1/2*c)^2 - a))/sqrt(a))*sgn(-tan(1/2*d*x + 1/2*c) + 1) - sqrt(-a)*log(abs(-
sqrt(-a)*tan(1/2*d*x + 1/2*c) + sqrt(-a*tan(1/2*d*x + 1/2*c)^2 - a)))*sgn(-
tan(1/2*d*x + 1/2*c) + 1) - (2*sqrt(a)*arctan((sqrt(2)*sqrt(-a) - sqrt(-a))
/sqrt(a)) - sqrt(-a)*log(abs(sqrt(2)*sqrt(-a) - sqrt(-a))))*sgn(-tan(1/2*d*
x + 1/2*c) + 1))/d
```

3.43 $\int \csc(c+dx)\sqrt{-a-a\sin(c+dx)}\,dx$

Optimal. Leaf size=40

$$\frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a(-\sin(c+dx))-a}}\right)}{d}$$

```
[Out] (2*Sqrt[a]*ArcTan[(Sqrt[a]*Cos[c + d*x])/Sqrt[-a - a*Sin[c + d*x]]])/d
```

Rubi [A] time = 0.0525074, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2773, 204}

$$\frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a(-\sin(c+dx))-a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*Sqrt[-a - a*Sin[c + d*x]],x]
```

```
[Out] (2*Sqrt[a]*ArcTan[(Sqrt[a]*Cos[c + d*x])/Sqrt[-a - a*Sin[c + d*x]]])/d
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \csc(c+dx)\sqrt{-a-a\sin(c+dx)}\,dx &= \frac{(2a)\operatorname{Subst}\left(\int \frac{1}{-a-x^2}\,dx, x, -\frac{a\cos(c+dx)}{\sqrt{-a-a\sin(c+dx)}}\right)}{d}\\ &= \frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{-a-a\sin(c+dx)}}\right)}{d}\end{aligned}$$

Mathematica [B] time = 0.0851717, size = 95, normalized size = 2.38

$$\frac{\sqrt{-a(\sin(c+dx)+1)}\left(\log\left(\sin\left(\frac{1}{2}(c+dx)\right)-\cos\left(\frac{1}{2}(c+dx)\right)+1\right)-\log\left(-\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)+1\right)\right)}{d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]*Sqrt[-a - a*Sin[c + d*x]],x]
```

```
[Out] ((-Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + Log[1 - Cos[(c + d*x)/2]
+ Sin[(c + d*x)/2]])*Sqrt[-(a*(1 + Sin[c + d*x]))])/(d*(Cos[(c + d*x)/2] +
Sin[(c + d*x)/2]))
```

Maple [A] time = 0.447, size = 69, normalized size = 1.7

$$-2\,\frac{(1+\sin(dx+c))\sqrt{a(\sin(dx+c)-1)}\sqrt{a}}{\cos(dx+c)\sqrt{-a-a\sin(dx+c)}d}\arctan\left(\frac{\sqrt{a(\sin(dx+c)-1)}}{\sqrt{a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*(-a-a*sin(d*x+c))^(1/2),x)
```

```
[Out] -2*(1+sin(d*x+c))*(a*(sin(d*x+c)-1))^(1/2)*a^(1/2)*arctan((a*(sin(d*x+c)-1)
)^(1/2)/a^(1/2))/cos(d*x+c)/(-a-a*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{-a\sin(dx+c)-a}\,\csc(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(-a-a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a*sin(d*x + c) - a)*csc(d*x + c), x)
```

Fricas [A] time = 1.52711, size = 589, normalized size = 14.72

$$\left[\frac{\sqrt{-a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2+4\left(\cos(dx+c)^2+(\cos(dx+c)+3)\sin(dx+c)-2\cos(dx+c)-3\right)\sqrt{-a\sin(dx+c)-a}\sqrt{-a}-9a\cos(dx+c)+\left(a\cos(dx+c)^2+8a\right.}{\cos(dx+c)^3+\cos(dx+c)^2+\left(\cos(dx+c)^2-1\right)\sin(dx+c)-\cos(dx+c)-1}\right.}{2d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(-a-a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(-a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 + 4*(cos(d*x + c)^
2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*cos(d*x + c) - 3)*sqrt(-a*sin(d*x +
 c) - a)*sqrt(-a) - 9*a*cos(d*x + c) + (a*cos(d*x + c)^2 + 8*a*cos(d*x + c)
 - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2
- 1)*sin(d*x + c) - cos(d*x + c) - 1))/d, sqrt(a)*arctan(1/2*sqrt(-a*sin(d*
x + c) - a)*(sin(d*x + c) - 2)/(sqrt(a)*cos(d*x + c)))/d]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{-a(\sin(c+dx)+1)}\,\csc(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(-a-a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(-a*(sin(c + d*x) + 1))*csc(c + d*x), x)
```

Giac [B] time = 2.13265, size = 258, normalized size = 6.45

$$2\sqrt{a}\arctan\left(-\frac{\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a}}{\sqrt{a}}\right)\mathrm{sgn}\left(-\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)+\sqrt{-a}\log\left(\left|-\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(-a-a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] (2*sqrt(a)*arctan(-(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1
/2*c)^2 - a))/sqrt(a))*sgn(-tan(1/2*d*x + 1/2*c) - 1) + sqrt(-a)*log(abs(-s
qrt(-a)*tan(1/2*d*x + 1/2*c) + sqrt(-a*tan(1/2*d*x + 1/2*c)^2 - a)))*sgn(-t
an(1/2*d*x + 1/2*c) - 1) - (2*sqrt(a)*arctan((sqrt(2)*sqrt(-a) + sqrt(-a))/
sqrt(a)) + sqrt(-a)*log(sqrt(2)*sqrt(-a) + sqrt(-a)))*sgn(-tan(1/2*d*x + 1/
2*c) - 1))/d
```

3.44 $\int \sin^3(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=162

$$-\frac{2a^2\sin^4(c+dx)\cos(c+dx)}{9d\sqrt{a\sin(c+dx)+a}}-\frac{34a^2\sin^3(c+dx)\cos(c+dx)}{63d\sqrt{a\sin(c+dx)+a}}-\frac{68a^2\cos(c+dx)}{45d\sqrt{a\sin(c+dx)+a}}-\frac{68\cos(c+dx)(a\sin(c+dx)+}{105d}$$

```
[Out] (-68*a^2*Cos[c + d*x])/(45*d*Sqrt[a + a*Sin[c + d*x]]) - (34*a^2*Cos[c + d*
x]*Sin[c + d*x]^3)/(63*d*Sqrt[a + a*Sin[c + d*x]]) - (2*a^2*Cos[c + d*x]*Si
n[c + d*x]^4)/(9*d*Sqrt[a + a*Sin[c + d*x]]) + (136*a*Cos[c + d*x]*Sqrt[a +
 a*Sin[c + d*x]])/(315*d) - (68*Cos[c + d*x]*(a + a*Sin[c + d*x])^(3/2))/(1
05*d)
```

Rubi [A] time = 0.241349, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2763, 21, 2770, 2759, 2751, 2646}

$$-\frac{2a^2\sin^4(c+dx)\cos(c+dx)}{9d\sqrt{a\sin(c+dx)+a}}-\frac{34a^2\sin^3(c+dx)\cos(c+dx)}{63d\sqrt{a\sin(c+dx)+a}}-\frac{68a^2\cos(c+dx)}{45d\sqrt{a\sin(c+dx)+a}}-\frac{68\cos(c+dx)(a\sin(c+dx)+}{105d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^3*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-68*a^2*Cos[c + d*x])/(45*d*Sqrt[a + a*Sin[c + d*x]]) - (34*a^2*Cos[c + d*
x]*Sin[c + d*x]^3)/(63*d*Sqrt[a + a*Sin[c + d*x]]) - (2*a^2*Cos[c + d*x]*Si
n[c + d*x]^4)/(9*d*Sqrt[a + a*Sin[c + d*x]]) + (136*a*Cos[c + d*x]*Sqrt[a +
 a*Sin[c + d*x]])/(315*d) - (68*Cos[c + d*x]*(a + a*Sin[c + d*x])^(3/2))/(1
05*d)
```

Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
```

```
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2759

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin[e
 + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ
[m, -2^(-1)]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \sin^3(c+dx)(a+a\sin(c+dx))^{3/2}\,dx &= -\frac{2a^2\cos(c+dx)\sin^4(c+dx)}{9d\sqrt{a+a\sin(c+dx)}}+\frac{2}{9}\int\frac{\sin^3(c+dx)\left(\frac{17a^2}{2}+\frac{17}{2}a^2\sin(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}\\
&= -\frac{2a^2\cos(c+dx)\sin^4(c+dx)}{9d\sqrt{a+a\sin(c+dx)}}+\frac{1}{9}(17a)\int\sin^3(c+dx)\sqrt{a+a\sin(c+dx)}\,d\\
&= -\frac{34a^2\cos(c+dx)\sin^3(c+dx)}{63d\sqrt{a+a\sin(c+dx)}}-\frac{2a^2\cos(c+dx)\sin^4(c+dx)}{9d\sqrt{a+a\sin(c+dx)}}+\frac{1}{21}(34a)\int\\
&= -\frac{34a^2\cos(c+dx)\sin^3(c+dx)}{63d\sqrt{a+a\sin(c+dx)}}-\frac{2a^2\cos(c+dx)\sin^4(c+dx)}{9d\sqrt{a+a\sin(c+dx)}}-\frac{68\cos(c+\\
&= -\frac{34a^2\cos(c+dx)\sin^3(c+dx)}{63d\sqrt{a+a\sin(c+dx)}}-\frac{2a^2\cos(c+dx)\sin^4(c+dx)}{9d\sqrt{a+a\sin(c+dx)}}+\frac{136a\cos(c\\
&= -\frac{68a^2\cos(c+dx)}{45d\sqrt{a+a\sin(c+dx)}}-\frac{34a^2\cos(c+dx)\sin^3(c+dx)}{63d\sqrt{a+a\sin(c+dx)}}-\frac{2a^2\cos(c+dx)s}{9d\sqrt{a+a\sin}}\end{aligned}$$

Mathematica [A] time = 0.529742, size = 165, normalized size = 1.02

$$\frac{(a(\sin(c+dx)+1))^{3/2}\left(3780\sin\left(\frac{1}{2}(c+dx)\right)-1050\sin\left(\frac{3}{2}(c+dx)\right)-378\sin\left(\frac{5}{2}(c+dx)\right)+135\sin\left(\frac{7}{2}(c+dx)\right)+35\,\text{s}\right.}{2520d\left(\sin\left(\frac{1}{2}(c\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^3*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] ((a*(1 + Sin[c + d*x]))^(3/2)*(-3780*Cos[(c + d*x)/2] - 1050*Cos[(3*(c + d*
x))/2] + 378*Cos[(5*(c + d*x))/2] + 135*Cos[(7*(c + d*x))/2] - 35*Cos[(9*(c
 + d*x))/2] + 3780*Sin[(c + d*x)/2] - 1050*Sin[(3*(c + d*x))/2] - 378*Sin[(
5*(c + d*x))/2] + 135*Sin[(7*(c + d*x))/2] + 35*Sin[(9*(c + d*x))/2]))/(252
```

```
0*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3)
```

Maple [A] time = 0.463, size = 85, normalized size = 0.5

$$\frac{(2+2\sin(dx+c))\,a^2(\sin(dx+c)-1)\left(35(\sin(dx+c))^4+85(\sin(dx+c))^3+102(\sin(dx+c))^2+136\sin(dx+c)+\right.}{315\,d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3*(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] 2/315*(1+sin(d*x+c))*a^2*(sin(d*x+c)-1)*(35*sin(d*x+c)^4+85*sin(d*x+c)^3+10
2*sin(d*x+c)^2+136*sin(d*x+c)+272)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx+c)+a)^{\frac{3}{2}}\sin(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(3/2)*sin(d*x + c)^3, x)
```

Fricas [A] time = 1.39264, size = 405, normalized size = 2.5

$$-\frac{2\left(35\,a\cos(dx+c)^5-50\,a\cos(dx+c)^4-172\,a\cos(dx+c)^3+134\,a\cos(dx+c)^2+409\,a\cos(dx+c)-\left(35\,a\cos(dx\right.\right.}{315\,(d\cos(dx+c)+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -2/315*(35*a*cos(d*x + c)^5 - 50*a*cos(d*x + c)^4 - 172*a*cos(d*x + c)^3 +
134*a*cos(d*x + c)^2 + 409*a*cos(d*x + c) - (35*a*cos(d*x + c)^4 + 85*a*cos
(d*x + c)^3 - 87*a*cos(d*x + c)^2 - 221*a*cos(d*x + c) + 188*a)*sin(d*x + c
) + 188*a)*sqrt(a*sin(d*x + c) + a)/(d*cos(d*x + c) + d*sin(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3*(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a \sin (dx + c) + a)^{\frac{3}{2}} \sin (dx + c)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(3/2)*sin(d*x + c)^3, x)
```

3.45 $\int \sin^2(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=116

$$-\frac{152a^2\cos(c+dx)}{105d\sqrt{a\sin(c+dx)+a}}-\frac{2\cos(c+dx)(a\sin(c+dx)+a)^{5/2}}{7ad}+\frac{4\cos(c+dx)(a\sin(c+dx)+a)^{3/2}}{35d}-\frac{38a\cos(c+dx)\sqrt{a}}{105}$$

```
[Out] (-152*a^2*Cos[c + d*x])/(105*d*Sqrt[a + a*Sin[c + d*x]]) - (38*a*Cos[c + d*
x]*Sqrt[a + a*Sin[c + d*x]])/(105*d) + (4*Cos[c + d*x]*(a + a*Sin[c + d*x])
^(3/2))/(35*d) - (2*Cos[c + d*x]*(a + a*Sin[c + d*x])^(5/2))/(7*a*d)
```

Rubi [A] time = 0.135378, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2759, 2751, 2647, 2646}

$$-\frac{152a^2\cos(c+dx)}{105d\sqrt{a\sin(c+dx)+a}}-\frac{2\cos(c+dx)(a\sin(c+dx)+a)^{5/2}}{7ad}+\frac{4\cos(c+dx)(a\sin(c+dx)+a)^{3/2}}{35d}-\frac{38a\cos(c+dx)\sqrt{a}}{105}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-152*a^2*Cos[c + d*x])/(105*d*Sqrt[a + a*Sin[c + d*x]]) - (38*a*Cos[c + d*
x]*Sqrt[a + a*Sin[c + d*x]])/(105*d) + (4*Cos[c + d*x]*(a + a*Sin[c + d*x])
^(3/2))/(35*d) - (2*Cos[c + d*x]*(a + a*Sin[c + d*x])^(5/2))/(7*a*d)
```

Rule 2759

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin[e
 + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ
[m, -2^(-1)]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int \sin^2(c+dx)(a+a\sin(c+dx))^{3/2}\,dx = -\frac{2\cos(c+dx)(a+a\sin(c+dx))^{5/2}}{7ad} + \frac{2\int\left(\frac{5a}{2}-a\sin(c+dx)\right)(a+a\sin(c\ldots}{7a}$$
$$= \frac{4\cos(c+dx)(a+a\sin(c+dx))^{3/2}}{35d} - \frac{2\cos(c+dx)(a+a\sin(c+dx))^{5/2}}{7ad} + \frac{1\ldots}{3\ldots}$$
$$= -\frac{38a\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{105d} + \frac{4\cos(c+dx)(a+a\sin(c+dx))^{3/2}}{35d} - \ldots$$
$$= -\frac{152a^2\cos(c+dx)}{105d\sqrt{a+a\sin(c+dx)}} - \frac{38a\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{105d} + \frac{4\cos(c+\ldots}{}$$

Mathematica [A] time = 0.352085, size = 141, normalized size = 1.22

$$\frac{(a(\sin(c+dx)+1))^{3/2}\left(735\sin\left(\frac{1}{2}(c+dx)\right)-175\sin\left(\frac{3}{2}(c+dx)\right)-63\sin\left(\frac{5}{2}(c+dx)\right)+15\sin\left(\frac{7}{2}(c+dx)\right)-735\cos\left(\ldots\right.\right.}{420d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] ((a*(1 + Sin[c + d*x]))^(3/2)*(-735*Cos[(c + d*x)/2] - 175*Cos[(3*(c + d*x)
)/2] + 63*Cos[(5*(c + d*x))/2] + 15*Cos[(7*(c + d*x))/2] + 735*Sin[(c + d*x
)/2] - 175*Sin[(3*(c + d*x))/2] - 63*Sin[(5*(c + d*x))/2] + 15*Sin[(7*(c +
d*x))/2]))/(420*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3)
```

Maple [A] time = 0.609, size = 75, normalized size = 0.7

$$\frac{(2+2\sin(dx+c))\,a^2(\sin(dx+c)-1)\left(15(\sin(dx+c))^3+39(\sin(dx+c))^2+52\sin(dx+c)+104\right)}{105\,d\cos(dx+c)}\frac{1}{\sqrt{a+a\sin(d\ldots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2*(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] 2/105*(1+sin(d*x+c))*a^2*(sin(d*x+c)-1)*(15*sin(d*x+c)^3+39*sin(d*x+c)^2+52
*sin(d*x+c)+104)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx+c)+a)^{\frac{3}{2}}\sin(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(3/2)*sin(d*x + c)^2, x)
```

Fricas [A] time = 1.51588, size = 338, normalized size = 2.91

$$\frac{2\left(15\,a\cos\left(dx+c\right)^{4}+39\,a\cos\left(dx+c\right)^{3}-43\,a\cos\left(dx+c\right)^{2}-143\,a\cos\left(dx+c\right)+\left(15\,a\cos\left(dx+c\right)^{3}-24\,a\cos\left(dx+c\right)^{2}\right.\right.}{105\left(d\cos\left(dx+c\right)+d\sin\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/105*(15*a*cos(d*x + c)^4 + 39*a*cos(d*x + c)^3 - 43*a*cos(d*x + c)^2 - 14
3*a*cos(d*x + c) + (15*a*cos(d*x + c)^3 - 24*a*cos(d*x + c)^2 - 67*a*cos(d*
x + c) + 76*a)*sin(d*x + c) - 76*a)*sqrt(a*sin(d*x + c) + a)/(d*cos(d*x + c
) + d*sin(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2*(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(dx+c\right)+a\right)^{\frac{3}{2}}\sin\left(dx+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(3/2)*sin(d*x + c)^2, x)
```

3.46 $\int \sin(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=86

$$-\frac{8a^2\cos(c+dx)}{5d\sqrt{a\sin(c+dx)+a}}-\frac{2a\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{5d}-\frac{2\cos(c+dx)(a\sin(c+dx)+a)^{3/2}}{5d}$$

```
[Out] (-8*a^2*Cos[c + d*x])/(5*d*Sqrt[a + a*Sin[c + d*x]]) - (2*a*Cos[c + d*x]*Sq
rt[a + a*Sin[c + d*x]])/(5*d) - (2*Cos[c + d*x]*(a + a*Sin[c + d*x])^(3/2))
/(5*d)
```

Rubi [A] time = 0.062433, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2751, 2647, 2646}

$$-\frac{8a^2\cos(c+dx)}{5d\sqrt{a\sin(c+dx)+a}}-\frac{2a\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{5d}-\frac{2\cos(c+dx)(a\sin(c+dx)+a)^{3/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-8*a^2*Cos[c + d*x])/(5*d*Sqrt[a + a*Sin[c + d*x]]) - (2*a*Cos[c + d*x]*Sq
rt[a + a*Sin[c + d*x]])/(5*d) - (2*Cos[c + d*x]*(a + a*Sin[c + d*x])^(3/2))
/(5*d)
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \sin(c+dx)(a+a\sin(c+dx))^{3/2}\,dx &= -\frac{2\cos(c+dx)(a+a\sin(c+dx))^{3/2}}{5d}+\frac{3}{5}\int (a+a\sin(c+dx))^{3/2}\,dx\\ &= -\frac{2a\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{5d}-\frac{2\cos(c+dx)(a+a\sin(c+dx))^{3/2}}{5d}+\frac{1}{5}\\ &= -\frac{8a^2\cos(c+dx)}{5d\sqrt{a+a\sin(c+dx)}}-\frac{2a\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{5d}-\frac{2\cos(c+dx)(a}{}\end{aligned}$$

Mathematica [A] time = 0.15917, size = 115, normalized size = 1.34

$$-\frac{(a(\sin(c+dx)+1))^{3/2}\left(-20\sin\left(\frac{1}{2}(c+dx)\right)+5\sin\left(\frac{3}{2}(c+dx)\right)+\sin\left(\frac{5}{2}(c+dx)\right)+20\cos\left(\frac{1}{2}(c+dx)\right)+5\cos\left(\frac{3}{2}(c+dx\right.\right.}{10d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] -((a*(1 + Sin[c + d*x]))^(3/2)*(20*Cos[(c + d*x)/2] + 5*Cos[(3*(c + d*x))/2
] - Cos[(5*(c + d*x))/2] - 20*Sin[(c + d*x)/2] + 5*Sin[(3*(c + d*x))/2] + S
in[(5*(c + d*x))/2]))/(10*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3)
```

Maple [A] time = 0.437, size = 63, normalized size = 0.7

$$\frac{(2+2\sin(dx+c))a^2(\sin(dx+c)-1)\left((\sin(dx+c))^2+3\sin(dx+c)+6\right)}{5d\cos(dx+c)}\frac{1}{\sqrt{a+a\sin(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)*(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] 2/5*(1+sin(d*x+c))*a^2*(sin(d*x+c)-1)*(sin(d*x+c)^2+3*sin(d*x+c)+6)/cos(d*x
+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int(a\sin(dx+c)+a)^{\frac{3}{2}}\sin(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(3/2)*sin(d*x + c), x)
```

Fricas [A] time = 1.52276, size = 259, normalized size = 3.01

$$\frac{2\left(a\cos(dx+c)^3-2a\cos(dx+c)^2-7a\cos(dx+c)-\left(a\cos(dx+c)^2+3a\cos(dx+c)-4a\right)\sin(dx+c)-4a\right)\sqrt{a\sin}}{5(d\cos(dx+c)+d\sin(dx+c)+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/5*(a*cos(d*x + c)^3 - 2*a*cos(d*x + c)^2 - 7*a*cos(d*x + c) - (a*cos(d*x
+ c)^2 + 3*a*cos(d*x + c) - 4*a)*sin(d*x + c) - 4*a)*sqrt(a*sin(d*x + c) +
a)/(d*cos(d*x + c) + d*sin(d*x + c) + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a (\sin (c + dx) + 1))^{\frac{3}{2}} \sin (c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Integral((a*(sin(c + d*x) + 1))**(3/2)*sin(c + d*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a \sin (dx + c) + a)^{\frac{3}{2}} \sin (dx + c)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(3/2)*sin(d*x + c), x)
```

3.47 $\int (a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=59

$$-\frac{8a^2\cos(c+dx)}{3d\sqrt{a\sin(c+dx)+a}}-\frac{2a\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{3d}$$

```
[Out] (-8*a^2*Cos[c + d*x])/(3*d*Sqrt[a + a*Sin[c + d*x]]) - (2*a*Cos[c + d*x]*Sq
rt[a + a*Sin[c + d*x]])/(3*d)
```

Rubi [A] time = 0.0281834, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2647, 2646}

$$-\frac{8a^2\cos(c+dx)}{3d\sqrt{a\sin(c+dx)+a}}-\frac{2a\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-8*a^2*Cos[c + d*x])/(3*d*Sqrt[a + a*Sin[c + d*x]]) - (2*a*Cos[c + d*x]*Sq
rt[a + a*Sin[c + d*x]])/(3*d)
```

Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int (a+a\sin(c+dx))^{3/2}\,dx &= -\frac{2a\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{3d}+\frac{1}{3}(4a)\int\sqrt{a+a\sin(c+dx)}\,dx\\ &= -\frac{8a^2\cos(c+dx)}{3d\sqrt{a+a\sin(c+dx)}}-\frac{2a\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{3d}\end{aligned}$$

Mathematica [A] time = 0.132218, size = 89, normalized size = 1.51

$$-\frac{(a(\sin(c+dx)+1))^{3/2}\left(-9\sin\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{3}{2}(c+dx)\right)+9\cos\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{3}{2}(c+dx)\right)\right)}{3d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] -((a*(1 + Sin[c + d*x]))^(3/2)*(9*Cos[(c + d*x)/2] + Cos[(3*(c + d*x))/2] -
 9*Sin[(c + d*x)/2] + Sin[(3*(c + d*x))/2]))/(3*d*(Cos[(c + d*x)/2] + Sin[(
c + d*x)/2])^3)
```

Maple [A] time = 0.371, size = 53, normalized size = 0.9

$$\frac{\left(2+2\,\sin\left(dx+c\right)\right)a^{2}\left(\sin\left(dx+c\right)-1\right)\left(\sin\left(dx+c\right)+5\right)}{3\,d\cos\left(dx+c\right)}\frac{1}{\sqrt{a+a\sin\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] 2/3*(1+sin(d*x+c))*a^2*(sin(d*x+c)-1)*(sin(d*x+c)+5)/cos(d*x+c)/(a+a*sin(d*
x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a\sin\left(dx+c\right)+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(3/2), x)
```

Fricas [A] time = 1.43832, size = 204, normalized size = 3.46

$$-\frac{2\left(a\cos\left(dx+c\right)^{2}+5\,a\cos\left(dx+c\right)+\left(a\cos\left(dx+c\right)-4\,a\right)\sin\left(dx+c\right)+4\,a\right)\sqrt{a\sin\left(dx+c\right)+a}}{3\left(d\cos\left(dx+c\right)+d\sin\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -2/3*(a*cos(d*x + c)^2 + 5*a*cos(d*x + c) + (a*cos(d*x + c) - 4*a)*sin(d*x
 + c) + 4*a)*sqrt(a*sin(d*x + c) + a)/(d*cos(d*x + c) + d*sin(d*x + c) + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a\sin\left(c+dx\right)+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Integral((a*sin(c + d*x) + a)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx+c)+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(3/2), x)
```

3.48 $\int \csc(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=66

$$-\frac{2a^2\cos(c+dx)}{d\sqrt{a\sin(c+dx)+a}}-\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{d}$$

```
[Out] (-2*a^(3/2)*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/d - (
2*a^2*Cos[c + d*x])/(d*Sqrt[a + a*Sin[c + d*x]])
```

Rubi [A] time = 0.0978839, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2763, 21, 2773, 206}

$$-\frac{2a^2\cos(c+dx)}{d\sqrt{a\sin(c+dx)+a}}-\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*a^(3/2)*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/d - (
2*a^2*Cos[c + d*x])/(d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \csc(c+dx)(a+a\sin(c+dx))^{3/2}\,dx &= -\frac{2a^2\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}}+2\int \frac{\csc(c+dx)\left(\frac{a^2}{2}+\frac{1}{2}a^2\sin(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}\,dx\\
&= -\frac{2a^2\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}}+a\int \csc(c+dx)\sqrt{a+a\sin(c+dx)}\,dx\\
&= -\frac{2a^2\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}}-\frac{\left(2a^2\right)\text{Subst}\left(\int \frac{1}{a-x^2}\,dx,x,\frac{a\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d}\\
&= -\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d}-\frac{2a^2\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}}
\end{aligned}
$$

Mathematica [A] time = 0.153641, size = 118, normalized size = 1.79

$$
\frac{(a(\sin(c+dx)+1))^{3/2}\left(2\sin\left(\frac{1}{2}(c+dx)\right)-2\cos\left(\frac{1}{2}(c+dx)\right)-\log\left(-\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)+1\right)+\log\left(\sin\left(\frac{1}{2}\right.\right.\right.}{d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] ((-2*Cos[(c + d*x)/2] - Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + Log[
1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 2*Sin[(c + d*x)/2])*(a*(1 + Sin[
c + d*x]))^(3/2))/(d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3)
```

Maple [A] time = 0.57, size = 84, normalized size = 1.3

$$
-2\,\frac{(1+\sin(dx+c))\sqrt{-a(\sin(dx+c)-1)}a}{\cos(dx+c)\sqrt{a+a\sin(dx+c)}d}\left(\sqrt{a-a\sin(dx+c)}+\sqrt{a}\text{Artanh}\left(\frac{\sqrt{a-a\sin(dx+c)}}{\sqrt{a}}\right)\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] -2*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*a*((a-a*sin(d*x+c))^(1/2)+a^(1/
2)*arctanh((a-a*sin(d*x+c))^(1/2)/a^(1/2)))/cos(d*x+c)/(a+a*sin(d*x+c))^(1/
2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$
\int (a\sin(dx+c)+a)^{\frac{3}{2}}\csc(dx+c)\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(3/2)*csc(d*x + c), x)
```

Fricas [B] time = 1.46189, size = 649, normalized size = 9.83

$$\frac{(a\cos(dx+c)+a\sin(dx+c)+a)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-4\left(\cos(dx+c)^2+(\cos(dx+c)+3)\sin(dx+c)-2\cos(dx+c)-3\right)\sqrt{a\sin(dx}}{\cos(dx+c)^3+\cos(dx+c)^2+\left(\cos(dx+c)^2-1\right)\sin(}\right.}{2\,(d\cos(dx+c)+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*((a*cos(d*x + c) + a*sin(d*x + c) + a)*sqrt(a)*log((a*cos(d*x + c)^3 -
7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) -
2*cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (
a*cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3
+ cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) -
 4*(a*cos(d*x + c) - a*sin(d*x + c) + a)*sqrt(a*sin(d*x + c) + a))/(d*cos(d
*x + c) + d*sin(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 2.41523, size = 350, normalized size = 5.3

$$\frac{\frac{2a^2\arctan\left(-\frac{\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{\sqrt{-a}}\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{\sqrt{-a}}-a^{\frac{3}{2}}\log\left(\left|-\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] (2*a^2*arctan(-(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^
2 + a))/sqrt(-a))*sgn(tan(1/2*d*x + 1/2*c) + 1)/sqrt(-a) - a^(3/2)*log(abs(
-sqrt(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)))*sgn(ta
n(1/2*d*x + 1/2*c) + 1) - (2*a^2*arctan((sqrt(2)*sqrt(a) + sqrt(a))/sqrt(-a
)) - sqrt(-a)*a^(3/2)*log(sqrt(2)*sqrt(a) + sqrt(a)) - 2*sqrt(2)*sqrt(-a)*a
^(3/2))*sgn(tan(1/2*d*x + 1/2*c) + 1)/sqrt(-a) + 2*(a^2*sgn(tan(1/2*d*x + 1
/2*c) + 1)*tan(1/2*d*x + 1/2*c) - a^2*sgn(tan(1/2*d*x + 1/2*c) + 1))/sqrt(a
*tan(1/2*d*x + 1/2*c)^2 + a))/d
```

3.49 $\int \csc^2(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=66

$$-\frac{a^2\cot(c+dx)}{d\sqrt{a\sin(c+dx)+a}}-\frac{3a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{d}$$

```
[Out] (-3*a^(3/2)*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/d - (
a^2*Cot[c + d*x])/(d*Sqrt[a + a*Sin[c + d*x]])
```

Rubi [A] time = 0.11004, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2762, 21, 2773, 206}

$$-\frac{a^2\cot(c+dx)}{d\sqrt{a\sin(c+dx)+a}}-\frac{3a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-3*a^(3/2)*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/d - (
a^2*Cot[c + d*x])/(d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \csc^2(c+dx)(a+a\sin(c+dx))^{3/2}\,dx &= -\frac{a^2\cot(c+dx)}{d\sqrt{a+a\sin(c+dx)}} - a\int \frac{\csc(c+dx)\left(-\frac{3a}{2}-\frac{3}{2}a\sin(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}\,dx \\
&= -\frac{a^2\cot(c+dx)}{d\sqrt{a+a\sin(c+dx)}} + \frac{1}{2}(3a)\int \csc(c+dx)\sqrt{a+a\sin(c+dx)}\,dx \\
&= -\frac{a^2\cot(c+dx)}{d\sqrt{a+a\sin(c+dx)}} - \frac{\left(3a^2\right)\text{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, \frac{a\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d} \\
&= -\frac{3a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d} - \frac{a^2\cot(c+dx)}{d\sqrt{a+a\sin(c+dx)}}
\end{aligned}$$

Mathematica [B] time = 0.646483, size = 180, normalized size = 2.73

$$-\frac{a\csc^4\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sin(c+dx)+1)}\left(-2\sin\left(\frac{1}{2}(c+dx)\right)+2\cos\left(\frac{1}{2}(c+dx)\right)+3\sin(c+dx)\left(\log\left(-\sin\left(\frac{1}{2}(c+dx)\right.\right.\right.\right.}{d\left(\cot\left(\frac{1}{2}(c+dx)\right)+1\right)\left(\csc\left(\frac{1}{4}(c+dx)\right)-\sec\left(\frac{1}{4}(c+dx)\right)\right)\left(\csc\left(\frac{1}{4}(c\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] -((a*Csc[(c + d*x)/2]^4*Sqrt[a*(1 + Sin[c + d*x])]*(2*Cos[(c + d*x)/2] - 2*
Sin[(c + d*x)/2] + 3*(Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[1
- Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])*Sin[c + d*x]))/(d*(1 + Cot[(c + d*x
)/2])*(Csc[(c + d*x)/4] - Sec[(c + d*x)/4])*(Csc[(c + d*x)/4] + Sec[(c + d*
x)/4])))
```

Maple [A] time = 0.628, size = 103, normalized size = 1.6

$$-\frac{1+\sin(dx+c)}{\cos(dx+c)\sin(dx+c)d}\sqrt{-a(\sin(dx+c)-1)}\sqrt{a}\left(3\,\text{Artanh}\left(\frac{\sqrt{a-a\sin(dx+c)}}{\sqrt{a}}\right)\sin(dx+c)a+\sqrt{a-a\sin(dx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] -(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*a^(1/2)*(3*arctanh((a-a*sin(d*x+c
))^(1/2)/a^(1/2))*sin(d*x+c)*a+(a-a*sin(d*x+c))^(1/2)*a^(1/2))/sin(d*x+c)/c
os(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx+c)+a)^{\frac{3}{2}}\csc(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(3/2)*csc(d*x + c)^2, x)
```

Fricas [B] time = 1.46665, size = 709, normalized size = 10.74

$$\frac{3\left(a\cos\left(dx+c\right)^{2}-\left(a\cos\left(dx+c\right)+a\right)\sin\left(dx+c\right)-a\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^{3}-7\,a\cos(dx+c)^{2}-4\left(\cos(dx+c)^{2}+(\cos(dx+c)+3)\sin(dx+c)-2\,c\right.}{\cos(dx+c)^{3}+\cos(dx+c)^{2}}\right.}{4\left(d\cos\left(dx+c\right)^{2}-\left(d\,c\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*(3*(a*cos(d*x + c)^2 - (a*cos(d*x + c) + a)*sin(d*x + c) - a)*sqrt(a)*l
og((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 + (cos(d*x +
c) + 3)*sin(d*x + c) - 2*cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a)
 - 9*a*cos(d*x + c) + (a*cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c
) - a)/(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c)
 - cos(d*x + c) - 1)) + 4*(a*cos(d*x + c) - a*sin(d*x + c) + a)*sqrt(a*sin(
d*x + c) + a))/(d*cos(d*x + c)^2 - (d*cos(d*x + c) + d)*sin(d*x + c) - d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 2.32806, size = 485, normalized size = 7.35

$$\frac{6\,a^{2}\arctan\left(-\frac{\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}}{\sqrt{-a}}\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{\sqrt{-a}}-3\,a^{\frac{3}{2}}\log\left(\left|-\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(6*a^2*arctan(-(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2
*c)^2 + a))/sqrt(-a))*sgn(tan(1/2*d*x + 1/2*c) + 1)/sqrt(-a) - 3*a^(3/2)*lo
g(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)))*
sgn(tan(1/2*d*x + 1/2*c) + 1) + sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)*a*sgn(ta
n(1/2*d*x + 1/2*c) + 1) + 2*a^(5/2)*sgn(tan(1/2*d*x + 1/2*c) + 1)/((sqrt(a)
*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a) - (6*sqr
t(2)*a^2*arctan((sqrt(2)*sqrt(a) + sqrt(a))/sqrt(-a)) - 3*sqrt(2)*sqrt(-a)*
a^(3/2)*log(sqrt(2)*sqrt(a) + sqrt(a)) + 6*a^2*arctan((sqrt(2)*sqrt(a) + sq
```

```
rt(a))/sqrt(-a)) - 3*sqrt(-a)*a^(3/2)*log(sqrt(2)*sqrt(a) + sqrt(a)) + sqrt
(2)*sqrt(-a)*a^(3/2) + 3*sqrt(-a)*a^(3/2))*sgn(tan(1/2*d*x + 1/2*c) + 1)/(s
qrt(2)*sqrt(-a) + sqrt(-a)))/d
```

3.50 $\int \csc^3(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=106

$$-\frac{7a^2\cot(c+dx)}{4d\sqrt{a\sin(c+dx)+a}}-\frac{7a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{4d}-\frac{a^2\cot(c+dx)\csc(c+dx)}{2d\sqrt{a\sin(c+dx)+a}}$$

```
[Out] (-7*a^(3/2)*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(4*d)
 - (7*a^2*Cot[c + d*x])/(4*d*Sqrt[a + a*Sin[c + d*x]]) - (a^2*Cot[c + d*x]*
Csc[c + d*x])/(2*d*Sqrt[a + a*Sin[c + d*x]])
```

Rubi [A] time = 0.165678, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2762, 21, 2772, 2773, 206}

$$-\frac{7a^2\cot(c+dx)}{4d\sqrt{a\sin(c+dx)+a}}-\frac{7a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{4d}-\frac{a^2\cot(c+dx)\csc(c+dx)}{2d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3*(a + a*Sin[c + d*x])^(3/2),x]
```


```
[Out] (-7*a^(3/2)*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(4*d)
 - (7*a^2*Cot[c + d*x])/(4*d*Sqrt[a + a*Sin[c + d*x]]) - (a^2*Cot[c + d*x]*
Csc[c + d*x])/(2*d*Sqrt[a + a*Sin[c + d*x]])
```


Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \csc^3(c+dx)(a+a\sin(c+dx))^{3/2}\,dx &= -\frac{a^2\cot(c+dx)\csc(c+dx)}{2d\sqrt{a+a\sin(c+dx)}}-\frac{1}{2}a\int\frac{\csc^2(c+dx)\left(-\frac{7a}{2}-\frac{7}{2}a\sin(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}\,dx\\ &= -\frac{a^2\cot(c+dx)\csc(c+dx)}{2d\sqrt{a+a\sin(c+dx)}}+\frac{1}{4}(7a)\int\csc^2(c+dx)\sqrt{a+a\sin(c+dx)}\,dx\\ &= -\frac{7a^2\cot(c+dx)}{4d\sqrt{a+a\sin(c+dx)}}-\frac{a^2\cot(c+dx)\csc(c+dx)}{2d\sqrt{a+a\sin(c+dx)}}+\frac{1}{8}(7a)\int\csc(c+dx)\\ &= -\frac{7a^2\cot(c+dx)}{4d\sqrt{a+a\sin(c+dx)}}-\frac{a^2\cot(c+dx)\csc(c+dx)}{2d\sqrt{a+a\sin(c+dx)}}-\frac{\left(7a^2\right)\mathrm{Subst}\left(\int\frac{1}{a-x^2}\right.}{4d}\\ &= -\frac{7a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{4d}-\frac{7a^2\cot(c+dx)}{4d\sqrt{a+a\sin(c+dx)}}-\frac{a^2\cot(c+dx)\csc(}{2d\sqrt{a+a\sin(c+}}\end{aligned}$$

Mathematica [B] time = 0.602813, size = 250, normalized size = 2.36

$$\frac{a\csc^7\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sin(c+dx)+1)}\left(-6\sin\left(\frac{1}{2}(c+dx)\right)-14\sin\left(\frac{3}{2}(c+dx)\right)+6\cos\left(\frac{1}{2}(c+dx)\right)-14\cos\left(\frac{3}{2}(c+dx\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (a*Csc[(c + d*x)/2]^7*Sqrt[a*(1 + Sin[c + d*x])]*(6*Cos[(c + d*x)/2] - 14*C
os[(3*(c + d*x))/2] - 7*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 7*Co
s[2*(c + d*x)]*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 7*Log[1 - Cos
[(c + d*x)/2] + Sin[(c + d*x)/2]] - 7*Cos[2*(c + d*x)]*Log[1 - Cos[(c + d*x
)/2] + Sin[(c + d*x)/2]] - 6*Sin[(c + d*x)/2] - 14*Sin[(3*(c + d*x))/2]))/(
4*d*(1 + Cot[(c + d*x)/2])*(Csc[(c + d*x)/4]^2 - Sec[(c + d*x)/4]^2)^2)
```

Maple [A] time = 0.656, size = 126, normalized size = 1.2

$$-\frac{1+\sin(dx+c)}{4(\sin(dx+c))^2\cos(dx+c)\,d}\sqrt{-a(\sin(dx+c)-1)}\left(9\sqrt{-a(\sin(dx+c)-1)}a^{5/2}-7\left(-a(\sin(dx+c)-1)\right)^{3/2}a^{3/2}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] -1/4*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(9*(-a*(sin(d*x+c)-1))^(1/2)*
a^(5/2)-7*(-a*(sin(d*x+c)-1))^(3/2)*a^(3/2)+7*arctanh((-a*(sin(d*x+c)-1))^(
1/2)/a^(1/2))*a^3*sin(d*x+c)^2)/sin(d*x+c)^2/a^(3/2)/cos(d*x+c)/(a+a*sin(d*
x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx+c)+a)^{\frac{3}{2}}\csc(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(3/2)*csc(d*x + c)^3, x)
```

Fricas [B] time = 1.5433, size = 878, normalized size = 8.28

$$\frac{7\left(a\cos(dx+c)^3+a\cos(dx+c)^2-a\cos(dx+c)+\left(a\cos(dx+c)^2-a\right)\sin(dx+c)-a\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2}{}\right.}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/16*(7*(a*cos(d*x + c)^3 + a*cos(d*x + c)^2 - a*cos(d*x + c) + (a*cos(d*x
+ c)^2 - a)*sin(d*x + c) - a)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x +
 c)^2 - 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*cos(d*x + c
) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*cos(d*x + c
)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + cos(d*x + c
)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) + 4*(7*a*cos(d
*x + c)^2 + 2*a*cos(d*x + c) + (7*a*cos(d*x + c) + 5*a)*sin(d*x + c) - 5*a)
*sqrt(a*sin(d*x + c) + a))/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2 - d*cos(d*x
 + c) + (d*cos(d*x + c)^2 - d)*sin(d*x + c) - d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 2.01454, size = 734, normalized size = 6.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*(14*a^2*arctan(-(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/
2*c)^2 + a))/sqrt(-a))*sgn(tan(1/2*d*x + 1/2*c) + 1)/sqrt(-a) - 7*a^(3/2)*l
og(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)))
*sgn(tan(1/2*d*x + 1/2*c) + 1) + (a*sgn(tan(1/2*d*x + 1/2*c) + 1)*tan(1/2*d
*x + 1/2*c) + 6*a*sgn(tan(1/2*d*x + 1/2*c) + 1))*sqrt(a*tan(1/2*d*x + 1/2*c
)^2 + a) - (28*sqrt(2)*a^2*arctan((sqrt(2)*sqrt(a) + sqrt(a))/sqrt(-a)) - 1
4*sqrt(2)*sqrt(-a)*a^(3/2)*log(sqrt(2)*sqrt(a) + sqrt(a)) + 42*a^2*arctan((
sqrt(2)*sqrt(a) + sqrt(a))/sqrt(-a)) - 21*sqrt(-a)*a^(3/2)*log(sqrt(2)*sqrt
(a) + sqrt(a)) + 18*sqrt(2)*sqrt(-a)*a^(3/2) + 22*sqrt(-a)*a^(3/2))*sgn(tan
(1/2*d*x + 1/2*c) + 1)/(2*sqrt(2)*sqrt(-a) + 3*sqrt(-a)) + 2*((sqrt(a)*tan(
1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^3*a^2*sgn(tan(1/2*d*
x + 1/2*c) + 1) + 6*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/
2*c)^2 + a))^2*a^(5/2)*sgn(tan(1/2*d*x + 1/2*c) + 1) + (sqrt(a)*tan(1/2*d*x
 + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))*a^3*sgn(tan(1/2*d*x + 1/2*c
) + 1) - 6*a^(7/2)*sgn(tan(1/2*d*x + 1/2*c) + 1))/((sqrt(a)*tan(1/2*d*x + 1
/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a)^2)/d
```

3.51 $\int \csc^4(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=144

$$-\frac{11a^2\cot(c+dx)}{8d\sqrt{a\sin(c+dx)+a}}-\frac{11a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{8d}-\frac{a^2\cot(c+dx)\csc^2(c+dx)}{3d\sqrt{a\sin(c+dx)+a}}-\frac{11a^2\cot(c+dx)\csc(c+dx)}{12d\sqrt{a\sin(c+dx)+a}}$$

```
[Out] (-11*a^(3/2)*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(8*d
) - (11*a^2*Cot[c + d*x])/(8*d*Sqrt[a + a*Sin[c + d*x]]) - (11*a^2*Cot[c +
d*x]*Csc[c + d*x])/(12*d*Sqrt[a + a*Sin[c + d*x]]) - (a^2*Cot[c + d*x]*Csc[
c + d*x]^2)/(3*d*Sqrt[a + a*Sin[c + d*x]])
```

Rubi [A] time = 0.231238, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2762, 21, 2772, 2773, 206}

$$-\frac{11a^2\cot(c+dx)}{8d\sqrt{a\sin(c+dx)+a}}-\frac{11a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{8d}-\frac{a^2\cot(c+dx)\csc^2(c+dx)}{3d\sqrt{a\sin(c+dx)+a}}-\frac{11a^2\cot(c+dx)\csc(c+dx)}{12d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-11*a^(3/2)*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(8*d
) - (11*a^2*Cot[c + d*x])/(8*d*Sqrt[a + a*Sin[c + d*x]]) - (11*a^2*Cot[c +
d*x]*Csc[c + d*x])/(12*d*Sqrt[a + a*Sin[c + d*x]]) - (a^2*Cot[c + d*x]*Csc[
c + d*x]^2)/(3*d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
```

```
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \csc^4(c+dx)(a+a\sin(c+dx))^{3/2}\,dx = -\frac{a^2\cot(c+dx)\csc^2(c+dx)}{3d\sqrt{a+a\sin(c+dx)}} - \frac{1}{3}a\int \frac{\csc^3(c+dx)\left(-\frac{11a}{2}-\frac{11}{2}a\sin(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}$$

$$= -\frac{a^2\cot(c+dx)\csc^2(c+dx)}{3d\sqrt{a+a\sin(c+dx)}} + \frac{1}{6}(11a)\int \csc^3(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$$

$$= -\frac{11a^2\cot(c+dx)\csc(c+dx)}{12d\sqrt{a+a\sin(c+dx)}} - \frac{a^2\cot(c+dx)\csc^2(c+dx)}{3d\sqrt{a+a\sin(c+dx)}} + \frac{1}{8}(11a)\int \csc$$

$$= -\frac{11a^2\cot(c+dx)}{8d\sqrt{a+a\sin(c+dx)}} - \frac{11a^2\cot(c+dx)\csc(c+dx)}{12d\sqrt{a+a\sin(c+dx)}} - \frac{a^2\cot(c+dx)\csc^2}{3d\sqrt{a+a\sin(c}$$

$$= -\frac{11a^2\cot(c+dx)}{8d\sqrt{a+a\sin(c+dx)}} - \frac{11a^2\cot(c+dx)\csc(c+dx)}{12d\sqrt{a+a\sin(c+dx)}} - \frac{a^2\cot(c+dx)\csc^2}{3d\sqrt{a+a\sin(c}$$

$$= -\frac{11a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{8d} - \frac{11a^2\cot(c+dx)}{8d\sqrt{a+a\sin(c+dx)}} - \frac{11a^2\cot(c+dx)\,c}{12d\sqrt{a+a\sin(}$$

Mathematica [A] time = 0.928437, size = 286, normalized size = 1.99

$$a\csc^{10}\left(\tfrac{1}{2}(c+dx)\right)\sqrt{a(\sin(c+dx)+1)}\left(108\sin\left(\tfrac{1}{2}(c+dx)\right)-22\sin\left(\tfrac{3}{2}(c+dx)\right)-66\sin\left(\tfrac{5}{2}(c+dx)\right)-108\cos\left(\tfrac{1}{2}(c+\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (a*Csc[(c + d*x)/2]^10*Sqrt[a*(1 + Sin[c + d*x])]*(-108*Cos[(c + d*x)/2] -
22*Cos[(3*(c + d*x))/2] + 66*Cos[(5*(c + d*x))/2] + 108*Sin[(c + d*x)/2] -
99*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[c + d*x] + 99*Log[1 - C
os[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[c + d*x] - 22*Sin[(3*(c + d*x))/2]
- 66*Sin[(5*(c + d*x))/2] + 33*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]
*Sin[3*(c + d*x)] - 33*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[3*(
c + d*x)]))/(24*d*(1 + Cot[(c + d*x)/2])*(Csc[(c + d*x)/4]^2 - Sec[(c + d*x
)/4]^2)^3)
```

Maple [A] time = 0.744, size = 144, normalized size = 1.

$$-\frac{1+\sin(dx+c)}{24\left(\sin(dx+c)\right)^3\cos(dx+c)\,d}\sqrt{-a\left(\sin(dx+c)-1\right)}\left(33\left(-a\left(\sin(dx+c)-1\right)\right)^{5/2}a^{5/2}-88\left(-a\left(\sin(dx+c)-1\right)\right)^{3/2}a^{7}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4*(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] -1/24*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)/a^(7/2)*(33*(-a*(sin(d*x+c)-
1))^(5/2)*a^(5/2)-88*(-a*(sin(d*x+c)-1))^(3/2)*a^(7/2)+33*arctanh((-a*(sin(
d*x+c)-1))^(1/2)/a^(1/2))*a^5*sin(d*x+c)^3+63*(-a*(sin(d*x+c)-1))^(1/2)*a^(
9/2))/sin(d*x+c)^3/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx+c)+a)^{\frac{3}{2}}\csc(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(3/2)*csc(d*x + c)^4, x)
```

Fricas [B] time = 1.5301, size = 1002, normalized size = 6.96

$$\frac{33\left(a\cos(dx+c)^4-2a\cos(dx+c)^2-\left(a\cos(dx+c)^3+a\cos(dx+c)^2-a\cos(dx+c)-a\right)\sin(dx+c)+a\right)\sqrt{a}\log\left(\frac{a\cos\cdots}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/96*(33*(a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^2 - (a*cos(d*x + c)^3 + a*cos
(d*x + c)^2 - a*cos(d*x + c) - a)*sin(d*x + c) + a)*sqrt(a)*log((a*cos(d*x
+ c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*
x + c) - 2*cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x
 + c) + (a*cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*
x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c
) - 1)) + 4*(33*a*cos(d*x + c)^3 + 11*a*cos(d*x + c)^2 - 41*a*cos(d*x + c)
- (33*a*cos(d*x + c)^2 + 22*a*cos(d*x + c) - 19*a)*sin(d*x + c) - 19*a)*sqr
t(a*sin(d*x + c) + a))/(d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 - (d*cos(d*x
+ c)^3 + d*cos(d*x + c)^2 - d*cos(d*x + c) - d)*sin(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4*(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 3.18432, size = 844, normalized size = 5.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/48*(66*a^2*arctan(-(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1
/2*c)^2 + a))/sqrt(-a))*sgn(tan(1/2*d*x + 1/2*c) + 1)/sqrt(-a) - 33*a^(3/2)
*log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)
))*sgn(tan(1/2*d*x + 1/2*c) + 1) + sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)*(26*a
*sgn(tan(1/2*d*x + 1/2*c) + 1) + (2*a*sgn(tan(1/2*d*x + 1/2*c) + 1)*tan(1/2
*d*x + 1/2*c) + 9*a*sgn(tan(1/2*d*x + 1/2*c) + 1))*tan(1/2*d*x + 1/2*c)) -
(330*sqrt(2)*a^2*arctan((sqrt(2)*sqrt(a) + sqrt(a))/sqrt(-a)) - 165*sqrt(2)
*sqrt(-a)*a^(3/2)*log(sqrt(2)*sqrt(a) + sqrt(a)) + 462*a^2*arctan((sqrt(2)*
sqrt(a) + sqrt(a))/sqrt(-a)) - 231*sqrt(-a)*a^(3/2)*log(sqrt(2)*sqrt(a) + s
qrt(a)) + 136*sqrt(2)*sqrt(-a)*a^(3/2) + 198*sqrt(-a)*a^(3/2))*sgn(tan(1/2*
d*x + 1/2*c) + 1)/(5*sqrt(2)*sqrt(-a) + 7*sqrt(-a)) + 2*(9*(sqrt(a)*tan(1/2
*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^5*a^2*sgn(tan(1/2*d*x +
 1/2*c) + 1) + 30*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*
c)^2 + a))^4*a^(5/2)*sgn(tan(1/2*d*x + 1/2*c) + 1) - 48*(sqrt(a)*tan(1/2*d*
x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a^(7/2)*sgn(tan(1/2*d*x
+ 1/2*c) + 1) - 9*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*
c)^2 + a))*a^4*sgn(tan(1/2*d*x + 1/2*c) + 1) + 26*a^(9/2)*sgn(tan(1/2*d*x +
 1/2*c) + 1))/((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^
2 + a))^2 - a)^3)/d
```

3.52 $\int \sin^3(c+dx)(a+a\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=203

$$-\frac{2a^2\sin^4(c+dx)\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{11d}-\frac{46a^3\sin^4(c+dx)\cos(c+dx)}{99d\sqrt{a\sin(c+dx)+a}}-\frac{710a^3\sin^3(c+dx)\cos(c+dx)}{693d\sqrt{a\sin(c+dx)+a}}+\frac{568}{}$$

```
[Out] (-284*a^3*Cos[c + d*x])/(99*d*Sqrt[a + a*Sin[c + d*x]]) - (710*a^3*Cos[c +
d*x]*Sin[c + d*x]^3)/(693*d*Sqrt[a + a*Sin[c + d*x]]) - (46*a^3*Cos[c + d*x
]*Sin[c + d*x]^4)/(99*d*Sqrt[a + a*Sin[c + d*x]]) + (568*a^2*Cos[c + d*x]*S
qrt[a + a*Sin[c + d*x]])/(693*d) - (2*a^2*Cos[c + d*x]*Sin[c + d*x]^4*Sqrt[
a + a*Sin[c + d*x]])/(11*d) - (284*a*Cos[c + d*x]*(a + a*Sin[c + d*x])^(3/2
))/(231*d)
```

Rubi [A] time = 0.351971, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2763, 2981, 2770, 2759, 2751, 2646}

$$-\frac{2a^2\sin^4(c+dx)\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{11d}-\frac{46a^3\sin^4(c+dx)\cos(c+dx)}{99d\sqrt{a\sin(c+dx)+a}}-\frac{710a^3\sin^3(c+dx)\cos(c+dx)}{693d\sqrt{a\sin(c+dx)+a}}+\frac{568}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^3*(a + a*Sin[c + d*x])^(5/2),x]
```


```
[Out] (-284*a^3*Cos[c + d*x])/(99*d*Sqrt[a + a*Sin[c + d*x]]) - (710*a^3*Cos[c +
d*x]*Sin[c + d*x]^3)/(693*d*Sqrt[a + a*Sin[c + d*x]]) - (46*a^3*Cos[c + d*x
]*Sin[c + d*x]^4)/(99*d*Sqrt[a + a*Sin[c + d*x]]) + (568*a^2*Cos[c + d*x]*S
qrt[a + a*Sin[c + d*x]])/(693*d) - (2*a^2*Cos[c + d*x]*Sin[c + d*x]^4*Sqrt[
a + a*Sin[c + d*x]])/(11*d) - (284*a*Cos[c + d*x]*(a + a*Sin[c + d*x])^(3/2
))/(231*d)
```


Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2759

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin[e
 + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ
[m, -2^(-1)]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int \sin^3(c+dx)(a+a\sin(c+dx))^{5/2}\,dx = -\frac{2a^2\cos(c+dx)\sin^4(c+dx)\sqrt{a+a\sin(c+dx)}}{11d} + \frac{2}{11}\int \sin^3(c+dx)\sqrt{a+
$$
$$= -\frac{46a^3\cos(c+dx)\sin^4(c+dx)}{99d\sqrt{a+a\sin(c+dx)}} - \frac{2a^2\cos(c+dx)\sin^4(c+dx)\sqrt{a+a\sin(c-}{11d}$$
$$= -\frac{710a^3\cos(c+dx)\sin^3(c+dx)}{693d\sqrt{a+a\sin(c+dx)}} - \frac{46a^3\cos(c+dx)\sin^4(c+dx)}{99d\sqrt{a+a\sin(c+dx)}} - \frac{2a^2\cos(}{}$$
$$= -\frac{710a^3\cos(c+dx)\sin^3(c+dx)}{693d\sqrt{a+a\sin(c+dx)}} - \frac{46a^3\cos(c+dx)\sin^4(c+dx)}{99d\sqrt{a+a\sin(c+dx)}} - \frac{2a^2\cos(}{}$$
$$= -\frac{710a^3\cos(c+dx)\sin^3(c+dx)}{693d\sqrt{a+a\sin(c+dx)}} - \frac{46a^3\cos(c+dx)\sin^4(c+dx)}{99d\sqrt{a+a\sin(c+dx)}} + \frac{568a^2\cos}{}$$
$$= -\frac{284a^3\cos(c+dx)}{99d\sqrt{a+a\sin(c+dx)}} - \frac{710a^3\cos(c+dx)\sin^3(c+dx)}{693d\sqrt{a+a\sin(c+dx)}} - \frac{46a^3\cos(c+d}{99d\sqrt{a+a\sin}}$$

Mathematica [A] time = 1.26886, size = 189, normalized size = 0.93

$$-\frac{(a(\sin(c+dx)+1))^{5/2}\left(-31878\sin\left(\frac{1}{2}(c+dx)\right)+8778\sin\left(\frac{3}{2}(c+dx)\right)+3465\sin\left(\frac{5}{2}(c+dx)\right)-1287\sin\left(\frac{7}{2}(c+dx)\right)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^3*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] -((a*(1 + Sin[c + d*x]))^(5/2)*(31878*Cos[(c + d*x)/2] + 8778*Cos[(3*(c + d
*x))/2] - 3465*Cos[(5*(c + d*x))/2] - 1287*Cos[(7*(c + d*x))/2] + 385*Cos[(
9*(c + d*x))/2] + 63*Cos[(11*(c + d*x))/2] - 31878*Sin[(c + d*x)/2] + 8778*
Sin[(3*(c + d*x))/2] + 3465*Sin[(5*(c + d*x))/2] - 1287*Sin[(7*(c + d*x))/2
] - 385*Sin[(9*(c + d*x))/2] + 63*Sin[(11*(c + d*x))/2]))/(11088*d*(Cos[(c
+ d*x)/2] + Sin[(c + d*x)/2])^5)
```

Maple [A] time = 0.628, size = 95, normalized size = 0.5

$$\frac{(2+2\sin(dx+c))\,a^3(\sin(dx+c)-1)\left(63(\sin(dx+c))^5+224(\sin(dx+c))^4+355(\sin(dx+c))^3+426(\sin(dx+c}{693\,d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3*(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] 2/693*(1+sin(d*x+c))*a^3*(sin(d*x+c)-1)*(63*sin(d*x+c)^5+224*sin(d*x+c)^4+3
55*sin(d*x+c)^3+426*sin(d*x+c)^2+568*sin(d*x+c)+1136)/cos(d*x+c)/(a+a*sin(d
*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int(a\sin(dx+c)+a)^{\frac{5}{2}}\sin(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(5/2)*sin(d*x + c)^3, x)
```

Fricas [A] time = 1.53847, size = 508, normalized size = 2.5

$$-\frac{2\left(63\,a^2\cos(dx+c)^6+224\,a^2\cos(dx+c)^5-320\,a^2\cos(dx+c)^4-874\,a^2\cos(dx+c)^3+593\,a^2\cos(dx+c)^2+1786\,a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/693*(63*a^2*cos(d*x + c)^6 + 224*a^2*cos(d*x + c)^5 - 320*a^2*cos(d*x +
c)^4 - 874*a^2*cos(d*x + c)^3 + 593*a^2*cos(d*x + c)^2 + 1786*a^2*cos(d*x +
 c) + 800*a^2 + (63*a^2*cos(d*x + c)^5 - 161*a^2*cos(d*x + c)^4 - 481*a^2*c
os(d*x + c)^3 + 393*a^2*cos(d*x + c)^2 + 986*a^2*cos(d*x + c) - 800*a^2)*si
n(d*x + c))*sqrt(a*sin(d*x + c) + a)/(d*cos(d*x + c) + d*sin(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3*(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx+c)+a)^{\frac{5}{2}}\sin(dx+c)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(5/2)*sin(d*x + c)^3, x)
```

3.53 $\int \sin^2(c+dx)(a+a\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=146

$$-\frac{208a^2\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{315d}-\frac{832a^3\cos(c+dx)}{315d\sqrt{a\sin(c+dx)+a}}-\frac{2\cos(c+dx)(a\sin(c+dx)+a)^{7/2}}{9ad}+\frac{4\cos(c+dx)(a}{6}$$

```
[Out] (-832*a^3*Cos[c + d*x])/(315*d*Sqrt[a + a*Sin[c + d*x]]) - (208*a^2*Cos[c +
 d*x]*Sqrt[a + a*Sin[c + d*x]])/(315*d) - (26*a*Cos[c + d*x]*(a + a*Sin[c +
 d*x])^(3/2))/(105*d) + (4*Cos[c + d*x]*(a + a*Sin[c + d*x])^(5/2))/(63*d)
- (2*Cos[c + d*x]*(a + a*Sin[c + d*x])^(7/2))/(9*a*d)
```

Rubi [A] time = 0.154681, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2759, 2751, 2647, 2646}

$$-\frac{208a^2\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{315d}-\frac{832a^3\cos(c+dx)}{315d\sqrt{a\sin(c+dx)+a}}-\frac{2\cos(c+dx)(a\sin(c+dx)+a)^{7/2}}{9ad}+\frac{4\cos(c+dx)(a}{6}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-832*a^3*Cos[c + d*x])/(315*d*Sqrt[a + a*Sin[c + d*x]]) - (208*a^2*Cos[c +
 d*x]*Sqrt[a + a*Sin[c + d*x]])/(315*d) - (26*a*Cos[c + d*x]*(a + a*Sin[c +
 d*x])^(3/2))/(105*d) + (4*Cos[c + d*x]*(a + a*Sin[c + d*x])^(5/2))/(63*d)
- (2*Cos[c + d*x]*(a + a*Sin[c + d*x])^(7/2))/(9*a*d)
```

Rule 2759

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin[e
 + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ
[m, -2^(-1)]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int \sin^2(c+dx)(a+a\sin(c+dx))^{5/2}\,dx = -\frac{2\cos(c+dx)(a+a\sin(c+dx))^{7/2}}{9ad} + \frac{2\int\left(\frac{7a}{2}-a\sin(c+dx)\right)(a+a\sin(c\ldots}{9a}$$
$$= \frac{4\cos(c+dx)(a+a\sin(c+dx))^{5/2}}{63d} - \frac{2\cos(c+dx)(a+a\sin(c+dx))^{7/2}}{9ad} + \frac{1}{2}\ldots$$
$$= -\frac{26a\cos(c+dx)(a+a\sin(c+dx))^{3/2}}{105d} + \frac{4\cos(c+dx)(a+a\sin(c+dx))^{5/2}}{63d}\ldots$$
$$= -\frac{208a^2\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{315d} - \frac{26a\cos(c+dx)(a+a\sin(c+dx))^{3}}{105d}\ldots$$
$$= -\frac{832a^3\cos(c+dx)}{315d\sqrt{a+a\sin(c+dx)}} - \frac{208a^2\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{315d} - \frac{26a\cos\ldots}{}$$

Mathematica [A] time = 1.01727, size = 165, normalized size = 1.13

$$\frac{(a(\sin(c+dx)+1))^{5/2}\left(8190\sin\left(\frac{1}{2}(c+dx)\right)-2100\sin\left(\frac{3}{2}(c+dx)\right)-756\sin\left(\frac{5}{2}(c+dx)\right)+225\sin\left(\frac{7}{2}(c+dx)\right)+35\,\mathrm{s}\ldots}{2520d\left(\sin\left(\frac{1}{2}(c\ldots\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] ((a*(1 + Sin[c + d*x]))^(5/2)*(-8190*Cos[(c + d*x)/2] - 2100*Cos[(3*(c + d*
x))/2] + 756*Cos[(5*(c + d*x))/2] + 225*Cos[(7*(c + d*x))/2] - 35*Cos[(9*(c
 + d*x))/2] + 8190*Sin[(c + d*x)/2] - 2100*Sin[(3*(c + d*x))/2] - 756*Sin[(
5*(c + d*x))/2] + 225*Sin[(7*(c + d*x))/2] + 35*Sin[(9*(c + d*x))/2]))/(252
0*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5)
```

Maple [A] time = 0.622, size = 85, normalized size = 0.6

$$\frac{(2+2\sin(dx+c))\,a^3(\sin(dx+c)-1)\left(35(\sin(dx+c))^4+130(\sin(dx+c))^3+219(\sin(dx+c))^2+292\sin(dx+\ldots}{315\,d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2*(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] 2/315*(1+sin(d*x+c))*a^3*(sin(d*x+c)-1)*(35*sin(d*x+c)^4+130*sin(d*x+c)^3+2
19*sin(d*x+c)^2+292*sin(d*x+c)+584)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx+c)+a)^{\frac{5}{2}}\sin(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(5/2)*sin(d*x + c)^2, x)
```

Fricas [A] time = 1.37957, size = 437, normalized size = 2.99

$$-\frac{2\left(35\,a^2\cos\left(dx+c\right)^5-95\,a^2\cos\left(dx+c\right)^4-289\,a^2\cos\left(dx+c\right)^3+263\,a^2\cos\left(dx+c\right)^2+838\,a^2\cos\left(dx+c\right)+416\,a^2-\right.}{315\left(d\cos\left(dx+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/315*(35*a^2*cos(d*x + c)^5 - 95*a^2*cos(d*x + c)^4 - 289*a^2*cos(d*x + c
)^3 + 263*a^2*cos(d*x + c)^2 + 838*a^2*cos(d*x + c) + 416*a^2 - (35*a^2*cos
(d*x + c)^4 + 130*a^2*cos(d*x + c)^3 - 159*a^2*cos(d*x + c)^2 - 422*a^2*cos
(d*x + c) + 416*a^2)*sin(d*x + c))*sqrt(a*sin(d*x + c) + a)/(d*cos(d*x + c)
 + d*sin(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2*(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(dx+c\right)+a\right)^{\frac{5}{2}}\sin\left(dx+c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(5/2)*sin(d*x + c)^2, x)
```

3.54 $\int \sin(c+dx)(a+a\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=116

$$-\frac{64a^3\cos(c+dx)}{21d\sqrt{a\sin(c+dx)+a}}-\frac{16a^2\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{21d}-\frac{2a\cos(c+dx)(a\sin(c+dx)+a)^{3/2}}{7d}-\frac{2\cos(c+dx)(}{}$$

```
[Out] (-64*a^3*Cos[c + d*x])/(21*d*Sqrt[a + a*Sin[c + d*x]]) - (16*a^2*Cos[c + d*
x]*Sqrt[a + a*Sin[c + d*x]])/(21*d) - (2*a*Cos[c + d*x]*(a + a*Sin[c + d*x]
)^(3/2))/(7*d) - (2*Cos[c + d*x]*(a + a*Sin[c + d*x])^(5/2))/(7*d)
```

Rubi [A] time = 0.0829704, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2751, 2647, 2646}

$$-\frac{64a^3\cos(c+dx)}{21d\sqrt{a\sin(c+dx)+a}}-\frac{16a^2\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{21d}-\frac{2a\cos(c+dx)(a\sin(c+dx)+a)^{3/2}}{7d}-\frac{2\cos(c+dx)(}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-64*a^3*Cos[c + d*x])/(21*d*Sqrt[a + a*Sin[c + d*x]]) - (16*a^2*Cos[c + d*
x]*Sqrt[a + a*Sin[c + d*x]])/(21*d) - (2*a*Cos[c + d*x]*(a + a*Sin[c + d*x]
)^(3/2))/(7*d) - (2*Cos[c + d*x]*(a + a*Sin[c + d*x])^(5/2))/(7*d)
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int \sin(c+dx)(a+a\sin(c+dx))^{5/2}\,dx = -\frac{2\cos(c+dx)(a+a\sin(c+dx))^{5/2}}{7d} + \frac{5}{7}\int (a+a\sin(c+dx))^{5/2}\,dx$$
$$= -\frac{2a\cos(c+dx)(a+a\sin(c+dx))^{3/2}}{7d} - \frac{2\cos(c+dx)(a+a\sin(c+dx))^{5/2}}{7d} + \frac{1}{7}(\ldots$$
$$= -\frac{16a^2\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{21d} - \frac{2a\cos(c+dx)(a+a\sin(c+dx))^{3/2}}{7d} - \frac{2}{\ldots}$$
$$= -\frac{64a^3\cos(c+dx)}{21d\sqrt{a+a\sin(c+dx)}} - \frac{16a^2\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{21d} - \frac{2a\cos(c+dx)(\ldots}{\ldots}$$

Mathematica [A] time = 0.613934, size = 141, normalized size = 1.22

$$\frac{(a(\sin(c+dx)+1))^{5/2}\left(315\sin\left(\frac{1}{2}(c+dx)\right) - 77\sin\left(\frac{3}{2}(c+dx)\right) - 21\sin\left(\frac{5}{2}(c+dx)\right) + 3\sin\left(\frac{7}{2}(c+dx)\right) - 315\cos\left(\frac{1}{2}(c+\ldots\right.\right.}{84d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] ((a*(1 + Sin[c + d*x]))^(5/2)*(-315*Cos[(c + d*x)/2] - 77*Cos[(3*(c + d*x))
/2] + 21*Cos[(5*(c + d*x))/2] + 3*Cos[(7*(c + d*x))/2] + 315*Sin[(c + d*x)/
2] - 77*Sin[(3*(c + d*x))/2] - 21*Sin[(5*(c + d*x))/2] + 3*Sin[(7*(c + d*x)
)/2]))/(84*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5)
```

Maple [A] time = 0.506, size = 75, normalized size = 0.7

$$\frac{(2+2\sin(dx+c))\,a^3(\sin(dx+c)-1)\left(3(\sin(dx+c))^3+12(\sin(dx+c))^2+23\sin(dx+c)+46\right)}{21\,d\cos(dx+c)}\frac{1}{\sqrt{a+a\sin(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)*(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] 2/21*(1+sin(d*x+c))*a^3*(sin(d*x+c)-1)*(3*sin(d*x+c)^3+12*sin(d*x+c)^2+23*s
in(d*x+c)+46)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx+c)+a)^{\frac{5}{2}}\sin(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(5/2)*sin(d*x + c), x)
```

Fricas [A] time = 1.38325, size = 355, normalized size = 3.06

$$\frac{2\left(3\,a^2\cos\left(dx+c\right)^4+12\,a^2\cos\left(dx+c\right)^3-17\,a^2\cos\left(dx+c\right)^2-58\,a^2\cos\left(dx+c\right)-32\,a^2+\left(3\,a^2\cos\left(dx+c\right)^3-9\,a^2\right.\right.}{21\left(d\cos\left(dx+c\right)+d\sin\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/21*(3*a^2*cos(d*x + c)^4 + 12*a^2*cos(d*x + c)^3 - 17*a^2*cos(d*x + c)^2
- 58*a^2*cos(d*x + c) - 32*a^2 + (3*a^2*cos(d*x + c)^3 - 9*a^2*cos(d*x + c)
^2 - 26*a^2*cos(d*x + c) + 32*a^2)*sin(d*x + c))*sqrt(a*sin(d*x + c) + a)/(
d*cos(d*x + c) + d*sin(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(dx+c\right)+a\right)^{\frac{5}{2}}\sin\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(5/2)*sin(d*x + c), x)
```

3.55 $\int (a+a\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=89

$$-\frac{64a^3\cos(c+dx)}{15d\sqrt{a\sin(c+dx)+a}}-\frac{16a^2\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{15d}-\frac{2a\cos(c+dx)(a\sin(c+dx)+a)^{3/2}}{5d}$$

```
[Out] (-64*a^3*Cos[c + d*x])/(15*d*Sqrt[a + a*Sin[c + d*x]]) - (16*a^2*Cos[c + d*
x]*Sqrt[a + a*Sin[c + d*x]])/(15*d) - (2*a*Cos[c + d*x]*(a + a*Sin[c + d*x]
)^(3/2))/(5*d)
```

Rubi [A] time = 0.0461202, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2647, 2646}

$$-\frac{64a^3\cos(c+dx)}{15d\sqrt{a\sin(c+dx)+a}}-\frac{16a^2\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{15d}-\frac{2a\cos(c+dx)(a\sin(c+dx)+a)^{3/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-64*a^3*Cos[c + d*x])/(15*d*Sqrt[a + a*Sin[c + d*x]]) - (16*a^2*Cos[c + d*
x]*Sqrt[a + a*Sin[c + d*x]])/(15*d) - (2*a*Cos[c + d*x]*(a + a*Sin[c + d*x]
)^(3/2))/(5*d)
```

Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int (a+a\sin(c+dx))^{5/2}\,dx &= -\frac{2a\cos(c+dx)(a+a\sin(c+dx))^{3/2}}{5d}+\frac{1}{5}(8a)\int (a+a\sin(c+dx))^{3/2}\,dx\\ &= -\frac{16a^2\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{15d}-\frac{2a\cos(c+dx)(a+a\sin(c+dx))^{3/2}}{5d}+\frac{1}{15}\left(32a^2\right)\int\\ &= -\frac{64a^3\cos(c+dx)}{15d\sqrt{a+a\sin(c+dx)}}-\frac{16a^2\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{15d}-\frac{2a\cos(c+dx)(a+a\sin(c}{5d}\end{aligned}$$

Mathematica [A] time = 0.30948, size = 117, normalized size = 1.31

$$-\frac{(a(\sin(c+dx)+1))^{5/2}\left(-150\sin\left(\frac{1}{2}(c+dx)\right)+25\sin\left(\frac{3}{2}(c+dx)\right)+3\sin\left(\frac{5}{2}(c+dx)\right)+150\cos\left(\frac{1}{2}(c+dx)\right)+25\cos\left(\frac{3}{2}\right.\right.}{30d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] -((a*(1 + Sin[c + d*x]))^(5/2)*(150*Cos[(c + d*x)/2] + 25*Cos[(3*(c + d*x))
/2] - 3*Cos[(5*(c + d*x))/2] - 150*Sin[(c + d*x)/2] + 25*Sin[(3*(c + d*x))/
2] + 3*Sin[(5*(c + d*x))/2]))/(30*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5
)
```

Maple [A] time = 0.506, size = 65, normalized size = 0.7

$$\frac{(2+2\sin(dx+c))\,a^3(\sin(dx+c)-1)\left(3(\sin(dx+c))^2+14\sin(dx+c)+43\right)}{15\,d\cos(dx+c)}\frac{1}{\sqrt{a+a\sin(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] 2/15*(1+sin(d*x+c))*a^3*(sin(d*x+c)-1)*(3*sin(d*x+c)^2+14*sin(d*x+c)+43)/co
s(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx+c)+a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(5/2), x)
```

Fricas [A] time = 1.40427, size = 292, normalized size = 3.28

$$\frac{2\left(3\,a^2\cos(dx+c)^3-11\,a^2\cos(dx+c)^2-46\,a^2\cos(dx+c)-32\,a^2-\left(3\,a^2\cos(dx+c)^2+14\,a^2\cos(dx+c)-32\,a^2\right)\right.}{15\,(d\cos(dx+c)+d\sin(dx+c)+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*a^2*cos(d*x + c)^3 - 11*a^2*cos(d*x + c)^2 - 46*a^2*cos(d*x + c) -
32*a^2 - (3*a^2*cos(d*x + c)^2 + 14*a^2*cos(d*x + c) - 32*a^2)*sin(d*x + c)
)*sqrt(a*sin(d*x + c) + a)/(d*cos(d*x + c) + d*sin(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx+c)+a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(5/2), x)
```

3.56 $\int \csc(c+dx)(a+a\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=98

$$-\frac{14a^3\cos(c+dx)}{3d\sqrt{a\sin(c+dx)+a}}-\frac{2a^2\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{3d}-\frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{d}$$

```
[Out] (-2*a^(5/2)*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/d - (
14*a^3*Cos[c + d*x])/(3*d*Sqrt[a + a*Sin[c + d*x]]) - (2*a^2*Cos[c + d*x]*S
qrt[a + a*Sin[c + d*x]])/(3*d)
```

Rubi [A] time = 0.192339, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2763, 2981, 2773, 206}

$$-\frac{14a^3\cos(c+dx)}{3d\sqrt{a\sin(c+dx)+a}}-\frac{2a^2\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{3d}-\frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-2*a^(5/2)*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/d - (
14*a^3*Cos[c + d*x])/(3*d*Sqrt[a + a*Sin[c + d*x]]) - (2*a^2*Cos[c + d*x]*S
qrt[a + a*Sin[c + d*x]])/(3*d)
```

Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \csc(c+dx)(a+a\sin(c+dx))^{5/2}\,dx = -\frac{2a^2\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{3d} + \frac{2}{3}\int \csc(c+dx)\sqrt{a+a\sin(c+dx)}\left(\frac{3a^2}{2}\right.$$

$$= -\frac{14a^3\cos(c+dx)}{3d\sqrt{a+a\sin(c+dx)}} - \frac{2a^2\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{3d} + a^2\int \csc(c+dx)$$

$$= -\frac{14a^3\cos(c+dx)}{3d\sqrt{a+a\sin(c+dx)}} - \frac{2a^2\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{3d} - \frac{(2a^3)\,\mathrm{Subst}\left(\int\right.}{}$$

$$= -\frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d} - \frac{14a^3\cos(c+dx)}{3d\sqrt{a+a\sin(c+dx)}} - \frac{2a^2\cos(c+dx)\sqrt{a+a}}{3d}$$

Mathematica [A] time = 0.37231, size = 143, normalized size = 1.46

$$-\frac{(a(\sin(c+dx)+1))^{5/2}\left(-15\sin\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{3}{2}(c+dx)\right)+15\cos\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{3}{2}(c+dx)\right)+3\log\left(-\sin\left(\frac{1}{2}(c\right.\right.\right.}{3d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] -((a*(1 + Sin[c + d*x]))^(5/2)*(15*Cos[(c + d*x)/2] + Cos[(3*(c + d*x))/2]
+ 3*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 3*Log[1 - Cos[(c + d*x)/
2] + Sin[(c + d*x)/2]] - 15*Sin[(c + d*x)/2] + Sin[(3*(c + d*x))/2]))/(3*d*
(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5)
```

Maple [A] time = 0.615, size = 103, normalized size = 1.1

$$-\frac{(2+2\sin(dx+c))\,a}{3\,d\cos(dx+c)}\sqrt{-a(\sin(dx+c)-1)}\left(3\,a^{3/2}\mathrm{Artanh}\left(\frac{\sqrt{a-a\sin(dx+c)}}{\sqrt{a}}\right)-(a-a\sin(dx+c))^{\frac{3}{2}}+9\,a\sqrt{a-a\sin}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] -2/3*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*a*(3*a^(3/2)*arctanh((a-a*sin
(d*x+c))^(1/2)/a^(1/2))-(a-a*sin(d*x+c))^(3/2)+9*a*(a-a*sin(d*x+c))^(1/2))/
cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx+c)+a)^{\frac{5}{2}}\csc(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(5/2)*csc(d*x + c), x)
```

Fricas [B] time = 1.44608, size = 733, normalized size = 7.48

$$3\left(a^2\cos(dx+c)+a^2\sin(dx+c)+a^2\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-4\left(\cos(dx+c)^2+(\cos(dx+c)+3)\sin(dx+c)-2\cos(dx+c)-3\right)\sqrt{a}}{\cos(dx+c)^3+\cos(dx+c)^2+\left(\cos(dx+c)^2-\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/6*(3*(a^2*cos(d*x + c) + a^2*sin(d*x + c) + a^2)*sqrt(a)*log((a*cos(d*x +
 c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x
 + c) - 2*cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x
+ c) + (a*cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x
 + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c)
 - 1)) - 4*(a^2*cos(d*x + c)^2 + 8*a^2*cos(d*x + c) + 7*a^2 + (a^2*cos(d*x
+ c) - 7*a^2)*sin(d*x + c))*sqrt(a*sin(d*x + c) + a))/(d*cos(d*x + c) + d*s
in(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 2.72137, size = 429, normalized size = 4.38

$$\frac{6a^3\arctan\left(-\frac{\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{\sqrt{-a}}\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{\sqrt{-a}}-3a^{\frac{5}{2}}\log\left(\left|-\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(6*a^3*arctan(-(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2
*c)^2 + a))/sqrt(-a))*sgn(tan(1/2*d*x + 1/2*c) + 1)/sqrt(-a) - 3*a^(5/2)*lo
g(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)))*
```

```
sgn(tan(1/2*d*x + 1/2*c) + 1) - (6*a^3*arctan((sqrt(2)*sqrt(a) + sqrt(a))/s
qrt(-a)) - 3*sqrt(-a)*a^(5/2)*log(sqrt(2)*sqrt(a) + sqrt(a)) - 14*sqrt(2)*s
qrt(-a)*a^(5/2))*sgn(tan(1/2*d*x + 1/2*c) + 1)/sqrt(-a) - 4*(4*a^4*sgn(tan(
1/2*d*x + 1/2*c) + 1) - (3*a^4*sgn(tan(1/2*d*x + 1/2*c) + 1) + (4*a^4*sgn(t
an(1/2*d*x + 1/2*c) + 1)*tan(1/2*d*x + 1/2*c) - 3*a^4*sgn(tan(1/2*d*x + 1/2
*c) + 1))*tan(1/2*d*x + 1/2*c))*tan(1/2*d*x + 1/2*c))/(a*tan(1/2*d*x + 1/2*
c)^2 + a)^(3/2))/d
```

3.57 $\int \csc^2(c+dx)(a+a\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=94

$$-\frac{a^3\cos(c+dx)}{d\sqrt{a\sin(c+dx)+a}}-\frac{a^2\cot(c+dx)\sqrt{a\sin(c+dx)+a}}{d}-\frac{5a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{d}$$

```
[Out] (-5*a^(5/2)*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/d - (
a^3*Cos[c + d*x])/(d*Sqrt[a + a*Sin[c + d*x]]) - (a^2*Cot[c + d*x]*Sqrt[a +
 a*Sin[c + d*x]])/d
```

Rubi [A] time = 0.193435, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2762, 2981, 2773, 206}

$$-\frac{a^3\cos(c+dx)}{d\sqrt{a\sin(c+dx)+a}}-\frac{a^2\cot(c+dx)\sqrt{a\sin(c+dx)+a}}{d}-\frac{5a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-5*a^(5/2)*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/d - (
a^3*Cos[c + d*x])/(d*Sqrt[a + a*Sin[c + d*x]]) - (a^2*Cot[c + d*x]*Sqrt[a +
 a*Sin[c + d*x]])/d
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \csc^2(c+dx)(a+a\sin(c+dx))^{5/2}\,dx = -\frac{a^2\cot(c+dx)\sqrt{a+a\sin(c+dx)}}{d} - a\int \csc(c+dx)\left(-\frac{5a}{2}-\frac{1}{2}a\sin(c+dx)\right)\sqrt{}$$

$$= -\frac{a^3\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}} - \frac{a^2\cot(c+dx)\sqrt{a+a\sin(c+dx)}}{d} + \frac{1}{2}\left(5a^2\right)\int \csc(c+$$

$$= -\frac{a^3\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}} - \frac{a^2\cot(c+dx)\sqrt{a+a\sin(c+dx)}}{d} - \frac{\left(5a^3\right)\text{Subst}\left(\int \frac{1}{a-}\right.}{}$$

$$= -\frac{5a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d} - \frac{a^3\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}} - \frac{a^2\cot(c+dx)\sqrt{a+a\,\mathrm{si}}}{d}$$

Mathematica [A] time = 0.801497, size = 182, normalized size = 1.94

$$-\frac{a^2\csc^4\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sin(c+dx)+1)}\left(2\sin\left(\frac{3}{2}(c+dx)\right)+2\cos\left(\frac{3}{2}(c+dx)\right)+5\sin(c+dx)\left(\log\left(-\sin\left(\frac{1}{2}(c+dx)\right)+\right.\right.\right.}{d\left(\cot\left(\frac{1}{2}(c+dx)\right)+1\right)\left(\csc\left(\frac{1}{4}(c+dx)\right)-\sec\left(\frac{1}{4}(c+dx)\right)\right)\left(\csc\left(\frac{1}{4}(c+d\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] -((a^2*Csc[(c + d*x)/2]^4*Sqrt[a*(1 + Sin[c + d*x])]*(2*Cos[(3*(c + d*x))/2
] + 5*(Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[1 - Cos[(c + d*x)
/2] + Sin[(c + d*x)/2]])*Sin[c + d*x] + 2*Sin[(3*(c + d*x))/2]))/(d*(1 + Co
t[(c + d*x)/2])*(Csc[(c + d*x)/4] - Sec[(c + d*x)/4])*(Csc[(c + d*x)/4] + S
ec[(c + d*x)/4])))
```

Maple [A] time = 0.703, size = 123, normalized size = 1.3

$$-\frac{1+\sin(dx+c)}{\cos(dx+c)\sin(dx+c)\,d}\sqrt{-a(\sin(dx+c)-1)}a^{\frac{3}{2}}\left(\sin(dx+c)\left(2\sqrt{a-a\sin(dx+c)}\sqrt{a}+5\,\mathrm{Artanh}\left(\frac{\sqrt{a-a\sin(dx+}}{\sqrt{a}}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] -(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*a^(3/2)*(sin(d*x+c)*(2*(a-a*sin(d
*x+c))^(1/2)*a^(1/2)+5*arctanh((a-a*sin(d*x+c))^(1/2)/a^(1/2))*a)+(a-a*sin(
d*x+c))^(1/2)*a^(1/2))/sin(d*x+c)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx+c)+a)^{\frac{5}{2}}\csc(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(5/2)*csc(d*x + c)^2, x)
```

Fricas [B] time = 1.4929, size = 790, normalized size = 8.4

$$\frac{5\left(a^2\cos(dx+c)^2 - a^2 - \left(a^2\cos(dx+c) + a^2\right)\sin(dx+c)\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3 - 7a\cos(dx+c)^2 - 4\left(\cos(dx+c)^2 + (\cos(dx+c)+3)\sin(\cdots\right.}{\cos(dx+c)^3 + \cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/4*(5*(a^2*cos(d*x + c)^2 - a^2 - (a^2*cos(d*x + c) + a^2)*sin(d*x + c))*s
qrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 + (co
s(d*x + c) + 3)*sin(d*x + c) - 2*cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)
*sqrt(a) - 9*a*cos(d*x + c) + (a*cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin
(d*x + c) - a)/(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(
d*x + c) - cos(d*x + c) - 1)) + 4*(2*a^2*cos(d*x + c)^2 + a^2*cos(d*x + c)
- a^2 + (2*a^2*cos(d*x + c) + a^2)*sin(d*x + c))*sqrt(a*sin(d*x + c) + a))/
(d*cos(d*x + c)^2 - (d*cos(d*x + c) + d)*sin(d*x + c) - d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 2.77994, size = 567, normalized size = 6.03

$$\frac{\frac{10\,a^3\arctan\left(-\frac{\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}}{\sqrt{-a}}\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{\sqrt{-a}} - 5\,a^{\frac{5}{2}}\log\left(\left|-\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 \cdots}\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 1/2*(10*a^3*arctan(-(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/
2*c)^2 + a))/sqrt(-a))*sgn(tan(1/2*d*x + 1/2*c) + 1)/sqrt(-a) - 5*a^(5/2)*l
og(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)))
```

```
*sgn(tan(1/2*d*x + 1/2*c) + 1) + 2*a^(7/2)*sgn(tan(1/2*d*x + 1/2*c) + 1)/((
sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a) -
 (10*sqrt(2)*a^3*arctan((sqrt(2)*sqrt(a) + sqrt(a))/sqrt(-a)) - 5*sqrt(2)*s
qrt(-a)*a^(5/2)*log(sqrt(2)*sqrt(a) + sqrt(a)) + 10*a^3*arctan((sqrt(2)*sqr
t(a) + sqrt(a))/sqrt(-a)) - 5*sqrt(-a)*a^(5/2)*log(sqrt(2)*sqrt(a) + sqrt(a
)) - 3*sqrt(2)*sqrt(-a)*a^(5/2) - 5*sqrt(-a)*a^(5/2))*sgn(tan(1/2*d*x + 1/2
*c) + 1)/(sqrt(2)*sqrt(-a) + sqrt(-a)) - (3*a^3*sgn(tan(1/2*d*x + 1/2*c) +
1) - (a^3*sgn(tan(1/2*d*x + 1/2*c) + 1)*tan(1/2*d*x + 1/2*c) + 4*a^3*sgn(ta
n(1/2*d*x + 1/2*c) + 1))*tan(1/2*d*x + 1/2*c))/sqrt(a*tan(1/2*d*x + 1/2*c)^
2 + a))/d
```

3.58 $\int \csc^3(c+dx)(a+a\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=106

$$-\frac{9a^3\cot(c+dx)}{4d\sqrt{a\sin(c+dx)+a}}-\frac{19a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{4d}-\frac{a^2\cot(c+dx)\csc(c+dx)\sqrt{a\sin(c+dx)+a}}{2d}$$

```
[Out] (-19*a^(5/2)*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(4*d
) - (9*a^3*Cot[c + d*x])/(4*d*Sqrt[a + a*Sin[c + d*x]]) - (a^2*Cot[c + d*x]
*Csc[c + d*x]*Sqrt[a + a*Sin[c + d*x]])/(2*d)
```

Rubi [A] time = 0.218477, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2762, 2980, 2773, 206}

$$-\frac{9a^3\cot(c+dx)}{4d\sqrt{a\sin(c+dx)+a}}-\frac{19a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{4d}-\frac{a^2\cot(c+dx)\csc(c+dx)\sqrt{a\sin(c+dx)+a}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-19*a^(5/2)*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(4*d
) - (9*a^3*Cot[c + d*x])/(4*d*Sqrt[a + a*Sin[c + d*x]]) - (a^2*Cot[c + d*x]
*Csc[c + d*x]*Sqrt[a + a*Sin[c + d*x]])/(2*d)
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \csc^3(c+dx)(a+a\sin(c+dx))^{5/2}\,dx = -\frac{a^2\cot(c+dx)\csc(c+dx)\sqrt{a+a\sin(c+dx)}}{2d} - \frac{1}{2}a\int \csc^2(c+dx)\left(-\frac{9a}{2} - \frac{5}{2}\right.$$

$$= -\frac{9a^3\cot(c+dx)}{4d\sqrt{a+a\sin(c+dx)}} - \frac{a^2\cot(c+dx)\csc(c+dx)\sqrt{a+a\sin(c+dx)}}{2d} + \frac{1}{8}(19$$

$$= -\frac{9a^3\cot(c+dx)}{4d\sqrt{a+a\sin(c+dx)}} - \frac{a^2\cot(c+dx)\csc(c+dx)\sqrt{a+a\sin(c+dx)}}{2d} - \frac{(19a^3}{}$$

$$= -\frac{19a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{4d} - \frac{9a^3\cot(c+dx)}{4d\sqrt{a+a\sin(c+dx)}} - \frac{a^2\cot(c+dx)\csc(c}{}$$

Mathematica [B] time = 0.749989, size = 252, normalized size = 2.38

$$\frac{a^2\csc^7\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sin(c+dx)+1)}\left(-14\sin\left(\frac{1}{2}(c+dx)\right) - 22\sin\left(\frac{3}{2}(c+dx)\right) + 14\cos\left(\frac{1}{2}(c+dx)\right) - 22\cos\left(\frac{3}{2}(c+dx\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (a^2*Csc[(c + d*x)/2]^7*Sqrt[a*(1 + Sin[c + d*x])]*(14*Cos[(c + d*x)/2] - 2
2*Cos[(3*(c + d*x))/2] - 19*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] +
19*Cos[2*(c + d*x)]*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 19*Log[1
 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 19*Cos[2*(c + d*x)]*Log[1 - Cos[(
c + d*x)/2] + Sin[(c + d*x)/2]] - 14*Sin[(c + d*x)/2] - 22*Sin[(3*(c + d*x)
)/2]))/(4*d*(1 + Cot[(c + d*x)/2])*(Csc[(c + d*x)/4]^2 - Sec[(c + d*x)/4]^2
)^2)
```

Maple [A] time = 0.623, size = 126, normalized size = 1.2

$$-\frac{1+\sin(dx+c)}{4(\sin(dx+c))^2\cos(dx+c)\,d}\sqrt{-a(\sin(dx+c)-1)}\sqrt{a}\left(19\,\mathrm{Artanh}\left(\frac{\sqrt{-a(\sin(dx+c)-1)}}{\sqrt{a}}\right)(\sin(dx+c))^2a^2 + 13\sqrt{-}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] -1/4*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*a^(1/2)*(19*arctanh((-a*(sin(
d*x+c)-1))^(1/2)/a^(1/2))*sin(d*x+c)^2*a^2+13*(-a*(sin(d*x+c)-1))^(1/2)*a^(
3/2)-11*(-a*(sin(d*x+c)-1))^(3/2)*a^(1/2))/sin(d*x+c)^2/cos(d*x+c)/(a+a*sin
(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx+c)+a)^{\frac{5}{2}}\csc(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(5/2)*csc(d*x + c)^3, x)
```

Fricas [B] time = 1.53422, size = 911, normalized size = 8.59

$$19\left(a^2\cos(dx+c)^3+a^2\cos(dx+c)^2-a^2\cos(dx+c)-a^2+\left(a^2\cos(dx+c)^2-a^2\right)\sin(dx+c)\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/16*(19*(a^2*cos(d*x + c)^3 + a^2*cos(d*x + c)^2 - a^2*cos(d*x + c) - a^2
+ (a^2*cos(d*x + c)^2 - a^2)*sin(d*x + c))*sqrt(a)*log((a*cos(d*x + c)^3 -
7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) -
2*cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (
a*cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3
+ cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) +
 4*(11*a^2*cos(d*x + c)^2 + 2*a^2*cos(d*x + c) - 9*a^2 + (11*a^2*cos(d*x +
c) + 9*a^2)*sin(d*x + c))*sqrt(a*sin(d*x + c) + a))/(d*cos(d*x + c)^3 + d*c
os(d*x + c)^2 - d*cos(d*x + c) + (d*cos(d*x + c)^2 - d)*sin(d*x + c) - d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 2.60091, size = 740, normalized size = 6.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 1/8*(38*a^3*arctan(-(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/
2*c)^2 + a))/sqrt(-a))*sgn(tan(1/2*d*x + 1/2*c) + 1)/sqrt(-a) - 19*a^(5/2)*
```

```
log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))
)*sgn(tan(1/2*d*x + 1/2*c) + 1) + (a^2*sgn(tan(1/2*d*x + 1/2*c) + 1)*tan(1/
2*d*x + 1/2*c) + 10*a^2*sgn(tan(1/2*d*x + 1/2*c) + 1))*sqrt(a*tan(1/2*d*x +
 1/2*c)^2 + a) - (76*sqrt(2)*a^3*arctan((sqrt(2)*sqrt(a) + sqrt(a))/sqrt(-a
)) - 38*sqrt(2)*sqrt(-a)*a^(5/2)*log(sqrt(2)*sqrt(a) + sqrt(a)) + 114*a^3*a
rctan((sqrt(2)*sqrt(a) + sqrt(a))/sqrt(-a)) - 57*sqrt(-a)*a^(5/2)*log(sqrt(
2)*sqrt(a) + sqrt(a)) + 34*sqrt(2)*sqrt(-a)*a^(5/2) + 42*sqrt(-a)*a^(5/2))*
sgn(tan(1/2*d*x + 1/2*c) + 1)/(2*sqrt(2)*sqrt(-a) + 3*sqrt(-a)) + 2*((sqrt(
a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^3*a^3*sgn(tan
(1/2*d*x + 1/2*c) + 1) + 10*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*
d*x + 1/2*c)^2 + a))^2*a^(7/2)*sgn(tan(1/2*d*x + 1/2*c) + 1) + (sqrt(a)*tan
(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))*a^4*sgn(tan(1/2*d*x
 + 1/2*c) + 1) - 10*a^(9/2)*sgn(tan(1/2*d*x + 1/2*c) + 1))/((sqrt(a)*tan(1/
2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a)^2)/d
```

3.59 $\int \csc^4(c+dx)(a+a\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=144

$$-\frac{25a^3\cot(c+dx)}{8d\sqrt{a\sin(c+dx)+a}}-\frac{25a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{8d}-\frac{a^2\cot(c+dx)\csc^2(c+dx)\sqrt{a\sin(c+dx)+a}}{3d}-\frac{13a^3\cot(c}{12d\sqrt{a\,s}}$$

```
[Out] (-25*a^(5/2)*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(8*d
) - (25*a^3*Cot[c + d*x])/(8*d*Sqrt[a + a*Sin[c + d*x]]) - (13*a^3*Cot[c +
d*x]*Csc[c + d*x])/(12*d*Sqrt[a + a*Sin[c + d*x]]) - (a^2*Cot[c + d*x]*Csc[
c + d*x]^2*Sqrt[a + a*Sin[c + d*x]])/(3*d)
```

Rubi [A] time = 0.275132, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2762, 2980, 2772, 2773, 206}

$$-\frac{25a^3\cot(c+dx)}{8d\sqrt{a\sin(c+dx)+a}}-\frac{25a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{8d}-\frac{a^2\cot(c+dx)\csc^2(c+dx)\sqrt{a\sin(c+dx)+a}}{3d}-\frac{13a^3\cot(c}{12d\sqrt{a\,s}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-25*a^(5/2)*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(8*d
) - (25*a^3*Cot[c + d*x])/(8*d*Sqrt[a + a*Sin[c + d*x]]) - (13*a^3*Cot[c +
d*x]*Csc[c + d*x])/(12*d*Sqrt[a + a*Sin[c + d*x]]) - (a^2*Cot[c + d*x]*Csc[
c + d*x]^2*Sqrt[a + a*Sin[c + d*x]])/(3*d)
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
```

```
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \csc^4(c+dx)(a+a\sin(c+dx))^{5/2}\,dx = -\frac{a^2\cot(c+dx)\csc^2(c+dx)\sqrt{a+a\sin(c+dx)}}{3d} - \frac{1}{3}a\int \csc^3(c+dx)\left(-\frac{13a}{2} - \right.$$

$$= -\frac{13a^3\cot(c+dx)\csc(c+dx)}{12d\sqrt{a+a\sin(c+dx)}} - \frac{a^2\cot(c+dx)\csc^2(c+dx)\sqrt{a+a\sin(c+dx)}}{3d}$$

$$= -\frac{25a^3\cot(c+dx)}{8d\sqrt{a+a\sin(c+dx)}} - \frac{13a^3\cot(c+dx)\csc(c+dx)}{12d\sqrt{a+a\sin(c+dx)}} - \frac{a^2\cot(c+dx)\csc^2(c}{}$$

$$= -\frac{25a^3\cot(c+dx)}{8d\sqrt{a+a\sin(c+dx)}} - \frac{13a^3\cot(c+dx)\csc(c+dx)}{12d\sqrt{a+a\sin(c+dx)}} - \frac{a^2\cot(c+dx)\csc^2(c}{}$$

$$= -\frac{25a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{8d} - \frac{25a^3\cot(c+dx)}{8d\sqrt{a+a\sin(c+dx)}} - \frac{13a^3\cot(c+dx)\csc(}{12d\sqrt{a+a\sin(c+}}$$

Mathematica [A] time = 1.15858, size = 288, normalized size = 2.

$$\frac{a^2\csc^{10}\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sin(c+dx)+1)}\left(228\sin\left(\frac{1}{2}(c+dx)\right)+14\sin\left(\frac{3}{2}(c+dx)\right)-150\sin\left(\frac{5}{2}(c+dx)\right)-228\cos\left(\frac{1}{2}(c+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (a^2*Csc[(c + d*x)/2]^10*Sqrt[a*(1 + Sin[c + d*x])]*(-228*Cos[(c + d*x)/2]
+ 14*Cos[(3*(c + d*x))/2] + 150*Cos[(5*(c + d*x))/2] + 228*Sin[(c + d*x)/2]
 - 225*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[c + d*x] + 225*Log[
1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[c + d*x] + 14*Sin[(3*(c + d*x)
)/2] - 150*Sin[(5*(c + d*x))/2] + 75*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*
x)/2]]*Sin[3*(c + d*x)] - 75*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*S
in[3*(c + d*x)]))/(24*d*(1 + Cot[(c + d*x)/2])*(Csc[(c + d*x)/4]^2 - Sec[(c
 + d*x)/4]^2)^3)
```

Maple [A] time = 0.732, size = 144, normalized size = 1.

$$-\frac{1+\sin(dx+c)}{24(\sin(dx+c))^3\cos(dx+c)d}\sqrt{-a(\sin(dx+c)-1)}\left(75(-a(\sin(dx+c)-1))^{5/2}a^{3/2}+75\,\mathrm{Artanh}\left(\frac{\sqrt{-a(\sin(dx}{\sqrt{a}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4*(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] -1/24*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(75*(-a*(sin(d*x+c)-1))^(5/2
)*a^(3/2)+75*arctanh((-a*(sin(d*x+c)-1))^(1/2)/a^(1/2))*a^4*sin(d*x+c)^3-18
4*(-a*(sin(d*x+c)-1))^(3/2)*a^(5/2)+117*(-a*(sin(d*x+c)-1))^(1/2)*a^(7/2))/
sin(d*x+c)^3/a^(3/2)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx+c)+a)^{\frac{5}{2}}\csc(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(5/2)*csc(d*x + c)^4, x)
```

Fricas [B] time = 1.91238, size = 1040, normalized size = 7.22

$$\frac{75\left(a^2\cos(dx+c)^4-2a^2\cos(dx+c)^2+a^2-\left(a^2\cos(dx+c)^3+a^2\cos(dx+c)^2-a^2\cos(dx+c)-a^2\right)\sin(dx+c)\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/96*(75*(a^2*cos(d*x + c)^4 - 2*a^2*cos(d*x + c)^2 + a^2 - (a^2*cos(d*x +
c)^3 + a^2*cos(d*x + c)^2 - a^2*cos(d*x + c) - a^2)*sin(d*x + c))*sqrt(a)*l
og((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 + (cos(d*x +
c) + 3)*sin(d*x + c) - 2*cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a)
 - 9*a*cos(d*x + c) + (a*cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c
) - a)/(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c)
 - cos(d*x + c) - 1)) + 4*(75*a^2*cos(d*x + c)^3 + 41*a^2*cos(d*x + c)^2 -
83*a^2*cos(d*x + c) - 49*a^2 - (75*a^2*cos(d*x + c)^2 + 34*a^2*cos(d*x + c)
 - 49*a^2)*sin(d*x + c))*sqrt(a*sin(d*x + c) + a))/(d*cos(d*x + c)^4 - 2*d*
cos(d*x + c)^2 - (d*cos(d*x + c)^3 + d*cos(d*x + c)^2 - d*cos(d*x + c) - d)
*sin(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4*(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 2.64298, size = 852, normalized size = 5.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 1/48*(150*a^3*arctan(-(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x +
1/2*c)^2 + a))/sqrt(-a))*sgn(tan(1/2*d*x + 1/2*c) + 1)/sqrt(-a) - 75*a^(5/2
)*log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a
)))*sgn(tan(1/2*d*x + 1/2*c) + 1) + (62*a^2*sgn(tan(1/2*d*x + 1/2*c) + 1) +
 (2*a^2*sgn(tan(1/2*d*x + 1/2*c) + 1)*tan(1/2*d*x + 1/2*c) + 15*a^2*sgn(tan
(1/2*d*x + 1/2*c) + 1))*tan(1/2*d*x + 1/2*c))*sqrt(a*tan(1/2*d*x + 1/2*c)^2
 + a) - (750*sqrt(2)*a^3*arctan((sqrt(2)*sqrt(a) + sqrt(a))/sqrt(-a)) - 375
*sqrt(2)*sqrt(-a)*a^(5/2)*log(sqrt(2)*sqrt(a) + sqrt(a)) + 1050*a^3*arctan(
(sqrt(2)*sqrt(a) + sqrt(a))/sqrt(-a)) - 525*sqrt(-a)*a^(5/2)*log(sqrt(2)*sq
rt(a) + sqrt(a)) + 376*sqrt(2)*sqrt(-a)*a^(5/2) + 546*sqrt(-a)*a^(5/2))*sgn
(tan(1/2*d*x + 1/2*c) + 1)/(5*sqrt(2)*sqrt(-a) + 7*sqrt(-a)) + 2*(15*(sqrt(
a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^5*a^3*sgn(tan
(1/2*d*x + 1/2*c) + 1) + 66*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*
d*x + 1/2*c)^2 + a))^4*a^(7/2)*sgn(tan(1/2*d*x + 1/2*c) + 1) - 120*(sqrt(a)
*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a^(9/2)*sgn(t
an(1/2*d*x + 1/2*c) + 1) - 15*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/
2*d*x + 1/2*c)^2 + a))*a^5*sgn(tan(1/2*d*x + 1/2*c) + 1) + 62*a^(11/2)*sgn(
tan(1/2*d*x + 1/2*c) + 1))/((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*
d*x + 1/2*c)^2 + a))^2 - a)^3)/d
```

3.60 $\int \csc^5(c+dx)(a+a\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=182

$$-\frac{163a^3\cot(c+dx)}{64d\sqrt{a\sin(c+dx)+a}}-\frac{163a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{64d}-\frac{17a^3\cot(c+dx)\csc^2(c+dx)}{24d\sqrt{a\sin(c+dx)+a}}-\frac{a^2\cot(c+dx)\csc^3(c+d}{4d}$$

```
[Out] (-163*a^(5/2)*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(64
*d) - (163*a^3*Cot[c + d*x])/(64*d*Sqrt[a + a*Sin[c + d*x]]) - (163*a^3*Cot
[c + d*x]*Csc[c + d*x])/(96*d*Sqrt[a + a*Sin[c + d*x]]) - (17*a^3*Cot[c + d
*x]*Csc[c + d*x]^2)/(24*d*Sqrt[a + a*Sin[c + d*x]]) - (a^2*Cot[c + d*x]*Csc
[c + d*x]^3*Sqrt[a + a*Sin[c + d*x]])/(4*d)
```

Rubi [A] time = 0.335321, antiderivative size = 182, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2762, 2980, 2772, 2773, 206}

$$-\frac{163a^3\cot(c+dx)}{64d\sqrt{a\sin(c+dx)+a}}-\frac{163a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{64d}-\frac{17a^3\cot(c+dx)\csc^2(c+dx)}{24d\sqrt{a\sin(c+dx)+a}}-\frac{a^2\cot(c+dx)\csc^3(c+d}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^5*(a + a*Sin[c + d*x])^(5/2),x]
```


```
[Out] (-163*a^(5/2)*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(64
*d) - (163*a^3*Cot[c + d*x])/(64*d*Sqrt[a + a*Sin[c + d*x]]) - (163*a^3*Cot
[c + d*x]*Csc[c + d*x])/(96*d*Sqrt[a + a*Sin[c + d*x]]) - (17*a^3*Cot[c + d
*x]*Csc[c + d*x]^2)/(24*d*Sqrt[a + a*Sin[c + d*x]]) - (a^2*Cot[c + d*x]*Csc
[c + d*x]^3*Sqrt[a + a*Sin[c + d*x]])/(4*d)
```


Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
```

```
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \csc^5(c+dx)(a+a\sin(c+dx))^{5/2}\,dx = -\frac{a^2\cot(c+dx)\csc^3(c+dx)\sqrt{a+a\sin(c+dx)}}{4d} - \frac{1}{4}a\int \csc^4(c+dx)\left(-\frac{17a}{2} - \right.$$

$$= -\frac{17a^3\cot(c+dx)\csc^2(c+dx)}{24d\sqrt{a+a\sin(c+dx)}} - \frac{a^2\cot(c+dx)\csc^3(c+dx)\sqrt{a+a\sin(c+dx)}}{4d}$$

$$= -\frac{163a^3\cot(c+dx)\csc(c+dx)}{96d\sqrt{a+a\sin(c+dx)}} - \frac{17a^3\cot(c+dx)\csc^2(c+dx)}{24d\sqrt{a+a\sin(c+dx)}} - \frac{a^2\cot(c+d}{}$$

$$= -\frac{163a^3\cot(c+dx)}{64d\sqrt{a+a\sin(c+dx)}} - \frac{163a^3\cot(c+dx)\csc(c+dx)}{96d\sqrt{a+a\sin(c+dx)}} - \frac{17a^3\cot(c+dx)\mathrm{cs}}{24d\sqrt{a+a\sin(}}$$

$$= -\frac{163a^3\cot(c+dx)}{64d\sqrt{a+a\sin(c+dx)}} - \frac{163a^3\cot(c+dx)\csc(c+dx)}{96d\sqrt{a+a\sin(c+dx)}} - \frac{17a^3\cot(c+dx)\mathrm{cs}}{24d\sqrt{a+a\sin(}}$$

$$= -\frac{163a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{64d} - \frac{163a^3\cot(c+dx)}{64d\sqrt{a+a\sin(c+dx)}} - \frac{163a^3\cot(c+dx)}{96d\sqrt{a+a\sin}}$$

Mathematica [B] time = 1.60193, size = 370, normalized size = 2.03

$$-\frac{a^2\csc^{13}\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sin(c+dx)+1)}\left(1030\sin\left(\frac{1}{2}(c+dx)\right)+3102\sin\left(\frac{3}{2}(c+dx)\right)+326\sin\left(\frac{5}{2}(c+dx)\right)-978\sin\left(\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^5*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] -(a^2*Csc[(c + d*x)/2]^13*Sqrt[a*(1 + Sin[c + d*x])]*(-1030*Cos[(c + d*x)/2
] + 3102*Cos[(3*(c + d*x))/2] - 326*Cos[(5*(c + d*x))/2] - 978*Cos[(7*(c +
d*x))/2] + 1467*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 1956*Cos[2*(
c + d*x)]*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 489*Cos[4*(c + d*x
)]*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 1467*Log[1 - Cos[(c + d*x
)/2] + Sin[(c + d*x)/2]] + 1956*Cos[2*(c + d*x)]*Log[1 - Cos[(c + d*x)/2] +
 Sin[(c + d*x)/2]] - 489*Cos[4*(c + d*x)]*Log[1 - Cos[(c + d*x)/2] + Sin[(c
 + d*x)/2]] + 1030*Sin[(c + d*x)/2] + 3102*Sin[(3*(c + d*x))/2] + 326*Sin[(
```

```
5*(c + d*x))/2] - 978*Sin[(7*(c + d*x))/2]))/(192*d*(1 + Cot[(c + d*x)/2])*
(Csc[(c + d*x)/4]^2 - Sec[(c + d*x)/4]^2)^4)
```

Maple [A] time = 0.727, size = 162, normalized size = 0.9

$$-\frac{1+\sin(dx+c)}{192\,(\sin(dx+c))^4\cos(dx+c)\,d}\sqrt{-a(\sin(dx+c)-1)}\left(1047\sqrt{-a(\sin(dx+c)-1)}a^{11/2}-2303\,(-a(\sin(dx+c)-1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^5*(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] -1/192*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(1047*(-a*(sin(d*x+c)-1))^(
1/2)*a^(11/2)-2303*(-a*(sin(d*x+c)-1))^(3/2)*a^(9/2)+1793*(-a*(sin(d*x+c)-1
))^(5/2)*a^(7/2)-489*(-a*(sin(d*x+c)-1))^(7/2)*a^(5/2)+489*arctanh((-a*(sin
(d*x+c)-1))^(1/2)/a^(1/2))*a^6*sin(d*x+c)^4)/a^(7/2)/sin(d*x+c)^4/cos(d*x+c
)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx+c)+a)^{\frac{5}{2}}\csc(dx+c)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5*(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(5/2)*csc(d*x + c)^5, x)
```

Fricas [B] time = 1.91775, size = 1238, normalized size = 6.8

$$\frac{489\left(a^2\cos(dx+c)^5+a^2\cos(dx+c)^4-2\,a^2\cos(dx+c)^3-2\,a^2\cos(dx+c)^2+a^2\cos(dx+c)+a^2+\left(a^2\cos(dx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5*(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/768*(489*(a^2*cos(d*x + c)^5 + a^2*cos(d*x + c)^4 - 2*a^2*cos(d*x + c)^3
- 2*a^2*cos(d*x + c)^2 + a^2*cos(d*x + c) + a^2 + (a^2*cos(d*x + c)^4 - 2*a
^2*cos(d*x + c)^2 + a^2)*sin(d*x + c))*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*
cos(d*x + c)^2 - 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*co
s(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*co
s(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + co
s(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) + 4*(
489*a^2*cos(d*x + c)^4 + 326*a^2*cos(d*x + c)^3 - 836*a^2*cos(d*x + c)^2 -
374*a^2*cos(d*x + c) + 299*a^2 + (489*a^2*cos(d*x + c)^3 + 163*a^2*cos(d*x
+ c)^2 - 673*a^2*cos(d*x + c) - 299*a^2)*sin(d*x + c))*sqrt(a*sin(d*x + c)
+ a))/(d*cos(d*x + c)^5 + d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^3 - 2*d*cos(d
*x + c)^2 + d*cos(d*x + c) + (d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)*si
```

```
n(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**5*(a+a*sin(d*x+c))**(5/2),x)

[Out] Timed out
```

Giac [B] time = 3.28643, size = 1102, normalized size = 6.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5*(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")

[Out] 1/384*(978*a^3*arctan(-(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x +
 1/2*c)^2 + a))/sqrt(-a))*sgn(tan(1/2*d*x + 1/2*c) + 1)/sqrt(-a) - 489*a^(5
/2)*log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*tan(1/2*d*x + 1/2*c)^2 +
 a)))*sgn(tan(1/2*d*x + 1/2*c) + 1) + (400*a^2*sgn(tan(1/2*d*x + 1/2*c) + 1
) + (135*a^2*sgn(tan(1/2*d*x + 1/2*c) + 1) + 2*(3*a^2*sgn(tan(1/2*d*x + 1/2
*c) + 1)*tan(1/2*d*x + 1/2*c) + 20*a^2*sgn(tan(1/2*d*x + 1/2*c) + 1))*tan(1
/2*d*x + 1/2*c))*tan(1/2*d*x + 1/2*c))*sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a) -
 (11736*sqrt(2)*a^3*arctan((sqrt(2)*sqrt(a) + sqrt(a))/sqrt(-a)) - 5868*sqr
t(2)*sqrt(-a)*a^(5/2)*log(sqrt(2)*sqrt(a) + sqrt(a)) + 16626*a^3*arctan((sq
rt(2)*sqrt(a) + sqrt(a))/sqrt(-a)) - 8313*sqrt(-a)*a^(5/2)*log(sqrt(2)*sqrt
(a) + sqrt(a)) + 5366*sqrt(2)*sqrt(-a)*a^(5/2) + 7552*sqrt(-a)*a^(5/2))*sgn
(tan(1/2*d*x + 1/2*c) + 1)/(12*sqrt(2)*sqrt(-a) + 17*sqrt(-a)) + 2*(135*(sq
rt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^7*a^3*sgn(
tan(1/2*d*x + 1/2*c) + 1) + 480*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(
1/2*d*x + 1/2*c)^2 + a))^6*a^(7/2)*sgn(tan(1/2*d*x + 1/2*c) + 1) - 111*(sqr
t(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^5*a^4*sgn(t
an(1/2*d*x + 1/2*c) + 1) - 1200*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(
1/2*d*x + 1/2*c)^2 + a))^4*a^(9/2)*sgn(tan(1/2*d*x + 1/2*c) + 1) - 111*(sqr
t(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^3*a^5*sgn(t
an(1/2*d*x + 1/2*c) + 1) + 1120*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(
1/2*d*x + 1/2*c)^2 + a))^2*a^(11/2)*sgn(tan(1/2*d*x + 1/2*c) + 1) + 135*(sq
rt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))*a^6*sgn(ta
n(1/2*d*x + 1/2*c) + 1) - 400*a^(13/2)*sgn(tan(1/2*d*x + 1/2*c) + 1))/((sqr
t(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a)^4)/d
```

3.61 $\int \frac{\sin^3(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=139

$$-\frac{2\sin^2(c+dx)\cos(c+dx)}{5d\sqrt{a\sin(c+dx)+a}}+\frac{2\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{15ad}-\frac{28\cos(c+dx)}{15d\sqrt{a\sin(c+dx)+a}}+\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{\sqrt{a}d}$$

```
[Out] (Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])]
)/(Sqrt[a]*d) - (28*Cos[c + d*x])/(15*d*Sqrt[a + a*Sin[c + d*x]]) - (2*Cos[
c + d*x]*Sin[c + d*x]^2)/(5*d*Sqrt[a + a*Sin[c + d*x]]) + (2*Cos[c + d*x]*S
qrt[a + a*Sin[c + d*x]])/(15*a*d)
```

Rubi [A] time = 0.232121, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2778, 2968, 3023, 2751, 2649, 206}

$$-\frac{2\sin^2(c+dx)\cos(c+dx)}{5d\sqrt{a\sin(c+dx)+a}}+\frac{2\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{15ad}-\frac{28\cos(c+dx)}{15d\sqrt{a\sin(c+dx)+a}}+\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^3/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])]
)/(Sqrt[a]*d) - (28*Cos[c + d*x])/(15*d*Sqrt[a + a*Sin[c + d*x]]) - (2*Cos[
c + d*x]*Sin[c + d*x]^2)/(5*d*Sqrt[a + a*Sin[c + d*x]]) + (2*Cos[c + d*x]*S
qrt[a + a*Sin[c + d*x]])/(15*a*d)
```

Rule 2778

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> Simp[(-2*d*Cos[e + f*x]*(c + d*Sin[e + f*x])
^(n - 1))/(f*(2*n - 1)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(b*(2*n - 1))
, Int[((c + d*Sin[e + f*x])^(n - 2)*Simp[a*c*d - b*(2*d^2*(n - 1) + c^2*(2*
n - 1)) + d*(a*d - b*c*(4*n - 3))*Sin[e + f*x], x])/Sqrt[a + b*Sin[e + f*x]
], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sin^3(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx = -\frac{2\cos(c+dx)\sin^2(c+dx)}{5d\sqrt{a+a\sin(c+dx)}} - \frac{\int \frac{\sin(c+dx)(-4a+a\sin(c+dx))}{\sqrt{a+a\sin(c+dx)}}\,dx}{5a}$$

$$= -\frac{2\cos(c+dx)\sin^2(c+dx)}{5d\sqrt{a+a\sin(c+dx)}} - \frac{\int \frac{-4a\sin(c+dx)+a\sin^2(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx}{5a}$$

$$= -\frac{2\cos(c+dx)\sin^2(c+dx)}{5d\sqrt{a+a\sin(c+dx)}} + \frac{2\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{15ad} - \frac{2\int \frac{\frac{a^2}{2}-7a^2\sin(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx}{15a^2}$$

$$= -\frac{28\cos(c+dx)}{15d\sqrt{a+a\sin(c+dx)}} - \frac{2\cos(c+dx)\sin^2(c+dx)}{5d\sqrt{a+a\sin(c+dx)}} + \frac{2\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{15ad} - \int$$

$$= -\frac{28\cos(c+dx)}{15d\sqrt{a+a\sin(c+dx)}} - \frac{2\cos(c+dx)\sin^2(c+dx)}{5d\sqrt{a+a\sin(c+dx)}} + \frac{2\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{15ad} + \frac{2}{}$$

$$= \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{ad}} - \frac{28\cos(c+dx)}{15d\sqrt{a+a\sin(c+dx)}} - \frac{2\cos(c+dx)\sin^2(c+dx)}{5d\sqrt{a+a\sin(c+dx)}} + \frac{2\cos(c}{}$$

Mathematica [C] time = 0.233201, size = 150, normalized size = 1.08

$$\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)\left(60\sin\left(\frac{1}{2}(c+dx)\right)+5\sin\left(\frac{3}{2}(c+dx)\right)-3\sin\left(\frac{5}{2}(c+dx)\right)-60\cos\left(\frac{1}{2}(c+dx)\right)+5c\right.}{30d\sqrt{a(\sin(c+dx)+}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^3/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*((-60 - 60*I)*(-1)^(3/4)*ArcTanh[(1/
2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)/4])] - 60*Cos[(c + d*x)/2] + 5*Cos[
(3*(c + d*x))/2] + 3*Cos[(5*(c + d*x))/2] + 60*Sin[(c + d*x)/2] + 5*Sin[(3*
(c + d*x))/2] - 3*Sin[(5*(c + d*x))/2]))/(30*d*Sqrt[a*(1 + Sin[c + d*x])])
```

Maple [A] time = 0.589, size = 130, normalized size = 0.9

$$\frac{1+\sin(dx+c)}{15\,a^{3}\cos(dx+c)\,d}\sqrt{-a\left(\sin(dx+c)-1\right)}\left(15\,a^{5/2}\sqrt{2}\mathrm{Artanh}\left(1/2\,\frac{\sqrt{a-a\sin(dx+c)}\sqrt{2}}{\sqrt{a}}\right)-6\,\left(a-a\sin(dx+c)\right)^{5/2}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3/(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] 1/15*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(15*a^(5/2)*2^(1/2)*arctanh(1
/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))-6*(a-a*sin(d*x+c))^(5/2)+10*(a-a
*sin(d*x+c))^(3/2)*a-30*a^2*(a-a*sin(d*x+c))^(1/2))/a^3/cos(d*x+c)/(a+a*sin
(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(dx+c)^{3}}{\sqrt{a\sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)^3/sqrt(a*sin(d*x + c) + a), x)
```

Fricas [A] time = 1.90647, size = 648, normalized size = 4.66

$$\frac{\frac{15\sqrt{2}(a\cos(dx+c)+a\sin(dx+c)+a)\log\left(-\frac{\cos(dx+c)^{2}-(\cos(dx+c)-2)\sin(dx+c)+\frac{2\sqrt{2}\sqrt{a\sin(dx+c)+a}(\cos(dx+c)-\sin(dx+c)+1)}{\sqrt{a}}+3\cos(dx+c)+2}{\cos(dx+c)^{2}-(\cos(dx+c)+2)\sin(dx+c)-\cos(dx+c)-2}\right)}{\sqrt{a}}+4\left(3\cos(dx+c\right.}{30\left(ad\cos(dx+c)+ac\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/30*(15*sqrt(2)*(a*cos(d*x + c) + a*sin(d*x + c) + a)*log(-(cos(d*x + c)^2
 - (cos(d*x + c) - 2)*sin(d*x + c) + 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*(co
s(d*x + c) - sin(d*x + c) + 1)/sqrt(a) + 3*cos(d*x + c) + 2)/(cos(d*x + c)^
2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x + c) - 2))/sqrt(a) + 4*(3*cos
(d*x + c)^3 + 4*cos(d*x + c)^2 - (3*cos(d*x + c)^2 - cos(d*x + c) - 17)*sin
(d*x + c) - 16*cos(d*x + c) - 17)*sqrt(a*sin(d*x + c) + a))/(a*d*cos(d*x +
c) + a*d*sin(d*x + c) + a*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 3.41578, size = 410, normalized size = 2.95

$$-\frac{\frac{120\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}+\sqrt{a}\right)}{2\sqrt{-a}}\right)}{\sqrt{-a}\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}-\frac{\left(\left(\left(\left(\frac{13\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^7}-\frac{15\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^7}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\frac{40\,\mathrm{s}}{}}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -1/60*(120*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt
(a*tan(1/2*d*x + 1/2*c)^2 + a) + sqrt(a))/sqrt(-a))/(sqrt(-a)*sgn(tan(1/2*d
*x + 1/2*c) + 1)) - (((((13*sgn(tan(1/2*d*x + 1/2*c) + 1)*tan(1/2*d*x + 1/2
*c)/a^7 - 15*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^7)*tan(1/2*d*x + 1/2*c) + 40*s
gn(tan(1/2*d*x + 1/2*c) + 1)/a^7)*tan(1/2*d*x + 1/2*c) - 40*sgn(tan(1/2*d*x
 + 1/2*c) + 1)/a^7)*tan(1/2*d*x + 1/2*c) + 15*sgn(tan(1/2*d*x + 1/2*c) + 1)
/a^7)*tan(1/2*d*x + 1/2*c) - 13*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^7)/(a*tan(1
/2*d*x + 1/2*c)^2 + a)^(5/2) - (120*sqrt(2)*a^(21/2)*arctan(sqrt(a)/sqrt(-a
)) + 17*sqrt(2)*sqrt(-a)*a)*sgn(tan(1/2*d*x + 1/2*c) + 1)/(sqrt(-a)*a^(21/2
)))/d
```

3.62 $\int \frac{\sin^2(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=105

$$-\frac{2\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{3ad}+\frac{4\cos(c+dx)}{3d\sqrt{a\sin(c+dx)+a}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{\sqrt{a}d}$$

```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]]
)])/(Sqrt[a]*d)) + (4*Cos[c + d*x])/(3*d*Sqrt[a + a*Sin[c + d*x]]) - (2*Cos
[c + d*x]*Sqrt[a + a*Sin[c + d*x]])/(3*a*d)
```

Rubi [A] time = 0.119386, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2759, 2751, 2649, 206}

$$-\frac{2\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{3ad}+\frac{4\cos(c+dx)}{3d\sqrt{a\sin(c+dx)+a}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]]
)])/(Sqrt[a]*d)) + (4*Cos[c + d*x])/(3*d*Sqrt[a + a*Sin[c + d*x]]) - (2*Cos
[c + d*x]*Sqrt[a + a*Sin[c + d*x]])/(3*a*d)
```

Rule 2759

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin[e
 + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ
[m, -2^(-1)]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sin^2(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx = -\frac{2\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{3ad} + \frac{2\int \frac{\frac{a}{2}-a\sin(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx}{3a}$$
$$= \frac{4\cos(c+dx)}{3d\sqrt{a+a\sin(c+dx)}} - \frac{2\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{3ad} + \int \frac{1}{\sqrt{a+a\sin(c+dx)}}\,dx$$
$$= \frac{4\cos(c+dx)}{3d\sqrt{a+a\sin(c+dx)}} - \frac{2\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{3ad} - \frac{2\,\mathrm{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \frac{a\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d}$$
$$= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}d} + \frac{4\cos(c+dx)}{3d\sqrt{a+a\sin(c+dx)}} - \frac{2\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{3ad}$$

Mathematica [C] time = 0.217577, size = 105, normalized size = 1.

$$-\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)\left(-2\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3-(6+6i)(-1)^{3/4}\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(\right.\right.\right.}{3d\sqrt{a(\sin(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] -((((-6 - 6*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)
/4])] - 2*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3)*(Cos[(c + d*x)/2] + Sin[
(c + d*x)/2]))/(3*d*Sqrt[a*(1 + Sin[c + d*x])])
```

Maple [A] time = 0.566, size = 96, normalized size = 0.9

$$-\frac{1+\sin(dx+c)}{3\cos(dx+c)a^2d}\sqrt{-a(\sin(dx+c)-1)}\left(3a^{3/2}\sqrt{2}\mathrm{Artanh}\left(1/2\frac{\sqrt{a-a\sin(dx+c)}\sqrt{2}}{\sqrt{a}}\right)-2(a-a\sin(dx+c))^{3/2}\right)\frac{1}{\sqrt{a+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2/(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] -1/3*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(3*a^(3/2)*2^(1/2)*arctanh(1/
2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))-2*(a-a*sin(d*x+c))^(3/2))/a^2/cos
(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(dx+c)^2}{\sqrt{a\sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)^2/sqrt(a*sin(d*x + c) + a), x)
```

Fricas [B] time = 1.73476, size = 585, normalized size = 5.57

$$\frac{\frac{3\sqrt{2}(a\cos(dx+c)+a\sin(dx+c)+a)\log\left(-\frac{\cos(dx+c)^2-(\cos(dx+c)-2)\sin(dx+c)-\frac{2\sqrt{2}\sqrt{a\sin(dx+c)+a}(\cos(dx+c)-\sin(dx+c)+1)}{\sqrt{a}}+3\cos(dx+c)+2}{\cos(dx+c)^2-(\cos(dx+c)+2)\sin(dx+c)-\cos(dx+c)-2}\right)}{\sqrt{a}}-4\left(\cos(dx+c)^2+\right.}{6\left(ad\cos(dx+c)+ad\sin(dx+c)+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*(3*sqrt(2)*(a*cos(d*x + c) + a*sin(d*x + c) + a)*log(-(cos(d*x + c)^2 -
 (cos(d*x + c) - 2)*sin(d*x + c) - 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*(cos(
d*x + c) - sin(d*x + c) + 1)/sqrt(a) + 3*cos(d*x + c) + 2)/(cos(d*x + c)^2
- (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x + c) - 2))/sqrt(a) - 4*(cos(d*x
 + c)^2 + (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x + c) - 2)*sqrt(a*sin(d*
x + c) + a))/(a*d*cos(d*x + c) + a*d*sin(d*x + c) + a*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin^2(c+dx)}{\sqrt{a(\sin(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sin(c + d*x)**2/sqrt(a*(sin(c + d*x) + 1)), x)
```

Giac [B] time = 3.32096, size = 332, normalized size = 3.16

$$\frac{\frac{6\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}+\sqrt{a}\right)}{2\sqrt{-a}}\right)}{\sqrt{-a}\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}-\frac{\left(\left(\frac{\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^5}-\frac{3\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^5}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\frac{3\,\mathrm{sgn}\left(\tan\right.}{}}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a\right)^{\frac{3}{2}}}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*(6*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*t
an(1/2*d*x + 1/2*c)^2 + a) + sqrt(a))/sqrt(-a))/(sqrt(-a)*sgn(tan(1/2*d*x +
 1/2*c) + 1)) - (((sgn(tan(1/2*d*x + 1/2*c) + 1)*tan(1/2*d*x + 1/2*c)/a^5 -
 3*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^5)*tan(1/2*d*x + 1/2*c) + 3*sgn(tan(1/2*
d*x + 1/2*c) + 1)/a^5)*tan(1/2*d*x + 1/2*c) - sgn(tan(1/2*d*x + 1/2*c) + 1)
/a^5)/(a*tan(1/2*d*x + 1/2*c)^2 + a)^(3/2) - 2*(3*sqrt(2)*a^(15/2)*arctan(s
qrt(a)/sqrt(-a)) + sqrt(2)*sqrt(-a)*a)*sgn(tan(1/2*d*x + 1/2*c) + 1)/(sqrt(
-a)*a^(15/2)))/d
```

3.63 $\int \frac{\sin(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=72

$$\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{2\cos(c+dx)}{d\sqrt{a\sin(c+dx)+a}}$$

```
[Out] (Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])]
)/(Sqrt[a]*d) - (2*Cos[c + d*x])/(d*Sqrt[a + a*Sin[c + d*x]])
```

Rubi [A] time = 0.0483382, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2751, 2649, 206}

$$\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{2\cos(c+dx)}{d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])]
)/(Sqrt[a]*d) - (2*Cos[c + d*x])/(d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sin(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx = -\frac{2\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}} - \int \frac{1}{\sqrt{a+a\sin(c+dx)}}\,dx$$
$$= -\frac{2\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}} + \frac{2\,\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \frac{a\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d}$$
$$= \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}d} - \frac{2\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}}$$

Mathematica [C] time = 0.105799, size = 98, normalized size = 1.36

$$-\frac{2\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)\left(-\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)+(1+i)(-1)^{3/4}\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(t\right.\right.\right.}{d\sqrt{a(\sin(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-2*((1 + I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)/
4])] + Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c + d*
x)/2]))/(d*Sqrt[a*(1 + Sin[c + d*x])])
```

Maple [A] time = 0.547, size = 94, normalized size = 1.3

$$\frac{1+\sin(dx+c)}{ad\cos(dx+c)}\sqrt{-a(\sin(dx+c)-1)}\left(\sqrt{a}\sqrt{2}\text{Artanh}\left(\frac{\sqrt{2}}{2}\sqrt{a-a\sin(dx+c)}\frac{1}{\sqrt{a}}\right)-2\sqrt{a-a\sin(dx+c)}\right)\frac{1}{\sqrt{a+a\sin\ldots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] (1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(a^(1/2)*2^(1/2)*arctanh(1/2*(a-a*
sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))-2*(a-a*sin(d*x+c))^(1/2))/a/cos(d*x+c)/(
a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(dx+c)}{\sqrt{a\sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)/sqrt(a*sin(d*x + c) + a), x)
```

Fricas [B] time = 1.76941, size = 533, normalized size = 7.4

$$\frac{\frac{\sqrt{2}(a\cos(dx+c)+a\sin(dx+c)+a)\log\left(-\frac{\cos(dx+c)^2-(\cos(dx+c)-2)\sin(dx+c)+\frac{2\sqrt{2}\sqrt{a\sin(dx+c)+a}(\cos(dx+c)-\sin(dx+c)+1)}{\sqrt{a}}+3\cos(dx+c)+2}{\cos(dx+c)^2-(\cos(dx+c)+2)\sin(dx+c)-\cos(dx+c)-2}\right)}{\sqrt{a}}-4\sqrt{a\sin(dx+c)+a}(}{2(ad\cos(dx+c)+ad\sin(dx+c)+ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(2)*(a*cos(d*x + c) + a*sin(d*x + c) + a)*log(-(cos(d*x + c)^2 - (
cos(d*x + c) - 2)*sin(d*x + c) + 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*(cos(d*
x + c) - sin(d*x + c) + 1)/sqrt(a) + 3*cos(d*x + c) + 2)/(cos(d*x + c)^2 -
(cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x + c) - 2))/sqrt(a) - 4*sqrt(a*sin
(d*x + c) + a)*(cos(d*x + c) - sin(d*x + c) + 1))/(a*d*cos(d*x + c) + a*d*s
in(d*x + c) + a*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(c+dx)}{\sqrt{a(\sin(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sin(c + d*x)/sqrt(a*(sin(c + d*x) + 1)), x)
```

Giac [B] time = 2.88987, size = 246, normalized size = 3.42

$$\frac{2\left(\frac{\sqrt{2}\left(a\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)+\sqrt{-a}\sqrt{a}\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{\sqrt{-a}a}+\frac{\frac{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}-\frac{1}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}}{\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}-\frac{\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}\right.}\right.}{2\sqrt{-a}}\right.}{\sqrt{-a}\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 2*(sqrt(2)*(a*arctan(sqrt(a)/sqrt(-a)) + sqrt(-a)*sqrt(a))*sgn(tan(1/2*d*x
+ 1/2*c) + 1)/(sqrt(-a)*a) + (tan(1/2*d*x + 1/2*c)/sgn(tan(1/2*d*x + 1/2*c)
 + 1) - 1/sgn(tan(1/2*d*x + 1/2*c) + 1))/sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)
 - sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1
/2*d*x + 1/2*c)^2 + a) + sqrt(a))/sqrt(-a))/(sqrt(-a)*sgn(tan(1/2*d*x + 1/2
*c) + 1)))/d
```

3.64 $\int \frac{1}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=47

$$-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{\sqrt{a}d}$$

```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]]
)])/(Sqrt[a]*d))
```

Rubi [A] time = 0.0200594, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2649, 206}

$$-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]]
)])/(Sqrt[a]*d))
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a+a\sin(c+dx)}}\,dx &= -\frac{2\,\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \frac{a\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d}\\ &= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}d}\end{aligned}$$

Mathematica [C] time = 0.0464152, size = 73, normalized size = 1.55

$$\frac{(2+2i)(-1)^{3/4}\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(\tan\left(\frac{1}{4}(c+dx)\right)-1\right)\right)}{d\sqrt{a(\sin(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] ((2 + 2*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)/4]
)]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))/(d*Sqrt[a*(1 + Sin[c + d*x])])
```

Maple [A] time = 0.336, size = 75, normalized size = 1.6

$$-\frac{(1+\sin(dx+c))\sqrt{2}}{d\cos(dx+c)}\sqrt{-a(\sin(dx+c)-1)}\operatorname{Artanh}\left(\frac{\sqrt{2}}{2}\sqrt{-a(\sin(dx+c)-1)}\frac{1}{\sqrt{a}}\right)\frac{1}{\sqrt{a}}\frac{1}{\sqrt{a+a\sin(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] -(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*2^(1/2)/a^(1/2)*arctanh(1/2*(-a*(
sin(d*x+c)-1))^(1/2)*2^(1/2)/a^(1/2))/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(a*sin(d*x + c) + a), x)
```

Fricas [A] time = 1.70395, size = 463, normalized size = 9.85

$$\left[\frac{\sqrt{2}\log\left(-\frac{\cos(dx+c)^2-(\cos(dx+c)-2)\sin(dx+c)-\frac{2\sqrt{2}\sqrt{a\sin(dx+c)+a}(\cos(dx+c)-\sin(dx+c)+1)}{\sqrt{a}}+3\cos(dx+c)+2}{\cos(dx+c)^2-(\cos(dx+c)+2)\sin(dx+c)-\cos(dx+c)-2}\right)}{2\sqrt{a}d}, \frac{\sqrt{2}\sqrt{-\frac{1}{a}}\arctan\left(\frac{\sqrt{2}\sqrt{a\sin(dx+c)+a}\sqrt{-}}{\cos(dx+c)}\right.}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(2)*log(-(cos(d*x + c)^2 - (cos(d*x + c) - 2)*sin(d*x + c) - 2*sqr
t(2)*sqrt(a*sin(d*x + c) + a)*(cos(d*x + c) - sin(d*x + c) + 1)/sqrt(a) + 3
*cos(d*x + c) + 2)/(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos(
d*x + c) - 2))/(sqrt(a)*d), sqrt(2)*sqrt(-1/a)*arctan(sqrt(2)*sqrt(a*sin(d*
x + c) + a)*sqrt(-1/a)/cos(d*x + c))/d]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\sin(c+dx)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*sin(c + d*x) + a), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(a*sin(d*x + c) + a), x)
```

3.65 $\int \frac{\csc(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=84

$$\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{\sqrt{a}d}$$

```
[Out] (-2*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(Sqrt[a]*d) +
 (Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])
])/(Sqrt[a]*d)
```

Rubi [A] time = 0.113939, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2780, 2649, 206, 2773}

$$\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-2*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(Sqrt[a]*d) +
 (Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])
])/(Sqrt[a]*d)
```

Rule 2780

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e +
f*x]], x], x] - Dist[d/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\csc(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx = \frac{\int \csc(c+dx)\sqrt{a+a\sin(c+dx)}\,dx}{a} - \int \frac{1}{\sqrt{a+a\sin(c+dx)}}\,dx$$
$$= -\frac{2\,\text{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, \frac{a\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d} + \frac{2\,\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \frac{a\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d}$$
$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}d} + \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}d}$$

Mathematica [C] time = 0.0987107, size = 128, normalized size = 1.52

$$-\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)\left((2+2i)(-1)^{3/4}\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(\tan\left(\frac{1}{4}(c+dx)\right)-1\right)\right)+\log\left(-\sin\left(\frac{1}{2}(c\right.\right.\right.}{d\sqrt{a(\sin(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] -((((2 + 2*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)
/4])] + Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[1 - Cos[(c + d*x
)/2] + Sin[(c + d*x)/2]])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))/(d*Sqrt[a*
(1 + Sin[c + d*x])]))
```

Maple [A] time = 0.506, size = 96, normalized size = 1.1

$$\frac{1+\sin(dx+c)}{d\cos(dx+c)}\sqrt{-a(\sin(dx+c)-1)}\left(\sqrt{2}\text{Artanh}\left(\frac{\sqrt{2}}{2}\sqrt{a-a\sin(dx+c)}\frac{1}{\sqrt{a}}\right)-2\,\text{Artanh}\left(\frac{\sqrt{a-a\sin(dx+c)}}{\sqrt{a}}\right)\right)\frac{1}{\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)/(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] (1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(2^(1/2)*arctanh(1/2*(a-a*sin(d*x+
c))^(1/2)*2^(1/2)/a^(1/2))-2*arctanh((a-a*sin(d*x+c))^(1/2)/a^(1/2)))/a^(1/
2)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(dx+c)}{\sqrt{a\sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(d*x + c)/sqrt(a*sin(d*x + c) + a), x)
```

Fricas [B] time = 1.90382, size = 795, normalized size = 9.46

$$\frac{\sqrt{2}\sqrt{a}\log\left(-\frac{\cos(dx+c)^2-(\cos(dx+c)-2)\sin(dx+c)+\frac{2\sqrt{2}\sqrt{a\sin(dx+c)+a}(\cos(dx+c)-\sin(dx+c)+1)}{\sqrt{a}}+3\cos(dx+c)+2}{\cos(dx+c)^2-(\cos(dx+c)+2)\sin(dx+c)-\cos(dx+c)-2}\right)+\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-}{}\right.}{2ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(2)*sqrt(a)*log(-(cos(d*x + c)^2 - (cos(d*x + c) - 2)*sin(d*x + c)
 + 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*(cos(d*x + c) - sin(d*x + c) + 1)/sqr
t(a) + 3*cos(d*x + c) + 2)/(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(d*x + c
) - cos(d*x + c) - 2)) + sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2
 - 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*cos(d*x + c) - 3
)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*cos(d*x + c)^2 +
 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + cos(d*x + c)^2 +
 (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)))/(a*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(c+dx)}{\sqrt{a(\sin(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(csc(c + d*x)/sqrt(a*(sin(c + d*x) + 1)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.66 $\int \frac{\csc^2(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=109

$$-\frac{\cot(c+dx)}{d\sqrt{a\sin(c+dx)+a}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{\sqrt{a}d}$$

```
[Out] ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]]/(Sqrt[a]*d) - (Sqr
t[2]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])])/(S
qrt[a]*d) - Cot[c + d*x]/(d*Sqrt[a + a*Sin[c + d*x]])
```

Rubi [A] time = 0.203946, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2779, 2985, 2649, 206, 2773}

$$-\frac{\cot(c+dx)}{d\sqrt{a\sin(c+dx)+a}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]]/(Sqrt[a]*d) - (Sqr
t[2]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])])/(S
qrt[a]*d) - Cot[c + d*x]/(d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 2779

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> -Simp[(d*Cos[e + f*x]*(c + d*Sin[e + f*x])^(
n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(2*b*
(n + 1)*(c^2 - d^2)), Int[((c + d*Sin[e + f*x])^(n + 1)*Simp[a*d - 2*b*c*(n
 + 1) + b*d*(2*n + 3)*Sin[e + f*x], x])/Sqrt[a + b*Sin[e + f*x]], x], x] /;
 FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^2(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx &= -\frac{\cot(c+dx)}{d\sqrt{a+a\sin(c+dx)}}-\frac{\int \frac{\csc(c+dx)(a-a\sin(c+dx))}{\sqrt{a+a\sin(c+dx)}}\,dx}{2a}\\ &= -\frac{\cot(c+dx)}{d\sqrt{a+a\sin(c+dx)}}-\frac{\int \csc(c+dx)\sqrt{a+a\sin(c+dx)}\,dx}{2a}+\int \frac{1}{\sqrt{a+a\sin(c+dx)}}\,dx\\ &= -\frac{\cot(c+dx)}{d\sqrt{a+a\sin(c+dx)}}+\frac{\mathrm{Subst}\left(\int \frac{1}{a-x^2}\,dx,x,\frac{a\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d}-\frac{2\,\mathrm{Subst}\left(\int \frac{1}{2a-x^2}\,dx,x,\frac{a\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d}\\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}d}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}d}-\frac{\cot(c+dx)}{d\sqrt{a+a\sin(c+dx)}}\end{aligned}$$

Mathematica [C] time = 1.22936, size = 168, normalized size = 1.54

$$\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)\left(-\tan\left(\frac{1}{4}(c+dx)\right)-\cot\left(\frac{1}{4}(c+dx)\right)+2\sec\left(\frac{1}{2}(c+dx)\right)+(8+8i)(-1)^{3/4}\tanh^{-1}\left(\left(\frac{1}{2}\right.\right.\right.}{4d\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*((8 + 8*I)*(-1)^(3/4)*ArcTanh[(1/2 +
 I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)/4])] - Cot[(c + d*x)/4] + 2*Log[1 + Co
s[(c + d*x)/2] - Sin[(c + d*x)/2]] - 2*Log[1 - Cos[(c + d*x)/2] + Sin[(c +
d*x)/2]] + 2*Sec[(c + d*x)/2] - Tan[(c + d*x)/4]))/(4*d*Sqrt[a*(1 + Sin[c +
 d*x])])
```

Maple [A] time = 0.526, size = 133, normalized size = 1.2

$$-\frac{1+\sin(dx+c)}{\cos(dx+c)\sin(dx+c)d}\sqrt{-a(\sin(dx+c)-1)}\left(\sin(dx+c)a^3\left(\sqrt{2}\mathrm{Artanh}\left(\frac{\sqrt{2}}{2}\sqrt{a-a\sin(dx+c)}\frac{1}{\sqrt{a}}\right)-\mathrm{Artanh}\left(\sqrt{}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2/(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] -(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)/a^(7/2)*(sin(d*x+c)*a^3*(2^(1/2)*
arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))-arctanh((a-a*sin(d*x+c)
)^(1/2)/a^(1/2)))+(a-a*sin(d*x+c))^(1/2)*a^(5/2))/sin(d*x+c)/cos(d*x+c)/(a+
a*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(dx+c)^2}{\sqrt{a\sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(d*x + c)^2/sqrt(a*sin(d*x + c) + a), x)
```

Fricas [B] time = 1.94389, size = 1119, normalized size = 10.27

$$\frac{\left(\cos(dx+c)^2-(\cos(dx+c)+1)\sin(dx+c)-1\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2+4\left(\cos(dx+c)^2+(\cos(dx+c)+3)\sin(dx+c)-2\cos\right.}{\cos(dx+c)^3+\cos(dx+c)^2+}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*((cos(d*x + c)^2 - (cos(d*x + c) + 1)*sin(d*x + c) - 1)*sqrt(a)*log((a*
cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3
)*sin(d*x + c) - 2*cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a
*cos(d*x + c) + (a*cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)
/(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos
(d*x + c) - 1)) + 2*sqrt(2)*(a*cos(d*x + c)^2 - (a*cos(d*x + c) + a)*sin(d*
x + c) - a)*log(-(cos(d*x + c)^2 - (cos(d*x + c) - 2)*sin(d*x + c) - 2*sqrt
(2)*sqrt(a*sin(d*x + c) + a)*(cos(d*x + c) - sin(d*x + c) + 1)/sqrt(a) + 3*
cos(d*x + c) + 2)/(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos(d
*x + c) - 2))/sqrt(a) + 4*sqrt(a*sin(d*x + c) + a)*(cos(d*x + c) - sin(d*x
+ c) + 1))/(a*d*cos(d*x + c)^2 - a*d - (a*d*cos(d*x + c) + a*d)*sin(d*x + c
))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc^2(c+dx)}{\sqrt{a(\sin(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(csc(c + d*x)**2/sqrt(a*(sin(c + d*x) + 1)), x)
```

Giac [B] time = 3.01113, size = 628, normalized size = 5.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*((2*sqrt(2)*sqrt(a)*arctan((sqrt(2)*sqrt(a) + sqrt(a))/sqrt(-a)) - 8*sq
rt(2)*sqrt(a)*arctan(sqrt(a)/sqrt(-a)) - sqrt(2)*sqrt(-a)*log(sqrt(2)*sqrt(
a) + sqrt(a)) + 4*sqrt(a)*arctan((sqrt(2)*sqrt(a) + sqrt(a))/sqrt(-a)) - 8*
sqrt(a)*arctan(sqrt(a)/sqrt(-a)) - 2*sqrt(-a)*log(sqrt(2)*sqrt(a) + sqrt(a)
) - 3*sqrt(2)*sqrt(-a) - 2*sqrt(-a))*sgn(tan(1/2*d*x + 1/2*c) + 1)/(sqrt(2)
*sqrt(-a)*sqrt(a) + 2*sqrt(-a)*sqrt(a)) + 4*sqrt(2)*arctan(-1/2*sqrt(2)*(sq
rt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a) + sqrt(a))/
sqrt(-a))/(sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c) + 1)) - 2*arctan(-(sqrt(a)*tan
(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))/sqrt(-a))/(sqrt(-a)
*sgn(tan(1/2*d*x + 1/2*c) + 1)) + log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + s
qrt(a*tan(1/2*d*x + 1/2*c)^2 + a)))/(sqrt(a)*sgn(tan(1/2*d*x + 1/2*c) + 1))
 + sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)/(a*sgn(tan(1/2*d*x + 1/2*c) + 1)) + 2
*sqrt(a)/(((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 +
a))^2 - a)*sgn(tan(1/2*d*x + 1/2*c) + 1)))/d
```

3.67 $\int \frac{\csc^3(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=146

$$\frac{\cot(c+dx)}{4d\sqrt{a\sin(c+dx)+a}}-\frac{7\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{4\sqrt{a}d}+\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{\cot(c+dx)\csc(c+dx)}{2d\sqrt{a\sin(c+dx)+a}}$$

```
[Out] (-7*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(4*Sqrt[a]*d)
 + (Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]
])])/(Sqrt[a]*d) + Cot[c + d*x]/(4*d*Sqrt[a + a*Sin[c + d*x]]) - (Cot[c + d
*x]*Csc[c + d*x])/(2*d*Sqrt[a + a*Sin[c + d*x]])
```

Rubi [A] time = 0.343468, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2779, 2984, 2985, 2649, 206, 2773}

$$\frac{\cot(c+dx)}{4d\sqrt{a\sin(c+dx)+a}}-\frac{7\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{4\sqrt{a}d}+\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{\cot(c+dx)\csc(c+dx)}{2d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-7*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(4*Sqrt[a]*d)
 + (Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]
])])/(Sqrt[a]*d) + Cot[c + d*x]/(4*d*Sqrt[a + a*Sin[c + d*x]]) - (Cot[c + d
*x]*Csc[c + d*x])/(2*d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 2779

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> -Simp[(d*Cos[e + f*x]*(c + d*Sin[e + f*x])^(
n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(2*b*
(n + 1)*(c^2 - d^2)), Int[((c + d*Sin[e + f*x])^(n + 1)*Simp[a*d - 2*b*c*(n
 + 1) + b*d*(2*n + 3)*Sin[e + f*x], x])/Sqrt[a + b*Sin[e + f*x]], x], x] /;
 FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
```

```
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\csc^3(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx = -\frac{\cot(c+dx)\csc(c+dx)}{2d\sqrt{a+a\sin(c+dx)}} - \frac{\int \frac{\csc^2(c+dx)(a-3a\sin(c+dx))}{\sqrt{a+a\sin(c+dx)}}\,dx}{4a}$$

$$= \frac{\cot(c+dx)}{4d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc(c+dx)}{2d\sqrt{a+a\sin(c+dx)}} - \frac{\int \frac{\csc(c+dx)\left(-\frac{7a^2}{2}+\frac{1}{2}a^2\sin(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}\,dx}{4a^2}$$

$$= \frac{\cot(c+dx)}{4d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc(c+dx)}{2d\sqrt{a+a\sin(c+dx)}} + \frac{7\int \csc(c+dx)\sqrt{a+a\sin(c+dx)}\,dx}{8a} - \int$$

$$= \frac{\cot(c+dx)}{4d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc(c+dx)}{2d\sqrt{a+a\sin(c+dx)}} - \frac{7\,\text{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, \frac{a\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{4d} + \frac{2\,\text{S}}{}$$

$$= -\frac{7\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{4\sqrt{a}d} + \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}d} + \frac{\cot(c+dx)}{4d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+}{2d\sqrt{a}}$$

Mathematica [C] time = 3.46478, size = 307, normalized size = 2.1

$$\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)\left(4\tan\left(\frac{1}{4}(c+dx)\right)+4\cot\left(\frac{1}{4}(c+dx)\right)-\csc^2\left(\frac{1}{4}(c+dx)\right)+\sec^2\left(\frac{1}{4}(c+dx)\right)-\frac{8}{\cos\left(\frac{1}{4}(\right.}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(-8 - (64 + 64*I)*(-1)^(3/4)*ArcTanh
[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)/4])] + 4*Cot[(c + d*x)/4] - Csc
[(c + d*x)/4]^2 - 28*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 28*Log[
```

```
1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + Sec[(c + d*x)/4]^2 + 2/(Cos[(c +
 d*x)/4] - Sin[(c + d*x)/4])^2 - (8*Sin[(c + d*x)/4])/(Cos[(c + d*x)/4] - S
in[(c + d*x)/4]) - 2/(Cos[(c + d*x)/4] + Sin[(c + d*x)/4])^2 + (8*Sin[(c +
d*x)/4])/(Cos[(c + d*x)/4] + Sin[(c + d*x)/4]) + 4*Tan[(c + d*x)/4]))/(32*d
*Sqrt[a*(1 + Sin[c + d*x])])
```

Maple [A] time = 0.619, size = 162, normalized size = 1.1

$$-\frac{1+\sin(dx+c)}{4\,(\sin(dx+c))^2\cos(dx+c)\,d}\sqrt{-a(\sin(dx+c)-1)}\left(7\,a^5\mathrm{Artanh}\left(\frac{\sqrt{-a(\sin(dx+c)-1)}}{\sqrt{a}}\right)(\sin(dx+c))^2+(-a(\sin$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3/(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] -1/4*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)/a^(11/2)*(7*a^5*arctanh((-a*(
sin(d*x+c)-1))^(1/2)/a^(1/2))*sin(d*x+c)^2+(-a*(sin(d*x+c)-1))^(3/2)*a^(7/2
)+(-a*(sin(d*x+c)-1))^(1/2)*a^(9/2)-4*2^(1/2)*arctanh(1/2*(-a*(sin(d*x+c)-1
))^(1/2)*2^(1/2)/a^(1/2))*a^5*sin(d*x+c)^2)/sin(d*x+c)^2/cos(d*x+c)/(a+a*si
n(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(dx+c)^3}{\sqrt{a\sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(d*x + c)^3/sqrt(a*sin(d*x + c) + a), x)
```

Fricas [B] time = 1.92476, size = 1328, normalized size = 9.1

$$7\left(\cos(dx+c)^3+\cos(dx+c)^2+\left(\cos(dx+c)^2-1\right)\sin(dx+c)-\cos(dx+c)-1\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-4\,(\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/16*(7*(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c
) - cos(d*x + c) - 1)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 -
4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*cos(d*x + c) - 3)*s
qrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*cos(d*x + c)^2 + 8*
a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + cos(d*x + c)^2 + (c
os(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) + 8*sqrt(2)*(a*cos(d*x
 + c)^3 + a*cos(d*x + c)^2 - a*cos(d*x + c) + (a*cos(d*x + c)^2 - a)*sin(d*
```

```
x + c) - a)*log(-(cos(d*x + c)^2 - (cos(d*x + c) - 2)*sin(d*x + c) + 2*sqrt
(2)*sqrt(a*sin(d*x + c) + a)*(cos(d*x + c) - sin(d*x + c) + 1)/sqrt(a) + 3*
cos(d*x + c) + 2)/(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos(d
*x + c) - 2))/sqrt(a) - 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c)
 - 2*cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a))/(a*d*cos(d*x + c)^3 + a*d*
cos(d*x + c)^2 - a*d*cos(d*x + c) - a*d + (a*d*cos(d*x + c)^2 - a*d)*sin(d*
x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc^3(c+dx)}{\sqrt{a(\sin(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(csc(c + d*x)**3/sqrt(a*(sin(c + d*x) + 1)), x)
```

Giac [B] time = 2.85814, size = 830, normalized size = 5.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*(sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)*(tan(1/2*d*x + 1/2*c)/(a*sgn(tan(1/
2*d*x + 1/2*c) + 1)) - 2/(a*sgn(tan(1/2*d*x + 1/2*c) + 1))) - (42*sqrt(2)*a
^(3/2)*arctan((sqrt(2)*sqrt(a) + sqrt(a))/sqrt(-a)) - 64*sqrt(2)*a^(3/2)*ar
ctan(sqrt(a)/sqrt(-a)) - 21*sqrt(2)*sqrt(-a)*a*log(sqrt(2)*sqrt(a) + sqrt(a
)) + 56*a^(3/2)*arctan((sqrt(2)*sqrt(a) + sqrt(a))/sqrt(-a)) - 96*a^(3/2)*a
rctan(sqrt(a)/sqrt(-a)) - 28*sqrt(-a)*a*log(sqrt(2)*sqrt(a) + sqrt(a)) - 18
*sqrt(2)*sqrt(-a)*a - 28*sqrt(-a)*a)*sgn(tan(1/2*d*x + 1/2*c) + 1)/(3*sqrt(
2)*sqrt(-a)*a^(3/2) + 4*sqrt(-a)*a^(3/2)) - 16*sqrt(2)*arctan(-1/2*sqrt(2)*
(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a) + sqrt(a
))/sqrt(-a))/(sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c) + 1)) + 14*arctan(-(sqrt(a)
*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))/sqrt(-a))/(sqrt
(-a)*sgn(tan(1/2*d*x + 1/2*c) + 1)) - 7*log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*
c) + sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)))/(sqrt(a)*sgn(tan(1/2*d*x + 1/2*c)
 + 1)) + 2*((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 +
 a))^3 - 2*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 +
a))^2*sqrt(a) + (sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)
^2 + a))*a + 2*a^(3/2))/(((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*
x + 1/2*c)^2 + a))^2 - a)^2*sgn(tan(1/2*d*x + 1/2*c) + 1)))/d
```

3.68 $\int \frac{\sin^4(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=183

$$\frac{13\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{10a^2d}+\frac{15\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{\sin^3(c+dx)\cos(c+dx)}{2d(a\sin(c+dx)+a)^{3/2}}-\frac{9\sin^2(c+dx)\cos(c+\ldots}{10ad\sqrt{a\sin(c+dx)+\ldots}}$$

```
[Out] (15*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])])/(2*
Sqrt[2]*a^(3/2)*d) + (Cos[c + d*x]*Sin[c + d*x]^3)/(2*d*(a + a*Sin[c + d*x]
)^(3/2)) - (31*Cos[c + d*x])/(5*a*d*Sqrt[a + a*Sin[c + d*x]]) - (9*Cos[c +
d*x]*Sin[c + d*x]^2)/(10*a*d*Sqrt[a + a*Sin[c + d*x]]) + (13*Cos[c + d*x]*S
qrt[a + a*Sin[c + d*x]])/(10*a^2*d)
```

Rubi [A] time = 0.382127, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2765, 2983, 2968, 3023, 2751, 2649, 206}

$$\frac{13\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{10a^2d}+\frac{15\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{\sin^3(c+dx)\cos(c+dx)}{2d(a\sin(c+dx)+a)^{3/2}}-\frac{9\sin^2(c+dx)\cos(c+\ldots}{10ad\sqrt{a\sin(c+dx)+\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^4/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (15*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])])/(2*
Sqrt[2]*a^(3/2)*d) + (Cos[c + d*x]*Sin[c + d*x]^3)/(2*d*(a + a*Sin[c + d*x]
)^(3/2)) - (31*Cos[c + d*x])/(5*a*d*Sqrt[a + a*Sin[c + d*x]]) - (9*Cos[c +
d*x]*Sin[c + d*x]^2)/(10*a*d*Sqrt[a + a*Sin[c + d*x]]) + (13*Cos[c + d*x]*S
qrt[a + a*Sin[c + d*x]])/(10*a^2*d)
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sin^4(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx = \frac{\cos(c+dx)\sin^3(c+dx)}{2d(a+a\sin(c+dx))^{3/2}} - \frac{\int \frac{\sin^2(c+dx)\left(3a-\frac{9}{2}a\sin(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}dx}{2a^2}$$

$$= \frac{\cos(c+dx)\sin^3(c+dx)}{2d(a+a\sin(c+dx))^{3/2}} - \frac{9\cos(c+dx)\sin^2(c+dx)}{10ad\sqrt{a+a\sin(c+dx)}} - \frac{\int \frac{\sin(c+dx)\left(-9a^2+\frac{39}{4}a^2\sin(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}dx}{5a^3}$$

$$= \frac{\cos(c+dx)\sin^3(c+dx)}{2d(a+a\sin(c+dx))^{3/2}} - \frac{9\cos(c+dx)\sin^2(c+dx)}{10ad\sqrt{a+a\sin(c+dx)}} - \frac{\int \frac{-9a^2\sin(c+dx)+\frac{39}{4}a^2\sin^2(c+dx)}{\sqrt{a+a\sin(c+dx)}}dx}{5a^3}$$

$$= \frac{\cos(c+dx)\sin^3(c+dx)}{2d(a+a\sin(c+dx))^{3/2}} - \frac{9\cos(c+dx)\sin^2(c+dx)}{10ad\sqrt{a+a\sin(c+dx)}} + \frac{13\cos(c+dx)\sqrt{a+a\sin(c+d}}{10a^2d}$$

$$= \frac{\cos(c+dx)\sin^3(c+dx)}{2d(a+a\sin(c+dx))^{3/2}} - \frac{31\cos(c+dx)}{5ad\sqrt{a+a\sin(c+dx)}} - \frac{9\cos(c+dx)\sin^2(c+dx)}{10ad\sqrt{a+a\sin(c+dx)}} + 13\cos$$

$$= \frac{\cos(c+dx)\sin^3(c+dx)}{2d(a+a\sin(c+dx))^{3/2}} - \frac{31\cos(c+dx)}{5ad\sqrt{a+a\sin(c+dx)}} - \frac{9\cos(c+dx)\sin^2(c+dx)}{10ad\sqrt{a+a\sin(c+dx)}} + 13\cos$$

$$= \frac{15\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} + \frac{\cos(c+dx)\sin^3(c+dx)}{2d(a+a\sin(c+dx))^{3/2}} - \frac{31\cos(c+dx)}{5ad\sqrt{a+a\sin(c+dx)}} - \frac{9\cos}{10}$$

Mathematica [C] time = 0.453447, size = 178, normalized size = 0.97

$$\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)\left(55\sin\left(\frac{1}{2}(c+dx)\right)-41\sin\left(\frac{3}{2}(c+dx)\right)+3\sin\left(\frac{5}{2}(c+dx)\right)+\sin\left(\frac{7}{2}(c+dx)\right)-55}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^4/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(-55*Cos[(c + d*x)/2] - 41*Cos[(3*(c
 + d*x))/2] - 3*Cos[(5*(c + d*x))/2] + Cos[(7*(c + d*x))/2] + 55*Sin[(c + d
*x)/2] - (150 + 150*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[
(c + d*x)/4])]*(1 + Sin[c + d*x]) - 41*Sin[(3*(c + d*x))/2] + 3*Sin[(5*(c +
 d*x))/2] + Sin[(7*(c + d*x))/2]))/(20*d*(a*(1 + Sin[c + d*x]))^(3/2))
```

Maple [A] time = 0.533, size = 183, normalized size = 1.

$$\frac{1}{20\,d\cos(dx+c)}\left(\sin(dx+c)\left(-80\sqrt{a-a\sin(dx+c)}a^{5/2}-8\,(a-a\sin(dx+c))^{5/2}\sqrt{a}+75\sqrt{2}\text{Artanh}\left(1/2\,\frac{\sqrt{a-a}}{}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^4/(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] 1/20*(sin(d*x+c)*(-80*(a-a*sin(d*x+c))^(1/2)*a^(5/2)-8*(a-a*sin(d*x+c))^(5/
2)*a^(1/2)+75*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a
^3)-90*(a-a*sin(d*x+c))^(1/2)*a^(5/2)-8*(a-a*sin(d*x+c))^(5/2)*a^(1/2)+75*2
^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a^3)*(-a*(sin(d*
x+c)-1))^(1/2)/a^(9/2)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(dx+c)^4}{(a\sin(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)^4/(a*sin(d*x + c) + a)^(3/2), x)
```

Fricas [B] time = 1.82819, size = 834, normalized size = 4.56

$$75\sqrt{2}\left(\cos(dx+c)^2-(\cos(dx+c)+2)\sin(dx+c)-\cos(dx+c)-2\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2+2\sqrt{2}\sqrt{a\sin(dx+c)+a}\sqrt{a}(\cos(dx+c)-\mathrm{s}}{\cos(dx+c)^2-(\cos(dx+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/40*(75*sqrt(2)*(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*
x + c) - 2)*sqrt(a)*log(-(a*cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a*sin(d*x + c)
+ a)*sqrt(a)*(cos(d*x + c) - sin(d*x + c) + 1) + 3*a*cos(d*x + c) - (a*cos(
d*x + c) - 2*a)*sin(d*x + c) + 2*a)/(cos(d*x + c)^2 - (cos(d*x + c) + 2)*si
n(d*x + c) - cos(d*x + c) - 2)) - 4*(4*cos(d*x + c)^4 - 4*cos(d*x + c)^3 -
48*cos(d*x + c)^2 + (4*cos(d*x + c)^3 + 8*cos(d*x + c)^2 - 40*cos(d*x + c)
+ 5)*sin(d*x + c) - 45*cos(d*x + c) - 5)*sqrt(a*sin(d*x + c) + a))/(a^2*d*c
os(d*x + c)^2 - a^2*d*cos(d*x + c) - 2*a^2*d - (a^2*d*cos(d*x + c) + 2*a^2*
d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**4/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] sage2
```

3.69 $\int \frac{\sin^3(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=145

$$-\frac{7\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{6a^2d}-\frac{11\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{\sin^2(c+dx)\cos(c+dx)}{2d(a\sin(c+dx)+a)^{3/2}}+\frac{13\cos(c+dx)}{3ad\sqrt{a\sin(c+dx)+a}}$$

```
[Out] (-11*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])])/(2
*Sqrt[2]*a^(3/2)*d) + (Cos[c + d*x]*Sin[c + d*x]^2)/(2*d*(a + a*Sin[c + d*x
])^(3/2)) + (13*Cos[c + d*x])/(3*a*d*Sqrt[a + a*Sin[c + d*x]]) - (7*Cos[c +
 d*x]*Sqrt[a + a*Sin[c + d*x]])/(6*a^2*d)
```

Rubi [A] time = 0.252776, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2765, 2968, 3023, 2751, 2649, 206}

$$-\frac{7\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{6a^2d}-\frac{11\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{\sin^2(c+dx)\cos(c+dx)}{2d(a\sin(c+dx)+a)^{3/2}}+\frac{13\cos(c+dx)}{3ad\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^3/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-11*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])])/(2
*Sqrt[2]*a^(3/2)*d) + (Cos[c + d*x]*Sin[c + d*x]^2)/(2*d*(a + a*Sin[c + d*x
])^(3/2)) + (13*Cos[c + d*x])/(3*a*d*Sqrt[a + a*Sin[c + d*x]]) - (7*Cos[c +
 d*x]*Sqrt[a + a*Sin[c + d*x]])/(6*a^2*d)
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sin^3(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx = \frac{\cos(c+dx)\sin^2(c+dx)}{2d(a+a\sin(c+dx))^{3/2}} - \frac{\int \frac{\sin(c+dx)\left(2a-\frac{7}{2}a\sin(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}\,dx}{2a^2}$$

$$= \frac{\cos(c+dx)\sin^2(c+dx)}{2d(a+a\sin(c+dx))^{3/2}} - \frac{\int \frac{2a\sin(c+dx)-\frac{7}{2}a\sin^2(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx}{2a^2}$$

$$= \frac{\cos(c+dx)\sin^2(c+dx)}{2d(a+a\sin(c+dx))^{3/2}} - \frac{7\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{6a^2d} - \frac{\int \frac{-\frac{7a^2}{4}+\frac{13}{2}a^2\sin(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx}{3a^3}$$

$$= \frac{\cos(c+dx)\sin^2(c+dx)}{2d(a+a\sin(c+dx))^{3/2}} + \frac{13\cos(c+dx)}{3ad\sqrt{a+a\sin(c+dx)}} - \frac{7\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{6a^2d} + \frac{11}{\ldots}$$

$$= \frac{\cos(c+dx)\sin^2(c+dx)}{2d(a+a\sin(c+dx))^{3/2}} + \frac{13\cos(c+dx)}{3ad\sqrt{a+a\sin(c+dx)}} - \frac{7\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{6a^2d} - \frac{11}{\ldots}$$

$$= -\frac{11\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} + \frac{\cos(c+dx)\sin^2(c+dx)}{2d(a+a\sin(c+dx))^{3/2}} + \frac{13\cos(c+dx)}{3ad\sqrt{a+a\sin(c+dx)}} - \frac{7\cos \ldots}{\ldots}$$

Mathematica [C] time = 0.267868, size = 156, normalized size = 1.08

$$\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)\left(-11\sin\left(\frac{1}{2}(c+dx)\right)+7\sin\left(\frac{3}{2}(c+dx)\right)-\sin\left(\frac{5}{2}(c+dx)\right)+11\cos\left(\frac{1}{2}(c+dx)\right)+7\,\mathrm{c}\ldots\right.}{6d(a(\sin(c+d\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^3/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(11*Cos[(c + d*x)/2] + 7*Cos[(3*(c +
 d*x))/2] + Cos[(5*(c + d*x))/2] - 11*Sin[(c + d*x)/2] + (33 + 33*I)*(-1)^(
3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)/4])]*(1 + Sin[c + d
*x]) + 7*Sin[(3*(c + d*x))/2] - Sin[(5*(c + d*x))/2]))/(6*d*(a*(1 + Sin[c +
 d*x]))^(3/2))
```

Maple [A] time = 0.589, size = 183, normalized size = 1.3

$$-\frac{1}{12\,d\cos\left(dx+c\right)}\left(\sin\left(dx+c\right)\left(33\,\sqrt{2}\mathrm{Artanh}\left(1/2\,\frac{\sqrt{a-a\sin\left(dx+c\right)}\sqrt{2}}{\sqrt{a}}\right)a^{2}-24\,\sqrt{a-a\sin\left(dx+c\right)}a^{3/2}-8\,\left(a-a\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3/(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] -1/12*(sin(d*x+c)*(33*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^
(1/2))*a^2-24*(a-a*sin(d*x+c))^(1/2)*a^(3/2)-8*(a-a*sin(d*x+c))^(3/2)*a^(1/
2))+33*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a^2-30*(
a-a*sin(d*x+c))^(1/2)*a^(3/2)-8*(a-a*sin(d*x+c))^(3/2)*a^(1/2))*(-a*(sin(d*
x+c)-1))^(1/2)/a^(7/2)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(dx+c\right)^{3}}{\left(a\sin\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)^3/(a*sin(d*x + c) + a)^(3/2), x)
```

Fricas [B] time = 1.88381, size = 783, normalized size = 5.4

$$\frac{33\,\sqrt{2}\left(\cos\left(dx+c\right)^{2}-\left(\cos\left(dx+c\right)+2\right)\sin\left(dx+c\right)-\cos\left(dx+c\right)-2\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^{2}-2\,\sqrt{2}\sqrt{a\sin(dx+c)+a}\sqrt{a}(\cos(dx+c}{\cos(dx+c)^{2}-(\cos}\right.}{24\left(a^{2}d\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/24*(33*sqrt(2)*(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*
x + c) - 2)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*sin(d*x + c)
+ a)*sqrt(a)*(cos(d*x + c) - sin(d*x + c) + 1) + 3*a*cos(d*x + c) - (a*cos(
d*x + c) - 2*a)*sin(d*x + c) + 2*a)/(cos(d*x + c)^2 - (cos(d*x + c) + 2)*si
n(d*x + c) - cos(d*x + c) - 2)) - 4*(4*cos(d*x + c)^3 + 16*cos(d*x + c)^2 -
 (4*cos(d*x + c)^2 - 12*cos(d*x + c) + 3)*sin(d*x + c) + 15*cos(d*x + c) +
3)*sqrt(a*sin(d*x + c) + a))/(a^2*d*cos(d*x + c)^2 - a^2*d*cos(d*x + c) - 2
*a^2*d - (a^2*d*cos(d*x + c) + 2*a^2*d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 2.98985, size = 552, normalized size = 3.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -1/6*(8*(((2*tan(1/2*d*x + 1/2*c)/sgn(tan(1/2*d*x + 1/2*c) + 1) - 3/sgn(tan
(1/2*d*x + 1/2*c) + 1))*tan(1/2*d*x + 1/2*c) + 3/sgn(tan(1/2*d*x + 1/2*c) +
 1))*tan(1/2*d*x + 1/2*c) - 2/sgn(tan(1/2*d*x + 1/2*c) + 1))/(a*tan(1/2*d*x
 + 1/2*c)^2 + a)^(3/2) - 33*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(a)*tan(1/2*d*
x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a) + sqrt(a))/sqrt(-a))/(sqrt(
-a)*a*sgn(tan(1/2*d*x + 1/2*c) + 1)) + 6*(3*(sqrt(a)*tan(1/2*d*x + 1/2*c) -
 sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^3 + (sqrt(a)*tan(1/2*d*x + 1/2*c) - sq
rt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*sqrt(a) - (sqrt(a)*tan(1/2*d*x + 1/2*c)
 - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))*a + a^(3/2))/(((sqrt(a)*tan(1/2*d*x
+ 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 2*(sqrt(a)*tan(1/2*d*x +
 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))*sqrt(a) - a)^2*a*sgn(tan(1/2*
d*x + 1/2*c) + 1)))/d
```

3.70 $\int \frac{\sin^2(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=105

$$\frac{7\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{2\cos(c+dx)}{ad\sqrt{a\sin(c+dx)+a}}-\frac{\cos(c+dx)}{2d(a\sin(c+dx)+a)^{3/2}}$$

```
[Out] (7*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])])/(2*S
qrt[2]*a^(3/2)*d) - Cos[c + d*x]/(2*d*(a + a*Sin[c + d*x])^(3/2)) - (2*Cos[
c + d*x])/(a*d*Sqrt[a + a*Sin[c + d*x]])
```

Rubi [A] time = 0.128016, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2758, 2751, 2649, 206}

$$\frac{7\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{2\cos(c+dx)}{ad\sqrt{a\sin(c+dx)+a}}-\frac{\cos(c+dx)}{2d(a\sin(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (7*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])])/(2*S
qrt[2]*a^(3/2)*d) - Cos[c + d*x]/(2*d*(a + a*Sin[c + d*x])^(3/2)) - (2*Cos[
c + d*x])/(a*d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 2758

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)),
x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(a*m - b*(2*m
 + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -2^(-1)]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^2(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx &= -\frac{\cos(c+dx)}{2d(a+a\sin(c+dx))^{3/2}}+\frac{\int \frac{-\frac{3a}{2}+2a\sin(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx}{2a^2}\\ &= -\frac{\cos(c+dx)}{2d(a+a\sin(c+dx))^{3/2}}-\frac{2\cos(c+dx)}{ad\sqrt{a+a\sin(c+dx)}}-\frac{7\int \frac{1}{\sqrt{a+a\sin(c+dx)}}\,dx}{4a}\\ &= -\frac{\cos(c+dx)}{2d(a+a\sin(c+dx))^{3/2}}-\frac{2\cos(c+dx)}{ad\sqrt{a+a\sin(c+dx)}}+\frac{7\,\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx,x,\frac{a\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{2ad}\\ &= \frac{7\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{\cos(c+dx)}{2d(a+a\sin(c+dx))^{3/2}}-\frac{2\cos(c+dx)}{ad\sqrt{a+a\sin(c+dx)}}\end{aligned}$$

Mathematica [C] time = 0.271054, size = 134, normalized size = 1.28

$$-\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)\left(-3\sin\left(\frac{1}{2}(c+dx)\right)+2\sin\left(\frac{3}{2}(c+dx)\right)+3\cos\left(\frac{1}{2}(c+dx)\right)+2\cos\left(\frac{3}{2}(c+dx)\right)+(7\right.}{2d(a(\sin(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] -((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(3*Cos[(c + d*x)/2] + 2*Cos[(3*(c +
 d*x))/2] - 3*Sin[(c + d*x)/2] + (7 + 7*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(
-1)^(3/4)*(-1 + Tan[(c + d*x)/4])]*(1 + Sin[c + d*x]) + 2*Sin[(3*(c + d*x))
/2]))/(2*d*(a*(1 + Sin[c + d*x]))^(3/2))
```

Maple [A] time = 0.549, size = 143, normalized size = 1.4

$$\frac{1}{4\,d\cos(dx+c)}\left(\sin(dx+c)\left(7\sqrt{2}\text{Artanh}\left(1/2\,\frac{\sqrt{a-a\sin(dx+c)}\sqrt{2}}{\sqrt{a}}\right)a-8\sqrt{a-a\sin(dx+c)}\sqrt{a}\right)+7\sqrt{2}\text{Artanh}\left(1/2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2/(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] 1/4*(sin(d*x+c)*(7*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/
2))*a-8*(a-a*sin(d*x+c))^(1/2)*a^(1/2))+7*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+
c))^(1/2)*2^(1/2)/a^(1/2))*a-10*(a-a*sin(d*x+c))^(1/2)*a^(1/2))*(-a*(sin(d*
x+c)-1))^(1/2)/a^(5/2)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(dx+c)^2}{(a\sin(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)^2/(a*sin(d*x + c) + a)^(3/2), x)
```

Fricas [B] time = 1.76778, size = 725, normalized size = 6.9

$$\frac{7\sqrt{2}\left(\cos(dx+c)^2-(\cos(dx+c)+2)\sin(dx+c)-\cos(dx+c)-2\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2+2\sqrt{2}\sqrt{a\sin(dx+c)+a}\sqrt{a}(\cos(dx+c)}{\cos(dx+c)^2-(\cos(}\right.}{8\left(a^2d\cos(dx+c)^2-a^2d\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/8*(7*sqrt(2)*(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x
+ c) - 2)*sqrt(a)*log(-(a*cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a*sin(d*x + c) +
a)*sqrt(a)*(cos(d*x + c) - sin(d*x + c) + 1) + 3*a*cos(d*x + c) - (a*cos(d*
x + c) - 2*a)*sin(d*x + c) + 2*a)/(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(
d*x + c) - cos(d*x + c) - 2)) + 4*(4*cos(d*x + c)^2 + (4*cos(d*x + c) - 1)*
sin(d*x + c) + 5*cos(d*x + c) + 1)*sqrt(a*sin(d*x + c) + a))/(a^2*d*cos(d*x
 + c)^2 - a^2*d*cos(d*x + c) - 2*a^2*d - (a^2*d*cos(d*x + c) + 2*a^2*d)*sin
(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin^2(c+dx)}{(a(\sin(c+dx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Integral(sin(c + d*x)**2/(a*(sin(c + d*x) + 1))**(3/2), x)
```

Giac [B] time = 2.30412, size = 486, normalized size = 4.63

$$\frac{\frac{4\left(\frac{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}-\frac{1}{a\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}\right)}{\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}-\frac{7\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}+\sqrt{a}\right)}{2\sqrt{-a}}\right)}{\sqrt{-a}\,a\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}+\frac{2\left(3\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\right.}\right.\right.}{\left(\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\right.\right.}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(4*(tan(1/2*d*x + 1/2*c)/(a*sgn(tan(1/2*d*x + 1/2*c) + 1)) - 1/(a*sgn(t
an(1/2*d*x + 1/2*c) + 1)))/sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a) - 7*sqrt(2)*a
rctan(-1/2*sqrt(2)*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2
*c)^2 + a) + sqrt(a))/sqrt(-a))/(sqrt(-a)*a*sgn(tan(1/2*d*x + 1/2*c) + 1))
+ 2*(3*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^
```

```
3 + (sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*s
qrt(a) - (sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)
)*a + a^(3/2))/(((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c
)^2 + a))^2 + 2*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)
^2 + a))*sqrt(a) - a)^2*a*sgn(tan(1/2*d*x + 1/2*c) + 1)))/d
```

3.71 $\int \frac{\sin(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=77

$$\frac{\cos(c+dx)}{2d(a\sin(c+dx)+a)^{3/2}}-\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}$$

```
[Out] (-3*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])])/(2*
Sqrt[2]*a^(3/2)*d) + Cos[c + d*x]/(2*d*(a + a*Sin[c + d*x])^(3/2))
```

Rubi [A] time = 0.0569845, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2750, 2649, 206}

$$\frac{\cos(c+dx)}{2d(a\sin(c+dx)+a)^{3/2}}-\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-3*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])])/(2*
Sqrt[2]*a^(3/2)*d) + Cos[c + d*x]/(2*d*(a + a*Sin[c + d*x])^(3/2))
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sin(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx = \frac{\cos(c+dx)}{2d(a+a\sin(c+dx))^{3/2}} + \frac{3\int \frac{1}{\sqrt{a+a\sin(c+dx)}}\,dx}{4a}$$
$$= \frac{\cos(c+dx)}{2d(a+a\sin(c+dx))^{3/2}} - \frac{3\,\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \frac{a\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{2ad}$$
$$= -\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} + \frac{\cos(c+dx)}{2d(a+a\sin(c+dx))^{3/2}}$$

Mathematica [C] time = 0.194569, size = 108, normalized size = 1.4

$$\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)\left(-\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)+(3+3i)(-1)^{3/4}(\sin(c+dx)+1)\tanh^{-1}\left(\left(\frac{1}{2}+\right.\right.\right.}{2d(a(\sin(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]
 + (3 + 3*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)/
4])]*(1 + Sin[c + d*x])))/(2*d*(a*(1 + Sin[c + d*x]))^(3/2))
```

Maple [A] time = 0.438, size = 123, normalized size = 1.6

$$-\frac{1}{4\,d\cos(dx+c)}\left(3\sqrt{2}\text{Artanh}\left(1/2\,\frac{\sqrt{a-a\sin(dx+c)}\sqrt{2}}{\sqrt{a}}\right)a\sin(dx+c)+3\sqrt{2}\text{Artanh}\left(1/2\,\frac{\sqrt{a-a\sin(dx+c)}\sqrt{2}}{\sqrt{a}}\right)a\,-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] -1/4*(3*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a*sin(d
*x+c)+3*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a-2*(a-
a*sin(d*x+c))^(1/2)*a^(1/2))*(-a*(sin(d*x+c)-1))^(1/2)/a^(5/2)/cos(d*x+c)/(
a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(dx+c)}{(a\sin(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)/(a*sin(d*x + c) + a)^(3/2), x)
```

Fricas [B] time = 1.80934, size = 668, normalized size = 8.68

$$\frac{3\,\sqrt{2}\left(\cos\left(dx+c\right)^{2}-\left(\cos\left(dx+c\right)+2\right)\sin\left(dx+c\right)-\cos\left(dx+c\right)-2\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^{2}-2\,\sqrt{2}\sqrt{a\sin(dx+c)+a}\sqrt{a}(\cos(dx+c)\cdots}{\cos(dx+c)^{2}-(\cos(d\cdots}\right.}{8\left(a^{2}d\cos\left(dx+c\right)^{2}-a^{2}d\cos\left(dx+c\right)-2\,a^{2}d-\left(a\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/8*(3*sqrt(2)*(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x
+ c) - 2)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*sin(d*x + c) +
a)*sqrt(a)*(cos(d*x + c) - sin(d*x + c) + 1) + 3*a*cos(d*x + c) - (a*cos(d*
x + c) - 2*a)*sin(d*x + c) + 2*a)/(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(
d*x + c) - cos(d*x + c) - 2)) - 4*sqrt(a*sin(d*x + c) + a)*(cos(d*x + c) -
sin(d*x + c) + 1))/(a^2*d*cos(d*x + c)^2 - a^2*d*cos(d*x + c) - 2*a^2*d - (
a^2*d*cos(d*x + c) + 2*a^2*d)*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(c+dx\right)}{\left(a\left(\sin\left(c+dx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Integral(sin(c + d*x)/(a*(sin(c + d*x) + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] sage2
```

3.72 $\int \frac{1}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=77

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{\cos(c+dx)}{2d(a\sin(c+dx)+a)^{3/2}}$$

```
[Out] -ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])]/(2*Sqrt
[2]*a^(3/2)*d) - Cos[c + d*x]/(2*d*(a + a*Sin[c + d*x])^(3/2))
```

Rubi [A] time = 0.0386647, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2650, 2649, 206}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{\cos(c+dx)}{2d(a\sin(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^(-3/2),x]
```

```
[Out] -ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])]/(2*Sqrt
[2]*a^(3/2)*d) - Cos[c + d*x]/(2*d*(a + a*Sin[c + d*x])^(3/2))
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+a\sin(c+dx))^{3/2}}\,dx &= -\frac{\cos(c+dx)}{2d(a+a\sin(c+dx))^{3/2}}+\frac{\int \frac{1}{\sqrt{a+a\sin(c+dx)}}\,dx}{4a}\\
&= -\frac{\cos(c+dx)}{2d(a+a\sin(c+dx))^{3/2}}-\frac{\operatorname{Subst}\left(\int \frac{1}{2a-x^2}\,dx,x,\frac{a\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{2ad}\\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{\cos(c+dx)}{2d(a+a\sin(c+dx))^{3/2}}
\end{aligned}$$

Mathematica [C] time = 0.153895, size = 108, normalized size = 1.4

$$\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)-\cos\left(\frac{1}{2}(c+dx)\right)+(1+i)(-1)^{3/4}(\sin(c+dx)+1)\tanh^{-1}\left(\left(\frac{1}{2}+\right.\right.\right.}{2d(a(\sin(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^(-3/2),x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(-Cos[(c + d*x)/2] + Sin[(c + d*x)/2
] + (1 + I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)/4
])]*(1 + Sin[c + d*x])))/(2*d*(a*(1 + Sin[c + d*x]))^(3/2))
```

Maple [A] time = 0.386, size = 125, normalized size = 1.6

$$-\frac{1}{4\,d\cos(dx+c)}\left(\sqrt{2}\mathrm{Artanh}\left(\frac{\sqrt{2}}{2}\sqrt{a-a\sin(dx+c)}\frac{1}{\sqrt{a}}\right)\sin(dx+c)\,a^2+2\sqrt{a-a\sin(dx+c)}a^{3/2}+\sqrt{2}\mathrm{Artanh}\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] -1/4/a^(7/2)*(2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*s
in(d*x+c)*a^2+2*(a-a*sin(d*x+c))^(1/2)*a^(3/2)+2^(1/2)*arctanh(1/2*(a-a*sin
(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a^2)*(-a*(sin(d*x+c)-1))^(1/2)/cos(d*x+c)/(
a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\sin(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(-3/2), x)
```

Fricas [B] time = 1.87012, size = 666, normalized size = 8.65

$$\frac{\sqrt{2}\left(\cos(dx+c)^2-(\cos(dx+c)+2)\sin(dx+c)-\cos(dx+c)-2\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{2}\sqrt{a\sin(dx+c)+a}\sqrt{a}(\cos(dx+c)-\mathrm{s}}{\cos(dx+c)^2-(\cos(dx+}\right.}{8\left(a^2d\cos(dx+c)^2-a^2d\cos(dx+c)-2a^2d-\left(a^2\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/8*(sqrt(2)*(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x +
c) - 2)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)
```

```
*sqrt(a)*(cos(d*x + c) - sin(d*x + c) + 1) + 3*a*cos(d*x + c) - (a*cos(d*x
+ c) - 2*a)*sin(d*x + c) + 2*a)/(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(d*
x + c) - cos(d*x + c) - 2)) + 4*sqrt(a*sin(d*x + c) + a)*(cos(d*x + c) - si
n(d*x + c) + 1))/(a^2*d*cos(d*x + c)^2 - a^2*d*cos(d*x + c) - 2*a^2*d - (a^
2*d*cos(d*x + c) + 2*a^2*d)*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\sin(c+dx)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Integral((a*sin(c + d*x) + a)**(-3/2), x)
```

Giac [B] time = 2.72988, size = 396, normalized size = 5.14

$$\frac{\frac{\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}+\sqrt{a}\right)}{2\sqrt{-a}}\right)}{\sqrt{-a}a\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}+\frac{2\left(3\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^3+\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2\sqrt{a}-\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)a+a^{\frac{3}{2}}\right)}{\left(\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2+2\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)\sqrt{a}-a\right)^2a\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan
(1/2*d*x + 1/2*c)^2 + a) + sqrt(a))/sqrt(-a))/(sqrt(-a)*a*sgn(tan(1/2*d*x +
 1/2*c) + 1)) + 2*(3*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1
/2*c)^2 + a))^3 + (sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*
c)^2 + a))^2*sqrt(a) - (sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x +
 1/2*c)^2 + a))*a + a^(3/2))/(((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1
/2*d*x + 1/2*c)^2 + a))^2 + 2*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/
2*d*x + 1/2*c)^2 + a))*sqrt(a) - a)^2*a*sgn(tan(1/2*d*x + 1/2*c) + 1)))/d
```

3.73 $\int \frac{\csc(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=114

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{a^{3/2}d}+\frac{5\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{\cos(c+dx)}{2d(a\sin(c+dx)+a)^{3/2}}$$

```
[Out] (-2*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(a^(3/2)*d) +
 (5*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])])/(2*
Sqrt[2]*a^(3/2)*d) + Cos[c + d*x]/(2*d*(a + a*Sin[c + d*x])^(3/2))
```

Rubi [A] time = 0.213451, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2766, 2985, 2649, 206, 2773}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{a^{3/2}d}+\frac{5\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{\cos(c+dx)}{2d(a\sin(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(a^(3/2)*d) +
 (5*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])])/(2*
Sqrt[2]*a^(3/2)*d) + Cos[c + d*x]/(2*d*(a + a*Sin[c + d*x])^(3/2))
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\csc(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx &= \frac{\cos(c+dx)}{2d(a+a\sin(c+dx))^{3/2}} + \frac{\int \frac{\csc(c+dx)\left(2a-\frac{1}{2}a\sin(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}\,dx}{2a^2} \\
&= \frac{\cos(c+dx)}{2d(a+a\sin(c+dx))^{3/2}} + \frac{\int \csc(c+dx)\sqrt{a+a\sin(c+dx)}\,dx}{a^2} - \frac{5\int \frac{1}{\sqrt{a+a\sin(c+dx)}}\,dx}{4a} \\
&= \frac{\cos(c+dx)}{2d(a+a\sin(c+dx))^{3/2}} - \frac{2\,\text{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, \frac{a\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{ad} + \frac{5\,\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, -\frac{}{\sqrt{}}\right.}{2ad} \\
&= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{a^{3/2}d} + \frac{5\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} + \frac{\cos(c+dx)}{2d(a+a\sin(c+dx))^{3/2}}
\end{aligned}
$$

Mathematica [C] time = 0.199759, size = 223, normalized size = 1.96

$$
\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)\left(-\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)-2\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^2\log\left(-\right.\right.}{}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]
 - (5 + 5*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)/
4])]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 - 2*Log[1 + Cos[(c + d*x)/2] -
 Sin[(c + d*x)/2]]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 + 2*Log[1 - Cos[
(c + d*x)/2] + Sin[(c + d*x)/2]]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2))/
(2*d*(a*(1 + Sin[c + d*x]))^(3/2))
```

Maple [A] time = 0.551, size = 173, normalized size = 1.5

$$
\frac{1}{4d\cos(dx+c)}\left(-\sin(dx+c)\,a^3\left(-5\sqrt{2}\text{Artanh}\left(1/2\,\frac{\sqrt{a-a\sin(dx+c)}\sqrt{2}}{\sqrt{a}}\right)+8\,\text{Artanh}\left(\frac{\sqrt{a-a\sin(dx+c)}}{\sqrt{a}}\right)\right)+2\sqrt{a}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)/(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] 1/4/a^(9/2)*(-sin(d*x+c)*a^3*(-5*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)
*2^(1/2)/a^(1/2))+8*arctanh((a-a*sin(d*x+c))^(1/2)/a^(1/2)))+2*(a-a*sin(d*x
```

```
+c))^(1/2)*a^(5/2)+5*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(
1/2))*a^3-8*arctanh((a-a*sin(d*x+c))^(1/2)/a^(1/2))*a^3)*(-a*(sin(d*x+c)-1)
)^(1/2)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc (dx+c)}{(a \sin (dx+c)+a)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(d*x + c)/(a*sin(d*x + c) + a)^(3/2), x)
```

Fricas [B] time = 1.97072, size = 1215, normalized size = 10.66

$$5 \sqrt{2}\left(\cos (dx+c)^2-(\cos (dx+c)+2)\sin (dx+c)-\cos (dx+c)-2\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2+2\sqrt{2}\sqrt{a\sin(dx+c)+a}\sqrt{a}(\cos(dx+c)}{\cos(dx+c)^2-(\cos(a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/8*(5*sqrt(2)*(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x
+ c) - 2)*sqrt(a)*log(-(a*cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a*sin(d*x + c) +
a)*sqrt(a)*(cos(d*x + c) - sin(d*x + c) + 1) + 3*a*cos(d*x + c) - (a*cos(d*
x + c) - 2*a)*sin(d*x + c) + 2*a)/(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(
d*x + c) - cos(d*x + c) - 2)) + 4*(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(
d*x + c) - cos(d*x + c) - 2)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x +
c)^2 - 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*cos(d*x + c)
 - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*cos(d*x + c)
^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + cos(d*x + c)
^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) - 4*sqrt(a*sin(
d*x + c) + a)*(cos(d*x + c) - sin(d*x + c) + 1))/(a^2*d*cos(d*x + c)^2 - a^
2*d*cos(d*x + c) - 2*a^2*d - (a^2*d*cos(d*x + c) + 2*a^2*d)*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc (c+dx)}{(a (\sin (c+dx)+1))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Integral(csc(c + d*x)/(a*(sin(c + d*x) + 1))**(3/2), x)
```

Giac [B] time = 2.09122, size = 556, normalized size = 4.88

$$-\frac{\frac{5\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}+\sqrt{a}\right)}{2\sqrt{-a}}\right)}{\sqrt{-a}a\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}-\frac{4\arctan\left(-\frac{\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}+\frac{2\log\left(\left|-\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\sqrt{a\tan}\right.\right.}{a^{\frac{3}{2}}\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*(5*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*
tan(1/2*d*x + 1/2*c)^2 + a) + sqrt(a))/sqrt(-a))/(sqrt(-a)*a*sgn(tan(1/2*d*
x + 1/2*c) + 1)) - 4*arctan(-(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2
*d*x + 1/2*c)^2 + a))/sqrt(-a))/(sqrt(-a)*a*sgn(tan(1/2*d*x + 1/2*c) + 1))
+ 2*log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*tan(1/2*d*x + 1/2*c)^2 +
 a)))/(a^(3/2)*sgn(tan(1/2*d*x + 1/2*c) + 1)) + 2*(3*(sqrt(a)*tan(1/2*d*x +
 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^3 + (sqrt(a)*tan(1/2*d*x + 1/
2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*sqrt(a) - (sqrt(a)*tan(1/2*d*x
 + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))*a + a^(3/2))/(((sqrt(a)*tan
(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 2*(sqrt(a)*tan(
1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))*sqrt(a) - a)^2*a*sgn
(tan(1/2*d*x + 1/2*c) + 1)))/d
```

3.74 $\int \frac{\csc^2(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=144

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{a^{3/2}d}-\frac{9\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{3\cot(c+dx)}{2ad\sqrt{a\sin(c+dx)+a}}+\frac{\cot(c+dx)}{2d(a\sin(c+dx)+a)^{3/2}}$$

```
[Out] (3*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(a^(3/2)*d) -
(9*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])])/(2*S
qrt[2]*a^(3/2)*d) + Cot[c + d*x]/(2*d*(a + a*Sin[c + d*x])^(3/2)) - (3*Cot[
c + d*x])/(2*a*d*Sqrt[a + a*Sin[c + d*x]])
```

Rubi [A] time = 0.357679, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2766, 2984, 2985, 2649, 206, 2773}

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{a^{3/2}d}-\frac{9\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{3\cot(c+dx)}{2ad\sqrt{a\sin(c+dx)+a}}+\frac{\cot(c+dx)}{2d(a\sin(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (3*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(a^(3/2)*d) -
(9*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])])/(2*S
qrt[2]*a^(3/2)*d) + Cot[c + d*x]/(2*d*(a + a*Sin[c + d*x])^(3/2)) - (3*Cot[
c + d*x])/(2*a*d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
```

```
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc^2(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx &= \frac{\cot(c+dx)}{2d(a+a\sin(c+dx))^{3/2}}+\frac{\int \frac{\csc^2(c+dx)\left(3a-\frac{3}{2}a\sin(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}\,dx}{2a^2}\\
&= \frac{\cot(c+dx)}{2d(a+a\sin(c+dx))^{3/2}}-\frac{3\cot(c+dx)}{2ad\sqrt{a+a\sin(c+dx)}}+\frac{\int \frac{\csc(c+dx)\left(-3a^2+\frac{3}{2}a^2\sin(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}\,dx}{2a^3}\\
&= \frac{\cot(c+dx)}{2d(a+a\sin(c+dx))^{3/2}}-\frac{3\cot(c+dx)}{2ad\sqrt{a+a\sin(c+dx)}}-\frac{3\int \csc(c+dx)\sqrt{a+a\sin(c+dx)}\,dx}{2a^2}+\\
&= \frac{\cot(c+dx)}{2d(a+a\sin(c+dx))^{3/2}}-\frac{3\cot(c+dx)}{2ad\sqrt{a+a\sin(c+dx)}}+\frac{3\,\text{Subst}\left(\int \frac{1}{a-x^2}\,dx,x,\frac{a\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{ad}-\\
&= \frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{a^{3/2}d}-\frac{9\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{\cot(c+dx)}{2d(a+a\sin(c+dx))^{3/2}}-\frac{3\,c}{2ad\sqrt{a}}
\end{aligned}$$

Mathematica [C] time = 0.627292, size = 449, normalized size = 3.12

$$\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)\left(4\sin\left(\frac{1}{2}(c+dx)\right)+\frac{2\sin\left(\frac{1}{4}(c+dx)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^2}{\cos\left(\frac{1}{4}(c+dx)\right)-\sin\left(\frac{1}{4}(c+dx)\right)}-\frac{2\sin\left(\frac{1}{4}(c+dx)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+c\right.}{\sin\left(\frac{1}{4}(c+dx)\right)+\cos\left(\frac{1}{4}(c\right.}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(4*Sin[(c + d*x)/2] - 2*(Cos[(c + d*
x)/2] + Sin[(c + d*x)/2]) + 2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 + (18
 + 18*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)/4])]
```

```
*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 - Cot[(c + d*x)/4]*(Cos[(c + d*x)/
2] + Sin[(c + d*x)/2])^2 + 6*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*(
Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 - 6*Log[1 - Cos[(c + d*x)/2] + Sin[(
c + d*x)/2]]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 + (2*Sin[(c + d*x)/4]*
(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2)/(Cos[(c + d*x)/4] - Sin[(c + d*x)/
4]) - (2*Sin[(c + d*x)/4]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2)/(Cos[(c
+ d*x)/4] + Sin[(c + d*x)/4]) - (Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2*Tan
[(c + d*x)/4]))/(4*d*(a*(1 + Sin[c + d*x]))^(3/2))
```

Maple [A] time = 0.599, size = 219, normalized size = 1.5

$$-\frac{1}{4\cos(dx+c)\sin(dx+c)d}\left(9\sqrt{2}\mathrm{Artanh}\left(1/2\,\frac{\sqrt{-a(\sin(dx+c)-1)}\sqrt{2}}{\sqrt{a}}\right)(\sin(dx+c))^2a-12\,\mathrm{Artanh}\left(\frac{\sqrt{-a(\sin(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2/(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] -1/4/a^(5/2)*(9*2^(1/2)*arctanh(1/2*(-a*(sin(d*x+c)-1))^(1/2)*2^(1/2)/a^(1/
2))*sin(d*x+c)^2*a-12*arctanh((-a*(sin(d*x+c)-1))^(1/2)/a^(1/2))*sin(d*x+c)
^2*a+9*2^(1/2)*arctanh(1/2*(-a*(sin(d*x+c)-1))^(1/2)*2^(1/2)/a^(1/2))*a*sin
(d*x+c)+6*(-a*(sin(d*x+c)-1))^(1/2)*sin(d*x+c)*a^(1/2)-12*arctanh((-a*(sin(
d*x+c)-1))^(1/2)/a^(1/2))*sin(d*x+c)*a+4*(-a*(sin(d*x+c)-1))^(1/2)*a^(1/2))
*(-a*(sin(d*x+c)-1))^(1/2)/sin(d*x+c)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(dx+c)^2}{(a\sin(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(d*x + c)^2/(a*sin(d*x + c) + a)^(3/2), x)
```

Fricas [B] time = 1.90998, size = 1434, normalized size = 9.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/8*(9*sqrt(2)*(cos(d*x + c)^3 + 2*cos(d*x + c)^2 + (cos(d*x + c)^2 - cos(d
*x + c) - 2)*sin(d*x + c) - cos(d*x + c) - 2)*sqrt(a)*log(-(a*cos(d*x + c)^
2 - 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - sin(d*x + c)
 + 1) + 3*a*cos(d*x + c) - (a*cos(d*x + c) - 2*a)*sin(d*x + c) + 2*a)/(cos(
d*x + c)^2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x + c) - 2)) + 6*(cos(
d*x + c)^3 + 2*cos(d*x + c)^2 + (cos(d*x + c)^2 - cos(d*x + c) - 2)*sin(d*x
 + c) - cos(d*x + c) - 2)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^
```

```
2 + 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*cos(d*x + c) -
3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*cos(d*x + c)^2
+ 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + cos(d*x + c)^2
+ (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) + 4*(3*cos(d*x + c
)^2 + (3*cos(d*x + c) + 1)*sin(d*x + c) + 2*cos(d*x + c) - 1)*sqrt(a*sin(d*
x + c) + a))/(a^2*d*cos(d*x + c)^3 + 2*a^2*d*cos(d*x + c)^2 - a^2*d*cos(d*x
 + c) - 2*a^2*d + (a^2*d*cos(d*x + c)^2 - a^2*d*cos(d*x + c) - 2*a^2*d)*sin
(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc^2(c+dx)}{(a(\sin(c+dx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Integral(csc(c + d*x)**2/(a*(sin(c + d*x) + 1))**(3/2), x)
```

Giac [B] time = 2.61023, size = 684, normalized size = 4.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(9*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*t
an(1/2*d*x + 1/2*c)^2 + a) + sqrt(a))/sqrt(-a))/(sqrt(-a)*a*sgn(tan(1/2*d*x
 + 1/2*c) + 1)) - 6*arctan(-(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*
d*x + 1/2*c)^2 + a))/sqrt(-a))/(sqrt(-a)*a*sgn(tan(1/2*d*x + 1/2*c) + 1)) +
 3*log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*tan(1/2*d*x + 1/2*c)^2 +
a)))/(a^(3/2)*sgn(tan(1/2*d*x + 1/2*c) + 1)) + sqrt(a*tan(1/2*d*x + 1/2*c)^
2 + a)/(a^2*sgn(tan(1/2*d*x + 1/2*c) + 1)) + 2/(((sqrt(a)*tan(1/2*d*x + 1/2
*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a)*sqrt(a)*sgn(tan(1/2*d*x +
1/2*c) + 1)) + 2*(3*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/
2*c)^2 + a))^3 + (sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c
)^2 + a))^2*sqrt(a) - (sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x +
1/2*c)^2 + a))*a + a^(3/2))/(((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/
2*d*x + 1/2*c)^2 + a))^2 + 2*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2
*d*x + 1/2*c)^2 + a))*sqrt(a) - a)^2*a*sgn(tan(1/2*d*x + 1/2*c) + 1)))/d
```

3.75 $\int \frac{\csc^3(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=186

$$-\frac{19\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{4a^{3/2}d}+\frac{13\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{7\cot(c+dx)}{4ad\sqrt{a\sin(c+dx)+a}}-\frac{\cot(c+dx)\csc(c+dx)}{ad\sqrt{a\sin(c+dx)+a}}+\frac{\cot(}{2d(a}$$

```
[Out] (-19*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(4*a^(3/2)*d
) + (13*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])])
/(2*Sqrt[2]*a^(3/2)*d) + (Cot[c + d*x]*Csc[c + d*x])/(2*d*(a + a*Sin[c + d*
x])^(3/2)) + (7*Cot[c + d*x])/(4*a*d*Sqrt[a + a*Sin[c + d*x]]) - (Cot[c + d
*x]*Csc[c + d*x])/(a*d*Sqrt[a + a*Sin[c + d*x]])
```

Rubi [A] time = 0.486477, antiderivative size = 186, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2766, 2984, 2985, 2649, 206, 2773}

$$-\frac{19\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{4a^{3/2}d}+\frac{13\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{7\cot(c+dx)}{4ad\sqrt{a\sin(c+dx)+a}}-\frac{\cot(c+dx)\csc(c+dx)}{ad\sqrt{a\sin(c+dx)+a}}+\frac{\cot(}{2d(a}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-19*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(4*a^(3/2)*d
) + (13*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])])
/(2*Sqrt[2]*a^(3/2)*d) + (Cot[c + d*x]*Csc[c + d*x])/(2*d*(a + a*Sin[c + d*
x])^(3/2)) + (7*Cot[c + d*x])/(4*a*d*Sqrt[a + a*Sin[c + d*x]]) - (Cot[c + d
*x]*Csc[c + d*x])/(a*d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\csc^3(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx = \frac{\cot(c+dx)\csc(c+dx)}{2d(a+a\sin(c+dx))^{3/2}} + \frac{\int \frac{\csc^3(c+dx)\left(4a-\frac{5}{2}a\sin(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}\,dx}{2a^2}$$

$$= \frac{\cot(c+dx)\csc(c+dx)}{2d(a+a\sin(c+dx))^{3/2}} - \frac{\cot(c+dx)\csc(c+dx)}{ad\sqrt{a+a\sin(c+dx)}} + \frac{\int \frac{\csc^2(c+dx)\left(-7a^2+6a^2\sin(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}\,dx}{4a^3}$$

$$= \frac{\cot(c+dx)\csc(c+dx)}{2d(a+a\sin(c+dx))^{3/2}} + \frac{7\cot(c+dx)}{4ad\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc(c+dx)}{ad\sqrt{a+a\sin(c+dx)}} + \frac{\int \frac{\csc(c+dx)\left(\frac{1}{\ldots}\right.}{\sqrt{a+\ldots}}}{\ldots}$$

$$= \frac{\cot(c+dx)\csc(c+dx)}{2d(a+a\sin(c+dx))^{3/2}} + \frac{7\cot(c+dx)}{4ad\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc(c+dx)}{ad\sqrt{a+a\sin(c+dx)}} + \frac{19\int \csc(c+\ldots}{\ldots}$$

$$= \frac{\cot(c+dx)\csc(c+dx)}{2d(a+a\sin(c+dx))^{3/2}} + \frac{7\cot(c+dx)}{4ad\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc(c+dx)}{ad\sqrt{a+a\sin(c+dx)}} - \frac{19\,\mathrm{Subst}\left(\int \ldots\right.}{\ldots}$$

$$= -\frac{19\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{4a^{3/2}d} + \frac{13\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} + \frac{\cot(c+dx)\csc(c+dx)}{2d(a+a\sin(c+dx))^{3/2}} + \frac{\ldots}{4a\ldots}$$

Mathematica [C] time = 4.70745, size = 620, normalized size = 3.33

$$\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)\left(-32\sin\left(\frac{1}{2}(c+dx)\right) - \frac{24\sin\left(\frac{1}{4}(c+dx)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^2}{\cos\left(\frac{1}{4}(c+dx)\right)-\sin\left(\frac{1}{4}(c+dx)\right)} + \frac{24\sin\left(\frac{1}{4}(c+dx)\right)\left(\sin\left(\frac{1}{2}(c+d\ldots\right.\right.}{\sin\left(\frac{1}{4}(c+dx)\right)+\cos\ldots} \ldots\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(-32*Sin[(c + d*x)/2] + 16*(Cos[(c +
 d*x)/2] + Sin[(c + d*x)/2]) - 24*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 -
 (208 + 208*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x
)/4])]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 + 12*Cot[(c + d*x)/4]*(Cos[(
c + d*x)/2] + Sin[(c + d*x)/2])^2 - Csc[(c + d*x)/4]^2*(Cos[(c + d*x)/2] +
Sin[(c + d*x)/2])^2 - 76*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*(Cos[
(c + d*x)/2] + Sin[(c + d*x)/2])^2 + 76*Log[1 - Cos[(c + d*x)/2] + Sin[(c +
 d*x)/2]]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 + Sec[(c + d*x)/4]^2*(Cos
[(c + d*x)/2] + Sin[(c + d*x)/2])^2 + (2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/
2])^2)/(Cos[(c + d*x)/4] - Sin[(c + d*x)/4])^2 - (24*Sin[(c + d*x)/4]*(Cos[
(c + d*x)/2] + Sin[(c + d*x)/2])^2)/(Cos[(c + d*x)/4] - Sin[(c + d*x)/4]) -
 (2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2)/(Cos[(c + d*x)/4] + Sin[(c + d
*x)/4])^2 + (24*Sin[(c + d*x)/4]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2)/(
Cos[(c + d*x)/4] + Sin[(c + d*x)/4]) + 12*(Cos[(c + d*x)/2] + Sin[(c + d*x)
/2])^2*Tan[(c + d*x)/4]))/(32*d*(a*(1 + Sin[c + d*x]))^(3/2))
```

Maple [A] time = 0.652, size = 299, normalized size = 1.6

$$\frac{1}{4\left(\sin\left(dx+c\right)\right)^{2}\cos\left(dx+c\right)d}\left(13\sqrt{2}\mathrm{Artanh}\left(1/2\,\frac{\sqrt{-a\left(\sin\left(dx+c\right)-1\right)}\sqrt{2}}{\sqrt{a}}\right)\left(\sin\left(dx+c\right)\right)^{3}a^{2}+2\sqrt{-a\left(\sin\left(dx+c\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3/(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] 1/4*(13*2^(1/2)*arctanh(1/2*(-a*(sin(d*x+c)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(
d*x+c)^3*a^2+2*(-a*(sin(d*x+c)-1))^(1/2)*a^(3/2)*sin(d*x+c)^2+13*2^(1/2)*ar
ctanh(1/2*(-a*(sin(d*x+c)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(d*x+c)^2*a^2-19*ar
ctanh((-a*(sin(d*x+c)-1))^(1/2)/a^(1/2))*sin(d*x+c)^3*a^2+3*(-a*(sin(d*x+c)
-1))^(1/2)*a^(3/2)*sin(d*x+c)-5*(-a*(sin(d*x+c)-1))^(3/2)*a^(1/2)*sin(d*x+c
)-19*arctanh((-a*(sin(d*x+c)-1))^(1/2)/a^(1/2))*sin(d*x+c)^2*a^2+3*(-a*(sin
(d*x+c)-1))^(1/2)*a^(3/2)-5*(-a*(sin(d*x+c)-1))^(3/2)*a^(1/2))*(-a*(sin(d*x
+c)-1))^(1/2)/a^(7/2)/sin(d*x+c)^2/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 1.95225, size = 1651, normalized size = 8.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/16*(26*sqrt(2)*(cos(d*x + c)^4 - cos(d*x + c)^3 - 3*cos(d*x + c)^2 - (cos
(d*x + c)^3 + 2*cos(d*x + c)^2 - cos(d*x + c) - 2)*sin(d*x + c) + cos(d*x +
 c) + 2)*sqrt(a)*log(-(a*cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a*sin(d*x + c) + a
)*sqrt(a)*(cos(d*x + c) - sin(d*x + c) + 1) + 3*a*cos(d*x + c) - (a*cos(d*x
 + c) - 2*a)*sin(d*x + c) + 2*a)/(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(d
*x + c) - cos(d*x + c) - 2)) + 19*(cos(d*x + c)^4 - cos(d*x + c)^3 - 3*cos(
d*x + c)^2 - (cos(d*x + c)^3 + 2*cos(d*x + c)^2 - cos(d*x + c) - 2)*sin(d*x
 + c) + cos(d*x + c) + 2)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^
2 - 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*cos(d*x + c) -
3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*cos(d*x + c)^2
+ 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + cos(d*x + c)^2
+ (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) - 4*(7*cos(d*x + c
)^3 + 4*cos(d*x + c)^2 - (7*cos(d*x + c)^2 + 3*cos(d*x + c) - 2)*sin(d*x +
c) - 5*cos(d*x + c) - 2)*sqrt(a*sin(d*x + c) + a))/(a^2*d*cos(d*x + c)^4 -
a^2*d*cos(d*x + c)^3 - 3*a^2*d*cos(d*x + c)^2 + a^2*d*cos(d*x + c) + 2*a^2*
d - (a^2*d*cos(d*x + c)^3 + 2*a^2*d*cos(d*x + c)^2 - a^2*d*cos(d*x + c) - 2
*a^2*d)*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc^3(c+dx)}{(a(\sin(c+dx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Integral(csc(c + d*x)**3/(a*(sin(c + d*x) + 1))**(3/2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.76 $\int \frac{\sin^5(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=221

$$-\frac{157\sin^2(c+dx)\cos(c+dx)}{80a^2d\sqrt{a\sin(c+dx)+a}}+\frac{787\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{240a^3d}-\frac{1729\cos(c+dx)}{120a^2d\sqrt{a\sin(c+dx)+a}}+\frac{283\tanh^{-1}\left(\frac{\sqrt{a}}{\sqrt{2}\sqrt{a}}\right.}{16\sqrt{2}a^{5/2}}$$

```
[Out] (283*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])])/(1
6*Sqrt[2]*a^(5/2)*d) + (Cos[c + d*x]*Sin[c + d*x]^4)/(4*d*(a + a*Sin[c + d*
x])^(5/2)) + (21*Cos[c + d*x]*Sin[c + d*x]^3)/(16*a*d*(a + a*Sin[c + d*x])^
(3/2)) - (1729*Cos[c + d*x])/(120*a^2*d*Sqrt[a + a*Sin[c + d*x]]) - (157*Co
s[c + d*x]*Sin[c + d*x]^2)/(80*a^2*d*Sqrt[a + a*Sin[c + d*x]]) + (787*Cos[c
 + d*x]*Sqrt[a + a*Sin[c + d*x]])/(240*a^3*d)
```

Rubi [A] time = 0.521103, antiderivative size = 221, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {2765, 2977, 2983, 2968, 3023, 2751, 2649, 206}

$$-\frac{157\sin^2(c+dx)\cos(c+dx)}{80a^2d\sqrt{a\sin(c+dx)+a}}+\frac{787\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{240a^3d}-\frac{1729\cos(c+dx)}{120a^2d\sqrt{a\sin(c+dx)+a}}+\frac{283\tanh^{-1}\left(\frac{\sqrt{a}}{\sqrt{2}\sqrt{a}}\right.}{16\sqrt{2}a^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^5/(a + a*Sin[c + d*x])^(5/2),x]
```


```
[Out] (283*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])])/(1
6*Sqrt[2]*a^(5/2)*d) + (Cos[c + d*x]*Sin[c + d*x]^4)/(4*d*(a + a*Sin[c + d*
x])^(5/2)) + (21*Cos[c + d*x]*Sin[c + d*x]^3)/(16*a*d*(a + a*Sin[c + d*x])^
(3/2)) - (1729*Cos[c + d*x])/(120*a^2*d*Sqrt[a + a*Sin[c + d*x]]) - (157*Co
s[c + d*x]*Sin[c + d*x]^2)/(80*a^2*d*Sqrt[a + a*Sin[c + d*x]]) + (787*Cos[c
 + d*x]*Sqrt[a + a*Sin[c + d*x]])/(240*a^3*d)
```


Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sin^5(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx = \frac{\cos(c+dx)\sin^4(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} - \frac{\int \frac{\sin^3(c+dx)\left(4a-\frac{13}{2}a\sin(c+dx)\right)}{(a+a\sin(c+dx))^{3/2}}dx}{4a^2}$$

$$= \frac{\cos(c+dx)\sin^4(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} + \frac{21\cos(c+dx)\sin^3(c+dx)}{16ad(a+a\sin(c+dx))^{3/2}} - \frac{\int \frac{\sin^2(c+dx)\left(\frac{63a^2}{2}-\frac{157}{4}a^2\sin(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}}{8a^4}$$

$$= \frac{\cos(c+dx)\sin^4(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} + \frac{21\cos(c+dx)\sin^3(c+dx)}{16ad(a+a\sin(c+dx))^{3/2}} - \frac{157\cos(c+dx)\sin^2(c+dx)}{80a^2d\sqrt{a+a\sin(c+dx)}} -$$

$$= \frac{\cos(c+dx)\sin^4(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} + \frac{21\cos(c+dx)\sin^3(c+dx)}{16ad(a+a\sin(c+dx))^{3/2}} - \frac{157\cos(c+dx)\sin^2(c+dx)}{80a^2d\sqrt{a+a\sin(c+dx)}} -$$

$$= \frac{\cos(c+dx)\sin^4(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} + \frac{21\cos(c+dx)\sin^3(c+dx)}{16ad(a+a\sin(c+dx))^{3/2}} - \frac{157\cos(c+dx)\sin^2(c+dx)}{80a^2d\sqrt{a+a\sin(c+dx)}} +$$

$$= \frac{\cos(c+dx)\sin^4(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} + \frac{21\cos(c+dx)\sin^3(c+dx)}{16ad(a+a\sin(c+dx))^{3/2}} - \frac{1729\cos(c+dx)}{120a^2d\sqrt{a+a\sin(c+dx)}} - 1$$

$$= \frac{\cos(c+dx)\sin^4(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} + \frac{21\cos(c+dx)\sin^3(c+dx)}{16ad(a+a\sin(c+dx))^{3/2}} - \frac{1729\cos(c+dx)}{120a^2d\sqrt{a+a\sin(c+dx)}} - 1$$

$$= \frac{283\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} + \frac{\cos(c+dx)\sin^4(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} + \frac{21\cos(c+dx)\sin^3(c+dx)}{16ad(a+a\sin(c+dx))^{3/2}}$$

Mathematica [C] time = 0.566465, size = 221, normalized size = 1.

$$-\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)\left(-2547\sin\left(\frac{1}{2}(c+dx)\right)+3603\sin\left(\frac{3}{2}(c+dx)\right)+872\sin\left(\frac{5}{2}(c+dx)\right)+52\sin\left(\frac{7}{2}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^5/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] -((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(2547*Cos[(c + d*x)/2] + 3603*Cos[(
3*(c + d*x))/2] - 872*Cos[(5*(c + d*x))/2] + 52*Cos[(7*(c + d*x))/2] + 12*C
os[(9*(c + d*x))/2] - 2547*Sin[(c + d*x)/2] + (8490 + 8490*I)*(-1)^(3/4)*Ar
cTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)/4])]*(Cos[(c + d*x)/2] + S
in[(c + d*x)/2])^4 + 3603*Sin[(3*(c + d*x))/2] + 872*Sin[(5*(c + d*x))/2] +
 52*Sin[(7*(c + d*x))/2] - 12*Sin[(9*(c + d*x))/2]))/(480*d*(a*(1 + Sin[c +
 d*x]))^(5/2))
```

Maple [A] time = 0.783, size = 323, normalized size = 1.5

$$-\frac{1}{(480+480\sin(dx+c))\cos(dx+c)d}\left(\sin(dx+c)\left(384\,(a-a\sin(dx+c))^{5/2}\sqrt{a}+640\,(a-a\sin(dx+c))^{3/2}a^{3/2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^5/(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] -1/480/a^(11/2)*(sin(d*x+c)*(384*(a-a*sin(d*x+c))^(5/2)*a^(1/2)+640*(a-a*si
n(d*x+c))^(3/2)*a^(3/2)+7680*(a-a*sin(d*x+c))^(1/2)*a^(5/2)-8490*2^(1/2)*ar
ctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a^3)+(-192*(a-a*sin(d*x+c
))^(5/2)*a^(1/2)-320*(a-a*sin(d*x+c))^(3/2)*a^(3/2)-3840*(a-a*sin(d*x+c))^(
1/2)*a^(5/2)+4245*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2
))*a^3)*cos(d*x+c)^2+384*(a-a*sin(d*x+c))^(5/2)*a^(1/2)-470*(a-a*sin(d*x+c)
)^(3/2)*a^(3/2)+9780*(a-a*sin(d*x+c))^(1/2)*a^(5/2)-8490*2^(1/2)*arctanh(1/
2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a^3)*(-a*(sin(d*x+c)-1))^(1/2)/(1
+sin(d*x+c))/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(dx+c)^5}{(a\sin(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)^5/(a*sin(d*x + c) + a)^(5/2), x)
```

Fricas [B] time = 1.87799, size = 1041, normalized size = 4.71

$$\frac{4245\sqrt{2}\left(\cos(dx+c)^3+3\cos(dx+c)^2+\left(\cos(dx+c)^2-2\cos(dx+c)-4\right)\sin(dx+c)-2\cos(dx+c)-4\right)\sqrt{a}\log\left(-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/960*(4245*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + (cos(d*x + c)^2 -
2*cos(d*x + c) - 4)*sin(d*x + c) - 2*cos(d*x + c) - 4)*sqrt(a)*log(-(a*cos(
d*x + c)^2 + 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - sin
(d*x + c) + 1) + 3*a*cos(d*x + c) - (a*cos(d*x + c) - 2*a)*sin(d*x + c) + 2
*a)/(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x + c) - 2))
+ 4*(96*cos(d*x + c)^5 + 256*cos(d*x + c)^4 - 1760*cos(d*x + c)^3 + 2475*co
s(d*x + c)^2 - (96*cos(d*x + c)^4 - 160*cos(d*x + c)^3 - 1920*cos(d*x + c)^
2 - 4395*cos(d*x + c) - 60)*sin(d*x + c) + 4335*cos(d*x + c) - 60)*sqrt(a*s
in(d*x + c) + a))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 - 2*a^3*d*
cos(d*x + c) - 4*a^3*d + (a^3*d*cos(d*x + c)^2 - 2*a^3*d*cos(d*x + c) - 4*a
^3*d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**5/(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] sage2
```

3.77 $\int \frac{\sin^4(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=183

$$-\frac{95\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{48a^3d}+\frac{197\cos(c+dx)}{24a^2d\sqrt{a\sin(c+dx)+a}}-\frac{163\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{\sin^3(c+dx)\cos(c+dx)}{4d(a\sin(c+dx)+a)^{5/2}}$$

```
[Out] (-163*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])])/(
16*Sqrt[2]*a^(5/2)*d) + (Cos[c + d*x]*Sin[c + d*x]^3)/(4*d*(a + a*Sin[c + d
*x])^(5/2)) + (17*Cos[c + d*x]*Sin[c + d*x]^2)/(16*a*d*(a + a*Sin[c + d*x])
^(3/2)) + (197*Cos[c + d*x])/(24*a^2*d*Sqrt[a + a*Sin[c + d*x]]) - (95*Cos[
c + d*x]*Sqrt[a + a*Sin[c + d*x]])/(48*a^3*d)
```

Rubi [A] time = 0.385496, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2765, 2977, 2968, 3023, 2751, 2649, 206}

$$-\frac{95\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{48a^3d}+\frac{197\cos(c+dx)}{24a^2d\sqrt{a\sin(c+dx)+a}}-\frac{163\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{\sin^3(c+dx)\cos(c+dx)}{4d(a\sin(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^4/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-163*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])])/(
16*Sqrt[2]*a^(5/2)*d) + (Cos[c + d*x]*Sin[c + d*x]^3)/(4*d*(a + a*Sin[c + d
*x])^(5/2)) + (17*Cos[c + d*x]*Sin[c + d*x]^2)/(16*a*d*(a + a*Sin[c + d*x])
^(3/2)) + (197*Cos[c + d*x])/(24*a^2*d*Sqrt[a + a*Sin[c + d*x]]) - (95*Cos[
c + d*x]*Sqrt[a + a*Sin[c + d*x]])/(48*a^3*d)
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sin^4(c+dx)}{(a+a\sin(c+dx))^{5/2}}dx = \frac{\cos(c+dx)\sin^3(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} - \frac{\int \frac{\sin^2(c+dx)\left(3a-\frac{11}{2}a\sin(c+dx)\right)}{(a+a\sin(c+dx))^{3/2}}dx}{4a^2}$$

$$= \frac{\cos(c+dx)\sin^3(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} + \frac{17\cos(c+dx)\sin^2(c+dx)}{16ad(a+a\sin(c+dx))^{3/2}} - \frac{\int \frac{\sin(c+dx)\left(17a^2-\frac{95}{4}a^2\sin(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}dx}{8a^4}$$

$$= \frac{\cos(c+dx)\sin^3(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} + \frac{17\cos(c+dx)\sin^2(c+dx)}{16ad(a+a\sin(c+dx))^{3/2}} - \frac{\int \frac{17a^2\sin(c+dx)-\frac{95}{4}a^2\sin^2(c+dx)}{\sqrt{a+a\sin(c+dx)}}dx}{8a^4}$$

$$= \frac{\cos(c+dx)\sin^3(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} + \frac{17\cos(c+dx)\sin^2(c+dx)}{16ad(a+a\sin(c+dx))^{3/2}} - \frac{95\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{48a^3d}$$

$$= \frac{\cos(c+dx)\sin^3(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} + \frac{17\cos(c+dx)\sin^2(c+dx)}{16ad(a+a\sin(c+dx))^{3/2}} + \frac{197\cos(c+dx)}{24a^2d\sqrt{a+a\sin(c+dx)}} - 95\,\mathrm{co}$$

$$= \frac{\cos(c+dx)\sin^3(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} + \frac{17\cos(c+dx)\sin^2(c+dx)}{16ad(a+a\sin(c+dx))^{3/2}} + \frac{197\cos(c+dx)}{24a^2d\sqrt{a+a\sin(c+dx)}} - 95\,\mathrm{co}$$

$$= -\frac{163\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} + \frac{\cos(c+dx)\sin^3(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} + \frac{17\cos(c+dx)\sin^2(c+dx)}{16ad(a+a\sin(c+dx))^{3/2}} +$$

Mathematica [C] time = 0.48224, size = 197, normalized size = 1.08

$$\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)\left(-279\sin\left(\frac{1}{2}(c+dx)\right)+399\sin\left(\frac{3}{2}(c+dx)\right)+88\sin\left(\frac{5}{2}(c+dx)\right)+8\sin\left(\frac{7}{2}(c+dx)\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^4/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(279*Cos[(c + d*x)/2] + 399*Cos[(3*(
c + d*x))/2] - 88*Cos[(5*(c + d*x))/2] + 8*Cos[(7*(c + d*x))/2] - 279*Sin[(
c + d*x)/2] + (978 + 978*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 +
 Tan[(c + d*x)/4])]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4 + 399*Sin[(3*(c
 + d*x))/2] + 88*Sin[(5*(c + d*x))/2] + 8*Sin[(7*(c + d*x))/2]))/(96*d*(a*(
1 + Sin[c + d*x]))^(5/2))
```

Maple [A] time = 0.787, size = 269, normalized size = 1.5

$$-\frac{1}{(96+96\sin(dx+c))\cos(dx+c)d}\left(\sin(dx+c)\left(978\sqrt{2}\mathrm{Artanh}\left(1/2\,\frac{\sqrt{a-a\sin(dx+c)}\sqrt{2}}{\sqrt{a}}\right)a^2-768\sqrt{a-a\sin(dx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^4/(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] -1/96*(sin(d*x+c)*(978*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a
^(1/2))*a^2-768*(a-a*sin(d*x+c))^(1/2)*a^(3/2)-128*(a-a*sin(d*x+c))^(3/2)*a
^(1/2))+(-489*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a
^2+384*(a-a*sin(d*x+c))^(1/2)*a^(3/2)+64*(a-a*sin(d*x+c))^(3/2)*a^(1/2))*co
```

```
s(d*x+c)^2+978*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*
a^2-1092*(a-a*sin(d*x+c))^(1/2)*a^(3/2)+46*(a-a*sin(d*x+c))^(3/2)*a^(1/2))*
(-a*(sin(d*x+c)-1))^(1/2)/a^(9/2)/(1+sin(d*x+c))/cos(d*x+c)/(a+a*sin(d*x+c)
)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(dx+c)^4}{(a\sin(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)^4/(a*sin(d*x + c) + a)^(5/2), x)
```

Fricas [B] time = 1.88903, size = 976, normalized size = 5.33

$$489\sqrt{2}\left(\cos(dx+c)^3+3\cos(dx+c)^2+\left(\cos(dx+c)^2-2\cos(dx+c)-4\right)\sin(dx+c)-2\cos(dx+c)-4\right)\sqrt{a}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/192*(489*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + (cos(d*x + c)^2 - 2
*cos(d*x + c) - 4)*sin(d*x + c) - 2*cos(d*x + c) - 4)*sqrt(a)*log(-(a*cos(d
*x + c)^2 - 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - sin(
d*x + c) + 1) + 3*a*cos(d*x + c) - (a*cos(d*x + c) - 2*a)*sin(d*x + c) + 2*
a)/(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x + c) - 2)) -
 4*(32*cos(d*x + c)^4 - 160*cos(d*x + c)^3 + 279*cos(d*x + c)^2 + (32*cos(d
*x + c)^3 + 192*cos(d*x + c)^2 + 471*cos(d*x + c) + 12)*sin(d*x + c) + 459*
cos(d*x + c) - 12)*sqrt(a*sin(d*x + c) + a))/(a^3*d*cos(d*x + c)^3 + 3*a^3*
d*cos(d*x + c)^2 - 2*a^3*d*cos(d*x + c) - 4*a^3*d + (a^3*d*cos(d*x + c)^2 -
 2*a^3*d*cos(d*x + c) - 4*a^3*d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**4/(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] sage2
```

3.78 $\int \frac{\sin^3(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=145

$$-\frac{9\cos(c+dx)}{4a^2d\sqrt{a\sin(c+dx)+a}}+\frac{75\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{\sin^2(c+dx)\cos(c+dx)}{4d(a\sin(c+dx)+a)^{5/2}}-\frac{13\cos(c+dx)}{16ad(a\sin(c+dx)+a)^{3/2}}$$

```
[Out] (75*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])])/(16
*Sqrt[2]*a^(5/2)*d) + (Cos[c + d*x]*Sin[c + d*x]^2)/(4*d*(a + a*Sin[c + d*x
])^(5/2)) - (13*Cos[c + d*x])/(16*a*d*(a + a*Sin[c + d*x])^(3/2)) - (9*Cos[
c + d*x])/(4*a^2*d*Sqrt[a + a*Sin[c + d*x]])
```

Rubi [A] time = 0.267828, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2765, 2968, 3019, 2751, 2649, 206}

$$-\frac{9\cos(c+dx)}{4a^2d\sqrt{a\sin(c+dx)+a}}+\frac{75\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{\sin^2(c+dx)\cos(c+dx)}{4d(a\sin(c+dx)+a)^{5/2}}-\frac{13\cos(c+dx)}{16ad(a\sin(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^3/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (75*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])])/(16
*Sqrt[2]*a^(5/2)*d) + (Cos[c + d*x]*Sin[c + d*x]^2)/(4*d*(a + a*Sin[c + d*x
])^(5/2)) - (13*Cos[c + d*x])/(16*a*d*(a + a*Sin[c + d*x])^(3/2)) - (9*Cos[
c + d*x])/(4*a^2*d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3019

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b - a
*B + b*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[1
/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*
B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sin^3(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx = \frac{\cos(c+dx)\sin^2(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} - \frac{\int \frac{\sin(c+dx)\left(2a-\frac{9}{2}a\sin(c+dx)\right)}{(a+a\sin(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= \frac{\cos(c+dx)\sin^2(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} - \frac{\int \frac{2a\sin(c+dx)-\frac{9}{2}a\sin^2(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= \frac{\cos(c+dx)\sin^2(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} - \frac{13\cos(c+dx)}{16ad(a+a\sin(c+dx))^{3/2}} + \frac{\int \frac{-\frac{39a^2}{4}+9a^2\sin(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx}{8a^4}$$

$$= \frac{\cos(c+dx)\sin^2(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} - \frac{13\cos(c+dx)}{16ad(a+a\sin(c+dx))^{3/2}} - \frac{9\cos(c+dx)}{4a^2d\sqrt{a+a\sin(c+dx)}} - \frac{75\int \frac{\quad}{\sqrt{a\cdots}}}{\cdots}$$

$$= \frac{\cos(c+dx)\sin^2(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} - \frac{13\cos(c+dx)}{16ad(a+a\sin(c+dx))^{3/2}} - \frac{9\cos(c+dx)}{4a^2d\sqrt{a+a\sin(c+dx)}} + \frac{75\,\mathrm{Subs}\cdots}{\cdots}$$

$$= \frac{75\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} + \frac{\cos(c+dx)\sin^2(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} - \frac{13\cos(c+dx)}{16ad(a+a\sin(c+dx))^{3/2}} - \frac{\cdots}{4a^2\cdots}$$

Mathematica [C] time = 0.321032, size = 173, normalized size = 1.19

$$\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)\left(45\sin\left(\frac{1}{2}(c+dx)\right)-69\sin\left(\frac{3}{2}(c+dx)\right)-16\sin\left(\frac{5}{2}(c+dx)\right)-45\cos\left(\frac{1}{2}(c+dx)\right)-6\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^3/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(-45*Cos[(c + d*x)/2] - 69*Cos[(3*(c
 + d*x))/2] + 16*Cos[(5*(c + d*x))/2] + 45*Sin[(c + d*x)/2] - (150 + 150*I)
*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)/4])]*(Cos[(c
 + d*x)/2] + Sin[(c + d*x)/2])^4 - 69*Sin[(3*(c + d*x))/2] - 16*Sin[(5*(c +
 d*x))/2]))/(32*d*(a*(1 + Sin[c + d*x]))^(5/2))
```

Maple [A] time = 0.742, size = 233, normalized size = 1.6

$$\frac{1}{(32+32\sin(dx+c))\cos(dx+c)d}\left(\sin(dx+c)\left(150\sqrt{2}\mathrm{Artanh}\left(1/2\frac{\sqrt{a-a\sin(dx+c)}\sqrt{2}}{\sqrt{a}}\right)a^2-128\sqrt{a-a\sin(dx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3/(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] 1/32/a^(9/2)*(sin(d*x+c)*(150*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^
(1/2)/a^(1/2))*a^2-128*(a-a*sin(d*x+c))^(1/2)*a^(3/2))+(-75*2^(1/2)*arctanh
(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a^2+64*(a-a*sin(d*x+c))^(1/2)*
a^(3/2))*cos(d*x+c)^2+150*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2
)/a^(1/2))*a^2-204*(a-a*sin(d*x+c))^(1/2)*a^(3/2)+42*(a-a*sin(d*x+c))^(3/2)
*a^(1/2))*(-a*(sin(d*x+c)-1))^(1/2)/(1+sin(d*x+c))/cos(d*x+c)/(a+a*sin(d*x+
c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(dx+c)^3}{(a\sin(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)^3/(a*sin(d*x + c) + a)^(5/2), x)
```

Fricas [B] time = 1.79555, size = 910, normalized size = 6.28

$$\frac{75\sqrt{2}\left(\cos(dx+c)^3+3\cos(dx+c)^2+\left(\cos(dx+c)^2-2\cos(dx+c)-4\right)\sin(dx+c)-2\cos(dx+c)-4\right)\sqrt{a}\log\left(\right.}{64\left(a^3d\cos(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/64*(75*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + (cos(d*x + c)^2 - 2*c
os(d*x + c) - 4)*sin(d*x + c) - 2*cos(d*x + c) - 4)*sqrt(a)*log(-(a*cos(d*x
 + c)^2 + 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - sin(d*
x + c) + 1) + 3*a*cos(d*x + c) - (a*cos(d*x + c) - 2*a)*sin(d*x + c) + 2*a)
/(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x + c) - 2)) - 4
*(32*cos(d*x + c)^3 - 53*cos(d*x + c)^2 - (32*cos(d*x + c)^2 + 85*cos(d*x +
 c) + 4)*sin(d*x + c) - 81*cos(d*x + c) + 4)*sqrt(a*sin(d*x + c) + a))/(a^3
*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 - 2*a^3*d*cos(d*x + c) - 4*a^3*d
 + (a^3*d*cos(d*x + c)^2 - 2*a^3*d*cos(d*x + c) - 4*a^3*d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3/(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] sage2
```

3.79 $\int \frac{\sin^2(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=107

$$-\frac{19\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{13\cos(c+dx)}{16ad(a\sin(c+dx)+a)^{3/2}}-\frac{\cos(c+dx)}{4d(a\sin(c+dx)+a)^{5/2}}$$

```
[Out] (-19*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])])/(1
6*Sqrt[2]*a^(5/2)*d) - Cos[c + d*x]/(4*d*(a + a*Sin[c + d*x])^(5/2)) + (13*
Cos[c + d*x])/(16*a*d*(a + a*Sin[c + d*x])^(3/2))
```

Rubi [A] time = 0.128479, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2758, 2750, 2649, 206}

$$-\frac{19\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{13\cos(c+dx)}{16ad(a\sin(c+dx)+a)^{3/2}}-\frac{\cos(c+dx)}{4d(a\sin(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-19*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])])/(1
6*Sqrt[2]*a^(5/2)*d) - Cos[c + d*x]/(4*d*(a + a*Sin[c + d*x])^(5/2)) + (13*
Cos[c + d*x])/(16*a*d*(a + a*Sin[c + d*x])^(3/2))
```

Rule 2758

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)),
x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(a*m - b*(2*m
 + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -2^(-1)]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sin^2(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx = -\frac{\cos(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} + \frac{\int \frac{-\frac{5a}{2}+4a\sin(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx}{4a^2}$$
$$= -\frac{\cos(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} + \frac{13\cos(c+dx)}{16ad(a+a\sin(c+dx))^{3/2}} + \frac{19\int \frac{1}{\sqrt{a+a\sin(c+dx)}}\,dx}{32a^2}$$
$$= -\frac{\cos(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} + \frac{13\cos(c+dx)}{16ad(a+a\sin(c+dx))^{3/2}} - \frac{19\,\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \frac{a\cos(c+d}{\sqrt{a+a\sin(c-}}\right.}{16a^2d}$$
$$= -\frac{19\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{\cos(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} + \frac{13\cos(c+dx)}{16ad(a+a\sin(c+dx))^{3/2}}$$

Mathematica [C] time = 0.205651, size = 196, normalized size = 1.83

$$\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)\left(8\sin\left(\frac{1}{2}(c+dx)\right)+13\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^3-26\sin\left(\frac{1}{2}(c+dx)\right)\left(\sin\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(8*Sin[(c + d*x)/2] - 4*(Cos[(c + d*
x)/2] + Sin[(c + d*x)/2]) - 26*Sin[(c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[(c
+ d*x)/2])^2 + 13*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3 + (19 + 19*I)*(-1
)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)/4])]*(Cos[(c + d
*x)/2] + Sin[(c + d*x)/2])^4))/(16*d*(a*(1 + Sin[c + d*x]))^(5/2))
```

Maple [B] time = 0.658, size = 193, normalized size = 1.8

$$-\frac{1}{(32+32\sin(dx+c))\cos(dx+c)d}\left(-19\sqrt{2}\text{Artanh}\left(1/2\frac{\sqrt{a-a\sin(dx+c)}\sqrt{2}}{\sqrt{a}}\right)a^2(\cos(dx+c))^2+38\sqrt{2}\text{Artanh}\left(1/\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2/(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] -1/32/a^(9/2)*(-19*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/
2))*a^2*cos(d*x+c)^2+38*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/
a^(1/2))*sin(d*x+c)*a^2+38*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/
2)/a^(1/2))*a^2+26*(a-a*sin(d*x+c))^(3/2)*a^(1/2)-44*(a-a*sin(d*x+c))^(1/2)
*a^(3/2))*(-a*(sin(d*x+c)-1))^(1/2)/(1+sin(d*x+c))/cos(d*x+c)/(a+a*sin(d*x+
c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(dx+c)^2}{(a\sin(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)^2/(a*sin(d*x + c) + a)^(5/2), x)
```

Fricas [B] time = 1.79337, size = 855, normalized size = 7.99

$$\frac{19\,\sqrt{2}\left(\cos\left(dx+c\right)^{3}+3\,\cos\left(dx+c\right)^{2}+\left(\cos\left(dx+c\right)^{2}-2\,\cos\left(dx+c\right)-4\right)\sin\left(dx+c\right)-2\,\cos\left(dx+c\right)-4\right)\sqrt{a}\log\left(\right.}{64\left(a^{3}d\cos\left(dx+c\right)^{3}+3\,a^{3}d\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/64*(19*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + (cos(d*x + c)^2 - 2*c
os(d*x + c) - 4)*sin(d*x + c) - 2*cos(d*x + c) - 4)*sqrt(a)*log(-(a*cos(d*x
 + c)^2 - 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - sin(d*
x + c) + 1) + 3*a*cos(d*x + c) - (a*cos(d*x + c) - 2*a)*sin(d*x + c) + 2*a)
/(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x + c) - 2)) - 4
*(13*cos(d*x + c)^2 + (13*cos(d*x + c) + 4)*sin(d*x + c) + 9*cos(d*x + c) -
 4)*sqrt(a*sin(d*x + c) + a))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^
2 - 2*a^3*d*cos(d*x + c) - 4*a^3*d + (a^3*d*cos(d*x + c)^2 - 2*a^3*d*cos(d*
x + c) - 4*a^3*d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2/(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

sage_2

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] sage2
```

3.80 $\int \frac{\sin(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=107

$$-\frac{5\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{5\cos(c+dx)}{16ad(a\sin(c+dx)+a)^{3/2}}+\frac{\cos(c+dx)}{4d(a\sin(c+dx)+a)^{5/2}}$$

```
[Out] (-5*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])])/(16
*Sqrt[2]*a^(5/2)*d) + Cos[c + d*x]/(4*d*(a + a*Sin[c + d*x])^(5/2)) - (5*Co
s[c + d*x])/(16*a*d*(a + a*Sin[c + d*x])^(3/2))
```

Rubi [A] time = 0.0738224, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2750, 2650, 2649, 206}

$$-\frac{5\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{5\cos(c+dx)}{16ad(a\sin(c+dx)+a)^{3/2}}+\frac{\cos(c+dx)}{4d(a\sin(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-5*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])])/(16
*Sqrt[2]*a^(5/2)*d) + Cos[c + d*x]/(4*d*(a + a*Sin[c + d*x])^(5/2)) - (5*Co
s[c + d*x])/(16*a*d*(a + a*Sin[c + d*x])^(3/2))
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sin(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx = \frac{\cos(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} + \frac{5\int \frac{1}{(a+a\sin(c+dx))^{3/2}}\,dx}{8a}$$
$$= \frac{\cos(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} - \frac{5\cos(c+dx)}{16ad(a+a\sin(c+dx))^{3/2}} + \frac{5\int \frac{1}{\sqrt{a+a\sin(c+dx)}}\,dx}{32a^2}$$
$$= \frac{\cos(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} - \frac{5\cos(c+dx)}{16ad(a+a\sin(c+dx))^{3/2}} - \frac{5\,\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \frac{a\cos(c+d}{\sqrt{a+a\sin(c}}\right.}{16a^2d}$$
$$= -\frac{5\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} + \frac{\cos(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} - \frac{5\cos(c+dx)}{16ad(a+a\sin(c+dx))^{3/2}}$$

Mathematica [C] time = 0.183147, size = 196, normalized size = 1.83

$$\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)\left(-8\sin\left(\frac{1}{2}(c+dx)\right)-5\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^3+10\sin\left(\frac{1}{2}(c+dx)\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(-8*Sin[(c + d*x)/2] + 4*(Cos[(c + d
*x)/2] + Sin[(c + d*x)/2]) + 10*Sin[(c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[(c
 + d*x)/2])^2 - 5*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3 + (5 + 5*I)*(-1)^
(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)/4])]*(Cos[(c + d*x
)/2] + Sin[(c + d*x)/2])^4))/(16*d*(a*(1 + Sin[c + d*x]))^(5/2))
```

Maple [B] time = 0.561, size = 193, normalized size = 1.8

$$-\frac{1}{(32+32\sin(dx+c))\cos(dx+c)d}\left(-5\sqrt{2}\text{Artanh}\left(1/2\frac{\sqrt{a-a\sin(dx+c)}\sqrt{2}}{\sqrt{a}}\right)a^3(\cos(dx+c))^2+10\sqrt{2}\text{Artanh}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] -1/32*(-5*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a^3*c
os(d*x+c)^2+10*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*
a^3*sin(d*x+c)-10*(a-a*sin(d*x+c))^(3/2)*a^(3/2)+12*(a-a*sin(d*x+c))^(1/2)*
a^(5/2)+10*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a^3)
*(-a*(sin(d*x+c)-1))^(1/2)/a^(11/2)/(1+sin(d*x+c))/cos(d*x+c)/(a+a*sin(d*x+
c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(dx+c)}{(a\sin(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)/(a*sin(d*x + c) + a)^(5/2), x)
```

Fricas [B] time = 1.81739, size = 848, normalized size = 7.93

$$\frac{5\sqrt{2}\left(\cos(dx+c)^3+3\cos(dx+c)^2+\left(\cos(dx+c)^2-2\cos(dx+c)-4\right)\sin(dx+c)-2\cos(dx+c)-4\right)\sqrt{a}\log\left(-\frac{a\,\mathrm{co}}{}\right.}{64\left(a^3d\cos(dx+c)^3+3\,a^3d\cos(dx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/64*(5*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + (cos(d*x + c)^2 - 2*co
s(d*x + c) - 4)*sin(d*x + c) - 2*cos(d*x + c) - 4)*sqrt(a)*log(-(a*cos(d*x
+ c)^2 - 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - sin(d*x
 + c) + 1) + 3*a*cos(d*x + c) - (a*cos(d*x + c) - 2*a)*sin(d*x + c) + 2*a)/
(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x + c) - 2)) + 4*
(5*cos(d*x + c)^2 + (5*cos(d*x + c) + 4)*sin(d*x + c) + cos(d*x + c) - 4)*s
qrt(a*sin(d*x + c) + a))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 - 2
*a^3*d*cos(d*x + c) - 4*a^3*d + (a^3*d*cos(d*x + c)^2 - 2*a^3*d*cos(d*x + c
) - 4*a^3*d)*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(c+dx)}{\left(a\left(\sin(c+dx)+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Integral(sin(c + d*x)/(a*(sin(c + d*x) + 1))**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] sage2
```

3.81 $\int \frac{1}{(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=107

$$-\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{3\cos(c+dx)}{16ad(a\sin(c+dx)+a)^{3/2}}-\frac{\cos(c+dx)}{4d(a\sin(c+dx)+a)^{5/2}}$$

```
[Out] (-3*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])])/(16
*Sqrt[2]*a^(5/2)*d) - Cos[c + d*x]/(4*d*(a + a*Sin[c + d*x])^(5/2)) - (3*Co
s[c + d*x])/(16*a*d*(a + a*Sin[c + d*x])^(3/2))
```

Rubi [A] time = 0.0578148, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2650, 2649, 206}

$$-\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{3\cos(c+dx)}{16ad(a\sin(c+dx)+a)^{3/2}}-\frac{\cos(c+dx)}{4d(a\sin(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^(-5/2),x]
```

```
[Out] (-3*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])])/(16
*Sqrt[2]*a^(5/2)*d) - Cos[c + d*x]/(4*d*(a + a*Sin[c + d*x])^(5/2)) - (3*Co
s[c + d*x])/(16*a*d*(a + a*Sin[c + d*x])^(3/2))
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(a+a\sin(c+dx))^{5/2}}\,dx = -\frac{\cos(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} + \frac{3\int \frac{1}{(a+a\sin(c+dx))^{3/2}}\,dx}{8a}$$

$$= -\frac{\cos(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} - \frac{3\cos(c+dx)}{16ad(a+a\sin(c+dx))^{3/2}} + \frac{3\int \frac{1}{\sqrt{a+a\sin(c+dx)}}\,dx}{32a^2}$$

$$= -\frac{\cos(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} - \frac{3\cos(c+dx)}{16ad(a+a\sin(c+dx))^{3/2}} - \frac{3\,\mathrm{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \frac{a\cos(c+dx)}{\sqrt{a+a\sin(c+a}}\right.}{16a^2d}$$

$$= -\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{\cos(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} - \frac{3\cos(c+dx)}{16ad(a+a\sin(c+dx))^{3/2}}$$

Mathematica [C] time = 0.158948, size = 196, normalized size = 1.83

$$\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)\left(8\sin\left(\frac{1}{2}(c+dx)\right)-3\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^3+6\sin\left(\frac{1}{2}(c+dx)\right)\left(\sin\left(\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^(-5/2),x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(8*Sin[(c + d*x)/2] - 4*(Cos[(c + d*
x)/2] + Sin[(c + d*x)/2]) + 6*Sin[(c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[(c +
 d*x)/2])^2 - 3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3 + (3 + 3*I)*(-1)^(3
/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)/4])]*(Cos[(c + d*x)/
2] + Sin[(c + d*x)/2])^4))/(16*d*(a*(1 + Sin[c + d*x]))^(5/2))
```

Maple [B] time = 0.768, size = 195, normalized size = 1.8

$$-\frac{1}{(32+32\sin(dx+c))\cos(dx+c)\,d}\left(\sin(dx+c)\left(6\sqrt{a-a\sin(dx+c)}a^{3/2}+6\sqrt{2}\mathrm{Artanh}\left(1/2\frac{\sqrt{a-a\sin(dx+c)}\sqrt{2}}{\sqrt{a}}\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] -1/32/a^(9/2)*(sin(d*x+c)*(6*(a-a*sin(d*x+c))^(1/2)*a^(3/2)+6*2^(1/2)*arcta
nh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a^2)-3*2^(1/2)*arctanh(1/2*(
a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a^2*cos(d*x+c)^2+14*(a-a*sin(d*x+c))
^(1/2)*a^(3/2)+6*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2)
)*a^2)*(-a*(sin(d*x+c)-1))^(1/2)/(1+sin(d*x+c))/cos(d*x+c)/(a+a*sin(d*x+c))
^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\sin(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(-5/2), x)
```

Fricas [B] time = 1.72917, size = 851, normalized size = 7.95

$$\frac{3\sqrt{2}\left(\cos(dx+c)^3 + 3\cos(dx+c)^2 + \left(\cos(dx+c)^2 - 2\cos(dx+c) - 4\right)\sin(dx+c) - 2\cos(dx+c) - 4\right)\sqrt{a}\log\left(-\right.}{64\left(a^3d\cos(dx+c)^3 + 3a^3d\cos(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/64*(3*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + (cos(d*x + c)^2 - 2*co
s(d*x + c) - 4)*sin(d*x + c) - 2*cos(d*x + c) - 4)*sqrt(a)*log(-(a*cos(d*x
+ c)^2 - 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - sin(d*x
 + c) + 1) + 3*a*cos(d*x + c) - (a*cos(d*x + c) - 2*a)*sin(d*x + c) + 2*a)/
(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x + c) - 2)) + 4*
(3*cos(d*x + c)^2 + (3*cos(d*x + c) - 4)*sin(d*x + c) + 7*cos(d*x + c) + 4)
*sqrt(a*sin(d*x + c) + a))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 -
 2*a^3*d*cos(d*x + c) - 4*a^3*d + (a^3*d*cos(d*x + c)^2 - 2*a^3*d*cos(d*x +
 c) - 4*a^3*d)*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\sin(c+dx)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Integral((a*sin(c + d*x) + a)**(-5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] sage2
```

3.82 $\int \frac{\csc(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=144

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{a^{5/2}d}+\frac{43\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{11\cos(c+dx)}{16ad(a\sin(c+dx)+a)^{3/2}}+\frac{\cos(c+dx)}{4d(a\sin(c+dx)+a)^{5/2}}$$

```
[Out] (-2*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(a^(5/2)*d) +
 (43*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])])/(1
6*Sqrt[2]*a^(5/2)*d) + Cos[c + d*x]/(4*d*(a + a*Sin[c + d*x])^(5/2)) + (11*
Cos[c + d*x])/(16*a*d*(a + a*Sin[c + d*x])^(3/2))
```

Rubi [A] time = 0.325342, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2766, 2978, 2985, 2649, 206, 2773}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{a^{5/2}d}+\frac{43\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{11\cos(c+dx)}{16ad(a\sin(c+dx)+a)^{3/2}}+\frac{\cos(c+dx)}{4d(a\sin(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(a^(5/2)*d) +
 (43*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])])/(1
6*Sqrt[2]*a^(5/2)*d) + Cos[c + d*x]/(4*d*(a + a*Sin[c + d*x])^(5/2)) + (11*
Cos[c + d*x])/(16*a*d*(a + a*Sin[c + d*x])^(3/2))
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
```

```
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx &= \frac{\cos(c+dx)}{4d(a+a\sin(c+dx))^{5/2}}+\frac{\int \frac{\csc(c+dx)\left(4a-\frac{3}{2}a\sin(c+dx)\right)}{(a+a\sin(c+dx))^{3/2}}\,dx}{4a^2}\\
&= \frac{\cos(c+dx)}{4d(a+a\sin(c+dx))^{5/2}}+\frac{11\cos(c+dx)}{16ad(a+a\sin(c+dx))^{3/2}}+\frac{\int \frac{\csc(c+dx)\left(8a^2-\frac{11}{4}a^2\sin(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}\,dx}{8a^4}\\
&= \frac{\cos(c+dx)}{4d(a+a\sin(c+dx))^{5/2}}+\frac{11\cos(c+dx)}{16ad(a+a\sin(c+dx))^{3/2}}+\frac{\int \csc(c+dx)\sqrt{a+a\sin(c+dx}}{a^3}\\
&= \frac{\cos(c+dx)}{4d(a+a\sin(c+dx))^{5/2}}+\frac{11\cos(c+dx)}{16ad(a+a\sin(c+dx))^{3/2}}-\frac{2\,\mathrm{Subst}\left(\int \frac{1}{a-x^2}\,dx,x,\frac{a\cos(c+d}{\sqrt{a+a\sin(c+}}\right.}{a^2d}\\
&= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{a^{5/2}d}+\frac{43\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{\cos(c+dx)}{4d(a+a\sin(c+dx))^{5/2}}+
\end{aligned}$$

Mathematica [C] time = 0.275614, size = 296, normalized size = 2.06

$$\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)\left(-8\sin\left(\frac{1}{2}(c+dx)\right)+11\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^3-22\sin\left(\frac{1}{2}(c+dx)\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(-8*Sin[(c + d*x)/2] + 4*(Cos[(c + d
*x)/2] + Sin[(c + d*x)/2]) - 22*Sin[(c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[(c
 + d*x)/2])^2 + 11*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3 - (43 + 43*I)*(-
1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)/4])]*(Cos[(c +
```

```
d*x)/2] + Sin[(c + d*x)/2])^4 - 16*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)
/2]]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4 + 16*Log[1 - Cos[(c + d*x)/2]
+ Sin[(c + d*x)/2]]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4))/(16*d*(a*(1 +
 Sin[c + d*x]))^(5/2))
```

Maple [B] time = 0.696, size = 262, normalized size = 1.8

$$\frac{1}{(32+32\sin(dx+c))\cos(dx+c)d}\left(2\sin(dx+c)a^5\left(43\sqrt{2}\mathrm{Artanh}\left(1/2\frac{\sqrt{a-a\sin(dx+c)}\sqrt{2}}{\sqrt{a}}\right)-64\,\mathrm{Artanh}\left(\frac{\sqrt{a-a}}{}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)/(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] 1/32/a^(15/2)*(2*sin(d*x+c)*a^5*(43*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1
/2)*2^(1/2)/a^(1/2))-64*arctanh((a-a*sin(d*x+c))^(1/2)/a^(1/2)))-a^5*(43*2^
(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))-64*arctanh((a-a*s
in(d*x+c))^(1/2)/a^(1/2)))*cos(d*x+c)^2+52*(a-a*sin(d*x+c))^(1/2)*a^(9/2)-2
2*(a-a*sin(d*x+c))^(3/2)*a^(7/2)+86*a^5*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c)
)^(1/2)*2^(1/2)/a^(1/2))-128*arctanh((a-a*sin(d*x+c))^(1/2)/a^(1/2))*a^5)*(
-a*(sin(d*x+c)-1))^(1/2)/(1+sin(d*x+c))/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(dx+c)}{(a\sin(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(d*x + c)/(a*sin(d*x + c) + a)^(5/2), x)
```

Fricas [B] time = 1.98025, size = 1458, normalized size = 10.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/64*(43*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + (cos(d*x + c)^2 - 2*c
os(d*x + c) - 4)*sin(d*x + c) - 2*cos(d*x + c) - 4)*sqrt(a)*log(-(a*cos(d*x
 + c)^2 + 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - sin(d*
x + c) + 1) + 3*a*cos(d*x + c) - (a*cos(d*x + c) - 2*a)*sin(d*x + c) + 2*a)
/(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x + c) - 2)) + 3
2*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + (cos(d*x + c)^2 - 2*cos(d*x + c) - 4
)*sin(d*x + c) - 2*cos(d*x + c) - 4)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*co
s(d*x + c)^2 - 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*cos(
d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*cos(
d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + cos(
```

```
d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) - 4*(11
*cos(d*x + c)^2 + (11*cos(d*x + c) - 4)*sin(d*x + c) + 15*cos(d*x + c) + 4)
*sqrt(a*sin(d*x + c) + a))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 -
 2*a^3*d*cos(d*x + c) - 4*a^3*d + (a^3*d*cos(d*x + c)^2 - 2*a^3*d*cos(d*x +
 c) - 4*a^3*d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+a*sin(d*x+c))**(5/2),x)

[Out] Timed out
```

Giac [B] time = 3.17235, size = 776, normalized size = 5.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")

[Out] -1/16*(43*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(
a*tan(1/2*d*x + 1/2*c)^2 + a) + sqrt(a))/sqrt(-a))/(sqrt(-a)*a^2*sgn(tan(1/
2*d*x + 1/2*c) + 1)) - 32*arctan(-(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*ta
n(1/2*d*x + 1/2*c)^2 + a))/sqrt(-a))/(sqrt(-a)*a^2*sgn(tan(1/2*d*x + 1/2*c)
 + 1)) + 16*log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*tan(1/2*d*x + 1/
2*c)^2 + a)))/(a^(5/2)*sgn(tan(1/2*d*x + 1/2*c) + 1)) + 2*(53*(sqrt(a)*tan(
1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^7 + 179*(sqrt(a)*tan
(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^6*sqrt(a) + 127*(sq
rt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^5*a - 195*
(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*a^(3/
2) + 7*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^
3*a^2 + 121*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 +
 a))^2*a^(5/2) - 67*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/
2*c)^2 + a))*a^3 + 15*a^(7/2))/(((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan
(1/2*d*x + 1/2*c)^2 + a))^2 + 2*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(
1/2*d*x + 1/2*c)^2 + a))*sqrt(a) - a)^4*a^2*sgn(tan(1/2*d*x + 1/2*c) + 1)))
/d
```

3.83 $\int \frac{\csc^2(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=174

$$-\frac{35\cot(c+dx)}{16a^2d\sqrt{a\sin(c+dx)+a}}+\frac{5\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{a^{5/2}d}-\frac{115\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{15\cot(c+dx)}{16ad(a\sin(c+dx)+a)^{3/2}}+\frac{}{4d(}$$

```
[Out] (5*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(a^(5/2)*d) -
(115*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])])/(1
6*Sqrt[2]*a^(5/2)*d) + Cot[c + d*x]/(4*d*(a + a*Sin[c + d*x])^(5/2)) + (15*
Cot[c + d*x])/(16*a*d*(a + a*Sin[c + d*x])^(3/2)) - (35*Cot[c + d*x])/(16*a
^2*d*Sqrt[a + a*Sin[c + d*x]])
```

Rubi [A] time = 0.506849, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2766, 2978, 2984, 2985, 2649, 206, 2773}

$$-\frac{35\cot(c+dx)}{16a^2d\sqrt{a\sin(c+dx)+a}}+\frac{5\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{a^{5/2}d}-\frac{115\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{15\cot(c+dx)}{16ad(a\sin(c+dx)+a)^{3/2}}+\frac{}{4d(}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (5*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(a^(5/2)*d) -
(115*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])])/(1
6*Sqrt[2]*a^(5/2)*d) + Cot[c + d*x]/(4*d*(a + a*Sin[c + d*x])^(5/2)) + (15*
Cot[c + d*x])/(16*a*d*(a + a*Sin[c + d*x])^(3/2)) - (35*Cot[c + d*x])/(16*a
^2*d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\csc^2(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx = \frac{\cot(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} + \frac{\int \frac{\csc^2(c+dx)\left(5a-\frac{5}{2}a\sin(c+dx)\right)}{(a+a\sin(c+dx))^{3/2}}dx}{4a^2}$$

$$= \frac{\cot(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} + \frac{15\cot(c+dx)}{16ad(a+a\sin(c+dx))^{3/2}} + \frac{\int \frac{\csc^2(c+dx)\left(\frac{35a^2}{2}-\frac{45}{4}a^2\sin(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}dx}{8a^4}$$

$$= \frac{\cot(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} + \frac{15\cot(c+dx)}{16ad(a+a\sin(c+dx))^{3/2}} - \frac{35\cot(c+dx)}{16a^2d\sqrt{a+a\sin(c+dx)}} + \frac{\int \frac{\csc(c+}{}}{}$$

$$= \frac{\cot(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} + \frac{15\cot(c+dx)}{16ad(a+a\sin(c+dx))^{3/2}} - \frac{35\cot(c+dx)}{16a^2d\sqrt{a+a\sin(c+dx)}} - \frac{5\int \csc}{}$$

$$= \frac{\cot(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} + \frac{15\cot(c+dx)}{16ad(a+a\sin(c+dx))^{3/2}} - \frac{35\cot(c+dx)}{16a^2d\sqrt{a+a\sin(c+dx)}} + \frac{5\,\mathrm{Subs}}{}$$

$$= \frac{5\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{a^{5/2}d} - \frac{115\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} + \frac{\cot(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} + \frac{}{16ad}$$

Mathematica [C] time = 0.588044, size = 509, normalized size = 2.93

$$\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)\left(8\sin\left(\frac{1}{2}(c+dx)\right)+\frac{8\sin\left(\frac{1}{4}(c+dx)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^4}{\cos\left(\frac{1}{4}(c+dx)\right)-\sin\left(\frac{1}{4}(c+dx)\right)} - \frac{8\sin\left(\frac{1}{4}(c+dx)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+c\right.}{\sin\left(\frac{1}{4}(c+dx)\right)+\cos\left(\frac{1}{4}(c\right.}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(8*Sin[(c + d*x)/2] - 4*(Cos[(c + d*
x)/2] + Sin[(c + d*x)/2]) + 38*Sin[(c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[(c
+ d*x)/2])^2 - 19*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3 + 8*(Cos[(c + d*x
)/2] + Sin[(c + d*x)/2])^4 + (115 + 115*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(
-1)^(3/4)*(-1 + Tan[(c + d*x)/4])]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4
- 4*Cot[(c + d*x)/4]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4 + 40*Log[1 + C
os[(c + d*x)/2] - Sin[(c + d*x)/2]]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4
 - 40*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*(Cos[(c + d*x)/2] + Sin[
(c + d*x)/2])^4 + (8*Sin[(c + d*x)/4]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])
^4)/(Cos[(c + d*x)/4] - Sin[(c + d*x)/4]) - (8*Sin[(c + d*x)/4]*(Cos[(c + d
*x)/2] + Sin[(c + d*x)/2])^4)/(Cos[(c + d*x)/4] + Sin[(c + d*x)/4]) - 4*(Co
s[(c + d*x)/2] + Sin[(c + d*x)/2])^4*Tan[(c + d*x)/4]))/(16*d*(a*(1 + Sin[c
 + d*x]))^(5/2))
```

Maple [B] time = 0.839, size = 356, normalized size = 2.1

$$-\frac{1}{(32+32\sin(dx+c))\sin(dx+c)\cos(dx+c)d}\left(115\sqrt{2}\mathrm{Artanh}\left(1/2\frac{\sqrt{-a(\sin(dx+c)-1)}\sqrt{2}}{\sqrt{a}}\right)(\sin(dx+c))^3a^2+32\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2/(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] -1/32*(115*2^(1/2)*arctanh(1/2*(-a*(sin(d*x+c)-1))^(1/2)*2^(1/2)/a^(1/2))*s
in(d*x+c)^3*a^2+32*(-a*(sin(d*x+c)-1))^(1/2)*a^(3/2)*sin(d*x+c)^2+230*2^(1/
2)*arctanh(1/2*(-a*(sin(d*x+c)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(d*x+c)^2*a^2-
160*arctanh((-a*(sin(d*x+c)-1))^(1/2)/a^(1/2))*sin(d*x+c)^3*a^2+148*(-a*(si
n(d*x+c)-1))^(1/2)*a^(3/2)*sin(d*x+c)-38*(-a*(sin(d*x+c)-1))^(3/2)*a^(1/2)*
sin(d*x+c)+115*2^(1/2)*arctanh(1/2*(-a*(sin(d*x+c)-1))^(1/2)*2^(1/2)/a^(1/2
))*sin(d*x+c)*a^2-320*arctanh((-a*(sin(d*x+c)-1))^(1/2)/a^(1/2))*sin(d*x+c)
^2*a^2+32*(-a*(sin(d*x+c)-1))^(1/2)*a^(3/2)-160*arctanh((-a*(sin(d*x+c)-1))
^(1/2)/a^(1/2))*sin(d*x+c)*a^2)*(-a*(sin(d*x+c)-1))^(1/2)/a^(9/2)/(1+sin(d*
x+c))/sin(d*x+c)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(dx+c)^2}{(a\sin(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(d*x + c)^2/(a*sin(d*x + c) + a)^(5/2), x)
```

Fricas [B] time = 2.02892, size = 1683, normalized size = 9.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/64*(115*sqrt(2)*(cos(d*x + c)^4 - 2*cos(d*x + c)^3 - 5*cos(d*x + c)^2 - (
cos(d*x + c)^3 + 3*cos(d*x + c)^2 - 2*cos(d*x + c) - 4)*sin(d*x + c) + 2*co
s(d*x + c) + 4)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*sin(d*x +
 c) + a)*sqrt(a)*(cos(d*x + c) - sin(d*x + c) + 1) + 3*a*cos(d*x + c) - (a*
cos(d*x + c) - 2*a)*sin(d*x + c) + 2*a)/(cos(d*x + c)^2 - (cos(d*x + c) + 2
)*sin(d*x + c) - cos(d*x + c) - 2)) + 80*(cos(d*x + c)^4 - 2*cos(d*x + c)^3
 - 5*cos(d*x + c)^2 - (cos(d*x + c)^3 + 3*cos(d*x + c)^2 - 2*cos(d*x + c) -
 4)*sin(d*x + c) + 2*cos(d*x + c) + 4)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*
cos(d*x + c)^2 + 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*co
s(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*co
s(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + co
s(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) + 4*(
35*cos(d*x + c)^3 - 20*cos(d*x + c)^2 - (35*cos(d*x + c)^2 + 55*cos(d*x + c
) + 4)*sin(d*x + c) - 51*cos(d*x + c) + 4)*sqrt(a*sin(d*x + c) + a))/(a^3*d
*cos(d*x + c)^4 - 2*a^3*d*cos(d*x + c)^3 - 5*a^3*d*cos(d*x + c)^2 + 2*a^3*d
*cos(d*x + c) + 4*a^3*d - (a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 -
2*a^3*d*cos(d*x + c) - 4*a^3*d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2/(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] sage2
```

3.84 $\int \frac{\csc^3(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=224

$$\frac{63\cot(c+dx)}{16a^2d\sqrt{a\sin(c+dx)+a}} - \frac{39\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{4a^{5/2}d} + \frac{219\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{31\cot(c+dx)\csc(c+dx)}{16a^2d\sqrt{a\sin(c+dx)+a}} +$$

```
[Out] (-39*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(4*a^(5/2)*d
) + (219*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])]
)/(16*Sqrt[2]*a^(5/2)*d) + (Cot[c + d*x]*Csc[c + d*x])/(4*d*(a + a*Sin[c +
d*x])^(5/2)) + (19*Cot[c + d*x]*Csc[c + d*x])/(16*a*d*(a + a*Sin[c + d*x])^
(3/2)) + (63*Cot[c + d*x])/(16*a^2*d*Sqrt[a + a*Sin[c + d*x]]) - (31*Cot[c
+ d*x]*Csc[c + d*x])/(16*a^2*d*Sqrt[a + a*Sin[c + d*x]])
```

Rubi [A] time = 0.660126, antiderivative size = 224, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2766, 2978, 2984, 2985, 2649, 206, 2773}

$$\frac{63\cot(c+dx)}{16a^2d\sqrt{a\sin(c+dx)+a}} - \frac{39\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{4a^{5/2}d} + \frac{219\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{31\cot(c+dx)\csc(c+dx)}{16a^2d\sqrt{a\sin(c+dx)+a}} +$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3/(a + a*Sin[c + d*x])^(5/2),x]
```


```
[Out] (-39*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(4*a^(5/2)*d
) + (219*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])]
)/(16*Sqrt[2]*a^(5/2)*d) + (Cot[c + d*x]*Csc[c + d*x])/(4*d*(a + a*Sin[c +
d*x])^(5/2)) + (19*Cot[c + d*x]*Csc[c + d*x])/(16*a*d*(a + a*Sin[c + d*x])^
(3/2)) + (63*Cot[c + d*x])/(16*a^2*d*Sqrt[a + a*Sin[c + d*x]]) - (31*Cot[c
+ d*x]*Csc[c + d*x])/(16*a^2*d*Sqrt[a + a*Sin[c + d*x]])
```


Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\csc^3(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx = \frac{\cot(c+dx)\csc(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} + \frac{\int \frac{\csc^3(c+dx)\left(6a-\frac{7}{2}a\sin(c+dx)\right)}{(a+a\sin(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= \frac{\cot(c+dx)\csc(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} + \frac{19\cot(c+dx)\csc(c+dx)}{16ad(a+a\sin(c+dx))^{3/2}} + \frac{\int \frac{\csc^3(c+dx)\left(31a^2-\frac{95}{4}a^2\sin(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}\,dx}{8a^4}$$

$$= \frac{\cot(c+dx)\csc(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} + \frac{19\cot(c+dx)\csc(c+dx)}{16ad(a+a\sin(c+dx))^{3/2}} - \frac{31\cot(c+dx)\csc(c+dx)}{16a^2d\sqrt{a+a\sin(c+dx)}} + \int$$

$$= \frac{\cot(c+dx)\csc(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} + \frac{19\cot(c+dx)\csc(c+dx)}{16ad(a+a\sin(c+dx))^{3/2}} + \frac{63\cot(c+dx)}{16a^2d\sqrt{a+a\sin(c+dx)}} - \frac{31\,c}{16a}$$

$$= \frac{\cot(c+dx)\csc(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} + \frac{19\cot(c+dx)\csc(c+dx)}{16ad(a+a\sin(c+dx))^{3/2}} + \frac{63\cot(c+dx)}{16a^2d\sqrt{a+a\sin(c+dx)}} - \frac{31\,c}{16a}$$

$$= \frac{\cot(c+dx)\csc(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} + \frac{19\cot(c+dx)\csc(c+dx)}{16ad(a+a\sin(c+dx))^{3/2}} + \frac{63\cot(c+dx)}{16a^2d\sqrt{a+a\sin(c+dx)}} - \frac{31\,c}{16a}$$

$$= -\frac{39\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{4a^{5/2}d} + \frac{219\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} + \frac{\cot(c+dx)\csc(c+dx)}{4d(a+a\sin(c+dx))^{5/2}}$$

Mathematica [C] time = 1.07375, size = 680, normalized size = 3.04

$$\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)\left(-16\sin\left(\frac{1}{2}(c+dx)\right) - \frac{40\sin\left(\frac{1}{4}(c+dx)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^4}{\cos\left(\frac{1}{4}(c+dx)\right)-\sin\left(\frac{1}{4}(c+dx)\right)} + \frac{40\sin\left(\frac{1}{4}(c+dx)\right)\left(\sin\left(\frac{1}{2}\right.\right.}{\sin\left(\frac{1}{4}(c+dx)\right)}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(-16*Sin[(c + d*x)/2] + 8*(Cos[(c +
d*x)/2] + Sin[(c + d*x)/2]) - 108*Sin[(c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[
(c + d*x)/2])^2 + 54*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3 - 40*(Cos[(c +
 d*x)/2] + Sin[(c + d*x)/2])^4 - (438 + 438*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/
2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)/4])]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]
)^4 + 20*Cot[(c + d*x)/4]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4 - Csc[(c
+ d*x)/4]^2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4 - 156*Log[1 + Cos[(c +
d*x)/2] - Sin[(c + d*x)/2]]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4 + 156*L
og[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*(Cos[(c + d*x)/2] + Sin[(c + d*
x)/2])^4 + Sec[(c + d*x)/4]^2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4 + (2*
(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4)/(Cos[(c + d*x)/4] - Sin[(c + d*x)/
4])^2 - (40*Sin[(c + d*x)/4]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4)/(Cos[
(c + d*x)/4] - Sin[(c + d*x)/4]) - (2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])
^4)/(Cos[(c + d*x)/4] + Sin[(c + d*x)/4])^2 + (40*Sin[(c + d*x)/4]*(Cos[(c
+ d*x)/2] + Sin[(c + d*x)/2])^4)/(Cos[(c + d*x)/4] + Sin[(c + d*x)/4]) + 20
*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4*Tan[(c + d*x)/4]))/(32*d*(a*(1 + S
in[c + d*x]))^(5/2))
```

Maple [B] time = 0.853, size = 404, normalized size = 1.8

$$-\frac{1}{(32+32\sin(dx+c))(\sin(dx+c))^{2}\cos(dx+c)\,d}\left(-219\sqrt{2}\mathrm{Artanh}\left(1/2\,\frac{\sqrt{-a(\sin(dx+c)-1)}\sqrt{2}}{\sqrt{a}}\right)(\sin(dx+c))^{4}a^{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3/(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] -1/32/a^(9/2)*(-219*2^(1/2)*arctanh(1/2*(-a*(sin(d*x+c)-1))^(1/2)*2^(1/2)/a
^(1/2))*sin(d*x+c)^4*a^2+312*arctanh((-a*(sin(d*x+c)-1))^(1/2)/a^(1/2))*sin
(d*x+c)^4*a^2-438*2^(1/2)*arctanh(1/2*(-a*(sin(d*x+c)-1))^(1/2)*2^(1/2)/a^(
1/2))*sin(d*x+c)^3*a^2+126*(-a*(sin(d*x+c)-1))^(3/2)*a^(1/2)*sin(d*x+c)^2+6
24*arctanh((-a*(sin(d*x+c)-1))^(1/2)/a^(1/2))*sin(d*x+c)^3*a^2-219*2^(1/2)*
arctanh(1/2*(-a*(sin(d*x+c)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(d*x+c)^2*a^2+144
*(-a*(sin(d*x+c)-1))^(3/2)*a^(1/2)*sin(d*x+c)-172*(-a*(sin(d*x+c)-1))^(1/2)
*a^(3/2)*sin(d*x+c)^2+312*arctanh((-a*(sin(d*x+c)-1))^(1/2)/a^(1/2))*sin(d*
x+c)^2*a^2+72*(-a*(sin(d*x+c)-1))^(3/2)*a^(1/2)-112*(-a*(sin(d*x+c)-1))^(1/
2)*a^(3/2)*sin(d*x+c)-56*(-a*(sin(d*x+c)-1))^(1/2)*a^(3/2))*(-a*(sin(d*x+c)
-1))^(1/2)/(1+sin(d*x+c))/sin(d*x+c)^2/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 2.07802, size = 1909, normalized size = 8.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/64*(219*sqrt(2)*(cos(d*x + c)^5 + 3*cos(d*x + c)^4 - 3*cos(d*x + c)^3 - 7
*cos(d*x + c)^2 + (cos(d*x + c)^4 - 2*cos(d*x + c)^3 - 5*cos(d*x + c)^2 + 2
*cos(d*x + c) + 4)*sin(d*x + c) + 2*cos(d*x + c) + 4)*sqrt(a)*log(-(a*cos(d
*x + c)^2 + 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - sin(
d*x + c) + 1) + 3*a*cos(d*x + c) - (a*cos(d*x + c) - 2*a)*sin(d*x + c) + 2*
a)/(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x + c) - 2)) +
 156*(cos(d*x + c)^5 + 3*cos(d*x + c)^4 - 3*cos(d*x + c)^3 - 7*cos(d*x + c)
^2 + (cos(d*x + c)^4 - 2*cos(d*x + c)^3 - 5*cos(d*x + c)^2 + 2*cos(d*x + c)
 + 4)*sin(d*x + c) + 2*cos(d*x + c) + 4)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*
a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*
cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*
cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 +
cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) - 4
*(63*cos(d*x + c)^4 + 95*cos(d*x + c)^3 - 51*cos(d*x + c)^2 + (63*cos(d*x +
 c)^3 - 32*cos(d*x + c)^2 - 83*cos(d*x + c) + 4)*sin(d*x + c) - 87*cos(d*x
```

```
+ c) - 4)*sqrt(a*sin(d*x + c) + a))/(a^3*d*cos(d*x + c)^5 + 3*a^3*d*cos(d*x
 + c)^4 - 3*a^3*d*cos(d*x + c)^3 - 7*a^3*d*cos(d*x + c)^2 + 2*a^3*d*cos(d*x
 + c) + 4*a^3*d + (a^3*d*cos(d*x + c)^4 - 2*a^3*d*cos(d*x + c)^3 - 5*a^3*d*
cos(d*x + c)^2 + 2*a^3*d*cos(d*x + c) + 4*a^3*d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3/(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] sage2
```

3.85 $\int \frac{\sqrt{a+a\sin(e+fx)}}{\sqrt{\sin(e+fx)}}\,dx$

Optimal. Leaf size=37

$$-\frac{2\sqrt{a}\sin^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}}\right)}{f}$$

```
[Out] (-2*Sqrt[a]*ArcSin[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + a*Sin[e + f*x]]])/f
```

Rubi [A] time = 0.0555735, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {2774, 216}

$$-\frac{2\sqrt{a}\sin^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]/Sqrt[Sin[e + f*x]],x]
```


```
[Out] (-2*Sqrt[a]*ArcSin[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + a*Sin[e + f*x]]])/f
```


Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+a\sin(e+fx)}}{\sqrt{\sin(e+fx)}}\,dx &= -\frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a}}}\,dx, x, \frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{f}\\ &= -\frac{2\sqrt{a}\sin^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{f}\end{aligned}$$

Mathematica [C] time = 0.51352, size = 164, normalized size = 4.43

$$\frac{(1+i)e^{\frac{1}{2}i(e+fx)}\sqrt{-ie^{-i(e+fx)}\left(-1+e^{2i(e+fx)}\right)}\sqrt{a(\sin(e+fx)+1)}\left(\tan^{-1}\left(\sqrt{-1+e^{2i(e+fx)}}\right)-i\tanh^{-1}\left(\frac{e^{i(e+fx)}}{\sqrt{-1+e^{2i(e+fx)}}}\right)\right)}{\sqrt{2}f\sqrt{-1+e^{2i(e+fx)}}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]/Sqrt[Sin[e + f*x]],x]
```

```
[Out] ((1 + I)*E^((I/2)*(e + f*x))*Sqrt[((-I)*(-1 + E^((2*I)*(e + f*x))))/E^(I*(e
 + f*x))]*(ArcTan[Sqrt[-1 + E^((2*I)*(e + f*x))]] - I*ArcTanh[E^(I*(e + f*x
))/Sqrt[-1 + E^((2*I)*(e + f*x))]])*Sqrt[a*(1 + Sin[e + f*x]]])/(Sqrt[2]*Sq
rt[-1 + E^((2*I)*(e + f*x))]*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))
```

Maple [B] time = 0.141, size = 320, normalized size = 8.7

$$\frac{\sqrt{2}}{2f\left(1-\cos\left(fx+e\right)+\sin\left(fx+e\right)\right)}\sqrt{-\frac{-1+\cos\left(fx+e\right)}{\sin\left(fx+e\right)}}\sqrt{a\left(1+\sin\left(fx+e\right)\right)}\sqrt{\sin\left(fx+e\right)}\left(\ln\left(-\left(\sqrt{2}\sqrt{-\frac{-1+\mathrm{co}}{\sin(}}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(1/2)/sin(f*x+e)^(1/2),x)
```

```
[Out] 1/2/f*(-(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*(a*(1+sin(f*x+e)))^(1/2)*sin(f*x+
e)^(1/2)*(ln(-(2^(1/2)*(-(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*sin(f*x+e)+sin(f
*x+e)-cos(f*x+e)+1)/(2^(1/2)*(-(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*sin(f*x+e)
-sin(f*x+e)+cos(f*x+e)-1))+4*arctan(2^(1/2)*(-(-1+cos(f*x+e))/sin(f*x+e))^(
1/2)+1)+4*arctan(2^(1/2)*(-(-1+cos(f*x+e))/sin(f*x+e))^(1/2)-1)+ln(-(2^(1/2
)*(-(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*sin(f*x+e)-sin(f*x+e)+cos(f*x+e)-1)/(
2^(1/2)*(-(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*sin(f*x+e)+sin(f*x+e)-cos(f*x+e
)+1)))*2^(1/2)/(1-cos(f*x+e)+sin(f*x+e))
```

Maxima [B] time = 2.09046, size = 284, normalized size = 7.68

$$-\frac{2\sqrt{2}\sqrt{a}\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)^{\frac{3}{2}}-3\sqrt{2}\left(\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\frac{\sin(fx+e)}{\cos(fx+e)+1}}\right)\right)+\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\frac{\sin(fx+e)}{\cos(fx+e)+1}}\right.\right.\right.}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/sin(f*x+e)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*(2*sqrt(2)*sqrt(a)*(sin(f*x + e)/(cos(f*x + e) + 1))^(3/2) - 3*sqrt(2)
*(sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(sin(f*x + e)/(cos(f*x + e) +
 1)))) + sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(sin(f*x + e)/(cos(f*
x + e) + 1)))))*sqrt(a) + 6*sqrt(2)*sqrt(a)*sqrt(sin(f*x + e)/(cos(f*x + e)
 + 1)) - 2*(3*sqrt(2)*sqrt(a)*sin(f*x + e)/(cos(f*x + e) + 1) + sqrt(2)*sqr
t(a)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2)/sqrt(sin(f*x + e)/(cos(f*x + e) +
 1)))/f
```

Fricas [B] time = 2.57388, size = 934, normalized size = 25.24

$$\left[\sqrt{-a}\log\left(\frac{128\,a\cos(fx+e)^{5}-128\,a\cos(fx+e)^{4}-416\,a\cos(fx+e)^{3}+128\,a\cos(fx+e)^{2}-8\left(16\cos(fx+e)^{4}-24\cos(fx+e)^{3}-66\cos(fx+e)^{2}+\left(16\cos(fx+e\right.\right.}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/sin(f*x+e)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(-a)*log((128*a*cos(f*x + e)^5 - 128*a*cos(f*x + e)^4 - 416*a*cos(
f*x + e)^3 + 128*a*cos(f*x + e)^2 - 8*(16*cos(f*x + e)^4 - 24*cos(f*x + e)^
3 - 66*cos(f*x + e)^2 + (16*cos(f*x + e)^3 + 40*cos(f*x + e)^2 - 26*cos(f*x
 + e) - 51)*sin(f*x + e) + 25*cos(f*x + e) + 51)*sqrt(a*sin(f*x + e) + a)*s
qrt(-a)*sqrt(sin(f*x + e)) + 289*a*cos(f*x + e) + (128*a*cos(f*x + e)^4 + 2
56*a*cos(f*x + e)^3 - 160*a*cos(f*x + e)^2 - 288*a*cos(f*x + e) + a)*sin(f*
x + e) + a)/(cos(f*x + e) + sin(f*x + e) + 1))/f, 1/2*sqrt(a)*arctan(1/4*(8
*cos(f*x + e)^2 + 8*sin(f*x + e) - 9)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*sqrt
(sin(f*x + e))/(2*a*cos(f*x + e)^3 + a*cos(f*x + e)*sin(f*x + e) - 2*a*cos(
f*x + e)))/f]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}}{\sqrt{\sin\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(1/2)/sin(f*x+e)**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))/sqrt(sin(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a\sin\left(fx+e\right)+a}}{\sqrt{\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/sin(f*x+e)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)/sqrt(sin(f*x + e)), x)
```

3.86 $\int \frac{\sqrt{a-a\sin(e+fx)}}{\sqrt{-\sin(e+fx)}}\,dx$

Optimal. Leaf size=38

$$\frac{2\sqrt{a}\sin^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a-a\sin(e+fx)}}\right)}{f}$$

```
[Out] (2*Sqrt[a]*ArcSin[(Sqrt[a]*Cos[e + f*x])/Sqrt[a - a*Sin[e + f*x]]])/f
```

Rubi [A] time = 0.0701887, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2774, 216}

$$\frac{2\sqrt{a}\sin^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a-a\sin(e+fx)}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - a*Sin[e + f*x]]/Sqrt[-Sin[e + f*x]],x]
```

```
[Out] (2*Sqrt[a]*ArcSin[(Sqrt[a]*Cos[e + f*x])/Sqrt[a - a*Sin[e + f*x]]])/f
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a-a\sin(e+fx)}}{\sqrt{-\sin(e+fx)}}\,dx &= -\frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a}}}\,dx, x, -\frac{a\cos(e+fx)}{\sqrt{a-a\sin(e+fx)}}\right)}{f}\\ &= \frac{2\sqrt{a}\sin^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a-a\sin(e+fx)}}\right)}{f}\end{aligned}$$

Mathematica [C] time = 0.468421, size = 119, normalized size = 3.13

$$-\frac{\sqrt{-1+e^{2i(e+fx)}}\sqrt{a-a\sin(e+fx)}\left(\tan^{-1}\left(\sqrt{-1+e^{2i(e+fx)}}\right)+i\tanh^{-1}\left(\frac{e^{i(e+fx)}}{\sqrt{-1+e^{2i(e+fx)}}}\right)\right)}{f\left(e^{i(e+fx)}-i\right)\sqrt{-\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a - a*Sin[e + f*x]]/Sqrt[-Sin[e + f*x]],x]
```

```
[Out] -((Sqrt[-1 + E^((2*I)*(e + f*x))]*(ArcTan[Sqrt[-1 + E^((2*I)*(e + f*x))]] +
 I*ArcTanh[E^(I*(e + f*x))/Sqrt[-1 + E^((2*I)*(e + f*x))]])*Sqrt[a - a*Sin[
e + f*x]])/((-I + E^(I*(e + f*x)))*f*Sqrt[-Sin[e + f*x]]))
```

Maple [B] time = 0.112, size = 271, normalized size = 7.1

$$\frac{\sqrt{2}\sin\left(fx+e\right)}{2f\left(-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)\right)}\sqrt{-\frac{-1+\cos\left(fx+e\right)}{\sin\left(fx+e\right)}}\sqrt{-a\left(-1+\sin\left(fx+e\right)\right)}\left(\ln\left(-\left(\sqrt{2}\sqrt{-\frac{-1+\cos\left(fx+e\right)}{\sin\left(fx+e\right)}}\,\mathrm{si}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*sin(f*x+e))^(1/2)/(-sin(f*x+e))^(1/2),x)
```

```
[Out] 1/2/f*2^(1/2)*(-(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*(-a*(-1+sin(f*x+e)))^(1/2
)*sin(f*x+e)*(ln(-(2^(1/2)*(-(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*sin(f*x+e)+s
in(f*x+e)-cos(f*x+e)+1)/(2^(1/2)*(-(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*sin(f*
x+e)-sin(f*x+e)+cos(f*x+e)-1))-ln(-(2^(1/2)*(-(-1+cos(f*x+e))/sin(f*x+e))^(
1/2)*sin(f*x+e)-sin(f*x+e)+cos(f*x+e)-1)/(2^(1/2)*(-(-1+cos(f*x+e))/sin(f*x
+e))^(1/2)*sin(f*x+e)+sin(f*x+e)-cos(f*x+e)+1)))/(-sin(f*x+e))^(1/2)/(-1+co
s(f*x+e)+sin(f*x+e))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a\sin\left(fx+e\right)+a}}{\sqrt{-\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))^(1/2)/(-sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a*sin(f*x + e) + a)/sqrt(-sin(f*x + e)), x)
```

Fricas [B] time = 2.63137, size = 941, normalized size = 24.76

$$\left[\frac{\sqrt{-a}\log\left(\frac{128a\cos\left(fx+e\right)^{5}-128a\cos\left(fx+e\right)^{4}-416a\cos\left(fx+e\right)^{3}+128a\cos\left(fx+e\right)^{2}+8\left(16\cos\left(fx+e\right)^{4}-24\cos\left(fx+e\right)^{3}-66\cos\left(fx+e\right)^{2}-\left(16\cos\left(fx+e\right)^{3}+\cdots\right.\right.}{\cdots}\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))^(1/2)/(-sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(-a)*log((128*a*cos(f*x + e)^5 - 128*a*cos(f*x + e)^4 - 416*a*cos(
f*x + e)^3 + 128*a*cos(f*x + e)^2 + 8*(16*cos(f*x + e)^4 - 24*cos(f*x + e)^
```

```
3 - 66*cos(f*x + e)^2 - (16*cos(f*x + e)^3 + 40*cos(f*x + e)^2 - 26*cos(f*x
 + e) - 51)*sin(f*x + e) + 25*cos(f*x + e) + 51)*sqrt(-a*sin(f*x + e) + a)*
sqrt(-a)*sqrt(-sin(f*x + e)) + 289*a*cos(f*x + e) - (128*a*cos(f*x + e)^4 +
 256*a*cos(f*x + e)^3 - 160*a*cos(f*x + e)^2 - 288*a*cos(f*x + e) + a)*sin(
f*x + e) + a)/(cos(f*x + e) - sin(f*x + e) + 1))/f, -1/2*sqrt(a)*arctan(1/4
*(8*cos(f*x + e)^2 - 8*sin(f*x + e) - 9)*sqrt(-a*sin(f*x + e) + a)*sqrt(a)*
sqrt(-sin(f*x + e))/(2*a*cos(f*x + e)^3 - a*cos(f*x + e)*sin(f*x + e) - 2*a
*cos(f*x + e)))/f]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a\left(\sin\left(e+fx\right)-1\right)}}{\sqrt{-\sin\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))**(1/2)/(-sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(-a*(sin(e + f*x) - 1))/sqrt(-sin(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a\sin\left(fx+e\right)+a}}{\sqrt{-\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))^(1/2)/(-sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a*sin(f*x + e) + a)/sqrt(-sin(f*x + e)), x)
```

3.87 $\int \frac{1}{\sqrt{\sin(x)}\sqrt{1+\sin(x)}}\,dx$

Optimal. Leaf size=17

$$-\sqrt{2}\sin^{-1}\left(\frac{\cos(x)}{\sin(x)+1}\right)$$

```
[Out] -(Sqrt[2]*ArcSin[Cos[x]/(1 + Sin[x])])
```

Rubi [A] time = 0.0390379, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2781, 216}

$$-\sqrt{2}\sin^{-1}\left(\frac{\cos(x)}{\sin(x)+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Sin[x]]*Sqrt[1 + Sin[x]]),x]

[Out] -(Sqrt[2]*ArcSin[Cos[x]/(1 + Sin[x])])
```

Rule 2781

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> -Dist[Sqrt[2]/(Sqrt[a]*f), Subst[Int[1/Sqrt[1 - x
^2], x], x, (b*Cos[e + f*x])/(a + b*Sin[e + f*x])], x] /; FreeQ[{a, b, d, e
, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d, a/b] && GtQ[a, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{\sin(x)}\sqrt{1+\sin(x)}}\,dx &= -\left(\sqrt{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, \frac{\cos(x)}{1+\sin(x)}\right)\right)\\ &= -\sqrt{2}\sin^{-1}\left(\frac{\cos(x)}{1+\sin(x)}\right)\end{aligned}$$

Mathematica [C] time = 2.43993, size = 123, normalized size = 7.24

$$\frac{2\sqrt{\sin(x)}\sec^2\left(\frac{x}{4}\right)\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)\left(F\left(\sin^{-1}\left(\frac{1}{\sqrt{\tan\left(\frac{x}{4}\right)}}\right)\middle|-1\right)+\Pi\left(1-\sqrt{2};-\sin^{-1}\left(\frac{1}{\sqrt{\tan\left(\frac{x}{4}\right)}}\right)\middle|-1\right)+\Pi\left(1+\sqrt{2};-\sin\right.\right.}{\sqrt{\sin(x)+1}\tan^{\frac{3}{2}}\left(\frac{x}{4}\right)\sqrt{1-\cot^2\left(\frac{x}{4}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Sin[x]]*Sqrt[1 + Sin[x]]),x]
```

```
[Out] (2*(EllipticF[ArcSin[1/Sqrt[Tan[x/4]]], -1] + EllipticPi[1 - Sqrt[2], -ArcS
in[1/Sqrt[Tan[x/4]]], -1] + EllipticPi[1 + Sqrt[2], -ArcSin[1/Sqrt[Tan[x/4]
]], -1])*Sec[x/4]^2*(Cos[x/2] + Sin[x/2])*Sqrt[Sin[x]])/(Sqrt[1 - Cot[x/4]^
2]*Sqrt[1 + Sin[x]]*Tan[x/4]^(3/2))
```

Maple [B] time = 0.086, size = 52, normalized size = 3.1

$$-2\,\frac{(1-\cos(x)+\sin(x))\sqrt{\sin(x)}}{\sqrt{1+\sin(x)}\,(-1+\cos(x))}\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\arctan\left(\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sin(x)^(1/2)/(1+sin(x))^(1/2),x)
```

```
[Out] -2*(-(-1+cos(x))/sin(x))^(1/2)*(1-cos(x)+sin(x))*sin(x)^(1/2)*arctan((-(-1+
cos(x))/sin(x))^(1/2))/(1+sin(x))^(1/2)/(-1+cos(x))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\sin(x)+1}\sqrt{\sin(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(x)^(1/2)/(1+sin(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(sin(x) + 1)*sqrt(sin(x))), x)
```

Fricas [A] time = 1.68873, size = 107, normalized size = 6.29

$$2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\sin(x)+1}\sqrt{\sin(x)}}{\cos(x)+\sin(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(x)^(1/2)/(1+sin(x))^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(2)*arctan(sqrt(2)*sqrt(sin(x) + 1)*sqrt(sin(x))/(cos(x) + sin(x) + 1
))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\sin(x)+1}\sqrt{\sin(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(x)**(1/2)/(1+sin(x))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(sin(x) + 1)*sqrt(sin(x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\sin(x)+1}\sqrt{\sin(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(x)^(1/2)/(1+sin(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(sin(x) + 1)*sqrt(sin(x))), x)
```

3.88 $\int \frac{1}{\sqrt{\sin(x)}\sqrt{a+a\sin(x)}}\,dx$

Optimal. Leaf size=42

$$-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\cos(x)}{\sqrt{2}\sqrt{\sin(x)}\sqrt{a\sin(x)+a}}\right)}{\sqrt{a}}$$

```
[Out] -((Sqrt[2]*ArcTan[(Sqrt[a]*Cos[x])/(Sqrt[2]*Sqrt[Sin[x]]*Sqrt[a + a*Sin[x]]
)])/Sqrt[a])
```

Rubi [A] time = 0.0574363, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2782, 205}

$$-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\cos(x)}{\sqrt{2}\sqrt{\sin(x)}\sqrt{a\sin(x)+a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Sin[x]]*Sqrt[a + a*Sin[x]]),x]
```

```
[Out] -((Sqrt[2]*ArcTan[(Sqrt[a]*Cos[x])/(Sqrt[2]*Sqrt[Sin[x]]*Sqrt[a + a*Sin[x]]
)])/Sqrt[a])
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{\sin(x)}\sqrt{a+a\sin(x)}}\,dx &= -\left((2a)\,\text{Subst}\left(\int \frac{1}{2a^2+ax^2}\,dx, x, \frac{a\cos(x)}{\sqrt{\sin(x)}\sqrt{a+a\sin(x)}}\right)\right)\\ &= -\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\cos(x)}{\sqrt{2}\sqrt{\sin(x)}\sqrt{a+a\sin(x)}}\right)}{\sqrt{a}}\end{aligned}$$

Mathematica [C] time = 0.0899767, size = 125, normalized size = 2.98

$$\frac{2\sqrt{\sin(x)}\sec^2\left(\frac{x}{4}\right)\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)\left(F\left(\sin^{-1}\left(\frac{1}{\sqrt{\tan\left(\frac{x}{4}\right)}}\right)\middle|-1\right)+\Pi\left(1-\sqrt{2};-\sin^{-1}\left(\frac{1}{\sqrt{\tan\left(\frac{x}{4}\right)}}\right)\middle|-1\right)+\Pi\left(1+\sqrt{2};-\right.\right.}{\tan^{\frac{3}{2}}\left(\frac{x}{4}\right)\sqrt{1-\cot^2\left(\frac{x}{4}\right)}\sqrt{a(\sin(x)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Sin[x]]*Sqrt[a + a*Sin[x]]),x]
```

```
[Out] (2*(EllipticF[ArcSin[1/Sqrt[Tan[x/4]]], -1] + EllipticPi[1 - Sqrt[2], -ArcS
in[1/Sqrt[Tan[x/4]]], -1] + EllipticPi[1 + Sqrt[2], -ArcSin[1/Sqrt[Tan[x/4]
]], -1])*Sec[x/4]^2*(Cos[x/2] + Sin[x/2])*Sqrt[Sin[x]])/(Sqrt[1 - Cot[x/4]^
2]*Sqrt[a*(1 + Sin[x])]*Tan[x/4]^(3/2))
```

Maple [A] time = 0.089, size = 54, normalized size = 1.3

$$-2\frac{(1-\cos(x)+\sin(x))\sqrt{\sin(x)}}{\sqrt{a(1+\sin(x))}(-1+\cos(x))}\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\arctan\left(\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sin(x)^(1/2)/(a+a*sin(x))^(1/2),x)
```

```
[Out] -2*(-(-1+cos(x))/sin(x))^(1/2)*(1-cos(x)+sin(x))*sin(x)^(1/2)*arctan((-(-1+
cos(x))/sin(x))^(1/2))/(a*(1+sin(x)))^(1/2)/(-1+cos(x))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{a\sin(x)+a}\sqrt{\sin(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(x)^(1/2)/(a+a*sin(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a*sin(x) + a)*sqrt(sin(x))), x)
```

Fricas [A] time = 2.00485, size = 539, normalized size = 12.83

$$\left[\frac{1}{4}\sqrt{2}\sqrt{-\frac{1}{a}}\log\left(\frac{17\cos(x)^3-4\sqrt{2}\left(3\cos(x)^2+(3\cos(x)+4)\sin(x)-\cos(x)-4\right)\sqrt{a\sin(x)+a}\sqrt{-\frac{1}{a}}\sqrt{\sin(x)}+3\cos}{\cos(x)^3+3\cos(x)^2+\left(\cos(x)^2-2\cos(x)-4\right)\sin(x)-}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(x)^(1/2)/(a+a*sin(x))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(2)*sqrt(-1/a)*log((17*cos(x)^3 - 4*sqrt(2)*(3*cos(x)^2 + (3*cos(x
) + 4)*sin(x) - cos(x) - 4)*sqrt(a*sin(x) + a)*sqrt(-1/a)*sqrt(sin(x)) + 3*
cos(x)^2 + (17*cos(x)^2 + 14*cos(x) - 4)*sin(x) - 18*cos(x) - 4)/(cos(x)^3
+ 3*cos(x)^2 + (cos(x)^2 - 2*cos(x) - 4)*sin(x) - 2*cos(x) - 4)), 1/2*sqrt(
2)*arctan(1/4*sqrt(2)*sqrt(a*sin(x) + a)*(3*sin(x) - 1)/(sqrt(a)*cos(x)*sqr
t(sin(x))))/sqrt(a)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\left(\sin\left(x\right)+1\right)}\sqrt{\sin\left(x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(x)**(1/2)/(a+a*sin(x))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a*(sin(x) + 1))*sqrt(sin(x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\sin\left(x\right)+a}\sqrt{\sin\left(x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(x)^(1/2)/(a+a*sin(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a*sin(x) + a)*sqrt(sin(x))), x)
```

3.89 $\int \frac{1}{\sqrt{1-\sin(x)}\sqrt{\sin(x)}}\,dx$

Optimal. Leaf size=31

$$\sqrt{2}\tanh^{-1}\left(\frac{\cos(x)}{\sqrt{2}\sqrt{1-\sin(x)}\sqrt{\sin(x)}}\right)$$

```
[Out] Sqrt[2]*ArcTanh[Cos[x]/(Sqrt[2]*Sqrt[1 - Sin[x]]*Sqrt[Sin[x]])]
```

Rubi [A] time = 0.0459595, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2782, 206}

$$\sqrt{2}\tanh^{-1}\left(\frac{\cos(x)}{\sqrt{2}\sqrt{1-\sin(x)}\sqrt{\sin(x)}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 - Sin[x]]*Sqrt[Sin[x]]),x]
```

```
[Out] Sqrt[2]*ArcTanh[Cos[x]/(Sqrt[2]*Sqrt[1 - Sin[x]]*Sqrt[Sin[x]])]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{1-\sin(x)}\sqrt{\sin(x)}}\,dx &= -\left(2\,\mathrm{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, -\frac{\cos(x)}{\sqrt{1-\sin(x)}\sqrt{\sin(x)}}\right)\right)\\ &= \sqrt{2}\tanh^{-1}\left(\frac{\cos(x)}{\sqrt{2}\sqrt{1-\sin(x)}\sqrt{\sin(x)}}\right)\end{aligned}$$

Mathematica [C] time = 2.402, size = 125, normalized size = 4.03

$$\frac{2\sin(x)\sec^2\left(\frac{x}{4}\right)\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)\left(F\left(\sin^{-1}\left(\frac{1}{\sqrt{\tan\left(\frac{x}{4}\right)}}\right)\middle|-1\right)+\Pi\left(-1-\sqrt{2};-\sin^{-1}\left(\frac{1}{\sqrt{\tan\left(\frac{x}{4}\right)}}\right)\middle|-1\right)+\Pi\left(-1+\sqrt{2};-\mathrm{si}\right.\right.}{\sqrt{-(\sin(x)-1)\sin(x)}\tan^{\frac{3}{2}}\left(\frac{x}{4}\right)\sqrt{1-\cot^2\left(\frac{x}{4}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 - Sin[x]]*Sqrt[Sin[x]]),x]
```

```
[Out] (2*(EllipticF[ArcSin[1/Sqrt[Tan[x/4]]], -1] + EllipticPi[-1 - Sqrt[2], -Arc
Sin[1/Sqrt[Tan[x/4]]], -1] + EllipticPi[-1 + Sqrt[2], -ArcSin[1/Sqrt[Tan[x/
4]]], -1])*Sec[x/4]^2*(Cos[x/2] - Sin[x/2])*Sin[x])/(Sqrt[1 - Cot[x/4]^2]*S
qrt[-((-1 + Sin[x])*Sin[x])]*Tan[x/4]^(3/2))
```

Maple [B] time = 0.098, size = 52, normalized size = 1.7

$$-2\frac{(-1+\cos(x)+\sin(x))\sqrt{\sin(x)}}{\sqrt{1-\sin(x)}(-1+\cos(x))}\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\mathrm{Artanh}\left(\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-sin(x))^(1/2)/sin(x)^(1/2),x)
```

```
[Out] -2*(-(-1+cos(x))/sin(x))^(1/2)*(-1+cos(x)+sin(x))*sin(x)^(1/2)*arctanh((-(-
1+cos(x))/sin(x))^(1/2))/(1-sin(x))^(1/2)/(-1+cos(x))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{-\sin(x)+1}\sqrt{\sin(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sin(x))^(1/2)/sin(x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-sin(x) + 1)*sqrt(sin(x))), x)
```

Fricas [A] time = 1.67378, size = 104, normalized size = 3.35

$$\sqrt{2}\log\left(\frac{\sqrt{2}\sqrt{-\sin(x)+1}\sqrt{\sin(x)}+\cos(x)}{\sin(x)-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sin(x))^(1/2)/sin(x)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(2)*log((sqrt(2)*sqrt(-sin(x) + 1)*sqrt(sin(x)) + cos(x))/(sin(x) - 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{1-\sin(x)}\sqrt{\sin(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sin(x))**(1/2)/sin(x)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(1 - sin(x))*sqrt(sin(x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-\sin(x)+1}\sqrt{\sin(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sin(x))^(1/2)/sin(x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-sin(x) + 1)*sqrt(sin(x))), x)
```

3.90 $\int \frac{1}{\sqrt{\sin(x)}\sqrt{a-a\sin(x)}}\,dx$

Optimal. Leaf size=42

$$\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(x)}{\sqrt{2}\sqrt{\sin(x)}\sqrt{a-a\sin(x)}}\right)}{\sqrt{a}}$$

```
[Out] (Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[x])/(Sqrt[2]*Sqrt[Sin[x]]*Sqrt[a - a*Sin[x]])
])/Sqrt[a]
```

Rubi [A] time = 0.064463, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2782, 208}

$$\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(x)}{\sqrt{2}\sqrt{\sin(x)}\sqrt{a-a\sin(x)}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Sin[x]]*Sqrt[a - a*Sin[x]]),x]
```

```
[Out] (Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[x])/(Sqrt[2]*Sqrt[Sin[x]]*Sqrt[a - a*Sin[x]])
])/Sqrt[a]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{\sin(x)}\sqrt{a-a\sin(x)}}\,dx &= -\left((2a)\operatorname{Subst}\left(\int \frac{1}{2a^2-ax^2}\,dx, x, -\frac{a\cos(x)}{\sqrt{\sin(x)}\sqrt{a-a\sin(x)}}\right)\right)\\ &= \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(x)}{\sqrt{2}\sqrt{\sin(x)}\sqrt{a-a\sin(x)}}\right)}{\sqrt{a}}\end{aligned}$$

Mathematica [C] time = 0.10248, size = 128, normalized size = 3.05

$$\frac{2\sqrt{\sin(x)}\sec^2\left(\frac{x}{4}\right)\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)\left(F\left(\sin^{-1}\left(\frac{1}{\sqrt{\tan\left(\frac{x}{4}\right)}}\right)\middle|-1\right)+\Pi\left(-1-\sqrt{2};-\sin^{-1}\left(\frac{1}{\sqrt{\tan\left(\frac{x}{4}\right)}}\right)\middle|-1\right)+\Pi\left(-1+\sqrt{2};\right.\right.}{\tan^{\frac{3}{2}}\left(\frac{x}{4}\right)\sqrt{1-\cot^2\left(\frac{x}{4}\right)}\sqrt{a-a\sin(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Sin[x]]*Sqrt[a - a*Sin[x]]),x]
```

```
[Out] (2*(EllipticF[ArcSin[1/Sqrt[Tan[x/4]]], -1] + EllipticPi[-1 - Sqrt[2], -Arc
Sin[1/Sqrt[Tan[x/4]]], -1] + EllipticPi[-1 + Sqrt[2], -ArcSin[1/Sqrt[Tan[x/
4]]], -1])*Sec[x/4]^2*(Cos[x/2] - Sin[x/2])*Sqrt[Sin[x]])/(Sqrt[1 - Cot[x/4
]^2]*Sqrt[a - a*Sin[x]]*Tan[x/4]^(3/2))
```

Maple [A] time = 0.093, size = 53, normalized size = 1.3

$$-2\,\frac{(-1+\cos(x)+\sin(x))\sqrt{\sin(x)}}{\sqrt{-a(-1+\sin(x))}\,(-1+\cos(x))}\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\operatorname{Artanh}\left(\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sin(x)^(1/2)/(a-a*sin(x))^(1/2),x)
```

```
[Out] -2*(-(-1+cos(x))/sin(x))^(1/2)*(-1+cos(x)+sin(x))*sin(x)^(1/2)*arctanh((-(-
1+cos(x))/sin(x))^(1/2))/(-a*(-1+sin(x)))^(1/2)/(-1+cos(x))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-a\sin(x)+a}\sqrt{\sin(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(x)^(1/2)/(a-a*sin(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-a*sin(x) + a)*sqrt(sin(x))), x)
```

Fricas [A] time = 2.09568, size = 543, normalized size = 12.93

$$\left[\frac{\sqrt{2}\log\left(\frac{17\cos(x)^3+3\cos(x)^2+\frac{4\sqrt{2}\left(3\cos(x)^2-(3\cos(x)+4)\sin(x)-\cos(x)-4\right)\sqrt{-a\sin(x)+a}\sqrt{\sin(x)}}{\sqrt{a}}-\left(17\cos(x)^2+14\cos(x)-4\right)\sin(x)-18\cos(x)-4}{\cos(x)^3+3\cos(x)^2-\left(\cos(x)^2-2\cos(x)-4\right)\sin(x)-2\cos(x)-4}\right)}{4\sqrt{a}},-\frac{1}{2}\sqrt{2}\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(x)^(1/2)/(a-a*sin(x))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(2)*log((17*cos(x)^3 + 3*cos(x)^2 + 4*sqrt(2)*(3*cos(x)^2 - (3*cos
(x) + 4)*sin(x) - cos(x) - 4)*sqrt(-a*sin(x) + a)*sqrt(sin(x))/sqrt(a) - (1
7*cos(x)^2 + 14*cos(x) - 4)*sin(x) - 18*cos(x) - 4)/(cos(x)^3 + 3*cos(x)^2
- (cos(x)^2 - 2*cos(x) - 4)*sin(x) - 2*cos(x) - 4))/sqrt(a), -1/2*sqrt(2)*s
qrt(-1/a)*arctan(1/4*sqrt(2)*sqrt(-a*sin(x) + a)*sqrt(-1/a)*(3*sin(x) + 1)/
```

```
(cos(x)*sqrt(sin(x))))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-a\left(\sin\left(x\right)-1\right)}\sqrt{\sin\left(x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(x)**(1/2)/(a-a*sin(x))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-a*(sin(x) - 1))*sqrt(sin(x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-a\sin\left(x\right)+a}\sqrt{\sin\left(x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(x)^(1/2)/(a-a*sin(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-a*sin(x) + a)*sqrt(sin(x))), x)
```

3.91 $\int \frac{\sqrt[3]{\sin(c+dx)}}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=184

$$-\frac{\sin^{\frac{4}{3}}(c+dx)\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\sin^2(c+dx)\right)}{36a^2d\sqrt{\cos^2(c+dx)}}+\frac{4\sqrt[3]{\sin(c+dx)}\cos(c+dx)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\sin^2(c+dx)\right)}{9a^2d\sqrt{\cos^2(c+dx)}}-\frac{\sqrt[3]{\sin(c+}}{9a^2d(\sin}$$

```
[Out] (4*Cos[c + d*x]*Hypergeometric2F1[1/6, 1/2, 7/6, Sin[c + d*x]^2]*Sin[c + d*
x]^(1/3))/(9*a^2*d*Sqrt[Cos[c + d*x]^2]) - (Cos[c + d*x]*Hypergeometric2F1[
1/2, 2/3, 5/3, Sin[c + d*x]^2]*Sin[c + d*x]^(4/3))/(36*a^2*d*Sqrt[Cos[c + d
*x]^2]) - (Cos[c + d*x]*Sin[c + d*x]^(1/3))/(9*a^2*d*(1 + Sin[c + d*x])) -
(Cos[c + d*x]*Sin[c + d*x]^(1/3))/(3*d*(a + a*Sin[c + d*x])^2)
```

Rubi [A] time = 0.213211, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2764, 2978, 2748, 2643}

$$-\frac{\sin^{\frac{4}{3}}(c+dx)\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\sin^2(c+dx)\right)}{36a^2d\sqrt{\cos^2(c+dx)}}+\frac{4\sqrt[3]{\sin(c+dx)}\cos(c+dx)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\sin^2(c+dx)\right)}{9a^2d\sqrt{\cos^2(c+dx)}}-\frac{\sqrt[3]{\sin(c+}}{9a^2d(\sin}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^(1/3)/(a + a*Sin[c + d*x])^2,x]
```



```
[Out] (4*Cos[c + d*x]*Hypergeometric2F1[1/6, 1/2, 7/6, Sin[c + d*x]^2]*Sin[c + d*
x]^(1/3))/(9*a^2*d*Sqrt[Cos[c + d*x]^2]) - (Cos[c + d*x]*Hypergeometric2F1[
1/2, 2/3, 5/3, Sin[c + d*x]^2]*Sin[c + d*x]^(4/3))/(36*a^2*d*Sqrt[Cos[c + d
*x]^2]) - (Cos[c + d*x]*Sin[c + d*x]^(1/3))/(9*a^2*d*(1 + Sin[c + d*x])) -
(Cos[c + d*x]*Sin[c + d*x]^(1/3))/(3*d*(a + a*Sin[c + d*x])^2)
```


Rule 2764

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m
*(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a*d*n - b*c*
(m + 1) - b*d*(m + n + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && L
tQ[m, -1] && LtQ[0, n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c,
 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{\sqrt[3]{\sin(c+dx)}}{(a+a\sin(c+dx))^2}\,dx = -\frac{\cos(c+dx)\sqrt[3]{\sin(c+dx)}}{3d(a+a\sin(c+dx))^2} + \frac{\int \frac{\frac{a}{3}+\frac{2}{3}a\sin(c+dx)}{\sin^{\frac{2}{3}}(c+dx)(a+a\sin(c+dx))}\,dx}{3a^2}$$

$$= -\frac{\cos(c+dx)\sqrt[3]{\sin(c+dx)}}{9a^2d(1+\sin(c+dx))} - \frac{\cos(c+dx)\sqrt[3]{\sin(c+dx)}}{3d(a+a\sin(c+dx))^2} + \frac{\int \frac{\frac{4a^2}{9}-\frac{1}{9}a^2\sin(c+dx)}{\sin^{\frac{2}{3}}(c+dx)}\,dx}{3a^4}$$

$$= -\frac{\cos(c+dx)\sqrt[3]{\sin(c+dx)}}{9a^2d(1+\sin(c+dx))} - \frac{\cos(c+dx)\sqrt[3]{\sin(c+dx)}}{3d(a+a\sin(c+dx))^2} - \frac{\int \sqrt[3]{\sin(c+dx)}\,dx}{27a^2} + \frac{4\int \frac{1}{\sin^{\frac{2}{3}}(c+}}{27a^2}$$

$$= \frac{4\cos(c+dx)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\sin^2(c+dx)\right)\sqrt[3]{\sin(c+dx)}}{9a^2d\sqrt{\cos^2(c+dx)}} - \frac{\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\sin^2(c+\right.}{36a^2d\sqrt{\cos^2(c+a}}$$

Mathematica [A] time = 0.403805, size = 121, normalized size = 0.66

$$\frac{\sqrt[3]{\sin(c+dx)}\sec^3(c+dx)\left(80\cos^2(c+dx)^{3/2}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\sin^2(c+dx)\right)+27\sin(c+dx)\cos^2(c+dx)^{3/2}\,{}_2F_1\left(\frac{2}{3},\frac{5}{2};\frac{5}{3};\text{si}\right.\right.}{180a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^(1/3)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (Sec[c + d*x]^3*Sin[c + d*x]^(1/3)*(80*(Cos[c + d*x]^2)^(3/2)*Hypergeometri
c2F1[1/6, 1/2, 7/6, Sin[c + d*x]^2] + 27*(Cos[c + d*x]^2)^(3/2)*Hypergeomet
ric2F1[2/3, 5/2, 5/3, Sin[c + d*x]^2]*Sin[c + d*x] + 4*(-25 + 5*Cos[2*(c +
d*x)] + 27*Sin[c + d*x])))/(180*a^2*d)
```

Maple [F] time = 0.219, size = 0, normalized size = 0.

$$\int \frac{1}{(a+a\sin(dx+c))^2}\sqrt[3]{\sin(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^(1/3)/(a+a*sin(d*x+c))^2,x)
```

```
[Out] int(sin(d*x+c)^(1/3)/(a+a*sin(d*x+c))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(dx+c)^{\frac{1}{3}}}{(a\sin(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^(1/3)/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)^(1/3)/(a*sin(d*x + c) + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sin(dx+c)^{\frac{1}{3}}}{a^2\cos(dx+c)^2-2\,a^2\sin(dx+c)-2\,a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^(1/3)/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(-sin(d*x + c)^(1/3)/(a^2*cos(d*x + c)^2 - 2*a^2*sin(d*x + c) - 2*a
^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**(1/3)/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(dx+c)^{\frac{1}{3}}}{(a\sin(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^(1/3)/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)^(1/3)/(a*sin(d*x + c) + a)^2, x)
```

3.92 $\int \sin^3(c+dx)(a+a\sin(c+dx))^{2/3}\,dx$

Optimal. Leaf size=161

$$-\frac{67\cos(c+dx)(a\sin(c+dx)+a)^{2/3}\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{55\;2^{5/6}d(\sin(c+dx)+1)^{7/6}}-\frac{3\sin^2(c+dx)\cos(c+dx)(a\sin(c+dx)+a)}{11d}$$

```
[Out] (-63*Cos[c + d*x]*(a + a*Sin[c + d*x])^(2/3))/(220*d) - (3*Cos[c + d*x]*Sin
[c + d*x]^2*(a + a*Sin[c + d*x])^(2/3))/(11*d) - (67*Cos[c + d*x]*Hypergeom
etric2F1[-1/6, 1/2, 3/2, (1 - Sin[c + d*x])/2]*(a + a*Sin[c + d*x])^(2/3))/
(55*2^(5/6)*d*(1 + Sin[c + d*x])^(7/6)) - (3*Cos[c + d*x]*(a + a*Sin[c + d*
x])^(5/3))/(44*a*d)
```

Rubi [A] time = 0.27803, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2783, 2968, 3023, 2751, 2652, 2651}

$$-\frac{67\cos(c+dx)(a\sin(c+dx)+a)^{2/3}\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{55\;2^{5/6}d(\sin(c+dx)+1)^{7/6}}-\frac{3\sin^2(c+dx)\cos(c+dx)(a\sin(c+dx)+a)}{11d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^3*(a + a*Sin[c + d*x])^(2/3),x]
```

```
[Out] (-63*Cos[c + d*x]*(a + a*Sin[c + d*x])^(2/3))/(220*d) - (3*Cos[c + d*x]*Sin
[c + d*x]^2*(a + a*Sin[c + d*x])^(2/3))/(11*d) - (67*Cos[c + d*x]*Hypergeom
etric2F1[-1/6, 1/2, 3/2, (1 - Sin[c + d*x])/2]*(a + a*Sin[c + d*x])^(2/3))/
(55*2^(5/6)*d*(1 + Sin[c + d*x])^(7/6)) - (3*Cos[c + d*x]*(a + a*Sin[c + d*
x])^(5/3))/(44*a*d)
```

Rule 2783

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*(c + d*Sin[e + f*x])^(n - 1))/(f*(m + n)), x] + Dist[1/(b*(m + n)), Int[(
a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n - 2)*Simp[d*(a*c*m + b*d*(n -
 1)) + b*c^2*(m + n) + d*(a*d*m + b*c*(m + 2*n - 1))*Sin[e + f*x], x], x],
x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && IntegerQ[n]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && !LtQ[m, -2^(-1)]
```

Rule 2652

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Sin[c + d*x])^FracPart[n])/(1 + (b*Sin[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Sin[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] && !IntegerQ[2*n] && !GtQ[a, 0]
```

Rule 2651

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(2^(n +
 1/2)*a^(n - 1/2)*b*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1*(1
 - (b*Sin[c + d*x])/a))/2])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[2*n] && GtQ[a, 0]
```

Rubi steps

$$\int \sin^3(c+dx)(a+a\sin(c+dx))^{2/3}\,dx = -\frac{3\cos(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^{2/3}}{11d} + \frac{3\int \sin(c+dx)\left(2a+\frac{2}{3}a\sin\right.}{}$$

$$= -\frac{3\cos(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^{2/3}}{11d} + \frac{3\int (a+a\sin(c+dx))^{2/3}\left(2a\right.}{}$$

$$= -\frac{3\cos(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^{2/3}}{11d} - \frac{3\cos(c+dx)(a+a\sin(c+}{44ad}$$

$$= -\frac{63\cos(c+dx)(a+a\sin(c+dx))^{2/3}}{220d} - \frac{3\cos(c+dx)\sin^2(c+dx)(a+a\sin(c+}{11d}$$

$$= -\frac{63\cos(c+dx)(a+a\sin(c+dx))^{2/3}}{220d} - \frac{3\cos(c+dx)\sin^2(c+dx)(a+a\sin(c+}{11d}$$

$$= -\frac{63\cos(c+dx)(a+a\sin(c+dx))^{2/3}}{220d} - \frac{3\cos(c+dx)\sin^2(c+dx)(a+a\sin(c+}{11d}$$

Mathematica [A] time = 0.695829, size = 160, normalized size = 0.99

$$\frac{3(a(\sin(c+dx)+1))^{2/3}\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(67\sqrt{2}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\sin^2\left(\frac{1}{4}(2c+2dx+\pi)\right)\right)+\sqrt{1-\sin(c+d}\right.}{440d\sqrt{1-\sin(c+dx)}\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^3*(a + a*Sin[c + d*x])^(2/3),x]
```

```
[Out] (3*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(a*(1 + Sin[c + d*x]))^(2/3)*(67*S
qrt[2]*Hypergeometric2F1[1/6, 1/2, 7/6, Sin[(2*c + Pi + 2*d*x)/4]^2] + Sqrt
[1 - Sin[c + d*x]]*(-144 + 25*Cos[2*(c + d*x)] - 92*Sin[c + d*x] + 10*Sin[3
*(c + d*x)])))/(440*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*Sqrt[1 - Sin[c
+ d*x]])
```

Maple [F] time = 0.193, size = 0, normalized size = 0.

$$\int \left(\sin \left(dx + c\right)\right)^{3} \left(a + a\sin \left(dx + c\right)\right)^{\frac{2}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3*(a+a*sin(d*x+c))^(2/3),x)
```

```
[Out] int(sin(d*x+c)^3*(a+a*sin(d*x+c))^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin \left(dx + c\right) + a\right)^{\frac{2}{3}} \sin \left(dx + c\right)^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+a*sin(d*x+c))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(2/3)*sin(d*x + c)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\left(\cos \left(dx + c\right)^{2} - 1\right)\left(a\sin \left(dx + c\right) + a\right)^{\frac{2}{3}} \sin \left(dx + c\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+a*sin(d*x+c))^(2/3),x, algorithm="fricas")
```

```
[Out] integral(-(cos(d*x + c)^2 - 1)*(a*sin(d*x + c) + a)^(2/3)*sin(d*x + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3*(a+a*sin(d*x+c))**(2/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin \left(dx + c\right) + a\right)^{\frac{2}{3}} \sin \left(dx + c\right)^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+a*sin(d*x+c))^(2/3),x, algorithm="giac")

[Out] integrate((a*sin(d*x + c) + a)^(2/3)*sin(d*x + c)^3, x)
```

3.93 $\int \sin^2(c+dx)(a+a\sin(c+dx))^{2/3}\,dx$

Optimal. Leaf size=126

$$-\frac{19\cos(c+dx)(a\sin(c+dx)+a)^{2/3}\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{10\ 2^{5/6}d(\sin(c+dx)+1)^{7/6}}-\frac{3\cos(c+dx)(a\sin(c+dx)+a)^{5/3}}{8ad}+\frac{9\cos(c$$

```
[Out] (9*Cos[c + d*x]*(a + a*Sin[c + d*x])^(2/3))/(40*d) - (19*Cos[c + d*x]*Hyper
geometric2F1[-1/6, 1/2, 3/2, (1 - Sin[c + d*x])/2]*(a + a*Sin[c + d*x])^(2/
3))/(10*2^(5/6)*d*(1 + Sin[c + d*x])^(7/6)) - (3*Cos[c + d*x]*(a + a*Sin[c
+ d*x])^(5/3))/(8*a*d)
```

Rubi [A] time = 0.145496, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2759, 2751, 2652, 2651}

$$-\frac{19\cos(c+dx)(a\sin(c+dx)+a)^{2/3}\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{10\ 2^{5/6}d(\sin(c+dx)+1)^{7/6}}-\frac{3\cos(c+dx)(a\sin(c+dx)+a)^{5/3}}{8ad}+\frac{9\cos(c$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2*(a + a*Sin[c + d*x])^(2/3),x]
```

```
[Out] (9*Cos[c + d*x]*(a + a*Sin[c + d*x])^(2/3))/(40*d) - (19*Cos[c + d*x]*Hyper
geometric2F1[-1/6, 1/2, 3/2, (1 - Sin[c + d*x])/2]*(a + a*Sin[c + d*x])^(2/
3))/(10*2^(5/6)*d*(1 + Sin[c + d*x])^(7/6)) - (3*Cos[c + d*x]*(a + a*Sin[c
+ d*x])^(5/3))/(8*a*d)
```

Rule 2759

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin[e
 + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ
[m, -2^(-1)]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2652

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Sin[c + d*x])^FracPart[n])/(1 + (b*Sin[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Sin[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2651

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(2^(n +
 1/2)*a^(n - 1/2)*b*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1*(1
```

```
 - (b*Sin[c + d*x])/a))/2])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rubi steps

$$\int \sin^2(c+dx)(a+a\sin(c+dx))^{2/3}\,dx = -\frac{3\cos(c+dx)(a+a\sin(c+dx))^{5/3}}{8ad} + \frac{3\int\left(\frac{5a}{3} - a\sin(c+dx)\right)(a+a\sin(c+d$$
$$= \frac{9\cos(c+dx)(a+a\sin(c+dx))^{2/3}}{40d} - \frac{3\cos(c+dx)(a+a\sin(c+dx))^{5/3}}{8ad} + \frac{19}{40}\int$$
$$= \frac{9\cos(c+dx)(a+a\sin(c+dx))^{2/3}}{40d} - \frac{3\cos(c+dx)(a+a\sin(c+dx))^{5/3}}{8ad} + (19($$
$$= \frac{9\cos(c+dx)(a+a\sin(c+dx))^{2/3}}{40d} - \frac{19\cos(c+dx)\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{3}{2};\frac{1}{2}(1-\sin(c+\right.}{10\,2^{5/6}d(1+\sin(c+}$$

Mathematica [A] time = 0.438797, size = 151, normalized size = 1.2

$$\frac{3(a(\sin(c+dx)+1))^{2/3}\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(19\sqrt{2}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\sin^2\left(\frac{1}{4}(2c+2dx+\pi)\right)\right)+\sqrt{1-\sin(c+d}\right.}{80d\sqrt{1-\sin(c+dx)}\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2*(a + a*Sin[c + d*x])^(2/3),x]
```

```
[Out] (3*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(a*(1 + Sin[c + d*x]))^(2/3)*(19*S
qrt[2]*Hypergeometric2F1[1/6, 1/2, 7/6, Sin[(2*c + Pi + 2*d*x)/4]^2] + Sqrt
[1 - Sin[c + d*x]]*(5*Cos[2*(c + d*x)] - 14*(2 + Sin[c + d*x]))))/(80*d*(Co
s[(c + d*x)/2] + Sin[(c + d*x)/2])*Sqrt[1 - Sin[c + d*x]])
```

Maple [F] time = 0.197, size = 0, normalized size = 0.

$$\int (\sin(dx+c))^2 (a+a\sin(dx+c))^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2*(a+a*sin(d*x+c))^(2/3),x)
```

```
[Out] int(sin(d*x+c)^2*(a+a*sin(d*x+c))^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx+c)+a)^{\frac{2}{3}}\sin(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+a*sin(d*x+c))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(2/3)*sin(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(\cos\left(dx+c\right)^{2}-1\right)\left(a\sin\left(dx+c\right)+a\right)^{\frac{2}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+a*sin(d*x+c))^(2/3),x, algorithm="fricas")
```

```
[Out] integral(-(cos(d*x + c)^2 - 1)*(a*sin(d*x + c) + a)^(2/3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\left(\sin\left(c+dx\right)+1\right)\right)^{\frac{2}{3}}\sin^{2}\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2*(a+a*sin(d*x+c))**(2/3),x)
```

```
[Out] Integral((a*(sin(c + d*x) + 1))**(2/3)*sin(c + d*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(dx+c\right)+a\right)^{\frac{2}{3}}\sin\left(dx+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+a*sin(d*x+c))^(2/3),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(2/3)*sin(d*x + c)^2, x)
```

3.94 $\int \sin(c+dx)(a+a\sin(c+dx))^{2/3}\,dx$

Optimal. Leaf size=96

$$-\frac{4\sqrt[6]{2}\cos(c+dx)(a\sin(c+dx)+a)^{2/3}\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{5d(\sin(c+dx)+1)^{7/6}}-\frac{3\cos(c+dx)(a\sin(c+dx)+a)^{2/3}}{5d}$$

```
[Out] (-3*Cos[c + d*x]*(a + a*Sin[c + d*x])^(2/3))/(5*d) - (4*2^(1/6)*Cos[c + d*x
]*Hypergeometric2F1[-1/6, 1/2, 3/2, (1 - Sin[c + d*x])/2]*(a + a*Sin[c + d*
x])^(2/3))/(5*d*(1 + Sin[c + d*x])^(7/6))
```

Rubi [A] time = 0.0682446, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2751, 2652, 2651}

$$-\frac{4\sqrt[6]{2}\cos(c+dx)(a\sin(c+dx)+a)^{2/3}\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{5d(\sin(c+dx)+1)^{7/6}}-\frac{3\cos(c+dx)(a\sin(c+dx)+a)^{2/3}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]*(a + a*Sin[c + d*x])^(2/3),x]
```

```
[Out] (-3*Cos[c + d*x]*(a + a*Sin[c + d*x])^(2/3))/(5*d) - (4*2^(1/6)*Cos[c + d*x
]*Hypergeometric2F1[-1/6, 1/2, 3/2, (1 - Sin[c + d*x])/2]*(a + a*Sin[c + d*
x])^(2/3))/(5*d*(1 + Sin[c + d*x])^(7/6))
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && !LtQ[m, -2^(-1)]
```

Rule 2652

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Sin[c + d*x])^FracPart[n])/(1 + (b*Sin[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Sin[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] && !IntegerQ[2*n] && !GtQ[a, 0]
```

Rule 2651

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(2^(n +
 1/2)*a^(n - 1/2)*b*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1*(1
 - (b*Sin[c + d*x])/a))/2])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[2*n] && GtQ[a, 0]
```

Rubi steps

$$\int \sin(c+dx)(a+a\sin(c+dx))^{2/3}\,dx = -\frac{3\cos(c+dx)(a+a\sin(c+dx))^{2/3}}{5d} + \frac{2}{5}\int (a+a\sin(c+dx))^{2/3}\,dx$$
$$= -\frac{3\cos(c+dx)(a+a\sin(c+dx))^{2/3}}{5d} + \frac{\left(2(a+a\sin(c+dx))^{2/3}\right)\int(1+\sin(c+}{5(1+\sin(c+dx))^{2/3}}$$
$$= -\frac{3\cos(c+dx)(a+a\sin(c+dx))^{2/3}}{5d} - \frac{4\sqrt[6]{2}\cos(c+dx)\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{3}{2};\frac{1}{2}(1-\mathrm{si}\right.}{5d(1+\sin(c}$$

Mathematica [A] time = 0.230909, size = 138, normalized size = 1.44

$$-\frac{3(a(\sin(c+dx)+1))^{2/3}\left(\sqrt{1-\sin(c+dx)}(\sin(c+dx)+2)-\sqrt{2}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\sin^2\left(\frac{1}{4}(2c+2dx+\pi)\right)\right)\right)\left(\cos\left(\frac{1}{2}(c+d\right.\right.}{5d\sqrt{1-\sin(c+dx)}\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]*(a + a*Sin[c + d*x])^(2/3),x]
```

```
[Out] (-3*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(a*(1 + Sin[c + d*x]))^(2/3)*(-(S
qrt[2]*Hypergeometric2F1[1/6, 1/2, 7/6, Sin[(2*c + Pi + 2*d*x)/4]^2]) + Sqr
t[1 - Sin[c + d*x]]*(2 + Sin[c + d*x])))/(5*d*(Cos[(c + d*x)/2] + Sin[(c +
d*x)/2])*Sqrt[1 - Sin[c + d*x]])
```

Maple [F] time = 0.075, size = 0, normalized size = 0.

$$\int \sin(dx+c)\,(a+a\sin(dx+c))^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)*(a+a*sin(d*x+c))^(2/3),x)
```

```
[Out] int(sin(d*x+c)*(a+a*sin(d*x+c))^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx+c)+a)^{\frac{2}{3}}\sin(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+a*sin(d*x+c))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(2/3)*sin(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((a\sin(dx+c)+a)^{\frac{2}{3}}\sin(dx+c),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+a*sin(d*x+c))^(2/3),x, algorithm="fricas")
```

```
[Out] integral((a*sin(d*x + c) + a)^(2/3)*sin(d*x + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a(\sin(c+dx)+1))^{\frac{2}{3}} \sin(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+a*sin(d*x+c))**(2/3),x)
```

```
[Out] Integral((a*(sin(c + d*x) + 1))**(2/3)*sin(c + d*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx+c)+a)^{\frac{2}{3}} \sin(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+a*sin(d*x+c))^(2/3),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(2/3)*sin(d*x + c), x)
```

3.95 $\int (a+a\sin(c+dx))^{2/3}\,dx$

Optimal. Leaf size=66

$$-\frac{2\sqrt[6]{2}\cos(c+dx)(a\sin(c+dx)+a)^{2/3}\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{d(\sin(c+dx)+1)^{7/6}}$$

```
[Out] (-2*2^(1/6)*Cos[c + d*x]*Hypergeometric2F1[-1/6, 1/2, 3/2, (1 - Sin[c + d*x
])/2]*(a + a*Sin[c + d*x])^(2/3))/(d*(1 + Sin[c + d*x])^(7/6))
```

Rubi [A] time = 0.0312848, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2652, 2651}

$$-\frac{2\sqrt[6]{2}\cos(c+dx)(a\sin(c+dx)+a)^{2/3}\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{d(\sin(c+dx)+1)^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^(2/3),x]
```

```
[Out] (-2*2^(1/6)*Cos[c + d*x]*Hypergeometric2F1[-1/6, 1/2, 3/2, (1 - Sin[c + d*x
])/2]*(a + a*Sin[c + d*x])^(2/3))/(d*(1 + Sin[c + d*x])^(7/6))
```

Rule 2652

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Sin[c + d*x])^FracPart[n])/(1 + (b*Sin[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Sin[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] && !IntegerQ[2*n] && !GtQ[a, 0]
```

Rule 2651

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(2^(n +
 1/2)*a^(n - 1/2)*b*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1*(1
 - (b*Sin[c + d*x])/a))/2])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[2*n] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int (a+a\sin(c+dx))^{2/3}\,dx &= \frac{(a+a\sin(c+dx))^{2/3}\int (1+\sin(c+dx))^{2/3}\,dx}{(1+\sin(c+dx))^{2/3}}\\ &= -\frac{2\sqrt[6]{2}\cos(c+dx)\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)(a+a\sin(c+dx))^{2/3}}{d(1+\sin(c+dx))^{7/6}}\end{aligned}$$

Mathematica [A] time = 0.202962, size = 124, normalized size = 1.88

$$-\frac{3(a(\sin(c+dx)+1))^{2/3}\left(\sqrt{2-2\sin(c+dx)}-2\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\sin^2\left(\frac{1}{4}(2c+2dx+\pi)\right)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+\right.\right.}{2d\sqrt{2-2\sin(c+dx)}\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^(2/3),x]
```

```
[Out] (-3*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(-2*Hypergeometric2F1[1/6, 1/2, 7
/6, Sin[(2*c + Pi + 2*d*x)/4]^2] + Sqrt[2 - 2*Sin[c + d*x]])*(a*(1 + Sin[c
+ d*x]))^(2/3))/(2*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*Sqrt[2 - 2*Sin[c
 + d*x]])
```

Maple [F] time = 0.003, size = 0, normalized size = 0.

$$\int (a + a\sin(dx + c))^{\frac{2}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^(2/3),x)
```

```
[Out] int((a+a*sin(d*x+c))^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx + c) + a)^{\frac{2}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(2/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((a\sin(dx + c) + a)^{\frac{2}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(2/3),x, algorithm="fricas")
```

```
[Out] integral((a*sin(d*x + c) + a)^(2/3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(c + dx) + a)^{\frac{2}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**(2/3),x)
```

```
[Out] Integral((a*sin(c + d*x) + a)**(2/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a \sin(dx + c) + a)^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(2/3),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(2/3), x)
```

3.96 $\int \csc(c+dx)(a+a\sin(c+dx))^{2/3}\,dx$

Optimal. Leaf size=77

$$-\frac{2\sqrt[6]{2}\cos(c+dx)(a\sin(c+dx)+a)^{2/3}F_1\left(\frac{1}{2};1,-\frac{1}{6};\frac{3}{2};1-\sin(c+dx),\frac{1}{2}(1-\sin(c+dx))\right)}{d(\sin(c+dx)+1)^{7/6}}$$

```
[Out] (-2*2^(1/6)*AppellF1[1/2, 1, -1/6, 3/2, 1 - Sin[c + d*x], (1 - Sin[c + d*x]
)/2]*Cos[c + d*x]*(a + a*Sin[c + d*x])^(2/3))/(d*(1 + Sin[c + d*x])^(7/6))
```

Rubi [A] time = 0.110812, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2787, 2785, 130, 429}

$$-\frac{2\sqrt[6]{2}\cos(c+dx)(a\sin(c+dx)+a)^{2/3}F_1\left(\frac{1}{2};1,-\frac{1}{6};\frac{3}{2};1-\sin(c+dx),\frac{1}{2}(1-\sin(c+dx))\right)}{d(\sin(c+dx)+1)^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*(a + a*Sin[c + d*x])^(2/3),x]
```


```
[Out] (-2*2^(1/6)*AppellF1[1/2, 1, -1/6, 3/2, 1 - Sin[c + d*x], (1 - Sin[c + d*x]
)/2]*Cos[c + d*x]*(a + a*Sin[c + d*x])^(2/3))/(d*(1 + Sin[c + d*x])^(7/6))
```


Rule 2787

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m
])/(1 + (b*Sin[e + f*x])/a)^FracPart[m], Int[(1 + (b*Sin[e + f*x])/a)^m*(d*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 2785

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Dist[(b*(d/b)^n*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a - x)^n*(2*a - x)^(m - 1/2)
)/Sqrt[x], x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

Rule 130

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + (b*x^k)/e)^m*(c + (d*x^k)/e)^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \csc(c+dx)(a+a\sin(c+dx))^{2/3}\,dx = \frac{(a+a\sin(c+dx))^{2/3}\int \csc(c+dx)(1+\sin(c+dx))^{2/3}\,dx}{(1+\sin(c+dx))^{2/3}}$$

$$= -\frac{\left(\cos(c+dx)(a+a\sin(c+dx))^{2/3}\right)\text{Subst}\left(\int \frac{\sqrt[6]{2-x}}{(1-x)\sqrt{x}}\,dx, x, 1-\sin(c+dx)\right)}{d\sqrt{1-\sin(c+dx)}(1+\sin(c+dx))^{7/6}}$$

$$= -\frac{\left(2\cos(c+dx)(a+a\sin(c+dx))^{2/3}\right)\text{Subst}\left(\int \frac{\sqrt[6]{2-x^2}}{1-x^2}\,dx, x, \sqrt{1-\sin(c+dx}\right.}{d\sqrt{1-\sin(c+dx)}(1+\sin(c+dx))^{7/6}}$$

$$= -\frac{2\sqrt[6]{2}F_1\left(\frac{1}{2};1,-\frac{1}{6};\frac{3}{2};1-\sin(c+dx),\frac{1}{2}(1-\sin(c+dx))\right)\cos(c+dx)(a+a\sin}{d(1+\sin(c+dx))^{7/6}}$$

Mathematica [F] time = 2.72419, size = 0, normalized size = 0.

$$\int \csc(c+dx)(a+a\sin(c+dx))^{2/3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Csc[c + d*x]*(a + a*Sin[c + d*x])^(2/3),x]
```

```
[Out] Integrate[Csc[c + d*x]*(a + a*Sin[c + d*x])^(2/3), x]
```

Maple [F] time = 0.096, size = 0, normalized size = 0.

$$\int \csc(dx+c)(a+a\sin(dx+c))^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*(a+a*sin(d*x+c))^(2/3),x)
```

```
[Out] int(csc(d*x+c)*(a+a*sin(d*x+c))^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx+c)+a)^{\frac{2}{3}}\csc(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sin(d*x+c))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(2/3)*csc(d*x + c), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sin(d*x+c))^(2/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sin(d*x+c))**(2/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx+c)+a)^{\frac{2}{3}}\csc(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sin(d*x+c))^(2/3),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(2/3)*csc(d*x + c), x)
```

3.97 $\int \csc^2(c+dx)(a+a\sin(c+dx))^{2/3}\,dx$

Optimal. Leaf size=77

$$-\frac{2\sqrt[6]{2}\cos(c+dx)(a\sin(c+dx)+a)^{2/3}F_1\left(\frac{1}{2};2,-\frac{1}{6};\frac{3}{2};1-\sin(c+dx),\frac{1}{2}(1-\sin(c+dx))\right)}{d(\sin(c+dx)+1)^{7/6}}$$

```
[Out] (-2*2^(1/6)*AppellF1[1/2, 2, -1/6, 3/2, 1 - Sin[c + d*x], (1 - Sin[c + d*x]
)/2]*Cos[c + d*x]*(a + a*Sin[c + d*x])^(2/3))/(d*(1 + Sin[c + d*x])^(7/6))
```

Rubi [A] time = 0.13149, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2787, 2785, 130, 429}

$$-\frac{2\sqrt[6]{2}\cos(c+dx)(a\sin(c+dx)+a)^{2/3}F_1\left(\frac{1}{2};2,-\frac{1}{6};\frac{3}{2};1-\sin(c+dx),\frac{1}{2}(1-\sin(c+dx))\right)}{d(\sin(c+dx)+1)^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2*(a + a*Sin[c + d*x])^(2/3),x]
```

```
[Out] (-2*2^(1/6)*AppellF1[1/2, 2, -1/6, 3/2, 1 - Sin[c + d*x], (1 - Sin[c + d*x]
)/2]*Cos[c + d*x]*(a + a*Sin[c + d*x])^(2/3))/(d*(1 + Sin[c + d*x])^(7/6))
```

Rule 2787

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m
])/(1 + (b*Sin[e + f*x])/a)^FracPart[m], Int[(1 + (b*Sin[e + f*x])/a)^m*(d*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 2785

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Dist[(b*(d/b)^n*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a - x)^n*(2*a - x)^(m - 1/2)
)/Sqrt[x], x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

Rule 130

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + (b*x^k)/e)^m*(c + (d*x^k)/e)^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \csc^2(c+dx)(a+a\sin(c+dx))^{2/3}\,dx = \frac{(a+a\sin(c+dx))^{2/3}\int \csc^2(c+dx)(1+\sin(c+dx))^{2/3}\,dx}{(1+\sin(c+dx))^{2/3}}$$
$$= -\frac{\left(\cos(c+dx)(a+a\sin(c+dx))^{2/3}\right)\text{Subst}\left(\int \frac{\sqrt[6]{2-x}}{(1-x)^2\sqrt{x}}\,dx, x, 1-\sin(c+dx)\right)}{d\sqrt{1-\sin(c+dx)}(1+\sin(c+dx))^{7/6}}$$
$$= -\frac{\left(2\cos(c+dx)(a+a\sin(c+dx))^{2/3}\right)\text{Subst}\left(\int \frac{\sqrt[6]{2-x^2}}{(1-x^2)^2}\,dx, x, \sqrt{1-\sin(c+dx)}\right)}{d\sqrt{1-\sin(c+dx)}(1+\sin(c+dx))^{7/6}}$$
$$= -\frac{2\sqrt[6]{2}F_1\left(\frac{1}{2};2,-\frac{1}{6};\frac{3}{2};1-\sin(c+dx),\frac{1}{2}(1-\sin(c+dx))\right)\cos(c+dx)(a+a\sin(}{d(1+\sin(c+dx))^{7/6}}$$

Mathematica [C] time = 14.6154, size = 143, normalized size = 1.86

$$-\frac{2e^{i(c+dx)}\left(\left(1+ie^{-i(c+dx)}\right)^{2/3}\left(e^{i(c+dx)}-i\right){}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-ie^{-i(c+dx)}\right)-e^{i(c+dx)}-i\right)(a(\sin(c+dx)+1))^{2/3}}{d\left(e^{i(c+dx)}-i\right)\left(e^{i(c+dx)}+i\right)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[c + d*x]^2*(a + a*Sin[c + d*x])^(2/3),x]
```

```
[Out] (-2*E^(I*(c + d*x))*(-I - E^(I*(c + d*x)) + (1 + I/E^(I*(c + d*x)))^(2/3)*(
-I + E^(I*(c + d*x)))*Hypergeometric2F1[1/3, 2/3, 4/3, (-I)/E^(I*(c + d*x))
])*(a*(1 + Sin[c + d*x]))^(2/3))/(d*(-I + E^(I*(c + d*x)))*(I + E^(I*(c + d
*x)))^2)
```

Maple [F] time = 0.098, size = 0, normalized size = 0.

$$\int (\csc(dx+c))^2 (a+a\sin(dx+c))^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*(a+a*sin(d*x+c))^(2/3),x)
```

```
[Out] int(csc(d*x+c)^2*(a+a*sin(d*x+c))^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx+c)+a)^{\frac{2}{3}}\csc(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+a*sin(d*x+c))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(2/3)*csc(d*x + c)^2, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+a*sin(d*x+c))^(2/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*(a+a*sin(d*x+c))**(2/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx+c)+a)^{\frac{2}{3}}\csc(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+a*sin(d*x+c))^(2/3),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(2/3)*csc(d*x + c)^2, x)
```

3.98 $\int \sin^3(c+dx)(a+a\sin(c+dx))^{4/3}\,dx$

Optimal. Leaf size=162

$$-\frac{388\ 2^{5/6} a\cos(c+dx)\sqrt[3]{a\sin(c+dx)+a}\,{}_2F_1\left(-\frac{5}{6},\frac{1}{2};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{455d(\sin(c+dx)+1)^{5/6}}-\frac{3\sin^2(c+dx)\cos(c+dx)(a\sin(c+dx)+a}{13d}$$

```
[Out] (-388*2^(5/6)*a*Cos[c + d*x]*Hypergeometric2F1[-5/6, 1/2, 3/2, (1 - Sin[c +
 d*x])/2]*(a + a*Sin[c + d*x])^(1/3))/(455*d*(1 + Sin[c + d*x])^(5/6)) - (7
2*Cos[c + d*x]*(a + a*Sin[c + d*x])^(4/3))/(455*d) - (3*Cos[c + d*x]*Sin[c
+ d*x]^2*(a + a*Sin[c + d*x])^(4/3))/(13*d) - (6*Cos[c + d*x]*(a + a*Sin[c
+ d*x])^(7/3))/(65*a*d)
```

Rubi [A] time = 0.278708, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2783, 2968, 3023, 2751, 2652, 2651}

$$-\frac{388\ 2^{5/6} a\cos(c+dx)\sqrt[3]{a\sin(c+dx)+a}\,{}_2F_1\left(-\frac{5}{6},\frac{1}{2};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{455d(\sin(c+dx)+1)^{5/6}}-\frac{3\sin^2(c+dx)\cos(c+dx)(a\sin(c+dx)+a}{13d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^3*(a + a*Sin[c + d*x])^(4/3),x]
```


```
[Out] (-388*2^(5/6)*a*Cos[c + d*x]*Hypergeometric2F1[-5/6, 1/2, 3/2, (1 - Sin[c +
 d*x])/2]*(a + a*Sin[c + d*x])^(1/3))/(455*d*(1 + Sin[c + d*x])^(5/6)) - (7
2*Cos[c + d*x]*(a + a*Sin[c + d*x])^(4/3))/(455*d) - (3*Cos[c + d*x]*Sin[c
+ d*x]^2*(a + a*Sin[c + d*x])^(4/3))/(13*d) - (6*Cos[c + d*x]*(a + a*Sin[c
+ d*x])^(7/3))/(65*a*d)
```


Rule 2783

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*(c + d*Sin[e + f*x])^(n - 1))/(f*(m + n)), x] + Dist[1/(b*(m + n)), Int[(
a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n - 2)*Simp[d*(a*c*m + b*d*(n -
 1)) + b*c^2*(m + n) + d*(a*d*m + b*c*(m + 2*n - 1))*Sin[e + f*x], x], x],
x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && IntegerQ[n]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2652

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Sin[c + d*x])^FracPart[n])/(1 + (b*Sin[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Sin[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2651

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(2^(n +
 1/2)*a^(n - 1/2)*b*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1*(1
 - (b*Sin[c + d*x])/a))/2])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rubi steps

$$\int \sin^3(c+dx)(a+a\sin(c+dx))^{4/3}\,dx = -\frac{3\cos(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^{4/3}}{13d} + \frac{3\int \sin(c+dx)(a+a\sin}{}$$

$$= -\frac{3\cos(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^{4/3}}{13d} + \frac{3\int (a+a\sin(c+dx))^{4/3}}{}$$

$$= -\frac{3\cos(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^{4/3}}{13d} - \frac{6\cos(c+dx)(a+a\sin(c}{65ad}$$

$$= -\frac{72\cos(c+dx)(a+a\sin(c+dx))^{4/3}}{455d} - \frac{3\cos(c+dx)\sin^2(c+dx)(a+a\sin}{13d}$$

$$= -\frac{72\cos(c+dx)(a+a\sin(c+dx))^{4/3}}{455d} - \frac{3\cos(c+dx)\sin^2(c+dx)(a+a\sin}{13d}$$

$$= -\frac{388\ 2^{5/6}a\cos(c+dx)\,{}_2F_1\left(-\frac{5}{6},\frac{1}{2};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)\sqrt[3]{a+a\sin(c+dx)}}{455d(1+\sin(c+dx))^{5/6}}$$

Mathematica [C] time = 2.57543, size = 373, normalized size = 2.3

$$\frac{(a(\sin(c+dx)+1))^{4/3}\left(-\frac{3}{40}\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)(278\sin(2(c+dx))-35\sin(4(c+dx))+790\cos(c+}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^3*(a + a*Sin[c + d*x])^(4/3),x]
```

```
[Out] ((a*(1 + Sin[c + d*x]))^(4/3)*((291*(-1)^(3/4)*(I + E^(I*(c + d*x)))*(20*E^
(I*(c + d*x))*Sqrt[Cos[(2*c + Pi + 2*d*x)/4]^2]*Hypergeometric2F1[-1/3, 1/3
, 2/3, (-I)/E^(I*(c + d*x))] - 2*(1 + I/E^(I*(c + d*x)))^(2/3)*(1 + E^((2*I
)*(c + d*x)))*Hypergeometric2F1[1/2, 5/6, 11/6, Sin[(2*c + Pi + 2*d*x)/4]^2
```

```
] + (5*I)*Hypergeometric2F1[1/3, 2/3, 5/3, (-I)/E^(I*(c + d*x))]*Sqrt[2 - 2
*Sin[c + d*x]]))/(20*Sqrt[2]*E^(((3*I)/2)*(c + d*x))*(1 + I/E^(I*(c + d*x))
)^(2/3)*Sqrt[(I*(-I + E^(I*(c + d*x)))^2)/E^(I*(c + d*x))]) - (3*(Cos[(c +
d*x)/2] + Sin[(c + d*x)/2])*(-1940 + 790*Cos[c + d*x] - 98*Cos[3*(c + d*x)]
 + 278*Sin[2*(c + d*x)] - 35*Sin[4*(c + d*x)]))/40))/(91*d*(Cos[(c + d*x)/2
] + Sin[(c + d*x)/2])^3)
```

Maple [F] time = 0.193, size = 0, normalized size = 0.

$$\int \left(\sin \left(dx+c\right)\right)^{3} \left(a+a\sin \left(dx+c\right)\right)^{\frac{4}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3*(a+a*sin(d*x+c))^(4/3),x)
```

```
[Out] int(sin(d*x+c)^3*(a+a*sin(d*x+c))^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin \left(dx+c\right)+a\right)^{\frac{4}{3}} \sin \left(dx+c\right)^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+a*sin(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(4/3)*sin(d*x + c)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a\cos \left(dx+c\right)^{4}-2\,a\cos \left(dx+c\right)^{2}-\left(a\cos \left(dx+c\right)^{2}-a\right)\sin \left(dx+c\right)+a\right)\left(a\sin \left(dx+c\right)+a\right)^{\frac{1}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+a*sin(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] integral((a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^2 - (a*cos(d*x + c)^2 - a)*si
n(d*x + c) + a)*(a*sin(d*x + c) + a)^(1/3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3*(a+a*sin(d*x+c))**(4/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx+c)+a)^{\frac{4}{3}}\sin(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+a*sin(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(4/3)*sin(d*x + c)^3, x)
```

3.99 $\int \sin^2(c+dx)(a+a\sin(c+dx))^{4/3}\,dx$

Optimal. Leaf size=127

$$-\frac{37\ 2^{5/6}a\cos(c+dx)\sqrt[3]{a\sin(c+dx)+a}\,{}_2F_1\left(-\frac{5}{6},\frac{1}{2};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{35d(\sin(c+dx)+1)^{5/6}}-\frac{3\cos(c+dx)(a\sin(c+dx)+a)^{7/3}}{10ad}+\frac{9\cos(c$$

```
[Out] (-37*2^(5/6)*a*Cos[c + d*x]*Hypergeometric2F1[-5/6, 1/2, 3/2, (1 - Sin[c +
d*x])/2]*(a + a*Sin[c + d*x])^(1/3))/(35*d*(1 + Sin[c + d*x])^(5/6)) + (9*C
os[c + d*x]*(a + a*Sin[c + d*x])^(4/3))/(70*d) - (3*Cos[c + d*x]*(a + a*Sin
[c + d*x])^(7/3))/(10*a*d)
```

Rubi [A] time = 0.142597, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2759, 2751, 2652, 2651}

$$-\frac{37\ 2^{5/6}a\cos(c+dx)\sqrt[3]{a\sin(c+dx)+a}\,{}_2F_1\left(-\frac{5}{6},\frac{1}{2};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{35d(\sin(c+dx)+1)^{5/6}}-\frac{3\cos(c+dx)(a\sin(c+dx)+a)^{7/3}}{10ad}+\frac{9\cos(c$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2*(a + a*Sin[c + d*x])^(4/3),x]
```

```
[Out] (-37*2^(5/6)*a*Cos[c + d*x]*Hypergeometric2F1[-5/6, 1/2, 3/2, (1 - Sin[c +
d*x])/2]*(a + a*Sin[c + d*x])^(1/3))/(35*d*(1 + Sin[c + d*x])^(5/6)) + (9*C
os[c + d*x]*(a + a*Sin[c + d*x])^(4/3))/(70*d) - (3*Cos[c + d*x]*(a + a*Sin
[c + d*x])^(7/3))/(10*a*d)
```

Rule 2759

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin[e
 + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ
[m, -2^(-1)]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2652

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Sin[c + d*x])^FracPart[n])/(1 + (b*Sin[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Sin[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2651

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(2^(n +
 1/2)*a^(n - 1/2)*b*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1*(1
```

```
- (b*Sin[c + d*x])/a))/2])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[2*n] && GtQ[a, 0]
```

Rubi steps

$$\int \sin^2(c+dx)(a+a\sin(c+dx))^{4/3}\,dx = -\frac{3\cos(c+dx)(a+a\sin(c+dx))^{7/3}}{10ad} + \frac{3\int\left(\frac{7a}{3}-a\sin(c+dx)\right)(a+a\sin(c\ldots}{10a}$$
$$= \frac{9\cos(c+dx)(a+a\sin(c+dx))^{4/3}}{70d} - \frac{3\cos(c+dx)(a+a\sin(c+dx))^{7/3}}{10ad} + \frac{3\ldots}{7\ldots}$$
$$= \frac{9\cos(c+dx)(a+a\sin(c+dx))^{4/3}}{70d} - \frac{3\cos(c+dx)(a+a\sin(c+dx))^{7/3}}{10ad} + \ldots$$
$$= -\frac{37\, 2^{5/6} a\cos(c+dx)\,{}_2F_1\left(-\frac{5}{6},\frac{1}{2};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)\sqrt[3]{a+a\sin(c+dx)}}{35d(1+\sin(c+dx))^{5/6}} - \ldots$$

Mathematica [C] time = 2.56345, size = 363, normalized size = 2.86

$$\frac{(a(\sin(c+dx)+1))^{4/3}\left(-\frac{3}{10}\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)(22\sin(2(c+dx))+60\cos(c+dx)-7\cos(3(c+dx))\ldots\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2*(a + a*Sin[c + d*x])^(4/3),x]
```

```
[Out] ((a*(1 + Sin[c + d*x]))^(4/3)*((111*(-1)^(3/4)*(I + E^(I*(c + d*x)))*(20*E^
(I*(c + d*x))*Sqrt[Cos[(2*c + Pi + 2*d*x)/4]^2]*Hypergeometric2F1[-1/3, 1/3
, 2/3, (-I)/E^(I*(c + d*x))] - 2*(1 + I/E^(I*(c + d*x)))^(2/3)*(1 + E^((2*I
)*(c + d*x)))*Hypergeometric2F1[1/2, 5/6, 11/6, Sin[(2*c + Pi + 2*d*x)/4]^2
] + (5*I)*Hypergeometric2F1[1/3, 2/3, 5/3, (-I)/E^(I*(c + d*x))]*Sqrt[2 - 2
*Sin[c + d*x]]))/(20*Sqrt[2]*E^(((3*I)/2)*(c + d*x))*(1 + I/E^(I*(c + d*x))
)^(2/3)*Sqrt[(I*(-I + E^(I*(c + d*x)))^2)/E^(I*(c + d*x))]) - (3*(Cos[(c +
d*x)/2] + Sin[(c + d*x)/2])*(-185 + 60*Cos[c + d*x] - 7*Cos[3*(c + d*x)] +
22*Sin[2*(c + d*x)]))/10))/(28*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3)
```

Maple [F] time = 0.201, size = 0, normalized size = 0.

$$\int (\sin(dx+c))^2 (a+a\sin(dx+c))^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2*(a+a*sin(d*x+c))^(4/3),x)
```

```
[Out] int(sin(d*x+c)^2*(a+a*sin(d*x+c))^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx+c)+a)^{\frac{4}{3}}\sin(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+a*sin(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(4/3)*sin(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(a\cos\left(dx+c\right)^{2}+\left(a\cos\left(dx+c\right)^{2}-a\right)\sin\left(dx+c\right)-a\right)\left(a\sin\left(dx+c\right)+a\right)^{\frac{1}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+a*sin(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] integral(-(a*cos(d*x + c)^2 + (a*cos(d*x + c)^2 - a)*sin(d*x + c) - a)*(a*s
in(d*x + c) + a)^(1/3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2*(a+a*sin(d*x+c))**(4/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a\sin\left(dx+c\right)+a\right)^{\frac{4}{3}}\sin\left(dx+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+a*sin(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(4/3)*sin(d*x + c)^2, x)
```

3.100 $\int \sin(c+dx)(a+a\sin(c+dx))^{4/3}\,dx$

Optimal. Leaf size=97

$$-\frac{8\ 2^{5/6}a\cos(c+dx)\sqrt[3]{a\sin(c+dx)+a}\ {}_2F_1\left(-\frac{5}{6},\frac{1}{2};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{7d(\sin(c+dx)+1)^{5/6}}-\frac{3\cos(c+dx)(a\sin(c+dx)+a)^{4/3}}{7d}$$

```
[Out] (-8*2^(5/6)*a*Cos[c + d*x]*Hypergeometric2F1[-5/6, 1/2, 3/2, (1 - Sin[c + d
*x])/2]*(a + a*Sin[c + d*x])^(1/3))/(7*d*(1 + Sin[c + d*x])^(5/6)) - (3*Cos
[c + d*x]*(a + a*Sin[c + d*x])^(4/3))/(7*d)
```

Rubi [A] time = 0.0671871, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2751, 2652, 2651}

$$-\frac{8\ 2^{5/6}a\cos(c+dx)\sqrt[3]{a\sin(c+dx)+a}\ {}_2F_1\left(-\frac{5}{6},\frac{1}{2};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{7d(\sin(c+dx)+1)^{5/6}}-\frac{3\cos(c+dx)(a\sin(c+dx)+a)^{4/3}}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]*(a + a*Sin[c + d*x])^(4/3),x]
```

```
[Out] (-8*2^(5/6)*a*Cos[c + d*x]*Hypergeometric2F1[-5/6, 1/2, 3/2, (1 - Sin[c + d
*x])/2]*(a + a*Sin[c + d*x])^(1/3))/(7*d*(1 + Sin[c + d*x])^(5/6)) - (3*Cos
[c + d*x]*(a + a*Sin[c + d*x])^(4/3))/(7*d)
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2652

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Sin[c + d*x])^FracPart[n])/(1 + (b*Sin[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Sin[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2651

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(2^(n +
 1/2)*a^(n - 1/2)*b*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1*(1
 - (b*Sin[c + d*x])/a))/2])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rubi steps

$$\int \sin(c+dx)(a+a\sin(c+dx))^{4/3}\,dx = -\frac{3\cos(c+dx)(a+a\sin(c+dx))^{4/3}}{7d} + \frac{4}{7}\int (a+a\sin(c+dx))^{4/3}\,dx$$
$$= -\frac{3\cos(c+dx)(a+a\sin(c+dx))^{4/3}}{7d} + \frac{\left(4a\sqrt[3]{a+a\sin(c+dx)}\right)\int(1+\sin(c+dx)}{7\sqrt[3]{1+\sin(c+dx)}}$$
$$= -\frac{8\,2^{5/6}a\cos(c+dx)\,{}_2F_1\left(-\frac{5}{6},\frac{1}{2};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)\sqrt[3]{a+a\sin(c+dx)}}{7d(1+\sin(c+dx))^{5/6}} - 3\,\mathrm{co}$$

Mathematica [C] time = 1.92383, size = 351, normalized size = 3.62

$$\frac{(a(\sin(c+dx)+1))^{4/3}\left(-\frac{3}{2}\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)(\sin(2(c+dx))+4\cos(c+dx)-10)+\frac{3(-1)^{3/4}e^{-\frac{3}{2}i(c+dx)}\left(e^{i(c+d}\right.}{}\right.}{7d\left(\sin\left(\frac{1}{2}(c+\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]*(a + a*Sin[c + d*x])^(4/3),x]
```

```
[Out] ((a*(1 + Sin[c + d*x]))^(4/3)*((3*(-1)^(3/4)*(I + E^(I*(c + d*x)))*(20*E^(I
*(c + d*x))*Sqrt[Cos[(2*c + Pi + 2*d*x)/4]^2]*Hypergeometric2F1[-1/3, 1/3,
2/3, (-I)/E^(I*(c + d*x))] - 2*(1 + I/E^(I*(c + d*x)))^(2/3)*(1 + E^((2*I)*
(c + d*x)))*Hypergeometric2F1[1/2, 5/6, 11/6, Sin[(2*c + Pi + 2*d*x)/4]^2]
+ (5*I)*Hypergeometric2F1[1/3, 2/3, 5/3, (-I)/E^(I*(c + d*x))]*Sqrt[2 - 2*S
in[c + d*x]]))/(2*Sqrt[2]*E^(((3*I)/2)*(c + d*x))*(1 + I/E^(I*(c + d*x)))^(
2/3)*Sqrt[(I*(-I + E^(I*(c + d*x)))^2)/E^(I*(c + d*x))]) - (3*(Cos[(c + d*x
)/2] + Sin[(c + d*x)/2])*(-10 + 4*Cos[c + d*x] + Sin[2*(c + d*x)]))/2))/(7*
d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3)
```

Maple [F] time = 0.075, size = 0, normalized size = 0.

$$\int \sin(dx+c)(a+a\sin(dx+c))^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)*(a+a*sin(d*x+c))^(4/3),x)
```

```
[Out] int(sin(d*x+c)*(a+a*sin(d*x+c))^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx+c)+a)^{\frac{4}{3}}\sin(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+a*sin(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(4/3)*sin(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(a\cos\left(dx+c\right)^{2}-a\sin\left(dx+c\right)-a\right)\left(a\sin\left(dx+c\right)+a\right)^{\frac{1}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+a*sin(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] integral(-(a*cos(d*x + c)^2 - a*sin(d*x + c) - a)*(a*sin(d*x + c) + a)^(1/3
), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+a*sin(d*x+c))**(4/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a\sin\left(dx+c\right)+a\right)^{\frac{4}{3}}\sin\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+a*sin(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(4/3)*sin(d*x + c), x)
```

3.101 $\int (a+a\sin(c+dx))^{4/3}\,dx$

Optimal. Leaf size=67

$$-\frac{2\,2^{5/6}a\cos(c+dx)\sqrt[3]{a\sin(c+dx)+a}\,{}_2F_1\left(-\frac{5}{6},\frac{1}{2};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{d(\sin(c+dx)+1)^{5/6}}$$

```
[Out] (-2*2^(5/6)*a*Cos[c + d*x]*Hypergeometric2F1[-5/6, 1/2, 3/2, (1 - Sin[c + d
*x])/2]*(a + a*Sin[c + d*x])^(1/3))/(d*(1 + Sin[c + d*x])^(5/6))
```

Rubi [A] time = 0.0300104, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2652, 2651}

$$-\frac{2\,2^{5/6}a\cos(c+dx)\sqrt[3]{a\sin(c+dx)+a}\,{}_2F_1\left(-\frac{5}{6},\frac{1}{2};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{d(\sin(c+dx)+1)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^(4/3),x]
```

```
[Out] (-2*2^(5/6)*a*Cos[c + d*x]*Hypergeometric2F1[-5/6, 1/2, 3/2, (1 - Sin[c + d
*x])/2]*(a + a*Sin[c + d*x])^(1/3))/(d*(1 + Sin[c + d*x])^(5/6))
```

Rule 2652

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Sin[c + d*x])^FracPart[n])/(1 + (b*Sin[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Sin[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] && !IntegerQ[2*n] && !GtQ[a, 0]
```

Rule 2651

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(2^(n +
 1/2)*a^(n - 1/2)*b*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1*(1
 - (b*Sin[c + d*x])/a))/2])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[2*n] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int (a+a\sin(c+dx))^{4/3}\,dx &= \frac{\left(a\sqrt[3]{a+a\sin(c+dx)}\right)\int (1+\sin(c+dx))^{4/3}\,dx}{\sqrt[3]{1+\sin(c+dx)}}\\ &= -\frac{2\,2^{5/6}a\cos(c+dx)\,{}_2F_1\left(-\frac{5}{6},\frac{1}{2};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)\sqrt[3]{a+a\sin(c+dx)}}{d(1+\sin(c+dx))^{5/6}}\end{aligned}$$

Mathematica [C] time = 1.6485, size = 341, normalized size = 5.09

$$\frac{(a(\sin(c+dx)+1))^{4/3}\left(-\frac{3}{2}(\cos(c+dx)-5)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)+\frac{3(-1)^{3/4}e^{-\frac{3}{2}i(c+dx)}\left(e^{i(c+dx)}+i\right)\left(-2\left(1+ie^{-i(c+dx)}\right)^{2/3}(\cdots}{\cdots}\right.}{2d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^(4/3),x]
```

```
[Out] (((-3*(-5 + Cos[c + d*x])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))/2 + (3*(-1
)^(3/4)*(I + E^(I*(c + d*x)))*(20*E^(I*(c + d*x))*Sqrt[Cos[(2*c + Pi + 2*d*
x)/4]^2]*Hypergeometric2F1[-1/3, 1/3, 2/3, (-I)/E^(I*(c + d*x))] - 2*(1 + I
/E^(I*(c + d*x)))^(2/3)*(1 + E^((2*I)*(c + d*x)))*Hypergeometric2F1[1/2, 5/
6, 11/6, Sin[(2*c + Pi + 2*d*x)/4]^2] + (5*I)*Hypergeometric2F1[1/3, 2/3, 5
/3, (-I)/E^(I*(c + d*x))]*Sqrt[2 - 2*Sin[c + d*x]]))/(4*Sqrt[2]*E^(((3*I)/2
)*(c + d*x))*(1 + I/E^(I*(c + d*x)))^(2/3)*Sqrt[(I*(-I + E^(I*(c + d*x)))^2
)/E^(I*(c + d*x))]))*(a*(1 + Sin[c + d*x]))^(4/3))/(2*d*(Cos[(c + d*x)/2] +
 Sin[(c + d*x)/2])^3)
```

Maple [F] time = 0.005, size = 0, normalized size = 0.

$$\int (a + a\sin(dx + c))^{\frac{4}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^(4/3),x)
```

```
[Out] int((a+a*sin(d*x+c))^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx + c) + a)^{\frac{4}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(4/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((a\sin(dx + c) + a)^{\frac{4}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] integral((a*sin(d*x + c) + a)^(4/3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**(4/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx+c)+a)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(4/3), x)
```

3.102 $\int \csc(c+dx)(a+a\sin(c+dx))^{4/3}\,dx$

Optimal. Leaf size=78

$$-\frac{2\ 2^{5/6}a\cos(c+dx)\sqrt[3]{a\sin(c+dx)+a}F_1\left(\frac{1}{2};1,-\frac{5}{6};\frac{3}{2};1-\sin(c+dx),\frac{1}{2}(1-\sin(c+dx))\right)}{d(\sin(c+dx)+1)^{5/6}}$$

```
[Out] (-2*2^(5/6)*a*AppellF1[1/2, 1, -5/6, 3/2, 1 - Sin[c + d*x], (1 - Sin[c + d*
x])/2]*Cos[c + d*x]*(a + a*Sin[c + d*x])^(1/3))/(d*(1 + Sin[c + d*x])^(5/6)
)
```

Rubi [A] time = 0.11369, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2787, 2785, 130, 429}

$$-\frac{2\ 2^{5/6}a\cos(c+dx)\sqrt[3]{a\sin(c+dx)+a}F_1\left(\frac{1}{2};1,-\frac{5}{6};\frac{3}{2};1-\sin(c+dx),\frac{1}{2}(1-\sin(c+dx))\right)}{d(\sin(c+dx)+1)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*(a + a*Sin[c + d*x])^(4/3),x]
```

```
[Out] (-2*2^(5/6)*a*AppellF1[1/2, 1, -5/6, 3/2, 1 - Sin[c + d*x], (1 - Sin[c + d*
x])/2]*Cos[c + d*x]*(a + a*Sin[c + d*x])^(1/3))/(d*(1 + Sin[c + d*x])^(5/6)
)
```

Rule 2787

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m
])/(1 + (b*Sin[e + f*x])/a)^FracPart[m], Int[(1 + (b*Sin[e + f*x])/a)^m*(d*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 2785

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Dist[(b*(d/b)^n*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a - x)^n*(2*a - x)^(m - 1/2)
)/Sqrt[x], x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

Rule 130

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + (b*x^k)/e)^m*(c + (d*x^k)/e)^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \csc(c+dx)(a+a\sin(c+dx))^{4/3}\,dx = \frac{\left(a\sqrt[3]{a+a\sin(c+dx)}\right)\int \csc(c+dx)(1+\sin(c+dx))^{4/3}\,dx}{\sqrt[3]{1+\sin(c+dx)}}$$
$$= -\frac{\left(a\cos(c+dx)\sqrt[3]{a+a\sin(c+dx)}\right)\text{Subst}\left(\int \frac{(2-x)^{5/6}}{(1-x)\sqrt{x}}\,dx,x,1-\sin(c+dx)\right)}{d\sqrt{1-\sin(c+dx)}(1+\sin(c+dx))^{5/6}}$$
$$= -\frac{\left(2a\cos(c+dx)\sqrt[3]{a+a\sin(c+dx)}\right)\text{Subst}\left(\int \frac{\left(2-x^2\right)^{5/6}}{1-x^2}\,dx,x,\sqrt{1-\sin(c+dx)}\right)}{d\sqrt{1-\sin(c+dx)}(1+\sin(c+dx))^{5/6}}$$
$$= -\frac{2\,2^{5/6}aF_1\left(\frac{1}{2};1,-\frac{5}{6};\frac{3}{2};1-\sin(c+dx),\frac{1}{2}(1-\sin(c+dx))\right)\cos(c+dx)\sqrt[3]{a+a\,\mathrm{si}}}{d(1+\sin(c+dx))^{5/6}}$$

Mathematica [C] time = 9.51563, size = 2791, normalized size = 35.78

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[c + d*x]*(a + a*Sin[c + d*x])^(4/3),x]
```

```
[Out] (3*(a*(1 + Sin[c + d*x]))^(4/3))/(d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2
) - ((15 + 15*I)*AppellF1[2/3, 1/3, 1/3, 5/3, (1/2 + I/2)*(1 + Cot[(c + d*x
)/2]), (1/2 - I/2)*(1 + Cot[(c + d*x)/2])]*(a*(1 + Sin[c + d*x]))^(4/3)*(1
+ Tan[(c + d*x)/2]))/(d*((5 + 5*I)*AppellF1[2/3, 1/3, 1/3, 5/3, (1/2 + I/2)
*(1 + Cot[(c + d*x)/2]), (1/2 - I/2)*(1 + Cot[(c + d*x)/2])]*Sec[(c + d*x)/
2] + AppellF1[5/3, 1/3, 4/3, 8/3, (1/2 + I/2)*(1 + Cot[(c + d*x)/2]), (1/2
- I/2)*(1 + Cot[(c + d*x)/2])]*(Csc[(c + d*x)/2] + Sec[(c + d*x)/2]) + I*Ap
pellF1[5/3, 4/3, 1/3, 8/3, (1/2 + I/2)*(1 + Cot[(c + d*x)/2]), (1/2 - I/2)*
(1 + Cot[(c + d*x)/2])]*(Csc[(c + d*x)/2] + Sec[(c + d*x)/2]))*(Cos[(c + d*
x)/2] + Sin[(c + d*x)/2])^3) + ((15/2 + (15*I)/2)*AppellF1[2/3, 1/3, 1/3, 5
/3, (1/2 + I/2)*(1 + Tan[(c + d*x)/2]), (1/2 - I/2)*(1 + Tan[(c + d*x)/2])]
*(a*(1 + Sin[c + d*x]))^(4/3))/(d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2*(
(5 + 5*I)*AppellF1[2/3, 1/3, 1/3, 5/3, (1/2 + I/2)*(1 + Tan[(c + d*x)/2]),
(1/2 - I/2)*(1 + Tan[(c + d*x)/2])] + (AppellF1[5/3, 1/3, 4/3, 8/3, (1/2 +
I/2)*(1 + Tan[(c + d*x)/2]), (1/2 - I/2)*(1 + Tan[(c + d*x)/2])] + I*Appell
F1[5/3, 4/3, 1/3, 8/3, (1/2 + I/2)*(1 + Tan[(c + d*x)/2]), (1/2 - I/2)*(1 +
 Tan[(c + d*x)/2])])*(1 + Tan[(c + d*x)/2]))) - (3*Cos[(3*(c + d*x))/2]*Csc
[c + d*x]*(a*(1 + Sin[c + d*x]))^(4/3)*((1 + Tan[(c + d*x)/2])/Sqrt[Sec[(c
+ d*x)/2]^2])^(2/3)*(8 + (1 + I)*2^(2/3)*(((1 - I)*(I + Cot[(c + d*x)/2]))/
(1 + Cot[(c + d*x)/2]))^(1/3)*Hypergeometric2F1[1/3, 2/3, 5/3, ((1 + I) + (
1 - I)*Tan[(c + d*x)/2])/(2 + 2*Tan[(c + d*x)/2])]*(I + Tan[(c + d*x)/2]) -
 AppellF1[2/3, 1/3, 1/3, 5/3, (1/2 + I/2)*(1 + Cot[(c + d*x)/2]), (1/2 - I/
2)*(1 + Cot[(c + d*x)/2])]*((2 + 2*I) - (2 - 2*I)*Cot[(c + d*x)/2])^(1/3)*(
(-1 - I)*(I + Cot[(c + d*x)/2]))^(1/3)*(1 + Tan[(c + d*x)/2])))/(4*d*(Cos[(
c + d*x)/2] + Sin[(c + d*x)/2])^3*(1 + Tan[(c + d*x)/2])*((-3*Sec[(c + d*x)
/2]^2*((1 + Tan[(c + d*x)/2])/Sqrt[Sec[(c + d*x)/2]^2])^(2/3)*(8 + (1 + I)*
2^(2/3)*(((1 - I)*(I + Cot[(c + d*x)/2]))/(1 + Cot[(c + d*x)/2]))^(1/3)*Hyp
ergeometric2F1[1/3, 2/3, 5/3, ((1 + I) + (1 - I)*Tan[(c + d*x)/2])/(2 + 2*T
an[(c + d*x)/2])]*(I + Tan[(c + d*x)/2]) - AppellF1[2/3, 1/3, 1/3, 5/3, (1/
2 + I/2)*(1 + Cot[(c + d*x)/2]), (1/2 - I/2)*(1 + Cot[(c + d*x)/2])]*((2 +
2*I) - (2 - 2*I)*Cot[(c + d*x)/2])^(1/3)*((-1 - I)*(I + Cot[(c + d*x)/2]))^
(1/3)*(1 + Tan[(c + d*x)/2])))/(8*(1 + Tan[(c + d*x)/2])^2) + ((8 + (1 + I)
*2^(2/3)*(((1 - I)*(I + Cot[(c + d*x)/2]))/(1 + Cot[(c + d*x)/2]))^(1/3)*Hy
pergeometric2F1[1/3, 2/3, 5/3, ((1 + I) + (1 - I)*Tan[(c + d*x)/2])/(2 + 2*
```

```
Tan[(c + d*x)/2])]*(I + Tan[(c + d*x)/2]) - AppellF1[2/3, 1/3, 1/3, 5/3, (1
/2 + I/2)*(1 + Cot[(c + d*x)/2]), (1/2 - I/2)*(1 + Cot[(c + d*x)/2])]*((2 +
 2*I) - (2 - 2*I)*Cot[(c + d*x)/2])^(1/3)*((-1 - I)*(I + Cot[(c + d*x)/2]))
^(1/3)*(1 + Tan[(c + d*x)/2]))*(Sqrt[Sec[(c + d*x)/2]^2]/2 - (Tan[(c + d*x)
/2]*(1 + Tan[(c + d*x)/2]))/(2*Sqrt[Sec[(c + d*x)/2]^2])))/(2*(1 + Tan[(c +
 d*x)/2])*((1 + Tan[(c + d*x)/2])/Sqrt[Sec[(c + d*x)/2]^2])^(1/3)) + (3*((1
 + Tan[(c + d*x)/2])/Sqrt[Sec[(c + d*x)/2]^2])^(2/3)*(-(AppellF1[2/3, 1/3,
1/3, 5/3, (1/2 + I/2)*(1 + Cot[(c + d*x)/2]), (1/2 - I/2)*(1 + Cot[(c + d*x
)/2])]*((2 + 2*I) - (2 - 2*I)*Cot[(c + d*x)/2])^(1/3)*((-1 - I)*(I + Cot[(c
 + d*x)/2]))^(1/3)*Sec[(c + d*x)/2]^2)/2 + ((1 + I)*(((1 - I)*(I + Cot[(c +
 d*x)/2]))/(1 + Cot[(c + d*x)/2]))^(1/3)*Hypergeometric2F1[1/3, 2/3, 5/3, (
(1 + I) + (1 - I)*Tan[(c + d*x)/2])/(2 + 2*Tan[(c + d*x)/2])]*Sec[(c + d*x)
/2]^2)/2^(1/3) + ((1/3 + I/3)*2^(2/3)*(((1/2 - I/2)*(I + Cot[(c + d*x)/2])*
Csc[(c + d*x)/2]^2)/(1 + Cot[(c + d*x)/2])^2 - ((1/2 - I/2)*Csc[(c + d*x)/2
]^2)/(1 + Cot[(c + d*x)/2]))*Hypergeometric2F1[1/3, 2/3, 5/3, ((1 + I) + (1
 - I)*Tan[(c + d*x)/2])/(2 + 2*Tan[(c + d*x)/2])]*(I + Tan[(c + d*x)/2]))/(
((1 - I)*(I + Cot[(c + d*x)/2]))/(1 + Cot[(c + d*x)/2]))^(2/3) - ((1/6 + I/
6)*AppellF1[2/3, 1/3, 1/3, 5/3, (1/2 + I/2)*(1 + Cot[(c + d*x)/2]), (1/2 -
I/2)*(1 + Cot[(c + d*x)/2])]*((2 + 2*I) - (2 - 2*I)*Cot[(c + d*x)/2])^(1/3)
*Csc[(c + d*x)/2]^2*(1 + Tan[(c + d*x)/2]))/((-1 - I)*(I + Cot[(c + d*x)/2]
))^(2/3) - ((1/3 - I/3)*AppellF1[2/3, 1/3, 1/3, 5/3, (1/2 + I/2)*(1 + Cot[(
c + d*x)/2]), (1/2 - I/2)*(1 + Cot[(c + d*x)/2])]*((-1 - I)*(I + Cot[(c + d
*x)/2]))^(1/3)*Csc[(c + d*x)/2]^2*(1 + Tan[(c + d*x)/2]))/((2 + 2*I) - (2 -
 2*I)*Cot[(c + d*x)/2])^(2/3) - ((2 + 2*I) - (2 - 2*I)*Cot[(c + d*x)/2])^(1
/3)*((-1 - I)*(I + Cot[(c + d*x)/2]))^(1/3)*((-1/30 + I/30)*AppellF1[5/3, 1
/3, 4/3, 8/3, (1/2 + I/2)*(1 + Cot[(c + d*x)/2]), (1/2 - I/2)*(1 + Cot[(c +
 d*x)/2])]*Csc[(c + d*x)/2]^2 - (1/30 + I/30)*AppellF1[5/3, 4/3, 1/3, 8/3,
(1/2 + I/2)*(1 + Cot[(c + d*x)/2]), (1/2 - I/2)*(1 + Cot[(c + d*x)/2])]*Csc
[(c + d*x)/2]^2)*(1 + Tan[(c + d*x)/2]) + ((2/3 + (2*I)/3)*2^(2/3)*(((1 - I
)*(I + Cot[(c + d*x)/2]))/(1 + Cot[(c + d*x)/2]))^(1/3)*(I + Tan[(c + d*x)/
2])*(2 + 2*Tan[(c + d*x)/2])*(-((Sec[(c + d*x)/2]^2*((1 + I) + (1 - I)*Tan[
(c + d*x)/2]))/(2 + 2*Tan[(c + d*x)/2])^2) + ((1/2 - I/2)*Sec[(c + d*x)/2]^
2)/(2 + 2*Tan[(c + d*x)/2]))*(-Hypergeometric2F1[1/3, 2/3, 5/3, ((1 + I) +
(1 - I)*Tan[(c + d*x)/2])/(2 + 2*Tan[(c + d*x)/2])] + (1 - ((1 + I) + (1 -
I)*Tan[(c + d*x)/2])/(2 + 2*Tan[(c + d*x)/2]))^(-1/3)))/((1 + I) + (1 - I)*
Tan[(c + d*x)/2])))/(4*(1 + Tan[(c + d*x)/2]))))
```

Maple [F] time = 0.096, size = 0, normalized size = 0.

$$\int \csc(dx + c)(a + a\sin(dx + c))^{\frac{4}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*(a+a*sin(d*x+c))^(4/3),x)
```

```
[Out] int(csc(d*x+c)*(a+a*sin(d*x+c))^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx + c) + a)^{\frac{4}{3}} \csc(dx + c)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sin(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(4/3)*csc(d*x + c), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sin(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sin(d*x+c))**(4/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx+c)+a)^{\frac{4}{3}}\csc(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sin(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(4/3)*csc(d*x + c), x)
```

3.103 $\int \csc^2(c+dx)(a+a\sin(c+dx))^{4/3}\,dx$

Optimal. Leaf size=78

$$-\frac{2\,2^{5/6}a\cos(c+dx)\sqrt[3]{a\sin(c+dx)+a}F_1\left(\frac{1}{2};2,-\frac{5}{6};\frac{3}{2};1-\sin(c+dx),\frac{1}{2}(1-\sin(c+dx))\right)}{d(\sin(c+dx)+1)^{5/6}}$$

```
[Out] (-2*2^(5/6)*a*AppellF1[1/2, 2, -5/6, 3/2, 1 - Sin[c + d*x], (1 - Sin[c + d*
x])/2]*Cos[c + d*x]*(a + a*Sin[c + d*x])^(1/3))/(d*(1 + Sin[c + d*x])^(5/6)
)
```

Rubi [A] time = 0.129967, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2787, 2785, 130, 429}

$$-\frac{2\,2^{5/6}a\cos(c+dx)\sqrt[3]{a\sin(c+dx)+a}F_1\left(\frac{1}{2};2,-\frac{5}{6};\frac{3}{2};1-\sin(c+dx),\frac{1}{2}(1-\sin(c+dx))\right)}{d(\sin(c+dx)+1)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2*(a + a*Sin[c + d*x])^(4/3),x]
```

```
[Out] (-2*2^(5/6)*a*AppellF1[1/2, 2, -5/6, 3/2, 1 - Sin[c + d*x], (1 - Sin[c + d*
x])/2]*Cos[c + d*x]*(a + a*Sin[c + d*x])^(1/3))/(d*(1 + Sin[c + d*x])^(5/6)
)
```

Rule 2787

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m
])/(1 + (b*Sin[e + f*x])/a)^FracPart[m], Int[(1 + (b*Sin[e + f*x])/a)^m*(d*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 2785

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Dist[(b*(d/b)^n*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a - x)^n*(2*a - x)^(m - 1/2)
)/Sqrt[x], x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

Rule 130

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + (b*x^k)/e)^m*(c + (d*x^k)/e)^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \csc^2(c+dx)(a+a\sin(c+dx))^{4/3}\,dx = \frac{\left(a\sqrt[3]{a+a\sin(c+dx)}\right)\int \csc^2(c+dx)(1+\sin(c+dx))^{4/3}\,dx}{\sqrt[3]{1+\sin(c+dx)}}$$

$$= -\frac{\left(a\cos(c+dx)\sqrt[3]{a+a\sin(c+dx)}\right)\text{Subst}\left(\int \frac{(2-x)^{5/6}}{(1-x)^2\sqrt{x}}\,dx, x, 1-\sin(c+dx)\right)}{d\sqrt{1-\sin(c+dx)}(1+\sin(c+dx))^{5/6}}$$

$$= -\frac{\left(2a\cos(c+dx)\sqrt[3]{a+a\sin(c+dx)}\right)\text{Subst}\left(\int \frac{\left(2-x^2\right)^{5/6}}{\left(1-x^2\right)^2}\,dx, x, \sqrt{1-\sin(c+dx)}\right)}{d\sqrt{1-\sin(c+dx)}(1+\sin(c+dx))^{5/6}}$$

$$= -\frac{2\,2^{5/6}aF_1\left(\frac{1}{2};2,-\frac{5}{6};\frac{3}{2};1-\sin(c+dx),\frac{1}{2}(1-\sin(c+dx))\right)\cos(c+dx)\sqrt[3]{a+a\,\mathrm{s}}}{d(1+\sin(c+dx))^{5/6}}$$

Mathematica [C] time = 10.5367, size = 2800, normalized size = 35.9

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[c + d*x]^2*(a + a*Sin[c + d*x])^(4/3),x]
```

```
[Out] ((-1 - Cot[c + d*x])*(a*(1 + Sin[c + d*x]))^(4/3))/(d*(Cos[(c + d*x)/2] + S
in[(c + d*x)/2])^2) - ((15/2 + (15*I)/2)*AppellF1[2/3, 1/3, 1/3, 5/3, (1/2
+ I/2)*(1 + Cot[(c + d*x)/2]), (1/2 - I/2)*(1 + Cot[(c + d*x)/2])]*(a*(1 +
Sin[c + d*x]))^(4/3)*(1 + Tan[(c + d*x)/2]))/(d*((5 + 5*I)*AppellF1[2/3, 1/
3, 1/3, 5/3, (1/2 + I/2)*(1 + Cot[(c + d*x)/2]), (1/2 - I/2)*(1 + Cot[(c +
d*x)/2])]*Sec[(c + d*x)/2] + AppellF1[5/3, 1/3, 4/3, 8/3, (1/2 + I/2)*(1 +
Cot[(c + d*x)/2]), (1/2 - I/2)*(1 + Cot[(c + d*x)/2])]*(Csc[(c + d*x)/2] +
Sec[(c + d*x)/2]) + I*AppellF1[5/3, 4/3, 1/3, 8/3, (1/2 + I/2)*(1 + Cot[(c
+ d*x)/2]), (1/2 - I/2)*(1 + Cot[(c + d*x)/2])]*(Csc[(c + d*x)/2] + Sec[(c
+ d*x)/2]))*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3) + ((10 + 10*I)*AppellF
1[2/3, 1/3, 1/3, 5/3, (1/2 + I/2)*(1 + Tan[(c + d*x)/2]), (1/2 - I/2)*(1 +
Tan[(c + d*x)/2])]*(a*(1 + Sin[c + d*x]))^(4/3))/(d*(Cos[(c + d*x)/2] + Sin
[(c + d*x)/2])^2*((5 + 5*I)*AppellF1[2/3, 1/3, 1/3, 5/3, (1/2 + I/2)*(1 + T
an[(c + d*x)/2]), (1/2 - I/2)*(1 + Tan[(c + d*x)/2])] + (AppellF1[5/3, 1/3,
 4/3, 8/3, (1/2 + I/2)*(1 + Tan[(c + d*x)/2]), (1/2 - I/2)*(1 + Tan[(c + d*
x)/2])] + I*AppellF1[5/3, 4/3, 1/3, 8/3, (1/2 + I/2)*(1 + Tan[(c + d*x)/2])
, (1/2 - I/2)*(1 + Tan[(c + d*x)/2])])*(1 + Tan[(c + d*x)/2]))) + (Cos[(3*(
c + d*x))/2]*Csc[c + d*x]*(a*(1 + Sin[c + d*x]))^(4/3)*((1 + Tan[(c + d*x)/
2])/Sqrt[Sec[(c + d*x)/2]^2])^(2/3)*(8 + (1 + I)*2^(2/3)*(((1 - I)*(I + Cot
[(c + d*x)/2]))/(1 + Cot[(c + d*x)/2]))^(1/3)*Hypergeometric2F1[1/3, 2/3, 5
/3, ((1 + I) + (1 - I)*Tan[(c + d*x)/2])/(2 + 2*Tan[(c + d*x)/2])]*(I + Tan
[(c + d*x)/2]) - AppellF1[2/3, 1/3, 1/3, 5/3, (1/2 + I/2)*(1 + Cot[(c + d*x
)/2]), (1/2 - I/2)*(1 + Cot[(c + d*x)/2])]*((2 + 2*I) - (2 - 2*I)*Cot[(c +
d*x)/2])^(1/3)*((-1 - I)*(I + Cot[(c + d*x)/2]))^(1/3)*(1 + Tan[(c + d*x)/2
])))/(4*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3*(1 + Tan[(c + d*x)/2])*((
-3*Sec[(c + d*x)/2]^2*((1 + Tan[(c + d*x)/2])/Sqrt[Sec[(c + d*x)/2]^2])^(2/
3)*(8 + (1 + I)*2^(2/3)*(((1 - I)*(I + Cot[(c + d*x)/2]))/(1 + Cot[(c + d*x
)/2]))^(1/3)*Hypergeometric2F1[1/3, 2/3, 5/3, ((1 + I) + (1 - I)*Tan[(c + d
*x)/2])/(2 + 2*Tan[(c + d*x)/2])]*(I + Tan[(c + d*x)/2]) - AppellF1[2/3, 1/
3, 1/3, 5/3, (1/2 + I/2)*(1 + Cot[(c + d*x)/2]), (1/2 - I/2)*(1 + Cot[(c +
d*x)/2])]*((2 + 2*I) - (2 - 2*I)*Cot[(c + d*x)/2])^(1/3)*((-1 - I)*(I + Cot
[(c + d*x)/2]))^(1/3)*(1 + Tan[(c + d*x)/2])))/(8*(1 + Tan[(c + d*x)/2])^2)
 + ((8 + (1 + I)*2^(2/3)*(((1 - I)*(I + Cot[(c + d*x)/2]))/(1 + Cot[(c + d*
x)/2]))^(1/3)*Hypergeometric2F1[1/3, 2/3, 5/3, ((1 + I) + (1 - I)*Tan[(c +
```

```
d*x)/2])/(2 + 2*Tan[(c + d*x)/2])]*(I + Tan[(c + d*x)/2]) - AppellF1[2/3, 1
/3, 1/3, 5/3, (1/2 + I/2)*(1 + Cot[(c + d*x)/2]), (1/2 - I/2)*(1 + Cot[(c +
 d*x)/2])]*((2 + 2*I) - (2 - 2*I)*Cot[(c + d*x)/2])^(1/3)*((-1 - I)*(I + Co
t[(c + d*x)/2]))^(1/3)*(1 + Tan[(c + d*x)/2]))*(Sqrt[Sec[(c + d*x)/2]^2]/2
- (Tan[(c + d*x)/2]*(1 + Tan[(c + d*x)/2]))/(2*Sqrt[Sec[(c + d*x)/2]^2])))/
(2*(1 + Tan[(c + d*x)/2])*((1 + Tan[(c + d*x)/2])/Sqrt[Sec[(c + d*x)/2]^2])
^(1/3)) + (3*((1 + Tan[(c + d*x)/2])/Sqrt[Sec[(c + d*x)/2]^2])^(2/3)*(-(App
ellF1[2/3, 1/3, 1/3, 5/3, (1/2 + I/2)*(1 + Cot[(c + d*x)/2]), (1/2 - I/2)*(
1 + Cot[(c + d*x)/2])]*((2 + 2*I) - (2 - 2*I)*Cot[(c + d*x)/2])^(1/3)*((-1
- I)*(I + Cot[(c + d*x)/2]))^(1/3)*Sec[(c + d*x)/2]^2)/2 + ((1 + I)*(((1 -
I)*(I + Cot[(c + d*x)/2]))/(1 + Cot[(c + d*x)/2]))^(1/3)*Hypergeometric2F1[
1/3, 2/3, 5/3, ((1 + I) + (1 - I)*Tan[(c + d*x)/2])/(2 + 2*Tan[(c + d*x)/2]
)]*Sec[(c + d*x)/2]^2)/2^(1/3) + ((1/3 + I/3)*2^(2/3)*(((1/2 - I/2)*(I + Co
t[(c + d*x)/2])*Csc[(c + d*x)/2]^2)/(1 + Cot[(c + d*x)/2])^2 - ((1/2 - I/2)
*Csc[(c + d*x)/2]^2)/(1 + Cot[(c + d*x)/2]))*Hypergeometric2F1[1/3, 2/3, 5/
3, ((1 + I) + (1 - I)*Tan[(c + d*x)/2])/(2 + 2*Tan[(c + d*x)/2])]*(I + Tan[
(c + d*x)/2]))/(((1 - I)*(I + Cot[(c + d*x)/2]))/(1 + Cot[(c + d*x)/2]))^(2
/3) - ((1/6 + I/6)*AppellF1[2/3, 1/3, 1/3, 5/3, (1/2 + I/2)*(1 + Cot[(c + d
*x)/2]), (1/2 - I/2)*(1 + Cot[(c + d*x)/2])]*((2 + 2*I) - (2 - 2*I)*Cot[(c
+ d*x)/2])^(1/3)*Csc[(c + d*x)/2]^2*(1 + Tan[(c + d*x)/2]))/((-1 - I)*(I +
Cot[(c + d*x)/2]))^(2/3) - ((1/3 - I/3)*AppellF1[2/3, 1/3, 1/3, 5/3, (1/2 +
 I/2)*(1 + Cot[(c + d*x)/2]), (1/2 - I/2)*(1 + Cot[(c + d*x)/2])]*((-1 - I)
*(I + Cot[(c + d*x)/2]))^(1/3)*Csc[(c + d*x)/2]^2*(1 + Tan[(c + d*x)/2]))/(
(2 + 2*I) - (2 - 2*I)*Cot[(c + d*x)/2])^(2/3) - ((2 + 2*I) - (2 - 2*I)*Cot[
(c + d*x)/2])^(1/3)*((-1 - I)*(I + Cot[(c + d*x)/2]))^(1/3)*((-1/30 + I/30)
*AppellF1[5/3, 1/3, 4/3, 8/3, (1/2 + I/2)*(1 + Cot[(c + d*x)/2]), (1/2 - I/
2)*(1 + Cot[(c + d*x)/2])]*Csc[(c + d*x)/2]^2 - (1/30 + I/30)*AppellF1[5/3,
 4/3, 1/3, 8/3, (1/2 + I/2)*(1 + Cot[(c + d*x)/2]), (1/2 - I/2)*(1 + Cot[(c
 + d*x)/2])]*Csc[(c + d*x)/2]^2)*(1 + Tan[(c + d*x)/2]) + ((2/3 + (2*I)/3)*
2^(2/3)*(((1 - I)*(I + Cot[(c + d*x)/2]))/(1 + Cot[(c + d*x)/2]))^(1/3)*(I
+ Tan[(c + d*x)/2])*(2 + 2*Tan[(c + d*x)/2])*(-((Sec[(c + d*x)/2]^2*((1 + I
) + (1 - I)*Tan[(c + d*x)/2]))/(2 + 2*Tan[(c + d*x)/2])^2) + ((1/2 - I/2)*S
ec[(c + d*x)/2]^2)/(2 + 2*Tan[(c + d*x)/2]))*(-Hypergeometric2F1[1/3, 2/3,
5/3, ((1 + I) + (1 - I)*Tan[(c + d*x)/2])/(2 + 2*Tan[(c + d*x)/2])] + (1 -
((1 + I) + (1 - I)*Tan[(c + d*x)/2])/(2 + 2*Tan[(c + d*x)/2]))^(-1/3)))/((1
 + I) + (1 - I)*Tan[(c + d*x)/2])))/(4*(1 + Tan[(c + d*x)/2]))))
```

Maple [F] time = 0.102, size = 0, normalized size = 0.

$$\int (\csc(dx+c))^2 (a + a\sin(dx+c))^{\frac{4}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*(a+a*sin(d*x+c))^(4/3),x)
```

```
[Out] int(csc(d*x+c)^2*(a+a*sin(d*x+c))^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx+c) + a)^{\frac{4}{3}} \csc(dx+c)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+a*sin(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(4/3)*csc(d*x + c)^2, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+a*sin(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*(a+a*sin(d*x+c))**(4/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx+c)+a)^{\frac{4}{3}}\csc(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+a*sin(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(4/3)*csc(d*x + c)^2, x)
```

3.104 $\int \frac{\sin^3(c+dx)}{\sqrt[3]{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=161

$$\frac{37\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{40\,2^{5/6}d\sqrt[6]{\sin(c+dx)+1}\sqrt[3]{a\sin(c+dx)+a}}-\frac{3\sin^2(c+dx)\cos(c+dx)}{8d\sqrt[3]{a\sin(c+dx)+a}}+\frac{3\cos(c+dx)(a\sin(c+dx)+a)^{2/3}}{40ad}-\frac{}{80}$$

```
[Out] (-99*Cos[c + d*x])/(80*d*(a + a*Sin[c + d*x])^(1/3)) - (3*Cos[c + d*x]*Sin[
c + d*x]^2)/(8*d*(a + a*Sin[c + d*x])^(1/3)) + (37*Cos[c + d*x]*Hypergeomet
ric2F1[1/2, 5/6, 3/2, (1 - Sin[c + d*x])/2])/(40*2^(5/6)*d*(1 + Sin[c + d*x
])^(1/6)*(a + a*Sin[c + d*x])^(1/3)) + (3*Cos[c + d*x]*(a + a*Sin[c + d*x])
^(2/3))/(40*a*d)
```

Rubi [A] time = 0.252884, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2783, 2968, 3023, 2751, 2652, 2651}

$$\frac{37\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{40\,2^{5/6}d\sqrt[6]{\sin(c+dx)+1}\sqrt[3]{a\sin(c+dx)+a}}-\frac{3\sin^2(c+dx)\cos(c+dx)}{8d\sqrt[3]{a\sin(c+dx)+a}}+\frac{3\cos(c+dx)(a\sin(c+dx)+a)^{2/3}}{40ad}-\frac{}{80}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^3/(a + a*Sin[c + d*x])^(1/3),x]
```

```
[Out] (-99*Cos[c + d*x])/(80*d*(a + a*Sin[c + d*x])^(1/3)) - (3*Cos[c + d*x]*Sin[
c + d*x]^2)/(8*d*(a + a*Sin[c + d*x])^(1/3)) + (37*Cos[c + d*x]*Hypergeomet
ric2F1[1/2, 5/6, 3/2, (1 - Sin[c + d*x])/2])/(40*2^(5/6)*d*(1 + Sin[c + d*x
])^(1/6)*(a + a*Sin[c + d*x])^(1/3)) + (3*Cos[c + d*x]*(a + a*Sin[c + d*x])
^(2/3))/(40*a*d)
```

Rule 2783

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*(c + d*Sin[e + f*x])^(n - 1))/(f*(m + n)), x] + Dist[1/(b*(m + n)), Int[(
a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n - 2)*Simp[d*(a*c*m + b*d*(n -
 1)) + b*c^2*(m + n) + d*(a*d*m + b*c*(m + 2*n - 1))*Sin[e + f*x], x], x],
x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && IntegerQ[n]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
```

```
!LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2652

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Sin[c + d*x])^FracPart[n])/(1 + (b*Sin[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Sin[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2651

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(2^(n +
 1/2)*a^(n - 1/2)*b*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1*(1
 - (b*Sin[c + d*x])/a))/2])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sin^3(c+dx)}{\sqrt[3]{a+a\sin(c+dx)}}\,dx = -\frac{3\cos(c+dx)\sin^2(c+dx)}{8d\sqrt[3]{a+a\sin(c+dx)}}+\frac{3\int \frac{\sin(c+dx)\left(2a-\frac{1}{3}a\sin(c+dx)\right)}{\sqrt[3]{a+a\sin(c+dx)}}\,dx}{8a}$$

$$= -\frac{3\cos(c+dx)\sin^2(c+dx)}{8d\sqrt[3]{a+a\sin(c+dx)}}+\frac{3\int \frac{2a\sin(c+dx)-\frac{1}{3}a\sin^2(c+dx)}{\sqrt[3]{a+a\sin(c+dx)}}\,dx}{8a}$$

$$= -\frac{3\cos(c+dx)\sin^2(c+dx)}{8d\sqrt[3]{a+a\sin(c+dx)}}+\frac{3\cos(c+dx)(a+a\sin(c+dx))^{2/3}}{40ad}+\frac{9\int \frac{-\frac{2a^2}{9}+\frac{11}{3}a^2\sin(c+dx)}{\sqrt[3]{a+a\sin(c+dx)}}\,dx}{40a^2}$$

$$= -\frac{99\cos(c+dx)}{80d\sqrt[3]{a+a\sin(c+dx)}}-\frac{3\cos(c+dx)\sin^2(c+dx)}{8d\sqrt[3]{a+a\sin(c+dx)}}+\frac{3\cos(c+dx)(a+a\sin(c+dx))^{2/3}}{40ad}-$$

$$= -\frac{99\cos(c+dx)}{80d\sqrt[3]{a+a\sin(c+dx)}}-\frac{3\cos(c+dx)\sin^2(c+dx)}{8d\sqrt[3]{a+a\sin(c+dx)}}+\frac{3\cos(c+dx)(a+a\sin(c+dx))^{2/3}}{40ad}-$$

$$= -\frac{99\cos(c+dx)}{80d\sqrt[3]{a+a\sin(c+dx)}}-\frac{3\cos(c+dx)\sin^2(c+dx)}{8d\sqrt[3]{a+a\sin(c+dx)}}+\frac{37\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2};\frac{1}{2}(1-\sin\right.}{40\,2^{5/6}d\sqrt[6]{1+\sin(c+dx)}\sqrt[3]{a+a\sin}}$$

Mathematica [A] time = 0.45825, size = 110, normalized size = 0.68

$$\frac{3\cos(c+dx)\left(\sqrt{1-\sin(c+dx)}(2\sin(c+dx)+5\cos(2(c+dx))-36)-37\sqrt{2}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\sin^2\left(\frac{1}{4}(2c+2dx+\pi)\right)\right)\right)}{80d\sqrt{1-\sin(c+dx)}\sqrt[3]{a(\sin(c+dx)+1)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[c + d*x]^3/(a + a*Sin[c + d*x])^(1/3),x]
```

```
[Out] (3*Cos[c + d*x]*(-37*Sqrt[2]*Hypergeometric2F1[1/6, 1/2, 7/6, Sin[(2*c + Pi
 + 2*d*x)/4]^2] + Sqrt[1 - Sin[c + d*x]]*(-36 + 5*Cos[2*(c + d*x)] + 2*Sin[
```

```
c + d*x])))/(80*d*Sqrt[1 - Sin[c + d*x]]*(a*(1 + Sin[c + d*x]))^(1/3))
```

Maple [F] time = 0.286, size = 0, normalized size = 0.

$$\int (\sin(dx+c))^3 \frac{1}{\sqrt[3]{a+a\sin(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3/(a+a*sin(d*x+c))^(1/3),x)
```

```
[Out] int(sin(d*x+c)^3/(a+a*sin(d*x+c))^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(dx+c)^3}{(a\sin(dx+c)+a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+a*sin(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)^3/(a*sin(d*x + c) + a)^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\left(\cos(dx+c)^2-1\right)\sin(dx+c)}{(a\sin(dx+c)+a)^{\frac{1}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+a*sin(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] integral(-(cos(d*x + c)^2 - 1)*sin(d*x + c)/(a*sin(d*x + c) + a)^(1/3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3/(a+a*sin(d*x+c))**(1/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin (dx+c)^3}{(a\sin (dx+c)+a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+a*sin(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)^3/(a*sin(d*x + c) + a)^(1/3), x)
```

3.105 $\int \frac{\sin^2(c+dx)}{\sqrt[3]{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=126

$$-\frac{7\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{5\,2^{5/6}d\sqrt[6]{\sin(c+dx)+1}\sqrt[3]{a\sin(c+dx)+a}}-\frac{3\cos(c+dx)(a\sin(c+dx)+a)^{2/3}}{5ad}+\frac{9\cos(c+dx)}{10d\sqrt[3]{a\sin(c+dx)+a}}$$

```
[Out] (9*Cos[c + d*x])/(10*d*(a + a*Sin[c + d*x])^(1/3)) - (7*Cos[c + d*x]*Hyperg
eometric2F1[1/2, 5/6, 3/2, (1 - Sin[c + d*x])/2])/(5*2^(5/6)*d*(1 + Sin[c +
 d*x])^(1/6)*(a + a*Sin[c + d*x])^(1/3)) - (3*Cos[c + d*x]*(a + a*Sin[c + d
*x])^(2/3))/(5*a*d)
```

Rubi [A] time = 0.131674, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2759, 2751, 2652, 2651}

$$-\frac{7\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{5\,2^{5/6}d\sqrt[6]{\sin(c+dx)+1}\sqrt[3]{a\sin(c+dx)+a}}-\frac{3\cos(c+dx)(a\sin(c+dx)+a)^{2/3}}{5ad}+\frac{9\cos(c+dx)}{10d\sqrt[3]{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2/(a + a*Sin[c + d*x])^(1/3),x]
```

```
[Out] (9*Cos[c + d*x])/(10*d*(a + a*Sin[c + d*x])^(1/3)) - (7*Cos[c + d*x]*Hyperg
eometric2F1[1/2, 5/6, 3/2, (1 - Sin[c + d*x])/2])/(5*2^(5/6)*d*(1 + Sin[c +
 d*x])^(1/6)*(a + a*Sin[c + d*x])^(1/3)) - (3*Cos[c + d*x]*(a + a*Sin[c + d
*x])^(2/3))/(5*a*d)
```

Rule 2759

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin[e
 + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ
[m, -2^(-1)]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2652

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Sin[c + d*x])^FracPart[n])/(1 + (b*Sin[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Sin[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2651

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(2^(n +
 1/2)*a^(n - 1/2)*b*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1*(1
```

```
 - (b*Sin[c + d*x])/a))/2])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^2(c+dx)}{\sqrt[3]{a+a\sin(c+dx)}}dx &= -\frac{3\cos(c+dx)(a+a\sin(c+dx))^{2/3}}{5ad}+\frac{3\int \frac{\frac{2a}{3}-a\sin(c+dx)}{\sqrt[3]{a+a\sin(c+dx)}}dx}{5a}\\ &= \frac{9\cos(c+dx)}{10d\sqrt[3]{a+a\sin(c+dx)}}-\frac{3\cos(c+dx)(a+a\sin(c+dx))^{2/3}}{5ad}+\frac{7}{10}\int \frac{1}{\sqrt[3]{a+a\sin(c+dx)}}dx\\ &= \frac{9\cos(c+dx)}{10d\sqrt[3]{a+a\sin(c+dx)}}-\frac{3\cos(c+dx)(a+a\sin(c+dx))^{2/3}}{5ad}+\frac{\left(7\sqrt[3]{1+\sin(c+dx)}\right)\int \frac{1}{\sqrt[3]{1+\sin(}}}{10\sqrt[3]{a+a\sin(c+dx)}}\\ &= \frac{9\cos(c+dx)}{10d\sqrt[3]{a+a\sin(c+dx)}}-\frac{7\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{5\ 2^{5/6}d\sqrt[6]{1+\sin(c+dx)}\sqrt[3]{a+a\sin(c+dx)}}-\frac{3\cos(c+dx)(a+a}{5ad}\end{aligned}$$

Mathematica [A] time = 0.296551, size = 95, normalized size = 0.75

$$-\frac{3\cos(c+dx)\left(\sqrt{2-2\sin(c+dx)}(2\sin(c+dx)-1)-14\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\sin^2\left(\frac{1}{4}(2c+2dx+\pi)\right)\right)\right)}{10d\sqrt{2-2\sin(c+dx)}\sqrt[3]{a(\sin(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2/(a + a*Sin[c + d*x])^(1/3),x]
```

```
[Out] (-3*Cos[c + d*x]*(-14*Hypergeometric2F1[1/6, 1/2, 7/6, Sin[(2*c + Pi + 2*d*
x)/4]^2] + Sqrt[2 - 2*Sin[c + d*x]]*(-1 + 2*Sin[c + d*x])))/(10*d*Sqrt[2 -
2*Sin[c + d*x]]*(a*(1 + Sin[c + d*x]))^(1/3))
```

Maple [F] time = 0.162, size = 0, normalized size = 0.

$$\int (\sin(dx+c))^2 \frac{1}{\sqrt[3]{a+a\sin(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2/(a+a*sin(d*x+c))^(1/3),x)
```

```
[Out] int(sin(d*x+c)^2/(a+a*sin(d*x+c))^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(dx+c)^2}{(a\sin(dx+c)+a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+a*sin(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)^2/(a*sin(d*x + c) + a)^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\cos\left(dx+c\right)^2-1}{\left(a\sin\left(dx+c\right)+a\right)^{\frac{1}{3}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+a*sin(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] integral(-(cos(d*x + c)^2 - 1)/(a*sin(d*x + c) + a)^(1/3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin^2\left(c+dx\right)}{\sqrt[3]{a\left(\sin\left(c+dx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2/(a+a*sin(d*x+c))**(1/3),x)
```

```
[Out] Integral(sin(c + d*x)**2/(a*(sin(c + d*x) + 1))**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(dx+c\right)^2}{\left(a\sin\left(dx+c\right)+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+a*sin(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)^2/(a*sin(d*x + c) + a)^(1/3), x)
```

3.106 $\int \frac{\sin(c+dx)}{\sqrt[3]{a+a\sin(c+dx)}} dx$

Optimal. Leaf size=93

$$\frac{\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{2^{5/6}d\sqrt[6]{\sin(c+dx)+1}\sqrt[3]{a\sin(c+dx)+a}}-\frac{3\cos(c+dx)}{2d\sqrt[3]{a\sin(c+dx)+a}}$$

```
[Out] (-3*Cos[c + d*x])/(2*d*(a + a*Sin[c + d*x])^(1/3)) + (Cos[c + d*x]*Hypergeo
metric2F1[1/2, 5/6, 3/2, (1 - Sin[c + d*x])/2])/(2^(5/6)*d*(1 + Sin[c + d*x
])^(1/6)*(a + a*Sin[c + d*x])^(1/3))
```

Rubi [A] time = 0.0626866, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2751, 2652, 2651}

$$\frac{\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{2^{5/6}d\sqrt[6]{\sin(c+dx)+1}\sqrt[3]{a\sin(c+dx)+a}}-\frac{3\cos(c+dx)}{2d\sqrt[3]{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(a + a*Sin[c + d*x])^(1/3),x]
```

```
[Out] (-3*Cos[c + d*x])/(2*d*(a + a*Sin[c + d*x])^(1/3)) + (Cos[c + d*x]*Hypergeo
metric2F1[1/2, 5/6, 3/2, (1 - Sin[c + d*x])/2])/(2^(5/6)*d*(1 + Sin[c + d*x
])^(1/6)*(a + a*Sin[c + d*x])^(1/3))
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2652

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Sin[c + d*x])^FracPart[n])/(1 + (b*Sin[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Sin[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2651

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(2^(n +
 1/2)*a^(n - 1/2)*b*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1*(1
 - (b*Sin[c + d*x])/a))/2])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sin(c+dx)}{\sqrt[3]{a+a\sin(c+dx)}}\,dx = -\frac{3\cos(c+dx)}{2d\sqrt[3]{a+a\sin(c+dx)}} - \frac{1}{2}\int \frac{1}{\sqrt[3]{a+a\sin(c+dx)}}\,dx$$
$$= -\frac{3\cos(c+dx)}{2d\sqrt[3]{a+a\sin(c+dx)}} - \frac{\sqrt[3]{1+\sin(c+dx)}\int \frac{1}{\sqrt[3]{1+\sin(c+dx)}}\,dx}{2\sqrt[3]{a+a\sin(c+dx)}}$$
$$= -\frac{3\cos(c+dx)}{2d\sqrt[3]{a+a\sin(c+dx)}} + \frac{\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{2^{5/6}d\sqrt[6]{1+\sin(c+dx)}\sqrt[3]{a+a\sin(c+dx)}}$$

Mathematica [A] time = 0.148261, size = 84, normalized size = 0.9

$$-\frac{3\cos(c+dx)\left(2\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\sin^2\left(\frac{1}{4}(2c+2dx+\pi)\right)\right)+\sqrt{2-2\sin(c+dx)}\right)}{2d\sqrt{2-2\sin(c+dx)}\sqrt[3]{a(\sin(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]/(a + a*Sin[c + d*x])^(1/3),x]
```

```
[Out] (-3*Cos[c + d*x]*(2*Hypergeometric2F1[1/6, 1/2, 7/6, Sin[(2*c + Pi + 2*d*x)
/4]^2] + Sqrt[2 - 2*Sin[c + d*x]]))/(2*d*Sqrt[2 - 2*Sin[c + d*x]]*(a*(1 + S
in[c + d*x]))^(1/3))
```

Maple [F] time = 0.132, size = 0, normalized size = 0.

$$\int \sin(dx+c)\frac{1}{\sqrt[3]{a+a\sin(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/(a+a*sin(d*x+c))^(1/3),x)
```

```
[Out] int(sin(d*x+c)/(a+a*sin(d*x+c))^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(dx+c)}{(a\sin(dx+c)+a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+a*sin(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)/(a*sin(d*x + c) + a)^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sin(dx+c)}{(a\sin(dx+c)+a)^{\frac{1}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+a*sin(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] integral(sin(d*x + c)/(a*sin(d*x + c) + a)^(1/3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(c+dx)}{\sqrt[3]{a(\sin(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+a*sin(d*x+c))**(1/3),x)
```

```
[Out] Integral(sin(c + d*x)/(a*(sin(c + d*x) + 1))**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(dx+c)}{(a\sin(dx+c)+a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+a*sin(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)/(a*sin(d*x + c) + a)^(1/3), x)
```

3.107 $\int \frac{1}{\sqrt[3]{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=66

$$-\frac{\sqrt[6]{2}\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{d\sqrt[6]{\sin(c+dx)+1}\sqrt[3]{a\sin(c+dx)+a}}$$

```
[Out] -((2^(1/6)*Cos[c + d*x]*Hypergeometric2F1[1/2, 5/6, 3/2, (1 - Sin[c + d*x])
/2])/(d*(1 + Sin[c + d*x])^(1/6)*(a + a*Sin[c + d*x])^(1/3)))
```

Rubi [A] time = 0.0309078, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2652, 2651}

$$-\frac{\sqrt[6]{2}\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{d\sqrt[6]{\sin(c+dx)+1}\sqrt[3]{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^(-1/3),x]
```

```
[Out] -((2^(1/6)*Cos[c + d*x]*Hypergeometric2F1[1/2, 5/6, 3/2, (1 - Sin[c + d*x])
/2])/(d*(1 + Sin[c + d*x])^(1/6)*(a + a*Sin[c + d*x])^(1/3)))
```

Rule 2652

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Sin[c + d*x])^FracPart[n])/(1 + (b*Sin[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Sin[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2651

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(2^(n +
 1/2)*a^(n - 1/2)*b*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1*(1
 - (b*Sin[c + d*x])/a))/2])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt[3]{a+a\sin(c+dx)}}\,dx &= \frac{\sqrt[3]{1+\sin(c+dx)}\int \frac{1}{\sqrt[3]{1+\sin(c+dx)}}\,dx}{\sqrt[3]{a+a\sin(c+dx)}}\\ &= -\frac{\sqrt[6]{2}\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{d\sqrt[6]{1+\sin(c+dx)}\sqrt[3]{a+a\sin(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0994563, size = 70, normalized size = 1.06

$$\frac{3\sqrt{2}\cos(c+dx)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\sin^2\left(\frac{1}{4}(2c+2dx+\pi)\right)\right)}{d\sqrt{1-\sin(c+dx)}\sqrt[3]{a(\sin(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^(-1/3),x]
```

```
[Out] (3*Sqrt[2]*Cos[c + d*x]*Hypergeometric2F1[1/6, 1/2, 7/6, Sin[(2*c + Pi + 2*
d*x)/4]^2])/(d*Sqrt[1 - Sin[c + d*x]]*(a*(1 + Sin[c + d*x]))^(1/3))
```

Maple [F] time = 0.006, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{a+a\sin(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(d*x+c))^(1/3),x)
```

```
[Out] int(1/(a+a*sin(d*x+c))^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\sin(dx+c)+a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(-1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(a\sin(dx+c)+a)^{\frac{1}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] integral((a*sin(d*x + c) + a)^(-1/3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{a\sin(c+dx)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))**(1/3),x)
```

```
[Out] Integral((a*sin(c + d*x) + a)**(-1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\sin(dx+c)+a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(-1/3), x)
```

3.108 $\int \frac{\csc(c+dx)}{\sqrt[3]{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=77

$$-\frac{\sqrt[6]{2}\cos(c+dx)F_1\left(\frac{1}{2};1,\frac{5}{6};\frac{3}{2};1-\sin(c+dx),\frac{1}{2}(1-\sin(c+dx))\right)}{d\sqrt[6]{\sin(c+dx)+1}\sqrt[3]{a\sin(c+dx)+a}}$$

```
[Out] -((2^(1/6)*AppellF1[1/2, 1, 5/6, 3/2, 1 - Sin[c + d*x], (1 - Sin[c + d*x])/
2]*Cos[c + d*x])/(d*(1 + Sin[c + d*x])^(1/6)*(a + a*Sin[c + d*x])^(1/3)))
```

Rubi [A] time = 0.106019, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2787, 2785, 130, 429}

$$-\frac{\sqrt[6]{2}\cos(c+dx)F_1\left(\frac{1}{2};1,\frac{5}{6};\frac{3}{2};1-\sin(c+dx),\frac{1}{2}(1-\sin(c+dx))\right)}{d\sqrt[6]{\sin(c+dx)+1}\sqrt[3]{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]/(a + a*Sin[c + d*x])^(1/3),x]
```

```
[Out] -((2^(1/6)*AppellF1[1/2, 1, 5/6, 3/2, 1 - Sin[c + d*x], (1 - Sin[c + d*x])/
2]*Cos[c + d*x])/(d*(1 + Sin[c + d*x])^(1/6)*(a + a*Sin[c + d*x])^(1/3)))
```

Rule 2787

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m
])/(1 + (b*Sin[e + f*x])/a)^FracPart[m], Int[(1 + (b*Sin[e + f*x])/a)^m*(d*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 2785

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Dist[(b*(d/b)^n*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a - x)^n*(2*a - x)^(m - 1/2)
)/Sqrt[x], x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

Rule 130

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + (b*x^k)/e)^m*(c + (d*x^k)/e)^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\csc(c+dx)}{\sqrt[3]{a+a\sin(c+dx)}}\,dx = \frac{\sqrt[3]{1+\sin(c+dx)}\int \frac{\csc(c+dx)}{\sqrt[3]{1+\sin(c+dx)}}\,dx}{\sqrt[3]{a+a\sin(c+dx)}}$$
$$= -\frac{\cos(c+dx)\,\text{Subst}\left(\int \frac{1}{(1-x)(2-x)^{5/6}\sqrt{x}}\,dx, x, 1-\sin(c+dx)\right)}{d\sqrt{1-\sin(c+dx)}\sqrt[6]{1+\sin(c+dx)}\sqrt[3]{a+a\sin(c+dx)}}$$
$$= -\frac{(2\cos(c+dx))\,\text{Subst}\left(\int \frac{1}{(1-x^2)(2-x^2)^{5/6}}\,dx, x, \sqrt{1-\sin(c+dx)}\right)}{d\sqrt{1-\sin(c+dx)}\sqrt[6]{1+\sin(c+dx)}\sqrt[3]{a+a\sin(c+dx)}}$$
$$= -\frac{\sqrt[6]{2}F_1\left(\frac{1}{2};1,\frac{5}{6};\frac{3}{2};1-\sin(c+dx),\frac{1}{2}(1-\sin(c+dx))\right)\cos(c+dx)}{d\sqrt[6]{1+\sin(c+dx)}\sqrt[3]{a+a\sin(c+dx)}}$$

Mathematica [F] time = 3.43862, size = 0, normalized size = 0.

$$\int \frac{\csc(c+dx)}{\sqrt[3]{a+a\sin(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Csc[c + d*x]/(a + a*Sin[c + d*x])^(1/3),x]
```

```
[Out] Integrate[Csc[c + d*x]/(a + a*Sin[c + d*x])^(1/3), x]
```

Maple [F] time = 0.096, size = 0, normalized size = 0.

$$\int \csc(dx+c)\frac{1}{\sqrt[3]{a+a\sin(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)/(a+a*sin(d*x+c))^(1/3),x)
```

```
[Out] int(csc(d*x+c)/(a+a*sin(d*x+c))^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(dx+c)}{(a\sin(dx+c)+a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+a*sin(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(csc(d*x + c)/(a*sin(d*x + c) + a)^(1/3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+a*sin(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(c+dx)}{\sqrt[3]{a(\sin(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+a*sin(d*x+c))**(1/3),x)
```

```
[Out] Integral(csc(c + d*x)/(a*(sin(c + d*x) + 1))**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(dx+c)}{(a\sin(dx+c)+a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+a*sin(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate(csc(d*x + c)/(a*sin(d*x + c) + a)^(1/3), x)
```

3.109 $\int \frac{\csc^2(c+dx)}{\sqrt[3]{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=77

$$-\frac{\sqrt[6]{2}\cos(c+dx)F_1\left(\frac{1}{2};2,\frac{5}{6};\frac{3}{2};1-\sin(c+dx),\frac{1}{2}(1-\sin(c+dx))\right)}{d\sqrt[6]{\sin(c+dx)+1}\sqrt[3]{a\sin(c+dx)+a}}$$

```
[Out] -((2^(1/6)*AppellF1[1/2, 2, 5/6, 3/2, 1 - Sin[c + d*x], (1 - Sin[c + d*x])/
2]*Cos[c + d*x])/(d*(1 + Sin[c + d*x])^(1/6)*(a + a*Sin[c + d*x])^(1/3)))
```

Rubi [A] time = 0.119338, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2787, 2785, 130, 429}

$$-\frac{\sqrt[6]{2}\cos(c+dx)F_1\left(\frac{1}{2};2,\frac{5}{6};\frac{3}{2};1-\sin(c+dx),\frac{1}{2}(1-\sin(c+dx))\right)}{d\sqrt[6]{\sin(c+dx)+1}\sqrt[3]{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2/(a + a*Sin[c + d*x])^(1/3),x]
```

```
[Out] -((2^(1/6)*AppellF1[1/2, 2, 5/6, 3/2, 1 - Sin[c + d*x], (1 - Sin[c + d*x])/
2]*Cos[c + d*x])/(d*(1 + Sin[c + d*x])^(1/6)*(a + a*Sin[c + d*x])^(1/3)))
```

Rule 2787

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m
])/(1 + (b*Sin[e + f*x])/a)^FracPart[m], Int[(1 + (b*Sin[e + f*x])/a)^m*(d*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] && !IntegerQ[m] && !GtQ[a, 0]
```

Rule 2785

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Dist[(b*(d/b)^n*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a - x)^n*(2*a - x)^(m - 1/2)
)/Sqrt[x], x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] && !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

Rule 130

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + (b*x^k)/e)^m*(c + (d*x^k)/e)^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\csc^2(c+dx)}{\sqrt[3]{a+a\sin(c+dx)}}\,dx &= \frac{\sqrt[3]{1+\sin(c+dx)}\int \frac{\csc^2(c+dx)}{\sqrt[3]{1+\sin(c+dx)}}\,dx}{\sqrt[3]{a+a\sin(c+dx)}}\\
&= -\frac{\cos(c+dx)\,\text{Subst}\left(\int \frac{1}{(1-x)^2(2-x)^{5/6}\sqrt{x}}\,dx,x,1-\sin(c+dx)\right)}{d\sqrt{1-\sin(c+dx)}\sqrt[6]{1+\sin(c+dx)}\sqrt[3]{a+a\sin(c+dx)}}\\
&= -\frac{(2\cos(c+dx))\,\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)^2\left(2-x^2\right)^{5/6}}\,dx,x,\sqrt{1-\sin(c+dx)}\right)}{d\sqrt{1-\sin(c+dx)}\sqrt[6]{1+\sin(c+dx)}\sqrt[3]{a+a\sin(c+dx)}}\\
&= -\frac{\sqrt[6]{2}F_1\left(\frac{1}{2};2,\frac{5}{6};\frac{3}{2};1-\sin(c+dx),\frac{1}{2}(1-\sin(c+dx))\right)\cos(c+dx)}{d\sqrt[6]{1+\sin(c+dx)}\sqrt[3]{a+a\sin(c+dx)}}
\end{aligned}
$$

Mathematica [C] time = 8.7488, size = 184, normalized size = 2.39

$$
\frac{2\,2^{2/3}\cos^{\frac{2}{3}}\left(\frac{1}{4}(2c+2dx-\pi)\right)(\cos(c+dx)+i\sin(c+dx))\left(4i\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-ie^{-i(c+dx)}\right)\cos(c+dx)(\sin(c+dx)+i\cos(c+\right.}{5d\left(-(-1)^{3/4}e^{-\frac{1}{2}i(c+dx)}\left(e^{i(c+dx)}+i\right)\right)^{2/3}\left(1+e^{2i(c+dx)}\right)\sqrt[3]{a(\sin(c+dx)+1)}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[c + d*x]^2/(a + a*Sin[c + d*x])^(1/3),x]
```

```
[Out] (2*2^(2/3)*Cos[(2*c - Pi + 2*d*x)/4]^(2/3)*(Cos[c + d*x] + I*Sin[c + d*x])*
(1 + 4*Sin[c + d*x] + (4*I)*Cos[c + d*x]*Hypergeometric2F1[1/3, 2/3, 4/3, (
-I)/E^(I*(c + d*x))]*(1 + I*Cos[c + d*x] + Sin[c + d*x])^(2/3)))/(5*d*(-(((
-1)^(3/4)*(I + E^(I*(c + d*x))))/E^((I/2)*(c + d*x))))^(2/3)*(1 + E^((2*I)*
(c + d*x)))*(a*(1 + Sin[c + d*x]))^(1/3))
```

Maple [F] time = 0.105, size = 0, normalized size = 0.

$$
\int (\csc(dx+c))^2 \frac{1}{\sqrt[3]{a+a\sin(dx+c)}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2/(a+a*sin(d*x+c))^(1/3),x)
```

```
[Out] int(csc(d*x+c)^2/(a+a*sin(d*x+c))^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$
\int \frac{\csc(dx+c)^2}{(a\sin(dx+c)+a)^{\frac{1}{3}}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+a*sin(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(csc(d*x + c)^2/(a*sin(d*x + c) + a)^(1/3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+a*sin(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc^2(c+dx)}{\sqrt[3]{a(\sin(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2/(a+a*sin(d*x+c))**(1/3),x)
```

```
[Out] Integral(csc(c + d*x)**2/(a*(sin(c + d*x) + 1))**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(dx+c)^2}{(a\sin(dx+c)+a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+a*sin(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate(csc(d*x + c)^2/(a*sin(d*x + c) + a)^(1/3), x)
```

3.110 $\int \frac{\sin^3(c+dx)}{(a+a\sin(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=162

$$-\frac{2\sqrt[6]{2}\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{ad\sqrt[6]{\sin(c+dx)+1}\sqrt[3]{a\sin(c+dx)+a}}-\frac{3\sin^2(c+dx)\cos(c+dx)}{5d(a\sin(c+dx)+a)^{4/3}}+\frac{6\cos(c+dx)}{5ad\sqrt[3]{a\sin(c+dx)+a}}+\frac{6\cos(c+dx)}{5d(a\sin(c+dx)+a)^{4/3}}$$

```
[Out] (6*Cos[c + d*x])/(5*d*(a + a*Sin[c + d*x])^(4/3)) - (3*Cos[c + d*x]*Sin[c +
 d*x]^2)/(5*d*(a + a*Sin[c + d*x])^(4/3)) + (6*Cos[c + d*x])/(5*a*d*(a + a*
Sin[c + d*x])^(1/3)) - (2*2^(1/6)*Cos[c + d*x]*Hypergeometric2F1[1/2, 5/6,
3/2, (1 - Sin[c + d*x])/2])/(a*d*(1 + Sin[c + d*x])^(1/6)*(a + a*Sin[c + d*
x])^(1/3))
```

Rubi [A] time = 0.26968, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2783, 2968, 3019, 2751, 2652, 2651}

$$-\frac{2\sqrt[6]{2}\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{ad\sqrt[6]{\sin(c+dx)+1}\sqrt[3]{a\sin(c+dx)+a}}-\frac{3\sin^2(c+dx)\cos(c+dx)}{5d(a\sin(c+dx)+a)^{4/3}}+\frac{6\cos(c+dx)}{5ad\sqrt[3]{a\sin(c+dx)+a}}+\frac{6\cos(c+dx)}{5d(a\sin(c+dx)+a)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^3/(a + a*Sin[c + d*x])^(4/3),x]
```

```
[Out] (6*Cos[c + d*x])/(5*d*(a + a*Sin[c + d*x])^(4/3)) - (3*Cos[c + d*x]*Sin[c +
 d*x]^2)/(5*d*(a + a*Sin[c + d*x])^(4/3)) + (6*Cos[c + d*x])/(5*a*d*(a + a*
Sin[c + d*x])^(1/3)) - (2*2^(1/6)*Cos[c + d*x]*Hypergeometric2F1[1/2, 5/6,
3/2, (1 - Sin[c + d*x])/2])/(a*d*(1 + Sin[c + d*x])^(1/6)*(a + a*Sin[c + d*
x])^(1/3))
```

Rule 2783

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*(c + d*Sin[e + f*x])^(n - 1))/(f*(m + n)), x] + Dist[1/(b*(m + n)), Int[(
a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n - 2)*Simp[d*(a*c*m + b*d*(n -
 1)) + b*c^2*(m + n) + d*(a*d*m + b*c*(m + 2*n - 1))*Sin[e + f*x], x], x],
x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && IntegerQ[n]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3019

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b - a
*B + b*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[1
/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*
B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
```

```
B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2652

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Sin[c + d*x])^FracPart[n])/(1 + (b*Sin[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Sin[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2651

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(2^(n +
 1/2)*a^(n - 1/2)*b*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1*(1
 - (b*Sin[c + d*x])/a))/2])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^3(c+dx)}{(a+a\sin(c+dx))^{4/3}}\,dx &= -\frac{3\cos(c+dx)\sin^2(c+dx)}{5d(a+a\sin(c+dx))^{4/3}}+\frac{3\int \frac{\sin(c+dx)\left(2a-\frac{4}{3}a\sin(c+dx)\right)}{(a+a\sin(c+dx))^{4/3}}\,dx}{5a}\\
&= -\frac{3\cos(c+dx)\sin^2(c+dx)}{5d(a+a\sin(c+dx))^{4/3}}+\frac{3\int \frac{2a\sin(c+dx)-\frac{4}{3}a\sin^2(c+dx)}{(a+a\sin(c+dx))^{4/3}}\,dx}{5a}\\
&= \frac{6\cos(c+dx)}{5d(a+a\sin(c+dx))^{4/3}}-\frac{3\cos(c+dx)\sin^2(c+dx)}{5d(a+a\sin(c+dx))^{4/3}}-\frac{9\int \frac{-\frac{40a^2}{9}+\frac{20}{9}a^2\sin(c+dx)}{\sqrt[3]{a+a\sin(c+dx)}}\,dx}{25a^3}\\
&= \frac{6\cos(c+dx)}{5d(a+a\sin(c+dx))^{4/3}}-\frac{3\cos(c+dx)\sin^2(c+dx)}{5d(a+a\sin(c+dx))^{4/3}}+\frac{6\cos(c+dx)}{5ad\sqrt[3]{a+a\sin(c+dx)}}+\frac{2\int \frac{}{\sqrt[3]{a}}\cdots}{}\\
&= \frac{6\cos(c+dx)}{5d(a+a\sin(c+dx))^{4/3}}-\frac{3\cos(c+dx)\sin^2(c+dx)}{5d(a+a\sin(c+dx))^{4/3}}+\frac{6\cos(c+dx)}{5ad\sqrt[3]{a+a\sin(c+dx)}}+\frac{\left(2\sqrt[3]{1}\cdots\right.}{}\\
&= \frac{6\cos(c+dx)}{5d(a+a\sin(c+dx))^{4/3}}-\frac{3\cos(c+dx)\sin^2(c+dx)}{5d(a+a\sin(c+dx))^{4/3}}+\frac{6\cos(c+dx)}{5ad\sqrt[3]{a+a\sin(c+dx)}}-\frac{2\sqrt[6]{2}\,c\cdots}{a\cdots}
\end{aligned}$$

Mathematica [A] time = 0.483215, size = 116, normalized size = 0.72

$$\frac{3\cos(c+dx)\left(20\sqrt{2}(\sin(c+dx)+1)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\sin^2\left(\frac{1}{4}(2c+2dx+\pi)\right)\right)+\sqrt{1-\sin(c+dx)}(4\sin(c+dx)+\cos(2(c\cdots}{10d\sqrt{1-\sin(c+dx)}(a(\sin(c+dx)+1))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^3/(a + a*Sin[c + d*x])^(4/3),x]
```

```
[Out] (3*Cos[c + d*x]*(20*Sqrt[2]*Hypergeometric2F1[1/6, 1/2, 7/6, Sin[(2*c + Pi
+ 2*d*x)/4]^2]*(1 + Sin[c + d*x]) + Sqrt[1 - Sin[c + d*x]]*(7 + Cos[2*(c +
```

```
d*x)] + 4*Sin[c + d*x])))/(10*d*Sqrt[1 - Sin[c + d*x]]*(a*(1 + Sin[c + d*x]
))^(4/3))
```

Maple [F] time = 0.284, size = 0, normalized size = 0.

$$\int (\sin (dx + c))^3 (a + a \sin (dx + c))^{-\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3/(a+a*sin(d*x+c))^(4/3),x)
```

```
[Out] int(sin(d*x+c)^3/(a+a*sin(d*x+c))^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin (dx + c)^3}{(a \sin (dx + c) + a)^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+a*sin(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)^3/(a*sin(d*x + c) + a)^(4/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(\cos (dx + c)^2 - 1\right)(a \sin (dx + c) + a)^{\frac{2}{3}} \sin (dx + c)}{a^2 \cos (dx + c)^2 - 2 a^2 \sin (dx + c) - 2 a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+a*sin(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] integral((cos(d*x + c)^2 - 1)*(a*sin(d*x + c) + a)^(2/3)*sin(d*x + c)/(a^2*
cos(d*x + c)^2 - 2*a^2*sin(d*x + c) - 2*a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3/(a+a*sin(d*x+c))**(4/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(dx+c\right)^3}{\left(a\sin\left(dx+c\right)+a\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+a*sin(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)^3/(a*sin(d*x + c) + a)^(4/3), x)
```

3.111 $\int \frac{\sin^2(c+dx)}{(a+a\sin(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=129

$$\frac{13\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{5\,2^{5/6}ad\sqrt[6]{\sin(c+dx)+1}\sqrt[3]{a\sin(c+dx)+a}}-\frac{3\cos(c+dx)}{2ad\sqrt[3]{a\sin(c+dx)+a}}-\frac{3\cos(c+dx)}{5d(a\sin(c+dx)+a)^{4/3}}$$

```
[Out] (-3*Cos[c + d*x])/(5*d*(a + a*Sin[c + d*x])^(4/3)) - (3*Cos[c + d*x])/(2*a*
d*(a + a*Sin[c + d*x])^(1/3)) + (13*Cos[c + d*x]*Hypergeometric2F1[1/2, 5/6
, 3/2, (1 - Sin[c + d*x])/2])/(5*2^(5/6)*a*d*(1 + Sin[c + d*x])^(1/6)*(a +
a*Sin[c + d*x])^(1/3))
```

Rubi [A] time = 0.136466, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2758, 2751, 2652, 2651}

$$\frac{13\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{5\,2^{5/6}ad\sqrt[6]{\sin(c+dx)+1}\sqrt[3]{a\sin(c+dx)+a}}-\frac{3\cos(c+dx)}{2ad\sqrt[3]{a\sin(c+dx)+a}}-\frac{3\cos(c+dx)}{5d(a\sin(c+dx)+a)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2/(a + a*Sin[c + d*x])^(4/3),x]
```

```
[Out] (-3*Cos[c + d*x])/(5*d*(a + a*Sin[c + d*x])^(4/3)) - (3*Cos[c + d*x])/(2*a*
d*(a + a*Sin[c + d*x])^(1/3)) + (13*Cos[c + d*x]*Hypergeometric2F1[1/2, 5/6
, 3/2, (1 - Sin[c + d*x])/2])/(5*2^(5/6)*a*d*(1 + Sin[c + d*x])^(1/6)*(a +
a*Sin[c + d*x])^(1/3))
```

Rule 2758

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)),
x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(a*m - b*(2*m
 + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -2^(-1)]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2652

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Sin[c + d*x])^FracPart[n])/(1 + (b*Sin[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Sin[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2651

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(2^(n +
 1/2)*a^(n - 1/2)*b*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1*(1
```

```
- (b*Sin[c + d*x])/a))/2])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[2*n] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^2(c+dx)}{(a+a\sin(c+dx))^{4/3}}\,dx &= -\frac{3\cos(c+dx)}{5d(a+a\sin(c+dx))^{4/3}}+\frac{3\int \frac{-\frac{4a}{3}+\frac{5}{3}a\sin(c+dx)}{\sqrt[3]{a+a\sin(c+dx)}}\,dx}{5a^2}\\ &= -\frac{3\cos(c+dx)}{5d(a+a\sin(c+dx))^{4/3}}-\frac{3\cos(c+dx)}{2ad\sqrt[3]{a+a\sin(c+dx)}}-\frac{13\int \frac{1}{\sqrt[3]{a+a\sin(c+dx)}}\,dx}{10a}\\ &= -\frac{3\cos(c+dx)}{5d(a+a\sin(c+dx))^{4/3}}-\frac{3\cos(c+dx)}{2ad\sqrt[3]{a+a\sin(c+dx)}}-\frac{\left(13\sqrt[3]{1+\sin(c+dx)}\right)\int \frac{1}{\sqrt[3]{1+\sin(c+a}}}{10a\sqrt[3]{a+a\sin(c+dx)}}\\ &= -\frac{3\cos(c+dx)}{5d(a+a\sin(c+dx))^{4/3}}-\frac{3\cos(c+dx)}{2ad\sqrt[3]{a+a\sin(c+dx)}}+\frac{13\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2};\frac{1}{2}(1-\mathrm{s}\right.}{5\,2^{5/6}ad\sqrt[6]{1+\sin(c+dx)}\sqrt[3]{a+a}}\end{aligned}$$

Mathematica [A] time = 0.327028, size = 108, normalized size = 0.84

$$-\frac{3\cos(c+dx)\left(13\sqrt{2}(\sin(c+dx)+1)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\sin^2\left(\frac{1}{4}(2c+2dx+\pi)\right)\right)+\sqrt{1-\sin(c+dx)}(5\sin(c+dx)+7)\right)}{10d\sqrt{1-\sin(c+dx)}(a(\sin(c+dx)+1))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2/(a + a*Sin[c + d*x])^(4/3),x]
```

```
[Out] (-3*Cos[c + d*x]*(13*Sqrt[2]*Hypergeometric2F1[1/6, 1/2, 7/6, Sin[(2*c + Pi
 + 2*d*x)/4]^2]*(1 + Sin[c + d*x]) + Sqrt[1 - Sin[c + d*x]]*(7 + 5*Sin[c +
d*x])))/(10*d*Sqrt[1 - Sin[c + d*x]]*(a*(1 + Sin[c + d*x]))^(4/3))
```

Maple [F] time = 0.162, size = 0, normalized size = 0.

$$\int (\sin(dx+c))^2 (a+a\sin(dx+c))^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2/(a+a*sin(d*x+c))^(4/3),x)
```

```
[Out] int(sin(d*x+c)^2/(a+a*sin(d*x+c))^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(dx+c)^2}{(a\sin(dx+c)+a)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+a*sin(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)^2/(a*sin(d*x + c) + a)^(4/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(\cos\left(dx+c\right)^{2}-1\right)\left(a\sin\left(dx+c\right)+a\right)^{\frac{2}{3}}}{a^{2}\cos\left(dx+c\right)^{2}-2\,a^{2}\sin\left(dx+c\right)-2\,a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+a*sin(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] integral((cos(d*x + c)^2 - 1)*(a*sin(d*x + c) + a)^(2/3)/(a^2*cos(d*x + c)^
2 - 2*a^2*sin(d*x + c) - 2*a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2/(a+a*sin(d*x+c))**(4/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(dx+c\right)^{2}}{\left(a\sin\left(dx+c\right)+a\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+a*sin(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)^2/(a*sin(d*x + c) + a)^(4/3), x)
```

3.112 $\int \frac{\sin(c+dx)}{(a+a\sin(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=99

$$\frac{3\cos(c+dx)}{5d(a\sin(c+dx)+a)^{4/3}}-\frac{4\sqrt[6]{2}\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{5ad\sqrt[6]{\sin(c+dx)+1}\sqrt[3]{a\sin(c+dx)+a}}$$

```
[Out] (3*Cos[c + d*x])/(5*d*(a + a*Sin[c + d*x])^(4/3)) - (4*2^(1/6)*Cos[c + d*x]
*Hypergeometric2F1[1/2, 5/6, 3/2, (1 - Sin[c + d*x])/2])/(5*a*d*(1 + Sin[c
+ d*x])^(1/6)*(a + a*Sin[c + d*x])^(1/3))
```

Rubi [A] time = 0.0789303, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2750, 2652, 2651}

$$\frac{3\cos(c+dx)}{5d(a\sin(c+dx)+a)^{4/3}}-\frac{4\sqrt[6]{2}\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{5ad\sqrt[6]{\sin(c+dx)+1}\sqrt[3]{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(a + a*Sin[c + d*x])^(4/3),x]
```

```
[Out] (3*Cos[c + d*x])/(5*d*(a + a*Sin[c + d*x])^(4/3)) - (4*2^(1/6)*Cos[c + d*x]
*Hypergeometric2F1[1/2, 5/6, 3/2, (1 - Sin[c + d*x])/2])/(5*a*d*(1 + Sin[c
+ d*x])^(1/6)*(a + a*Sin[c + d*x])^(1/3))
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2652

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Sin[c + d*x])^FracPart[n])/(1 + (b*Sin[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Sin[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] && !IntegerQ[2*n] && !GtQ[a, 0]
```

Rule 2651

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(2^(n +
 1/2)*a^(n - 1/2)*b*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1*(1
 - (b*Sin[c + d*x])/a))/2])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[2*n] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sin(c+dx)}{(a+a\sin(c+dx))^{4/3}}\,dx = \frac{3\cos(c+dx)}{5d(a+a\sin(c+dx))^{4/3}} + \frac{4\int \frac{1}{\sqrt[3]{a+a\sin(c+dx)}}\,dx}{5a}$$
$$= \frac{3\cos(c+dx)}{5d(a+a\sin(c+dx))^{4/3}} + \frac{\left(4\sqrt[3]{1+\sin(c+dx)}\right)\int \frac{1}{\sqrt[3]{1+\sin(c+dx)}}\,dx}{5a\sqrt[3]{a+a\sin(c+dx)}}$$
$$= \frac{3\cos(c+dx)}{5d(a+a\sin(c+dx))^{4/3}} - \frac{4\sqrt[6]{2}\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{5ad\sqrt[6]{1+\sin(c+dx)}\sqrt[3]{a+a\sin(c+dx)}}$$

Mathematica [A] time = 0.295461, size = 130, normalized size = 1.31

$$\frac{3\left(8(\sin(c+dx)+1)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\sin^2\left(\frac{1}{4}(2c+2dx+\pi)\right)\right)+\sqrt{2-2\sin(c+dx)}\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\sin\right.}{5d\sqrt{2-2\sin(c+dx)}(a(\sin(c+dx)+1))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]/(a + a*Sin[c + d*x])^(4/3),x]
```

```
[Out] (3*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/
2])*(Sqrt[2 - 2*Sin[c + d*x]] + 8*Hypergeometric2F1[1/6, 1/2, 7/6, Sin[(2*c
 + Pi + 2*d*x)/4]^2]*(1 + Sin[c + d*x])))/(5*d*Sqrt[2 - 2*Sin[c + d*x]]*(a*
(1 + Sin[c + d*x]))^(4/3))
```

Maple [F] time = 0.082, size = 0, normalized size = 0.

$$\int \sin(dx+c)\,(a+a\sin(dx+c))^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/(a+a*sin(d*x+c))^(4/3),x)
```

```
[Out] int(sin(d*x+c)/(a+a*sin(d*x+c))^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(dx+c)}{(a\sin(dx+c)+a)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+a*sin(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)/(a*sin(d*x + c) + a)^(4/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{(a\sin(dx+c)+a)^{\frac{2}{3}}\sin(dx+c)}{a^2\cos(dx+c)^2-2a^2\sin(dx+c)-2a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+a*sin(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] integral(-(a*sin(d*x + c) + a)^(2/3)*sin(d*x + c)/(a^2*cos(d*x + c)^2 - 2*a
^2*sin(d*x + c) - 2*a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(c+dx)}{(a(\sin(c+dx)+1))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+a*sin(d*x+c))**(4/3),x)
```

```
[Out] Integral(sin(c + d*x)/(a*(sin(c + d*x) + 1))**(4/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(dx+c)}{(a\sin(dx+c)+a)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+a*sin(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)/(a*sin(d*x + c) + a)^(4/3), x)
```

3.113 $\int \frac{1}{(a+a\sin(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=69

$$-\frac{\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{11}{6};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{2^{5/6}ad\sqrt[6]{\sin(c+dx)+1}\sqrt[3]{a\sin(c+dx)+a}}$$

```
[Out] -((Cos[c + d*x]*Hypergeometric2F1[1/2, 11/6, 3/2, (1 - Sin[c + d*x])/2])/(2
^(5/6)*a*d*(1 + Sin[c + d*x])^(1/6)*(a + a*Sin[c + d*x])^(1/3)))
```

Rubi [A] time = 0.0307922, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2652, 2651}

$$-\frac{\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{11}{6};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{2^{5/6}ad\sqrt[6]{\sin(c+dx)+1}\sqrt[3]{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^(-4/3),x]
```

```
[Out] -((Cos[c + d*x]*Hypergeometric2F1[1/2, 11/6, 3/2, (1 - Sin[c + d*x])/2])/(2
^(5/6)*a*d*(1 + Sin[c + d*x])^(1/6)*(a + a*Sin[c + d*x])^(1/3)))
```

Rule 2652

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Sin[c + d*x])^FracPart[n])/(1 + (b*Sin[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Sin[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] && !IntegerQ[2*n] && !GtQ[a, 0]
```

Rule 2651

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(2^(n +
 1/2)*a^(n - 1/2)*b*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1*(1
 - (b*Sin[c + d*x])/a))/2])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[2*n] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+a\sin(c+dx))^{4/3}}\,dx &= \frac{\sqrt[3]{1+\sin(c+dx)}\int \frac{1}{(1+\sin(c+dx))^{4/3}}\,dx}{a\sqrt[3]{a+a\sin(c+dx)}}\\ &= -\frac{\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{11}{6};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{2^{5/6}ad\sqrt[6]{1+\sin(c+dx)}\sqrt[3]{a+a\sin(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.196842, size = 130, normalized size = 1.88

$$-\frac{3\left(\sqrt{2-2\sin(c+dx)}-2(\sin(c+dx)+1)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\sin^2\left(\frac{1}{4}(2c+2dx+\pi)\right)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\mathrm{si}\right.}{5d\sqrt{2-2\sin(c+dx)}(a(\sin(c+dx)+1))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^(-4/3),x]
```

```
[Out] (-3*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)
/2])*(Sqrt[2 - 2*Sin[c + d*x]] - 2*Hypergeometric2F1[1/6, 1/2, 7/6, Sin[(2*
c + Pi + 2*d*x)/4]^2]*(1 + Sin[c + d*x])))/(5*d*Sqrt[2 - 2*Sin[c + d*x]]*(a
*(1 + Sin[c + d*x]))^(4/3))
```

Maple [F] time = 0.005, size = 0, normalized size = 0.

$$\int (a + a\sin(dx + c))^{-\frac{4}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(d*x+c))^(4/3),x)
```

```
[Out] int(1/(a+a*sin(d*x+c))^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\sin(dx + c) + a)^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(-4/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{(a\sin(dx + c) + a)^{\frac{2}{3}}}{a^2\cos(dx + c)^2 - 2a^2\sin(dx + c) - 2a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] integral(-(a*sin(d*x + c) + a)^(2/3)/(a^2*cos(d*x + c)^2 - 2*a^2*sin(d*x +
c) - 2*a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\sin(c + dx) + a)^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))**(4/3),x)
```

```
[Out] Integral((a*sin(c + d*x) + a)**(-4/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\sin(dx+c)+a)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(-4/3), x)
```

3.114 $\int \frac{\csc(c+dx)}{(a+a\sin(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=80

$$-\frac{\cos(c+dx)F_1\left(\frac{1}{2};1,\frac{11}{6};\frac{3}{2};1-\sin(c+dx),\frac{1}{2}(1-\sin(c+dx))\right)}{2^{5/6}ad\sqrt[6]{\sin(c+dx)+1}\sqrt[3]{a\sin(c+dx)+a}}$$

```
[Out] -((AppellF1[1/2, 1, 11/6, 3/2, 1 - Sin[c + d*x], (1 - Sin[c + d*x])/2]*Cos[
c + d*x])/(2^(5/6)*a*d*(1 + Sin[c + d*x])^(1/6)*(a + a*Sin[c + d*x])^(1/3))
)
```

Rubi [A] time = 0.117159, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2787, 2785, 130, 429}

$$-\frac{\cos(c+dx)F_1\left(\frac{1}{2};1,\frac{11}{6};\frac{3}{2};1-\sin(c+dx),\frac{1}{2}(1-\sin(c+dx))\right)}{2^{5/6}ad\sqrt[6]{\sin(c+dx)+1}\sqrt[3]{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]/(a + a*Sin[c + d*x])^(4/3),x]
```

```
[Out] -((AppellF1[1/2, 1, 11/6, 3/2, 1 - Sin[c + d*x], (1 - Sin[c + d*x])/2]*Cos[
c + d*x])/(2^(5/6)*a*d*(1 + Sin[c + d*x])^(1/6)*(a + a*Sin[c + d*x])^(1/3))
)
```

Rule 2787

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m
])/(1 + (b*Sin[e + f*x])/a)^FracPart[m], Int[(1 + (b*Sin[e + f*x])/a)^m*(d*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] && !IntegerQ[m] && !GtQ[a, 0]
```

Rule 2785

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Dist[(b*(d/b)^n*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a - x)^n*(2*a - x)^(m - 1/2)
)/Sqrt[x], x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] && !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

Rule 130

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + (b*x^k)/e)^m*(c + (d*x^k)/e)^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
```

```
&& (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc(c+dx)}{(a+a\sin(c+dx))^{4/3}}\,dx &= \frac{\sqrt[3]{1+\sin(c+dx)}\int \frac{\csc(c+dx)}{(1+\sin(c+dx))^{4/3}}\,dx}{a\sqrt[3]{a+a\sin(c+dx)}}\\
&= -\frac{\cos(c+dx)\,\text{Subst}\left(\int \frac{1}{(1-x)(2-x)^{11/6}\sqrt{x}}\,dx, x, 1-\sin(c+dx)\right)}{ad\sqrt{1-\sin(c+dx)}\sqrt[6]{1+\sin(c+dx)}\sqrt[3]{a+a\sin(c+dx)}}\\
&= -\frac{(2\cos(c+dx))\,\text{Subst}\left(\int \frac{1}{(1-x^2)(2-x^2)^{11/6}}\,dx, x, \sqrt{1-\sin(c+dx)}\right)}{ad\sqrt{1-\sin(c+dx)}\sqrt[6]{1+\sin(c+dx)}\sqrt[3]{a+a\sin(c+dx)}}\\
&= -\frac{F_1\left(\frac{1}{2};1,\frac{11}{6};\frac{3}{2};1-\sin(c+dx),\frac{1}{2}(1-\sin(c+dx))\right)\cos(c+dx)}{2^{5/6}ad\sqrt[6]{1+\sin(c+dx)}\sqrt[3]{a+a\sin(c+dx)}}
\end{aligned}$$

Mathematica [F] time = 10.1668, size = 0, normalized size = 0.

$$\int \frac{\csc(c+dx)}{(a+a\sin(c+dx))^{4/3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Csc[c + d*x]/(a + a*Sin[c + d*x])^(4/3),x]
```

```
[Out] Integrate[Csc[c + d*x]/(a + a*Sin[c + d*x])^(4/3), x]
```

Maple [F] time = 0.099, size = 0, normalized size = 0.

$$\int \csc(dx+c)\,(a+a\sin(dx+c))^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)/(a+a*sin(d*x+c))^(4/3),x)
```

```
[Out] int(csc(d*x+c)/(a+a*sin(d*x+c))^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(dx+c)}{(a\sin(dx+c)+a)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+a*sin(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate(csc(d*x + c)/(a*sin(d*x + c) + a)^(4/3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+a*sin(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(c+dx)}{(a(\sin(c+dx)+1))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+a*sin(d*x+c))**(4/3),x)
```

```
[Out] Integral(csc(c + d*x)/(a*(sin(c + d*x) + 1))**(4/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(dx+c)}{(a\sin(dx+c)+a)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+a*sin(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] integrate(csc(d*x + c)/(a*sin(d*x + c) + a)^(4/3), x)
```

3.115 $\int \frac{\csc^2(c+dx)}{(a+a\sin(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=80

$$-\frac{\cos(c+dx)F_1\left(\frac{1}{2};2,\frac{11}{6};\frac{3}{2};1-\sin(c+dx),\frac{1}{2}(1-\sin(c+dx))\right)}{2^{5/6}ad\sqrt[6]{\sin(c+dx)+1}\sqrt[3]{a\sin(c+dx)+a}}$$

```
[Out] -((AppellF1[1/2, 2, 11/6, 3/2, 1 - Sin[c + d*x], (1 - Sin[c + d*x])/2]*Cos[
c + d*x])/(2^(5/6)*a*d*(1 + Sin[c + d*x])^(1/6)*(a + a*Sin[c + d*x])^(1/3))
)
```

Rubi [A] time = 0.137083, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2787, 2785, 130, 429}

$$-\frac{\cos(c+dx)F_1\left(\frac{1}{2};2,\frac{11}{6};\frac{3}{2};1-\sin(c+dx),\frac{1}{2}(1-\sin(c+dx))\right)}{2^{5/6}ad\sqrt[6]{\sin(c+dx)+1}\sqrt[3]{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2/(a + a*Sin[c + d*x])^(4/3),x]
```

```
[Out] -((AppellF1[1/2, 2, 11/6, 3/2, 1 - Sin[c + d*x], (1 - Sin[c + d*x])/2]*Cos[
c + d*x])/(2^(5/6)*a*d*(1 + Sin[c + d*x])^(1/6)*(a + a*Sin[c + d*x])^(1/3))
)
```

Rule 2787

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m
])/(1 + (b*Sin[e + f*x])/a)^FracPart[m], Int[(1 + (b*Sin[e + f*x])/a)^m*(d*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] && !IntegerQ[m] && !GtQ[a, 0]
```

Rule 2785

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Dist[(b*(d/b)^n*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a - x)^n*(2*a - x)^(m - 1/2)
)/Sqrt[x], x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] && !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

Rule 130

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + (b*x^k)/e)^m*(c + (d*x^k)/e)^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
```

```
&& (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^2(c+dx)}{(a+a\sin(c+dx))^{4/3}}\,dx &= \frac{\sqrt[3]{1+\sin(c+dx)}\int \frac{\csc^2(c+dx)}{(1+\sin(c+dx))^{4/3}}\,dx}{a\sqrt[3]{a+a\sin(c+dx)}}\\ &= -\frac{\cos(c+dx)\,\text{Subst}\left(\int \frac{1}{(1-x)^2(2-x)^{11/6}\sqrt{x}}\,dx,x,1-\sin(c+dx)\right)}{ad\sqrt{1-\sin(c+dx)}\sqrt[6]{1+\sin(c+dx)}\sqrt[3]{a+a\sin(c+dx)}}\\ &= -\frac{(2\cos(c+dx))\,\text{Subst}\left(\int \frac{1}{(1-x^2)^2(2-x^2)^{11/6}}\,dx,x,\sqrt{1-\sin(c+dx)}\right)}{ad\sqrt{1-\sin(c+dx)}\sqrt[6]{1+\sin(c+dx)}\sqrt[3]{a+a\sin(c+dx)}}\\ &= -\frac{F_1\left(\frac{1}{2};2,\frac{11}{6};\frac{3}{2};1-\sin(c+dx),\frac{1}{2}(1-\sin(c+dx))\right)\cos(c+dx)}{2^{5/6}ad\sqrt[6]{1+\sin(c+dx)}\sqrt[3]{a+a\sin(c+dx)}}\end{aligned}$$

Mathematica [C] time = 14.0285, size = 230, normalized size = 2.88

$$\frac{8\,2^{2/3}\cos^{\frac{8}{3}}\left(\frac{1}{4}(2c+2dx-\pi)\right)(\cos(2(c+dx))+i\sin(2(c+dx)))\left(14i\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-ie^{-i(c+dx)}\right)(\sin(c+dx)+i\cos(c+\ldots\right.}{55d\left(-1+ie^{i(c+dx)}\right)^3\left(e^{i(c+dx)}-i\right)\left(-(-1)^{3/4}e^{-\frac{1}{2}i(c+dx)}\left(e^{i(c+dx)}+\ldots\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[c + d*x]^2/(a + a*Sin[c + d*x])^(4/3),x]
```

```
[Out] (8*2^(2/3)*Cos[(2*c - Pi + 2*d*x)/4]^(8/3)*(Cos[2*(c + d*x)] + I*Sin[2*(c +
 d*x)])*(6 - 14*Cos[2*(c + d*x)] + 35*Sin[c + d*x] + (14*I)*Hypergeometric2
F1[1/3, 2/3, 4/3, (-I)/E^(I*(c + d*x))]*(1 + I*Cos[c + d*x] + Sin[c + d*x])
^(2/3)*(2*Cos[c + d*x] + Sin[2*(c + d*x)])))/(55*d*(-1 + I*E^(I*(c + d*x)))
^3*(-I + E^(I*(c + d*x)))*(-(((-1)^(3/4)*(I + E^(I*(c + d*x))))/E^((I/2)*(c
 + d*x))))^(2/3)*(a*(1 + Sin[c + d*x]))^(4/3))
```

Maple [F] time = 0.105, size = 0, normalized size = 0.

$$\int (\csc(dx+c))^2 (a+a\sin(dx+c))^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2/(a+a*sin(d*x+c))^(4/3),x)
```

```
[Out] int(csc(d*x+c)^2/(a+a*sin(d*x+c))^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(dx+c)^2}{(a\sin(dx+c)+a)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+a*sin(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate(csc(d*x + c)^2/(a*sin(d*x + c) + a)^(4/3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+a*sin(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2/(a+a*sin(d*x+c))**(4/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc (dx + c)^2}{(a \sin (dx + c) + a)^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+a*sin(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] integrate(csc(d*x + c)^2/(a*sin(d*x + c) + a)^(4/3), x)
```

3.116 $\int \sin^n(e+fx)(1+\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=96

$$-\frac{2(4n+5)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};1-\sin(e+fx)\right)}{f(2n+3)\sqrt{\sin(e+fx)+1}}-\frac{2\cos(e+fx)\sin^{n+1}(e+fx)}{f(2n+3)\sqrt{\sin(e+fx)+1}}$$

```
[Out] (-2*(5 + 4*n)*Cos[e + f*x]*Hypergeometric2F1[1/2, -n, 3/2, 1 - Sin[e + f*x]
])/(f*(3 + 2*n)*Sqrt[1 + Sin[e + f*x]]) - (2*Cos[e + f*x]*Sin[e + f*x]^(1 +
 n))/(f*(3 + 2*n)*Sqrt[1 + Sin[e + f*x]])
```

Rubi [A] time = 0.11055, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2763, 21, 2776, 65}

$$-\frac{2(4n+5)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};1-\sin(e+fx)\right)}{f(2n+3)\sqrt{\sin(e+fx)+1}}-\frac{2\cos(e+fx)\sin^{n+1}(e+fx)}{f(2n+3)\sqrt{\sin(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^n*(1 + Sin[e + f*x])^(3/2),x]
```

```
[Out] (-2*(5 + 4*n)*Cos[e + f*x]*Hypergeometric2F1[1/2, -n, 3/2, 1 - Sin[e + f*x]
])/(f*(3 + 2*n)*Sqrt[1 + Sin[e + f*x]]) - (2*Cos[e + f*x]*Sin[e + f*x]^(1 +
 n))/(f*(3 + 2*n)*Sqrt[1 + Sin[e + f*x]])
```

Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2776

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[(c + d*x)^n/Sqrt[a - b*x], x],
 x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !IntegerQ[2*n]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\int \sin^n(e+fx)(1+\sin(e+fx))^{3/2}\,dx = -\frac{2\cos(e+fx)\sin^{1+n}(e+fx)}{f(3+2n)\sqrt{1+\sin(e+fx)}}+\frac{2\int \frac{\sin^n(e+fx)\left(\frac{1}{2}(5+4n)+\frac{1}{2}(5+4n)\sin(e+fx)\right)}{\sqrt{1+\sin(e+fx)}}\,dx}{3+2n}$$

$$= -\frac{2\cos(e+fx)\sin^{1+n}(e+fx)}{f(3+2n)\sqrt{1+\sin(e+fx)}}+\frac{(5+4n)\int \sin^n(e+fx)\sqrt{1+\sin(e+fx)}\,dx}{3+2n}$$

$$= -\frac{2\cos(e+fx)\sin^{1+n}(e+fx)}{f(3+2n)\sqrt{1+\sin(e+fx)}}+\frac{((5+4n)\cos(e+fx))\,\mathrm{Subst}\left(\int \frac{x^n}{\sqrt{1-x}}\,dx,x,\sin\right.}{f(3+2n)\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+}$$

$$= -\frac{2(5+4n)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};1-\sin(e+fx)\right)}{f(3+2n)\sqrt{1+\sin(e+fx)}}-\frac{2\cos(e+fx)\sin^{1+n}(e+}{f(3+2n)\sqrt{1+\sin(e+}$$

Mathematica [C] time = 23.2207, size = 5109, normalized size = 53.22

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^n*(1 + Sin[e + f*x])^(3/2),x]
```

```
[Out] Result too large to show
```

Maple [F] time = 0.118, size = 0, normalized size = 0.

$$\int \left(\sin\left(fx+e\right)\right)^n \left(1+\sin\left(fx+e\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^n*(1+sin(f*x+e))^(3/2),x)
```

```
[Out] int(sin(f*x+e)^n*(1+sin(f*x+e))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sin\left(fx+e\right)^n \left(\sin\left(fx+e\right)+1\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^n*(1+sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(f*x + e)^n*(sin(f*x + e) + 1)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sin\left(fx+e\right)^{n}\left(\sin\left(fx+e\right)+1\right)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^n*(1+sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sin(f*x + e)^n*(sin(f*x + e) + 1)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**n*(1+sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sin\left(fx+e\right)^{n}\left(\sin\left(fx+e\right)+1\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^n*(1+sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^n*(sin(f*x + e) + 1)^(3/2), x)
```

3.117 $\int \sin^n(e+fx)\sqrt{1+\sin(e+fx)}\,dx$

Optimal. Leaf size=43

$$-\frac{2\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};1-\sin(e+fx)\right)}{f\sqrt{\sin(e+fx)+1}}$$

```
[Out] (-2*Cos[e + f*x]*Hypergeometric2F1[1/2, -n, 3/2, 1 - Sin[e + f*x]])/(f*Sqrt
[1 + Sin[e + f*x]])
```

Rubi [A] time = 0.047783, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2776, 65}

$$-\frac{2\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};1-\sin(e+fx)\right)}{f\sqrt{\sin(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^n*Sqrt[1 + Sin[e + f*x]],x]
```

```
[Out] (-2*Cos[e + f*x]*Hypergeometric2F1[1/2, -n, 3/2, 1 - Sin[e + f*x]])/(f*Sqrt
[1 + Sin[e + f*x]])
```

Rule 2776

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[(c + d*x)^n/Sqrt[a - b*x], x],
 x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !IntegerQ[2*n]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\begin{aligned}\int \sin^n(e+fx)\sqrt{1+\sin(e+fx)}\,dx &= \frac{\cos(e+fx)\,\text{Subst}\left(\int \frac{x^n}{\sqrt{1-x}}\,dx,x,\sin(e+fx)\right)}{f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}\\ &= -\frac{2\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};1-\sin(e+fx)\right)}{f\sqrt{1+\sin(e+fx)}}\end{aligned}$$

Mathematica [C] time = 0.408344, size = 186, normalized size = 4.33

$$\frac{2^{1-n}e^{i(e+fx)}\left(-ie^{-i(e+fx)}\left(-1+e^{2i(e+fx)}\right)\right)^{n+1}\sqrt{\sin(e+fx)+1}\left(i(2n-1)\,{}_2F_1\left(1,\frac{1}{4}(2n+3);\frac{1}{4}(3-2n);e^{2i(e+fx)}\right)+(2n+1)e^{i(e}\right.}{f(2n-1)(2n+1)\left(e^{i(e+fx)}+i\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^n*Sqrt[1 + Sin[e + f*x]],x]
```

```
[Out] (2^(1 - n)*E^(I*(e + f*x))*(((-I)*(-1 + E^((2*I)*(e + f*x))))/E^(I*(e + f*x
)))^(1 + n)*(I*(-1 + 2*n)*Hypergeometric2F1[1, (3 + 2*n)/4, (3 - 2*n)/4, E^
((2*I)*(e + f*x))] + E^(I*(e + f*x))*(1 + 2*n)*Hypergeometric2F1[1, (5 + 2*
n)/4, (5 - 2*n)/4, E^((2*I)*(e + f*x))])*Sqrt[1 + Sin[e + f*x]])/((I + E^(I
*(e + f*x)))*f*(-1 + 2*n)*(1 + 2*n))
```

Maple [F] time = 0.088, size = 0, normalized size = 0.

$$\int \left(\sin \left(fx+e\right)\right)^{n} \sqrt{1+\sin \left(fx+e\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^n*(1+sin(f*x+e))^(1/2),x)
```

```
[Out] int(sin(f*x+e)^n*(1+sin(f*x+e))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sin \left(fx+e\right)^{n} \sqrt{\sin \left(fx+e\right)+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^n*(1+sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(f*x + e)^n*sqrt(sin(f*x + e) + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sin \left(fx+e\right)^{n} \sqrt{\sin \left(fx+e\right)+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^n*(1+sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sin(f*x + e)^n*sqrt(sin(f*x + e) + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\sin \left(e+fx\right)+1} \sin^{n} \left(e+fx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**n*(1+sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(sin(e + f*x) + 1)*sin(e + f*x)**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sin\left(fx+e\right)^{n}\sqrt{\sin\left(fx+e\right)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^n*(1+sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^n*sqrt(sin(f*x + e) + 1), x)
```

3.118 $\int \frac{\sin^n(e+fx)}{\sqrt{1+\sin(e+fx)}}\,dx$

Optimal. Leaf size=58

$$-\frac{\cos(e+fx)F_1\left(\frac{1}{2};-n,1;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)}{f\sqrt{\sin(e+fx)+1}}$$

```
[Out] -((AppellF1[1/2, -n, 1, 3/2, 1 - Sin[e + f*x], (1 - Sin[e + f*x])/2]*Cos[e
+ f*x])/(f*Sqrt[1 + Sin[e + f*x]]))
```

Rubi [A] time = 0.0676877, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2785, 130, 429}

$$-\frac{\cos(e+fx)F_1\left(\frac{1}{2};-n,1;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)}{f\sqrt{\sin(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^n/Sqrt[1 + Sin[e + f*x]],x]
```

```
[Out] -((AppellF1[1/2, -n, 1, 3/2, 1 - Sin[e + f*x], (1 - Sin[e + f*x])/2]*Cos[e
+ f*x])/(f*Sqrt[1 + Sin[e + f*x]]))
```

Rule 2785

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Dist[(b*(d/b)^n*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a - x)^n*(2*a - x)^(m - 1/2)
)/Sqrt[x], x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

Rule 130

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + (b*x^k)/e)^m*(c + (d*x^k)/e)^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\sin^n(e+fx)}{\sqrt{1+\sin(e+fx)}}\,dx = -\frac{\cos(e+fx)\,\text{Subst}\left(\int \frac{(1-x)^n}{(2-x)\sqrt{x}}\,dx, x, 1-\sin(e+fx)\right)}{f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}$$
$$= -\frac{(2\cos(e+fx))\,\text{Subst}\left(\int \frac{\left(1-x^2\right)^n}{2-x^2}\,dx, x, \sqrt{1-\sin(e+fx)}\right)}{f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}$$
$$= -\frac{F_1\left(\frac{1}{2};-n,1;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)\cos(e+fx)}{f\sqrt{1+\sin(e+fx)}}$$

Mathematica [B] time = 1.53659, size = 225, normalized size = 3.88

$$\frac{\sqrt{\sin(e+fx)+1}\cos(e+fx)(-\sin(e+fx))^{-n}\sin^n(e+fx)\left(1-\frac{1}{\sin(e+fx)+1}\right)^{-n}\left(4\sqrt{\frac{\sin(e+fx)-1}{\sin(e+fx)+1}}(-\sin(e+fx))^nF_1\left(-n-\frac{1}{2};\right.\right.}{4f(2n}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^n/Sqrt[1 + Sin[e + f*x]],x]
```

```
[Out] (Cos[e + f*x]*Sin[e + f*x]^n*Sqrt[1 + Sin[e + f*x]]*(4*AppellF1[-1/2 - n, -
1/2, -n, 1/2 - n, 2/(1 + Sin[e + f*x]), (1 + Sin[e + f*x])^(-1)]*(-Sin[e +
f*x])^n*Sqrt[(-1 + Sin[e + f*x])/(1 + Sin[e + f*x])] - (1 + 2*n)*AppellF1[1
, 1/2, -n, 2, (1 + Sin[e + f*x])/2, 1 + Sin[e + f*x]]*Sqrt[2 - 2*Sin[e + f*
x]]*(1 - (1 + Sin[e + f*x])^(-1))^n))/(4*f*(1 + 2*n)*(-1 + Sin[e + f*x])*(-
Sin[e + f*x])^n*(1 - (1 + Sin[e + f*x])^(-1))^n)
```

Maple [F] time = 0.088, size = 0, normalized size = 0.

$$\int \left(\sin\left(fx+e\right)\right)^{n}\frac{1}{\sqrt{1+\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^n/(1+sin(f*x+e))^(1/2),x)
```

```
[Out] int(sin(f*x+e)^n/(1+sin(f*x+e))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(fx+e\right)^{n}}{\sqrt{\sin\left(fx+e\right)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^n/(1+sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(f*x + e)^n/sqrt(sin(f*x + e) + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sin\left(fx+e\right)^{n}}{\sqrt{\sin\left(fx+e\right)+1}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^n/(1+sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sin(f*x + e)^n/sqrt(sin(f*x + e) + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sin^{n}\left(e+fx\right)}{\sqrt{\sin\left(e+fx\right)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**n/(1+sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(sin(e + f*x)**n/sqrt(sin(e + f*x) + 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sin\left(fx+e\right)^{n}}{\sqrt{\sin\left(fx+e\right)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^n/(1+sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^n/sqrt(sin(f*x + e) + 1), x)
```

3.119 $\int \frac{\sin^n(e+fx)}{(1+\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=60

$$-\frac{\cos(e+fx)F_1\left(\frac{1}{2};-n,2;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)}{2f\sqrt{\sin(e+fx)+1}}$$

```
[Out] -(AppellF1[1/2, -n, 2, 3/2, 1 - Sin[e + f*x], (1 - Sin[e + f*x])/2]*Cos[e +
 f*x])/(2*f*Sqrt[1 + Sin[e + f*x]])
```

Rubi [A] time = 0.0722906, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2785, 130, 429}

$$-\frac{\cos(e+fx)F_1\left(\frac{1}{2};-n,2;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)}{2f\sqrt{\sin(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^n/(1 + Sin[e + f*x])^(3/2),x]
```

```
[Out] -(AppellF1[1/2, -n, 2, 3/2, 1 - Sin[e + f*x], (1 - Sin[e + f*x])/2]*Cos[e +
 f*x])/(2*f*Sqrt[1 + Sin[e + f*x]])
```

Rule 2785

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Dist[(b*(d/b)^n*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a - x)^n*(2*a - x)^(m - 1/2)
)/Sqrt[x], x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

Rule 130

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + (b*x^k)/e)^m*(c + (d*x^k)/e)^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\sin^n(e+fx)}{(1+\sin(e+fx))^{3/2}}\,dx = -\frac{\cos(e+fx)\,\text{Subst}\left(\int \frac{(1-x)^n}{(2-x)^2\sqrt{x}}\,dx, x, 1-\sin(e+fx)\right)}{f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}$$
$$= -\frac{(2\cos(e+fx))\,\text{Subst}\left(\int \frac{(1-x^2)^n}{(2-x^2)^2}\,dx, x, \sqrt{1-\sin(e+fx)}\right)}{f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}$$
$$= -\frac{F_1\left(\frac{1}{2}; -n, 2; \frac{3}{2}; 1-\sin(e+fx), \frac{1}{2}(1-\sin(e+fx))\right)\cos(e+fx)}{2f\sqrt{1+\sin(e+fx)}}$$

Mathematica [B] time = 3.65186, size = 263, normalized size = 4.38

$$\frac{\sec(e+fx)\sin^n(e+fx)\left(\sqrt{2-2\sin(e+fx)}(\sin(e+fx)+1)^2(-\sin(e+fx))^{-n}F_1\left(1; \frac{1}{2}, -n; 2; \frac{1}{2}(\sin(e+fx)+1), \sin(\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^n/(1 + Sin[e + f*x])^(3/2),x]
```

```
[Out] (Sec[e + f*x]*Sin[e + f*x]^n*((AppellF1[1, 1/2, -n, 2, (1 + Sin[e + f*x])/2
, 1 + Sin[e + f*x]]*Sqrt[2 - 2*Sin[e + f*x]]*(1 + Sin[e + f*x])^2)/(-Sin[e
+ f*x])^n - (4*(1 + Sin[e + f*x])*Sqrt[1 - 2/(1 + Sin[e + f*x])]*(2*(1 + 2*
n)*AppellF1[1/2 - n, -1/2, -n, 3/2 - n, 2/(1 + Sin[e + f*x]), (1 + Sin[e +
f*x])^(-1)] + (-1 + 2*n)*AppellF1[-1/2 - n, -1/2, -n, 1/2 - n, 2/(1 + Sin[e
 + f*x]), (1 + Sin[e + f*x])^(-1)]*(1 + Sin[e + f*x])))/((-1 + 4*n^2)*(1 -
(1 + Sin[e + f*x])^(-1))^n)))/(8*f*Sqrt[1 + Sin[e + f*x]])
```

Maple [F] time = 0.088, size = 0, normalized size = 0.

$$\int \left(\sin\left(fx+e\right)\right)^n\left(1+\sin\left(fx+e\right)\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^n/(1+sin(f*x+e))^(3/2),x)
```

```
[Out] int(sin(f*x+e)^n/(1+sin(f*x+e))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(fx+e\right)^n}{\left(\sin\left(fx+e\right)+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^n/(1+sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(f*x + e)^n/(sin(f*x + e) + 1)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sin\left(fx+e\right)^{n}\sqrt{\sin\left(fx+e\right)+1}}{\cos\left(fx+e\right)^{2}-2\,\sin\left(fx+e\right)-2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^n/(1+sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-sin(f*x + e)^n*sqrt(sin(f*x + e) + 1)/(cos(f*x + e)^2 - 2*sin(f*x
 + e) - 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin^{n}\left(e+fx\right)}{\left(\sin\left(e+fx\right)+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**n/(1+sin(f*x+e))**(3/2),x)
```

```
[Out] Integral(sin(e + f*x)**n/(sin(e + f*x) + 1)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(fx+e\right)^{n}}{\left(\sin\left(fx+e\right)+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^n/(1+sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^n/(sin(f*x + e) + 1)^(3/2), x)
```

3.120 $\int \sin^n(e+fx)(a+a\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=106

$$-\frac{2a^2(4n+5)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};1-\sin(e+fx)\right)}{f(2n+3)\sqrt{a\sin(e+fx)+a}}-\frac{2a^2\cos(e+fx)\sin^{n+1}(e+fx)}{f(2n+3)\sqrt{a\sin(e+fx)+a}}$$

```
[Out] (-2*a^2*(5 + 4*n)*Cos[e + f*x]*Hypergeometric2F1[1/2, -n, 3/2, 1 - Sin[e +
f*x]])/(f*(3 + 2*n)*Sqrt[a + a*Sin[e + f*x]]) - (2*a^2*Cos[e + f*x]*Sin[e +
 f*x]^(1 + n))/(f*(3 + 2*n)*Sqrt[a + a*Sin[e + f*x]])
```

Rubi [A] time = 0.140269, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2763, 21, 2776, 65}

$$-\frac{2a^2(4n+5)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};1-\sin(e+fx)\right)}{f(2n+3)\sqrt{a\sin(e+fx)+a}}-\frac{2a^2\cos(e+fx)\sin^{n+1}(e+fx)}{f(2n+3)\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^n*(a + a*Sin[e + f*x])^(3/2),x]
```


```
[Out] (-2*a^2*(5 + 4*n)*Cos[e + f*x]*Hypergeometric2F1[1/2, -n, 3/2, 1 - Sin[e +
f*x]])/(f*(3 + 2*n)*Sqrt[a + a*Sin[e + f*x]]) - (2*a^2*Cos[e + f*x]*Sin[e +
 f*x]^(1 + n))/(f*(3 + 2*n)*Sqrt[a + a*Sin[e + f*x]])
```


Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2776

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[(c + d*x)^n/Sqrt[a - b*x], x],
 x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !IntegerQ[2*n]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\int \sin^n(e+fx)(a+a\sin(e+fx))^{3/2}\,dx = -\frac{2a^2\cos(e+fx)\sin^{1+n}(e+fx)}{f(3+2n)\sqrt{a+a\sin(e+fx)}}+\frac{2\int\frac{\sin^n(e+fx)\left(\frac{1}{2}a^2(5+4n)+\frac{1}{2}a^2(5+4n)\sin(e+fx)\right)}{\sqrt{a+a\sin(e+fx)}}\,dx}{3+2n}$$

$$= -\frac{2a^2\cos(e+fx)\sin^{1+n}(e+fx)}{f(3+2n)\sqrt{a+a\sin(e+fx)}}+\frac{(a(5+4n))\int\sin^n(e+fx)\sqrt{a+a\sin(e+f}}{3+2n}$$

$$= -\frac{2a^2\cos(e+fx)\sin^{1+n}(e+fx)}{f(3+2n)\sqrt{a+a\sin(e+fx)}}+\frac{\left(a^3(5+4n)\cos(e+fx)\right)\operatorname{Subst}\left(\int\frac{x^n}{\sqrt{a-ax}}\,dx\right.}{f(3+2n)\sqrt{a-a\sin(e+fx)}\sqrt{a+a\,s}}$$

$$= -\frac{2a^2(5+4n)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};1-\sin(e+fx)\right)}{f(3+2n)\sqrt{a+a\sin(e+fx)}}-\frac{2a^2\cos(e+fx)\sin^1}{f(3+2n)\sqrt{a+a\,\mathrm{si}}}$$

Mathematica [C] time = 6.33124, size = 5111, normalized size = 48.22

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^n*(a + a*Sin[e + f*x])^(3/2),x]

[Out] Result too large to show
```

Maple [F] time = 0.125, size = 0, normalized size = 0.

$$\int\left(\sin\left(fx+e\right)\right)^n\left(a+a\sin\left(fx+e\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^n*(a+a*sin(f*x+e))^(3/2),x)

[Out] int(sin(f*x+e)^n*(a+a*sin(f*x+e))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\sin\left(fx+e\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^n*(a+a*sin(f*x+e))^(3/2),x, algorithm="maxima")

[Out] integrate((a*sin(f*x + e) + a)^(3/2)*sin(f*x + e)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\sin\left(fx+e\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^n*(a+a*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)^(3/2)*sin(f*x + e)^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**n*(a+a*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\sin\left(fx+e\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^n*(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)*sin(f*x + e)^n, x)
```

3.121 $\int \sin^n(e+fx)\sqrt{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=46

$$-\frac{2a\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};1-\sin(e+fx)\right)}{f\sqrt{a\sin(e+fx)+a}}$$

```
[Out] (-2*a*Cos[e + f*x]*Hypergeometric2F1[1/2, -n, 3/2, 1 - Sin[e + f*x]])/(f*Sq
rt[a + a*Sin[e + f*x]])
```

Rubi [A] time = 0.0615039, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2776, 65}

$$-\frac{2a\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};1-\sin(e+fx)\right)}{f\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^n*Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] (-2*a*Cos[e + f*x]*Hypergeometric2F1[1/2, -n, 3/2, 1 - Sin[e + f*x]])/(f*Sq
rt[a + a*Sin[e + f*x]])
```

Rule 2776

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[(c + d*x)^n/Sqrt[a - b*x], x],
 x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !IntegerQ[2*n]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\begin{aligned}\int \sin^n(e+fx)\sqrt{a+a\sin(e+fx)}\,dx &= \frac{\left(a^2\cos(e+fx)\right)\text{Subst}\left(\int \frac{x^n}{\sqrt{a-ax}}\,dx,x,\sin(e+fx)\right)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}\\ &= -\frac{2a\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};1-\sin(e+fx)\right)}{f\sqrt{a+a\sin(e+fx)}}\end{aligned}$$

Mathematica [C] time = 4.08545, size = 264, normalized size = 5.74

$$\frac{(1+i)e^{-\frac{1}{2}ifx}\sqrt{a(\sin(e+fx)+1)}\sin^n(e+fx)\left(\sin^2(e)e^{2ifx}-i\sin(2e)e^{2ifx}+\cos^2(e)\left(-e^{2ifx}\right)+1\right)^{-n}\left((2n+1)e^{ifx}\left(\cos\left(\frac{e}{2}\right)\right.\right.}{f(2n-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^n*Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] ((1 + I)*(E^(I*f*x)*(1 + 2*n)*Hypergeometric2F1[(1 - 2*n)/4, -n, (5 - 2*n)/
4, E^((2*I)*f*x)*(Cos[e] + I*Sin[e])^2]*(Cos[e/2] + I*Sin[e/2]) + (-1 + 2*n
)*Hypergeometric2F1[(-1 - 2*n)/4, -n, (3 - 2*n)/4, E^((2*I)*f*x)*(Cos[e] +
I*Sin[e])^2]*(I*Cos[e/2] + Sin[e/2]))*Sin[e + f*x]^n*Sqrt[a*(1 + Sin[e + f*
x])])/(E^((I/2)*f*x)*f*(-1 + 2*n)*(1 + 2*n)*(1 - E^((2*I)*f*x)*Cos[e]^2 + E
^((2*I)*f*x)*Sin[e]^2 - I*E^((2*I)*f*x)*Sin[2*e])^n*(Cos[(e + f*x)/2] + Sin
[(e + f*x)/2]))
```

Maple [F] time = 0.138, size = 0, normalized size = 0.

$$\int \left(\sin \left(fx+e\right)\right)^{n} \sqrt{a+a\sin \left(fx+e\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^n*(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] int(sin(f*x+e)^n*(a+a*sin(f*x+e))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\sin \left(fx+e\right)+a}\sin \left(fx+e\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^n*(a+a*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)*sin(f*x + e)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{a\sin \left(fx+e\right)+a}\sin \left(fx+e\right)^{n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^n*(a+a*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*sin(f*x + e) + a)*sin(f*x + e)^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\left(\sin \left(e+fx\right)+1\right)}\sin^{n}\left(e+fx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**n*(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))*sin(e + f*x)**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\sin\left(fx+e\right)+a}\sin\left(fx+e\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^n*(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)*sin(f*x + e)^n, x)
```

3.122 $\int \frac{\sin^n(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx$

Optimal. Leaf size=60

$$-\frac{\cos(e+fx)F_1\left(\frac{1}{2};-n,1;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)}{f\sqrt{a\sin(e+fx)+a}}$$

```
[Out] -((AppellF1[1/2, -n, 1, 3/2, 1 - Sin[e + f*x], (1 - Sin[e + f*x])/2]*Cos[e
+ f*x])/(f*Sqrt[a + a*Sin[e + f*x]]))
```

Rubi [A] time = 0.124268, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2787, 2785, 130, 429}

$$-\frac{\cos(e+fx)F_1\left(\frac{1}{2};-n,1;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)}{f\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^n/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] -((AppellF1[1/2, -n, 1, 3/2, 1 - Sin[e + f*x], (1 - Sin[e + f*x])/2]*Cos[e
+ f*x])/(f*Sqrt[a + a*Sin[e + f*x]]))
```

Rule 2787

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m
])/(1 + (b*Sin[e + f*x])/a)^FracPart[m], Int[(1 + (b*Sin[e + f*x])/a)^m*(d*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 2785

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Dist[(b*(d/b)^n*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a - x)^n*(2*a - x)^(m - 1/2)
)/Sqrt[x], x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

Rule 130

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + (b*x^k)/e)^m*(c + (d*x^k)/e)^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\sin^n(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx = \frac{\sqrt{1+\sin(e+fx)}\int \frac{\sin^n(e+fx)}{\sqrt{1+\sin(e+fx)}}\,dx}{\sqrt{a+a\sin(e+fx)}}$$
$$= -\frac{\cos(e+fx)\,\text{Subst}\left(\int \frac{(1-x)^n}{(2-x)\sqrt{x}}\,dx, x, 1-\sin(e+fx)\right)}{f\sqrt{1-\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}$$
$$= -\frac{(2\cos(e+fx))\,\text{Subst}\left(\int \frac{\left(1-x^2\right)^n}{2-x^2}\,dx, x, \sqrt{1-\sin(e+fx)}\right)}{f\sqrt{1-\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}$$
$$= -\frac{F_1\left(\frac{1}{2};-n,1;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)\cos(e+fx)}{f\sqrt{a+a\sin(e+fx)}}$$

Mathematica [B] time = 1.31282, size = 234, normalized size = 3.9

$$\frac{\cos(e+fx)\sqrt{a(\sin(e+fx)+1)}\sin^{2n}(e+fx)\left(-\sin^2(e+fx)\right)^{-n}\left(1-\frac{1}{\sin(e+fx)+1}\right)^{-n}\left(4\sqrt{\frac{\sin(e+fx)-1}{\sin(e+fx)+1}}(-\sin(e+fx))^nF_1\left(-n\right.\right.}{4af}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^n/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] (Cos[e + f*x]*Sin[e + f*x]^(2*n)*Sqrt[a*(1 + Sin[e + f*x])]*(4*AppellF1[-1/
2 - n, -1/2, -n, 1/2 - n, 2/(1 + Sin[e + f*x]), (1 + Sin[e + f*x])^(-1)]*(-
Sin[e + f*x])^n*Sqrt[(-1 + Sin[e + f*x])/(1 + Sin[e + f*x])] - (1 + 2*n)*Ap
pellF1[1, 1/2, -n, 2, (1 + Sin[e + f*x])/2, 1 + Sin[e + f*x]]*Sqrt[2 - 2*Si
n[e + f*x]]*(1 - (1 + Sin[e + f*x])^(-1))^n))/(4*a*f*(1 + 2*n)*(-1 + Sin[e
+ f*x])*(-Sin[e + f*x]^2)^n*(1 - (1 + Sin[e + f*x])^(-1))^n)
```

Maple [F] time = 0.124, size = 0, normalized size = 0.

$$\int \left(\sin\left(fx+e\right)\right)^n \frac{1}{\sqrt{a+a\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^n/(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] int(sin(f*x+e)^n/(a+a*sin(f*x+e))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(fx+e\right)^n}{\sqrt{a\sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^n/(a+a*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(f*x + e)^n/sqrt(a*sin(f*x + e) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sin\left(fx+e\right)^{n}}{\sqrt{a\sin\left(fx+e\right)+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^n/(a+a*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sin(f*x + e)^n/sqrt(a*sin(f*x + e) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin^{n}\left(e+fx\right)}{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**n/(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(sin(e + f*x)**n/sqrt(a*(sin(e + f*x) + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(fx+e\right)^{n}}{\sqrt{a\sin\left(fx+e\right)+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^n/(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^n/sqrt(a*sin(f*x + e) + a), x)
```

3.123 $\int \frac{\sin^n(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=65

$$-\frac{\cos(e+fx)F_1\left(\frac{1}{2};-n,2;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)}{2af\sqrt{a\sin(e+fx)+a}}$$

```
[Out] -(AppellF1[1/2, -n, 2, 3/2, 1 - Sin[e + f*x], (1 - Sin[e + f*x])/2]*Cos[e +
 f*x])/(2*a*f*Sqrt[a + a*Sin[e + f*x]])
```

Rubi [A] time = 0.136025, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2787, 2785, 130, 429}

$$-\frac{\cos(e+fx)F_1\left(\frac{1}{2};-n,2;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)}{2af\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^n/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] -(AppellF1[1/2, -n, 2, 3/2, 1 - Sin[e + f*x], (1 - Sin[e + f*x])/2]*Cos[e +
 f*x])/(2*a*f*Sqrt[a + a*Sin[e + f*x]])
```

Rule 2787

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m
])/(1 + (b*Sin[e + f*x])/a)^FracPart[m], Int[(1 + (b*Sin[e + f*x])/a)^m*(d*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] && !IntegerQ[m] && !GtQ[a, 0]
```

Rule 2785

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Dist[(b*(d/b)^n*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a - x)^n*(2*a - x)^(m - 1/2)
)/Sqrt[x], x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] && !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

Rule 130

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + (b*x^k)/e)^m*(c + (d*x^k)/e)^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\sin^n(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx = \frac{\sqrt{1+\sin(e+fx)}\int \frac{\sin^n(e+fx)}{(1+\sin(e+fx))^{3/2}}\,dx}{a\sqrt{a+a\sin(e+fx)}}$$
$$= -\frac{\cos(e+fx)\,\text{Subst}\left(\int \frac{(1-x)^n}{(2-x)^2\sqrt{x}}\,dx, x, 1-\sin(e+fx)\right)}{af\sqrt{1-\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}$$
$$= -\frac{(2\cos(e+fx))\,\text{Subst}\left(\int \frac{\left(1-x^2\right)^n}{\left(2-x^2\right)^2}\,dx, x, \sqrt{1-\sin(e+fx)}\right)}{af\sqrt{1-\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}$$
$$= -\frac{F_1\left(\frac{1}{2};-n,2;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)\cos(e+fx)}{2af\sqrt{a+a\sin(e+fx)}}$$

Mathematica [B] time = 2.2251, size = 274, normalized size = 4.22

$$\frac{\sec(e+fx)\sin^n(e+fx)\left(a^2\sqrt{2-2\sin(e+fx)}(\sin(e+fx)+1)^2(-\sin(e+fx))^{-n}F_1\left(1;\frac{1}{2},-n;2;\frac{1}{2}(\sin(e+fx)+1),\text{si}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^n/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] (Sec[e + f*x]*Sin[e + f*x]^n*((a^2*AppellF1[1, 1/2, -n, 2, (1 + Sin[e + f*x
])/2, 1 + Sin[e + f*x]]*Sqrt[2 - 2*Sin[e + f*x]]*(1 + Sin[e + f*x])^2)/(-Si
n[e + f*x])^n - (4*a*(-1 + Sin[e + f*x])*(2*a*(1 + 2*n)*AppellF1[1/2 - n, -
1/2, -n, 3/2 - n, 2/(1 + Sin[e + f*x]), (1 + Sin[e + f*x])^(-1)] + a*(-1 +
2*n)*AppellF1[-1/2 - n, -1/2, -n, 1/2 - n, 2/(1 + Sin[e + f*x]), (1 + Sin[e
 + f*x])^(-1)]*(1 + Sin[e + f*x])))/((-1 + 4*n^2)*Sqrt[1 - 2/(1 + Sin[e + f
*x])]*(1 - (1 + Sin[e + f*x])^(-1))^n)))/(8*a^3*f*Sqrt[a*(1 + Sin[e + f*x])
])
```

Maple [F] time = 0.123, size = 0, normalized size = 0.

$$\int \left(\sin\left(fx+e\right)\right)^n\left(a+a\sin\left(fx+e\right)\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^n/(a+a*sin(f*x+e))^(3/2),x)
```

```
[Out] int(sin(f*x+e)^n/(a+a*sin(f*x+e))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(fx+e\right)^n}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^n/(a+a*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(f*x + e)^n/(a*sin(f*x + e) + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{a\sin\left(fx+e\right)+a}\sin\left(fx+e\right)^{n}}{a^{2}\cos\left(fx+e\right)^{2}-2\,a^{2}\sin\left(fx+e\right)-2\,a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^n/(a+a*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(a*sin(f*x + e) + a)*sin(f*x + e)^n/(a^2*cos(f*x + e)^2 - 2*a
^2*sin(f*x + e) - 2*a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sin^{n}\left(e+fx\right)}{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**n/(a+a*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral(sin(e + f*x)**n/(a*(sin(e + f*x) + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sin\left(fx+e\right)^{n}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^n/(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^n/(a*sin(f*x + e) + a)^(3/2), x)
```

3.124 $\int (d\sin(e+fx))^n(1+\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=130

$$\frac{(4n+5)\cos(e+fx)(d\sin(e+fx))^{n+1}\,{}_2F_1\left(\frac{1}{2},n+1;n+2;\sin(e+fx)\right)}{df(n+1)(2n+3)\sqrt{1-\sin(e+fx)}\sqrt{\sin(e+fx)+1}}-\frac{2\cos(e+fx)(d\sin(e+fx))^{n+1}}{df(2n+3)\sqrt{\sin(e+fx)+1}}$$

```
[Out] (-2*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n))/(d*f*(3 + 2*n)*Sqrt[1 + Sin[e +
f*x]]) + ((5 + 4*n)*Cos[e + f*x]*Hypergeometric2F1[1/2, 1 + n, 2 + n, Sin[e
 + f*x]]*(d*Sin[e + f*x])^(1 + n))/(d*f*(1 + n)*(3 + 2*n)*Sqrt[1 - Sin[e +
f*x]]*Sqrt[1 + Sin[e + f*x]])
```

Rubi [A] time = 0.140899, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2763, 21, 2776, 64}

$$\frac{(4n+5)\cos(e+fx)(d\sin(e+fx))^{n+1}\,{}_2F_1\left(\frac{1}{2},n+1;n+2;\sin(e+fx)\right)}{df(n+1)(2n+3)\sqrt{1-\sin(e+fx)}\sqrt{\sin(e+fx)+1}}-\frac{2\cos(e+fx)(d\sin(e+fx))^{n+1}}{df(2n+3)\sqrt{\sin(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sin[e + f*x])^n*(1 + Sin[e + f*x])^(3/2),x]
```

```
[Out] (-2*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n))/(d*f*(3 + 2*n)*Sqrt[1 + Sin[e +
f*x]]) + ((5 + 4*n)*Cos[e + f*x]*Hypergeometric2F1[1/2, 1 + n, 2 + n, Sin[e
 + f*x]]*(d*Sin[e + f*x])^(1 + n))/(d*f*(1 + n)*(3 + 2*n)*Sqrt[1 - Sin[e +
f*x]]*Sqrt[1 + Sin[e + f*x]])
```

Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2776

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[(c + d*x)^n/Sqrt[a - b*x], x],
 x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !IntegerQ[2*n]
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rubi steps

$$\int (d\sin(e+fx))^n(1+\sin(e+fx))^{3/2}\,dx = -\frac{2\cos(e+fx)(d\sin(e+fx))^{1+n}}{df(3+2n)\sqrt{1+\sin(e+fx)}} + \frac{2\int \frac{(d\sin(e+fx))^n\left(\frac{1}{2}d(5+4n)+\frac{1}{2}d(5+4n)\sin(e+fx)\right)}{\sqrt{1+\sin(e+fx)}}}{d(3+2n)}$$

$$= -\frac{2\cos(e+fx)(d\sin(e+fx))^{1+n}}{df(3+2n)\sqrt{1+\sin(e+fx)}} + \frac{(5+4n)\int (d\sin(e+fx))^n\sqrt{1+\sin(e+f}{3+2n}$$

$$= -\frac{2\cos(e+fx)(d\sin(e+fx))^{1+n}}{df(3+2n)\sqrt{1+\sin(e+fx)}} + \frac{((5+4n)\cos(e+fx))\,\mathrm{Subst}\left(\int \frac{(dx)^n}{\sqrt{1-x}}\,dx,\right.}{f(3+2n)\sqrt{1-\sin(e+fx)}\sqrt{1+\sin}$$

$$= -\frac{2\cos(e+fx)(d\sin(e+fx))^{1+n}}{df(3+2n)\sqrt{1+\sin(e+fx)}} + \frac{(5+4n)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},1+n;2+n;\right.}{df(1+n)(3+2n)\sqrt{1-\sin(e+}$$

Mathematica [C] time = 6.30222, size = 5129, normalized size = 39.45

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sin[e + f*x])^n*(1 + Sin[e + f*x])^(3/2),x]
```

```
[Out] Result too large to show
```

Maple [F] time = 0.116, size = 0, normalized size = 0.

$$\int \left(d\sin\left(fx+e\right)\right)^n\left(1+\sin\left(fx+e\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^n*(1+sin(f*x+e))^(3/2),x)
```

```
[Out] int((d*sin(f*x+e))^n*(1+sin(f*x+e))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\sin\left(fx+e\right)\right)^n\left(\sin\left(fx+e\right)+1\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(1+sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*sin(f*x + e))^n*(sin(f*x + e) + 1)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(d\sin\left(fx+e\right)\right)^{n}\left(\sin\left(fx+e\right)+1\right)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(1+sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((d*sin(f*x + e))^n*(sin(f*x + e) + 1)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**n*(1+sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(d\sin\left(fx+e\right)\right)^{n}\left(\sin\left(fx+e\right)+1\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(1+sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*sin(f*x + e))^n*(sin(f*x + e) + 1)^(3/2), x)
```

3.125 $\int (d\sin(e+fx))^n\sqrt{1+\sin(e+fx)}\,dx$

Optimal. Leaf size=72

$$\frac{\cos(e+fx)(d\sin(e+fx))^{n+1}\,{}_2F_1\left(\frac{1}{2},n+1;n+2;\sin(e+fx)\right)}{df(n+1)\sqrt{1-\sin(e+fx)}\sqrt{\sin(e+fx)+1}}$$

```
[Out] (Cos[e + f*x]*Hypergeometric2F1[1/2, 1 + n, 2 + n, Sin[e + f*x]]*(d*Sin[e +
 f*x])^(1 + n))/(d*f*(1 + n)*Sqrt[1 - Sin[e + f*x]]*Sqrt[1 + Sin[e + f*x]])
```

Rubi [A] time = 0.055852, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2776, 64}

$$\frac{\cos(e+fx)(d\sin(e+fx))^{n+1}\,{}_2F_1\left(\frac{1}{2},n+1;n+2;\sin(e+fx)\right)}{df(n+1)\sqrt{1-\sin(e+fx)}\sqrt{\sin(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sin[e + f*x])^n*Sqrt[1 + Sin[e + f*x]],x]
```

```
[Out] (Cos[e + f*x]*Hypergeometric2F1[1/2, 1 + n, 2 + n, Sin[e + f*x]]*(d*Sin[e +
 f*x])^(1 + n))/(d*f*(1 + n)*Sqrt[1 - Sin[e + f*x]]*Sqrt[1 + Sin[e + f*x]])
```

Rule 2776

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[(c + d*x)^n/Sqrt[a - b*x], x],
 x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !IntegerQ[2*n]
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rubi steps

$$\begin{aligned}\int (d\sin(e+fx))^n\sqrt{1+\sin(e+fx)}\,dx &= \frac{\cos(e+fx)\,\text{Subst}\left(\int\frac{(dx)^n}{\sqrt{1-x}}\,dx,x,\sin(e+fx)\right)}{f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}\\ &= \frac{\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},1+n;2+n;\sin(e+fx)\right)(d\sin(e+fx))^{1+n}}{df(1+n)\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}\end{aligned}$$

Mathematica [C] time = 0.306138, size = 215, normalized size = 2.99

$$\frac{(1-i)2^{-n}e^{\frac{1}{2}i(e+fx)}\left(-ie^{-i(e+fx)}\left(-1+e^{2i(e+fx)}\right)\right)^{n+1}\sqrt{\sin(e+fx)+1}\left(i(2n-1)\,{}_2F_1\left(1,\frac{1}{4}(2n+3);\frac{1}{4}(3-2n);e^{2i(e+fx)}\right)+(2n\right.}{f(2n-1)(2n+1)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sin[e + f*x])^n*Sqrt[1 + Sin[e + f*x]],x]
```

```
[Out] ((1 - I)*E^((I/2)*(e + f*x))*(((-I)*(-1 + E^((2*I)*(e + f*x))))/E^(I*(e + f
*x)))^(1 + n)*(I*(-1 + 2*n)*Hypergeometric2F1[1, (3 + 2*n)/4, (3 - 2*n)/4,
E^((2*I)*(e + f*x))] + E^(I*(e + f*x))*(1 + 2*n)*Hypergeometric2F1[1, (5 +
2*n)/4, (5 - 2*n)/4, E^((2*I)*(e + f*x))])*(d*Sin[e + f*x])^n*Sqrt[1 + Sin[
e + f*x]])/(2^n*f*(-1 + 2*n)*(1 + 2*n)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)*Sin[e + f*x]^n)
```

Maple [F] time = 0.109, size = 0, normalized size = 0.

$$\int \left(d \sin \left(f x+e\right)\right)^{n} \sqrt{1+\sin \left(f x+e\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^n*(1+sin(f*x+e))^(1/2),x)
```

```
[Out] int((d*sin(f*x+e))^n*(1+sin(f*x+e))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d \sin \left(f x+e\right)\right)^{n} \sqrt{\sin \left(f x+e\right)+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(1+sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*sin(f*x + e))^n*sqrt(sin(f*x + e) + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(d \sin \left(f x+e\right)\right)^{n} \sqrt{\sin \left(f x+e\right)+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(1+sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((d*sin(f*x + e))^n*sqrt(sin(f*x + e) + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d \sin \left(e+f x\right)\right)^{n} \sqrt{\sin \left(e+f x\right)+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**n*(1+sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((d*sin(e + f*x))**n*sqrt(sin(e + f*x) + 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d \sin \left(fx+e\right)\right)^{n} \sqrt{\sin \left(fx+e\right)+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(1+sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*sin(f*x + e))^n*sqrt(sin(f*x + e) + 1), x)
```

3.126 $\int \frac{(d\sin(e+fx))^n}{\sqrt{1+\sin(e+fx)}}\,dx$

Optimal. Leaf size=78

$$-\frac{\cos(e+fx)\sin^{-n}(e+fx)F_1\left(\frac{1}{2};-n,1;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)(d\sin(e+fx))^n}{f\sqrt{\sin(e+fx)+1}}$$

```
[Out] -((AppellF1[1/2, -n, 1, 3/2, 1 - Sin[e + f*x], (1 - Sin[e + f*x])/2]*Cos[e
+ f*x]*(d*Sin[e + f*x])^n)/(f*Sin[e + f*x]^n*Sqrt[1 + Sin[e + f*x]]))
```

Rubi [A] time = 0.118393, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2786, 2785, 130, 429}

$$-\frac{\cos(e+fx)\sin^{-n}(e+fx)F_1\left(\frac{1}{2};-n,1;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)(d\sin(e+fx))^n}{f\sqrt{\sin(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sin[e + f*x])^n/Sqrt[1 + Sin[e + f*x]],x]
```

```
[Out] -((AppellF1[1/2, -n, 1, 3/2, 1 - Sin[e + f*x], (1 - Sin[e + f*x])/2]*Cos[e
+ f*x]*(d*Sin[e + f*x])^n)/(f*Sin[e + f*x]^n*Sqrt[1 + Sin[e + f*x]]))
```

Rule 2786

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[((d/b)^IntPart[n]*(d*Sin[e + f*x])^FracPart[n
])/(b*Sin[e + f*x])^FracPart[n], Int[(a + b*Sin[e + f*x])^m*(b*Sin[e + f*x]
)^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !In
tegerQ[m] && GtQ[a, 0] &&  !GtQ[d/b, 0]
```

Rule 2785

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Dist[(b*(d/b)^n*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a - x)^n*(2*a - x)^(m - 1/2)
)/Sqrt[x], x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

Rule 130

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + (b*x^k)/e)^m*(c + (d*x^k)/e)^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{(d\sin(e+fx))^n}{\sqrt{1+\sin(e+fx)}}\,dx = \left(\sin^{-n}(e+fx)(d\sin(e+fx))^n\right)\int \frac{\sin^n(e+fx)}{\sqrt{1+\sin(e+fx)}}\,dx$$

$$= -\frac{\left(\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n\right)\text{Subst}\left(\int \frac{(1-x)^n}{(2-x)\sqrt{x}}\,dx, x, 1-\sin(e+fx)\right)}{f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}$$

$$= -\frac{\left(2\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n\right)\text{Subst}\left(\int \frac{\left(1-x^2\right)^n}{2-x^2}\,dx, x, \sqrt{1-\sin(e+fx)}\right)}{f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}$$

$$= -\frac{F_1\left(\frac{1}{2};-n,1;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n}{f\sqrt{1+\sin(e+fx)}}$$

Mathematica [B] time = 0.404747, size = 227, normalized size = 2.91

$$\frac{\sqrt{\sin(e+fx)+1}\cos(e+fx)(-\sin(e+fx))^{-n}\left(1-\frac{1}{\sin(e+fx)+1}\right)^{-n}\left(4\sqrt{\frac{\sin(e+fx)-1}{\sin(e+fx)+1}}(-\sin(e+fx))^n F_1\left(-n-\frac{1}{2};-\frac{1}{2},-n;\frac{1}{2}-\right.\right.}{4f(2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sin[e + f*x])^n/Sqrt[1 + Sin[e + f*x]],x]
```

```
[Out] (Cos[e + f*x]*(d*Sin[e + f*x])^n*Sqrt[1 + Sin[e + f*x]]*(4*AppellF1[-1/2 -
n, -1/2, -n, 1/2 - n, 2/(1 + Sin[e + f*x]), (1 + Sin[e + f*x])^(-1)]*(-Sin[
e + f*x])^n*Sqrt[(-1 + Sin[e + f*x])/(1 + Sin[e + f*x])] - (1 + 2*n)*Appell
F1[1, 1/2, -n, 2, (1 + Sin[e + f*x])/2, 1 + Sin[e + f*x]]*Sqrt[2 - 2*Sin[e
+ f*x]]*(1 - (1 + Sin[e + f*x])^(-1))^n))/(4*f*(1 + 2*n)*(-1 + Sin[e + f*x]
)*(-Sin[e + f*x])^n*(1 - (1 + Sin[e + f*x])^(-1))^n)
```

Maple [F] time = 0.109, size = 0, normalized size = 0.

$$\int \left(d\sin\left(fx+e\right)\right)^n \frac{1}{\sqrt{1+\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^n/(1+sin(f*x+e))^(1/2),x)
```

```
[Out] int((d*sin(f*x+e))^n/(1+sin(f*x+e))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sin\left(fx+e\right)\right)^n}{\sqrt{\sin\left(fx+e\right)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n/(1+sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*sin(f*x + e))^n/sqrt(sin(f*x + e) + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(d\sin\left(fx+e\right)\right)^{n}}{\sqrt{\sin\left(fx+e\right)+1}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n/(1+sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((d*sin(f*x + e))^n/sqrt(sin(f*x + e) + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(d\sin\left(e+fx\right)\right)^{n}}{\sqrt{\sin\left(e+fx\right)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**n/(1+sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((d*sin(e + f*x))**n/sqrt(sin(e + f*x) + 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(d\sin\left(fx+e\right)\right)^{n}}{\sqrt{\sin\left(fx+e\right)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n/(1+sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*sin(f*x + e))^n/sqrt(sin(f*x + e) + 1), x)
```

3.127 $\int \frac{(d\sin(e+fx))^n}{(1+\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=80

$$-\frac{\cos(e+fx)\sin^{-n}(e+fx)F_1\left(\frac{1}{2};-n,2;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)(d\sin(e+fx))^n}{2f\sqrt{\sin(e+fx)+1}}$$

```
[Out] -(AppellF1[1/2, -n, 2, 3/2, 1 - Sin[e + f*x], (1 - Sin[e + f*x])/2]*Cos[e +
 f*x]*(d*Sin[e + f*x])^n)/(2*f*Sin[e + f*x]^n*Sqrt[1 + Sin[e + f*x]])
```

Rubi [A] time = 0.135221, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2786, 2785, 130, 429}

$$-\frac{\cos(e+fx)\sin^{-n}(e+fx)F_1\left(\frac{1}{2};-n,2;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)(d\sin(e+fx))^n}{2f\sqrt{\sin(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sin[e + f*x])^n/(1 + Sin[e + f*x])^(3/2),x]
```

```
[Out] -(AppellF1[1/2, -n, 2, 3/2, 1 - Sin[e + f*x], (1 - Sin[e + f*x])/2]*Cos[e +
 f*x]*(d*Sin[e + f*x])^n)/(2*f*Sin[e + f*x]^n*Sqrt[1 + Sin[e + f*x]])
```

Rule 2786

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[((d/b)^IntPart[n]*(d*Sin[e + f*x])^FracPart[n
])/(b*Sin[e + f*x])^FracPart[n], Int[(a + b*Sin[e + f*x])^m*(b*Sin[e + f*x]
)^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !In
tegerQ[m] && GtQ[a, 0] &&  !GtQ[d/b, 0]
```

Rule 2785

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Dist[(b*(d/b)^n*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a - x)^n*(2*a - x)^(m - 1/2)
)/Sqrt[x], x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

Rule 130

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + (b*x^k)/e)^m*(c + (d*x^k)/e)^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{(d\sin(e+fx))^n}{(1+\sin(e+fx))^{3/2}}\,dx = \left(\sin^{-n}(e+fx)(d\sin(e+fx))^n\right)\int \frac{\sin^n(e+fx)}{(1+\sin(e+fx))^{3/2}}\,dx$$
$$= -\frac{\left(\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n\right)\text{Subst}\left(\int \frac{(1-x)^n}{(2-x)^2\sqrt{x}}\,dx, x, 1-\sin(e+fx)\right)}{f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}$$
$$= -\frac{\left(2\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n\right)\text{Subst}\left(\int \frac{(1-x^2)^n}{(2-x^2)^2}\,dx, x, \sqrt{1-\sin(e+fx)}\right)}{f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}$$
$$= -\frac{F_1\left(\frac{1}{2};-n,2;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx)}{2f\sqrt{1+\sin(e+fx)}}$$

Mathematica [B] time = 0.857067, size = 265, normalized size = 3.31

$$\frac{\sec(e+fx)\left(\sqrt{2-2\sin(e+fx)}(\sin(e+fx)+1)^2(-\sin(e+fx))^{-n}F_1\left(1;\frac{1}{2},-n;2;\frac{1}{2}(\sin(e+fx)+1),\sin(e+fx)+1\right)\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sin[e + f*x])^n/(1 + Sin[e + f*x])^(3/2),x]
```

```
[Out] (Sec[e + f*x]*(d*Sin[e + f*x])^n*((AppellF1[1, 1/2, -n, 2, (1 + Sin[e + f*x
])/2, 1 + Sin[e + f*x]]*Sqrt[2 - 2*Sin[e + f*x]]*(1 + Sin[e + f*x])^2)/(-Si
n[e + f*x])^n - (4*(1 + Sin[e + f*x])*Sqrt[1 - 2/(1 + Sin[e + f*x])]*(2*(1
+ 2*n)*AppellF1[1/2 - n, -1/2, -n, 3/2 - n, 2/(1 + Sin[e + f*x]), (1 + Sin[
e + f*x])^(-1)] + (-1 + 2*n)*AppellF1[-1/2 - n, -1/2, -n, 1/2 - n, 2/(1 + S
in[e + f*x]), (1 + Sin[e + f*x])^(-1)]*(1 + Sin[e + f*x])))/((-1 + 4*n^2)*(
1 - (1 + Sin[e + f*x])^(-1))^n)))/(8*f*Sqrt[1 + Sin[e + f*x]])
```

Maple [F] time = 0.109, size = 0, normalized size = 0.

$$\int \left(d\sin\left(fx+e\right)\right)^n\left(1+\sin\left(fx+e\right)\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^n/(1+sin(f*x+e))^(3/2),x)
```

```
[Out] int((d*sin(f*x+e))^n/(1+sin(f*x+e))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sin\left(fx+e\right)\right)^n}{\left(\sin\left(fx+e\right)+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n/(1+sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*sin(f*x + e))^n/(sin(f*x + e) + 1)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-\frac{\left(d\sin\left(fx+e\right)\right)^{n}\sqrt{\sin\left(fx+e\right)+1}}{\cos\left(fx+e\right)^{2}-2\sin\left(fx+e\right)-2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n/(1+sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-(d*sin(f*x + e))^n*sqrt(sin(f*x + e) + 1)/(cos(f*x + e)^2 - 2*sin
(f*x + e) - 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(d\sin\left(e+fx\right)\right)^{n}}{\left(\sin\left(e+fx\right)+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**n/(1+sin(f*x+e))**(3/2),x)
```

```
[Out] Integral((d*sin(e + f*x))**n/(sin(e + f*x) + 1)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(d\sin\left(fx+e\right)\right)^{n}}{\left(\sin\left(fx+e\right)+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n/(1+sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*sin(f*x + e))^n/(sin(f*x + e) + 1)^(3/2), x)
```

3.128 $\int (d\sin(e+fx))^n (a+a\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=131

$$-\frac{2a^2(4n+5)\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};1-\sin(e+fx)\right)}{f(2n+3)\sqrt{a\sin(e+fx)+a}}-\frac{2a^2\cos(e+fx)(d\sin(e+fx)}{df(2n+3)\sqrt{a\sin(e+fx)+}}$$

```
[Out] (-2*a^2*(5 + 4*n)*Cos[e + f*x]*Hypergeometric2F1[1/2, -n, 3/2, 1 - Sin[e +
f*x]]*(d*Sin[e + f*x])^n)/(f*(3 + 2*n)*Sin[e + f*x]^n*Sqrt[a + a*Sin[e + f*
x]]) - (2*a^2*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n))/(d*f*(3 + 2*n)*Sqrt[a
+ a*Sin[e + f*x]])
```

Rubi [A] time = 0.170092, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2763, 21, 2776, 67, 65}

$$-\frac{2a^2(4n+5)\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};1-\sin(e+fx)\right)}{f(2n+3)\sqrt{a\sin(e+fx)+a}}-\frac{2a^2\cos(e+fx)(d\sin(e+fx)}{df(2n+3)\sqrt{a\sin(e+fx)+}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sin[e + f*x])^n*(a + a*Sin[e + f*x])^(3/2),x]
```


```
[Out] (-2*a^2*(5 + 4*n)*Cos[e + f*x]*Hypergeometric2F1[1/2, -n, 3/2, 1 - Sin[e +
f*x]]*(d*Sin[e + f*x])^n)/(f*(3 + 2*n)*Sin[e + f*x]^n*Sqrt[a + a*Sin[e + f*
x]]) - (2*a^2*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n))/(d*f*(3 + 2*n)*Sqrt[a
+ a*Sin[e + f*x]])
```


Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2776

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[(c + d*x)^n/Sqrt[a - b*x], x],
 x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !IntegerQ[2*n]
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-((b*c)
/d))^IntPart[m]*(b*x)^FracPart[m])/(-((d*x)/c))^FracPart[m], Int[(-((d*x)/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
!IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-(d/(b*c)), 0]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\int (d\sin(e+fx))^n (a+a\sin(e+fx))^{3/2}\,dx = -\frac{2a^2\cos(e+fx)(d\sin(e+fx))^{1+n}}{df(3+2n)\sqrt{a+a\sin(e+fx)}} + \frac{2\int \frac{(d\sin(e+fx))^n\left(\frac{1}{2}a^2d(5+4n)+\frac{1}{2}a^2d(5+4n)\sin \cdots\right.}{\sqrt{a+a\sin(e+fx)}}}{d(3+2n)}$$

$$= -\frac{2a^2\cos(e+fx)(d\sin(e+fx))^{1+n}}{df(3+2n)\sqrt{a+a\sin(e+fx)}} + \frac{(a(5+4n))\int (d\sin(e+fx))^n\sqrt{a+a\,\mathrm{s}\cdots}}{3+2n}$$

$$= -\frac{2a^2\cos(e+fx)(d\sin(e+fx))^{1+n}}{df(3+2n)\sqrt{a+a\sin(e+fx)}} + \frac{\left(a^3(5+4n)\cos(e+fx)\right)\mathrm{Subst}\left(\int \frac{(d\cdots}{\sqrt{a\cdots}}\right.}{f(3+2n)\sqrt{a-a\sin(e+fx)}\sqrt{a\cdots}}$$

$$= -\frac{2a^2\cos(e+fx)(d\sin(e+fx))^{1+n}}{df(3+2n)\sqrt{a+a\sin(e+fx)}} + \frac{\left(a^3(5+4n)\cos(e+fx)\sin^{-n}(e+fx)\cdots\right.}{f(3+2n)\sqrt{a-a\cdots}}$$

$$= -\frac{2a^2(5+4n)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};1-\sin(e+fx)\right)\sin^{-n}(e+fx)(d\sin\cdots}{f(3+2n)\sqrt{a+a\sin(e+fx)}}$$

Mathematica [C] time = 6.32103, size = 5131, normalized size = 39.17

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sin[e + f*x])^n*(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] Result too large to show
```

Maple [F] time = 0.145, size = 0, normalized size = 0.

$$\int \left(d\sin\left(fx+e\right)\right)^n \left(a+a\sin\left(fx+e\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(3/2),x)
```

```
[Out] int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(d\sin\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e))^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(d\sin\left(fx+e\right)\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e))^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**n*(a+a*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(d\sin\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e))^n, x)
```

3.129 $\int (d\sin(e+fx))^n\sqrt{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=66

$$-\frac{2a\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};1-\sin(e+fx)\right)}{f\sqrt{a\sin(e+fx)+a}}$$

```
[Out] (-2*a*Cos[e + f*x]*Hypergeometric2F1[1/2, -n, 3/2, 1 - Sin[e + f*x]]*(d*Sin
[e + f*x])^n)/(f*Sin[e + f*x]^n*Sqrt[a + a*Sin[e + f*x]])
```

Rubi [A] time = 0.0745947, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2776, 67, 65}

$$-\frac{2a\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};1-\sin(e+fx)\right)}{f\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sin[e + f*x])^n*Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] (-2*a*Cos[e + f*x]*Hypergeometric2F1[1/2, -n, 3/2, 1 - Sin[e + f*x]]*(d*Sin
[e + f*x])^n)/(f*Sin[e + f*x]^n*Sqrt[a + a*Sin[e + f*x]])
```

Rule 2776

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[(c + d*x)^n/Sqrt[a - b*x], x],
 x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !IntegerQ[2*n]
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-((b*c)
/d))^IntPart[m]*(b*x)^FracPart[m])/(-((d*x)/c))^FracPart[m], Int[(-((d*x)/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
!IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-(d/(b*c)), 0]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\int (d\sin(e+fx))^n\sqrt{a+a\sin(e+fx)}\,dx = \frac{\left(a^2\cos(e+fx)\right)\text{Subst}\left(\int \frac{(dx)^n}{\sqrt{a-ax}}dx, x, \sin(e+fx)\right)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}$$
$$= \frac{\left(a^2\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n\right)\text{Subst}\left(\int \frac{x^n}{\sqrt{a-ax}}dx, x, \sin(e\right.}{f\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}$$
$$= -\frac{2a\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};1-\sin(e+fx)\right)\sin^{-n}(e+fx)(d\sin(e+fx)}{f\sqrt{a+a\sin(e+fx)}}$$

Mathematica [C] time = 0.408685, size = 266, normalized size = 4.03

$$\frac{(1+i)e^{-\frac{1}{2}ifx}\sqrt{a(\sin(e+fx)+1)}(d\sin(e+fx))^n\left(\sin^2(e)e^{2ifx}-i\sin(2e)e^{2ifx}+\cos^2(e)\left(-e^{2ifx}\right)+1\right)^{-n}\left((2n+1)e^{ifx}\left(\text{c}\right.\right.}{f(2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sin[e + f*x])^n*Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] ((1 + I)*(E^(I*f*x)*(1 + 2*n)*Hypergeometric2F1[(1 - 2*n)/4, -n, (5 - 2*n)/
4, E^((2*I)*f*x)*(Cos[e] + I*Sin[e])^2]*(Cos[e/2] + I*Sin[e/2]) + (-1 + 2*n
)*Hypergeometric2F1[(-1 - 2*n)/4, -n, (3 - 2*n)/4, E^((2*I)*f*x)*(Cos[e] +
I*Sin[e])^2]*(I*Cos[e/2] + Sin[e/2]))*(d*Sin[e + f*x])^n*Sqrt[a*(1 + Sin[e
+ f*x])])/(E^((I/2)*f*x)*f*(-1 + 2*n)*(1 + 2*n)*(1 - E^((2*I)*f*x)*Cos[e]^2
 + E^((2*I)*f*x)*Sin[e]^2 - I*E^((2*I)*f*x)*Sin[2*e])^n*(Cos[(e + f*x)/2] +
 Sin[(e + f*x)/2]))
```

Maple [F] time = 0.161, size = 0, normalized size = 0.

$$\int \left(d\sin\left(fx+e\right)\right)^n\sqrt{a+a\sin\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\sin\left(fx+e\right)+a}\left(d\sin\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e))^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{a\sin\left(fx+e\right)+a}\left(d\sin\left(fx+e\right)\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e))^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a\left(\sin\left(e+fx\right)+1\right)}\left(d\sin\left(e+fx\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**n*(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))*(d*sin(e + f*x))**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a\sin\left(fx+e\right)+a}\left(d\sin\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e))^n, x)
```

3.130 $\int \frac{(d\sin(e+fx))^n}{\sqrt{a+a\sin(e+fx)}}\,dx$

Optimal. Leaf size=80

$$-\frac{\cos(e+fx)\sin^{-n}(e+fx)F_1\left(\frac{1}{2};-n,1;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)(d\sin(e+fx))^n}{f\sqrt{a\sin(e+fx)+a}}$$

```
[Out] -((AppellF1[1/2, -n, 1, 3/2, 1 - Sin[e + f*x], (1 - Sin[e + f*x])/2]*Cos[e
+ f*x]*(d*Sin[e + f*x])^n)/(f*Sin[e + f*x]^n*Sqrt[a + a*Sin[e + f*x]]))
```

Rubi [A] time = 0.18048, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2787, 2786, 2785, 130, 429}

$$-\frac{\cos(e+fx)\sin^{-n}(e+fx)F_1\left(\frac{1}{2};-n,1;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)(d\sin(e+fx))^n}{f\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sin[e + f*x])^n/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] -((AppellF1[1/2, -n, 1, 3/2, 1 - Sin[e + f*x], (1 - Sin[e + f*x])/2]*Cos[e
+ f*x]*(d*Sin[e + f*x])^n)/(f*Sin[e + f*x]^n*Sqrt[a + a*Sin[e + f*x]]))
```

Rule 2787

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m
])/(1 + (b*Sin[e + f*x])/a)^FracPart[m], Int[(1 + (b*Sin[e + f*x])/a)^m*(d*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] && !IntegerQ[m] && !GtQ[a, 0]
```

Rule 2786

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[((d/b)^IntPart[n]*(d*Sin[e + f*x])^FracPart[n
])/(b*Sin[e + f*x])^FracPart[n], Int[(a + b*Sin[e + f*x])^m*(b*Sin[e + f*x]
)^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && !In
tegerQ[m] && GtQ[a, 0] && !GtQ[d/b, 0]
```

Rule 2785

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Dist[(b*(d/b)^n*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a - x)^n*(2*a - x)^(m - 1/2)
)/Sqrt[x], x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] && !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

Rule 130

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + (b*x^k)/e)^m*(c + (d*x^k)/e)^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{(d\sin(e+fx))^n}{\sqrt{a+a\sin(e+fx)}}\,dx = \frac{\sqrt{1+\sin(e+fx)}\int \frac{(d\sin(e+fx))^n}{\sqrt{1+\sin(e+fx)}}\,dx}{\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\left(\sin^{-n}(e+fx)(d\sin(e+fx))^n\sqrt{1+\sin(e+fx)}\right)\int \frac{\sin^n(e+fx)}{\sqrt{1+\sin(e+fx)}}\,dx}{\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{\left(\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n\right)\text{Subst}\left(\int \frac{(1-x)^n}{(2-x)\sqrt{x}}\,dx, x, 1-\sin(e+fx)\right)}{f\sqrt{1-\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{\left(2\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n\right)\text{Subst}\left(\int \frac{(1-x^2)^n}{2-x^2}\,dx, x, \sqrt{1-\sin(e+fx)}\right)}{f\sqrt{1-\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{F_1\left(\frac{1}{2};-n,1;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^{}}{f\sqrt{a+a\sin(e+fx)}}$$

Mathematica [B] time = 0.696504, size = 242, normalized size = 3.02

$$\frac{\cos(e+fx)\sqrt{a(\sin(e+fx)+1)}\sin^n(e+fx)\left(-\sin^2(e+fx)\right)^{-n}\left(1-\frac{1}{\sin(e+fx)+1}\right)^{-n}\left(4\sqrt{\frac{\sin(e+fx)-1}{\sin(e+fx)+1}}(-\sin(e+fx))^nF_1\left(-n\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sin[e + f*x])^n/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] (Cos[e + f*x]*Sin[e + f*x]^n*(d*Sin[e + f*x])^n*Sqrt[a*(1 + Sin[e + f*x])]*
(4*AppellF1[-1/2 - n, -1/2, -n, 1/2 - n, 2/(1 + Sin[e + f*x]), (1 + Sin[e +
 f*x])^(-1)]*(-Sin[e + f*x])^n*Sqrt[(-1 + Sin[e + f*x])/(1 + Sin[e + f*x])]
 - (1 + 2*n)*AppellF1[1, 1/2, -n, 2, (1 + Sin[e + f*x])/2, 1 + Sin[e + f*x]
]*Sqrt[2 - 2*Sin[e + f*x]]*(1 - (1 + Sin[e + f*x])^(-1))^n))/(4*a*f*(1 + 2*
n)*(-1 + Sin[e + f*x])*(-Sin[e + f*x]^2)^n*(1 - (1 + Sin[e + f*x])^(-1))^n)
```

Maple [F] time = 0.148, size = 0, normalized size = 0.

$$\int \left(d\sin\left(fx+e\right)\right)^n \frac{1}{\sqrt{a+a\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] int((d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sin\left(fx+e\right)\right)^n}{\sqrt{a\sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*sin(f*x + e))^n/sqrt(a*sin(f*x + e) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(d\sin\left(fx+e\right)\right)^n}{\sqrt{a\sin\left(fx+e\right)+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((d*sin(f*x + e))^n/sqrt(a*sin(f*x + e) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sin\left(e+fx\right)\right)^n}{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**n/(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((d*sin(e + f*x))**n/sqrt(a*(sin(e + f*x) + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sin\left(fx+e\right)\right)^n}{\sqrt{a\sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*sin(f*x + e))^n/sqrt(a*sin(f*x + e) + a), x)
```

3.131 $\int \frac{(d\sin(e+fx))^n}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=85

$$-\frac{\cos(e+fx)\sin^{-n}(e+fx)F_1\left(\frac{1}{2};-n,2;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)(d\sin(e+fx))^n}{2af\sqrt{a\sin(e+fx)+a}}$$

```
[Out] -(AppellF1[1/2, -n, 2, 3/2, 1 - Sin[e + f*x], (1 - Sin[e + f*x])/2]*Cos[e +
 f*x]*(d*Sin[e + f*x])^n)/(2*a*f*Sin[e + f*x]^n*Sqrt[a + a*Sin[e + f*x]])
```

Rubi [A] time = 0.202223, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2787, 2786, 2785, 130, 429}

$$-\frac{\cos(e+fx)\sin^{-n}(e+fx)F_1\left(\frac{1}{2};-n,2;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)(d\sin(e+fx))^n}{2af\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sin[e + f*x])^n/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] -(AppellF1[1/2, -n, 2, 3/2, 1 - Sin[e + f*x], (1 - Sin[e + f*x])/2]*Cos[e +
 f*x]*(d*Sin[e + f*x])^n)/(2*a*f*Sin[e + f*x]^n*Sqrt[a + a*Sin[e + f*x]])
```

Rule 2787

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m
])/(1 + (b*Sin[e + f*x])/a)^FracPart[m], Int[(1 + (b*Sin[e + f*x])/a)^m*(d*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] && !IntegerQ[m] && !GtQ[a, 0]
```

Rule 2786

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[((d/b)^IntPart[n]*(d*Sin[e + f*x])^FracPart[n
])/(b*Sin[e + f*x])^FracPart[n], Int[(a + b*Sin[e + f*x])^m*(b*Sin[e + f*x]
)^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && !In
tegerQ[m] && GtQ[a, 0] && !GtQ[d/b, 0]
```

Rule 2785

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Dist[(b*(d/b)^n*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a - x)^n*(2*a - x)^(m - 1/2)
)/Sqrt[x], x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] && !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

Rule 130

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + (b*x^k)/e)^m*(c + (d*x^k)/e)^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{(d\sin(e+fx))^n}{(a+a\sin(e+fx))^{3/2}}\,dx = \frac{\sqrt{1+\sin(e+fx)}\int \frac{(d\sin(e+fx))^n}{(1+\sin(e+fx))^{3/2}}\,dx}{a\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\left(\sin^{-n}(e+fx)(d\sin(e+fx))^n\sqrt{1+\sin(e+fx)}\right)\int \frac{\sin^n(e+fx)}{(1+\sin(e+fx))^{3/2}}\,dx}{a\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{\left(\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n\right)\operatorname{Subst}\left(\int \frac{(1-x)^n}{(2-x)^2\sqrt{x}}\,dx, x, 1-\sin(e+fx)\right)}{af\sqrt{1-\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{\left(2\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n\right)\operatorname{Subst}\left(\int \frac{\left(1-x^2\right)^n}{\left(2-x^2\right)^2}\,dx, x, \sqrt{1-\sin(e+fx)}\right)}{af\sqrt{1-\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{F_1\left(\frac{1}{2};-n,2;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx)}{2af\sqrt{a+a\sin(e+fx)}}$$

Mathematica [B] time = 1.02376, size = 276, normalized size = 3.25

$$\frac{\sec(e+fx)(d\sin(e+fx))^n\left(a^2\sqrt{2-2\sin(e+fx)}(\sin(e+fx)+1)^2(-\sin(e+fx))^{-n}F_1\left(1;\frac{1}{2},-n;2;\frac{1}{2}(\sin(e+fx)+1)\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sin[e + f*x])^n/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] (Sec[e + f*x]*(d*Sin[e + f*x])^n*((a^2*AppellF1[1, 1/2, -n, 2, (1 + Sin[e +
 f*x])/2, 1 + Sin[e + f*x]]*Sqrt[2 - 2*Sin[e + f*x]]*(1 + Sin[e + f*x])^2)/
(-Sin[e + f*x])^n - (4*a*(-1 + Sin[e + f*x])*(2*a*(1 + 2*n)*AppellF1[1/2 -
n, -1/2, -n, 3/2 - n, 2/(1 + Sin[e + f*x]), (1 + Sin[e + f*x])^(-1)] + a*(-
1 + 2*n)*AppellF1[-1/2 - n, -1/2, -n, 1/2 - n, 2/(1 + Sin[e + f*x]), (1 + S
in[e + f*x])^(-1)]*(1 + Sin[e + f*x])))/((-1 + 4*n^2)*Sqrt[1 - 2/(1 + Sin[e
 + f*x])]*(1 - (1 + Sin[e + f*x])^(-1))^n)))/(8*a^3*f*Sqrt[a*(1 + Sin[e + f
*x])])
```

Maple [F] time = 0.142, size = 0, normalized size = 0.

$$\int \left(d\sin\left(fx+e\right)\right)^n\left(a+a\sin\left(fx+e\right)\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(3/2),x)
```

```
[Out] int((d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d \sin \left(fx+e\right)\right)^{n}}{\left(a \sin \left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*sin(f*x + e))^n/(a*sin(f*x + e) + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{a \sin \left(fx+e\right)+a}\left(d \sin \left(fx+e\right)\right)^{n}}{a^{2} \cos \left(fx+e\right)^{2}-2\,a^{2} \sin \left(fx+e\right)-2\,a^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e))^n/(a^2*cos(f*x + e)^2 -
 2*a^2*sin(f*x + e) - 2*a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d \sin \left(e+fx\right)\right)^{n}}{\left(a\left(\sin \left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**n/(a+a*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral((d*sin(e + f*x))**n/(a*(sin(e + f*x) + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d \sin \left(fx+e\right)\right)^{n}}{\left(a \sin \left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*sin(f*x + e))^n/(a*sin(f*x + e) + a)^(3/2), x)
```

3.132 $\int \sin^n(e+fx)(1+\sin(e+fx))^m\,dx$

Optimal. Leaf size=71

$$-\frac{2^{m+\frac{1}{2}}\cos(e+fx)F_1\left(\frac{1}{2};-n,\frac{1}{2}-m;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)}{f\sqrt{\sin(e+fx)+1}}$$

```
[Out] -((2^(1/2 + m)*AppellF1[1/2, -n, 1/2 - m, 3/2, 1 - Sin[e + f*x], (1 - Sin[e
 + f*x])/2]*Cos[e + f*x])/(f*Sqrt[1 + Sin[e + f*x]]))
```

Rubi [A] time = 0.0599366, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2785, 133}

$$-\frac{2^{m+\frac{1}{2}}\cos(e+fx)F_1\left(\frac{1}{2};-n,\frac{1}{2}-m;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)}{f\sqrt{\sin(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^n*(1 + Sin[e + f*x])^m,x]
```

```
[Out] -((2^(1/2 + m)*AppellF1[1/2, -n, 1/2 - m, 3/2, 1 - Sin[e + f*x], (1 - Sin[e
 + f*x])/2]*Cos[e + f*x])/(f*Sqrt[1 + Sin[e + f*x]]))
```

Rule 2785

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Dist[(b*(d/b)^n*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a - x)^n*(2*a - x)^(m - 1/2)
)/Sqrt[x], x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\begin{aligned}\int \sin^n(e+fx)(1+\sin(e+fx))^m\,dx &= -\frac{\cos(e+fx)\,\text{Subst}\left(\int \frac{(1-x)^n(2-x)^{-\frac{1}{2}+m}}{\sqrt{x}}\,dx,x,1-\sin(e+fx)\right)}{f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}\\ &= -\frac{2^{\frac{1}{2}+m}F_1\left(\frac{1}{2};-n,\frac{1}{2}-m;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)\cos(e+fx)}{f\sqrt{1+\sin(e+fx)}}\end{aligned}$$

Mathematica [B] time = 14.8043, size = 2805, normalized size = 39.51

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^n*(1 + Sin[e + f*x])^m,x]
```

```
[Out] (-3*AppellF1[1/2, -n, 1 + m + n, 3/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e
+ Pi/2 - f*x)/2]^2]*Cos[e + f*x]*Sin[e + f*x]^(2*n)*(1 + Sin[e + f*x])^m)/(
f*(Sec[(-e + Pi/2 - f*x)/2]^2)^m*(3*AppellF1[1/2, -n, 1 + m + n, 3/2, Tan[(
-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2] - 2*(n*AppellF1[3/2, 1
- n, 1 + m + n, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^
2] + (1 + m + n)*AppellF1[3/2, -n, 2 + m + n, 5/2, Tan[(-e + Pi/2 - f*x)/2]
^2, -Tan[(-e + Pi/2 - f*x)/2]^2])*Tan[(-e + Pi/2 - f*x)/2]^2)*((-3*n*Appell
F1[1/2, -n, 1 + m + n, 3/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f
*x)/2]^2]*Cos[e + f*x]^2*Sin[e + f*x]^(-1 + n))/((Sec[(-e + Pi/2 - f*x)/2]^
2)^m*(3*AppellF1[1/2, -n, 1 + m + n, 3/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[
(-e + Pi/2 - f*x)/2]^2] - 2*(n*AppellF1[3/2, 1 - n, 1 + m + n, 5/2, Tan[(-e
 + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2] + (1 + m + n)*AppellF1[3/
2, -n, 2 + m + n, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2
]^2])*Tan[(-e + Pi/2 - f*x)/2]^2)) + (3*AppellF1[1/2, -n, 1 + m + n, 3/2, T
an[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2]*Sin[e + f*x]^(1 + n
))/((Sec[(-e + Pi/2 - f*x)/2]^2)^m*(3*AppellF1[1/2, -n, 1 + m + n, 3/2, Tan
[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2] - 2*(n*AppellF1[3/2,
1 - n, 1 + m + n, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2
]^2] + (1 + m + n)*AppellF1[3/2, -n, 2 + m + n, 5/2, Tan[(-e + Pi/2 - f*x)/
2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2])*Tan[(-e + Pi/2 - f*x)/2]^2)) - (3*m*App
ellF1[1/2, -n, 1 + m + n, 3/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2
- f*x)/2]^2]*Cos[e + f*x]*Sin[e + f*x]^n*Tan[(-e + Pi/2 - f*x)/2])/((Sec[(-
e + Pi/2 - f*x)/2]^2)^m*(3*AppellF1[1/2, -n, 1 + m + n, 3/2, Tan[(-e + Pi/2
 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2] - 2*(n*AppellF1[3/2, 1 - n, 1 +
m + n, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2] + (1 +
 m + n)*AppellF1[3/2, -n, 2 + m + n, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[
(-e + Pi/2 - f*x)/2]^2])*Tan[(-e + Pi/2 - f*x)/2]^2)) + (3*Cos[e + f*x]*Sin
[e + f*x]^n*(-(n*AppellF1[3/2, 1 - n, 1 + m + n, 5/2, Tan[(-e + Pi/2 - f*x)
/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2]*Sec[(-e + Pi/2 - f*x)/2]^2*Tan[(-e + Pi
/2 - f*x)/2])/3 - ((1 + m + n)*AppellF1[3/2, -n, 2 + m + n, 5/2, Tan[(-e +
Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2]*Sec[(-e + Pi/2 - f*x)/2]^2*T
an[(-e + Pi/2 - f*x)/2])/3))/((Sec[(-e + Pi/2 - f*x)/2]^2)^m*(3*AppellF1[1/
2, -n, 1 + m + n, 3/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2
]^2] - 2*(n*AppellF1[3/2, 1 - n, 1 + m + n, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2
, -Tan[(-e + Pi/2 - f*x)/2]^2] + (1 + m + n)*AppellF1[3/2, -n, 2 + m + n, 5
/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2])*Tan[(-e + Pi/
2 - f*x)/2]^2)) - (3*AppellF1[1/2, -n, 1 + m + n, 3/2, Tan[(-e + Pi/2 - f*x
)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2]*Cos[e + f*x]*Sin[e + f*x]^n*(-2*(n*App
ellF1[3/2, 1 - n, 1 + m + n, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi
/2 - f*x)/2]^2] + (1 + m + n)*AppellF1[3/2, -n, 2 + m + n, 5/2, Tan[(-e + P
i/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2])*Sec[(-e + Pi/2 - f*x)/2]^2*T
an[(-e + Pi/2 - f*x)/2] + 3*(-(n*AppellF1[3/2, 1 - n, 1 + m + n, 5/2, Tan[(
-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2]*Sec[(-e + Pi/2 - f*x)/2
]^2*Tan[(-e + Pi/2 - f*x)/2])/3 - ((1 + m + n)*AppellF1[3/2, -n, 2 + m + n,
 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2]*Sec[(-e + Pi
/2 - f*x)/2]^2*Tan[(-e + Pi/2 - f*x)/2])/3) - 2*Tan[(-e + Pi/2 - f*x)/2]^2*
(n*((-3*(1 + m + n)*AppellF1[5/2, 1 - n, 2 + m + n, 7/2, Tan[(-e + Pi/2 - f
*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2]*Sec[(-e + Pi/2 - f*x)/2]^2*Tan[(-e +
 Pi/2 - f*x)/2])/5 + (3*(1 - n)*AppellF1[5/2, 2 - n, 1 + m + n, 7/2, Tan[(-
e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2]*Sec[(-e + Pi/2 - f*x)/2]
^2*Tan[(-e + Pi/2 - f*x)/2])/5) + (1 + m + n)*((-3*n*AppellF1[5/2, 1 - n, 2
 + m + n, 7/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2]*Sec
[(-e + Pi/2 - f*x)/2]^2*Tan[(-e + Pi/2 - f*x)/2])/5 - (3*(2 + m + n)*Appell
F1[5/2, -n, 3 + m + n, 7/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f
*x)/2]^2]*Sec[(-e + Pi/2 - f*x)/2]^2*Tan[(-e + Pi/2 - f*x)/2])/5))))/((Sec[
```

```
(-e + Pi/2 - f*x)/2]^2)^m*(3*AppellF1[1/2, -n, 1 + m + n, 3/2, Tan[(-e + Pi
/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2] - 2*(n*AppellF1[3/2, 1 - n, 1
+ m + n, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2] + (1
 + m + n)*AppellF1[3/2, -n, 2 + m + n, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Ta
n[(-e + Pi/2 - f*x)/2]^2])*Tan[(-e + Pi/2 - f*x)/2]^2)^2)))
```

Maple [F] time = 0.547, size = 0, normalized size = 0.

$$\int \left(\sin \left(fx+e\right)\right)^{n} \left(1+\sin \left(fx+e\right)\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^n*(1+sin(f*x+e))^m,x)
```

```
[Out] int(sin(f*x+e)^n*(1+sin(f*x+e))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\sin \left(fx+e\right)+1\right)^{m} \sin \left(fx+e\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^n*(1+sin(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((sin(f*x + e) + 1)^m*sin(f*x + e)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(\sin \left(fx+e\right)+1\right)^{m} \sin \left(fx+e\right)^{n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^n*(1+sin(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((sin(f*x + e) + 1)^m*sin(f*x + e)^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\sin \left(e+fx\right)+1\right)^{m} \sin^{n}\left(e+fx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**n*(1+sin(f*x+e))**m,x)
```

```
[Out] Integral((sin(e + f*x) + 1)**m*sin(e + f*x)**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\sin\left(fx+e\right)+1\right)^{m} \sin\left(fx+e\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^n*(1+sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((sin(f*x + e) + 1)^m*sin(f*x + e)^n, x)
```

3.133 $\int (1-\sin(e+fx))^m(-\sin(e+fx))^n\,dx$

Optimal. Leaf size=68

$$\frac{2^{m+\frac{1}{2}}\cos(e+fx)F_1\left(\frac{1}{2};-n,\frac{1}{2}-m;\frac{3}{2};\sin(e+fx)+1,\frac{1}{2}(\sin(e+fx)+1)\right)}{f\sqrt{1-\sin(e+fx)}}$$

```
[Out] (2^(1/2 + m)*AppellF1[1/2, -n, 1/2 - m, 3/2, 1 + Sin[e + f*x], (1 + Sin[e +
 f*x])/2]*Cos[e + f*x])/(f*Sqrt[1 - Sin[e + f*x]])
```

Rubi [A] time = 0.0607976, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2785, 133}

$$\frac{2^{m+\frac{1}{2}}\cos(e+fx)F_1\left(\frac{1}{2};-n,\frac{1}{2}-m;\frac{3}{2};\sin(e+fx)+1,\frac{1}{2}(\sin(e+fx)+1)\right)}{f\sqrt{1-\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sin[e + f*x])^m*(-Sin[e + f*x])^n,x]
```

```
[Out] (2^(1/2 + m)*AppellF1[1/2, -n, 1/2 - m, 3/2, 1 + Sin[e + f*x], (1 + Sin[e +
 f*x])/2]*Cos[e + f*x])/(f*Sqrt[1 - Sin[e + f*x]])
```

Rule 2785

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Dist[(b*(d/b)^n*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a - x)^n*(2*a - x)^(m - 1/2)
)/Sqrt[x], x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\begin{aligned}\int (1-\sin(e+fx))^m(-\sin(e+fx))^n\,dx &= \frac{\cos(e+fx)\,\text{Subst}\left(\int \frac{(1-x)^n(2-x)^{-\frac{1}{2}+m}}{\sqrt{x}}\,dx,x,1+\sin(e+fx)\right)}{f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}\\ &= \frac{2^{\frac{1}{2}+m}F_1\left(\frac{1}{2};-n,\frac{1}{2}-m;\frac{3}{2};1+\sin(e+fx),\frac{1}{2}(1+\sin(e+fx))\right)\cos(e+fx)}{f\sqrt{1-\sin(e+fx)}}\end{aligned}$$

Mathematica [B] time = 2.25331, size = 300, normalized size = 4.41

$$-\frac{(2m+3)\cos(e+fx)(1}{f(2m+1)\left((2m+3)F_1\left(m+\frac{1}{2};-n,m+n+1;m+\frac{3}{2};\cot^2\left(\frac{1}{4}(2e+2fx+\pi)\right),-\tan^2\left(\frac{1}{4}(2e+2fx-\pi)\right)\right)-2\tan^2\left(\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - Sin[e + f*x])^m*(-Sin[e + f*x])^n,x]
```

```
[Out] -(((3 + 2*m)*AppellF1[1/2 + m, -n, 1 + m + n, 3/2 + m, Cot[(2*e + Pi + 2*f*
x)/4]^2, -Tan[(2*e - Pi + 2*f*x)/4]^2]*Cos[e + f*x]*(1 - Sin[e + f*x])^m*(-
Sin[e + f*x])^n)/(f*(1 + 2*m)*((3 + 2*m)*AppellF1[1/2 + m, -n, 1 + m + n, 3
/2 + m, Cot[(2*e + Pi + 2*f*x)/4]^2, -Tan[(2*e - Pi + 2*f*x)/4]^2] - 2*(n*A
ppellF1[3/2 + m, 1 - n, 1 + m + n, 5/2 + m, Cot[(2*e + Pi + 2*f*x)/4]^2, -T
an[(2*e - Pi + 2*f*x)/4]^2] + (1 + m + n)*AppellF1[3/2 + m, -n, 2 + m + n,
5/2 + m, Cot[(2*e + Pi + 2*f*x)/4]^2, -Tan[(2*e - Pi + 2*f*x)/4]^2])*Tan[(2
*e - Pi + 2*f*x)/4]^2)))
```

Maple [F] time = 0.62, size = 0, normalized size = 0.

$$\int (1-\sin (fx+e))^{m} (-\sin (fx+e))^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-sin(f*x+e))^m*(-sin(f*x+e))^n,x)
```

```
[Out] int((1-sin(f*x+e))^m*(-sin(f*x+e))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (-\sin (fx+e))^{n} (-\sin (fx+e)+1)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sin(f*x+e))^m*(-sin(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate((-sin(f*x + e))^n*(-sin(f*x + e) + 1)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((-\sin (fx+e))^{n} (-\sin (fx+e)+1)^{m}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sin(f*x+e))^m*(-sin(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral((-sin(f*x + e))^n*(-sin(f*x + e) + 1)^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (-\sin (e+fx))^{n} (1-\sin (e+fx))^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sin(f*x+e))**m*(-sin(f*x+e))**n,x)
```

```
[Out] Integral((-sin(e + f*x))**n*(1 - sin(e + f*x))**m, x)
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int (-\sin(fx+e))^n (-\sin(fx+e)+1)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sin(f*x+e))^m*(-sin(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((-sin(f*x + e))^n*(-sin(f*x + e) + 1)^m, x)
```

3.134 $\int (d\sin(e+fx))^n(1+\sin(e+fx))^m\,dx$

Optimal. Leaf size=91

$$-\frac{2^{m+\frac{1}{2}}\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n F_1\left(\frac{1}{2};-n,\frac{1}{2}-m;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)}{f\sqrt{\sin(e+fx)+1}}$$

```
[Out] -((2^(1/2 + m)*AppellF1[1/2, -n, 1/2 - m, 3/2, 1 - Sin[e + f*x], (1 - Sin[e
 + f*x])/2]*Cos[e + f*x]*(d*Sin[e + f*x])^n)/(f*Sin[e + f*x]^n*Sqrt[1 + Sin
[e + f*x]]))
```

Rubi [A] time = 0.0997954, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2786, 2785, 133}

$$-\frac{2^{m+\frac{1}{2}}\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n F_1\left(\frac{1}{2};-n,\frac{1}{2}-m;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)}{f\sqrt{\sin(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sin[e + f*x])^n*(1 + Sin[e + f*x])^m,x]
```

```
[Out] -((2^(1/2 + m)*AppellF1[1/2, -n, 1/2 - m, 3/2, 1 - Sin[e + f*x], (1 - Sin[e
 + f*x])/2]*Cos[e + f*x]*(d*Sin[e + f*x])^n)/(f*Sin[e + f*x]^n*Sqrt[1 + Sin
[e + f*x]]))
```

Rule 2786

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[((d/b)^IntPart[n]*(d*Sin[e + f*x])^FracPart[n
])/(b*Sin[e + f*x])^FracPart[n], Int[(a + b*Sin[e + f*x])^m*(b*Sin[e + f*x]
)^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !In
tegerQ[m] && GtQ[a, 0] &&  !GtQ[d/b, 0]
```

Rule 2785

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Dist[(b*(d/b)^n*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a - x)^n*(2*a - x)^(m - 1/2)
)/Sqrt[x], x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int (d\sin(e+fx))^n(1+\sin(e+fx))^m\,dx = \left(\sin^{-n}(e+fx)(d\sin(e+fx))^n\right)\int \sin^n(e+fx)(1+\sin(e+fx))^m\,dx$$
$$= -\frac{\left(\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n\right)\text{Subst}\left(\int \frac{(1-x)^n(2-x)^{-\frac{1}{2}+m}}{\sqrt{x}}\,dx, x, \right.}{f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}$$
$$= -\frac{2^{\frac{1}{2}+m}F_1\left(\frac{1}{2};-n,\frac{1}{2}-m;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)\cos(e+fx)\,\mathrm{s}}{f\sqrt{1+\sin(e+fx)}}$$

Mathematica [B] time = 6.21041, size = 2813, normalized size = 30.91

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sin[e + f*x])^n*(1 + Sin[e + f*x])^m,x]
```

```
[Out] (-3*AppellF1[1/2, -n, 1 + m + n, 3/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e
+ Pi/2 - f*x)/2]^2]*Cos[e + f*x]*Sin[e + f*x]^n*(d*Sin[e + f*x])^n*(1 + Sin
[e + f*x])^m)/(f*(Sec[(-e + Pi/2 - f*x)/2]^2)^m*(3*AppellF1[1/2, -n, 1 + m
+ n, 3/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2] - 2*(n*A
ppellF1[3/2, 1 - n, 1 + m + n, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e +
Pi/2 - f*x)/2]^2] + (1 + m + n)*AppellF1[3/2, -n, 2 + m + n, 5/2, Tan[(-e +
 Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2])*Tan[(-e + Pi/2 - f*x)/2]^2
)*((-3*n*AppellF1[1/2, -n, 1 + m + n, 3/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan
[(-e + Pi/2 - f*x)/2]^2]*Cos[e + f*x]^2*Sin[e + f*x]^(-1 + n))/((Sec[(-e +
Pi/2 - f*x)/2]^2)^m*(3*AppellF1[1/2, -n, 1 + m + n, 3/2, Tan[(-e + Pi/2 - f
*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2] - 2*(n*AppellF1[3/2, 1 - n, 1 + m +
n, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2] + (1 + m +
 n)*AppellF1[3/2, -n, 2 + m + n, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e
+ Pi/2 - f*x)/2]^2])*Tan[(-e + Pi/2 - f*x)/2]^2)) + (3*AppellF1[1/2, -n, 1
+ m + n, 3/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2]*Sin[
e + f*x]^(1 + n))/((Sec[(-e + Pi/2 - f*x)/2]^2)^m*(3*AppellF1[1/2, -n, 1 +
m + n, 3/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2] - 2*(n
*AppellF1[3/2, 1 - n, 1 + m + n, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e
+ Pi/2 - f*x)/2]^2] + (1 + m + n)*AppellF1[3/2, -n, 2 + m + n, 5/2, Tan[(-e
 + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2])*Tan[(-e + Pi/2 - f*x)/2]
^2)) - (3*m*AppellF1[1/2, -n, 1 + m + n, 3/2, Tan[(-e + Pi/2 - f*x)/2]^2, -
Tan[(-e + Pi/2 - f*x)/2]^2]*Cos[e + f*x]*Sin[e + f*x]^n*Tan[(-e + Pi/2 - f*
x)/2])/((Sec[(-e + Pi/2 - f*x)/2]^2)^m*(3*AppellF1[1/2, -n, 1 + m + n, 3/2,
 Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2] - 2*(n*AppellF1[3
/2, 1 - n, 1 + m + n, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*
x)/2]^2] + (1 + m + n)*AppellF1[3/2, -n, 2 + m + n, 5/2, Tan[(-e + Pi/2 - f
*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2])*Tan[(-e + Pi/2 - f*x)/2]^2)) + (3*C
os[e + f*x]*Sin[e + f*x]^n*(-(n*AppellF1[3/2, 1 - n, 1 + m + n, 5/2, Tan[(-
e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2]*Sec[(-e + Pi/2 - f*x)/2]
^2*Tan[(-e + Pi/2 - f*x)/2])/3 - ((1 + m + n)*AppellF1[3/2, -n, 2 + m + n,
5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2]*Sec[(-e + Pi/
2 - f*x)/2]^2*Tan[(-e + Pi/2 - f*x)/2])/3))/((Sec[(-e + Pi/2 - f*x)/2]^2)^m
*(3*AppellF1[1/2, -n, 1 + m + n, 3/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e
+ Pi/2 - f*x)/2]^2] - 2*(n*AppellF1[3/2, 1 - n, 1 + m + n, 5/2, Tan[(-e + P
i/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2] + (1 + m + n)*AppellF1[3/2, -
n, 2 + m + n, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2]
)*Tan[(-e + Pi/2 - f*x)/2]^2)) - (3*AppellF1[1/2, -n, 1 + m + n, 3/2, Tan[(
-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2]*Cos[e + f*x]*Sin[e + f*
x]^n*(-2*(n*AppellF1[3/2, 1 - n, 1 + m + n, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2
```

```
, -Tan[(-e + Pi/2 - f*x)/2]^2] + (1 + m + n)*AppellF1[3/2, -n, 2 + m + n, 5
/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2])*Sec[(-e + Pi/
2 - f*x)/2]^2*Tan[(-e + Pi/2 - f*x)/2] + 3*(-(n*AppellF1[3/2, 1 - n, 1 + m
+ n, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2]*Sec[(-e
+ Pi/2 - f*x)/2]^2*Tan[(-e + Pi/2 - f*x)/2])/3 - ((1 + m + n)*AppellF1[3/2,
 -n, 2 + m + n, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^
2]*Sec[(-e + Pi/2 - f*x)/2]^2*Tan[(-e + Pi/2 - f*x)/2])/3) - 2*Tan[(-e + Pi
/2 - f*x)/2]^2*(n*((-3*(1 + m + n)*AppellF1[5/2, 1 - n, 2 + m + n, 7/2, Tan
[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2]*Sec[(-e + Pi/2 - f*x)
/2]^2*Tan[(-e + Pi/2 - f*x)/2])/5 + (3*(1 - n)*AppellF1[5/2, 2 - n, 1 + m +
 n, 7/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2]*Sec[(-e +
 Pi/2 - f*x)/2]^2*Tan[(-e + Pi/2 - f*x)/2])/5) + (1 + m + n)*((-3*n*AppellF
1[5/2, 1 - n, 2 + m + n, 7/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 -
 f*x)/2]^2]*Sec[(-e + Pi/2 - f*x)/2]^2*Tan[(-e + Pi/2 - f*x)/2])/5 - (3*(2
+ m + n)*AppellF1[5/2, -n, 3 + m + n, 7/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan
[(-e + Pi/2 - f*x)/2]^2]*Sec[(-e + Pi/2 - f*x)/2]^2*Tan[(-e + Pi/2 - f*x)/2
])/5))))/((Sec[(-e + Pi/2 - f*x)/2]^2)^m*(3*AppellF1[1/2, -n, 1 + m + n, 3/
2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2] - 2*(n*AppellF1
[3/2, 1 - n, 1 + m + n, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 -
f*x)/2]^2] + (1 + m + n)*AppellF1[3/2, -n, 2 + m + n, 5/2, Tan[(-e + Pi/2 -
 f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2])*Tan[(-e + Pi/2 - f*x)/2]^2)^2)))
```

Maple [F] time = 0.607, size = 0, normalized size = 0.

$$\int \left(d\sin\left(fx+e\right)\right)^{n}\left(1+\sin\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^n*(1+sin(f*x+e))^m,x)
```

```
[Out] int((d*sin(f*x+e))^n*(1+sin(f*x+e))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\sin\left(fx+e\right)\right)^{n}\left(\sin\left(fx+e\right)+1\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(1+sin(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((d*sin(f*x + e))^n*(sin(f*x + e) + 1)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(d\sin\left(fx+e\right)\right)^{n}\left(\sin\left(fx+e\right)+1\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(1+sin(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((d*sin(f*x + e))^n*(sin(f*x + e) + 1)^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d \sin \left(e + fx\right)\right)^{n} \left(\sin \left(e + fx\right) + 1\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**n*(1+sin(f*x+e))**m,x)
```

```
[Out] Integral((d*sin(e + f*x))**n*(sin(e + f*x) + 1)**m, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d \sin \left(fx + e\right)\right)^{n} \left(\sin \left(fx + e\right) + 1\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(1+sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((d*sin(f*x + e))^n*(sin(f*x + e) + 1)^m, x)
```

3.135 $\int (1-\sin(e+fx))^m (d\sin(e+fx))^n\,dx$

Optimal. Leaf size=90

$$\frac{2^{m+\frac{1}{2}}\cos(e+fx)(-\sin(e+fx))^{-n}(d\sin(e+fx))^n F_1\left(\frac{1}{2};-n,\frac{1}{2}-m;\frac{3}{2};\sin(e+fx)+1,\frac{1}{2}(\sin(e+fx)+1)\right)}{f\sqrt{1-\sin(e+fx)}}$$

```
[Out] (2^(1/2 + m)*AppellF1[1/2, -n, 1/2 - m, 3/2, 1 + Sin[e + f*x], (1 + Sin[e +
 f*x])/2]*Cos[e + f*x]*(d*Sin[e + f*x])^n)/(f*Sqrt[1 - Sin[e + f*x]]*(-Sin[
e + f*x])^n)
```

Rubi [A] time = 0.109557, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {2786, 2785, 133}

$$\frac{2^{m+\frac{1}{2}}\cos(e+fx)(-\sin(e+fx))^{-n}(d\sin(e+fx))^n F_1\left(\frac{1}{2};-n,\frac{1}{2}-m;\frac{3}{2};\sin(e+fx)+1,\frac{1}{2}(\sin(e+fx)+1)\right)}{f\sqrt{1-\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sin[e + f*x])^m*(d*Sin[e + f*x])^n,x]
```

```
[Out] (2^(1/2 + m)*AppellF1[1/2, -n, 1/2 - m, 3/2, 1 + Sin[e + f*x], (1 + Sin[e +
 f*x])/2]*Cos[e + f*x]*(d*Sin[e + f*x])^n)/(f*Sqrt[1 - Sin[e + f*x]]*(-Sin[
e + f*x])^n)
```

Rule 2786

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[((d/b)^IntPart[n]*(d*Sin[e + f*x])^FracPart[n
])/(b*Sin[e + f*x])^FracPart[n], Int[(a + b*Sin[e + f*x])^m*(b*Sin[e + f*x]
)^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !In
tegerQ[m] && GtQ[a, 0] &&  !GtQ[d/b, 0]
```

Rule 2785

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Dist[(b*(d/b)^n*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a - x)^n*(2*a - x)^(m - 1/2)
)/Sqrt[x], x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int (1-\sin(e+fx))^m(d\sin(e+fx))^n\,dx = \left((-\sin(e+fx))^{-n}(d\sin(e+fx))^n\right)\int (1-\sin(e+fx))^m(-\sin(e+fx))^n\,dx$$

$$= \frac{\left(\cos(e+fx)(-\sin(e+fx))^{-n}(d\sin(e+fx))^n\right)\text{Subst}\left(\int \frac{(1-x)^n(2-x)^{-\frac{1}{2}+m}}{\sqrt{x}}\,dx,\right.}{f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}$$

$$= \frac{2^{\frac{1}{2}+m}F_1\left(\frac{1}{2};-n,\frac{1}{2}-m;\frac{3}{2};1+\sin(e+fx),\frac{1}{2}(1+\sin(e+fx))\right)\cos(e+fx)(-}{f\sqrt{1-\sin(e+fx)}}$$

Mathematica [B] time = 0.712451, size = 300, normalized size = 3.33

$$-\frac{(2m+3)\cos(e+fx)(1}{f(2m+1)\left((2m+3)F_1\left(m+\frac{1}{2};-n,m+n+1;m+\frac{3}{2};\cot^2\left(\frac{1}{4}(2e+2fx+\pi)\right),-\tan^2\left(\frac{1}{4}(2e+2fx-\pi)\right)\right)-2\tan^2\left(\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - Sin[e + f*x])^m*(d*Sin[e + f*x])^n,x]
```

```
[Out] -(((3 + 2*m)*AppellF1[1/2 + m, -n, 1 + m + n, 3/2 + m, Cot[(2*e + Pi + 2*f*
x)/4]^2, -Tan[(2*e - Pi + 2*f*x)/4]^2]*Cos[e + f*x]*(1 - Sin[e + f*x])^m*(d
*Sin[e + f*x])^n)/(f*(1 + 2*m)*((3 + 2*m)*AppellF1[1/2 + m, -n, 1 + m + n,
3/2 + m, Cot[(2*e + Pi + 2*f*x)/4]^2, -Tan[(2*e - Pi + 2*f*x)/4]^2] - 2*(n*
AppellF1[3/2 + m, 1 - n, 1 + m + n, 5/2 + m, Cot[(2*e + Pi + 2*f*x)/4]^2, -
Tan[(2*e - Pi + 2*f*x)/4]^2] + (1 + m + n)*AppellF1[3/2 + m, -n, 2 + m + n,
 5/2 + m, Cot[(2*e + Pi + 2*f*x)/4]^2, -Tan[(2*e - Pi + 2*f*x)/4]^2])*Tan[(
2*e - Pi + 2*f*x)/4]^2)))
```

Maple [F] time = 0.597, size = 0, normalized size = 0.

$$\int \left(1-\sin\left(fx+e\right)\right)^m\left(d\sin\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-sin(f*x+e))^m*(d*sin(f*x+e))^n,x)
```

```
[Out] int((1-sin(f*x+e))^m*(d*sin(f*x+e))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\sin\left(fx+e\right)\right)^n\left(-\sin\left(fx+e\right)+1\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sin(f*x+e))^m*(d*sin(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate((d*sin(f*x + e))^n*(-sin(f*x + e) + 1)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(d\sin\left(fx+e\right)\right)^n\left(-\sin\left(fx+e\right)+1\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sin(f*x+e))^m*(d*sin(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral((d*sin(f*x + e))^n*(-sin(f*x + e) + 1)^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(d\sin\left(e+fx\right)\right)^n\left(1-\sin\left(e+fx\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sin(f*x+e))**m*(d*sin(f*x+e))**n,x)
```

```
[Out] Integral((d*sin(e + f*x))**n*(1 - sin(e + f*x))**m, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(d\sin\left(fx+e\right)\right)^n\left(-\sin\left(fx+e\right)+1\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sin(f*x+e))^m*(d*sin(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((d*sin(f*x + e))^n*(-sin(f*x + e) + 1)^m, x)
```

3.136 $\int \sin^n(e+fx)(a+a\sin(e+fx))^m\,dx$

Optimal. Leaf size=87

$$-\frac{2^{m+\frac{1}{2}}\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^m F_1\left(\frac{1}{2};-n,\frac{1}{2}-m;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)}{f}$$

```
[Out] -((2^(1/2 + m)*AppellF1[1/2, -n, 1/2 - m, 3/2, 1 - Sin[e + f*x], (1 - Sin[e
 + f*x])/2]*Cos[e + f*x]*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Sin[e + f*x])
^m)/f)
```

Rubi [A] time = 0.0981528, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2787, 2785, 133}

$$-\frac{2^{m+\frac{1}{2}}\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^m F_1\left(\frac{1}{2};-n,\frac{1}{2}-m;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^n*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] -((2^(1/2 + m)*AppellF1[1/2, -n, 1/2 - m, 3/2, 1 - Sin[e + f*x], (1 - Sin[e
 + f*x])/2]*Cos[e + f*x]*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Sin[e + f*x])
^m)/f)
```

Rule 2787

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m
])/(1 + (b*Sin[e + f*x])/a)^FracPart[m], Int[(1 + (b*Sin[e + f*x])/a)^m*(d*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 2785

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Dist[(b*(d/b)^n*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a - x)^n*(2*a - x)^(m - 1/2)
)/Sqrt[x], x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int \sin^n(e+fx)(a+a\sin(e+fx))^m\,dx = \left((1+\sin(e+fx))^{-m}(a+a\sin(e+fx))^m\right)\int \sin^n(e+fx)(1+\sin(e+fx))^m\,dx$$

$$= -\frac{\left(\cos(e+fx)(1+\sin(e+fx))^{-\frac{1}{2}-m}(a+a\sin(e+fx))^m\right)\text{Subst}\left(\int \frac{(1-x)^n(2-x)^{-\frac{1}{2}}}{\sqrt{x}}\right.}{f\sqrt{1-\sin(e+fx)}}$$

$$= -\frac{2^{\frac{1}{2}+m}F_1\left(\frac{1}{2};-n,\frac{1}{2}-m;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)\cos(e+fx)(1+\text{s}}{f}$$

Mathematica [B] time = 6.30142, size = 2807, normalized size = 32.26

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^n*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] (-3*AppellF1[1/2, -n, 1 + m + n, 3/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e
+ Pi/2 - f*x)/2]^2]*Cos[e + f*x]*Sin[e + f*x]^(2*n)*(a + a*Sin[e + f*x])^m)
/(f*(Sec[(-e + Pi/2 - f*x)/2]^2)^m*(3*AppellF1[1/2, -n, 1 + m + n, 3/2, Tan
[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2] - 2*(n*AppellF1[3/2,
1 - n, 1 + m + n, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2
]^2] + (1 + m + n)*AppellF1[3/2, -n, 2 + m + n, 5/2, Tan[(-e + Pi/2 - f*x)/
2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2])*Tan[(-e + Pi/2 - f*x)/2]^2)*((-3*n*Appe
llF1[1/2, -n, 1 + m + n, 3/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 -
 f*x)/2]^2]*Cos[e + f*x]^2*Sin[e + f*x]^(-1 + n))/((Sec[(-e + Pi/2 - f*x)/2
]^2)^m*(3*AppellF1[1/2, -n, 1 + m + n, 3/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Ta
n[(-e + Pi/2 - f*x)/2]^2] - 2*(n*AppellF1[3/2, 1 - n, 1 + m + n, 5/2, Tan[(
-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2] + (1 + m + n)*AppellF1[
3/2, -n, 2 + m + n, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)
/2]^2])*Tan[(-e + Pi/2 - f*x)/2]^2)) + (3*AppellF1[1/2, -n, 1 + m + n, 3/2,
 Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2]*Sin[e + f*x]^(1 +
 n))/((Sec[(-e + Pi/2 - f*x)/2]^2)^m*(3*AppellF1[1/2, -n, 1 + m + n, 3/2, T
an[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2] - 2*(n*AppellF1[3/2
, 1 - n, 1 + m + n, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)
/2]^2] + (1 + m + n)*AppellF1[3/2, -n, 2 + m + n, 5/2, Tan[(-e + Pi/2 - f*x
)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2])*Tan[(-e + Pi/2 - f*x)/2]^2)) - (3*m*A
ppellF1[1/2, -n, 1 + m + n, 3/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/
2 - f*x)/2]^2]*Cos[e + f*x]*Sin[e + f*x]^n*Tan[(-e + Pi/2 - f*x)/2])/((Sec[
(-e + Pi/2 - f*x)/2]^2)^m*(3*AppellF1[1/2, -n, 1 + m + n, 3/2, Tan[(-e + Pi
/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2] - 2*(n*AppellF1[3/2, 1 - n, 1
+ m + n, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2] + (1
 + m + n)*AppellF1[3/2, -n, 2 + m + n, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Ta
n[(-e + Pi/2 - f*x)/2]^2])*Tan[(-e + Pi/2 - f*x)/2]^2)) + (3*Cos[e + f*x]*S
in[e + f*x]^n*(-(n*AppellF1[3/2, 1 - n, 1 + m + n, 5/2, Tan[(-e + Pi/2 - f*
x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2]*Sec[(-e + Pi/2 - f*x)/2]^2*Tan[(-e +
Pi/2 - f*x)/2])/3 - ((1 + m + n)*AppellF1[3/2, -n, 2 + m + n, 5/2, Tan[(-e
+ Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2]*Sec[(-e + Pi/2 - f*x)/2]^2
*Tan[(-e + Pi/2 - f*x)/2])/3))/((Sec[(-e + Pi/2 - f*x)/2]^2)^m*(3*AppellF1[
1/2, -n, 1 + m + n, 3/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)
/2]^2] - 2*(n*AppellF1[3/2, 1 - n, 1 + m + n, 5/2, Tan[(-e + Pi/2 - f*x)/2]
^2, -Tan[(-e + Pi/2 - f*x)/2]^2] + (1 + m + n)*AppellF1[3/2, -n, 2 + m + n,
 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2])*Tan[(-e + P
i/2 - f*x)/2]^2)) - (3*AppellF1[1/2, -n, 1 + m + n, 3/2, Tan[(-e + Pi/2 - f
*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2]*Cos[e + f*x]*Sin[e + f*x]^n*(-2*(n*A
ppellF1[3/2, 1 - n, 1 + m + n, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e +
```

```
Pi/2 - f*x)/2]^2] + (1 + m + n)*AppellF1[3/2, -n, 2 + m + n, 5/2, Tan[(-e +
 Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2])*Sec[(-e + Pi/2 - f*x)/2]^2
*Tan[(-e + Pi/2 - f*x)/2] + 3*(-(n*AppellF1[3/2, 1 - n, 1 + m + n, 5/2, Tan
[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2]*Sec[(-e + Pi/2 - f*x)
/2]^2*Tan[(-e + Pi/2 - f*x)/2])/3 - ((1 + m + n)*AppellF1[3/2, -n, 2 + m +
n, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2]*Sec[(-e +
Pi/2 - f*x)/2]^2*Tan[(-e + Pi/2 - f*x)/2])/3) - 2*Tan[(-e + Pi/2 - f*x)/2]^
2*(n*((-3*(1 + m + n)*AppellF1[5/2, 1 - n, 2 + m + n, 7/2, Tan[(-e + Pi/2 -
 f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2]*Sec[(-e + Pi/2 - f*x)/2]^2*Tan[(-e
 + Pi/2 - f*x)/2])/5 + (3*(1 - n)*AppellF1[5/2, 2 - n, 1 + m + n, 7/2, Tan[
(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2]*Sec[(-e + Pi/2 - f*x)/
2]^2*Tan[(-e + Pi/2 - f*x)/2])/5) + (1 + m + n)*((-3*n*AppellF1[5/2, 1 - n,
 2 + m + n, 7/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2]*S
ec[(-e + Pi/2 - f*x)/2]^2*Tan[(-e + Pi/2 - f*x)/2])/5 - (3*(2 + m + n)*Appe
llF1[5/2, -n, 3 + m + n, 7/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 -
 f*x)/2]^2]*Sec[(-e + Pi/2 - f*x)/2]^2*Tan[(-e + Pi/2 - f*x)/2])/5))))/((Se
c[(-e + Pi/2 - f*x)/2]^2)^m*(3*AppellF1[1/2, -n, 1 + m + n, 3/2, Tan[(-e +
Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2] - 2*(n*AppellF1[3/2, 1 - n,
1 + m + n, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2] +
(1 + m + n)*AppellF1[3/2, -n, 2 + m + n, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -
Tan[(-e + Pi/2 - f*x)/2]^2])*Tan[(-e + Pi/2 - f*x)/2]^2)^2)))
```

Maple [F] time = 0.636, size = 0, normalized size = 0.

$$\int \left(\sin \left(fx+e\right)\right)^{n} \left(a+a\sin \left(fx+e\right)\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^n*(a+a*sin(f*x+e))^m,x)
```

```
[Out] int(sin(f*x+e)^n*(a+a*sin(f*x+e))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin \left(fx+e\right)+a\right)^{m} \sin \left(fx+e\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^n*(a+a*sin(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*sin(f*x + e)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a\sin \left(fx+e\right)+a\right)^{m} \sin \left(fx+e\right)^{n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^n*(a+a*sin(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)^m*sin(f*x + e)^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\left(\sin\left(e+fx\right)+1\right)\right)^{m}\sin^{n}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**n*(a+a*sin(f*x+e))**m,x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**m*sin(e + f*x)**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^{m}\sin\left(fx+e\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^n*(a+a*sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*sin(f*x + e)^n, x)
```

3.137 $\int (-\sin(e+fx))^n (a-a\sin(e+fx))^m \, dx$

Optimal. Leaf size=85

$$\frac{2^{m+\frac{1}{2}}\cos(e+fx)(1-\sin(e+fx))^{-m-\frac{1}{2}}(a-a\sin(e+fx))^m F_1\left(\frac{1}{2};-n,\frac{1}{2}-m;\frac{3}{2};\sin(e+fx)+1,\frac{1}{2}(\sin(e+fx)+1)\right)}{f}$$

```
[Out] (2^(1/2 + m)*AppellF1[1/2, -n, 1/2 - m, 3/2, 1 + Sin[e + f*x], (1 + Sin[e +
 f*x])/2]*Cos[e + f*x]*(1 - Sin[e + f*x])^(-1/2 - m)*(a - a*Sin[e + f*x])^m
)/f
```

Rubi [A] time = 0.109815, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2787, 2785, 133}

$$\frac{2^{m+\frac{1}{2}}\cos(e+fx)(1-\sin(e+fx))^{-m-\frac{1}{2}}(a-a\sin(e+fx))^m F_1\left(\frac{1}{2};-n,\frac{1}{2}-m;\frac{3}{2};\sin(e+fx)+1,\frac{1}{2}(\sin(e+fx)+1)\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(-Sin[e + f*x])^n*(a - a*Sin[e + f*x])^m,x]
```

```
[Out] (2^(1/2 + m)*AppellF1[1/2, -n, 1/2 - m, 3/2, 1 + Sin[e + f*x], (1 + Sin[e +
 f*x])/2]*Cos[e + f*x]*(1 - Sin[e + f*x])^(-1/2 - m)*(a - a*Sin[e + f*x])^m
)/f
```

Rule 2787

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m
])/(1 + (b*Sin[e + f*x])/a)^FracPart[m], Int[(1 + (b*Sin[e + f*x])/a)^m*(d*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 2785

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Dist[(b*(d/b)^n*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a - x)^n*(2*a - x)^(m - 1/2)
)/Sqrt[x], x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int (-\sin(e+fx))^n(a-a\sin(e+fx))^m\,dx = \left((1-\sin(e+fx))^{-m}(a-a\sin(e+fx))^m\right)\int (1-\sin(e+fx))^m(-\sin(e+fx)$$

$$= \frac{\left(\cos(e+fx)(1-\sin(e+fx))^{-\frac{1}{2}-m}(a-a\sin(e+fx))^m\right)\operatorname{Subst}\left(\int \frac{(1-x)^n(2-x)^-}{\sqrt{x}}\right.}{f\sqrt{1+\sin(e+fx)}}$$

$$= \frac{2^{\frac{1}{2}+m}F_1\left(\frac{1}{2};-n,\frac{1}{2}-m;\frac{3}{2};1+\sin(e+fx),\frac{1}{2}(1+\sin(e+fx))\right)\cos(e+fx)(1-}{f}$$

Mathematica [B] time = 0.604021, size = 301, normalized size = 3.54

$$-\frac{(2m+3)\cos(e+fx)(-\sin(}{f(2m+1)\left((2m+3)F_1\left(m+\frac{1}{2};-n,m+n+1;m+\frac{3}{2};\cot^2\left(\frac{1}{4}(2e+2fx+\pi)\right),-\tan^2\left(\frac{1}{4}(2e+2fx-\pi)\right)\right)-2\tan^2\left(\frac{1}{4}(2\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-Sin[e + f*x])^n*(a - a*Sin[e + f*x])^m,x]
```

```
[Out] -(((3 + 2*m)*AppellF1[1/2 + m, -n, 1 + m + n, 3/2 + m, Cot[(2*e + Pi + 2*f*
x)/4]^2, -Tan[(2*e - Pi + 2*f*x)/4]^2]*Cos[e + f*x]*(-Sin[e + f*x])^n*(a -
a*Sin[e + f*x])^m)/(f*(1 + 2*m)*((3 + 2*m)*AppellF1[1/2 + m, -n, 1 + m + n,
 3/2 + m, Cot[(2*e + Pi + 2*f*x)/4]^2, -Tan[(2*e - Pi + 2*f*x)/4]^2] - 2*(n
*AppellF1[3/2 + m, 1 - n, 1 + m + n, 5/2 + m, Cot[(2*e + Pi + 2*f*x)/4]^2,
-Tan[(2*e - Pi + 2*f*x)/4]^2] + (1 + m + n)*AppellF1[3/2 + m, -n, 2 + m + n
, 5/2 + m, Cot[(2*e + Pi + 2*f*x)/4]^2, -Tan[(2*e - Pi + 2*f*x)/4]^2])*Tan[
(2*e - Pi + 2*f*x)/4]^2)))
```

Maple [F] time = 0.675, size = 0, normalized size = 0.

$$\int \left(-\sin\left(fx+e\right)\right)^{n}\left(a-a\sin\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-sin(f*x+e))^n*(a-a*sin(f*x+e))^m,x)
```

```
[Out] int((-sin(f*x+e))^n*(a-a*sin(f*x+e))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-a\sin\left(fx+e\right)+a\right)^{m}\left(-\sin\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sin(f*x+e))^n*(a-a*sin(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((-a*sin(f*x + e) + a)^m*(-sin(f*x + e))^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(-a\sin\left(fx+e\right)+a\right)^{m}\left(-\sin\left(fx+e\right)\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sin(f*x+e))^n*(a-a*sin(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((-a*sin(f*x + e) + a)^m*(-sin(f*x + e))^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(-\sin\left(e+fx\right)\right)^{n}\left(-a\left(\sin\left(e+fx\right)-1\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sin(f*x+e))**n*(a-a*sin(f*x+e))**m,x)
```

```
[Out] Integral((-sin(e + f*x))**n*(-a*(sin(e + f*x) - 1))**m, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(-a\sin\left(fx+e\right)+a\right)^{m}\left(-\sin\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sin(f*x+e))^n*(a-a*sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((-a*sin(f*x + e) + a)^m*(-sin(f*x + e))^n, x)
```

3.138 $\int (d\sin(e+fx))^n(a+a\sin(e+fx))^m\,dx$

Optimal. Leaf size=107

$$-\frac{2^{m+\frac{1}{2}}\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}\sin^{-n}(e+fx)(a\sin(e+fx)+a)^m(d\sin(e+fx))^nF_1\left(\frac{1}{2};-n,\frac{1}{2}-m;\frac{3}{2};1-\sin(e+\right.}{f}$$

```
[Out] -((2^(1/2 + m)*AppellF1[1/2, -n, 1/2 - m, 3/2, 1 - Sin[e + f*x], (1 - Sin[e
 + f*x])/2]*Cos[e + f*x]*(d*Sin[e + f*x])^n*(1 + Sin[e + f*x])^(-1/2 - m)*(
a + a*Sin[e + f*x])^m)/(f*Sin[e + f*x]^n))
```

Rubi [A] time = 0.149901, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2787, 2786, 2785, 133}

$$-\frac{2^{m+\frac{1}{2}}\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}\sin^{-n}(e+fx)(a\sin(e+fx)+a)^m(d\sin(e+fx))^nF_1\left(\frac{1}{2};-n,\frac{1}{2}-m;\frac{3}{2};1-\sin(e+\right.}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sin[e + f*x])^n*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] -((2^(1/2 + m)*AppellF1[1/2, -n, 1/2 - m, 3/2, 1 - Sin[e + f*x], (1 - Sin[e
 + f*x])/2]*Cos[e + f*x]*(d*Sin[e + f*x])^n*(1 + Sin[e + f*x])^(-1/2 - m)*(
a + a*Sin[e + f*x])^m)/(f*Sin[e + f*x]^n))
```

Rule 2787

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m
])/(1 + (b*Sin[e + f*x])/a)^FracPart[m], Int[(1 + (b*Sin[e + f*x])/a)^m*(d*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 2786

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[((d/b)^IntPart[n]*(d*Sin[e + f*x])^FracPart[n
])/(b*Sin[e + f*x])^FracPart[n], Int[(a + b*Sin[e + f*x])^m*(b*Sin[e + f*x]
)^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !In
tegerQ[m] && GtQ[a, 0] &&  !GtQ[d/b, 0]
```

Rule 2785

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Dist[(b*(d/b)^n*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a - x)^n*(2*a - x)^(m - 1/2)
)/Sqrt[x], x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
```

```
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^m\,dx = \left((1+\sin(e+fx))^{-m}(a+a\sin(e+fx))^m\right)\int (d\sin(e+fx))^n(1+\sin(e+$$

$$= \left(\sin^{-n}(e+fx)(d\sin(e+fx))^n(1+\sin(e+fx))^{-m}(a+a\sin(e+fx))^m\right)\int$$

$$= -\frac{\left(\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n(1+\sin(e+fx))^{-\frac{1}{2}-m}(a+a\,\mathrm{s}\right.}{f\sqrt{1-\sin(e+}}$$

$$= -\frac{2^{\frac{1}{2}+m}F_1\left(\frac{1}{2};-n,\frac{1}{2}-m;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)\cos(e+fx}{}$$

Mathematica [B] time = 6.22707, size = 2815, normalized size = 26.31

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sin[e + f*x])^n*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] (-3*AppellF1[1/2, -n, 1 + m + n, 3/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e
+ Pi/2 - f*x)/2]^2]*Cos[e + f*x]*Sin[e + f*x]^n*(d*Sin[e + f*x])^n*(a + a*S
in[e + f*x])^m)/(f*(Sec[(-e + Pi/2 - f*x)/2]^2)^m*(3*AppellF1[1/2, -n, 1 +
m + n, 3/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2] - 2*(n
*AppellF1[3/2, 1 - n, 1 + m + n, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e
+ Pi/2 - f*x)/2]^2] + (1 + m + n)*AppellF1[3/2, -n, 2 + m + n, 5/2, Tan[(-e
 + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2])*Tan[(-e + Pi/2 - f*x)/2]
^2)*((-3*n*AppellF1[1/2, -n, 1 + m + n, 3/2, Tan[(-e + Pi/2 - f*x)/2]^2, -T
an[(-e + Pi/2 - f*x)/2]^2]*Cos[e + f*x]^2*Sin[e + f*x]^(-1 + n))/((Sec[(-e
+ Pi/2 - f*x)/2]^2)^m*(3*AppellF1[1/2, -n, 1 + m + n, 3/2, Tan[(-e + Pi/2 -
 f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2] - 2*(n*AppellF1[3/2, 1 - n, 1 + m
+ n, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2] + (1 + m
 + n)*AppellF1[3/2, -n, 2 + m + n, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-
e + Pi/2 - f*x)/2]^2])*Tan[(-e + Pi/2 - f*x)/2]^2)) + (3*AppellF1[1/2, -n,
1 + m + n, 3/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2]*Si
n[e + f*x]^(1 + n))/((Sec[(-e + Pi/2 - f*x)/2]^2)^m*(3*AppellF1[1/2, -n, 1
+ m + n, 3/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2] - 2*
(n*AppellF1[3/2, 1 - n, 1 + m + n, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-
e + Pi/2 - f*x)/2]^2] + (1 + m + n)*AppellF1[3/2, -n, 2 + m + n, 5/2, Tan[(
-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2])*Tan[(-e + Pi/2 - f*x)/
2]^2)) - (3*m*AppellF1[1/2, -n, 1 + m + n, 3/2, Tan[(-e + Pi/2 - f*x)/2]^2,
 -Tan[(-e + Pi/2 - f*x)/2]^2]*Cos[e + f*x]*Sin[e + f*x]^n*Tan[(-e + Pi/2 -
f*x)/2])/((Sec[(-e + Pi/2 - f*x)/2]^2)^m*(3*AppellF1[1/2, -n, 1 + m + n, 3/
2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2] - 2*(n*AppellF1
[3/2, 1 - n, 1 + m + n, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 -
f*x)/2]^2] + (1 + m + n)*AppellF1[3/2, -n, 2 + m + n, 5/2, Tan[(-e + Pi/2 -
 f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2])*Tan[(-e + Pi/2 - f*x)/2]^2)) + (3
*Cos[e + f*x]*Sin[e + f*x]^n*(-(n*AppellF1[3/2, 1 - n, 1 + m + n, 5/2, Tan[
(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2]*Sec[(-e + Pi/2 - f*x)/
2]^2*Tan[(-e + Pi/2 - f*x)/2])/3 - ((1 + m + n)*AppellF1[3/2, -n, 2 + m + n
, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2]*Sec[(-e + P
i/2 - f*x)/2]^2*Tan[(-e + Pi/2 - f*x)/2])/3))/((Sec[(-e + Pi/2 - f*x)/2]^2)
^m*(3*AppellF1[1/2, -n, 1 + m + n, 3/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-
```

```
e + Pi/2 - f*x)/2]^2] - 2*(n*AppellF1[3/2, 1 - n, 1 + m + n, 5/2, Tan[(-e +
 Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2] + (1 + m + n)*AppellF1[3/2,
 -n, 2 + m + n, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^
2])*Tan[(-e + Pi/2 - f*x)/2]^2)) - (3*AppellF1[1/2, -n, 1 + m + n, 3/2, Tan
[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2]*Cos[e + f*x]*Sin[e +
f*x]^n*(-2*(n*AppellF1[3/2, 1 - n, 1 + m + n, 5/2, Tan[(-e + Pi/2 - f*x)/2]
^2, -Tan[(-e + Pi/2 - f*x)/2]^2] + (1 + m + n)*AppellF1[3/2, -n, 2 + m + n,
 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2])*Sec[(-e + P
i/2 - f*x)/2]^2*Tan[(-e + Pi/2 - f*x)/2] + 3*(-(n*AppellF1[3/2, 1 - n, 1 +
m + n, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2]*Sec[(-
e + Pi/2 - f*x)/2]^2*Tan[(-e + Pi/2 - f*x)/2])/3 - ((1 + m + n)*AppellF1[3/
2, -n, 2 + m + n, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2
]^2]*Sec[(-e + Pi/2 - f*x)/2]^2*Tan[(-e + Pi/2 - f*x)/2])/3) - 2*Tan[(-e +
Pi/2 - f*x)/2]^2*(n*((-3*(1 + m + n)*AppellF1[5/2, 1 - n, 2 + m + n, 7/2, T
an[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2]*Sec[(-e + Pi/2 - f*
x)/2]^2*Tan[(-e + Pi/2 - f*x)/2])/5 + (3*(1 - n)*AppellF1[5/2, 2 - n, 1 + m
 + n, 7/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2]*Sec[(-e
 + Pi/2 - f*x)/2]^2*Tan[(-e + Pi/2 - f*x)/2])/5) + (1 + m + n)*((-3*n*Appel
lF1[5/2, 1 - n, 2 + m + n, 7/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2
 - f*x)/2]^2]*Sec[(-e + Pi/2 - f*x)/2]^2*Tan[(-e + Pi/2 - f*x)/2])/5 - (3*(
2 + m + n)*AppellF1[5/2, -n, 3 + m + n, 7/2, Tan[(-e + Pi/2 - f*x)/2]^2, -T
an[(-e + Pi/2 - f*x)/2]^2]*Sec[(-e + Pi/2 - f*x)/2]^2*Tan[(-e + Pi/2 - f*x)
/2])/5))))/((Sec[(-e + Pi/2 - f*x)/2]^2)^m*(3*AppellF1[1/2, -n, 1 + m + n,
3/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2] - 2*(n*Appell
F1[3/2, 1 - n, 1 + m + n, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2
- f*x)/2]^2] + (1 + m + n)*AppellF1[3/2, -n, 2 + m + n, 5/2, Tan[(-e + Pi/2
 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2])*Tan[(-e + Pi/2 - f*x)/2]^2)^2))
)
```

Maple [F] time = 0.658, size = 0, normalized size = 0.

$$\int \left(d\sin\left(fx+e\right)\right)^{n}\left(a+a\sin\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^m,x)

[Out] int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^{m}\left(d\sin\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^m,x, algorithm="maxima")

[Out] integrate((a*sin(f*x + e) + a)^m*(d*sin(f*x + e))^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a\sin\left(fx+e\right)+a\right)^{m}\left(d\sin\left(fx+e\right)\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)^m*(d*sin(f*x + e))^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\left(\sin\left(e+fx\right)+1\right)\right)^{m}\left(d\sin\left(e+fx\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**n*(a+a*sin(f*x+e))**m,x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**m*(d*sin(e + f*x))**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^{m}\left(d\sin\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(d*sin(f*x + e))^n, x)
```

3.139 $\int (d\sin(e+fx))^n (a-a\sin(e+fx))^m\,dx$

Optimal. Leaf size=107

$$\frac{2^{m+\frac{1}{2}}\cos(e+fx)(1-\sin(e+fx))^{-m-\frac{1}{2}}(-\sin(e+fx))^{-n}(a-a\sin(e+fx))^m(d\sin(e+fx))^n F_1\left(\frac{1}{2};-n,\frac{1}{2}-m;\frac{3}{2};\sin(e+f\right.}{f}$$

```
[Out] (2^(1/2 + m)*AppellF1[1/2, -n, 1/2 - m, 3/2, 1 + Sin[e + f*x], (1 + Sin[e +
 f*x])/2]*Cos[e + f*x]*(1 - Sin[e + f*x])^(-1/2 - m)*(d*Sin[e + f*x])^n*(a
- a*Sin[e + f*x])^m)/(f*(-Sin[e + f*x])^n)
```

Rubi [A] time = 0.159756, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2787, 2786, 2785, 133}

$$\frac{2^{m+\frac{1}{2}}\cos(e+fx)(1-\sin(e+fx))^{-m-\frac{1}{2}}(-\sin(e+fx))^{-n}(a-a\sin(e+fx))^m(d\sin(e+fx))^n F_1\left(\frac{1}{2};-n,\frac{1}{2}-m;\frac{3}{2};\sin(e+f\right.}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sin[e + f*x])^n*(a - a*Sin[e + f*x])^m,x]
```

```
[Out] (2^(1/2 + m)*AppellF1[1/2, -n, 1/2 - m, 3/2, 1 + Sin[e + f*x], (1 + Sin[e +
 f*x])/2]*Cos[e + f*x]*(1 - Sin[e + f*x])^(-1/2 - m)*(d*Sin[e + f*x])^n*(a
- a*Sin[e + f*x])^m)/(f*(-Sin[e + f*x])^n)
```

Rule 2787

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m
])/(1 + (b*Sin[e + f*x])/a)^FracPart[m], Int[(1 + (b*Sin[e + f*x])/a)^m*(d*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 2786

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[((d/b)^IntPart[n]*(d*Sin[e + f*x])^FracPart[n
])/(b*Sin[e + f*x])^FracPart[n], Int[(a + b*Sin[e + f*x])^m*(b*Sin[e + f*x]
)^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !In
tegerQ[m] && GtQ[a, 0] &&  !GtQ[d/b, 0]
```

Rule 2785

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Dist[(b*(d/b)^n*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a - x)^n*(2*a - x)^(m - 1/2)
)/Sqrt[x], x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
```

```
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int (d\sin(e+fx))^n(a-a\sin(e+fx))^m\,dx = \left((1-\sin(e+fx))^{-m}(a-a\sin(e+fx))^m\right)\int (1-\sin(e+fx))^m(d\sin(e+f$$

$$= \left((1-\sin(e+fx))^{-m}(-\sin(e+fx))^{-n}(d\sin(e+fx))^n(a-a\sin(e+fx))^m\right.$$

$$= \frac{\left(\cos(e+fx)(1-\sin(e+fx))^{-\frac{1}{2}-m}(-\sin(e+fx))^{-n}(d\sin(e+fx))^n(a-a\right.}{f\sqrt{1+\sin(e+}}$$

$$= \frac{2^{\frac{1}{2}+m}F_1\left(\frac{1}{2};-n,\frac{1}{2}-m;\frac{3}{2};1+\sin(e+fx),\frac{1}{2}(1+\sin(e+fx))\right)\cos(e+fx)(}{}$$

Mathematica [B] time = 0.380462, size = 301, normalized size = 2.81

$$-\frac{(2m+3)\cos(e+fx)(a-}{f(2m+1)\left((2m+3)F_1\left(m+\frac{1}{2};-n,m+n+1;m+\frac{3}{2};\cot^2\left(\frac{1}{4}(2e+2fx+\pi)\right),-\tan^2\left(\frac{1}{4}(2e+2fx-\pi)\right)\right)-2\tan^2\left(\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sin[e + f*x])^n*(a - a*Sin[e + f*x])^m,x]
```

```
[Out] -(((3 + 2*m)*AppellF1[1/2 + m, -n, 1 + m + n, 3/2 + m, Cot[(2*e + Pi + 2*f*
x)/4]^2, -Tan[(2*e - Pi + 2*f*x)/4]^2]*Cos[e + f*x]*(d*Sin[e + f*x])^n*(a -
 a*Sin[e + f*x])^m)/(f*(1 + 2*m)*((3 + 2*m)*AppellF1[1/2 + m, -n, 1 + m + n
, 3/2 + m, Cot[(2*e + Pi + 2*f*x)/4]^2, -Tan[(2*e - Pi + 2*f*x)/4]^2] - 2*(
n*AppellF1[3/2 + m, 1 - n, 1 + m + n, 5/2 + m, Cot[(2*e + Pi + 2*f*x)/4]^2,
 -Tan[(2*e - Pi + 2*f*x)/4]^2] + (1 + m + n)*AppellF1[3/2 + m, -n, 2 + m +
n, 5/2 + m, Cot[(2*e + Pi + 2*f*x)/4]^2, -Tan[(2*e - Pi + 2*f*x)/4]^2])*Tan
[(2*e - Pi + 2*f*x)/4]^2)))
```

Maple [F] time = 0.655, size = 0, normalized size = 0.

$$\int \left(d\sin\left(fx+e\right)\right)^n\left(a-a\sin\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^n*(a-a*sin(f*x+e))^m,x)
```

```
[Out] int((d*sin(f*x+e))^n*(a-a*sin(f*x+e))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-a\sin\left(fx+e\right)+a\right)^m\left(d\sin\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a-a*sin(f*x+e))^m,x, algorithm="maxima")

[Out] integrate((-a*sin(f*x + e) + a)^m*(d*sin(f*x + e))^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(-a\sin\left(fx+e\right)+a\right)^{m}\left(d\sin\left(fx+e\right)\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a-a*sin(f*x+e))^m,x, algorithm="fricas")

[Out] integral((-a*sin(f*x + e) + a)^m*(d*sin(f*x + e))^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\sin\left(e+fx\right)\right)^{n}\left(-a\left(\sin\left(e+fx\right)-1\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**n*(a-a*sin(f*x+e))**m,x)

[Out] Integral((d*sin(e + f*x))**n*(-a*(sin(e + f*x) - 1))**m, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-a\sin\left(fx+e\right)+a\right)^{m}\left(d\sin\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a-a*sin(f*x+e))^m,x, algorithm="giac")

[Out] integrate((-a*sin(f*x + e) + a)^m*(d*sin(f*x + e))^n, x)
```

3.140 $\int \sin^4(c+dx)(a+a\sin(c+dx))^n\,dx$

Optimal. Leaf size=294

$$-\frac{2^{n+\frac{1}{2}}\left(n^4+6n^3+17n^2+12n+9\right)\cos(c+dx)(\sin(c+dx)+1)^{-n-\frac{1}{2}}(a\sin(c+dx)+a)^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-n;\frac{3}{2};\frac{1}{2}(1-\sin(c\right.}{d(n+1)(n+2)(n+3)(n+4)}$$

```
[Out] ((9 - n - n^2)*Cos[c + d*x]*(a + a*Sin[c + d*x])^n)/(d*(1 + n)*(2 + n)*(3 +
 n)*(4 + n)) - (n*Cos[c + d*x]*Sin[c + d*x]^2*(a + a*Sin[c + d*x])^n)/(d*(3
 + n)*(4 + n)) - (Cos[c + d*x]*Sin[c + d*x]^3*(a + a*Sin[c + d*x])^n)/(d*(4
 + n)) - (2^(1/2 + n)*(9 + 12*n + 17*n^2 + 6*n^3 + n^4)*Cos[c + d*x]*Hyperg
eometric2F1[1/2, 1/2 - n, 3/2, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^(-1
/2 - n)*(a + a*Sin[c + d*x])^n)/(d*(1 + n)*(2 + n)*(3 + n)*(4 + n)) - ((9 +
 3*n + n^2)*Cos[c + d*x]*(a + a*Sin[c + d*x])^(1 + n))/(a*d*(2 + n)*(3 + n)
*(4 + n))
```

Rubi [A] time = 0.509731, antiderivative size = 294, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2783, 2983, 2968, 3023, 2751, 2652, 2651}

$$-\frac{2^{n+\frac{1}{2}}\left(n^4+6n^3+17n^2+12n+9\right)\cos(c+dx)(\sin(c+dx)+1)^{-n-\frac{1}{2}}(a\sin(c+dx)+a)^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-n;\frac{3}{2};\frac{1}{2}(1-\sin(c\right.}{d(n+1)(n+2)(n+3)(n+4)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^4*(a + a*Sin[c + d*x])^n,x]
```


```
[Out] ((9 - n - n^2)*Cos[c + d*x]*(a + a*Sin[c + d*x])^n)/(d*(1 + n)*(2 + n)*(3 +
 n)*(4 + n)) - (n*Cos[c + d*x]*Sin[c + d*x]^2*(a + a*Sin[c + d*x])^n)/(d*(3
 + n)*(4 + n)) - (Cos[c + d*x]*Sin[c + d*x]^3*(a + a*Sin[c + d*x])^n)/(d*(4
 + n)) - (2^(1/2 + n)*(9 + 12*n + 17*n^2 + 6*n^3 + n^4)*Cos[c + d*x]*Hyperg
eometric2F1[1/2, 1/2 - n, 3/2, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^(-1
/2 - n)*(a + a*Sin[c + d*x])^n)/(d*(1 + n)*(2 + n)*(3 + n)*(4 + n)) - ((9 +
 3*n + n^2)*Cos[c + d*x]*(a + a*Sin[c + d*x])^(1 + n))/(a*d*(2 + n)*(3 + n)
*(4 + n))
```


Rule 2783

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*(c + d*Sin[e + f*x])^(n - 1))/(f*(m + n)), x] + Dist[1/(b*(m + n)), Int[(
a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n - 2)*Simp[d*(a*c*m + b*d*(n -
 1)) + b*c^2*(m + n) + d*(a*d*m + b*c*(m + 2*n - 1))*Sin[e + f*x], x], x],
x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && IntegerQ[n]
```

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
```

```
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2652

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Sin[c + d*x])^FracPart[n])/(1 + (b*Sin[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Sin[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2651

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(2^(n +
 1/2)*a^(n - 1/2)*b*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1*(1
 - (b*Sin[c + d*x])/a))/2])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rubi steps

$$\int \sin^4(c+dx)(a+a\sin(c+dx))^n\,dx = -\frac{\cos(c+dx)\sin^3(c+dx)(a+a\sin(c+dx))^n}{d(4+n)} + \frac{\int \sin^2(c+dx)(a+a\sin(c+}{a(4}$$
$$= -\frac{n\cos(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^n}{d(3+n)(4+n)} - \frac{\cos(c+dx)\sin^3(c+dx)(a}{d(4+n)}$$
$$= -\frac{n\cos(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^n}{d(3+n)(4+n)} - \frac{\cos(c+dx)\sin^3(c+dx)(a}{d(4+n)}$$
$$= -\frac{n\cos(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^n}{d(3+n)(4+n)} - \frac{\cos(c+dx)\sin^3(c+dx)(a}{d(4+n)}$$
$$= \frac{(9-n-n^2)\cos(c+dx)(a+a\sin(c+dx))^n}{d(1+n)(2+n)(3+n)(4+n)} - \frac{n\cos(c+dx)\sin^2(c+dx)(a+}{d(3+n)(4+n)}$$
$$= \frac{(9-n-n^2)\cos(c+dx)(a+a\sin(c+dx))^n}{d(1+n)(2+n)(3+n)(4+n)} - \frac{n\cos(c+dx)\sin^2(c+dx)(a+}{d(3+n)(4+n)}$$
$$= \frac{(9-n-n^2)\cos(c+dx)(a+a\sin(c+dx))^n}{d(1+n)(2+n)(3+n)(4+n)} - \frac{n\cos(c+dx)\sin^2(c+dx)(a+}{d(3+n)(4+n)}$$

Mathematica [F] time = 180.089, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[Sin[c + d*x]^4*(a + a*Sin[c + d*x])^n,x]
```

```
[Out] $Aborted
```

Maple [F] time = 1.428, size = 0, normalized size = 0.

$$\int (\sin(dx+c))^4 (a+a\sin(dx+c))^n\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^4*(a+a*sin(d*x+c))^n,x)
```

```
[Out] int(sin(d*x+c)^4*(a+a*sin(d*x+c))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx+c)+a)^n \sin(dx+c)^4\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4*(a+a*sin(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^n*sin(d*x + c)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(\cos\left(dx+c\right)^{4}-2\,\cos\left(dx+c\right)^{2}+1\right)\left(a\sin\left(dx+c\right)+a\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4*(a+a*sin(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*(a*sin(d*x + c) + a)^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**4*(a+a*sin(d*x+c))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(dx+c\right)+a\right)^{n}\sin\left(dx+c\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4*(a+a*sin(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^n*sin(d*x + c)^4, x)
```

3.141 $\int \sin^3(c+dx)(a+a\sin(c+dx))^n\,dx$

Optimal. Leaf size=215

$$-\frac{2^{n+\frac{1}{2}}n\left(n^2+3n+5\right)\cos(c+dx)(\sin(c+dx)+1)^{-n-\frac{1}{2}}(a\sin(c+dx)+a)^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-n;\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{d(n+1)(n+2)(n+3)}-\frac{n\,\mathrm{co}}{}$$

```
[Out] -(((4 + n)*Cos[c + d*x]*(a + a*Sin[c + d*x])^n)/(d*(1 + n)*(2 + n)*(3 + n))
) - (Cos[c + d*x]*Sin[c + d*x]^2*(a + a*Sin[c + d*x])^n)/(d*(3 + n)) - (2^(
1/2 + n)*n*(5 + 3*n + n^2)*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2
, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^(-1/2 - n)*(a + a*Sin[c + d*x])^
n)/(d*(1 + n)*(2 + n)*(3 + n)) - (n*Cos[c + d*x]*(a + a*Sin[c + d*x])^(1 +
n))/(a*d*(6 + 5*n + n^2))
```

Rubi [A] time = 0.294057, antiderivative size = 215, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2783, 2968, 3023, 2751, 2652, 2651}

$$-\frac{2^{n+\frac{1}{2}}n\left(n^2+3n+5\right)\cos(c+dx)(\sin(c+dx)+1)^{-n-\frac{1}{2}}(a\sin(c+dx)+a)^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-n;\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{d(n+1)(n+2)(n+3)}-\frac{n\,\mathrm{co}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^3*(a + a*Sin[c + d*x])^n,x]
```


```
[Out] -(((4 + n)*Cos[c + d*x]*(a + a*Sin[c + d*x])^n)/(d*(1 + n)*(2 + n)*(3 + n))
) - (Cos[c + d*x]*Sin[c + d*x]^2*(a + a*Sin[c + d*x])^n)/(d*(3 + n)) - (2^(
1/2 + n)*n*(5 + 3*n + n^2)*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2
, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^(-1/2 - n)*(a + a*Sin[c + d*x])^
n)/(d*(1 + n)*(2 + n)*(3 + n)) - (n*Cos[c + d*x]*(a + a*Sin[c + d*x])^(1 +
n))/(a*d*(6 + 5*n + n^2))
```


Rule 2783

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*(c + d*Sin[e + f*x])^(n - 1))/(f*(m + n)), x] + Dist[1/(b*(m + n)), Int[(
a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n - 2)*Simp[d*(a*c*m + b*d*(n -
 1)) + b*c^2*(m + n) + d*(a*d*m + b*c*(m + 2*n - 1))*Sin[e + f*x], x], x],
x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && IntegerQ[n]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
```

```
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 !LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2652

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Sin[c + d*x])^FracPart[n])/(1 + (b*Sin[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Sin[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2651

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(2^(n +
 1/2)*a^(n - 1/2)*b*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1*(1
 - (b*Sin[c + d*x])/a))/2])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rubi steps

$$\int \sin^3(c+dx)(a+a\sin(c+dx))^n\,dx = -\frac{\cos(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^n}{d(3+n)}+\frac{\int \sin(c+dx)(a+a\sin(c+dx))}{a(3+n}$$

$$= -\frac{\cos(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^n}{d(3+n)}+\frac{\int (a+a\sin(c+dx))^n\left(2a\sin(c\right.}{a(3+}$$

$$= -\frac{\cos(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^n}{d(3+n)}-\frac{n\cos(c+dx)(a+a\sin(c+dx))}{ad\left(6+5n+n^2\right)}$$

$$= -\frac{(4+n)\cos(c+dx)(a+a\sin(c+dx))^n}{d(1+n)(2+n)(3+n)}-\frac{\cos(c+dx)\sin^2(c+dx)(a+a\sin(c+}{d(3+n)}$$

$$= -\frac{(4+n)\cos(c+dx)(a+a\sin(c+dx))^n}{d(1+n)(2+n)(3+n)}-\frac{\cos(c+dx)\sin^2(c+dx)(a+a\sin(c+}{d(3+n)}$$

$$= -\frac{(4+n)\cos(c+dx)(a+a\sin(c+dx))^n}{d(1+n)(2+n)(3+n)}-\frac{\cos(c+dx)\sin^2(c+dx)(a+a\sin(c+}{d(3+n)}$$

Mathematica [C] time = 127.957, size = 60244, normalized size = 280.2

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[c + d*x]^3*(a + a*Sin[c + d*x])^n,x]
```

```
[Out] Result too large to show
```

Maple [F] time = 1.111, size = 0, normalized size = 0.

$$\int (\sin (dx+c))^3 (a+a\sin (dx+c))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3*(a+a*sin(d*x+c))^n,x)
```

```
[Out] int(sin(d*x+c)^3*(a+a*sin(d*x+c))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin (dx+c)+a)^n \sin (dx+c)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+a*sin(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^n*sin(d*x + c)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(\cos (dx+c)^2-1\right)(a\sin (dx+c)+a)^n \sin (dx+c), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+a*sin(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral(-(cos(d*x + c)^2 - 1)*(a*sin(d*x + c) + a)^n*sin(d*x + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3*(a+a*sin(d*x+c))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin (dx+c)+a)^n \sin (dx+c)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+a*sin(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^n*sin(d*x + c)^3, x)
```

3.142 $\int \sin^2(c+dx)(a+a\sin(c+dx))^n\,dx$

Optimal. Leaf size=156

$$-\frac{2^{n+\frac{1}{2}}\left(n^2+n+1\right)\cos(c+dx)(\sin(c+dx)+1)^{-n-\frac{1}{2}}(a\sin(c+dx)+a)^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-n;\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{d(n+1)(n+2)}+\frac{\cos(c+d}{d}$$

```
[Out] (Cos[c + d*x]*(a + a*Sin[c + d*x])^n)/(d*(2 + 3*n + n^2)) - (2^(1/2 + n)*(1
 + n + n^2)*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1 - Sin[c +
d*x])/2]*(1 + Sin[c + d*x])^(-1/2 - n)*(a + a*Sin[c + d*x])^n)/(d*(1 + n)*(
2 + n)) - (Cos[c + d*x]*(a + a*Sin[c + d*x])^(1 + n))/(a*d*(2 + n))
```

Rubi [A] time = 0.142928, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2759, 2751, 2652, 2651}

$$-\frac{2^{n+\frac{1}{2}}\left(n^2+n+1\right)\cos(c+dx)(\sin(c+dx)+1)^{-n-\frac{1}{2}}(a\sin(c+dx)+a)^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-n;\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{d(n+1)(n+2)}+\frac{\cos(c+d}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2*(a + a*Sin[c + d*x])^n,x]
```


```
[Out] (Cos[c + d*x]*(a + a*Sin[c + d*x])^n)/(d*(2 + 3*n + n^2)) - (2^(1/2 + n)*(1
 + n + n^2)*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1 - Sin[c +
d*x])/2]*(1 + Sin[c + d*x])^(-1/2 - n)*(a + a*Sin[c + d*x])^n)/(d*(1 + n)*(
2 + n)) - (Cos[c + d*x]*(a + a*Sin[c + d*x])^(1 + n))/(a*d*(2 + n))
```


Rule 2759

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin[e
 + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ
[m, -2^(-1)]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2652

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Sin[c + d*x])^FracPart[n])/(1 + (b*Sin[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Sin[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2651

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(2^(n +
 1/2)*a^(n - 1/2)*b*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1*(1
```

```
 - (b*Sin[c + d*x])/a))/2])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rubi steps

$$\int \sin^2(c+dx)(a+a\sin(c+dx))^n\,dx = -\frac{\cos(c+dx)(a+a\sin(c+dx))^{1+n}}{ad(2+n)} + \frac{\int (a(1+n)-a\sin(c+dx))(a+a\sin(c}{a(2+n)}$$

$$= \frac{\cos(c+dx)(a+a\sin(c+dx))^n}{d\left(2+3n+n^2\right)} - \frac{\cos(c+dx)(a+a\sin(c+dx))^{1+n}}{ad(2+n)} + \frac{\left(1+n\right.}{}$$

$$= \frac{\cos(c+dx)(a+a\sin(c+dx))^n}{d\left(2+3n+n^2\right)} - \frac{\cos(c+dx)(a+a\sin(c+dx))^{1+n}}{ad(2+n)} + \frac{\left(\left(1+\right.\right.}{}$$

$$= \frac{\cos(c+dx)(a+a\sin(c+dx))^n}{d\left(2+3n+n^2\right)} - \frac{2^{\frac{1}{2}+n}\left(1+n+n^2\right)\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-n\right.}{}$$

Mathematica [C] time = 54.1764, size = 28439, normalized size = 182.3

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[c + d*x]^2*(a + a*Sin[c + d*x])^n,x]
```

```
[Out] Result too large to show
```

Maple [F] time = 0.907, size = 0, normalized size = 0.

$$\int (\sin(dx+c))^2(a+a\sin(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2*(a+a*sin(d*x+c))^n,x)
```

```
[Out] int(sin(d*x+c)^2*(a+a*sin(d*x+c))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx+c)+a)^n\sin(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+a*sin(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^n*sin(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(\cos\left(dx+c\right)^{2}-1\right)\left(a\sin\left(dx+c\right)+a\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+a*sin(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral(-(cos(d*x + c)^2 - 1)*(a*sin(d*x + c) + a)^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a\left(\sin\left(c+dx\right)+1\right)\right)^{n}\sin^{2}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2*(a+a*sin(d*x+c))**n,x)
```

```
[Out] Integral((a*(sin(c + d*x) + 1))**n*sin(c + d*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a\sin\left(dx+c\right)+a\right)^{n}\sin\left(dx+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+a*sin(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^n*sin(d*x + c)^2, x)
```

3.143 $\int \sin(c+dx)(a+a\sin(c+dx))^n \, dx$

Optimal. Leaf size=109

$$-\frac{2^{n+\frac{1}{2}} n \cos(c+dx)(\sin(c+dx)+1)^{-n-\frac{1}{2}}(a\sin(c+dx)+a)^n \, {}_2F_1\left(\frac{1}{2},\frac{1}{2}-n;\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{d(n+1)} - \frac{\cos(c+dx)(a\sin(\cdots}{d(n+\cdots}$$

```
[Out] -((Cos[c + d*x]*(a + a*Sin[c + d*x])^n)/(d*(1 + n))) - (2^(1/2 + n)*n*Cos[c
 + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1 - Sin[c + d*x])/2]*(1 + Sin
[c + d*x])^(-1/2 - n)*(a + a*Sin[c + d*x])^n)/(d*(1 + n))
```

Rubi [A] time = 0.0638162, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2751, 2652, 2651}

$$-\frac{2^{n+\frac{1}{2}} n \cos(c+dx)(\sin(c+dx)+1)^{-n-\frac{1}{2}}(a\sin(c+dx)+a)^n \, {}_2F_1\left(\frac{1}{2},\frac{1}{2}-n;\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{d(n+1)} - \frac{\cos(c+dx)(a\sin(\cdots}{d(n+\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]*(a + a*Sin[c + d*x])^n,x]
```

```
[Out] -((Cos[c + d*x]*(a + a*Sin[c + d*x])^n)/(d*(1 + n))) - (2^(1/2 + n)*n*Cos[c
 + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1 - Sin[c + d*x])/2]*(1 + Sin
[c + d*x])^(-1/2 - n)*(a + a*Sin[c + d*x])^n)/(d*(1 + n))
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2652

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Sin[c + d*x])^FracPart[n])/(1 + (b*Sin[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Sin[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2651

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(2^(n +
 1/2)*a^(n - 1/2)*b*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1*(1
 - (b*Sin[c + d*x])/a))/2])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rubi steps

$$\int \sin(c+dx)(a+a\sin(c+dx))^n\,dx = -\frac{\cos(c+dx)(a+a\sin(c+dx))^n}{d(1+n)} + \frac{n\int(a+a\sin(c+dx))^n\,dx}{1+n}$$
$$= -\frac{\cos(c+dx)(a+a\sin(c+dx))^n}{d(1+n)} + \frac{(n(1+\sin(c+dx))^{-n}(a+a\sin(c+dx))^n)\int(1}{1+n}$$
$$= -\frac{\cos(c+dx)(a+a\sin(c+dx))^n}{d(1+n)} - \frac{2^{\frac{1}{2}+n}n\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-n;\frac{3}{2};\frac{1}{2}(1-\sin(c}{d(}$$

Mathematica [C] time = 0.443593, size = 178, normalized size = 1.63

$$-\frac{\sqrt[4]{-1}2^{-2n-1}e^{-\frac{3}{2}i(c+dx)}\left(-(-1)^{3/4}e^{-\frac{1}{2}i(c+dx)}\left(e^{i(c+dx)}+i\right)\right)^{2n+1}\left((n-1)e^{2i(c+dx)}\,{}_2F_1\left(1,n;-n;-ie^{-i(c+dx)}\right)-(n+1)\,{}_2F_1\left(1,n+2\right.\right.}{d(n-1)(n+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[c + d*x]*(a + a*Sin[c + d*x])^n,x]
```

```
[Out] -(((-1)^(1/4)*2^(-1 - 2*n)*(-(((-1)^(3/4)*(I + E^(I*(c + d*x))))/E^((I/2)*(
c + d*x))))^(1 + 2*n)*(E^((2*I)*(c + d*x))*(-1 + n)*Hypergeometric2F1[1, n,
 -n, (-I)/E^(I*(c + d*x))] - (1 + n)*Hypergeometric2F1[1, 2 + n, 2 - n, (-I
)/E^(I*(c + d*x))])*(a*(1 + Sin[c + d*x]))^n)/(d*E^(((3*I)/2)*(c + d*x))*(-
1 + n)*(1 + n)*Sin[(2*c + Pi + 2*d*x)/4]^(2*n)))
```

Maple [F] time = 0.875, size = 0, normalized size = 0.

$$\int \sin(dx+c)(a+a\sin(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)*(a+a*sin(d*x+c))^n,x)
```

```
[Out] int(sin(d*x+c)*(a+a*sin(d*x+c))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx+c)+a)^n\sin(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+a*sin(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^n*sin(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((a\sin(dx+c)+a)^n\sin(dx+c),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+a*sin(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((a*sin(d*x + c) + a)^n*sin(d*x + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a (\sin (c + dx) + 1))^n \sin (c + dx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+a*sin(d*x+c))**n,x)
```

```
[Out] Integral((a*(sin(c + d*x) + 1))**n*sin(c + d*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a \sin (dx + c) + a)^n \sin (dx + c) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+a*sin(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^n*sin(d*x + c), x)
```

3.144 $\int (a+a\sin(c+dx))^n\,dx$

Optimal. Leaf size=74

$$-\frac{2^{n+\frac{1}{2}}\cos(c+dx)(\sin(c+dx)+1)^{-n-\frac{1}{2}}(a\sin(c+dx)+a)^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-n;\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{d}$$

```
[Out] -((2^(1/2 + n)*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1 - Sin[c
 + d*x])/2]*(1 + Sin[c + d*x])^(-1/2 - n)*(a + a*Sin[c + d*x])^n)/d)
```

Rubi [A] time = 0.0303929, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2652, 2651}

$$-\frac{2^{n+\frac{1}{2}}\cos(c+dx)(\sin(c+dx)+1)^{-n-\frac{1}{2}}(a\sin(c+dx)+a)^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-n;\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^n,x]
```

```
[Out] -((2^(1/2 + n)*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1 - Sin[c
 + d*x])/2]*(1 + Sin[c + d*x])^(-1/2 - n)*(a + a*Sin[c + d*x])^n)/d)
```

Rule 2652

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Sin[c + d*x])^FracPart[n])/(1 + (b*Sin[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Sin[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] && !IntegerQ[2*n] && !GtQ[a, 0]
```

Rule 2651

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(2^(n +
 1/2)*a^(n - 1/2)*b*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1*(1
 - (b*Sin[c + d*x])/a))/2])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[2*n] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int (a+a\sin(c+dx))^n\,dx &= \left((1+\sin(c+dx))^{-n}(a+a\sin(c+dx))^n\right)\int (1+\sin(c+dx))^n\,dx\\ &= -\frac{2^{\frac{1}{2}+n}\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-n;\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))^{-\frac{1}{2}-n}(a+a\sin(c+dx))^n}{d}\end{aligned}$$

Mathematica [A] time = 0.155195, size = 90, normalized size = 1.22

$$\frac{\sqrt{2}\cos(c+dx)(a(\sin(c+dx)+1))^n\,{}_2F_1\left(\frac{1}{2},n+\frac{1}{2};n+\frac{3}{2};\frac{1}{4}\cos^2(c+dx)\csc^2\left(\frac{1}{4}(2c+2dx-\pi)\right)\right)}{(2dn+d)\sqrt{1-\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^n,x]
```

```
[Out] (Sqrt[2]*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 + n, 3/2 + n, (Cos[c + d*x
]^2*Csc[(2*c - Pi + 2*d*x)/4]^2)/4]*(a*(1 + Sin[c + d*x]))^n)/((d + 2*d*n)*
Sqrt[1 - Sin[c + d*x]])
```

Maple [F] time = 0.003, size = 0, normalized size = 0.

$$\int (a + a \sin(dx + c))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^n,x)
```

```
[Out] int((a+a*sin(d*x+c))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a \sin(dx + c) + a)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((a \sin(dx + c) + a)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((a*sin(d*x + c) + a)^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a \sin(c + dx) + a)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**n,x)
```

```
[Out] Integral((a*sin(c + d*x) + a)**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx+c)+a)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^n, x)
```

3.145 $\int \csc(c+dx)(a+a\sin(c+dx))^n\,dx$

Optimal. Leaf size=85

$$-\frac{2^{n+\frac{1}{2}}\cos(c+dx)(\sin(c+dx)+1)^{-n-\frac{1}{2}}(a\sin(c+dx)+a)^n F_1\left(\frac{1}{2};1,\frac{1}{2}-n;\frac{3}{2};1-\sin(c+dx),\frac{1}{2}(1-\sin(c+dx))\right)}{d}$$

```
[Out] -((2^(1/2 + n)*AppellF1[1/2, 1, 1/2 - n, 3/2, 1 - Sin[c + d*x], (1 - Sin[c
+ d*x])/2]*Cos[c + d*x]*(1 + Sin[c + d*x])^(-1/2 - n)*(a + a*Sin[c + d*x])^
n)/d)
```

Rubi [A] time = 0.110809, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2787, 2785, 130, 429}

$$-\frac{2^{n+\frac{1}{2}}\cos(c+dx)(\sin(c+dx)+1)^{-n-\frac{1}{2}}(a\sin(c+dx)+a)^n F_1\left(\frac{1}{2};1,\frac{1}{2}-n;\frac{3}{2};1-\sin(c+dx),\frac{1}{2}(1-\sin(c+dx))\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*(a + a*Sin[c + d*x])^n,x]
```

```
[Out] -((2^(1/2 + n)*AppellF1[1/2, 1, 1/2 - n, 3/2, 1 - Sin[c + d*x], (1 - Sin[c
+ d*x])/2]*Cos[c + d*x]*(1 + Sin[c + d*x])^(-1/2 - n)*(a + a*Sin[c + d*x])^
n)/d)
```

Rule 2787

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m
])/(1 + (b*Sin[e + f*x])/a)^FracPart[m], Int[(1 + (b*Sin[e + f*x])/a)^m*(d*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 2785

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Dist[(b*(d/b)^n*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a - x)^n*(2*a - x)^(m - 1/2)
)/Sqrt[x], x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

Rule 130

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + (b*x^k)/e)^m*(c + (d*x^k)/e)^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \csc(c+dx)(a+a\sin(c+dx))^n\,dx = \left((1+\sin(c+dx))^{-n}(a+a\sin(c+dx))^n\right)\int \csc(c+dx)(1+\sin(c+dx))^n\,dx$$

$$= -\frac{\left(\cos(c+dx)(1+\sin(c+dx))^{-\frac{1}{2}-n}(a+a\sin(c+dx))^n\right)\operatorname{Subst}\left(\int \frac{(2-x)^{-\frac{1}{2}+n}}{(1-x)\sqrt{x}}\,dx, x,\right.}{d\sqrt{1-\sin(c+dx)}}$$

$$= -\frac{\left(2\cos(c+dx)(1+\sin(c+dx))^{-\frac{1}{2}-n}(a+a\sin(c+dx))^n\right)\operatorname{Subst}\left(\int \frac{\left(2-x^2\right)^{-\frac{1}{2}+n}}{1-x^2}\,dx\right.}{d\sqrt{1-\sin(c+dx)}}$$

$$= -\frac{2^{\frac{1}{2}+n}F_1\left(\frac{1}{2};1,\frac{1}{2}-n;\frac{3}{2};1-\sin(c+dx),\frac{1}{2}(1-\sin(c+dx))\right)\cos(c+dx)(1+\sin(c}{d}$$

Mathematica [C] time = 15.841, size = 2560, normalized size = 30.12

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[c + d*x]*(a + a*Sin[c + d*x])^n,x]
```

```
[Out] -(Csc[c + d*x]*(a + a*Sin[c + d*x])^n*(AppellF1[2*n, n, n, 1 + 2*n, (-1 - I
)/(-1 + Tan[(-c + Pi/2 - d*x)/2]), (-1 + I)/(-1 + Tan[(-c + Pi/2 - d*x)/2])
]*((-I + Tan[(-c + Pi/2 - d*x)/2])/(-1 + Tan[(-c + Pi/2 - d*x)/2]))^n*((I +
 Tan[(-c + Pi/2 - d*x)/2])/(-1 + Tan[(-c + Pi/2 - d*x)/2]))^n - AppellF1[2*
n, n, n, 1 + 2*n, (1 - I)/(1 + Tan[(-c + Pi/2 - d*x)/2]), (1 + I)/(1 + Tan[
(-c + Pi/2 - d*x)/2])]*((-I + Tan[(-c + Pi/2 - d*x)/2])/(1 + Tan[(-c + Pi/2
 - d*x)/2]))^n*((I + Tan[(-c + Pi/2 - d*x)/2])/(1 + Tan[(-c + Pi/2 - d*x)/2
]))^n))/(2*d*n*(Sec[(-c + Pi/2 - d*x)/2]^2)^n*(-(Tan[(-c + Pi/2 - d*x)/2]*(
AppellF1[2*n, n, n, 1 + 2*n, (-1 - I)/(-1 + Tan[(-c + Pi/2 - d*x)/2]), (-1
+ I)/(-1 + Tan[(-c + Pi/2 - d*x)/2])]*((-I + Tan[(-c + Pi/2 - d*x)/2])/(-1
+ Tan[(-c + Pi/2 - d*x)/2]))^n*((I + Tan[(-c + Pi/2 - d*x)/2])/(-1 + Tan[(-
c + Pi/2 - d*x)/2]))^n - AppellF1[2*n, n, n, 1 + 2*n, (1 - I)/(1 + Tan[(-c
+ Pi/2 - d*x)/2]), (1 + I)/(1 + Tan[(-c + Pi/2 - d*x)/2])]*((-I + Tan[(-c +
 Pi/2 - d*x)/2])/(1 + Tan[(-c + Pi/2 - d*x)/2]))^n*((I + Tan[(-c + Pi/2 - d
*x)/2])/(1 + Tan[(-c + Pi/2 - d*x)/2]))^n))/(2*(Sec[(-c + Pi/2 - d*x)/2]^2)
^n) + ((((1 - I)*n^2*AppellF1[1 + 2*n, n, 1 + n, 2 + 2*n, (-1 - I)/(-1 + Ta
n[(-c + Pi/2 - d*x)/2]), (-1 + I)/(-1 + Tan[(-c + Pi/2 - d*x)/2])]*Sec[(-c
+ Pi/2 - d*x)/2]^2)/((1 + 2*n)*(-1 + Tan[(-c + Pi/2 - d*x)/2])^2) + ((1 + I
)*n^2*AppellF1[1 + 2*n, 1 + n, n, 2 + 2*n, (-1 - I)/(-1 + Tan[(-c + Pi/2 -
d*x)/2]), (-1 + I)/(-1 + Tan[(-c + Pi/2 - d*x)/2])]*Sec[(-c + Pi/2 - d*x)/2
]^2)/((1 + 2*n)*(-1 + Tan[(-c + Pi/2 - d*x)/2])^2))*((-I + Tan[(-c + Pi/2 -
 d*x)/2])/(-1 + Tan[(-c + Pi/2 - d*x)/2]))^n*((I + Tan[(-c + Pi/2 - d*x)/2]
)/(-1 + Tan[(-c + Pi/2 - d*x)/2]))^n + n*AppellF1[2*n, n, n, 1 + 2*n, (-1 -
 I)/(-1 + Tan[(-c + Pi/2 - d*x)/2]), (-1 + I)/(-1 + Tan[(-c + Pi/2 - d*x)/2
])]*((-I + Tan[(-c + Pi/2 - d*x)/2])/(-1 + Tan[(-c + Pi/2 - d*x)/2]))^(-1 +
 n)*((I + Tan[(-c + Pi/2 - d*x)/2])/(-1 + Tan[(-c + Pi/2 - d*x)/2]))^n*(Sec
[(-c + Pi/2 - d*x)/2]^2/(2*(-1 + Tan[(-c + Pi/2 - d*x)/2])) - (Sec[(-c + Pi
/2 - d*x)/2]^2*(-I + Tan[(-c + Pi/2 - d*x)/2]))/(2*(-1 + Tan[(-c + Pi/2 - d
*x)/2])^2)) + n*AppellF1[2*n, n, n, 1 + 2*n, (-1 - I)/(-1 + Tan[(-c + Pi/2
- d*x)/2]), (-1 + I)/(-1 + Tan[(-c + Pi/2 - d*x)/2])]*((-I + Tan[(-c + Pi/2
 - d*x)/2])/(-1 + Tan[(-c + Pi/2 - d*x)/2]))^n*((I + Tan[(-c + Pi/2 - d*x)/
2])/(-1 + Tan[(-c + Pi/2 - d*x)/2]))^(-1 + n)*(Sec[(-c + Pi/2 - d*x)/2]^2/(
2*(-1 + Tan[(-c + Pi/2 - d*x)/2])) - (Sec[(-c + Pi/2 - d*x)/2]^2*(I + Tan[(
-c + Pi/2 - d*x)/2]))/(2*(-1 + Tan[(-c + Pi/2 - d*x)/2])^2)) - ((-I + Tan[(
```

```
-c + Pi/2 - d*x)/2])/(1 + Tan[(-c + Pi/2 - d*x)/2]))^n*((I + Tan[(-c + Pi/2
 - d*x)/2])/(1 + Tan[(-c + Pi/2 - d*x)/2]))^n*(((-1 - I)*n^2*AppellF1[1 + 2
*n, n, 1 + n, 2 + 2*n, (1 - I)/(1 + Tan[(-c + Pi/2 - d*x)/2]), (1 + I)/(1 +
 Tan[(-c + Pi/2 - d*x)/2])]*Sec[(-c + Pi/2 - d*x)/2]^2)/((1 + 2*n)*(1 + Tan
[(-c + Pi/2 - d*x)/2])^2) - ((1 - I)*n^2*AppellF1[1 + 2*n, 1 + n, n, 2 + 2*
n, (1 - I)/(1 + Tan[(-c + Pi/2 - d*x)/2]), (1 + I)/(1 + Tan[(-c + Pi/2 - d*
x)/2])]*Sec[(-c + Pi/2 - d*x)/2]^2)/((1 + 2*n)*(1 + Tan[(-c + Pi/2 - d*x)/2
])^2)) - n*AppellF1[2*n, n, n, 1 + 2*n, (1 - I)/(1 + Tan[(-c + Pi/2 - d*x)/
2]), (1 + I)/(1 + Tan[(-c + Pi/2 - d*x)/2])]*((-I + Tan[(-c + Pi/2 - d*x)/2
])/(1 + Tan[(-c + Pi/2 - d*x)/2]))^(-1 + n)*((I + Tan[(-c + Pi/2 - d*x)/2])
/(1 + Tan[(-c + Pi/2 - d*x)/2]))^n*(-(Sec[(-c + Pi/2 - d*x)/2]^2*(-I + Tan[
(-c + Pi/2 - d*x)/2]))/(2*(1 + Tan[(-c + Pi/2 - d*x)/2])^2) + Sec[(-c + Pi/
2 - d*x)/2]^2/(2*(1 + Tan[(-c + Pi/2 - d*x)/2]))) - n*AppellF1[2*n, n, n, 1
 + 2*n, (1 - I)/(1 + Tan[(-c + Pi/2 - d*x)/2]), (1 + I)/(1 + Tan[(-c + Pi/2
 - d*x)/2])]*((-I + Tan[(-c + Pi/2 - d*x)/2])/(1 + Tan[(-c + Pi/2 - d*x)/2]
))^n*((I + Tan[(-c + Pi/2 - d*x)/2])/(1 + Tan[(-c + Pi/2 - d*x)/2]))^(-1 +
n)*(-(Sec[(-c + Pi/2 - d*x)/2]^2*(I + Tan[(-c + Pi/2 - d*x)/2]))/(2*(1 + Ta
n[(-c + Pi/2 - d*x)/2])^2) + Sec[(-c + Pi/2 - d*x)/2]^2/(2*(1 + Tan[(-c + P
i/2 - d*x)/2]))))/(2*n*(Sec[(-c + Pi/2 - d*x)/2]^2)^n)))
```

Maple [F] time = 0.736, size = 0, normalized size = 0.

$$\int \csc\left(dx+c\right)\left(a+a\sin\left(dx+c\right)\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*(a+a*sin(d*x+c))^n,x)
```

```
[Out] int(csc(d*x+c)*(a+a*sin(d*x+c))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(dx+c\right)+a\right)^{n}\csc\left(dx+c\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sin(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^n*csc(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a\sin\left(dx+c\right)+a\right)^{n}\csc\left(dx+c\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sin(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((a*sin(d*x + c) + a)^n*csc(d*x + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a(\sin(c+dx)+1))^n \csc(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sin(d*x+c))**n,x)
```

```
[Out] Integral((a*(sin(c + d*x) + 1))**n*csc(c + d*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx+c)+a)^n \csc(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sin(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^n*csc(d*x + c), x)
```

3.146 $\int \csc^2(c+dx)(a+a\sin(c+dx))^n\,dx$

Optimal. Leaf size=85

$$-\frac{2^{n+\frac{1}{2}}\cos(c+dx)(\sin(c+dx)+1)^{-n-\frac{1}{2}}(a\sin(c+dx)+a)^n F_1\left(\frac{1}{2};2,\frac{1}{2}-n;\frac{3}{2};1-\sin(c+dx),\frac{1}{2}(1-\sin(c+dx))\right)}{d}$$

```
[Out] -((2^(1/2 + n)*AppellF1[1/2, 2, 1/2 - n, 3/2, 1 - Sin[c + d*x], (1 - Sin[c
+ d*x])/2]*Cos[c + d*x]*(1 + Sin[c + d*x])^(-1/2 - n)*(a + a*Sin[c + d*x])^
n)/d)
```

Rubi [A] time = 0.115223, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2787, 2785, 130, 429}

$$-\frac{2^{n+\frac{1}{2}}\cos(c+dx)(\sin(c+dx)+1)^{-n-\frac{1}{2}}(a\sin(c+dx)+a)^n F_1\left(\frac{1}{2};2,\frac{1}{2}-n;\frac{3}{2};1-\sin(c+dx),\frac{1}{2}(1-\sin(c+dx))\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2*(a + a*Sin[c + d*x])^n,x]
```

```
[Out] -((2^(1/2 + n)*AppellF1[1/2, 2, 1/2 - n, 3/2, 1 - Sin[c + d*x], (1 - Sin[c
+ d*x])/2]*Cos[c + d*x]*(1 + Sin[c + d*x])^(-1/2 - n)*(a + a*Sin[c + d*x])^
n)/d)
```

Rule 2787

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m
])/(1 + (b*Sin[e + f*x])/a)^FracPart[m], Int[(1 + (b*Sin[e + f*x])/a)^m*(d*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 2785

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Dist[(b*(d/b)^n*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a - x)^n*(2*a - x)^(m - 1/2)
)/Sqrt[x], x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

Rule 130

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + (b*x^k)/e)^m*(c + (d*x^k)/e)^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \csc^2(c+dx)(a+a\sin(c+dx))^n\,dx = \left((1+\sin(c+dx))^{-n}(a+a\sin(c+dx))^n\right)\int \csc^2(c+dx)(1+\sin(c+dx))^n\,dx$$

$$= -\frac{\left(\cos(c+dx)(1+\sin(c+dx))^{-\frac{1}{2}-n}(a+a\sin(c+dx))^n\right)\operatorname{Subst}\left(\int \frac{(2-x)^{-\frac{1}{2}+n}}{(1-x)^2\sqrt{x}}\,dx, x\right.}{d\sqrt{1-\sin(c+dx)}}$$

$$= -\frac{\left(2\cos(c+dx)(1+\sin(c+dx))^{-\frac{1}{2}-n}(a+a\sin(c+dx))^n\right)\operatorname{Subst}\left(\int \frac{(2-x^2)^{-\frac{1}{2}+n}}{(1-x^2)^2}\,d\right.}{d\sqrt{1-\sin(c+dx)}}$$

$$= -\frac{2^{\frac{1}{2}+n}F_1\left(\frac{1}{2};2,\frac{1}{2}-n;\frac{3}{2};1-\sin(c+dx),\frac{1}{2}(1-\sin(c+dx))\right)\cos(c+dx)(1+\sin(}{d}$$

Mathematica [C] time = 21.2805, size = 4206, normalized size = 49.48

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[c + d*x]^2*(a + a*Sin[c + d*x])^n,x]
```

```
[Out] -((Csc[c + d*x]^2*(a + a*Sin[c + d*x])^n*(-(AppellF1[1 + 2*n, n, n, 2 + 2*n
, (1 - I)/(1 + Tan[(-c + Pi/2 - d*x)/2]), (1 + I)/(1 + Tan[(-c + Pi/2 - d*x
)/2])]*(-1 + Tan[(-c + Pi/2 - d*x)/2])*((-I + Tan[(-c + Pi/2 - d*x)/2])/(1
+ Tan[(-c + Pi/2 - d*x)/2]))^n*((I + Tan[(-c + Pi/2 - d*x)/2])/(1 + Tan[(-c
 + Pi/2 - d*x)/2]))^n) - AppellF1[1 + 2*n, n, n, 2*(1 + n), (-1 - I)/(-1 +
Tan[(-c + Pi/2 - d*x)/2]), (-1 + I)/(-1 + Tan[(-c + Pi/2 - d*x)/2])]*((-I +
 Tan[(-c + Pi/2 - d*x)/2])/(-1 + Tan[(-c + Pi/2 - d*x)/2]))^n*((I + Tan[(-c
 + Pi/2 - d*x)/2])/(-1 + Tan[(-c + Pi/2 - d*x)/2]))^n*(1 + Tan[(-c + Pi/2 -
 d*x)/2])))/(d*(1 + 2*n)*(Sec[(-c + Pi/2 - d*x)/2]^2)^n*(-1 + Tan[(-c + Pi/
2 - d*x)/2])*(1 + Tan[(-c + Pi/2 - d*x)/2])*(-((Sec[(-c + Pi/2 - d*x)/2]^2)
^(1 - n)*(-(AppellF1[1 + 2*n, n, n, 2 + 2*n, (1 - I)/(1 + Tan[(-c + Pi/2 -
d*x)/2]), (1 + I)/(1 + Tan[(-c + Pi/2 - d*x)/2])]*(-1 + Tan[(-c + Pi/2 - d*
x)/2])*((-I + Tan[(-c + Pi/2 - d*x)/2])/(1 + Tan[(-c + Pi/2 - d*x)/2]))^n*(
(I + Tan[(-c + Pi/2 - d*x)/2])/(1 + Tan[(-c + Pi/2 - d*x)/2]))^n) - AppellF
1[1 + 2*n, n, n, 2*(1 + n), (-1 - I)/(-1 + Tan[(-c + Pi/2 - d*x)/2]), (-1 +
 I)/(-1 + Tan[(-c + Pi/2 - d*x)/2])]*((-I + Tan[(-c + Pi/2 - d*x)/2])/(-1 +
 Tan[(-c + Pi/2 - d*x)/2]))^n*((I + Tan[(-c + Pi/2 - d*x)/2])/(-1 + Tan[(-c
 + Pi/2 - d*x)/2]))^n*(1 + Tan[(-c + Pi/2 - d*x)/2])))/(2*(1 + 2*n)*(-1 + T
an[(-c + Pi/2 - d*x)/2])*(1 + Tan[(-c + Pi/2 - d*x)/2])^2) - ((Sec[(-c + Pi
/2 - d*x)/2]^2)^(1 - n)*(-(AppellF1[1 + 2*n, n, n, 2 + 2*n, (1 - I)/(1 + Ta
n[(-c + Pi/2 - d*x)/2]), (1 + I)/(1 + Tan[(-c + Pi/2 - d*x)/2])]*(-1 + Tan[
(-c + Pi/2 - d*x)/2])*((-I + Tan[(-c + Pi/2 - d*x)/2])/(1 + Tan[(-c + Pi/2
- d*x)/2]))^n*((I + Tan[(-c + Pi/2 - d*x)/2])/(1 + Tan[(-c + Pi/2 - d*x)/2]
))^n) - AppellF1[1 + 2*n, n, n, 2*(1 + n), (-1 - I)/(-1 + Tan[(-c + Pi/2 -
d*x)/2]), (-1 + I)/(-1 + Tan[(-c + Pi/2 - d*x)/2])]*((-I + Tan[(-c + Pi/2 -
 d*x)/2])/(-1 + Tan[(-c + Pi/2 - d*x)/2]))^n*((I + Tan[(-c + Pi/2 - d*x)/2]
)/(-1 + Tan[(-c + Pi/2 - d*x)/2]))^n*(1 + Tan[(-c + Pi/2 - d*x)/2])))/(2*(1
 + 2*n)*(-1 + Tan[(-c + Pi/2 - d*x)/2])^2*(1 + Tan[(-c + Pi/2 - d*x)/2])) -
 (n*Tan[(-c + Pi/2 - d*x)/2]*(-(AppellF1[1 + 2*n, n, n, 2 + 2*n, (1 - I)/(1
 + Tan[(-c + Pi/2 - d*x)/2]), (1 + I)/(1 + Tan[(-c + Pi/2 - d*x)/2])]*(-1 +
 Tan[(-c + Pi/2 - d*x)/2])*((-I + Tan[(-c + Pi/2 - d*x)/2])/(1 + Tan[(-c +
Pi/2 - d*x)/2]))^n*((I + Tan[(-c + Pi/2 - d*x)/2])/(1 + Tan[(-c + Pi/2 - d*
x)/2]))^n) - AppellF1[1 + 2*n, n, n, 2*(1 + n), (-1 - I)/(-1 + Tan[(-c + Pi
/2 - d*x)/2]), (-1 + I)/(-1 + Tan[(-c + Pi/2 - d*x)/2])]*((-I + Tan[(-c + P
```

```
i/2 - d*x)/2])/(-1 + Tan[(-c + Pi/2 - d*x)/2]))^n*((I + Tan[(-c + Pi/2 - d*
x)/2])/(-1 + Tan[(-c + Pi/2 - d*x)/2]))^n*(1 + Tan[(-c + Pi/2 - d*x)/2])))/
((1 + 2*n)*(Sec[(-c + Pi/2 - d*x)/2]^2)^n*(-1 + Tan[(-c + Pi/2 - d*x)/2])*(
1 + Tan[(-c + Pi/2 - d*x)/2])) + (-(AppellF1[1 + 2*n, n, n, 2*(1 + n), (-1
- I)/(-1 + Tan[(-c + Pi/2 - d*x)/2]), (-1 + I)/(-1 + Tan[(-c + Pi/2 - d*x)/
2])]*Sec[(-c + Pi/2 - d*x)/2]^2*((-I + Tan[(-c + Pi/2 - d*x)/2])/(-1 + Tan[
(-c + Pi/2 - d*x)/2]))^n*((I + Tan[(-c + Pi/2 - d*x)/2])/(-1 + Tan[(-c + Pi
/2 - d*x)/2]))^n)/2 - (AppellF1[1 + 2*n, n, n, 2 + 2*n, (1 - I)/(1 + Tan[(-
c + Pi/2 - d*x)/2]), (1 + I)/(1 + Tan[(-c + Pi/2 - d*x)/2])]*Sec[(-c + Pi/2
 - d*x)/2]^2*((-I + Tan[(-c + Pi/2 - d*x)/2])/(1 + Tan[(-c + Pi/2 - d*x)/2]
))^n*((I + Tan[(-c + Pi/2 - d*x)/2])/(1 + Tan[(-c + Pi/2 - d*x)/2]))^n)/2 -
 (((1/4 - I/4)*n*(1 + 2*n)*AppellF1[2 + 2*n, n, 1 + n, 1 + 2*(1 + n), (-1 -
 I)/(-1 + Tan[(-c + Pi/2 - d*x)/2]), (-1 + I)/(-1 + Tan[(-c + Pi/2 - d*x)/2
])]*Sec[(-c + Pi/2 - d*x)/2]^2)/((1 + n)*(-1 + Tan[(-c + Pi/2 - d*x)/2])^2)
 + ((1/4 + I/4)*n*(1 + 2*n)*AppellF1[2 + 2*n, 1 + n, n, 1 + 2*(1 + n), (-1
- I)/(-1 + Tan[(-c + Pi/2 - d*x)/2]), (-1 + I)/(-1 + Tan[(-c + Pi/2 - d*x)/
2])]*Sec[(-c + Pi/2 - d*x)/2]^2)/((1 + n)*(-1 + Tan[(-c + Pi/2 - d*x)/2])^2
))*((-I + Tan[(-c + Pi/2 - d*x)/2])/(-1 + Tan[(-c + Pi/2 - d*x)/2]))^n*((I
+ Tan[(-c + Pi/2 - d*x)/2])/(-1 + Tan[(-c + Pi/2 - d*x)/2]))^n*(1 + Tan[(-c
 + Pi/2 - d*x)/2]) - n*AppellF1[1 + 2*n, n, n, 2*(1 + n), (-1 - I)/(-1 + Ta
n[(-c + Pi/2 - d*x)/2]), (-1 + I)/(-1 + Tan[(-c + Pi/2 - d*x)/2])]*((-I + T
an[(-c + Pi/2 - d*x)/2])/(-1 + Tan[(-c + Pi/2 - d*x)/2]))^(-1 + n)*((I + Ta
n[(-c + Pi/2 - d*x)/2])/(-1 + Tan[(-c + Pi/2 - d*x)/2]))^n*(1 + Tan[(-c + P
i/2 - d*x)/2])*(Sec[(-c + Pi/2 - d*x)/2]^2/(2*(-1 + Tan[(-c + Pi/2 - d*x)/2
])) - (Sec[(-c + Pi/2 - d*x)/2]^2*(-I + Tan[(-c + Pi/2 - d*x)/2]))/(2*(-1 +
 Tan[(-c + Pi/2 - d*x)/2])^2)) - n*AppellF1[1 + 2*n, n, n, 2*(1 + n), (-1 -
 I)/(-1 + Tan[(-c + Pi/2 - d*x)/2]), (-1 + I)/(-1 + Tan[(-c + Pi/2 - d*x)/2
])]*((-I + Tan[(-c + Pi/2 - d*x)/2])/(-1 + Tan[(-c + Pi/2 - d*x)/2]))^n*((I
 + Tan[(-c + Pi/2 - d*x)/2])/(-1 + Tan[(-c + Pi/2 - d*x)/2]))^(-1 + n)*(1 +
 Tan[(-c + Pi/2 - d*x)/2])*(Sec[(-c + Pi/2 - d*x)/2]^2/(2*(-1 + Tan[(-c + P
i/2 - d*x)/2])) - (Sec[(-c + Pi/2 - d*x)/2]^2*(I + Tan[(-c + Pi/2 - d*x)/2]
))/(2*(-1 + Tan[(-c + Pi/2 - d*x)/2])^2)) - (-1 + Tan[(-c + Pi/2 - d*x)/2])
*((-I + Tan[(-c + Pi/2 - d*x)/2])/(1 + Tan[(-c + Pi/2 - d*x)/2]))^n*((I + T
an[(-c + Pi/2 - d*x)/2])/(1 + Tan[(-c + Pi/2 - d*x)/2]))^n*(((-1/2 - I/2)*n
*(1 + 2*n)*AppellF1[2 + 2*n, n, 1 + n, 3 + 2*n, (1 - I)/(1 + Tan[(-c + Pi/2
 - d*x)/2]), (1 + I)/(1 + Tan[(-c + Pi/2 - d*x)/2])]*Sec[(-c + Pi/2 - d*x)/
2]^2)/((2 + 2*n)*(1 + Tan[(-c + Pi/2 - d*x)/2])^2) - ((1/2 - I/2)*n*(1 + 2*
n)*AppellF1[2 + 2*n, 1 + n, n, 3 + 2*n, (1 - I)/(1 + Tan[(-c + Pi/2 - d*x)/
2]), (1 + I)/(1 + Tan[(-c + Pi/2 - d*x)/2])]*Sec[(-c + Pi/2 - d*x)/2]^2)/((
2 + 2*n)*(1 + Tan[(-c + Pi/2 - d*x)/2])^2)) - n*AppellF1[1 + 2*n, n, n, 2 +
 2*n, (1 - I)/(1 + Tan[(-c + Pi/2 - d*x)/2]), (1 + I)/(1 + Tan[(-c + Pi/2 -
 d*x)/2])]*(-1 + Tan[(-c + Pi/2 - d*x)/2])*((-I + Tan[(-c + Pi/2 - d*x)/2])
/(1 + Tan[(-c + Pi/2 - d*x)/2]))^(-1 + n)*((I + Tan[(-c + Pi/2 - d*x)/2])/(
1 + Tan[(-c + Pi/2 - d*x)/2]))^n*(-(Sec[(-c + Pi/2 - d*x)/2]^2*(-I + Tan[(-
c + Pi/2 - d*x)/2]))/(2*(1 + Tan[(-c + Pi/2 - d*x)/2])^2) + Sec[(-c + Pi/2
- d*x)/2]^2/(2*(1 + Tan[(-c + Pi/2 - d*x)/2]))) - n*AppellF1[1 + 2*n, n, n,
 2 + 2*n, (1 - I)/(1 + Tan[(-c + Pi/2 - d*x)/2]), (1 + I)/(1 + Tan[(-c + Pi
/2 - d*x)/2])]*(-1 + Tan[(-c + Pi/2 - d*x)/2])*((-I + Tan[(-c + Pi/2 - d*x)
/2])/(1 + Tan[(-c + Pi/2 - d*x)/2]))^n*((I + Tan[(-c + Pi/2 - d*x)/2])/(1 +
 Tan[(-c + Pi/2 - d*x)/2]))^(-1 + n)*(-(Sec[(-c + Pi/2 - d*x)/2]^2*(I + Tan
[(-c + Pi/2 - d*x)/2]))/(2*(1 + Tan[(-c + Pi/2 - d*x)/2])^2) + Sec[(-c + Pi
/2 - d*x)/2]^2/(2*(1 + Tan[(-c + Pi/2 - d*x)/2]))))/((1 + 2*n)*(Sec[(-c + P
i/2 - d*x)/2]^2)^n*(-1 + Tan[(-c + Pi/2 - d*x)/2])*(1 + Tan[(-c + Pi/2 - d*
x)/2])))))
```

Maple [F] time = 0.355, size = 0, normalized size = 0.

$$\int \left(\csc \left(dx+c\right)\right)^{2} \left(a+a \sin \left(dx+c\right)\right)^{n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*(a+a*sin(d*x+c))^n,x)
```

```
[Out] int(csc(d*x+c)^2*(a+a*sin(d*x+c))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a \sin \left(dx+c\right)+a\right)^{n} \csc \left(dx+c\right)^{2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+a*sin(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^n*csc(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\left(a \sin \left(dx+c\right)+a\right)^{n} \csc \left(dx+c\right)^{2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+a*sin(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((a*sin(d*x + c) + a)^n*csc(d*x + c)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*(a+a*sin(d*x+c))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a \sin \left(dx+c\right)+a\right)^{n} \csc \left(dx+c\right)^{2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+a*sin(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^n*csc(d*x + c)^2, x)
```

3.147 $\int (1+\sin(c+dx))^n\,dx$

Optimal. Leaf size=58

$$-\frac{2^{n+\frac{1}{2}}\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-n;\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{d\sqrt{\sin(c+dx)+1}}$$

```
[Out] -((2^(1/2 + n)*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1 - Sin[c
 + d*x])/2])/(d*Sqrt[1 + Sin[c + d*x]]))
```

Rubi [A] time = 0.0139806, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {2651}

$$-\frac{2^{n+\frac{1}{2}}\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-n;\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{d\sqrt{\sin(c+dx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sin[c + d*x])^n,x]
```

```
[Out] -((2^(1/2 + n)*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1 - Sin[c
 + d*x])/2])/(d*Sqrt[1 + Sin[c + d*x]]))
```

Rule 2651

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(2^(n +
 1/2)*a^(n - 1/2)*b*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1*(1
 - (b*Sin[c + d*x])/a))/2])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rubi steps

$$\int (1+\sin(c+dx))^n\,dx=-\frac{2^{\frac{1}{2}+n}\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-n;\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{d\sqrt{1+\sin(c+dx)}}$$

Mathematica [A] time = 0.12066, size = 88, normalized size = 1.52

$$\frac{\sqrt{2}\cos(c+dx)(\sin(c+dx)+1)^n\,{}_2F_1\left(\frac{1}{2},n+\frac{1}{2};n+\frac{3}{2};\frac{1}{4}\cos^2(c+dx)\csc^2\left(\frac{1}{4}(2c+2dx-\pi)\right)\right)}{(2dn+d)\sqrt{1-\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sin[c + d*x])^n,x]
```

```
[Out] (Sqrt[2]*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 + n, 3/2 + n, (Cos[c + d*x
]^2*Csc[(2*c - Pi + 2*d*x)/4]^2)/4]*(1 + Sin[c + d*x])^n)/((d + 2*d*n)*Sqrt
[1 - Sin[c + d*x]])
```

Maple [F] time = 0.26, size = 0, normalized size = 0.

$$\int (1+\sin (dx+c))^{n} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+sin(d*x+c))^n,x)
```

```
[Out] int((1+sin(d*x+c))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (\sin (dx+c)+1)^{n} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((sin(d*x + c) + 1)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((\sin (dx+c)+1)^{n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((sin(d*x + c) + 1)^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (\sin (c+dx)+1)^{n} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(d*x+c))**n,x)
```

```
[Out] Integral((sin(c + d*x) + 1)**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (\sin (dx+c)+1)^{n} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((sin(d*x + c) + 1)^n, x)
```

3.148 $\int (1-\sin(c+dx))^n\,dx$

Optimal. Leaf size=57

$$\frac{2^{n+\frac{1}{2}}\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-n;\frac{3}{2};\frac{1}{2}(\sin(c+dx)+1)\right)}{d\sqrt{1-\sin(c+dx)}}$$

```
[Out] (2^(1/2 + n)*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1 + Sin[c +
 d*x])/2])/(d*Sqrt[1 - Sin[c + d*x]])
```

Rubi [A] time = 0.0182336, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2651}

$$\frac{2^{n+\frac{1}{2}}\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-n;\frac{3}{2};\frac{1}{2}(\sin(c+dx)+1)\right)}{d\sqrt{1-\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sin[c + d*x])^n,x]
```

```
[Out] (2^(1/2 + n)*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1 + Sin[c +
 d*x])/2])/(d*Sqrt[1 - Sin[c + d*x]])
```

Rule 2651

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(2^(n +
 1/2)*a^(n - 1/2)*b*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1*(1
 - (b*Sin[c + d*x])/a))/2])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rubi steps

$$\int (1-\sin(c+dx))^n\,dx=\frac{2^{\frac{1}{2}+n}\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-n;\frac{3}{2};\frac{1}{2}(1+\sin(c+dx))\right)}{d\sqrt{1-\sin(c+dx)}}$$

Mathematica [A] time = 0.104671, size = 90, normalized size = 1.58

$$\frac{\cos(c+dx)(1-\sin(c+dx))^n\cos^2\left(\frac{1}{4}(2c+2dx+\pi)\right)^{-n-\frac{1}{2}}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-n;\frac{3}{2};\frac{1}{4}\cos^2(c+dx)\csc^2\left(\frac{1}{4}(2c+2dx-\pi)\right)\right)}{d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - Sin[c + d*x])^n,x]
```

```
[Out] (Cos[c + d*x]*(Cos[(2*c + Pi + 2*d*x)/4]^2)^(-1/2 - n)*Hypergeometric2F1[1/
2, 1/2 - n, 3/2, (Cos[c + d*x]^2*Csc[(2*c - Pi + 2*d*x)/4]^2)/4]*(1 - Sin[c
 + d*x])^n)/d
```

Maple [F] time = 0.283, size = 0, normalized size = 0.

$$\int (1-\sin (dx+c))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-sin(d*x+c))^n,x)
```

```
[Out] int((1-sin(d*x+c))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (-\sin (dx+c)+1)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sin(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((-sin(d*x + c) + 1)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((-\sin (dx+c)+1)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sin(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((-sin(d*x + c) + 1)^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (1-\sin (c+dx))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sin(d*x+c))**n,x)
```

```
[Out] Integral((1 - sin(c + d*x))**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (-\sin (dx+c)+1)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sin(d*x+c))^n,x, algorithm="giac")

[Out] integrate((-sin(d*x + c) + 1)^n, x)
```

3.149 $\int \sin^3(e+fx)(a+b\sin(e+fx))\,dx$

Optimal. Leaf size=77

$$\frac{a\cos^3(e+fx)}{3f}-\frac{a\cos(e+fx)}{f}-\frac{b\sin^3(e+fx)\cos(e+fx)}{4f}-\frac{3b\sin(e+fx)\cos(e+fx)}{8f}+\frac{3bx}{8}$$

```
[Out] (3*b*x)/8 - (a*Cos[e + f*x])/f + (a*Cos[e + f*x]^3)/(3*f) - (3*b*Cos[e + f*
x]*Sin[e + f*x])/(8*f) - (b*Cos[e + f*x]*Sin[e + f*x]^3)/(4*f)
```

Rubi [A] time = 0.0583029, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2748, 2633, 2635, 8}

$$\frac{a\cos^3(e+fx)}{3f}-\frac{a\cos(e+fx)}{f}-\frac{b\sin^3(e+fx)\cos(e+fx)}{4f}-\frac{3b\sin(e+fx)\cos(e+fx)}{8f}+\frac{3bx}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^3*(a + b*Sin[e + f*x]),x]
```

```
[Out] (3*b*x)/8 - (a*Cos[e + f*x])/f + (a*Cos[e + f*x]^3)/(3*f) - (3*b*Cos[e + f*
x]*Sin[e + f*x])/(8*f) - (b*Cos[e + f*x]*Sin[e + f*x]^3)/(4*f)
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \sin^3(e+fx)(a+b\sin(e+fx))\,dx = a\int \sin^3(e+fx)\,dx + b\int \sin^4(e+fx)\,dx$$
$$= -\frac{b\cos(e+fx)\sin^3(e+fx)}{4f} + \frac{1}{4}(3b)\int \sin^2(e+fx)\,dx - \frac{a\,\mathrm{Subst}\left(\int \left(1-x^2\right)\right.}{f}$$
$$= -\frac{a\cos(e+fx)}{f} + \frac{a\cos^3(e+fx)}{3f} - \frac{3b\cos(e+fx)\sin(e+fx)}{8f} - \frac{b\cos(e+fx)}{4}$$
$$= \frac{3bx}{8} - \frac{a\cos(e+fx)}{f} + \frac{a\cos^3(e+fx)}{3f} - \frac{3b\cos(e+fx)\sin(e+fx)}{8f} - \frac{b\cos(e}{}$$

Mathematica [A] time = 0.164443, size = 76, normalized size = 0.99

$$-\frac{3a\cos(e+fx)}{4f} + \frac{a\cos(3(e+fx))}{12f} + \frac{3b(e+fx)}{8f} - \frac{b\sin(2(e+fx))}{4f} + \frac{b\sin(4(e+fx))}{32f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^3*(a + b*Sin[e + f*x]),x]
```

```
[Out] (3*b*(e + f*x))/(8*f) - (3*a*Cos[e + f*x])/(4*f) + (a*Cos[3*(e + f*x)])/(12
*f) - (b*Sin[2*(e + f*x)])/(4*f) + (b*Sin[4*(e + f*x)])/(32*f)
```

Maple [A] time = 0.014, size = 60, normalized size = 0.8

$$\frac{1}{f}\left(b\left(-\frac{\cos(fx+e)}{4}\left(\left(\sin(fx+e)\right)^3+\frac{3\sin(fx+e)}{2}\right)+\frac{3fx}{8}+\frac{3e}{8}\right)-\frac{a\left(2+\left(\sin(fx+e)\right)^2\right)\cos(fx+e)}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3*(a+b*sin(f*x+e)),x)
```

```
[Out] 1/f*(b*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-1/3*a*
(2+sin(f*x+e)^2)*cos(f*x+e))
```

Maxima [A] time = 1.66268, size = 77, normalized size = 1.

$$\frac{32\left(\cos(fx+e)^3-3\cos(fx+e)\right)a+3\left(12fx+12e+\sin(4fx+4e)-8\sin(2fx+2e)\right)b}{96f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+b*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/96*(32*(cos(f*x + e)^3 - 3*cos(f*x + e))*a + 3*(12*f*x + 12*e + sin(4*f*x
 + 4*e) - 8*sin(2*f*x + 2*e))*b)/f
```

Fricas [A] time = 1.61581, size = 157, normalized size = 2.04

$$\frac{8\,a\cos\left(fx+e\right)^3+9\,bfx-24\,a\cos\left(fx+e\right)+3\left(2\,b\cos\left(fx+e\right)^3-5\,b\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+b*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/24*(8*a*cos(f*x + e)^3 + 9*b*f*x - 24*a*cos(f*x + e) + 3*(2*b*cos(f*x + e
)^3 - 5*b*cos(f*x + e))*sin(f*x + e))/f
```

Sympy [A] time = 1.74778, size = 144, normalized size = 1.87

$$\begin{cases} -\frac{a\sin^2\left(e+fx\right)\cos\left(e+fx\right)}{f}-\frac{2a\cos^3\left(e+fx\right)}{3f}+\frac{3bx\sin^4\left(e+fx\right)}{8}+\frac{3bx\sin^2\left(e+fx\right)\cos^2\left(e+fx\right)}{4}+\frac{3bx\cos^4\left(e+fx\right)}{8}-\frac{5b\sin^3\left(e+fx\right)\cos\left(e+fx\right)}{8f}-\frac{3b\sin \\ x\left(a+b\sin\left(e\right)\right)\sin^3\left(e\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3*(a+b*sin(f*x+e)),x)
```

```
[Out] Piecewise((-a*sin(e + f*x)**2*cos(e + f*x)/f - 2*a*cos(e + f*x)**3/(3*f) +
3*b*x*sin(e + f*x)**4/8 + 3*b*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 3*b*x*c
os(e + f*x)**4/8 - 5*b*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 3*b*sin(e + f*x
)*cos(e + f*x)**3/(8*f), Ne(f, 0)), (x*(a + b*sin(e))*sin(e)**3, True))
```

Giac [A] time = 1.68039, size = 89, normalized size = 1.16

$$\frac{3}{8}\,bx+\frac{a\cos\left(3\,fx+3\,e\right)}{12\,f}-\frac{3\,a\cos\left(fx+e\right)}{4\,f}+\frac{b\sin\left(4\,fx+4\,e\right)}{32\,f}-\frac{b\sin\left(2\,fx+2\,e\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+b*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] 3/8*b*x + 1/12*a*cos(3*f*x + 3*e)/f - 3/4*a*cos(f*x + e)/f + 1/32*b*sin(4*f
*x + 4*e)/f - 1/4*b*sin(2*f*x + 2*e)/f
```

3.150 $\int \sin^2(e+fx)(a+b\sin(e+fx))\,dx$

Optimal. Leaf size=55

$$-\frac{a\sin(e+fx)\cos(e+fx)}{2f}+\frac{ax}{2}+\frac{b\cos^3(e+fx)}{3f}-\frac{b\cos(e+fx)}{f}$$

```
[Out] (a*x)/2 - (b*Cos[e + f*x])/f + (b*Cos[e + f*x]^3)/(3*f) - (a*Cos[e + f*x]*S
in[e + f*x])/(2*f)
```

Rubi [A] time = 0.044637, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2748, 2635, 8, 2633}

$$-\frac{a\sin(e+fx)\cos(e+fx)}{2f}+\frac{ax}{2}+\frac{b\cos^3(e+fx)}{3f}-\frac{b\cos(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^2*(a + b*Sin[e + f*x]),x]
```

```
[Out] (a*x)/2 - (b*Cos[e + f*x])/f + (b*Cos[e + f*x]^3)/(3*f) - (a*Cos[e + f*x]*S
in[e + f*x])/(2*f)
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sin^2(e+fx)(a+b\sin(e+fx))\,dx &= a\int \sin^2(e+fx)\,dx+b\int \sin^3(e+fx)\,dx\\ &= -\frac{a\cos(e+fx)\sin(e+fx)}{2f}+\frac{1}{2}a\int 1\,dx-\frac{b\,\text{Subst}\left(\int \left(1-x^2\right)dx,x,\cos(e+f\right.}{f}\\ &= \frac{ax}{2}-\frac{b\cos(e+fx)}{f}+\frac{b\cos^3(e+fx)}{3f}-\frac{a\cos(e+fx)\sin(e+fx)}{2f}\end{aligned}$$

Mathematica [A] time = 0.0578861, size = 60, normalized size = 1.09

$$\frac{a(e+fx)}{2f}-\frac{a\sin(2(e+fx))}{4f}-\frac{3b\cos(e+fx)}{4f}+\frac{b\cos(3(e+fx))}{12f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^2*(a + b*Sin[e + f*x]),x]
```

```
[Out] (a*(e + f*x))/(2*f) - (3*b*Cos[e + f*x])/(4*f) + (b*Cos[3*(e + f*x)])/(12*f
) - (a*Sin[2*(e + f*x)])/(4*f)
```

Maple [A] time = 0.014, size = 49, normalized size = 0.9

$$\frac{1}{f}\left(-\frac{b\left(2+\left(\sin\left(fx+e\right)\right)^{2}\right)\cos\left(fx+e\right)}{3}+a\left(-\frac{\sin\left(fx+e\right)\cos\left(fx+e\right)}{2}+\frac{fx}{2}+\frac{e}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2*(a+b*sin(f*x+e)),x)
```

```
[Out] 1/f*(-1/3*b*(2+sin(f*x+e)^2)*cos(f*x+e)+a*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f
*x+1/2*e))
```

Maxima [A] time = 1.62178, size = 65, normalized size = 1.18

$$\frac{3\left(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right)\right)a+4\left(\cos\left(fx+e\right)^{3}-3\,\cos\left(fx+e\right)\right)b}{12\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(a+b*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/12*(3*(2*f*x + 2*e - sin(2*f*x + 2*e))*a + 4*(cos(f*x + e)^3 - 3*cos(f*x
+ e))*b)/f
```

Fricas [A] time = 1.59012, size = 120, normalized size = 2.18

$$\frac{2\,b\cos\left(fx+e\right)^{3}+3\,afx-3\,a\cos\left(fx+e\right)\sin\left(fx+e\right)-6\,b\cos\left(fx+e\right)}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(a+b*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*b*cos(f*x + e)^3 + 3*a*f*x - 3*a*cos(f*x + e)*sin(f*x + e) - 6*b*cos
(f*x + e))/f
```

Sympy [A] time = 0.918767, size = 92, normalized size = 1.67

$$\begin{cases} \frac{ax\sin^2(e+fx)}{2} + \frac{ax\cos^2(e+fx)}{2} - \frac{a\sin(e+fx)\cos(e+fx)}{2f} - \frac{b\sin^2(e+fx)\cos(e+fx)}{f} - \frac{2b\cos^3(e+fx)}{3f} & \text{for } f \neq 0 \\ x(a+b\sin(e))\sin^2(e) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2*(a+b*sin(f*x+e)),x)
```

```
[Out] Piecewise((a*x*sin(e + f*x)**2/2 + a*x*cos(e + f*x)**2/2 - a*sin(e + f*x)*c
os(e + f*x)/(2*f) - b*sin(e + f*x)**2*cos(e + f*x)/f - 2*b*cos(e + f*x)**3/
(3*f), Ne(f, 0)), (x*(a + b*sin(e))*sin(e)**2, True))
```

Giac [A] time = 1.68803, size = 68, normalized size = 1.24

$$\frac{1}{2}ax + \frac{b\cos(3fx+3e)}{12f} - \frac{3b\cos(fx+e)}{4f} - \frac{a\sin(2fx+2e)}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(a+b*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/2*a*x + 1/12*b*cos(3*f*x + 3*e)/f - 3/4*b*cos(f*x + e)/f - 1/4*a*sin(2*f*
x + 2*e)/f
```

3.151 $\int \sin(e+fx)(a+b\sin(e+fx))\,dx$

Optimal. Leaf size=39

$$-\frac{a\cos(e+fx)}{f}-\frac{b\sin(e+fx)\cos(e+fx)}{2f}+\frac{bx}{2}$$

```
[Out] (b*x)/2 - (a*Cos[e + f*x])/f - (b*Cos[e + f*x]*Sin[e + f*x])/(2*f)
```

Rubi [A] time = 0.0140097, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2734}

$$-\frac{a\cos(e+fx)}{f}-\frac{b\sin(e+fx)\cos(e+fx)}{2f}+\frac{bx}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]*(a + b*Sin[e + f*x]),x]
```

```
[Out] (b*x)/2 - (a*Cos[e + f*x])/f - (b*Cos[e + f*x]*Sin[e + f*x])/(2*f)
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \sin(e+fx)(a+b\sin(e+fx))\,dx=\frac{bx}{2}-\frac{a\cos(e+fx)}{f}-\frac{b\cos(e+fx)\sin(e+fx)}{2f}$$

Mathematica [A] time = 0.0897402, size = 35, normalized size = 0.9

$$-\frac{4a\cos(e+fx)+b(\sin(2(e+fx))-2(e+fx))}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]*(a + b*Sin[e + f*x]),x]
```

```
[Out] -(4*a*Cos[e + f*x] + b*(-2*(e + f*x) + Sin[2*(e + f*x)]))/(4*f)
```

Maple [A] time = 0.013, size = 39, normalized size = 1.

$$\frac{1}{f}\left(b\left(-\frac{\sin\left(fx+e\right)\cos\left(fx+e\right)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-\cos\left(fx+e\right)a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)*(a+b*sin(f*x+e)),x)
```

```
[Out] 1/f*(b*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-cos(f*x+e)*a)
```

Maxima [A] time = 1.73938, size = 49, normalized size = 1.26

$$\frac{\left(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right)\right)b-4\,a\cos\left(fx+e\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/4*((2*f*x + 2*e - sin(2*f*x + 2*e))*b - 4*a*cos(f*x + e))/f
```

Fricas [A] time = 1.60564, size = 86, normalized size = 2.21

$$\frac{bfx-b\cos\left(fx+e\right)\sin\left(fx+e\right)-2\,a\cos\left(fx+e\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/2*(b*f*x - b*cos(f*x + e)*sin(f*x + e) - 2*a*cos(f*x + e))/f
```

Sympy [A] time = 0.360071, size = 66, normalized size = 1.69

$$\begin{cases} -\frac{a\cos\left(e+fx\right)}{f}+\frac{bx\sin^{2}\left(e+fx\right)}{2}+\frac{bx\cos^{2}\left(e+fx\right)}{2}-\frac{b\sin\left(e+fx\right)\cos\left(e+fx\right)}{2f} & \text{for } f\neq 0\\ x\left(a+b\sin\left(e\right)\right)\sin\left(e\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*sin(f*x+e)),x)
```

```
[Out] Piecewise((-a*cos(e + f*x)/f + b*x*sin(e + f*x)**2/2 + b*x*cos(e + f*x)**2/
2 - b*sin(e + f*x)*cos(e + f*x)/(2*f), Ne(f, 0)), (x*(a + b*sin(e))*sin(e),
 True))
```

Giac [A] time = 1.68234, size = 46, normalized size = 1.18

$$\frac{1}{2}\,bx-\frac{a\cos\left(fx+e\right)}{f}-\frac{b\sin\left(2\,fx+2\,e\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/2*b*x - a*cos(f*x + e)/f - 1/4*b*sin(2*f*x + 2*e)/f
```

3.152 $\int (a+b\sin(e+fx))\,dx$

Optimal. Leaf size=16

$$ax-\frac{b\cos(e+fx)}{f}$$

```
[Out] a*x - (b*Cos[e + f*x])/f
```

Rubi [A] time = 0.0081813, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {2638}

$$ax-\frac{b\cos(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*Sin[e + f*x],x]

[Out] a*x - (b*Cos[e + f*x])/f
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a+b\sin(e+fx))\,dx &= ax+b\int \sin(e+fx)\,dx\\ &= ax-\frac{b\cos(e+fx)}{f}\end{aligned}$$

Mathematica [A] time = 0.0062939, size = 27, normalized size = 1.69

$$ax+\frac{b\sin(e)\sin(fx)}{f}-\frac{b\cos(e)\cos(fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*Sin[e + f*x],x]

[Out] a*x - (b*Cos[e]*Cos[f*x])/f + (b*Sin[e]*Sin[f*x])/f
```

Maple [A] time = 0.009, size = 17, normalized size = 1.1

$$ax-\frac{b\cos\left(fx+e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*sin(f*x+e),x)
```

```
[Out] a*x-b*cos(f*x+e)/f
```

Maxima [A] time = 1.64819, size = 22, normalized size = 1.38

$$ax-\frac{b\cos\left(fx+e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sin(f*x+e),x, algorithm="maxima")
```

```
[Out] a*x - b*cos(f*x + e)/f
```

Fricas [A] time = 1.54762, size = 38, normalized size = 2.38

$$\frac{afx-b\cos\left(fx+e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sin(f*x+e),x, algorithm="fricas")
```

```
[Out] (a*f*x - b*cos(f*x + e))/f
```

Sympy [A] time = 0.236832, size = 19, normalized size = 1.19

$$ax+b\left(\begin{cases}-\frac{\cos\left(e+fx\right)}{f} & \text{for}\ f\neq 0\\ x\sin\left(e\right) & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sin(f*x+e),x)
```

```
[Out] a*x + b*Piecewise((-cos(e + f*x)/f, Ne(f, 0)), (x*sin(e), True))
```

Giac [A] time = 1.64106, size = 23, normalized size = 1.44

$$ax-\frac{b\cos\left(fx+e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sin(f*x+e),x, algorithm="giac")
```

```
[Out] a*x - b*cos(f*x + e)/f
```

3.153 $\int \csc(e+fx)(a+b\sin(e+fx))\,dx$

Optimal. Leaf size=17

$$bx-\frac{a\tanh^{-1}(\cos(e+fx))}{f}$$

```
[Out] b*x - (a*ArcTanh[Cos[e + f*x]])/f
```

Rubi [A] time = 0.0215194, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2735, 3770}

$$bx-\frac{a\tanh^{-1}(\cos(e+fx))}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]*(a + b*Sin[e + f*x]),x]
```

```
[Out] b*x - (a*ArcTanh[Cos[e + f*x]])/f
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \csc(e+fx)(a+b\sin(e+fx))\,dx&=bx+a\int \csc(e+fx)\,dx\\&=bx-\frac{a\tanh^{-1}(\cos(e+fx))}{f}\end{aligned}$$

Mathematica [B] time = 0.0204587, size = 43, normalized size = 2.53

$$\frac{a\log\left(\sin\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{f}-\frac{a\log\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{f}+bx$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]*(a + b*Sin[e + f*x]),x]
```

```
[Out] b*x - (a*Log[Cos[e/2 + (f*x)/2]])/f + (a*Log[Sin[e/2 + (f*x)/2]])/f
```

Maple [A] time = 0.027, size = 32, normalized size = 1.9

$$bx+\frac{a\ln\left(\csc\left(fx+e\right)-\cot\left(fx+e\right)\right)}{f}+\frac{be}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)*(a+b*sin(f*x+e)),x)
```

```
[Out] b*x+1/f*a*ln(csc(f*x+e)-cot(f*x+e))+1/f*b*e
```

Maxima [A] time = 1.74925, size = 39, normalized size = 2.29

$$\frac{\left(fx+e\right)b-a\log\left(\cot\left(fx+e\right)+\csc\left(fx+e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] ((f*x + e)*b - a*log(cot(f*x + e) + csc(f*x + e)))/f
```

Fricas [B] time = 1.75978, size = 111, normalized size = 6.53

$$\frac{2\,bfx-a\log\left(\frac{1}{2}\,\cos\left(fx+e\right)+\frac{1}{2}\right)+a\log\left(-\frac{1}{2}\,\cos\left(fx+e\right)+\frac{1}{2}\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*f*x - a*log(1/2*cos(f*x + e) + 1/2) + a*log(-1/2*cos(f*x + e) + 1/
2))/f
```

Sympy [B] time = 7.32409, size = 51, normalized size = 3.

$$a\left(\begin{cases}\frac{x\cot(e)\csc(e)}{\cot(e)+\csc(e)}+\frac{x\csc^2(e)}{\cot(e)+\csc(e)} & \text{for } f=0\\ -\frac{\log\left(\cot\left(e+fx\right)+\csc\left(e+fx\right)\right)}{f} & \text{otherwise}\end{cases}\right)+bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sin(f*x+e)),x)
```

```
[Out] a*Piecewise((x*cot(e)*csc(e)/(cot(e) + csc(e)) + x*csc(e)**2/(cot(e) + csc(
e)), Eq(f, 0)), (-log(cot(e + f*x) + csc(e + f*x))/f, True)) + b*x
```

<u>Giac</u> [A] time = 2.15185, size = 36, normalized size = 2.12

$$\frac{(fx+e)b+a\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right|\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] ((f*x + e)*b + a*log(abs(tan(1/2*f*x + 1/2*e))))/f
```

3.154 $\int \csc^2(e+fx)(a+b\sin(e+fx))\,dx$

Optimal. Leaf size=26

$$-\frac{a\cot(e+fx)}{f}-\frac{b\tanh^{-1}(\cos(e+fx))}{f}$$

```
[Out] -((b*ArcTanh[Cos[e + f*x]])/f) - (a*Cot[e + f*x])/f
```

Rubi [A] time = 0.0370161, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2748, 3767, 8, 3770}

$$-\frac{a\cot(e+fx)}{f}-\frac{b\tanh^{-1}(\cos(e+fx))}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2*(a + b*Sin[e + f*x]),x]
```

```
[Out] -((b*ArcTanh[Cos[e + f*x]])/f) - (a*Cot[e + f*x])/f
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \csc^2(e+fx)(a+b\sin(e+fx))\,dx &= a\int \csc^2(e+fx)\,dx+b\int \csc(e+fx)\,dx\\ &= -\frac{b\tanh^{-1}(\cos(e+fx))}{f}-\frac{a\,\text{Subst}\left(\int 1\,dx,x,\cot(e+fx)\right)}{f}\\ &= -\frac{b\tanh^{-1}(\cos(e+fx))}{f}-\frac{a\cot(e+fx)}{f}\end{aligned}$$

Mathematica [A] time = 0.0244442, size = 52, normalized size = 2.

$$-\frac{a\cot(e+fx)}{f}+\frac{b\log\left(\sin\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{f}-\frac{b\log\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2*(a + b*Sin[e + f*x]),x]
```

```
[Out] -((a*Cot[e + f*x])/f) - (b*Log[Cos[e/2 + (f*x)/2]])/f + (b*Log[Sin[e/2 + (f
*x)/2]])/f
```

Maple [A] time = 0.029, size = 35, normalized size = 1.4

$$\frac{b\ln\left(\csc\left(fx+e\right)-\cot\left(fx+e\right)\right)}{f}-\frac{\cot\left(fx+e\right)a}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2*(a+b*sin(f*x+e)),x)
```

```
[Out] 1/f*b*ln(csc(f*x+e)-cot(f*x+e))-a*cot(f*x+e)/f
```

Maxima [A] time = 1.76716, size = 54, normalized size = 2.08

$$-\frac{b\left(\log\left(\cos\left(fx+e\right)+1\right)-\log\left(\cos\left(fx+e\right)-1\right)\right)+\frac{2a}{\tan\left(fx+e\right)}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] -1/2*(b*(log(cos(f*x + e) + 1) - log(cos(f*x + e) - 1)) + 2*a/tan(f*x + e))
/f
```

Fricas [B] time = 1.59715, size = 180, normalized size = 6.92

$$-\frac{b\log\left(\frac{1}{2}\cos\left(fx+e\right)+\frac{1}{2}\right)\sin\left(fx+e\right)-b\log\left(-\frac{1}{2}\cos\left(fx+e\right)+\frac{1}{2}\right)\sin\left(fx+e\right)+2a\cos\left(fx+e\right)}{2f\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] -1/2*(b*log(1/2*cos(f*x + e) + 1/2)*sin(f*x + e) - b*log(-1/2*cos(f*x + e)
+ 1/2)*sin(f*x + e) + 2*a*cos(f*x + e))/(f*sin(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a+b\sin\left(e+fx\right)\right)\csc^{2}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2*(a+b*sin(f*x+e)),x)
```

```
[Out] Integral((a + b*sin(e + f*x))*csc(e + f*x)**2, x)
```

Giac [B] time = 2.18197, size = 84, normalized size = 3.23

$$\frac{2\,b\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right|\right)+a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-\frac{2\,b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+a}{\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/2*(2*b*log(abs(tan(1/2*f*x + 1/2*e))) + a*tan(1/2*f*x + 1/2*e) - (2*b*tan
(1/2*f*x + 1/2*e) + a)/tan(1/2*f*x + 1/2*e))/f
```

3.155 $\int \csc^3(e+fx)(a+b\sin(e+fx))\,dx$

Optimal. Leaf size=48

$$-\frac{a\tanh^{-1}(\cos(e+fx))}{2f}-\frac{a\cot(e+fx)\csc(e+fx)}{2f}-\frac{b\cot(e+fx)}{f}$$

```
[Out] -(a*ArcTanh[Cos[e + f*x]])/(2*f) - (b*Cot[e + f*x])/f - (a*Cot[e + f*x]*Csc
[e + f*x])/(2*f)
```

Rubi [A] time = 0.0470495, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2748, 3768, 3770, 3767, 8}

$$-\frac{a\tanh^{-1}(\cos(e+fx))}{2f}-\frac{a\cot(e+fx)\csc(e+fx)}{2f}-\frac{b\cot(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3*(a + b*Sin[e + f*x]),x]
```

```
[Out] -(a*ArcTanh[Cos[e + f*x]])/(2*f) - (b*Cot[e + f*x])/f - (a*Cot[e + f*x]*Csc
[e + f*x])/(2*f)
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \csc^3(e+fx)(a+b\sin(e+fx))\,dx = a\int \csc^3(e+fx)\,dx + b\int \csc^2(e+fx)\,dx$$
$$= -\frac{a\cot(e+fx)\csc(e+fx)}{2f} + \frac{1}{2}a\int \csc(e+fx)\,dx - \frac{b\,\mathrm{Subst}(\int 1\,dx, x, \cot(e$$
$$= -\frac{a\tanh^{-1}(\cos(e+fx))}{2f} - \frac{b\cot(e+fx)}{f} - \frac{a\cot(e+fx)\csc(e+fx)}{2f}$$

Mathematica [A] time = 0.0279821, size = 91, normalized size = 1.9

$$-\frac{a\csc^2\left(\frac{1}{2}(e+fx)\right)}{8f} + \frac{a\sec^2\left(\frac{1}{2}(e+fx)\right)}{8f} + \frac{a\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)}{2f} - \frac{a\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)}{2f} - \frac{b\cot(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^3*(a + b*Sin[e + f*x]),x]
```

```
[Out] -((b*Cot[e + f*x])/f) - (a*Csc[(e + f*x)/2]^2)/(8*f) - (a*Log[Cos[(e + f*x)
/2]])/(2*f) + (a*Log[Sin[(e + f*x)/2]])/(2*f) + (a*Sec[(e + f*x)/2]^2)/(8*f
)
```

Maple [A] time = 0.089, size = 54, normalized size = 1.1

$$-\frac{\cot(fx+e)\,a\csc(fx+e)}{2f} + \frac{a\ln(\csc(fx+e)-\cot(fx+e))}{2f} - \frac{b\cot(fx+e)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3*(a+b*sin(f*x+e)),x)
```

```
[Out] -1/2*a*cot(f*x+e)*csc(f*x+e)/f+1/2/f*a*ln(csc(f*x+e)-cot(f*x+e))-b*cot(f*x+
e)/f
```

Maxima [A] time = 1.61466, size = 81, normalized size = 1.69

$$\frac{a\left(\frac{2\cos(fx+e)}{\cos(fx+e)^2-1} - \log(\cos(fx+e)+1) + \log(\cos(fx+e)-1)\right) - \frac{4b}{\tan(fx+e)}}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/4*(a*(2*cos(f*x + e)/(cos(f*x + e)^2 - 1) - log(cos(f*x + e) + 1) + log(c
os(f*x + e) - 1)) - 4*b/tan(f*x + e))/f
```

Fricas [B] time = 1.74508, size = 251, normalized size = 5.23

$$\frac{4b\cos(fx+e)\sin(fx+e) + 2a\cos(fx+e) - \left(a\cos(fx+e)^2 - a\right)\log\left(\frac{1}{2}\cos(fx+e) + \frac{1}{2}\right) + \left(a\cos(fx+e)^2 - a\right.}{4\left(f\cos(fx+e)^2 - f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/4*(4*b*cos(f*x + e)*sin(f*x + e) + 2*a*cos(f*x + e) - (a*cos(f*x + e)^2 -
 a)*log(1/2*cos(f*x + e) + 1/2) + (a*cos(f*x + e)^2 - a)*log(-1/2*cos(f*x +
 e) + 1/2))/(f*cos(f*x + e)^2 - f)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\sin\left(e + fx\right)\right)\csc^3\left(e + fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3*(a+b*sin(f*x+e)),x)
```

```
[Out] Integral((a + b*sin(e + f*x))*csc(e + f*x)**3, x)
```

Giac [B] time = 2.05635, size = 124, normalized size = 2.58

$$\frac{a\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 + 4a\log\left(\left|\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)\right|\right) + 4b\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - \frac{6a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2 + 4b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right) + a}{\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2}}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/8*(a*tan(1/2*f*x + 1/2*e)^2 + 4*a*log(abs(tan(1/2*f*x + 1/2*e))) + 4*b*ta
n(1/2*f*x + 1/2*e) - (6*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)
 + a)/tan(1/2*f*x + 1/2*e)^2)/f
```

3.156 $\int \csc^4(e+fx)(a+b\sin(e+fx))\,dx$

Optimal. Leaf size=64

$$-\frac{a\cot^3(e+fx)}{3f}-\frac{a\cot(e+fx)}{f}-\frac{b\tanh^{-1}(\cos(e+fx))}{2f}-\frac{b\cot(e+fx)\csc(e+fx)}{2f}$$

```
[Out] -(b*ArcTanh[Cos[e + f*x]])/(2*f) - (a*Cot[e + f*x])/f - (a*Cot[e + f*x]^3)/
(3*f) - (b*Cot[e + f*x]*Csc[e + f*x])/(2*f)
```

Rubi [A] time = 0.050834, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2748, 3767, 3768, 3770}

$$-\frac{a\cot^3(e+fx)}{3f}-\frac{a\cot(e+fx)}{f}-\frac{b\tanh^{-1}(\cos(e+fx))}{2f}-\frac{b\cot(e+fx)\csc(e+fx)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^4*(a + b*Sin[e + f*x]),x]
```

```
[Out] -(b*ArcTanh[Cos[e + f*x]])/(2*f) - (a*Cot[e + f*x])/f - (a*Cot[e + f*x]^3)/
(3*f) - (b*Cot[e + f*x]*Csc[e + f*x])/(2*f)
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc^4(e+fx)(a+b\sin(e+fx))\,dx = a\int \csc^4(e+fx)\,dx + b\int \csc^3(e+fx)\,dx$$
$$= -\frac{b\cot(e+fx)\csc(e+fx)}{2f} + \frac{1}{2}b\int \csc(e+fx)\,dx - \frac{a\,\text{Subst}\left(\int (1+x^2)\,dx, x, c\right.}{f}$$
$$= -\frac{b\tanh^{-1}(\cos(e+fx))}{2f} - \frac{a\cot(e+fx)}{f} - \frac{a\cot^3(e+fx)}{3f} - \frac{b\cot(e+fx)\csc(e+}{2f}$$

Mathematica [A] time = 0.0272745, size = 115, normalized size = 1.8

$$-\frac{2a\cot(e+fx)}{3f} - \frac{a\cot(e+fx)\csc^2(e+fx)}{3f} - \frac{b\csc^2\left(\frac{1}{2}(e+fx)\right)}{8f} + \frac{b\sec^2\left(\frac{1}{2}(e+fx)\right)}{8f} + \frac{b\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)}{2f} - \frac{b\log}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^4*(a + b*Sin[e + f*x]),x]
```

```
[Out] (-2*a*Cot[e + f*x])/(3*f) - (b*Csc[(e + f*x)/2]^2)/(8*f) - (a*Cot[e + f*x]*
Csc[e + f*x]^2)/(3*f) - (b*Log[Cos[(e + f*x)/2]])/(2*f) + (b*Log[Sin[(e + f
*x)/2]])/(2*f) + (b*Sec[(e + f*x)/2]^2)/(8*f)
```

Maple [A] time = 0.084, size = 74, normalized size = 1.2

$$-\frac{2\cot(fx+e)a}{3f} - \frac{\cot(fx+e)a(\csc(fx+e))^2}{3f} - \frac{b\cot(fx+e)\csc(fx+e)}{2f} + \frac{b\ln(\csc(fx+e)-\cot(fx+e))}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^4*(a+b*sin(f*x+e)),x)
```

```
[Out] -2/3*a*cot(f*x+e)/f-1/3/f*a*cot(f*x+e)*csc(f*x+e)^2-1/2*b*cot(f*x+e)*csc(f*
x+e)/f+1/2/f*b*ln(csc(f*x+e)-cot(f*x+e))
```

Maxima [A] time = 1.72575, size = 99, normalized size = 1.55

$$\frac{3b\left(\frac{2\cos(fx+e)}{\cos(fx+e)^2-1} - \log(\cos(fx+e)+1) + \log(\cos(fx+e)-1)\right) - \frac{4(3\tan(fx+e)^2+1)a}{\tan(fx+e)^3}}{12f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/12*(3*b*(2*cos(f*x + e)/(cos(f*x + e)^2 - 1) - log(cos(f*x + e) + 1) + lo
g(cos(f*x + e) - 1)) - 4*(3*tan(f*x + e)^2 + 1)*a/tan(f*x + e)^3)/f
```

Fricas [B] time = 2.01722, size = 344, normalized size = 5.38

$$-\frac{8\,a\cos\left(fx+e\right)^{3}-6\,b\cos\left(fx+e\right)\sin\left(fx+e\right)+3\left(b\cos\left(fx+e\right)^{2}-b\right)\log\left(\frac{1}{2}\,\cos\left(fx+e\right)+\frac{1}{2}\right)\sin\left(fx+e\right)-3\,(}{12\left(f\cos\left(fx+e\right)^{2}-f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] -1/12*(8*a*cos(f*x + e)^3 - 6*b*cos(f*x + e)*sin(f*x + e) + 3*(b*cos(f*x +
e)^2 - b)*log(1/2*cos(f*x + e) + 1/2)*sin(f*x + e) - 3*(b*cos(f*x + e)^2 -
b)*log(-1/2*cos(f*x + e) + 1/2)*sin(f*x + e) - 12*a*cos(f*x + e))/((f*cos(f
*x + e)^2 - f)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**4*(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [B] time = 2.33545, size = 165, normalized size = 2.58

$$\frac{a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+3\,b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+12\,b\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right|\right)+9\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-\frac{22\,b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+9\,a}{}}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/24*(a*tan(1/2*f*x + 1/2*e)^3 + 3*b*tan(1/2*f*x + 1/2*e)^2 + 12*b*log(abs(
tan(1/2*f*x + 1/2*e))) + 9*a*tan(1/2*f*x + 1/2*e) - (22*b*tan(1/2*f*x + 1/2
*e)^3 + 9*a*tan(1/2*f*x + 1/2*e)^2 + 3*b*tan(1/2*f*x + 1/2*e) + a)/tan(1/2*
f*x + 1/2*e)^3)/f
```

3.157 $\int \sin^3(e+fx)(a+b\sin(e+fx))^2\,dx$

Optimal. Leaf size=112

$$\frac{\left(a^2+2b^2\right)\cos^3(e+fx)}{3f}-\frac{\left(a^2+b^2\right)\cos(e+fx)}{f}-\frac{ab\sin^3(e+fx)\cos(e+fx)}{2f}-\frac{3ab\sin(e+fx)\cos(e+fx)}{4f}+\frac{3abx}{4}-b$$

```
[Out] (3*a*b*x)/4 - ((a^2 + b^2)*Cos[e + f*x])/f + ((a^2 + 2*b^2)*Cos[e + f*x]^3)
/(3*f) - (b^2*Cos[e + f*x]^5)/(5*f) - (3*a*b*Cos[e + f*x]*Sin[e + f*x])/(4*
f) - (a*b*Cos[e + f*x]*Sin[e + f*x]^3)/(2*f)
```

Rubi [A] time = 0.104428, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2789, 2635, 8, 3013, 373}

$$\frac{\left(a^2+2b^2\right)\cos^3(e+fx)}{3f}-\frac{\left(a^2+b^2\right)\cos(e+fx)}{f}-\frac{ab\sin^3(e+fx)\cos(e+fx)}{2f}-\frac{3ab\sin(e+fx)\cos(e+fx)}{4f}+\frac{3abx}{4}-b$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^3*(a + b*Sin[e + f*x])^2,x]
```

```
[Out] (3*a*b*x)/4 - ((a^2 + b^2)*Cos[e + f*x])/f + ((a^2 + 2*b^2)*Cos[e + f*x]^3)
/(3*f) - (b^2*Cos[e + f*x]^5)/(5*f) - (3*a*b*Cos[e + f*x]*Sin[e + f*x])/(4*
f) - (a*b*Cos[e + f*x]*Sin[e + f*x]^3)/(2*f)
```

Rule 2789

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)])^2, x_Symbol] :> Dist[(2*c*d)/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] +
 Int[(b*Sin[e + f*x])^m*(c^2 + d^2*Sin[e + f*x]^2), x] /; FreeQ[{b, c, d, e
, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3013

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2)
, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \sin^3(e+fx)(a+b\sin(e+fx))^2\,dx = (2ab)\int \sin^4(e+fx)\,dx + \int \sin^3(e+fx)\left(a^2+b^2\sin^2(e+fx)\right)\,dx$$
$$= -\frac{ab\cos(e+fx)\sin^3(e+fx)}{2f} + \frac{1}{2}(3ab)\int \sin^2(e+fx)\,dx - \frac{\text{Subst}\left(\int \left(1-x^2\right.\right.}{}$$
$$= -\frac{3ab\cos(e+fx)\sin(e+fx)}{4f} - \frac{ab\cos(e+fx)\sin^3(e+fx)}{2f} + \frac{1}{4}(3ab)\int 1\,dx$$
$$= \frac{3abx}{4} - \frac{\left(a^2+b^2\right)\cos(e+fx)}{f} + \frac{\left(a^2+2b^2\right)\cos^3(e+fx)}{3f} - \frac{b^2\cos^5(e+fx)}{5f} -$$

Mathematica [A] time = 0.34122, size = 91, normalized size = 0.81

$$\frac{-30\left(6a^2+5b^2\right)\cos(e+fx)+5\left(4a^2+5b^2\right)\cos(3(e+fx))-3b(b\cos(5(e+fx))-5a(12(e+fx)-8\sin(2(e+fx))+}{240f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^3*(a + b*Sin[e + f*x])^2,x]
```

```
[Out] (-30*(6*a^2 + 5*b^2)*Cos[e + f*x] + 5*(4*a^2 + 5*b^2)*Cos[3*(e + f*x)] - 3*
b*(b*Cos[5*(e + f*x)] - 5*a*(12*(e + f*x) - 8*Sin[2*(e + f*x)] + Sin[4*(e +
 f*x)])))/(240*f)
```

Maple [A] time = 0.023, size = 95, normalized size = 0.9

$$\frac{1}{f}\left(-\frac{b^2\cos\left(fx+e\right)}{5}\left(\frac{8}{3}+\left(\sin\left(fx+e\right)\right)^4+\frac{4\left(\sin\left(fx+e\right)\right)^2}{3}\right)+2ab\left(-1/4\left(\left(\sin\left(fx+e\right)\right)^3+3/2\sin\left(fx+e\right)\right)\cos\left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3*(a+b*sin(f*x+e))^2,x)
```

```
[Out] 1/f*(-1/5*b^2*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)+2*a*b*(-1/4*(s
in(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-1/3*a^2*(2+sin(f*x+e)
^2)*cos(f*x+e))
```

Maxima [A] time = 1.85758, size = 127, normalized size = 1.13

$$\frac{80\left(\cos\left(fx+e\right)^3-3\cos\left(fx+e\right)\right)a^2+15\left(12fx+12e+\sin\left(4fx+4e\right)-8\sin\left(2fx+2e\right)\right)ab-16\left(3\cos\left(fx+e\right.\right.}{240f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+b*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/240*(80*(cos(f*x + e)^3 - 3*cos(f*x + e))*a^2 + 15*(12*f*x + 12*e + sin(4
*f*x + 4*e) - 8*sin(2*f*x + 2*e))*a*b - 16*(3*cos(f*x + e)^5 - 10*cos(f*x +
```

```
e)^3 + 15*cos(f*x + e))*b^2)/f
```

Fricas [A] time = 1.912, size = 232, normalized size = 2.07

$$-\frac{12\,b^2\cos\left(fx+e\right)^5 - 45\,abfx - 20\left(a^2+2\,b^2\right)\cos\left(fx+e\right)^3 + 60\left(a^2+b^2\right)\cos\left(fx+e\right) - 15\left(2\,ab\cos\left(fx+e\right)^3 - 5\,ab\,c\right.}{60\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+b*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -1/60*(12*b^2*cos(f*x + e)^5 - 45*a*b*f*x - 20*(a^2 + 2*b^2)*cos(f*x + e)^3
 + 60*(a^2 + b^2)*cos(f*x + e) - 15*(2*a*b*cos(f*x + e)^3 - 5*a*b*cos(f*x +
 e))*sin(f*x + e))/f
```

Sympy [A] time = 3.37078, size = 221, normalized size = 1.97

$$\begin{cases} -\frac{a^2\sin^2\left(e+fx\right)\cos\left(e+fx\right)}{f} - \frac{2a^2\cos^3\left(e+fx\right)}{3f} + \frac{3abx\sin^4\left(e+fx\right)}{4} + \frac{3abx\sin^2\left(e+fx\right)\cos^2\left(e+fx\right)}{2} + \frac{3abx\cos^4\left(e+fx\right)}{4} - \frac{5ab\sin^3\left(e+fx\right)\cos\left(e+fx\right)}{4f} - \\ x\left(a+b\sin\left(e\right)\right)^2\sin^3\left(e\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3*(a+b*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((-a**2*sin(e + f*x)**2*cos(e + f*x)/f - 2*a**2*cos(e + f*x)**3/(3
*f) + 3*a*b*x*sin(e + f*x)**4/4 + 3*a*b*x*sin(e + f*x)**2*cos(e + f*x)**2/2
 + 3*a*b*x*cos(e + f*x)**4/4 - 5*a*b*sin(e + f*x)**3*cos(e + f*x)/(4*f) - 3
*a*b*sin(e + f*x)*cos(e + f*x)**3/(4*f) - b**2*sin(e + f*x)**4*cos(e + f*x)
/f - 4*b**2*sin(e + f*x)**2*cos(e + f*x)**3/(3*f) - 8*b**2*cos(e + f*x)**5/
(15*f), Ne(f, 0)), (x*(a + b*sin(e))**2*sin(e)**3, True))
```

Giac [A] time = 1.56763, size = 173, normalized size = 1.54

$$\frac{3}{4}\,abx - \frac{b^2\cos\left(5\,fx+5\,e\right)}{80\,f} + \frac{ab\sin\left(4\,fx+4\,e\right)}{16\,f} - \frac{ab\sin\left(2\,fx+2\,e\right)}{2\,f} + \frac{\left(4\,a^2+5\,b^2\right)\cos\left(3\,fx+3\,e\right)}{48\,f} - \frac{\left(2\,a^2+3\,b^2\right)\cos}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+b*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 3/4*a*b*x - 1/80*b^2*cos(5*f*x + 5*e)/f + 1/16*a*b*sin(4*f*x + 4*e)/f - 1/2
*a*b*sin(2*f*x + 2*e)/f + 1/48*(4*a^2 + 5*b^2)*cos(3*f*x + 3*e)/f - 1/8*(2*
a^2 + 3*b^2)*cos(f*x + e)/f - 1/4*(2*a^2 + b^2)*cos(f*x + e)/f
```

3.158 $\int \sin^2(e+fx)(a+b\sin(e+fx))^2\,dx$

Optimal. Leaf size=101

$$-\frac{(4a^2+3b^2)\sin(e+fx)\cos(e+fx)}{8f}+\frac{1}{8}x\left(4a^2+3b^2\right)+\frac{2ab\cos^3(e+fx)}{3f}-\frac{2ab\cos(e+fx)}{f}-\frac{b^2\sin^3(e+fx)\cos(e}{4f}$$

```
[Out] ((4*a^2 + 3*b^2)*x)/8 - (2*a*b*Cos[e + f*x])/f + (2*a*b*Cos[e + f*x]^3)/(3*
f) - ((4*a^2 + 3*b^2)*Cos[e + f*x]*Sin[e + f*x])/(8*f) - (b^2*Cos[e + f*x]*
Sin[e + f*x]^3)/(4*f)
```

Rubi [A] time = 0.0890224, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2789, 2633, 3014, 2635, 8}

$$-\frac{(4a^2+3b^2)\sin(e+fx)\cos(e+fx)}{8f}+\frac{1}{8}x\left(4a^2+3b^2\right)+\frac{2ab\cos^3(e+fx)}{3f}-\frac{2ab\cos(e+fx)}{f}-\frac{b^2\sin^3(e+fx)\cos(e}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^2*(a + b*Sin[e + f*x])^2,x]
```

```
[Out] ((4*a^2 + 3*b^2)*x)/8 - (2*a*b*Cos[e + f*x])/f + (2*a*b*Cos[e + f*x]^3)/(3*
f) - ((4*a^2 + 3*b^2)*Cos[e + f*x]*Sin[e + f*x])/(8*f) - (b^2*Cos[e + f*x]*
Sin[e + f*x]^3)/(4*f)
```

Rule 2789

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)])^2, x_Symbol] :> Dist[(2*c*d)/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] +
 Int[(b*Sin[e + f*x])^m*(c^2 + d^2*Sin[e + f*x]^2), x] /; FreeQ[{b, c, d, e
, f, m}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3014

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*
(m + 2)), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])^m
, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \sin^2(e+fx)(a+b\sin(e+fx))^2\,dx = (2ab)\int \sin^3(e+fx)\,dx + \int \sin^2(e+fx)\left(a^2+b^2\sin^2(e+fx)\right)dx$$
$$= -\frac{b^2\cos(e+fx)\sin^3(e+fx)}{4f} + \frac{1}{4}\left(4a^2+3b^2\right)\int \sin^2(e+fx)\,dx - \frac{(2ab)\,\text{Subst}\,(\ldots}{\ldots}$$
$$= -\frac{2ab\cos(e+fx)}{f} + \frac{2ab\cos^3(e+fx)}{3f} - \frac{\left(4a^2+3b^2\right)\cos(e+fx)\sin(e+fx)}{8f} - \frac{b\ldots}{\ldots}$$
$$= \frac{1}{8}\left(4a^2+3b^2\right)x - \frac{2ab\cos(e+fx)}{f} + \frac{2ab\cos^3(e+fx)}{3f} - \frac{\left(4a^2+3b^2\right)\cos(e+fx\ldots}{8f}$$

Mathematica [A] time = 0.152915, size = 117, normalized size = 1.16

$$\frac{a^2(e+fx)}{2f} - \frac{a^2\sin(2(e+fx))}{4f} - \frac{3ab\cos(e+fx)}{2f} + \frac{ab\cos(3(e+fx))}{6f} + \frac{3b^2(e+fx)}{8f} - \frac{b^2\sin(2(e+fx))}{4f} + \frac{b^2\sin(4(e+\ldots}{32f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^2*(a + b*Sin[e + f*x])^2,x]
```

```
[Out] (a^2*(e + f*x))/(2*f) + (3*b^2*(e + f*x))/(8*f) - (3*a*b*Cos[e + f*x])/(2*f
) + (a*b*Cos[3*(e + f*x)])/(6*f) - (a^2*Sin[2*(e + f*x)])/(4*f) - (b^2*Sin[
2*(e + f*x)])/(4*f) + (b^2*Sin[4*(e + f*x)])/(32*f)
```

Maple [A] time = 0.023, size = 89, normalized size = 0.9

$$\frac{1}{f}\left(b^2\left(-\frac{\cos(fx+e)}{4}\left((\sin(fx+e))^3 + \frac{3\sin(fx+e)}{2}\right) + \frac{3fx}{8} + \frac{3e}{8}\right) - \frac{2ab\left(2+(\sin(fx+e))^2\right)\cos(fx+e)}{3} + a^2\left(-\ldots\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2*(a+b*sin(f*x+e))^2,x)
```

```
[Out] 1/f*(b^2*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-2/3*
a*b*(2+sin(f*x+e)^2)*cos(f*x+e)+a^2*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2
*e))
```

Maxima [A] time = 1.7117, size = 113, normalized size = 1.12

$$\frac{24\left(2fx+2e-\sin(2fx+2e)\right)a^2 + 64\left(\cos(fx+e)^3 - 3\cos(fx+e)\right)ab + 3\left(12fx+12e+\sin(4fx+4e) - 8\sin(\ldots\right.}{96f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(a+b*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/96*(24*(2*f*x + 2*e - sin(2*f*x + 2*e))*a^2 + 64*(cos(f*x + e)^3 - 3*cos(
f*x + e))*a*b + 3*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*b
```

```
^2)/f
```

Fricas [A] time = 1.86773, size = 201, normalized size = 1.99

$$\frac{16\,ab\cos\left(fx+e\right)^{3}+3\left(4\,a^{2}+3\,b^{2}\right)fx-48\,ab\cos\left(fx+e\right)+3\left(2\,b^{2}\cos\left(fx+e\right)^{3}-\left(4\,a^{2}+5\,b^{2}\right)\cos\left(fx+e\right)\right)\sin\left(f\right.}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(a+b*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/24*(16*a*b*cos(f*x + e)^3 + 3*(4*a^2 + 3*b^2)*f*x - 48*a*b*cos(f*x + e) +
 3*(2*b^2*cos(f*x + e)^3 - (4*a^2 + 5*b^2)*cos(f*x + e))*sin(f*x + e))/f
```

Sympy [A] time = 2.10618, size = 211, normalized size = 2.09

$$\begin{cases} \frac{a^2x\sin^2\left(e+fx\right)}{2}+\frac{a^2x\cos^2\left(e+fx\right)}{2}-\frac{a^2\sin\left(e+fx\right)\cos\left(e+fx\right)}{2f}-\frac{2ab\sin^2\left(e+fx\right)\cos\left(e+fx\right)}{f}-\frac{4ab\cos^3\left(e+fx\right)}{3f}+\frac{3b^2x\sin^4\left(e+fx\right)}{8}+\frac{3b^2x\sin^2\left(e\right.}{} \\ x\left(a+b\sin\left(e\right)\right)^2\sin^2\left(e\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2*(a+b*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((a**2*x*sin(e + f*x)**2/2 + a**2*x*cos(e + f*x)**2/2 - a**2*sin(e
 + f*x)*cos(e + f*x)/(2*f) - 2*a*b*sin(e + f*x)**2*cos(e + f*x)/f - 4*a*b*c
os(e + f*x)**3/(3*f) + 3*b**2*x*sin(e + f*x)**4/8 + 3*b**2*x*sin(e + f*x)**
2*cos(e + f*x)**2/4 + 3*b**2*x*cos(e + f*x)**4/8 - 5*b**2*sin(e + f*x)**3*c
os(e + f*x)/(8*f) - 3*b**2*sin(e + f*x)*cos(e + f*x)**3/(8*f), Ne(f, 0)), (
x*(a + b*sin(e))**2*sin(e)**2, True))
```

Giac [A] time = 2.1186, size = 116, normalized size = 1.15

$$\frac{1}{8}\left(4\,a^{2}+3\,b^{2}\right)x+\frac{ab\cos\left(3\,fx+3\,e\right)}{6\,f}-\frac{3\,ab\cos\left(fx+e\right)}{2\,f}+\frac{b^{2}\sin\left(4\,fx+4\,e\right)}{32\,f}-\frac{\left(a^{2}+b^{2}\right)\sin\left(2\,fx+2\,e\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(a+b*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/8*(4*a^2 + 3*b^2)*x + 1/6*a*b*cos(3*f*x + 3*e)/f - 3/2*a*b*cos(f*x + e)/f
 + 1/32*b^2*sin(4*f*x + 4*e)/f - 1/4*(a^2 + b^2)*sin(2*f*x + 2*e)/f
```

3.159 $\int \sin(e+fx)(a+b\sin(e+fx))^2\,dx$

Optimal. Leaf size=71

$$-\frac{2\left(a^2+b^2\right)\cos(e+fx)}{3f}-\frac{\cos(e+fx)(a+b\sin(e+fx))^2}{3f}-\frac{ab\sin(e+fx)\cos(e+fx)}{3f}+abx$$

```
[Out] a*b*x - (2*(a^2 + b^2)*Cos[e + f*x])/(3*f) - (a*b*Cos[e + f*x]*Sin[e + f*x]
)/(3*f) - (Cos[e + f*x]*(a + b*Sin[e + f*x])^2)/(3*f)
```

Rubi [A] time = 0.048996, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2753, 2734}

$$-\frac{2\left(a^2+b^2\right)\cos(e+fx)}{3f}-\frac{\cos(e+fx)(a+b\sin(e+fx))^2}{3f}-\frac{ab\sin(e+fx)\cos(e+fx)}{3f}+abx$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]*(a + b*Sin[e + f*x])^2,x]
```

```
[Out] a*b*x - (2*(a^2 + b^2)*Cos[e + f*x])/(3*f) - (a*b*Cos[e + f*x]*Sin[e + f*x]
)/(3*f) - (Cos[e + f*x]*(a + b*Sin[e + f*x])^2)/(3*f)
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \sin(e+fx)(a+b\sin(e+fx))^2\,dx &= -\frac{\cos(e+fx)(a+b\sin(e+fx))^2}{3f}+\frac{1}{3}\int (2b+2a\sin(e+fx))(a+b\sin(e+fx))\,d\\ &= abx-\frac{2\left(a^2+b^2\right)\cos(e+fx)}{3f}-\frac{ab\cos(e+fx)\sin(e+fx)}{3f}-\frac{\cos(e+fx)(a+b\sin}{3f}\end{aligned}$$

Mathematica [A] time = 0.210484, size = 59, normalized size = 0.83

$$\frac{b(12a(e+fx)-6a\sin(2(e+fx))+b\cos(3(e+fx)))-3\left(4a^2+3b^2\right)\cos(e+fx)}{12f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]*(a + b*Sin[e + f*x])^2,x]
```

```
[Out] (-3*(4*a^2 + 3*b^2)*Cos[e + f*x] + b*(12*a*(e + f*x) + b*Cos[3*(e + f*x)] -
 6*a*Sin[2*(e + f*x)]))/(12*f)
```

Maple [A] time = 0.019, size = 64, normalized size = 0.9

$$\frac{1}{f}\left(-\frac{b^2\left(2+\left(\sin\left(fx+e\right)\right)^2\right)\cos\left(fx+e\right)}{3}+2\,ab\left(-1/2\,\sin\left(fx+e\right)\cos\left(fx+e\right)+1/2\,fx+e/2\right)-\cos\left(fx+e\right)a^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)*(a+b*sin(f*x+e))^2,x)
```

```
[Out] 1/f*(-1/3*b^2*(2+sin(f*x+e)^2)*cos(f*x+e)+2*a*b*(-1/2*sin(f*x+e)*cos(f*x+e)
+1/2*f*x+1/2*e)-cos(f*x+e)*a^2)
```

Maxima [A] time = 1.60699, size = 84, normalized size = 1.18

$$\frac{3\left(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right)\right)ab+2\left(\cos\left(fx+e\right)^3-3\,\cos\left(fx+e\right)\right)b^2-6\,a^2\cos\left(fx+e\right)}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/6*(3*(2*f*x + 2*e - sin(2*f*x + 2*e))*a*b + 2*(cos(f*x + e)^3 - 3*cos(f*x
 + e))*b^2 - 6*a^2*cos(f*x + e))/f
```

Fricas [A] time = 1.97532, size = 139, normalized size = 1.96

$$\frac{b^2\cos\left(fx+e\right)^3+3\,abfx-3\,ab\cos\left(fx+e\right)\sin\left(fx+e\right)-3\left(a^2+b^2\right)\cos\left(fx+e\right)}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(b^2*cos(f*x + e)^3 + 3*a*b*f*x - 3*a*b*cos(f*x + e)*sin(f*x + e) - 3*(
a^2 + b^2)*cos(f*x + e))/f
```

Sympy [A] time = 1.03553, size = 107, normalized size = 1.51

$$\begin{cases}-\frac{a^2\cos\left(e+fx\right)}{f}+abx\sin^2\left(e+fx\right)+abx\cos^2\left(e+fx\right)-\frac{ab\sin\left(e+fx\right)\cos\left(e+fx\right)}{f}-\frac{b^2\sin^2\left(e+fx\right)\cos\left(e+fx\right)}{f}-\frac{2b^2\cos^3\left(e+fx\right)}{3f} & \text{fo} \\ x\left(a+b\sin\left(e\right)\right)^2\sin\left(e\right) & \text{o}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((-a**2*cos(e + f*x)/f + a*b*x*sin(e + f*x)**2 + a*b*x*cos(e + f*x
)**2 - a*b*sin(e + f*x)*cos(e + f*x)/f - b**2*sin(e + f*x)**2*cos(e + f*x)/
f - 2*b**2*cos(e + f*x)**3/(3*f), Ne(f, 0)), (x*(a + b*sin(e))**2*sin(e), T
rue))
```

Giac [A] time = 2.03036, size = 103, normalized size = 1.45

$$abx + \frac{b^2 \cos\left(3 fx + 3 e\right)}{12 f} - \frac{b^2 \cos\left(fx + e\right)}{4 f} - \frac{ab \sin\left(2 fx + 2 e\right)}{2 f} - \frac{\left(2 a^2 + b^2\right) \cos\left(fx + e\right)}{2 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] a*b*x + 1/12*b^2*cos(3*f*x + 3*e)/f - 1/4*b^2*cos(f*x + e)/f - 1/2*a*b*sin(
2*f*x + 2*e)/f - 1/2*(2*a^2 + b^2)*cos(f*x + e)/f
```

3.160 $\int (a+b\sin(e+fx))^2\,dx$

Optimal. Leaf size=50

$$\frac{1}{2}x\left(2a^2+b^2\right)-\frac{2ab\cos(e+fx)}{f}-\frac{b^2\sin(e+fx)\cos(e+fx)}{2f}$$

```
[Out] ((2*a^2 + b^2)*x)/2 - (2*a*b*Cos[e + f*x])/f - (b^2*Cos[e + f*x]*Sin[e + f*
x])/(2*f)
```

Rubi [A] time = 0.0149389, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2644}

$$\frac{1}{2}x\left(2a^2+b^2\right)-\frac{2ab\cos(e+fx)}{f}-\frac{b^2\sin(e+fx)\cos(e+fx)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^2,x]
```

```
[Out] ((2*a^2 + b^2)*x)/2 - (2*a*b*Cos[e + f*x])/f - (b^2*Cos[e + f*x]*Sin[e + f*
x])/(2*f)
```

Rule 2644

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[((2*a^2 + b
^2)*x)/2, x] + (-Simp[(2*a*b*Cos[c + d*x])/d, x] - Simp[(b^2*Cos[c + d*x]*S
in[c + d*x])/(2*d), x]) /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$\int (a+b\sin(e+fx))^2\,dx=\frac{1}{2}\left(2a^2+b^2\right)x-\frac{2ab\cos(e+fx)}{f}-\frac{b^2\cos(e+fx)\sin(e+fx)}{2f}$$

Mathematica [A] time = 0.0980773, size = 46, normalized size = 0.92

$$-\frac{-2\left(2a^2+b^2\right)(e+fx)+8ab\cos(e+fx)+b^2\sin(2(e+fx))}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^2,x]
```

```
[Out] -(-2*(2*a^2 + b^2)*(e + f*x) + 8*a*b*Cos[e + f*x] + b^2*Sin[2*(e + f*x)])/(
4*f)
```

Maple [A] time = 0.017, size = 51, normalized size = 1.

$$\frac{1}{f}\left(b^2\left(-\frac{\sin\left(fx+e\right)\cos\left(fx+e\right)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-2\,ab\cos\left(fx+e\right)+a^2\left(fx+e\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^2,x)
```

```
[Out] 1/f*(b^2*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-2*a*b*cos(f*x+e)+a^2*(f
*x+e))
```

Maxima [A] time = 1.83984, size = 62, normalized size = 1.24

$$a^2x + \frac{\left(2\,fx + 2\,e - \sin\left(2\,fx + 2\,e\right)\right)b^2}{4\,f} - \frac{2\,ab\cos\left(fx + e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] a^2*x + 1/4*(2*f*x + 2*e - sin(2*f*x + 2*e))*b^2/f - 2*a*b*cos(f*x + e)/f
```

Fricas [A] time = 1.88579, size = 109, normalized size = 2.18

$$-\frac{b^2\cos\left(fx + e\right)\sin\left(fx + e\right) - \left(2\,a^2 + b^2\right)fx + 4\,ab\cos\left(fx + e\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(b^2*cos(f*x + e)*sin(f*x + e) - (2*a^2 + b^2)*f*x + 4*a*b*cos(f*x + e
))/f
```

Sympy [A] time = 0.459778, size = 78, normalized size = 1.56

$$\begin{cases} a^2x - \frac{2ab\cos\left(e+fx\right)}{f} + \frac{b^2x\sin^2\left(e+fx\right)}{2} + \frac{b^2x\cos^2\left(e+fx\right)}{2} - \frac{b^2\sin\left(e+fx\right)\cos\left(e+fx\right)}{2f} & \text{for } f \neq 0 \\ x\left(a + b\sin\left(e\right)\right)^2 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((a**2*x - 2*a*b*cos(e + f*x)/f + b**2*x*sin(e + f*x)**2/2 + b**2*
x*cos(e + f*x)**2/2 - b**2*sin(e + f*x)*cos(e + f*x)/(2*f), Ne(f, 0)), (x*(
a + b*sin(e))**2, True))
```

Giac [A] time = 1.95581, size = 61, normalized size = 1.22

$$\frac{1}{2}\left(2\,a^2 + b^2\right)x - \frac{2\,ab\cos\left(fx + e\right)}{f} - \frac{b^2\sin\left(2\,fx + 2\,e\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*a^2 + b^2)*x - 2*a*b*cos(f*x + e)/f - 1/4*b^2*sin(2*f*x + 2*e)/f
```

3.161 $\int \csc(e+fx)(a+b\sin(e+fx))^2\,dx$

Optimal. Leaf size=35

$$-\frac{a^2\tanh^{-1}(\cos(e+fx))}{f}+2abx-\frac{b^2\cos(e+fx)}{f}$$

```
[Out] 2*a*b*x - (a^2*ArcTanh[Cos[e + f*x]])/f - (b^2*Cos[e + f*x])/f
```

Rubi [A] time = 0.0589407, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2746, 2735, 3770}

$$-\frac{a^2\tanh^{-1}(\cos(e+fx))}{f}+2abx-\frac{b^2\cos(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]*(a + b*Sin[e + f*x])^2,x]
```

```
[Out] 2*a*b*x - (a^2*ArcTanh[Cos[e + f*x]])/f - (b^2*Cos[e + f*x])/f
```

Rule 2746

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2/((c_.) + (d_.)*sin[(e_.) + (f
_.)*(x_)]), x_Symbol] :> -Simp[(b^2*Cos[e + f*x])/(d*f), x] + Dist[1/d, Int
[Simp[a^2*d - b*(b*c - 2*a*d)*Sin[e + f*x], x]/(c + d*Sin[e + f*x]), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \csc(e+fx)(a+b\sin(e+fx))^2\,dx &= -\frac{b^2\cos(e+fx)}{f}+\int \csc(e+fx)\left(a^2+2ab\sin(e+fx)\right)dx\\
&= 2abx-\frac{b^2\cos(e+fx)}{f}+a^2\int \csc(e+fx)\,dx\\
&= 2abx-\frac{a^2\tanh^{-1}(\cos(e+fx))}{f}-\frac{b^2\cos(e+fx)}{f}
\end{aligned}$$

Mathematica [B] time = 0.0222213, size = 76, normalized size = 2.17

$$\frac{a^2\log\left(\sin\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{f}-\frac{a^2\log\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{f}+2abx+\frac{b^2\sin(e)\sin(fx)}{f}-\frac{b^2\cos(e)\cos(fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]*(a + b*Sin[e + f*x])^2,x]
```

```
[Out] 2*a*b*x - (b^2*Cos[e]*Cos[f*x])/f - (a^2*Log[Cos[e/2 + (f*x)/2]])/f + (a^2*
Log[Sin[e/2 + (f*x)/2]])/f + (b^2*Sin[e]*Sin[f*x])/f
```

Maple [A] time = 0.044, size = 52, normalized size = 1.5

$$2\,abx-\frac{b^2\cos\left(fx+e\right)}{f}+\frac{a^2\ln\left(\csc\left(fx+e\right)-\cot\left(fx+e\right)\right)}{f}+2\,\frac{abe}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)*(a+b*sin(f*x+e))^2,x)
```

```
[Out] 2*a*b*x-b^2*cos(f*x+e)/f+1/f*a^2*ln(csc(f*x+e)-cot(f*x+e))+2/f*a*b*e
```

Maxima [A] time = 1.64878, size = 59, normalized size = 1.69

$$\frac{2\left(fx+e\right)ab-b^2\cos\left(fx+e\right)-a^2\log\left(\cot\left(fx+e\right)+\csc\left(fx+e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] (2*(f*x + e)*a*b - b^2*cos(f*x + e) - a^2*log(cot(f*x + e) + csc(f*x + e)))
/f
```

Fricas [A] time = 2.07163, size = 147, normalized size = 4.2

$$\frac{4\,abfx-2\,b^2\cos\left(fx+e\right)-a^2\log\left(\frac{1}{2}\cos\left(fx+e\right)+\frac{1}{2}\right)+a^2\log\left(-\frac{1}{2}\cos\left(fx+e\right)+\frac{1}{2}\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(4*a*b*f*x - 2*b^2*cos(f*x + e) - a^2*log(1/2*cos(f*x + e) + 1/2) + a^2
*log(-1/2*cos(f*x + e) + 1/2))/f
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\sin\left(e+fx\right)\right)^2\csc\left(e+fx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sin(f*x+e))**2,x)
```

```
[Out] Integral((a + b*sin(e + f*x))**2*csc(e + f*x), x)
```

Giac [A] time = 1.80073, size = 70, normalized size = 2.

$$\frac{2\left(fx+e\right)ab+a^{2}\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right|\right)-\frac{2b^{2}}{\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+1}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] (2*(f*x + e)*a*b + a^2*log(abs(tan(1/2*f*x + 1/2*e))) - 2*b^2/(tan(1/2*f*x
+ 1/2*e)^2 + 1))/f
```

3.162 $\int \csc^2(e+fx)(a+b\sin(e+fx))^2\,dx$

Optimal. Leaf size=34

$$-\frac{a^2\cot(e+fx)}{f}-\frac{2ab\tanh^{-1}(\cos(e+fx))}{f}+b^2x$$

```
[Out] b^2*x - (2*a*b*ArcTanh[Cos[e + f*x]])/f - (a^2*Cot[e + f*x])/f
```

Rubi [A] time = 0.0655697, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2789, 3770, 3012, 8}

$$-\frac{a^2\cot(e+fx)}{f}-\frac{2ab\tanh^{-1}(\cos(e+fx))}{f}+b^2x$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2*(a + b*Sin[e + f*x])^2,x]
```

```
[Out] b^2*x - (2*a*b*ArcTanh[Cos[e + f*x]])/f - (a^2*Cot[e + f*x])/f
```

Rule 2789

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)])^2, x_Symbol] :> Dist[(2*c*d)/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] +
 Int[(b*Sin[e + f*x])^m*(c^2 + d^2*Sin[e + f*x]^2), x] /; FreeQ[{b, c, d, e
, f, m}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3012

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[(A*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m
 + 1)), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \csc^2(e+fx)(a+b\sin(e+fx))^2\,dx &= (2ab)\int \csc(e+fx)\,dx+\int \csc^2(e+fx)\left(a^2+b^2\sin^2(e+fx)\right)dx\\ &= -\frac{2ab\tanh^{-1}(\cos(e+fx))}{f}-\frac{a^2\cot(e+fx)}{f}+b^2\int 1\,dx\\ &= b^2x-\frac{2ab\tanh^{-1}(\cos(e+fx))}{f}-\frac{a^2\cot(e+fx)}{f}\end{aligned}$$

Mathematica [B] time = 0.220298, size = 76, normalized size = 2.24

$$\frac{a^2 \tan \left(\frac{1}{2}(e+fx)\right)+a^2 \left(-\cot \left(\frac{1}{2}(e+fx)\right)\right)+2b\left(2a \log \left(\sin \left(\frac{1}{2}(e+fx)\right)\right)-2a \log \left(\cos \left(\frac{1}{2}(e+fx)\right)\right)+be+bfx\right)}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2*(a + b*Sin[e + f*x])^2,x]
```

```
[Out] (-(a^2*Cot[(e + f*x)/2]) + 2*b*(b*e + b*f*x - 2*a*Log[Cos[(e + f*x)/2]] + 2
*a*Log[Sin[(e + f*x)/2]]) + a^2*Tan[(e + f*x)/2])/(2*f)
```

Maple [A] time = 0.042, size = 52, normalized size = 1.5

$$b^2 x-\frac{a^2 \cot \left(fx+e\right)}{f}+2\frac{ab \ln \left(\csc \left(fx+e\right)-\cot \left(fx+e\right)\right)}{f}+\frac{b^2 e}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2*(a+b*sin(f*x+e))^2,x)
```

```
[Out] b^2*x-a^2*cot(f*x+e)/f+2/f*a*b*ln(csc(f*x+e)-cot(f*x+e))+1/f*b^2*e
```

Maxima [A] time = 1.65617, size = 70, normalized size = 2.06

$$\frac{\left(fx+e\right)b^2-ab\left(\log \left(\cos \left(fx+e\right)+1\right)-\log \left(\cos \left(fx+e\right)-1\right)\right)-\frac{a^2}{\tan\left(fx+e\right)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] ((f*x + e)*b^2 - a*b*(log(cos(f*x + e) + 1) - log(cos(f*x + e) - 1)) - a^2/
tan(f*x + e))/f
```

Fricas [B] time = 1.99554, size = 209, normalized size = 6.15

$$\frac{b^2 fx \sin \left(fx+e\right)-ab \log \left(\frac{1}{2}\cos \left(fx+e\right)+\frac{1}{2}\right)\sin \left(fx+e\right)+ab \log \left(-\frac{1}{2}\cos \left(fx+e\right)+\frac{1}{2}\right)\sin \left(fx+e\right)-a^2 \cos \left(fx+e\right)}{f \sin \left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] (b^2*f*x*sin(f*x + e) - a*b*log(1/2*cos(f*x + e) + 1/2)*sin(f*x + e) + a*b*
log(-1/2*cos(f*x + e) + 1/2)*sin(f*x + e) - a^2*cos(f*x + e))/(f*sin(f*x +
e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a+b\sin\left(e+fx\right)\right)^2\csc^2\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2*(a+b*sin(f*x+e))**2,x)
```

```
[Out] Integral((a + b*sin(e + f*x))**2*csc(e + f*x)**2, x)
```

Giac [B] time = 1.75793, size = 107, normalized size = 3.15

$$\frac{2\left(fx+e\right)b^2+4\,ab\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right|\right)+a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-\frac{4\,ab\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+a^2}{\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*(f*x + e)*b^2 + 4*a*b*log(abs(tan(1/2*f*x + 1/2*e))) + a^2*tan(1/2*f
*x + 1/2*e) - (4*a*b*tan(1/2*f*x + 1/2*e) + a^2)/tan(1/2*f*x + 1/2*e))/f
```

3.163 $\int \csc^3(e+fx)(a+b\sin(e+fx))^2\,dx$

Optimal. Leaf size=59

$$-\frac{(a^2+2b^2)\tanh^{-1}(\cos(e+fx))}{2f}-\frac{a^2\cot(e+fx)\csc(e+fx)}{2f}-\frac{2ab\cot(e+fx)}{f}$$

```
[Out] -((a^2 + 2*b^2)*ArcTanh[Cos[e + f*x]])/(2*f) - (2*a*b*Cot[e + f*x])/f - (a^
2*Cot[e + f*x]*Csc[e + f*x])/(2*f)
```

Rubi [A] time = 0.0756538, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2789, 3767, 8, 3012, 3770}

$$-\frac{(a^2+2b^2)\tanh^{-1}(\cos(e+fx))}{2f}-\frac{a^2\cot(e+fx)\csc(e+fx)}{2f}-\frac{2ab\cot(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3*(a + b*Sin[e + f*x])^2,x]
```

```
[Out] -((a^2 + 2*b^2)*ArcTanh[Cos[e + f*x]])/(2*f) - (2*a*b*Cot[e + f*x])/f - (a^
2*Cot[e + f*x]*Csc[e + f*x])/(2*f)
```

Rule 2789

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)])^2, x_Symbol] :> Dist[(2*c*d)/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] +
 Int[(b*Sin[e + f*x])^m*(c^2 + d^2*Sin[e + f*x]^2), x] /; FreeQ[{b, c, d, e
, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3012

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[(A*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m
 + 1)), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc^3(e+fx)(a+b\sin(e+fx))^2\,dx = (2ab)\int \csc^2(e+fx)\,dx + \int \csc^3(e+fx)\left(a^2+b^2\sin^2(e+fx)\right)dx$$
$$= -\frac{a^2\cot(e+fx)\csc(e+fx)}{2f} + \frac{1}{2}\left(a^2+2b^2\right)\int \csc(e+fx)\,dx - \frac{(2ab)\,\text{Subst}(}{}$$
$$= -\frac{\left(a^2+2b^2\right)\tanh^{-1}(\cos(e+fx))}{2f} - \frac{2ab\cot(e+fx)}{f} - \frac{a^2\cot(e+fx)\csc(e+}{2f}$$

Mathematica [B] time = 0.458003, size = 133, normalized size = 2.25

$$\frac{a^2\left(-\csc^2\left(\frac{1}{2}(e+fx)\right)\right)+a^2\sec^2\left(\frac{1}{2}(e+fx)\right)+4a^2\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)-4a^2\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)+8ab\tan\left(\frac{1}{2}(e+\right.}{8f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^3*(a + b*Sin[e + f*x])^2,x]
```

```
[Out] (-8*a*b*Cot[(e + f*x)/2] - a^2*Csc[(e + f*x)/2]^2 - 4*a^2*Log[Cos[(e + f*x)
/2]] - 8*b^2*Log[Cos[(e + f*x)/2]] + 4*a^2*Log[Sin[(e + f*x)/2]] + 8*b^2*Lo
g[Sin[(e + f*x)/2]] + a^2*Sec[(e + f*x)/2]^2 + 8*a*b*Tan[(e + f*x)/2])/(8*f
)
```

Maple [A] time = 0.052, size = 82, normalized size = 1.4

$$-\frac{a^2\cot(fx+e)\csc(fx+e)}{2f}+\frac{a^2\ln(\csc(fx+e)-\cot(fx+e))}{2f}-2\frac{ab\cot(fx+e)}{f}+\frac{b^2\ln(\csc(fx+e)-\cot(f}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3*(a+b*sin(f*x+e))^2,x)
```

```
[Out] -1/2*a^2*cot(f*x+e)*csc(f*x+e)/f+1/2/f*a^2*ln(csc(f*x+e)-cot(f*x+e))-2*a*b*
cot(f*x+e)/f+1/f*b^2*ln(csc(f*x+e)-cot(f*x+e))
```

Maxima [A] time = 1.973, size = 120, normalized size = 2.03

$$\frac{a^2\left(\frac{2\cos(fx+e)}{\cos(fx+e)^2-1}-\log(\cos(fx+e)+1)+\log(\cos(fx+e)-1)\right)-2b^2(\log(\cos(fx+e)+1)-\log(\cos(fx+e)-1}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/4*(a^2*(2*cos(f*x + e)/(cos(f*x + e)^2 - 1) - log(cos(f*x + e) + 1) + log
(cos(f*x + e) - 1)) - 2*b^2*(log(cos(f*x + e) + 1) - log(cos(f*x + e) - 1))
 - 8*a*b/tan(f*x + e))/f
```

Fricas [B] time = 1.92625, size = 316, normalized size = 5.36

$$\frac{8\,ab\cos\left(fx+e\right)\sin\left(fx+e\right)+2\,a^2\cos\left(fx+e\right)-\left(\left(a^2+2\,b^2\right)\cos\left(fx+e\right)^2-a^2-2\,b^2\right)\log\left(\frac{1}{2}\cos\left(fx+e\right)+\frac{1}{2}\right)+\left(\left(a^2\right.\right.}{4\left(f\cos\left(fx+e\right)^2-f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/4*(8*a*b*cos(f*x + e)*sin(f*x + e) + 2*a^2*cos(f*x + e) - ((a^2 + 2*b^2)*
cos(f*x + e)^2 - a^2 - 2*b^2)*log(1/2*cos(f*x + e) + 1/2) + ((a^2 + 2*b^2)*
cos(f*x + e)^2 - a^2 - 2*b^2)*log(-1/2*cos(f*x + e) + 1/2))/(f*cos(f*x + e)
^2 - f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3*(a+b*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.66352, size = 169, normalized size = 2.86

$$\frac{a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+8\,ab\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+4\left(a^2+2\,b^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right|\right)-\frac{6\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+12\,b^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+}{\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/8*(a^2*tan(1/2*f*x + 1/2*e)^2 + 8*a*b*tan(1/2*f*x + 1/2*e) + 4*(a^2 + 2*b
^2)*log(abs(tan(1/2*f*x + 1/2*e))) - (6*a^2*tan(1/2*f*x + 1/2*e)^2 + 12*b^2
*tan(1/2*f*x + 1/2*e)^2 + 8*a*b*tan(1/2*f*x + 1/2*e) + a^2)/tan(1/2*f*x + 1
/2*e)^2)/f
```

3.164 $\int \csc^4(e+fx)(a+b\sin(e+fx))^2\,dx$

Optimal. Leaf size=82

$$-\frac{(2a^2+3b^2)\cot(e+fx)}{3f}-\frac{a^2\cot(e+fx)\csc^2(e+fx)}{3f}-\frac{ab\tanh^{-1}(\cos(e+fx))}{f}-\frac{ab\cot(e+fx)\csc(e+fx)}{f}$$

```
[Out] -((a*b*ArcTanh[Cos[e + f*x]])/f) - ((2*a^2 + 3*b^2)*Cot[e + f*x])/(3*f) - (
a*b*Cot[e + f*x]*Csc[e + f*x])/f - (a^2*Cot[e + f*x]*Csc[e + f*x]^2)/(3*f)
```

Rubi [A] time = 0.0872102, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2789, 3768, 3770, 3012, 3767, 8}

$$-\frac{(2a^2+3b^2)\cot(e+fx)}{3f}-\frac{a^2\cot(e+fx)\csc^2(e+fx)}{3f}-\frac{ab\tanh^{-1}(\cos(e+fx))}{f}-\frac{ab\cot(e+fx)\csc(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^4*(a + b*Sin[e + f*x])^2,x]
```

```
[Out] -((a*b*ArcTanh[Cos[e + f*x]])/f) - ((2*a^2 + 3*b^2)*Cot[e + f*x])/(3*f) - (
a*b*Cot[e + f*x]*Csc[e + f*x])/f - (a^2*Cot[e + f*x]*Csc[e + f*x]^2)/(3*f)
```

Rule 2789

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)])^2, x_Symbol] :> Dist[(2*c*d)/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] +
 Int[(b*Sin[e + f*x])^m*(c^2 + d^2*Sin[e + f*x]^2), x] /; FreeQ[{b, c, d, e
, f, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3012

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[(A*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m
 + 1)), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \csc^4(e+fx)(a+b\sin(e+fx))^2\,dx = (2ab)\int \csc^3(e+fx)\,dx + \int \csc^4(e+fx)\left(a^2+b^2\sin^2(e+fx)\right)dx$$
$$= -\frac{ab\cot(e+fx)\csc(e+fx)}{f} - \frac{a^2\cot(e+fx)\csc^2(e+fx)}{3f} + (ab)\int \csc(e+fx)$$
$$= -\frac{ab\tanh^{-1}(\cos(e+fx))}{f} - \frac{ab\cot(e+fx)\csc(e+fx)}{f} - \frac{a^2\cot(e+fx)\csc^2(e+}{3f}$$
$$= -\frac{ab\tanh^{-1}(\cos(e+fx))}{f} - \frac{\left(2a^2+3b^2\right)\cot(e+fx)}{3f} - \frac{ab\cot(e+fx)\csc(e+fx}{f}$$

Mathematica [A] time = 0.0392186, size = 132, normalized size = 1.61

$$-\frac{2a^2\cot(e+fx)}{3f} - \frac{a^2\cot(e+fx)\csc^2(e+fx)}{3f} - \frac{ab\csc^2\left(\frac{1}{2}(e+fx)\right)}{4f} + \frac{ab\sec^2\left(\frac{1}{2}(e+fx)\right)}{4f} + \frac{ab\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^4*(a + b*Sin[e + f*x])^2,x]
```

```
[Out] (-2*a^2*Cot[e + f*x])/(3*f) - (b^2*Cot[e + f*x])/f - (a*b*Csc[(e + f*x)/2]^
2)/(4*f) - (a^2*Cot[e + f*x]*Csc[e + f*x]^2)/(3*f) - (a*b*Log[Cos[(e + f*x)
/2]])/f + (a*b*Log[Sin[(e + f*x)/2]])/f + (a*b*Sec[(e + f*x)/2]^2)/(4*f)
```

Maple [A] time = 0.052, size = 93, normalized size = 1.1

$$-\frac{2a^2\cot(fx+e)}{3f} - \frac{a^2\cot(fx+e)\left(\csc(fx+e)\right)^2}{3f} - \frac{ab\cot(fx+e)\csc(fx+e)}{f} + \frac{ab\ln\left(\csc(fx+e)-\cot(fx+e)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^4*(a+b*sin(f*x+e))^2,x)
```

```
[Out] -2/3*a^2*cot(f*x+e)/f-1/3*a^2*cot(f*x+e)*csc(f*x+e)^2/f-a*b*cot(f*x+e)*csc(
f*x+e)/f+1/f*a*b*ln(csc(f*x+e)-cot(f*x+e))-1/f*b^2*cot(f*x+e)
```

Maxima [A] time = 1.99123, size = 120, normalized size = 1.46

$$\frac{3ab\left(\frac{2\cos(fx+e)}{\cos(fx+e)^2-1} - \log\left(\cos(fx+e)+1\right) + \log\left(\cos(fx+e)-1\right)\right) - \frac{6b^2}{\tan(fx+e)} - \frac{2\left(3\tan(fx+e)^2+1\right)a^2}{\tan(fx+e)^3}}{6f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/6*(3*a*b*(2*cos(f*x + e)/(cos(f*x + e)^2 - 1) - log(cos(f*x + e) + 1) + l
og(cos(f*x + e) - 1)) - 6*b^2/tan(f*x + e) - 2*(3*tan(f*x + e)^2 + 1)*a^2/t
an(f*x + e)^3)/f
```

Fricas [A] time = 1.89331, size = 387, normalized size = 4.72

$$-\frac{2\left(2a^2+3b^2\right)\cos\left(fx+e\right)^3-6ab\cos\left(fx+e\right)\sin\left(fx+e\right)+3\left(ab\cos\left(fx+e\right)^2-ab\right)\log\left(\frac{1}{2}\cos\left(fx+e\right)+\frac{1}{2}\right)\sin}{6\left(f\cos\left(fx+e\right)^2-f\right)\mathrm{si}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -1/6*(2*(2*a^2 + 3*b^2)*cos(f*x + e)^3 - 6*a*b*cos(f*x + e)*sin(f*x + e) +
3*(a*b*cos(f*x + e)^2 - a*b)*log(1/2*cos(f*x + e) + 1/2)*sin(f*x + e) - 3*(
a*b*cos(f*x + e)^2 - a*b)*log(-1/2*cos(f*x + e) + 1/2)*sin(f*x + e) - 6*(a^
2 + b^2)*cos(f*x + e))/((f*cos(f*x + e)^2 - f)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**4*(a+b*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.85136, size = 224, normalized size = 2.73

$$\frac{a^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+6ab\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+24ab\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right|\right)+9a^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+12b^2\tan\left(\frac{1}{2}f\right.}{24f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/24*(a^2*tan(1/2*f*x + 1/2*e)^3 + 6*a*b*tan(1/2*f*x + 1/2*e)^2 + 24*a*b*lo
g(abs(tan(1/2*f*x + 1/2*e))) + 9*a^2*tan(1/2*f*x + 1/2*e) + 12*b^2*tan(1/2*
f*x + 1/2*e) - (44*a*b*tan(1/2*f*x + 1/2*e)^3 + 9*a^2*tan(1/2*f*x + 1/2*e)^
2 + 12*b^2*tan(1/2*f*x + 1/2*e)^2 + 6*a*b*tan(1/2*f*x + 1/2*e) + a^2)/tan(1
/2*f*x + 1/2*e)^3)/f
```

3.165 $\int \csc^5(e+fx)(a+b\sin(e+fx))^2\,dx$

Optimal. Leaf size=110

$$-\frac{(3a^2+4b^2)\tanh^{-1}(\cos(e+fx))}{8f}-\frac{(3a^2+4b^2)\cot(e+fx)\csc(e+fx)}{8f}-\frac{a^2\cot(e+fx)\csc^3(e+fx)}{4f}-\frac{2ab\cot^3(e+f}{3f}$$

```
[Out] -((3*a^2 + 4*b^2)*ArcTanh[Cos[e + f*x]])/(8*f) - (2*a*b*Cot[e + f*x])/f - (
2*a*b*Cot[e + f*x]^3)/(3*f) - ((3*a^2 + 4*b^2)*Cot[e + f*x]*Csc[e + f*x])/(
8*f) - (a^2*Cot[e + f*x]*Csc[e + f*x]^3)/(4*f)
```

Rubi [A] time = 0.0939383, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2789, 3767, 3012, 3768, 3770}

$$-\frac{(3a^2+4b^2)\tanh^{-1}(\cos(e+fx))}{8f}-\frac{(3a^2+4b^2)\cot(e+fx)\csc(e+fx)}{8f}-\frac{a^2\cot(e+fx)\csc^3(e+fx)}{4f}-\frac{2ab\cot^3(e+f}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^5*(a + b*Sin[e + f*x])^2,x]
```


```
[Out] -((3*a^2 + 4*b^2)*ArcTanh[Cos[e + f*x]])/(8*f) - (2*a*b*Cot[e + f*x])/f - (
2*a*b*Cot[e + f*x]^3)/(3*f) - ((3*a^2 + 4*b^2)*Cot[e + f*x]*Csc[e + f*x])/(
8*f) - (a^2*Cot[e + f*x]*Csc[e + f*x]^3)/(4*f)
```


Rule 2789

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)])^2, x_Symbol] :> Dist[(2*c*d)/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] +
 Int[(b*Sin[e + f*x])^m*(c^2 + d^2*Sin[e + f*x]^2), x] /; FreeQ[{b, c, d, e
, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3012

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[(A*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m
 + 1)), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc^5(e+fx)(a+b\sin(e+fx))^2\,dx = (2ab)\int \csc^4(e+fx)\,dx + \int \csc^5(e+fx)\left(a^2+b^2\sin^2(e+fx)\right)\,dx$$
$$= -\frac{a^2\cot(e+fx)\csc^3(e+fx)}{4f} + \frac{1}{4}\left(3a^2+4b^2\right)\int \csc^3(e+fx)\,dx - \frac{(2ab)\,\text{Sub}}{}$$
$$= -\frac{2ab\cot(e+fx)}{f} - \frac{2ab\cot^3(e+fx)}{3f} - \frac{\left(3a^2+4b^2\right)\cot(e+fx)\csc(e+fx)}{8f}$$
$$= -\frac{\left(3a^2+4b^2\right)\tanh^{-1}(\cos(e+fx))}{8f} - \frac{2ab\cot(e+fx)}{f} - \frac{2ab\cot^3(e+fx)}{3f} - \left(\right.$$

Mathematica [B] time = 0.0412662, size = 255, normalized size = 2.32

$$-\frac{a^2\csc^4\left(\frac{1}{2}(e+fx)\right)}{64f} - \frac{3a^2\csc^2\left(\frac{1}{2}(e+fx)\right)}{32f} + \frac{a^2\sec^4\left(\frac{1}{2}(e+fx)\right)}{64f} + \frac{3a^2\sec^2\left(\frac{1}{2}(e+fx)\right)}{32f} + \frac{3a^2\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right.}{8f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[e + f*x]^5*(a + b*Sin[e + f*x])^2,x]
```

```
[Out] (-4*a*b*Cot[e + f*x])/(3*f) - (3*a^2*Csc[(e + f*x)/2]^2)/(32*f) - (b^2*Csc[
(e + f*x)/2]^2)/(8*f) - (a^2*Csc[(e + f*x)/2]^4)/(64*f) - (2*a*b*Cot[e + f*
x]*Csc[e + f*x]^2)/(3*f) - (3*a^2*Log[Cos[(e + f*x)/2]])/(8*f) - (b^2*Log[C
os[(e + f*x)/2]])/(2*f) + (3*a^2*Log[Sin[(e + f*x)/2]])/(8*f) + (b^2*Log[Si
n[(e + f*x)/2]])/(2*f) + (3*a^2*Sec[(e + f*x)/2]^2)/(32*f) + (b^2*Sec[(e +
f*x)/2]^2)/(8*f) + (a^2*Sec[(e + f*x)/2]^4)/(64*f)
```

Maple [A] time = 0.111, size = 146, normalized size = 1.3

$$-\frac{a^2\cot(fx+e)\left(\csc(fx+e)\right)^3}{4f} - \frac{3a^2\cot(fx+e)\csc(fx+e)}{8f} + \frac{3a^2\ln\left(\csc(fx+e)-\cot(fx+e)\right)}{8f} - \frac{4ab\cot(f}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^5*(a+b*sin(f*x+e))^2,x)
```

```
[Out] -1/4*a^2*cot(f*x+e)*csc(f*x+e)^3/f-3/8*a^2*cot(f*x+e)*csc(f*x+e)/f+3/8/f*a^
2*ln(csc(f*x+e)-cot(f*x+e))-4/3*a*b*cot(f*x+e)/f-2/3/f*a*b*cot(f*x+e)*csc(f
*x+e)^2-1/2/f*b^2*cot(f*x+e)*csc(f*x+e)+1/2/f*b^2*ln(csc(f*x+e)-cot(f*x+e))
```

Maxima [A] time = 1.99729, size = 198, normalized size = 1.8

$$\frac{3a^2\left(\frac{2\left(3\cos(fx+e)^3-5\cos(fx+e)\right)}{\cos(fx+e)^4-2\cos(fx+e)^2+1} - 3\log\left(\cos(fx+e)+1\right) + 3\log\left(\cos(fx+e)-1\right)\right) + 12b^2\left(\frac{2\cos(fx+e)}{\cos(fx+e)^2-1} - \log\left(\cos(f\right.\right.}{48f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/48*(3*a^2*(2*(3*cos(f*x + e)^3 - 5*cos(f*x + e))/(cos(f*x + e)^4 - 2*cos(
f*x + e)^2 + 1) - 3*log(cos(f*x + e) + 1) + 3*log(cos(f*x + e) - 1)) + 12*b
^2*(2*cos(f*x + e)/(cos(f*x + e)^2 - 1) - log(cos(f*x + e) + 1) + log(cos(f
*x + e) - 1)) - 32*(3*tan(f*x + e)^2 + 1)*a*b/tan(f*x + e)^3)/f
```

Fricas [B] time = 1.66156, size = 558, normalized size = 5.07

$$\frac{6\left(3a^2+4b^2\right)\cos\left(fx+e\right)^3-6\left(5a^2+4b^2\right)\cos\left(fx+e\right)-3\left(\left(3a^2+4b^2\right)\cos\left(fx+e\right)^4-2\left(3a^2+4b^2\right)\cos\left(fx+e\right)^2\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/48*(6*(3*a^2 + 4*b^2)*cos(f*x + e)^3 - 6*(5*a^2 + 4*b^2)*cos(f*x + e) - 3
*((3*a^2 + 4*b^2)*cos(f*x + e)^4 - 2*(3*a^2 + 4*b^2)*cos(f*x + e)^2 + 3*a^2
 + 4*b^2)*log(1/2*cos(f*x + e) + 1/2) + 3*((3*a^2 + 4*b^2)*cos(f*x + e)^4 -
 2*(3*a^2 + 4*b^2)*cos(f*x + e)^2 + 3*a^2 + 4*b^2)*log(-1/2*cos(f*x + e) +
1/2) + 32*(2*a*b*cos(f*x + e)^3 - 3*a*b*cos(f*x + e))*sin(f*x + e))/(f*cos(
f*x + e)^4 - 2*f*cos(f*x + e)^2 + f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**5*(a+b*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.82501, size = 311, normalized size = 2.83

$$\frac{3a^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+16ab\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+24a^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+24b^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+144ab\tan\left(\frac{1}{2}fx+\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/192*(3*a^2*tan(1/2*f*x + 1/2*e)^4 + 16*a*b*tan(1/2*f*x + 1/2*e)^3 + 24*a^
2*tan(1/2*f*x + 1/2*e)^2 + 24*b^2*tan(1/2*f*x + 1/2*e)^2 + 144*a*b*tan(1/2*
f*x + 1/2*e) + 24*(3*a^2 + 4*b^2)*log(abs(tan(1/2*f*x + 1/2*e))) - (150*a^2
*tan(1/2*f*x + 1/2*e)^4 + 200*b^2*tan(1/2*f*x + 1/2*e)^4 + 144*a*b*tan(1/2*
```

```
f*x + 1/2*e)^3 + 24*a^2*tan(1/2*f*x + 1/2*e)^2 + 24*b^2*tan(1/2*f*x + 1/2*e
)^2 + 16*a*b*tan(1/2*f*x + 1/2*e) + 3*a^2)/tan(1/2*f*x + 1/2*e)^4)/f
```

3.166 $\int \sin^3(e+fx)(a+b\sin(e+fx))^3\,dx$

Optimal. Leaf size=171

$$\frac{a\left(a^2+6b^2\right)\cos^3(e+fx)}{3f}-\frac{a\left(a^2+3b^2\right)\cos(e+fx)}{f}-\frac{b\left(18a^2+5b^2\right)\sin^3(e+fx)\cos(e+fx)}{24f}-\frac{b\left(18a^2+5b^2\right)\sin(e+$$

```
[Out] (b*(18*a^2 + 5*b^2)*x)/16 - (a*(a^2 + 3*b^2)*Cos[e + f*x])/f + (a*(a^2 + 6*
b^2)*Cos[e + f*x]^3)/(3*f) - (3*a*b^2*Cos[e + f*x]^5)/(5*f) - (b*(18*a^2 +
5*b^2)*Cos[e + f*x]*Sin[e + f*x])/(16*f) - (b*(18*a^2 + 5*b^2)*Cos[e + f*x]
*Sin[e + f*x]^3)/(24*f) - (b^3*Cos[e + f*x]*Sin[e + f*x]^5)/(6*f)
```

Rubi [A] time = 0.209467, antiderivative size = 193, normalized size of antiderivative = 1.13, number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2793, 3023, 2748, 2633, 2635, 8}

$$\frac{a\left(5a^2+12b^2\right)\cos^3(e+fx)}{15f}-\frac{a\left(5a^2+12b^2\right)\cos(e+fx)}{5f}-\frac{b\left(18a^2+5b^2\right)\sin^3(e+fx)\cos(e+fx)}{24f}-\frac{b\left(18a^2+5b^2\right)\sin($$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^3*(a + b*Sin[e + f*x])^3,x]
```

```
[Out] (b*(18*a^2 + 5*b^2)*x)/16 - (a*(5*a^2 + 12*b^2)*Cos[e + f*x])/(5*f) + (a*(5
*a^2 + 12*b^2)*Cos[e + f*x]^3)/(15*f) - (b*(18*a^2 + 5*b^2)*Cos[e + f*x]*Si
n[e + f*x])/(16*f) - (b*(18*a^2 + 5*b^2)*Cos[e + f*x]*Sin[e + f*x]^3)/(24*f
) - (13*a*b^2*Cos[e + f*x]*Sin[e + f*x]^4)/(30*f) - (b^2*Cos[e + f*x]*Sin[e
 + f*x]^4*(a + b*Sin[e + f*x]))/(6*f)
```

Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \sin^3(e+fx)(a+b\sin(e+fx))^3\,dx = -\frac{b^2\cos(e+fx)\sin^4(e+fx)(a+b\sin(e+fx))}{6f}+\frac{1}{6}\int \sin^3(e+fx)\left(2a\left(3a^2\right.\right.$$

$$= -\frac{13ab^2\cos(e+fx)\sin^4(e+fx)}{30f}-\frac{b^2\cos(e+fx)\sin^4(e+fx)(a+b\sin(e+$$

$$= -\frac{13ab^2\cos(e+fx)\sin^4(e+fx)}{30f}-\frac{b^2\cos(e+fx)\sin^4(e+fx)(a+b\sin(e+$$

$$= -\frac{b\left(18a^2+5b^2\right)\cos(e+fx)\sin^3(e+fx)}{24f}-\frac{13ab^2\cos(e+fx)\sin^4(e+fx)}{30f}-$$

$$= -\frac{a\left(5a^2+12b^2\right)\cos(e+fx)}{5f}+\frac{a\left(5a^2+12b^2\right)\cos^3(e+fx)}{15f}-\frac{b\left(18a^2+5b^2\right)}{}$$

$$= \frac{1}{16}b\left(18a^2+5b^2\right)x-\frac{a\left(5a^2+12b^2\right)\cos(e+fx)}{5f}+\frac{a\left(5a^2+12b^2\right)\cos^3(e+f}{15f}$$

Mathematica [A] time = 0.700237, size = 147, normalized size = 0.86

$$\frac{-360a\left(2a^2+5b^2\right)\cos(e+fx)+20\left(4a^3+15ab^2\right)\cos(3(e+fx))+b\left(5\left(-9\left(16a^2+5b^2\right)\sin(2(e+fx))+9\left(2a^2+b^2\right.\right.\right.}{960f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^3*(a + b*Sin[e + f*x])^3,x]
```

```
[Out] (-360*a*(2*a^2 + 5*b^2)*Cos[e + f*x] + 20*(4*a^3 + 15*a*b^2)*Cos[3*(e + f*x
)] + b*(-36*a*b*Cos[5*(e + f*x)] + 5*(216*a^2*e + 60*b^2*e + 216*a^2*f*x +
60*b^2*f*x - 9*(16*a^2 + 5*b^2)*Sin[2*(e + f*x)] + 9*(2*a^2 + b^2)*Sin[4*(e
 + f*x)] - b^2*Sin[6*(e + f*x)])))/(960*f)
```

Maple [A] time = 0.025, size = 145, normalized size = 0.9

$$\frac{1}{f}\left(b^3\left(-\frac{\cos(fx+e)}{6}\left(\left(\sin(fx+e)\right)^5+\frac{5\left(\sin(fx+e)\right)^3}{4}+\frac{15\sin(fx+e)}{8}\right)+\frac{5fx}{16}+\frac{5e}{16}\right)-\frac{3ab^2\cos(fx+e)}{5}\left(\frac{8}{3}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3*(a+b*sin(f*x+e))^3,x)
```

```
[Out] 1/f*(b^3*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+e)+5
/16*f*x+5/16*e)-3/5*a*b^2*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)+3*
a^2*b*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-1/3*a^3
*(2+sin(f*x+e)^2)*cos(f*x+e))
```

Maxima [A] time = 2.55476, size = 196, normalized size = 1.15

$$320\left(\cos\left(fx+e\right)^3-3\cos\left(fx+e\right)\right)a^3+90\left(12fx+12e+\sin\left(4fx+4e\right)-8\sin\left(2fx+2e\right)\right)a^2b-192\left(3\cos\left(fx+e\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+b*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 1/960*(320*(cos(f*x + e)^3 - 3*cos(f*x + e))*a^3 + 90*(12*f*x + 12*e + sin(
4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*a^2*b - 192*(3*cos(f*x + e)^5 - 10*cos(f
*x + e)^3 + 15*cos(f*x + e))*a*b^2 + 5*(4*sin(2*f*x + 2*e)^3 + 60*f*x + 60*
e + 9*sin(4*f*x + 4*e) - 48*sin(2*f*x + 2*e))*b^3)/f
```

Fricas [A] time = 1.73995, size = 340, normalized size = 1.99

$$-\frac{144\,ab^2\cos\left(fx+e\right)^5-80\left(a^3+6\,ab^2\right)\cos\left(fx+e\right)^3-15\left(18\,a^2b+5\,b^3\right)fx+240\left(a^3+3\,ab^2\right)\cos\left(fx+e\right)+5\left(8\,b^3\cos\right.}{240\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+b*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -1/240*(144*a*b^2*cos(f*x + e)^5 - 80*(a^3 + 6*a*b^2)*cos(f*x + e)^3 - 15*(
18*a^2*b + 5*b^3)*f*x + 240*(a^3 + 3*a*b^2)*cos(f*x + e) + 5*(8*b^3*cos(f*x
 + e)^5 - 2*(18*a^2*b + 13*b^3)*cos(f*x + e)^3 + 3*(30*a^2*b + 11*b^3)*cos(
f*x + e))*sin(f*x + e))/f
```

Sympy [A] time = 6.07667, size = 393, normalized size = 2.3

$$\begin{cases} -\frac{a^3\sin^2\left(e+fx\right)\cos\left(e+fx\right)}{f}-\frac{2a^3\cos^3\left(e+fx\right)}{3f}+\frac{9a^2bx\sin^4\left(e+fx\right)}{8}+\frac{9a^2bx\sin^2\left(e+fx\right)\cos^2\left(e+fx\right)}{4}+\frac{9a^2bx\cos^4\left(e+fx\right)}{8}-\frac{15a^2b\sin^3\left(e+fx\right)\cos\left(e+\right.}{8f} \\ x\left(a+b\sin\left(e\right)\right)^3\sin^3\left(e\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3*(a+b*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((-a**3*sin(e + f*x)**2*cos(e + f*x)/f - 2*a**3*cos(e + f*x)**3/(3
*f) + 9*a**2*b*x*sin(e + f*x)**4/8 + 9*a**2*b*x*sin(e + f*x)**2*cos(e + f*x
)**2/4 + 9*a**2*b*x*cos(e + f*x)**4/8 - 15*a**2*b*sin(e + f*x)**3*cos(e + f
```

```
*x)/(8*f) - 9*a**2*b*sin(e + f*x)*cos(e + f*x)**3/(8*f) - 3*a*b**2*sin(e +
f*x)**4*cos(e + f*x)/f - 4*a*b**2*sin(e + f*x)**2*cos(e + f*x)**3/f - 8*a*b
**2*cos(e + f*x)**5/(5*f) + 5*b**3*x*sin(e + f*x)**6/16 + 15*b**3*x*sin(e +
 f*x)**4*cos(e + f*x)**2/16 + 15*b**3*x*sin(e + f*x)**2*cos(e + f*x)**4/16
+ 5*b**3*x*cos(e + f*x)**6/16 - 11*b**3*sin(e + f*x)**5*cos(e + f*x)/(16*f)
 - 5*b**3*sin(e + f*x)**3*cos(e + f*x)**3/(6*f) - 5*b**3*sin(e + f*x)*cos(e
 + f*x)**5/(16*f), Ne(f, 0)), (x*(a + b*sin(e))**3*sin(e)**3, True))
```

Giac [A] time = 2.21464, size = 243, normalized size = 1.42

$$-\frac{3\,ab^2\cos\left(5\,fx+5\,e\right)}{80\,f}-\frac{b^3\sin\left(6\,fx+6\,e\right)}{192\,f}+\frac{1}{16}\left(18\,a^2b+5\,b^3\right)x+\frac{\left(4\,a^3+15\,ab^2\right)\cos\left(3\,fx+3\,e\right)}{48\,f}-\frac{\left(2\,a^3+9\,ab^2\right)\,\cdots}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+b*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -3/80*a*b^2*cos(5*f*x + 5*e)/f - 1/192*b^3*sin(6*f*x + 6*e)/f + 1/16*(18*a^
2*b + 5*b^3)*x + 1/48*(4*a^3 + 15*a*b^2)*cos(3*f*x + 3*e)/f - 1/8*(2*a^3 +
9*a*b^2)*cos(f*x + e)/f - 1/4*(2*a^3 + 3*a*b^2)*cos(f*x + e)/f + 3/64*(2*a^
2*b + b^3)*sin(4*f*x + 4*e)/f - 3/64*(16*a^2*b + 5*b^3)*sin(2*f*x + 2*e)/f
```

3.167 $\int \sin^2(e+fx)(a+b\sin(e+fx))^3\,dx$

Optimal. Leaf size=160

$$\frac{b\left(15a^2+4b^2\right)\cos^3(e+fx)}{15f}-\frac{b\left(15a^2+4b^2\right)\cos(e+fx)}{5f}-\frac{a\left(4a^2+9b^2\right)\sin(e+fx)\cos(e+fx)}{8f}+\frac{1}{8}ax\left(4a^2+9b^2\right)-\frac{11}{}$$

```
[Out] (a*(4*a^2 + 9*b^2)*x)/8 - (b*(15*a^2 + 4*b^2)*Cos[e + f*x])/(5*f) + (b*(15*
a^2 + 4*b^2)*Cos[e + f*x]^3)/(15*f) - (a*(4*a^2 + 9*b^2)*Cos[e + f*x]*Sin[e
 + f*x])/(8*f) - (11*a*b^2*Cos[e + f*x]*Sin[e + f*x]^3)/(20*f) - (b^2*Cos[e
 + f*x]*Sin[e + f*x]^3*(a + b*Sin[e + f*x]))/(5*f)
```

Rubi [A] time = 0.215738, antiderivative size = 180, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2791, 2753, 2734}

$$\frac{\left(-52a^2b^2+3a^4-16b^4\right)\cos(e+fx)}{30bf}+\frac{\left(3a^2-16b^2\right)\cos(e+fx)(a+b\sin(e+fx))^2}{60bf}+\frac{a\left(6a^2-71b^2\right)\sin(e+fx)\cos(e+}{120f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^2*(a + b*Sin[e + f*x])^3,x]
```

```
[Out] (a*(4*a^2 + 9*b^2)*x)/8 + ((3*a^4 - 52*a^2*b^2 - 16*b^4)*Cos[e + f*x])/(30*
b*f) + (a*(6*a^2 - 71*b^2)*Cos[e + f*x]*Sin[e + f*x])/(120*f) + ((3*a^2 - 1
6*b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^2)/(60*b*f) + (a*Cos[e + f*x]*(a +
 b*Sin[e + f*x])^3)/(20*b*f) - (Cos[e + f*x]*(a + b*Sin[e + f*x])^4)/(5*b*f
)
```

Rule 2791

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> -Simp[(d^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*
Simp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x],
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a
^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \sin^2(e+fx)(a+b\sin(e+fx))^3\,dx = -\frac{\cos(e+fx)(a+b\sin(e+fx))^4}{5bf} + \frac{\int (4b-a\sin(e+fx))(a+b\sin(e+fx))^3}{5b}$$
$$= \frac{a\cos(e+fx)(a+b\sin(e+fx))^3}{20bf} - \frac{\cos(e+fx)(a+b\sin(e+fx))^4}{5bf} + \frac{\int (a+}{}$$
$$= \frac{\left(3a^2-16b^2\right)\cos(e+fx)(a+b\sin(e+fx))^2}{60bf} + \frac{a\cos(e+fx)(a+b\sin(e+f}{20bf}$$
$$= \frac{1}{8}a\left(4a^2+9b^2\right)x + \frac{\left(3a^4-52a^2b^2-16b^4\right)\cos(e+fx)}{30bf} + \frac{a\left(6a^2-71b^2\right)\cos(e}{120}$$

Mathematica [A] time = 0.635079, size = 117, normalized size = 0.73

$$\frac{15a\left(4\left(4a^2+9b^2\right)(e+fx)-8\left(a^2+3b^2\right)\sin(2(e+fx))+3b^2\sin(4(e+fx))\right)-60b\left(18a^2+5b^2\right)\cos(e+fx)+10\left(1\right.}{480f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^2*(a + b*Sin[e + f*x])^3,x]
```

```
[Out] (-60*b*(18*a^2 + 5*b^2)*Cos[e + f*x] + 10*(12*a^2*b + 5*b^3)*Cos[3*(e + f*x
)] - 6*b^3*Cos[5*(e + f*x)] + 15*a*(4*(4*a^2 + 9*b^2)*(e + f*x) - 8*(a^2 +
3*b^2)*Sin[2*(e + f*x)] + 3*b^2*Sin[4*(e + f*x)]))/(480*f)
```

Maple [A] time = 0.027, size = 124, normalized size = 0.8

$$\frac{1}{f}\left(-\frac{b^3\cos\left(fx+e\right)}{5}\left(\frac{8}{3}+\left(\sin\left(fx+e\right)\right)^4+\frac{4\left(\sin\left(fx+e\right)\right)^2}{3}\right)+3\,ab^2\left(-1/4\left(\left(\sin\left(fx+e\right)\right)^3+3/2\sin\left(fx+e\right)\right)\cos\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2*(a+b*sin(f*x+e))^3,x)
```

```
[Out] 1/f*(-1/5*b^3*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)+3*a*b^2*(-1/4*
(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-a^2*b*(2+sin(f*x+e)
^2)*cos(f*x+e)+a^3*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e))
```

Maxima [A] time = 1.73668, size = 163, normalized size = 1.02

$$\frac{120\left(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right)\right)a^3+480\left(\cos\left(fx+e\right)^3-3\,\cos\left(fx+e\right)\right)a^2b+45\left(12\,fx+12\,e+\sin\left(4\,fx+4\,e\right)-\right.}{480\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(a+b*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 1/480*(120*(2*f*x + 2*e - sin(2*f*x + 2*e))*a^3 + 480*(cos(f*x + e)^3 - 3*c
os(f*x + e))*a^2*b + 45*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2
*e))*a*b^2 - 32*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*b^
```

3)/f

Fricas [A] time = 1.67325, size = 285, normalized size = 1.78

$$-\frac{24\,b^3\cos\left(fx+e\right)^5-40\left(3\,a^2b+2\,b^3\right)\cos\left(fx+e\right)^3-15\left(4\,a^3+9\,ab^2\right)fx+120\left(3\,a^2b+b^3\right)\cos\left(fx+e\right)-15\left(6\,ab^2\cos\ldots\right.}{120\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(a+b*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -1/120*(24*b^3*cos(f*x + e)^5 - 40*(3*a^2*b + 2*b^3)*cos(f*x + e)^3 - 15*(4
*a^3 + 9*a*b^2)*f*x + 120*(3*a^2*b + b^3)*cos(f*x + e) - 15*(6*a*b^2*cos(f*
x + e)^3 - (4*a^3 + 15*a*b^2)*cos(f*x + e))*sin(f*x + e))/f
```

Sympy [A] time = 3.52925, size = 284, normalized size = 1.78

$$\begin{cases}\frac{a^3x\sin^2(e+fx)}{2}+\frac{a^3x\cos^2(e+fx)}{2}-\frac{a^3\sin(e+fx)\cos(e+fx)}{2f}-\frac{3a^2b\sin^2(e+fx)\cos(e+fx)}{f}-\frac{2a^2b\cos^3(e+fx)}{f}+\frac{9ab^2x\sin^4(e+fx)}{8}+\frac{9ab^2x\sin^2(\ldots}{\ldots} \\ x\left(a+b\sin(e)\right)^3\sin^2(e)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2*(a+b*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((a**3*x*sin(e + f*x)**2/2 + a**3*x*cos(e + f*x)**2/2 - a**3*sin(e
 + f*x)*cos(e + f*x)/(2*f) - 3*a**2*b*sin(e + f*x)**2*cos(e + f*x)/f - 2*a*
*2*b*cos(e + f*x)**3/f + 9*a*b**2*x*sin(e + f*x)**4/8 + 9*a*b**2*x*sin(e +
f*x)**2*cos(e + f*x)**2/4 + 9*a*b**2*x*cos(e + f*x)**4/8 - 15*a*b**2*sin(e
+ f*x)**3*cos(e + f*x)/(8*f) - 9*a*b**2*sin(e + f*x)*cos(e + f*x)**3/(8*f)
- b**3*sin(e + f*x)**4*cos(e + f*x)/f - 4*b**3*sin(e + f*x)**2*cos(e + f*x)
**3/(3*f) - 8*b**3*cos(e + f*x)**5/(15*f), Ne(f, 0)), (x*(a + b*sin(e))**3*
sin(e)**2, True))
```

Giac [A] time = 1.71505, size = 174, normalized size = 1.09

$$-\frac{b^3\cos\left(5\,fx+5\,e\right)}{80\,f}+\frac{3\,ab^2\sin\left(4\,fx+4\,e\right)}{32\,f}+\frac{1}{8}\left(4\,a^3+9\,ab^2\right)x+\frac{\left(12\,a^2b+5\,b^3\right)\cos\left(3\,fx+3\,e\right)}{48\,f}-\frac{\left(18\,a^2b+5\,b^3\right)\cos\left(\ldots\right)}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(a+b*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -1/80*b^3*cos(5*f*x + 5*e)/f + 3/32*a*b^2*sin(4*f*x + 4*e)/f + 1/8*(4*a^3 +
 9*a*b^2)*x + 1/48*(12*a^2*b + 5*b^3)*cos(3*f*x + 3*e)/f - 1/8*(18*a^2*b +
5*b^3)*cos(f*x + e)/f - 1/4*(a^3 + 3*a*b^2)*sin(2*f*x + 2*e)/f
```

3.168 $\int \sin(e+fx)(a+b\sin(e+fx))^3\,dx$

Optimal. Leaf size=121

$$-\frac{a\left(a^2+4b^2\right)\cos(e+fx)}{2f}-\frac{b\left(2a^2+3b^2\right)\sin(e+fx)\cos(e+fx)}{8f}+\frac{3}{8}bx\left(4a^2+b^2\right)-\frac{\cos(e+fx)(a+b\sin(e+fx))^3}{4f}$$

```
[Out] (3*b*(4*a^2 + b^2)*x)/8 - (a*(a^2 + 4*b^2)*Cos[e + f*x])/(2*f) - (b*(2*a^2
+ 3*b^2)*Cos[e + f*x]*Sin[e + f*x])/(8*f) - (a*Cos[e + f*x]*(a + b*Sin[e +
f*x])^2)/(4*f) - (Cos[e + f*x]*(a + b*Sin[e + f*x])^3)/(4*f)
```

Rubi [A] time = 0.11403, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2753, 2734}

$$-\frac{a\left(a^2+4b^2\right)\cos(e+fx)}{2f}-\frac{b\left(2a^2+3b^2\right)\sin(e+fx)\cos(e+fx)}{8f}+\frac{3}{8}bx\left(4a^2+b^2\right)-\frac{\cos(e+fx)(a+b\sin(e+fx))^3}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]*(a + b*Sin[e + f*x])^3,x]
```

```
[Out] (3*b*(4*a^2 + b^2)*x)/8 - (a*(a^2 + 4*b^2)*Cos[e + f*x])/(2*f) - (b*(2*a^2
+ 3*b^2)*Cos[e + f*x]*Sin[e + f*x])/(8*f) - (a*Cos[e + f*x]*(a + b*Sin[e +
f*x])^2)/(4*f) - (Cos[e + f*x]*(a + b*Sin[e + f*x])^3)/(4*f)
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \sin(e+fx)(a+b\sin(e+fx))^3\,dx=-\frac{\cos(e+fx)(a+b\sin(e+fx))^3}{4f}+\frac{1}{4}\int(3b+3a\sin(e+fx))(a+b\sin(e+fx$$

$$=-\frac{a\cos(e+fx)(a+b\sin(e+fx))^2}{4f}-\frac{\cos(e+fx)(a+b\sin(e+fx))^3}{4f}+\frac{1}{12}\int($$

$$=\frac{3}{8}b\left(4a^2+b^2\right)x-\frac{a\left(a^2+4b^2\right)\cos(e+fx)}{2f}-\frac{b\left(2a^2+3b^2\right)\cos(e+fx)\sin(e+$$

Mathematica [A] time = 0.338307, size = 100, normalized size = 0.83

$$\frac{b\left(-8\left(3a^2+b^2\right)\sin(2(e+fx))+48a^2e+48a^2fx+8ab\cos(3(e+fx))+b^2\sin(4(e+fx))+12b^2e+12b^2fx\right)-8a\left(4\right.}{32f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]*(a + b*Sin[e + f*x])^3,x]
```

```
[Out] (-8*a*(4*a^2 + 9*b^2)*Cos[e + f*x] + b*(48*a^2*e + 12*b^2*e + 48*a^2*f*x +
12*b^2*f*x + 8*a*b*Cos[3*(e + f*x)] - 8*(3*a^2 + b^2)*Sin[2*(e + f*x)] + b^
2*Sin[4*(e + f*x)]))/(32*f)
```

Maple [A] time = 0.022, size = 104, normalized size = 0.9

$$\frac{1}{f}\Bigg(b^3\Bigg(-\frac{\cos(fx+e)}{4}\Big((\sin(fx+e))^3+\frac{3\sin(fx+e)}{2}\Big)+\frac{3fx}{8}+\frac{3e}{8}\Bigg)-ab^2\big(2+(\sin(fx+e))^2\big)\cos(fx+e)+3a^2b(\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)*(a+b*sin(f*x+e))^3,x)
```

```
[Out] 1/f*(b^3*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-a*b^
2*(2+sin(f*x+e)^2)*cos(f*x+e)+3*a^2*b*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1
/2*e)-a^3*cos(f*x+e))
```

Maxima [A] time = 1.42315, size = 131, normalized size = 1.08

$$\frac{24\,(2\,fx+2\,e-\sin(2\,fx+2\,e))a^2b+32\,\big(\cos(fx+e)^3-3\,\cos(fx+e)\big)ab^2+\big(12\,fx+12\,e+\sin(4\,fx+4\,e)-8\,\sin(\cdots}{32\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 1/32*(24*(2*f*x + 2*e - sin(2*f*x + 2*e))*a^2*b + 32*(cos(f*x + e)^3 - 3*co
s(f*x + e))*a*b^2 + (12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))
*b^3 - 32*a^3*cos(f*x + e))/f
```

Fricas [A] time = 1.6102, size = 217, normalized size = 1.79

$$\frac{8\,ab^2\cos(fx+e)^3+3\,(4\,a^2b+b^3)fx-8\,(a^3+3\,ab^2)\cos(fx+e)+\big(2\,b^3\cos(fx+e)^3-(12\,a^2b+5\,b^3)\cos(fx+e)\big)\,\mathrm{s}\cdots}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/8*(8*a*b^2*cos(f*x + e)^3 + 3*(4*a^2*b + b^3)*f*x - 8*(a^3 + 3*a*b^2)*cos
(f*x + e) + (2*b^3*cos(f*x + e)^3 - (12*a^2*b + 5*b^3)*cos(f*x + e))*sin(f*
x + e))/f
```

Sympy [A] time = 2.01214, size = 233, normalized size = 1.93

$$\begin{cases} -\frac{a^3\cos(e+fx)}{f} + \frac{3a^2bx\sin^2(e+fx)}{2} + \frac{3a^2bx\cos^2(e+fx)}{2} - \frac{3a^2b\sin(e+fx)\cos(e+fx)}{2f} - \frac{3ab^2\sin^2(e+fx)\cos(e+fx)}{f} - \frac{2ab^2\cos^3(e+fx)}{f} + \frac{3b^3 \\ x(a+b\sin(e))^3\sin(e) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((-a**3*cos(e + f*x)/f + 3*a**2*b*x*sin(e + f*x)**2/2 + 3*a**2*b*x
*cos(e + f*x)**2/2 - 3*a**2*b*sin(e + f*x)*cos(e + f*x)/(2*f) - 3*a*b**2*si
n(e + f*x)**2*cos(e + f*x)/f - 2*a*b**2*cos(e + f*x)**3/f + 3*b**3*x*sin(e
+ f*x)**4/8 + 3*b**3*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 3*b**3*x*cos(e +
 f*x)**4/8 - 5*b**3*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 3*b**3*sin(e + f*x
)*cos(e + f*x)**3/(8*f), Ne(f, 0)), (x*(a + b*sin(e))**3*sin(e), True))
```

Giac [A] time = 1.6002, size = 157, normalized size = 1.3

$$\frac{ab^2\cos(3fx+3e)}{4f} - \frac{3ab^2\cos(fx+e)}{4f} + \frac{b^3\sin(4fx+4e)}{32f} + \frac{3}{8}\left(4a^2b+b^3\right)x - \frac{\left(2a^3+3ab^2\right)\cos(fx+e)}{2f} - \frac{(3a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/4*a*b^2*cos(3*f*x + 3*e)/f - 3/4*a*b^2*cos(f*x + e)/f + 1/32*b^3*sin(4*f*
x + 4*e)/f + 3/8*(4*a^2*b + b^3)*x - 1/2*(2*a^3 + 3*a*b^2)*cos(f*x + e)/f -
 1/4*(3*a^2*b + b^3)*sin(2*f*x + 2*e)/f
```

3.169 $\int (a+b\sin(e+fx))^3\,dx$

Optimal. Leaf size=90

$$-\frac{2b\left(4a^2+b^2\right)\cos(e+fx)}{3f}+\frac{1}{2}ax\left(2a^2+3b^2\right)-\frac{5ab^2\sin(e+fx)\cos(e+fx)}{6f}-\frac{b\cos(e+fx)(a+b\sin(e+fx))^2}{3f}$$

```
[Out] (a*(2*a^2 + 3*b^2)*x)/2 - (2*b*(4*a^2 + b^2)*Cos[e + f*x])/(3*f) - (5*a*b^2
*Cos[e + f*x]*Sin[e + f*x])/(6*f) - (b*Cos[e + f*x]*(a + b*Sin[e + f*x])^2)
/(3*f)
```

Rubi [A] time = 0.0656265, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2656, 2734}

$$-\frac{2b\left(4a^2+b^2\right)\cos(e+fx)}{3f}+\frac{1}{2}ax\left(2a^2+3b^2\right)-\frac{5ab^2\sin(e+fx)\cos(e+fx)}{6f}-\frac{b\cos(e+fx)(a+b\sin(e+fx))^2}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^3,x]
```

```
[Out] (a*(2*a^2 + 3*b^2)*x)/2 - (2*b*(4*a^2 + b^2)*Cos[e + f*x])/(3*f) - (5*a*b^2
*Cos[e + f*x]*Sin[e + f*x])/(6*f) - (b*Cos[e + f*x]*(a + b*Sin[e + f*x])^2)
/(3*f)
```

Rule 2656

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[1/n, Int[(a + b*Sin
[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x], x
], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] && In
tegerQ[2*n]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int (a+b\sin(e+fx))^3\,dx=-\frac{b\cos(e+fx)(a+b\sin(e+fx))^2}{3f}+\frac{1}{3}\int (a+b\sin(e+fx))\left(3a^2+2b^2+5ab\sin(e+fx)\right)\,d$$
$$=\frac{1}{2}a\left(2a^2+3b^2\right)x-\frac{2b\left(4a^2+b^2\right)\cos(e+fx)}{3f}-\frac{5ab^2\cos(e+fx)\sin(e+fx)}{6f}-\frac{b\cos(e+fx)(}{}$$

Mathematica [A] time = 0.157422, size = 71, normalized size = 0.79

$$\frac{6a\left(2a^2+3b^2\right)(e+fx)-9b\left(4a^2+b^2\right)\cos(e+fx)-9ab^2\sin(2(e+fx))+b^3\cos(3(e+fx))}{12f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^3,x]
```

```
[Out] (6*a*(2*a^2 + 3*b^2)*(e + f*x) - 9*b*(4*a^2 + b^2)*Cos[e + f*x] + b^3*Cos[3
*(e + f*x)] - 9*a*b^2*Sin[2*(e + f*x)])/(12*f)
```

Maple [A] time = 0.022, size = 76, normalized size = 0.8

$$\frac{1}{f}\left(-\frac{b^3\left(2+\left(\sin\left(fx+e\right)\right)^2\right)\cos\left(fx+e\right)}{3}+3\,ab^2\left(-1/2\,\sin\left(fx+e\right)\cos\left(fx+e\right)+1/2\,fx+e/2\right)-3\,a^2b\cos\left(fx+e\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^3,x)
```

```
[Out] 1/f*(-1/3*b^3*(2+sin(f*x+e)^2)*cos(f*x+e)+3*a*b^2*(-1/2*sin(f*x+e)*cos(f*x+
e)+1/2*f*x+1/2*e)-3*a^2*b*cos(f*x+e)+(f*x+e)*a^3)
```

Maxima [A] time = 1.96226, size = 100, normalized size = 1.11

$$a^3x+\frac{3\left(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right)\right)ab^2}{4\,f}+\frac{\left(\cos\left(fx+e\right)^3-3\,\cos\left(fx+e\right)\right)b^3}{3\,f}-\frac{3\,a^2b\cos\left(fx+e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] a^3*x + 3/4*(2*f*x + 2*e - sin(2*f*x + 2*e))*a*b^2/f + 1/3*(cos(f*x + e)^3
- 3*cos(f*x + e))*b^3/f - 3*a^2*b*cos(f*x + e)/f
```

Fricas [A] time = 1.74884, size = 169, normalized size = 1.88

$$\frac{2\,b^3\cos\left(fx+e\right)^3-9\,ab^2\cos\left(fx+e\right)\sin\left(fx+e\right)+3\left(2\,a^3+3\,ab^2\right)fx-6\left(3\,a^2b+b^3\right)\cos\left(fx+e\right)}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^3*cos(f*x + e)^3 - 9*a*b^2*cos(f*x + e)*sin(f*x + e) + 3*(2*a^3 +
3*a*b^2)*f*x - 6*(3*a^2*b + b^3)*cos(f*x + e))/f
```

Sympy [A] time = 1.32388, size = 128, normalized size = 1.42

$$\begin{cases} a^3x-\frac{3a^2b\cos\left(e+fx\right)}{f}+\frac{3ab^2x\sin^2\left(e+fx\right)}{2}+\frac{3ab^2x\cos^2\left(e+fx\right)}{2}-\frac{3ab^2\sin\left(e+fx\right)\cos\left(e+fx\right)}{2f}-\frac{b^3\sin^2\left(e+fx\right)\cos\left(e+fx\right)}{f}-\frac{2b^3\cos^3\left(e+fx\right)}{3f} \\ x\left(a+b\sin\left(e\right)\right)^3 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((a**3*x - 3*a**2*b*cos(e + f*x)/f + 3*a*b**2*x*sin(e + f*x)**2/2
+ 3*a*b**2*x*cos(e + f*x)**2/2 - 3*a*b**2*sin(e + f*x)*cos(e + f*x)/(2*f) -
 b**3*sin(e + f*x)**2*cos(e + f*x)/f - 2*b**3*cos(e + f*x)**3/(3*f), Ne(f,
0)), (x*(a + b*sin(e))**3, True))
```

Giac [A] time = 1.71127, size = 101, normalized size = 1.12

$$\frac{b^3\cos\left(3\,fx+3\,e\right)}{12\,f}-\frac{3\,ab^2\sin\left(2\,fx+2\,e\right)}{4\,f}+\frac{1}{2}\left(2\,a^3+3\,ab^2\right)x-\frac{3\left(4\,a^2b+b^3\right)\cos\left(fx+e\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/12*b^3*cos(3*f*x + 3*e)/f - 3/4*a*b^2*sin(2*f*x + 2*e)/f + 1/2*(2*a^3 + 3
*a*b^2)*x - 3/4*(4*a^2*b + b^3)*cos(f*x + e)/f
```

3.170 $\int \csc(e+fx)(a+b\sin(e+fx))^3\,dx$

Optimal. Leaf size=74

$$\frac{1}{2}bx\left(6a^2+b^2\right)-\frac{a^3\tanh^{-1}(\cos(e+fx))}{f}-\frac{5ab^2\cos(e+fx)}{2f}-\frac{b^2\cos(e+fx)(a+b\sin(e+fx))}{2f}$$

```
[Out] (b*(6*a^2 + b^2)*x)/2 - (a^3*ArcTanh[Cos[e + f*x]])/f - (5*a*b^2*Cos[e + f*
x])/(2*f) - (b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]))/(2*f)
```

Rubi [A] time = 0.114637, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2793, 3023, 2735, 3770}

$$\frac{1}{2}bx\left(6a^2+b^2\right)-\frac{a^3\tanh^{-1}(\cos(e+fx))}{f}-\frac{5ab^2\cos(e+fx)}{2f}-\frac{b^2\cos(e+fx)(a+b\sin(e+fx))}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]*(a + b*Sin[e + f*x])^3,x]
```

```
[Out] (b*(6*a^2 + b^2)*x)/2 - (a^3*ArcTanh[Cos[e + f*x]])/f - (5*a*b^2*Cos[e + f*
x])/(2*f) - (b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]))/(2*f)
```

Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc(e+fx)(a+b\sin(e+fx))^3\,dx = -\frac{b^2\cos(e+fx)(a+b\sin(e+fx))}{2f}+\frac{1}{2}\int \csc(e+fx)\left(2a^3+b\left(6a^2+b^2\right)\sin(e+\right.$$
$$= -\frac{5ab^2\cos(e+fx)}{2f}-\frac{b^2\cos(e+fx)(a+b\sin(e+fx))}{2f}+\frac{1}{2}\int \csc(e+fx)\left(2a^3+\right.$$
$$= \frac{1}{2}b\left(6a^2+b^2\right)x-\frac{5ab^2\cos(e+fx)}{2f}-\frac{b^2\cos(e+fx)(a+b\sin(e+fx))}{2f}+a^3\int \mathrm{cs}$$
$$= \frac{1}{2}b\left(6a^2+b^2\right)x-\frac{a^3\tanh^{-1}(\cos(e+fx))}{f}-\frac{5ab^2\cos(e+fx)}{2f}-\frac{b^2\cos(e+fx)(a}{2}$$

Mathematica [A] time = 0.159826, size = 81, normalized size = 1.09

$$-\frac{-2b\left(6a^2+b^2\right)(e+fx)-4a^3\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)+4a^3\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)+12ab^2\cos(e+fx)+b^3\sin(2(e+fx))}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]*(a + b*Sin[e + f*x])^3,x]
```

```
[Out] -(-2*b*(6*a^2 + b^2)*(e + f*x) + 12*a*b^2*Cos[e + f*x] + 4*a^3*Log[Cos[(e +
 f*x)/2]] - 4*a^3*Log[Sin[(e + f*x)/2]] + b^3*Sin[2*(e + f*x)])/(4*f)
```

Maple [A] time = 0.056, size = 92, normalized size = 1.2

$$\frac{a^3\ln\left(\csc\left(fx+e\right)-\cot\left(fx+e\right)\right)}{f}+3\,a^2bx+3\,\frac{a^2be}{f}-3\,\frac{ab^2\cos\left(fx+e\right)}{f}-\frac{b^3\sin\left(fx+e\right)\cos\left(fx+e\right)}{2\,f}+\frac{b^3x}{2}+\frac{b^3e}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)*(a+b*sin(f*x+e))^3,x)
```

```
[Out] 1/f*a^3*ln(csc(f*x+e)-cot(f*x+e))+3*a^2*b*x+3/f*a^2*b*e-3*a*b^2*cos(f*x+e)/
f-1/2/f*b^3*sin(f*x+e)*cos(f*x+e)+1/2*b^3*x+1/2/f*b^3*e
```

Maxima [A] time = 1.72161, size = 96, normalized size = 1.3

$$\frac{12\left(fx+e\right)a^2b+\left(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right)\right)b^3-12\,ab^2\cos\left(fx+e\right)-4\,a^3\log\left(\cot\left(fx+e\right)+\csc\left(fx+e\right)\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 1/4*(12*(f*x + e)*a^2*b + (2*f*x + 2*e - sin(2*f*x + 2*e))*b^3 - 12*a*b^2*c
os(f*x + e) - 4*a^3*log(cot(f*x + e) + csc(f*x + e)))/f
```

Fricas [A] time = 1.85242, size = 208, normalized size = 2.81

$$-\frac{b^3\cos\left(fx+e\right)\sin\left(fx+e\right)+6\,ab^2\cos\left(fx+e\right)+a^3\log\left(\frac{1}{2}\cos\left(fx+e\right)+\frac{1}{2}\right)-a^3\log\left(-\frac{1}{2}\cos\left(fx+e\right)+\frac{1}{2}\right)-\left(6\,a\right.}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -1/2*(b^3*cos(f*x + e)*sin(f*x + e) + 6*a*b^2*cos(f*x + e) + a^3*log(1/2*co
s(f*x + e) + 1/2) - a^3*log(-1/2*cos(f*x + e) + 1/2) - (6*a^2*b + b^3)*f*x)
/f
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\sin\left(e + fx\right)\right)^3 \csc\left(e + fx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sin(f*x+e))**3,x)
```

```
[Out] Integral((a + b*sin(e + f*x))**3*csc(e + f*x), x)
```

Giac [A] time = 2.25302, size = 154, normalized size = 2.08

$$\frac{2\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right|\right)+\left(6\,a^2b+b^3\right)\left(fx+e\right)+\frac{2\left(b^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-6\,ab^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-b^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-6\,ab^2\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*a^3*log(abs(tan(1/2*f*x + 1/2*e))) + (6*a^2*b + b^3)*(f*x + e) + 2*(
b^3*tan(1/2*f*x + 1/2*e)^3 - 6*a*b^2*tan(1/2*f*x + 1/2*e)^2 - b^3*tan(1/2*f
*x + 1/2*e) - 6*a*b^2)/(tan(1/2*f*x + 1/2*e)^2 + 1)^2)/f
```

3.171 $\int \csc^2(e+fx)(a+b\sin(e+fx))^3\,dx$

Optimal. Leaf size=68

$$\frac{b\left(a^2-b^2\right)\cos(e+fx)}{f}-\frac{3a^2b\tanh^{-1}(\cos(e+fx))}{f}-\frac{a^2\cot(e+fx)(a+b\sin(e+fx))}{f}+3ab^2x$$

```
[Out] 3*a*b^2*x - (3*a^2*b*ArcTanh[Cos[e + f*x]])/f + (b*(a^2 - b^2)*Cos[e + f*x]
)/f - (a^2*Cot[e + f*x]*(a + b*Sin[e + f*x]))/f
```

Rubi [A] time = 0.119885, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2792, 3023, 2735, 3770}

$$\frac{b\left(a^2-b^2\right)\cos(e+fx)}{f}-\frac{3a^2b\tanh^{-1}(\cos(e+fx))}{f}-\frac{a^2\cot(e+fx)(a+b\sin(e+fx))}{f}+3ab^2x$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2*(a + b*Sin[e + f*x])^3,x]
```

```
[Out] 3*a*b^2*x - (3*a^2*b*ArcTanh[Cos[e + f*x]])/f + (b*(a^2 - b^2)*Cos[e + f*x]
)/f - (a^2*Cot[e + f*x]*(a + b*Sin[e + f*x]))/f
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc^2(e+fx)(a+b\sin(e+fx))^3\,dx = -\frac{a^2\cot(e+fx)(a+b\sin(e+fx))}{f} + \int \csc(e+fx)\left(3a^2b+3ab^2\sin(e+fx)\right.$$
$$= \frac{b\left(a^2-b^2\right)\cos(e+fx)}{f} - \frac{a^2\cot(e+fx)(a+b\sin(e+fx))}{f} + \int \csc(e+fx)$$
$$= 3ab^2x + \frac{b\left(a^2-b^2\right)\cos(e+fx)}{f} - \frac{a^2\cot(e+fx)(a+b\sin(e+fx))}{f} + \left(3a^2b\right)$$
$$= 3ab^2x - \frac{3a^2b\tanh^{-1}(\cos(e+fx))}{f} + \frac{b\left(a^2-b^2\right)\cos(e+fx)}{f} - \frac{a^2\cot(e+fx}{}$$

Mathematica [A] time = 0.517057, size = 87, normalized size = 1.28

$$\frac{a^3\tan\left(\frac{1}{2}(e+fx)\right)+a^3\left(-\cot\left(\frac{1}{2}(e+fx)\right)\right)+6ab\left(a\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)-a\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)+b(e+fx)\right)-2b}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2*(a + b*Sin[e + f*x])^3,x]
```

```
[Out] (-2*b^3*Cos[e + f*x] - a^3*Cot[(e + f*x)/2] + 6*a*b*(b*(e + f*x) - a*Log[Co
s[(e + f*x)/2]] + a*Log[Sin[(e + f*x)/2]]) + a^3*Tan[(e + f*x)/2])/(2*f)
```

Maple [A] time = 0.052, size = 72, normalized size = 1.1

$$3\,ab^2x-\frac{a^3\cot\left(fx+e\right)}{f}-\frac{b^3\cos\left(fx+e\right)}{f}+3\,\frac{a^2b\ln\left(\csc\left(fx+e\right)-\cot\left(fx+e\right)\right)}{f}+3\,\frac{ab^2e}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2*(a+b*sin(f*x+e))^3,x)
```

```
[Out] 3*a*b^2*x-1/f*a^3*cot(f*x+e)-1/f*b^3*cos(f*x+e)+3/f*a^2*b*ln(csc(f*x+e)-cot
(f*x+e))+3/f*a*b^2*e
```

Maxima [A] time = 1.65011, size = 92, normalized size = 1.35

$$\frac{6\left(fx+e\right)ab^2-3\,a^2b\left(\log\left(\cos\left(fx+e\right)+1\right)-\log\left(\cos\left(fx+e\right)-1\right)\right)-2\,b^3\cos\left(fx+e\right)-\frac{2\,a^3}{\tan\left(fx+e\right)}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(6*(f*x + e)*a*b^2 - 3*a^2*b*(log(cos(f*x + e) + 1) - log(cos(f*x + e)
- 1)) - 2*b^3*cos(f*x + e) - 2*a^3/tan(f*x + e))/f
```

Fricas [A] time = 1.73625, size = 266, normalized size = 3.91

$$-\frac{3\,a^2b\log\left(\frac{1}{2}\,\cos\left(fx+e\right)+\frac{1}{2}\right)\sin\left(fx+e\right)-3\,a^2b\log\left(-\frac{1}{2}\,\cos\left(fx+e\right)+\frac{1}{2}\right)\sin\left(fx+e\right)+2\,a^3\cos\left(fx+e\right)-2\left(3\,ab^2\right.}{2\,f\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -1/2*(3*a^2*b*log(1/2*cos(f*x + e) + 1/2)*sin(f*x + e) - 3*a^2*b*log(-1/2*c
os(f*x + e) + 1/2)*sin(f*x + e) + 2*a^3*cos(f*x + e) - 2*(3*a*b^2*f*x - b^3
*cos(f*x + e))*sin(f*x + e))/(f*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2*(a+b*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 2.33008, size = 197, normalized size = 2.9

$$\frac{6\left(fx+e\right)ab^2+6\,a^2b\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right|\right)+a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-\frac{2\,a^2b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+2\,a^2b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+4}{\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/2*(6*(f*x + e)*a*b^2 + 6*a^2*b*log(abs(tan(1/2*f*x + 1/2*e))) + a^3*tan(1
/2*f*x + 1/2*e) - (2*a^2*b*tan(1/2*f*x + 1/2*e)^3 + a^3*tan(1/2*f*x + 1/2*e
)^2 + 2*a^2*b*tan(1/2*f*x + 1/2*e) + 4*b^3*tan(1/2*f*x + 1/2*e) + a^3)/(tan
(1/2*f*x + 1/2*e)^3 + tan(1/2*f*x + 1/2*e)))/f
```

3.172 $\int \csc^3(e+fx)(a+b\sin(e+fx))^3\,dx$

Optimal. Leaf size=79

$$-\frac{a\left(a^2+6b^2\right)\tanh^{-1}(\cos(e+fx))}{2f}-\frac{5a^2b\cot(e+fx)}{2f}-\frac{a^2\cot(e+fx)\csc(e+fx)(a+b\sin(e+fx))}{2f}+b^3x$$

```
[Out] b^3*x - (a*(a^2 + 6*b^2)*ArcTanh[Cos[e + f*x]])/(2*f) - (5*a^2*b*Cot[e + f*
x])/(2*f) - (a^2*Cot[e + f*x]*Csc[e + f*x]*(a + b*Sin[e + f*x]))/(2*f)
```

Rubi [A] time = 0.133165, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2792, 3021, 2735, 3770}

$$-\frac{a\left(a^2+6b^2\right)\tanh^{-1}(\cos(e+fx))}{2f}-\frac{5a^2b\cot(e+fx)}{2f}-\frac{a^2\cot(e+fx)\csc(e+fx)(a+b\sin(e+fx))}{2f}+b^3x$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3*(a + b*Sin[e + f*x])^3,x]
```

```
[Out] b^3*x - (a*(a^2 + 6*b^2)*ArcTanh[Cos[e + f*x]])/(2*f) - (5*a^2*b*Cot[e + f*
x])/(2*f) - (a^2*Cot[e + f*x]*Csc[e + f*x]*(a + b*Sin[e + f*x]))/(2*f)
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc^3(e+fx)(a+b\sin(e+fx))^3\,dx = -\frac{a^2\cot(e+fx)\csc(e+fx)(a+b\sin(e+fx))}{2f}+\frac{1}{2}\int \csc^2(e+fx)\left(5a^2b+a\left(a^2\right.\right.$$

$$= -\frac{5a^2b\cot(e+fx)}{2f}-\frac{a^2\cot(e+fx)\csc(e+fx)(a+b\sin(e+fx))}{2f}+\frac{1}{2}\int \csc(e$$

$$= b^3x-\frac{5a^2b\cot(e+fx)}{2f}-\frac{a^2\cot(e+fx)\csc(e+fx)(a+b\sin(e+fx))}{2f}+\frac{1}{2}\left(a\left(a\right.\right.$$

$$= b^3x-\frac{a\left(a^2+6b^2\right)\tanh^{-1}(\cos(e+fx))}{2f}-\frac{5a^2b\cot(e+fx)}{2f}-\frac{a^2\cot(e+fx)\csc}{}$$

Mathematica [A] time = 0.64197, size = 152, normalized size = 1.92

$$\frac{12a^2b\tan\left(\frac{1}{2}(e+fx)\right)-12a^2b\cot\left(\frac{1}{2}(e+fx)\right)+a^3\left(-\csc^2\left(\frac{1}{2}(e+fx)\right)\right)+a^3\sec^2\left(\frac{1}{2}(e+fx)\right)+4a^3\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right.}{8f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^3*(a + b*Sin[e + f*x])^3,x]
```

```
[Out] (8*b^3*e + 8*b^3*f*x - 12*a^2*b*Cot[(e + f*x)/2] - a^3*Csc[(e + f*x)/2]^2 -
 4*a^3*Log[Cos[(e + f*x)/2]] - 24*a*b^2*Log[Cos[(e + f*x)/2]] + 4*a^3*Log[S
in[(e + f*x)/2]] + 24*a*b^2*Log[Sin[(e + f*x)/2]] + a^3*Sec[(e + f*x)/2]^2
+ 12*a^2*b*Tan[(e + f*x)/2])/(8*f)
```

Maple [A] time = 0.063, size = 99, normalized size = 1.3

$$-\frac{a^3\csc\left(fx+e\right)\cot\left(fx+e\right)}{2f}+\frac{a^3\ln\left(\csc\left(fx+e\right)-\cot\left(fx+e\right)\right)}{2f}-3\frac{a^2b\cot\left(fx+e\right)}{f}+3\frac{ab^2\ln\left(\csc\left(fx+e\right)-\cot\left(f\right.\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3*(a+b*sin(f*x+e))^3,x)
```

```
[Out] -1/2/f*a^3*csc(f*x+e)*cot(f*x+e)+1/2/f*a^3*ln(csc(f*x+e)-cot(f*x+e))-3*a^2*
b*cot(f*x+e)/f+3/f*a*b^2*ln(csc(f*x+e)-cot(f*x+e))+b^3*x+1/f*b^3*e
```

Maxima [A] time = 2.37047, size = 138, normalized size = 1.75

$$\frac{4\left(fx+e\right)b^3+a^3\left(\frac{2\cos\left(fx+e\right)}{\cos\left(fx+e\right)^2-1}-\log\left(\cos\left(fx+e\right)+1\right)+\log\left(\cos\left(fx+e\right)-1\right)\right)-6ab^2\left(\log\left(\cos\left(fx+e\right)+1\right)-\log\left(\cos\right.\right.}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 1/4*(4*(f*x + e)*b^3 + a^3*(2*cos(f*x + e)/(cos(f*x + e)^2 - 1) - log(cos(f
*x + e) + 1) + log(cos(f*x + e) - 1)) - 6*a*b^2*(log(cos(f*x + e) + 1) - lo
g(cos(f*x + e) - 1)) - 12*a^2*b/tan(f*x + e))/f
```

Fricas [B] time = 1.72809, size = 383, normalized size = 4.85

$$\frac{4\,b^3 fx\cos\left(fx+e\right)^2 - 4\,b^3 fx + 12\,a^2 b\cos\left(fx+e\right)\sin\left(fx+e\right) + 2\,a^3\cos\left(fx+e\right) + \left(a^3+6\,ab^2 - \left(a^3+6\,ab^2\right)\cos\left(\right.\right.}{4\left(f\cos\left(fx+e\right)^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(4*b^3*f*x*cos(f*x + e)^2 - 4*b^3*f*x + 12*a^2*b*cos(f*x + e)*sin(f*x +
 e) + 2*a^3*cos(f*x + e) + (a^3 + 6*a*b^2 - (a^3 + 6*a*b^2)*cos(f*x + e)^2)
*log(1/2*cos(f*x + e) + 1/2) - (a^3 + 6*a*b^2 - (a^3 + 6*a*b^2)*cos(f*x + e
)^2)*log(-1/2*cos(f*x + e) + 1/2))/(f*cos(f*x + e)^2 - f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3*(a+b*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 2.01914, size = 192, normalized size = 2.43

$$\frac{a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2 + 8\left(fx+e\right)b^3 + 12\,a^2 b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right) + 4\left(a^3+6\,ab^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right|\right) - \frac{6\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\right.}{}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/8*(a^3*tan(1/2*f*x + 1/2*e)^2 + 8*(f*x + e)*b^3 + 12*a^2*b*tan(1/2*f*x +
1/2*e) + 4*(a^3 + 6*a*b^2)*log(abs(tan(1/2*f*x + 1/2*e))) - (6*a^3*tan(1/2*
f*x + 1/2*e)^2 + 36*a*b^2*tan(1/2*f*x + 1/2*e)^2 + 12*a^2*b*tan(1/2*f*x + 1
/2*e) + a^3)/tan(1/2*f*x + 1/2*e)^2)/f
```

3.173 $\int \csc^4(e+fx)(a+b\sin(e+fx))^3\,dx$

Optimal. Leaf size=109

$$-\frac{a\left(2a^2+9b^2\right)\cot(e+fx)}{3f}-\frac{b\left(3a^2+2b^2\right)\tanh^{-1}(\cos(e+fx))}{2f}-\frac{7a^2b\cot(e+fx)\csc(e+fx)}{6f}-\frac{a^2\cot(e+fx)\csc^2(e\cdots}{\cdots}$$

```
[Out] -(b*(3*a^2 + 2*b^2)*ArcTanh[Cos[e + f*x]])/(2*f) - (a*(2*a^2 + 9*b^2)*Cot[e
 + f*x])/(3*f) - (7*a^2*b*Cot[e + f*x]*Csc[e + f*x])/(6*f) - (a^2*Cot[e + f
*x]*Csc[e + f*x]^2*(a + b*Sin[e + f*x]))/(3*f)
```

Rubi [A] time = 0.181392, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2792, 3021, 2748, 3767, 8, 3770}

$$-\frac{a\left(2a^2+9b^2\right)\cot(e+fx)}{3f}-\frac{b\left(3a^2+2b^2\right)\tanh^{-1}(\cos(e+fx))}{2f}-\frac{7a^2b\cot(e+fx)\csc(e+fx)}{6f}-\frac{a^2\cot(e+fx)\csc^2(e\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^4*(a + b*Sin[e + f*x])^3,x]
```


```
[Out] -(b*(3*a^2 + 2*b^2)*ArcTanh[Cos[e + f*x]])/(2*f) - (a*(2*a^2 + 9*b^2)*Cot[e
 + f*x])/(3*f) - (7*a^2*b*Cot[e + f*x]*Csc[e + f*x])/(6*f) - (a^2*Cot[e + f
*x]*Csc[e + f*x]^2*(a + b*Sin[e + f*x]))/(3*f)
```


Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc^4(e+fx)(a+b\sin(e+fx))^3\,dx = -\frac{a^2\cot(e+fx)\csc^2(e+fx)(a+b\sin(e+fx))}{3f}+\frac{1}{3}\int \csc^3(e+fx)\left(7a^2b+\right.$$
$$= -\frac{7a^2b\cot(e+fx)\csc(e+fx)}{6f}-\frac{a^2\cot(e+fx)\csc^2(e+fx)(a+b\sin(e+fx$$
$$= -\frac{7a^2b\cot(e+fx)\csc(e+fx)}{6f}-\frac{a^2\cot(e+fx)\csc^2(e+fx)(a+b\sin(e+fx$$
$$= -\frac{b\left(3a^2+2b^2\right)\tanh^{-1}(\cos(e+fx))}{2f}-\frac{7a^2b\cot(e+fx)\csc(e+fx)}{6f}-\frac{a^2\cot}{}$$
$$= -\frac{b\left(3a^2+2b^2\right)\tanh^{-1}(\cos(e+fx))}{2f}-\frac{a\left(2a^2+9b^2\right)\cot(e+fx)}{3f}-\frac{7a^2b\cot(}{}$$

Mathematica [B] time = 6.183, size = 525, normalized size = 4.82

$$\frac{\sin^3(e+fx)\csc\left(\frac{1}{2}(e+fx)\right)\left(-2a^3\cos\left(\frac{1}{2}(e+fx)\right)-9ab^2\cos\left(\frac{1}{2}(e+fx)\right)\right)(a\csc(e+fx)+b)^3}{6f(a+b\sin(e+fx))^3}+\frac{\left(3a^2b+2b^3\right)\sin^3(e}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^4*(a + b*Sin[e + f*x])^3,x]
```

```
[Out] ((-2*a^3*Cos[(e + f*x)/2] - 9*a*b^2*Cos[(e + f*x)/2])*Csc[(e + f*x)/2]*(b +
 a*Csc[e + f*x])^3*Sin[e + f*x]^3)/(6*f*(a + b*Sin[e + f*x])^3) - (3*a^2*b*
Csc[(e + f*x)/2]^2*(b + a*Csc[e + f*x])^3*Sin[e + f*x]^3)/(8*f*(a + b*Sin[e
 + f*x])^3) - (a^3*Cot[(e + f*x)/2]*Csc[(e + f*x)/2]^2*(b + a*Csc[e + f*x])
^3*Sin[e + f*x]^3)/(24*f*(a + b*Sin[e + f*x])^3) + ((-3*a^2*b - 2*b^3)*(b +
 a*Csc[e + f*x])^3*Log[Cos[(e + f*x)/2]]*Sin[e + f*x]^3)/(2*f*(a + b*Sin[e
+ f*x])^3) + ((3*a^2*b + 2*b^3)*(b + a*Csc[e + f*x])^3*Log[Sin[(e + f*x)/2]
]*Sin[e + f*x]^3)/(2*f*(a + b*Sin[e + f*x])^3) + (3*a^2*b*(b + a*Csc[e + f*
x])^3*Sec[(e + f*x)/2]^2*Sin[e + f*x]^3)/(8*f*(a + b*Sin[e + f*x])^3) + ((b
 + a*Csc[e + f*x])^3*Sec[(e + f*x)/2]*(2*a^3*Sin[(e + f*x)/2] + 9*a*b^2*Sin
[(e + f*x)/2])*Sin[e + f*x]^3)/(6*f*(a + b*Sin[e + f*x])^3) + (a^3*(b + a*C
sc[e + f*x])^3*Sec[(e + f*x)/2]^2*Sin[e + f*x]^3*Tan[(e + f*x)/2])/(24*f*(a
 + b*Sin[e + f*x])^3)
```

Maple [A] time = 0.063, size = 122, normalized size = 1.1

$$-\frac{2 a^3 \cot\left(fx+e\right)}{3 f}-\frac{a^3 \cot\left(fx+e\right)\left(\csc\left(fx+e\right)\right)^2}{3 f}-\frac{3 a^2 b \cot\left(fx+e\right) \csc\left(fx+e\right)}{2 f}+\frac{3 a^2 b \ln\left(\csc\left(fx+e\right)-\cot\left(fx\right.\right.}{2 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^4*(a+b*sin(f*x+e))^3,x)
```

```
[Out] -2/3/f*a^3*cot(f*x+e)-1/3/f*a^3*cot(f*x+e)*csc(f*x+e)^2-3/2*a^2*b*cot(f*x+e
)*csc(f*x+e)/f+3/2/f*a^2*b*ln(csc(f*x+e)-cot(f*x+e))-3/f*a*b^2*cot(f*x+e)+1
/f*b^3*ln(csc(f*x+e)-cot(f*x+e))
```

Maxima [A] time = 2.64138, size = 159, normalized size = 1.46

$$\frac{9 a^2 b\left(\frac{2 \cos\left(fx+e\right)}{\cos\left(fx+e\right)^2-1}-\log\left(\cos\left(fx+e\right)+1\right)+\log\left(\cos\left(fx+e\right)-1\right)\right)-6 b^3\left(\log\left(\cos\left(fx+e\right)+1\right)-\log\left(\cos\left(fx+e\right)-1\right.\right.}{12 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 1/12*(9*a^2*b*(2*cos(f*x + e)/(cos(f*x + e)^2 - 1) - log(cos(f*x + e) + 1)
+ log(cos(f*x + e) - 1)) - 6*b^3*(log(cos(f*x + e) + 1) - log(cos(f*x + e)
- 1)) - 36*a*b^2/tan(f*x + e) - 4*(3*tan(f*x + e)^2 + 1)*a^3/tan(f*x + e)^3
)/f
```

Fricas [A] time = 1.72114, size = 471, normalized size = 4.32

$$\frac{18 a^2 b \cos\left(fx+e\right) \sin\left(fx+e\right)-4\left(2 a^3+9 a b^2\right) \cos\left(fx+e\right)^3+3\left(3 a^2 b+2 b^3-\left(3 a^2 b+2 b^3\right) \cos\left(fx+e\right)^2\right) \log\left(\frac{1}{2} \cos\right.}{12\left(f\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/12*(18*a^2*b*cos(f*x + e)*sin(f*x + e) - 4*(2*a^3 + 9*a*b^2)*cos(f*x + e)
^3 + 3*(3*a^2*b + 2*b^3 - (3*a^2*b + 2*b^3)*cos(f*x + e)^2)*log(1/2*cos(f*x
 + e) + 1/2)*sin(f*x + e) - 3*(3*a^2*b + 2*b^3 - (3*a^2*b + 2*b^3)*cos(f*x
+ e)^2)*log(-1/2*cos(f*x + e) + 1/2)*sin(f*x + e) + 12*(a^3 + 3*a*b^2)*cos(
f*x + e))/((f*cos(f*x + e)^2 - f)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**4*(a+b*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 2.11076, size = 271, normalized size = 2.49

$$\frac{a^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+9a^2b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+9a^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+36ab^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+12\left(3a^2b+2b^3\right)\log}{24f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/24*(a^3*tan(1/2*f*x + 1/2*e)^3 + 9*a^2*b*tan(1/2*f*x + 1/2*e)^2 + 9*a^3*t
an(1/2*f*x + 1/2*e) + 36*a*b^2*tan(1/2*f*x + 1/2*e) + 12*(3*a^2*b + 2*b^3)*
log(abs(tan(1/2*f*x + 1/2*e))) - (66*a^2*b*tan(1/2*f*x + 1/2*e)^3 + 44*b^3*
tan(1/2*f*x + 1/2*e)^3 + 9*a^3*tan(1/2*f*x + 1/2*e)^2 + 36*a*b^2*tan(1/2*f*
x + 1/2*e)^2 + 9*a^2*b*tan(1/2*f*x + 1/2*e) + a^3)/tan(1/2*f*x + 1/2*e)^3)/
f
```

3.174 $\int \csc^5(e+fx)(a+b\sin(e+fx))^3\,dx$

Optimal. Leaf size=134

$$-\frac{b\left(2a^2+b^2\right)\cot(e+fx)}{f}-\frac{3a\left(a^2+4b^2\right)\tanh^{-1}(\cos(e+fx))}{8f}-\frac{3a\left(a^2+4b^2\right)\cot(e+fx)\csc(e+fx)}{8f}-\frac{3a^2b\cot(e+f}{4}$$

```
[Out] (-3*a*(a^2 + 4*b^2)*ArcTanh[Cos[e + f*x]])/(8*f) - (b*(2*a^2 + b^2)*Cot[e +
 f*x])/f - (3*a*(a^2 + 4*b^2)*Cot[e + f*x]*Csc[e + f*x])/(8*f) - (3*a^2*b*C
ot[e + f*x]*Csc[e + f*x]^2)/(4*f) - (a^2*Cot[e + f*x]*Csc[e + f*x]^3*(a + b
*Sin[e + f*x]))/(4*f)
```

Rubi [A] time = 0.205392, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2792, 3021, 2748, 3768, 3770, 3767, 8}

$$-\frac{b\left(2a^2+b^2\right)\cot(e+fx)}{f}-\frac{3a\left(a^2+4b^2\right)\tanh^{-1}(\cos(e+fx))}{8f}-\frac{3a\left(a^2+4b^2\right)\cot(e+fx)\csc(e+fx)}{8f}-\frac{3a^2b\cot(e+f}{4}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^5*(a + b*Sin[e + f*x])^3,x]
```


```
[Out] (-3*a*(a^2 + 4*b^2)*ArcTanh[Cos[e + f*x]])/(8*f) - (b*(2*a^2 + b^2)*Cot[e +
 f*x])/f - (3*a*(a^2 + 4*b^2)*Cot[e + f*x]*Csc[e + f*x])/(8*f) - (3*a^2*b*C
ot[e + f*x]*Csc[e + f*x]^2)/(4*f) - (a^2*Cot[e + f*x]*Csc[e + f*x]^3*(a + b
*Sin[e + f*x]))/(4*f)
```


Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \csc^5(e+fx)(a+b\sin(e+fx))^3\,dx = -\frac{a^2\cot(e+fx)\csc^3(e+fx)(a+b\sin(e+fx))}{4f}+\frac{1}{4}\int \csc^4(e+fx)\left(9a^2b+\right.$$
$$= -\frac{3a^2b\cot(e+fx)\csc^2(e+fx)}{4f}-\frac{a^2\cot(e+fx)\csc^3(e+fx)(a+b\sin(e+f}{4f}$$
$$= -\frac{3a^2b\cot(e+fx)\csc^2(e+fx)}{4f}-\frac{a^2\cot(e+fx)\csc^3(e+fx)(a+b\sin(e+f}{4f}$$
$$= -\frac{3a\left(a^2+4b^2\right)\cot(e+fx)\csc(e+fx)}{8f}-\frac{3a^2b\cot(e+fx)\csc^2(e+fx)}{4f}-\frac{a^2}{}$$
$$= -\frac{3a\left(a^2+4b^2\right)\tanh^{-1}(\cos(e+fx))}{8f}-\frac{b\left(2a^2+b^2\right)\cot(e+fx)}{f}-\frac{3a\left(a^2+4b\right.}{}$$

Mathematica [B] time = 6.17099, size = 322, normalized size = 2.4

$$-\frac{3\left(a^3+4ab^2\right)\csc^2\left(\frac{1}{2}(e+fx)\right)}{32f}+\frac{3\left(a^3+4ab^2\right)\sec^2\left(\frac{1}{2}(e+fx)\right)}{32f}+\frac{3\left(a^3+4ab^2\right)\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)}{8f}-\frac{3\left(a^3+4ab\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[e + f*x]^5*(a + b*Sin[e + f*x])^3,x]
```

```
[Out] ((-2*a^2*b*Cos[(e + f*x)/2] - b^3*Cos[(e + f*x)/2])*Csc[(e + f*x)/2])/(2*f)
 - (3*(a^3 + 4*a*b^2)*Csc[(e + f*x)/2]^2)/(32*f) - (a^2*b*Cot[(e + f*x)/2]*
Csc[(e + f*x)/2]^2)/(8*f) - (a^3*Csc[(e + f*x)/2]^4)/(64*f) - (3*(a^3 + 4*a
*b^2)*Log[Cos[(e + f*x)/2]])/(8*f) + (3*(a^3 + 4*a*b^2)*Log[Sin[(e + f*x)/2
]])/(8*f) + (3*(a^3 + 4*a*b^2)*Sec[(e + f*x)/2]^2)/(32*f) + (a^3*Sec[(e + f
*x)/2]^4)/(64*f) + (Sec[(e + f*x)/2]*(2*a^2*b*Sin[(e + f*x)/2] + b^3*Sin[(e
 + f*x)/2]))/(2*f) + (a^2*b*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/(8*f)
```

Maple [A] time = 0.067, size = 166, normalized size = 1.2

$$-\frac{a^3\cot\left(fx+e\right)\left(\csc\left(fx+e\right)\right)^3}{4f}-\frac{3a^3\csc\left(fx+e\right)\cot\left(fx+e\right)}{8f}+\frac{3a^3\ln\left(\csc\left(fx+e\right)-\cot\left(fx+e\right)\right)}{8f}-2\frac{a^2b\cot\left(fx\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^5*(a+b*sin(f*x+e))^3,x)
```

```
[Out] -1/4/f*a^3*cot(f*x+e)*csc(f*x+e)^3-3/8/f*a^3*csc(f*x+e)*cot(f*x+e)+3/8/f*a^
3*ln(csc(f*x+e)-cot(f*x+e))-2*a^2*b*cot(f*x+e)/f-a^2*b*cot(f*x+e)*csc(f*x+e
)^2/f-3/2/f*a*b^2*cot(f*x+e)*csc(f*x+e)+3/2/f*a*b^2*ln(csc(f*x+e)-cot(f*x+e
))-1/f*b^3*cot(f*x+e)
```

Maxima [A] time = 1.65598, size = 219, normalized size = 1.63

$$\frac{a^3\left(\frac{2\left(3\cos\left(fx+e\right)^3-5\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)^4-2\cos\left(fx+e\right)^2+1}-3\log\left(\cos\left(fx+e\right)+1\right)+3\log\left(\cos\left(fx+e\right)-1\right)\right)+12ab^2\left(\frac{2\cos\left(fx+e\right)}{\cos\left(fx+e\right)^2-1}-\log\left(\cos\left(fx+\right.\right.\right.}{16f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 1/16*(a^3*(2*(3*cos(f*x + e)^3 - 5*cos(f*x + e))/(cos(f*x + e)^4 - 2*cos(f*
x + e)^2 + 1) - 3*log(cos(f*x + e) + 1) + 3*log(cos(f*x + e) - 1)) + 12*a*b
^2*(2*cos(f*x + e)/(cos(f*x + e)^2 - 1) - log(cos(f*x + e) + 1) + log(cos(f
*x + e) - 1)) - 16*b^3/tan(f*x + e) - 16*(3*tan(f*x + e)^2 + 1)*a^2*b/tan(f
*x + e)^3)/f
```

Fricas [A] time = 1.79595, size = 589, normalized size = 4.4

$$6\left(a^3+4ab^2\right)\cos\left(fx+e\right)^3-2\left(5a^3+12ab^2\right)\cos\left(fx+e\right)-3\left(\left(a^3+4ab^2\right)\cos\left(fx+e\right)^4+a^3+4ab^2-2\left(a^3+4ab^2\right)\mathrm{c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/16*(6*(a^3 + 4*a*b^2)*cos(f*x + e)^3 - 2*(5*a^3 + 12*a*b^2)*cos(f*x + e)
- 3*((a^3 + 4*a*b^2)*cos(f*x + e)^4 + a^3 + 4*a*b^2 - 2*(a^3 + 4*a*b^2)*cos
(f*x + e)^2)*log(1/2*cos(f*x + e) + 1/2) + 3*((a^3 + 4*a*b^2)*cos(f*x + e)^
4 + a^3 + 4*a*b^2 - 2*(a^3 + 4*a*b^2)*cos(f*x + e)^2)*log(-1/2*cos(f*x + e)
 + 1/2) + 16*((2*a^2*b + b^3)*cos(f*x + e)^3 - (3*a^2*b + b^3)*cos(f*x + e)
)*sin(f*x + e))/(f*cos(f*x + e)^4 - 2*f*cos(f*x + e)^2 + f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**5*(a+b*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.90712, size = 363, normalized size = 2.71

$$a^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+8a^2b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+8a^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+24ab^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+72a^2b\tan\left(\frac{1}{2}fx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/64*(a^3*tan(1/2*f*x + 1/2*e)^4 + 8*a^2*b*tan(1/2*f*x + 1/2*e)^3 + 8*a^3*t
an(1/2*f*x + 1/2*e)^2 + 24*a*b^2*tan(1/2*f*x + 1/2*e)^2 + 72*a^2*b*tan(1/2*
f*x + 1/2*e) + 32*b^3*tan(1/2*f*x + 1/2*e) + 24*(a^3 + 4*a*b^2)*log(abs(tan
(1/2*f*x + 1/2*e))) - (50*a^3*tan(1/2*f*x + 1/2*e)^4 + 200*a*b^2*tan(1/2*f*
x + 1/2*e)^4 + 72*a^2*b*tan(1/2*f*x + 1/2*e)^3 + 32*b^3*tan(1/2*f*x + 1/2*e
)^3 + 8*a^3*tan(1/2*f*x + 1/2*e)^2 + 24*a*b^2*tan(1/2*f*x + 1/2*e)^2 + 8*a^
2*b*tan(1/2*f*x + 1/2*e) + a^3)/tan(1/2*f*x + 1/2*e)^4)/f
```

3.175 $\int (a+b\sin(e+fx))^4 \, dx$

Optimal. Leaf size=137

$$-\frac{ab\left(19a^2+16b^2\right)\cos(e+fx)}{6f}-\frac{b^2\left(26a^2+9b^2\right)\sin(e+fx)\cos(e+fx)}{24f}+\frac{1}{8}x\left(24a^2b^2+8a^4+3b^4\right)-\frac{b\cos(e+fx)(a+}{4f}$$

```
[Out] ((8*a^4 + 24*a^2*b^2 + 3*b^4)*x)/8 - (a*b*(19*a^2 + 16*b^2)*Cos[e + f*x])/(
6*f) - (b^2*(26*a^2 + 9*b^2)*Cos[e + f*x]*Sin[e + f*x])/(24*f) - (7*a*b*Cos
[e + f*x]*(a + b*Sin[e + f*x])^2)/(12*f) - (b*Cos[e + f*x]*(a + b*Sin[e + f
*x])^3)/(4*f)
```

Rubi [A] time = 0.145565, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2656, 2753, 2734}

$$-\frac{ab\left(19a^2+16b^2\right)\cos(e+fx)}{6f}-\frac{b^2\left(26a^2+9b^2\right)\sin(e+fx)\cos(e+fx)}{24f}+\frac{1}{8}x\left(24a^2b^2+8a^4+3b^4\right)-\frac{b\cos(e+fx)(a+}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^4,x]
```

```
[Out] ((8*a^4 + 24*a^2*b^2 + 3*b^4)*x)/8 - (a*b*(19*a^2 + 16*b^2)*Cos[e + f*x])/(
6*f) - (b^2*(26*a^2 + 9*b^2)*Cos[e + f*x]*Sin[e + f*x])/(24*f) - (7*a*b*Cos
[e + f*x]*(a + b*Sin[e + f*x])^2)/(12*f) - (b*Cos[e + f*x]*(a + b*Sin[e + f
*x])^3)/(4*f)
```

Rule 2656

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[1/n, Int[(a + b*Sin
[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x], x
], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] && In
tegerQ[2*n]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int (a+b\sin(e+fx))^4\,dx = -\frac{b\cos(e+fx)(a+b\sin(e+fx))^3}{4f} + \frac{1}{4}\int (a+b\sin(e+fx))^2\left(4a^2+3b^2+7ab\sin(e+fx\right.$$
$$= -\frac{7ab\cos(e+fx)(a+b\sin(e+fx))^2}{12f} - \frac{b\cos(e+fx)(a+b\sin(e+fx))^3}{4f} + \frac{1}{12}\int (a+b\,\mathrm{si}$$
$$= \frac{1}{8}\left(8a^4+24a^2b^2+3b^4\right)x - \frac{ab\left(19a^2+16b^2\right)\cos(e+fx)}{6f} - \frac{b^2\left(26a^2+9b^2\right)\cos(e+fx)\,\mathrm{si}}{24f}$$

Mathematica [A] time = 0.358763, size = 106, normalized size = 0.77

$$\frac{3\left(4\left(24a^2b^2+8a^4+3b^4\right)(e+fx)-8\left(6a^2b^2+b^4\right)\sin(2(e+fx))+b^4\sin(4(e+fx))\right)-96ab\left(4a^2+3b^2\right)\cos(e+fx)}{96f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^4,x]
```

```
[Out] (-96*a*b*(4*a^2 + 3*b^2)*Cos[e + f*x] + 32*a*b^3*Cos[3*(e + f*x)] + 3*(4*(8
*a^4 + 24*a^2*b^2 + 3*b^4)*(e + f*x) - 8*(6*a^2*b^2 + b^4)*Sin[2*(e + f*x)]
 + b^4*Sin[4*(e + f*x)]))/(96*f)
```

Maple [A] time = 0.023, size = 116, normalized size = 0.9

$$\frac{1}{f}\left(b^4\left(-\frac{\cos(fx+e)}{4}\left(\left(\sin(fx+e)\right)^3+\frac{3\sin(fx+e)}{2}\right)+\frac{3fx}{8}+\frac{3e}{8}\right)-\frac{4ab^3\left(2+\left(\sin(fx+e)\right)^2\right)\cos(fx+e)}{3}+6\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^4,x)
```

```
[Out] 1/f*(b^4*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-4/3*
a*b^3*(2+sin(f*x+e)^2)*cos(f*x+e)+6*a^2*b^2*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2
*f*x+1/2*e)-4*a^3*b*cos(f*x+e)+a^4*(f*x+e))
```

Maxima [A] time = 2.40591, size = 153, normalized size = 1.12

$$a^4x + \frac{3\left(2fx+2e-\sin(2fx+2e)\right)a^2b^2}{2f} + \frac{4\left(\cos(fx+e)^3-3\cos(fx+e)\right)ab^3}{3f} + \frac{\left(12fx+12e+\sin(4fx+4e)\right.}{32f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^4,x, algorithm="maxima")
```

```
[Out] a^4*x + 3/2*(2*f*x + 2*e - sin(2*f*x + 2*e))*a^2*b^2/f + 4/3*(cos(f*x + e)^
3 - 3*cos(f*x + e))*a*b^3/f + 1/32*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*si
n(2*f*x + 2*e))*b^4/f - 4*a^3*b*cos(f*x + e)/f
```

Fricas [A] time = 1.78019, size = 244, normalized size = 1.78

$$\frac{32\,ab^3\cos\left(fx+e\right)^3+3\left(8\,a^4+24\,a^2b^2+3\,b^4\right)fx-96\left(a^3b+ab^3\right)\cos\left(fx+e\right)+3\left(2\,b^4\cos\left(fx+e\right)^3-\left(24\,a^2b^2+5\,b^4\right)\right.}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^4,x, algorithm="fricas")
```

```
[Out] 1/24*(32*a*b^3*cos(f*x + e)^3 + 3*(8*a^4 + 24*a^2*b^2 + 3*b^4)*f*x - 96*(a^
3*b + a*b^3)*cos(f*x + e) + 3*(2*b^4*cos(f*x + e)^3 - (24*a^2*b^2 + 5*b^4)*
cos(f*x + e))*sin(f*x + e))/f
```

Sympy [A] time = 2.11111, size = 240, normalized size = 1.75

$$\begin{cases} a^4x-\frac{4a^3b\cos\left(e+fx\right)}{f}+3a^2b^2x\sin^2\left(e+fx\right)+3a^2b^2x\cos^2\left(e+fx\right)-\frac{3a^2b^2\sin\left(e+fx\right)\cos\left(e+fx\right)}{f}-\frac{4ab^3\sin^2\left(e+fx\right)\cos\left(e+fx\right)}{f}-\frac{8ab}{} \\ x\left(a+b\sin\left(e\right)\right)^4 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**4,x)
```

```
[Out] Piecewise((a**4*x - 4*a**3*b*cos(e + f*x)/f + 3*a**2*b**2*x*sin(e + f*x)**2
 + 3*a**2*b**2*x*cos(e + f*x)**2 - 3*a**2*b**2*sin(e + f*x)*cos(e + f*x)/f
- 4*a*b**3*sin(e + f*x)**2*cos(e + f*x)/f - 8*a*b**3*cos(e + f*x)**3/(3*f)
+ 3*b**4*x*sin(e + f*x)**4/8 + 3*b**4*x*sin(e + f*x)**2*cos(e + f*x)**2/4 +
 3*b**4*x*cos(e + f*x)**4/8 - 5*b**4*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 3
*b**4*sin(e + f*x)*cos(e + f*x)**3/(8*f), Ne(f, 0)), (x*(a + b*sin(e))**4,
True))
```

Giac [A] time = 1.5568, size = 151, normalized size = 1.1

$$\frac{ab^3\cos\left(3\,fx+3\,e\right)}{3\,f}+\frac{b^4\sin\left(4\,fx+4\,e\right)}{32\,f}+\frac{1}{8}\left(8\,a^4+24\,a^2b^2+3\,b^4\right)x-\frac{\left(4\,a^3b+3\,ab^3\right)\cos\left(fx+e\right)}{f}-\frac{\left(6\,a^2b^2+b^4\right)\sin\left(\right.}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^4,x, algorithm="giac")
```

```
[Out] 1/3*a*b^3*cos(3*f*x + 3*e)/f + 1/32*b^4*sin(4*f*x + 4*e)/f + 1/8*(8*a^4 + 2
4*a^2*b^2 + 3*b^4)*x - (4*a^3*b + 3*a*b^3)*cos(f*x + e)/f - 1/4*(6*a^2*b^2
+ b^4)*sin(2*f*x + 2*e)/f
```

3.176 $\int \frac{\sin^4(x)}{a+b\sin(x)}\,dx$

Optimal. Leaf size=110

$$-\frac{ax\left(2a^2+b^2\right)}{2b^4}-\frac{\left(3a^2+2b^2\right)\cos(x)}{3b^3}+\frac{2a^4\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{b^4\sqrt{a^2-b^2}}+\frac{a\sin(x)\cos(x)}{2b^2}-\frac{\sin^2(x)\cos(x)}{3b}$$

```
[Out] -(a*(2*a^2 + b^2)*x)/(2*b^4) + (2*a^4*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^
2]])/(b^4*Sqrt[a^2 - b^2]) - ((3*a^2 + 2*b^2)*Cos[x])/(3*b^3) + (a*Cos[x]*S
in[x])/(2*b^2) - (Cos[x]*Sin[x]^2)/(3*b)
```

Rubi [A] time = 0.277666, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {2793, 3049, 3023, 2735, 2660, 618, 204}

$$-\frac{ax\left(2a^2+b^2\right)}{2b^4}-\frac{\left(3a^2+2b^2\right)\cos(x)}{3b^3}+\frac{2a^4\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{b^4\sqrt{a^2-b^2}}+\frac{a\sin(x)\cos(x)}{2b^2}-\frac{\sin^2(x)\cos(x)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^4/(a + b*Sin[x]),x]
```

```
[Out] -(a*(2*a^2 + b^2)*x)/(2*b^4) + (2*a^4*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^
2]])/(b^4*Sqrt[a^2 - b^2]) - ((3*a^2 + 2*b^2)*Cos[x])/(3*b^3) + (a*Cos[x]*S
in[x])/(2*b^2) - (Cos[x]*Sin[x]^2)/(3*b)
```

Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sin^4(x)}{a+b\sin(x)}\,dx &= -\frac{\cos(x)\sin^2(x)}{3b}+\frac{\int \frac{\sin(x)\left(2a+2b\sin(x)-3a\sin^2(x)\right)}{a+b\sin(x)}\,dx}{3b}\\
&= \frac{a\cos(x)\sin(x)}{2b^2}-\frac{\cos(x)\sin^2(x)}{3b}+\frac{\int \frac{-3a^2+ab\sin(x)+2\left(3a^2+2b^2\right)\sin^2(x)}{a+b\sin(x)}\,dx}{6b^2}\\
&= -\frac{\left(3a^2+2b^2\right)\cos(x)}{3b^3}+\frac{a\cos(x)\sin(x)}{2b^2}-\frac{\cos(x)\sin^2(x)}{3b}+\frac{\int \frac{-3a^2b-3a\left(2a^2+b^2\right)\sin(x)}{a+b\sin(x)}\,dx}{6b^3}\\
&= -\frac{a\left(2a^2+b^2\right)x}{2b^4}-\frac{\left(3a^2+2b^2\right)\cos(x)}{3b^3}+\frac{a\cos(x)\sin(x)}{2b^2}-\frac{\cos(x)\sin^2(x)}{3b}+\frac{a^4\int \frac{1}{a+b\sin(x)}\,dx}{b^4}\\
&= -\frac{a\left(2a^2+b^2\right)x}{2b^4}-\frac{\left(3a^2+2b^2\right)\cos(x)}{3b^3}+\frac{a\cos(x)\sin(x)}{2b^2}-\frac{\cos(x)\sin^2(x)}{3b}+\frac{\left(2a^4\right)\operatorname{Subst}\left(\int \frac{1}{a+2bx+ax^2}\right.}{b^4}\\
&= -\frac{a\left(2a^2+b^2\right)x}{2b^4}-\frac{\left(3a^2+2b^2\right)\cos(x)}{3b^3}+\frac{a\cos(x)\sin(x)}{2b^2}-\frac{\cos(x)\sin^2(x)}{3b}-\frac{\left(4a^4\right)\operatorname{Subst}\left(\int \frac{1}{-4\left(a^2-b^2\right)-}\right.}{b}\\
&= -\frac{a\left(2a^2+b^2\right)x}{2b^4}+\frac{2a^4\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{b^4\sqrt{a^2-b^2}}-\frac{\left(3a^2+2b^2\right)\cos(x)}{3b^3}+\frac{a\cos(x)\sin(x)}{2b^2}-\frac{\cos(x)\sin^2(x)}{3b}
\end{aligned}
$$

Mathematica [A] time = 0.255606, size = 98, normalized size = 0.89

$$\frac{-6ax\left(2a^2+b^2\right)-3b\left(4a^2+3b^2\right)\cos(x)+\frac{24a^4\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+3ab^2\sin(2x)+b^3\cos(3x)}{12b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^4/(a + b*Sin[x]),x]
```

```
[Out] (-6*a*(2*a^2 + b^2)*x + (24*a^4*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/S
qrt[a^2 - b^2] - 3*b*(4*a^2 + 3*b^2)*Cos[x] + b^3*Cos[3*x] + 3*a*b^2*Sin[2*
x])/(12*b^4)
```

Maple [B] time = 0.039, size = 213, normalized size = 1.9

$$-\frac{a}{b^2}\left(\tan\left(\frac{x}{2}\right)\right)^5\left(\left(\tan\left(\frac{x}{2}\right)\right)^2+1\right)^{-3}-2\frac{a^2(\tan(x/2))^4}{b^3\left((\tan(x/2))^2+1\right)^3}-4\frac{(\tan(x/2))^2a^2}{b^3\left((\tan(x/2))^2+1\right)^3}-4\frac{(\tan(x/2))^2}{b\left((\tan(x/2))^2+1\right)^3}+\frac{a}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^4/(a+b*sin(x)),x)
```

```
[Out] -1/b^2/(tan(1/2*x)^2+1)^3*a*tan(1/2*x)^5-2/b^3/(tan(1/2*x)^2+1)^3*a^2*tan(1
/2*x)^4-4/b^3/(tan(1/2*x)^2+1)^3*tan(1/2*x)^2*a^2-4/b/(tan(1/2*x)^2+1)^3*ta
n(1/2*x)^2+1/b^2/(tan(1/2*x)^2+1)^3*a*tan(1/2*x)-2/b^3/(tan(1/2*x)^2+1)^3*a
^2-4/3/b/(tan(1/2*x)^2+1)^3-2/b^4*arctan(tan(1/2*x))*a^3-1/b^2*arctan(tan(1
/2*x))*a+2*a^4/b^4/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*x)+2*b)/(a^2-b^2
)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^4/(a+b*sin(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.85976, size = 728, normalized size = 6.62

$$\left[-\frac{3\sqrt{-a^2+b^2}a^4\log\left(\frac{\left(2a^2-b^2\right)\cos(x)^2-2ab\sin(x)-a^2-b^2+2\left(a\cos(x)\sin(x)+b\cos(x)\right)\sqrt{-a^2+b^2}}{b^2\cos(x)^2-2ab\sin(x)-a^2-b^2}\right)-2\left(a^2b^3-b^5\right)\cos(x)^3-3\left(a^3b^2-a\right.}{6\left(a^2b^4-b^6\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^4/(a+b*sin(x)),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*sqrt(-a^2 + b^2)*a^4*log(((2*a^2 - b^2)*cos(x)^2 - 2*a*b*sin(x) -
a^2 - b^2 + 2*(a*cos(x)*sin(x) + b*cos(x))*sqrt(-a^2 + b^2))/(b^2*cos(x)^2
- 2*a*b*sin(x) - a^2 - b^2)) - 2*(a^2*b^3 - b^5)*cos(x)^3 - 3*(a^3*b^2 - a*
b^4)*cos(x)*sin(x) + 3*(2*a^5 - a^3*b^2 - a*b^4)*x + 6*(a^4*b - b^5)*cos(x)
)/(a^2*b^4 - b^6), -1/6*(6*sqrt(a^2 - b^2)*a^4*arctan(-(a*sin(x) + b)/(sqrt
(a^2 - b^2)*cos(x))) - 2*(a^2*b^3 - b^5)*cos(x)^3 - 3*(a^3*b^2 - a*b^4)*cos
(x)*sin(x) + 3*(2*a^5 - a^3*b^2 - a*b^4)*x + 6*(a^4*b - b^5)*cos(x))/(a^2*b
^4 - b^6)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**4/(a+b*sin(x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.56705, size = 201, normalized size = 1.83

$$\frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}x\right)+b}{\sqrt{a^2-b^2}}\right)\right)a^4}{\sqrt{a^2-b^2}b^4}-\frac{\left(2a^3+ab^2\right)x}{2b^4}-\frac{3ab\tan\left(\frac{1}{2}x\right)^5+6a^2\tan\left(\frac{1}{2}x\right)^4+12a^2\tan\left(\frac{1}{2}x\right)^2+}{3\left(\tan\left(\frac{1}{2}x\right)^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^4/(a+b*sin(x)),x, algorithm="giac")
```

```
[Out] 2*(pi*floor(1/2*x/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*x) + b)/sqrt(a^2 - b
^2)))*a^4/(sqrt(a^2 - b^2)*b^4) - 1/2*(2*a^3 + a*b^2)*x/b^4 - 1/3*(3*a*b*ta
n(1/2*x)^5 + 6*a^2*tan(1/2*x)^4 + 12*a^2*tan(1/2*x)^2 + 12*b^2*tan(1/2*x)^2
 - 3*a*b*tan(1/2*x) + 6*a^2 + 4*b^2)/((tan(1/2*x)^2 + 1)^3*b^3)
```

3.177 $\int \frac{\sin^3(x)}{a+b\sin(x)}\,dx$

Optimal. Leaf size=82

$$\frac{x(2a^2+b^2)}{2b^3}-\frac{2a^3\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{b^3\sqrt{a^2-b^2}}+\frac{a\cos(x)}{b^2}-\frac{\sin(x)\cos(x)}{2b}$$

```
[Out] ((2*a^2 + b^2)*x)/(2*b^3) - (2*a^3*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]]
)/(b^3*Sqrt[a^2 - b^2]) + (a*Cos[x])/b^2 - (Cos[x]*Sin[x])/(2*b)
```

Rubi [A] time = 0.163522, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2793, 3023, 2735, 2660, 618, 204}

$$\frac{x(2a^2+b^2)}{2b^3}-\frac{2a^3\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{b^3\sqrt{a^2-b^2}}+\frac{a\cos(x)}{b^2}-\frac{\sin(x)\cos(x)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^3/(a + b*Sin[x]),x]
```

```
[Out] ((2*a^2 + b^2)*x)/(2*b^3) - (2*a^3*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]]
)/(b^3*Sqrt[a^2 - b^2]) + (a*Cos[x])/b^2 - (Cos[x]*Sin[x])/(2*b)
```

Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sin^3(x)}{a+b\sin(x)}\,dx &= -\frac{\cos(x)\sin(x)}{2b}+\frac{\int \frac{a+b\sin(x)-2a\sin^2(x)}{a+b\sin(x)}\,dx}{2b}\\
&= \frac{a\cos(x)}{b^2}-\frac{\cos(x)\sin(x)}{2b}+\frac{\int \frac{ab+\left(2a^2+b^2\right)\sin(x)}{a+b\sin(x)}\,dx}{2b^2}\\
&= \frac{\left(2a^2+b^2\right)x}{2b^3}+\frac{a\cos(x)}{b^2}-\frac{\cos(x)\sin(x)}{2b}-\frac{a^3\int \frac{1}{a+b\sin(x)}\,dx}{b^3}\\
&= \frac{\left(2a^2+b^2\right)x}{2b^3}+\frac{a\cos(x)}{b^2}-\frac{\cos(x)\sin(x)}{2b}-\frac{\left(2a^3\right)\text{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx,x,\tan\left(\frac{x}{2}\right)\right)}{b^3}\\
&= \frac{\left(2a^2+b^2\right)x}{2b^3}+\frac{a\cos(x)}{b^2}-\frac{\cos(x)\sin(x)}{2b}+\frac{\left(4a^3\right)\text{Subst}\left(\int \frac{1}{-4\left(a^2-b^2\right)-x^2}\,dx,x,2b+2a\tan\left(\frac{x}{2}\right)\right)}{b^3}\\
&= \frac{\left(2a^2+b^2\right)x}{2b^3}-\frac{2a^3\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{b^3\sqrt{a^2-b^2}}+\frac{a\cos(x)}{b^2}-\frac{\cos(x)\sin(x)}{2b}
\end{aligned}
$$

Mathematica [A] time = 0.107992, size = 78, normalized size = 0.95

$$
\frac{-\frac{8a^3\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+4a^2x+4ab\cos(x)+2b^2x-b^2\sin(2x)}{4b^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^3/(a + b*Sin[x]),x]
```

```
[Out] (4*a^2*x + 2*b^2*x - (8*a^3*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/Sqrt[
a^2 - b^2] + 4*a*b*Cos[x] - b^2*Sin[2*x])/(4*b^3)
```

Maple [A] time = 0.036, size = 142, normalized size = 1.7

$$
\frac{1}{b}\left(\tan\left(\frac{x}{2}\right)\right)^3\left(\left(\tan\left(\frac{x}{2}\right)\right)^2+1\right)^{-2}+2\frac{(\tan(x/2))^2a}{b^2\left((\tan(x/2))^2+1\right)^2}-\frac{1}{b}\tan\left(\frac{x}{2}\right)\left(\left(\tan\left(\frac{x}{2}\right)\right)^2+1\right)^{-2}+2\frac{a}{b^2\left((\tan(x/2))^2+1\right)^2}+
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3/(a+b*sin(x)),x)
```

```
[Out] 1/b/(tan(1/2*x)^2+1)^2*tan(1/2*x)^3+2/b^2/(tan(1/2*x)^2+1)^2*tan(1/2*x)^2*a
-1/b/(tan(1/2*x)^2+1)^2*tan(1/2*x)+2/b^2/(tan(1/2*x)^2+1)^2*a+2/b^3*arctan(
tan(1/2*x))*a^2-2*a^3/b^3/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*x)+2*b)/(
a^2-b^2)^(1/2))+1/2*x/b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(a+b*sin(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.78392, size = 630, normalized size = 7.68

$$\left[-\frac{\sqrt{-a^2+b^2}a^3\log\left(-\frac{\left(2a^2-b^2\right)\cos(x)^2-2ab\sin(x)-a^2-b^2-2\left(a\cos(x)\sin(x)+b\cos(x)\right)\sqrt{-a^2+b^2}}{b^2\cos(x)^2-2ab\sin(x)-a^2-b^2}\right)+\left(a^2b^2-b^4\right)\cos\left(x\right)\sin\left(x\right)-\left(2a^4-\right.}{2\left(a^2b^3-b^5\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(a+b*sin(x)),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(-a^2 + b^2)*a^3*log(-((2*a^2 - b^2)*cos(x)^2 - 2*a*b*sin(x) - a
^2 - b^2 - 2*(a*cos(x)*sin(x) + b*cos(x))*sqrt(-a^2 + b^2))/(b^2*cos(x)^2 -
 2*a*b*sin(x) - a^2 - b^2)) + (a^2*b^2 - b^4)*cos(x)*sin(x) - (2*a^4 - a^2*
b^2 - b^4)*x - 2*(a^3*b - a*b^3)*cos(x))/(a^2*b^3 - b^5), 1/2*(2*sqrt(a^2 -
 b^2)*a^3*arctan(-(a*sin(x) + b)/(sqrt(a^2 - b^2)*cos(x))) - (a^2*b^2 - b^4
)*cos(x)*sin(x) + (2*a^4 - a^2*b^2 - b^4)*x + 2*(a^3*b - a*b^3)*cos(x))/(a^
2*b^3 - b^5)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**3/(a+b*sin(x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.82965, size = 151, normalized size = 1.84

$$-\frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}\left(a\right)+\arctan\left(\frac{a\tan\left(\frac{1}{2}x\right)+b}{\sqrt{a^2-b^2}}\right)\right)a^3}{\sqrt{a^2-b^2}b^3}+\frac{\left(2a^2+b^2\right)x}{2b^3}+\frac{b\tan\left(\frac{1}{2}x\right)^3+2a\tan\left(\frac{1}{2}x\right)^2-b\tan\left(\frac{1}{2}x\right)+2a}{\left(\tan\left(\frac{1}{2}x\right)^2+1\right)^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(a+b*sin(x)),x, algorithm="giac")
```

```
[Out] -2*(pi*floor(1/2*x/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*x) + b)/sqrt(a^2 -
b^2)))*a^3/(sqrt(a^2 - b^2)*b^3) + 1/2*(2*a^2 + b^2)*x/b^3 + (b*tan(1/2*x)^
3 + 2*a*tan(1/2*x)^2 - b*tan(1/2*x) + 2*a)/((tan(1/2*x)^2 + 1)^2*b^2)
```

3.178 $\int \frac{\sin^2(x)}{a+b\sin(x)}\,dx$

Optimal. Leaf size=61

$$\frac{2a^2\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{b^2\sqrt{a^2-b^2}}-\frac{ax}{b^2}-\frac{\cos(x)}{b}$$

```
[Out] -((a*x)/b^2) + (2*a^2*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(b^2*Sqrt[a
^2 - b^2]) - Cos[x]/b
```

Rubi [A] time = 0.103534, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2746, 12, 2735, 2660, 618, 204}

$$\frac{2a^2\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{b^2\sqrt{a^2-b^2}}-\frac{ax}{b^2}-\frac{\cos(x)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^2/(a + b*Sin[x]),x]
```

```
[Out] -((a*x)/b^2) + (2*a^2*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(b^2*Sqrt[a
^2 - b^2]) - Cos[x]/b
```

Rule 2746

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2/((c_.) + (d_.)*sin[(e_.) + (f
_.)*(x_)]), x_Symbol] :> -Simp[(b^2*Cos[e + f*x])/(d*f), x] + Dist[1/d, Int
[Simp[a^2*d - b*(b*c - 2*a*d)*Sin[e + f*x], x]/(c + d*Sin[e + f*x]), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^2(x)}{a+b\sin(x)}\,dx &= -\frac{\cos(x)}{b} - \frac{\int \frac{a\sin(x)}{a+b\sin(x)}\,dx}{b} \\
&= -\frac{\cos(x)}{b} - \frac{a\int \frac{\sin(x)}{a+b\sin(x)}\,dx}{b} \\
&= -\frac{ax}{b^2} - \frac{\cos(x)}{b} + \frac{a^2\int \frac{1}{a+b\sin(x)}\,dx}{b^2} \\
&= -\frac{ax}{b^2} - \frac{\cos(x)}{b} + \frac{\left(2a^2\right)\text{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx, x, \tan\left(\frac{x}{2}\right)\right)}{b^2} \\
&= -\frac{ax}{b^2} - \frac{\cos(x)}{b} - \frac{\left(4a^2\right)\text{Subst}\left(\int \frac{1}{-4\left(a^2-b^2\right)-x^2}\,dx, x, 2b+2a\tan\left(\frac{x}{2}\right)\right)}{b^2} \\
&= -\frac{ax}{b^2} + \frac{2a^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{b^2\sqrt{a^2-b^2}} - \frac{\cos(x)}{b}
\end{aligned}$$

Mathematica [A] time = 0.0890538, size = 56, normalized size = 0.92

$$-\frac{-\frac{2a^2\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+ax+b\cos(x)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^2/(a + b*Sin[x]),x]
```

```
[Out] -((a*x - (2*a^2*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] +
 b*Cos[x])/b^2)
```

Maple [A] time = 0.033, size = 72, normalized size = 1.2

$$-2\,\frac{1}{b\left((\tan(x/2))^2+1\right)}-2\,\frac{\arctan(\tan(x/2))\,a}{b^2}+2\,\frac{a^2}{b^2\sqrt{a^2-b^2}}\arctan\left(1/2\,\frac{2\,a\tan(x/2)+2\,b}{\sqrt{a^2-b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2/(a+b*sin(x)),x)
```

```
[Out] -2/b/(tan(1/2*x)^2+1)-2/b^2*arctan(tan(1/2*x))*a+2*a^2/b^2/(a^2-b^2)^(1/2)*
arctan(1/2*(2*a*tan(1/2*x)+2*b)/(a^2-b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a+b*sin(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.80041, size = 504, normalized size = 8.26

$$\left[-\frac{\sqrt{-a^2+b^2}a^2\log\left(\frac{\left(2a^2-b^2\right)\cos(x)^2-2ab\sin(x)-a^2-b^2+2(a\cos(x)\sin(x)+b\cos(x))\sqrt{-a^2+b^2}}{b^2\cos(x)^2-2ab\sin(x)-a^2-b^2}\right)+2\left(a^3-ab^2\right)x+2\left(a^2b-b^3\right)\cos(x)}{2\left(a^2b^2-b^4\right)},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a+b*sin(x)),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(-a^2 + b^2)*a^2*log(((2*a^2 - b^2)*cos(x)^2 - 2*a*b*sin(x) - a^
2 - b^2 + 2*(a*cos(x)*sin(x) + b*cos(x))*sqrt(-a^2 + b^2))/(b^2*cos(x)^2 -
2*a*b*sin(x) - a^2 - b^2)) + 2*(a^3 - a*b^2)*x + 2*(a^2*b - b^3)*cos(x))/(a
^2*b^2 - b^4), -(sqrt(a^2 - b^2)*a^2*arctan(-(a*sin(x) + b)/(sqrt(a^2 - b^2
)*cos(x))) + (a^3 - a*b^2)*x + (a^2*b - b^3)*cos(x))/(a^2*b^2 - b^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**2/(a+b*sin(x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.87935, size = 104, normalized size = 1.7

$$\frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}x\right)+b}{\sqrt{a^2-b^2}}\right)\right)a^2}{\sqrt{a^2-b^2}b^2}-\frac{ax}{b^2}-\frac{2}{\left(\tan\left(\frac{1}{2}x\right)^2+1\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a+b*sin(x)),x, algorithm="giac")
```

```
[Out] 2*(pi*floor(1/2*x/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*x) + b)/sqrt(a^2 - b
^2)))*a^2/(sqrt(a^2 - b^2)*b^2) - a*x/b^2 - 2/((tan(1/2*x)^2 + 1)*b)
```

3.179 $\int \frac{\sin(x)}{a+b\sin(x)}\,dx$

Optimal. Leaf size=50

$$\frac{x}{b}-\frac{2a\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}}$$

```
[Out] x/b - (2*a*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(b*Sqrt[a^2 - b^2])
```

Rubi [A] time = 0.0552835, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2735, 2660, 618, 204}

$$\frac{x}{b}-\frac{2a\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]/(a + b*Sin[x]),x]
```

```
[Out] x/b - (2*a*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(b*Sqrt[a^2 - b^2])
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sin(x)}{a+b\sin(x)}\,dx = \frac{x}{b} - \frac{a\int \frac{1}{a+b\sin(x)}\,dx}{b}$$
$$= \frac{x}{b} - \frac{(2a)\,\text{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx, x, \tan\left(\frac{x}{2}\right)\right)}{b}$$
$$= \frac{x}{b} + \frac{(4a)\,\text{Subst}\left(\int \frac{1}{-4(a^2-b^2)-x^2}\,dx, x, 2b+2a\tan\left(\frac{x}{2}\right)\right)}{b}$$
$$= \frac{x}{b} - \frac{2a\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}}$$

Mathematica [A] time = 0.0398464, size = 47, normalized size = 0.94

$$\frac{x - \frac{2a\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/(a + b*Sin[x]),x]
```

```
[Out] (x - (2*a*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2])/b
```

Maple [A] time = 0.029, size = 54, normalized size = 1.1

$$2\,\frac{\arctan\left(\tan\left(x/2\right)\right)}{b} - 2\,\frac{a}{b\sqrt{a^2-b^2}}\arctan\left(1/2\,\frac{2\,a\tan\left(x/2\right)+2\,b}{\sqrt{a^2-b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(a+b*sin(x)),x)
```

```
[Out] 2/b*arctan(tan(1/2*x))-2*a/b/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*x)+2*b
)/(a^2-b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+b*sin(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.80871, size = 423, normalized size = 8.46

$$\left[-\frac{\sqrt{-a^2+b^2}\,a\log\left(-\frac{\left(2\,a^2-b^2\right)\cos(x)^2-2\,ab\sin(x)-a^2-b^2-2\,(a\cos(x)\sin(x)+b\cos(x))\sqrt{-a^2+b^2}}{b^2\cos(x)^2-2\,ab\sin(x)-a^2-b^2}\right)-2\left(a^2-b^2\right)x}{2\left(a^2b-b^3\right)},\ \frac{\sqrt{a^2-b^2}\,a\arctan\left(-\frac{a\sin}{\sqrt{a^2-b}}\right.}{a^2b-b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+b*sin(x)),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(-a^2 + b^2)*a*log(-((2*a^2 - b^2)*cos(x)^2 - 2*a*b*sin(x) - a^2
 - b^2 - 2*(a*cos(x)*sin(x) + b*cos(x))*sqrt(-a^2 + b^2))/(b^2*cos(x)^2 - 2
*a*b*sin(x) - a^2 - b^2)) - 2*(a^2 - b^2)*x)/(a^2*b - b^3), (sqrt(a^2 - b^2
)*a*arctan(-(a*sin(x) + b)/(sqrt(a^2 - b^2)*cos(x))) + (a^2 - b^2)*x)/(a^2*
b - b^3)]
```

Sympy [A] time = 99.1242, size = 202, normalized size = 4.04

$$\begin{cases} \tilde{\infty}x & \text{for } a=0\wedge b=0 \\ \frac{x}{b} & \text{for } a=0 \\ \frac{x\tan\left(\frac{x}{2}\right)}{b\tan\left(\frac{x}{2}\right)-b}-\frac{x}{b\tan\left(\frac{x}{2}\right)-b}+\frac{2\tan\left(\frac{x}{2}\right)}{b\tan\left(\frac{x}{2}\right)-b} & \text{for } a=-b \\ \frac{x\tan\left(\frac{x}{2}\right)}{b\tan\left(\frac{x}{2}\right)+b}+\frac{x}{b\tan\left(\frac{x}{2}\right)+b}-\frac{2\tan\left(\frac{x}{2}\right)}{b\tan\left(\frac{x}{2}\right)+b} & \text{for } a=b \\ -\frac{\cos(x)}{a} & \text{for } b=0 \\ \frac{a^2x}{a^2b-b^3}+\frac{a\sqrt{-a^2+b^2}\log\left(\tan\left(\frac{x}{2}\right)+\frac{b}{a}-\frac{\sqrt{-a^2+b^2}}{a}\right)}{a^2b-b^3}-\frac{a\sqrt{-a^2+b^2}\log\left(\tan\left(\frac{x}{2}\right)+\frac{b}{a}+\frac{\sqrt{-a^2+b^2}}{a}\right)}{a^2b-b^3}-\frac{b^2x}{a^2b-b^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+b*sin(x)),x)
```

```
[Out] Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0)), (x/b, Eq(a, 0)), (x*tan(x/2)/(b*tan
(x/2) - b) - x/(b*tan(x/2) - b) + 2*tan(x/2)/(b*tan(x/2) - b), Eq(a, -b)),
(x*tan(x/2)/(b*tan(x/2) + b) + x/(b*tan(x/2) + b) - 2*tan(x/2)/(b*tan(x/2)
+ b), Eq(a, b)), (-cos(x)/a, Eq(b, 0)), (a**2*x/(a**2*b - b**3) + a*sqrt(-a
**2 + b**2)*log(tan(x/2) + b/a - sqrt(-a**2 + b**2)/a)/(a**2*b - b**3) - a*
sqrt(-a**2 + b**2)*log(tan(x/2) + b/a + sqrt(-a**2 + b**2)/a)/(a**2*b - b**
3) - b**2*x/(a**2*b - b**3), True))
```

Giac [A] time = 1.85884, size = 78, normalized size = 1.56

$$-\frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}x\right)+b}{\sqrt{a^2-b^2}}\right)\right)a}{\sqrt{a^2-b^2}b}+\frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+b*sin(x)),x, algorithm="giac")
```

```
[Out] -2*(pi*floor(1/2*x/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*x) + b)/sqrt(a^2 -
b^2)))*a/(sqrt(a^2 - b^2)*b) + x/b
```

3.180 $\int \frac{1}{a+b\sin(x)}\,dx$

Optimal. Leaf size=40

$$\frac{2\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

```
[Out] (2*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2]
```

Rubi [A] time = 0.0307481, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2660, 618, 204}

$$\frac{2\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[x])^(-1),x]
```

```
[Out] (2*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a+b\sin(x)}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx,x,\tan\left(\frac{x}{2}\right)\right)\\ &= -\left(4\,\text{Subst}\left(\int \frac{1}{-4\left(a^2-b^2\right)-x^2}\,dx,x,2b+2a\tan\left(\frac{x}{2}\right)\right)\right)\\ &= \frac{2\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}\end{aligned}$$

Mathematica [A] time = 0.0215806, size = 40, normalized size = 1.

$$\frac{2\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[x])^(-1),x]
```

```
[Out] (2*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2]
```

Maple [A] time = 0.023, size = 39, normalized size = 1.

$$2\,\frac{1}{\sqrt{a^2-b^2}}\arctan\left(1/2\,\frac{2\,a\tan(x/2)+2\,b}{\sqrt{a^2-b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(x)),x)
```

```
[Out] 2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*x)+2*b)/(a^2-b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.68665, size = 344, normalized size = 8.6

$$\left[-\frac{\sqrt{-a^2+b^2}\log\left(\frac{\left(2\,a^2-b^2\right)\cos(x)^2-2\,ab\sin(x)-a^2-b^2+2\,(a\cos(x)\sin(x)+b\cos(x))\sqrt{-a^2+b^2}}{b^2\cos(x)^2-2\,ab\sin(x)-a^2-b^2}\right)}{2\left(a^2-b^2\right)},-\frac{\arctan\left(-\frac{a\sin(x)+b}{\sqrt{a^2-b^2}\cos(x)}\right)}{\sqrt{a^2-b^2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(x)),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(x)^2 - 2*a*b*sin(x) - a^2 - b
^2 + 2*(a*cos(x)*sin(x) + b*cos(x))*sqrt(-a^2 + b^2))/(b^2*cos(x)^2 - 2*a*b
*sin(x) - a^2 - b^2))/(a^2 - b^2), -arctan(-(a*sin(x) + b)/(sqrt(a^2 - b^2)
*cos(x)))/sqrt(a^2 - b^2)]
```

Sympy [A] time = 8.71263, size = 114, normalized size = 2.85

$$\begin{cases} \tilde{\infty} \log\left(\tan\left(\frac{x}{2}\right)\right) & \text{for } a = 0 \wedge b = 0 \\ \frac{\log\left(\tan\left(\frac{x}{2}\right)\right)}{b} & \text{for } a = 0 \\ \frac{2}{b\tan\left(\frac{x}{2}\right) - b} & \text{for } a = -b \\ -\frac{2}{b\tan\left(\frac{x}{2}\right) + b} & \text{for } a = b \\ -\frac{\sqrt{-a^2+b^2}\log\left(\tan\left(\frac{x}{2}\right) + \frac{b}{a} - \frac{\sqrt{-a^2+b^2}}{a}\right)}{a^2-b^2} + \frac{\sqrt{-a^2+b^2}\log\left(\tan\left(\frac{x}{2}\right) + \frac{b}{a} + \frac{\sqrt{-a^2+b^2}}{a}\right)}{a^2-b^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(x)),x)
```

```
[Out] Piecewise((zoo*log(tan(x/2)), Eq(a, 0) & Eq(b, 0)), (log(tan(x/2))/b, Eq(a,
 0)), (2/(b*tan(x/2) - b), Eq(a, -b)), (-2/(b*tan(x/2) + b), Eq(a, b)), (-s
qrt(-a**2 + b**2)*log(tan(x/2) + b/a - sqrt(-a**2 + b**2)/a)/(a**2 - b**2)
+ sqrt(-a**2 + b**2)*log(tan(x/2) + b/a + sqrt(-a**2 + b**2)/a)/(a**2 - b**
2), True))
```

Giac [A] time = 1.8605, size = 65, normalized size = 1.62

$$\frac{2\left(\pi\left\lfloor\frac{x}{2\pi} + \frac{1}{2}\right\rfloor\operatorname{sgn}(a) + \arctan\left(\frac{a\tan\left(\frac{1}{2}x\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2 - b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(x)),x, algorithm="giac")
```

```
[Out] 2*(pi*floor(1/2*x/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*x) + b)/sqrt(a^2 - b
^2)))/sqrt(a^2 - b^2)
```

3.181 $\int \frac{\csc(x)}{a+b\sin(x)}\,dx$

Optimal. Leaf size=53

$$-\frac{2b\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}}-\frac{\tanh^{-1}(\cos(x))}{a}$$

```
[Out] (-2*b*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(a*Sqrt[a^2 - b^2]) - ArcTa
nh[Cos[x]]/a
```

Rubi [A] time = 0.0662638, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {2747, 3770, 2660, 618, 204}

$$-\frac{2b\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}}-\frac{\tanh^{-1}(\cos(x))}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]/(a + b*Sin[x]),x]
```

```
[Out] (-2*b*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(a*Sqrt[a^2 - b^2]) - ArcTa
nh[Cos[x]]/a
```

Rule 2747

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]),
x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc(x)}{a+b\sin(x)}\,dx &= \frac{\int \csc(x)\,dx}{a}-\frac{b\int \frac{1}{a+b\sin(x)}\,dx}{a}\\
&= -\frac{\tanh^{-1}(\cos(x))}{a}-\frac{(2b)\,\text{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx,x,\tan\left(\frac{x}{2}\right)\right)}{a}\\
&= -\frac{\tanh^{-1}(\cos(x))}{a}+\frac{(4b)\,\text{Subst}\left(\int \frac{1}{-4(a^2-b^2)-x^2}\,dx,x,2b+2a\tan\left(\frac{x}{2}\right)\right)}{a}\\
&= -\frac{2b\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}}-\frac{\tanh^{-1}(\cos(x))}{a}
\end{aligned}$$

Mathematica [A] time = 0.0577555, size = 62, normalized size = 1.17

$$\frac{-\frac{2b\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+\log\left(\sin\left(\frac{x}{2}\right)\right)-\log\left(\cos\left(\frac{x}{2}\right)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]/(a + b*Sin[x]),x]
```

```
[Out] ((-2*b*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] - Log[Cos[
x/2]] + Log[Sin[x/2]])/a
```

Maple [A] time = 0.041, size = 53, normalized size = 1.

$$-2\,{\frac{b}{a\sqrt{a^2-b^2}}}\arctan\left(1/2\,{\frac{2\,a\tan(x/2)+2\,b}{\sqrt{a^2-b^2}}}\right)+{\frac{1}{a}}\ln\left(\tan\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)/(a+b*sin(x)),x)
```

```
[Out] -2/a*b/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*x)+2*b)/(a^2-b^2)^(1/2))+1/a
*ln(tan(1/2*x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a+b*sin(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.09574, size = 582, normalized size = 10.98

$$\left[-\frac{\sqrt{-a^2+b^2}b\log\left(-\frac{\left(2\,a^2-b^2\right)\cos(x)^2-2\,ab\sin(x)-a^2-b^2-2\,(a\cos(x)\sin(x)+b\cos(x))\sqrt{-a^2+b^2}}{b^2\cos(x)^2-2\,ab\sin(x)-a^2-b^2}\right)+\left(a^2-b^2\right)\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)-\left(a^2-b^2\right.}{2\left(a^3-ab^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a+b*sin(x)),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(-a^2 + b^2)*b*log(-((2*a^2 - b^2)*cos(x)^2 - 2*a*b*sin(x) - a^2
 - b^2 - 2*(a*cos(x)*sin(x) + b*cos(x))*sqrt(-a^2 + b^2))/(b^2*cos(x)^2 - 2
*a*b*sin(x) - a^2 - b^2)) + (a^2 - b^2)*log(1/2*cos(x) + 1/2) - (a^2 - b^2)
*log(-1/2*cos(x) + 1/2))/(a^3 - a*b^2), 1/2*(2*sqrt(a^2 - b^2)*b*arctan(-(a
*sin(x) + b)/(sqrt(a^2 - b^2)*cos(x))) - (a^2 - b^2)*log(1/2*cos(x) + 1/2)
+ (a^2 - b^2)*log(-1/2*cos(x) + 1/2))/(a^3 - a*b^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(x)}{a+b\sin(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a+b*sin(x)),x)
```

```
[Out] Integral(csc(x)/(a + b*sin(x)), x)
```

Giac [A] time = 1.29672, size = 85, normalized size = 1.6

$$-\frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}x\right)+b}{\sqrt{a^2-b^2}}\right)\right)b}{\sqrt{a^2-b^2}a}+\frac{\log\left(\left|\tan\left(\frac{1}{2}x\right)\right|\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a+b*sin(x)),x, algorithm="giac")
```

```
[Out] -2*(pi*floor(1/2*x/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*x) + b)/sqrt(a^2 -
b^2)))*b/(sqrt(a^2 - b^2)*a) + log(abs(tan(1/2*x)))/a
```

3.182 $\int \frac{\csc^2(x)}{a+b\sin(x)}\,dx$

Optimal. Leaf size=62

$$\frac{2b^2\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{a^2\sqrt{a^2-b^2}}+\frac{b\tanh^{-1}(\cos(x))}{a^2}-\frac{\cot(x)}{a}$$

```
[Out] (2*b^2*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(a^2*Sqrt[a^2 - b^2]) + (b
*ArcTanh[Cos[x]])/a^2 - Cot[x]/a
```

Rubi [A] time = 0.111467, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {2802, 12, 2747, 3770, 2660, 618, 204}

$$\frac{2b^2\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{a^2\sqrt{a^2-b^2}}+\frac{b\tanh^{-1}(\cos(x))}{a^2}-\frac{\cot(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^2/(a + b*Sin[x]),x]
```

```
[Out] (2*b^2*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(a^2*Sqrt[a^2 - b^2]) + (b
*ArcTanh[Cos[x]])/a^2 - Cot[x]/a
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2747

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]),
x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc^2(x)}{a+b\sin(x)}\,dx &= -\frac{\cot(x)}{a}-\frac{\int \frac{b\csc(x)}{a+b\sin(x)}\,dx}{a}\\
&= -\frac{\cot(x)}{a}-\frac{b\int \frac{\csc(x)}{a+b\sin(x)}\,dx}{a}\\
&= -\frac{\cot(x)}{a}-\frac{b\int \csc(x)\,dx}{a^2}+\frac{b^2\int \frac{1}{a+b\sin(x)}\,dx}{a^2}\\
&= \frac{b\tanh^{-1}(\cos(x))}{a^2}-\frac{\cot(x)}{a}+\frac{\left(2b^2\right)\text{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx,x,\tan\left(\frac{x}{2}\right)\right)}{a^2}\\
&= \frac{b\tanh^{-1}(\cos(x))}{a^2}-\frac{\cot(x)}{a}-\frac{\left(4b^2\right)\text{Subst}\left(\int \frac{1}{-4\left(a^2-b^2\right)-x^2}\,dx,x,2b+2a\tan\left(\frac{x}{2}\right)\right)}{a^2}\\
&= \frac{2b^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{a^2\sqrt{a^2-b^2}}+\frac{b\tanh^{-1}(\cos(x))}{a^2}-\frac{\cot(x)}{a}
\end{aligned}$$

Mathematica [A] time = 0.238603, size = 91, normalized size = 1.47

$$\frac{\csc\left(\frac{x}{2}\right)\sec\left(\frac{x}{2}\right)\left(\frac{2b^2\sin(x)\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}-a\cos(x)+b\sin(x)\left(\log\left(\cos\left(\frac{x}{2}\right)\right)-\log\left(\sin\left(\frac{x}{2}\right)\right)\right)\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^2/(a + b*Sin[x]),x]
```

```
[Out] (Csc[x/2]*Sec[x/2]*(-(a*Cos[x]) + (2*b^2*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 -
 b^2]]*Sin[x])/Sqrt[a^2 - b^2] + b*(Log[Cos[x/2]] - Log[Sin[x/2]])*Sin[x]))
/(2*a^2)
```

Maple [A] time = 0.043, size = 77, normalized size = 1.2

$$\frac{1}{2a}\tan\left(\frac{x}{2}\right)+2\,\frac{b^2}{a^2\sqrt{a^2-b^2}}\arctan\left(1/2\,\frac{2\,a\tan(x/2)+2\,b}{\sqrt{a^2-b^2}}\right)-\frac{1}{2a}\left(\tan\left(\frac{x}{2}\right)\right)^{-1}-\frac{b}{a^2}\ln\left(\tan\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^2/(a+b*sin(x)),x)
```

```
[Out] 1/2/a*tan(1/2*x)+2/a^2*b^2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*x)+2*b)/
(a^2-b^2)^(1/2))-1/2/a/tan(1/2*x)-1/a^2*b*ln(tan(1/2*x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a+b*sin(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.18704, size = 752, normalized size = 12.13

$$\left[-\frac{\sqrt{-a^2+b^2}b^2\log\left(\frac{\left(2\,a^2-b^2\right)\cos(x)^2-2\,ab\sin(x)-a^2-b^2+2\,(a\cos(x)\sin(x)+b\cos(x))\sqrt{-a^2+b^2}}{b^2\cos(x)^2-2\,ab\sin(x)-a^2-b^2}\right)\sin(x)-\left(a^2b-b^3\right)\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)}{2\left(a^4-a^2b^2\right)\sin(x)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a+b*sin(x)),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(-a^2 + b^2)*b^2*log(((2*a^2 - b^2)*cos(x)^2 - 2*a*b*sin(x) - a^
2 - b^2 + 2*(a*cos(x)*sin(x) + b*cos(x))*sqrt(-a^2 + b^2))/(b^2*cos(x)^2 -
2*a*b*sin(x) - a^2 - b^2))*sin(x) - (a^2*b - b^3)*log(1/2*cos(x) + 1/2)*sin
(x) + (a^2*b - b^3)*log(-1/2*cos(x) + 1/2)*sin(x) + 2*(a^3 - a*b^2)*cos(x))
/((a^4 - a^2*b^2)*sin(x)), -1/2*(2*sqrt(a^2 - b^2)*b^2*arctan(-(a*sin(x) +
b)/(sqrt(a^2 - b^2)*cos(x)))*sin(x) - (a^2*b - b^3)*log(1/2*cos(x) + 1/2)*s
in(x) + (a^2*b - b^3)*log(-1/2*cos(x) + 1/2)*sin(x) + 2*(a^3 - a*b^2)*cos(x
))/((a^4 - a^2*b^2)*sin(x))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\csc^2(x)}{a+b\sin(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**2/(a+b*sin(x)),x)
```

```
[Out] Integral(csc(x)**2/(a + b*sin(x)), x)
```

Giac [A] time = 1.2401, size = 132, normalized size = 2.13

$$\frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}x\right)+b}{\sqrt{a^2-b^2}}\right)\right)b^2}{\sqrt{a^2-b^2}a^2}-\frac{b\log\left(\left|\tan\left(\frac{1}{2}x\right)\right|\right)}{a^2}+\frac{\tan\left(\frac{1}{2}x\right)}{2a}+\frac{2b\tan\left(\frac{1}{2}x\right)-a}{2a^2\tan\left(\frac{1}{2}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a+b*sin(x)),x, algorithm="giac")
```

```
[Out] 2*(pi*floor(1/2*x/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*x) + b)/sqrt(a^2 - b
^2)))*b^2/(sqrt(a^2 - b^2)*a^2) - b*log(abs(tan(1/2*x)))/a^2 + 1/2*tan(1/2*
x)/a + 1/2*(2*b*tan(1/2*x) - a)/(a^2*tan(1/2*x))
```

3.183 $\int \frac{\csc^3(x)}{a+b\sin(x)}\,dx$

Optimal. Leaf size=84

$$-\frac{2b^3\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{a^3\sqrt{a^2-b^2}}-\frac{\left(a^2+2b^2\right)\tanh^{-1}(\cos(x))}{2a^3}+\frac{b\cot(x)}{a^2}-\frac{\cot(x)\csc(x)}{2a}$$

```
[Out] (-2*b^3*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(a^3*Sqrt[a^2 - b^2]) - (
(a^2 + 2*b^2)*ArcTanh[Cos[x]])/(2*a^3) + (b*Cot[x])/a^2 - (Cot[x]*Csc[x])/(
2*a)
```

Rubi [A] time = 0.271398, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {2802, 3055, 3001, 3770, 2660, 618, 204}

$$-\frac{2b^3\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{a^3\sqrt{a^2-b^2}}-\frac{\left(a^2+2b^2\right)\tanh^{-1}(\cos(x))}{2a^3}+\frac{b\cot(x)}{a^2}-\frac{\cot(x)\csc(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^3/(a + b*Sin[x]),x]
```

```
[Out] (-2*b^3*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(a^3*Sqrt[a^2 - b^2]) - (
(a^2 + 2*b^2)*ArcTanh[Cos[x]])/(2*a^3) + (b*Cot[x])/a^2 - (Cot[x]*Csc[x])/(
2*a)
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
```

```
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\csc^3(x)}{a+b\sin(x)}\,dx &= -\frac{\cot(x)\csc(x)}{2a}+\frac{\int \frac{\csc^2(x)\left(-2b+a\sin(x)+b\sin^2(x)\right)}{a+b\sin(x)}\,dx}{2a}\\
&= \frac{b\cot(x)}{a^2}-\frac{\cot(x)\csc(x)}{2a}+\frac{\int \frac{\csc(x)\left(a^2+2b^2+ab\sin(x)\right)}{a+b\sin(x)}\,dx}{2a^2}\\
&= \frac{b\cot(x)}{a^2}-\frac{\cot(x)\csc(x)}{2a}-\frac{b^3\int \frac{1}{a+b\sin(x)}\,dx}{a^3}+\frac{\left(a^2+2b^2\right)\int \csc(x)\,dx}{2a^3}\\
&= -\frac{\left(a^2+2b^2\right)\tanh^{-1}(\cos(x))}{2a^3}+\frac{b\cot(x)}{a^2}-\frac{\cot(x)\csc(x)}{2a}-\frac{\left(2b^3\right)\text{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx,x,\tan\left(\frac{x}{2}\right)\right)}{a^3}\\
&= -\frac{\left(a^2+2b^2\right)\tanh^{-1}(\cos(x))}{2a^3}+\frac{b\cot(x)}{a^2}-\frac{\cot(x)\csc(x)}{2a}+\frac{\left(4b^3\right)\text{Subst}\left(\int \frac{1}{-4\left(a^2-b^2\right)-x^2}\,dx,x,2b+2a
\end{aligned}
$$

$$
= -\frac{2b^3\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{a^3\sqrt{a^2-b^2}}-\frac{\left(a^2+2b^2\right)\tanh^{-1}(\cos(x))}{2a^3}+\frac{b\cot(x)}{a^2}-\frac{\cot(x)\csc(x)}{2a}
$$

Mathematica [A] time = 0.47448, size = 144, normalized size = 1.71

$$\frac{-\frac{16b^3\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} - a^2\csc^2\left(\frac{x}{2}\right) + a^2\sec^2\left(\frac{x}{2}\right) + 4a^2\log\left(\sin\left(\frac{x}{2}\right)\right) - 4a^2\log\left(\cos\left(\frac{x}{2}\right)\right) - 4ab\tan\left(\frac{x}{2}\right) + 4ab\cot\left(\frac{x}{2}\right) +}{8a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^3/(a + b*Sin[x]),x]
```

```
[Out] ((-16*b^3*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] + 4*a*b
*Cot[x/2] - a^2*Csc[x/2]^2 - 4*a^2*Log[Cos[x/2]] - 8*b^2*Log[Cos[x/2]] + 4*
a^2*Log[Sin[x/2]] + 8*b^2*Log[Sin[x/2]] + a^2*Sec[x/2]^2 - 4*a*b*Tan[x/2])/
(8*a^3)
```

Maple [A] time = 0.05, size = 112, normalized size = 1.3

$$\frac{1}{8a}\left(\tan\left(\frac{x}{2}\right)\right)^2 - \frac{b}{2a^2}\tan\left(\frac{x}{2}\right) - 2\frac{b^3}{a^3\sqrt{a^2-b^2}}\arctan\left(1/2\,\frac{2a\tan(x/2)+2b}{\sqrt{a^2-b^2}}\right) - \frac{1}{8a}\left(\tan\left(\frac{x}{2}\right)\right)^{-2} + \frac{1}{2a}\ln\left(\tan\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^3/(a+b*sin(x)),x)
```

```
[Out] 1/8/a*tan(1/2*x)^2-1/2/a^2*tan(1/2*x)*b-2/a^3*b^3/(a^2-b^2)^(1/2)*arctan(1/
2*(2*a*tan(1/2*x)+2*b)/(a^2-b^2)^(1/2))-1/8/a/tan(1/2*x)^2+1/2/a*ln(tan(1/2
*x))+1/a^3*ln(tan(1/2*x))*b^2+1/2*b/a^2/tan(1/2*x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(a+b*sin(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.95383, size = 1131, normalized size = 13.46

$$\left[\frac{4\left(a^3b-ab^3\right)\cos(x)\sin(x) + 2\left(b^3\cos(x)^2-b^3\right)\sqrt{-a^2+b^2}\log\left(-\frac{\left(2a^2-b^2\right)\cos(x)^2-2ab\sin(x)-a^2-b^2-2\left(a\cos(x)\sin(x)+b\cos(x)\right)\sqrt{}}{b^2\cos(x)^2-2ab\sin(x)-a^2-b^2}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(a+b*sin(x)),x, algorithm="fricas")
```

```
[Out] [1/4*(4*(a^3*b - a*b^3)*cos(x)*sin(x) + 2*(b^3*cos(x)^2 - b^3)*sqrt(-a^2 +
b^2)*log(-((2*a^2 - b^2)*cos(x)^2 - 2*a*b*sin(x) - a^2 - b^2 - 2*(a*cos(x)*
```

```
sin(x) + b*cos(x))*sqrt(-a^2 + b^2))/(b^2*cos(x)^2 - 2*a*b*sin(x) - a^2 - b
^2)) - 2*(a^4 - a^2*b^2)*cos(x) - (a^4 + a^2*b^2 - 2*b^4 - (a^4 + a^2*b^2 -
 2*b^4)*cos(x)^2)*log(1/2*cos(x) + 1/2) + (a^4 + a^2*b^2 - 2*b^4 - (a^4 + a
^2*b^2 - 2*b^4)*cos(x)^2)*log(-1/2*cos(x) + 1/2))/(a^5 - a^3*b^2 - (a^5 - a
^3*b^2)*cos(x)^2), 1/4*(4*(a^3*b - a*b^3)*cos(x)*sin(x) - 4*(b^3*cos(x)^2 -
 b^3)*sqrt(a^2 - b^2)*arctan(-(a*sin(x) + b)/(sqrt(a^2 - b^2)*cos(x))) - 2*
(a^4 - a^2*b^2)*cos(x) - (a^4 + a^2*b^2 - 2*b^4 - (a^4 + a^2*b^2 - 2*b^4)*c
os(x)^2)*log(1/2*cos(x) + 1/2) + (a^4 + a^2*b^2 - 2*b^4 - (a^4 + a^2*b^2 -
2*b^4)*cos(x)^2)*log(-1/2*cos(x) + 1/2))/(a^5 - a^3*b^2 - (a^5 - a^3*b^2)*c
os(x)^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc^3(x)}{a + b\sin(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**3/(a+b*sin(x)),x)
```

```
[Out] Integral(csc(x)**3/(a + b*sin(x)), x)
```

Giac [A] time = 1.24654, size = 190, normalized size = 2.26

$$-\frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}x\right)+b}{\sqrt{a^2-b^2}}\right)\right)b^3}{\sqrt{a^2-b^2}a^3}+\frac{a\tan\left(\frac{1}{2}x\right)^2-4\,b\tan\left(\frac{1}{2}x\right)}{8\,a^2}+\frac{\left(a^2+2\,b^2\right)\log\left(\left|\tan\left(\frac{1}{2}x\right)\right|\right)}{2\,a^3}-\frac{6\,a^2\,\mathrm{ta}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(a+b*sin(x)),x, algorithm="giac")
```

```
[Out] -2*(pi*floor(1/2*x/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*x) + b)/sqrt(a^2 -
b^2)))*b^3/(sqrt(a^2 - b^2)*a^3) + 1/8*(a*tan(1/2*x)^2 - 4*b*tan(1/2*x))/a^
2 + 1/2*(a^2 + 2*b^2)*log(abs(tan(1/2*x)))/a^3 - 1/8*(6*a^2*tan(1/2*x)^2 +
12*b^2*tan(1/2*x)^2 - 4*a*b*tan(1/2*x) + a^2)/(a^3*tan(1/2*x)^2)
```

3.184 $\int \frac{\csc^4(x)}{a+b\sin(x)}\,dx$

Optimal. Leaf size=112

$$\frac{2b^4\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{a^4\sqrt{a^2-b^2}}-\frac{\left(2a^2+3b^2\right)\cot(x)}{3a^3}+\frac{b\left(a^2+2b^2\right)\tanh^{-1}(\cos(x))}{2a^4}+\frac{b\cot(x)\csc(x)}{2a^2}-\frac{\cot(x)\csc^2(x)}{3a}$$

```
[Out] (2*b^4*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(a^4*Sqrt[a^2 - b^2]) + (b
*(a^2 + 2*b^2)*ArcTanh[Cos[x]])/(2*a^4) - ((2*a^2 + 3*b^2)*Cot[x])/(3*a^3)
+ (b*Cot[x]*Csc[x])/(2*a^2) - (Cot[x]*Csc[x]^2)/(3*a)
```

Rubi [A] time = 0.433336, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {2802, 3055, 3001, 3770, 2660, 618, 204}

$$\frac{2b^4\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{a^4\sqrt{a^2-b^2}}-\frac{\left(2a^2+3b^2\right)\cot(x)}{3a^3}+\frac{b\left(a^2+2b^2\right)\tanh^{-1}(\cos(x))}{2a^4}+\frac{b\cot(x)\csc(x)}{2a^2}-\frac{\cot(x)\csc^2(x)}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^4/(a + b*Sin[x]),x]
```

```
[Out] (2*b^4*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(a^4*Sqrt[a^2 - b^2]) + (b
*(a^2 + 2*b^2)*ArcTanh[Cos[x]])/(2*a^4) - ((2*a^2 + 3*b^2)*Cot[x])/(3*a^3)
+ (b*Cot[x]*Csc[x])/(2*a^2) - (Cot[x]*Csc[x]^2)/(3*a)
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
```

```
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\csc^4(x)}{a+b\sin(x)}\,dx = -\frac{\cot(x)\csc^2(x)}{3a} + \frac{\int \frac{\csc^3(x)\left(-3b+2a\sin(x)+2b\sin^2(x)\right)}{a+b\sin(x)}\,dx}{3a}$$

$$= \frac{b\cot(x)\csc(x)}{2a^2} - \frac{\cot(x)\csc^2(x)}{3a} + \frac{\int \frac{\csc^2(x)\left(2\left(2a^2+3b^2\right)+ab\sin(x)-3b^2\sin^2(x)\right)}{a+b\sin(x)}\,dx}{6a^2}$$

$$= -\frac{\left(2a^2+3b^2\right)\cot(x)}{3a^3} + \frac{b\cot(x)\csc(x)}{2a^2} - \frac{\cot(x)\csc^2(x)}{3a} + \frac{\int \frac{\csc(x)\left(-3b\left(a^2+2b^2\right)-3ab^2\sin(x)\right)}{a+b\sin(x)}\,dx}{6a^3}$$

$$= -\frac{\left(2a^2+3b^2\right)\cot(x)}{3a^3} + \frac{b\cot(x)\csc(x)}{2a^2} - \frac{\cot(x)\csc^2(x)}{3a} + \frac{b^4\int \frac{1}{a+b\sin(x)}\,dx}{a^4} - \frac{\left(b\left(a^2+2b^2\right)\right)\int \csc(x}{2a^4}$$

$$= \frac{b\left(a^2+2b^2\right)\tanh^{-1}(\cos(x))}{2a^4} - \frac{\left(2a^2+3b^2\right)\cot(x)}{3a^3} + \frac{b\cot(x)\csc(x)}{2a^2} - \frac{\cot(x)\csc^2(x)}{3a} + \frac{\left(2b^4\right)\operatorname{Subst}}{}$$

$$= \frac{b\left(a^2+2b^2\right)\tanh^{-1}(\cos(x))}{2a^4} - \frac{\left(2a^2+3b^2\right)\cot(x)}{3a^3} + \frac{b\cot(x)\csc(x)}{2a^2} - \frac{\cot(x)\csc^2(x)}{3a} - \frac{\left(4b^4\right)\operatorname{Subst}}{}$$

$$= \frac{2b^4\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{a^4\sqrt{a^2-b^2}} + \frac{b\left(a^2+2b^2\right)\tanh^{-1}(\cos(x))}{2a^4} - \frac{\left(2a^2+3b^2\right)\cot(x)}{3a^3} + \frac{b\cot(x)\csc(x)}{2a^2} - \cot($$

Mathematica [A] time = 1.54879, size = 125, normalized size = 1.12

$$\frac{\frac{24b^4\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+a\left(2a^2+3b^2\right)\cos(3x)\csc^3(x)-3a\cot(x)\csc(x)\left(\left(2a^2+b^2\right)\csc(x)-2ab\right)+6b\left(a^2+2b^2\right)\left(\log\left(\right.\right.}{12a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^4/(a + b*Sin[x]),x]
```

```
[Out] ((24*b^4*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] + a*(2*a
^2 + 3*b^2)*Cos[3*x]*Csc[x]^3 - 3*a*Cot[x]*Csc[x]*(-2*a*b + (2*a^2 + b^2)*C
sc[x]) + 6*b*(a^2 + 2*b^2)*(Log[Cos[x/2]] - Log[Sin[x/2]]))/(12*a^4)
```

Maple [A] time = 0.051, size = 162, normalized size = 1.5

$$\frac{1}{24\,a}\left(\tan\left(\frac{x}{2}\right)\right)^3-\frac{b}{8\,a^2}\left(\tan\left(\frac{x}{2}\right)\right)^2+\frac{3}{8\,a}\tan\left(\frac{x}{2}\right)+\frac{b^2}{2\,a^3}\tan\left(\frac{x}{2}\right)+2\,\frac{b^4}{a^4\sqrt{a^2-b^2}}\arctan\left(1/2\,\frac{2\,a\tan(x/2)+2\,b}{\sqrt{a^2-b^2}}\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^4/(a+b*sin(x)),x)
```

```
[Out] 1/24/a*tan(1/2*x)^3-1/8/a^2*tan(1/2*x)^2*b+3/8/a*tan(1/2*x)+1/2/a^3*b^2*tan
(1/2*x)+2/a^4*b^4/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*x)+2*b)/(a^2-b^2)
^(1/2))-1/24/a/tan(1/2*x)^3-3/8/a/tan(1/2*x)-1/2/a^3/tan(1/2*x)*b^2+1/8/a^2
*b/tan(1/2*x)^2-1/2/a^2*b*ln(tan(1/2*x))-1/a^4*b^3*ln(tan(1/2*x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^4/(a+b*sin(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.05268, size = 1361, normalized size = 12.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^4/(a+b*sin(x)),x, algorithm="fricas")
```

```
[Out] [1/12*(4*(2*a^5 + a^3*b^2 - 3*a*b^4)*cos(x)^3 + 6*(b^4*cos(x)^2 - b^4)*sqrt
(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(x)^2 - 2*a*b*sin(x) - a^2 - b^2 + 2*(a*
cos(x)*sin(x) + b*cos(x))*sqrt(-a^2 + b^2))/(b^2*cos(x)^2 - 2*a*b*sin(x) -
a^2 - b^2))*sin(x) + 6*(a^4*b - a^2*b^3)*cos(x)*sin(x) + 3*(a^4*b + a^2*b^3
 - 2*b^5 - (a^4*b + a^2*b^3 - 2*b^5)*cos(x)^2)*log(1/2*cos(x) + 1/2)*sin(x)
```

```
 - 3*(a^4*b + a^2*b^3 - 2*b^5 - (a^4*b + a^2*b^3 - 2*b^5)*cos(x)^2)*log(-1/
2*cos(x) + 1/2)*sin(x) - 12*(a^5 - a*b^4)*cos(x))/((a^6 - a^4*b^2 - (a^6 -
a^4*b^2)*cos(x)^2)*sin(x)), 1/12*(4*(2*a^5 + a^3*b^2 - 3*a*b^4)*cos(x)^3 +
12*(b^4*cos(x)^2 - b^4)*sqrt(a^2 - b^2)*arctan(-(a*sin(x) + b)/(sqrt(a^2 -
b^2)*cos(x)))*sin(x) + 6*(a^4*b - a^2*b^3)*cos(x)*sin(x) + 3*(a^4*b + a^2*b
^3 - 2*b^5 - (a^4*b + a^2*b^3 - 2*b^5)*cos(x)^2)*log(1/2*cos(x) + 1/2)*sin(
x) - 3*(a^4*b + a^2*b^3 - 2*b^5 - (a^4*b + a^2*b^3 - 2*b^5)*cos(x)^2)*log(-
1/2*cos(x) + 1/2)*sin(x) - 12*(a^5 - a*b^4)*cos(x))/((a^6 - a^4*b^2 - (a^6
- a^4*b^2)*cos(x)^2)*sin(x))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc^4(x)}{a + b\sin(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**4/(a+b*sin(x)),x)
```

```
[Out] Integral(csc(x)**4/(a + b*sin(x)), x)
```

Giac [A] time = 1.50635, size = 262, normalized size = 2.34

$$\frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}x\right)+b}{\sqrt{a^2-b^2}}\right)\right)b^4}{\sqrt{a^2-b^2}\,a^4}+\frac{a^2\tan\left(\frac{1}{2}x\right)^3-3\,ab\tan\left(\frac{1}{2}x\right)^2+9\,a^2\tan\left(\frac{1}{2}x\right)+12\,b^2\tan\left(\frac{1}{2}x\right)}{24\,a^3}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^4/(a+b*sin(x)),x, algorithm="giac")
```

```
[Out] 2*(pi*floor(1/2*x/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*x) + b)/sqrt(a^2 - b
^2)))*b^4/(sqrt(a^2 - b^2)*a^4) + 1/24*(a^2*tan(1/2*x)^3 - 3*a*b*tan(1/2*x)
^2 + 9*a^2*tan(1/2*x) + 12*b^2*tan(1/2*x))/a^3 - 1/2*(a^2*b + 2*b^3)*log(ab
s(tan(1/2*x)))/a^4 + 1/24*(22*a^2*b*tan(1/2*x)^3 + 44*b^3*tan(1/2*x)^3 - 9*
a^3*tan(1/2*x)^2 - 12*a*b^2*tan(1/2*x)^2 + 3*a^2*b*tan(1/2*x) - a^3)/(a^4*t
an(1/2*x)^3)
```

3.185 $\int \frac{\sin^4(x)}{(a+b\sin(x))^2}\,dx$

Optimal. Leaf size=169

$$\frac{x\left(6a^2+b^2\right)}{2b^4}+\frac{a\left(3a^2-2b^2\right)\cos(x)}{b^3\left(a^2-b^2\right)}-\frac{2a^3\left(3a^2-4b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{b^4\left(a^2-b^2\right)^{3/2}}+\frac{a^2\sin^2(x)\cos(x)}{b\left(a^2-b^2\right)(a+b\sin(x))}-\frac{\left(3a^2-b^2\right)\sin(}{2b^2\left(a^2-\right.}$$

```
[Out] ((6*a^2 + b^2)*x)/(2*b^4) - (2*a^3*(3*a^2 - 4*b^2)*ArcTan[(b + a*Tan[x/2])/
Sqrt[a^2 - b^2]])/(b^4*(a^2 - b^2)^(3/2)) + (a*(3*a^2 - 2*b^2)*Cos[x])/(b^3
*(a^2 - b^2)) - ((3*a^2 - b^2)*Cos[x]*Sin[x])/(2*b^2*(a^2 - b^2)) + (a^2*Co
s[x]*Sin[x]^2)/(b*(a^2 - b^2)*(a + b*Sin[x]))
```

Rubi [A] time = 0.382792, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {2792, 3049, 3023, 2735, 2660, 618, 204}

$$\frac{x\left(6a^2+b^2\right)}{2b^4}+\frac{a\left(3a^2-2b^2\right)\cos(x)}{b^3\left(a^2-b^2\right)}-\frac{2a^3\left(3a^2-4b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{b^4\left(a^2-b^2\right)^{3/2}}+\frac{a^2\sin^2(x)\cos(x)}{b\left(a^2-b^2\right)(a+b\sin(x))}-\frac{\left(3a^2-b^2\right)\sin(}{2b^2\left(a^2-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^4/(a + b*Sin[x])^2,x]
```

```
[Out] ((6*a^2 + b^2)*x)/(2*b^4) - (2*a^3*(3*a^2 - 4*b^2)*ArcTan[(b + a*Tan[x/2])/
Sqrt[a^2 - b^2]])/(b^4*(a^2 - b^2)^(3/2)) + (a*(3*a^2 - 2*b^2)*Cos[x])/(b^3
*(a^2 - b^2)) - ((3*a^2 - b^2)*Cos[x]*Sin[x])/(2*b^2*(a^2 - b^2)) + (a^2*Co
s[x]*Sin[x]^2)/(b*(a^2 - b^2)*(a + b*Sin[x]))
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
```

```
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sin^4(x)}{(a+b\sin(x))^2}\,dx = \frac{a^2\cos(x)\sin^2(x)}{b\left(a^2-b^2\right)(a+b\sin(x))} - \frac{\int \frac{\sin(x)\left(2a^2-ab\sin(x)-\left(3a^2-b^2\right)\sin^2(x)\right)}{a+b\sin(x)}dx}{b\left(a^2-b^2\right)}$$

$$= -\frac{\left(3a^2-b^2\right)\cos(x)\sin(x)}{2b^2\left(a^2-b^2\right)} + \frac{a^2\cos(x)\sin^2(x)}{b\left(a^2-b^2\right)(a+b\sin(x))} - \frac{\int \frac{-a\left(3a^2-b^2\right)+b\left(a^2+b^2\right)\sin(x)+2a\left(3a^2-2b^2\right)\sin\cdots}{a+b\sin(x)}}{2b^2\left(a^2-b^2\right)}$$

$$= \frac{a\left(3a^2-2b^2\right)\cos(x)}{b^3\left(a^2-b^2\right)} - \frac{\left(3a^2-b^2\right)\cos(x)\sin(x)}{2b^2\left(a^2-b^2\right)} + \frac{a^2\cos(x)\sin^2(x)}{b\left(a^2-b^2\right)(a+b\sin(x))} - \frac{\int \frac{-ab\left(3a^2-b^2\right)-\left(a^2-\cdots\right.}{a+b\cdots}}{2b^3\left(a\cdots\right.}$$

$$= \frac{\left(6a^2+b^2\right)x}{2b^4} + \frac{a\left(3a^2-2b^2\right)\cos(x)}{b^3\left(a^2-b^2\right)} - \frac{\left(3a^2-b^2\right)\cos(x)\sin(x)}{2b^2\left(a^2-b^2\right)} + \frac{a^2\cos(x)\sin^2(x)}{b\left(a^2-b^2\right)(a+b\sin(x))} - \cdots$$

$$= \frac{\left(6a^2+b^2\right)x}{2b^4} + \frac{a\left(3a^2-2b^2\right)\cos(x)}{b^3\left(a^2-b^2\right)} - \frac{\left(3a^2-b^2\right)\cos(x)\sin(x)}{2b^2\left(a^2-b^2\right)} + \frac{a^2\cos(x)\sin^2(x)}{b\left(a^2-b^2\right)(a+b\sin(x))} - \cdots$$

$$= \frac{\left(6a^2+b^2\right)x}{2b^4} + \frac{a\left(3a^2-2b^2\right)\cos(x)}{b^3\left(a^2-b^2\right)} - \frac{\left(3a^2-b^2\right)\cos(x)\sin(x)}{2b^2\left(a^2-b^2\right)} + \frac{a^2\cos(x)\sin^2(x)}{b\left(a^2-b^2\right)(a+b\sin(x))} + \cdots$$

$$= \frac{\left(6a^2+b^2\right)x}{2b^4} - \frac{2a^3\left(3a^2-4b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{b^4\left(a^2-b^2\right)^{3/2}} + \frac{a\left(3a^2-2b^2\right)\cos(x)}{b^3\left(a^2-b^2\right)} - \frac{\left(3a^2-b^2\right)\cos(x)\,\mathrm{s}\cdots}{2b^2\left(a^2-b^2\right)}$$

Mathematica [A] time = 0.528877, size = 115, normalized size = 0.68

$$\frac{-\frac{8a^3\left(3a^2-4b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}} + 4ab\cos(x)\left(\frac{a^3}{(a-b)(a+b)(a+b\sin(x))}+2\right) + 12a^2x + 2b^2x - b^2\sin(2x)}{4b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^4/(a + b*Sin[x])^2,x]
```

```
[Out] (12*a^2*x + 2*b^2*x - (8*a^3*(3*a^2 - 4*b^2)*ArcTan[(b + a*Tan[x/2])/Sqrt[a
^2 - b^2]])/(a^2 - b^2)^(3/2) + 4*a*b*Cos[x]*(2 + a^3/((a - b)*(a + b)*(a +
 b*Sin[x]))) - b^2*Sin[2*x])/(4*b^4)
```

Maple [A] time = 0.051, size = 266, normalized size = 1.6

$$\frac{1}{b^2}\left(\tan\left(\frac{x}{2}\right)\right)^3\left(\left(\tan\left(\frac{x}{2}\right)\right)^2+1\right)^{-2} + 4\frac{(\tan(x/2))^2 a}{b^3\left((\tan(x/2))^2+1\right)^2} - \frac{1}{b^2}\tan\left(\frac{x}{2}\right)\left(\left(\tan\left(\frac{x}{2}\right)\right)^2+1\right)^{-2} + 4\frac{a}{b^3\left((\tan(x/2))^2+1\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^4/(a+b*sin(x))^2,x)
```

```
[Out] 1/b^2/(tan(1/2*x)^2+1)^2*tan(1/2*x)^3+4/b^3/(tan(1/2*x)^2+1)^2*tan(1/2*x)^2
*a-1/b^2/(tan(1/2*x)^2+1)^2*tan(1/2*x)+4/b^3/(tan(1/2*x)^2+1)^2*a+6/b^4*arc
tan(tan(1/2*x))*a^2+2*a^3/b^2/(tan(1/2*x)^2*a+2*tan(1/2*x)*b+a)/(a^2-b^2)*t
an(1/2*x)+2*a^4/b^3/(tan(1/2*x)^2*a+2*tan(1/2*x)*b+a)/(a^2-b^2)-6*a^5/b^4/(
```

```
a^2-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*x)+2*b)/(a^2-b^2)^(1/2))+8*a^3/b^2/(
a^2-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*x)+2*b)/(a^2-b^2)^(1/2))+1/2*x/b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^4/(a+b*sin(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.01208, size = 1277, normalized size = 7.56

$$\left[\frac{\left(a^4b^3 - 2\,a^2b^5 + b^7\right)\cos\left(x\right)^3 - \left(3\,a^6 - 4\,a^4b^2 + \left(3\,a^5b - 4\,a^3b^3\right)\sin\left(x\right)\right)\sqrt{-a^2 + b^2}\,\log\left(-\frac{\left(2\,a^2 - b^2\right)\cos\left(x\right)^2 - 2\,ab\sin\left(x\right) - a^2 - b^2 - 2\,(a \cdots}{b^2\cos\left(x\right)^2 - 2\,ab\sin \cdots}\right. \cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^4/(a+b*sin(x))^2,x, algorithm="fricas")
```

```
[Out] [1/2*((a^4*b^3 - 2*a^2*b^5 + b^7)*cos(x)^3 - (3*a^6 - 4*a^4*b^2 + (3*a^5*b
- 4*a^3*b^3)*sin(x))*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(x)^2 - 2*a*b*
sin(x) - a^2 - b^2 - 2*(a*cos(x)*sin(x) + b*cos(x))*sqrt(-a^2 + b^2))/(b^2*
cos(x)^2 - 2*a*b*sin(x) - a^2 - b^2)) + (6*a^7 - 11*a^5*b^2 + 4*a^3*b^4 + a
*b^6)*x + (6*a^6*b - 11*a^4*b^3 + 6*a^2*b^5 - b^7)*cos(x) + ((6*a^6*b - 11*
a^4*b^3 + 4*a^2*b^5 + b^7)*x + 3*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*cos(x))*sin(
x))/(a^5*b^4 - 2*a^3*b^6 + a*b^8 + (a^4*b^5 - 2*a^2*b^7 + b^9)*sin(x)), 1/2
*((a^4*b^3 - 2*a^2*b^5 + b^7)*cos(x)^3 + 2*(3*a^6 - 4*a^4*b^2 + (3*a^5*b -
4*a^3*b^3)*sin(x))*sqrt(a^2 - b^2)*arctan(-(a*sin(x) + b)/(sqrt(a^2 - b^2)*
cos(x))) + (6*a^7 - 11*a^5*b^2 + 4*a^3*b^4 + a*b^6)*x + (6*a^6*b - 11*a^4*b
^3 + 6*a^2*b^5 - b^7)*cos(x) + ((6*a^6*b - 11*a^4*b^3 + 4*a^2*b^5 + b^7)*x
+ 3*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*cos(x))*sin(x))/(a^5*b^4 - 2*a^3*b^6 + a*
b^8 + (a^4*b^5 - 2*a^2*b^7 + b^9)*sin(x))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**4/(a+b*sin(x))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.76866, size = 248, normalized size = 1.47

$$-\frac{2\left(3\,a^{5}-4\,a^{3}b^{2}\right)\left(\pi\left\lfloor\frac{x}{2\,\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}\left(a\right)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,x\right)+b}{\sqrt{a^{2}-b^{2}}}\right)\right)}{\left(a^{2}b^{4}-b^{6}\right)\sqrt{a^{2}-b^{2}}}+\frac{2\left(a^{3}b\tan\left(\frac{1}{2}\,x\right)+a^{4}\right)}{\left(a^{2}b^{3}-b^{5}\right)\left(a\tan\left(\frac{1}{2}\,x\right)^{2}+2\,b\tan\left(\frac{1}{2}\,x\right)+a\right)}+\frac{\left(6\,a^{2}\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^4/(a+b*sin(x))^2,x, algorithm="giac")
```

```
[Out] -2*(3*a^5 - 4*a^3*b^2)*(pi*floor(1/2*x/pi + 1/2)*sgn(a) + arctan((a*tan(1/2
*x) + b)/sqrt(a^2 - b^2)))/((a^2*b^4 - b^6)*sqrt(a^2 - b^2)) + 2*(a^3*b*tan
(1/2*x) + a^4)/((a^2*b^3 - b^5)*(a*tan(1/2*x)^2 + 2*b*tan(1/2*x) + a)) + 1/
2*(6*a^2 + b^2)*x/b^4 + (b*tan(1/2*x)^3 + 4*a*tan(1/2*x)^2 - b*tan(1/2*x) +
 4*a)/((tan(1/2*x)^2 + 1)^2*b^3)
```

3.186 $\int \frac{\sin^3(x)}{(a+b\sin(x))^2}\,dx$

Optimal. Leaf size=124

$$-\frac{(2a^2-b^2)\cos(x)}{b^2(a^2-b^2)}+\frac{2a^2(2a^2-3b^2)\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{b^3(a^2-b^2)^{3/2}}+\frac{a^2\sin(x)\cos(x)}{b(a^2-b^2)(a+b\sin(x))}-\frac{2ax}{b^3}$$

```
[Out] (-2*a*x)/b^3 + (2*a^2*(2*a^2 - 3*b^2)*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^
2]])/(b^3*(a^2 - b^2)^(3/2)) - ((2*a^2 - b^2)*Cos[x])/(b^2*(a^2 - b^2)) + (
a^2*Cos[x]*Sin[x])/(b*(a^2 - b^2)*(a + b*Sin[x]))
```

Rubi [A] time = 0.218285, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2792, 3023, 2735, 2660, 618, 204}

$$-\frac{(2a^2-b^2)\cos(x)}{b^2(a^2-b^2)}+\frac{2a^2(2a^2-3b^2)\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{b^3(a^2-b^2)^{3/2}}+\frac{a^2\sin(x)\cos(x)}{b(a^2-b^2)(a+b\sin(x))}-\frac{2ax}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^3/(a + b*Sin[x])^2,x]
```

```
[Out] (-2*a*x)/b^3 + (2*a^2*(2*a^2 - 3*b^2)*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^
2]])/(b^3*(a^2 - b^2)^(3/2)) - ((2*a^2 - b^2)*Cos[x])/(b^2*(a^2 - b^2)) + (
a^2*Cos[x]*Sin[x])/(b*(a^2 - b^2)*(a + b*Sin[x]))
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
```

```
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^3(x)}{(a+b\sin(x))^2}\,dx &= \frac{a^2\cos(x)\sin(x)}{b\left(a^2-b^2\right)(a+b\sin(x))} - \frac{\int \frac{a^2-ab\sin(x)-\left(2a^2-b^2\right)\sin^2(x)}{a+b\sin(x)}\,dx}{b\left(a^2-b^2\right)}\\
&= -\frac{\left(2a^2-b^2\right)\cos(x)}{b^2\left(a^2-b^2\right)} + \frac{a^2\cos(x)\sin(x)}{b\left(a^2-b^2\right)(a+b\sin(x))} - \frac{\int \frac{a^2b+2a\left(a^2-b^2\right)\sin(x)}{a+b\sin(x)}\,dx}{b^2\left(a^2-b^2\right)}\\
&= -\frac{2ax}{b^3} - \frac{\left(2a^2-b^2\right)\cos(x)}{b^2\left(a^2-b^2\right)} + \frac{a^2\cos(x)\sin(x)}{b\left(a^2-b^2\right)(a+b\sin(x))} + \frac{\left(a^2\left(2a^2-3b^2\right)\right)\int \frac{1}{a+b\sin(x)}\,dx}{b^3\left(a^2-b^2\right)}\\
&= -\frac{2ax}{b^3} - \frac{\left(2a^2-b^2\right)\cos(x)}{b^2\left(a^2-b^2\right)} + \frac{a^2\cos(x)\sin(x)}{b\left(a^2-b^2\right)(a+b\sin(x))} + \frac{\left(2a^2\left(2a^2-3b^2\right)\right)\text{Subst}\left(\int \frac{1}{a+2bx+ax^2}\right.}{b^3\left(a^2-b^2\right)}\\
&= -\frac{2ax}{b^3} - \frac{\left(2a^2-b^2\right)\cos(x)}{b^2\left(a^2-b^2\right)} + \frac{a^2\cos(x)\sin(x)}{b\left(a^2-b^2\right)(a+b\sin(x))} - \frac{\left(4a^2\left(2a^2-3b^2\right)\right)\text{Subst}\left(\int \frac{1}{-4\left(a^2-b^2\right)-}\right.}{b^3\left(a^2-b^2\right)}\\
&= -\frac{2ax}{b^3} + \frac{2a^2\left(2a^2-3b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{b^3\left(a^2-b^2\right)^{3/2}} - \frac{\left(2a^2-b^2\right)\cos(x)}{b^2\left(a^2-b^2\right)} + \frac{a^2\cos(x)\sin(x)}{b\left(a^2-b^2\right)(a+b\sin(x))}
\end{aligned}$$

Mathematica [A] time = 0.391117, size = 94, normalized size = 0.76

$$\frac{\frac{2a^2\left(2a^2-3b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}} + b\cos(x)\left(-\frac{a^3}{(a-b)(a+b)(a+b\sin(x))}-1\right) - 2ax}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^3/(a + b*Sin[x])^2,x]
```

```
[Out] (-2*a*x + (2*a^2*(2*a^2 - 3*b^2)*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/
(a^2 - b^2)^(3/2) + b*Cos[x]*(-1 - a^3/((a - b)*(a + b)*(a + b*Sin[x]))))/b
```

^3

Maple [A] time = 0.05, size = 196, normalized size = 1.6

$$-2\frac{1}{b^2\left((\tan(x/2))^2+1\right)}-4\frac{a\arctan(\tan(x/2))}{b^3}-2\frac{a^2\tan(x/2)}{b\left((\tan(x/2))^2a+2\tan(x/2)b+a\right)(a^2-b^2)}-2\frac{}{b^2\left((\tan(x/2))^2a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3/(a+b*sin(x))^2,x)
```

```
[Out] -2/b^2/(tan(1/2*x)^2+1)-4/b^3*a*arctan(tan(1/2*x))-2*a^2/b/(tan(1/2*x)^2*a+
2*tan(1/2*x)*b+a)/(a^2-b^2)*tan(1/2*x)-2*a^3/b^2/(tan(1/2*x)^2*a+2*tan(1/2*
x)*b+a)/(a^2-b^2)+4*a^4/b^3/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*x)+2*b)
/(a^2-b^2)^(1/2))-6*a^2/b/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*x)+2*b)/(
a^2-b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(a+b*sin(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.94299, size = 1071, normalized size = 8.64

$$\left[-\frac{\left(2a^5-3a^3b^2+\left(2a^4b-3a^2b^3\right)\sin(x)\right)\sqrt{-a^2+b^2}\log\left(\frac{(2a^2-b^2)\cos(x)^2-2ab\sin(x)-a^2-b^2+2(a\cos(x)\sin(x)+b\cos(x))\sqrt{-a^2+b^2}}{b^2\cos(x)^2-2ab\sin(x)-a^2-b^2}\right)+4}{2\left(a^5b^3-2a^3b^5+ab^7+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(a+b*sin(x))^2,x, algorithm="fricas")
```

```
[Out] [-1/2*((2*a^5 - 3*a^3*b^2 + (2*a^4*b - 3*a^2*b^3)*sin(x))*sqrt(-a^2 + b^2)*
log(((2*a^2 - b^2)*cos(x)^2 - 2*a*b*sin(x) - a^2 - b^2 + 2*(a*cos(x)*sin(x)
 + b*cos(x))*sqrt(-a^2 + b^2))/(b^2*cos(x)^2 - 2*a*b*sin(x) - a^2 - b^2)) +
 4*(a^6 - 2*a^4*b^2 + a^2*b^4)*x + 2*(2*a^5*b - 3*a^3*b^3 + a*b^5)*cos(x) +
 2*(2*(a^5*b - 2*a^3*b^3 + a*b^5)*x + (a^4*b^2 - 2*a^2*b^4 + b^6)*cos(x))*s
in(x))/(a^5*b^3 - 2*a^3*b^5 + a*b^7 + (a^4*b^4 - 2*a^2*b^6 + b^8)*sin(x)),
-((2*a^5 - 3*a^3*b^2 + (2*a^4*b - 3*a^2*b^3)*sin(x))*sqrt(a^2 - b^2)*arctan
(-(a*sin(x) + b)/(sqrt(a^2 - b^2)*cos(x))) + 2*(a^6 - 2*a^4*b^2 + a^2*b^4)*
x + (2*a^5*b - 3*a^3*b^3 + a*b^5)*cos(x) + (2*(a^5*b - 2*a^3*b^3 + a*b^5)*x
 + (a^4*b^2 - 2*a^2*b^4 + b^6)*cos(x))*sin(x))/(a^5*b^3 - 2*a^3*b^5 + a*b^7
 + (a^4*b^4 - 2*a^2*b^6 + b^8)*sin(x))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**3/(a+b*sin(x))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.86384, size = 275, normalized size = 2.22

$$\frac{2\left(2a^4-3a^2b^2\right)\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}x\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^2b^3-b^5\right)\sqrt{a^2-b^2}}-\frac{2\left(a^2b\tan\left(\frac{1}{2}x\right)^3+2a^3\tan\left(\frac{1}{2}x\right)^2-ab^2\tan\left(\frac{1}{2}x\right)^2+\right.}{\left(a\tan\left(\frac{1}{2}x\right)^4+2b\tan\left(\frac{1}{2}x\right)^3+2a\tan\left(\frac{1}{2}x\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(a+b*sin(x))^2,x, algorithm="giac")
```

```
[Out] 2*(2*a^4 - 3*a^2*b^2)*(pi*floor(1/2*x/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*
x) + b)/sqrt(a^2 - b^2)))/((a^2*b^3 - b^5)*sqrt(a^2 - b^2)) - 2*(a^2*b*tan(
1/2*x)^3 + 2*a^3*tan(1/2*x)^2 - a*b^2*tan(1/2*x)^2 + 3*a^2*b*tan(1/2*x) - 2
*b^3*tan(1/2*x) + 2*a^3 - a*b^2)/((a*tan(1/2*x)^4 + 2*b*tan(1/2*x)^3 + 2*a*
tan(1/2*x)^2 + 2*b*tan(1/2*x) + a)*(a^2*b^2 - b^4)) - 2*a*x/b^3
```

3.187 $\int \frac{\sin^2(x)}{(a+b\sin(x))^2}\,dx$

Optimal. Leaf size=87

$$-\frac{2a\left(a^2-2b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{b^2\left(a^2-b^2\right)^{3/2}}+\frac{a^2\cos(x)}{b\left(a^2-b^2\right)(a+b\sin(x))}+\frac{x}{b^2}$$

```
[Out] x/b^2 - (2*a*(a^2 - 2*b^2)*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(b^2*(
a^2 - b^2)^(3/2)) + (a^2*Cos[x])/(b*(a^2 - b^2)*(a + b*Sin[x]))
```

Rubi [A] time = 0.125101, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2790, 2735, 2660, 618, 204}

$$-\frac{2a\left(a^2-2b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{b^2\left(a^2-b^2\right)^{3/2}}+\frac{a^2\cos(x)}{b\left(a^2-b^2\right)(a+b\sin(x))}+\frac{x}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^2/(a + b*Sin[x])^2,x]
```

```
[Out] x/b^2 - (2*a*(a^2 - 2*b^2)*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(b^2*(
a^2 - b^2)^(3/2)) + (a^2*Cos[x])/(b*(a^2 - b^2)*(a + b*Sin[x]))
```

Rule 2790

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos[e +
 f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] - Dist[1/
(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m + 1)*(2
*b*c*d - a*(c^2 + d^2)) + (a^2*d^2 - 2*a*b*c*d*(m + 2) + b^2*(d^2*(m + 1) +
 c^2*(m + 2)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sin^2(x)}{(a+b\sin(x))^2}\,dx &= \frac{a^2\cos(x)}{b\left(a^2-b^2\right)(a+b\sin(x))}+\frac{\int \frac{ab+\left(a^2-b^2\right)\sin(x)}{a+b\sin(x)}\,dx}{b\left(a^2-b^2\right)}\\
&= \frac{x}{b^2}+\frac{a^2\cos(x)}{b\left(a^2-b^2\right)(a+b\sin(x))}-\frac{\left(a\left(a^2-2b^2\right)\right)\int \frac{1}{a+b\sin(x)}\,dx}{b^2\left(a^2-b^2\right)}\\
&= \frac{x}{b^2}+\frac{a^2\cos(x)}{b\left(a^2-b^2\right)(a+b\sin(x))}-\frac{\left(2a\left(a^2-2b^2\right)\right)\mathrm{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx,x,\tan\left(\frac{x}{2}\right)\right)}{b^2\left(a^2-b^2\right)}\\
&= \frac{x}{b^2}+\frac{a^2\cos(x)}{b\left(a^2-b^2\right)(a+b\sin(x))}+\frac{\left(4a\left(a^2-2b^2\right)\right)\mathrm{Subst}\left(\int \frac{1}{-4\left(a^2-b^2\right)-x^2}\,dx,x,2b+2a\tan\left(\frac{x}{2}\right)\right)}{b^2\left(a^2-b^2\right)}\\
&= \frac{x}{b^2}-\frac{2a\left(a^2-2b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{b^2\left(a^2-b^2\right)^{3/2}}+\frac{a^2\cos(x)}{b\left(a^2-b^2\right)(a+b\sin(x))}
\end{aligned}
$$

Mathematica [A] time = 0.214287, size = 83, normalized size = 0.95

$$
\frac{-\frac{2a\left(a^2-2b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}}+\frac{a^2b\cos(x)}{(a-b)(a+b)(a+b\sin(x))}+x}{b^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^2/(a + b*Sin[x])^2,x]
```

```
[Out] (x - (2*a*(a^2 - 2*b^2)*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(a^2 - b^
2)^(3/2) + (a^2*b*Cos[x])/((a - b)*(a + b)*(a + b*Sin[x])))/b^2
```

Maple [B] time = 0.045, size = 170, normalized size = 2.

$$
2\,\frac{\arctan\left(\tan\left(x/2\right)\right)}{b^2}+2\,\frac{a\tan\left(x/2\right)}{\left(\left(\tan\left(x/2\right)\right)^2a+2\,\tan\left(x/2\right)b+a\right)\left(a^2-b^2\right)}+2\,\frac{a^2}{b\left(\left(\tan\left(x/2\right)\right)^2a+2\,\tan\left(x/2\right)b+a\right)\left(a^2-b\right.}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2/(a+b*sin(x))^2,x)
```

```
[Out] 2/b^2*arctan(tan(1/2*x))+2*a/(tan(1/2*x)^2*a+2*tan(1/2*x)*b+a)/(a^2-b^2)*ta
n(1/2*x)+2*a^2/b/(tan(1/2*x)^2*a+2*tan(1/2*x)*b+a)/(a^2-b^2)-2*a^3/b^2/(a^2
-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*x)+2*b)/(a^2-b^2)^(1/2))+4*a/(a^2-b^2)^
(3/2)*arctan(1/2*(2*a*tan(1/2*x)+2*b)/(a^2-b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a+b*sin(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.85131, size = 896, normalized size = 10.3

$$\left[\frac{2\left(a^4b-2a^2b^3+b^5\right)x\sin(x)-\left(a^4-2a^2b^2+\left(a^3b-2ab^3\right)\sin(x)\right)\sqrt{-a^2+b^2}\log\left(-\frac{\left(2a^2-b^2\right)\cos(x)^2-2ab\sin(x)-a^2-b^2-2(a\cos(\ldots}{b^2\cos(x)^2-2ab\sin(x)-\ldots}\right.}{2\left(a^5b^2-2a^3b^4+ab^6+\left(a^4b^3-2a^2b^5+b^7\right)\sin(x\ldots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a+b*sin(x))^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*(a^4*b - 2*a^2*b^3 + b^5)*x*sin(x) - (a^4 - 2*a^2*b^2 + (a^3*b - 2*
a*b^3)*sin(x))*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(x)^2 - 2*a*b*sin(x)
 - a^2 - b^2 - 2*(a*cos(x)*sin(x) + b*cos(x))*sqrt(-a^2 + b^2))/(b^2*cos(x)
^2 - 2*a*b*sin(x) - a^2 - b^2)) + 2*(a^5 - 2*a^3*b^2 + a*b^4)*x + 2*(a^4*b
- a^2*b^3)*cos(x))/(a^5*b^2 - 2*a^3*b^4 + a*b^6 + (a^4*b^3 - 2*a^2*b^5 + b^
7)*sin(x)), ((a^4*b - 2*a^2*b^3 + b^5)*x*sin(x) + (a^4 - 2*a^2*b^2 + (a^3*b
 - 2*a*b^3)*sin(x))*sqrt(a^2 - b^2)*arctan(-(a*sin(x) + b)/(sqrt(a^2 - b^2)
*cos(x))) + (a^5 - 2*a^3*b^2 + a*b^4)*x + (a^4*b - a^2*b^3)*cos(x))/(a^5*b^
2 - 2*a^3*b^4 + a*b^6 + (a^4*b^3 - 2*a^2*b^5 + b^7)*sin(x))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**2/(a+b*sin(x))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.93779, size = 167, normalized size = 1.92

$$-\frac{2\left(a^3-2ab^2\right)\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}x\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^2b^2-b^4\right)\sqrt{a^2-b^2}}+\frac{2\left(ab\tan\left(\frac{1}{2}x\right)+a^2\right)}{\left(a^2b-b^3\right)\left(a\tan\left(\frac{1}{2}x\right)^2+2b\tan\left(\frac{1}{2}x\right)+a\right)}+\frac{x}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a+b*sin(x))^2,x, algorithm="giac")
```

```
[Out] -2*(a^3 - 2*a*b^2)*(pi*floor(1/2*x/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*x)
+ b)/sqrt(a^2 - b^2)))/((a^2*b^2 - b^4)*sqrt(a^2 - b^2)) + 2*(a*b*tan(1/2*x
) + a^2)/((a^2*b - b^3)*(a*tan(1/2*x)^2 + 2*b*tan(1/2*x) + a)) + x/b^2
```

3.188 $\int \frac{\sin(x)}{(a+b\sin(x))^2}\,dx$

Optimal. Leaf size=66

$$-\frac{2b\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}}-\frac{a\cos(x)}{\left(a^2-b^2\right)(a+b\sin(x))}$$

```
[Out] (-2*b*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2) - (a*Cos[
x])/((a^2 - b^2)*(a + b*Sin[x]))
```

Rubi [A] time = 0.0633352, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {2754, 12, 2660, 618, 204}

$$-\frac{2b\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}}-\frac{a\cos(x)}{\left(a^2-b^2\right)(a+b\sin(x))}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]/(a + b*Sin[x])^2,x]
```

```
[Out] (-2*b*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2) - (a*Cos[
x])/((a^2 - b^2)*(a + b*Sin[x]))
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sin(x)}{(a+b\sin(x))^2}\,dx &= -\frac{a\cos(x)}{(a^2-b^2)(a+b\sin(x))}+\frac{\int \frac{b}{a+b\sin(x)}\,dx}{-a^2+b^2}\\
&= -\frac{a\cos(x)}{(a^2-b^2)(a+b\sin(x))}-\frac{b\int \frac{1}{a+b\sin(x)}\,dx}{a^2-b^2}\\
&= -\frac{a\cos(x)}{(a^2-b^2)(a+b\sin(x))}-\frac{(2b)\,\text{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx,x,\tan\left(\frac{x}{2}\right)\right)}{a^2-b^2}\\
&= -\frac{a\cos(x)}{(a^2-b^2)(a+b\sin(x))}+\frac{(4b)\,\text{Subst}\left(\int \frac{1}{-4(a^2-b^2)-x^2}\,dx,x,2b+2a\tan\left(\frac{x}{2}\right)\right)}{a^2-b^2}\\
&= -\frac{2b\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}}-\frac{a\cos(x)}{(a^2-b^2)(a+b\sin(x))}
\end{aligned}
$$

Mathematica [A] time = 0.105275, size = 67, normalized size = 1.02

$$
-\frac{2b\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}}-\frac{a\cos(x)}{(a-b)(a+b)(a+b\sin(x))}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/(a + b*Sin[x])^2,x]
```

```
[Out] (-2*b*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2) - (a*Cos[
x])/((a - b)*(a + b)*(a + b*Sin[x]))
```

Maple [A] time = 0.039, size = 99, normalized size = 1.5

$$
4\,\frac{-2\,\tan(x/2)\,b-2\,a}{(4\,a^2-4\,b^2)\left((\tan(x/2))^2\,a+2\,\tan(x/2)\,b+a\right)}-8\,\frac{b}{(4\,a^2-4\,b^2)\sqrt{a^2-b^2}}\arctan\left(1/2\,\frac{2\,a\tan(x/2)+2\,b}{\sqrt{a^2-b^2}}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(a+b*sin(x))^2,x)
```

```
[Out] 4*(-2*tan(1/2*x)*b-2*a)/(4*a^2-4*b^2)/(tan(1/2*x)^2*a+2*tan(1/2*x)*b+a)-8*b
/(4*a^2-4*b^2)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*x)+2*b)/(a^2-b^2)^(1
/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+b*sin(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.72511, size = 612, normalized size = 9.27

$$\left[\frac{\left(b^2\sin(x)+ab\right)\sqrt{-a^2+b^2}\log\left(\frac{\left(2a^2-b^2\right)\cos(x)^2-2ab\sin(x)-a^2-b^2+2\left(a\cos(x)\sin(x)+b\cos(x)\right)\sqrt{-a^2+b^2}}{b^2\cos(x)^2-2ab\sin(x)-a^2-b^2}\right)-2\left(a^3-ab^2\right)\cos(x)}{2\left(a^5-2a^3b^2+ab^4+\left(a^4b-2a^2b^3+b^5\right)\sin(x)\right)}, \frac{\left(b^2\sin\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+b*sin(x))^2,x, algorithm="fricas")
```

```
[Out] [1/2*((b^2*sin(x) + a*b)*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(x)^2 - 2*a
*b*sin(x) - a^2 - b^2 + 2*(a*cos(x)*sin(x) + b*cos(x))*sqrt(-a^2 + b^2))/(b
^2*cos(x)^2 - 2*a*b*sin(x) - a^2 - b^2)) - 2*(a^3 - a*b^2)*cos(x))/(a^5 - 2
*a^3*b^2 + a*b^4 + (a^4*b - 2*a^2*b^3 + b^5)*sin(x)), ((b^2*sin(x) + a*b)*s
qrt(a^2 - b^2)*arctan(-(a*sin(x) + b)/(sqrt(a^2 - b^2)*cos(x))) - (a^3 - a*
b^2)*cos(x))/(a^5 - 2*a^3*b^2 + a*b^4 + (a^4*b - 2*a^2*b^3 + b^5)*sin(x))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+b*sin(x))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.84589, size = 122, normalized size = 1.85

$$-\frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}x\right)+b}{\sqrt{a^2-b^2}}\right)\right)b}{\left(a^2-b^2\right)^{\frac{3}{2}}}-\frac{2\left(b\tan\left(\frac{1}{2}x\right)+a\right)}{\left(a\tan\left(\frac{1}{2}x\right)^2+2b\tan\left(\frac{1}{2}x\right)+a\right)\left(a^2-b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+b*sin(x))^2,x, algorithm="giac")
```

```
[Out] -2*(pi*floor(1/2*x/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*x) + b)/sqrt(a^2 -
b^2)))*b/(a^2 - b^2)^(3/2) - 2*(b*tan(1/2*x) + a)/((a*tan(1/2*x)^2 + 2*b*ta
n(1/2*x) + a)*(a^2 - b^2))
```

3.189 $\int \frac{1}{(a+b\sin(x))^2}\,dx$

Optimal. Leaf size=65

$$\frac{2a\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}}+\frac{b\cos(x)}{\left(a^2-b^2\right)(a+b\sin(x))}$$

```
[Out] (2*a*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2) + (b*Cos[x
])/((a^2 - b^2)*(a + b*Sin[x]))
```

Rubi [A] time = 0.0508134, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {2664, 12, 2660, 618, 204}

$$\frac{2a\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}}+\frac{b\cos(x)}{\left(a^2-b^2\right)(a+b\sin(x))}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[x])^(-2),x]
```

```
[Out] (2*a*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2) + (b*Cos[x
])/((a^2 - b^2)*(a + b*Sin[x]))
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+b\sin(x))^2}\,dx &= \frac{b\cos(x)}{\left(a^2-b^2\right)(a+b\sin(x))}-\frac{\int \frac{a}{a+b\sin(x)}\,dx}{-a^2+b^2}\\
&= \frac{b\cos(x)}{\left(a^2-b^2\right)(a+b\sin(x))}+\frac{a\int \frac{1}{a+b\sin(x)}\,dx}{a^2-b^2}\\
&= \frac{b\cos(x)}{\left(a^2-b^2\right)(a+b\sin(x))}+\frac{(2a)\,\mathrm{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx,x,\tan\left(\frac{x}{2}\right)\right)}{a^2-b^2}\\
&= \frac{b\cos(x)}{\left(a^2-b^2\right)(a+b\sin(x))}-\frac{(4a)\,\mathrm{Subst}\left(\int \frac{1}{-4\left(a^2-b^2\right)-x^2}\,dx,x,2b+2a\tan\left(\frac{x}{2}\right)\right)}{a^2-b^2}\\
&= \frac{2a\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}}+\frac{b\cos(x)}{\left(a^2-b^2\right)(a+b\sin(x))}
\end{aligned}$$

Mathematica [A] time = 0.0959064, size = 66, normalized size = 1.02

$$\frac{2a\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}}+\frac{b\cos(x)}{(a-b)(a+b)(a+b\sin(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[x])^(-2),x]
```

```
[Out] (2*a*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2) + (b*Cos[x
])/((a - b)*(a + b)*(a + b*Sin[x]))
```

Maple [A] time = 0.038, size = 98, normalized size = 1.5

$$2\,\frac{1}{(\tan(x/2))^2\,a+2\,\tan(x/2)\,b+a}\left(\frac{b^2\tan(x/2)}{a\left(a^2-b^2\right)}+\frac{b}{a^2-b^2}\right)+2\,\frac{a}{\left(a^2-b^2\right)^{3/2}}\arctan\left(1/2\,\frac{2\,a\tan(x/2)+2\,b}{\sqrt{a^2-b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(x))^2,x)
```

```
[Out] 2*(b^2/a/(a^2-b^2)*tan(1/2*x)+1/(a^2-b^2)*b)/(tan(1/2*x)^2*a+2*tan(1/2*x)*b
+a)+2*a/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*x)+2*b)/(a^2-b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.81296, size = 614, normalized size = 9.45

$$\left[\frac{\left(ab\sin\left(x\right)+a^2\right)\sqrt{-a^2+b^2}\log\left(-\frac{\left(2\,a^2-b^2\right)\cos\left(x\right)^2-2\,ab\sin\left(x\right)-a^2-b^2-2\left(a\cos\left(x\right)\sin\left(x\right)+b\cos\left(x\right)\right)\sqrt{-a^2+b^2}}{b^2\cos\left(x\right)^2-2\,ab\sin\left(x\right)-a^2-b^2}\right)+2\left(a^2b-b^3\right)\cos\left(x\right)}{2\left(a^5-2\,a^3b^2+ab^4+\left(a^4b-2\,a^2b^3+b^5\right)\sin\left(x\right)\right)},-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(x))^2,x, algorithm="fricas")
```

```
[Out] [1/2*((a*b*sin(x) + a^2)*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(x)^2 - 2*
a*b*sin(x) - a^2 - b^2 - 2*(a*cos(x)*sin(x) + b*cos(x))*sqrt(-a^2 + b^2))/(
b^2*cos(x)^2 - 2*a*b*sin(x) - a^2 - b^2)) + 2*(a^2*b - b^3)*cos(x))/(a^5 -
2*a^3*b^2 + a*b^4 + (a^4*b - 2*a^2*b^3 + b^5)*sin(x)), -((a*b*sin(x) + a^2)
*sqrt(a^2 - b^2)*arctan(-(a*sin(x) + b)/(sqrt(a^2 - b^2)*cos(x))) - (a^2*b
- b^3)*cos(x))/(a^5 - 2*a^3*b^2 + a*b^4 + (a^4*b - 2*a^2*b^3 + b^5)*sin(x))
]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(x))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.66169, size = 128, normalized size = 1.97

$$\frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}\left(a\right)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,x\right)+b}{\sqrt{a^2-b^2}}\right)\right)a}{\left(a^2-b^2\right)^{\frac{3}{2}}}+\frac{2\left(b^2\tan\left(\frac{1}{2}\,x\right)+ab\right)}{\left(a^3-ab^2\right)\left(a\tan\left(\frac{1}{2}\,x\right)^2+2\,b\tan\left(\frac{1}{2}\,x\right)+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(x))^2,x, algorithm="giac")
```

```
[Out] 2*(pi*floor(1/2*x/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*x) + b)/sqrt(a^2 - b
^2)))*a/(a^2 - b^2)^(3/2) + 2*(b^2*tan(1/2*x) + a*b)/((a^3 - a*b^2)*(a*tan(
1/2*x)^2 + 2*b*tan(1/2*x) + a))
```

3.190 $\int \frac{\csc(x)}{(a+b\sin(x))^2}\,dx$

Optimal. Leaf size=93

$$-\frac{2b\left(2a^2-b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{a^2\left(a^2-b^2\right)^{3/2}}-\frac{b^2\cos(x)}{a\left(a^2-b^2\right)(a+b\sin(x))}-\frac{\tanh^{-1}(\cos(x))}{a^2}$$

```
[Out] (-2*b*(2*a^2 - b^2)*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(a^2*(a^2 - b
^2)^(3/2)) - ArcTanh[Cos[x]]/a^2 - (b^2*Cos[x])/(a*(a^2 - b^2)*(a + b*Sin[x
]))
```

Rubi [A] time = 0.179824, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {2802, 3001, 3770, 2660, 618, 204}

$$-\frac{2b\left(2a^2-b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{a^2\left(a^2-b^2\right)^{3/2}}-\frac{b^2\cos(x)}{a\left(a^2-b^2\right)(a+b\sin(x))}-\frac{\tanh^{-1}(\cos(x))}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]/(a + b*Sin[x])^2,x]
```

```
[Out] (-2*b*(2*a^2 - b^2)*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(a^2*(a^2 - b
^2)^(3/2)) - ArcTanh[Cos[x]]/a^2 - (b^2*Cos[x])/(a*(a^2 - b^2)*(a + b*Sin[x
]))
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\csc(x)}{(a+b\sin(x))^2}\,dx = -\frac{b^2\cos(x)}{a\left(a^2-b^2\right)(a+b\sin(x))}+\frac{\int \frac{\csc(x)\left(a^2-b^2-ab\sin(x)\right)}{a+b\sin(x)}\,dx}{a\left(a^2-b^2\right)}$$

$$= -\frac{b^2\cos(x)}{a\left(a^2-b^2\right)(a+b\sin(x))}+\frac{\int \csc(x)\,dx}{a^2}-\frac{\left(b\left(2a^2-b^2\right)\right)\int \frac{1}{a+b\sin(x)}\,dx}{a^2\left(a^2-b^2\right)}$$

$$= -\frac{\tanh^{-1}(\cos(x))}{a^2}-\frac{b^2\cos(x)}{a\left(a^2-b^2\right)(a+b\sin(x))}-\frac{\left(2b\left(2a^2-b^2\right)\right)\text{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx,x,\tan\left(\right.\right.}{a^2\left(a^2-b^2\right)}$$

$$= -\frac{\tanh^{-1}(\cos(x))}{a^2}-\frac{b^2\cos(x)}{a\left(a^2-b^2\right)(a+b\sin(x))}+\frac{\left(4b\left(2a^2-b^2\right)\right)\text{Subst}\left(\int \frac{1}{-4\left(a^2-b^2\right)-x^2}\,dx,x,2b\right.}{a^2\left(a^2-b^2\right)}$$

$$= -\frac{2b\left(2a^2-b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{a^2\left(a^2-b^2\right)^{3/2}}-\frac{\tanh^{-1}(\cos(x))}{a^2}-\frac{b^2\cos(x)}{a\left(a^2-b^2\right)(a+b\sin(x))}$$

Mathematica [A] time = 0.239066, size = 99, normalized size = 1.06

$$\frac{\frac{2b\left(b^2-2a^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}}-\frac{ab^2\cos(x)}{(a-b)(a+b)(a+b\sin(x))}+\log\left(\sin\left(\frac{x}{2}\right)\right)-\log\left(\cos\left(\frac{x}{2}\right)\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]/(a + b*Sin[x])^2,x]
```

```
[Out] ((2*b*(-2*a^2 + b^2)*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^
(3/2) - Log[Cos[x/2]] + Log[Sin[x/2]] - (a*b^2*Cos[x])/((a - b)*(a + b)*(a
+ b*Sin[x])))/a^2
```

Maple [A] time = 0.062, size = 174, normalized size = 1.9

$$-2\frac{b^3\tan(x/2)}{a^2\left((\tan(x/2))^2a+2\tan(x/2)b+a\right)\left(a^2-b^2\right)}-2\frac{b^2}{a\left((\tan(x/2))^2a+2\tan(x/2)b+a\right)\left(a^2-b^2\right)}-4\frac{b}{\left(a^2-b^2\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)/(a+b*sin(x))^2,x)
```

```
[Out] -2/a^2*b^3/(tan(1/2*x)^2*a+2*tan(1/2*x)*b+a)/(a^2-b^2)*tan(1/2*x)-2/a*b^2/(
tan(1/2*x)^2*a+2*tan(1/2*x)*b+a)/(a^2-b^2)-4*b/(a^2-b^2)^(3/2)*arctan(1/2*(
2*a*tan(1/2*x)+2*b)/(a^2-b^2)^(1/2))+2/a^2*b^3/(a^2-b^2)^(3/2)*arctan(1/2*(
2*a*tan(1/2*x)+2*b)/(a^2-b^2)^(1/2))+1/a^2*ln(tan(1/2*x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a+b*sin(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 4.09561, size = 1184, normalized size = 12.73

$$\left[-\frac{\left(2a^3b - ab^3 + \left(2a^2b^2 - b^4\right)\sin(x)\right)\sqrt{-a^2+b^2}\log\left(-\frac{\left(2a^2-b^2\right)\cos(x)^2 - 2ab\sin(x) - a^2 - b^2 - 2(a\cos(x)\sin(x) + b\cos(x))\sqrt{-a^2+b^2}}{b^2\cos(x)^2 - 2ab\sin(x) - a^2 - b^2}\right) + 2\left(a^3\right.}{2\left(a^7 -\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a+b*sin(x))^2,x, algorithm="fricas")
```

```
[Out] [-1/2*((2*a^3*b - a*b^3 + (2*a^2*b^2 - b^4)*sin(x))*sqrt(-a^2 + b^2)*log(-(
(2*a^2 - b^2)*cos(x)^2 - 2*a*b*sin(x) - a^2 - b^2 - 2*(a*cos(x)*sin(x) + b*
cos(x))*sqrt(-a^2 + b^2))/(b^2*cos(x)^2 - 2*a*b*sin(x) - a^2 - b^2)) + 2*(a
^3*b^2 - a*b^4)*cos(x) + (a^5 - 2*a^3*b^2 + a*b^4 + (a^4*b - 2*a^2*b^3 + b^
5)*sin(x))*log(1/2*cos(x) + 1/2) - (a^5 - 2*a^3*b^2 + a*b^4 + (a^4*b - 2*a^
2*b^3 + b^5)*sin(x))*log(-1/2*cos(x) + 1/2))/(a^7 - 2*a^5*b^2 + a^3*b^4 + (
a^6*b - 2*a^4*b^3 + a^2*b^5)*sin(x)), 1/2*(2*(2*a^3*b - a*b^3 + (2*a^2*b^2
- b^4)*sin(x))*sqrt(a^2 - b^2)*arctan(-(a*sin(x) + b)/(sqrt(a^2 - b^2)*cos(
x))) - 2*(a^3*b^2 - a*b^4)*cos(x) - (a^5 - 2*a^3*b^2 + a*b^4 + (a^4*b - 2*a
^2*b^3 + b^5)*sin(x))*log(1/2*cos(x) + 1/2) + (a^5 - 2*a^3*b^2 + a*b^4 + (a
^4*b - 2*a^2*b^3 + b^5)*sin(x))*log(-1/2*cos(x) + 1/2))/(a^7 - 2*a^5*b^2 +
a^3*b^4 + (a^6*b - 2*a^4*b^3 + a^2*b^5)*sin(x))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(x)}{(a + b\sin(x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a+b*sin(x))**2,x)
```

```
[Out] Integral(csc(x)/(a + b*sin(x))**2, x)
```

Giac [A] time = 1.60205, size = 181, normalized size = 1.95

$$-\frac{2\left(2a^2b-b^3\right)\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}x\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^4-a^2b^2\right)\sqrt{a^2-b^2}}-\frac{2\left(b^3\tan\left(\frac{1}{2}x\right)+ab^2\right)}{\left(a^4-a^2b^2\right)\left(a\tan\left(\frac{1}{2}x\right)^2+2b\tan\left(\frac{1}{2}x\right)+a\right)}+\frac{\log\left(\left|\mathrm{ta}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a+b*sin(x))^2,x, algorithm="giac")
```

```
[Out] -2*(2*a^2*b - b^3)*(pi*floor(1/2*x/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*x)
+ b)/sqrt(a^2 - b^2)))/((a^4 - a^2*b^2)*sqrt(a^2 - b^2)) - 2*(b^3*tan(1/2*x
) + a*b^2)/((a^4 - a^2*b^2)*(a*tan(1/2*x)^2 + 2*b*tan(1/2*x) + a)) + log(ab
s(tan(1/2*x)))/a^2
```

3.191 $\int \frac{\csc^2(x)}{(a+b\sin(x))^2}\,dx$

Optimal. Leaf size=123

$$\frac{2b^2\left(3a^2-2b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{a^3\left(a^2-b^2\right)^{3/2}}-\frac{\left(a^2-2b^2\right)\cot(x)}{a^2\left(a^2-b^2\right)}-\frac{b^2\cot(x)}{a\left(a^2-b^2\right)(a+b\sin(x))}+\frac{2b\tanh^{-1}(\cos(x))}{a^3}$$

```
[Out] (2*b^2*(3*a^2 - 2*b^2)*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(a^3*(a^2
- b^2)^(3/2)) + (2*b*ArcTanh[Cos[x]])/a^3 - ((a^2 - 2*b^2)*Cot[x])/(a^2*(a^
2 - b^2)) - (b^2*Cot[x])/(a*(a^2 - b^2)*(a + b*Sin[x]))
```

Rubi [A] time = 0.334821, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {2802, 3055, 3001, 3770, 2660, 618, 204}

$$\frac{2b^2\left(3a^2-2b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{a^3\left(a^2-b^2\right)^{3/2}}-\frac{\left(a^2-2b^2\right)\cot(x)}{a^2\left(a^2-b^2\right)}-\frac{b^2\cot(x)}{a\left(a^2-b^2\right)(a+b\sin(x))}+\frac{2b\tanh^{-1}(\cos(x))}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^2/(a + b*Sin[x])^2,x]
```

```
[Out] (2*b^2*(3*a^2 - 2*b^2)*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(a^3*(a^2
- b^2)^(3/2)) + (2*b*ArcTanh[Cos[x]])/a^3 - ((a^2 - 2*b^2)*Cot[x])/(a^2*(a^
2 - b^2)) - (b^2*Cot[x])/(a*(a^2 - b^2)*(a + b*Sin[x]))
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
```

```
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\csc^2(x)}{(a+b\sin(x))^2}\,dx = -\frac{b^2\cot(x)}{a\left(a^2-b^2\right)(a+b\sin(x))}+\frac{\int \frac{\csc^2(x)\left(a^2-2b^2-ab\sin(x)+b^2\sin^2(x)\right)}{a+b\sin(x)}\,dx}{a\left(a^2-b^2\right)}$$

$$= -\frac{\left(a^2-2b^2\right)\cot(x)}{a^2\left(a^2-b^2\right)}-\frac{b^2\cot(x)}{a\left(a^2-b^2\right)(a+b\sin(x))}+\frac{\int \frac{\csc(x)\left(-2b\left(a^2-b^2\right)+ab^2\sin(x)\right)}{a+b\sin(x)}\,dx}{a^2\left(a^2-b^2\right)}$$

$$= -\frac{\left(a^2-2b^2\right)\cot(x)}{a^2\left(a^2-b^2\right)}-\frac{b^2\cot(x)}{a\left(a^2-b^2\right)(a+b\sin(x))}-\frac{(2b)\int \csc(x)\,dx}{a^3}+\frac{\left(b^2\left(3a^2-2b^2\right)\right)\int \frac{1}{a+b\sin(\ldots}}{a^3\left(a^2-b^2\right)}$$

$$= \frac{2b\tanh^{-1}(\cos(x))}{a^3}-\frac{\left(a^2-2b^2\right)\cot(x)}{a^2\left(a^2-b^2\right)}-\frac{b^2\cot(x)}{a\left(a^2-b^2\right)(a+b\sin(x))}+\frac{\left(2b^2\left(3a^2-2b^2\right)\right)\text{Subst}\ldots}{a^3\left(\ldots\right.}$$

$$= \frac{2b\tanh^{-1}(\cos(x))}{a^3}-\frac{\left(a^2-2b^2\right)\cot(x)}{a^2\left(a^2-b^2\right)}-\frac{b^2\cot(x)}{a\left(a^2-b^2\right)(a+b\sin(x))}-\frac{\left(4b^2\left(3a^2-2b^2\right)\right)\text{Subst}\ldots}{\ldots}$$

$$= \frac{2b^2\left(3a^2-2b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{a^3\left(a^2-b^2\right)^{3/2}}+\frac{2b\tanh^{-1}(\cos(x))}{a^3}-\frac{\left(a^2-2b^2\right)\cot(x)}{a^2\left(a^2-b^2\right)}-\frac{b^2\cot(\ldots}{a\left(a^2-b^2\right)(a+\ldots}$$

Mathematica [A] time = 0.675647, size = 127, normalized size = 1.03

$$\frac{\frac{4b^2(3a^2-2b^2)\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}}+\frac{2ab^3\cos(x)}{(a-b)(a+b)(a+b\sin(x))}+a\tan\left(\frac{x}{2}\right)-a\cot\left(\frac{x}{2}\right)-4b\log\left(\sin\left(\frac{x}{2}\right)\right)+4b\log\left(\cos\left(\frac{x}{2}\right)\right)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^2/(a + b*Sin[x])^2,x]
```

```
[Out] ((4*b^2*(3*a^2 - 2*b^2)*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(a^2 - b^
2)^(3/2) - a*Cot[x/2] + 4*b*Log[Cos[x/2]] - 4*b*Log[Sin[x/2]] + (2*a*b^3*Co
s[x])/((a - b)*(a + b)*(a + b*Sin[x])) + a*Tan[x/2])/(2*a^3)
```

Maple [A] time = 0.064, size = 201, normalized size = 1.6

$$\frac{1}{2\,a^2}\tan\left(\frac{x}{2}\right)+2\,\frac{b^4\tan(x/2)}{a^3\left((\tan(x/2))^2\,a+2\,\tan(x/2)\,b+a\right)\left(a^2-b^2\right)}+2\,\frac{b^3}{a^2\left((\tan(x/2))^2\,a+2\,\tan(x/2)\,b+a\right)\left(a^2-b^2\right)}+6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^2/(a+b*sin(x))^2,x)
```

```
[Out] 1/2/a^2*tan(1/2*x)+2/a^3*b^4/(tan(1/2*x)^2*a+2*tan(1/2*x)*b+a)/(a^2-b^2)*ta
n(1/2*x)+2/a^2*b^3/(tan(1/2*x)^2*a+2*tan(1/2*x)*b+a)/(a^2-b^2)+6/a*b^2/(a^2
-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*x)+2*b)/(a^2-b^2)^(1/2))-4/a^3*b^4/(a^2
-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*x)+2*b)/(a^2-b^2)^(1/2))-1/2/a^2/tan(1/
2*x)-2/a^3*b*ln(tan(1/2*x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a+b*sin(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 4.12804, size = 1775, normalized size = 14.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a+b*sin(x))^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(2*(a^5*b - 3*a^3*b^3 + 2*a*b^5)*cos(x)*sin(x) + (3*a^2*b^3 - 2*b^5 -
 (3*a^2*b^3 - 2*b^5)*cos(x)^2 + (3*a^3*b^2 - 2*a*b^4)*sin(x))*sqrt(-a^2 + b
^2)*log(((2*a^2 - b^2)*cos(x)^2 - 2*a*b*sin(x) - a^2 - b^2 + 2*(a*cos(x)*si
```

```
n(x) + b*cos(x))*sqrt(-a^2 + b^2))/(b^2*cos(x)^2 - 2*a*b*sin(x) - a^2 - b^2
)) + 2*(a^6 - 2*a^4*b^2 + a^2*b^4)*cos(x) - 2*(a^4*b^2 - 2*a^2*b^4 + b^6 -
(a^4*b^2 - 2*a^2*b^4 + b^6)*cos(x)^2 + (a^5*b - 2*a^3*b^3 + a*b^5)*sin(x))*
log(1/2*cos(x) + 1/2) + 2*(a^4*b^2 - 2*a^2*b^4 + b^6 - (a^4*b^2 - 2*a^2*b^4
 + b^6)*cos(x)^2 + (a^5*b - 2*a^3*b^3 + a*b^5)*sin(x))*log(-1/2*cos(x) + 1/
2))/(a^7*b - 2*a^5*b^3 + a^3*b^5 - (a^7*b - 2*a^5*b^3 + a^3*b^5)*cos(x)^2 +
 (a^8 - 2*a^6*b^2 + a^4*b^4)*sin(x)), -((a^5*b - 3*a^3*b^3 + 2*a*b^5)*cos(x
)*sin(x) + (3*a^2*b^3 - 2*b^5 - (3*a^2*b^3 - 2*b^5)*cos(x)^2 + (3*a^3*b^2 -
 2*a*b^4)*sin(x))*sqrt(a^2 - b^2)*arctan(-(a*sin(x) + b)/(sqrt(a^2 - b^2)*c
os(x))) + (a^6 - 2*a^4*b^2 + a^2*b^4)*cos(x) - (a^4*b^2 - 2*a^2*b^4 + b^6 -
 (a^4*b^2 - 2*a^2*b^4 + b^6)*cos(x)^2 + (a^5*b - 2*a^3*b^3 + a*b^5)*sin(x))
*log(1/2*cos(x) + 1/2) + (a^4*b^2 - 2*a^2*b^4 + b^6 - (a^4*b^2 - 2*a^2*b^4
+ b^6)*cos(x)^2 + (a^5*b - 2*a^3*b^3 + a*b^5)*sin(x))*log(-1/2*cos(x) + 1/2
))/(a^7*b - 2*a^5*b^3 + a^3*b^5 - (a^7*b - 2*a^5*b^3 + a^3*b^5)*cos(x)^2 +
(a^8 - 2*a^6*b^2 + a^4*b^4)*sin(x))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc^2(x)}{(a+b\sin(x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**2/(a+b*sin(x))**2,x)
```

```
[Out] Integral(csc(x)**2/(a + b*sin(x))**2, x)
```

Giac [A] time = 1.70443, size = 316, normalized size = 2.57

$$\frac{2\left(3a^2b^2-2b^4\right)\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}x\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^5-a^3b^2\right)\sqrt{a^2-b^2}}+\frac{4a^3b\tan\left(\frac{1}{2}x\right)^3-4ab^3\tan\left(\frac{1}{2}x\right)^3-3a^4\tan\left(\frac{1}{2}x\right)^2+\ldots}{6\left(a^5-a^3b^2\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a+b*sin(x))^2,x, algorithm="giac")
```

```
[Out] 2*(3*a^2*b^2 - 2*b^4)*(pi*floor(1/2*x/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*
x) + b)/sqrt(a^2 - b^2)))/((a^5 - a^3*b^2)*sqrt(a^2 - b^2)) + 1/6*(4*a^3*b*
tan(1/2*x)^3 - 4*a*b^3*tan(1/2*x)^3 - 3*a^4*tan(1/2*x)^2 + 11*a^2*b^2*tan(1
/2*x)^2 + 4*b^4*tan(1/2*x)^2 - 2*a^3*b*tan(1/2*x) + 14*a*b^3*tan(1/2*x) - 3
*a^4 + 3*a^2*b^2)/((a^5 - a^3*b^2)*(a*tan(1/2*x)^3 + 2*b*tan(1/2*x)^2 + a*t
an(1/2*x))) - 2*b*log(abs(tan(1/2*x)))/a^3 + 1/2*tan(1/2*x)/a^2
```

3.192 $\int \frac{\csc^3(x)}{(a+b\sin(x))^2}\,dx$

Optimal. Leaf size=168

$$-\frac{2b^3\left(4a^2-3b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{a^4\left(a^2-b^2\right)^{3/2}}+\frac{b\left(2a^2-3b^2\right)\cot(x)}{a^3\left(a^2-b^2\right)}-\frac{\left(a^2+6b^2\right)\tanh^{-1}(\cos(x))}{2a^4}-\frac{\left(a^2-3b^2\right)\cot(x)\csc(x)}{2a^2\left(a^2-b^2\right)}-$$

```
[Out] (-2*b^3*(4*a^2 - 3*b^2)*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(a^4*(a^2
 - b^2)^(3/2)) - ((a^2 + 6*b^2)*ArcTanh[Cos[x]])/(2*a^4) + (b*(2*a^2 - 3*b^
2)*Cot[x])/(a^3*(a^2 - b^2)) - ((a^2 - 3*b^2)*Cot[x]*Csc[x])/(2*a^2*(a^2 -
b^2)) - (b^2*Cot[x]*Csc[x])/(a*(a^2 - b^2)*(a + b*Sin[x]))
```

Rubi [A] time = 0.574429, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {2802, 3055, 3001, 3770, 2660, 618, 204}

$$-\frac{2b^3\left(4a^2-3b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{a^4\left(a^2-b^2\right)^{3/2}}+\frac{b\left(2a^2-3b^2\right)\cot(x)}{a^3\left(a^2-b^2\right)}-\frac{\left(a^2+6b^2\right)\tanh^{-1}(\cos(x))}{2a^4}-\frac{\left(a^2-3b^2\right)\cot(x)\csc(x)}{2a^2\left(a^2-b^2\right)}-$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^3/(a + b*Sin[x])^2,x]
```

```
[Out] (-2*b^3*(4*a^2 - 3*b^2)*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(a^4*(a^2
 - b^2)^(3/2)) - ((a^2 + 6*b^2)*ArcTanh[Cos[x]])/(2*a^4) + (b*(2*a^2 - 3*b^
2)*Cot[x])/(a^3*(a^2 - b^2)) - ((a^2 - 3*b^2)*Cot[x]*Csc[x])/(2*a^2*(a^2 -
b^2)) - (b^2*Cot[x]*Csc[x])/(a*(a^2 - b^2)*(a + b*Sin[x]))
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
```

```
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\csc^3(x)}{(a+b\sin(x))^2}\,dx = -\frac{b^2\cot(x)\csc(x)}{a\left(a^2-b^2\right)(a+b\sin(x))} + \frac{\int \frac{\csc^3(x)\left(a^2-3b^2-ab\sin(x)+2b^2\sin^2(x)\right)}{a+b\sin(x)}dx}{a\left(a^2-b^2\right)}$$

$$= -\frac{\left(a^2-3b^2\right)\cot(x)\csc(x)}{2a^2\left(a^2-b^2\right)} - \frac{b^2\cot(x)\csc(x)}{a\left(a^2-b^2\right)(a+b\sin(x))} + \frac{\int \frac{\csc^2(x)\left(-2b\left(2a^2-3b^2\right)+a\left(a^2+b^2\right)\sin(x)+b\left(a^2-3b^2\right)\right.}{a+b\sin(x)}}{2a^2\left(a^2-b^2\right)}$$

$$= \frac{b\left(2a^2-3b^2\right)\cot(x)}{a^3\left(a^2-b^2\right)} - \frac{\left(a^2-3b^2\right)\cot(x)\csc(x)}{2a^2\left(a^2-b^2\right)} - \frac{b^2\cot(x)\csc(x)}{a\left(a^2-b^2\right)(a+b\sin(x))} + \frac{\int \frac{\csc(x)\left(a^4+5a^2b^2-6b^4\right.}{a+b\sin}}{2a^3\left(a^2\right.}$$

$$= \frac{b\left(2a^2-3b^2\right)\cot(x)}{a^3\left(a^2-b^2\right)} - \frac{\left(a^2-3b^2\right)\cot(x)\csc(x)}{2a^2\left(a^2-b^2\right)} - \frac{b^2\cot(x)\csc(x)}{a\left(a^2-b^2\right)(a+b\sin(x))} - \frac{\left(b^3\left(4a^2-3b^2\right)\right)\int}{a^4\left(a^2-b^2\right.}$$

$$= -\frac{\left(a^2+6b^2\right)\tanh^{-1}(\cos(x))}{2a^4} + \frac{b\left(2a^2-3b^2\right)\cot(x)}{a^3\left(a^2-b^2\right)} - \frac{\left(a^2-3b^2\right)\cot(x)\csc(x)}{2a^2\left(a^2-b^2\right)} - \frac{b^2\cot(x)\,\mathrm{cs}}{a\left(a^2-b^2\right)(a+}$$

$$= -\frac{\left(a^2+6b^2\right)\tanh^{-1}(\cos(x))}{2a^4} + \frac{b\left(2a^2-3b^2\right)\cot(x)}{a^3\left(a^2-b^2\right)} - \frac{\left(a^2-3b^2\right)\cot(x)\csc(x)}{2a^2\left(a^2-b^2\right)} - \frac{b^2\cot(x)\,\mathrm{cs}}{a\left(a^2-b^2\right)(a+}$$

$$= -\frac{2b^3\left(4a^2-3b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{a^4\left(a^2-b^2\right)^{3/2}} - \frac{\left(a^2+6b^2\right)\tanh^{-1}(\cos(x))}{2a^4} + \frac{b\left(2a^2-3b^2\right)\cot(x)}{a^3\left(a^2-b^2\right)} - \frac{\left(a^2-\right.}{2}$$

Mathematica [A] time = 0.864321, size = 171, normalized size = 1.02

$$\frac{\frac{16b^3\left(3b^2-4a^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}} + 4\left(a^2+6b^2\right)\log\left(\sin\left(\frac{x}{2}\right)\right) - 4\left(a^2+6b^2\right)\log\left(\cos\left(\frac{x}{2}\right)\right) - a^2\csc^2\left(\frac{x}{2}\right) + a^2\sec^2\left(\frac{x}{2}\right) - \frac{8ab^4}{(a-b)(a+b)}}{8a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^3/(a + b*Sin[x])^2,x]
```

```
[Out] ((16*b^3*(-4*a^2 + 3*b^2)*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(a^2 -
b^2)^(3/2) + 8*a*b*Cot[x/2] - a^2*Csc[x/2]^2 - 4*(a^2 + 6*b^2)*Log[Cos[x/2]
] + 4*(a^2 + 6*b^2)*Log[Sin[x/2]] + a^2*Sec[x/2]^2 - (8*a*b^4*Cos[x])/((a -
 b)*(a + b)*(a + b*Sin[x])) - 8*a*b*Tan[x/2])/(8*a^4)
```

Maple [A] time = 0.075, size = 236, normalized size = 1.4

$$\frac{1}{8a^2}\left(\tan\left(\frac{x}{2}\right)\right)^2 - \frac{b}{a^3}\tan\left(\frac{x}{2}\right) - 2\frac{b^5\tan(x/2)}{a^4\left((\tan(x/2))^2a+2\tan(x/2)b+a\right)\left(a^2-b^2\right)} - 2\frac{b^4}{a^3\left((\tan(x/2))^2a+2\tan(x/2)b+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^3/(a+b*sin(x))^2,x)
```

```
[Out] 1/8/a^2*tan(1/2*x)^2-1/a^3*tan(1/2*x)*b-2/a^4*b^5/(tan(1/2*x)^2*a+2*tan(1/2
*x)*b+a)/(a^2-b^2)*tan(1/2*x)-2/a^3*b^4/(tan(1/2*x)^2*a+2*tan(1/2*x)*b+a)/(
a^2-b^2)-8/a^2*b^3/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*x)+2*b)/(a^2-b^2
```

```
)^(1/2))+6/a^4*b^5/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*x)+2*b)/(a^2-b^2
)^(1/2))-1/8/a^2/tan(1/2*x)^2+1/2/a^2*ln(tan(1/2*x))+3/a^4*ln(tan(1/2*x))*b
^2+b/a^3/tan(1/2*x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(a+b*sin(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 6.92116, size = 2635, normalized size = 15.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(a+b*sin(x))^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*(2*a^5*b^2 - 5*a^3*b^4 + 3*a*b^6)*cos(x)^3 - 6*(a^6*b - 2*a^4*b^3
+ a^2*b^5)*cos(x)*sin(x) + 2*(4*a^3*b^3 - 3*a*b^5 - (4*a^3*b^3 - 3*a*b^5)*c
os(x)^2 + (4*a^2*b^4 - 3*b^6 - (4*a^2*b^4 - 3*b^6)*cos(x)^2)*sin(x))*sqrt(-
a^2 + b^2)*log(-((2*a^2 - b^2)*cos(x)^2 - 2*a*b*sin(x) - a^2 - b^2 - 2*(a*c
os(x)*sin(x) + b*cos(x))*sqrt(-a^2 + b^2))/(b^2*cos(x)^2 - 2*a*b*sin(x) - a
^2 - b^2)) + 2*(a^7 - 6*a^5*b^2 + 11*a^3*b^4 - 6*a*b^6)*cos(x) + (a^7 + 4*a
^5*b^2 - 11*a^3*b^4 + 6*a*b^6 - (a^7 + 4*a^5*b^2 - 11*a^3*b^4 + 6*a*b^6)*co
s(x)^2 + (a^6*b + 4*a^4*b^3 - 11*a^2*b^5 + 6*b^7 - (a^6*b + 4*a^4*b^3 - 11*
a^2*b^5 + 6*b^7)*cos(x)^2)*sin(x))*log(1/2*cos(x) + 1/2) - (a^7 + 4*a^5*b^2
 - 11*a^3*b^4 + 6*a*b^6 - (a^7 + 4*a^5*b^2 - 11*a^3*b^4 + 6*a*b^6)*cos(x)^2
 + (a^6*b + 4*a^4*b^3 - 11*a^2*b^5 + 6*b^7 - (a^6*b + 4*a^4*b^3 - 11*a^2*b^
5 + 6*b^7)*cos(x)^2)*sin(x))*log(-1/2*cos(x) + 1/2))/(a^9 - 2*a^7*b^2 + a^5
*b^4 - (a^9 - 2*a^7*b^2 + a^5*b^4)*cos(x)^2 + (a^8*b - 2*a^6*b^3 + a^4*b^5
- (a^8*b - 2*a^6*b^3 + a^4*b^5)*cos(x)^2)*sin(x)), -1/4*(4*(2*a^5*b^2 - 5*a
^3*b^4 + 3*a*b^6)*cos(x)^3 - 6*(a^6*b - 2*a^4*b^3 + a^2*b^5)*cos(x)*sin(x)
- 4*(4*a^3*b^3 - 3*a*b^5 - (4*a^3*b^3 - 3*a*b^5)*cos(x)^2 + (4*a^2*b^4 - 3*
b^6 - (4*a^2*b^4 - 3*b^6)*cos(x)^2)*sin(x))*sqrt(a^2 - b^2)*arctan(-(a*sin(
x) + b)/(sqrt(a^2 - b^2)*cos(x))) + 2*(a^7 - 6*a^5*b^2 + 11*a^3*b^4 - 6*a*b
^6)*cos(x) + (a^7 + 4*a^5*b^2 - 11*a^3*b^4 + 6*a*b^6 - (a^7 + 4*a^5*b^2 - 1
1*a^3*b^4 + 6*a*b^6)*cos(x)^2 + (a^6*b + 4*a^4*b^3 - 11*a^2*b^5 + 6*b^7 - (
a^6*b + 4*a^4*b^3 - 11*a^2*b^5 + 6*b^7)*cos(x)^2)*sin(x))*log(1/2*cos(x) +
1/2) - (a^7 + 4*a^5*b^2 - 11*a^3*b^4 + 6*a*b^6 - (a^7 + 4*a^5*b^2 - 11*a^3*
b^4 + 6*a*b^6)*cos(x)^2 + (a^6*b + 4*a^4*b^3 - 11*a^2*b^5 + 6*b^7 - (a^6*b
+ 4*a^4*b^3 - 11*a^2*b^5 + 6*b^7)*cos(x)^2)*sin(x))*log(-1/2*cos(x) + 1/2))
/(a^9 - 2*a^7*b^2 + a^5*b^4 - (a^9 - 2*a^7*b^2 + a^5*b^4)*cos(x)^2 + (a^8*b
 - 2*a^6*b^3 + a^4*b^5 - (a^8*b - 2*a^6*b^3 + a^4*b^5)*cos(x)^2)*sin(x))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc^3(x)}{(a + b\sin(x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**3/(a+b*sin(x))**2,x)
```

```
[Out] Integral(csc(x)**3/(a + b*sin(x))**2, x)
```

Giac [A] time = 1.60396, size = 290, normalized size = 1.73

$$-\frac{2\left(4\,a^2b^3-3\,b^5\right)\left(\pi\left\lfloor\frac{x}{2\,\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,x\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^6-a^4b^2\right)\sqrt{a^2-b^2}}-\frac{2\left(b^5\tan\left(\frac{1}{2}\,x\right)+ab^4\right)}{\left(a^6-a^4b^2\right)\left(a\tan\left(\frac{1}{2}\,x\right)^2+2\,b\tan\left(\frac{1}{2}\,x\right)+a\right)}+\frac{\left(a^2+6\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(a+b*sin(x))^2,x, algorithm="giac")
```

```
[Out] -2*(4*a^2*b^3 - 3*b^5)*(pi*floor(1/2*x/pi + 1/2)*sgn(a) + arctan((a*tan(1/2
*x) + b)/sqrt(a^2 - b^2)))/((a^6 - a^4*b^2)*sqrt(a^2 - b^2)) - 2*(b^5*tan(1
/2*x) + a*b^4)/((a^6 - a^4*b^2)*(a*tan(1/2*x)^2 + 2*b*tan(1/2*x) + a)) + 1/
2*(a^2 + 6*b^2)*log(abs(tan(1/2*x)))/a^4 + 1/8*(a^2*tan(1/2*x)^2 - 8*a*b*ta
n(1/2*x))/a^4 - 1/8*(6*a^2*tan(1/2*x)^2 + 36*b^2*tan(1/2*x)^2 - 8*a*b*tan(1
/2*x) + a^2)/(a^4*tan(1/2*x)^2)
```

3.193 $\int \frac{\sin^5(x)}{(a+b\sin(x))^3}\,dx$

Optimal. Leaf size=243

$$\frac{x\left(12a^2+b^2\right)}{2b^5}+\frac{3a\left(-7a^2b^2+4a^4+2b^4\right)\cos(x)}{2b^4\left(a^2-b^2\right)^2}-\frac{a^3\left(-29a^2b^2+12a^4+20b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{b^5\left(a^2-b^2\right)^{5/2}}+\frac{a^2\left(4a^2-7b^2\right)\sin}{2b^2\left(a^2-b^2\right)^2(a}$$

```
[Out] ((12*a^2 + b^2)*x)/(2*b^5) - (a^3*(12*a^4 - 29*a^2*b^2 + 20*b^4)*ArcTan[(b
+ a*Tan[x/2])/Sqrt[a^2 - b^2]])/(b^5*(a^2 - b^2)^(5/2)) + (3*a*(4*a^4 - 7*a
^2*b^2 + 2*b^4)*Cos[x])/(2*b^4*(a^2 - b^2)^2) - ((6*a^4 - 10*a^2*b^2 + b^4)
*Cos[x]*Sin[x])/(2*b^3*(a^2 - b^2)^2) + (a^2*Cos[x]*Sin[x]^3)/(2*b*(a^2 - b
^2)*(a + b*Sin[x])^2) + (a^2*(4*a^2 - 7*b^2)*Cos[x]*Sin[x]^2)/(2*b^2*(a^2 -
 b^2)^2*(a + b*Sin[x]))
```

Rubi [A] time = 0.661085, antiderivative size = 243, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {2792, 3047, 3049, 3023, 2735, 2660, 618, 204}

$$\frac{x\left(12a^2+b^2\right)}{2b^5}+\frac{3a\left(-7a^2b^2+4a^4+2b^4\right)\cos(x)}{2b^4\left(a^2-b^2\right)^2}-\frac{a^3\left(-29a^2b^2+12a^4+20b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{b^5\left(a^2-b^2\right)^{5/2}}+\frac{a^2\left(4a^2-7b^2\right)\sin}{2b^2\left(a^2-b^2\right)^2(a}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^5/(a + b*Sin[x])^3,x]
```

```
[Out] ((12*a^2 + b^2)*x)/(2*b^5) - (a^3*(12*a^4 - 29*a^2*b^2 + 20*b^4)*ArcTan[(b
+ a*Tan[x/2])/Sqrt[a^2 - b^2]])/(b^5*(a^2 - b^2)^(5/2)) + (3*a*(4*a^4 - 7*a
^2*b^2 + 2*b^4)*Cos[x])/(2*b^4*(a^2 - b^2)^2) - ((6*a^4 - 10*a^2*b^2 + b^4)
*Cos[x]*Sin[x])/(2*b^3*(a^2 - b^2)^2) + (a^2*Cos[x]*Sin[x]^3)/(2*b*(a^2 - b
^2)*(a + b*Sin[x])^2) + (a^2*(4*a^2 - 7*b^2)*Cos[x]*Sin[x]^2)/(2*b^2*(a^2 -
 b^2)^2*(a + b*Sin[x]))
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
```

```
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sin^5(x)}{(a+b\sin(x))^3}\,dx = \frac{a^2\cos(x)\sin^3(x)}{2b\left(a^2-b^2\right)(a+b\sin(x))^2} - \frac{\int \frac{\sin^2(x)\left(3a^2-2ab\sin(x)-2\left(2a^2-b^2\right)\sin^2(x)\right)}{(a+b\sin(x))^2}dx}{2b\left(a^2-b^2\right)}$$

$$= \frac{a^2\cos(x)\sin^3(x)}{2b\left(a^2-b^2\right)(a+b\sin(x))^2} + \frac{a^2\left(4a^2-7b^2\right)\cos(x)\sin^2(x)}{2b^2\left(a^2-b^2\right)^2(a+b\sin(x))} + \frac{\int \frac{\sin(x)\left(-2a^2\left(4a^2-7b^2\right)+ab\left(a^2-4b^2\right)\sin(\ldots\right.}{a+b\sin(x\ldots}}{2b^2\left(a^2-\ldots\right.}$$

$$= -\frac{\left(6a^4-10a^2b^2+b^4\right)\cos(x)\sin(x)}{2b^3\left(a^2-b^2\right)^2} + \frac{a^2\cos(x)\sin^3(x)}{2b\left(a^2-b^2\right)(a+b\sin(x))^2} + \frac{a^2\left(4a^2-7b^2\right)\cos(x)\sin^2\ldots}{2b^2\left(a^2-b^2\right)^2(a+b\sin(x\ldots}$$

$$= \frac{3a\left(4a^4-7a^2b^2+2b^4\right)\cos(x)}{2b^4\left(a^2-b^2\right)^2} - \frac{\left(6a^4-10a^2b^2+b^4\right)\cos(x)\sin(x)}{2b^3\left(a^2-b^2\right)^2} + \frac{a^2\cos(x)\sin^3(x)}{2b\left(a^2-b^2\right)(a+b\sin(x)\ldots}$$

$$= \frac{\left(12a^2+b^2\right)x}{2b^5} + \frac{3a\left(4a^4-7a^2b^2+2b^4\right)\cos(x)}{2b^4\left(a^2-b^2\right)^2} - \frac{\left(6a^4-10a^2b^2+b^4\right)\cos(x)\sin(x)}{2b^3\left(a^2-b^2\right)^2} + \frac{a^2\ldots}{2b\left(a^2-\ldots\right.}$$

$$= \frac{\left(12a^2+b^2\right)x}{2b^5} + \frac{3a\left(4a^4-7a^2b^2+2b^4\right)\cos(x)}{2b^4\left(a^2-b^2\right)^2} - \frac{\left(6a^4-10a^2b^2+b^4\right)\cos(x)\sin(x)}{2b^3\left(a^2-b^2\right)^2} + \frac{a^2\ldots}{2b\left(a^2-\ldots\right.}$$

$$= \frac{\left(12a^2+b^2\right)x}{2b^5} + \frac{3a\left(4a^4-7a^2b^2+2b^4\right)\cos(x)}{2b^4\left(a^2-b^2\right)^2} - \frac{\left(6a^4-10a^2b^2+b^4\right)\cos(x)\sin(x)}{2b^3\left(a^2-b^2\right)^2} + \frac{a^2\ldots}{2b\left(a^2-\ldots\right.}$$

$$= \frac{\left(12a^2+b^2\right)x}{2b^5} - \frac{a^3\left(12a^4-29a^2b^2+20b^4\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{b^5\left(a^2-b^2\right)^{5/2}} + \frac{3a\left(4a^4-7a^2b^2+2b^4\right)\cos(x)}{2b^4\left(a^2-b^2\right)^2}$$

Mathematica [A] time = 0.9175, size = 164, normalized size = 0.67

$$\frac{2x\left(12a^2+b^2\right) - \frac{4a^3\left(-29a^2b^2+12a^4+20b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}} + \frac{2a^4b\left(7a^2-10b^2\right)\cos(x)}{(a-b)^2(a+b)^2(a+b\sin(x))} - \frac{2a^5b\cos(x)}{(a-b)(a+b)(a+b\sin(x))^2} + 12ab\cos(x) - b^2\sin\ldots}{4b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^5/(a + b*Sin[x])^3,x]
```

```
[Out] (2*(12*a^2 + b^2)*x - (4*a^3*(12*a^4 - 29*a^2*b^2 + 20*b^4)*ArcTan[(b + a*T
an[x/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2) + 12*a*b*Cos[x] - (2*a^5*b*Cos
[x])/((a - b)*(a + b)*(a + b*Sin[x])^2) + (2*a^4*b*(7*a^2 - 10*b^2)*Cos[x])
/((a - b)^2*(a + b)^2*(a + b*Sin[x])) - b^2*Sin[2*x])/(4*b^5)
```

Maple [B] time = 0.066, size = 712, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^5/(a+b*sin(x))^3,x)
```

```
[Out] 1/b^3/(tan(1/2*x)^2+1)^2*tan(1/2*x)^3+6/b^4/(tan(1/2*x)^2+1)^2*tan(1/2*x)^2
*a-1/b^3/(tan(1/2*x)^2+1)^2*tan(1/2*x)+6/b^4/(tan(1/2*x)^2+1)^2*a+12/b^5*ar
ctan(tan(1/2*x))*a^2+5*a^6/b^3/(tan(1/2*x)^2*a+2*tan(1/2*x)*b+a)^2/(a^4-2*a
^2*b^2+b^4)*tan(1/2*x)^3-8*a^4/b/(tan(1/2*x)^2*a+2*tan(1/2*x)*b+a)^2/(a^4-2
*a^2*b^2+b^4)*tan(1/2*x)^3+6*a^7/b^4/(tan(1/2*x)^2*a+2*tan(1/2*x)*b+a)^2/(a
^4-2*a^2*b^2+b^4)*tan(1/2*x)^2+3*a^5/b^2/(tan(1/2*x)^2*a+2*tan(1/2*x)*b+a)^
2/(a^4-2*a^2*b^2+b^4)*tan(1/2*x)^2-18*a^3/(tan(1/2*x)^2*a+2*tan(1/2*x)*b+a)
^2/(a^4-2*a^2*b^2+b^4)*tan(1/2*x)^2+19*a^6/b^3/(tan(1/2*x)^2*a+2*tan(1/2*x)
*b+a)^2/(a^4-2*a^2*b^2+b^4)*tan(1/2*x)-28*a^4/b/(tan(1/2*x)^2*a+2*tan(1/2*x
)*b+a)^2/(a^4-2*a^2*b^2+b^4)*tan(1/2*x)+6*a^7/b^4/(tan(1/2*x)^2*a+2*tan(1/2
*x)*b+a)^2/(a^4-2*a^2*b^2+b^4)-9*a^5/b^2/(tan(1/2*x)^2*a+2*tan(1/2*x)*b+a)^
2/(a^4-2*a^2*b^2+b^4)-12*a^7/b^5/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan
(1/2*(2*a*tan(1/2*x)+2*b)/(a^2-b^2)^(1/2))+29*a^5/b^3/(a^4-2*a^2*b^2+b^4)/(
a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*x)+2*b)/(a^2-b^2)^(1/2))-20*a^3/b/(a
^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*x)+2*b)/(a^2-b^2)
^(1/2))+1/2*x/b^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^5/(a+b*sin(x))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.445, size = 2384, normalized size = 9.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^5/(a+b*sin(x))^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*(12*a^8*b^2 - 35*a^6*b^4 + 33*a^4*b^6 - 9*a^2*b^8 - b^10)*x*cos(x)
^2 + 8*(a^7*b^3 - 3*a^5*b^5 + 3*a^3*b^7 - a*b^9)*cos(x)^3 + (12*a^9 - 17*a^
7*b^2 - 9*a^5*b^4 + 20*a^3*b^6 - (12*a^7*b^2 - 29*a^5*b^4 + 20*a^3*b^6)*cos
(x)^2 + 2*(12*a^8*b - 29*a^6*b^3 + 20*a^4*b^5)*sin(x))*sqrt(-a^2 + b^2)*log
(-((2*a^2 - b^2)*cos(x)^2 - 2*a*b*sin(x) - a^2 - b^2 - 2*(a*cos(x)*sin(x) +
 b*cos(x))*sqrt(-a^2 + b^2))/(b^2*cos(x)^2 - 2*a*b*sin(x) - a^2 - b^2)) - 2
*(12*a^10 - 23*a^8*b^2 - 2*a^6*b^4 + 24*a^4*b^6 - 10*a^2*b^8 - b^10)*x - 2*
(12*a^9*b - 29*a^7*b^3 + 15*a^5*b^5 + 6*a^3*b^7 - 4*a*b^9)*cos(x) - 2*((a^6
*b^4 - 3*a^4*b^6 + 3*a^2*b^8 - b^10)*cos(x)^3 + 2*(12*a^9*b - 35*a^7*b^3 +
33*a^5*b^5 - 9*a^3*b^7 - a*b^9)*x + (18*a^8*b^2 - 51*a^6*b^4 + 46*a^4*b^6 -
 14*a^2*b^8 + b^10)*cos(x))*sin(x))/(a^8*b^5 - 2*a^6*b^7 + 2*a^2*b^11 - b^1
3 - (a^6*b^7 - 3*a^4*b^9 + 3*a^2*b^11 - b^13)*cos(x)^2 + 2*(a^7*b^6 - 3*a^5
*b^8 + 3*a^3*b^10 - a*b^12)*sin(x)), -1/2*((12*a^8*b^2 - 35*a^6*b^4 + 33*a^
4*b^6 - 9*a^2*b^8 - b^10)*x*cos(x)^2 + 4*(a^7*b^3 - 3*a^5*b^5 + 3*a^3*b^7 -
 a*b^9)*cos(x)^3 - (12*a^9 - 17*a^7*b^2 - 9*a^5*b^4 + 20*a^3*b^6 - (12*a^7*
b^2 - 29*a^5*b^4 + 20*a^3*b^6)*cos(x)^2 + 2*(12*a^8*b - 29*a^6*b^3 + 20*a^4
*b^5)*sin(x))*sqrt(a^2 - b^2)*arctan(-(a*sin(x) + b)/(sqrt(a^2 - b^2)*cos(x
))) - (12*a^10 - 23*a^8*b^2 - 2*a^6*b^4 + 24*a^4*b^6 - 10*a^2*b^8 - b^10)*x
 - (12*a^9*b - 29*a^7*b^3 + 15*a^5*b^5 + 6*a^3*b^7 - 4*a*b^9)*cos(x) - ((a^
6*b^4 - 3*a^4*b^6 + 3*a^2*b^8 - b^10)*cos(x)^3 + 2*(12*a^9*b - 35*a^7*b^3 +
```

```
 33*a^5*b^5 - 9*a^3*b^7 - a*b^9)*x + (18*a^8*b^2 - 51*a^6*b^4 + 46*a^4*b^6
- 14*a^2*b^8 + b^10)*cos(x))*sin(x))/(a^8*b^5 - 2*a^6*b^7 + 2*a^2*b^11 - b^
13 - (a^6*b^7 - 3*a^4*b^9 + 3*a^2*b^11 - b^13)*cos(x)^2 + 2*(a^7*b^6 - 3*a^
5*b^8 + 3*a^3*b^10 - a*b^12)*sin(x))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**5/(a+b*sin(x))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.70847, size = 697, normalized size = 2.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^5/(a+b*sin(x))^3,x, algorithm="giac")
```

```
[Out] -(12*a^7 - 29*a^5*b^2 + 20*a^3*b^4)*(pi*floor(1/2*x/pi + 1/2)*sgn(a) + arct
an((a*tan(1/2*x) + b)/sqrt(a^2 - b^2)))/((a^4*b^5 - 2*a^2*b^7 + b^9)*sqrt(a
^2 - b^2)) + (6*a^6*b*tan(1/2*x)^7 - 10*a^4*b^3*tan(1/2*x)^7 + a^2*b^5*tan(
1/2*x)^7 + 12*a^7*tan(1/2*x)^6 - 5*a^5*b^2*tan(1/2*x)^6 - 20*a^3*b^4*tan(1/
2*x)^6 + 4*a*b^6*tan(1/2*x)^6 + 54*a^6*b*tan(1/2*x)^5 - 90*a^4*b^3*tan(1/2*
x)^5 + 17*a^2*b^5*tan(1/2*x)^5 + 4*b^7*tan(1/2*x)^5 + 36*a^7*tan(1/2*x)^4 -
 15*a^5*b^2*tan(1/2*x)^4 - 66*a^3*b^4*tan(1/2*x)^4 + 24*a*b^6*tan(1/2*x)^4
+ 90*a^6*b*tan(1/2*x)^3 - 162*a^4*b^3*tan(1/2*x)^3 + 55*a^2*b^5*tan(1/2*x)^
3 - 4*b^7*tan(1/2*x)^3 + 36*a^7*tan(1/2*x)^2 - 31*a^5*b^2*tan(1/2*x)^2 - 40
*a^3*b^4*tan(1/2*x)^2 + 20*a*b^6*tan(1/2*x)^2 + 42*a^6*b*tan(1/2*x) - 74*a^
4*b^3*tan(1/2*x) + 23*a^2*b^5*tan(1/2*x) + 12*a^7 - 21*a^5*b^2 + 6*a^3*b^4)
/((a^4*b^4 - 2*a^2*b^6 + b^8)*(a*tan(1/2*x)^4 + 2*b*tan(1/2*x)^3 + 2*a*tan(
1/2*x)^2 + 2*b*tan(1/2*x) + a)^2) + 1/2*(12*a^2 + b^2)*x/b^5
```

3.194 $\int \frac{\sin^4(x)}{(a+b\sin(x))^3}\,dx$

Optimal. Leaf size=179

$$-\frac{\left(3a^2-2b^2\right)\cos(x)}{2b^3\left(a^2-b^2\right)}+\frac{3a^2\left(-5a^2b^2+2a^4+4b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{b^4\left(a^2-b^2\right)^{5/2}}+\frac{a^2\sin^2(x)\cos(x)}{2b\left(a^2-b^2\right)(a+b\sin(x))^2}-\frac{3a^3\left(a^2-2b^2\right)\cos}{2b^3\left(a^2-b^2\right)^2(a+b}$$

```
[Out] (-3*a*x)/b^4 + (3*a^2*(2*a^4 - 5*a^2*b^2 + 4*b^4)*ArcTan[(b + a*Tan[x/2])/S
qrt[a^2 - b^2]])/(b^4*(a^2 - b^2)^(5/2)) - ((3*a^2 - 2*b^2)*Cos[x])/(2*b^3*
(a^2 - b^2)) + (a^2*Cos[x]*Sin[x]^2)/(2*b*(a^2 - b^2)*(a + b*Sin[x])^2) - (
3*a^3*(a^2 - 2*b^2)*Cos[x])/(2*b^3*(a^2 - b^2)^2*(a + b*Sin[x]))
```

Rubi [A] time = 0.411073, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {2792, 3031, 3023, 2735, 2660, 618, 204}

$$-\frac{\left(3a^2-2b^2\right)\cos(x)}{2b^3\left(a^2-b^2\right)}+\frac{3a^2\left(-5a^2b^2+2a^4+4b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{b^4\left(a^2-b^2\right)^{5/2}}+\frac{a^2\sin^2(x)\cos(x)}{2b\left(a^2-b^2\right)(a+b\sin(x))^2}-\frac{3a^3\left(a^2-2b^2\right)\cos}{2b^3\left(a^2-b^2\right)^2(a+b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^4/(a + b*Sin[x])^3,x]
```

```
[Out] (-3*a*x)/b^4 + (3*a^2*(2*a^4 - 5*a^2*b^2 + 4*b^4)*ArcTan[(b + a*Tan[x/2])/S
qrt[a^2 - b^2]])/(b^4*(a^2 - b^2)^(5/2)) - ((3*a^2 - 2*b^2)*Cos[x])/(2*b^3*
(a^2 - b^2)) + (a^2*Cos[x]*Sin[x]^2)/(2*b*(a^2 - b^2)*(a + b*Sin[x])^2) - (
3*a^3*(a^2 - 2*b^2)*Cos[x])/(2*b^3*(a^2 - b^2)^2*(a + b*Sin[x]))
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

```
 && LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sin^4(x)}{(a+b\sin(x))^3}\,dx = \frac{a^2\cos(x)\sin^2(x)}{2b\left(a^2-b^2\right)(a+b\sin(x))^2} - \frac{\int \frac{\sin(x)\left(2a^2-2ab\sin(x)-\left(3a^2-2b^2\right)\sin^2(x)\right)}{(a+b\sin(x))^2}dx}{2b\left(a^2-b^2\right)}$$

$$= \frac{a^2\cos(x)\sin^2(x)}{2b\left(a^2-b^2\right)(a+b\sin(x))^2} - \frac{3a^3\left(a^2-2b^2\right)\cos(x)}{2b^3\left(a^2-b^2\right)^2(a+b\sin(x))} - \frac{\int \frac{3a^2b\left(a^2-2b^2\right)+a\left(3a^2-4b^2\right)\left(a^2-b^2\right)\sin(x)-b(3}{a+b\sin(x)}}{2b^3\left(a^2-b^2\right)^2}$$

$$= -\frac{\left(3a^2-2b^2\right)\cos(x)}{2b^3\left(a^2-b^2\right)} + \frac{a^2\cos(x)\sin^2(x)}{2b\left(a^2-b^2\right)(a+b\sin(x))^2} - \frac{3a^3\left(a^2-2b^2\right)\cos(x)}{2b^3\left(a^2-b^2\right)^2(a+b\sin(x))} - \frac{\int \frac{3a^2b^2\left(a^2-2b^2\right)}{a+}}{2b^4(}$$

$$= -\frac{3ax}{b^4} - \frac{\left(3a^2-2b^2\right)\cos(x)}{2b^3\left(a^2-b^2\right)} + \frac{a^2\cos(x)\sin^2(x)}{2b\left(a^2-b^2\right)(a+b\sin(x))^2} - \frac{3a^3\left(a^2-2b^2\right)\cos(x)}{2b^3\left(a^2-b^2\right)^2(a+b\sin(x))} + \frac{\left(3a^2\right.(}{}$$

$$= -\frac{3ax}{b^4} - \frac{\left(3a^2-2b^2\right)\cos(x)}{2b^3\left(a^2-b^2\right)} + \frac{a^2\cos(x)\sin^2(x)}{2b\left(a^2-b^2\right)(a+b\sin(x))^2} - \frac{3a^3\left(a^2-2b^2\right)\cos(x)}{2b^3\left(a^2-b^2\right)^2(a+b\sin(x))} + \frac{\left(3a^2\right.(}{}$$

$$= -\frac{3ax}{b^4} - \frac{\left(3a^2-2b^2\right)\cos(x)}{2b^3\left(a^2-b^2\right)} + \frac{a^2\cos(x)\sin^2(x)}{2b\left(a^2-b^2\right)(a+b\sin(x))^2} - \frac{3a^3\left(a^2-2b^2\right)\cos(x)}{2b^3\left(a^2-b^2\right)^2(a+b\sin(x))} - \frac{\left(6a^2\right.(}{}$$

$$= -\frac{3ax}{b^4} + \frac{3a^2\left(2a^4-5a^2b^2+4b^4\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{b^4\left(a^2-b^2\right)^{5/2}} - \frac{\left(3a^2-2b^2\right)\cos(x)}{2b^3\left(a^2-b^2\right)} + \frac{a^2\cos(x)\sin^2(x)}{2b\left(a^2-b^2\right)(a+b\sin}$$

Mathematica [A] time = 0.756829, size = 144, normalized size = 0.8

$$\frac{\frac{6a^2\left(-5a^2b^2+2a^4+4b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}} + \frac{a^3b\left(8b^2-5a^2\right)\cos(x)}{(a-b)^2(a+b)^2(a+b\sin(x))} + \frac{a^4b\cos(x)}{(a-b)(a+b)(a+b\sin(x))^2} - 6ax - 2b\cos(x)}{2b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^4/(a + b*Sin[x])^3,x]
```

```
[Out] (-6*a*x + (6*a^2*(2*a^4 - 5*a^2*b^2 + 4*b^4)*ArcTan[(b + a*Tan[x/2])/Sqrt[a
^2 - b^2]])/(a^2 - b^2)^(5/2) - 2*b*Cos[x] + (a^4*b*Cos[x])/((a - b)*(a + b
)*(a + b*Sin[x])^2) + (a^3*b*(-5*a^2 + 8*b^2)*Cos[x])/((a - b)^2*(a + b)^2*
(a + b*Sin[x])))/(2*b^4)
```

Maple [B] time = 0.062, size = 634, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^4/(a+b*sin(x))^3,x)
```

```
[Out] -2/b^3/(tan(1/2*x)^2+1)-6/b^4*a*arctan(tan(1/2*x))-3*a^5/b^2/(tan(1/2*x)^2*
a+2*tan(1/2*x)*b+a)^2/(a^4-2*a^2*b^2+b^4)*tan(1/2*x)^3+6*a^3/(tan(1/2*x)^2*
a+2*tan(1/2*x)*b+a)^2/(a^4-2*a^2*b^2+b^4)*tan(1/2*x)^3-4*a^6/b^3/(tan(1/2*x
)^2*a+2*tan(1/2*x)*b+a)^2/(a^4-2*a^2*b^2+b^4)*tan(1/2*x)^2-a^4/b/(tan(1/2*x
```

```
)^2*a+2*tan(1/2*x)*b+a)^2/(a^4-2*a^2*b^2+b^4)*tan(1/2*x)^2+14*a^2*b/(tan(1/
2*x)^2*a+2*tan(1/2*x)*b+a)^2/(a^4-2*a^2*b^2+b^4)*tan(1/2*x)^2-13*a^5/b^2/(t
an(1/2*x)^2*a+2*tan(1/2*x)*b+a)^2/(a^4-2*a^2*b^2+b^4)*tan(1/2*x)+22*a^3/(ta
n(1/2*x)^2*a+2*tan(1/2*x)*b+a)^2/(a^4-2*a^2*b^2+b^4)*tan(1/2*x)-4*a^6/b^3/(
tan(1/2*x)^2*a+2*tan(1/2*x)*b+a)^2/(a^4-2*a^2*b^2+b^4)+7*a^4/b/(tan(1/2*x)^
2*a+2*tan(1/2*x)*b+a)^2/(a^4-2*a^2*b^2+b^4)+6*a^6/b^4/(a^4-2*a^2*b^2+b^4)/(
a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*x)+2*b)/(a^2-b^2)^(1/2))-15*a^4/b^2/
(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*x)+2*b)/(a^2-b^
2)^(1/2))+12*a^2/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/
2*x)+2*b)/(a^2-b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^4/(a+b*sin(x))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.28055, size = 2013, normalized size = 11.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^4/(a+b*sin(x))^3,x, algorithm="fricas")
```

```
[Out] [1/4*(12*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*x*cos(x)^2 + 4*(a^6*b^3
- 3*a^4*b^5 + 3*a^2*b^7 - b^9)*cos(x)^3 - 3*(2*a^8 - 3*a^6*b^2 - a^4*b^4 +
4*a^2*b^6 - (2*a^6*b^2 - 5*a^4*b^4 + 4*a^2*b^6)*cos(x)^2 + 2*(2*a^7*b - 5*a
^5*b^3 + 4*a^3*b^5)*sin(x))*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(x)^2 -
2*a*b*sin(x) - a^2 - b^2 + 2*(a*cos(x)*sin(x) + b*cos(x))*sqrt(-a^2 + b^2))
/(b^2*cos(x)^2 - 2*a*b*sin(x) - a^2 - b^2)) - 12*(a^9 - 2*a^7*b^2 + 2*a^3*b
^6 - a*b^8)*x - 2*(6*a^8*b - 15*a^6*b^3 + 7*a^4*b^5 + 4*a^2*b^7 - 2*b^9)*co
s(x) - 2*(12*(a^8*b - 3*a^6*b^3 + 3*a^4*b^5 - a^2*b^7)*x + (9*a^7*b^2 - 25*
a^5*b^4 + 20*a^3*b^6 - 4*a*b^8)*cos(x))*sin(x))/(a^8*b^4 - 2*a^6*b^6 + 2*a^
2*b^10 - b^12 - (a^6*b^6 - 3*a^4*b^8 + 3*a^2*b^10 - b^12)*cos(x)^2 + 2*(a^7
*b^5 - 3*a^5*b^7 + 3*a^3*b^9 - a*b^11)*sin(x)), 1/2*(6*(a^7*b^2 - 3*a^5*b^4
 + 3*a^3*b^6 - a*b^8)*x*cos(x)^2 + 2*(a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9
)*cos(x)^3 - 3*(2*a^8 - 3*a^6*b^2 - a^4*b^4 + 4*a^2*b^6 - (2*a^6*b^2 - 5*a^
4*b^4 + 4*a^2*b^6)*cos(x)^2 + 2*(2*a^7*b - 5*a^5*b^3 + 4*a^3*b^5)*sin(x))*s
qrt(a^2 - b^2)*arctan(-(a*sin(x) + b)/(sqrt(a^2 - b^2)*cos(x))) - 6*(a^9 -
2*a^7*b^2 + 2*a^3*b^6 - a*b^8)*x - (6*a^8*b - 15*a^6*b^3 + 7*a^4*b^5 + 4*a^
2*b^7 - 2*b^9)*cos(x) - (12*(a^8*b - 3*a^6*b^3 + 3*a^4*b^5 - a^2*b^7)*x + (
9*a^7*b^2 - 25*a^5*b^4 + 20*a^3*b^6 - 4*a*b^8)*cos(x))*sin(x))/(a^8*b^4 - 2
*a^6*b^6 + 2*a^2*b^10 - b^12 - (a^6*b^6 - 3*a^4*b^8 + 3*a^2*b^10 - b^12)*co
s(x)^2 + 2*(a^7*b^5 - 3*a^5*b^7 + 3*a^3*b^9 - a*b^11)*sin(x))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**4/(a+b*sin(x))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.98173, size = 346, normalized size = 1.93

$$\frac{3\left(2a^6 - 5a^4b^2 + 4a^2b^4\right)\left(\pi\left\lfloor \frac{x}{2\pi} + \frac{1}{2}\right\rfloor \operatorname{sgn}(a) + \arctan\left(\frac{a\tan\left(\frac{1}{2}x\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^4b^4 - 2a^2b^6 + b^8\right)\sqrt{a^2-b^2}} - \frac{3a^5b\tan\left(\frac{1}{2}x\right)^3 - 6a^3b^3\tan\left(\frac{1}{2}x\right)^3 + 4a^6\tan\left(\frac{1}{2}\right.}{\left(a^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^4/(a+b*sin(x))^3,x, algorithm="giac")
```

```
[Out] 3*(2*a^6 - 5*a^4*b^2 + 4*a^2*b^4)*(pi*floor(1/2*x/pi + 1/2)*sgn(a) + arctan
((a*tan(1/2*x) + b)/sqrt(a^2 - b^2)))/((a^4*b^4 - 2*a^2*b^6 + b^8)*sqrt(a^2
 - b^2)) - (3*a^5*b*tan(1/2*x)^3 - 6*a^3*b^3*tan(1/2*x)^3 + 4*a^6*tan(1/2*x
)^2 + a^4*b^2*tan(1/2*x)^2 - 14*a^2*b^4*tan(1/2*x)^2 + 13*a^5*b*tan(1/2*x)
- 22*a^3*b^3*tan(1/2*x) + 4*a^6 - 7*a^4*b^2)/((a^4*b^3 - 2*a^2*b^5 + b^7)*(
a*tan(1/2*x)^2 + 2*b*tan(1/2*x) + a)^2) - 3*a*x/b^4 - 2/((tan(1/2*x)^2 + 1)
*b^3)
```

3.195 $\int \frac{\sin^3(x)}{(a+b\sin(x))^3}\,dx$

Optimal. Leaf size=144

$$-\frac{a\left(-5a^2b^2+2a^4+6b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{b^3\left(a^2-b^2\right)^{5/2}}+\frac{a^2\left(2a^2-5b^2\right)\cos(x)}{2b^2\left(a^2-b^2\right)^2(a+b\sin(x))}+\frac{a^2\sin(x)\cos(x)}{2b\left(a^2-b^2\right)(a+b\sin(x))^2}+\frac{x}{b^3}$$

```
[Out] x/b^3 - (a*(2*a^4 - 5*a^2*b^2 + 6*b^4)*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b
^2]])/(b^3*(a^2 - b^2)^(5/2)) + (a^2*Cos[x]*Sin[x])/(2*b*(a^2 - b^2)*(a + b
*Sin[x])^2) + (a^2*(2*a^2 - 5*b^2)*Cos[x])/(2*b^2*(a^2 - b^2)^2*(a + b*Sin[
x]))
```

Rubi [A] time = 0.244593, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2792, 3021, 2735, 2660, 618, 204}

$$-\frac{a\left(-5a^2b^2+2a^4+6b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{b^3\left(a^2-b^2\right)^{5/2}}+\frac{a^2\left(2a^2-5b^2\right)\cos(x)}{2b^2\left(a^2-b^2\right)^2(a+b\sin(x))}+\frac{a^2\sin(x)\cos(x)}{2b\left(a^2-b^2\right)(a+b\sin(x))^2}+\frac{x}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^3/(a + b*Sin[x])^3,x]
```


```
[Out] x/b^3 - (a*(2*a^4 - 5*a^2*b^2 + 6*b^4)*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b
^2]])/(b^3*(a^2 - b^2)^(5/2)) + (a^2*Cos[x]*Sin[x])/(2*b*(a^2 - b^2)*(a + b
*Sin[x])^2) + (a^2*(2*a^2 - 5*b^2)*Cos[x])/(2*b^2*(a^2 - b^2)^2*(a + b*Sin[
x]))
```


Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sin^3(x)}{(a+b\sin(x))^3}\,dx = \frac{a^2\cos(x)\sin(x)}{2b\left(a^2-b^2\right)(a+b\sin(x))^2} - \frac{\int \frac{a^2-2ab\sin(x)-2\left(a^2-b^2\right)\sin^2(x)}{(a+b\sin(x))^2}\,dx}{2b\left(a^2-b^2\right)}$$

$$= \frac{a^2\cos(x)\sin(x)}{2b\left(a^2-b^2\right)(a+b\sin(x))^2} + \frac{a^2\left(2a^2-5b^2\right)\cos(x)}{2b^2\left(a^2-b^2\right)^2(a+b\sin(x))} + \frac{\int \frac{ab\left(a^2-4b^2\right)+2\left(a^2-b^2\right)^2\sin(x)}{a+b\sin(x)}\,dx}{2b^2\left(a^2-b^2\right)^2}$$

$$= \frac{x}{b^3} + \frac{a^2\cos(x)\sin(x)}{2b\left(a^2-b^2\right)(a+b\sin(x))^2} + \frac{a^2\left(2a^2-5b^2\right)\cos(x)}{2b^2\left(a^2-b^2\right)^2(a+b\sin(x))} - \frac{\left(a\left(2a^4-5a^2b^2+6b^4\right)\right)\int \frac{1}{a+b\sin}}{2b^3\left(a^2-b^2\right)^2}$$

$$= \frac{x}{b^3} + \frac{a^2\cos(x)\sin(x)}{2b\left(a^2-b^2\right)(a+b\sin(x))^2} + \frac{a^2\left(2a^2-5b^2\right)\cos(x)}{2b^2\left(a^2-b^2\right)^2(a+b\sin(x))} - \frac{\left(a\left(2a^4-5a^2b^2+6b^4\right)\right)\text{Subst}}{b^3\left(a^2-\right.}$$

$$= \frac{x}{b^3} + \frac{a^2\cos(x)\sin(x)}{2b\left(a^2-b^2\right)(a+b\sin(x))^2} + \frac{a^2\left(2a^2-5b^2\right)\cos(x)}{2b^2\left(a^2-b^2\right)^2(a+b\sin(x))} + \frac{\left(2a\left(2a^4-5a^2b^2+6b^4\right)\right)\text{Subs}}{b^3}$$

$$= \frac{x}{b^3} - \frac{a\left(2a^4-5a^2b^2+6b^4\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{b^3\left(a^2-b^2\right)^{5/2}} + \frac{a^2\cos(x)\sin(x)}{2b\left(a^2-b^2\right)(a+b\sin(x))^2} + \frac{a^2\left(2a^2-5b^2\right)\text{c}}{2b^2\left(a^2-b^2\right)^2(a+}$$

Mathematica [A] time = 0.513841, size = 136, normalized size = 0.94

$$\frac{-\frac{2a\left(-5a^2b^2+2a^4+6b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}} + \frac{3a^2b\left(a^2-2b^2\right)\cos(x)}{(a-b)^2(a+b)^2(a+b\sin(x))} - \frac{a^3b\cos(x)}{(a-b)(a+b)(a+b\sin(x))^2} + 2x}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^3/(a + b*Sin[x])^3,x]
```

```
[Out] (2*x - (2*a*(2*a^4 - 5*a^2*b^2 + 6*b^4)*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 -
b^2]])/(a^2 - b^2)^(5/2) - (a^3*b*Cos[x])/((a - b)*(a + b)*(a + b*Sin[x])^2
) + (3*a^2*b*(a^2 - 2*b^2)*Cos[x])/((a - b)^2*(a + b)^2*(a + b*Sin[x])))/(2
*b^3)
```

Maple [B] time = 0.057, size = 612, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3/(a+b*sin(x))^3,x)
```

```
[Out] 2/b^3*arctan(tan(1/2*x))+a^4/b/(tan(1/2*x)^2*a+2*tan(1/2*x)*b+a)^2/(a^4-2*a
^2*b^2+b^4)*tan(1/2*x)^3-4*b*a^2/(tan(1/2*x)^2*a+2*tan(1/2*x)*b+a)^2/(a^4-2
*a^2*b^2+b^4)*tan(1/2*x)^3+2*a^5/b^2/(tan(1/2*x)^2*a+2*tan(1/2*x)*b+a)^2/(a
^4-2*a^2*b^2+b^4)*tan(1/2*x)^2-a^3/(tan(1/2*x)^2*a+2*tan(1/2*x)*b+a)^2/(a^4
-2*a^2*b^2+b^4)*tan(1/2*x)^2-10*b^2*a/(tan(1/2*x)^2*a+2*tan(1/2*x)*b+a)^2/(
a^4-2*a^2*b^2+b^4)*tan(1/2*x)^2+7*a^4/b/(tan(1/2*x)^2*a+2*tan(1/2*x)*b+a)^2
/(a^4-2*a^2*b^2+b^4)*tan(1/2*x)-16*b*a^2/(tan(1/2*x)^2*a+2*tan(1/2*x)*b+a)^
2/(a^4-2*a^2*b^2+b^4)*tan(1/2*x)+2*a^5/b^2/(tan(1/2*x)^2*a+2*tan(1/2*x)*b+a
)^2/(a^4-2*a^2*b^2+b^4)-5*a^3/(tan(1/2*x)^2*a+2*tan(1/2*x)*b+a)^2/(a^4-2*a^
2*b^2+b^4)-2*a^5/b^3/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*ta
n(1/2*x)+2*b)/(a^2-b^2)^(1/2))+5*a^3/b/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*
arctan(1/2*(2*a*tan(1/2*x)+2*b)/(a^2-b^2)^(1/2))-6*b*a/(a^4-2*a^2*b^2+b^4)/
(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*x)+2*b)/(a^2-b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(a+b*sin(x))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.16055, size = 1742, normalized size = 12.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(a+b*sin(x))^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*x*cos(x)^2 + (2*a^7 - 3*a^
5*b^2 + a^3*b^4 + 6*a*b^6 - (2*a^5*b^2 - 5*a^3*b^4 + 6*a*b^6)*cos(x)^2 + 2*
(2*a^6*b - 5*a^4*b^3 + 6*a^2*b^5)*sin(x))*sqrt(-a^2 + b^2)*log(-((2*a^2 - b
^2)*cos(x)^2 - 2*a*b*sin(x) - a^2 - b^2 - 2*(a*cos(x)*sin(x) + b*cos(x))*sq
```

```
rt(-a^2 + b^2))/(b^2*cos(x)^2 - 2*a*b*sin(x) - a^2 - b^2)) - 4*(a^8 - 2*a^6
*b^2 + 2*a^2*b^6 - b^8)*x - 2*(2*a^7*b - 7*a^5*b^3 + 5*a^3*b^5)*cos(x) - 2*
(4*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*x + 3*(a^6*b^2 - 3*a^4*b^4 + 2*a
^2*b^6)*cos(x))*sin(x))/(a^8*b^3 - 2*a^6*b^5 + 2*a^2*b^9 - b^11 - (a^6*b^5
- 3*a^4*b^7 + 3*a^2*b^9 - b^11)*cos(x)^2 + 2*(a^7*b^4 - 3*a^5*b^6 + 3*a^3*b
^8 - a*b^10)*sin(x)), -1/2*(2*(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*x*cos
(x)^2 - (2*a^7 - 3*a^5*b^2 + a^3*b^4 + 6*a*b^6 - (2*a^5*b^2 - 5*a^3*b^4 + 6
*a*b^6)*cos(x)^2 + 2*(2*a^6*b - 5*a^4*b^3 + 6*a^2*b^5)*sin(x))*sqrt(a^2 - b
^2)*arctan(-(a*sin(x) + b)/(sqrt(a^2 - b^2)*cos(x))) - 2*(a^8 - 2*a^6*b^2 +
 2*a^2*b^6 - b^8)*x - (2*a^7*b - 7*a^5*b^3 + 5*a^3*b^5)*cos(x) - (4*(a^7*b
- 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*x + 3*(a^6*b^2 - 3*a^4*b^4 + 2*a^2*b^6)*co
s(x))*sin(x))/(a^8*b^3 - 2*a^6*b^5 + 2*a^2*b^9 - b^11 - (a^6*b^5 - 3*a^4*b^
7 + 3*a^2*b^9 - b^11)*cos(x)^2 + 2*(a^7*b^4 - 3*a^5*b^6 + 3*a^3*b^8 - a*b^1
0)*sin(x))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**3/(a+b*sin(x))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.64813, size = 316, normalized size = 2.19

$$-\frac{\left(2\,a^{5}-5\,a^{3}b^{2}+6\,ab^{4}\right)\left(\pi\left\lfloor\frac{x}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(a\right)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,x\right)+b}{\sqrt{a^{2}-b^{2}}}\right)\right)}{\left(a^{4}b^{3}-2\,a^{2}b^{5}+b^{7}\right)\sqrt{a^{2}-b^{2}}}+\frac{a^{4}b\tan\left(\frac{1}{2}\,x\right)^{3}-4\,a^{2}b^{3}\tan\left(\frac{1}{2}\,x\right)^{3}+2\,a^{5}\tan\left(\frac{1}{2}\,x\right.}{\left(a^{4}b^{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(a+b*sin(x))^3,x, algorithm="giac")
```

```
[Out] -(2*a^5 - 5*a^3*b^2 + 6*a*b^4)*(pi*floor(1/2*x/pi + 1/2)*sgn(a) + arctan((a
*tan(1/2*x) + b)/sqrt(a^2 - b^2)))/((a^4*b^3 - 2*a^2*b^5 + b^7)*sqrt(a^2 -
b^2)) + (a^4*b*tan(1/2*x)^3 - 4*a^2*b^3*tan(1/2*x)^3 + 2*a^5*tan(1/2*x)^2 -
 a^3*b^2*tan(1/2*x)^2 - 10*a*b^4*tan(1/2*x)^2 + 7*a^4*b*tan(1/2*x) - 16*a^2
*b^3*tan(1/2*x) + 2*a^5 - 5*a^3*b^2)/((a^4*b^2 - 2*a^2*b^4 + b^6)*(a*tan(1/
2*x)^2 + 2*b*tan(1/2*x) + a)^2) + x/b^3
```

3.196 $\int \frac{\sin^2(x)}{(a+b\sin(x))^3}\,dx$

Optimal. Leaf size=118

$$\frac{(a^2+2b^2)\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}}+\frac{a^2\cos(x)}{2b(a^2-b^2)(a+b\sin(x))^2}-\frac{a(a^2-4b^2)\cos(x)}{2b(a^2-b^2)^2(a+b\sin(x))}$$

```
[Out] ((a^2 + 2*b^2)*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2)
+ (a^2*Cos[x])/(2*b*(a^2 - b^2)*(a + b*Sin[x])^2) - (a*(a^2 - 4*b^2)*Cos[x]
)/(2*b*(a^2 - b^2)^2*(a + b*Sin[x]))
```

Rubi [A] time = 0.143604, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2790, 2754, 12, 2660, 618, 204}

$$\frac{(a^2+2b^2)\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}}+\frac{a^2\cos(x)}{2b(a^2-b^2)(a+b\sin(x))^2}-\frac{a(a^2-4b^2)\cos(x)}{2b(a^2-b^2)^2(a+b\sin(x))}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^2/(a + b*Sin[x])^3,x]
```

```
[Out] ((a^2 + 2*b^2)*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2)
+ (a^2*Cos[x])/(2*b*(a^2 - b^2)*(a + b*Sin[x])^2) - (a*(a^2 - 4*b^2)*Cos[x]
)/(2*b*(a^2 - b^2)^2*(a + b*Sin[x]))
```

Rule 2790

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos[e +
 f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] - Dist[1/
(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m + 1)*(2
*b*c*d - a*(c^2 + d^2)) + (a^2*d^2 - 2*a*b*c*d*(m + 2) + b^2*(d^2*(m + 1) +
 c^2*(m + 2)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sin^2(x)}{(a+b\sin(x))^3}\,dx = \frac{a^2\cos(x)}{2b\left(a^2-b^2\right)(a+b\sin(x))^2}+\frac{\int \frac{2ab+\left(a^2-2b^2\right)\sin(x)}{(a+b\sin(x))^2}\,dx}{2b\left(a^2-b^2\right)}$$

$$= \frac{a^2\cos(x)}{2b\left(a^2-b^2\right)(a+b\sin(x))^2}-\frac{a\left(a^2-4b^2\right)\cos(x)}{2b\left(a^2-b^2\right)^2(a+b\sin(x))}+\frac{\int \frac{b\left(a^2+2b^2\right)}{a+b\sin(x)}\,dx}{2b\left(a^2-b^2\right)^2}$$

$$= \frac{a^2\cos(x)}{2b\left(a^2-b^2\right)(a+b\sin(x))^2}-\frac{a\left(a^2-4b^2\right)\cos(x)}{2b\left(a^2-b^2\right)^2(a+b\sin(x))}+\frac{\left(a^2+2b^2\right)\int \frac{1}{a+b\sin(x)}\,dx}{2\left(a^2-b^2\right)^2}$$

$$= \frac{a^2\cos(x)}{2b\left(a^2-b^2\right)(a+b\sin(x))^2}-\frac{a\left(a^2-4b^2\right)\cos(x)}{2b\left(a^2-b^2\right)^2(a+b\sin(x))}+\frac{\left(a^2+2b^2\right)\text{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx,x,\text{t}\right.}{\left(a^2-b^2\right)^2}$$

$$= \frac{a^2\cos(x)}{2b\left(a^2-b^2\right)(a+b\sin(x))^2}-\frac{a\left(a^2-4b^2\right)\cos(x)}{2b\left(a^2-b^2\right)^2(a+b\sin(x))}-\frac{\left(2\left(a^2+2b^2\right)\right)\text{Subst}\left(\int \frac{1}{-4\left(a^2-b^2\right)-x^2}\,d\right.}{\left(a^2-b^2\right)^2}$$

$$= \frac{\left(a^2+2b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}}+\frac{a^2\cos(x)}{2b\left(a^2-b^2\right)(a+b\sin(x))^2}-\frac{a\left(a^2-4b^2\right)\cos(x)}{2b\left(a^2-b^2\right)^2(a+b\sin(x))}$$

Mathematica [A] time = 0.351667, size = 94, normalized size = 0.8

$$\frac{\left(a^2+2b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}}+\frac{a\cos(x)\left(3ab-\left(a^2-4b^2\right)\sin(x)\right)}{2(a-b)^2(a+b)^2(a+b\sin(x))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^2/(a + b*Sin[x])^3,x]
```

```
[Out] ((a^2 + 2*b^2)*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2)
+ (a*Cos[x]*(3*a*b - (a^2 - 4*b^2)*Sin[x]))/(2*(a - b)^2*(a + b)^2*(a + b*S
in[x])^2)
```

Maple [B] time = 0.051, size = 265, normalized size = 2.3

$$8\frac{1}{\left((\tan(x/2))^2 a + 2\tan(x/2) b + a\right)^2}\left(1/8\frac{\left(a^2+2b^2\right)a(\tan(x/2))^3}{a^4-2a^2b^2+b^4} + 3/8\frac{b\left(a^2+2b^2\right)(\tan(x/2))^2}{a^4-2a^2b^2+b^4} - 1/8\frac{a\left(a^2-10b^2\right.}{a^4-2a^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2/(a+b*sin(x))^3,x)
```

```
[Out] 8*(1/8*(a^2+2*b^2)*a/(a^4-2*a^2*b^2+b^4)*tan(1/2*x)^3+3/8*b*(a^2+2*b^2)/(a^
4-2*a^2*b^2+b^4)*tan(1/2*x)^2-1/8*a*(a^2-10*b^2)/(a^4-2*a^2*b^2+b^4)*tan(1/
2*x)+3/8*a^2*b/(a^4-2*a^2*b^2+b^4))/(tan(1/2*x)^2*a+2*tan(1/2*x)*b+a)^2+a^2
/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*x)+2*b)/(a^2-b
^2)^(1/2))+2/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*x)
+2*b)/(a^2-b^2)^(1/2))*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a+b*sin(x))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.10686, size = 1138, normalized size = 9.64

$$\left[-\frac{2\left(a^5-5a^3b^2+4ab^4\right)\cos(x)\sin(x)+\left(a^4+3a^2b^2+2b^4-\left(a^2b^2+2b^4\right)\cos(x)^2+2\left(a^3b+2ab^3\right)\sin(x)\right)\sqrt{-a^2+}}{4\left(a^8-2a^6b^2+2a^2b^6-b^8-\left(a^6b^2-3a^4b^4+3a^2b^6-b^8\right)\cos(x)^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a+b*sin(x))^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*(a^5 - 5*a^3*b^2 + 4*a*b^4)*cos(x)*sin(x) + (a^4 + 3*a^2*b^2 + 2*b
^4 - (a^2*b^2 + 2*b^4)*cos(x)^2 + 2*(a^3*b + 2*a*b^3)*sin(x))*sqrt(-a^2 + b
^2)*log(((2*a^2 - b^2)*cos(x)^2 - 2*a*b*sin(x) - a^2 - b^2 + 2*(a*cos(x)*si
n(x) + b*cos(x))*sqrt(-a^2 + b^2))/(b^2*cos(x)^2 - 2*a*b*sin(x) - a^2 - b^2
)) - 6*(a^4*b - a^2*b^3)*cos(x))/(a^8 - 2*a^6*b^2 + 2*a^2*b^6 - b^8 - (a^6*
b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*cos(x)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*
b^5 - a*b^7)*sin(x)), -1/2*((a^5 - 5*a^3*b^2 + 4*a*b^4)*cos(x)*sin(x) + (a^
4 + 3*a^2*b^2 + 2*b^4 - (a^2*b^2 + 2*b^4)*cos(x)^2 + 2*(a^3*b + 2*a*b^3)*si
n(x))*sqrt(a^2 - b^2)*arctan(-(a*sin(x) + b)/(sqrt(a^2 - b^2)*cos(x))) - 3*
(a^4*b - a^2*b^3)*cos(x))/(a^8 - 2*a^6*b^2 + 2*a^2*b^6 - b^8 - (a^6*b^2 - 3
*a^4*b^4 + 3*a^2*b^6 - b^8)*cos(x)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a
*b^7)*sin(x))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**2/(a+b*sin(x))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.80612, size = 246, normalized size = 2.08

$$\frac{\left(\pi \left\lfloor \frac{x}{2\pi} + \frac{1}{2} \right\rfloor \operatorname{sgn}(a) + \arctan\left(\frac{a\tan\left(\frac{1}{2}x\right)+b}{\sqrt{a^2-b^2}}\right)\right)\left(a^2+2b^2\right)}{\left(a^4-2a^2b^2+b^4\right)\sqrt{a^2-b^2}} + \frac{a^3\tan\left(\frac{1}{2}x\right)^3 + 2ab^2\tan\left(\frac{1}{2}x\right)^3 + 3a^2b\tan\left(\frac{1}{2}x\right)^2 + 6b^3\tan(\ldots}{\left(a^4-2a^2b^2+b^4\right)\left(a\tan\left(\frac{1}{2}x\right)^2 + \ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a+b*sin(x))^3,x, algorithm="giac")
```

```
[Out] (pi*floor(1/2*x/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*x) + b)/sqrt(a^2 - b^2
)))*(a^2 + 2*b^2)/((a^4 - 2*a^2*b^2 + b^4)*sqrt(a^2 - b^2)) + (a^3*tan(1/2*
x)^3 + 2*a*b^2*tan(1/2*x)^3 + 3*a^2*b*tan(1/2*x)^2 + 6*b^3*tan(1/2*x)^2 - a
^3*tan(1/2*x) + 10*a*b^2*tan(1/2*x) + 3*a^2*b)/((a^4 - 2*a^2*b^2 + b^4)*(a*
tan(1/2*x)^2 + 2*b*tan(1/2*x) + a)^2)
```

3.197 $\int \frac{\sin(x)}{(a+b\sin(x))^3}\,dx$

Optimal. Leaf size=103

$$-\frac{3ab\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}}-\frac{\left(a^2+2b^2\right)\cos(x)}{2\left(a^2-b^2\right)^2(a+b\sin(x))}-\frac{a\cos(x)}{2\left(a^2-b^2\right)(a+b\sin(x))^2}$$

```
[Out] (-3*a*b*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2) - (a*Co
s[x])/(2*(a^2 - b^2)*(a + b*Sin[x])^2) - ((a^2 + 2*b^2)*Cos[x])/(2*(a^2 - b
^2)^2*(a + b*Sin[x]))
```

Rubi [A] time = 0.102521, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {2754, 12, 2660, 618, 204}

$$-\frac{3ab\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}}-\frac{\left(a^2+2b^2\right)\cos(x)}{2\left(a^2-b^2\right)^2(a+b\sin(x))}-\frac{a\cos(x)}{2\left(a^2-b^2\right)(a+b\sin(x))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]/(a + b*Sin[x])^3,x]
```

```
[Out] (-3*a*b*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2) - (a*Co
s[x])/(2*(a^2 - b^2)*(a + b*Sin[x])^2) - ((a^2 + 2*b^2)*Cos[x])/(2*(a^2 - b
^2)^2*(a + b*Sin[x]))
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin(x)}{(a+b\sin(x))^3}\,dx &= -\frac{a\cos(x)}{2\left(a^2-b^2\right)(a+b\sin(x))^2}-\frac{\int \frac{2b-a\sin(x)}{(a+b\sin(x))^2}\,dx}{2\left(a^2-b^2\right)}\\
&= -\frac{a\cos(x)}{2\left(a^2-b^2\right)(a+b\sin(x))^2}-\frac{\left(a^2+2b^2\right)\cos(x)}{2\left(a^2-b^2\right)^2(a+b\sin(x))}+\frac{\int -\frac{3ab}{a+b\sin(x)}\,dx}{2\left(a^2-b^2\right)^2}\\
&= -\frac{a\cos(x)}{2\left(a^2-b^2\right)(a+b\sin(x))^2}-\frac{\left(a^2+2b^2\right)\cos(x)}{2\left(a^2-b^2\right)^2(a+b\sin(x))}-\frac{(3ab)\int \frac{1}{a+b\sin(x)}\,dx}{2\left(a^2-b^2\right)^2}\\
&= -\frac{a\cos(x)}{2\left(a^2-b^2\right)(a+b\sin(x))^2}-\frac{\left(a^2+2b^2\right)\cos(x)}{2\left(a^2-b^2\right)^2(a+b\sin(x))}-\frac{(3ab)\,\mathrm{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx,x,\tan\left(\frac{x}{2}\right)\right)}{\left(a^2-b^2\right)^2}\\
&= -\frac{a\cos(x)}{2\left(a^2-b^2\right)(a+b\sin(x))^2}-\frac{\left(a^2+2b^2\right)\cos(x)}{2\left(a^2-b^2\right)^2(a+b\sin(x))}+\frac{(6ab)\,\mathrm{Subst}\left(\int \frac{1}{-4\left(a^2-b^2\right)-x^2}\,dx,x,2b+\ldots\right)}{\left(a^2-b^2\right)^2}\\
&= -\frac{3ab\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}}-\frac{a\cos(x)}{2\left(a^2-b^2\right)(a+b\sin(x))^2}-\frac{\left(a^2+2b^2\right)\cos(x)}{2\left(a^2-b^2\right)^2(a+b\sin(x))}
\end{aligned}$$

Mathematica [A] time = 0.278169, size = 94, normalized size = 0.91

$$-\frac{3ab\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}}-\frac{\cos(x)\left(b\left(a^2+2b^2\right)\sin(x)+a\left(2a^2+b^2\right)\right)}{2(a-b)^2(a+b)^2(a+b\sin(x))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/(a + b*Sin[x])^3,x]
```

```
[Out] (-3*a*b*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2) - (Cos[
x]*(a*(2*a^2 + b^2) + b*(a^2 + 2*b^2)*Sin[x]))/(2*(a - b)^2*(a + b)^2*(a +
b*Sin[x])^2)
```

Maple [B] time = 0.049, size = 221, normalized size = 2.2

$$4\,\frac{1}{\left(\left(\tan(x/2)\right)^2a+2\,\tan(x/2)\,b+a\right)^2}\left(-3/4\,\frac{a^2b\left(\tan(x/2)\right)^3}{a^4-2\,a^2b^2+b^4}-1/4\,\frac{\left(2\,a^4+5\,a^2b^2+2\,b^4\right)\left(\tan(x/2)\right)^2}{a\left(a^4-2\,a^2b^2+b^4\right)}-1/4\,\frac{\left(5\,a^2+4\,b^2\right)b\ldots}{a^4-2\,a^2b\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(a+b*sin(x))^3,x)
```

```
[Out] 4*(-3/4*a^2*b/(a^4-2*a^2*b^2+b^4)*tan(1/2*x)^3-1/4*(2*a^4+5*a^2*b^2+2*b^4)/
a/(a^4-2*a^2*b^2+b^4)*tan(1/2*x)^2-1/4*(5*a^2+4*b^2)*b/(a^4-2*a^2*b^2+b^4)*
tan(1/2*x)-1/4*(2*a^2+b^2)*a/(a^4-2*a^2*b^2+b^4))/(tan(1/2*x)^2*a+2*tan(1/2
*x)*b+a)^2-3*b*a/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/
2*x)+2*b)/(a^2-b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+b*sin(x))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.20117, size = 1072, normalized size = 10.41

$$\left[-\frac{2\left(a^4b+a^2b^3-2b^5\right)\cos(x)\sin(x)-3\left(ab^3\cos(x)^2-2a^2b^2\sin(x)-a^3b-ab^3\right)\sqrt{-a^2+b^2}\log\left(-\frac{\left(2a^2-b^2\right)\cos(x)^2-2ab}{\cdots}\right.}{4\left(a^8-2a^6b^2+2a^2b^6-b^8-\left(a^6b^2-3a^4b^4+3a^2b^6-b^8\right)\cos(x)^2+2\left(a^7b\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+b*sin(x))^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*(a^4*b + a^2*b^3 - 2*b^5)*cos(x)*sin(x) - 3*(a*b^3*cos(x)^2 - 2*a^
2*b^2*sin(x) - a^3*b - a*b^3)*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(x)^2
 - 2*a*b*sin(x) - a^2 - b^2 - 2*(a*cos(x)*sin(x) + b*cos(x))*sqrt(-a^2 + b^
2))/(b^2*cos(x)^2 - 2*a*b*sin(x) - a^2 - b^2)) + 2*(2*a^5 - a^3*b^2 - a*b^4
)*cos(x))/(a^8 - 2*a^6*b^2 + 2*a^2*b^6 - b^8 - (a^6*b^2 - 3*a^4*b^4 + 3*a^2
*b^6 - b^8)*cos(x)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*sin(x)), -
1/2*((a^4*b + a^2*b^3 - 2*b^5)*cos(x)*sin(x) + 3*(a*b^3*cos(x)^2 - 2*a^2*b^
2*sin(x) - a^3*b - a*b^3)*sqrt(a^2 - b^2)*arctan(-(a*sin(x) + b)/(sqrt(a^2
- b^2)*cos(x))) + (2*a^5 - a^3*b^2 - a*b^4)*cos(x))/(a^8 - 2*a^6*b^2 + 2*a^
2*b^6 - b^8 - (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*cos(x)^2 + 2*(a^7*b -
 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*sin(x))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+b*sin(x))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.69427, size = 255, normalized size = 2.48

$$-\frac{3\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}x\right)+b}{\sqrt{a^2-b^2}}\right)\right)ab}{\left(a^4-2a^2b^2+b^4\right)\sqrt{a^2-b^2}}-\frac{3a^3b\tan\left(\frac{1}{2}x\right)^3+2a^4\tan\left(\frac{1}{2}x\right)^2+5a^2b^2\tan\left(\frac{1}{2}x\right)^2+2b^4\tan\left(\frac{1}{2}x\right.}{\left(a^5-2a^3b^2+ab^4\right)\left(a\tan\left(\frac{1}{2}x\right)^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+b*sin(x))^3,x, algorithm="giac")
```

```
[Out] -3*(pi*floor(1/2*x/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*x) + b)/sqrt(a^2 -
b^2)))*a*b/((a^4 - 2*a^2*b^2 + b^4)*sqrt(a^2 - b^2)) - (3*a^3*b*tan(1/2*x)^
3 + 2*a^4*tan(1/2*x)^2 + 5*a^2*b^2*tan(1/2*x)^2 + 2*b^4*tan(1/2*x)^2 + 5*a^
3*b*tan(1/2*x) + 4*a*b^3*tan(1/2*x) + 2*a^4 + a^2*b^2)/((a^5 - 2*a^3*b^2 +
a*b^4)*(a*tan(1/2*x)^2 + 2*b*tan(1/2*x) + a)^2)
```

3.198 $\int \frac{1}{(a+b\sin(x))^3}\,dx$

Optimal. Leaf size=102

$$\frac{(2a^2+b^2)\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}}+\frac{3ab\cos(x)}{2(a^2-b^2)^2(a+b\sin(x))}+\frac{b\cos(x)}{2(a^2-b^2)(a+b\sin(x))^2}$$

```
[Out] ((2*a^2 + b^2)*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2)
+ (b*Cos[x])/(2*(a^2 - b^2)*(a + b*Sin[x])^2) + (3*a*b*Cos[x])/(2*(a^2 - b^
2)^2*(a + b*Sin[x]))
```

Rubi [A] time = 0.0946794, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.75, Rules used = {2664, 2754, 12, 2660, 618, 204}

$$\frac{(2a^2+b^2)\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}}+\frac{3ab\cos(x)}{2(a^2-b^2)^2(a+b\sin(x))}+\frac{b\cos(x)}{2(a^2-b^2)(a+b\sin(x))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[x])^(-3),x]
```

```
[Out] ((2*a^2 + b^2)*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2)
+ (b*Cos[x])/(2*(a^2 - b^2)*(a + b*Sin[x])^2) + (3*a*b*Cos[x])/(2*(a^2 - b^
2)^2*(a + b*Sin[x]))
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
```

```
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(a+b\sin(x))^3}\,dx = \frac{b\cos(x)}{2\left(a^2-b^2\right)(a+b\sin(x))^2} - \frac{\int \frac{-2a+b\sin(x)}{(a+b\sin(x))^2}\,dx}{2\left(a^2-b^2\right)}$$

$$= \frac{b\cos(x)}{2\left(a^2-b^2\right)(a+b\sin(x))^2} + \frac{3ab\cos(x)}{2\left(a^2-b^2\right)^2(a+b\sin(x))} + \frac{\int \frac{2a^2+b^2}{a+b\sin(x)}\,dx}{2\left(a^2-b^2\right)^2}$$

$$= \frac{b\cos(x)}{2\left(a^2-b^2\right)(a+b\sin(x))^2} + \frac{3ab\cos(x)}{2\left(a^2-b^2\right)^2(a+b\sin(x))} + \frac{\left(2a^2+b^2\right)\int \frac{1}{a+b\sin(x)}\,dx}{2\left(a^2-b^2\right)^2}$$

$$= \frac{b\cos(x)}{2\left(a^2-b^2\right)(a+b\sin(x))^2} + \frac{3ab\cos(x)}{2\left(a^2-b^2\right)^2(a+b\sin(x))} + \frac{\left(2a^2+b^2\right)\mathrm{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx, x, \tan\right.}{\left(a^2-b^2\right)^2}$$

$$= \frac{b\cos(x)}{2\left(a^2-b^2\right)(a+b\sin(x))^2} + \frac{3ab\cos(x)}{2\left(a^2-b^2\right)^2(a+b\sin(x))} - \frac{\left(2\left(2a^2+b^2\right)\right)\mathrm{Subst}\left(\int \frac{1}{-4\left(a^2-b^2\right)-x^2}\,dx,\right.}{\left(a^2-b^2\right)^2}$$

$$= \frac{\left(2a^2+b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}} + \frac{b\cos(x)}{2\left(a^2-b^2\right)(a+b\sin(x))^2} + \frac{3ab\cos(x)}{2\left(a^2-b^2\right)^2(a+b\sin(x))}$$

Mathematica [A] time = 0.181512, size = 93, normalized size = 0.91

$$\frac{\left(2a^2+b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}} + \frac{b\cos(x)\left(4a^2+3ab\sin(x)-b^2\right)}{2(a-b)^2(a+b)^2(a+b\sin(x))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[x])^(-3),x]
```

```
[Out] ((2*a^2 + b^2)*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2)
+ (b*Cos[x]*(4*a^2 - b^2 + 3*a*b*Sin[x]))/(2*(a - b)^2*(a + b)^2*(a + b*Sin
[x])^2)
```

Maple [B] time = 0.052, size = 300, normalized size = 2.9

$$2\frac{1}{\left((\tan(x/2))^2 a+2\tan(x/2)b+a\right)^2}\left(1/2\frac{b^2\left(5a^2-2b^2\right)(\tan(x/2))^3}{a\left(a^4-2a^2b^2+b^4\right)}+1/2\frac{b\left(4a^4+7a^2b^2-2b^4\right)(\tan(x/2))^2}{\left(a^4-2a^2b^2+b^4\right)a^2}+1/2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(x))^3,x)
```

```
[Out] 2*(1/2*b^2*(5*a^2-2*b^2)/(a^4-2*a^2*b^2+b^4)/a*tan(1/2*x)^3+1/2*b*(4*a^4+7*
a^2*b^2-2*b^4)/(a^4-2*a^2*b^2+b^4)/a^2*tan(1/2*x)^2+1/2*b^2*(11*a^2-2*b^2)/
a/(a^4-2*a^2*b^2+b^4)*tan(1/2*x)+1/2*b*(4*a^2-b^2)/(a^4-2*a^2*b^2+b^4))/(ta
n(1/2*x)^2*a+2*tan(1/2*x)*b+a)^2+2*a^2/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*
arctan(1/2*(2*a*tan(1/2*x)+2*b)/(a^2-b^2)^(1/2))+1/(a^4-2*a^2*b^2+b^4)/(a^2
-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*x)+2*b)/(a^2-b^2)^(1/2))*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(x))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.12588, size = 1135, normalized size = 11.13

$$\left[\frac{6\left(a^3b^2-ab^4\right)\cos(x)\sin(x)-\left(2a^4+3a^2b^2+b^4-\left(2a^2b^2+b^4\right)\cos(x)^2+2\left(2a^3b+ab^3\right)\sin(x)\right)\sqrt{-a^2+b^2}\log\left(\left(2\right.\right.}{4\left(a^8-2a^6b^2+2a^2b^6-b^8-\left(a^6b^2-3a^4b^4+3a^2b^6-b^8\right)\cos(x)^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(x))^3,x, algorithm="fricas")
```

```
[Out] [1/4*(6*(a^3*b^2 - a*b^4)*cos(x)*sin(x) - (2*a^4 + 3*a^2*b^2 + b^4 - (2*a^2
*b^2 + b^4)*cos(x)^2 + 2*(2*a^3*b + a*b^3)*sin(x))*sqrt(-a^2 + b^2)*log(((2
*a^2 - b^2)*cos(x)^2 - 2*a*b*sin(x) - a^2 - b^2 + 2*(a*cos(x)*sin(x) + b*co
s(x))*sqrt(-a^2 + b^2))/(b^2*cos(x)^2 - 2*a*b*sin(x) - a^2 - b^2)) + 2*(4*a
^4*b - 5*a^2*b^3 + b^5)*cos(x))/(a^8 - 2*a^6*b^2 + 2*a^2*b^6 - b^8 - (a^6*b
^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*cos(x)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b
^5 - a*b^7)*sin(x)), 1/2*(3*(a^3*b^2 - a*b^4)*cos(x)*sin(x) - (2*a^4 + 3*a^
2*b^2 + b^4 - (2*a^2*b^2 + b^4)*cos(x)^2 + 2*(2*a^3*b + a*b^3)*sin(x))*sqrt
(a^2 - b^2)*arctan(-(a*sin(x) + b)/(sqrt(a^2 - b^2)*cos(x))) + (4*a^4*b - 5
*a^2*b^3 + b^5)*cos(x))/(a^8 - 2*a^6*b^2 + 2*a^2*b^6 - b^8 - (a^6*b^2 - 3*a
^4*b^4 + 3*a^2*b^6 - b^8)*cos(x)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b
^7)*sin(x))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(x))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.58914, size = 290, normalized size = 2.84

$$\frac{\left(\pi \left\lfloor \frac{x}{2\pi} + \frac{1}{2} \right\rfloor \operatorname{sgn}(a) + \arctan\left(\frac{a\tan\left(\frac{1}{2}x\right)+b}{\sqrt{a^2-b^2}}\right)\right)\left(2a^2+b^2\right)}{\left(a^4-2a^2b^2+b^4\right)\sqrt{a^2-b^2}} + \frac{5a^3b^2\tan\left(\frac{1}{2}x\right)^3 - 2ab^4\tan\left(\frac{1}{2}x\right)^3 + 4a^4b\tan\left(\frac{1}{2}x\right)^2 + 7a^2b^3}{\left(a^6-2a^4b^2+a^2b^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(x))^3,x, algorithm="giac")
```

```
[Out] (pi*floor(1/2*x/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*x) + b)/sqrt(a^2 - b^2
)))*(2*a^2 + b^2)/((a^4 - 2*a^2*b^2 + b^4)*sqrt(a^2 - b^2)) + (5*a^3*b^2*ta
n(1/2*x)^3 - 2*a*b^4*tan(1/2*x)^3 + 4*a^4*b*tan(1/2*x)^2 + 7*a^2*b^3*tan(1/
2*x)^2 - 2*b^5*tan(1/2*x)^2 + 11*a^3*b^2*tan(1/2*x) - 2*a*b^4*tan(1/2*x) +
4*a^4*b - a^2*b^3)/((a^6 - 2*a^4*b^2 + a^2*b^4)*(a*tan(1/2*x)^2 + 2*b*tan(1
/2*x) + a)^2)
```

3.199 $\int \frac{\csc(x)}{(a+b\sin(x))^3}\,dx$

Optimal. Leaf size=145

$$-\frac{b\left(-5a^2b^2+6a^4+2b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{a^3\left(a^2-b^2\right)^{5/2}}-\frac{b^2\left(5a^2-2b^2\right)\cos(x)}{2a^2\left(a^2-b^2\right)^2(a+b\sin(x))}-\frac{b^2\cos(x)}{2a\left(a^2-b^2\right)(a+b\sin(x))^2}-\frac{\tanh^{-1}(\mathrm{co}}{a^3}$$

```
[Out] -((b*(6*a^4 - 5*a^2*b^2 + 2*b^4)*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/
(a^3*(a^2 - b^2)^(5/2))) - ArcTanh[Cos[x]]/a^3 - (b^2*Cos[x])/(2*a*(a^2 - b
^2)*(a + b*Sin[x])^2) - (b^2*(5*a^2 - 2*b^2)*Cos[x])/(2*a^2*(a^2 - b^2)^2*(
a + b*Sin[x]))
```

Rubi [A] time = 0.370485, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {2802, 3055, 3001, 3770, 2660, 618, 204}

$$-\frac{b\left(-5a^2b^2+6a^4+2b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{a^3\left(a^2-b^2\right)^{5/2}}-\frac{b^2\left(5a^2-2b^2\right)\cos(x)}{2a^2\left(a^2-b^2\right)^2(a+b\sin(x))}-\frac{b^2\cos(x)}{2a\left(a^2-b^2\right)(a+b\sin(x))^2}-\frac{\tanh^{-1}(\mathrm{co}}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]/(a + b*Sin[x])^3,x]
```

```
[Out] -((b*(6*a^4 - 5*a^2*b^2 + 2*b^4)*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/
(a^3*(a^2 - b^2)^(5/2))) - ArcTanh[Cos[x]]/a^3 - (b^2*Cos[x])/(2*a*(a^2 - b
^2)*(a + b*Sin[x])^2) - (b^2*(5*a^2 - 2*b^2)*Cos[x])/(2*a^2*(a^2 - b^2)^2*(
a + b*Sin[x]))
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
```

```
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\csc(x)}{(a+b\sin(x))^3}\,dx = -\frac{b^2\cos(x)}{2a\left(a^2-b^2\right)(a+b\sin(x))^2} + \frac{\int \frac{\csc(x)\left(2\left(a^2-b^2\right)-2ab\sin(x)+b^2\sin^2(x)\right)}{(a+b\sin(x))^2}dx}{2a\left(a^2-b^2\right)}$$

$$= -\frac{b^2\cos(x)}{2a\left(a^2-b^2\right)(a+b\sin(x))^2} - \frac{b^2\left(5a^2-2b^2\right)\cos(x)}{2a^2\left(a^2-b^2\right)^2(a+b\sin(x))} + \frac{\int \frac{\csc(x)\left(2\left(a^2-b^2\right)^2-ab\left(4a^2-b^2\right)\sin(x)\right)}{a+b\sin(x)}}{2a^2\left(a^2-b^2\right)^2}$$

$$= -\frac{b^2\cos(x)}{2a\left(a^2-b^2\right)(a+b\sin(x))^2} - \frac{b^2\left(5a^2-2b^2\right)\cos(x)}{2a^2\left(a^2-b^2\right)^2(a+b\sin(x))} + \frac{\int \csc(x)\,dx}{a^3} - \frac{\left(b\left(6a^4-5a^2b^2+\right.\right.}{2a^3\left(\right.}$$

$$= -\frac{\tanh^{-1}(\cos(x))}{a^3} - \frac{b^2\cos(x)}{2a\left(a^2-b^2\right)(a+b\sin(x))^2} - \frac{b^2\left(5a^2-2b^2\right)\cos(x)}{2a^2\left(a^2-b^2\right)^2(a+b\sin(x))} - \frac{\left(b\left(6a^4-5a^2\right.\right.}{}$$

$$= -\frac{\tanh^{-1}(\cos(x))}{a^3} - \frac{b^2\cos(x)}{2a\left(a^2-b^2\right)(a+b\sin(x))^2} - \frac{b^2\left(5a^2-2b^2\right)\cos(x)}{2a^2\left(a^2-b^2\right)^2(a+b\sin(x))} + \frac{\left(2b\left(6a^4-5a\right.\right.}{}$$

$$= -\frac{b\left(6a^4-5a^2b^2+2b^4\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{a^3\left(a^2-b^2\right)^{5/2}} - \frac{\tanh^{-1}(\cos(x))}{a^3} - \frac{b^2\cos(x)}{2a\left(a^2-b^2\right)(a+b\sin(x))^2} - \frac{}{2}$$

Mathematica [A] time = 0.795476, size = 140, normalized size = 0.97

$$-\frac{\frac{2b\left(-5a^2b^2+6a^4+2b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}} + \frac{ab^2\cos(x)\left(b\left(5a^2-2b^2\right)\sin(x)+6a^3-3ab^2\right)}{(a-b)^2(a+b)^2(a+b\sin(x))^2} - 2\log\left(\sin\left(\frac{x}{2}\right)\right) + 2\log\left(\cos\left(\frac{x}{2}\right)\right)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]/(a + b*Sin[x])^3,x]
```

```
[Out] -((2*b*(6*a^4 - 5*a^2*b^2 + 2*b^4)*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]]
)/(a^2 - b^2)^(5/2) + 2*Log[Cos[x/2]] - 2*Log[Sin[x/2]] + (a*b^2*Cos[x]*(6*
a^3 - 3*a*b^2 + b*(5*a^2 - 2*b^2)*Sin[x]))/((a - b)^2*(a + b)^2*(a + b*Sin[
x])^2))/(2*a^3)
```

Maple [B] time = 0.075, size = 614, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)/(a+b*sin(x))^3,x)
```

```
[Out] -7*b^3/(tan(1/2*x)^2*a+2*tan(1/2*x)*b+a)^2/(a^4-2*a^2*b^2+b^4)*tan(1/2*x)^3
+4/a^2*b^5/(tan(1/2*x)^2*a+2*tan(1/2*x)*b+a)^2/(a^4-2*a^2*b^2+b^4)*tan(1/2*
x)^3-6*b^2*a/(tan(1/2*x)^2*a+2*tan(1/2*x)*b+a)^2/(a^4-2*a^2*b^2+b^4)*tan(1/
2*x)^2-9/a*b^4/(tan(1/2*x)^2*a+2*tan(1/2*x)*b+a)^2/(a^4-2*a^2*b^2+b^4)*tan(
1/2*x)^2+6/a^3*b^6/(tan(1/2*x)^2*a+2*tan(1/2*x)*b+a)^2/(a^4-2*a^2*b^2+b^4)*
tan(1/2*x)^2-17*b^3/(tan(1/2*x)^2*a+2*tan(1/2*x)*b+a)^2/(a^4-2*a^2*b^2+b^4)
*tan(1/2*x)+8/a^2*b^5/(tan(1/2*x)^2*a+2*tan(1/2*x)*b+a)^2/(a^4-2*a^2*b^2+b^
```

```
4)*tan(1/2*x)-6*a*b^2/(tan(1/2*x)^2*a+2*tan(1/2*x)*b+a)^2/(a^4-2*a^2*b^2+b^
4)+3/a*b^4/(tan(1/2*x)^2*a+2*tan(1/2*x)*b+a)^2/(a^4-2*a^2*b^2+b^4)-6*b*a/(a
^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*x)+2*b)/(a^2-b^2)
^(1/2))+5/a*b^3/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2
*x)+2*b)/(a^2-b^2)^(1/2))-2/a^3*b^5/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arc
tan(1/2*(2*a*tan(1/2*x)+2*b)/(a^2-b^2)^(1/2))+1/a^3*ln(tan(1/2*x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a+b*sin(x))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 8.20914, size = 2233, normalized size = 15.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a+b*sin(x))^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*(5*a^5*b^3 - 7*a^3*b^5 + 2*a*b^7)*cos(x)*sin(x) + (6*a^6*b + a^4*b
^3 - 3*a^2*b^5 + 2*b^7 - (6*a^4*b^3 - 5*a^2*b^5 + 2*b^7)*cos(x)^2 + 2*(6*a^
5*b^2 - 5*a^3*b^4 + 2*a*b^6)*sin(x))*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*c
os(x)^2 - 2*a*b*sin(x) - a^2 - b^2 - 2*(a*cos(x)*sin(x) + b*cos(x))*sqrt(-a
^2 + b^2))/(b^2*cos(x)^2 - 2*a*b*sin(x) - a^2 - b^2)) + 6*(2*a^6*b^2 - 3*a^
4*b^4 + a^2*b^6)*cos(x) + 2*(a^8 - 2*a^6*b^2 + 2*a^2*b^6 - b^8 - (a^6*b^2 -
 3*a^4*b^4 + 3*a^2*b^6 - b^8)*cos(x)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 -
 a*b^7)*sin(x))*log(1/2*cos(x) + 1/2) - 2*(a^8 - 2*a^6*b^2 + 2*a^2*b^6 - b^
8 - (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*cos(x)^2 + 2*(a^7*b - 3*a^5*b^3
 + 3*a^3*b^5 - a*b^7)*sin(x))*log(-1/2*cos(x) + 1/2))/(a^11 - 2*a^9*b^2 + 2
*a^5*b^6 - a^3*b^8 - (a^9*b^2 - 3*a^7*b^4 + 3*a^5*b^6 - a^3*b^8)*cos(x)^2 +
 2*(a^10*b - 3*a^8*b^3 + 3*a^6*b^5 - a^4*b^7)*sin(x)), -1/2*((5*a^5*b^3 - 7
*a^3*b^5 + 2*a*b^7)*cos(x)*sin(x) - (6*a^6*b + a^4*b^3 - 3*a^2*b^5 + 2*b^7
- (6*a^4*b^3 - 5*a^2*b^5 + 2*b^7)*cos(x)^2 + 2*(6*a^5*b^2 - 5*a^3*b^4 + 2*a
*b^6)*sin(x))*sqrt(a^2 - b^2)*arctan(-(a*sin(x) + b)/(sqrt(a^2 - b^2)*cos(x
))) + 3*(2*a^6*b^2 - 3*a^4*b^4 + a^2*b^6)*cos(x) + (a^8 - 2*a^6*b^2 + 2*a^2
*b^6 - b^8 - (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*cos(x)^2 + 2*(a^7*b -
3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*sin(x))*log(1/2*cos(x) + 1/2) - (a^8 - 2*a^6
*b^2 + 2*a^2*b^6 - b^8 - (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*cos(x)^2 +
 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*sin(x))*log(-1/2*cos(x) + 1/2))/
(a^11 - 2*a^9*b^2 + 2*a^5*b^6 - a^3*b^8 - (a^9*b^2 - 3*a^7*b^4 + 3*a^5*b^6
- a^3*b^8)*cos(x)^2 + 2*(a^10*b - 3*a^8*b^3 + 3*a^6*b^5 - a^4*b^7)*sin(x))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(x)}{(a + b\sin(x))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a+b*sin(x))**3,x)
```

```
[Out] Integral(csc(x)/(a + b*sin(x))**3, x)
```

Giac [A] time = 1.62522, size = 332, normalized size = 2.29

$$-\frac{\left(6\,a^{4}b-5\,a^{2}b^{3}+2\,b^{5}\right)\left(\pi\left\lfloor\frac{x}{2\,\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}\left(a\right)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,x\right)+b}{\sqrt{a^{2}-b^{2}}}\right)\right)}{\left(a^{7}-2\,a^{5}b^{2}+a^{3}b^{4}\right)\sqrt{a^{2}-b^{2}}}-\frac{7\,a^{3}b^{3}\tan\left(\frac{1}{2}\,x\right)^{3}-4\,ab^{5}\tan\left(\frac{1}{2}\,x\right)^{3}+6\,a^{4}b^{2}\,\mathrm{ta}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a+b*sin(x))^3,x, algorithm="giac")
```

```
[Out] -(6*a^4*b - 5*a^2*b^3 + 2*b^5)*(pi*floor(1/2*x/pi + 1/2)*sgn(a) + arctan((a
*tan(1/2*x) + b)/sqrt(a^2 - b^2)))/((a^7 - 2*a^5*b^2 + a^3*b^4)*sqrt(a^2 -
b^2)) - (7*a^3*b^3*tan(1/2*x)^3 - 4*a*b^5*tan(1/2*x)^3 + 6*a^4*b^2*tan(1/2*
x)^2 + 9*a^2*b^4*tan(1/2*x)^2 - 6*b^6*tan(1/2*x)^2 + 17*a^3*b^3*tan(1/2*x)
- 8*a*b^5*tan(1/2*x) + 6*a^4*b^2 - 3*a^2*b^4)/((a^7 - 2*a^5*b^2 + a^3*b^4)*
(a*tan(1/2*x)^2 + 2*b*tan(1/2*x) + a)^2) + log(abs(tan(1/2*x)))/a^3
```

3.200 $\int \frac{\csc^2(x)}{(a+b\sin(x))^3}\,dx$

Optimal. Leaf size=187

$$\frac{3b^2\left(-5a^2b^2+4a^4+2b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{a^4\left(a^2-b^2\right)^{5/2}}-\frac{\left(-11a^2b^2+2a^4+6b^4\right)\cot(x)}{2a^3\left(a^2-b^2\right)^2}-\frac{3b^2\left(2a^2-b^2\right)\cot(x)}{2a^2\left(a^2-b^2\right)^2(a+b\sin(x))}-\frac{b^2}{2a\left(a^2-b^2\right.}$$

```
[Out] (3*b^2*(4*a^4 - 5*a^2*b^2 + 2*b^4)*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]]
)/(a^4*(a^2 - b^2)^(5/2)) + (3*b*ArcTanh[Cos[x]])/a^4 - ((2*a^4 - 11*a^2*b^
2 + 6*b^4)*Cot[x])/(2*a^3*(a^2 - b^2)^2) - (b^2*Cot[x])/(2*a*(a^2 - b^2)*(a
 + b*Sin[x])^2) - (3*b^2*(2*a^2 - b^2)*Cot[x])/(2*a^2*(a^2 - b^2)^2*(a + b*
Sin[x]))
```

Rubi [A] time = 0.639396, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {2802, 3055, 3001, 3770, 2660, 618, 204}

$$\frac{3b^2\left(-5a^2b^2+4a^4+2b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{a^4\left(a^2-b^2\right)^{5/2}}-\frac{\left(-11a^2b^2+2a^4+6b^4\right)\cot(x)}{2a^3\left(a^2-b^2\right)^2}-\frac{3b^2\left(2a^2-b^2\right)\cot(x)}{2a^2\left(a^2-b^2\right)^2(a+b\sin(x))}-\frac{b^2}{2a\left(a^2-b^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^2/(a + b*Sin[x])^3,x]
```

```
[Out] (3*b^2*(4*a^4 - 5*a^2*b^2 + 2*b^4)*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]]
)/(a^4*(a^2 - b^2)^(5/2)) + (3*b*ArcTanh[Cos[x]])/a^4 - ((2*a^4 - 11*a^2*b^
2 + 6*b^4)*Cot[x])/(2*a^3*(a^2 - b^2)^2) - (b^2*Cot[x])/(2*a*(a^2 - b^2)*(a
 + b*Sin[x])^2) - (3*b^2*(2*a^2 - b^2)*Cot[x])/(2*a^2*(a^2 - b^2)^2*(a + b*
Sin[x]))
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
```

```
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\csc^2(x)}{(a+b\sin(x))^3}\,dx = -\frac{b^2\cot(x)}{2a\left(a^2-b^2\right)(a+b\sin(x))^2} + \frac{\int \frac{\csc^2(x)\left(2a^2-3b^2-2ab\sin(x)+2b^2\sin^2(x)\right)}{(a+b\sin(x))^2}dx}{2a\left(a^2-b^2\right)}$$

$$= -\frac{b^2\cot(x)}{2a\left(a^2-b^2\right)(a+b\sin(x))^2} - \frac{3b^2\left(2a^2-b^2\right)\cot(x)}{2a^2\left(a^2-b^2\right)^2(a+b\sin(x))} + \frac{\int \frac{\csc^2(x)\left(2a^4-11a^2b^2+6b^4-ab\left(4a^2-b^2\right)\sin(x\ldots\right.}{a+b\sin(x)}}{2a^2\left(a^2-b^2\right)^2}$$

$$= -\frac{\left(2a^4-11a^2b^2+6b^4\right)\cot(x)}{2a^3\left(a^2-b^2\right)^2} - \frac{b^2\cot(x)}{2a\left(a^2-b^2\right)(a+b\sin(x))^2} - \frac{3b^2\left(2a^2-b^2\right)\cot(x)}{2a^2\left(a^2-b^2\right)^2(a+b\sin(x))} + \ldots$$

$$= -\frac{\left(2a^4-11a^2b^2+6b^4\right)\cot(x)}{2a^3\left(a^2-b^2\right)^2} - \frac{b^2\cot(x)}{2a\left(a^2-b^2\right)(a+b\sin(x))^2} - \frac{3b^2\left(2a^2-b^2\right)\cot(x)}{2a^2\left(a^2-b^2\right)^2(a+b\sin(x))} - \frac{(3b)\ldots}{}$$

$$= \frac{3b\tanh^{-1}(\cos(x))}{a^4} - \frac{\left(2a^4-11a^2b^2+6b^4\right)\cot(x)}{2a^3\left(a^2-b^2\right)^2} - \frac{b^2\cot(x)}{2a\left(a^2-b^2\right)(a+b\sin(x))^2} - \frac{3b^2\left(2a^2-b\ldots\right.}{2a^2\left(a^2-b^2\right)^2(\ldots}$$

$$= \frac{3b\tanh^{-1}(\cos(x))}{a^4} - \frac{\left(2a^4-11a^2b^2+6b^4\right)\cot(x)}{2a^3\left(a^2-b^2\right)^2} - \frac{b^2\cot(x)}{2a\left(a^2-b^2\right)(a+b\sin(x))^2} - \frac{3b^2\left(2a^2-b\ldots\right.}{2a^2\left(a^2-b^2\right)^2(\ldots}$$

$$= \frac{3b^2\left(4a^4-5a^2b^2+2b^4\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{a^4\left(a^2-b^2\right)^{5/2}} + \frac{3b\tanh^{-1}(\cos(x))}{a^4} - \frac{\left(2a^4-11a^2b^2+6b^4\right)\cot(x)}{2a^3\left(a^2-b^2\right)^2} - \ldots$$

Mathematica [A] time = 1.24753, size = 174, normalized size = 0.93

$$\frac{\frac{6b^2\left(-5a^2b^2+4a^4+2b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}} + \frac{ab^3\left(7a^2-4b^2\right)\cos(x)}{(a-b)^2(a+b)^2(a+b\sin(x))} + \frac{a^2b^3\cos(x)}{(a-b)(a+b)(a+b\sin(x))^2} + a\tan\left(\frac{x}{2}\right) - a\cot\left(\frac{x}{2}\right) - 6b\log\left(\sin\left(\frac{x}{2}\right)\right) + \ldots}{2a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^2/(a + b*Sin[x])^3,x]
```

```
[Out] ((6*b^2*(4*a^4 - 5*a^2*b^2 + 2*b^4)*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]
])/(a^2 - b^2)^(5/2) - a*Cot[x/2] + 6*b*Log[Cos[x/2]] - 6*b*Log[Sin[x/2]] +
 (a^2*b^3*Cos[x])/((a - b)*(a + b)*(a + b*Sin[x])^2) + (a*b^3*(7*a^2 - 4*b^
2)*Cos[x])/((a - b)^2*(a + b)^2*(a + b*Sin[x])) + a*Tan[x/2])/(2*a^4)
```

Maple [B] time = 0.085, size = 641, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^2/(a+b*sin(x))^3,x)
```

```
[Out] 1/2/a^3*tan(1/2*x)+9/a*b^4/(tan(1/2*x)^2*a+2*tan(1/2*x)*b+a)^2/(a^4-2*a^2*b
^2+b^4)*tan(1/2*x)^3-6/a^3*b^6/(tan(1/2*x)^2*a+2*tan(1/2*x)*b+a)^2/(a^4-2*a
^2*b^2+b^4)*tan(1/2*x)^3+8*b^3/(tan(1/2*x)^2*a+2*tan(1/2*x)*b+a)^2/(a^4-2*a
```

```
^2*b^2+b^4)*tan(1/2*x)^2+11/a^2*b^5/(tan(1/2*x)^2*a+2*tan(1/2*x)*b+a)^2/(a^
4-2*a^2*b^2+b^4)*tan(1/2*x)^2-10/a^4*b^7/(tan(1/2*x)^2*a+2*tan(1/2*x)*b+a)^
2/(a^4-2*a^2*b^2+b^4)*tan(1/2*x)^2+23/a*b^4/(tan(1/2*x)^2*a+2*tan(1/2*x)*b+
a)^2/(a^4-2*a^2*b^2+b^4)*tan(1/2*x)-14/a^3*b^6/(tan(1/2*x)^2*a+2*tan(1/2*x)
*b+a)^2/(a^4-2*a^2*b^2+b^4)*tan(1/2*x)+8*b^3/(tan(1/2*x)^2*a+2*tan(1/2*x)*b
+a)^2/(a^4-2*a^2*b^2+b^4)-5/a^2*b^5/(tan(1/2*x)^2*a+2*tan(1/2*x)*b+a)^2/(a^
4-2*a^2*b^2+b^4)+12/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan
(1/2*x)+2*b)/(a^2-b^2)^(1/2))*b^2-15/a^2*b^4/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^
(1/2)*arctan(1/2*(2*a*tan(1/2*x)+2*b)/(a^2-b^2)^(1/2))+6/a^4*b^6/(a^4-2*a^2
*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*x)+2*b)/(a^2-b^2)^(1/2))-
1/2/a^3/tan(1/2*x)-3/a^4*b*ln(tan(1/2*x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a+b*sin(x))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 8.01966, size = 3114, normalized size = 16.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a+b*sin(x))^3,x, algorithm="fricas")
```

```
[Out] [1/4*(2*(2*a^7*b^2 - 13*a^5*b^4 + 17*a^3*b^6 - 6*a*b^8)*cos(x)^3 - 2*(4*a^8
*b - 20*a^6*b^3 + 25*a^4*b^5 - 9*a^2*b^7)*cos(x)*sin(x) - 3*(8*a^5*b^3 - 10
*a^3*b^5 + 4*a*b^7 - 2*(4*a^5*b^3 - 5*a^3*b^5 + 2*a*b^7)*cos(x)^2 + (4*a^6*
b^2 - a^4*b^4 - 3*a^2*b^6 + 2*b^8 - (4*a^4*b^4 - 5*a^2*b^6 + 2*b^8)*cos(x)^
2)*sin(x))*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(x)^2 - 2*a*b*sin(x) - a^
2 - b^2 + 2*(a*cos(x)*sin(x) + b*cos(x))*sqrt(-a^2 + b^2))/(b^2*cos(x)^2 -
2*a*b*sin(x) - a^2 - b^2)) - 2*(2*a^9 - 4*a^7*b^2 - 7*a^5*b^4 + 15*a^3*b^6
- 6*a*b^8)*cos(x) + 6*(2*a^7*b^2 - 6*a^5*b^4 + 6*a^3*b^6 - 2*a*b^8 - 2*(a^7
*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*cos(x)^2 + (a^8*b - 2*a^6*b^3 + 2*a^2
*b^7 - b^9 - (a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*cos(x)^2)*sin(x))*log(
1/2*cos(x) + 1/2) - 6*(2*a^7*b^2 - 6*a^5*b^4 + 6*a^3*b^6 - 2*a*b^8 - 2*(a^7
*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*cos(x)^2 + (a^8*b - 2*a^6*b^3 + 2*a^2
*b^7 - b^9 - (a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*cos(x)^2)*sin(x))*log(
-1/2*cos(x) + 1/2))/(2*a^11*b - 6*a^9*b^3 + 6*a^7*b^5 - 2*a^5*b^7 - 2*(a^11
*b - 3*a^9*b^3 + 3*a^7*b^5 - a^5*b^7)*cos(x)^2 + (a^12 - 2*a^10*b^2 + 2*a^6
*b^6 - a^4*b^8 - (a^10*b^2 - 3*a^8*b^4 + 3*a^6*b^6 - a^4*b^8)*cos(x)^2)*sin
(x)), 1/2*((2*a^7*b^2 - 13*a^5*b^4 + 17*a^3*b^6 - 6*a*b^8)*cos(x)^3 - (4*a^
8*b - 20*a^6*b^3 + 25*a^4*b^5 - 9*a^2*b^7)*cos(x)*sin(x) - 3*(8*a^5*b^3 - 1
0*a^3*b^5 + 4*a*b^7 - 2*(4*a^5*b^3 - 5*a^3*b^5 + 2*a*b^7)*cos(x)^2 + (4*a^6
*b^2 - a^4*b^4 - 3*a^2*b^6 + 2*b^8 - (4*a^4*b^4 - 5*a^2*b^6 + 2*b^8)*cos(x)
^2)*sin(x))*sqrt(a^2 - b^2)*arctan(-(a*sin(x) + b)/(sqrt(a^2 - b^2)*cos(x))
) - (2*a^9 - 4*a^7*b^2 - 7*a^5*b^4 + 15*a^3*b^6 - 6*a*b^8)*cos(x) + 3*(2*a^
7*b^2 - 6*a^5*b^4 + 6*a^3*b^6 - 2*a*b^8 - 2*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^
6 - a*b^8)*cos(x)^2 + (a^8*b - 2*a^6*b^3 + 2*a^2*b^7 - b^9 - (a^6*b^3 - 3*a
^4*b^5 + 3*a^2*b^7 - b^9)*cos(x)^2)*sin(x))*log(1/2*cos(x) + 1/2) - 3*(2*a^
```

```
7*b^2 - 6*a^5*b^4 + 6*a^3*b^6 - 2*a*b^8 - 2*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^
6 - a*b^8)*cos(x)^2 + (a^8*b - 2*a^6*b^3 + 2*a^2*b^7 - b^9 - (a^6*b^3 - 3*a
^4*b^5 + 3*a^2*b^7 - b^9)*cos(x)^2)*sin(x))*log(-1/2*cos(x) + 1/2))/(2*a^11
*b - 6*a^9*b^3 + 6*a^7*b^5 - 2*a^5*b^7 - 2*(a^11*b - 3*a^9*b^3 + 3*a^7*b^5
- a^5*b^7)*cos(x)^2 + (a^12 - 2*a^10*b^2 + 2*a^6*b^6 - a^4*b^8 - (a^10*b^2
- 3*a^8*b^4 + 3*a^6*b^6 - a^4*b^8)*cos(x)^2)*sin(x))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc^2(x)}{(a + b\sin(x))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**2/(a+b*sin(x))**3,x)
```

```
[Out] Integral(csc(x)**2/(a + b*sin(x))**3, x)
```

Giac [A] time = 1.62451, size = 378, normalized size = 2.02

$$\frac{3\left(4\,a^4b^2 - 5\,a^2b^4 + 2\,b^6\right)\left(\pi\left\lfloor\frac{x}{2\pi} + \frac{1}{2}\right\rfloor\operatorname{sgn}(a) + \arctan\left(\frac{a\tan\left(\frac{1}{2}x\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^8 - 2\,a^6b^2 + a^4b^4\right)\sqrt{a^2 - b^2}} + \frac{9\,a^3b^4\tan\left(\frac{1}{2}x\right)^3 - 6\,ab^6\tan\left(\frac{1}{2}x\right)^3 + 8\,a^4b^3\tan}{(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a+b*sin(x))^3,x, algorithm="giac")
```

```
[Out] 3*(4*a^4*b^2 - 5*a^2*b^4 + 2*b^6)*(pi*floor(1/2*x/pi + 1/2)*sgn(a) + arctan
((a*tan(1/2*x) + b)/sqrt(a^2 - b^2)))/((a^8 - 2*a^6*b^2 + a^4*b^4)*sqrt(a^2
 - b^2)) + (9*a^3*b^4*tan(1/2*x)^3 - 6*a*b^6*tan(1/2*x)^3 + 8*a^4*b^3*tan(1
/2*x)^2 + 11*a^2*b^5*tan(1/2*x)^2 - 10*b^7*tan(1/2*x)^2 + 23*a^3*b^4*tan(1/
2*x) - 14*a*b^6*tan(1/2*x) + 8*a^4*b^3 - 5*a^2*b^5)/((a^8 - 2*a^6*b^2 + a^4
*b^4)*(a*tan(1/2*x)^2 + 2*b*tan(1/2*x) + a)^2) - 3*b*log(abs(tan(1/2*x)))/a
^4 + 1/2*tan(1/2*x)/a^3 + 1/2*(6*b*tan(1/2*x) - a)/(a^4*tan(1/2*x))
```

3.201 $\int \frac{\csc^3(x)}{(a+b\sin(x))^3}\,dx$

Optimal. Leaf size=241

$$-\frac{b^3\left(-29a^2b^2+20a^4+12b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{a^5\left(a^2-b^2\right)^{5/2}}+\frac{3b\left(-7a^2b^2+2a^4+4b^4\right)\cot(x)}{2a^4\left(a^2-b^2\right)^2}-\frac{\left(a^2+12b^2\right)\tanh^{-1}(\cos(x))}{2a^5}-\left(\right.$$

```
[Out] -((b^3*(20*a^4 - 29*a^2*b^2 + 12*b^4)*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^
2]])/(a^5*(a^2 - b^2)^(5/2))) - ((a^2 + 12*b^2)*ArcTanh[Cos[x]])/(2*a^5) +
(3*b*(2*a^4 - 7*a^2*b^2 + 4*b^4)*Cot[x])/(2*a^4*(a^2 - b^2)^2) - ((a^4 - 10
*a^2*b^2 + 6*b^4)*Cot[x]*Csc[x])/(2*a^3*(a^2 - b^2)^2) - (b^2*Cot[x]*Csc[x]
)/(2*a*(a^2 - b^2)*(a + b*Sin[x])^2) - (b^2*(7*a^2 - 4*b^2)*Cot[x]*Csc[x])/
(2*a^2*(a^2 - b^2)^2*(a + b*Sin[x]))
```

Rubi [A] time = 0.873067, antiderivative size = 241, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {2802, 3055, 3001, 3770, 2660, 618, 204}

$$-\frac{b^3\left(-29a^2b^2+20a^4+12b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{a^5\left(a^2-b^2\right)^{5/2}}+\frac{3b\left(-7a^2b^2+2a^4+4b^4\right)\cot(x)}{2a^4\left(a^2-b^2\right)^2}-\frac{\left(a^2+12b^2\right)\tanh^{-1}(\cos(x))}{2a^5}-\left(\right.$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^3/(a + b*Sin[x])^3,x]
```

```
[Out] -((b^3*(20*a^4 - 29*a^2*b^2 + 12*b^4)*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^
2]])/(a^5*(a^2 - b^2)^(5/2))) - ((a^2 + 12*b^2)*ArcTanh[Cos[x]])/(2*a^5) +
(3*b*(2*a^4 - 7*a^2*b^2 + 4*b^4)*Cot[x])/(2*a^4*(a^2 - b^2)^2) - ((a^4 - 10
*a^2*b^2 + 6*b^4)*Cot[x]*Csc[x])/(2*a^3*(a^2 - b^2)^2) - (b^2*Cot[x]*Csc[x]
)/(2*a*(a^2 - b^2)*(a + b*Sin[x])^2) - (b^2*(7*a^2 - 4*b^2)*Cot[x]*Csc[x])/
(2*a^2*(a^2 - b^2)^2*(a + b*Sin[x]))
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
```

```
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\csc^3(x)}{(a+b\sin(x))^3}\,dx = -\frac{b^2\cot(x)\csc(x)}{2a\left(a^2-b^2\right)(a+b\sin(x))^2} + \frac{\int \frac{\csc^3(x)\left(2\left(a^2-2b^2\right)-2ab\sin(x)+3b^2\sin^2(x)\right)}{(a+b\sin(x))^2}dx}{2a\left(a^2-b^2\right)}$$

$$= -\frac{b^2\cot(x)\csc(x)}{2a\left(a^2-b^2\right)(a+b\sin(x))^2} - \frac{b^2\left(7a^2-4b^2\right)\cot(x)\csc(x)}{2a^2\left(a^2-b^2\right)^2(a+b\sin(x))} + \frac{\int \frac{\csc^3(x)\left(2\left(a^4-10a^2b^2+6b^4\right)-ab\left(4a^2-b^2\right.\right.}{a+b\sin(}}{2a^2\left(a^2-\right.}$$

$$= -\frac{\left(a^4-10a^2b^2+6b^4\right)\cot(x)\csc(x)}{2a^3\left(a^2-b^2\right)^2} - \frac{b^2\cot(x)\csc(x)}{2a\left(a^2-b^2\right)(a+b\sin(x))^2} - \frac{b^2\left(7a^2-4b^2\right)\cot(x)\csc(}{2a^2\left(a^2-b^2\right)^2(a+b\sin(x}$$

$$= \frac{3b\left(2a^4-7a^2b^2+4b^4\right)\cot(x)}{2a^4\left(a^2-b^2\right)^2} - \frac{\left(a^4-10a^2b^2+6b^4\right)\cot(x)\csc(x)}{2a^3\left(a^2-b^2\right)^2} - \frac{b^2\cot(x)\csc(x)}{2a\left(a^2-b^2\right)(a+b\sin(x)}$$

$$= \frac{3b\left(2a^4-7a^2b^2+4b^4\right)\cot(x)}{2a^4\left(a^2-b^2\right)^2} - \frac{\left(a^4-10a^2b^2+6b^4\right)\cot(x)\csc(x)}{2a^3\left(a^2-b^2\right)^2} - \frac{b^2\cot(x)\csc(x)}{2a\left(a^2-b^2\right)(a+b\sin(x)}$$

$$= -\frac{\left(a^2+12b^2\right)\tanh^{-1}(\cos(x))}{2a^5} + \frac{3b\left(2a^4-7a^2b^2+4b^4\right)\cot(x)}{2a^4\left(a^2-b^2\right)^2} - \frac{\left(a^4-10a^2b^2+6b^4\right)\cot(x)\,\mathrm{cs}}{2a^3\left(a^2-b^2\right)^2}$$

$$= -\frac{\left(a^2+12b^2\right)\tanh^{-1}(\cos(x))}{2a^5} + \frac{3b\left(2a^4-7a^2b^2+4b^4\right)\cot(x)}{2a^4\left(a^2-b^2\right)^2} - \frac{\left(a^4-10a^2b^2+6b^4\right)\cot(x)\,\mathrm{cs}}{2a^3\left(a^2-b^2\right)^2}$$

$$= -\frac{b^3\left(20a^4-29a^2b^2+12b^4\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{a^5\left(a^2-b^2\right)^{5/2}} - \frac{\left(a^2+12b^2\right)\tanh^{-1}(\cos(x))}{2a^5} + \frac{3b\left(2a^4-7a^2b\right.}{2a^4\left(a\right.}$$

Mathematica [A] time = 1.88916, size = 220, normalized size = 0.91

$$\frac{-\frac{8b^3\left(-29a^2b^2+20a^4+12b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}} + 4\left(a^2+12b^2\right)\log\left(\sin\left(\frac{x}{2}\right)\right) - 4\left(a^2+12b^2\right)\log\left(\cos\left(\frac{x}{2}\right)\right) + \frac{12ab^4\left(2b^2-3a^2\right)\cos(x)}{(a-b)^2(a+b)^2(a+b\sin(x))}}{8a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^3/(a + b*Sin[x])^3,x]
```

```
[Out] ((-8*b^3*(20*a^4 - 29*a^2*b^2 + 12*b^4)*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 -
b^2]])/(a^2 - b^2)^(5/2) + 12*a*b*Cot[x/2] - a^2*Csc[x/2]^2 - 4*(a^2 + 12*b
^2)*Log[Cos[x/2]] + 4*(a^2 + 12*b^2)*Log[Sin[x/2]] + a^2*Sec[x/2]^2 - (4*a^
2*b^4*Cos[x])/((a - b)*(a + b)*(a + b*Sin[x])^2) + (12*a*b^4*(-3*a^2 + 2*b^
2)*Cos[x])/((a - b)^2*(a + b)^2*(a + b*Sin[x])) - 12*a*b*Tan[x/2])/(8*a^5)
```

Maple [B] time = 0.088, size = 686, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^3/(a+b*sin(x))^3,x)
```

```
[Out] 1/8/a^3*tan(1/2*x)^2-3/2/a^4*tan(1/2*x)*b-11/a^2*b^5/(tan(1/2*x)^2*a+2*tan(
1/2*x)*b+a)^2/(a^4-2*a^2*b^2+b^4)*tan(1/2*x)^3+8/a^4*b^7/(tan(1/2*x)^2*a+2*
tan(1/2*x)*b+a)^2/(a^4-2*a^2*b^2+b^4)*tan(1/2*x)^3-10/a*b^4/(tan(1/2*x)^2*a
+2*tan(1/2*x)*b+a)^2/(a^4-2*a^2*b^2+b^4)*tan(1/2*x)^2-13/a^3*b^6/(tan(1/2*x
)^2*a+2*tan(1/2*x)*b+a)^2/(a^4-2*a^2*b^2+b^4)*tan(1/2*x)^2+14/a^5*b^8/(tan(
1/2*x)^2*a+2*tan(1/2*x)*b+a)^2/(a^4-2*a^2*b^2+b^4)*tan(1/2*x)^2-29/a^2*b^5/
(tan(1/2*x)^2*a+2*tan(1/2*x)*b+a)^2/(a^4-2*a^2*b^2+b^4)*tan(1/2*x)+20/a^4*b
^7/(tan(1/2*x)^2*a+2*tan(1/2*x)*b+a)^2/(a^4-2*a^2*b^2+b^4)*tan(1/2*x)-10/a*
b^4/(tan(1/2*x)^2*a+2*tan(1/2*x)*b+a)^2/(a^4-2*a^2*b^2+b^4)+7/a^3*b^6/(tan(
1/2*x)^2*a+2*tan(1/2*x)*b+a)^2/(a^4-2*a^2*b^2+b^4)-20/a*b^3/(a^4-2*a^2*b^2+
b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*x)+2*b)/(a^2-b^2)^(1/2))+29/a^
3*b^5/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*x)+2*b)/(
a^2-b^2)^(1/2))-12/a^5*b^7/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(
2*a*tan(1/2*x)+2*b)/(a^2-b^2)^(1/2))-1/8/a^3/tan(1/2*x)^2+1/2/a^3*ln(tan(1/
2*x))+6/a^5*ln(tan(1/2*x))*b^2+3/2*b/a^4/tan(1/2*x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(a+b*sin(x))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 15.1431, size = 4528, normalized size = 18.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(a+b*sin(x))^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*(11*a^8*b^2 - 43*a^6*b^4 + 50*a^4*b^6 - 18*a^2*b^8)*cos(x)^3 + (20
*a^6*b^3 - 9*a^4*b^5 - 17*a^2*b^7 + 12*b^9 + (20*a^4*b^5 - 29*a^2*b^7 + 12*
b^9)*cos(x)^4 - (20*a^6*b^3 + 11*a^4*b^5 - 46*a^2*b^7 + 24*b^9)*cos(x)^2 +
2*(20*a^5*b^4 - 29*a^3*b^6 + 12*a*b^8 - (20*a^5*b^4 - 29*a^3*b^6 + 12*a*b^8
)*cos(x)^2)*sin(x))*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(x)^2 - 2*a*b*s
in(x) - a^2 - b^2 - 2*(a*cos(x)*sin(x) + b*cos(x))*sqrt(-a^2 + b^2))/(b^2*c
os(x)^2 - 2*a*b*sin(x) - a^2 - b^2)) + 2*(a^10 - 14*a^8*b^2 + 46*a^6*b^4 -
51*a^4*b^6 + 18*a^2*b^8)*cos(x) + (a^10 + 10*a^8*b^2 - 24*a^6*b^4 + 2*a^4*b
^6 + 23*a^2*b^8 - 12*b^10 + (a^8*b^2 + 9*a^6*b^4 - 33*a^4*b^6 + 35*a^2*b^8
- 12*b^10)*cos(x)^4 - (a^10 + 11*a^8*b^2 - 15*a^6*b^4 - 31*a^4*b^6 + 58*a^2
*b^8 - 24*b^10)*cos(x)^2 + 2*(a^9*b + 9*a^7*b^3 - 33*a^5*b^5 + 35*a^3*b^7 -
 12*a*b^9 - (a^9*b + 9*a^7*b^3 - 33*a^5*b^5 + 35*a^3*b^7 - 12*a*b^9)*cos(x)
^2)*sin(x))*log(1/2*cos(x) + 1/2) - (a^10 + 10*a^8*b^2 - 24*a^6*b^4 + 2*a^4
*b^6 + 23*a^2*b^8 - 12*b^10 + (a^8*b^2 + 9*a^6*b^4 - 33*a^4*b^6 + 35*a^2*b^
8 - 12*b^10)*cos(x)^4 - (a^10 + 11*a^8*b^2 - 15*a^6*b^4 - 31*a^4*b^6 + 58*a
^2*b^8 - 24*b^10)*cos(x)^2 + 2*(a^9*b + 9*a^7*b^3 - 33*a^5*b^5 + 35*a^3*b^7
 - 12*a*b^9 - (a^9*b + 9*a^7*b^3 - 33*a^5*b^5 + 35*a^3*b^7 - 12*a*b^9)*cos(
x)^2)*sin(x))*log(-1/2*cos(x) + 1/2) + 2*(3*(2*a^7*b^3 - 9*a^5*b^5 + 11*a^3
*b^7 - 4*a*b^9)*cos(x)^3 - (4*a^9*b - 6*a^7*b^3 - 15*a^5*b^5 + 29*a^3*b^7 -
 12*a*b^9)*cos(x))*sin(x))/(a^13 - 2*a^11*b^2 + 2*a^7*b^6 - a^5*b^8 + (a^11
*b^2 - 3*a^9*b^4 + 3*a^7*b^6 - a^5*b^8)*cos(x)^4 - (a^13 - a^11*b^2 - 3*a^9
```

```
*b^4 + 5*a^7*b^6 - 2*a^5*b^8)*cos(x)^2 + 2*(a^12*b - 3*a^10*b^3 + 3*a^8*b^5
 - a^6*b^7 - (a^12*b - 3*a^10*b^3 + 3*a^8*b^5 - a^6*b^7)*cos(x)^2)*sin(x)),
 -1/4*(2*(11*a^8*b^2 - 43*a^6*b^4 + 50*a^4*b^6 - 18*a^2*b^8)*cos(x)^3 - 2*(
20*a^6*b^3 - 9*a^4*b^5 - 17*a^2*b^7 + 12*b^9 + (20*a^4*b^5 - 29*a^2*b^7 + 1
2*b^9)*cos(x)^4 - (20*a^6*b^3 + 11*a^4*b^5 - 46*a^2*b^7 + 24*b^9)*cos(x)^2
+ 2*(20*a^5*b^4 - 29*a^3*b^6 + 12*a*b^8 - (20*a^5*b^4 - 29*a^3*b^6 + 12*a*b
^8)*cos(x)^2)*sin(x))*sqrt(a^2 - b^2)*arctan(-(a*sin(x) + b)/(sqrt(a^2 - b^
2)*cos(x))) + 2*(a^10 - 14*a^8*b^2 + 46*a^6*b^4 - 51*a^4*b^6 + 18*a^2*b^8)*
cos(x) + (a^10 + 10*a^8*b^2 - 24*a^6*b^4 + 2*a^4*b^6 + 23*a^2*b^8 - 12*b^10
 + (a^8*b^2 + 9*a^6*b^4 - 33*a^4*b^6 + 35*a^2*b^8 - 12*b^10)*cos(x)^4 - (a^
10 + 11*a^8*b^2 - 15*a^6*b^4 - 31*a^4*b^6 + 58*a^2*b^8 - 24*b^10)*cos(x)^2
+ 2*(a^9*b + 9*a^7*b^3 - 33*a^5*b^5 + 35*a^3*b^7 - 12*a*b^9 - (a^9*b + 9*a^
7*b^3 - 33*a^5*b^5 + 35*a^3*b^7 - 12*a*b^9)*cos(x)^2)*sin(x))*log(1/2*cos(x
) + 1/2) - (a^10 + 10*a^8*b^2 - 24*a^6*b^4 + 2*a^4*b^6 + 23*a^2*b^8 - 12*b^
10 + (a^8*b^2 + 9*a^6*b^4 - 33*a^4*b^6 + 35*a^2*b^8 - 12*b^10)*cos(x)^4 - (
a^10 + 11*a^8*b^2 - 15*a^6*b^4 - 31*a^4*b^6 + 58*a^2*b^8 - 24*b^10)*cos(x)^
2 + 2*(a^9*b + 9*a^7*b^3 - 33*a^5*b^5 + 35*a^3*b^7 - 12*a*b^9 - (a^9*b + 9*
a^7*b^3 - 33*a^5*b^5 + 35*a^3*b^7 - 12*a*b^9)*cos(x)^2)*sin(x))*log(-1/2*co
s(x) + 1/2) + 2*(3*(2*a^7*b^3 - 9*a^5*b^5 + 11*a^3*b^7 - 4*a*b^9)*cos(x)^3
- (4*a^9*b - 6*a^7*b^3 - 15*a^5*b^5 + 29*a^3*b^7 - 12*a*b^9)*cos(x))*sin(x)
)/(a^13 - 2*a^11*b^2 + 2*a^7*b^6 - a^5*b^8 + (a^11*b^2 - 3*a^9*b^4 + 3*a^7*
b^6 - a^5*b^8)*cos(x)^4 - (a^13 - a^11*b^2 - 3*a^9*b^4 + 5*a^7*b^6 - 2*a^5*
b^8)*cos(x)^2 + 2*(a^12*b - 3*a^10*b^3 + 3*a^8*b^5 - a^6*b^7 - (a^12*b - 3*
a^10*b^3 + 3*a^8*b^5 - a^6*b^7)*cos(x)^2)*sin(x))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc^3(x)}{(a + b\sin(x))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**3/(a+b*sin(x))**3,x)
```

```
[Out] Integral(csc(x)**3/(a + b*sin(x))**3, x)
```

Giac [B] time = 1.64417, size = 694, normalized size = 2.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(a+b*sin(x))^3,x, algorithm="giac")
```

```
[Out] -(20*a^4*b^3 - 29*a^2*b^5 + 12*b^7)*(pi*floor(1/2*x/pi + 1/2)*sgn(a) + arct
an((a*tan(1/2*x) + b)/sqrt(a^2 - b^2)))/((a^9 - 2*a^7*b^2 + a^5*b^4)*sqrt(a
^2 - b^2)) - 1/8*(2*a^8*tan(1/2*x)^6 + 20*a^6*b^2*tan(1/2*x)^6 - 46*a^4*b^4
*tan(1/2*x)^6 + 24*a^2*b^6*tan(1/2*x)^6 - 4*a^7*b*tan(1/2*x)^5 + 104*a^5*b^
3*tan(1/2*x)^5 - 108*a^3*b^5*tan(1/2*x)^5 + 32*a*b^7*tan(1/2*x)^5 + 5*a^8*t
an(1/2*x)^4 - 2*a^6*b^2*tan(1/2*x)^4 + 165*a^4*b^4*tan(1/2*x)^4 - 80*a^2*b^
6*tan(1/2*x)^4 - 16*b^8*tan(1/2*x)^4 - 12*a^7*b*tan(1/2*x)^3 + 72*a^5*b^3*t
an(1/2*x)^3 + 124*a^3*b^5*tan(1/2*x)^3 - 112*a*b^7*tan(1/2*x)^3 + 4*a^8*tan
(1/2*x)^2 - 28*a^6*b^2*tan(1/2*x)^2 + 124*a^4*b^4*tan(1/2*x)^2 - 76*a^2*b^6
*tan(1/2*x)^2 - 8*a^7*b*tan(1/2*x) + 16*a^5*b^3*tan(1/2*x) - 8*a^3*b^5*tan(
```

```
1/2*x) + a^8 - 2*a^6*b^2 + a^4*b^4)/((a^9 - 2*a^7*b^2 + a^5*b^4)*(a*tan(1/2
*x)^3 + 2*b*tan(1/2*x)^2 + a*tan(1/2*x))^2) + 1/2*(a^2 + 12*b^2)*log(abs(ta
n(1/2*x)))/a^5 + 1/8*(a^3*tan(1/2*x)^2 - 12*a^2*b*tan(1/2*x))/a^6
```

3.202 $\int \frac{1}{(a+b\sin(c+dx))^4}\,dx$

Optimal. Leaf size=182

$$\frac{a\left(2a^2+3b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{7/2}}+\frac{b\left(11a^2+4b^2\right)\cos(c+dx)}{6d\left(a^2-b^2\right)^3(a+b\sin(c+dx))}+\frac{5ab\cos(c+dx)}{6d\left(a^2-b^2\right)^2(a+b\sin(c+dx))^2}+\frac{}{3d\,(a^2}$$

```
[Out] (a*(2*a^2 + 3*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2
- b^2)^(7/2)*d) + (b*Cos[c + d*x])/(3*(a^2 - b^2)*d*(a + b*Sin[c + d*x])^3)
 + (5*a*b*Cos[c + d*x])/(6*(a^2 - b^2)^2*d*(a + b*Sin[c + d*x])^2) + (b*(11
*a^2 + 4*b^2)*Cos[c + d*x])/(6*(a^2 - b^2)^3*d*(a + b*Sin[c + d*x]))
```

Rubi [A] time = 0.224564, antiderivative size = 182, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {2664, 2754, 12, 2660, 618, 204}

$$\frac{a\left(2a^2+3b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{7/2}}+\frac{b\left(11a^2+4b^2\right)\cos(c+dx)}{6d\left(a^2-b^2\right)^3(a+b\sin(c+dx))}+\frac{5ab\cos(c+dx)}{6d\left(a^2-b^2\right)^2(a+b\sin(c+dx))^2}+\frac{}{3d\,(a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])^(-4),x]
```

```
[Out] (a*(2*a^2 + 3*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2
- b^2)^(7/2)*d) + (b*Cos[c + d*x])/(3*(a^2 - b^2)*d*(a + b*Sin[c + d*x])^3)
 + (5*a*b*Cos[c + d*x])/(6*(a^2 - b^2)^2*d*(a + b*Sin[c + d*x])^2) + (b*(11
*a^2 + 4*b^2)*Cos[c + d*x])/(6*(a^2 - b^2)^3*d*(a + b*Sin[c + d*x]))
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(a+b\sin(c+dx))^4}\,dx &= \frac{b\cos(c+dx)}{3\left(a^2-b^2\right)d(a+b\sin(c+dx))^3}-\frac{\int \frac{-3a+2b\sin(c+dx)}{(a+b\sin(c+dx))^3}\,dx}{3\left(a^2-b^2\right)}\\
&= \frac{b\cos(c+dx)}{3\left(a^2-b^2\right)d(a+b\sin(c+dx))^3}+\frac{5ab\cos(c+dx)}{6\left(a^2-b^2\right)^2d(a+b\sin(c+dx))^2}+\frac{\int \frac{2\left(3a^2+2b^2\right)-5ab\sin(c+d\ldots}{(a+b\sin(c+dx))^2}}{6\left(a^2-b^2\right)^2}\\
&= \frac{b\cos(c+dx)}{3\left(a^2-b^2\right)d(a+b\sin(c+dx))^3}+\frac{5ab\cos(c+dx)}{6\left(a^2-b^2\right)^2d(a+b\sin(c+dx))^2}+\frac{b\left(11a^2+4b^2\right)\cos\ldots}{6\left(a^2-b^2\right)^3d(a+b\,\mathrm{s}\ldots}\\
&= \frac{b\cos(c+dx)}{3\left(a^2-b^2\right)d(a+b\sin(c+dx))^3}+\frac{5ab\cos(c+dx)}{6\left(a^2-b^2\right)^2d(a+b\sin(c+dx))^2}+\frac{b\left(11a^2+4b^2\right)\cos\ldots}{6\left(a^2-b^2\right)^3d(a+b\,\mathrm{s}\ldots}\\
&= \frac{b\cos(c+dx)}{3\left(a^2-b^2\right)d(a+b\sin(c+dx))^3}+\frac{5ab\cos(c+dx)}{6\left(a^2-b^2\right)^2d(a+b\sin(c+dx))^2}+\frac{b\left(11a^2+4b^2\right)\cos\ldots}{6\left(a^2-b^2\right)^3d(a+b\,\mathrm{s}\ldots}\\
&= \frac{b\cos(c+dx)}{3\left(a^2-b^2\right)d(a+b\sin(c+dx))^3}+\frac{5ab\cos(c+dx)}{6\left(a^2-b^2\right)^2d(a+b\sin(c+dx))^2}+\frac{b\left(11a^2+4b^2\right)\cos\ldots}{6\left(a^2-b^2\right)^3d(a+b\,\mathrm{s}\ldots}\\
&= \frac{a\left(2a^2+3b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{7/2}d}+\frac{b\cos(c+dx)}{3\left(a^2-b^2\right)d(a+b\sin(c+dx))^3}+\frac{5ab\cos(\ldots}{6\left(a^2-b^2\right)^2d(a+\ldots}
\end{aligned}
$$

Mathematica [A] time = 1.06478, size = 157, normalized size = 0.86

$$
\frac{\frac{6a\left(2a^2+3b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{7/2}}+\frac{b\cos(c+dx)\left(b^2\left(11a^2+4b^2\right)\sin^2(c+dx)+3ab\left(9a^2+b^2\right)\sin(c+dx)-5a^2b^2+18a^4+2b^4\right)}{(a-b)^3(a+b)^3(a+b\sin(c+dx))^3}}{6d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])^(-4),x]
```

```
[Out] ((6*a*(2*a^2 + 3*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^
2 - b^2)^(7/2) + (b*Cos[c + d*x]*(18*a^4 - 5*a^2*b^2 + 2*b^4 + 3*a*b*(9*a^2
 + b^2)*Sin[c + d*x] + b^2*(11*a^2 + 4*b^2)*Sin[c + d*x]^2))/((a - b)^3*(a
+ b)^3*(a + b*Sin[c + d*x])^3))/(6*d)
```

Maple [B] time = 0.076, size = 1733, normalized size = 9.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(d*x+c))^4,x)
```

```
[Out] 9/d/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^3*b^2*a^3/(a^6-3*a^4*
b^2+3*a^2*b^4-b^6)*tan(1/2*d*x+1/2*c)^5-6/d/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1
/2*d*x+1/2*c)*b+a)^3*b^4*a/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)*tan(1/2*d*x+1/2*c)
^5+2/d/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^3*b^6/a/(a^6-3*a^4
*b^2+3*a^2*b^4-b^6)*tan(1/2*d*x+1/2*c)^5+6/d/(tan(1/2*d*x+1/2*c)^2*a+2*tan(
1/2*d*x+1/2*c)*b+a)^3*b*a^4/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)*tan(1/2*d*x+1/2*c
)^4+27/d/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^3*b^3*a^2/(a^6-3
*a^4*b^2+3*a^2*b^4-b^6)*tan(1/2*d*x+1/2*c)^4-12/d/(tan(1/2*d*x+1/2*c)^2*a+2
*tan(1/2*d*x+1/2*c)*b+a)^3*b^5/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)*tan(1/2*d*x+1/
2*c)^4+4/d/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^3*b^7/a^2/(a^6
-3*a^4*b^2+3*a^2*b^4-b^6)*tan(1/2*d*x+1/2*c)^4+36/d/(tan(1/2*d*x+1/2*c)^2*a
+2*tan(1/2*d*x+1/2*c)*b+a)^3*a^3*b^2/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)*tan(1/2*
d*x+1/2*c)^3+14/d/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^3*a*b^4
/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)*tan(1/2*d*x+1/2*c)^3-8/3/d/(tan(1/2*d*x+1/2*
c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^3/a*b^6/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)*tan(
1/2*d*x+1/2*c)^3+8/3/d/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^3/
a^3*b^8/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)*tan(1/2*d*x+1/2*c)^3+12/d/(tan(1/2*d*
x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^3*a^4*b/(a^6-3*a^4*b^2+3*a^2*b^4-b^6
)*tan(1/2*d*x+1/2*c)^2+40/d/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+
a)^3*a^2*b^3/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)*tan(1/2*d*x+1/2*c)^2-6/d/(tan(1/
2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^3*b^5/(a^6-3*a^4*b^2+3*a^2*b^4-b
^6)*tan(1/2*d*x+1/2*c)^2+4/d/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b
+a)^3/a^2*b^7/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)*tan(1/2*d*x+1/2*c)^2+27/d/(tan(
1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^3*b^2*a^3/(a^6-3*a^4*b^2+3*a^2
*b^4-b^6)*tan(1/2*d*x+1/2*c)-4/d/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*
c)*b+a)^3*b^4*a/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)*tan(1/2*d*x+1/2*c)+2/d/(tan(1
/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^3*b^6/a/(a^6-3*a^4*b^2+3*a^2*b^
4-b^6)*tan(1/2*d*x+1/2*c)+6/d/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*
b+a)^3*b/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)*a^4-5/3/d/(tan(1/2*d*x+1/2*c)^2*a+2*
tan(1/2*d*x+1/2*c)*b+a)^3*b^3/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)*a^2+2/3/d/(tan(
1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^3*b^5/(a^6-3*a^4*b^2+3*a^2*b^4
-b^6)+2/d*a^3/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a
*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+3/d*a/(a^6-3*a^4*b^2+3*a^2*b^4-b^
6)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))
*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.03495, size = 2101, normalized size = 11.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] [1/12*(2*(11*a^4*b^3 - 7*a^2*b^5 - 4*b^7)*cos(d*x + c)^3 - 6*(9*a^5*b^2 - 8
*a^3*b^4 - a*b^6)*cos(d*x + c)*sin(d*x + c) - 3*(2*a^6 + 9*a^4*b^2 + 9*a^2*
b^4 - 3*(2*a^4*b^2 + 3*a^2*b^4)*cos(d*x + c)^2 + (6*a^5*b + 11*a^3*b^3 + 3*
a*b^5 - (2*a^3*b^3 + 3*a*b^5)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(-a^2 + b^2
)*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(
a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*
x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - 12*(3*a^6*b - 2*a^4*b^3 - b^7
)*cos(d*x + c))/(3*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d
*cos(d*x + c)^2 - (a^11 - a^9*b^2 - 6*a^7*b^4 + 14*a^5*b^6 - 11*a^3*b^8 + 3
*a*b^10)*d + ((a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d*cos(d*
x + c)^2 - (3*a^10*b - 11*a^8*b^3 + 14*a^6*b^5 - 6*a^4*b^7 - a^2*b^9 + b^11
)*d)*sin(d*x + c)), 1/6*((11*a^4*b^3 - 7*a^2*b^5 - 4*b^7)*cos(d*x + c)^3 -
3*(9*a^5*b^2 - 8*a^3*b^4 - a*b^6)*cos(d*x + c)*sin(d*x + c) + 3*(2*a^6 + 9*
a^4*b^2 + 9*a^2*b^4 - 3*(2*a^4*b^2 + 3*a^2*b^4)*cos(d*x + c)^2 + (6*a^5*b +
 11*a^3*b^3 + 3*a*b^5 - (2*a^3*b^3 + 3*a*b^5)*cos(d*x + c)^2)*sin(d*x + c))
*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c)
)) - 6*(3*a^6*b - 2*a^4*b^3 - b^7)*cos(d*x + c))/(3*(a^9*b^2 - 4*a^7*b^4 +
6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cos(d*x + c)^2 - (a^11 - a^9*b^2 - 6*a^7*
b^4 + 14*a^5*b^6 - 11*a^3*b^8 + 3*a*b^10)*d + ((a^8*b^3 - 4*a^6*b^5 + 6*a^4
*b^7 - 4*a^2*b^9 + b^11)*d*cos(d*x + c)^2 - (3*a^10*b - 11*a^8*b^3 + 14*a^6
*b^5 - 6*a^4*b^7 - a^2*b^9 + b^11)*d)*sin(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.23768, size = 689, normalized size = 3.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/3*(3*(2*a^3 + 3*a*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan(
(a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/((a^6 - 3*a^4*b^2 + 3*a^2*b^
4 - b^6)*sqrt(a^2 - b^2)) + (27*a^6*b^2*tan(1/2*d*x + 1/2*c)^5 - 18*a^4*b^4
*tan(1/2*d*x + 1/2*c)^5 + 6*a^2*b^6*tan(1/2*d*x + 1/2*c)^5 + 18*a^7*b*tan(1
/2*d*x + 1/2*c)^4 + 81*a^5*b^3*tan(1/2*d*x + 1/2*c)^4 - 36*a^3*b^5*tan(1/2*
d*x + 1/2*c)^4 + 12*a*b^7*tan(1/2*d*x + 1/2*c)^4 + 108*a^6*b^2*tan(1/2*d*x
+ 1/2*c)^3 + 42*a^4*b^4*tan(1/2*d*x + 1/2*c)^3 - 8*a^2*b^6*tan(1/2*d*x + 1/
2*c)^3 + 8*b^8*tan(1/2*d*x + 1/2*c)^3 + 36*a^7*b*tan(1/2*d*x + 1/2*c)^2 + 1
20*a^5*b^3*tan(1/2*d*x + 1/2*c)^2 - 18*a^3*b^5*tan(1/2*d*x + 1/2*c)^2 + 12*
a*b^7*tan(1/2*d*x + 1/2*c)^2 + 81*a^6*b^2*tan(1/2*d*x + 1/2*c) - 12*a^4*b^4
*tan(1/2*d*x + 1/2*c) + 6*a^2*b^6*tan(1/2*d*x + 1/2*c) + 18*a^7*b - 5*a^5*b
^3 + 2*a^3*b^5)/((a^9 - 3*a^7*b^2 + 3*a^5*b^4 - a^3*b^6)*(a*tan(1/2*d*x + 1
/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)^3))/d
```

3.203 $\int \sin(e+fx)\sqrt{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=172

$$-\frac{2\left(a^2-b^2\right)\sqrt{\frac{a+b\sin(e+fx)}{a+b}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{3bf\sqrt{a+b\sin(e+fx)}}-\frac{2\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{3f}+\frac{2a\sqrt{a+b\sin(e+fx)}E\left(\frac{1}{2}\left(e+fx\right.\right.}{3bf\sqrt{\frac{a+b\sin(e+fx)}{a+b}}}$$

```
[Out] (-2*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]])/(3*f) + (2*a*EllipticE[(e - Pi/2
 + f*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[e + f*x]])/(3*b*f*Sqrt[(a + b*Sin[
e + f*x])/(a + b)]) - (2*(a^2 - b^2)*EllipticF[(e - Pi/2 + f*x)/2, (2*b)/(a
 + b)]*Sqrt[(a + b*Sin[e + f*x])/(a + b)])/(3*b*f*Sqrt[a + b*Sin[e + f*x]])
```

Rubi [A] time = 0.169782, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2753, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2\left(a^2-b^2\right)\sqrt{\frac{a+b\sin(e+fx)}{a+b}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{3bf\sqrt{a+b\sin(e+fx)}}-\frac{2\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{3f}+\frac{2a\sqrt{a+b\sin(e+fx)}E\left(\frac{1}{2}\left(e+fx\right.\right.}{3bf\sqrt{\frac{a+b\sin(e+fx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]],x]
```

```
[Out] (-2*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]])/(3*f) + (2*a*EllipticE[(e - Pi/2
 + f*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[e + f*x]])/(3*b*f*Sqrt[(a + b*Sin[
e + f*x])/(a + b)]) - (2*(a^2 - b^2)*EllipticF[(e - Pi/2 + f*x)/2, (2*b)/(a
 + b)]*Sqrt[(a + b*Sin[e + f*x])/(a + b)])/(3*b*f*Sqrt[a + b*Sin[e + f*x]])
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \sin(e+fx)\sqrt{a+b\sin(e+fx)}\,dx = -\frac{2\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{3f} + \frac{2}{3}\int \frac{\frac{b}{2}+\frac{1}{2}a\sin(e+fx)}{\sqrt{a+b\sin(e+fx)}}\,dx$$

$$= -\frac{2\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{3f} + \frac{a\int \sqrt{a+b\sin(e+fx)}\,dx}{3b} - \frac{\left(a^2-b^2\right)\int \cdots}{3\cdots}$$

$$= -\frac{2\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{3f} + \frac{\left(a\sqrt{a+b\sin(e+fx)}\right)\int \sqrt{\frac{a}{a+b}+\frac{b\sin(e+fx)}{a+b}}\cdots}{3b\sqrt{\frac{a+b\sin(e+fx)}{a+b}}}$$

$$= -\frac{2\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{3f} + \frac{2aE\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2b}{a+b}\right)\sqrt{a+b\sin(e+\cdots}}{3bf\sqrt{\frac{a+b\sin(e+fx)}{a+b}}}$$

Mathematica [A] time = 3.15589, size = 143, normalized size = 0.83

$$-\frac{2\left(-\left(a^2-b^2\right)\sqrt{\frac{a+b\sin(e+fx)}{a+b}}F\left(\frac{1}{4}(-2e-2fx+\pi)|\frac{2b}{a+b}\right)+b\cos(e+fx)(a+b\sin(e+fx))+a(a+b)\sqrt{\frac{a+b\sin(e+fx)}{a+b}}E\left(\frac{1}{4}\cdots\right.\right.}{3bf\sqrt{a+b\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]],x]
```

```
[Out] (-2*(b*Cos[e + f*x]*(a + b*Sin[e + f*x]) + a*(a + b)*EllipticE[(-2*e + Pi -
 2*f*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[e + f*x])/(a + b)] - (a^2 - b^2)*
EllipticF[(-2*e + Pi - 2*f*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[e + f*x])/(
a + b)]))/(3*b*f*Sqrt[a + b*Sin[e + f*x]])
```

Maple [B] time = 0.918, size = 460, normalized size = 2.7

$$\frac{2}{3b^2\cos(fx+e)f}\left(\sqrt{\frac{a+b\sin(fx+e)}{a-b}}\sqrt{-\frac{(-1+\sin(fx+e))b}{a+b}}\sqrt{-\frac{(1+\sin(fx+e))b}{a-b}}\mathrm{EllipticF}\left(\sqrt{\frac{a+b\sin(fx\cdots}{a-b}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)*(a+b*sin(f*x+e))^(1/2),x)
```

```
[Out] 2/3*(((a+b*sin(f*x+e))/(a-b))^(1/2)*(-(-1+sin(f*x+e))*b/(a+b))^(1/2)*(-(1+s
in(f*x+e))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(f*x+e))/(a-b))^(1/2),((a-b)/(
a+b))^(1/2))*a^2*b-((a+b*sin(f*x+e))/(a-b))^(1/2)*(-(-1+sin(f*x+e))*b/(a+b)
)^(1/2)*(-(1+sin(f*x+e))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(f*x+e))/(a-b))^
(1/2),((a-b)/(a+b))^(1/2))*b^3-((a+b*sin(f*x+e))/(a-b))^(1/2)*(-(-1+sin(f*x
+e))*b/(a+b))^(1/2)*(-(1+sin(f*x+e))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(f*x
+e))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3+((a+b*sin(f*x+e))/(a-b))^(1/2)*(
-(-1+sin(f*x+e))*b/(a+b))^(1/2)*(-(1+sin(f*x+e))*b/(a-b))^(1/2)*EllipticE((
(a+b*sin(f*x+e))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^2+sin(f*x+e)^3*b^3+s
in(f*x+e)^2*a*b^2-b^3*sin(f*x+e)-a*b^2)/b^2/cos(f*x+e)/(a+b*sin(f*x+e))^(1/
2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sin\left(fx+e\right)+a}\sin\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sin(f*x + e) + a)*sin(f*x + e), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b \sin\left(fx+e\right)+a}\sin\left(fx+e\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sin(f*x + e) + a)*sin(f*x + e), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a+b\sin\left(e+fx\right)}\sin\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sin(e + f*x))*sin(e + f*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sin\left(fx+e\right)+a}\sin\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sin(f*x + e) + a)*sin(f*x + e), x)
```

3.204 $\int \sqrt{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=62

$$\frac{2\sqrt{a+b\sin(e+fx)}E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{f\sqrt{\frac{a+b\sin(e+fx)}{a+b}}}$$

```
[Out] (2*EllipticE[(e - Pi/2 + f*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[e + f*x]])/(
f*Sqrt[(a + b*Sin[e + f*x])/(a + b)])
```

Rubi [A] time = 0.0365646, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2655, 2653}

$$\frac{2\sqrt{a+b\sin(e+fx)}E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{f\sqrt{\frac{a+b\sin(e+fx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sin[e + f*x]],x]
```

```
[Out] (2*EllipticE[(e - Pi/2 + f*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[e + f*x]])/(
f*Sqrt[(a + b*Sin[e + f*x])/(a + b)])
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+b\sin(e+fx)}\,dx &= \frac{\sqrt{a+b\sin(e+fx)}\int \sqrt{\frac{a}{a+b}+\frac{b\sin(e+fx)}{a+b}}\,dx}{\sqrt{\frac{a+b\sin(e+fx)}{a+b}}}\\ &= \frac{2E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2b}{a+b}\right)\sqrt{a+b\sin(e+fx)}}{f\sqrt{\frac{a+b\sin(e+fx)}{a+b}}}\end{aligned}$$

Mathematica [A] time = 0.0735071, size = 61, normalized size = 0.98

$$-\frac{2\sqrt{a+b\sin(e+fx)}E\left(\frac{1}{4}(-2e-2fx+\pi)|\frac{2b}{a+b}\right)}{f\sqrt{\frac{a+b\sin(e+fx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sin[e + f*x]],x]
```

```
[Out] (-2*EllipticE[(-2*e + Pi - 2*f*x)/4, (2*b)/(a + b)]*Sqrt[a + b*Sin[e + f*x]
])/(f*Sqrt[(a + b*Sin[e + f*x])/(a + b)])
```

Maple [B] time = 0.882, size = 239, normalized size = 3.9

$$-2\frac{a-b}{b\cos\left(fx+e\right)\sqrt{a+b\sin\left(fx+e\right)}f}\sqrt{\frac{a+b\sin\left(fx+e\right)}{a-b}}\sqrt{-\frac{\left(-1+\sin\left(fx+e\right)\right)b}{a+b}}\sqrt{-\frac{\left(1+\sin\left(fx+e\right)\right)b}{a-b}}\left(\text{Ellipti}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^(1/2),x)
```

```
[Out] -2/b*(a-b)*((a+b*sin(f*x+e))/(a-b))^(1/2)*(-(-1+sin(f*x+e))*b/(a+b))^(1/2)*
(-(1+sin(f*x+e))*b/(a-b))^(1/2)*(EllipticE(((a+b*sin(f*x+e))/(a-b))^(1/2),(
(a-b)/(a+b))^(1/2))*a+EllipticE(((a+b*sin(f*x+e))/(a-b))^(1/2),((a-b)/(a+b)
)^(1/2))*b-a*EllipticF(((a+b*sin(f*x+e))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))-
EllipticF(((a+b*sin(f*x+e))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*b)/cos(f*x+e)
/(a+b*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sin(f*x + e) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b\sin\left(fx+e\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sin(f*x + e) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a+b\sin\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sin(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sin(f*x + e) + a), x)
```

3.205 $\int \csc(e+fx)\sqrt{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=128

$$\frac{2b\sqrt{\frac{a+b\sin(e+fx)}{a+b}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{f\sqrt{a+b\sin(e+fx)}}+\frac{2a\sqrt{\frac{a+b\sin(e+fx)}{a+b}}\Pi\left(2;\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{f\sqrt{a+b\sin(e+fx)}}$$

```
[Out] (2*b*EllipticF[(e - Pi/2 + f*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[e + f*x])
/(a + b)])/(f*Sqrt[a + b*Sin[e + f*x]]) + (2*a*EllipticPi[2, (e - Pi/2 + f*
x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[e + f*x])/(a + b)])/(f*Sqrt[a + b*Sin[
e + f*x]])
```

Rubi [A] time = 0.235212, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2803, 2663, 2661, 2807, 2805}

$$\frac{2b\sqrt{\frac{a+b\sin(e+fx)}{a+b}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{f\sqrt{a+b\sin(e+fx)}}+\frac{2a\sqrt{\frac{a+b\sin(e+fx)}{a+b}}\Pi\left(2;\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{f\sqrt{a+b\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]*Sqrt[a + b*Sin[e + f*x]],x]
```


```
[Out] (2*b*EllipticF[(e - Pi/2 + f*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[e + f*x])
/(a + b)])/(f*Sqrt[a + b*Sin[e + f*x]]) + (2*a*EllipticPi[2, (e - Pi/2 + f*
x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[e + f*x])/(a + b)])/(f*Sqrt[a + b*Sin[
e + f*x]])
```


Rule 2803

```
Int[Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_.) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[1/Sqrt[c + d*Sin[e + f*x]], x], x]
 + Dist[(b*c - a*d)/b, Int[1/((a + b*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
```

```
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \csc(e+fx)\sqrt{a+b\sin(e+fx)}\,dx = a\int \frac{\csc(e+fx)}{\sqrt{a+b\sin(e+fx)}}\,dx + b\int \frac{1}{\sqrt{a+b\sin(e+fx)}}\,dx$$

$$= \frac{\left(a\sqrt{\frac{a+b\sin(e+fx)}{a+b}}\right)\int \frac{\csc(e+fx)}{\sqrt{\frac{a}{a+b}+\frac{b\sin(e+fx)}{a+b}}}\,dx}{\sqrt{a+b\sin(e+fx)}} + \frac{\left(b\sqrt{\frac{a+b\sin(e+fx)}{a+b}}\right)\int \frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin(e+fx)}{a+b}}}\,dx}{\sqrt{a+b\sin(e+fx)}}$$

$$= \frac{2bF\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(e+fx)}{a+b}}}{f\sqrt{a+b\sin(e+fx)}} + \frac{2a\Pi\left(2;\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(e+}{a+b}}}{f\sqrt{a+b\sin(e+fx)}}$$

Mathematica [A] time = 15.4153, size = 89, normalized size = 0.7

$$-\frac{2\sqrt{\frac{a+b\sin(e+fx)}{a+b}}\left(bF\left(\frac{1}{4}(-2e-2fx+\pi)|\frac{2b}{a+b}\right)+a\Pi\left(2;\frac{1}{4}(-2e-2fx+\pi)|\frac{2b}{a+b}\right)\right)}{f\sqrt{a+b\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]*Sqrt[a + b*Sin[e + f*x]],x]
```

```
[Out] (-2*(b*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*b)/(a + b)] + a*EllipticPi[2, (-
2*e + Pi - 2*f*x)/4, (2*b)/(a + b)])*Sqrt[(a + b*Sin[e + f*x])/(a + b)])/(f
*Sqrt[a + b*Sin[e + f*x]])
```

Maple [A] time = 0.832, size = 169, normalized size = 1.3

$$2\frac{a-b}{\cos\left(fx+e\right)\sqrt{a+b\sin\left(fx+e\right)}f}\sqrt{\frac{a+b\sin\left(fx+e\right)}{a-b}}\sqrt{-\frac{\left(-1+\sin\left(fx+e\right)\right)b}{a+b}}\sqrt{-\frac{\left(1+\sin\left(fx+e\right)\right)b}{a-b}}\left(\mathrm{EllipticF}\left(\sqrt{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)*(a+b*sin(f*x+e))^(1/2),x)
```

```
[Out] 2*(a-b)*((a+b*sin(f*x+e))/(a-b))^(1/2)*(-(-1+sin(f*x+e))*b/(a+b))^(1/2)*(-(
1+sin(f*x+e))*b/(a-b))^(1/2)*(EllipticF(((a+b*sin(f*x+e))/(a-b))^(1/2),((a-
b)/(a+b))^(1/2))-EllipticPi(((a+b*sin(f*x+e))/(a-b))^(1/2),(a-b)/a,((a-b)/(
a+b))^(1/2)))/cos(f*x+e)/(a+b*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sin\left(fx+e\right)+a} \csc\left(fx+e\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sin(f*x + e) + a)*csc(f*x + e), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a+b \sin\left(e+fx\right)} \csc\left(e+fx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sin(e + f*x))*csc(e + f*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sin\left(fx+e\right)+a} \csc\left(fx+e\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sin(f*x + e) + a)*csc(f*x + e), x)
```

3.206 $\int \csc^2(e+fx)\sqrt{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=213

$$-\frac{\cot(e+fx)\sqrt{a+b\sin(e+fx)}}{f}+\frac{a\sqrt{\frac{a+b\sin(e+fx)}{a+b}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{f\sqrt{a+b\sin(e+fx)}}-\frac{\sqrt{a+b\sin(e+fx)}E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{f\sqrt{\frac{a+b\sin(e+fx)}{a+b}}}+$$

```
[Out] -((Cot[e + f*x]*Sqrt[a + b*Sin[e + f*x]])/f) - (EllipticE[(e - Pi/2 + f*x)/
2, (2*b)/(a + b)]*Sqrt[a + b*Sin[e + f*x]])/(f*Sqrt[(a + b*Sin[e + f*x])/(a
 + b)]) + (a*EllipticF[(e - Pi/2 + f*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[e
 + f*x])/(a + b)])/(f*Sqrt[a + b*Sin[e + f*x]]) + (b*EllipticPi[2, (e - Pi/
2 + f*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[e + f*x])/(a + b)])/(f*Sqrt[a +
b*Sin[e + f*x]])
```

Rubi [A] time = 0.480974, antiderivative size = 213, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {2796, 3060, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$-\frac{\cot(e+fx)\sqrt{a+b\sin(e+fx)}}{f}+\frac{a\sqrt{\frac{a+b\sin(e+fx)}{a+b}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{f\sqrt{a+b\sin(e+fx)}}-\frac{\sqrt{a+b\sin(e+fx)}E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{f\sqrt{\frac{a+b\sin(e+fx)}{a+b}}}+$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]],x]
```

```
[Out] -((Cot[e + f*x]*Sqrt[a + b*Sin[e + f*x]])/f) - (EllipticE[(e - Pi/2 + f*x)/
2, (2*b)/(a + b)]*Sqrt[a + b*Sin[e + f*x]])/(f*Sqrt[(a + b*Sin[e + f*x])/(a
 + b)]) + (a*EllipticF[(e - Pi/2 + f*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[e
 + f*x])/(a + b)])/(f*Sqrt[a + b*Sin[e + f*x]]) + (b*EllipticPi[2, (e - Pi/
2 + f*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[e + f*x])/(a + b)])/(f*Sqrt[a +
b*Sin[e + f*x]])
```

Rule 2796

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m + 1)*(c + d*Sin[e + f*x])^n)/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m +
 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n
- 1)*Simp[a*c*(m + 1) + b*d*n + (a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] -
b*d*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1
] && LtQ[0, n, 1] && IntegersQ[2*m, 2*n]
```

Rule 3060

```
Int[((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2)/(Sqrt[(a_.) + (b_.)*sin[(e_.)
 + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist
[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x], x] - Dist[1/(b*d), Int[Simp[a*c
*C - A*b*d + (b*c*C + a*C*d)*Sin[e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c
+ d*Sin[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \csc^2(e+fx)\sqrt{a+b\sin(e+fx)}\,dx = -\frac{\cot(e+fx)\sqrt{a+b\sin(e+fx)}}{f} + \int \frac{\csc(e+fx)\left(\frac{b}{2}-\frac{1}{2}b\sin^2(e+fx)\right)}{\sqrt{a+b\sin(e+fx)}}\,dx$$

$$= -\frac{\cot(e+fx)\sqrt{a+b\sin(e+fx)}}{f} - \frac{1}{2}\int \sqrt{a+b\sin(e+fx)}\,dx - \frac{\int \frac{\csc(e+fx)\left(-\frac{b^2}{2}-\frac{1}{2}\right.}{\sqrt{a+b\sin(}}}{b}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b\sin(e+fx)}}{f} + \frac{1}{2}a\int \frac{1}{\sqrt{a+b\sin(e+fx)}}\,dx + \frac{1}{2}b\int \frac{\csc(}{\sqrt{a+b\sin}}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b\sin(e+fx)}}{f} - \frac{E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2b}{a+b}\right)\sqrt{a+b\sin(e+fx)}}{f\sqrt{\frac{a+b\sin(e+fx)}{a+b}}} +$$

$$= -\frac{\cot(e+fx)\sqrt{a+b\sin(e+fx)}}{f} - \frac{E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2b}{a+b}\right)\sqrt{a+b\sin(e+fx)}}{f\sqrt{\frac{a+b\sin(e+fx)}{a+b}}} +$$

Mathematica [C] time = 8.8882, size = 312, normalized size = 1.46

$$\frac{-4\cot(e+fx)\sqrt{a+b\sin(e+fx)} - \frac{2b\sqrt{\frac{a+b\sin(e+fx)}{a+b}}\Pi\left(2;\frac{1}{4}(-2e-2fx+\pi)|\frac{2b}{a+b}\right)}{\sqrt{a+b\sin(e+fx)}} + \frac{2i\sec(e+fx)\sqrt{-\frac{b(\sin(e+fx)-1)}{a+b}}\sqrt{-\frac{b(\sin(e+fx)+1)}{a-b}}\left(b\left(b\Pi\left(\frac{a+b}{a};i\sinh\right.\right.\right.}{}}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]],x]
```

```
[Out] (((2*I)*(-2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Si
n[e + f*x]]], (a + b)/(a - b)] + b*(-2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^
(-1)]*Sqrt[a + b*Sin[e + f*x]]], (a + b)/(a - b)] + b*EllipticPi[(a + b)/a,
 I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[e + f*x]]], (a + b)/(a - b)])
)*Sec[e + f*x]*Sqrt[-((b*(-1 + Sin[e + f*x]))/(a + b))]*Sqrt[-((b*(1 + Sin[
e + f*x]))/(a - b))])/(a*b*Sqrt[-(a + b)^(-1)]) - 4*Cot[e + f*x]*Sqrt[a + b
*Sin[e + f*x]] - (2*b*EllipticPi[2, (-2*e + Pi - 2*f*x)/4, (2*b)/(a + b)]*S
qrt[(a + b*Sin[e + f*x])/(a + b)])/Sqrt[a + b*Sin[e + f*x]])/(4*f)
```

Maple [A] time = 1.047, size = 456, normalized size = 2.1

$$-\frac{1}{ab\sin(fx+e)\cos(fx+e)f}\left(ab^2\sin(fx+e)(\cos(fx+e))^2 + \sqrt{\frac{b\sin(fx+e)}{a-b}+\frac{a}{a-b}}\sqrt{-\frac{b\sin(fx+e)}{a+b}+\frac{b}{a+b}}\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2*(a+b*sin(f*x+e))^(1/2),x)
```

```
[Out] -(a*b^2*sin(f*x+e)*cos(f*x+e)^2+(b/(a-b)*sin(f*x+e)+1/(a-b)*a)^(1/2)*(-b/(a
+b)*sin(f*x+e)+b/(a+b))^(1/2)*(-b/(a-b)*sin(f*x+e)-b/(a-b))^(1/2)*(Elliptic
F((b/(a-b)*sin(f*x+e)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^2*b-EllipticF
((b/(a-b)*sin(f*x+e)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a*b^2+EllipticPi
```

```
((b/(a-b)*sin(f*x+e)+1/(a-b)*a)^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a*b^2-El
lipticPi((b/(a-b)*sin(f*x+e)+1/(a-b)*a)^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*
b^3-EllipticE((b/(a-b)*sin(f*x+e)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^3
+EllipticE((b/(a-b)*sin(f*x+e)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a*b^2)
*sin(f*x+e)+a^2*b*cos(f*x+e)^2)/a/b/sin(f*x+e)/cos(f*x+e)/(a+b*sin(f*x+e))^
(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sin\left(fx+e\right)+a}\csc\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sin(f*x + e) + a)*csc(f*x + e)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{b\sin\left(fx+e\right)+a}\csc\left(fx+e\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sin(f*x + e) + a)*csc(f*x + e)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a+b\sin\left(e+fx\right)}\csc^2\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2*(a+b*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sin(e + f*x))*csc(e + f*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sin\left(fx+e\right)+a}\csc\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sin(f*x + e) + a)*csc(f*x + e)^2, x)
```

3.207 $\int \frac{\sin(e+fx)}{\sqrt{a+b\sin(e+fx)}}\,dx$

Optimal. Leaf size=132

$$\frac{2\sqrt{a+b\sin(e+fx)}E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{bf\sqrt{\frac{a+b\sin(e+fx)}{a+b}}}-\frac{2a\sqrt{\frac{a+b\sin(e+fx)}{a+b}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{bf\sqrt{a+b\sin(e+fx)}}$$

```
[Out] (2*EllipticE[(e - Pi/2 + f*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[e + f*x]])/(
b*f*Sqrt[(a + b*Sin[e + f*x])/(a + b)]) - (2*a*EllipticF[(e - Pi/2 + f*x)/2
, (2*b)/(a + b)]*Sqrt[(a + b*Sin[e + f*x])/(a + b)])/(b*f*Sqrt[a + b*Sin[e
+ f*x]])
```

Rubi [A] time = 0.1069, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2752, 2663, 2661, 2655, 2653}

$$\frac{2\sqrt{a+b\sin(e+fx)}E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{bf\sqrt{\frac{a+b\sin(e+fx)}{a+b}}}-\frac{2a\sqrt{\frac{a+b\sin(e+fx)}{a+b}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{bf\sqrt{a+b\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]/Sqrt[a + b*Sin[e + f*x]],x]
```

```
[Out] (2*EllipticE[(e - Pi/2 + f*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[e + f*x]])/(
b*f*Sqrt[(a + b*Sin[e + f*x])/(a + b)]) - (2*a*EllipticF[(e - Pi/2 + f*x)/2
, (2*b)/(a + b)]*Sqrt[(a + b*Sin[e + f*x])/(a + b)])/(b*f*Sqrt[a + b*Sin[e
+ f*x]])
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
```

```
0] && !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\sin(e+fx)}{\sqrt{a+b\sin(e+fx)}}\,dx = \frac{\int \sqrt{a+b\sin(e+fx)}\,dx}{b} - \frac{a\int \frac{1}{\sqrt{a+b\sin(e+fx)}}\,dx}{b}$$

$$= \frac{\sqrt{a+b\sin(e+fx)}\int \sqrt{\frac{a}{a+b}+\frac{b\sin(e+fx)}{a+b}}\,dx}{b\sqrt{\frac{a+b\sin(e+fx)}{a+b}}} - \frac{\left(a\sqrt{\frac{a+b\sin(e+fx)}{a+b}}\right)\int \frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin(e+fx)}{a+b}}}\,dx}{b\sqrt{a+b\sin(e+fx)}}$$

$$= \frac{2E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2b}{a+b}\right)\sqrt{a+b\sin(e+fx)}}{bf\sqrt{\frac{a+b\sin(e+fx)}{a+b}}} - \frac{2aF\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(e+fx)}{a+b}}}{bf\sqrt{a+b\sin(e+fx)}}$$

Mathematica [A] time = 2.40032, size = 94, normalized size = 0.71

$$-\frac{2\sqrt{\frac{a+b\sin(e+fx)}{a+b}}\left((a+b)E\left(\frac{1}{4}(-2e-2fx+\pi)|\frac{2b}{a+b}\right)-aF\left(\frac{1}{4}(-2e-2fx+\pi)|\frac{2b}{a+b}\right)\right)}{bf\sqrt{a+b\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]/Sqrt[a + b*Sin[e + f*x]],x]
```

```
[Out] (-2*((a + b)*EllipticE[(-2*e + Pi - 2*f*x)/4, (2*b)/(a + b)] - a*EllipticF[
(-2*e + Pi - 2*f*x)/4, (2*b)/(a + b)])*Sqrt[(a + b*Sin[e + f*x])/(a + b)])/
(b*f*Sqrt[a + b*Sin[e + f*x]])
```

Maple [A] time = 0.819, size = 202, normalized size = 1.5

$$-2\frac{a-b}{b^2\cos\left(fx+e\right)\sqrt{a+b\sin\left(fx+e\right)}f}\sqrt{\frac{a+b\sin\left(fx+e\right)}{a-b}}\sqrt{-\frac{\left(-1+\sin\left(fx+e\right)\right)b}{a+b}}\sqrt{-\frac{\left(1+\sin\left(fx+e\right)\right)b}{a-b}}\left(\text{Ellipt}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)/(a+b*sin(f*x+e))^(1/2),x)
```

```
[Out] -2*(a-b)*((a+b*sin(f*x+e))/(a-b))^(1/2)*(-(-1+sin(f*x+e))*b/(a+b))^(1/2)*(-
(1+sin(f*x+e))*b/(a-b))^(1/2)*(EllipticE(((a+b*sin(f*x+e))/(a-b))^(1/2),((a
-b)/(a+b))^(1/2))*a+EllipticE(((a+b*sin(f*x+e))/(a-b))^(1/2),((a-b)/(a+b))^
(1/2))*b-EllipticF(((a+b*sin(f*x+e))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*b)/b
^2/cos(f*x+e)/(a+b*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(fx+e)}{\sqrt{b\sin(fx+e)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(f*x + e)/sqrt(b*sin(f*x + e) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sin(fx+e)}{\sqrt{b\sin(fx+e)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sin(f*x + e)/sqrt(b*sin(f*x + e) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(e+fx)}{\sqrt{a+b\sin(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(sin(e + f*x)/sqrt(a + b*sin(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(fx+e)}{\sqrt{b\sin(fx+e)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)/sqrt(b*sin(f*x + e) + a), x)
```

3.208 $\int \frac{1}{\sqrt{a+b\sin(e+fx)}}\,dx$

Optimal. Leaf size=62

$$\frac{2\sqrt{\frac{a+b\sin(e+fx)}{a+b}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{f\sqrt{a+b\sin(e+fx)}}$$

```
[Out] (2*EllipticF[(e - Pi/2 + f*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[e + f*x])/(
a + b)])/(f*Sqrt[a + b*Sin[e + f*x]])
```

Rubi [A] time = 0.0367861, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2663, 2661}

$$\frac{2\sqrt{\frac{a+b\sin(e+fx)}{a+b}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{f\sqrt{a+b\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*Sin[e + f*x]],x]
```

```
[Out] (2*EllipticF[(e - Pi/2 + f*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[e + f*x])/(
a + b)])/(f*Sqrt[a + b*Sin[e + f*x]])
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a+b\sin(e+fx)}}\,dx &= \frac{\sqrt{\frac{a+b\sin(e+fx)}{a+b}}\int \frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin(e+fx)}{a+b}}}\,dx}{\sqrt{a+b\sin(e+fx)}}\\ &= \frac{2F\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(e+fx)}{a+b}}}{f\sqrt{a+b\sin(e+fx)}}\end{aligned}$$

Mathematica [A] time = 0.0560489, size = 61, normalized size = 0.98

$$-\frac{2\sqrt{\frac{a+b\sin(e+fx)}{a+b}}F\left(\frac{1}{4}(-2e-2fx+\pi)|\frac{2b}{a+b}\right)}{f\sqrt{a+b\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*Sin[e + f*x]],x]
```

```
[Out] (-2*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[e + f*x
])/(a + b)])/(f*Sqrt[a + b*Sin[e + f*x]])
```

Maple [A] time = 0.695, size = 126, normalized size = 2.

$$2\frac{a-b}{b\cos\left(fx+e\right)\sqrt{a+b\sin\left(fx+e\right)}f}\sqrt{\frac{a+b\sin\left(fx+e\right)}{a-b}}\sqrt{-\frac{\left(-1+\sin\left(fx+e\right)\right)b}{a+b}}\sqrt{-\frac{\left(1+\sin\left(fx+e\right)\right)b}{a-b}}\mathrm{EllipticF}\left(\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e))^(1/2),x)
```

```
[Out] 2*(a-b)*((a+b*sin(f*x+e))/(a-b))^(1/2)*(-(-1+sin(f*x+e))*b/(a+b))^(1/2)*(-(
1+sin(f*x+e))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(f*x+e))/(a-b))^(1/2),((a-b
)/(a+b))^(1/2))/b/cos(f*x+e)/(a+b*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*sin(f*x + e) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\sqrt{b\sin\left(fx+e\right)+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(b*sin(f*x + e) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+b\sin\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*sin(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*sin(f*x + e) + a), x)
```

3.209 $\int \frac{\csc(e+fx)}{\sqrt{a+b\sin(e+fx)}}\,dx$

Optimal. Leaf size=63

$$\frac{2\sqrt{\frac{a+b\sin(e+fx)}{a+b}}\Pi\left(2;\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{f\sqrt{a+b\sin(e+fx)}}$$

```
[Out] (2*EllipticPi[2, (e - Pi/2 + f*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[e + f*x
])/(a + b)])/(f*Sqrt[a + b*Sin[e + f*x]])
```

Rubi [A] time = 0.129918, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2807, 2805}

$$\frac{2\sqrt{\frac{a+b\sin(e+fx)}{a+b}}\Pi\left(2;\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{f\sqrt{a+b\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]/Sqrt[a + b*Sin[e + f*x]],x]
```

```
[Out] (2*EllipticPi[2, (e - Pi/2 + f*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[e + f*x
])/(a + b)])/(f*Sqrt[a + b*Sin[e + f*x]])
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc(e+fx)}{\sqrt{a+b\sin(e+fx)}}\,dx &= \frac{\sqrt{\frac{a+b\sin(e+fx)}{a+b}}\int \frac{\csc(e+fx)}{\sqrt{\frac{a}{a+b}+\frac{b\sin(e+fx)}{a+b}}}\,dx}{\sqrt{a+b\sin(e+fx)}}\\
&= \frac{2\Pi\left(2;\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(e+fx)}{a+b}}}{f\sqrt{a+b\sin(e+fx)}}
\end{aligned}$$

Mathematica [A] time = 0.0771257, size = 62, normalized size = 0.98

$$-\frac{2\sqrt{\frac{a+b\sin(e+fx)}{a+b}}\Pi\left(2;\frac{1}{4}(-2e-2fx+\pi)|\frac{2b}{a+b}\right)}{f\sqrt{a+b\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]/Sqrt[a + b*Sin[e + f*x]],x]
```

```
[Out] (-2*EllipticPi[2, (-2*e + Pi - 2*f*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[e +
 f*x])/(a + b)])/(f*Sqrt[a + b*Sin[e + f*x]])
```

Maple [A] time = 0.589, size = 135, normalized size = 2.1

$$-2\frac{a-b}{\cos\left(fx+e\right)a\sqrt{a+b\sin\left(fx+e\right)}f}\sqrt{\frac{a+b\sin\left(fx+e\right)}{a-b}}\sqrt{-\frac{\left(-1+\sin\left(fx+e\right)\right)b}{a+b}}\sqrt{-\frac{\left(1+\sin\left(fx+e\right)\right)b}{a-b}}\mathrm{EllipticI}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)/(a+b*sin(f*x+e))^(1/2),x)
```

```
[Out] -2*(a-b)*((a+b*sin(f*x+e))/(a-b))^(1/2)*(-(-1+sin(f*x+e))*b/(a+b))^(1/2)*(-
(1+sin(f*x+e))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(f*x+e))/(a-b))^(1/2),(a-
b)/a,((a-b)/(a+b))^(1/2))/a/cos(f*x+e)/(a+b*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(fx+e\right)}{\sqrt{b\sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(f*x + e)/sqrt(b*sin(f*x + e) + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(e+fx)}{\sqrt{a+b\sin(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(csc(e + f*x)/sqrt(a + b*sin(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(fx+e)}{\sqrt{b\sin(fx+e)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)/sqrt(b*sin(f*x + e) + a), x)
```

3.210 $\int \frac{\csc^2(e+fx)}{\sqrt{a+b\sin(e+fx)}}\,dx$

Optimal. Leaf size=222

$$-\frac{\cot(e+fx)\sqrt{a+b\sin(e+fx)}}{af}+\frac{\sqrt{\frac{a+b\sin(e+fx)}{a+b}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{f\sqrt{a+b\sin(e+fx)}}-\frac{\sqrt{a+b\sin(e+fx)}E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{af\sqrt{\frac{a+b\sin(e+fx)}{a+b}}}$$

```
[Out] -((Cot[e + f*x]*Sqrt[a + b*Sin[e + f*x]])/(a*f)) - (EllipticE[(e - Pi/2 + f
*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[e + f*x]])/(a*f*Sqrt[(a + b*Sin[e + f*
x])/(a + b)]) + (EllipticF[(e - Pi/2 + f*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*S
in[e + f*x])/(a + b)])/(f*Sqrt[a + b*Sin[e + f*x]]) - (b*EllipticPi[2, (e -
 Pi/2 + f*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[e + f*x])/(a + b)])/(a*f*Sqr
t[a + b*Sin[e + f*x]])
```

Rubi [A] time = 0.495332, antiderivative size = 222, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {2802, 3060, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$-\frac{\cot(e+fx)\sqrt{a+b\sin(e+fx)}}{af}+\frac{\sqrt{\frac{a+b\sin(e+fx)}{a+b}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{f\sqrt{a+b\sin(e+fx)}}-\frac{\sqrt{a+b\sin(e+fx)}E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{af\sqrt{\frac{a+b\sin(e+fx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2/Sqrt[a + b*Sin[e + f*x]],x]
```

```
[Out] -((Cot[e + f*x]*Sqrt[a + b*Sin[e + f*x]])/(a*f)) - (EllipticE[(e - Pi/2 + f
*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[e + f*x]])/(a*f*Sqrt[(a + b*Sin[e + f*
x])/(a + b)]) + (EllipticF[(e - Pi/2 + f*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*S
in[e + f*x])/(a + b)])/(f*Sqrt[a + b*Sin[e + f*x]]) - (b*EllipticPi[2, (e -
 Pi/2 + f*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[e + f*x])/(a + b)])/(a*f*Sqr
t[a + b*Sin[e + f*x]])
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3060

```
Int[((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2)/(Sqrt[(a_.) + (b_.)*sin[(e_.)
 + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist
[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x], x] - Dist[1/(b*d), Int[Simp[a*c
*C - A*b*d + (b*c*C + a*C*d)*Sin[e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c
+ d*Sin[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c
```

```
 - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\csc^2(e+fx)}{\sqrt{a+b\sin(e+fx)}}\,dx = -\frac{\cot(e+fx)\sqrt{a+b\sin(e+fx)}}{af} + \frac{\int \frac{\csc(e+fx)\left(-\frac{b}{2}-\frac{1}{2}b\sin^2(e+fx)\right)}{\sqrt{a+b\sin(e+fx)}}dx}{a}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b\sin(e+fx)}}{af} - \frac{\int \sqrt{a+b\sin(e+fx)}\,dx}{2a} - \frac{\int \frac{\csc(e+fx)\left(\frac{b^2}{2}-\frac{1}{2}ab\sin(e+fx)\right)}{\sqrt{a+b\sin(e+fx)}}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b\sin(e+fx)}}{af} + \frac{1}{2}\int \frac{1}{\sqrt{a+b\sin(e+fx)}}dx - \frac{b\int \frac{\csc(e+fx)}{\sqrt{a+b\sin(e+fx)}}dx}{2a} - $$

$$= -\frac{\cot(e+fx)\sqrt{a+b\sin(e+fx)}}{af} - \frac{E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2b}{a+b}\right)\sqrt{a+b\sin(e+fx)}}{af\sqrt{\frac{a+b\sin(e+fx)}{a+b}}} + $$

$$= -\frac{\cot(e+fx)\sqrt{a+b\sin(e+fx)}}{af} - \frac{E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2b}{a+b}\right)\sqrt{a+b\sin(e+fx)}}{af\sqrt{\frac{a+b\sin(e+fx)}{a+b}}} + \frac{F\left(\frac{1}{2}\left(e-\right.\right.}{}$$

Mathematica [C] time = 10.0247, size = 315, normalized size = 1.42

$$\frac{-4\cot(e+fx)\sqrt{a+b\sin(e+fx)} + \frac{6b\sqrt{\frac{a+b\sin(e+fx)}{a+b}}\Pi\left(2;\frac{1}{4}(-2e-2fx+\pi)|\frac{2b}{a+b}\right)}{\sqrt{a+b\sin(e+fx)}} + \frac{2i\sec(e+fx)\sqrt{-\frac{b(\sin(e+fx)-1)}{a+b}}\sqrt{-\frac{b(\sin(e+fx)+1)}{a-b}}\left(b\left(b\Pi\left(\frac{a+b}{a};i\right.\right.\right.}{}}{4af}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2/Sqrt[a + b*Sin[e + f*x]],x]
```

```
[Out] (((2*I)*(-2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Si
n[e + f*x]]], (a + b)/(a - b)] + b*(-2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^
(-1)]*Sqrt[a + b*Sin[e + f*x]]], (a + b)/(a - b)] + b*EllipticPi[(a + b)/a,
 I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[e + f*x]]], (a + b)/(a - b)])
)*Sec[e + f*x]*Sqrt[-((b*(-1 + Sin[e + f*x]))/(a + b))]*Sqrt[-((b*(1 + Sin[
e + f*x]))/(a - b))])/(a*b*Sqrt[-(a + b)^(-1)]) - 4*Cot[e + f*x]*Sqrt[a + b
*Sin[e + f*x]] + (6*b*EllipticPi[2, (-2*e + Pi - 2*f*x)/4, (2*b)/(a + b)]*S
qrt[(a + b*Sin[e + f*x])/(a + b)])/Sqrt[a + b*Sin[e + f*x]])/(4*a*f)
```

Maple [A] time = 1.823, size = 412, normalized size = 1.9

$$\frac{1}{f\cos(fx+e)}\sqrt{-(-b\sin(fx+e)-a)(\cos(fx+e))^2}\left(-\frac{1}{a\sin(fx+e)}\sqrt{-(-b\sin(fx+e)-a)(\cos(fx+e))^2} - \frac{b}{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2/(a+b*sin(f*x+e))^(1/2),x)
```

```
[Out] (-(-b*sin(f*x+e)-a)*cos(f*x+e)^2)^(1/2)*(-1/a*(-(-b*sin(f*x+e)-a)*cos(f*x+e
)^2)^(1/2)/sin(f*x+e)-1/a*b*(a/b-1)*((a+b*sin(f*x+e))/(a-b))^(1/2)*(b*(1-si
n(f*x+e))/(a+b))^(1/2)*((-sin(f*x+e)-1)*b/(a-b))^(1/2)/(-(-b*sin(f*x+e)-a)*
cos(f*x+e)^2)^(1/2)*((-a/b-1)*EllipticE(((a+b*sin(f*x+e))/(a-b))^(1/2),((a-
```

```
b)/(a+b))^(1/2))+EllipticF(((a+b*sin(f*x+e))/(a-b))^(1/2),((a-b)/(a+b))^(1/
2)))+1/a^2*b^2*(a/b-1)*((a+b*sin(f*x+e))/(a-b))^(1/2)*(b*(1-sin(f*x+e))/(a+
b))^(1/2)*((-sin(f*x+e)-1)*b/(a-b))^(1/2)/(-(-b*sin(f*x+e)-a)*cos(f*x+e)^2)
^(1/2)*EllipticPi(((a+b*sin(f*x+e))/(a-b))^(1/2),-(-a/b+1)/a*b,((a-b)/(a+b)
)^(1/2)))/cos(f*x+e)/(a+b*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(fx+e\right)^2}{\sqrt{b\sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(f*x + e)^2/sqrt(b*sin(f*x + e) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc^2\left(e+fx\right)}{\sqrt{a+b\sin\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2/(a+b*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(csc(e + f*x)**2/sqrt(a + b*sin(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(fx+e\right)^2}{\sqrt{b\sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^2/sqrt(b*sin(f*x + e) + a), x)
```

3.211 $\int \sqrt{\sin(c+dx)}\sqrt{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=371

$$-\frac{\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}-\frac{\sqrt{a+b}\tan(c+dx)\sqrt{\frac{a(1-\csc(c+dx))}{a+b}}\sqrt{\frac{a(\csc(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}\sqrt{\sin(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{d}+$$

```
[Out] -((Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(d*Sqrt[Sin[c + d*x]])) + ((a - b
)*Sqrt[a + b]*Sqrt[(a*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Csc[c + d*x
]))/(a - b)]*EllipticE[ArcSin[Sqrt[a + b*Sin[c + d*x]]/(Sqrt[a + b]*Sqrt[Si
n[c + d*x]])], -((a + b)/(a - b))]*Tan[c + d*x])/(a*d) - (Sqrt[a + b]*Sqrt[
(a*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Csc[c + d*x]))/(a - b)]*Ellipt
icF[ArcSin[Sqrt[a + b*Sin[c + d*x]]/(Sqrt[a + b]*Sqrt[Sin[c + d*x]])], -((a
 + b)/(a - b))]*Tan[c + d*x])/d + (a*Sqrt[a + b]*Sqrt[(a*(1 - Csc[c + d*x])
)/(a + b)]*Sqrt[(a*(1 + Csc[c + d*x]))/(a - b)]*EllipticPi[(a + b)/b, ArcSi
n[Sqrt[a + b*Sin[c + d*x]]/(Sqrt[a + b]*Sqrt[Sin[c + d*x]])], -((a + b)/(a
- b))]*Tan[c + d*x])/(b*d)
```

Rubi [A] time = 0.563021, antiderivative size = 371, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {2821, 3054, 2809, 12, 2801, 2816, 2994}

$$-\frac{\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}-\frac{\sqrt{a+b}\tan(c+dx)\sqrt{\frac{a(1-\csc(c+dx))}{a+b}}\sqrt{\frac{a(\csc(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}\sqrt{\sin(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{d}+$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sin[c + d*x]]*Sqrt[a + b*Sin[c + d*x]],x]
```


```
[Out] -((Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(d*Sqrt[Sin[c + d*x]])) + ((a - b
)*Sqrt[a + b]*Sqrt[(a*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Csc[c + d*x
]))/(a - b)]*EllipticE[ArcSin[Sqrt[a + b*Sin[c + d*x]]/(Sqrt[a + b]*Sqrt[Si
n[c + d*x]])], -((a + b)/(a - b))]*Tan[c + d*x])/(a*d) - (Sqrt[a + b]*Sqrt[
(a*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Csc[c + d*x]))/(a - b)]*Ellipt
icF[ArcSin[Sqrt[a + b*Sin[c + d*x]]/(Sqrt[a + b]*Sqrt[Sin[c + d*x]])], -((a
 + b)/(a - b))]*Tan[c + d*x])/d + (a*Sqrt[a + b]*Sqrt[(a*(1 - Csc[c + d*x])
)/(a + b)]*Sqrt[(a*(1 + Csc[c + d*x]))/(a - b)]*EllipticPi[(a + b)/b, ArcSi
n[Sqrt[a + b*Sin[c + d*x]]/(Sqrt[a + b]*Sqrt[Sin[c + d*x]])], -((a + b)/(a
- b))]*Tan[c + d*x])/(b*d)
```


Rule 2821

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[1/(d*(m + n)), Int[
(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a^2*c*d*(m +
 n) + b*d*(b*c*(m - 1) + a*d*n) + (a*d*(2*b*c + a*d)*(m + n) - b*d*(a*c - b
*d*(m + n - 1)))*Sin[e + f*x] + b*d*(b*c*n + a*d*(2*m + n - 1))*Sin[e + f*x
]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[0, m, 2] && LtQ[-1, n, 2] && NeQ
[m + n, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3054

```
Int[((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2)/(((a_.) + (b_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :
```

```
> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]], x], x]
 + Dist[1/b^2, Int[(A*b^2 - a^2*C - 2*a*b*C*Sin[e + f*x])/((a + b*Sin[e + f
*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
 C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2801

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin
[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[1/(a - b), Int[1/(Sqrt[a + b*Sin[
e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[b/(a - b), Int[(1 + Sin[
e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \sqrt{\sin(c+dx)}\sqrt{a+b\sin(c+dx)}\,dx = -\frac{\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{d\sqrt{\sin(c+dx)}} + \frac{\int \frac{-\frac{ab}{2}+\frac{1}{2}ab\sin^2(c+dx)}{\sin^{\frac{3}{2}}(c+dx)\sqrt{a+b\sin(c+dx)}}\,dx}{b}$$

$$= -\frac{\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{d\sqrt{\sin(c+dx)}} + \frac{1}{2}a\int \frac{\sqrt{\sin(c+dx)}}{\sqrt{a+b\sin(c+dx)}}\,dx + \frac{\int -\frac{\cdots}{2\sin^{\frac{3}{2}}(c\cdots}}{\cdots}$$

$$= -\frac{\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{d\sqrt{\sin(c+dx)}} + \frac{a\sqrt{a+b}\sqrt{\frac{a(1-\csc(c+dx))}{a+b}}\sqrt{\frac{a(1+\csc(c+dx))}{a-b}}\Pi\left(\cdots\right.}{\cdots}$$

$$= -\frac{\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{d\sqrt{\sin(c+dx)}} + \frac{a\sqrt{a+b}\sqrt{\frac{a(1-\csc(c+dx))}{a+b}}\sqrt{\frac{a(1+\csc(c+dx))}{a-b}}\Pi\left(\cdots\right.}{\cdots}$$

$$= -\frac{\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{d\sqrt{\sin(c+dx)}} + \frac{(a-b)\sqrt{a+b}\sqrt{\frac{a(1-\csc(c+dx))}{a+b}}\sqrt{\frac{a(1+\csc(c+dx))}{a-b}}\cdots}{\cdots}$$

Mathematica [C] time = 26.7561, size = 10847, normalized size = 29.24

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Sin[c + d*x]]*Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] Result too large to show
```

Maple [C] time = 0.568, size = 9567, normalized size = 25.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^(1/2)*(a+b*sin(d*x+c))^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sin(dx+c)+a}\sqrt{\sin(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^(1/2)*(a+b*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sin(d*x + c) + a)*sqrt(sin(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b\sin\left(dx+c\right)+a}\sqrt{\sin\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^(1/2)*(a+b*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sin(d*x + c) + a)*sqrt(sin(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a+b\sin\left(c+dx\right)}\sqrt{\sin\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**(1/2)*(a+b*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sin(c + d*x))*sqrt(sin(c + d*x)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^(1/2)*(a+b*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.212 $\int \frac{1}{\sqrt{\sin(c+dx)}\sqrt{a+b\sin(c+dx)}}\,dx$

Optimal. Leaf size=109

$$-\frac{2\sqrt{a+b}\tan(c+dx)\sqrt{\frac{a(1-\csc(c+dx))}{a+b}}\sqrt{\frac{a(\csc(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}\sqrt{\sin(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{ad}$$

```
[Out] (-2*Sqrt[a + b]*Sqrt[(a*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Csc[c + d
*x]))/(a - b)]*EllipticF[ArcSin[Sqrt[a + b*Sin[c + d*x]]/(Sqrt[a + b]*Sqrt[
Sin[c + d*x]])], -((a + b)/(a - b))]*Tan[c + d*x])/(a*d)
```

Rubi [A] time = 0.0677167, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {2816}

$$-\frac{2\sqrt{a+b}\tan(c+dx)\sqrt{\frac{a(1-\csc(c+dx))}{a+b}}\sqrt{\frac{a(\csc(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}\sqrt{\sin(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Sin[c + d*x]]*Sqrt[a + b*Sin[c + d*x]]),x]
```

```
[Out] (-2*Sqrt[a + b]*Sqrt[(a*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Csc[c + d
*x]))/(a - b)]*EllipticF[ArcSin[Sqrt[a + b*Sin[c + d*x]]/(Sqrt[a + b]*Sqrt[
Sin[c + d*x]])], -((a + b)/(a - b))]*Tan[c + d*x])/(a*d)
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rubi steps

$$\int \frac{1}{\sqrt{\sin(c+dx)}\sqrt{a+b\sin(c+dx)}}\,dx = -\frac{2\sqrt{a+b}\sqrt{\frac{a(1-\csc(c+dx))}{a+b}}\sqrt{\frac{a(1+\csc(c+dx))}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}\sqrt{\sin(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\tan(}{ad}$$

Mathematica [A] time = 3.26843, size = 172, normalized size = 1.58

$$\frac{8a\sin^4\left(\frac{1}{4}(2c+2dx-\pi)\right)\sec(c+dx)\sqrt{-\frac{(a+b)\sin(c+dx)(a+b\sin(c+dx))}{a^2(\sin(c+dx)-1)^2}}\sqrt{-\frac{(a+b)\cot^2\left(\frac{1}{4}(2c+2dx-\pi)\right)}{a-b}}F\left(\sin^{-1}\left(\sqrt{-\frac{a+b\sin(c+dx)}{a(\sin(c+dx)-1)}}\right.\right.}{d(a+b)\sqrt{\sin(c+dx)}\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Sin[c + d*x]]*Sqrt[a + b*Sin[c + d*x]]),x]
```

```
[Out] (8*a*Sqrt[-(((a + b)*Cot[(2*c - Pi + 2*d*x)/4]^2)/(a - b))]*EllipticF[ArcSi
n[Sqrt[-((a + b*Sin[c + d*x])/(a*(-1 + Sin[c + d*x])))]], (2*a)/(a - b)]*Se
c[c + d*x]*Sqrt[-(((a + b)*Sin[c + d*x]*(a + b*Sin[c + d*x]))/(a^2*(-1 + Si
n[c + d*x])^2))]*Sin[(2*c - Pi + 2*d*x)/4]^4)/((a + b)*d*Sqrt[Sin[c + d*x]]
*Sqrt[a + b*Sin[c + d*x]])
```

Maple [B] time = 0.207, size = 310, normalized size = 2.8

$$-\frac{\sqrt{2}}{da\left(-1+\cos\left(dx+c\right)\right)}\left(b+\sqrt{-a^2+b^2}\right)\sqrt{\frac{1}{\sin\left(dx+c\right)}\left(\sqrt{-a^2+b^2}\sin\left(dx+c\right)+b\sin\left(dx+c\right)-\cos\left(dx+c\right)a+a\right)\left(b+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sin(d*x+c)^(1/2)/(a+b*sin(d*x+c))^(1/2),x)
```

```
[Out] -1/d*(b+(-a^2+b^2)^(1/2))/a/(a+b*sin(d*x+c))^(1/2)*(((-a^2+b^2)^(1/2)*sin(d
*x+c)+b*sin(d*x+c)-cos(d*x+c)*a+a)/(b+(-a^2+b^2)^(1/2))/sin(d*x+c))^(1/2)*(
((-a^2+b^2)^(1/2)*sin(d*x+c)-b*sin(d*x+c)+cos(d*x+c)*a-a)/(-a^2+b^2)^(1/2)/
sin(d*x+c))^(1/2)*((-1+cos(d*x+c))*a/(b+(-a^2+b^2)^(1/2))/sin(d*x+c))^(1/2)
*EllipticF((((-a^2+b^2)^(1/2)*sin(d*x+c)+b*sin(d*x+c)-cos(d*x+c)*a+a)/(b+(-
a^2+b^2)^(1/2))/sin(d*x+c))^(1/2),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b
^2)^(1/2))^(1/2))*sin(d*x+c)^(3/2)*2^(1/2)/(-1+cos(d*x+c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\sin\left(dx+c\right)+a}\sqrt{\sin\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(d*x+c)^(1/2)/(a+b*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*sin(d*x + c) + a)*sqrt(sin(d*x + c))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{b\sin\left(dx+c\right)+a}\sqrt{\sin\left(dx+c\right)}}{b\cos\left(dx+c\right)^2-a\sin\left(dx+c\right)-b},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(d*x+c)^(1/2)/(a+b*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(b*sin(d*x + c) + a)*sqrt(sin(d*x + c))/(b*cos(d*x + c)^2 - a
*sin(d*x + c) - b), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+b\sin\left(c+dx\right)}\sqrt{\sin\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(d*x+c)**(1/2)/(a+b*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*sin(c + d*x))*sqrt(sin(c + d*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\sin\left(dx+c\right)+a}\sqrt{\sin\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(d*x+c)^(1/2)/(a+b*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*sin(d*x + c) + a)*sqrt(sin(d*x + c))), x)
```

3.213 $\int (d\sin(e+fx))^m (a+b\sin(e+fx))^3\,dx$

Optimal. Leaf size=270

$$\frac{b\left(3a^2(m+3)+b^2(m+2)\right)\cos(e+fx)(d\sin(e+fx))^{m+2}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};\sin^2(e+fx)\right)}{d^2f(m+2)(m+3)\sqrt{\cos^2(e+fx)}}+\frac{a\left(a^2(m+2)+3b^2(m+1)\right)c}{}$$

```
[Out] -((a*b^2*(7 + 2*m)*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + m))/(d*f*(2 + m)*(3 +
 m))) + (a*(3*b^2*(1 + m) + a^2*(2 + m))*Cos[e + f*x]*Hypergeometric2F1[1/2
, (1 + m)/2, (3 + m)/2, Sin[e + f*x]^2]*(d*Sin[e + f*x])^(1 + m))/(d*f*(1 +
 m)*(2 + m)*Sqrt[Cos[e + f*x]^2]) + (b*(b^2*(2 + m) + 3*a^2*(3 + m))*Cos[e
+ f*x]*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, Sin[e + f*x]^2]*(d*Sin[
e + f*x])^(2 + m))/(d^2*f*(2 + m)*(3 + m)*Sqrt[Cos[e + f*x]^2]) - (b^2*Cos[
e + f*x]*(d*Sin[e + f*x])^(1 + m)*(a + b*Sin[e + f*x]))/(d*f*(3 + m))
```

Rubi [A] time = 0.379773, antiderivative size = 270, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2793, 3023, 2748, 2643}

$$\frac{b\left(3a^2(m+3)+b^2(m+2)\right)\cos(e+fx)(d\sin(e+fx))^{m+2}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};\sin^2(e+fx)\right)}{d^2f(m+2)(m+3)\sqrt{\cos^2(e+fx)}}+\frac{a\left(a^2(m+2)+3b^2(m+1)\right)c}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sin[e + f*x])^m*(a + b*Sin[e + f*x])^3,x]
```

```
[Out] -((a*b^2*(7 + 2*m)*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + m))/(d*f*(2 + m)*(3 +
 m))) + (a*(3*b^2*(1 + m) + a^2*(2 + m))*Cos[e + f*x]*Hypergeometric2F1[1/2
, (1 + m)/2, (3 + m)/2, Sin[e + f*x]^2]*(d*Sin[e + f*x])^(1 + m))/(d*f*(1 +
 m)*(2 + m)*Sqrt[Cos[e + f*x]^2]) + (b*(b^2*(2 + m) + 3*a^2*(3 + m))*Cos[e
+ f*x]*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, Sin[e + f*x]^2]*(d*Sin[
e + f*x])^(2 + m))/(d^2*f*(2 + m)*(3 + m)*Sqrt[Cos[e + f*x]^2]) - (b^2*Cos[
e + f*x]*(d*Sin[e + f*x])^(1 + m)*(a + b*Sin[e + f*x]))/(d*f*(3 + m))
```

Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
```

```
!LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int (d\sin(e+fx))^m(a+b\sin(e+fx))^3\,dx = -\frac{b^2\cos(e+fx)(d\sin(e+fx))^{1+m}(a+b\sin(e+fx))}{df(3+m)} + \frac{\int (d\sin(e+fx))^m}{}$$

$$= -\frac{ab^2(7+2m)\cos(e+fx)(d\sin(e+fx))^{1+m}}{df(2+m)(3+m)} - \frac{b^2\cos(e+fx)(d\sin(e+f}{df(3}$$

$$= -\frac{ab^2(7+2m)\cos(e+fx)(d\sin(e+fx))^{1+m}}{df(2+m)(3+m)} - \frac{b^2\cos(e+fx)(d\sin(e+f}{df(3}$$

$$= -\frac{ab^2(7+2m)\cos(e+fx)(d\sin(e+fx))^{1+m}}{df(2+m)(3+m)} + \frac{a\left(a^2+\frac{3b^2(1+m)}{2+m}\right)\cos(e+f}{}$$

Mathematica [A] time = 0.770993, size = 199, normalized size = 0.74

$$\frac{\sin(e+fx)\cos(e+fx)(d\sin(e+fx))^m\left(\frac{b\left(3a^2(m+3)+b^2(m+2)\right)\sin(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};\sin^2(e+fx)\right)}{(m+2)\sqrt{\cos^2(e+fx)}}+\frac{a(m+3)\left(a^2(m+2)+3b^2(m+1)\right){}_2F_1(}{(m+1)(m+2)\sqrt{\cos}}\right.}{f(m+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sin[e + f*x])^m*(a + b*Sin[e + f*x])^3,x]
```

```
[Out] (Cos[e + f*x]*Sin[e + f*x]*(d*Sin[e + f*x])^m*(-((a*b^2*(7 + 2*m))/(2 + m))
 + (a*(3 + m)*(3*b^2*(1 + m) + a^2*(2 + m))*Hypergeometric2F1[1/2, (1 + m)/
2, (3 + m)/2, Sin[e + f*x]^2])/((1 + m)*(2 + m)*Sqrt[Cos[e + f*x]^2]) + (b*
(b^2*(2 + m) + 3*a^2*(3 + m))*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2,
Sin[e + f*x]^2]*Sin[e + f*x])/((2 + m)*Sqrt[Cos[e + f*x]^2]) - b^2*(a + b*S
in[e + f*x])))/(f*(3 + m))
```

Maple [F] time = 2.731, size = 0, normalized size = 0.

$$\int \left(d\sin\left(fx+e\right)\right)^m\left(a+b\sin\left(fx+e\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^m*(a+b*sin(f*x+e))^3,x)
```

```
[Out] int((d*sin(f*x+e))^m*(a+b*sin(f*x+e))^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)+a\right)^{3}\left(d\sin\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^m*(a+b*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)^3*(d*sin(f*x + e))^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(3\,ab^{2}\cos\left(fx+e\right)^{2}-a^{3}-3\,ab^{2}+\left(b^{3}\cos\left(fx+e\right)^{2}-3\,a^{2}b-b^{3}\right)\sin\left(fx+e\right)\right)\left(d\sin\left(fx+e\right)\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^m*(a+b*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] integral(-(3*a*b^2*cos(f*x + e)^2 - a^3 - 3*a*b^2 + (b^3*cos(f*x + e)^2 - 3
*a^2*b - b^3)*sin(f*x + e))*(d*sin(f*x + e))^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**m*(a+b*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)+a\right)^{3}\left(d\sin\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^m*(a+b*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)^3*(d*sin(f*x + e))^m, x)
```

3.214 $\int (d\sin(e+fx))^m(a+b\sin(e+fx))^2\,dx$

Optimal. Leaf size=194

$$\frac{\left(a^2(m+2)+b^2(m+1)\right)\cos(e+fx)(d\sin(e+fx))^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\sin^2(e+fx)\right)}{df(m+1)(m+2)\sqrt{\cos^2(e+fx)}}+\frac{2ab\cos(e+fx)(d\sin(e+fx}{d^2f(m+}$$

```
[Out] -((b^2*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + m))/(d*f*(2 + m))) + ((b^2*(1 + m
) + a^2*(2 + m))*Cos[e + f*x]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2,
Sin[e + f*x]^2]*(d*Sin[e + f*x])^(1 + m))/(d*f*(1 + m)*(2 + m)*Sqrt[Cos[e +
 f*x]^2]) + (2*a*b*Cos[e + f*x]*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2
, Sin[e + f*x]^2]*(d*Sin[e + f*x])^(2 + m))/(d^2*f*(2 + m)*Sqrt[Cos[e + f*x
]^2])
```

Rubi [A] time = 0.134654, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {2789, 2643, 3014}

$$\frac{\left(a^2(m+2)+b^2(m+1)\right)\cos(e+fx)(d\sin(e+fx))^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\sin^2(e+fx)\right)}{df(m+1)(m+2)\sqrt{\cos^2(e+fx)}}+\frac{2ab\cos(e+fx)(d\sin(e+fx}{d^2f(m+}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sin[e + f*x])^m*(a + b*Sin[e + f*x])^2,x]
```

```
[Out] -((b^2*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + m))/(d*f*(2 + m))) + ((b^2*(1 + m
) + a^2*(2 + m))*Cos[e + f*x]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2,
Sin[e + f*x]^2]*(d*Sin[e + f*x])^(1 + m))/(d*f*(1 + m)*(2 + m)*Sqrt[Cos[e +
 f*x]^2]) + (2*a*b*Cos[e + f*x]*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2
, Sin[e + f*x]^2]*(d*Sin[e + f*x])^(2 + m))/(d^2*f*(2 + m)*Sqrt[Cos[e + f*x
]^2])
```

Rule 2789

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)])^2, x_Symbol] :> Dist[(2*c*d)/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] +
 Int[(b*Sin[e + f*x])^m*(c^2 + d^2*Sin[e + f*x]^2), x] /; FreeQ[{b, c, d, e
, f, m}, x]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3014

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*
(m + 2)), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])^m
, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]
```

Rubi steps

$$\int (d\sin(e+fx))^m(a+b\sin(e+fx))^2\,dx = \frac{(2ab)\int (d\sin(e+fx))^{1+m}\,dx}{d} + \int (d\sin(e+fx))^m\left(a^2+b^2\sin^2(e+fx)\right)dx$$

$$= -\frac{b^2\cos(e+fx)(d\sin(e+fx))^{1+m}}{df(2+m)} + \frac{2ab\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{2+m}{2};\frac{4+m}{2};\sin^2(e\right.}{d^2f(2+m)\sqrt{\cos^2(e}}$$

$$= -\frac{b^2\cos(e+fx)(d\sin(e+fx))^{1+m}}{df(2+m)} + \frac{\left(a^2+\frac{b^2(1+m)}{2+m}\right)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\right.}{df(1+m)\sqrt{c}}$$

Mathematica [A] time = 0.305454, size = 144, normalized size = 0.74

$$-\frac{\cos(e+fx)\sin^2(e+fx)^{\frac{1}{2}(-m-1)}(d\sin(e+fx))^m\left(a\left(a\sin(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1-m}{2};\frac{3}{2};\cos^2(e+fx)\right)+2b\sqrt{\sin^2(e+fx)}\,{}_2F_1\left(\right.\right.\right.}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sin[e + f*x])^m*(a + b*Sin[e + f*x])^2,x]
```

```
[Out] -((Cos[e + f*x]*(d*Sin[e + f*x])^m*(Sin[e + f*x]^2)^((-1 - m)/2)*(b^2*Hyper
geometric2F1[1/2, (-1 - m)/2, 3/2, Cos[e + f*x]^2]*Sin[e + f*x] + a*(a*Hype
rgeometric2F1[1/2, (1 - m)/2, 3/2, Cos[e + f*x]^2]*Sin[e + f*x] + 2*b*Hyper
geometric2F1[1/2, -m/2, 3/2, Cos[e + f*x]^2]*Sqrt[Sin[e + f*x]^2])))/f)
```

Maple [F] time = 3.307, size = 0, normalized size = 0.

$$\int \left(d\sin\left(fx+e\right)\right)^m\left(a+b\sin\left(fx+e\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^m*(a+b*sin(f*x+e))^2,x)
```

```
[Out] int((d*sin(f*x+e))^m*(a+b*sin(f*x+e))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)+a\right)^2\left(d\sin\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^m*(a+b*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)^2*(d*sin(f*x + e))^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(b^2\cos\left(fx+e\right)^2-2\,ab\sin\left(fx+e\right)-a^2-b^2\right)\left(d\sin\left(fx+e\right)\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^m*(a+b*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral(-(b^2*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2)*(d*sin(f*x
+ e))^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**m*(a+b*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)+a\right)^{2}\left(d\sin\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^m*(a+b*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)^2*(d*sin(f*x + e))^m, x)
```

3.215 $\int (d\sin(e+fx))^m(a+b\sin(e+fx))\,dx$

Optimal. Leaf size=139

$$\frac{a\cos(e+fx)(d\sin(e+fx))^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\sin^2(e+fx)\right)}{df(m+1)\sqrt{\cos^2(e+fx)}}+\frac{b\cos(e+fx)(d\sin(e+fx))^{m+2}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};\sin^2(\right.}{d^2f(m+2)\sqrt{\cos^2(e+fx)}}$$

```
[Out] (a*Cos[e + f*x]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sin[e + f*x]^2
]*(d*Sin[e + f*x])^(1 + m))/(d*f*(1 + m)*Sqrt[Cos[e + f*x]^2]) + (b*Cos[e +
 f*x]*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, Sin[e + f*x]^2]*(d*Sin[e
 + f*x])^(2 + m))/(d^2*f*(2 + m)*Sqrt[Cos[e + f*x]^2])
```

Rubi [A] time = 0.0722351, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2748, 2643}

$$\frac{a\cos(e+fx)(d\sin(e+fx))^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\sin^2(e+fx)\right)}{df(m+1)\sqrt{\cos^2(e+fx)}}+\frac{b\cos(e+fx)(d\sin(e+fx))^{m+2}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};\sin^2(\right.}{d^2f(m+2)\sqrt{\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sin[e + f*x])^m*(a + b*Sin[e + f*x]),x]
```

```
[Out] (a*Cos[e + f*x]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sin[e + f*x]^2
]*(d*Sin[e + f*x])^(1 + m))/(d*f*(1 + m)*Sqrt[Cos[e + f*x]^2]) + (b*Cos[e +
 f*x]*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, Sin[e + f*x]^2]*(d*Sin[e
 + f*x])^(2 + m))/(d^2*f*(2 + m)*Sqrt[Cos[e + f*x]^2])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int (d\sin(e+fx))^m(a+b\sin(e+fx))\,dx &= a\int (d\sin(e+fx))^m\,dx+\frac{b\int (d\sin(e+fx))^{1+m}\,dx}{d}\\ &= \frac{a\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\sin^2(e+fx)\right)(d\sin(e+fx))^{1+m}}{df(1+m)\sqrt{\cos^2(e+fx)}}+\frac{b\cos(e+f}{}\end{aligned}$$

Mathematica [A] time = 0.156493, size = 111, normalized size = 0.8

$$\frac{\sqrt{\cos^2(e+fx)}\tan(e+fx)(d\sin(e+fx))^m\left(a(m+2)\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\sin^2(e+fx)\right)+b(m+1)\sin(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\right.\right.}{f(m+1)(m+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sin[e + f*x])^m*(a + b*Sin[e + f*x]),x]
```

```
[Out] (Sqrt[Cos[e + f*x]^2]*(d*Sin[e + f*x])^m*(a*(2 + m)*Hypergeometric2F1[1/2,
(1 + m)/2, (3 + m)/2, Sin[e + f*x]^2] + b*(1 + m)*Hypergeometric2F1[1/2, (2
 + m)/2, (4 + m)/2, Sin[e + f*x]^2]*Sin[e + f*x])*Tan[e + f*x])/(f*(1 + m)*
(2 + m))
```

Maple [F] time = 1.076, size = 0, normalized size = 0.

$$\int \left(d\sin\left(fx+e\right)\right)^{m}\left(a+b\sin\left(fx+e\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^m*(a+b*sin(f*x+e)),x)
```

```
[Out] int((d*sin(f*x+e))^m*(a+b*sin(f*x+e)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)\right)^{m}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^m*(a+b*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)*(d*sin(f*x + e))^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^m*(a+b*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral((b*sin(f*x + e) + a)*(d*sin(f*x + e))^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**m*(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sin \left(fx+e\right)+a\right) \left(d \sin \left(fx+e\right)\right)^{m} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^m*(a+b*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)*(d*sin(f*x + e))^m, x)
```

3.216 $\int \frac{(d\sin(e+fx))^m}{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=195

$$\frac{b\cos(e+fx)\sin^2(e+fx)^{-m/2}(d\sin(e+fx))^m F_1\left(\frac{1}{2};-\frac{m}{2},1;\frac{3}{2};\cos^2(e+fx),-\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)}{f\left(a^2-b^2\right)}-\frac{ad\cos(e+fx)\sin^2(e+f$$

```
[Out] -((a*d*AppellF1[1/2, (1 - m)/2, 1, 3/2, Cos[e + f*x]^2, -((b^2*Cos[e + f*x]
^2)/(a^2 - b^2))]*Cos[e + f*x]*(d*Sin[e + f*x])^(-1 + m)*(Sin[e + f*x]^2)^(
(1 - m)/2))/((a^2 - b^2)*f)) + (b*AppellF1[1/2, -m/2, 1, 3/2, Cos[e + f*x]^
2, -((b^2*Cos[e + f*x]^2)/(a^2 - b^2))]*Cos[e + f*x]*(d*Sin[e + f*x])^m)/((
a^2 - b^2)*f*(Sin[e + f*x]^2)^(m/2))
```

Rubi [A] time = 0.248497, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {2823, 3189, 429}

$$\frac{b\cos(e+fx)\sin^2(e+fx)^{-m/2}(d\sin(e+fx))^m F_1\left(\frac{1}{2};-\frac{m}{2},1;\frac{3}{2};\cos^2(e+fx),-\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)}{f\left(a^2-b^2\right)}-\frac{ad\cos(e+fx)\sin^2(e+f$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sin[e + f*x])^m/(a + b*Sin[e + f*x]),x]
```

```
[Out] -((a*d*AppellF1[1/2, (1 - m)/2, 1, 3/2, Cos[e + f*x]^2, -((b^2*Cos[e + f*x]
^2)/(a^2 - b^2))]*Cos[e + f*x]*(d*Sin[e + f*x])^(-1 + m)*(Sin[e + f*x]^2)^(
(1 - m)/2))/((a^2 - b^2)*f)) + (b*AppellF1[1/2, -m/2, 1, 3/2, Cos[e + f*x]^
2, -((b^2*Cos[e + f*x]^2)/(a^2 - b^2))]*Cos[e + f*x]*(d*Sin[e + f*x])^m)/((
a^2 - b^2)*f*(Sin[e + f*x]^2)^(m/2))
```

Rule 2823

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[a, Int[(d*Sin[e + f*x])^n/(a^2 - b^2*Sin[e + f*x]^
2), x], x] - Dist[b/d, Int[(d*Sin[e + f*x])^(n + 1)/(a^2 - b^2*Sin[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3189

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[
(ff*d^(2*IntPart[(m - 1)/2] + 1)*(d*Sin[e + f*x])^(2*FracPart[(m - 1)/2]))/
(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2]), Subst[Int[(1 - ff^2*x^2)^((m - 1)
/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, d,
 e, f, m, p}, x] &&  !IntegerQ[m]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{(d\sin(e+fx))^m}{a+b\sin(e+fx)}\,dx = a\int \frac{(d\sin(e+fx))^m}{a^2-b^2\sin^2(e+fx)}\,dx - \frac{b\int \frac{(d\sin(e+fx))^{1+m}}{a^2-b^2\sin^2(e+fx)}\,dx}{d}$$

$$= -\frac{\left(ad(d\sin(e+fx))^{2\left(-\frac{1}{2}+\frac{m}{2}\right)}\sin^2(e+fx)^{\frac{1}{2}-\frac{m}{2}}\right)\text{Subst}\left(\int \frac{(1-x^2)^{\frac{1}{2}(-1+m)}}{a^2-b^2+b^2x^2}\,dx, x, \cos(e+fx)\right)}{f} + \frac{\left(b(d\right.}{}$$

$$= -\frac{adF_1\left(\frac{1}{2};\frac{1-m}{2},1;\frac{3}{2};\cos^2(e+fx),-\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)\cos(e+fx)(d\sin(e+fx))^{-1+m}\sin^2(e+fx)^{\frac{1-m}{2}}}{(a^2-b^2)f}$$

Mathematica [B] time = 17.5526, size = 1593, normalized size = 8.17

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sin[e + f*x])^m/(a + b*Sin[e + f*x]),x]
```

```
[Out] ((Sec[e + f*x]^2)^(m/2)*(d*Sin[e + f*x])^m*Tan[e + f*x]*(Tan[e + f*x]/Sqrt[
Sec[e + f*x]^2])^m*(a*b*(2 + m)*AppellF1[(1 + m)/2, m/2, 1, (3 + m)/2, -Tan
[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2] + (1 + m)*((a^2 - b^2)*AppellF1
[(2 + m)/2, (-1 + m)/2, 1, (4 + m)/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e
+ f*x]^2)/a^2] - a^2*Hypergeometric2F1[(1 + m)/2, (2 + m)/2, (4 + m)/2, -Ta
n[e + f*x]^2])*Tan[e + f*x]))/(a^2*b*f*(1 + m)*(2 + m)*(a + b*Sin[e + f*x])
*(((Sec[e + f*x]^2)^(1 + m/2)*(Tan[e + f*x]/Sqrt[Sec[e + f*x]^2])^m*(a*b*(2
 + m)*AppellF1[(1 + m)/2, m/2, 1, (3 + m)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2
)*Tan[e + f*x]^2] + (1 + m)*((a^2 - b^2)*AppellF1[(2 + m)/2, (-1 + m)/2, 1,
 (4 + m)/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2] - a^2*Hyper
geometric2F1[(1 + m)/2, (2 + m)/2, (4 + m)/2, -Tan[e + f*x]^2])*Tan[e + f*x
]))/(a^2*b*(1 + m)*(2 + m)) + (m*(Sec[e + f*x]^2)^(m/2)*Tan[e + f*x]^2*(Tan
[e + f*x]/Sqrt[Sec[e + f*x]^2])^m*(a*b*(2 + m)*AppellF1[(1 + m)/2, m/2, 1,
(3 + m)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2] + (1 + m)*((a^2
- b^2)*AppellF1[(2 + m)/2, (-1 + m)/2, 1, (4 + m)/2, -Tan[e + f*x]^2, ((-a^
2 + b^2)*Tan[e + f*x]^2)/a^2] - a^2*Hypergeometric2F1[(1 + m)/2, (2 + m)/2,
 (4 + m)/2, -Tan[e + f*x]^2])*Tan[e + f*x]))/(a^2*b*(1 + m)*(2 + m)) + (m*(
Sec[e + f*x]^2)^(m/2)*Tan[e + f*x]*(Tan[e + f*x]/Sqrt[Sec[e + f*x]^2])^(-1
+ m)*(a*b*(2 + m)*AppellF1[(1 + m)/2, m/2, 1, (3 + m)/2, -Tan[e + f*x]^2, (
-1 + b^2/a^2)*Tan[e + f*x]^2] + (1 + m)*((a^2 - b^2)*AppellF1[(2 + m)/2, (-
1 + m)/2, 1, (4 + m)/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]
 - a^2*Hypergeometric2F1[(1 + m)/2, (2 + m)/2, (4 + m)/2, -Tan[e + f*x]^2])
*Tan[e + f*x])*(Sqrt[Sec[e + f*x]^2] - Tan[e + f*x]^2/Sqrt[Sec[e + f*x]^2])
)/(a^2*b*(1 + m)*(2 + m)) + ((Sec[e + f*x]^2)^(m/2)*Tan[e + f*x]*(Tan[e + f
*x]/Sqrt[Sec[e + f*x]^2])^m*((1 + m)*((a^2 - b^2)*AppellF1[(2 + m)/2, (-1 +
 m)/2, 1, (4 + m)/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2] -
a^2*Hypergeometric2F1[(1 + m)/2, (2 + m)/2, (4 + m)/2, -Tan[e + f*x]^2])*Se
c[e + f*x]^2 + a*b*(2 + m)*(-((m*(1 + m)*AppellF1[1 + (1 + m)/2, 1 + m/2, 1
, 1 + (3 + m)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Sec[e + f*
x]^2*Tan[e + f*x])/(3 + m)) + (2*(-1 + b^2/a^2)*(1 + m)*AppellF1[1 + (1 + m
)/2, m/2, 2, 1 + (3 + m)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]
*Sec[e + f*x]^2*Tan[e + f*x])/(3 + m)) + (1 + m)*Tan[e + f*x]*((a^2 - b^2)*
(-(((-1 + m)*(2 + m)*AppellF1[1 + (2 + m)/2, 1 + (-1 + m)/2, 1, 1 + (4 + m)
/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]*Sec[e + f*x]^2*Tan[
e + f*x])/(4 + m)) + (2*(-a^2 + b^2)*(2 + m)*AppellF1[1 + (2 + m)/2, (-1 +
m)/2, 2, 1 + (4 + m)/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]
*Sec[e + f*x]^2*Tan[e + f*x])/(a^2*(4 + m))) - a^2*(2 + m)*Csc[e + f*x]*Sec
```

```
[e + f*x]*(-Hypergeometric2F1[(1 + m)/2, (2 + m)/2, (4 + m)/2, -Tan[e + f*x
]^2] + (1 + Tan[e + f*x]^2)^((-1 - m)/2)))))/(a^2*b*(1 + m)*(2 + m))))
```

Maple [F] time = 0.399, size = 0, normalized size = 0.

$$\int \frac{(d\sin(fx+e))^m}{a+b\sin(fx+e)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^m/(a+b*sin(f*x+e)),x)
```

```
[Out] int((d*sin(f*x+e))^m/(a+b*sin(f*x+e)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d\sin(fx+e))^m}{b\sin(fx+e)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^m/(a+b*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((d*sin(f*x + e))^m/(b*sin(f*x + e) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(d\sin(fx+e))^m}{b\sin(fx+e)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^m/(a+b*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral((d*sin(f*x + e))^m/(b*sin(f*x + e) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**m/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sin\left(fx+e\right)\right)^m}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^m/(a+b*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*sin(f*x + e))^m/(b*sin(f*x + e) + a), x)
```

3.217 $\int \frac{(d\sin(e+fx))^m}{(a+b\sin(e+fx))^2}\,dx$

Optimal. Leaf size=306

$$-\frac{b^2\cos(e+fx)\sin^2(e+fx)^{\frac{1}{2}(-m-1)}(d\sin(e+fx))^{m+1}F_1\left(\frac{1}{2};\frac{1}{2}(-m-1),2;\frac{3}{2};\cos^2(e+fx),-\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)}{df\left(a^2-b^2\right)^2}-\frac{a^2d\cos(e+}{}$$

```
[Out] -((b^2*AppellF1[1/2, (-1 - m)/2, 2, 3/2, Cos[e + f*x]^2, -((b^2*Cos[e + f*x
]^2)/(a^2 - b^2))]*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + m)*(Sin[e + f*x]^2)^(
(-1 - m)/2))/((a^2 - b^2)^2*d*f)) - (a^2*d*AppellF1[1/2, (1 - m)/2, 2, 3/2,
 Cos[e + f*x]^2, -((b^2*Cos[e + f*x]^2)/(a^2 - b^2))]*Cos[e + f*x]*(d*Sin[e
 + f*x])^(-1 + m)*(Sin[e + f*x]^2)^((1 - m)/2))/((a^2 - b^2)^2*f) + (2*a*b*
AppellF1[1/2, -m/2, 2, 3/2, Cos[e + f*x]^2, -((b^2*Cos[e + f*x]^2)/(a^2 - b
^2))]*Cos[e + f*x]*(d*Sin[e + f*x])^m)/((a^2 - b^2)^2*f*(Sin[e + f*x]^2)^(m
/2))
```

Rubi [A] time = 0.420374, antiderivative size = 306, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2824, 3189, 429, 16}

$$-\frac{b^2\cos(e+fx)\sin^2(e+fx)^{\frac{1}{2}(-m-1)}(d\sin(e+fx))^{m+1}F_1\left(\frac{1}{2};\frac{1}{2}(-m-1),2;\frac{3}{2};\cos^2(e+fx),-\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)}{df\left(a^2-b^2\right)^2}-\frac{a^2d\cos(e+}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sin[e + f*x])^m/(a + b*Sin[e + f*x])^2,x]
```

```
[Out] -((b^2*AppellF1[1/2, (-1 - m)/2, 2, 3/2, Cos[e + f*x]^2, -((b^2*Cos[e + f*x
]^2)/(a^2 - b^2))]*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + m)*(Sin[e + f*x]^2)^(
(-1 - m)/2))/((a^2 - b^2)^2*d*f)) - (a^2*d*AppellF1[1/2, (1 - m)/2, 2, 3/2,
 Cos[e + f*x]^2, -((b^2*Cos[e + f*x]^2)/(a^2 - b^2))]*Cos[e + f*x]*(d*Sin[e
 + f*x])^(-1 + m)*(Sin[e + f*x]^2)^((1 - m)/2))/((a^2 - b^2)^2*f) + (2*a*b*
AppellF1[1/2, -m/2, 2, 3/2, Cos[e + f*x]^2, -((b^2*Cos[e + f*x]^2)/(a^2 - b
^2))]*Cos[e + f*x]*(d*Sin[e + f*x])^m)/((a^2 - b^2)^2*f*(Sin[e + f*x]^2)^(m
/2))
```

Rule 2824

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(d*sin[e + f*x])^n/((a - b*sin[e +
 f*x])^m/(a^2 - b^2*sin[e + f*x]^2)^m), x], x] /; FreeQ[{a, b, d, e, f, n},
 x] && NeQ[a^2 - b^2, 0] && ILtQ[m, -1]
```

Rule 3189

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[
(ff*d^(2*IntPart[(m - 1)/2] + 1)*(d*Sin[e + f*x])^(2*FracPart[(m - 1)/2]))/
(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2]), Subst[Int[(1 - ff^2*x^2)^((m - 1)
/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, d,
 e, f, m, p}, x] &&  !IntegerQ[m]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{(d\sin(e+fx))^m}{(a+b\sin(e+fx))^2}\,dx = \int \left(\frac{a^2(d\sin(e+fx))^m}{\left(a^2-b^2\sin^2(e+fx)\right)^2} - \frac{2ab\sin(e+fx)(d\sin(e+fx))^m}{\left(a^2-b^2\sin^2(e+fx)\right)^2} + \frac{b^2\sin^2(e+fx)(d\sin(e+f$$

$$= a^2\int \frac{(d\sin(e+fx))^m}{\left(a^2-b^2\sin^2(e+fx)\right)^2}\,dx - (2ab)\int \frac{\sin(e+fx)(d\sin(e+fx))^m}{\left(a^2-b^2\sin^2(e+fx)\right)^2}\,dx + b^2\int \frac{\sin^2(e+}{(-a^2+}$$

$$= \frac{b^2\int \frac{(d\sin(e+fx))^{2+m}}{\left(-a^2+b^2\sin^2(e+fx)\right)^2}\,dx}{d^2} - \frac{(2ab)\int \frac{(d\sin(e+fx))^{1+m}}{\left(a^2-b^2\sin^2(e+fx)\right)^2}\,dx}{d} - \frac{\left(a^2d(d\sin(e+fx))^{2\left(-\frac{1}{2}+\frac{m}{2}\right)}\sin^2(e}{}$$

$$= -\frac{a^2dF_1\left(\frac{1}{2};\frac{1-m}{2},2;\frac{3}{2};\cos^2(e+fx),-\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)\cos(e+fx)(d\sin(e+fx))^{-1+m}\sin^2(e+fx}{\left(a^2-b^2\right)^2 f}$$

$$= -\frac{b^2F_1\left(\frac{1}{2};\frac{1}{2}(-1-m),2;\frac{3}{2};\cos^2(e+fx),-\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)\cos(e+fx)(d\sin(e+fx))^{1+m}\sin^2(e+}{\left(a^2-b^2\right)^2 df}$$

Mathematica [B] time = 18.8918, size = 1856, normalized size = 6.07

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sin[e + f*x])^m/(a + b*Sin[e + f*x])^2,x]
```

```
[Out] -(((Sec[e + f*x]^2)^(m/2)*(d*Sin[e + f*x])^m*Tan[e + f*x]*(Tan[e + f*x]/Sqr
t[Sec[e + f*x]^2])^m*(-(a*(a^2 + b^2)*(2 + m)*AppellF1[(1 + m)/2, m/2, 1, (
3 + m)/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]) + 2*b*(a*b*(
2 + m)*AppellF1[(1 + m)/2, m/2, 2, (3 + m)/2, -Tan[e + f*x]^2, ((-a^2 + b^2
)*Tan[e + f*x]^2)/a^2] + (a^2 - b^2)*(1 + m)*AppellF1[(2 + m)/2, (-1 + m)/2
, 2, (4 + m)/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]*Tan[e +
 f*x])))/(a^3*(a^2 - b^2)*f*(1 + m)*(2 + m)*(a + b*Sin[e + f*x])^2*(-(((Sec
[e + f*x]^2)^(1 + m/2)*(Tan[e + f*x]/Sqrt[Sec[e + f*x]^2])^m*(-(a*(a^2 + b^
2)*(2 + m)*AppellF1[(1 + m)/2, m/2, 1, (3 + m)/2, -Tan[e + f*x]^2, ((-a^2 +
 b^2)*Tan[e + f*x]^2)/a^2]) + 2*b*(a*b*(2 + m)*AppellF1[(1 + m)/2, m/2, 2,
(3 + m)/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2] + (a^2 - b^2
)*(1 + m)*AppellF1[(2 + m)/2, (-1 + m)/2, 2, (4 + m)/2, -Tan[e + f*x]^2, ((
-a^2 + b^2)*Tan[e + f*x]^2)/a^2]*Tan[e + f*x])))/(a^3*(a^2 - b^2)*(1 + m)*(
2 + m))) - (m*(Sec[e + f*x]^2)^(m/2)*Tan[e + f*x]^2*(Tan[e + f*x]/Sqrt[Sec[
e + f*x]^2])^m*(-(a*(a^2 + b^2)*(2 + m)*AppellF1[(1 + m)/2, m/2, 1, (3 + m)
/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]) + 2*b*(a*b*(2 + m)
*AppellF1[(1 + m)/2, m/2, 2, (3 + m)/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[
```

```
e + f*x]^2)/a^2] + (a^2 - b^2)*(1 + m)*AppellF1[(2 + m)/2, (-1 + m)/2, 2, (
4 + m)/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]*Tan[e + f*x])
))/(a^3*(a^2 - b^2)*(1 + m)*(2 + m)) - (m*(Sec[e + f*x]^2)^(m/2)*Tan[e + f*
x]*(Tan[e + f*x]/Sqrt[Sec[e + f*x]^2])^(-1 + m)*(Sqrt[Sec[e + f*x]^2] - Tan
[e + f*x]^2/Sqrt[Sec[e + f*x]^2])*(-(a*(a^2 + b^2)*(2 + m)*AppellF1[(1 + m)
/2, m/2, 1, (3 + m)/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2])
 + 2*b*(a*b*(2 + m)*AppellF1[(1 + m)/2, m/2, 2, (3 + m)/2, -Tan[e + f*x]^2,
 ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2] + (a^2 - b^2)*(1 + m)*AppellF1[(2 + m)/
2, (-1 + m)/2, 2, (4 + m)/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)
/a^2]*Tan[e + f*x])))/(a^3*(a^2 - b^2)*(1 + m)*(2 + m)) - ((Sec[e + f*x]^2)
^(m/2)*Tan[e + f*x]*(Tan[e + f*x]/Sqrt[Sec[e + f*x]^2])^m*(-(a*(a^2 + b^2)*
(2 + m)*(-((m*(1 + m)*AppellF1[1 + (1 + m)/2, 1 + m/2, 1, 1 + (3 + m)/2, -T
an[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]*Sec[e + f*x]^2*Tan[e + f*
x])/(3 + m)) + (2*(-a^2 + b^2)*(1 + m)*AppellF1[1 + (1 + m)/2, m/2, 2, 1 +
(3 + m)/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]*Sec[e + f*x]
^2*Tan[e + f*x])/(a^2*(3 + m)))) + 2*b*((a^2 - b^2)*(1 + m)*AppellF1[(2 + m
)/2, (-1 + m)/2, 2, (4 + m)/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^
2)/a^2]*Sec[e + f*x]^2 + a*b*(2 + m)*(-((m*(1 + m)*AppellF1[1 + (1 + m)/2,
1 + m/2, 2, 1 + (3 + m)/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a
^2]*Sec[e + f*x]^2*Tan[e + f*x])/(3 + m)) + (4*(-a^2 + b^2)*(1 + m)*AppellF
1[1 + (1 + m)/2, m/2, 3, 1 + (3 + m)/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[
e + f*x]^2)/a^2]*Sec[e + f*x]^2*Tan[e + f*x])/(a^2*(3 + m))) + (a^2 - b^2)*
(1 + m)*Tan[e + f*x]*(-(((-1 + m)*(2 + m)*AppellF1[1 + (2 + m)/2, 1 + (-1 +
 m)/2, 2, 1 + (4 + m)/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2
]*Sec[e + f*x]^2*Tan[e + f*x])/(4 + m)) + (4*(-a^2 + b^2)*(2 + m)*AppellF1[
1 + (2 + m)/2, (-1 + m)/2, 3, 1 + (4 + m)/2, -Tan[e + f*x]^2, ((-a^2 + b^2)
*Tan[e + f*x]^2)/a^2]*Sec[e + f*x]^2*Tan[e + f*x])/(a^2*(4 + m))))))/(a^3*(
a^2 - b^2)*(1 + m)*(2 + m)))))
```

Maple [F] time = 0.66, size = 0, normalized size = 0.

$$\int \frac{\left(d\sin\left(fx+e\right)\right)^{m}}{\left(a+b\sin\left(fx+e\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^m/(a+b*sin(f*x+e))^2,x)
```

```
[Out] int((d*sin(f*x+e))^m/(a+b*sin(f*x+e))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sin\left(fx+e\right)\right)^{m}}{\left(b\sin\left(fx+e\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^m/(a+b*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate((d*sin(f*x + e))^m/(b*sin(f*x + e) + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(d\sin\left(fx+e\right)\right)^{m}}{b^{2}\cos\left(fx+e\right)^{2}-2\,ab\sin\left(fx+e\right)-a^{2}-b^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^m/(a+b*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral(-(d*sin(f*x + e))^m/(b^2*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2
 - b^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**m/(a+b*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sin\left(fx+e\right)\right)^{m}}{\left(b\sin\left(fx+e\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^m/(a+b*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*sin(f*x + e))^m/(b*sin(f*x + e) + a)^2, x)
```

3.218 $\int \frac{(d\sin(e+fx))^m}{(a+b\sin(e+fx))^3}\,dx$

Optimal. Leaf size=406

$$-\frac{3ab^2\cos(e+fx)\sin^2(e+fx)^{\frac{1}{2}(-m-1)}(d\sin(e+fx))^{m+1}F_1\left(\frac{1}{2};\frac{1}{2}(-m-1),3;\frac{3}{2};\cos^2(e+fx),-\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)}{df\left(a^2-b^2\right)^3}-\frac{a^3d\cos(}{}$$

```
[Out] (-3*a*b^2*AppellF1[1/2, (-1 - m)/2, 3, 3/2, Cos[e + f*x]^2, -((b^2*Cos[e +
f*x]^2)/(a^2 - b^2))]*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + m)*(Sin[e + f*x]^2
)^((-1 - m)/2))/((a^2 - b^2)^3*d*f) - (a^3*d*AppellF1[1/2, (1 - m)/2, 3, 3/
2, Cos[e + f*x]^2, -((b^2*Cos[e + f*x]^2)/(a^2 - b^2))]*Cos[e + f*x]*(d*Sin
[e + f*x])^(-1 + m)*(Sin[e + f*x]^2)^((1 - m)/2))/((a^2 - b^2)^3*f) + (b^3*
AppellF1[1/2, (-2 - m)/2, 3, 3/2, Cos[e + f*x]^2, -((b^2*Cos[e + f*x]^2)/(a
^2 - b^2))]*Cos[e + f*x]*(d*Sin[e + f*x])^m)/((a^2 - b^2)^3*f*(Sin[e + f*x]
^2)^(m/2)) + (3*a^2*b*AppellF1[1/2, -m/2, 3, 3/2, Cos[e + f*x]^2, -((b^2*Co
s[e + f*x]^2)/(a^2 - b^2))]*Cos[e + f*x]*(d*Sin[e + f*x])^m)/((a^2 - b^2)^3
*f*(Sin[e + f*x]^2)^(m/2))
```

Rubi [A] time = 0.54851, antiderivative size = 406, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2824, 3189, 429, 16}

$$-\frac{3ab^2\cos(e+fx)\sin^2(e+fx)^{\frac{1}{2}(-m-1)}(d\sin(e+fx))^{m+1}F_1\left(\frac{1}{2};\frac{1}{2}(-m-1),3;\frac{3}{2};\cos^2(e+fx),-\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)}{df\left(a^2-b^2\right)^3}-\frac{a^3d\cos(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sin[e + f*x])^m/(a + b*Sin[e + f*x])^3,x]
```


```
[Out] (-3*a*b^2*AppellF1[1/2, (-1 - m)/2, 3, 3/2, Cos[e + f*x]^2, -((b^2*Cos[e +
f*x]^2)/(a^2 - b^2))]*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + m)*(Sin[e + f*x]^2
)^((-1 - m)/2))/((a^2 - b^2)^3*d*f) - (a^3*d*AppellF1[1/2, (1 - m)/2, 3, 3/
2, Cos[e + f*x]^2, -((b^2*Cos[e + f*x]^2)/(a^2 - b^2))]*Cos[e + f*x]*(d*Sin
[e + f*x])^(-1 + m)*(Sin[e + f*x]^2)^((1 - m)/2))/((a^2 - b^2)^3*f) + (b^3*
AppellF1[1/2, (-2 - m)/2, 3, 3/2, Cos[e + f*x]^2, -((b^2*Cos[e + f*x]^2)/(a
^2 - b^2))]*Cos[e + f*x]*(d*Sin[e + f*x])^m)/((a^2 - b^2)^3*f*(Sin[e + f*x]
^2)^(m/2)) + (3*a^2*b*AppellF1[1/2, -m/2, 3, 3/2, Cos[e + f*x]^2, -((b^2*Co
s[e + f*x]^2)/(a^2 - b^2))]*Cos[e + f*x]*(d*Sin[e + f*x])^m)/((a^2 - b^2)^3
*f*(Sin[e + f*x]^2)^(m/2))
```


Rule 2824

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(d*sin[e + f*x])^n/((a - b*sin[e +
 f*x])^m/(a^2 - b^2*sin[e + f*x]^2)^m), x], x] /; FreeQ[{a, b, d, e, f, n},
 x] && NeQ[a^2 - b^2, 0] && ILtQ[m, -1]
```

Rule 3189

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[
(ff*d^(2*IntPart[(m - 1)/2] + 1)*(d*Sin[e + f*x])^(2*FracPart[(m - 1)/2]))/
(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2]), Subst[Int[(1 - ff^2*x^2)^((m - 1)
/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, d,
```

```
e, f, m, p}, x] && !IntegerQ[m]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{(d\sin(e+fx))^m}{(a+b\sin(e+fx))^3}\,dx = \int \left(\frac{a^3(d\sin(e+fx))^m}{\left(a^2-b^2\sin^2(e+fx)\right)^3} - \frac{3a^2b\sin(e+fx)(d\sin(e+fx))^m}{\left(a^2-b^2\sin^2(e+fx)\right)^3} + \frac{3ab^2\sin^2(e+fx)(d\sin(e\right.$$

$$= a^3\int \frac{(d\sin(e+fx))^m}{\left(a^2-b^2\sin^2(e+fx)\right)^3}\,dx - \left(3a^2b\right)\int \frac{\sin(e+fx)(d\sin(e+fx))^m}{\left(a^2-b^2\sin^2(e+fx)\right)^3}\,dx + \left(3ab^2\right)\int \frac{\sin^2}{(}$$

$$= \frac{b^3\int \frac{(d\sin(e+fx))^{3+m}}{\left(-a^2+b^2\sin^2(e+fx)\right)^3}\,dx}{d^3} + \frac{\left(3ab^2\right)\int \frac{(d\sin(e+fx))^{2+m}}{\left(a^2-b^2\sin^2(e+fx)\right)^3}\,dx}{d^2} - \frac{\left(3a^2b\right)\int \frac{(d\sin(e+fx))^{1+m}}{\left(a^2-b^2\sin^2(e+fx)\right)^3}\,dx}{d} - $$

$$= -\frac{a^3dF_1\left(\frac{1}{2};\frac{1-m}{2},3;\frac{3}{2};\cos^2(e+fx),-\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)\cos(e+fx)(d\sin(e+fx))^{-1+m}\sin^2(e+fx}{\left(a^2-b^2\right)^3f}$$

$$= -\frac{3ab^2F_1\left(\frac{1}{2};\frac{1}{2}(-1-m),3;\frac{3}{2};\cos^2(e+fx),-\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)\cos(e+fx)(d\sin(e+fx))^{1+m}\sin^2(}{\left(a^2-b^2\right)^3df}$$

Mathematica [B] time = 18.5914, size = 2388, normalized size = 5.88

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sin[e + f*x])^m/(a + b*Sin[e + f*x])^3,x]
```

```
[Out] -(((Sec[e + f*x]^2)^(m/2)*(d*Sin[e + f*x])^m*Tan[e + f*x]*(Tan[e + f*x]/Sqr
t[Sec[e + f*x]^2])^m*(-(a*(a^2 + 3*b^2)*(2 + m)*AppellF1[(1 + m)/2, (-2 + m
)/2, 2, (3 + m)/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]) + b
*(4*a*b*(2 + m)*AppellF1[(1 + m)/2, (-2 + m)/2, 3, (3 + m)/2, -Tan[e + f*x]
^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2] + (1 + m)*((3*a^2 + b^2)*AppellF1[(2
 + m)/2, (-1 + m)/2, 2, (4 + m)/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f
*x]^2)/a^2] - 4*b^2*AppellF1[(2 + m)/2, (-1 + m)/2, 3, (4 + m)/2, -Tan[e +
f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2])*Tan[e + f*x])))/(a^4*(a^2 - b^2
)*f*(1 + m)*(2 + m)*(a + b*Sin[e + f*x])^3*(-(((Sec[e + f*x]^2)^(1 + m/2)*(
Tan[e + f*x]/Sqrt[Sec[e + f*x]^2])^m*(-(a*(a^2 + 3*b^2)*(2 + m)*AppellF1[(1
 + m)/2, (-2 + m)/2, 2, (3 + m)/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f
*x]^2)/a^2]) + b*(4*a*b*(2 + m)*AppellF1[(1 + m)/2, (-2 + m)/2, 3, (3 + m)/
2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2] + (1 + m)*((3*a^2 +
```

```
b^2)*AppellF1[(2 + m)/2, (-1 + m)/2, 2, (4 + m)/2, -Tan[e + f*x]^2, ((-a^2
+ b^2)*Tan[e + f*x]^2)/a^2] - 4*b^2*AppellF1[(2 + m)/2, (-1 + m)/2, 3, (4 +
 m)/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2])*Tan[e + f*x])))
/(a^4*(a^2 - b^2)*(1 + m)*(2 + m))) - (m*(Sec[e + f*x]^2)^(m/2)*Tan[e + f*x
]^2*(Tan[e + f*x]/Sqrt[Sec[e + f*x]^2])^m*(-(a*(a^2 + 3*b^2)*(2 + m)*Appell
F1[(1 + m)/2, (-2 + m)/2, 2, (3 + m)/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[
e + f*x]^2)/a^2]) + b*(4*a*b*(2 + m)*AppellF1[(1 + m)/2, (-2 + m)/2, 3, (3
+ m)/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2] + (1 + m)*((3*a
^2 + b^2)*AppellF1[(2 + m)/2, (-1 + m)/2, 2, (4 + m)/2, -Tan[e + f*x]^2, ((
-a^2 + b^2)*Tan[e + f*x]^2)/a^2] - 4*b^2*AppellF1[(2 + m)/2, (-1 + m)/2, 3,
 (4 + m)/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2])*Tan[e + f*
x])))/(a^4*(a^2 - b^2)*(1 + m)*(2 + m)) - (m*(Sec[e + f*x]^2)^(m/2)*Tan[e +
 f*x]*(Tan[e + f*x]/Sqrt[Sec[e + f*x]^2])^(-1 + m)*(Sqrt[Sec[e + f*x]^2] -
Tan[e + f*x]^2/Sqrt[Sec[e + f*x]^2])*(-(a*(a^2 + 3*b^2)*(2 + m)*AppellF1[(1
 + m)/2, (-2 + m)/2, 2, (3 + m)/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f
*x]^2)/a^2]) + b*(4*a*b*(2 + m)*AppellF1[(1 + m)/2, (-2 + m)/2, 3, (3 + m)/
2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2] + (1 + m)*((3*a^2 +
b^2)*AppellF1[(2 + m)/2, (-1 + m)/2, 2, (4 + m)/2, -Tan[e + f*x]^2, ((-a^2
+ b^2)*Tan[e + f*x]^2)/a^2] - 4*b^2*AppellF1[(2 + m)/2, (-1 + m)/2, 3, (4 +
 m)/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2])*Tan[e + f*x])))
/(a^4*(a^2 - b^2)*(1 + m)*(2 + m)) - ((Sec[e + f*x]^2)^(m/2)*Tan[e + f*x]*(
Tan[e + f*x]/Sqrt[Sec[e + f*x]^2])^m*(-(a*(a^2 + 3*b^2)*(2 + m)*(-(((-2 + m
)*(1 + m)*AppellF1[1 + (1 + m)/2, 1 + (-2 + m)/2, 2, 1 + (3 + m)/2, -Tan[e
+ f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]*Sec[e + f*x]^2*Tan[e + f*x])/(
3 + m)) + (4*(-a^2 + b^2)*(1 + m)*AppellF1[1 + (1 + m)/2, (-2 + m)/2, 3, 1
+ (3 + m)/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]*Sec[e + f*
x]^2*Tan[e + f*x])/(a^2*(3 + m)))) + b*((1 + m)*((3*a^2 + b^2)*AppellF1[(2
+ m)/2, (-1 + m)/2, 2, (4 + m)/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*
x]^2)/a^2] - 4*b^2*AppellF1[(2 + m)/2, (-1 + m)/2, 3, (4 + m)/2, -Tan[e + f
*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2])*Sec[e + f*x]^2 + 4*a*b*(2 + m)*(
-(((-2 + m)*(1 + m)*AppellF1[1 + (1 + m)/2, 1 + (-2 + m)/2, 3, 1 + (3 + m)/
2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]*Sec[e + f*x]^2*Tan[e
 + f*x])/(3 + m)) + (6*(-a^2 + b^2)*(1 + m)*AppellF1[1 + (1 + m)/2, (-2 + m
)/2, 4, 1 + (3 + m)/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]*
Sec[e + f*x]^2*Tan[e + f*x])/(a^2*(3 + m))) + (1 + m)*Tan[e + f*x]*((3*a^2
+ b^2)*(-(((-1 + m)*(2 + m)*AppellF1[1 + (2 + m)/2, 1 + (-1 + m)/2, 2, 1 +
(4 + m)/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]*Sec[e + f*x]
^2*Tan[e + f*x])/(4 + m)) + (4*(-a^2 + b^2)*(2 + m)*AppellF1[1 + (2 + m)/2,
 (-1 + m)/2, 3, 1 + (4 + m)/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^
2)/a^2]*Sec[e + f*x]^2*Tan[e + f*x])/(a^2*(4 + m))) - 4*b^2*(-(((-1 + m)*(2
 + m)*AppellF1[1 + (2 + m)/2, 1 + (-1 + m)/2, 3, 1 + (4 + m)/2, -Tan[e + f*
x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]*Sec[e + f*x]^2*Tan[e + f*x])/(4 +
m)) + (6*(-a^2 + b^2)*(2 + m)*AppellF1[1 + (2 + m)/2, (-1 + m)/2, 4, 1 + (4
 + m)/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]*Sec[e + f*x]^2
*Tan[e + f*x])/(a^2*(4 + m)))))))/(a^4*(a^2 - b^2)*(1 + m)*(2 + m)))))
```

Maple [F] time = 0.835, size = 0, normalized size = 0.

$$\int \frac{\left(d \sin \left(f x+e\right)\right)^{m}}{\left(a +b \sin \left(f x+e\right)\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^m/(a+b*sin(f*x+e))^3,x)
```

```
[Out] int((d*sin(f*x+e))^m/(a+b*sin(f*x+e))^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sin\left(fx+e\right)\right)^m}{\left(b\sin\left(fx+e\right)+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^m/(a+b*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] integrate((d*sin(f*x + e))^m/(b*sin(f*x + e) + a)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\left(d\sin\left(fx+e\right)\right)^m}{3\,ab^2\cos\left(fx+e\right)^2-a^3-3\,ab^2+\left(b^3\cos\left(fx+e\right)^2-3\,a^2b-b^3\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^m/(a+b*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] integral(-(d*sin(f*x + e))^m/(3*a*b^2*cos(f*x + e)^2 - a^3 - 3*a*b^2 + (b^3
*cos(f*x + e)^2 - 3*a^2*b - b^3)*sin(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**m/(a+b*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sin\left(fx+e\right)\right)^m}{\left(b\sin\left(fx+e\right)+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^m/(a+b*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate((d*sin(f*x + e))^m/(b*sin(f*x + e) + a)^3, x)
```

3.219 $\int \sin^{-1-\frac{a^2}{a^2+b^2}}(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=142

$$\frac{2a\left(a^2+b^2\right)\cos(c+dx)\sin^{\frac{b^2}{a^2+b^2}}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{b^2}{2\left(a^2+b^2\right)};\frac{1}{2}\left(3-\frac{a^2}{a^2+b^2}\right);\sin^2(c+dx)\right)}{bd\sqrt{\cos^2(c+dx)}}-\frac{\left(a^2+b^2\right)\cos(c+dx)\sin^{-\frac{a^2}{a^2+b^2}}}{d}$$

```
[Out] -(((a^2 + b^2)*Cos[c + d*x])/(d*Sin[c + d*x]^(a^2/(a^2 + b^2)))) + (2*a*(a^
2 + b^2)*Cos[c + d*x]*Hypergeometric2F1[1/2, b^2/(2*(a^2 + b^2)), (3 - a^2/
(a^2 + b^2))/2, Sin[c + d*x]^2]*Sin[c + d*x]^(b^2/(a^2 + b^2)))/(b*d*Sqrt[C
os[c + d*x]^2])
```

Rubi [A] time = 0.12323, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2789, 2643, 3011}

$$\frac{2a\left(a^2+b^2\right)\cos(c+dx)\sin^{\frac{b^2}{a^2+b^2}}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{b^2}{2\left(a^2+b^2\right)};\frac{1}{2}\left(3-\frac{a^2}{a^2+b^2}\right);\sin^2(c+dx)\right)}{bd\sqrt{\cos^2(c+dx)}}-\frac{\left(a^2+b^2\right)\cos(c+dx)\sin^{-\frac{a^2}{a^2+b^2}}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^(-1 - a^2/(a^2 + b^2))*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -(((a^2 + b^2)*Cos[c + d*x])/(d*Sin[c + d*x]^(a^2/(a^2 + b^2)))) + (2*a*(a^
2 + b^2)*Cos[c + d*x]*Hypergeometric2F1[1/2, b^2/(2*(a^2 + b^2)), (3 - a^2/
(a^2 + b^2))/2, Sin[c + d*x]^2]*Sin[c + d*x]^(b^2/(a^2 + b^2)))/(b*d*Sqrt[C
os[c + d*x]^2])
```

Rule 2789

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)])^2, x_Symbol] :> Dist[(2*c*d)/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] +
 Int[(b*Sin[e + f*x])^m*(c^2 + d^2*Sin[e + f*x]^2), x] /; FreeQ[{b, c, d, e
, f, m}, x]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3011

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(A*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(
m + 1)), x] /; FreeQ[{b, e, f, A, C, m}, x] && EqQ[A*(m + 2) + C*(m + 1), 0
]
```

Rubi steps

$$\int \sin^{-1-\frac{a^2}{a^2+b^2}}(c+dx)(a+b\sin(c+dx))^2\,dx = (2ab)\int \sin^{-\frac{a^2}{a^2+b^2}}(c+dx)\,dx + \int \sin^{-1-\frac{a^2}{a^2+b^2}}(c+dx)\left(a^2+b^2\sin^2(c+dx\right.$$

$$= -\frac{\left(a^2+b^2\right)\cos(c+dx)\sin^{-\frac{a^2}{a^2+b^2}}(c+dx)}{d} + \frac{2a\left(a^2+b^2\right)\cos(c+dx)\,{}_2F_1\left(\frac{1}{2}\right.}{}$$

Mathematica [A] time = 0.310594, size = 188, normalized size = 1.32

$$-\frac{\cos(c+dx)\sin^{-\frac{a^2}{a^2+b^2}}(c+dx)\sin^2(c+dx)^{-\frac{b^2}{2\left(a^2+b^2\right)}}\left(\sqrt{\sin^2(c+dx)}\left(a^2\,{}_2F_1\left(\frac{1}{2},\frac{a^2}{2\left(a^2+b^2\right)}+1;\frac{3}{2};\cos^2(c+dx)\right)+b^2\,{}_2F_1\left(\frac{1}{2},\right.\right.\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^(-1 - a^2/(a^2 + b^2))*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -((Cos[c + d*x]*(2*a*b*Hypergeometric2F1[1/2, (1 + a^2/(a^2 + b^2))/2, 3/2,
 Cos[c + d*x]^2]*Sin[c + d*x] + (b^2*Hypergeometric2F1[1/2, a^2/(2*(a^2 + b
^2)), 3/2, Cos[c + d*x]^2] + a^2*Hypergeometric2F1[1/2, 1 + a^2/(2*(a^2 + b
^2)), 3/2, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2]))/(d*Sin[c + d*x]^(a^2/(a^
2 + b^2))*(Sin[c + d*x]^2)^(b^2/(2*(a^2 + b^2)))))
```

Maple [F] time = 5.474, size = 0, normalized size = 0.

$$\int (\sin(dx+c))^{-1-\frac{a^2}{a^2+b^2}}(a+b\sin(dx+c))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^(-1-a^2/(a^2+b^2))*(a+b*sin(d*x+c))^2,x)
```

```
[Out] int(sin(d*x+c)^(-1-a^2/(a^2+b^2))*(a+b*sin(d*x+c))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sin(dx+c)+a)^2\sin(dx+c)^{-\frac{a^2}{a^2+b^2}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^(-1-a^2/(a^2+b^2))*(a+b*sin(d*x+c))^2,x, algorithm="ma
xima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^2*sin(d*x + c)^(-a^2/(a^2 + b^2) - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(b^2\cos(dx+c)^2-2\,ab\sin(dx+c)-a^2-b^2\right)\sin(dx+c)^{-\frac{2\,a^2+b^2}{a^2+b^2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^(-1-a^2/(a^2+b^2))*(a+b*sin(d*x+c))^2,x, algorithm="fr
icas")
```

```
[Out] integral(-(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)*sin(d*x + c
)^(-(2*a^2 + b^2)/(a^2 + b^2)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**(-1-a**2/(a**2+b**2))*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sin(dx+c)+a)^2 \sin(dx+c)^{-\frac{a^2}{a^2+b^2}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^(-1-a^2/(a^2+b^2))*(a+b*sin(d*x+c))^2,x, algorithm="gi
ac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^2*sin(d*x + c)^(-a^2/(a^2 + b^2) - 1), x)
```

3.220 $\int \frac{(1+2\sin(c+dx))^2}{\sin^{\frac{6}{5}}(c+dx)}\,dx$

Optimal. Leaf size=73

$$\frac{5\sin^{\frac{4}{5}}(c+dx)\cos(c+dx)\,{}_2F_1\left(\frac{2}{5},\frac{1}{2};\frac{7}{5};\sin^2(c+dx)\right)}{d\sqrt{\cos^2(c+dx)}}-\frac{5\cos(c+dx)}{d\sqrt[5]{\sin(c+dx)}}$$

```
[Out] (-5*Cos[c + d*x])/(d*Sin[c + d*x]^(1/5)) + (5*Cos[c + d*x]*Hypergeometric2F
1[2/5, 1/2, 7/5, Sin[c + d*x]^2]*Sin[c + d*x]^(4/5))/(d*Sqrt[Cos[c + d*x]^2
])
```

Rubi [A] time = 0.0738084, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {2789, 2643, 3011}

$$\frac{5\sin^{\frac{4}{5}}(c+dx)\cos(c+dx)\,{}_2F_1\left(\frac{2}{5},\frac{1}{2};\frac{7}{5};\sin^2(c+dx)\right)}{d\sqrt{\cos^2(c+dx)}}-\frac{5\cos(c+dx)}{d\sqrt[5]{\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*Sin[c + d*x])^2/Sin[c + d*x]^(6/5),x]
```

```
[Out] (-5*Cos[c + d*x])/(d*Sin[c + d*x]^(1/5)) + (5*Cos[c + d*x]*Hypergeometric2F
1[2/5, 1/2, 7/5, Sin[c + d*x]^2]*Sin[c + d*x]^(4/5))/(d*Sqrt[Cos[c + d*x]^2
])
```

Rule 2789

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)])^2, x_Symbol] :> Dist[(2*c*d)/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] +
 Int[(b*Sin[e + f*x])^m*(c^2 + d^2*Sin[e + f*x]^2), x] /; FreeQ[{b, c, d, e
, f, m}, x]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3011

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(A*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(
m + 1)), x] /; FreeQ[{b, e, f, A, C, m}, x] && EqQ[A*(m + 2) + C*(m + 1), 0
]
```

Rubi steps

$$\int \frac{(1+2\sin(c+dx))^2}{\sin^{\frac{6}{5}}(c+dx)}\,dx = 4\int \frac{1}{\sqrt[5]{\sin(c+dx)}}\,dx + \int \frac{1+4\sin^2(c+dx)}{\sin^{\frac{6}{5}}(c+dx)}\,dx$$
$$= -\frac{5\cos(c+dx)}{d\sqrt[5]{\sin(c+dx)}} + \frac{5\cos(c+dx)\,{}_2F_1\left(\frac{2}{5},\frac{1}{2};\frac{7}{5};\sin^2(c+dx)\right)\sin^{\frac{4}{5}}(c+dx)}{d\sqrt{\cos^2(c+dx)}}$$

Mathematica [A] time = 0.104947, size = 73, normalized size = 1.

$$-\frac{4\sin^{\frac{4}{5}}(c+dx)\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{3}{5};\frac{3}{2};\cos^2(c+dx)\right)}{d\sin^2(c+dx)^{2/5}} - \frac{5\cos(c+dx)}{d\sqrt[5]{\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*Sin[c + d*x])^2/Sin[c + d*x]^(6/5),x]
```

```
[Out] (-5*Cos[c + d*x])/(d*Sin[c + d*x]^(1/5)) - (4*Cos[c + d*x]*Hypergeometric2F
1[1/2, 3/5, 3/2, Cos[c + d*x]^2]*Sin[c + d*x]^(4/5))/(d*(Sin[c + d*x]^2)^(2
/5))
```

Maple [F] time = 0.183, size = 0, normalized size = 0.

$$\int (1+2\,\sin(dx+c))^2 (\sin(dx+c))^{-\frac{6}{5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*sin(d*x+c))^2/sin(d*x+c)^(6/5),x)
```

```
[Out] int((1+2*sin(d*x+c))^2/sin(d*x+c)^(6/5),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(2\,\sin(dx+c)+1)^2}{\sin(dx+c)^{\frac{6}{5}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*sin(d*x+c))^2/sin(d*x+c)^(6/5),x, algorithm="maxima")
```

```
[Out] integrate((2*sin(d*x + c) + 1)^2/sin(d*x + c)^(6/5), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(4\,\cos(dx+c)^2 - 4\,\sin(dx+c) - 5\right)\sin(dx+c)^{\frac{4}{5}}}{\cos(dx+c)^2 - 1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*sin(d*x+c))^2/sin(d*x+c)^(6/5),x, algorithm="fricas")
```

```
[Out] integral((4*cos(d*x + c)^2 - 4*sin(d*x + c) - 5)*sin(d*x + c)^(4/5)/(cos(d*
x + c)^2 - 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*sin(d*x+c))**2/sin(d*x+c)**(6/5),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(2 \sin(dx + c) + 1)^2}{\sin(dx + c)^{\frac{6}{5}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*sin(d*x+c))^2/sin(d*x+c)^(6/5),x, algorithm="giac")
```

```
[Out] integrate((2*sin(d*x + c) + 1)^2/sin(d*x + c)^(6/5), x)
```

3.221 $\int \sin^m(c+dx)(a+b\sin(c+dx))^n\,dx$

Optimal. Leaf size=23

$$\text{Unintegrable}\left(\sin^m(c+dx)(a+b\sin(c+dx))^n,x\right)$$

```
[Out] Unintegrable[Sin[c + d*x]^m*(a + b*Sin[c + d*x])^n, x]
```

Rubi [A] time = 0.0397752, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \sin^m(c+dx)(a+b\sin(c+dx))^n\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sin[c + d*x]^m*(a + b*Sin[c + d*x])^n,x]

[Out] Defer[Int][Sin[c + d*x]^m*(a + b*Sin[c + d*x])^n, x]
```

Rubi steps

$$\int \sin^m(c+dx)(a+b\sin(c+dx))^n\,dx = \int \sin^m(c+dx)(a+b\sin(c+dx))^n\,dx$$

Mathematica [A] time = 2.27161, size = 0, normalized size = 0.

$$\int \sin^m(c+dx)(a+b\sin(c+dx))^n\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sin[c + d*x]^m*(a + b*Sin[c + d*x])^n,x]

[Out] Integrate[Sin[c + d*x]^m*(a + b*Sin[c + d*x])^n, x]
```

Maple [A] time = 0.602, size = 0, normalized size = 0.

$$\int (\sin(dx+c))^m (a+b\sin(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^m*(a+b*sin(d*x+c))^n,x)

[Out] int(sin(d*x+c)^m*(a+b*sin(d*x+c))^n,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (b\sin(dx+c)+a)^n \sin(dx+c)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^m*(a+b*sin(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^n*sin(d*x + c)^m, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b\sin(dx+c)+a)^n\sin(dx+c)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^m*(a+b*sin(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((b*sin(d*x + c) + a)^n*sin(d*x + c)^m, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int (a+b\sin(c+dx))^n\sin^m(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**m*(a+b*sin(d*x+c))**n,x)
```

```
[Out] Integral((a + b*sin(c + d*x))**n*sin(c + d*x)**m, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (b\sin(dx+c)+a)^n\sin(dx+c)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^m*(a+b*sin(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^n*sin(d*x + c)^m, x)
```

3.222 $\int \sin^3(c+dx)(a+b\sin(c+dx))^n\,dx$

Optimal. Leaf size=351

$$\frac{\sqrt{2}a\left(2a^2+b^2\left(n^2+5n+4\right)\right)\cos(c+dx)(a+b\sin(c+dx))^n\left(\frac{a+b\sin(c+dx)}{a+b}\right)^{-n}F_1\left(\frac{1}{2};\frac{1}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(c+dx)),\frac{b(1-\sin\cdots}{a\cdots}\right.}{b^3d(n+2)(n+3)\sqrt{\sin(c+dx)+1}}$$

```
[Out] (2*a*Cos[c + d*x]*(a + b*Sin[c + d*x])^(1 + n))/(b^2*d*(2 + n)*(3 + n)) - (
Cos[c + d*x]*Sin[c + d*x]*(a + b*Sin[c + d*x])^(1 + n))/(b*d*(3 + n)) - (Sq
rt[2]*(a + b)*(2*a^2 + b^2*(2 + n)^2)*AppellF1[1/2, 1/2, -1 - n, 3/2, (1 -
Sin[c + d*x])/2, (b*(1 - Sin[c + d*x]))/(a + b)]*Cos[c + d*x]*(a + b*Sin[c
+ d*x])^n)/(b^3*d*(2 + n)*(3 + n)*Sqrt[1 + Sin[c + d*x]]*((a + b*Sin[c + d*
x])/(a + b))^n) + (Sqrt[2]*a*(2*a^2 + b^2*(4 + 5*n + n^2))*AppellF1[1/2, 1/
2, -n, 3/2, (1 - Sin[c + d*x])/2, (b*(1 - Sin[c + d*x]))/(a + b)]*Cos[c + d
*x]*(a + b*Sin[c + d*x])^n)/(b^3*d*(2 + n)*(3 + n)*Sqrt[1 + Sin[c + d*x]]*(
(a + b*Sin[c + d*x])/(a + b))^n)
```

Rubi [A] time = 0.485529, antiderivative size = 351, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2793, 3023, 2756, 2665, 139, 138}

$$\frac{\sqrt{2}a\left(2a^2+b^2\left(n^2+5n+4\right)\right)\cos(c+dx)(a+b\sin(c+dx))^n\left(\frac{a+b\sin(c+dx)}{a+b}\right)^{-n}F_1\left(\frac{1}{2};\frac{1}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(c+dx)),\frac{b(1-\sin\cdots}{a\cdots}\right.}{b^3d(n+2)(n+3)\sqrt{\sin(c+dx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^3*(a + b*Sin[c + d*x])^n,x]
```

```
[Out] (2*a*Cos[c + d*x]*(a + b*Sin[c + d*x])^(1 + n))/(b^2*d*(2 + n)*(3 + n)) - (
Cos[c + d*x]*Sin[c + d*x]*(a + b*Sin[c + d*x])^(1 + n))/(b*d*(3 + n)) - (Sq
rt[2]*(a + b)*(2*a^2 + b^2*(2 + n)^2)*AppellF1[1/2, 1/2, -1 - n, 3/2, (1 -
Sin[c + d*x])/2, (b*(1 - Sin[c + d*x]))/(a + b)]*Cos[c + d*x]*(a + b*Sin[c
+ d*x])^n)/(b^3*d*(2 + n)*(3 + n)*Sqrt[1 + Sin[c + d*x]]*((a + b*Sin[c + d*
x])/(a + b))^n) + (Sqrt[2]*a*(2*a^2 + b^2*(4 + 5*n + n^2))*AppellF1[1/2, 1/
2, -n, 3/2, (1 - Sin[c + d*x])/2, (b*(1 - Sin[c + d*x]))/(a + b)]*Cos[c + d
*x]*(a + b*Sin[c + d*x])^n)/(b^3*d*(2 + n)*(3 + n)*Sqrt[1 + Sin[c + d*x]]*(
(a + b*Sin[c + d*x])/(a + b))^n)
```

Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
```

```
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2756

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[(b*c - a*d)/b, Int[(a + b*Sin[e + f*x])^m,
x], x] + Dist[d/b, Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2665

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)
^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*n]
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int \sin^3(c+dx)(a+b\sin(c+dx))^n\,dx = -\frac{\cos(c+dx)\sin(c+dx)(a+b\sin(c+dx))^{1+n}}{bd(3+n)} + \frac{\int (a+b\sin(c+dx))^n \left(a+b\right.}{}$$

$$= \frac{2a\cos(c+dx)(a+b\sin(c+dx))^{1+n}}{b^2d(2+n)(3+n)} - \frac{\cos(c+dx)\sin(c+dx)(a+b\sin(c+}{bd(3+n)}$$

$$= \frac{2a\cos(c+dx)(a+b\sin(c+dx))^{1+n}}{b^2d(2+n)(3+n)} - \frac{\cos(c+dx)\sin(c+dx)(a+b\sin(c+}{bd(3+n)}$$

$$= \frac{2a\cos(c+dx)(a+b\sin(c+dx))^{1+n}}{b^2d(2+n)(3+n)} - \frac{\cos(c+dx)\sin(c+dx)(a+b\sin(c+}{bd(3+n)}$$

$$= \frac{2a\cos(c+dx)(a+b\sin(c+dx))^{1+n}}{b^2d(2+n)(3+n)} - \frac{\cos(c+dx)\sin(c+dx)(a+b\sin(c+}{bd(3+n)}$$

$$= \frac{2a\cos(c+dx)(a+b\sin(c+dx))^{1+n}}{b^2d(2+n)(3+n)} - \frac{\cos(c+dx)\sin(c+dx)(a+b\sin(c+}{bd(3+n)}$$

Mathematica [F] time = 4.27136, size = 0, normalized size = 0.

$$\int \sin^3(c+dx)(a+b\sin(c+dx))^n\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sin[c + d*x]^3*(a + b*Sin[c + d*x])^n,x]
```

```
[Out] Integrate[Sin[c + d*x]^3*(a + b*Sin[c + d*x])^n, x]
```

Maple [F] time = 0.282, size = 0, normalized size = 0.

$$\int (\sin(dx+c))^3 (a+b\sin(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3*(a+b*sin(d*x+c))^n,x)
```

```
[Out] int(sin(d*x+c)^3*(a+b*sin(d*x+c))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sin(dx+c)+a)^n \sin(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+b*sin(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^n*sin(d*x + c)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(\cos\left(dx+c\right)^2-1\right)\left(b\sin\left(dx+c\right)+a\right)^n\sin\left(dx+c\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+b*sin(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral(-(cos(d*x + c)^2 - 1)*(b*sin(d*x + c) + a)^n*sin(d*x + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3*(a+b*sin(d*x+c))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(dx+c\right)+a\right)^n\sin\left(dx+c\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+b*sin(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^n*sin(d*x + c)^3, x)
```

3.223 $\int \sin^2(c+dx)(a+b\sin(c+dx))^n\,dx$

Optimal. Leaf size=274

$$-\frac{\sqrt{2}\left(a^2+b^2(n+1)\right)\cos(c+dx)(a+b\sin(c+dx))^n\left(\frac{a+b\sin(c+dx)}{a+b}\right)^{-n}F_1\left(\frac{1}{2};\frac{1}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(c+dx)),\frac{b(1-\sin(c+dx))}{a+b}\right)}{b^2d(n+2)\sqrt{\sin(c+dx)+1}}$$

```
[Out] -((Cos[c + d*x]*(a + b*Sin[c + d*x])^(1 + n))/(b*d*(2 + n))) + (Sqrt[2]*a*(
a + b)*AppellF1[1/2, 1/2, -1 - n, 3/2, (1 - Sin[c + d*x])/2, (b*(1 - Sin[c
+ d*x]))/(a + b)]*Cos[c + d*x]*(a + b*Sin[c + d*x])^n)/(b^2*d*(2 + n)*Sqrt[
1 + Sin[c + d*x]]*((a + b*Sin[c + d*x])/(a + b))^n) - (Sqrt[2]*(a^2 + b^2*(
1 + n))*AppellF1[1/2, 1/2, -n, 3/2, (1 - Sin[c + d*x])/2, (b*(1 - Sin[c + d
*x]))/(a + b)]*Cos[c + d*x]*(a + b*Sin[c + d*x])^n)/(b^2*d*(2 + n)*Sqrt[1 +
 Sin[c + d*x]]*((a + b*Sin[c + d*x])/(a + b))^n)
```

Rubi [A] time = 0.298298, antiderivative size = 274, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2791, 2756, 2665, 139, 138}

$$-\frac{\sqrt{2}\left(a^2+b^2(n+1)\right)\cos(c+dx)(a+b\sin(c+dx))^n\left(\frac{a+b\sin(c+dx)}{a+b}\right)^{-n}F_1\left(\frac{1}{2};\frac{1}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(c+dx)),\frac{b(1-\sin(c+dx))}{a+b}\right)}{b^2d(n+2)\sqrt{\sin(c+dx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2*(a + b*Sin[c + d*x])^n,x]
```


```
[Out] -((Cos[c + d*x]*(a + b*Sin[c + d*x])^(1 + n))/(b*d*(2 + n))) + (Sqrt[2]*a*(
a + b)*AppellF1[1/2, 1/2, -1 - n, 3/2, (1 - Sin[c + d*x])/2, (b*(1 - Sin[c
+ d*x]))/(a + b)]*Cos[c + d*x]*(a + b*Sin[c + d*x])^n)/(b^2*d*(2 + n)*Sqrt[
1 + Sin[c + d*x]]*((a + b*Sin[c + d*x])/(a + b))^n) - (Sqrt[2]*(a^2 + b^2*(
1 + n))*AppellF1[1/2, 1/2, -n, 3/2, (1 - Sin[c + d*x])/2, (b*(1 - Sin[c + d
*x]))/(a + b)]*Cos[c + d*x]*(a + b*Sin[c + d*x])^n)/(b^2*d*(2 + n)*Sqrt[1 +
 Sin[c + d*x]]*((a + b*Sin[c + d*x])/(a + b))^n)
```


Rule 2791

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> -Simp[(d^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*
Simp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x],
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a
^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 2756

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[(b*c - a*d)/b, Int[(a + b*Sin[e + f*x])^m,
x], x] + Dist[d/b, Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2665

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)
^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d
```

```
, n}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*n]
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int \sin^2(c+dx)(a+b\sin(c+dx))^n\,dx = -\frac{\cos(c+dx)(a+b\sin(c+dx))^{1+n}}{bd(2+n)} + \frac{\int (b(1+n)-a\sin(c+dx))(a+b\sin(c+d}{b(2+n)}$$

$$= -\frac{\cos(c+dx)(a+b\sin(c+dx))^{1+n}}{bd(2+n)} - \frac{a\int (a+b\sin(c+dx))^{1+n}\,dx}{b^2(2+n)} + \frac{(a^2+b^2(1}{}$$

$$= -\frac{\cos(c+dx)(a+b\sin(c+dx))^{1+n}}{bd(2+n)} - \frac{(a\cos(c+dx))\,\mathrm{Subst}\left(\int \frac{(a+bx)^{1+n}}{\sqrt{1-x}\sqrt{1+x}}\,dx, x, \mathrm{si}\right.}{b^2d(2+n)\sqrt{1-\sin(c+dx)}\sqrt{1+\sin(c}$$

$$= -\frac{\cos(c+dx)(a+b\sin(c+dx))^{1+n}}{bd(2+n)} + \frac{\left(a(-a-b)\cos(c+dx)(a+b\sin(c+dx))^n\right.}{b^2d(2+n)\sqrt{}}$$

$$= -\frac{\cos(c+dx)(a+b\sin(c+dx))^{1+n}}{bd(2+n)} + \frac{\sqrt{2}a(a+b)F_1\left(\frac{1}{2};\frac{1}{2},-1-n;\frac{3}{2};\frac{1}{2}(1-\sin(c\right.}{b}$$

Mathematica [F] time = 6.78741, size = 0, normalized size = 0.

$$\int \sin^2(c+dx)(a+b\sin(c+dx))^n\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sin[c + d*x]^2*(a + b*Sin[c + d*x])^n,x]
```

```
[Out] Integrate[Sin[c + d*x]^2*(a + b*Sin[c + d*x])^n, x]
```

Maple [F] time = 0.222, size = 0, normalized size = 0.

$$\int (\sin(dx+c))^2 (a+b\sin(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2*(a+b*sin(d*x+c))^n,x)
```

```
[Out] int(sin(d*x+c)^2*(a+b*sin(d*x+c))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sin(dx+c)+a)^n \sin(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+b*sin(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^n*sin(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\left(\cos(dx+c)^2-1\right)(b\sin(dx+c)+a)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+b*sin(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral(-(cos(d*x + c)^2 - 1)*(b*sin(d*x + c) + a)^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2*(a+b*sin(d*x+c))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sin(dx+c)+a)^n \sin(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+b*sin(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^n*sin(d*x + c)^2, x)
```

3.224 $\int \sin(c+dx)(a+b\sin(c+dx))^n\,dx$

Optimal. Leaf size=220

$$\frac{\sqrt{2}a\cos(c+dx)(a+b\sin(c+dx))^n\left(\frac{a+b\sin(c+dx)}{a+b}\right)^{-n}F_1\left(\frac{1}{2};\frac{1}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(c+dx)),\frac{b(1-\sin(c+dx))}{a+b}\right)}{bd\sqrt{\sin(c+dx)+1}}-\frac{\sqrt{2}(a+b)\cos(c+$$

```
[Out] -((Sqrt[2]*(a + b)*AppellF1[1/2, 1/2, -1 - n, 3/2, (1 - Sin[c + d*x])/2, (b
*(1 - Sin[c + d*x]))/(a + b)]*Cos[c + d*x]*(a + b*Sin[c + d*x])^n)/(b*d*Sqr
t[1 + Sin[c + d*x]]*((a + b*Sin[c + d*x])/(a + b))^n)) + (Sqrt[2]*a*AppellF
1[1/2, 1/2, -n, 3/2, (1 - Sin[c + d*x])/2, (b*(1 - Sin[c + d*x]))/(a + b)]*
Cos[c + d*x]*(a + b*Sin[c + d*x])^n)/(b*d*Sqrt[1 + Sin[c + d*x]]*((a + b*Si
n[c + d*x])/(a + b))^n)
```

Rubi [A] time = 0.188408, antiderivative size = 220, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2756, 2665, 139, 138}

$$\frac{\sqrt{2}a\cos(c+dx)(a+b\sin(c+dx))^n\left(\frac{a+b\sin(c+dx)}{a+b}\right)^{-n}F_1\left(\frac{1}{2};\frac{1}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(c+dx)),\frac{b(1-\sin(c+dx))}{a+b}\right)}{bd\sqrt{\sin(c+dx)+1}}-\frac{\sqrt{2}(a+b)\cos(c+$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]*(a + b*Sin[c + d*x])^n,x]
```

```
[Out] -((Sqrt[2]*(a + b)*AppellF1[1/2, 1/2, -1 - n, 3/2, (1 - Sin[c + d*x])/2, (b
*(1 - Sin[c + d*x]))/(a + b)]*Cos[c + d*x]*(a + b*Sin[c + d*x])^n)/(b*d*Sqr
t[1 + Sin[c + d*x]]*((a + b*Sin[c + d*x])/(a + b))^n)) + (Sqrt[2]*a*AppellF
1[1/2, 1/2, -n, 3/2, (1 - Sin[c + d*x])/2, (b*(1 - Sin[c + d*x]))/(a + b)]*
Cos[c + d*x]*(a + b*Sin[c + d*x])^n)/(b*d*Sqrt[1 + Sin[c + d*x]]*((a + b*Si
n[c + d*x])/(a + b))^n)
```

Rule 2756

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[(b*c - a*d)/b, Int[(a + b*Sin[e + f*x])^m,
x], x] + Dist[d/b, Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2665

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)
^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*n]
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int \sin(c+dx)(a+b\sin(c+dx))^n\,dx = \frac{\int (a+b\sin(c+dx))^{1+n}\,dx}{b} - \frac{a\int (a+b\sin(c+dx))^n\,dx}{b}$$

$$= \frac{\cos(c+dx)\,\mathrm{Subst}\left(\int \frac{(a+bx)^{1+n}}{\sqrt{1-x}\sqrt{1+x}}\,dx, x, \sin(c+dx)\right)}{bd\sqrt{1-\sin(c+dx)}\sqrt{1+\sin(c+dx)}} - \frac{(a\cos(c+dx))\,\mathrm{Subst}\left(\int \cdots\right.}{bd\sqrt{1-\sin(c+\cdots}}$$

$$= -\frac{\left(a\cos(c+dx)(a+b\sin(c+dx))^n\left(-\frac{a+b\sin(c+dx)}{-a-b}\right)^{-n}\right)\mathrm{Subst}\left(\int \frac{\left(-\frac{a}{-a-b}-\frac{bx}{-a-b}\right)^n}{\sqrt{1-x}\sqrt{1+x}} \cdots\right.}{bd\sqrt{1-\sin(c+dx)}\sqrt{1+\sin(c+dx)}}$$

$$= -\frac{\sqrt{2}(a+b)F_1\left(\frac{1}{2};\frac{1}{2},-1-n;\frac{3}{2};\frac{1}{2}(1-\sin(c+dx)),\frac{b(1-\sin(c+dx))}{a+b}\right)\cos(c+dx)(a+\cdots}{bd\sqrt{1+\sin(c+dx)}}$$

Mathematica [A] time = 0.470251, size = 193, normalized size = 0.88

$$\frac{\sec(c+dx)\sqrt{-\frac{b(\sin(c+dx)-1)}{a+b}}\sqrt{\frac{b(\sin(c+dx)+1)}{b-a}}(a+b\sin(c+dx))^{n+1}\left((n+1)(a+b\sin(c+dx))F_1\left(n+2;\frac{1}{2},\frac{1}{2};n+3;\frac{a+b\sin\cdots}{a-\cdots}\right.\right.}{b^2d(n+1)(n+2)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[c + d*x]*(a + b*Sin[c + d*x])^n,x]
```

```
[Out] (Sec[c + d*x]*Sqrt[-((b*(-1 + Sin[c + d*x]))/(a + b))]*Sqrt[(b*(1 + Sin[c +
 d*x]))/(-a + b)]*(a + b*Sin[c + d*x])^(1 + n)*(-(a*(2 + n)*AppellF1[1 + n,
 1/2, 1/2, 2 + n, (a + b*Sin[c + d*x])/(a - b), (a + b*Sin[c + d*x])/(a + b
)]) + (1 + n)*AppellF1[2 + n, 1/2, 1/2, 3 + n, (a + b*Sin[c + d*x])/(a - b)
, (a + b*Sin[c + d*x])/(a + b)]*(a + b*Sin[c + d*x])))/(b^2*d*(1 + n)*(2 +
n))
```

Maple [F] time = 0.145, size = 0, normalized size = 0.

$$\int \sin(dx+c)\,(a+b\sin(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)*(a+b*sin(d*x+c))^n,x)
```

```
[Out] int(sin(d*x+c)*(a+b*sin(d*x+c))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sin (dx+c)+a)^n \sin (dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+b*sin(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^n*sin(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b\sin (dx+c)+a)^n \sin (dx+c)\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+b*sin(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((b*sin(d*x + c) + a)^n*sin(d*x + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+b*sin(d*x+c))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sin (dx+c)+a)^n \sin (dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+b*sin(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^n*sin(d*x + c), x)
```

3.225 $\int (a+b\sin(c+dx))^n\,dx$

Optimal. Leaf size=104

$$-\frac{\sqrt{2}\cos(c+dx)(a+b\sin(c+dx))^n\left(\frac{a+b\sin(c+dx)}{a+b}\right)^{-n}F_1\left(\frac{1}{2};\frac{1}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(c+dx)),\frac{b(1-\sin(c+dx))}{a+b}\right)}{d\sqrt{\sin(c+dx)+1}}$$

```
[Out] -((Sqrt[2]*AppellF1[1/2, 1/2, -n, 3/2, (1 - Sin[c + d*x])/2, (b*(1 - Sin[c
+ d*x]))/(a + b)]*Cos[c + d*x]*(a + b*Sin[c + d*x])^n)/(d*Sqrt[1 + Sin[c +
d*x]]*((a + b*Sin[c + d*x])/(a + b))^n))
```

Rubi [A] time = 0.0647978, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2665, 139, 138}

$$-\frac{\sqrt{2}\cos(c+dx)(a+b\sin(c+dx))^n\left(\frac{a+b\sin(c+dx)}{a+b}\right)^{-n}F_1\left(\frac{1}{2};\frac{1}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(c+dx)),\frac{b(1-\sin(c+dx))}{a+b}\right)}{d\sqrt{\sin(c+dx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])^n,x]
```

```
[Out] -((Sqrt[2]*AppellF1[1/2, 1/2, -n, 3/2, (1 - Sin[c + d*x])/2, (b*(1 - Sin[c
+ d*x]))/(a + b)]*Cos[c + d*x]*(a + b*Sin[c + d*x])^n)/(d*Sqrt[1 + Sin[c +
d*x]]*((a + b*Sin[c + d*x])/(a + b))^n))
```

Rule 2665

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)
^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*n]
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int (a+b\sin(c+dx))^n\,dx = \frac{\cos(c+dx)\operatorname{Subst}\left(\int \frac{(a+bx)^n}{\sqrt{1-x}\sqrt{1+x}}\,dx, x, \sin(c+dx)\right)}{d\sqrt{1-\sin(c+dx)}\sqrt{1+\sin(c+dx)}}$$
$$= \frac{\left(\cos(c+dx)(a+b\sin(c+dx))^n\left(-\frac{a+b\sin(c+dx)}{-a-b}\right)^{-n}\right)\operatorname{Subst}\left(\int \frac{\left(-\frac{a}{-a-b}-\frac{bx}{-a-b}\right)^n}{\sqrt{1-x}\sqrt{1+x}}\,dx, x, \sin(c+dx)\right)}{d\sqrt{1-\sin(c+dx)}\sqrt{1+\sin(c+dx)}}$$
$$= -\frac{\sqrt{2}F_1\left(\frac{1}{2};\frac{1}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(c+dx)),\frac{b(1-\sin(c+dx))}{a+b}\right)\cos(c+dx)(a+b\sin(c+dx))^n\left(\frac{a+b\sin(c-}{a+b}\right.}{d\sqrt{1+\sin(c+dx)}}$$

Mathematica [A] time = 0.240716, size = 120, normalized size = 1.15

$$\frac{\sec(c+dx)\sqrt{-\frac{b(\sin(c+dx)-1)}{a+b}}\sqrt{\frac{b(\sin(c+dx)+1)}{b-a}}(a+b\sin(c+dx))^{n+1}F_1\left(n+1;\frac{1}{2},\frac{1}{2};n+2;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)}{bd(n+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sin[c + d*x])^n,x]
```

```
[Out] (AppellF1[1 + n, 1/2, 1/2, 2 + n, (a + b*Sin[c + d*x])/(a - b), (a + b*Sin[
c + d*x])/(a + b)]*Sec[c + d*x]*Sqrt[-((b*(-1 + Sin[c + d*x]))/(a + b))]*Sq
rt[(b*(1 + Sin[c + d*x]))/(-a + b)]*(a + b*Sin[c + d*x])^(1 + n))/(b*d*(1 +
 n))
```

Maple [F] time = 0.006, size = 0, normalized size = 0.

$$\int (a+b\sin(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))^n,x)
```

```
[Out] int((a+b*sin(d*x+c))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sin(dx+c)+a)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left((b\sin(dx+c)+a)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((b*sin(d*x + c) + a)^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b\sin(c + dx))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))**n,x)
```

```
[Out] Integral((a + b*sin(c + d*x))**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sin(dx + c) + a)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^n, x)
```

3.226 $\int \csc(c+dx)(a+b\sin(c+dx))^n\,dx$

Optimal. Leaf size=21

$$\text{Unintegrable}\left(\csc(c+dx)(a+b\sin(c+dx))^n,x\right)$$

```
[Out] Unintegrable[Csc[c + d*x]*(a + b*Sin[c + d*x])^n, x]
```

Rubi [A] time = 0.0314893, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \csc(c+dx)(a+b\sin(c+dx))^n\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Csc[c + d*x]*(a + b*Sin[c + d*x])^n,x]
```

```
[Out] Defer[Int][Csc[c + d*x]*(a + b*Sin[c + d*x])^n, x]
```

Rubi steps

$$\int \csc(c+dx)(a+b\sin(c+dx))^n\,dx=\int \csc(c+dx)(a+b\sin(c+dx))^n\,dx$$

Mathematica [A] time = 2.40316, size = 0, normalized size = 0.

$$\int \csc(c+dx)(a+b\sin(c+dx))^n\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Csc[c + d*x]*(a + b*Sin[c + d*x])^n,x]
```

```
[Out] Integrate[Csc[c + d*x]*(a + b*Sin[c + d*x])^n, x]
```

Maple [A] time = 0.824, size = 0, normalized size = 0.

$$\int \csc\left(dx+c\right)\left(a+b\sin\left(dx+c\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*(a+b*sin(d*x+c))^n,x)
```

```
[Out] int(csc(d*x+c)*(a+b*sin(d*x+c))^n,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(dx+c\right)+a\right)^{n}\csc\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*sin(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^n*csc(d*x + c), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b\sin(dx+c)+a)^n\csc(dx+c),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*sin(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((b*sin(d*x + c) + a)^n*csc(d*x + c), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int(a+b\sin(c+dx))^n\csc(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*sin(d*x+c))**n,x)
```

```
[Out] Integral((a + b*sin(c + d*x))**n*csc(c + d*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int(b\sin(dx+c)+a)^n\csc(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*sin(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^n*csc(d*x + c), x)
```

3.227 $\int (a + a\sin(e + fx))(c - c\sin(e + fx))^4\,dx$

Optimal. Leaf size=116

$$\frac{7ac^4\cos^3(e+fx)}{12f} + \frac{a\cos^3(e+fx)\left(c^2-c^2\sin(e+fx)\right)^2}{5f} + \frac{7a\cos^3(e+fx)\left(c^4-c^4\sin(e+fx)\right)}{20f} + \frac{7ac^4\sin(e+fx)\cos(}{8f}$$

```
[Out] (7*a*c^4*x)/8 + (7*a*c^4*Cos[e + f*x]^3)/(12*f) + (7*a*c^4*Cos[e + f*x]*Sin
[e + f*x])/(8*f) + (a*Cos[e + f*x]^3*(c^2 - c^2*Sin[e + f*x])^2)/(5*f) + (7
*a*Cos[e + f*x]^3*(c^4 - c^4*Sin[e + f*x]))/(20*f)
```

Rubi [A] time = 0.162034, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {2736, 2678, 2669, 2635, 8}

$$\frac{7ac^4\cos^3(e+fx)}{12f} + \frac{a\cos^3(e+fx)\left(c^2-c^2\sin(e+fx)\right)^2}{5f} + \frac{7a\cos^3(e+fx)\left(c^4-c^4\sin(e+fx)\right)}{20f} + \frac{7ac^4\sin(e+fx)\cos(}{8f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^4,x]
```

```
[Out] (7*a*c^4*x)/8 + (7*a*c^4*Cos[e + f*x]^3)/(12*f) + (7*a*c^4*Cos[e + f*x]*Sin
[e + f*x])/(8*f) + (a*Cos[e + f*x]^3*(c^2 - c^2*Sin[e + f*x])^2)/(5*f) + (7
*a*Cos[e + f*x]^3*(c^4 - c^4*Sin[e + f*x]))/(20*f)
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
```

```
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (a+a\sin(e+fx))(c-c\sin(e+fx))^4\,dx = (ac)\int \cos^2(e+fx)(c-c\sin(e+fx))^3\,dx$$
$$= \frac{a\cos^3(e+fx)\left(c^2-c^2\sin(e+fx)\right)^2}{5f} + \frac{1}{5}\left(7ac^2\right)\int \cos^2(e+fx)(c-c\sin$$
$$= \frac{a\cos^3(e+fx)\left(c^2-c^2\sin(e+fx)\right)^2}{5f} + \frac{7a\cos^3(e+fx)\left(c^4-c^4\sin(e+\right.}{20f}$$
$$= \frac{7ac^4\cos^3(e+fx)}{12f} + \frac{a\cos^3(e+fx)\left(c^2-c^2\sin(e+fx)\right)^2}{5f} + \frac{7a\cos^3(e+}{}$$
$$= \frac{7ac^4\cos^3(e+fx)}{12f} + \frac{7ac^4\cos(e+fx)\sin(e+fx)}{8f} + \frac{a\cos^3(e+fx)\left(c^2-\right.}{5f}$$
$$= \frac{7}{8}ac^4x + \frac{7ac^4\cos^3(e+fx)}{12f} + \frac{7ac^4\cos(e+fx)\sin(e+fx)}{8f} + \frac{a\cos^3(e+}{}$$

Mathematica [A] time = 0.548112, size = 64, normalized size = 0.55

$$\frac{ac^4(120\sin(2(e+fx)) - 45\sin(4(e+fx)) + 420\cos(e+fx) + 130\cos(3(e+fx)) - 6\cos(5(e+fx)) + 420fx)}{480f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^4,x]
```

```
[Out] (a*c^4*(420*f*x + 420*Cos[e + f*x] + 130*Cos[3*(e + f*x)] - 6*Cos[5*(e + f*
x)] + 120*Sin[2*(e + f*x)] - 45*Sin[4*(e + f*x)]))/(480*f)
```

Maple [A] time = 0.021, size = 149, normalized size = 1.3

$$\frac{1}{f}\left(-\frac{ac^4\cos\left(fx+e\right)}{5}\left(\frac{8}{3}+\left(\sin\left(fx+e\right)\right)^4+\frac{4\left(\sin\left(fx+e\right)\right)^2}{3}\right)-3\,ac^4\left(-1/4\,\left(\left(\sin\left(fx+e\right)\right)^3+3/2\,\sin\left(fx+e\right)\right)\cos\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(c-c*sin(f*x+e))^4,x)
```

```
[Out] 1/f*(-1/5*a*c^4*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)-3*a*c^4*(-1/
4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-2/3*a*c^4*(2+sin(
f*x+e)^2)*cos(f*x+e)+2*a*c^4*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)+3*a
*c^4*cos(f*x+e)+a*c^4*(f*x+e))
```

Maxima [A] time = 1.20619, size = 197, normalized size = 1.7

$$-\frac{32\left(3\cos\left(fx+e\right)^{5}-10\cos\left(fx+e\right)^{3}+15\cos\left(fx+e\right)\right)ac^{4}-320\left(\cos\left(fx+e\right)^{3}-3\cos\left(fx+e\right)\right)ac^{4}+45\left(12fx+1\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c-c*sin(f*x+e))^4,x, algorithm="maxima")
```

```
[Out] -1/480*(32*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*a*c^4 -
 320*(cos(f*x + e)^3 - 3*cos(f*x + e))*a*c^4 + 45*(12*f*x + 12*e + sin(4*f*
x + 4*e) - 8*sin(2*f*x + 2*e))*a*c^4 - 240*(2*f*x + 2*e - sin(2*f*x + 2*e))
*a*c^4 - 480*(f*x + e)*a*c^4 - 1440*a*c^4*cos(f*x + e))/f
```

Fricas [A] time = 1.64101, size = 196, normalized size = 1.69

$$-\frac{24\,ac^{4}\cos\left(fx+e\right)^{5}-160\,ac^{4}\cos\left(fx+e\right)^{3}-105\,ac^{4}fx+15\left(6\,ac^{4}\cos\left(fx+e\right)^{3}-7\,ac^{4}\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{120\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c-c*sin(f*x+e))^4,x, algorithm="fricas")
```

```
[Out] -1/120*(24*a*c^4*cos(f*x + e)^5 - 160*a*c^4*cos(f*x + e)^3 - 105*a*c^4*f*x
+ 15*(6*a*c^4*cos(f*x + e)^3 - 7*a*c^4*cos(f*x + e))*sin(f*x + e))/f
```

Sympy [A] time = 4.43833, size = 314, normalized size = 2.71

$$\begin{cases}-\frac{9ac^{4}x\sin^{4}\left(e+fx\right)}{8}-\frac{9ac^{4}x\sin^{2}\left(e+fx\right)\cos^{2}\left(e+fx\right)}{4}+ac^{4}x\sin^{2}\left(e+fx\right)-\frac{9ac^{4}x\cos^{4}\left(e+fx\right)}{8}+ac^{4}x\cos^{2}\left(e+fx\right)+ac^{4}x-\frac{ac^{4}\sin^{4}\left(e+\right.}{} \\ x\left(a\sin\left(e\right)+a\right)\left(-c\sin\left(e\right)+c\right)^{4}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c-c*sin(f*x+e))**4,x)
```

```
[Out] Piecewise((-9*a*c**4*x*sin(e + f*x)**4/8 - 9*a*c**4*x*sin(e + f*x)**2*cos(e
 + f*x)**2/4 + a*c**4*x*sin(e + f*x)**2 - 9*a*c**4*x*cos(e + f*x)**4/8 + a*
c**4*x*cos(e + f*x)**2 + a*c**4*x - a*c**4*sin(e + f*x)**4*cos(e + f*x)/f +
 15*a*c**4*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 4*a*c**4*sin(e + f*x)**2*co
s(e + f*x)**3/(3*f) - 2*a*c**4*sin(e + f*x)**2*cos(e + f*x)/f + 9*a*c**4*si
n(e + f*x)*cos(e + f*x)**3/(8*f) - a*c**4*sin(e + f*x)*cos(e + f*x)/f - 8*a
*c**4*cos(e + f*x)**5/(15*f) - 4*a*c**4*cos(e + f*x)**3/(3*f) + 3*a*c**4*co
s(e + f*x)/f, Ne(f, 0)), (x*(a*sin(e) + a)*(-c*sin(e) + c)**4, True))
```

Giac [A] time = 1.71468, size = 135, normalized size = 1.16

$$\frac{7}{8}\,ac^{4}x-\frac{ac^{4}\cos\left(5\,fx+5\,e\right)}{80\,f}+\frac{13\,ac^{4}\cos\left(3\,fx+3\,e\right)}{48\,f}+\frac{7\,ac^{4}\cos\left(fx+e\right)}{8\,f}-\frac{3\,ac^{4}\sin\left(4\,fx+4\,e\right)}{32\,f}+\frac{ac^{4}\sin\left(2\,fx+2\,e\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c-c*sin(f*x+e))^4,x, algorithm="giac")
```

```
[Out] 7/8*a*c^4*x - 1/80*a*c^4*cos(5*f*x + 5*e)/f + 13/48*a*c^4*cos(3*f*x + 3*e)/
f + 7/8*a*c^4*cos(f*x + e)/f - 3/32*a*c^4*sin(4*f*x + 4*e)/f + 1/4*a*c^4*si
n(2*f*x + 2*e)/f
```

3.228 $\int (a+a\sin(e+fx))(c-c\sin(e+fx))^3\,dx$

Optimal. Leaf size=83

$$\frac{5ac^3\cos^3(e+fx)}{12f}+\frac{a\cos^3(e+fx)\left(c^3-c^3\sin(e+fx)\right)}{4f}+\frac{5ac^3\sin(e+fx)\cos(e+fx)}{8f}+\frac{5}{8}ac^3x$$

```
[Out] (5*a*c^3*x)/8 + (5*a*c^3*Cos[e + f*x]^3)/(12*f) + (5*a*c^3*Cos[e + f*x]*Sin
[e + f*x])/(8*f) + (a*Cos[e + f*x]^3*(c^3 - c^3*Sin[e + f*x]))/(4*f)
```

Rubi [A] time = 0.112533, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {2736, 2678, 2669, 2635, 8}

$$\frac{5ac^3\cos^3(e+fx)}{12f}+\frac{a\cos^3(e+fx)\left(c^3-c^3\sin(e+fx)\right)}{4f}+\frac{5ac^3\sin(e+fx)\cos(e+fx)}{8f}+\frac{5}{8}ac^3x$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^3,x]
```

```
[Out] (5*a*c^3*x)/8 + (5*a*c^3*Cos[e + f*x]^3)/(12*f) + (5*a*c^3*Cos[e + f*x]*Sin
[e + f*x])/(8*f) + (a*Cos[e + f*x]^3*(c^3 - c^3*Sin[e + f*x]))/(4*f)
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (a+a\sin(e+fx))(c-c\sin(e+fx))^3\,dx = (ac)\int \cos^2(e+fx)(c-c\sin(e+fx))^2\,dx$$
$$= \frac{a\cos^3(e+fx)\left(c^3-c^3\sin(e+fx)\right)}{4f} + \frac{1}{4}\left(5ac^2\right)\int \cos^2(e+fx)(c-c\sin$$
$$= \frac{5ac^3\cos^3(e+fx)}{12f} + \frac{a\cos^3(e+fx)\left(c^3-c^3\sin(e+fx)\right)}{4f} + \frac{1}{4}\left(5ac^3\right)\int$$
$$= \frac{5ac^3\cos^3(e+fx)}{12f} + \frac{5ac^3\cos(e+fx)\sin(e+fx)}{8f} + \frac{a\cos^3(e+fx)\left(c^3-\right.}{4f}$$
$$= \frac{5}{8}ac^3x + \frac{5ac^3\cos^3(e+fx)}{12f} + \frac{5ac^3\cos(e+fx)\sin(e+fx)}{8f} + \frac{a\cos^3(e+}{}$$

Mathematica [A] time = 0.371495, size = 54, normalized size = 0.65

$$\frac{ac^3(24\sin(2(e+fx))-3\sin(4(e+fx))+48\cos(e+fx)+16\cos(3(e+fx))+60fx)}{96f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^3,x]
```

```
[Out] (a*c^3*(60*f*x + 48*Cos[e + f*x] + 16*Cos[3*(e + f*x)] + 24*Sin[2*(e + f*x)
] - 3*Sin[4*(e + f*x)]))/(96*f)
```

Maple [A] time = 0.017, size = 89, normalized size = 1.1

$$\frac{1}{f}\left(-ac^3\left(-\frac{\cos(fx+e)}{4}\left(\left(\sin(fx+e)\right)^3+\frac{3\sin(fx+e)}{2}\right)+\frac{3fx}{8}+\frac{3e}{8}\right)-\frac{2ac^3\left(2+\left(\sin(fx+e)\right)^2\right)\cos(fx+e)}{3}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(c-c*sin(f*x+e))^3,x)
```

```
[Out] 1/f*(-a*c^3*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-2
/3*a*c^3*(2+sin(f*x+e)^2)*cos(f*x+e)+2*a*c^3*cos(f*x+e)+a*c^3*(f*x+e))
```

Maxima [A] time = 1.21324, size = 116, normalized size = 1.4

$$\frac{64\left(\cos(fx+e)^3-3\cos(fx+e)\right)ac^3-3\left(12fx+12e+\sin(4fx+4e)-8\sin(2fx+2e)\right)ac^3+96(fx+e)ac^3+}{96f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c-c*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 1/96*(64*(cos(f*x + e)^3 - 3*cos(f*x + e))*a*c^3 - 3*(12*f*x + 12*e + sin(4
*f*x + 4*e) - 8*sin(2*f*x + 2*e))*a*c^3 + 96*(f*x + e)*a*c^3 + 192*a*c^3*co
s(f*x + e))/f
```

Fricas [A] time = 1.50477, size = 154, normalized size = 1.86

$$\frac{16\,ac^3\cos\left(fx+e\right)^3+15\,ac^3fx-3\left(2\,ac^3\cos\left(fx+e\right)^3-5\,ac^3\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c-c*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/24*(16*a*c^3*cos(f*x + e)^3 + 15*a*c^3*f*x - 3*(2*a*c^3*cos(f*x + e)^3 -
5*a*c^3*cos(f*x + e))*sin(f*x + e))/f
```

Sympy [A] time = 2.18679, size = 196, normalized size = 2.36

$$\begin{cases} -\frac{3ac^3x\sin^4\left(e+fx\right)}{8}-\frac{3ac^3x\sin^2\left(e+fx\right)\cos^2\left(e+fx\right)}{4}-\frac{3ac^3x\cos^4\left(e+fx\right)}{8}+ac^3x+\frac{5ac^3\sin^3\left(e+fx\right)\cos\left(e+fx\right)}{8f}-\frac{2ac^3\sin^2\left(e+fx\right)\cos\left(e+fx\right)}{f}+\frac{3a \\ x\left(a\sin\left(e\right)+a\right)\left(-c\sin\left(e\right)+c\right)^3 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c-c*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((-3*a*c**3*x*sin(e + f*x)**4/8 - 3*a*c**3*x*sin(e + f*x)**2*cos(e
 + f*x)**2/4 - 3*a*c**3*x*cos(e + f*x)**4/8 + a*c**3*x + 5*a*c**3*sin(e + f
*x)**3*cos(e + f*x)/(8*f) - 2*a*c**3*sin(e + f*x)**2*cos(e + f*x)/f + 3*a*c
**3*sin(e + f*x)*cos(e + f*x)**3/(8*f) - 4*a*c**3*cos(e + f*x)**3/(3*f) + 2
*a*c**3*cos(e + f*x)/f, Ne(f, 0)), (x*(a*sin(e) + a)*(-c*sin(e) + c)**3, Tr
ue))
```

Giac [A] time = 1.90293, size = 109, normalized size = 1.31

$$\frac{5}{8}\,ac^3x+\frac{ac^3\cos\left(3\,fx+3\,e\right)}{6\,f}+\frac{ac^3\cos\left(fx+e\right)}{2\,f}-\frac{ac^3\sin\left(4\,fx+4\,e\right)}{32\,f}+\frac{ac^3\sin\left(2\,fx+2\,e\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c-c*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 5/8*a*c^3*x + 1/6*a*c^3*cos(3*f*x + 3*e)/f + 1/2*a*c^3*cos(f*x + e)/f - 1/3
2*a*c^3*sin(4*f*x + 4*e)/f + 1/4*a*c^3*sin(2*f*x + 2*e)/f
```

3.229 $\int (a+a\sin(e+fx))(c-c\sin(e+fx))^2\,dx$

Optimal. Leaf size=52

$$\frac{ac^2\cos^3(e+fx)}{3f}+\frac{ac^2\sin(e+fx)\cos(e+fx)}{2f}+\frac{1}{2}ac^2x$$

```
[Out] (a*c^2*x)/2 + (a*c^2*Cos[e + f*x]^3)/(3*f) + (a*c^2*Cos[e + f*x]*Sin[e + f*
x])/(2*f)
```

Rubi [A] time = 0.0649879, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2736, 2669, 2635, 8}

$$\frac{ac^2\cos^3(e+fx)}{3f}+\frac{ac^2\sin(e+fx)\cos(e+fx)}{2f}+\frac{1}{2}ac^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^2,x]
```

```
[Out] (a*c^2*x)/2 + (a*c^2*Cos[e + f*x]^3)/(3*f) + (a*c^2*Cos[e + f*x]*Sin[e + f*
x])/(2*f)
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (a+a\sin(e+fx))(c-c\sin(e+fx))^2\,dx = (ac)\int \cos^2(e+fx)(c-c\sin(e+fx))\,dx$$
$$= \frac{ac^2\cos^3(e+fx)}{3f} + \left(ac^2\right)\int \cos^2(e+fx)\,dx$$
$$= \frac{ac^2\cos^3(e+fx)}{3f} + \frac{ac^2\cos(e+fx)\sin(e+fx)}{2f} + \frac{1}{2}\left(ac^2\right)\int 1\,dx$$
$$= \frac{1}{2}ac^2x + \frac{ac^2\cos^3(e+fx)}{3f} + \frac{ac^2\cos(e+fx)\sin(e+fx)}{2f}$$

Mathematica [A] time = 0.287442, size = 42, normalized size = 0.81

$$\frac{ac^2(3\sin(2(e+fx)) + 3\cos(e+fx) + \cos(3(e+fx)) + 6fx)}{12f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^2,x]
```

```
[Out] (a*c^2*(6*f*x + 3*Cos[e + f*x] + Cos[3*(e + f*x)] + 3*Sin[2*(e + f*x)]))/(1
2*f)
```

Maple [A] time = 0.015, size = 77, normalized size = 1.5

$$\frac{1}{f}\left(-\frac{ac^2\left(2+\left(\sin\left(fx+e\right)\right)^2\right)\cos\left(fx+e\right)}{3} - ac^2\left(-\frac{\sin\left(fx+e\right)\cos\left(fx+e\right)}{2} + \frac{fx}{2} + \frac{e}{2}\right) + ac^2\cos\left(fx+e\right) + ac^2\left(fx+e\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(c-c*sin(f*x+e))^2,x)
```

```
[Out] 1/f*(-1/3*a*c^2*(2+sin(f*x+e)^2)*cos(f*x+e)-a*c^2*(-1/2*sin(f*x+e)*cos(f*x+
e)+1/2*f*x+1/2*e)+a*c^2*cos(f*x+e)+a*c^2*(f*x+e))
```

Maxima [A] time = 1.12239, size = 104, normalized size = 2.

$$\frac{4\left(\cos\left(fx+e\right)^3 - 3\cos\left(fx+e\right)\right)ac^2 - 3\left(2fx+2e-\sin\left(2fx+2e\right)\right)ac^2 + 12\left(fx+e\right)ac^2 + 12ac^2\cos\left(fx+e\right)}{12f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c-c*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/12*(4*(cos(f*x + e)^3 - 3*cos(f*x + e))*a*c^2 - 3*(2*f*x + 2*e - sin(2*f*
x + 2*e))*a*c^2 + 12*(f*x + e)*a*c^2 + 12*a*c^2*cos(f*x + e))/f
```

Fricas [A] time = 1.33194, size = 111, normalized size = 2.13

$$\frac{2\,ac^2\cos\left(fx+e\right)^3+3\,ac^2fx+3\,ac^2\cos\left(fx+e\right)\sin\left(fx+e\right)}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c-c*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/6*(2*a*c^2*cos(f*x + e)^3 + 3*a*c^2*f*x + 3*a*c^2*cos(f*x + e)*sin(f*x +
e))/f
```

Sympy [A] time = 0.699654, size = 133, normalized size = 2.56

$$\begin{cases}-\frac{ac^2x\sin^2\left(e+fx\right)}{2}-\frac{ac^2x\cos^2\left(e+fx\right)}{2}+ac^2x-\frac{ac^2\sin^2\left(e+fx\right)\cos\left(e+fx\right)}{f}+\frac{ac^2\sin\left(e+fx\right)\cos\left(e+fx\right)}{2f}-\frac{2ac^2\cos^3\left(e+fx\right)}{3f}+\frac{ac^2\cos\left(e+fx\right)}{f}\\ x\left(a\sin\left(e\right)+a\right)\left(-c\sin\left(e\right)+c\right)^2\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c-c*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((-a*c**2*x*sin(e + f*x)**2/2 - a*c**2*x*cos(e + f*x)**2/2 + a*c**
2*x - a*c**2*sin(e + f*x)**2*cos(e + f*x)/f + a*c**2*sin(e + f*x)*cos(e + f
*x)/(2*f) - 2*a*c**2*cos(e + f*x)**3/(3*f) + a*c**2*cos(e + f*x)/f, Ne(f, 0
)), (x*(a*sin(e) + a)*(-c*sin(e) + c)**2, True))
```

Giac [A] time = 1.91836, size = 84, normalized size = 1.62

$$\frac{1}{2}\,ac^2x+\frac{ac^2\cos\left(3\,fx+3\,e\right)}{12\,f}+\frac{ac^2\cos\left(fx+e\right)}{4\,f}+\frac{ac^2\sin\left(2\,fx+2\,e\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c-c*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/2*a*c^2*x + 1/12*a*c^2*cos(3*f*x + 3*e)/f + 1/4*a*c^2*cos(f*x + e)/f + 1/
4*a*c^2*sin(2*f*x + 2*e)/f
```

3.230 $\int (a+a\sin(e+fx))(c-c\sin(e+fx))\,dx$

Optimal. Leaf size=29

$$\frac{ac\sin(e+fx)\cos(e+fx)}{2f}+\frac{acx}{2}$$

```
[Out] (a*c*x)/2 + (a*c*Cos[e + f*x]*Sin[e + f*x])/(2*f)
```

Rubi [A] time = 0.0183067, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {2734}

$$\frac{ac\sin(e+fx)\cos(e+fx)}{2f}+\frac{acx}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])*(c - c*Sin[e + f*x]),x]
```

```
[Out] (a*c*x)/2 + (a*c*Cos[e + f*x]*Sin[e + f*x])/(2*f)
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))(c-c\sin(e+fx))\,dx=\frac{acx}{2}+\frac{ac\cos(e+fx)\sin(e+fx)}{2f}$$

Mathematica [A] time = 0.0241253, size = 25, normalized size = 0.86

$$\frac{ac(2(e+fx)+\sin(2(e+fx)))}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])*(c - c*Sin[e + f*x]),x]
```

```
[Out] (a*c*(2*(e + f*x) + Sin[2*(e + f*x)]))/(4*f)
```

Maple [A] time = 0.013, size = 40, normalized size = 1.4

$$\frac{1}{f}\left(-ca\left(-\frac{\sin\left(fx+e\right)\cos\left(fx+e\right)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+ca\left(fx+e\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(c-c*sin(f*x+e)),x)
```

```
[Out] 1/f*(-c*a*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)+c*a*(f*x+e))
```

Maxima [A] time = 1.18982, size = 50, normalized size = 1.72

$$-\frac{\left(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right)\right)ac-4\left(fx+e\right)ac}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c-c*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] -1/4*((2*f*x + 2*e - sin(2*f*x + 2*e))*a*c - 4*(f*x + e)*a*c)/f
```

Fricas [A] time = 1.50889, size = 66, normalized size = 2.28

$$\frac{acfx+ac\cos\left(fx+e\right)\sin\left(fx+e\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c-c*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/2*(a*c*f*x + a*c*cos(f*x + e)*sin(f*x + e))/f
```

Sympy [A] time = 0.406081, size = 70, normalized size = 2.41

$$\begin{cases} -\frac{acx\sin^{2}\left(e+fx\right)}{2}-\frac{acx\cos^{2}\left(e+fx\right)}{2}+acx+\frac{ac\sin\left(e+fx\right)\cos\left(e+fx\right)}{2f} & \text{for}\: f\neq 0 \\ x\left(a\sin\left(e\right)+a\right)\left(-c\sin\left(e\right)+c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c-c*sin(f*x+e)),x)
```

```
[Out] Piecewise((-a*c*x*sin(e + f*x)**2/2 - a*c*x*cos(e + f*x)**2/2 + a*c*x + a*c
*sin(e + f*x)*cos(e + f*x)/(2*f), Ne(f, 0)), (x*(a*sin(e) + a)*(-c*sin(e) +
 c), True))
```

Giac [A] time = 1.9982, size = 31, normalized size = 1.07

$$\frac{1}{2}\,acx+\frac{ac\sin\left(2\,fx+2\,e\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c-c*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/2*a*c*x + 1/4*a*c*sin(2*f*x + 2*e)/f
```

3.231 $\int \frac{a+a\sin(e+fx)}{c-c\sin(e+fx)}\,dx$

Optimal. Leaf size=33

$$\frac{2a\cos(e+fx)}{f(c-c\sin(e+fx))}-\frac{ax}{c}$$

```
[Out] -((a*x)/c) + (2*a*Cos[e + f*x])/(f*(c - c*Sin[e + f*x]))
```

Rubi [A] time = 0.0484159, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2735, 2648}

$$\frac{2a\cos(e+fx)}{f(c-c\sin(e+fx))}-\frac{ax}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])/(c - c*Sin[e + f*x]),x]

[Out] -((a*x)/c) + (2*a*Cos[e + f*x])/(f*(c - c*Sin[e + f*x]))
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a+a\sin(e+fx)}{c-c\sin(e+fx)}\,dx &= -\frac{ax}{c}+(2a)\int \frac{1}{c-c\sin(e+fx)}\,dx\\ &= -\frac{ax}{c}+\frac{2a\cos(e+fx)}{f(c-c\sin(e+fx))}\end{aligned}$$

Mathematica [B] time = 0.182072, size = 83, normalized size = 2.52

$$\frac{a\left(fx\sin\left(e+\frac{fx}{2}\right)+4\sin\left(\frac{fx}{2}\right)-fx\cos\left(\frac{fx}{2}\right)\right)}{cf\left(\cos\left(\frac{e}{2}\right)-\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])/(c - c*Sin[e + f*x]),x]
```

```
[Out] (a*(-(f*x*Cos[(f*x)/2]) + 4*Sin[(f*x)/2] + f*x*Sin[e + (f*x)/2]))/(c*f*(Cos
[e/2] - Sin[e/2])*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))
```

Maple [A] time = 0.057, size = 43, normalized size = 1.3

$$-2\,{\frac {a\arctan \left(\tan \left(1/2\,fx+e/2 \right) \right) }{cf}}-4\,{\frac {a}{cf \left(\tan \left(1/2\,fx+e/2 \right) -1 \right) }}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))/(c-c*sin(f*x+e)),x)
```

```
[Out] -2/f*a/c*arctan(tan(1/2*f*x+1/2*e))-4/f*a/c/(tan(1/2*f*x+1/2*e)-1)
```

Maxima [B] time = 2.23758, size = 111, normalized size = 3.36

$$-\frac{2 \left(a \left(\frac{\arctan\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}\right)}{c} - \frac{1}{c - \frac{c \sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}}\right) - \frac{a}{c - \frac{c \sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] -2*(a*(arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c - 1/(c - c*sin(f*x + e)/(c
os(f*x + e) + 1))) - a/(c - c*sin(f*x + e)/(cos(f*x + e) + 1)))/f
```

Fricas [A] time = 1.63003, size = 159, normalized size = 4.82

$$-\frac{afx + \left(afx - 2\,a\right)\cos\left(fx + e\right) - \left(afx + 2\,a\right)\sin\left(fx + e\right) - 2\,a}{cf\cos\left(fx + e\right) - cf\sin\left(fx + e\right) + cf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] -(a*f*x + (a*f*x - 2*a)*cos(f*x + e) - (a*f*x + 2*a)*sin(f*x + e) - 2*a)/(c
*f*cos(f*x + e) - c*f*sin(f*x + e) + c*f)
```

Sympy [A] time = 1.91624, size = 88, normalized size = 2.67

$$\begin{cases} - \frac{a f x \tan{\left(\frac{e}{2} + \frac{f x}{2} \right)}}{c f \tan{\left(\frac{e}{2} + \frac{f x}{2} \right)} - c f} + \frac{a f x}{c f \tan{\left(\frac{e}{2} + \frac{f x}{2} \right)} - c f} - \frac{4 a}{c f \tan{\left(\frac{e}{2} + \frac{f x}{2} \right)} - c f} & \text{for}\: f \neq 0 \\\frac{x \left(a \sin{\left(e \right)} + a\right)}{- c \sin{\left(e \right)} + c} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e)),x)
```

```
[Out] Piecewise((-a*f*x*tan(e/2 + f*x/2)/(c*f*tan(e/2 + f*x/2) - c*f) + a*f*x/(c*
f*tan(e/2 + f*x/2) - c*f) - 4*a/(c*f*tan(e/2 + f*x/2) - c*f), Ne(f, 0)), (x
*(a*sin(e) + a)/(-c*sin(e) + c), True))
```

Giac [A] time = 1.672, size = 50, normalized size = 1.52

$$-\frac{\frac{(fx+e)a}{c}+\frac{4a}{c\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] -((f*x + e)*a/c + 4*a/(c*(tan(1/2*f*x + 1/2*e) - 1)))/f
```

3.232 $\int \frac{a+a\sin(e+fx)}{(c-c\sin(e+fx))^2}\,dx$

Optimal. Leaf size=30

$$\frac{ac\cos^3(e+fx)}{3f(c-c\sin(e+fx))^3}$$

```
[Out] (a*c*Cos[e + f*x]^3)/(3*f*(c - c*Sin[e + f*x])^3)
```

Rubi [A] time = 0.0729124, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2736, 2671}

$$\frac{ac\cos^3(e+fx)}{3f(c-c\sin(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])/(c - c*Sin[e + f*x])^2,x]

[Out] (a*c*Cos[e + f*x]^3)/(3*f*(c - c*Sin[e + f*x])^3)
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a+a\sin(e+fx)}{(c-c\sin(e+fx))^2}\,dx &= (ac)\int \frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^3}\,dx\\ &= \frac{ac\cos^3(e+fx)}{3f(c-c\sin(e+fx))^3}\end{aligned}$$

Mathematica [B] time = 0.270363, size = 74, normalized size = 2.47

$$-\frac{a\left(\cos\left(e+\frac{3fx}{2}\right)-3\cos\left(e+\frac{fx}{2}\right)\right)}{3c^2f\left(\cos\left(\frac{e}{2}\right)-\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])/(c - c*Sin[e + f*x])^2,x]
```

```
[Out] -(a*(-3*Cos[e + (f*x)/2] + Cos[e + (3*f*x)/2]))/(3*c^2*f*(Cos[e/2] - Sin[e/
2])*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3)
```

Maple [A] time = 0.067, size = 56, normalized size = 1.9

$$2\,{\frac {a}{f{c}^{2}}\left(-2\, \left(\tan \left(1/2\,fx+e/2 \right) -1 \right) ^{-2}-4/3\, \left(\tan \left(1/2\,fx+e/2 \right) -1 \right) ^{-3}- \left(\tan \left(1/2\,fx+e/2 \right) -1 \right) ^{-1} \right) }$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))/(c-c*sin(f*x+e))^2,x)
```

```
[Out] 2/f*a/c^2*(-2/(tan(1/2*f*x+1/2*e)-1)^2-4/3/(tan(1/2*f*x+1/2*e)-1)^3-1/(tan(
1/2*f*x+1/2*e)-1))
```

Maxima [B] time = 1.24675, size = 293, normalized size = 9.77

$$-\frac{2\left(\frac{a\left(\frac{3\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-\frac{3\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}-2\right)}{c^{2}-\frac{3c^{2}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{3c^{2}\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}-\frac{c^{2}\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}}-\frac{a\left(\frac{3\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-1\right)}{c^{2}-\frac{3c^{2}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{3c^{2}\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}-\frac{c^{2}\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}}\right)}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] -2/3*(a*(3*sin(f*x + e)/(cos(f*x + e) + 1) - 3*sin(f*x + e)^2/(cos(f*x + e)
 + 1)^2 - 2)/(c^2 - 3*c^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*c^2*sin(f*x +
 e)^2/(cos(f*x + e) + 1)^2 - c^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) - a*(
3*sin(f*x + e)/(cos(f*x + e) + 1) - 1)/(c^2 - 3*c^2*sin(f*x + e)/(cos(f*x +
 e) + 1) + 3*c^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - c^2*sin(f*x + e)^3/(
cos(f*x + e) + 1)^3))/f
```

Fricas [B] time = 1.25734, size = 250, normalized size = 8.33

$$\frac{a\cos\left(fx+e\right)^{2}-a\cos\left(fx+e\right)-\left(a\cos\left(fx+e\right)+2\,a\right)\sin\left(fx+e\right)-2\,a}{3\left(c^{2}f\cos\left(fx+e\right)^{2}-c^{2}f\cos\left(fx+e\right)-2\,c^{2}f+\left(c^{2}f\cos\left(fx+e\right)+2\,c^{2}f\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(a*cos(f*x + e)^2 - a*cos(f*x + e) - (a*cos(f*x + e) + 2*a)*sin(f*x + e
) - 2*a)/(c^2*f*cos(f*x + e)^2 - c^2*f*cos(f*x + e) - 2*c^2*f + (c^2*f*cos(
f*x + e) + 2*c^2*f)*sin(f*x + e))
```

Sympy [A] time = 6.83427, size = 158, normalized size = 5.27

$$\begin{cases} -\frac{6a\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3c^2f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)-9c^2f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+9c^2f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-3c^2f} - \frac{2a}{3c^2f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)-9c^2f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+9c^2f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-3c^2f} & \text{for } f \neq 0 \\ \frac{x(a\sin(e)+a)}{(-c\sin(e)+c)^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((-6*a*tan(e/2 + f*x/2)**2/(3*c**2*f*tan(e/2 + f*x/2)**3 - 9*c**2*
f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) - 2*a/(3*c**2
*f*tan(e/2 + f*x/2)**3 - 9*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 +
f*x/2) - 3*c**2*f), Ne(f, 0)), (x*(a*sin(e) + a)/(-c*sin(e) + c)**2, True))
```

Giac [A] time = 1.93706, size = 53, normalized size = 1.77

$$-\frac{2\left(3\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+a\right)}{3\,c^2f\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] -2/3*(3*a*tan(1/2*f*x + 1/2*e)^2 + a)/(c^2*f*(tan(1/2*f*x + 1/2*e) - 1)^3)
```

3.233 $\int \frac{a+a\sin(e+fx)}{(c-c\sin(e+fx))^3}\,dx$

Optimal. Leaf size=60

$$\frac{a\cos^3(e+fx)}{15f(c-c\sin(e+fx))^3}+\frac{ac\cos^3(e+fx)}{5f(c-c\sin(e+fx))^4}$$

```
[Out] (a*c*Cos[e + f*x]^3)/(5*f*(c - c*Sin[e + f*x])^4) + (a*Cos[e + f*x]^3)/(15*
f*(c - c*Sin[e + f*x])^3)
```

Rubi [A] time = 0.116896, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2736, 2672, 2671}

$$\frac{a\cos^3(e+fx)}{15f(c-c\sin(e+fx))^3}+\frac{ac\cos^3(e+fx)}{5f(c-c\sin(e+fx))^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])/(c - c*Sin[e + f*x])^3,x]
```

```
[Out] (a*c*Cos[e + f*x]^3)/(5*f*(c - c*Sin[e + f*x])^4) + (a*Cos[e + f*x]^3)/(15*
f*(c - c*Sin[e + f*x])^3)
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rubi steps

$$\int \frac{a+a\sin(e+fx)}{(c-c\sin(e+fx))^3}\,dx = (ac)\int \frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^4}\,dx$$
$$= \frac{ac\cos^3(e+fx)}{5f(c-c\sin(e+fx))^4} + \frac{1}{5}a\int \frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^3}\,dx$$
$$= \frac{ac\cos^3(e+fx)}{5f(c-c\sin(e+fx))^4} + \frac{a\cos^3(e+fx)}{15f(c-c\sin(e+fx))^3}$$

Mathematica [A] time = 0.32822, size = 96, normalized size = 1.6

$$\frac{a\left(\sin\left(2e+\frac{5fx}{2}\right)+15\cos\left(e+\frac{fx}{2}\right)-5\cos\left(e+\frac{3fx}{2}\right)+5\sin\left(\frac{fx}{2}\right)\right)}{30c^3f\left(\cos\left(\frac{e}{2}\right)-\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])/(c - c*Sin[e + f*x])^3,x]
```

```
[Out] (a*(15*Cos[e + (f*x)/2] - 5*Cos[e + (3*f*x)/2] + 5*Sin[(f*x)/2] + Sin[2*e +
 (5*f*x)/2]))/(30*c^3*f*(Cos[e/2] - Sin[e/2])*(Cos[(e + f*x)/2] - Sin[(e +
f*x)/2])^5)
```

Maple [A] time = 0.076, size = 86, normalized size = 1.4

$$2\frac{a}{fc^3}\left(-4\left(\tan\left(1/2\,fx+e/2\right)-1\right)^{-4}-14/3\left(\tan\left(1/2\,fx+e/2\right)-1\right)^{-3}-3\left(\tan\left(1/2\,fx+e/2\right)-1\right)^{-2}-8/5\left(\tan\left(1/2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))/(c-c*sin(f*x+e))^3,x)
```

```
[Out] 2/f*a/c^3*(-4/(tan(1/2*f*x+1/2*e)-1)^4-14/3/(tan(1/2*f*x+1/2*e)-1)^3-3/(tan
(1/2*f*x+1/2*e)-1)^2-8/5/(tan(1/2*f*x+1/2*e)-1)^5-1/(tan(1/2*f*x+1/2*e)-1))
```

Maxima [B] time = 1.25825, size = 525, normalized size = 8.75

$$-\frac{2\left(\frac{a\left(\frac{20\sin(fx+e)}{\cos(fx+e)+1}-\frac{40\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{30\sin(fx+e)^3}{(\cos(fx+e)+1)^3}-\frac{15\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-7\right)}{c^3-\frac{5c^3\sin(fx+e)}{\cos(fx+e)+1}+\frac{10c^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{10c^3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{5c^3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{c^3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}-\frac{3a\left(\frac{5\sin(fx+e)}{\cos(fx+e)+1}-\frac{5\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{5\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right.}{c^3-\frac{5c^3\sin(fx+e)}{\cos(fx+e)+1}+\frac{10c^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{10c^3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{5c^3\sin(fx\ldots}{(\cos(fx+e)+\ldots}}\right.}{15f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] -2/15*(a*(20*sin(f*x + e)/(cos(f*x + e) + 1) - 40*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 + 30*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 15*sin(f*x + e)^4/(co
s(f*x + e) + 1)^4 - 7)/(c^3 - 5*c^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*c^
3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 10*c^3*sin(f*x + e)^3/(cos(f*x + e)
 + 1)^3 + 5*c^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - c^3*sin(f*x + e)^5/(c
```

```
os(f*x + e) + 1)^5) - 3*a*(5*sin(f*x + e)/(cos(f*x + e) + 1) - 5*sin(f*x +
e)^2/(cos(f*x + e) + 1)^2 + 5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 1)/(c^3
 - 5*c^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*c^3*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 - 10*c^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*c^3*sin(f*x + e
)^4/(cos(f*x + e) + 1)^4 - c^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5))/f
```

Fricas [B] time = 1.30051, size = 382, normalized size = 6.37

$$-\frac{a\cos\left(fx+e\right)^{3}-2\,a\cos\left(fx+e\right)^{2}+3\,a\cos\left(fx+e\right)+\left(a\cos\left(fx+e\right)^{2}+3\,a\cos\left(fx+e\right)+6\,a\right)\sin\left(fx+e\right)}{15\left(c^{3}f\cos\left(fx+e\right)^{3}+3\,c^{3}f\cos\left(fx+e\right)^{2}-2\,c^{3}f\cos\left(fx+e\right)-4\,c^{3}f-\left(c^{3}f\cos\left(fx+e\right)^{2}-2\,c^{3}f\cos\left(fx+e\right)-4\,c^{3}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -1/15*(a*cos(f*x + e)^3 - 2*a*cos(f*x + e)^2 + 3*a*cos(f*x + e) + (a*cos(f*
x + e)^2 + 3*a*cos(f*x + e) + 6*a)*sin(f*x + e) + 6*a)/(c^3*f*cos(f*x + e)^
3 + 3*c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x + e) - 4*c^3*f - (c^3*f*cos(f*
x + e)^2 - 2*c^3*f*cos(f*x + e) - 4*c^3*f)*sin(f*x + e))
```

Sympy [A] time = 15.0005, size = 573, normalized size = 9.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((-2*a*tan(e/2 + f*x/2)**5/(15*c**3*f*tan(e/2 + f*x/2)**5 - 75*c**
3*f*tan(e/2 + f*x/2)**4 + 150*c**3*f*tan(e/2 + f*x/2)**3 - 150*c**3*f*tan(e
/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) - 20*a*tan(e/2 + f
*x/2)**4/(15*c**3*f*tan(e/2 + f*x/2)**5 - 75*c**3*f*tan(e/2 + f*x/2)**4 + 1
50*c**3*f*tan(e/2 + f*x/2)**3 - 150*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*
tan(e/2 + f*x/2) - 15*c**3*f) + 10*a*tan(e/2 + f*x/2)**3/(15*c**3*f*tan(e/2
 + f*x/2)**5 - 75*c**3*f*tan(e/2 + f*x/2)**4 + 150*c**3*f*tan(e/2 + f*x/2)*
*3 - 150*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*
f) - 30*a*tan(e/2 + f*x/2)**2/(15*c**3*f*tan(e/2 + f*x/2)**5 - 75*c**3*f*ta
n(e/2 + f*x/2)**4 + 150*c**3*f*tan(e/2 + f*x/2)**3 - 150*c**3*f*tan(e/2 + f
*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) - 6*a/(15*c**3*f*tan(e/2
 + f*x/2)**5 - 75*c**3*f*tan(e/2 + f*x/2)**4 + 150*c**3*f*tan(e/2 + f*x/2)*
*3 - 150*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*
f), Ne(f, 0)), (x*(a*sin(e) + a)/(-c*sin(e) + c)**3, True))
```

Giac [A] time = 1.63448, size = 113, normalized size = 1.88

$$-\frac{2\left(15\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-15\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+25\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-5\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+4\,a\right)}{15\,c^{3}f\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -2/15*(15*a*tan(1/2*f*x + 1/2*e)^4 - 15*a*tan(1/2*f*x + 1/2*e)^3 + 25*a*tan
(1/2*f*x + 1/2*e)^2 - 5*a*tan(1/2*f*x + 1/2*e) + 4*a)/(c^3*f*(tan(1/2*f*x +
 1/2*e) - 1)^5)
```

3.234 $\int \frac{a+a\sin(e+fx)}{(c-c\sin(e+fx))^4}\,dx$

Optimal. Leaf size=92

$$\frac{2a\cos^3(e+fx)}{105cf(c-c\sin(e+fx))^3}+\frac{2a\cos^3(e+fx)}{35f(c-c\sin(e+fx))^4}+\frac{ac\cos^3(e+fx)}{7f(c-c\sin(e+fx))^5}$$

```
[Out] (a*c*Cos[e + f*x]^3)/(7*f*(c - c*Sin[e + f*x])^5) + (2*a*Cos[e + f*x]^3)/(3
5*f*(c - c*Sin[e + f*x])^4) + (2*a*Cos[e + f*x]^3)/(105*c*f*(c - c*Sin[e +
f*x])^3)
```

Rubi [A] time = 0.17018, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2736, 2672, 2671}

$$\frac{2a\cos^3(e+fx)}{105cf(c-c\sin(e+fx))^3}+\frac{2a\cos^3(e+fx)}{35f(c-c\sin(e+fx))^4}+\frac{ac\cos^3(e+fx)}{7f(c-c\sin(e+fx))^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])/(c - c*Sin[e + f*x])^4,x]
```

```
[Out] (a*c*Cos[e + f*x]^3)/(7*f*(c - c*Sin[e + f*x])^5) + (2*a*Cos[e + f*x]^3)/(3
5*f*(c - c*Sin[e + f*x])^4) + (2*a*Cos[e + f*x]^3)/(105*c*f*(c - c*Sin[e +
f*x])^3)
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] && !IGtQ[m, 0]
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] && !ILtQ[p, 0]
```

Rubi steps

$$\int \frac{a+a\sin(e+fx)}{(c-c\sin(e+fx))^4}\,dx = (ac)\int \frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^5}\,dx$$
$$= \frac{ac\cos^3(e+fx)}{7f(c-c\sin(e+fx))^5} + \frac{1}{7}(2a)\int \frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^4}\,dx$$
$$= \frac{ac\cos^3(e+fx)}{7f(c-c\sin(e+fx))^5} + \frac{2a\cos^3(e+fx)}{35f(c-c\sin(e+fx))^4} + \frac{(2a)\int \frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^3}\,dx}{35c}$$
$$= \frac{ac\cos^3(e+fx)}{7f(c-c\sin(e+fx))^5} + \frac{2a\cos^3(e+fx)}{35f(c-c\sin(e+fx))^4} + \frac{2a\cos^3(e+fx)}{105cf(c-c\sin(e+fx))^3}$$

Mathematica [A] time = 0.466238, size = 109, normalized size = 1.18

$$\frac{a\left(7\sin\left(2e+\frac{5fx}{2}\right)+70\cos\left(e+\frac{fx}{2}\right)-21\cos\left(e+\frac{3fx}{2}\right)+\cos\left(3e+\frac{7fx}{2}\right)+35\sin\left(\frac{fx}{2}\right)\right)}{210c^4f\left(\cos\left(\frac{e}{2}\right)-\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])/(c - c*Sin[e + f*x])^4,x]
```

```
[Out] (a*(70*Cos[e + (f*x)/2] - 21*Cos[e + (3*f*x)/2] + Cos[3*e + (7*f*x)/2] + 35
*Sin[(f*x)/2] + 7*Sin[2*e + (5*f*x)/2]))/(210*c^4*f*(Cos[e/2] - Sin[e/2])*(
Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7)
```

Maple [A] time = 0.086, size = 116, normalized size = 1.3

$$2\,\frac{a}{fc^4}\left(-4\left(\tan\left(1/2\,fx+e/2\right)-1\right)^{-2}-8\left(\tan\left(1/2\,fx+e/2\right)-1\right)^{-6}-14\left(\tan\left(1/2\,fx+e/2\right)-1\right)^{-4}-\frac{28}{3\left(\tan\left(1/2\,fx+\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))/(c-c*sin(f*x+e))^4,x)
```

```
[Out] 2/f*a/c^4*(-4/(tan(1/2*f*x+1/2*e)-1)^2-8/(tan(1/2*f*x+1/2*e)-1)^6-14/(tan(1
/2*f*x+1/2*e)-1)^4-28/3/(tan(1/2*f*x+1/2*e)-1)^3-68/5/(tan(1/2*f*x+1/2*e)-1
)^5-1/(tan(1/2*f*x+1/2*e)-1)-16/7/(tan(1/2*f*x+1/2*e)-1)^7)
```

Maxima [B] time = 1.25502, size = 757, normalized size = 8.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e))^4,x, algorithm="maxima")
```

```
[Out] 2/105*(a*(91*sin(f*x + e)/(cos(f*x + e) + 1) - 168*sin(f*x + e)^2/(cos(f*x
+ e) + 1)^2 + 280*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 175*sin(f*x + e)^4/
(cos(f*x + e) + 1)^4 + 105*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 13)/(c^4 -
 7*c^4*sin(f*x + e)/(cos(f*x + e) + 1) + 21*c^4*sin(f*x + e)^2/(cos(f*x + e
```

```
) + 1)^2 - 35*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 35*c^4*sin(f*x + e)
^4/(cos(f*x + e) + 1)^4 - 21*c^4*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 7*c^
4*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - c^4*sin(f*x + e)^7/(cos(f*x + e) +
1)^7) - 3*a*(49*sin(f*x + e)/(cos(f*x + e) + 1) - 147*sin(f*x + e)^2/(cos(f
*x + e) + 1)^2 + 210*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 210*sin(f*x + e)
^4/(cos(f*x + e) + 1)^4 + 105*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 35*sin(
f*x + e)^6/(cos(f*x + e) + 1)^6 - 12)/(c^4 - 7*c^4*sin(f*x + e)/(cos(f*x +
e) + 1) + 21*c^4*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 35*c^4*sin(f*x + e)^
3/(cos(f*x + e) + 1)^3 + 35*c^4*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 21*c^
4*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 7*c^4*sin(f*x + e)^6/(cos(f*x + e)
+ 1)^6 - c^4*sin(f*x + e)^7/(cos(f*x + e) + 1)^7))/f
```

Fricas [B] time = 1.40681, size = 517, normalized size = 5.62

$$\frac{2\,a\cos\left(fx+e\right)^{4}+8\,a\cos\left(fx+e\right)^{3}-9\,a\cos\left(fx+e\right)^{2}+15\,a\cos\left(fx+e\right)-\left(2\,a\cos\left(fx+e\right)^{3}-6\,a\cos\left(fx+\right.\right.}{105\left(c^{4}f\cos\left(fx+e\right)^{4}-3\,c^{4}f\cos\left(fx+e\right)^{3}-8\,c^{4}f\cos\left(fx+e\right)^{2}+4\,c^{4}f\cos\left(fx+e\right)+8\,c^{4}f+\left(c^{4}f\cos\left(fx+e\right)^{3}+4\,c^{4}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e))^4,x, algorithm="fricas")
```

```
[Out] 1/105*(2*a*cos(f*x + e)^4 + 8*a*cos(f*x + e)^3 - 9*a*cos(f*x + e)^2 + 15*a*
cos(f*x + e) - (2*a*cos(f*x + e)^3 - 6*a*cos(f*x + e)^2 - 15*a*cos(f*x + e)
 - 30*a)*sin(f*x + e) + 30*a)/(c^4*f*cos(f*x + e)^4 - 3*c^4*f*cos(f*x + e)^
3 - 8*c^4*f*cos(f*x + e)^2 + 4*c^4*f*cos(f*x + e) + 8*c^4*f + (c^4*f*cos(f*
x + e)^3 + 4*c^4*f*cos(f*x + e)^2 - 4*c^4*f*cos(f*x + e) - 8*c^4*f)*sin(f*x
 + e))
```

Sympy [A] time = 48.9284, size = 1061, normalized size = 11.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e))**4,x)
```

```
[Out] Piecewise((-30*a*tan(e/2 + f*x/2)**7/(105*c**4*f*tan(e/2 + f*x/2)**7 - 735*
c**4*f*tan(e/2 + f*x/2)**6 + 2205*c**4*f*tan(e/2 + f*x/2)**5 - 3675*c**4*f*
tan(e/2 + f*x/2)**4 + 3675*c**4*f*tan(e/2 + f*x/2)**3 - 2205*c**4*f*tan(e/2
 + f*x/2)**2 + 735*c**4*f*tan(e/2 + f*x/2) - 105*c**4*f) - 210*a*tan(e/2 +
f*x/2)**5/(105*c**4*f*tan(e/2 + f*x/2)**7 - 735*c**4*f*tan(e/2 + f*x/2)**6
+ 2205*c**4*f*tan(e/2 + f*x/2)**5 - 3675*c**4*f*tan(e/2 + f*x/2)**4 + 3675*
c**4*f*tan(e/2 + f*x/2)**3 - 2205*c**4*f*tan(e/2 + f*x/2)**2 + 735*c**4*f*t
an(e/2 + f*x/2) - 105*c**4*f) + 140*a*tan(e/2 + f*x/2)**4/(105*c**4*f*tan(e
/2 + f*x/2)**7 - 735*c**4*f*tan(e/2 + f*x/2)**6 + 2205*c**4*f*tan(e/2 + f*x
/2)**5 - 3675*c**4*f*tan(e/2 + f*x/2)**4 + 3675*c**4*f*tan(e/2 + f*x/2)**3
- 2205*c**4*f*tan(e/2 + f*x/2)**2 + 735*c**4*f*tan(e/2 + f*x/2) - 105*c**4*
f) - 350*a*tan(e/2 + f*x/2)**3/(105*c**4*f*tan(e/2 + f*x/2)**7 - 735*c**4*f
*tan(e/2 + f*x/2)**6 + 2205*c**4*f*tan(e/2 + f*x/2)**5 - 3675*c**4*f*tan(e/
2 + f*x/2)**4 + 3675*c**4*f*tan(e/2 + f*x/2)**3 - 2205*c**4*f*tan(e/2 + f*x
/2)**2 + 735*c**4*f*tan(e/2 + f*x/2) - 105*c**4*f) + 84*a*tan(e/2 + f*x/2)*
*2/(105*c**4*f*tan(e/2 + f*x/2)**7 - 735*c**4*f*tan(e/2 + f*x/2)**6 + 2205*
c**4*f*tan(e/2 + f*x/2)**5 - 3675*c**4*f*tan(e/2 + f*x/2)**4 + 3675*c**4*f*
```

```
tan(e/2 + f*x/2)**3 - 2205*c**4*f*tan(e/2 + f*x/2)**2 + 735*c**4*f*tan(e/2
+ f*x/2) - 105*c**4*f) - 98*a*tan(e/2 + f*x/2)/(105*c**4*f*tan(e/2 + f*x/2)
**7 - 735*c**4*f*tan(e/2 + f*x/2)**6 + 2205*c**4*f*tan(e/2 + f*x/2)**5 - 36
75*c**4*f*tan(e/2 + f*x/2)**4 + 3675*c**4*f*tan(e/2 + f*x/2)**3 - 2205*c**4
*f*tan(e/2 + f*x/2)**2 + 735*c**4*f*tan(e/2 + f*x/2) - 105*c**4*f) - 16*a/(
105*c**4*f*tan(e/2 + f*x/2)**7 - 735*c**4*f*tan(e/2 + f*x/2)**6 + 2205*c**4
*f*tan(e/2 + f*x/2)**5 - 3675*c**4*f*tan(e/2 + f*x/2)**4 + 3675*c**4*f*tan(
e/2 + f*x/2)**3 - 2205*c**4*f*tan(e/2 + f*x/2)**2 + 735*c**4*f*tan(e/2 + f*
x/2) - 105*c**4*f), Ne(f, 0)), (x*(a*sin(e) + a)/(-c*sin(e) + c)**4, True))
```

Giac [A] time = 1.7361, size = 154, normalized size = 1.67

$$-\frac{2\left(105\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{6}-210\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{5}+455\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-350\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+273\,a\tan\right.}{105\,c^{4}f\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e))^4,x, algorithm="giac")
```

```
[Out] -2/105*(105*a*tan(1/2*f*x + 1/2*e)^6 - 210*a*tan(1/2*f*x + 1/2*e)^5 + 455*a
*tan(1/2*f*x + 1/2*e)^4 - 350*a*tan(1/2*f*x + 1/2*e)^3 + 273*a*tan(1/2*f*x
+ 1/2*e)^2 - 56*a*tan(1/2*f*x + 1/2*e) + 23*a)/(c^4*f*(tan(1/2*f*x + 1/2*e)
 - 1)^7)
```

3.235 $\int \frac{a+a\sin(e+fx)}{(c-c\sin(e+fx))^5}\,dx$

Optimal. Leaf size=126

$$\frac{2ac\cos^3(e+fx)}{315f\left(c^2-c^2\sin(e+fx)\right)^3}+\frac{2a\cos^3(e+fx)}{105cf(c-c\sin(e+fx))^4}+\frac{a\cos^3(e+fx)}{21f(c-c\sin(e+fx))^5}+\frac{ac\cos^3(e+fx)}{9f(c-c\sin(e+fx))^6}$$

```
[Out] (a*c*Cos[e + f*x]^3)/(9*f*(c - c*Sin[e + f*x])^6) + (a*Cos[e + f*x]^3)/(21*
f*(c - c*Sin[e + f*x])^5) + (2*a*Cos[e + f*x]^3)/(105*c*f*(c - c*Sin[e + f*
x])^4) + (2*a*c*Cos[e + f*x]^3)/(315*f*(c^2 - c^2*Sin[e + f*x])^3)
```

Rubi [A] time = 0.219617, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2736, 2672, 2671}

$$\frac{2ac\cos^3(e+fx)}{315f\left(c^2-c^2\sin(e+fx)\right)^3}+\frac{2a\cos^3(e+fx)}{105cf(c-c\sin(e+fx))^4}+\frac{a\cos^3(e+fx)}{21f(c-c\sin(e+fx))^5}+\frac{ac\cos^3(e+fx)}{9f(c-c\sin(e+fx))^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])/(c - c*Sin[e + f*x])^5,x]
```


```
[Out] (a*c*Cos[e + f*x]^3)/(9*f*(c - c*Sin[e + f*x])^6) + (a*Cos[e + f*x]^3)/(21*
f*(c - c*Sin[e + f*x])^5) + (2*a*Cos[e + f*x]^3)/(105*c*f*(c - c*Sin[e + f*
x])^4) + (2*a*c*Cos[e + f*x]^3)/(315*f*(c^2 - c^2*Sin[e + f*x])^3)
```


Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rubi steps

$$\int \frac{a+a\sin(e+fx)}{(c-c\sin(e+fx))^5}\,dx = (ac)\int \frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^6}\,dx$$
$$= \frac{ac\cos^3(e+fx)}{9f(c-c\sin(e+fx))^6} + \frac{1}{3}a\int \frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^5}\,dx$$
$$= \frac{ac\cos^3(e+fx)}{9f(c-c\sin(e+fx))^6} + \frac{a\cos^3(e+fx)}{21f(c-c\sin(e+fx))^5} + \frac{(2a)\int \frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^4}\,dx}{21c}$$
$$= \frac{ac\cos^3(e+fx)}{9f(c-c\sin(e+fx))^6} + \frac{a\cos^3(e+fx)}{21f(c-c\sin(e+fx))^5} + \frac{2a\cos^3(e+fx)}{105cf(c-c\sin(e+fx))^4} + \frac{(2a)\int \frac{\cos}{(c-}}{1}$$
$$= \frac{ac\cos^3(e+fx)}{9f(c-c\sin(e+fx))^6} + \frac{a\cos^3(e+fx)}{21f(c-c\sin(e+fx))^5} + \frac{2a\cos^3(e+fx)}{105cf(c-c\sin(e+fx))^4} + \frac{2a\,\mathrm{c}}{315c^2f(c}$$

Mathematica [A] time = 0.584732, size = 124, normalized size = 0.98

$$\frac{a\left(36\sin\left(2e+\frac{5fx}{2}\right)-\sin\left(4e+\frac{9fx}{2}\right)+315\cos\left(e+\frac{fx}{2}\right)-84\cos\left(e+\frac{3fx}{2}\right)+9\cos\left(3e+\frac{7fx}{2}\right)+189\sin\left(\frac{fx}{2}\right)\right)}{1260c^5f\left(\cos\left(\frac{e}{2}\right)-\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])/(c - c*Sin[e + f*x])^5,x]
```

```
[Out] (a*(315*Cos[e + (f*x)/2] - 84*Cos[e + (3*f*x)/2] + 9*Cos[3*e + (7*f*x)/2] +
 189*Sin[(f*x)/2] + 36*Sin[2*e + (5*f*x)/2] - Sin[4*e + (9*f*x)/2]))/(1260*
c^5*f*(Cos[e/2] - Sin[e/2])*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9)
```

Maple [A] time = 0.096, size = 146, normalized size = 1.2

$$2\,{\frac{a}{f{c}^{5}}}\left(-{\frac{148}{3\left(\tan\left(1/2\,fx+e/2\right)-1\right)^{6}}}-16\,\left(\tan\left(1/2\,fx+e/2\right)-1\right)^{-8}-{\frac{46}{3\left(\tan\left(1/2\,fx+e/2\right)-1\right)^{3}}}-5\,\left(\tan\left(1/2\,fx+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))/(c-c*sin(f*x+e))^5,x)
```

```
[Out] 2/f*a/c^5*(-148/3/(tan(1/2*f*x+1/2*e)-1)^6-16/(tan(1/2*f*x+1/2*e)-1)^8-46/3
/(tan(1/2*f*x+1/2*e)-1)^3-5/(tan(1/2*f*x+1/2*e)-1)^2-236/5/(tan(1/2*f*x+1/2
*e)-1)^5-1/(tan(1/2*f*x+1/2*e)-1)-32/9/(tan(1/2*f*x+1/2*e)-1)^9-32/(tan(1/2
*f*x+1/2*e)-1)^4-248/7/(tan(1/2*f*x+1/2*e)-1)^7)
```

Maxima [B] time = 1.26822, size = 990, normalized size = 7.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e))^5,x, algorithm="maxima")
```

```
[Out] -2/315*(a*(432*sin(f*x + e)/(cos(f*x + e) + 1) - 1728*sin(f*x + e)^2/(cos(f
*x + e) + 1)^2 + 3612*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 5418*sin(f*x +
e)^4/(cos(f*x + e) + 1)^4 + 5040*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 3360
*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 1260*sin(f*x + e)^7/(cos(f*x + e) +
1)^7 - 315*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 83)/(c^5 - 9*c^5*sin(f*x +
 e)/(cos(f*x + e) + 1) + 36*c^5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 84*c^
5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 126*c^5*sin(f*x + e)^4/(cos(f*x + e
) + 1)^4 - 126*c^5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 84*c^5*sin(f*x + e
)^6/(cos(f*x + e) + 1)^6 - 36*c^5*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 9*c
^5*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - c^5*sin(f*x + e)^9/(cos(f*x + e) +
 1)^9) - 5*a*(45*sin(f*x + e)/(cos(f*x + e) + 1) - 117*sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 + 273*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 315*sin(f*x + e
)^4/(cos(f*x + e) + 1)^4 + 315*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 147*si
n(f*x + e)^6/(cos(f*x + e) + 1)^6 + 63*sin(f*x + e)^7/(cos(f*x + e) + 1)^7
- 5)/(c^5 - 9*c^5*sin(f*x + e)/(cos(f*x + e) + 1) + 36*c^5*sin(f*x + e)^2/(
cos(f*x + e) + 1)^2 - 84*c^5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 126*c^5*
sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 126*c^5*sin(f*x + e)^5/(cos(f*x + e)
+ 1)^5 + 84*c^5*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 36*c^5*sin(f*x + e)^7
/(cos(f*x + e) + 1)^7 + 9*c^5*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - c^5*sin
(f*x + e)^9/(cos(f*x + e) + 1)^9))/f
```

Fricas [B] time = 1.38339, size = 653, normalized size = 5.18

$$-\frac{2\,a\cos\left(fx+e\right)^{5}-8\,a\cos\left(fx+e\right)^{4}-25\,a\cos\left(fx+e\right)^{3}+20\,a\cos\left(fx+e\right)^{2}-35\,a\cos\left(fx+e\right)+(2\,a\cos(}{315\,(c^{5}f\cos\left(fx+e\right)^{5}+5\,c^{5}f\cos\left(fx+e\right)^{4}-8\,c^{5}f\cos\left(fx+e\right)^{3}-20\,c^{5}f\cos\left(fx+e\right)^{2}+8\,c^{5}f\cos\left(fx+e\right)+16\,c^{5}f-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e))^5,x, algorithm="fricas")
```

```
[Out] -1/315*(2*a*cos(f*x + e)^5 - 8*a*cos(f*x + e)^4 - 25*a*cos(f*x + e)^3 + 20*
a*cos(f*x + e)^2 - 35*a*cos(f*x + e) + (2*a*cos(f*x + e)^4 + 10*a*cos(f*x +
 e)^3 - 15*a*cos(f*x + e)^2 - 35*a*cos(f*x + e) - 70*a)*sin(f*x + e) - 70*a
)/(c^5*f*cos(f*x + e)^5 + 5*c^5*f*cos(f*x + e)^4 - 8*c^5*f*cos(f*x + e)^3 -
 20*c^5*f*cos(f*x + e)^2 + 8*c^5*f*cos(f*x + e) + 16*c^5*f - (c^5*f*cos(f*x
 + e)^4 - 4*c^5*f*cos(f*x + e)^3 - 12*c^5*f*cos(f*x + e)^2 + 8*c^5*f*cos(f*
x + e) + 16*c^5*f)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e))**5,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.68009, size = 194, normalized size = 1.54

$$-\frac{2\left(315\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{8}-945\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{7}+2625\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{6}-3465\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{5}+3843\,a\right.}{315\,c^{5}f\left(\tan\left(\frac{1}{2}\,fx+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e))^5,x, algorithm="giac")
```

```
[Out] -2/315*(315*a*tan(1/2*f*x + 1/2*e)^8 - 945*a*tan(1/2*f*x + 1/2*e)^7 + 2625*
a*tan(1/2*f*x + 1/2*e)^6 - 3465*a*tan(1/2*f*x + 1/2*e)^5 + 3843*a*tan(1/2*f
*x + 1/2*e)^4 - 2247*a*tan(1/2*f*x + 1/2*e)^3 + 1143*a*tan(1/2*f*x + 1/2*e)
^2 - 207*a*tan(1/2*f*x + 1/2*e) + 58*a)/(c^5*f*(tan(1/2*f*x + 1/2*e) - 1)^9
)
```

3.236 $\int (a+a\sin(e+fx))^2(c-c\sin(e+fx))^5\,dx$

Optimal. Leaf size=152

$$\frac{3a^2c^5\cos^5(e+fx)}{10f}+\frac{3a^2c^5\sin(e+fx)\cos^3(e+fx)}{8f}+\frac{a^2c^3\cos^5(e+fx)(c-c\sin(e+fx))^2}{7f}+\frac{3a^2\cos^5(e+fx)\left(c^5-c^5\,\mathrm{s}\right.}{14f}$$

```
[Out] (9*a^2*c^5*x)/16 + (3*a^2*c^5*Cos[e + f*x]^5)/(10*f) + (9*a^2*c^5*Cos[e + f
*x]*Sin[e + f*x])/(16*f) + (3*a^2*c^5*Cos[e + f*x]^3*Sin[e + f*x])/(8*f) +
(a^2*c^3*Cos[e + f*x]^5*(c - c*Sin[e + f*x])^2)/(7*f) + (3*a^2*Cos[e + f*x]
^5*(c^5 - c^5*Sin[e + f*x]))/(14*f)
```

Rubi [A] time = 0.196606, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {2736, 2678, 2669, 2635, 8}

$$\frac{3a^2c^5\cos^5(e+fx)}{10f}+\frac{3a^2c^5\sin(e+fx)\cos^3(e+fx)}{8f}+\frac{a^2c^3\cos^5(e+fx)(c-c\sin(e+fx))^2}{7f}+\frac{3a^2\cos^5(e+fx)\left(c^5-c^5\,\mathrm{s}\right.}{14f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^5,x]
```

```
[Out] (9*a^2*c^5*x)/16 + (3*a^2*c^5*Cos[e + f*x]^5)/(10*f) + (9*a^2*c^5*Cos[e + f
*x]*Sin[e + f*x])/(16*f) + (3*a^2*c^5*Cos[e + f*x]^3*Sin[e + f*x])/(8*f) +
(a^2*c^3*Cos[e + f*x]^5*(c - c*Sin[e + f*x])^2)/(7*f) + (3*a^2*Cos[e + f*x]
^5*(c^5 - c^5*Sin[e + f*x]))/(14*f)
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
```

```
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (a+a\sin(e+fx))^2(c-c\sin(e+fx))^5\,dx = \left(a^2c^2\right)\int \cos^4(e+fx)(c-c\sin(e+fx))^3\,dx$$
$$= \frac{a^2c^3\cos^5(e+fx)(c-c\sin(e+fx))^2}{7f} + \frac{1}{7}\left(9a^2c^3\right)\int \cos^4(e+fx)(c-c$$
$$= \frac{a^2c^3\cos^5(e+fx)(c-c\sin(e+fx))^2}{7f} + \frac{3a^2\cos^5(e+fx)\left(c^5-c^5\sin(e\right.}{14f}$$
$$= \frac{3a^2c^5\cos^5(e+fx)}{10f} + \frac{a^2c^3\cos^5(e+fx)(c-c\sin(e+fx))^2}{7f} + \frac{3a^2\cos^5(}{}$$
$$= \frac{3a^2c^5\cos^5(e+fx)}{10f} + \frac{3a^2c^5\cos^3(e+fx)\sin(e+fx)}{8f} + \frac{a^2c^3\cos^5(e+f}{}$$
$$= \frac{3a^2c^5\cos^5(e+fx)}{10f} + \frac{9a^2c^5\cos(e+fx)\sin(e+fx)}{16f} + \frac{3a^2c^5\cos^3(e+f}{8f}$$
$$= \frac{9}{16}a^2c^5x + \frac{3a^2c^5\cos^5(e+fx)}{10f} + \frac{9a^2c^5\cos(e+fx)\sin(e+fx)}{16f} + \frac{3a^2c^5}{}$$

Mathematica [A] time = 1.12437, size = 89, normalized size = 0.59

$$\frac{a^2c^5(665\sin(2(e+fx))-35\sin(4(e+fx))-35\sin(6(e+fx))+945\cos(e+fx)+455\cos(3(e+fx))+77\cos(5(e+}{2240f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^5,x]
```

```
[Out] (a^2*c^5*(1260*e + 1260*f*x + 945*Cos[e + f*x] + 455*Cos[3*(e + f*x)] + 77*
Cos[5*(e + f*x)] - 5*Cos[7*(e + f*x)] + 665*Sin[2*(e + f*x)] - 35*Sin[4*(e
+ f*x)] - 35*Sin[6*(e + f*x)]))/(2240*f)
```

Maple [A] time = 0.022, size = 255, normalized size = 1.7

$$\frac{1}{f}\left(\frac{c^5a^2\cos(fx+e)}{7}\left(\frac{16}{5}+(\sin(fx+e))^6+\frac{6(\sin(fx+e))^4}{5}+\frac{8(\sin(fx+e))^2}{5}\right)+3c^5a^2\left(-1/6\left((\sin(fx+e))^5\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e))^5,x)
```

```
[Out] 1/f*(1/7*c^5*a^2*(16/5+sin(f*x+e)^6+6/5*sin(f*x+e)^4+8/5*sin(f*x+e)^2)*cos(
f*x+e)+3*c^5*a^2*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(
f*x+e)+5/16*f*x+5/16*e)+1/5*c^5*a^2*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos
(f*x+e)-5*c^5*a^2*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/
```

```
8*e)-5/3*c^5*a^2*(2+sin(f*x+e)^2)*cos(f*x+e)+c^5*a^2*(-1/2*sin(f*x+e)*cos(f
*x+e)+1/2*f*x+1/2*e)+3*c^5*a^2*cos(f*x+e)+c^5*a^2*(f*x+e))
```

Maxima [A] time = 1.18755, size = 346, normalized size = 2.28

$$-\frac{192\left(5\cos\left(fx+e\right)^{7}-21\cos\left(fx+e\right)^{5}+35\cos\left(fx+e\right)^{3}-35\cos\left(fx+e\right)\right)a^{2}c^{5}-448\left(3\cos\left(fx+e\right)^{5}-10\cos\left(fx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e))^5,x, algorithm="maxima")
```

```
[Out] -1/6720*(192*(5*cos(f*x + e)^7 - 21*cos(f*x + e)^5 + 35*cos(f*x + e)^3 - 35
*cos(f*x + e))*a^2*c^5 - 448*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos
(f*x + e))*a^2*c^5 - 11200*(cos(f*x + e)^3 - 3*cos(f*x + e))*a^2*c^5 - 105*
(4*sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*sin(4*f*x + 4*e) - 48*sin(2*f*x +
 2*e))*a^2*c^5 + 1050*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e
))*a^2*c^5 - 1680*(2*f*x + 2*e - sin(2*f*x + 2*e))*a^2*c^5 - 6720*(f*x + e)
*a^2*c^5 - 20160*a^2*c^5*cos(f*x + e))/f
```

Fricas [A] time = 1.45072, size = 246, normalized size = 1.62

$$-\frac{80\,a^{2}c^{5}\cos\left(fx+e\right)^{7}-448\,a^{2}c^{5}\cos\left(fx+e\right)^{5}-315\,a^{2}c^{5}fx+35\left(8\,a^{2}c^{5}\cos\left(fx+e\right)^{5}-6\,a^{2}c^{5}\cos\left(fx+e\right)^{3}-9\,a^{2}c^{5}\cos\right.}{560\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e))^5,x, algorithm="fricas")
```

```
[Out] -1/560*(80*a^2*c^5*cos(f*x + e)^7 - 448*a^2*c^5*cos(f*x + e)^5 - 315*a^2*c^
5*f*x + 35*(8*a^2*c^5*cos(f*x + e)^5 - 6*a^2*c^5*cos(f*x + e)^3 - 9*a^2*c^5
*cos(f*x + e))*sin(f*x + e))/f
```

Sympy [A] time = 19.7392, size = 629, normalized size = 4.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(c-c*sin(f*x+e))**5,x)
```

```
[Out] Piecewise((15*a**2*c**5*x*sin(e + f*x)**6/16 + 45*a**2*c**5*x*sin(e + f*x)*
*4*cos(e + f*x)**2/16 - 15*a**2*c**5*x*sin(e + f*x)**4/8 + 45*a**2*c**5*x*s
in(e + f*x)**2*cos(e + f*x)**4/16 - 15*a**2*c**5*x*sin(e + f*x)**2*cos(e +
f*x)**2/4 + a**2*c**5*x*sin(e + f*x)**2/2 + 15*a**2*c**5*x*cos(e + f*x)**6/
16 - 15*a**2*c**5*x*cos(e + f*x)**4/8 + a**2*c**5*x*cos(e + f*x)**2/2 + a**
2*c**5*x + a**2*c**5*sin(e + f*x)**6*cos(e + f*x)/f - 33*a**2*c**5*sin(e +
f*x)**5*cos(e + f*x)/(16*f) + 2*a**2*c**5*sin(e + f*x)**4*cos(e + f*x)**3/f
 + a**2*c**5*sin(e + f*x)**4*cos(e + f*x)/f - 5*a**2*c**5*sin(e + f*x)**3*c
os(e + f*x)**3/(2*f) + 25*a**2*c**5*sin(e + f*x)**3*cos(e + f*x)/(8*f) + 8*
a**2*c**5*sin(e + f*x)**2*cos(e + f*x)**5/(5*f) + 4*a**2*c**5*sin(e + f*x)*
```

```
*2*cos(e + f*x)**3/(3*f) - 5*a**2*c**5*sin(e + f*x)**2*cos(e + f*x)/f - 15*
a**2*c**5*sin(e + f*x)*cos(e + f*x)**5/(16*f) + 15*a**2*c**5*sin(e + f*x)*c
os(e + f*x)**3/(8*f) - a**2*c**5*sin(e + f*x)*cos(e + f*x)/(2*f) + 16*a**2*
c**5*cos(e + f*x)**7/(35*f) + 8*a**2*c**5*cos(e + f*x)**5/(15*f) - 10*a**2*
c**5*cos(e + f*x)**3/(3*f) + 3*a**2*c**5*cos(e + f*x)/f, Ne(f, 0)), (x*(a*s
in(e) + a)**2*(-c*sin(e) + c)**5, True))
```

Giac [A] time = 1.65352, size = 208, normalized size = 1.37

$$\frac{9}{16}\,a^2c^5x-\frac{a^2c^5\cos\left(7\,fx+7\,e\right)}{448\,f}+\frac{11\,a^2c^5\cos\left(5\,fx+5\,e\right)}{320\,f}+\frac{13\,a^2c^5\cos\left(3\,fx+3\,e\right)}{64\,f}+\frac{27\,a^2c^5\cos\left(fx+e\right)}{64\,f}-\frac{a^2c^5\sin}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e))^5,x, algorithm="giac")
```

```
[Out] 9/16*a^2*c^5*x - 1/448*a^2*c^5*cos(7*f*x + 7*e)/f + 11/320*a^2*c^5*cos(5*f*
x + 5*e)/f + 13/64*a^2*c^5*cos(3*f*x + 3*e)/f + 27/64*a^2*c^5*cos(f*x + e)/
f - 1/64*a^2*c^5*sin(6*f*x + 6*e)/f - 1/64*a^2*c^5*sin(4*f*x + 4*e)/f + 19/
64*a^2*c^5*sin(2*f*x + 2*e)/f
```

3.237 $\int (a+a\sin(e+fx))^2(c-c\sin(e+fx))^4\,dx$

Optimal. Leaf size=118

$$\frac{7a^2c^4\cos^5(e+fx)}{30f}+\frac{a^2\cos^5(e+fx)\left(c^4-c^4\sin(e+fx)\right)}{6f}+\frac{7a^2c^4\sin(e+fx)\cos^3(e+fx)}{24f}+\frac{7a^2c^4\sin(e+fx)\cos(e+\ldots}{16f}$$

```
[Out] (7*a^2*c^4*x)/16 + (7*a^2*c^4*Cos[e + f*x]^5)/(30*f) + (7*a^2*c^4*Cos[e + f
*x]*Sin[e + f*x])/(16*f) + (7*a^2*c^4*Cos[e + f*x]^3*Sin[e + f*x])/(24*f) +
 (a^2*Cos[e + f*x]^5*(c^4 - c^4*Sin[e + f*x]))/(6*f)
```

Rubi [A] time = 0.1447, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {2736, 2678, 2669, 2635, 8}

$$\frac{7a^2c^4\cos^5(e+fx)}{30f}+\frac{a^2\cos^5(e+fx)\left(c^4-c^4\sin(e+fx)\right)}{6f}+\frac{7a^2c^4\sin(e+fx)\cos^3(e+fx)}{24f}+\frac{7a^2c^4\sin(e+fx)\cos(e+\ldots}{16f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^4,x]
```

```
[Out] (7*a^2*c^4*x)/16 + (7*a^2*c^4*Cos[e + f*x]^5)/(30*f) + (7*a^2*c^4*Cos[e + f
*x]*Sin[e + f*x])/(16*f) + (7*a^2*c^4*Cos[e + f*x]^3*Sin[e + f*x])/(24*f) +
 (a^2*Cos[e + f*x]^5*(c^4 - c^4*Sin[e + f*x]))/(6*f)
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (a + a\sin(e+fx))^2(c - c\sin(e+fx))^4\,dx = (a^2c^2)\int \cos^4(e+fx)(c - c\sin(e+fx))^2\,dx$$
$$= \frac{a^2\cos^5(e+fx)\left(c^4 - c^4\sin(e+fx)\right)}{6f} + \frac{1}{6}\left(7a^2c^3\right)\int \cos^4(e+fx)(c - c$$
$$= \frac{7a^2c^4\cos^5(e+fx)}{30f} + \frac{a^2\cos^5(e+fx)\left(c^4 - c^4\sin(e+fx)\right)}{6f} + \frac{1}{6}\left(7a^2c^4\right)$$
$$= \frac{7a^2c^4\cos^5(e+fx)}{30f} + \frac{7a^2c^4\cos^3(e+fx)\sin(e+fx)}{24f} + \frac{a^2\cos^5(e+fx)}{}$$
$$= \frac{7a^2c^4\cos^5(e+fx)}{30f} + \frac{7a^2c^4\cos(e+fx)\sin(e+fx)}{16f} + \frac{7a^2c^4\cos^3(e+f}{24}$$
$$= \frac{7}{16}a^2c^4x + \frac{7a^2c^4\cos^5(e+fx)}{30f} + \frac{7a^2c^4\cos(e+fx)\sin(e+fx)}{16f} + \frac{7a^2c^4}{}$$

Mathematica [A] time = 0.745638, size = 79, normalized size = 0.67

$$\frac{a^2c^4(255\sin(2(e+fx)) + 15\sin(4(e+fx)) - 5\sin(6(e+fx)) + 240\cos(e+fx) + 120\cos(3(e+fx)) + 24\cos(5(e+}{960f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^4,x]
```

```
[Out] (a^2*c^4*(420*e + 420*f*x + 240*Cos[e + f*x] + 120*Cos[3*(e + f*x)] + 24*Co
s[5*(e + f*x)] + 255*Sin[2*(e + f*x)] + 15*Sin[4*(e + f*x)] - 5*Sin[6*(e +
f*x)]))/(960*f)
```

Maple [A] time = 0.022, size = 211, normalized size = 1.8

$$\frac{1}{f}\left(c^4a^2\left(-\frac{\cos(fx+e)}{6}\left((\sin(fx+e))^5 + \frac{5(\sin(fx+e))^3}{4} + \frac{15\sin(fx+e)}{8}\right) + \frac{5fx}{16} + \frac{5e}{16}\right) + \frac{2c^4a^2\cos(fx+e)}{5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e))^4,x)
```

```
[Out] 1/f*(c^4*a^2*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+
e)+5/16*f*x+5/16*e)+2/5*c^4*a^2*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x
+e)-c^4*a^2*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-4
/3*c^4*a^2*(2+sin(f*x+e)^2)*cos(f*x+e)-c^4*a^2*(-1/2*sin(f*x+e)*cos(f*x+e)+
1/2*f*x+1/2*e)+2*c^4*a^2*cos(f*x+e)+c^4*a^2*(f*x+e))
```

Maxima [A] time = 1.14419, size = 282, normalized size = 2.39

$$\frac{128\left(3\cos\left(fx+e\right)^{5}-10\cos\left(fx+e\right)^{3}+15\cos\left(fx+e\right)\right)a^{2}c^{4}+1280\left(\cos\left(fx+e\right)^{3}-3\cos\left(fx+e\right)\right)a^{2}c^{4}+5\left(4\sin\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e))^4,x, algorithm="maxima")
```

```
[Out] 1/960*(128*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*a^2*c^4
 + 1280*(cos(f*x + e)^3 - 3*cos(f*x + e))*a^2*c^4 + 5*(4*sin(2*f*x + 2*e)^3
 + 60*f*x + 60*e + 9*sin(4*f*x + 4*e) - 48*sin(2*f*x + 2*e))*a^2*c^4 - 30*(
12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*a^2*c^4 - 240*(2*f*x
 + 2*e - sin(2*f*x + 2*e))*a^2*c^4 + 960*(f*x + e)*a^2*c^4 + 1920*a^2*c^4*c
os(f*x + e))/f
```

Fricas [A] time = 1.44834, size = 207, normalized size = 1.75

$$\frac{96\,a^{2}c^{4}\cos\left(fx+e\right)^{5}+105\,a^{2}c^{4}fx-5\left(8\,a^{2}c^{4}\cos\left(fx+e\right)^{5}-14\,a^{2}c^{4}\cos\left(fx+e\right)^{3}-21\,a^{2}c^{4}\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{240\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e))^4,x, algorithm="fricas")
```

```
[Out] 1/240*(96*a^2*c^4*cos(f*x + e)^5 + 105*a^2*c^4*f*x - 5*(8*a^2*c^4*cos(f*x +
 e)^5 - 14*a^2*c^4*cos(f*x + e)^3 - 21*a^2*c^4*cos(f*x + e))*sin(f*x + e))/
f
```

Sympy [A] time = 10.0939, size = 530, normalized size = 4.49

$$\begin{cases} \frac{5a^{2}c^{4}x\sin^{6}\left(e+fx\right)}{16}+\frac{15a^{2}c^{4}x\sin^{4}\left(e+fx\right)\cos^{2}\left(e+fx\right)}{16}-\frac{3a^{2}c^{4}x\sin^{4}\left(e+fx\right)}{8}+\frac{15a^{2}c^{4}x\sin^{2}\left(e+fx\right)\cos^{4}\left(e+fx\right)}{16}-\frac{3a^{2}c^{4}x\sin^{2}\left(e+fx\right)\cos^{2}\left(e+fx\right)}{4}-\frac{a^{2}}{} \\ x\left(a\sin\left(e\right)+a\right)^{2}\left(-c\sin\left(e\right)+c\right)^{4} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(c-c*sin(f*x+e))**4,x)
```

```
[Out] Piecewise((5*a**2*c**4*x*sin(e + f*x)**6/16 + 15*a**2*c**4*x*sin(e + f*x)**
4*cos(e + f*x)**2/16 - 3*a**2*c**4*x*sin(e + f*x)**4/8 + 15*a**2*c**4*x*sin
(e + f*x)**2*cos(e + f*x)**4/16 - 3*a**2*c**4*x*sin(e + f*x)**2*cos(e + f*x
)**2/4 - a**2*c**4*x*sin(e + f*x)**2/2 + 5*a**2*c**4*x*cos(e + f*x)**6/16 -
 3*a**2*c**4*x*cos(e + f*x)**4/8 - a**2*c**4*x*cos(e + f*x)**2/2 + a**2*c**
4*x - 11*a**2*c**4*sin(e + f*x)**5*cos(e + f*x)/(16*f) + 2*a**2*c**4*sin(e
+ f*x)**4*cos(e + f*x)/f - 5*a**2*c**4*sin(e + f*x)**3*cos(e + f*x)**3/(6*f
) + 5*a**2*c**4*sin(e + f*x)**3*cos(e + f*x)/(8*f) + 8*a**2*c**4*sin(e + f*
x)**2*cos(e + f*x)**3/(3*f) - 4*a**2*c**4*sin(e + f*x)**2*cos(e + f*x)/f -
5*a**2*c**4*sin(e + f*x)*cos(e + f*x)**5/(16*f) + 3*a**2*c**4*sin(e + f*x)*
cos(e + f*x)**3/(8*f) + a**2*c**4*sin(e + f*x)*cos(e + f*x)/(2*f) + 16*a**2
*c**4*cos(e + f*x)**5/(15*f) - 8*a**2*c**4*cos(e + f*x)**3/(3*f) + 2*a**2*c
**4*cos(e + f*x)/f, Ne(f, 0)), (x*(a*sin(e) + a)**2*(-c*sin(e) + c)**4, Tru
```

```
e))
```

Giac [A] time = 1.66635, size = 180, normalized size = 1.53

$$\frac{7}{16}\,a^2c^4x + \frac{a^2c^4\cos\left(5\,fx+5\,e\right)}{40\,f} + \frac{a^2c^4\cos\left(3\,fx+3\,e\right)}{8\,f} + \frac{a^2c^4\cos\left(fx+e\right)}{4\,f} - \frac{a^2c^4\sin\left(6\,fx+6\,e\right)}{192\,f} + \frac{a^2c^4\sin\left(4\,fx+\right.}{64\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e))^4,x, algorithm="giac")
```

```
[Out] 7/16*a^2*c^4*x + 1/40*a^2*c^4*cos(5*f*x + 5*e)/f + 1/8*a^2*c^4*cos(3*f*x +
3*e)/f + 1/4*a^2*c^4*cos(f*x + e)/f - 1/192*a^2*c^4*sin(6*f*x + 6*e)/f + 1/
64*a^2*c^4*sin(4*f*x + 4*e)/f + 17/64*a^2*c^4*sin(2*f*x + 2*e)/f
```

3.238 $\int (a+a\sin(e+fx))^2(c-c\sin(e+fx))^3\,dx$

Optimal. Leaf size=85

$$\frac{a^2c^3\cos^5(e+fx)}{5f}+\frac{a^2c^3\sin(e+fx)\cos^3(e+fx)}{4f}+\frac{3a^2c^3\sin(e+fx)\cos(e+fx)}{8f}+\frac{3}{8}a^2c^3x$$

```
[Out] (3*a^2*c^3*x)/8 + (a^2*c^3*Cos[e + f*x]^5)/(5*f) + (3*a^2*c^3*Cos[e + f*x]*
Sin[e + f*x])/(8*f) + (a^2*c^3*Cos[e + f*x]^3*Sin[e + f*x])/(4*f)
```

Rubi [A] time = 0.0974407, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2736, 2669, 2635, 8}

$$\frac{a^2c^3\cos^5(e+fx)}{5f}+\frac{a^2c^3\sin(e+fx)\cos^3(e+fx)}{4f}+\frac{3a^2c^3\sin(e+fx)\cos(e+fx)}{8f}+\frac{3}{8}a^2c^3x$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^3,x]
```

```
[Out] (3*a^2*c^3*x)/8 + (a^2*c^3*Cos[e + f*x]^5)/(5*f) + (3*a^2*c^3*Cos[e + f*x]*
Sin[e + f*x])/(8*f) + (a^2*c^3*Cos[e + f*x]^3*Sin[e + f*x])/(4*f)
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (a+a\sin(e+fx))^2(c-c\sin(e+fx))^3\,dx = \left(a^2c^2\right)\int \cos^4(e+fx)(c-c\sin(e+fx))\,dx$$
$$= \frac{a^2c^3\cos^5(e+fx)}{5f} + \left(a^2c^3\right)\int \cos^4(e+fx)\,dx$$
$$= \frac{a^2c^3\cos^5(e+fx)}{5f} + \frac{a^2c^3\cos^3(e+fx)\sin(e+fx)}{4f} + \frac{1}{4}\left(3a^2c^3\right)\int \cos^2($$
$$= \frac{a^2c^3\cos^5(e+fx)}{5f} + \frac{3a^2c^3\cos(e+fx)\sin(e+fx)}{8f} + \frac{a^2c^3\cos^3(e+fx)}{4f}$$
$$= \frac{3}{8}a^2c^3x + \frac{a^2c^3\cos^5(e+fx)}{5f} + \frac{3a^2c^3\cos(e+fx)\sin(e+fx)}{8f} + \frac{a^2c^3\text{co}}{}$$

Mathematica [A] time = 1.54283, size = 69, normalized size = 0.81

$$\frac{a^2c^3(40\sin(2(e+fx)) + 5\sin(4(e+fx)) + 20\cos(e+fx) + 10\cos(3(e+fx)) + 2\cos(5(e+fx)) + 60e + 60fx)}{160f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^3,x]
```

```
[Out] (a^2*c^3*(60*e + 60*f*x + 20*Cos[e + f*x] + 10*Cos[3*(e + f*x)] + 2*Cos[5*(
e + f*x)] + 40*Sin[2*(e + f*x)] + 5*Sin[4*(e + f*x)]))/(160*f)
```

Maple [B] time = 0.016, size = 159, normalized size = 1.9

$$\frac{1}{f}\left(\frac{c^3a^2\cos(fx+e)}{5}\left(\frac{8}{3}+(\sin(fx+e))^4+\frac{4(\sin(fx+e))^2}{3}\right)+c^3a^2\left(-\frac{\cos(fx+e)}{4}\left((\sin(fx+e))^3+\frac{3\sin(fx+}{2}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e))^3,x)
```

```
[Out] 1/f*(1/5*c^3*a^2*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)+c^3*a^2*(-1
/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-2/3*c^3*a^2*(2+s
in(f*x+e)^2)*cos(f*x+e)-2*c^3*a^2*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e
)+c^3*a^2*cos(f*x+e)+c^3*a^2*(f*x+e))
```

Maxima [B] time = 1.15238, size = 213, normalized size = 2.51

$$\frac{32\left(3\cos(fx+e)^5-10\cos(fx+e)^3+15\cos(fx+e)\right)a^2c^3+320\left(\cos(fx+e)^3-3\cos(fx+e)\right)a^2c^3+15\left(12fx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 1/480*(32*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*a^2*c^3
+ 320*(cos(f*x + e)^3 - 3*cos(f*x + e))*a^2*c^3 + 15*(12*f*x + 12*e + sin(4
```

```
*f*x + 4*e) - 8*sin(2*f*x + 2*e))*a^2*c^3 - 240*(2*f*x + 2*e - sin(2*f*x +
2*e))*a^2*c^3 + 480*(f*x + e)*a^2*c^3 + 480*a^2*c^3*cos(f*x + e))/f
```

Fricas [A] time = 1.45246, size = 163, normalized size = 1.92

$$\frac{8\,a^2c^3\cos\left(fx+e\right)^5+15\,a^2c^3fx+5\left(2\,a^2c^3\cos\left(fx+e\right)^3+3\,a^2c^3\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{40\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/40*(8*a^2*c^3*cos(f*x + e)^5 + 15*a^2*c^3*f*x + 5*(2*a^2*c^3*cos(f*x + e)
^3 + 3*a^2*c^3*cos(f*x + e))*sin(f*x + e))/f
```

Sympy [A] time = 5.08104, size = 340, normalized size = 4.

$$\begin{cases}\frac{3a^2c^3x\sin^4\left(e+fx\right)}{8}+\frac{3a^2c^3x\sin^2\left(e+fx\right)\cos^2\left(e+fx\right)}{4}-a^2c^3x\sin^2\left(e+fx\right)+\frac{3a^2c^3x\cos^4\left(e+fx\right)}{8}-a^2c^3x\cos^2\left(e+fx\right)+a^2c^3x+\frac{a^2c^3\sin}{} \\ x\left(a\sin\left(e\right)+a\right)^2\left(-c\sin\left(e\right)+c\right)^3\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(c-c*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((3*a**2*c**3*x*sin(e + f*x)**4/8 + 3*a**2*c**3*x*sin(e + f*x)**2*
cos(e + f*x)**2/4 - a**2*c**3*x*sin(e + f*x)**2 + 3*a**2*c**3*x*cos(e + f*x
)**4/8 - a**2*c**3*x*cos(e + f*x)**2 + a**2*c**3*x + a**2*c**3*sin(e + f*x)
**4*cos(e + f*x)/f - 5*a**2*c**3*sin(e + f*x)**3*cos(e + f*x)/(8*f) + 4*a**
2*c**3*sin(e + f*x)**2*cos(e + f*x)**3/(3*f) - 2*a**2*c**3*sin(e + f*x)**2*
cos(e + f*x)/f - 3*a**2*c**3*sin(e + f*x)*cos(e + f*x)**3/(8*f) + a**2*c**3
*sin(e + f*x)*cos(e + f*x)/f + 8*a**2*c**3*cos(e + f*x)**5/(15*f) - 4*a**2*
c**3*cos(e + f*x)**3/(3*f) + a**2*c**3*cos(e + f*x)/f, Ne(f, 0)), (x*(a*sin
(e) + a)**2*(-c*sin(e) + c)**3, True))
```

Giac [A] time = 1.87252, size = 151, normalized size = 1.78

$$\frac{3}{8}\,a^2c^3x+\frac{a^2c^3\cos\left(5\,fx+5\,e\right)}{80\,f}+\frac{a^2c^3\cos\left(3\,fx+3\,e\right)}{16\,f}+\frac{a^2c^3\cos\left(fx+e\right)}{8\,f}+\frac{a^2c^3\sin\left(4\,fx+4\,e\right)}{32\,f}+\frac{a^2c^3\sin\left(2\,fx+2\,e\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 3/8*a^2*c^3*x + 1/80*a^2*c^3*cos(5*f*x + 5*e)/f + 1/16*a^2*c^3*cos(3*f*x +
3*e)/f + 1/8*a^2*c^3*cos(f*x + e)/f + 1/32*a^2*c^3*sin(4*f*x + 4*e)/f + 1/4
*a^2*c^3*sin(2*f*x + 2*e)/f
```

3.239 $\int (a+a\sin (e+fx))^2(c-c\sin (e+fx))^2\,dx$

Optimal. Leaf size=64

$$\frac{a^2c^2\sin(e+fx)\cos^3(e+fx)}{4f}+\frac{3a^2c^2\sin(e+fx)\cos(e+fx)}{8f}+\frac{3}{8}a^2c^2x$$

```
[Out] (3*a^2*c^2*x)/8 + (3*a^2*c^2*Cos[e + f*x]*Sin[e + f*x])/(8*f) + (a^2*c^2*Co
s[e + f*x]^3*Sin[e + f*x])/(4*f)
```

Rubi [A] time = 0.0687816, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2736, 2635, 8}

$$\frac{a^2c^2\sin(e+fx)\cos^3(e+fx)}{4f}+\frac{3a^2c^2\sin(e+fx)\cos(e+fx)}{8f}+\frac{3}{8}a^2c^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^2,x]
```

```
[Out] (3*a^2*c^2*x)/8 + (3*a^2*c^2*Cos[e + f*x]*Sin[e + f*x])/(8*f) + (a^2*c^2*Co
s[e + f*x]^3*Sin[e + f*x])/(4*f)
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int (a+a\sin(e+fx))^2(c-c\sin(e+fx))^2\,dx &= \left(a^2c^2\right)\int \cos^4(e+fx)\,dx\\ &= \frac{a^2c^2\cos^3(e+fx)\sin(e+fx)}{4f}+\frac{1}{4}\left(3a^2c^2\right)\int \cos^2(e+fx)\,dx\\ &= \frac{3a^2c^2\cos(e+fx)\sin(e+fx)}{8f}+\frac{a^2c^2\cos^3(e+fx)\sin(e+fx)}{4f}+\frac{1}{8}\left(3a\right.\\ &= \frac{3}{8}a^2c^2x+\frac{3a^2c^2\cos(e+fx)\sin(e+fx)}{8f}+\frac{a^2c^2\cos^3(e+fx)\sin(e+fx}{4f}\end{aligned}$$

Mathematica [A] time = 0.0484535, size = 39, normalized size = 0.61

$$\frac{a^2c^2(12(e+fx)+8\sin(2(e+fx))+\sin(4(e+fx)))}{32f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^2,x]
```

```
[Out] (a^2*c^2*(12*(e + f*x) + 8*Sin[2*(e + f*x)] + Sin[4*(e + f*x)]))/(32*f)
```

Maple [A] time = 0.014, size = 88, normalized size = 1.4

$$\frac{1}{f}\left(c^2a^2\left(-\frac{\cos\left(fx+e\right)}{4}\left(\left(\sin\left(fx+e\right)\right)^3+\frac{3\sin\left(fx+e\right)}{2}\right)+\frac{3fx}{8}+\frac{3e}{8}\right)-2c^2a^2\left(-1/2\sin\left(fx+e\right)\cos\left(fx+e\right)+1/2f\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e))^2,x)
```

```
[Out] 1/f*(c^2*a^2*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-
2*c^2*a^2*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)+c^2*a^2*(f*x+e))
```

Maxima [A] time = 1.11695, size = 109, normalized size = 1.7

$$\frac{\left(12fx+12e+\sin\left(4fx+4e\right)-8\sin\left(2fx+2e\right)\right)a^2c^2-16\left(2fx+2e-\sin\left(2fx+2e\right)\right)a^2c^2+32\left(fx+e\right)a^2c^2}{32f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/32*((12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*a^2*c^2 - 16*
(2*f*x + 2*e - sin(2*f*x + 2*e))*a^2*c^2 + 32*(f*x + e)*a^2*c^2)/f
```

Fricas [A] time = 1.42943, size = 122, normalized size = 1.91

$$\frac{3a^2c^2fx+\left(2a^2c^2\cos\left(fx+e\right)^3+3a^2c^2\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/8*(3*a^2*c^2*f*x + (2*a^2*c^2*cos(f*x + e)^3 + 3*a^2*c^2*cos(f*x + e))*si
n(f*x + e))/f
```

Sympy [A] time = 2.32217, size = 206, normalized size = 3.22

$$\begin{cases} \frac{3a^2c^2x\sin^4\left(e+fx\right)}{8} + \frac{3a^2c^2x\sin^2\left(e+fx\right)\cos^2\left(e+fx\right)}{4} - a^2c^2x\sin^2\left(e+fx\right) + \frac{3a^2c^2x\cos^4\left(e+fx\right)}{8} - a^2c^2x\cos^2\left(e+fx\right) + a^2c^2x - \frac{5a \\ x\left(a\sin\left(e\right)+a\right)^2\left(-c\sin\left(e\right)+c\right)^2 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(c-c*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((3*a**2*c**2*x*sin(e + f*x)**4/8 + 3*a**2*c**2*x*sin(e + f*x)**2*
cos(e + f*x)**2/4 - a**2*c**2*x*sin(e + f*x)**2 + 3*a**2*c**2*x*cos(e + f*x
)**4/8 - a**2*c**2*x*cos(e + f*x)**2 + a**2*c**2*x - 5*a**2*c**2*sin(e + f*
x)**3*cos(e + f*x)/(8*f) - 3*a**2*c**2*sin(e + f*x)*cos(e + f*x)**3/(8*f) +
 a**2*c**2*sin(e + f*x)*cos(e + f*x)/f, Ne(f, 0)), (x*(a*sin(e) + a)**2*(-c
*sin(e) + c)**2, True))
```

Giac [A] time = 1.97903, size = 70, normalized size = 1.09

$$\frac{3}{8}\,a^2c^2x + \frac{a^2c^2\sin\left(4\,fx+4\,e\right)}{32\,f} + \frac{a^2c^2\sin\left(2\,fx+2\,e\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 3/8*a^2*c^2*x + 1/32*a^2*c^2*sin(4*f*x + 4*e)/f + 1/4*a^2*c^2*sin(2*f*x + 2
*e)/f
```

3.240 $\int (a+a\sin(e+fx))^2(c-c\sin(e+fx))\,dx$

Optimal. Leaf size=52

$$-\frac{a^2c\cos^3(e+fx)}{3f}+\frac{a^2c\sin(e+fx)\cos(e+fx)}{2f}+\frac{1}{2}a^2cx$$

```
[Out] (a^2*c*x)/2 - (a^2*c*Cos[e + f*x]^3)/(3*f) + (a^2*c*Cos[e + f*x]*Sin[e + f*
x])/(2*f)
```

Rubi [A] time = 0.0767365, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2736, 2669, 2635, 8}

$$-\frac{a^2c\cos^3(e+fx)}{3f}+\frac{a^2c\sin(e+fx)\cos(e+fx)}{2f}+\frac{1}{2}a^2cx$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x]),x]
```

```
[Out] (a^2*c*x)/2 - (a^2*c*Cos[e + f*x]^3)/(3*f) + (a^2*c*Cos[e + f*x]*Sin[e + f*
x])/(2*f)
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (a+a\sin(e+fx))^2(c-c\sin(e+fx))\,dx = (ac)\int \cos^2(e+fx)(a+a\sin(e+fx))\,dx$$
$$= -\frac{a^2c\cos^3(e+fx)}{3f} + \left(a^2c\right)\int \cos^2(e+fx)\,dx$$
$$= -\frac{a^2c\cos^3(e+fx)}{3f} + \frac{a^2c\cos(e+fx)\sin(e+fx)}{2f} + \frac{1}{2}\left(a^2c\right)\int 1\,dx$$
$$= \frac{1}{2}a^2cx - \frac{a^2c\cos^3(e+fx)}{3f} + \frac{a^2c\cos(e+fx)\sin(e+fx)}{2f}$$

Mathematica [A] time = 0.339411, size = 43, normalized size = 0.83

$$-\frac{a^2c(-3(\sin(2(e+fx))+2fx)+3\cos(e+fx)+\cos(3(e+fx)))}{12f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x]),x]
```

```
[Out] -(a^2*c*(3*Cos[e + f*x] + Cos[3*(e + f*x)] - 3*(2*f*x + Sin[2*(e + f*x)])))
/(12*f)
```

Maple [A] time = 0.014, size = 78, normalized size = 1.5

$$\frac{1}{f}\left(\frac{a^2c\left(2+\left(\sin\left(fx+e\right)\right)^2\right)\cos\left(fx+e\right)}{3} - a^2c\left(-\frac{\sin\left(fx+e\right)\cos\left(fx+e\right)}{2}+\frac{fx}{2}+\frac{e}{2}\right) - a^2c\cos\left(fx+e\right) + a^2c\left(fx+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e)),x)
```

```
[Out] 1/f*(1/3*a^2*c*(2+sin(f*x+e)^2)*cos(f*x+e)-a^2*c*(-1/2*sin(f*x+e)*cos(f*x+e
)+1/2*f*x+1/2*e)-a^2*c*cos(f*x+e)+a^2*c*(f*x+e))
```

Maxima [A] time = 1.12922, size = 104, normalized size = 2.

$$-\frac{4\left(\cos\left(fx+e\right)^3-3\cos\left(fx+e\right)\right)a^2c+3\left(2fx+2e-\sin\left(2fx+2e\right)\right)a^2c-12\left(fx+e\right)a^2c+12a^2c\cos\left(fx+e\right)}{12f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] -1/12*(4*(cos(f*x + e)^3 - 3*cos(f*x + e))*a^2*c + 3*(2*f*x + 2*e - sin(2*f
*x + 2*e))*a^2*c - 12*(f*x + e)*a^2*c + 12*a^2*c*cos(f*x + e))/f
```

Fricas [A] time = 1.23752, size = 112, normalized size = 2.15

$$-\frac{2\,a^2c\cos\left(fx+e\right)^3-3\,a^2cfx-3\,a^2c\cos\left(fx+e\right)\sin\left(fx+e\right)}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] -1/6*(2*a^2*c*cos(f*x + e)^3 - 3*a^2*c*f*x - 3*a^2*c*cos(f*x + e)*sin(f*x +
 e))/f
```

Sympy [A] time = 1.54697, size = 133, normalized size = 2.56

$$\begin{cases}-\frac{a^2cx\sin^2(e+fx)}{2}-\frac{a^2cx\cos^2(e+fx)}{2}+a^2cx+\frac{a^2c\sin^2(e+fx)\cos(e+fx)}{f}+\frac{a^2c\sin(e+fx)\cos(e+fx)}{2f}+\frac{2a^2c\cos^3(e+fx)}{3f}-\frac{a^2c\cos(e+fx)}{f} & \text{fo}\\ x\left(a\sin(e)+a\right)^2\left(-c\sin(e)+c\right) & \text{ot}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(c-c*sin(f*x+e)),x)
```

```
[Out] Piecewise((-a**2*c*x*sin(e + f*x)**2/2 - a**2*c*x*cos(e + f*x)**2/2 + a**2*
c*x + a**2*c*sin(e + f*x)**2*cos(e + f*x)/f + a**2*c*sin(e + f*x)*cos(e + f
*x)/(2*f) + 2*a**2*c*cos(e + f*x)**3/(3*f) - a**2*c*cos(e + f*x)/f, Ne(f, 0
)), (x*(a*sin(e) + a)**2*(-c*sin(e) + c), True))
```

Giac [A] time = 1.91144, size = 84, normalized size = 1.62

$$\frac{1}{2}\,a^2cx-\frac{a^2c\cos\left(3\,fx+3\,e\right)}{12\,f}-\frac{a^2c\cos\left(fx+e\right)}{4\,f}+\frac{a^2c\sin\left(2\,fx+2\,e\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/2*a^2*c*x - 1/12*a^2*c*cos(3*f*x + 3*e)/f - 1/4*a^2*c*cos(f*x + e)/f + 1/
4*a^2*c*sin(2*f*x + 2*e)/f
```

3.241 $\int \frac{(a+a\sin(e+fx))^2}{c-c\sin(e+fx)}\,dx$

Optimal. Leaf size=57

$$\frac{3a^2\cos(e+fx)}{cf}+\frac{2a^2c\cos^3(e+fx)}{f(c-c\sin(e+fx))^2}-\frac{3a^2x}{c}$$

```
[Out] (-3*a^2*x)/c + (3*a^2*Cos[e + f*x])/(c*f) + (2*a^2*c*Cos[e + f*x]^3)/(f*(c
- c*Sin[e + f*x])^2)
```

Rubi [A] time = 0.140906, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2736, 2680, 2682, 8}

$$\frac{3a^2\cos(e+fx)}{cf}+\frac{2a^2c\cos^3(e+fx)}{f(c-c\sin(e+fx))^2}-\frac{3a^2x}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2/(c - c*Sin[e + f*x]),x]
```

```
[Out] (-3*a^2*x)/c + (3*a^2*Cos[e + f*x])/(c*f) + (2*a^2*c*Cos[e + f*x]^3)/(f*(c
- c*Sin[e + f*x])^2)
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^2}{c-c\sin(e+fx)}\,dx = \left(a^2c^2\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^3}\,dx$$
$$= \frac{2a^2c\cos^3(e+fx)}{f(c-c\sin(e+fx))^2} - \left(3a^2\right)\int \frac{\cos^2(e+fx)}{c-c\sin(e+fx)}\,dx$$
$$= \frac{3a^2\cos(e+fx)}{cf} + \frac{2a^2c\cos^3(e+fx)}{f(c-c\sin(e+fx))^2} - \frac{\left(3a^2\right)\int 1\,dx}{c}$$
$$= -\frac{3a^2x}{c} + \frac{3a^2\cos(e+fx)}{cf} + \frac{2a^2c\cos^3(e+fx)}{f(c-c\sin(e+fx))^2}$$

Mathematica [B] time = 0.371187, size = 130, normalized size = 2.28

$$\frac{a^2(\sin(e+fx)+1)^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)(3(e+fx)-\cos(e+fx))+\sin\left(\frac{1}{2}(e+fx)\right)(\cos\right.}{cf(\sin(e+fx)-1)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2/(c - c*Sin[e + f*x]),x]
```

```
[Out] (a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2]*(3*(e + f*x) -
 Cos[e + f*x]) + (-8 - 3*e - 3*f*x + Cos[e + f*x])*Sin[(e + f*x)/2])*(1 + S
in[e + f*x])^2)/(c*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*(-1 + Sin[e +
f*x]))
```

Maple [A] time = 0.073, size = 73, normalized size = 1.3

$$-8\,\frac{a^2}{cf\left(\tan\left(1/2\,fx+e/2\right)-1\right)}+2\,\frac{a^2}{cf\left(1+\left(\tan\left(1/2\,fx+e/2\right)\right)^2\right)}-6\,\frac{a^2\arctan\left(\tan\left(1/2\,fx+e/2\right)\right)}{cf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e)),x)
```

```
[Out] -8/f*a^2/c/(tan(1/2*f*x+1/2*e)-1)+2/f*a^2/c/(1+tan(1/2*f*x+1/2*e)^2)-6/f*a^
2/c*arctan(tan(1/2*f*x+1/2*e))
```

Maxima [B] time = 1.70054, size = 294, normalized size = 5.16

$$-\frac{2\left(a^2\left(\frac{\frac{\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-2}{c-\frac{c\sin(fx+e)}{\cos(fx+e)+1}+\frac{c\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{c\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}+\frac{\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c}\right)+2\,a^2\left(\frac{\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c}-\frac{1}{c-\frac{c\sin(fx+e)}{\cos(fx+e)+1}}\right)-\frac{a^2}{c-\frac{c\sin(fx+e)}{\cos(fx+e)+1}}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] -2*(a^2*((sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^2/(cos(f*x + e) +
1)^2 - 2)/(c - c*sin(f*x + e)/(cos(f*x + e) + 1) + c*sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 - c*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + arctan(sin(f*x + e
)/(cos(f*x + e) + 1))/c) + 2*a^2*(arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c
 - 1/(c - c*sin(f*x + e)/(cos(f*x + e) + 1))) - a^2/(c - c*sin(f*x + e)/(co
s(f*x + e) + 1)))/f
```

Fricas [A] time = 1.32484, size = 238, normalized size = 4.18

$$-\frac{3\,a^2fx - a^2\cos\left(fx+e\right)^2 - 4\,a^2 + \left(3\,a^2fx - 5\,a^2\right)\cos\left(fx+e\right) - \left(3\,a^2fx - a^2\cos\left(fx+e\right) + 4\,a^2\right)\sin\left(fx+e\right)}{cf\cos\left(fx+e\right) - cf\sin\left(fx+e\right) + cf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] -(3*a^2*f*x - a^2*cos(f*x + e)^2 - 4*a^2 + (3*a^2*f*x - 5*a^2)*cos(f*x + e)
 - (3*a^2*f*x - a^2*cos(f*x + e) + 4*a^2)*sin(f*x + e))/(c*f*cos(f*x + e) -
 c*f*sin(f*x + e) + c*f)
```

Sympy [A] time = 7.49521, size = 456, normalized size = 8.

$$\begin{cases} -\frac{3a^2fx\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)}{cf\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)-cf\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+cf\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-cf} + \frac{3a^2fx\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{cf\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)-cf\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+cf\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-cf} - \frac{3a^2fx\tan\left(\frac{e}{2}+\ldots\right)}{cf\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)-cf\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)\ldots} \\ \frac{x(a\sin(e)+a)^2}{-c\sin(e)+c} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2/(c-c*sin(f*x+e)),x)
```

```
[Out] Piecewise((-3*a**2*f*x*tan(e/2 + f*x/2)**3/(c*f*tan(e/2 + f*x/2)**3 - c*f*t
an(e/2 + f*x/2)**2 + c*f*tan(e/2 + f*x/2) - c*f) + 3*a**2*f*x*tan(e/2 + f*x
/2)**2/(c*f*tan(e/2 + f*x/2)**3 - c*f*tan(e/2 + f*x/2)**2 + c*f*tan(e/2 + f
*x/2) - c*f) - 3*a**2*f*x*tan(e/2 + f*x/2)/(c*f*tan(e/2 + f*x/2)**3 - c*f*t
an(e/2 + f*x/2)**2 + c*f*tan(e/2 + f*x/2) - c*f) + 3*a**2*f*x/(c*f*tan(e/2
+ f*x/2)**3 - c*f*tan(e/2 + f*x/2)**2 + c*f*tan(e/2 + f*x/2) - c*f) - 2*a**
2*tan(e/2 + f*x/2)**3/(c*f*tan(e/2 + f*x/2)**3 - c*f*tan(e/2 + f*x/2)**2 +
c*f*tan(e/2 + f*x/2) - c*f) - 6*a**2*tan(e/2 + f*x/2)**2/(c*f*tan(e/2 + f*x
/2)**3 - c*f*tan(e/2 + f*x/2)**2 + c*f*tan(e/2 + f*x/2) - c*f) - 8*a**2/(c*
f*tan(e/2 + f*x/2)**3 - c*f*tan(e/2 + f*x/2)**2 + c*f*tan(e/2 + f*x/2) - c*
f), Ne(f, 0)), (x*(a*sin(e) + a)**2/(-c*sin(e) + c), True))
```

Giac [A] time = 2.06178, size = 139, normalized size = 2.44

$$-\frac{\frac{3\left(fx+e\right)a^2}{c} + \frac{2\left(4\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2 - a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right) + 5\,a^2\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3 - \tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2 + \tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right) - 1\right)c}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] -(3*(f*x + e)*a^2/c + 2*(4*a^2*tan(1/2*f*x + 1/2*e)^2 - a^2*tan(1/2*f*x + 1
/2*e) + 5*a^2)/((tan(1/2*f*x + 1/2*e)^3 - tan(1/2*f*x + 1/2*e)^2 + tan(1/2*
f*x + 1/2*e) - 1)*c))/f
```

3.242 $\int \frac{(a+a\sin(e+fx))^2}{(c-c\sin(e+fx))^2}\,dx$

Optimal. Leaf size=72

$$-\frac{2a^2\cos(e+fx)}{f\left(c^2-c^2\sin(e+fx)\right)}+\frac{a^2x}{c^2}+\frac{2a^2c\cos^3(e+fx)}{3f(c-c\sin(e+fx))^3}$$

```
[Out] (a^2*x)/c^2 + (2*a^2*c*Cos[e + f*x]^3)/(3*f*(c - c*Sin[e + f*x])^3) - (2*a^
2*Cos[e + f*x])/(f*(c^2 - c^2*Sin[e + f*x]))
```

Rubi [A] time = 0.136745, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2736, 2680, 8}

$$-\frac{2a^2\cos(e+fx)}{f\left(c^2-c^2\sin(e+fx)\right)}+\frac{a^2x}{c^2}+\frac{2a^2c\cos^3(e+fx)}{3f(c-c\sin(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2/(c - c*Sin[e + f*x])^2,x]
```

```
[Out] (a^2*x)/c^2 + (2*a^2*c*Cos[e + f*x]^3)/(3*f*(c - c*Sin[e + f*x])^3) - (2*a^
2*Cos[e + f*x])/(f*(c^2 - c^2*Sin[e + f*x]))
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^2}{(c-c\sin(e+fx))^2}\,dx = \left(a^2c^2\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^4}\,dx$$
$$= \frac{2a^2c\cos^3(e+fx)}{3f(c-c\sin(e+fx))^3} - a^2\int \frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^2}\,dx$$
$$= \frac{2a^2c\cos^3(e+fx)}{3f(c-c\sin(e+fx))^3} - \frac{2a^2\cos(e+fx)}{f\left(c^2-c^2\sin(e+fx)\right)} + \frac{a^2\int 1\,dx}{c^2}$$
$$= \frac{a^2x}{c^2} + \frac{2a^2c\cos^3(e+fx)}{3f(c-c\sin(e+fx))^3} - \frac{2a^2\cos(e+fx)}{f\left(c^2-c^2\sin(e+fx)\right)}$$

Mathematica [A] time = 0.613442, size = 121, normalized size = 1.68

$$-\frac{a^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(-3(3e+3fx+8)\cos\left(\frac{1}{2}(e+fx)\right)+(3e+3fx+16)\cos\left(\frac{3}{2}(e+fx)\right)+6\sin\left(\frac{1}{2}(\right.\right.}{6c^2f(\sin(e+fx)-1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2/(c - c*Sin[e + f*x])^2,x]
```

```
[Out] -(a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(-3*(8 + 3*e + 3*f*x)*Cos[(e +
f*x)/2] + (16 + 3*e + 3*f*x)*Cos[(3*(e + f*x))/2] + 6*(2*(2 + e + f*x) + (e
 + f*x)*Cos[e + f*x])*Sin[(e + f*x)/2]))/(6*c^2*f*(-1 + Sin[e + f*x])^2)
```

Maple [A] time = 0.075, size = 71, normalized size = 1.

$$-\frac{16\,a^2}{3\,fc^2}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{-3} - 8\,\frac{a^2}{fc^2\left(\tan\left(1/2\,fx+e/2\right)-1\right)^2} + 2\,\frac{a^2\arctan\left(\tan\left(1/2\,fx+e/2\right)\right)}{fc^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^2,x)
```

```
[Out] -16/3/f*a^2/c^2/(tan(1/2*f*x+1/2*e)-1)^3-8/f*a^2/c^2/(tan(1/2*f*x+1/2*e)-1)
^2+2/f*a^2/c^2*arctan(tan(1/2*f*x+1/2*e))
```

Maxima [B] time = 1.77773, size = 491, normalized size = 6.82

$$\frac{2\left(a^2\left(\frac{\frac{9\sin(fx+e)}{\cos(fx+e)+1}-\frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-4}{c^2-\frac{3c^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{3c^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{c^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}+\frac{3\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c^2}\right)-\frac{a^2\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}-\frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-2\right)}{c^2-\frac{3c^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{3c^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{c^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}+\frac{2}{c^2-\frac{3c^2\sin(fx+e)}{\cos(fx+e)+1}\cdots}\right.}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 2/3*(a^2*((9*sin(f*x + e)/(cos(f*x + e) + 1) - 3*sin(f*x + e)^2/(cos(f*x +
e) + 1)^2 - 4)/(c^2 - 3*c^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*c^2*sin(f*x
```

```
 + e)^2/(cos(f*x + e) + 1)^2 - c^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 3
*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c^2) - a^2*(3*sin(f*x + e)/(cos(f*
x + e) + 1) - 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 2)/(c^2 - 3*c^2*sin(f
*x + e)/(cos(f*x + e) + 1) + 3*c^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - c^
2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 2*a^2*(3*sin(f*x + e)/(cos(f*x + e
) + 1) - 1)/(c^2 - 3*c^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*c^2*sin(f*x +
e)^2/(cos(f*x + e) + 1)^2 - c^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3))/f
```

Fricas [B] time = 1.35024, size = 362, normalized size = 5.03

$$-\frac{6\,a^2fx-\left(3\,a^2fx+8\,a^2\right)\cos\left(fx+e\right)^2+4\,a^2+\left(3\,a^2fx-4\,a^2\right)\cos\left(fx+e\right)-\left(6\,a^2fx-4\,a^2+\left(3\,a^2fx-8\,a^2\right)\cos\left(\cdots\right.\right.}{3\left(c^2f\cos\left(fx+e\right)^2-c^2f\cos\left(fx+e\right)-2\,c^2f+\left(c^2f\cos\left(fx+e\right)+2\,c^2f\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -1/3*(6*a^2*f*x - (3*a^2*f*x + 8*a^2)*cos(f*x + e)^2 + 4*a^2 + (3*a^2*f*x -
 4*a^2)*cos(f*x + e) - (6*a^2*f*x - 4*a^2 + (3*a^2*f*x - 8*a^2)*cos(f*x + e
))*sin(f*x + e))/(c^2*f*cos(f*x + e)^2 - c^2*f*cos(f*x + e) - 2*c^2*f + (c^
2*f*cos(f*x + e) + 2*c^2*f)*sin(f*x + e))
```

Sympy [A] time = 16.866, size = 486, normalized size = 6.75

$$\begin{cases} \frac{3a^2fx\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)}{3c^2f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)-9c^2f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+9c^2f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-3c^2f} - \frac{9a^2fx\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3c^2f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)-9c^2f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+9c^2f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-3c^2f} + \frac{\cdots}{3c^2f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\cdots\right.} \\ \frac{x(a\sin(e)+a)^2}{(-c\sin(e)+c)^2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2/(c-c*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((3*a**2*f*x*tan(e/2 + f*x/2)**3/(3*c**2*f*tan(e/2 + f*x/2)**3 - 9
*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) - 9*a**
2*f*x*tan(e/2 + f*x/2)**2/(3*c**2*f*tan(e/2 + f*x/2)**3 - 9*c**2*f*tan(e/2
+ f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) + 9*a**2*f*x*tan(e/2 +
f*x/2)/(3*c**2*f*tan(e/2 + f*x/2)**3 - 9*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**
2*f*tan(e/2 + f*x/2) - 3*c**2*f) - 3*a**2*f*x/(3*c**2*f*tan(e/2 + f*x/2)**3
 - 9*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) + 8
*a**2*tan(e/2 + f*x/2)**3/(3*c**2*f*tan(e/2 + f*x/2)**3 - 9*c**2*f*tan(e/2
+ f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) - 24*a**2*tan(e/2 + f*x
/2)**2/(3*c**2*f*tan(e/2 + f*x/2)**3 - 9*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**
2*f*tan(e/2 + f*x/2) - 3*c**2*f), Ne(f, 0)), (x*(a*sin(e) + a)**2/(-c*sin(e
) + c)**2, True))
```

Giac [A] time = 2.00909, size = 81, normalized size = 1.12

$$\frac{\frac{3\left(fx+e\right)a^2}{c^2}-\frac{8\left(3\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-a^2\right)}{c^2\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^3}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/3*(3*(f*x + e)*a^2/c^2 - 8*(3*a^2*tan(1/2*f*x + 1/2*e) - a^2)/(c^2*(tan(1
/2*f*x + 1/2*e) - 1)^3))/f
```

3.243 $\int \frac{(a+a\sin(e+fx))^2}{(c-c\sin(e+fx))^3}\,dx$

Optimal. Leaf size=34

$$\frac{a^2c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^5}$$

```
[Out] (a^2*c^2*Cos[e + f*x]^5)/(5*f*(c - c*Sin[e + f*x])^5)
```

Rubi [A] time = 0.0938423, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2736, 2671}

$$\frac{a^2c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2/(c - c*Sin[e + f*x])^3,x]
```

```
[Out] (a^2*c^2*Cos[e + f*x]^5)/(5*f*(c - c*Sin[e + f*x])^5)
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+a\sin(e+fx))^2}{(c-c\sin(e+fx))^3}\,dx &= \left(a^2c^2\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^5}\,dx\\ &= \frac{a^2c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^5}\end{aligned}$$

Mathematica [B] time = 0.401749, size = 81, normalized size = 2.38

$$\frac{a^2\left(-10\sin\left(\frac{1}{2}(e+fx)\right)-5\sin\left(\frac{3}{2}(e+fx)\right)+\sin\left(\frac{5}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}{10c^3f(\sin(e+fx)-1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2/(c - c*Sin[e + f*x])^3,x]
```

```
[Out] (a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(-10*Sin[(e + f*x)/2] - 5*Sin[(3
*(e + f*x))/2] + Sin[(5*(e + f*x))/2]))/(10*c^3*f*(-1 + Sin[e + f*x])^3)
```

Maple [B] time = 0.088, size = 88, normalized size = 2.6

$$2\,\frac{a^2}{fc^3}\left(-\frac{16}{5\left(\tan\left(1/2\,fx+e/2\right)-1\right)^5}-8\,\left(\tan\left(1/2\,fx+e/2\right)-1\right)^{-4}-8\,\left(\tan\left(1/2\,fx+e/2\right)-1\right)^{-3}-\left(\tan\left(1/2\,fx+e/2\right)-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^3,x)
```

```
[Out] 2/f*a^2/c^3*(-16/5/(tan(1/2*f*x+1/2*e)-1)^5-8/(tan(1/2*f*x+1/2*e)-1)^4-8/(t
an(1/2*f*x+1/2*e)-1)^3-1/(tan(1/2*f*x+1/2*e)-1)-4/(tan(1/2*f*x+1/2*e)-1)^2)
```

Maxima [B] time = 1.2695, size = 752, normalized size = 22.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] -2/15*(a^2*(20*sin(f*x + e)/(cos(f*x + e) + 1) - 40*sin(f*x + e)^2/(cos(f*x
 + e) + 1)^2 + 30*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 15*sin(f*x + e)^4/(
cos(f*x + e) + 1)^4 - 7)/(c^3 - 5*c^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*
c^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 10*c^3*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3 + 5*c^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - c^3*sin(f*x + e)^5/
(cos(f*x + e) + 1)^5) - 6*a^2*(5*sin(f*x + e)/(cos(f*x + e) + 1) - 5*sin(f*
x + e)^2/(cos(f*x + e) + 1)^2 + 5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 1)/
(c^3 - 5*c^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*c^3*sin(f*x + e)^2/(cos(f
*x + e) + 1)^2 - 10*c^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*c^3*sin(f*x
 + e)^4/(cos(f*x + e) + 1)^4 - c^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 2
*a^2*(5*sin(f*x + e)/(cos(f*x + e) + 1) - 10*sin(f*x + e)^2/(cos(f*x + e) +
 1)^2 - 1)/(c^3 - 5*c^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*c^3*sin(f*x +
e)^2/(cos(f*x + e) + 1)^2 - 10*c^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*
c^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - c^3*sin(f*x + e)^5/(cos(f*x + e)
+ 1)^5))/f
```

Fricas [B] time = 1.28247, size = 398, normalized size = 11.71

$$\frac{a^2\cos\left(fx+e\right)^3+3\,a^2\cos\left(fx+e\right)^2-2\,a^2\cos\left(fx+e\right)-4\,a^2+\left(a^2\cos\left(fx+e\right)^2-2\,a^2\cos\left(fx+e\right)-4\,a^2\right)\sin\left(f\right.}{5\left(c^3f\cos\left(fx+e\right)^3+3\,c^3f\cos\left(fx+e\right)^2-2\,c^3f\cos\left(fx+e\right)-4\,c^3f-\left(c^3f\cos\left(fx+e\right)^2-2\,c^3f\cos\left(fx+e\right)-4\,c^3f\right)\mathrm{s}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/5*(a^2*cos(f*x + e)^3 + 3*a^2*cos(f*x + e)^2 - 2*a^2*cos(f*x + e) - 4*a^2
 + (a^2*cos(f*x + e)^2 - 2*a^2*cos(f*x + e) - 4*a^2)*sin(f*x + e))/(c^3*f*c
```

```
os(f*x + e)^3 + 3*c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x + e) - 4*c^3*f - (
c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x + e) - 4*c^3*f)*sin(f*x + e))
```

Sympy [A] time = 49.3753, size = 364, normalized size = 10.71

$$\begin{cases} -\frac{2a^2\tan^5\left(\frac{e}{2}+\frac{fx}{2}\right)}{5c^3f\tan^5\left(\frac{e}{2}+\frac{fx}{2}\right)-25c^3f\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)+50c^3f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)-50c^3f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+25c^3f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-5c^3f} - \frac{}{5c^3f\tan^5\left(\frac{e}{2}+\frac{fx}{2}\right)-25c^3f\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)} \\ \frac{x(a\sin(e)+a)^2}{(-c\sin(e)+c)^3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2/(c-c*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((-2*a**2*tan(e/2 + f*x/2)**5/(5*c**3*f*tan(e/2 + f*x/2)**5 - 25*c
**3*f*tan(e/2 + f*x/2)**4 + 50*c**3*f*tan(e/2 + f*x/2)**3 - 50*c**3*f*tan(e
/2 + f*x/2)**2 + 25*c**3*f*tan(e/2 + f*x/2) - 5*c**3*f) - 20*a**2*tan(e/2 +
 f*x/2)**3/(5*c**3*f*tan(e/2 + f*x/2)**5 - 25*c**3*f*tan(e/2 + f*x/2)**4 +
50*c**3*f*tan(e/2 + f*x/2)**3 - 50*c**3*f*tan(e/2 + f*x/2)**2 + 25*c**3*f*t
an(e/2 + f*x/2) - 5*c**3*f) - 10*a**2*tan(e/2 + f*x/2)/(5*c**3*f*tan(e/2 +
f*x/2)**5 - 25*c**3*f*tan(e/2 + f*x/2)**4 + 50*c**3*f*tan(e/2 + f*x/2)**3 -
 50*c**3*f*tan(e/2 + f*x/2)**2 + 25*c**3*f*tan(e/2 + f*x/2) - 5*c**3*f), Ne
(f, 0)), (x*(a*sin(e) + a)**2/(-c*sin(e) + c)**3, True))
```

Giac [A] time = 2.17096, size = 81, normalized size = 2.38

$$-\frac{2\left(5\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+10\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+a^2\right)}{5\,c^3f\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -2/5*(5*a^2*tan(1/2*f*x + 1/2*e)^4 + 10*a^2*tan(1/2*f*x + 1/2*e)^2 + a^2)/(
c^3*f*(tan(1/2*f*x + 1/2*e) - 1)^5)
```

3.244 $\int \frac{(a+a\sin(e+fx))^2}{(c-c\sin(e+fx))^4}\,dx$

Optimal. Leaf size=67

$$\frac{a^2c^2\cos^5(e+fx)}{7f(c-c\sin(e+fx))^6}+\frac{a^2c\cos^5(e+fx)}{35f(c-c\sin(e+fx))^5}$$

```
[Out] (a^2*c^2*Cos[e + f*x]^5)/(7*f*(c - c*Sin[e + f*x])^6) + (a^2*c*Cos[e + f*x]
^5)/(35*f*(c - c*Sin[e + f*x])^5)
```

Rubi [A] time = 0.135929, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2736, 2672, 2671}

$$\frac{a^2c^2\cos^5(e+fx)}{7f(c-c\sin(e+fx))^6}+\frac{a^2c\cos^5(e+fx)}{35f(c-c\sin(e+fx))^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2/(c - c*Sin[e + f*x])^4,x]
```


```
[Out] (a^2*c^2*Cos[e + f*x]^5)/(7*f*(c - c*Sin[e + f*x])^6) + (a^2*c*Cos[e + f*x]
^5)/(35*f*(c - c*Sin[e + f*x])^5)
```


Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+a\sin(e+fx))^2}{(c-c\sin(e+fx))^4}\,dx &= \left(a^2c^2\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^6}\,dx\\ &= \frac{a^2c^2\cos^5(e+fx)}{7f(c-c\sin(e+fx))^6}+\frac{1}{7}\left(a^2c\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^5}\,dx\\ &= \frac{a^2c^2\cos^5(e+fx)}{7f(c-c\sin(e+fx))^6}+\frac{a^2c\cos^5(e+fx)}{35f(c-c\sin(e+fx))^5}\end{aligned}$$

Mathematica [A] time = 0.600629, size = 117, normalized size = 1.75

$$-\frac{a^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(-70\sin\left(\frac{1}{2}(e+fx)\right)-35\sin\left(\frac{3}{2}(e+fx)\right)+7\sin\left(\frac{5}{2}(e+fx)\right)-35\cos\left(\frac{1}{2}(e+\right.\right.}{140c^4f(\sin(e+fx)-1)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2/(c - c*Sin[e + f*x])^4,x]
```

```
[Out] -(a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(-35*Cos[(e + f*x)/2] + 14*Cos[
(3*(e + f*x))/2] + Cos[(7*(e + f*x))/2] - 70*Sin[(e + f*x)/2] - 35*Sin[(3*(
e + f*x))/2] + 7*Sin[(5*(e + f*x))/2]))/(140*c^4*f*(-1 + Sin[e + f*x])^4)
```

Maple [A] time = 0.092, size = 118, normalized size = 1.8

$$2\,\frac{a^2}{fc^4}\left(-\frac{128}{5\left(\tan\left(1/2\,fx+e/2\right)-1\right)^5}-16\left(\tan\left(1/2\,fx+e/2\right)-1\right)^{-6}-5\left(\tan\left(1/2\,fx+e/2\right)-1\right)^{-2}-14\left(\tan\left(1/2\,fx\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^4,x)
```

```
[Out] 2/f*a^2/c^4*(-128/5/(tan(1/2*f*x+1/2*e)-1)^5-16/(tan(1/2*f*x+1/2*e)-1)^6-5/
(tan(1/2*f*x+1/2*e)-1)^2-14/(tan(1/2*f*x+1/2*e)-1)^3-24/(tan(1/2*f*x+1/2*e)
-1)^4-1/(tan(1/2*f*x+1/2*e)-1)-32/7/(tan(1/2*f*x+1/2*e)-1)^7)
```

Maxima [B] time = 1.27222, size = 1102, normalized size = 16.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^4,x, algorithm="maxima")
```

```
[Out] 2/105*(2*a^2*(91*sin(f*x + e)/(cos(f*x + e) + 1) - 168*sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 + 280*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 175*sin(f*x + e
)^4/(cos(f*x + e) + 1)^4 + 105*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 13)/(c
^4 - 7*c^4*sin(f*x + e)/(cos(f*x + e) + 1) + 21*c^4*sin(f*x + e)^2/(cos(f*x
 + e) + 1)^2 - 35*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 35*c^4*sin(f*x
+ e)^4/(cos(f*x + e) + 1)^4 - 21*c^4*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 +
7*c^4*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - c^4*sin(f*x + e)^7/(cos(f*x + e
) + 1)^7) - 3*a^2*(49*sin(f*x + e)/(cos(f*x + e) + 1) - 147*sin(f*x + e)^2/
```

```
(cos(f*x + e) + 1)^2 + 210*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 210*sin(f*
x + e)^4/(cos(f*x + e) + 1)^4 + 105*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 3
5*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 12)/(c^4 - 7*c^4*sin(f*x + e)/(cos(
f*x + e) + 1) + 21*c^4*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 35*c^4*sin(f*x
 + e)^3/(cos(f*x + e) + 1)^3 + 35*c^4*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 -
 21*c^4*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 7*c^4*sin(f*x + e)^6/(cos(f*x
 + e) + 1)^6 - c^4*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) - 4*a^2*(14*sin(f*x
 + e)/(cos(f*x + e) + 1) - 42*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 35*sin(
f*x + e)^3/(cos(f*x + e) + 1)^3 - 35*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 -
2)/(c^4 - 7*c^4*sin(f*x + e)/(cos(f*x + e) + 1) + 21*c^4*sin(f*x + e)^2/(co
s(f*x + e) + 1)^2 - 35*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 35*c^4*sin
(f*x + e)^4/(cos(f*x + e) + 1)^4 - 21*c^4*sin(f*x + e)^5/(cos(f*x + e) + 1)
^5 + 7*c^4*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - c^4*sin(f*x + e)^7/(cos(f*
x + e) + 1)^7))/f
```

Fricas [B] time = 1.33499, size = 537, normalized size = 8.01

$$-\frac{a^2\cos\left(fx+e\right)^4+4\,a^2\cos\left(fx+e\right)^3+13\,a^2\cos\left(fx+e\right)^2-10\,a^2\cos\left(fx+e\right)-20\,a^2-\left(a^2\cos\left(fx+e\right)^3-3\,a^2\cos\right.}{35\left(c^4f\cos\left(fx+e\right)^4-3\,c^4f\cos\left(fx+e\right)^3-8\,c^4f\cos\left(fx+e\right)^2+4\,c^4f\cos\left(fx+e\right)+8\,c^4f+\left(c^4f\cos\left(fx+e\right)^3+4\,c\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^4,x, algorithm="fricas")
```

```
[Out] -1/35*(a^2*cos(f*x + e)^4 + 4*a^2*cos(f*x + e)^3 + 13*a^2*cos(f*x + e)^2 -
10*a^2*cos(f*x + e) - 20*a^2 - (a^2*cos(f*x + e)^3 - 3*a^2*cos(f*x + e)^2 +
 10*a^2*cos(f*x + e) + 20*a^2)*sin(f*x + e))/(c^4*f*cos(f*x + e)^4 - 3*c^4*
f*cos(f*x + e)^3 - 8*c^4*f*cos(f*x + e)^2 + 4*c^4*f*cos(f*x + e) + 8*c^4*f
+ (c^4*f*cos(f*x + e)^3 + 4*c^4*f*cos(f*x + e)^2 - 4*c^4*f*cos(f*x + e) - 8
*c^4*f)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2/(c-c*sin(f*x+e))**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 2.12167, size = 173, normalized size = 2.58

$$-\frac{2\left(35\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6-35\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+140\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-70\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+91\,a^2\tan\left(\frac{1}{2}\right.\right.}{35\,c^4f\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^4,x, algorithm="giac")
```

```
[Out] -2/35*(35*a^2*tan(1/2*f*x + 1/2*e)^6 - 35*a^2*tan(1/2*f*x + 1/2*e)^5 + 140*
a^2*tan(1/2*f*x + 1/2*e)^4 - 70*a^2*tan(1/2*f*x + 1/2*e)^3 + 91*a^2*tan(1/2
*f*x + 1/2*e)^2 - 7*a^2*tan(1/2*f*x + 1/2*e) + 6*a^2)/(c^4*f*(tan(1/2*f*x +
 1/2*e) - 1)^7)
```

3.245 $\int \frac{(a+a\sin(e+fx))^2}{(c-c\sin(e+fx))^5}\,dx$

Optimal. Leaf size=98

$$\frac{a^2c^2\cos^5(e+fx)}{9f(c-c\sin(e+fx))^7}+\frac{2a^2\cos^5(e+fx)}{315f(c-c\sin(e+fx))^5}+\frac{2a^2c\cos^5(e+fx)}{63f(c-c\sin(e+fx))^6}$$

```
[Out] (a^2*c^2*Cos[e + f*x]^5)/(9*f*(c - c*Sin[e + f*x])^7) + (2*a^2*c*Cos[e + f*
x]^5)/(63*f*(c - c*Sin[e + f*x])^6) + (2*a^2*Cos[e + f*x]^5)/(315*f*(c - c*
Sin[e + f*x])^5)
```

Rubi [A] time = 0.182284, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2736, 2672, 2671}

$$\frac{a^2c^2\cos^5(e+fx)}{9f(c-c\sin(e+fx))^7}+\frac{2a^2\cos^5(e+fx)}{315f(c-c\sin(e+fx))^5}+\frac{2a^2c\cos^5(e+fx)}{63f(c-c\sin(e+fx))^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2/(c - c*Sin[e + f*x])^5,x]
```

```
[Out] (a^2*c^2*Cos[e + f*x]^5)/(9*f*(c - c*Sin[e + f*x])^7) + (2*a^2*c*Cos[e + f*
x]^5)/(63*f*(c - c*Sin[e + f*x])^6) + (2*a^2*Cos[e + f*x]^5)/(315*f*(c - c*
Sin[e + f*x])^5)
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^2}{(c-c\sin(e+fx))^5}\,dx = \left(a^2c^2\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^7}\,dx$$
$$= \frac{a^2c^2\cos^5(e+fx)}{9f(c-c\sin(e+fx))^7} + \frac{1}{9}\left(2a^2c\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^6}\,dx$$
$$= \frac{a^2c^2\cos^5(e+fx)}{9f(c-c\sin(e+fx))^7} + \frac{2a^2c\cos^5(e+fx)}{63f(c-c\sin(e+fx))^6} + \frac{1}{63}\left(2a^2\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^5}\,dx$$
$$= \frac{a^2c^2\cos^5(e+fx)}{9f(c-c\sin(e+fx))^7} + \frac{2a^2c\cos^5(e+fx)}{63f(c-c\sin(e+fx))^6} + \frac{2a^2\cos^5(e+fx)}{315f(c-c\sin(e+fx))^5}$$

Mathematica [A] time = 0.536023, size = 121, normalized size = 1.23

$$\frac{a^2\left(441\sin\left(\frac{1}{2}(e+fx)\right)+210\sin\left(\frac{3}{2}(e+fx)\right)-36\sin\left(\frac{5}{2}(e+fx)\right)+\sin\left(\frac{9}{2}(e+fx)\right)+315\cos\left(\frac{1}{2}(e+fx)\right)-126\cos\left(\right.\right.}{1260c^5f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2/(c - c*Sin[e + f*x])^5,x]
```

```
[Out] (a^2*(315*Cos[(e + f*x)/2] - 126*Cos[(3*(e + f*x))/2] - 9*Cos[(7*(e + f*x))
/2] + 441*Sin[(e + f*x)/2] + 210*Sin[(3*(e + f*x))/2] - 36*Sin[(5*(e + f*x)
)/2] + Sin[(9*(e + f*x))/2]))/(1260*c^5*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)
/2])^9)
```

Maple [A] time = 0.104, size = 148, normalized size = 1.5

$$2\,{\frac{a^2}{fc^5}}\left(-{\frac{404}{5\left(\tan\left(1/2\,fx+e/2\right)-1\right)^5}}-{\frac{272}{3\left(\tan\left(1/2\,fx+e/2\right)-1\right)^6}}-50\,\left(\tan\left(1/2\,fx+e/2\right)-1\right)^{-4}-{\frac{64}{3\left(\tan\left(1/2\,fx+\right.\right.}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^5,x)
```

```
[Out] 2/f*a^2/c^5*(-404/5/(tan(1/2*f*x+1/2*e)-1)^5-272/3/(tan(1/2*f*x+1/2*e)-1)^6
-50/(tan(1/2*f*x+1/2*e)-1)^4-64/3/(tan(1/2*f*x+1/2*e)-1)^3-32/(tan(1/2*f*x+
1/2*e)-1)^8-6/(tan(1/2*f*x+1/2*e)-1)^2-1/(tan(1/2*f*x+1/2*e)-1)-64/9/(tan(1
/2*f*x+1/2*e)-1)^9-480/7/(tan(1/2*f*x+1/2*e)-1)^7)
```

Maxima [B] time = 1.40376, size = 1449, normalized size = 14.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^5,x, algorithm="maxima")
```

```
[Out] -2/315*(a^2*(432*sin(f*x + e)/(cos(f*x + e) + 1) - 1728*sin(f*x + e)^2/(cos
(f*x + e) + 1)^2 + 3612*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 5418*sin(f*x
+ e)^4/(cos(f*x + e) + 1)^4 + 5040*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 33
```

```
60*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 1260*sin(f*x + e)^7/(cos(f*x + e)
+ 1)^7 - 315*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 83)/(c^5 - 9*c^5*sin(f*x
 + e)/(cos(f*x + e) + 1) + 36*c^5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 84*
c^5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 126*c^5*sin(f*x + e)^4/(cos(f*x +
 e) + 1)^4 - 126*c^5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 84*c^5*sin(f*x +
 e)^6/(cos(f*x + e) + 1)^6 - 36*c^5*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 9
*c^5*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - c^5*sin(f*x + e)^9/(cos(f*x + e)
 + 1)^9) - 10*a^2*(45*sin(f*x + e)/(cos(f*x + e) + 1) - 117*sin(f*x + e)^2/
(cos(f*x + e) + 1)^2 + 273*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 315*sin(f*
x + e)^4/(cos(f*x + e) + 1)^4 + 315*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 1
47*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 63*sin(f*x + e)^7/(cos(f*x + e) +
1)^7 - 5)/(c^5 - 9*c^5*sin(f*x + e)/(cos(f*x + e) + 1) + 36*c^5*sin(f*x + e
)^2/(cos(f*x + e) + 1)^2 - 84*c^5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 126
*c^5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 126*c^5*sin(f*x + e)^5/(cos(f*x
+ e) + 1)^5 + 84*c^5*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 36*c^5*sin(f*x +
 e)^7/(cos(f*x + e) + 1)^7 + 9*c^5*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - c^
5*sin(f*x + e)^9/(cos(f*x + e) + 1)^9) + 14*a^2*(9*sin(f*x + e)/(cos(f*x +
e) + 1) - 36*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 54*sin(f*x + e)^3/(cos(f
*x + e) + 1)^3 - 81*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 45*sin(f*x + e)^5
/(cos(f*x + e) + 1)^5 - 30*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 1)/(c^5 -
9*c^5*sin(f*x + e)/(cos(f*x + e) + 1) + 36*c^5*sin(f*x + e)^2/(cos(f*x + e)
 + 1)^2 - 84*c^5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 126*c^5*sin(f*x + e)
^4/(cos(f*x + e) + 1)^4 - 126*c^5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 84*
c^5*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 36*c^5*sin(f*x + e)^7/(cos(f*x +
e) + 1)^7 + 9*c^5*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - c^5*sin(f*x + e)^9/
(cos(f*x + e) + 1)^9))/f
```

Fricas [B] time = 1.31428, size = 684, normalized size = 6.98

$$\frac{2\,a^2\cos\left(fx+e\right)^5-8\,a^2\cos\left(fx+e\right)^4-25\,a^2\cos\left(fx+e\right)^3-85\,a^2\cos\left(fx+e\right)^2+70\,a^2\cos\left(fx+e\right)+140\,a^2+\left(2\,\cdots\right.}{315\left(c^5f\cos\left(fx+e\right)^5+5\,c^5f\cos\left(fx+e\right)^4-8\,c^5f\cos\left(fx+e\right)^3-20\,c^5f\cos\left(fx+e\right)^2+8\,c^5f\cos\left(fx+e\right)+16\,c^5f-\left(\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^5,x, algorithm="fricas")
```

```
[Out] 1/315*(2*a^2*cos(f*x + e)^5 - 8*a^2*cos(f*x + e)^4 - 25*a^2*cos(f*x + e)^3
- 85*a^2*cos(f*x + e)^2 + 70*a^2*cos(f*x + e) + 140*a^2 + (2*a^2*cos(f*x +
e)^4 + 10*a^2*cos(f*x + e)^3 - 15*a^2*cos(f*x + e)^2 + 70*a^2*cos(f*x + e)
+ 140*a^2)*sin(f*x + e))/(c^5*f*cos(f*x + e)^5 + 5*c^5*f*cos(f*x + e)^4 - 8
*c^5*f*cos(f*x + e)^3 - 20*c^5*f*cos(f*x + e)^2 + 8*c^5*f*cos(f*x + e) + 16
*c^5*f - (c^5*f*cos(f*x + e)^4 - 4*c^5*f*cos(f*x + e)^3 - 12*c^5*f*cos(f*x
+ e)^2 + 8*c^5*f*cos(f*x + e) + 16*c^5*f)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2/(c-c*sin(f*x+e))**5,x)
```

```
[Out] Timed out
```

Giac [A] time = 2.20265, size = 219, normalized size = 2.23

$$-\frac{2\left(315\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^8-630\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^7+2310\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6-2520\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+340\right.}{315\,c^5f\left(\tan\left(\frac{1}{2}\,f\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^5,x, algorithm="giac")
```

```
[Out] -2/315*(315*a^2*tan(1/2*f*x + 1/2*e)^8 - 630*a^2*tan(1/2*f*x + 1/2*e)^7 + 2
310*a^2*tan(1/2*f*x + 1/2*e)^6 - 2520*a^2*tan(1/2*f*x + 1/2*e)^5 + 3402*a^2
*tan(1/2*f*x + 1/2*e)^4 - 1638*a^2*tan(1/2*f*x + 1/2*e)^3 + 1062*a^2*tan(1/
2*f*x + 1/2*e)^2 - 108*a^2*tan(1/2*f*x + 1/2*e) + 47*a^2)/(c^5*f*(tan(1/2*f
*x + 1/2*e) - 1)^9)
```

3.246 $\int \frac{(a+a\sin(e+fx))^2}{(c-c\sin(e+fx))^6}\,dx$

Optimal. Leaf size=132

$$\frac{a^2c^2\cos^5(e+fx)}{11f(c-c\sin(e+fx))^8}+\frac{2a^2\cos^5(e+fx)}{1155cf(c-c\sin(e+fx))^5}+\frac{2a^2\cos^5(e+fx)}{231f(c-c\sin(e+fx))^6}+\frac{a^2c\cos^5(e+fx)}{33f(c-c\sin(e+fx))^7}$$

```
[Out] (a^2*c^2*Cos[e + f*x]^5)/(11*f*(c - c*Sin[e + f*x])^8) + (a^2*c*Cos[e + f*x
]^5)/(33*f*(c - c*Sin[e + f*x])^7) + (2*a^2*Cos[e + f*x]^5)/(231*f*(c - c*S
in[e + f*x])^6) + (2*a^2*Cos[e + f*x]^5)/(1155*c*f*(c - c*Sin[e + f*x])^5)
```

Rubi [A] time = 0.232794, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2736, 2672, 2671}

$$\frac{a^2c^2\cos^5(e+fx)}{11f(c-c\sin(e+fx))^8}+\frac{2a^2\cos^5(e+fx)}{1155cf(c-c\sin(e+fx))^5}+\frac{2a^2\cos^5(e+fx)}{231f(c-c\sin(e+fx))^6}+\frac{a^2c\cos^5(e+fx)}{33f(c-c\sin(e+fx))^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2/(c - c*Sin[e + f*x])^6,x]
```


```
[Out] (a^2*c^2*Cos[e + f*x]^5)/(11*f*(c - c*Sin[e + f*x])^8) + (a^2*c*Cos[e + f*x
]^5)/(33*f*(c - c*Sin[e + f*x])^7) + (2*a^2*Cos[e + f*x]^5)/(231*f*(c - c*S
in[e + f*x])^6) + (2*a^2*Cos[e + f*x]^5)/(1155*c*f*(c - c*Sin[e + f*x])^5)
```


Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^2}{(c-c\sin(e+fx))^6}\,dx = \left(a^2c^2\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^8}\,dx$$
$$= \frac{a^2c^2\cos^5(e+fx)}{11f(c-c\sin(e+fx))^8} + \frac{1}{11}\left(3a^2c\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^7}\,dx$$
$$= \frac{a^2c^2\cos^5(e+fx)}{11f(c-c\sin(e+fx))^8} + \frac{a^2c\cos^5(e+fx)}{33f(c-c\sin(e+fx))^7} + \frac{1}{33}\left(2a^2\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^6}\,dx$$
$$= \frac{a^2c^2\cos^5(e+fx)}{11f(c-c\sin(e+fx))^8} + \frac{a^2c\cos^5(e+fx)}{33f(c-c\sin(e+fx))^7} + \frac{2a^2\cos^5(e+fx)}{231f(c-c\sin(e+fx))^6} + \frac{\left(2a^2\right)\int}{}$$
$$= \frac{a^2c^2\cos^5(e+fx)}{11f(c-c\sin(e+fx))^8} + \frac{a^2c\cos^5(e+fx)}{33f(c-c\sin(e+fx))^7} + \frac{2a^2\cos^5(e+fx)}{231f(c-c\sin(e+fx))^6} + \frac{2a^2}{1155cf(c}$$

Mathematica [A] time = 0.672689, size = 133, normalized size = 1.01

$$\frac{a^2\left(2541\sin\left(\frac{1}{2}(e+fx)\right)+1155\sin\left(\frac{3}{2}(e+fx)\right)-165\sin\left(\frac{5}{2}(e+fx)\right)+11\sin\left(\frac{9}{2}(e+fx)\right)+2079\cos\left(\frac{1}{2}(e+fx)\right)-8\right.}{9240c^6f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^{11}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2/(c - c*Sin[e + f*x])^6,x]
```

```
[Out] (a^2*(2079*Cos[(e + f*x)/2] - 825*Cos[(3*(e + f*x))/2] - 55*Cos[(7*(e + f*x
))/2] + Cos[(11*(e + f*x))/2] + 2541*Sin[(e + f*x)/2] + 1155*Sin[(3*(e + f*
x))/2] - 165*Sin[(5*(e + f*x))/2] + 11*Sin[(9*(e + f*x))/2]))/(9240*c^6*f*(
Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^11)
```

Maple [A] time = 0.107, size = 178, normalized size = 1.4

$$2\,\frac{a^2}{fc^6}\left(-288\,\left(\tan\left(1/2\,fx+e/2\right)-1\right)^{-8}-\frac{932}{5\,\left(\tan\left(1/2\,fx+e/2\right)-1\right)^5}-88\,\left(\tan\left(1/2\,fx+e/2\right)-1\right)^{-4}-\frac{1}{11\,\left(\tan\left(1/2\,f\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^6,x)
```

```
[Out] 2/f*a^2/c^6*(-288/(tan(1/2*f*x+1/2*e)-1)^8-932/5/(tan(1/2*f*x+1/2*e)-1)^5-8
8/(tan(1/2*f*x+1/2*e)-1)^4-128/11/(tan(1/2*f*x+1/2*e)-1)^11-292/(tan(1/2*f*
x+1/2*e)-1)^6-30/(tan(1/2*f*x+1/2*e)-1)^3-7/(tan(1/2*f*x+1/2*e)-1)^2-64/(ta
n(1/2*f*x+1/2*e)-1)^10-2376/7/(tan(1/2*f*x+1/2*e)-1)^7-1/(tan(1/2*f*x+1/2*e
)-1)-512/3/(tan(1/2*f*x+1/2*e)-1)^9)
```

Maxima [B] time = 1.456, size = 1798, normalized size = 13.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^6,x, algorithm="maxima")
```

```
[Out] -2/3465*(5*a^2*(913*sin(f*x + e)/(cos(f*x + e) + 1) - 4565*sin(f*x + e)^2/(
cos(f*x + e) + 1)^2 + 12540*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 25080*sin
(f*x + e)^4/(cos(f*x + e) + 1)^4 + 33726*sin(f*x + e)^5/(cos(f*x + e) + 1)^
5 - 33726*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 23100*sin(f*x + e)^7/(cos(f
*x + e) + 1)^7 - 11550*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 3465*sin(f*x +
 e)^9/(cos(f*x + e) + 1)^9 - 693*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 14
6)/(c^6 - 11*c^6*sin(f*x + e)/(cos(f*x + e) + 1) + 55*c^6*sin(f*x + e)^2/(c
os(f*x + e) + 1)^2 - 165*c^6*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 330*c^6*
sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 462*c^6*sin(f*x + e)^5/(cos(f*x + e)
+ 1)^5 + 462*c^6*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 330*c^6*sin(f*x + e)
^7/(cos(f*x + e) + 1)^7 + 165*c^6*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 55*
c^6*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 11*c^6*sin(f*x + e)^10/(cos(f*x +
 e) + 1)^10 - c^6*sin(f*x + e)^11/(cos(f*x + e) + 1)^11) - 6*a^2*(671*sin(f
*x + e)/(cos(f*x + e) + 1) - 2200*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 660
0*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 10890*sin(f*x + e)^4/(cos(f*x + e)
+ 1)^4 + 15246*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 12936*sin(f*x + e)^6/(
cos(f*x + e) + 1)^6 + 9240*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 3465*sin(f
*x + e)^8/(cos(f*x + e) + 1)^8 + 1155*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 -
 61)/(c^6 - 11*c^6*sin(f*x + e)/(cos(f*x + e) + 1) + 55*c^6*sin(f*x + e)^2/
(cos(f*x + e) + 1)^2 - 165*c^6*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 330*c^
6*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 462*c^6*sin(f*x + e)^5/(cos(f*x + e
) + 1)^5 + 462*c^6*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 330*c^6*sin(f*x +
e)^7/(cos(f*x + e) + 1)^7 + 165*c^6*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 5
5*c^6*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 11*c^6*sin(f*x + e)^10/(cos(f*x
 + e) + 1)^10 - c^6*sin(f*x + e)^11/(cos(f*x + e) + 1)^11) + 4*a^2*(253*sin
(f*x + e)/(cos(f*x + e) + 1) - 1265*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2
640*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 5280*sin(f*x + e)^4/(cos(f*x + e)
 + 1)^4 + 5313*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 5313*sin(f*x + e)^6/(c
os(f*x + e) + 1)^6 + 2310*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 1155*sin(f*
x + e)^8/(cos(f*x + e) + 1)^8 - 23)/(c^6 - 11*c^6*sin(f*x + e)/(cos(f*x + e
) + 1) + 55*c^6*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 165*c^6*sin(f*x + e)^
3/(cos(f*x + e) + 1)^3 + 330*c^6*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 462*
c^6*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 462*c^6*sin(f*x + e)^6/(cos(f*x +
 e) + 1)^6 - 330*c^6*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 165*c^6*sin(f*x
+ e)^8/(cos(f*x + e) + 1)^8 - 55*c^6*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 +
11*c^6*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - c^6*sin(f*x + e)^11/(cos(f*x
 + e) + 1)^11))/f
```

Fricas [B] time = 1.39429, size = 832, normalized size = 6.3

$$-\frac{2\,a^2\cos\left(fx+e\right)^6+12\,a^2\cos\left(fx+e\right)^5-25\,a^2\cos\left(fx+e\right)^4-70\,a^2\cos\left(fx+e\right)^3-245\,a^2\cos\left(fx+e\right)^2+210\,a^2}{1155\left(c^6f\cos\left(fx+e\right)^6-5\,c^6f\cos\left(fx+e\right)^5-18\,c^6f\cos\left(fx+e\right)^4+20\,c^6f\cos\left(fx+e\right)^3+48\,c^6f\cos\left(fx+e\right)^2-16\,c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^6,x, algorithm="fricas")
```

```
[Out] -1/1155*(2*a^2*cos(f*x + e)^6 + 12*a^2*cos(f*x + e)^5 - 25*a^2*cos(f*x + e)
^4 - 70*a^2*cos(f*x + e)^3 - 245*a^2*cos(f*x + e)^2 + 210*a^2*cos(f*x + e)
+ 420*a^2 - (2*a^2*cos(f*x + e)^5 - 10*a^2*cos(f*x + e)^4 - 35*a^2*cos(f*x
+ e)^3 + 35*a^2*cos(f*x + e)^2 - 210*a^2*cos(f*x + e) - 420*a^2)*sin(f*x +
e))/(c^6*f*cos(f*x + e)^6 - 5*c^6*f*cos(f*x + e)^5 - 18*c^6*f*cos(f*x + e)^
4 + 20*c^6*f*cos(f*x + e)^3 + 48*c^6*f*cos(f*x + e)^2 - 16*c^6*f*cos(f*x +
e) - 32*c^6*f + (c^6*f*cos(f*x + e)^5 + 6*c^6*f*cos(f*x + e)^4 - 12*c^6*f*c
os(f*x + e)^3 - 32*c^6*f*cos(f*x + e)^2 + 16*c^6*f*cos(f*x + e) + 32*c^6*f)
```

```
*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2/(c-c*sin(f*x+e))**6,x)
```

```
[Out] Timed out
```

Giac [A] time = 2.1786, size = 265, normalized size = 2.01

$$-\frac{2\left(1155\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{10}-3465\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{9}+13860\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{8}-23100\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{7}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^6,x, algorithm="giac")
```

```
[Out] -2/1155*(1155*a^2*tan(1/2*f*x + 1/2*e)^10 - 3465*a^2*tan(1/2*f*x + 1/2*e)^9
 + 13860*a^2*tan(1/2*f*x + 1/2*e)^8 - 23100*a^2*tan(1/2*f*x + 1/2*e)^7 + 37
422*a^2*tan(1/2*f*x + 1/2*e)^6 - 32802*a^2*tan(1/2*f*x + 1/2*e)^5 + 27060*a
^2*tan(1/2*f*x + 1/2*e)^4 - 11220*a^2*tan(1/2*f*x + 1/2*e)^3 + 4895*a^2*tan
(1/2*f*x + 1/2*e)^2 - 517*a^2*tan(1/2*f*x + 1/2*e) + 152*a^2)/(c^6*f*(tan(1
/2*f*x + 1/2*e) - 1)^11)
```

3.247 $\int (a+a\sin(e+fx))^3(c-c\sin(e+fx))^6\,dx$

Optimal. Leaf size=180

$$\frac{11a^3c^6\cos^7(e+fx)}{56f}+\frac{a^3\cos^7(e+fx)\left(c^3-c^3\sin(e+fx)\right)^2}{9f}+\frac{11a^3\cos^7(e+fx)\left(c^6-c^6\sin(e+fx)\right)}{72f}+\frac{11a^3c^6\sin(e+f}{48}$$

```
[Out] (55*a^3*c^6*x)/128 + (11*a^3*c^6*Cos[e + f*x]^7)/(56*f) + (55*a^3*c^6*Cos[e
 + f*x]*Sin[e + f*x])/(128*f) + (55*a^3*c^6*Cos[e + f*x]^3*Sin[e + f*x])/(1
92*f) + (11*a^3*c^6*Cos[e + f*x]^5*Sin[e + f*x])/(48*f) + (a^3*Cos[e + f*x]
^7*(c^3 - c^3*Sin[e + f*x])^2)/(9*f) + (11*a^3*Cos[e + f*x]^7*(c^6 - c^6*Si
n[e + f*x]))/(72*f)
```

Rubi [A] time = 0.205928, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {2736, 2678, 2669, 2635, 8}

$$\frac{11a^3c^6\cos^7(e+fx)}{56f}+\frac{a^3\cos^7(e+fx)\left(c^3-c^3\sin(e+fx)\right)^2}{9f}+\frac{11a^3\cos^7(e+fx)\left(c^6-c^6\sin(e+fx)\right)}{72f}+\frac{11a^3c^6\sin(e+f}{48}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^6,x]
```

```
[Out] (55*a^3*c^6*x)/128 + (11*a^3*c^6*Cos[e + f*x]^7)/(56*f) + (55*a^3*c^6*Cos[e
 + f*x]*Sin[e + f*x])/(128*f) + (55*a^3*c^6*Cos[e + f*x]^3*Sin[e + f*x])/(1
92*f) + (11*a^3*c^6*Cos[e + f*x]^5*Sin[e + f*x])/(48*f) + (a^3*Cos[e + f*x]
^7*(c^3 - c^3*Sin[e + f*x])^2)/(9*f) + (11*a^3*Cos[e + f*x]^7*(c^6 - c^6*Si
n[e + f*x]))/(72*f)
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (a+a\sin(e+fx))^3(c-c\sin(e+fx))^6\,dx = \left(a^3c^3\right)\int \cos^6(e+fx)(c-c\sin(e+fx))^3\,dx$$
$$= \frac{a^3\cos^7(e+fx)\left(c^3-c^3\sin(e+fx)\right)^2}{9f}+\frac{1}{9}\left(11a^3c^4\right)\int \cos^6(e+fx)(c-$$
$$= \frac{a^3\cos^7(e+fx)\left(c^3-c^3\sin(e+fx)\right)^2}{9f}+\frac{11a^3\cos^7(e+fx)\left(c^6-c^6\sin(\right.}{72f}$$
$$= \frac{11a^3c^6\cos^7(e+fx)}{56f}+\frac{a^3\cos^7(e+fx)\left(c^3-c^3\sin(e+fx)\right)^2}{9f}+\frac{11a^3\text{co}}{}$$
$$= \frac{11a^3c^6\cos^7(e+fx)}{56f}+\frac{11a^3c^6\cos^5(e+fx)\sin(e+fx)}{48f}+\frac{a^3\cos^7(e+f}{}$$
$$= \frac{11a^3c^6\cos^7(e+fx)}{56f}+\frac{55a^3c^6\cos^3(e+fx)\sin(e+fx)}{192f}+\frac{11a^3c^6\cos^5(e}{}$$
$$= \frac{11a^3c^6\cos^7(e+fx)}{56f}+\frac{55a^3c^6\cos(e+fx)\sin(e+fx)}{128f}+\frac{55a^3c^6\cos^3(e}{1}$$
$$= \frac{55}{128}a^3c^6x+\frac{11a^3c^6\cos^7(e+fx)}{56f}+\frac{55a^3c^6\cos(e+fx)\sin(e+fx)}{128f}+\frac{5}{}$$

Mathematica [A] time = 2.14343, size = 109, normalized size = 0.61

$$\frac{a^3c^6(18144\sin(2(e+fx))+1512\sin(4(e+fx))-672\sin(6(e+fx))-189\sin(8(e+fx))+16632\cos(e+fx)+974}{64512f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^6,x]
```

```
[Out] (a^3*c^6*(27720*e + 27720*f*x + 16632*Cos[e + f*x] + 9744*Cos[3*(e + f*x)]
+ 3024*Cos[5*(e + f*x)] + 324*Cos[7*(e + f*x)] - 28*Cos[9*(e + f*x)] + 1814
4*Sin[2*(e + f*x)] + 1512*Sin[4*(e + f*x)] - 672*Sin[6*(e + f*x)] - 189*Sin
[8*(e + f*x)]))/(64512*f)
```

Maple [A] time = 0.023, size = 297, normalized size = 1.7

$$\frac{1}{f}\left(-\frac{c^6a^3\cos\left(fx+e\right)}{9}\left(\frac{128}{35}+\left(\sin\left(fx+e\right)\right)^8+\frac{8\left(\sin\left(fx+e\right)\right)^6}{7}+\frac{48\left(\sin\left(fx+e\right)\right)^4}{35}+\frac{64\left(\sin\left(fx+e\right)\right)^2}{35}\right)-3c^6\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^6,x)
```

```
[Out] 1/f*(-1/9*c^6*a^3*(128/35+sin(f*x+e)^8+8/7*sin(f*x+e)^6+48/35*sin(f*x+e)^4+
64/35*sin(f*x+e)^2)*cos(f*x+e)-3*c^6*a^3*(-1/8*(sin(f*x+e)^7+7/6*sin(f*x+e)
^5+35/24*sin(f*x+e)^3+35/16*sin(f*x+e))*cos(f*x+e)+35/128*f*x+35/128*e)+8*c
^6*a^3*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+e)+5/1
6*f*x+5/16*e)+6/5*c^6*a^3*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)-6*
c^6*a^3*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-8/3*c
^6*a^3*(2+sin(f*x+e)^2)*cos(f*x+e)+3*c^6*a^3*cos(f*x+e)+c^6*a^3*(f*x+e))
```

Maxima [A] time = 1.22534, size = 406, normalized size = 2.26

$$-\frac{1024\left(35\cos\left(fx+e\right)^{9}-180\cos\left(fx+e\right)^{7}+378\cos\left(fx+e\right)^{5}-420\cos\left(fx+e\right)^{3}+315\cos\left(fx+e\right)\right)a^{3}c^{6}-129024}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^6,x, algorithm="maxima")
```

```
[Out] -1/322560*(1024*(35*cos(f*x + e)^9 - 180*cos(f*x + e)^7 + 378*cos(f*x + e)^
5 - 420*cos(f*x + e)^3 + 315*cos(f*x + e))*a^3*c^6 - 129024*(3*cos(f*x + e)
^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*a^3*c^6 - 860160*(cos(f*x + e)^3
- 3*cos(f*x + e))*a^3*c^6 + 315*(128*sin(2*f*x + 2*e)^3 + 840*f*x + 840*e +
 3*sin(8*f*x + 8*e) + 168*sin(4*f*x + 4*e) - 768*sin(2*f*x + 2*e))*a^3*c^6
- 13440*(4*sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*sin(4*f*x + 4*e) - 48*sin
(2*f*x + 2*e))*a^3*c^6 + 60480*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*
f*x + 2*e))*a^3*c^6 - 322560*(f*x + e)*a^3*c^6 - 967680*a^3*c^6*cos(f*x + e
))/f
```

Fricas [A] time = 1.47667, size = 297, normalized size = 1.65

$$-\frac{896\,a^{3}c^{6}\cos\left(fx+e\right)^{9}-4608\,a^{3}c^{6}\cos\left(fx+e\right)^{7}-3465\,a^{3}c^{6}fx+21\left(144\,a^{3}c^{6}\cos\left(fx+e\right)^{7}-88\,a^{3}c^{6}\cos\left(fx+e\right)^{5}-1\right.}{8064\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^6,x, algorithm="fricas")
```

```
[Out] -1/8064*(896*a^3*c^6*cos(f*x + e)^9 - 4608*a^3*c^6*cos(f*x + e)^7 - 3465*a^
3*c^6*f*x + 21*(144*a^3*c^6*cos(f*x + e)^7 - 88*a^3*c^6*cos(f*x + e)^5 - 11
0*a^3*c^6*cos(f*x + e)^3 - 165*a^3*c^6*cos(f*x + e))*sin(f*x + e))/f
```

Sympy [A] time = 47.784, size = 838, normalized size = 4.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(c-c*sin(f*x+e))**6,x)
```

```
[Out] Piecewise((-105*a**3*c**6*x*sin(e + f*x)**8/128 - 105*a**3*c**6*x*sin(e + f
*x)**6*cos(e + f*x)**2/32 + 5*a**3*c**6*x*sin(e + f*x)**6/2 - 315*a**3*c**6
*x*sin(e + f*x)**4*cos(e + f*x)**4/64 + 15*a**3*c**6*x*sin(e + f*x)**4*cos(
e + f*x)**2/2 - 9*a**3*c**6*x*sin(e + f*x)**4/4 - 105*a**3*c**6*x*sin(e + f
*x)**2*cos(e + f*x)**6/32 + 15*a**3*c**6*x*sin(e + f*x)**2*cos(e + f*x)**4/
2 - 9*a**3*c**6*x*sin(e + f*x)**2*cos(e + f*x)**2/2 - 105*a**3*c**6*x*cos(e
 + f*x)**8/128 + 5*a**3*c**6*x*cos(e + f*x)**6/2 - 9*a**3*c**6*x*cos(e + f*
x)**4/4 + a**3*c**6*x - a**3*c**6*sin(e + f*x)**8*cos(e + f*x)/f + 279*a**3
*c**6*sin(e + f*x)**7*cos(e + f*x)/(128*f) - 8*a**3*c**6*sin(e + f*x)**6*co
s(e + f*x)**3/(3*f) + 511*a**3*c**6*sin(e + f*x)**5*cos(e + f*x)**3/(128*f)
 - 11*a**3*c**6*sin(e + f*x)**5*cos(e + f*x)/(2*f) - 16*a**3*c**6*sin(e + f
*x)**4*cos(e + f*x)**5/(5*f) + 6*a**3*c**6*sin(e + f*x)**4*cos(e + f*x)/f +
 385*a**3*c**6*sin(e + f*x)**3*cos(e + f*x)**5/(128*f) - 20*a**3*c**6*sin(e
 + f*x)**3*cos(e + f*x)**3/(3*f) + 15*a**3*c**6*sin(e + f*x)**3*cos(e + f*x
)/(4*f) - 64*a**3*c**6*sin(e + f*x)**2*cos(e + f*x)**7/(35*f) + 8*a**3*c**6
*sin(e + f*x)**2*cos(e + f*x)**3/f - 8*a**3*c**6*sin(e + f*x)**2*cos(e + f*
x)/f + 105*a**3*c**6*sin(e + f*x)*cos(e + f*x)**7/(128*f) - 5*a**3*c**6*sin
(e + f*x)*cos(e + f*x)**5/(2*f) + 9*a**3*c**6*sin(e + f*x)*cos(e + f*x)**3/
(4*f) - 128*a**3*c**6*cos(e + f*x)**9/(315*f) + 16*a**3*c**6*cos(e + f*x)**
5/(5*f) - 16*a**3*c**6*cos(e + f*x)**3/(3*f) + 3*a**3*c**6*cos(e + f*x)/f,
Ne(f, 0)), (x*(a*sin(e) + a)**3*(-c*sin(e) + c)**6, True))
```

Giac [A] time = 2.06995, size = 265, normalized size = 1.47

$$\frac{55}{128}a^3c^6x - \frac{a^3c^6\cos\left(9fx+9e\right)}{2304f} + \frac{9a^3c^6\cos\left(7fx+7e\right)}{1792f} + \frac{3a^3c^6\cos\left(5fx+5e\right)}{64f} + \frac{29a^3c^6\cos\left(3fx+3e\right)}{192f} + \frac{33a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^6,x, algorithm="giac")
```

```
[Out] 55/128*a^3*c^6*x - 1/2304*a^3*c^6*cos(9*f*x + 9*e)/f + 9/1792*a^3*c^6*cos(7
*f*x + 7*e)/f + 3/64*a^3*c^6*cos(5*f*x + 5*e)/f + 29/192*a^3*c^6*cos(3*f*x
+ 3*e)/f + 33/128*a^3*c^6*cos(f*x + e)/f - 3/1024*a^3*c^6*sin(8*f*x + 8*e)/
f - 1/96*a^3*c^6*sin(6*f*x + 6*e)/f + 3/128*a^3*c^6*sin(4*f*x + 4*e)/f + 9/
32*a^3*c^6*sin(2*f*x + 2*e)/f
```

3.248 $\int (a+a\sin(e+fx))^3(c-c\sin(e+fx))^5\,dx$

Optimal. Leaf size=145

$$\frac{9a^3c^5\cos^7(e+fx)}{56f}+\frac{a^3\cos^7(e+fx)\left(c^5-c^5\sin(e+fx)\right)}{8f}+\frac{3a^3c^5\sin(e+fx)\cos^5(e+fx)}{16f}+\frac{15a^3c^5\sin(e+fx)\cos^3(e\cdot}{64f}$$

```
[Out] (45*a^3*c^5*x)/128 + (9*a^3*c^5*Cos[e + f*x]^7)/(56*f) + (45*a^3*c^5*Cos[e
+ f*x]*Sin[e + f*x])/(128*f) + (15*a^3*c^5*Cos[e + f*x]^3*Sin[e + f*x])/(64
*f) + (3*a^3*c^5*Cos[e + f*x]^5*Sin[e + f*x])/(16*f) + (a^3*Cos[e + f*x]^7*
(c^5 - c^5*Sin[e + f*x]))/(8*f)
```

Rubi [A] time = 0.157084, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {2736, 2678, 2669, 2635, 8}

$$\frac{9a^3c^5\cos^7(e+fx)}{56f}+\frac{a^3\cos^7(e+fx)\left(c^5-c^5\sin(e+fx)\right)}{8f}+\frac{3a^3c^5\sin(e+fx)\cos^5(e+fx)}{16f}+\frac{15a^3c^5\sin(e+fx)\cos^3(e\cdot}{64f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^5,x]
```



```
[Out] (45*a^3*c^5*x)/128 + (9*a^3*c^5*Cos[e + f*x]^7)/(56*f) + (45*a^3*c^5*Cos[e
+ f*x]*Sin[e + f*x])/(128*f) + (15*a^3*c^5*Cos[e + f*x]^3*Sin[e + f*x])/(64
*f) + (3*a^3*c^5*Cos[e + f*x]^5*Sin[e + f*x])/(16*f) + (a^3*Cos[e + f*x]^7*
(c^5 - c^5*Sin[e + f*x]))/(8*f)
```


Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
```

```
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (a+a\sin(e+fx))^3(c-c\sin(e+fx))^5\,dx = \left(a^3c^3\right)\int \cos^6(e+fx)(c-c\sin(e+fx))^2\,dx$$
$$= \frac{a^3\cos^7(e+fx)\left(c^5-c^5\sin(e+fx)\right)}{8f} + \frac{1}{8}\left(9a^3c^4\right)\int \cos^6(e+fx)(c-c$$
$$= \frac{9a^3c^5\cos^7(e+fx)}{56f} + \frac{a^3\cos^7(e+fx)\left(c^5-c^5\sin(e+fx)\right)}{8f} + \frac{1}{8}\left(9a^3c^5\right)$$
$$= \frac{9a^3c^5\cos^7(e+fx)}{56f} + \frac{3a^3c^5\cos^5(e+fx)\sin(e+fx)}{16f} + \frac{a^3\cos^7(e+fx)}{}$$
$$= \frac{9a^3c^5\cos^7(e+fx)}{56f} + \frac{15a^3c^5\cos^3(e+fx)\sin(e+fx)}{64f} + \frac{3a^3c^5\cos^5(e+}{1}$$
$$= \frac{9a^3c^5\cos^7(e+fx)}{56f} + \frac{45a^3c^5\cos(e+fx)\sin(e+fx)}{128f} + \frac{15a^3c^5\cos^3(e+}{6}$$
$$= \frac{45}{128}a^3c^5x + \frac{9a^3c^5\cos^7(e+fx)}{56f} + \frac{45a^3c^5\cos(e+fx)\sin(e+fx)}{128f} + 15$$

Mathematica [A] time = 1.23014, size = 89, normalized size = 0.61

$$\frac{a^3c^5(1792\sin(2(e+fx))+280\sin(4(e+fx))-7\sin(8(e+fx))+1120\cos(e+fx)+672\cos(3(e+fx))+224\cos(5}{7168f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^5,x]
```

```
[Out] (a^3*c^5*(2520*e + 2520*f*x + 1120*Cos[e + f*x] + 672*Cos[3*(e + f*x)] + 22
4*Cos[5*(e + f*x)] + 32*Cos[7*(e + f*x)] + 1792*Sin[2*(e + f*x)] + 280*Sin[
4*(e + f*x)] - 7*Sin[8*(e + f*x)]))/(7168*f)
```

Maple [B] time = 0.02, size = 276, normalized size = 1.9

$$\frac{1}{f}\left(-c^5a^3\left(-\frac{\cos(fx+e)}{8}\left((\sin(fx+e))^7+\frac{7(\sin(fx+e))^5}{6}+\frac{35(\sin(fx+e))^3}{24}+\frac{35\sin(fx+e)}{16}\right)+\frac{35fx}{128}+\frac{35}{12}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^5,x)
```

```
[Out] 1/f*(-c^5*a^3*(-1/8*(sin(f*x+e)^7+7/6*sin(f*x+e)^5+35/24*sin(f*x+e)^3+35/16
*sin(f*x+e))*cos(f*x+e)+35/128*f*x+35/128*e)-2/7*c^5*a^3*(16/5+sin(f*x+e)^6
+6/5*sin(f*x+e)^4+8/5*sin(f*x+e)^2)*cos(f*x+e)+2*c^5*a^3*(-1/6*(sin(f*x+e)^
5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+e)+5/16*f*x+5/16*e)+6/5*c^5*a^3
```

```
*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)-2*c^5*a^3*(2+sin(f*x+e)^2)*
cos(f*x+e)-2*c^5*a^3*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)+2*c^5*a^3*c
os(f*x+e)+c^5*a^3*(f*x+e))
```

Maxima [B] time = 1.21141, size = 381, normalized size = 2.63

$$\frac{6144\left(5\cos\left(fx+e\right)^{7}-21\cos\left(fx+e\right)^{5}+35\cos\left(fx+e\right)^{3}-35\cos\left(fx+e\right)\right)a^{3}c^{5}+43008\left(3\cos\left(fx+e\right)^{5}-10\cos\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^5,x, algorithm="maxima")
```

```
[Out] 1/107520*(6144*(5*cos(f*x + e)^7 - 21*cos(f*x + e)^5 + 35*cos(f*x + e)^3 -
35*cos(f*x + e))*a^3*c^5 + 43008*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15
*cos(f*x + e))*a^3*c^5 + 215040*(cos(f*x + e)^3 - 3*cos(f*x + e))*a^3*c^5 -
 35*(128*sin(2*f*x + 2*e)^3 + 840*f*x + 840*e + 3*sin(8*f*x + 8*e) + 168*si
n(4*f*x + 4*e) - 768*sin(2*f*x + 2*e))*a^3*c^5 + 1120*(4*sin(2*f*x + 2*e)^3
 + 60*f*x + 60*e + 9*sin(4*f*x + 4*e) - 48*sin(2*f*x + 2*e))*a^3*c^5 - 5376
0*(2*f*x + 2*e - sin(2*f*x + 2*e))*a^3*c^5 + 107520*(f*x + e)*a^3*c^5 + 215
040*a^3*c^5*cos(f*x + e))/f
```

Fricas [A] time = 1.35618, size = 247, normalized size = 1.7

$$\frac{256\,a^{3}c^{5}\cos\left(fx+e\right)^{7}+315\,a^{3}c^{5}fx-7\left(16\,a^{3}c^{5}\cos\left(fx+e\right)^{7}-24\,a^{3}c^{5}\cos\left(fx+e\right)^{5}-30\,a^{3}c^{5}\cos\left(fx+e\right)^{3}-45\,a^{3}c^{5}\cos\right.}{896\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^5,x, algorithm="fricas")
```

```
[Out] 1/896*(256*a^3*c^5*cos(f*x + e)^7 + 315*a^3*c^5*f*x - 7*(16*a^3*c^5*cos(f*x
 + e)^7 - 24*a^3*c^5*cos(f*x + e)^5 - 30*a^3*c^5*cos(f*x + e)^3 - 45*a^3*c^
5*cos(f*x + e))*sin(f*x + e))/f
```

Sympy [A] time = 29.9373, size = 740, normalized size = 5.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(c-c*sin(f*x+e))**5,x)
```

```
[Out] Piecewise((-35*a**3*c**5*x*sin(e + f*x)**8/128 - 35*a**3*c**5*x*sin(e + f*x
)**6*cos(e + f*x)**2/32 + 5*a**3*c**5*x*sin(e + f*x)**6/8 - 105*a**3*c**5*x
*sin(e + f*x)**4*cos(e + f*x)**4/64 + 15*a**3*c**5*x*sin(e + f*x)**4*cos(e
+ f*x)**2/8 - 35*a**3*c**5*x*sin(e + f*x)**2*cos(e + f*x)**6/32 + 15*a**3*c
**5*x*sin(e + f*x)**2*cos(e + f*x)**4/8 - a**3*c**5*x*sin(e + f*x)**2 - 35*
a**3*c**5*x*cos(e + f*x)**8/128 + 5*a**3*c**5*x*cos(e + f*x)**6/8 - a**3*c*
*5*x*cos(e + f*x)**2 + a**3*c**5*x + 93*a**3*c**5*sin(e + f*x)**7*cos(e + f
*x)/(128*f) - 2*a**3*c**5*sin(e + f*x)**6*cos(e + f*x)/f + 511*a**3*c**5*si
```

```
n(e + f*x)**5*cos(e + f*x)**3/(384*f) - 11*a**3*c**5*sin(e + f*x)**5*cos(e
+ f*x)/(8*f) - 4*a**3*c**5*sin(e + f*x)**4*cos(e + f*x)**3/f + 6*a**3*c**5*
sin(e + f*x)**4*cos(e + f*x)/f + 385*a**3*c**5*sin(e + f*x)**3*cos(e + f*x)
**5/(384*f) - 5*a**3*c**5*sin(e + f*x)**3*cos(e + f*x)**3/(3*f) - 16*a**3*c
**5*sin(e + f*x)**2*cos(e + f*x)**5/(5*f) + 8*a**3*c**5*sin(e + f*x)**2*cos
(e + f*x)**3/f - 6*a**3*c**5*sin(e + f*x)**2*cos(e + f*x)/f + 35*a**3*c**5*
sin(e + f*x)*cos(e + f*x)**7/(128*f) - 5*a**3*c**5*sin(e + f*x)*cos(e + f*x
)**5/(8*f) + a**3*c**5*sin(e + f*x)*cos(e + f*x)/f - 32*a**3*c**5*cos(e + f
*x)**7/(35*f) + 16*a**3*c**5*cos(e + f*x)**5/(5*f) - 4*a**3*c**5*cos(e + f*
x)**3/f + 2*a**3*c**5*cos(e + f*x)/f, Ne(f, 0)), (x*(a*sin(e) + a)**3*(-c*s
in(e) + c)**5, True))
```

Giac [A] time = 1.69469, size = 208, normalized size = 1.43

$$\frac{45}{128}\,a^3c^5x + \frac{a^3c^5\cos\left(7\,fx + 7\,e\right)}{224\,f} + \frac{a^3c^5\cos\left(5\,fx + 5\,e\right)}{32\,f} + \frac{3\,a^3c^5\cos\left(3\,fx + 3\,e\right)}{32\,f} + \frac{5\,a^3c^5\cos\left(fx + e\right)}{32\,f} - \frac{a^3c^5\sin\left(8\right.}{102}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^5,x, algorithm="giac")
```

```
[Out] 45/128*a^3*c^5*x + 1/224*a^3*c^5*cos(7*f*x + 7*e)/f + 1/32*a^3*c^5*cos(5*f*
x + 5*e)/f + 3/32*a^3*c^5*cos(3*f*x + 3*e)/f + 5/32*a^3*c^5*cos(f*x + e)/f
- 1/1024*a^3*c^5*sin(8*f*x + 8*e)/f + 5/128*a^3*c^5*sin(4*f*x + 4*e)/f + 1/
4*a^3*c^5*sin(2*f*x + 2*e)/f
```

3.249 $\int (a+a\sin(e+fx))^3(c-c\sin(e+fx))^4\,dx$

Optimal. Leaf size=112

$$\frac{a^3c^4\cos^7(e+fx)}{7f}+\frac{a^3c^4\sin(e+fx)\cos^5(e+fx)}{6f}+\frac{5a^3c^4\sin(e+fx)\cos^3(e+fx)}{24f}+\frac{5a^3c^4\sin(e+fx)\cos(e+fx)}{16f}+\frac{5}{16}\ldots$$

```
[Out] (5*a^3*c^4*x)/16 + (a^3*c^4*Cos[e + f*x]^7)/(7*f) + (5*a^3*c^4*Cos[e + f*x]
*Sin[e + f*x])/(16*f) + (5*a^3*c^4*Cos[e + f*x]^3*Sin[e + f*x])/(24*f) + (a
^3*c^4*Cos[e + f*x]^5*Sin[e + f*x])/(6*f)
```

Rubi [A] time = 0.10777, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2736, 2669, 2635, 8}

$$\frac{a^3c^4\cos^7(e+fx)}{7f}+\frac{a^3c^4\sin(e+fx)\cos^5(e+fx)}{6f}+\frac{5a^3c^4\sin(e+fx)\cos^3(e+fx)}{24f}+\frac{5a^3c^4\sin(e+fx)\cos(e+fx)}{16f}+\frac{5}{16}\ldots$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^4,x]
```


```
[Out] (5*a^3*c^4*x)/16 + (a^3*c^4*Cos[e + f*x]^7)/(7*f) + (5*a^3*c^4*Cos[e + f*x]
*Sin[e + f*x])/(16*f) + (5*a^3*c^4*Cos[e + f*x]^3*Sin[e + f*x])/(24*f) + (a
^3*c^4*Cos[e + f*x]^5*Sin[e + f*x])/(6*f)
```


Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (a+a\sin(e+fx))^3(c-c\sin(e+fx))^4\,dx = \left(a^3c^3\right)\int \cos^6(e+fx)(c-c\sin(e+fx))\,dx$$
$$= \frac{a^3c^4\cos^7(e+fx)}{7f} + \left(a^3c^4\right)\int \cos^6(e+fx)\,dx$$
$$= \frac{a^3c^4\cos^7(e+fx)}{7f} + \frac{a^3c^4\cos^5(e+fx)\sin(e+fx)}{6f} + \frac{1}{6}\left(5a^3c^4\right)\int \cos^4($$
$$= \frac{a^3c^4\cos^7(e+fx)}{7f} + \frac{5a^3c^4\cos^3(e+fx)\sin(e+fx)}{24f} + \frac{a^3c^4\cos^5(e+fx}{6f}$$
$$= \frac{a^3c^4\cos^7(e+fx)}{7f} + \frac{5a^3c^4\cos(e+fx)\sin(e+fx)}{16f} + \frac{5a^3c^4\cos^3(e+fx}{24f}$$
$$= \frac{5}{16}a^3c^4x + \frac{a^3c^4\cos^7(e+fx)}{7f} + \frac{5a^3c^4\cos(e+fx)\sin(e+fx)}{16f} + \frac{5a^3c^4}{}$$

Mathematica [A] time = 1.05507, size = 89, normalized size = 0.79

$$\frac{a^3c^4(315\sin(2(e+fx)) + 63\sin(4(e+fx)) + 7\sin(6(e+fx)) + 105\cos(e+fx) + 63\cos(3(e+fx)) + 21\cos(5(e+f}{1344f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^4,x]
```

```
[Out] (a^3*c^4*(420*e + 420*f*x + 105*Cos[e + f*x] + 63*Cos[3*(e + f*x)] + 21*Cos
[5*(e + f*x)] + 3*Cos[7*(e + f*x)] + 315*Sin[2*(e + f*x)] + 63*Sin[4*(e + f
*x)] + 7*Sin[6*(e + f*x)]))/(1344*f)
```

Maple [B] time = 0.016, size = 255, normalized size = 2.3

$$\frac{1}{f}\left(-\frac{c^4a^3\cos(fx+e)}{7}\left(\frac{16}{5}+(\sin(fx+e))^6+\frac{6(\sin(fx+e))^4}{5}+\frac{8(\sin(fx+e))^2}{5}\right)-c^4a^3\left(-\frac{\cos(fx+e)}{6}\left((\sin(f\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^4,x)
```

```
[Out] 1/f*(-1/7*c^4*a^3*(16/5+sin(f*x+e)^6+6/5*sin(f*x+e)^4+8/5*sin(f*x+e)^2)*cos
(f*x+e)-c^4*a^3*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f
*x+e)+5/16*f*x+5/16*e)+3/5*c^4*a^3*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(
f*x+e)+3*c^4*a^3*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8
*e)-c^4*a^3*(2+sin(f*x+e)^2)*cos(f*x+e)-3*c^4*a^3*(-1/2*sin(f*x+e)*cos(f*x+
e)+1/2*f*x+1/2*e)+c^4*a^3*cos(f*x+e)+c^4*a^3*(f*x+e))
```

Maxima [B] time = 1.19201, size = 346, normalized size = 3.09

$$\frac{192\left(5\cos(fx+e)^7-21\cos(fx+e)^5+35\cos(fx+e)^3-35\cos(fx+e)\right)a^3c^4+1344\left(3\cos(fx+e)^5-10\cos(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^4,x, algorithm="maxima")
```

```
[Out] 1/6720*(192*(5*cos(f*x + e)^7 - 21*cos(f*x + e)^5 + 35*cos(f*x + e)^3 - 35*
cos(f*x + e))*a^3*c^4 + 1344*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos
(f*x + e))*a^3*c^4 + 6720*(cos(f*x + e)^3 - 3*cos(f*x + e))*a^3*c^4 - 35*(4
*sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*sin(4*f*x + 4*e) - 48*sin(2*f*x + 2
*e))*a^3*c^4 + 630*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*
a^3*c^4 - 5040*(2*f*x + 2*e - sin(2*f*x + 2*e))*a^3*c^4 + 6720*(f*x + e)*a^
3*c^4 + 6720*a^3*c^4*cos(f*x + e))/f
```

Fricas [A] time = 1.38752, size = 207, normalized size = 1.85

$$\frac{48\,a^3c^4\cos\left(fx+e\right)^7+105\,a^3c^4fx+7\left(8\,a^3c^4\cos\left(fx+e\right)^5+10\,a^3c^4\cos\left(fx+e\right)^3+15\,a^3c^4\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{336\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^4,x, algorithm="fricas")
```

```
[Out] 1/336*(48*a^3*c^4*cos(f*x + e)^7 + 105*a^3*c^4*f*x + 7*(8*a^3*c^4*cos(f*x +
 e)^5 + 10*a^3*c^4*cos(f*x + e)^3 + 15*a^3*c^4*cos(f*x + e))*sin(f*x + e))/
f
```

Sympy [A] time = 18.6363, size = 631, normalized size = 5.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(c-c*sin(f*x+e))**4,x)
```

```
[Out] Piecewise((-5*a**3*c**4*x*sin(e + f*x)**6/16 - 15*a**3*c**4*x*sin(e + f*x)*
*4*cos(e + f*x)**2/16 + 9*a**3*c**4*x*sin(e + f*x)**4/8 - 15*a**3*c**4*x*si
n(e + f*x)**2*cos(e + f*x)**4/16 + 9*a**3*c**4*x*sin(e + f*x)**2*cos(e + f*
x)**2/4 - 3*a**3*c**4*x*sin(e + f*x)**2/2 - 5*a**3*c**4*x*cos(e + f*x)**6/1
6 + 9*a**3*c**4*x*cos(e + f*x)**4/8 - 3*a**3*c**4*x*cos(e + f*x)**2/2 + a**
3*c**4*x - a**3*c**4*sin(e + f*x)**6*cos(e + f*x)/f + 11*a**3*c**4*sin(e +
f*x)**5*cos(e + f*x)/(16*f) - 2*a**3*c**4*sin(e + f*x)**4*cos(e + f*x)**3/f
 + 3*a**3*c**4*sin(e + f*x)**4*cos(e + f*x)/f + 5*a**3*c**4*sin(e + f*x)**3
*cos(e + f*x)**3/(6*f) - 15*a**3*c**4*sin(e + f*x)**3*cos(e + f*x)/(8*f) -
8*a**3*c**4*sin(e + f*x)**2*cos(e + f*x)**5/(5*f) + 4*a**3*c**4*sin(e + f*x
)**2*cos(e + f*x)**3/f - 3*a**3*c**4*sin(e + f*x)**2*cos(e + f*x)/f + 5*a**
3*c**4*sin(e + f*x)*cos(e + f*x)**5/(16*f) - 9*a**3*c**4*sin(e + f*x)*cos(e
 + f*x)**3/(8*f) + 3*a**3*c**4*sin(e + f*x)*cos(e + f*x)/(2*f) - 16*a**3*c*
*4*cos(e + f*x)**7/(35*f) + 8*a**3*c**4*cos(e + f*x)**5/(5*f) - 2*a**3*c**4
*cos(e + f*x)**3/f + a**3*c**4*cos(e + f*x)/f, Ne(f, 0)), (x*(a*sin(e) + a)
**3*(-c*sin(e) + c)**4, True))
```

Giac [A] time = 1.72489, size = 208, normalized size = 1.86

$$\frac{5}{16}\,a^3c^4x+\frac{a^3c^4\cos\left(7\,fx+7\,e\right)}{448\,f}+\frac{a^3c^4\cos\left(5\,fx+5\,e\right)}{64\,f}+\frac{3\,a^3c^4\cos\left(3\,fx+3\,e\right)}{64\,f}+\frac{5\,a^3c^4\cos\left(fx+e\right)}{64\,f}+\frac{a^3c^4\sin\left(6\,fx\right.}{192\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^4,x, algorithm="giac")
```

```
[Out] 5/16*a^3*c^4*x + 1/448*a^3*c^4*cos(7*f*x + 7*e)/f + 1/64*a^3*c^4*cos(5*f*x
+ 5*e)/f + 3/64*a^3*c^4*cos(3*f*x + 3*e)/f + 5/64*a^3*c^4*cos(f*x + e)/f +
1/192*a^3*c^4*sin(6*f*x + 6*e)/f + 3/64*a^3*c^4*sin(4*f*x + 4*e)/f + 15/64*
a^3*c^4*sin(2*f*x + 2*e)/f
```

3.250 $\int (a+a\sin(e+fx))^3(c-c\sin(e+fx))^3\,dx$

Optimal. Leaf size=91

$$\frac{a^3c^3\sin(e+fx)\cos^5(e+fx)}{6f}+\frac{5a^3c^3\sin(e+fx)\cos^3(e+fx)}{24f}+\frac{5a^3c^3\sin(e+fx)\cos(e+fx)}{16f}+\frac{5}{16}a^3c^3x$$

```
[Out] (5*a^3*c^3*x)/16 + (5*a^3*c^3*Cos[e + f*x]*Sin[e + f*x])/(16*f) + (5*a^3*c^
3*Cos[e + f*x]^3*Sin[e + f*x])/(24*f) + (a^3*c^3*Cos[e + f*x]^5*Sin[e + f*x
])/(6*f)
```

Rubi [A] time = 0.0793481, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2736, 2635, 8}

$$\frac{a^3c^3\sin(e+fx)\cos^5(e+fx)}{6f}+\frac{5a^3c^3\sin(e+fx)\cos^3(e+fx)}{24f}+\frac{5a^3c^3\sin(e+fx)\cos(e+fx)}{16f}+\frac{5}{16}a^3c^3x$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^3,x]
```

```
[Out] (5*a^3*c^3*x)/16 + (5*a^3*c^3*Cos[e + f*x]*Sin[e + f*x])/(16*f) + (5*a^3*c^
3*Cos[e + f*x]^3*Sin[e + f*x])/(24*f) + (a^3*c^3*Cos[e + f*x]^5*Sin[e + f*x
])/(6*f)
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (a+a\sin(e+fx))^3(c-c\sin(e+fx))^3\,dx = \left(a^3c^3\right)\int \cos^6(e+fx)\,dx$$
$$= \frac{a^3c^3\cos^5(e+fx)\sin(e+fx)}{6f} + \frac{1}{6}\left(5a^3c^3\right)\int \cos^4(e+fx)\,dx$$
$$= \frac{5a^3c^3\cos^3(e+fx)\sin(e+fx)}{24f} + \frac{a^3c^3\cos^5(e+fx)\sin(e+fx)}{6f} + \frac{1}{8}\left(5a\right.$$
$$= \frac{5a^3c^3\cos(e+fx)\sin(e+fx)}{16f} + \frac{5a^3c^3\cos^3(e+fx)\sin(e+fx)}{24f} + \frac{a^3c^3}{}$$
$$= \frac{5}{16}a^3c^3x + \frac{5a^3c^3\cos(e+fx)\sin(e+fx)}{16f} + \frac{5a^3c^3\cos^3(e+fx)\sin(e+}{24f}$$

Mathematica [A] time = 0.0512375, size = 49, normalized size = 0.54

$$\frac{a^3c^3(45\sin(2(e+fx))+9\sin(4(e+fx))+\sin(6(e+fx))+60e+60fx)}{192f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^3,x]
```

```
[Out] (a^3*c^3*(60*e + 60*f*x + 45*Sin[2*(e + f*x)] + 9*Sin[4*(e + f*x)] + Sin[6*
(e + f*x)]))/(192*f)
```

Maple [A] time = 0.013, size = 140, normalized size = 1.5

$$\frac{1}{f}\left(-c^3a^3\left(-\frac{\cos(fx+e)}{6}\left(\left(\sin(fx+e)\right)^5+\frac{5\left(\sin(fx+e)\right)^3}{4}+\frac{15\sin(fx+e)}{8}\right)+\frac{5fx}{16}+\frac{5e}{16}\right)+3c^3a^3\left(-1/4\left(\left(\sin\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^3,x)
```

```
[Out] 1/f*(-c^3*a^3*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x
+e)+5/16*f*x+5/16*e)+3*c^3*a^3*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+
e)+3/8*f*x+3/8*e)-3*c^3*a^3*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)+c^3*
a^3*(f*x+e))
```

Maxima [A] time = 1.22467, size = 178, normalized size = 1.96

$$-\frac{\left(4\sin\left(2fx+2e\right)^3+60fx+60e+9\sin\left(4fx+4e\right)-48\sin\left(2fx+2e\right)\right)a^3c^3-18\left(12fx+12e+\sin\left(4fx+4\right.\right.}{192f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] -1/192*((4*sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*sin(4*f*x + 4*e) - 48*sin
(2*f*x + 2*e))*a^3*c^3 - 18*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x
 + 2*e))*a^3*c^3 + 144*(2*f*x + 2*e - sin(2*f*x + 2*e))*a^3*c^3 - 192*(f*x
```

```
+ e)*a^3*c^3)/f
```

Fricas [A] time = 1.40205, size = 163, normalized size = 1.79

$$\frac{15\,a^3c^3fx + \left(8\,a^3c^3\cos\left(fx+e\right)^5 + 10\,a^3c^3\cos\left(fx+e\right)^3 + 15\,a^3c^3\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{48\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/48*(15*a^3*c^3*f*x + (8*a^3*c^3*cos(f*x + e)^5 + 10*a^3*c^3*cos(f*x + e)^
3 + 15*a^3*c^3*cos(f*x + e))*sin(f*x + e))/f
```

Sympy [A] time = 10.1909, size = 398, normalized size = 4.37

$$\begin{cases} -\frac{5a^3c^3x\sin^6\left(e+fx\right)}{16} - \frac{15a^3c^3x\sin^4\left(e+fx\right)\cos^2\left(e+fx\right)}{16} + \frac{9a^3c^3x\sin^4\left(e+fx\right)}{8} - \frac{15a^3c^3x\sin^2\left(e+fx\right)\cos^4\left(e+fx\right)}{16} + \frac{9a^3c^3x\sin^2\left(e+fx\right)\cos^2\left(e+fx\right)}{4} - \\ x\left(a\sin\left(e\right)+a\right)^3\left(-c\sin\left(e\right)+c\right)^3 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(c-c*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((-5*a**3*c**3*x*sin(e + f*x)**6/16 - 15*a**3*c**3*x*sin(e + f*x)*
*4*cos(e + f*x)**2/16 + 9*a**3*c**3*x*sin(e + f*x)**4/8 - 15*a**3*c**3*x*si
n(e + f*x)**2*cos(e + f*x)**4/16 + 9*a**3*c**3*x*sin(e + f*x)**2*cos(e + f*
x)**2/4 - 3*a**3*c**3*x*sin(e + f*x)**2/2 - 5*a**3*c**3*x*cos(e + f*x)**6/1
6 + 9*a**3*c**3*x*cos(e + f*x)**4/8 - 3*a**3*c**3*x*cos(e + f*x)**2/2 + a**
3*c**3*x + 11*a**3*c**3*sin(e + f*x)**5*cos(e + f*x)/(16*f) + 5*a**3*c**3*s
in(e + f*x)**3*cos(e + f*x)**3/(6*f) - 15*a**3*c**3*sin(e + f*x)**3*cos(e +
 f*x)/(8*f) + 5*a**3*c**3*sin(e + f*x)*cos(e + f*x)**5/(16*f) - 9*a**3*c**3
*sin(e + f*x)*cos(e + f*x)**3/(8*f) + 3*a**3*c**3*sin(e + f*x)*cos(e + f*x)
/(2*f), Ne(f, 0)), (x*(a*sin(e) + a)**3*(-c*sin(e) + c)**3, True))
```

Giac [A] time = 1.77461, size = 99, normalized size = 1.09

$$\frac{5}{16}\,a^3c^3x + \frac{a^3c^3\sin\left(6\,fx+6\,e\right)}{192\,f} + \frac{3\,a^3c^3\sin\left(4\,fx+4\,e\right)}{64\,f} + \frac{15\,a^3c^3\sin\left(2\,fx+2\,e\right)}{64\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 5/16*a^3*c^3*x + 1/192*a^3*c^3*sin(6*f*x + 6*e)/f + 3/64*a^3*c^3*sin(4*f*x
+ 4*e)/f + 15/64*a^3*c^3*sin(2*f*x + 2*e)/f
```

3.251 $\int (a+a\sin(e+fx))^3(c-c\sin(e+fx))^2\,dx$

Optimal. Leaf size=85

$$-\frac{a^3c^2\cos^5(e+fx)}{5f}+\frac{a^3c^2\sin(e+fx)\cos^3(e+fx)}{4f}+\frac{3a^3c^2\sin(e+fx)\cos(e+fx)}{8f}+\frac{3}{8}a^3c^2x$$

```
[Out] (3*a^3*c^2*x)/8 - (a^3*c^2*Cos[e + f*x]^5)/(5*f) + (3*a^3*c^2*Cos[e + f*x]*
Sin[e + f*x])/(8*f) + (a^3*c^2*Cos[e + f*x]^3*Sin[e + f*x])/(4*f)
```

Rubi [A] time = 0.0924749, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2736, 2669, 2635, 8}

$$-\frac{a^3c^2\cos^5(e+fx)}{5f}+\frac{a^3c^2\sin(e+fx)\cos^3(e+fx)}{4f}+\frac{3a^3c^2\sin(e+fx)\cos(e+fx)}{8f}+\frac{3}{8}a^3c^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^2,x]
```

```
[Out] (3*a^3*c^2*x)/8 - (a^3*c^2*Cos[e + f*x]^5)/(5*f) + (3*a^3*c^2*Cos[e + f*x]*
Sin[e + f*x])/(8*f) + (a^3*c^2*Cos[e + f*x]^3*Sin[e + f*x])/(4*f)
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (a+a\sin(e+fx))^3(c-c\sin(e+fx))^2\,dx = \left(a^2c^2\right)\int \cos^4(e+fx)(a+a\sin(e+fx))\,dx$$
$$= -\frac{a^3c^2\cos^5(e+fx)}{5f} + \left(a^3c^2\right)\int \cos^4(e+fx)\,dx$$
$$= -\frac{a^3c^2\cos^5(e+fx)}{5f} + \frac{a^3c^2\cos^3(e+fx)\sin(e+fx)}{4f} + \frac{1}{4}\left(3a^3c^2\right)\int \cos^2(e$$
$$= -\frac{a^3c^2\cos^5(e+fx)}{5f} + \frac{3a^3c^2\cos(e+fx)\sin(e+fx)}{8f} + \frac{a^3c^2\cos^3(e+fx)\mathrm{si}}{4f}$$
$$= \frac{3}{8}a^3c^2x - \frac{a^3c^2\cos^5(e+fx)}{5f} + \frac{3a^3c^2\cos(e+fx)\sin(e+fx)}{8f} + \frac{a^3c^2\cos^3(e}{}$$

Mathematica [A] time = 1.53295, size = 69, normalized size = 0.81

$$\frac{a^3c^2(40\sin(2(e+fx))+5\sin(4(e+fx))-20\cos(e+fx)-10\cos(3(e+fx))-2\cos(5(e+fx))+60e+60fx)}{160f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^2,x]
```

```
[Out] (a^3*c^2*(60*e + 60*f*x - 20*Cos[e + f*x] - 10*Cos[3*(e + f*x)] - 2*Cos[5*(
e + f*x)] + 40*Sin[2*(e + f*x)] + 5*Sin[4*(e + f*x)]))/(160*f)
```

Maple [B] time = 0.015, size = 160, normalized size = 1.9

$$\frac{1}{f}\left(-\frac{c^2a^3\cos\left(fx+e\right)}{5}\left(\frac{8}{3}+\left(\sin\left(fx+e\right)\right)^4+\frac{4\left(\sin\left(fx+e\right)\right)^2}{3}\right)+c^2a^3\left(-\frac{\cos\left(fx+e\right)}{4}\left(\left(\sin\left(fx+e\right)\right)^3+\frac{3\sin\left(fx+e\right)}{2}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^2,x)
```

```
[Out] 1/f*(-1/5*c^2*a^3*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)+c^2*a^3*(-
1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)+2/3*c^2*a^3*(2+
sin(f*x+e)^2)*cos(f*x+e)-2*c^2*a^3*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*
e)-c^2*a^3*cos(f*x+e)+c^2*a^3*(f*x+e))
```

Maxima [B] time = 1.59996, size = 213, normalized size = 2.51

$$-\frac{32\left(3\cos\left(fx+e\right)^5-10\cos\left(fx+e\right)^3+15\cos\left(fx+e\right)\right)a^3c^2+320\left(\cos\left(fx+e\right)^3-3\cos\left(fx+e\right)\right)a^3c^2-15\left(12fx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] -1/480*(32*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*a^3*c^2
 + 320*(cos(f*x + e)^3 - 3*cos(f*x + e))*a^3*c^2 - 15*(12*f*x + 12*e + sin(
```

```
4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*a^3*c^2 + 240*(2*f*x + 2*e - sin(2*f*x +
 2*e))*a^3*c^2 - 480*(f*x + e)*a^3*c^2 + 480*a^3*c^2*cos(f*x + e))/f
```

Fricas [A] time = 1.39263, size = 165, normalized size = 1.94

$$-\frac{8\,a^3c^2\cos\left(fx+e\right)^5-15\,a^3c^2fx-5\left(2\,a^3c^2\cos\left(fx+e\right)^3+3\,a^3c^2\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{40\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -1/40*(8*a^3*c^2*cos(f*x + e)^5 - 15*a^3*c^2*f*x - 5*(2*a^3*c^2*cos(f*x + e
)^3 + 3*a^3*c^2*cos(f*x + e))*sin(f*x + e))/f
```

Sympy [A] time = 5.52163, size = 340, normalized size = 4.

$$\begin{cases}\frac{3a^3c^2x\sin^4\left(e+fx\right)}{8}+\frac{3a^3c^2x\sin^2\left(e+fx\right)\cos^2\left(e+fx\right)}{4}-a^3c^2x\sin^2\left(e+fx\right)+\frac{3a^3c^2x\cos^4\left(e+fx\right)}{8}-a^3c^2x\cos^2\left(e+fx\right)+a^3c^2x-\frac{a^3 \\ x\left(a\sin\left(e\right)+a\right)^3\left(-c\sin\left(e\right)+c\right)^2\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(c-c*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((3*a**3*c**2*x*sin(e + f*x)**4/8 + 3*a**3*c**2*x*sin(e + f*x)**2*
cos(e + f*x)**2/4 - a**3*c**2*x*sin(e + f*x)**2 + 3*a**3*c**2*x*cos(e + f*x
)**4/8 - a**3*c**2*x*cos(e + f*x)**2 + a**3*c**2*x - a**3*c**2*sin(e + f*x)
**4*cos(e + f*x)/f - 5*a**3*c**2*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 4*a**
3*c**2*sin(e + f*x)**2*cos(e + f*x)**3/(3*f) + 2*a**3*c**2*sin(e + f*x)**2*
cos(e + f*x)/f - 3*a**3*c**2*sin(e + f*x)*cos(e + f*x)**3/(8*f) + a**3*c**2
*sin(e + f*x)*cos(e + f*x)/f - 8*a**3*c**2*cos(e + f*x)**5/(15*f) + 4*a**3*
c**2*cos(e + f*x)**3/(3*f) - a**3*c**2*cos(e + f*x)/f, Ne(f, 0)), (x*(a*sin
(e) + a)**3*(-c*sin(e) + c)**2, True))
```

Giac [A] time = 2.4822, size = 151, normalized size = 1.78

$$\frac{3}{8}\,a^3c^2x-\frac{a^3c^2\cos\left(5\,fx+5\,e\right)}{80\,f}-\frac{a^3c^2\cos\left(3\,fx+3\,e\right)}{16\,f}-\frac{a^3c^2\cos\left(fx+e\right)}{8\,f}+\frac{a^3c^2\sin\left(4\,fx+4\,e\right)}{32\,f}+\frac{a^3c^2\sin\left(2\,fx+\right.}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 3/8*a^3*c^2*x - 1/80*a^3*c^2*cos(5*f*x + 5*e)/f - 1/16*a^3*c^2*cos(3*f*x +
3*e)/f - 1/8*a^3*c^2*cos(f*x + e)/f + 1/32*a^3*c^2*sin(4*f*x + 4*e)/f + 1/4
*a^3*c^2*sin(2*f*x + 2*e)/f
```

3.252 $\int (a + a\sin(e + fx))^3(c - c\sin(e + fx))\,dx$

Optimal. Leaf size=82

$$-\frac{5a^3c\cos^3(e+fx)}{12f} - \frac{c\cos^3(e+fx)\left(a^3\sin(e+fx)+a^3\right)}{4f} + \frac{5a^3c\sin(e+fx)\cos(e+fx)}{8f} + \frac{5}{8}a^3cx$$

```
[Out] (5*a^3*c*x)/8 - (5*a^3*c*Cos[e + f*x]^3)/(12*f) + (5*a^3*c*Cos[e + f*x]*Sin
[e + f*x])/(8*f) - (c*Cos[e + f*x]^3*(a^3 + a^3*Sin[e + f*x]))/(4*f)
```

Rubi [A] time = 0.0984802, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {2736, 2678, 2669, 2635, 8}

$$-\frac{5a^3c\cos^3(e+fx)}{12f} - \frac{c\cos^3(e+fx)\left(a^3\sin(e+fx)+a^3\right)}{4f} + \frac{5a^3c\sin(e+fx)\cos(e+fx)}{8f} + \frac{5}{8}a^3cx$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x]),x]
```

```
[Out] (5*a^3*c*x)/8 - (5*a^3*c*Cos[e + f*x]^3)/(12*f) + (5*a^3*c*Cos[e + f*x]*Sin
[e + f*x])/(8*f) - (c*Cos[e + f*x]^3*(a^3 + a^3*Sin[e + f*x]))/(4*f)
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (a+a\sin(e+fx))^3(c-c\sin(e+fx))\,dx=(ac)\int \cos^2(e+fx)(a+a\sin(e+fx))^2\,dx$$
$$=-\frac{c\cos^3(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{4f}+\frac{1}{4}\left(5a^2c\right)\int \cos^2(e+fx)(a+a\,\mathrm{s}$$
$$=-\frac{5a^3c\cos^3(e+fx)}{12f}-\frac{c\cos^3(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{4f}+\frac{1}{4}\left(5a^3c\right)\int$$
$$=-\frac{5a^3c\cos^3(e+fx)}{12f}+\frac{5a^3c\cos(e+fx)\sin(e+fx)}{8f}-\frac{c\cos^3(e+fx)\left(a^3\right.}{4}$$
$$=\frac{5}{8}a^3cx-\frac{5a^3c\cos^3(e+fx)}{12f}+\frac{5a^3c\cos(e+fx)\sin(e+fx)}{8f}-\frac{c\cos^3(e+}{}$$

Mathematica [A] time = 0.385545, size = 54, normalized size = 0.66

$$\frac{a^3c(24\sin(2(e+fx))-3\sin(4(e+fx))-48\cos(e+fx)-16\cos(3(e+fx))+60fx)}{96f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x]),x]
```

```
[Out] (a^3*c*(60*f*x - 48*Cos[e + f*x] - 16*Cos[3*(e + f*x)] + 24*Sin[2*(e + f*x)
] - 3*Sin[4*(e + f*x)]))/(96*f)
```

Maple [A] time = 0.017, size = 89, normalized size = 1.1

$$\frac{1}{f}\left(-a^3c\left(-\frac{\cos(fx+e)}{4}\left((\sin(fx+e))^3+\frac{3\sin(fx+e)}{2}\right)+\frac{3fx}{8}+\frac{3e}{8}\right)+\frac{2a^3c\left(2+(\sin(fx+e))^2\right)\cos(fx+e)}{3}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e)),x)
```

```
[Out] 1/f*(-a^3*c*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)+2
/3*a^3*c*(2+sin(f*x+e)^2)*cos(f*x+e)-2*a^3*c*cos(f*x+e)+a^3*c*(f*x+e))
```

Maxima [A] time = 1.29211, size = 116, normalized size = 1.41

$$-\frac{64\left(\cos(fx+e)^3-3\cos(fx+e)\right)a^3c+3\left(12fx+12e+\sin(4fx+4e)-8\sin(2fx+2e)\right)a^3c-96(fx+e)a^3c}{96f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] -1/96*(64*(cos(f*x + e)^3 - 3*cos(f*x + e))*a^3*c + 3*(12*f*x + 12*e + sin(
4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*a^3*c - 96*(f*x + e)*a^3*c + 192*a^3*c*c
os(f*x + e))/f
```

Fricas [A] time = 1.46137, size = 155, normalized size = 1.89

$$-\frac{16\,a^3c\cos\left(fx+e\right)^3-15\,a^3cfx+3\left(2\,a^3c\cos\left(fx+e\right)^3-5\,a^3c\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] -1/24*(16*a^3*c*cos(f*x + e)^3 - 15*a^3*c*f*x + 3*(2*a^3*c*cos(f*x + e)^3 -
 5*a^3*c*cos(f*x + e))*sin(f*x + e))/f
```

Sympy [A] time = 3.19206, size = 196, normalized size = 2.39

$$\begin{cases} -\frac{3a^3cx\sin^4\left(e+fx\right)}{8}-\frac{3a^3cx\sin^2\left(e+fx\right)\cos^2\left(e+fx\right)}{4}-\frac{3a^3cx\cos^4\left(e+fx\right)}{8}+a^3cx+\frac{5a^3c\sin^3\left(e+fx\right)\cos\left(e+fx\right)}{8f}+\frac{2a^3c\sin^2\left(e+fx\right)\cos\left(e+fx\right)}{f}+\frac{3a}{} \\ x\left(a\sin\left(e\right)+a\right)^3\left(-c\sin\left(e\right)+c\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(c-c*sin(f*x+e)),x)
```

```
[Out] Piecewise((-3*a**3*c*x*sin(e + f*x)**4/8 - 3*a**3*c*x*sin(e + f*x)**2*cos(e
 + f*x)**2/4 - 3*a**3*c*x*cos(e + f*x)**4/8 + a**3*c*x + 5*a**3*c*sin(e + f
*x)**3*cos(e + f*x)/(8*f) + 2*a**3*c*sin(e + f*x)**2*cos(e + f*x)/f + 3*a**
3*c*sin(e + f*x)*cos(e + f*x)**3/(8*f) + 4*a**3*c*cos(e + f*x)**3/(3*f) - 2
*a**3*c*cos(e + f*x)/f, Ne(f, 0)), (x*(a*sin(e) + a)**3*(-c*sin(e) + c), Tr
ue))
```

Giac [A] time = 2.44191, size = 109, normalized size = 1.33

$$\frac{5}{8}\,a^3cx-\frac{a^3c\cos\left(3\,fx+3\,e\right)}{6\,f}-\frac{a^3c\cos\left(fx+e\right)}{2\,f}-\frac{a^3c\sin\left(4\,fx+4\,e\right)}{32\,f}+\frac{a^3c\sin\left(2\,fx+2\,e\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] 5/8*a^3*c*x - 1/6*a^3*c*cos(3*f*x + 3*e)/f - 1/2*a^3*c*cos(f*x + e)/f - 1/3
2*a^3*c*sin(4*f*x + 4*e)/f + 1/4*a^3*c*sin(2*f*x + 2*e)/f
```

3.253 $\int \frac{(a+a\sin(e+fx))^3}{c-c\sin(e+fx)}\,dx$

Optimal. Leaf size=94

$$\frac{2a^3c^2\cos^5(e+fx)}{f(c-c\sin(e+fx))^3}+\frac{15a^3\cos(e+fx)}{2cf}+\frac{5a^3\cos^3(e+fx)}{2f(c-c\sin(e+fx))}-\frac{15a^3x}{2c}$$

```
[Out] (-15*a^3*x)/(2*c) + (15*a^3*Cos[e + f*x])/(2*c*f) + (2*a^3*c^2*Cos[e + f*x]
^5)/(f*(c - c*Sin[e + f*x])^3) + (5*a^3*Cos[e + f*x]^3)/(2*f*(c - c*Sin[e +
 f*x]))
```

Rubi [A] time = 0.182553, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {2736, 2680, 2679, 2682, 8}

$$\frac{2a^3c^2\cos^5(e+fx)}{f(c-c\sin(e+fx))^3}+\frac{15a^3\cos(e+fx)}{2cf}+\frac{5a^3\cos^3(e+fx)}{2f(c-c\sin(e+fx))}-\frac{15a^3x}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3/(c - c*Sin[e + f*x]),x]
```

```
[Out] (-15*a^3*x)/(2*c) + (15*a^3*Cos[e + f*x])/(2*c*f) + (2*a^3*c^2*Cos[e + f*x]
^5)/(f*(c - c*Sin[e + f*x])^3) + (5*a^3*Cos[e + f*x]^3)/(2*f*(c - c*Sin[e +
 f*x]))
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2679

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(a*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+a\sin(e+fx))^3}{c-c\sin(e+fx)}\,dx &= \left(a^3c^3\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^4}\,dx\\
&= \frac{2a^3c^2\cos^5(e+fx)}{f(c-c\sin(e+fx))^3}-\left(5a^3c\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^2}\,dx\\
&= \frac{2a^3c^2\cos^5(e+fx)}{f(c-c\sin(e+fx))^3}+\frac{5a^3\cos^3(e+fx)}{2f(c-c\sin(e+fx))}-\frac{1}{2}\left(15a^3\right)\int \frac{\cos^2(e+fx)}{c-c\sin(e+fx)}\,dx\\
&= \frac{15a^3\cos(e+fx)}{2cf}+\frac{2a^3c^2\cos^5(e+fx)}{f(c-c\sin(e+fx))^3}+\frac{5a^3\cos^3(e+fx)}{2f(c-c\sin(e+fx))}-\frac{\left(15a^3\right)\int 1\,dx}{2c}\\
&= -\frac{15a^3x}{2c}+\frac{15a^3\cos(e+fx)}{2cf}+\frac{2a^3c^2\cos^5(e+fx)}{f(c-c\sin(e+fx))^3}+\frac{5a^3\cos^3(e+fx)}{2f(c-c\sin(e+fx))}\end{aligned}$$

Mathematica [A] time = 0.50658, size = 153, normalized size = 1.63

$$\frac{a^3(\sin(e+fx)+1)^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)(30(e+fx)-\sin(2(e+fx))-16\cos(e+fx))\right.}{4cf(\sin(e+fx)-1)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3/(c - c*Sin[e + f*x]),x]
```

```
[Out] (a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^3*(Cos[(e + f
*x)/2]*(30*(e + f*x) - 16*Cos[e + f*x] - Sin[2*(e + f*x)]) + Sin[(e + f*x)/
2]*(-64 - 30*e - 30*f*x + 16*Cos[e + f*x] + Sin[2*(e + f*x)])))/(4*c*f*(Cos
[(e + f*x)/2] + Sin[(e + f*x)/2])^6*(-1 + Sin[e + f*x]))
```

Maple [B] time = 0.08, size = 181, normalized size = 1.9

$$-16\frac{a^3}{cf\left(\tan\left(1/2\,fx+e/2\right)-1\right)}-\frac{a^3}{cf}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3\left(1+\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2\right)^{-2}+8\frac{a^3\left(\tan\left(1/2\,fx+e/2\right)\right)^2}{cf\left(1+\left(\tan\left(1/2\,fx+e/2\right)\right)^2\right)^2}+\frac{a}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e)),x)
```

```
[Out] -16/f*a^3/c/(tan(1/2*f*x+1/2*e)-1)-1/f*a^3/c/(1+tan(1/2*f*x+1/2*e)^2)^2*tan
(1/2*f*x+1/2*e)^3+8/f*a^3/c/(1+tan(1/2*f*x+1/2*e)^2)^2*tan(1/2*f*x+1/2*e)^2
+1/f*a^3/c/(1+tan(1/2*f*x+1/2*e)^2)^2*tan(1/2*f*x+1/2*e)+8/f*a^3/c/(1+tan(1
/2*f*x+1/2*e)^2)^2-15/f*a^3/c*arctan(tan(1/2*f*x+1/2*e))
```

Maxima [B] time = 2.1269, size = 585, normalized size = 6.22

$$-\frac{6\,a^3\left(\frac{\frac{\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-2}{c-\frac{c\sin(fx+e)}{\cos(fx+e)+1}+\frac{c\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{c\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}+\frac{\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c}\right)+a^3\left(\frac{\frac{\sin(fx+e)}{\cos(fx+e)+1}-\frac{5\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}-\frac{3\sin(fx}{(\cos(fx+e}}{c-\frac{c\sin(fx+e)}{\cos(fx+e)+1}+\frac{2c\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{2c\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{c\sin(fx+e)^4}{(\cos(fx+e)+1)^4}}\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] -(6*a^3*((sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^2/(cos(f*x + e) +
1)^2 - 2)/(c - c*sin(f*x + e)/(cos(f*x + e) + 1) + c*sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 - c*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + arctan(sin(f*x + e
)/(cos(f*x + e) + 1))/c) + a^3*((sin(f*x + e)/(cos(f*x + e) + 1) - 5*sin(f*
x + e)^2/(cos(f*x + e) + 1)^2 + 3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 3*s
in(f*x + e)^4/(cos(f*x + e) + 1)^4 - 4)/(c - c*sin(f*x + e)/(cos(f*x + e) +
 1) + 2*c*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 2*c*sin(f*x + e)^3/(cos(f*x
 + e) + 1)^3 + c*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - c*sin(f*x + e)^5/(co
s(f*x + e) + 1)^5) + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c) + 6*a^3*(
arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c - 1/(c - c*sin(f*x + e)/(cos(f*x
+ e) + 1))) - 2*a^3/(c - c*sin(f*x + e)/(cos(f*x + e) + 1)))/f
```

Fricas [A] time = 1.409, size = 312, normalized size = 3.32

$$\frac{a^3\cos\left(fx+e\right)^3-15\,a^3fx+8\,a^3\cos\left(fx+e\right)^2+16\,a^3-\left(15\,a^3fx-23\,a^3\right)\cos\left(fx+e\right)+\left(15\,a^3fx+a^3\cos\left(fx+e\right)^2\right.}{2\left(cf\cos\left(fx+e\right)-cf\sin\left(fx+e\right)+cf\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/2*(a^3*cos(f*x + e)^3 - 15*a^3*f*x + 8*a^3*cos(f*x + e)^2 + 16*a^3 - (15*
a^3*f*x - 23*a^3)*cos(f*x + e) + (15*a^3*f*x + a^3*cos(f*x + e)^2 - 7*a^3*c
os(f*x + e) + 16*a^3)*sin(f*x + e))/(c*f*cos(f*x + e) - c*f*sin(f*x + e) +
c*f)
```

Sympy [A] time = 17.9005, size = 1170, normalized size = 12.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3/(c-c*sin(f*x+e)),x)
```

```
[Out] Piecewise((-15*a**3*f*x*tan(e/2 + f*x/2)**5/(2*c*f*tan(e/2 + f*x/2)**5 - 2*
c*f*tan(e/2 + f*x/2)**4 + 4*c*f*tan(e/2 + f*x/2)**3 - 4*c*f*tan(e/2 + f*x/2
)**2 + 2*c*f*tan(e/2 + f*x/2) - 2*c*f) + 15*a**3*f*x*tan(e/2 + f*x/2)**4/(2
*c*f*tan(e/2 + f*x/2)**5 - 2*c*f*tan(e/2 + f*x/2)**4 + 4*c*f*tan(e/2 + f*x/
2)**3 - 4*c*f*tan(e/2 + f*x/2)**2 + 2*c*f*tan(e/2 + f*x/2) - 2*c*f) - 30*a*
```

```
*3*f*x*tan(e/2 + f*x/2)**3/(2*c*f*tan(e/2 + f*x/2)**5 - 2*c*f*tan(e/2 + f*x
/2)**4 + 4*c*f*tan(e/2 + f*x/2)**3 - 4*c*f*tan(e/2 + f*x/2)**2 + 2*c*f*tan(
e/2 + f*x/2) - 2*c*f) + 30*a**3*f*x*tan(e/2 + f*x/2)**2/(2*c*f*tan(e/2 + f*
x/2)**5 - 2*c*f*tan(e/2 + f*x/2)**4 + 4*c*f*tan(e/2 + f*x/2)**3 - 4*c*f*tan
(e/2 + f*x/2)**2 + 2*c*f*tan(e/2 + f*x/2) - 2*c*f) - 15*a**3*f*x*tan(e/2 +
f*x/2)/(2*c*f*tan(e/2 + f*x/2)**5 - 2*c*f*tan(e/2 + f*x/2)**4 + 4*c*f*tan(e
/2 + f*x/2)**3 - 4*c*f*tan(e/2 + f*x/2)**2 + 2*c*f*tan(e/2 + f*x/2) - 2*c*f
) + 15*a**3*f*x/(2*c*f*tan(e/2 + f*x/2)**5 - 2*c*f*tan(e/2 + f*x/2)**4 + 4*
c*f*tan(e/2 + f*x/2)**3 - 4*c*f*tan(e/2 + f*x/2)**2 + 2*c*f*tan(e/2 + f*x/2
) - 2*c*f) - 14*a**3*tan(e/2 + f*x/2)**5/(2*c*f*tan(e/2 + f*x/2)**5 - 2*c*f
*tan(e/2 + f*x/2)**4 + 4*c*f*tan(e/2 + f*x/2)**3 - 4*c*f*tan(e/2 + f*x/2)**
2 + 2*c*f*tan(e/2 + f*x/2) - 2*c*f) - 20*a**3*tan(e/2 + f*x/2)**4/(2*c*f*ta
n(e/2 + f*x/2)**5 - 2*c*f*tan(e/2 + f*x/2)**4 + 4*c*f*tan(e/2 + f*x/2)**3 -
 4*c*f*tan(e/2 + f*x/2)**2 + 2*c*f*tan(e/2 + f*x/2) - 2*c*f) - 10*a**3*tan(
e/2 + f*x/2)**3/(2*c*f*tan(e/2 + f*x/2)**5 - 2*c*f*tan(e/2 + f*x/2)**4 + 4*
c*f*tan(e/2 + f*x/2)**3 - 4*c*f*tan(e/2 + f*x/2)**2 + 2*c*f*tan(e/2 + f*x/2
) - 2*c*f) - 50*a**3*tan(e/2 + f*x/2)**2/(2*c*f*tan(e/2 + f*x/2)**5 - 2*c*f
*tan(e/2 + f*x/2)**4 + 4*c*f*tan(e/2 + f*x/2)**3 - 4*c*f*tan(e/2 + f*x/2)**
2 + 2*c*f*tan(e/2 + f*x/2) - 2*c*f) - 34*a**3/(2*c*f*tan(e/2 + f*x/2)**5 -
2*c*f*tan(e/2 + f*x/2)**4 + 4*c*f*tan(e/2 + f*x/2)**3 - 4*c*f*tan(e/2 + f*x
/2)**2 + 2*c*f*tan(e/2 + f*x/2) - 2*c*f), Ne(f, 0)), (x*(a*sin(e) + a)**3/(
-c*sin(e) + c), True))
```

Giac [A] time = 2.29381, size = 158, normalized size = 1.68

$$-\frac{\frac{15\left(fx+e\right)a^3}{c}+\frac{32\,a^3}{c\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)}+\frac{2\left(a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-8\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-8\,a^3\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)^2c}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] -1/2*(15*(f*x + e)*a^3/c + 32*a^3/(c*(tan(1/2*f*x + 1/2*e) - 1)) + 2*(a^3*t
an(1/2*f*x + 1/2*e)^3 - 8*a^3*tan(1/2*f*x + 1/2*e)^2 - a^3*tan(1/2*f*x + 1/
2*e) - 8*a^3)/((tan(1/2*f*x + 1/2*e)^2 + 1)^2*c))/f
```

3.254 $\int \frac{(a+a\sin(e+fx))^3}{(c-c\sin(e+fx))^2}\,dx$

Optimal. Leaf size=92

$$-\frac{5a^3\cos(e+fx)}{c^2f}+\frac{2a^3c^2\cos^5(e+fx)}{3f(c-c\sin(e+fx))^4}+\frac{5a^3x}{c^2}-\frac{10a^3\cos^3(e+fx)}{3f(c-c\sin(e+fx))^2}$$

```
[Out] (5*a^3*x)/c^2 - (5*a^3*Cos[e + f*x])/(c^2*f) + (2*a^3*c^2*Cos[e + f*x]^5)/(
3*f*(c - c*Sin[e + f*x])^4) - (10*a^3*Cos[e + f*x]^3)/(3*f*(c - c*Sin[e + f
*x])^2)
```

Rubi [A] time = 0.18605, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2736, 2680, 2682, 8}

$$-\frac{5a^3\cos(e+fx)}{c^2f}+\frac{2a^3c^2\cos^5(e+fx)}{3f(c-c\sin(e+fx))^4}+\frac{5a^3x}{c^2}-\frac{10a^3\cos^3(e+fx)}{3f(c-c\sin(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3/(c - c*Sin[e + f*x])^2,x]
```

```
[Out] (5*a^3*x)/c^2 - (5*a^3*Cos[e + f*x])/(c^2*f) + (2*a^3*c^2*Cos[e + f*x]^5)/(
3*f*(c - c*Sin[e + f*x])^4) - (10*a^3*Cos[e + f*x]^3)/(3*f*(c - c*Sin[e + f
*x])^2)
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^3}{(c-c\sin(e+fx))^2}\,dx = \left(a^3c^3\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^5}\,dx$$
$$= \frac{2a^3c^2\cos^5(e+fx)}{3f(c-c\sin(e+fx))^4} - \frac{1}{3}\left(5a^3c\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^3}\,dx$$
$$= \frac{2a^3c^2\cos^5(e+fx)}{3f(c-c\sin(e+fx))^4} - \frac{10a^3\cos^3(e+fx)}{3f(c-c\sin(e+fx))^2} + \frac{\left(5a^3\right)\int \frac{\cos^2(e+fx)}{c-c\sin(e+fx)}\,dx}{c}$$
$$= -\frac{5a^3\cos(e+fx)}{c^2f} + \frac{2a^3c^2\cos^5(e+fx)}{3f(c-c\sin(e+fx))^4} - \frac{10a^3\cos^3(e+fx)}{3f(c-c\sin(e+fx))^2} + \frac{\left(5a^3\right)\int 1\,dx}{c^2}$$
$$= \frac{5a^3x}{c^2} - \frac{5a^3\cos(e+fx)}{c^2f} + \frac{2a^3c^2\cos^5(e+fx)}{3f(c-c\sin(e+fx))^4} - \frac{10a^3\cos^3(e+fx)}{3f(c-c\sin(e+fx))^2}$$

Mathematica [A] time = 0.960648, size = 149, normalized size = 1.62

$$\frac{a^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(6(15e+15fx+23)\cos\left(\frac{1}{2}(e+fx)\right)-(30e+30fx+121)\cos\left(\frac{3}{2}(e+fx)\right)+3\cos\right.}{12c^2f(\sin(e+fx)-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3/(c - c*Sin[e + f*x])^2,x]
```

```
[Out] (a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(6*(23 + 15*e + 15*f*x)*Cos[(e +
 f*x)/2] - (121 + 30*e + 30*f*x)*Cos[(3*(e + f*x))/2] + 3*Cos[(5*(e + f*x))
/2] - 6*(31 + 20*e + 20*f*x + 2*(-2 + 5*e + 5*f*x)*Cos[e + f*x] - Cos[2*(e
+ f*x)])*Sin[(e + f*x)/2]))/(12*c^2*f*(-1 + Sin[e + f*x])^2)
```

Maple [A] time = 0.086, size = 121, normalized size = 1.3

$$-\frac{32\,a^3}{3\,fc^2}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{-3}-16\,\frac{a^3}{fc^2\left(\tan\left(1/2\,fx+e/2\right)-1\right)^2}+8\,\frac{a^3}{fc^2\left(\tan\left(1/2\,fx+e/2\right)-1\right)}-2\,\frac{a^3}{fc^2\left(1+\left(\tan\left(1/2\,f\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^2,x)
```

```
[Out] -32/3/f*a^3/c^2/(tan(1/2*f*x+1/2*e)-1)^3-16/f*a^3/c^2/(tan(1/2*f*x+1/2*e)-1
)^2+8/f*a^3/c^2/(tan(1/2*f*x+1/2*e)-1)-2/f*a^3/c^2/(1+tan(1/2*f*x+1/2*e)^2)
+10/f*a^3/c^2*arctan(tan(1/2*f*x+1/2*e))
```

Maxima [B] time = 1.80274, size = 802, normalized size = 8.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 2/3*(2*a^3*((12*sin(f*x + e)/(cos(f*x + e) + 1) - 11*sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 + 9*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 3*sin(f*x + e)^4/(c
```

```
os(f*x + e) + 1)^4 - 5)/(c^2 - 3*c^2*sin(f*x + e)/(cos(f*x + e) + 1) + 4*c^
2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 4*c^2*sin(f*x + e)^3/(cos(f*x + e)
+ 1)^3 + 3*c^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - c^2*sin(f*x + e)^5/(co
s(f*x + e) + 1)^5) + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c^2) + 3*a^3
*((9*sin(f*x + e)/(cos(f*x + e) + 1) - 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^
2 - 4)/(c^2 - 3*c^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*c^2*sin(f*x + e)^2/
(cos(f*x + e) + 1)^2 - c^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 3*arctan(
sin(f*x + e)/(cos(f*x + e) + 1))/c^2) - a^3*(3*sin(f*x + e)/(cos(f*x + e) +
 1) - 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 2)/(c^2 - 3*c^2*sin(f*x + e)/
(cos(f*x + e) + 1) + 3*c^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - c^2*sin(f*
x + e)^3/(cos(f*x + e) + 1)^3) + 3*a^3*(3*sin(f*x + e)/(cos(f*x + e) + 1) -
 1)/(c^2 - 3*c^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*c^2*sin(f*x + e)^2/(co
s(f*x + e) + 1)^2 - c^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3))/f
```

Fricas [B] time = 1.30488, size = 435, normalized size = 4.73

$$-\frac{3\,a^3\cos\left(fx+e\right)^3+30\,a^3fx+8\,a^3-\left(15\,a^3fx+31\,a^3\right)\cos\left(fx+e\right)^2+\left(15\,a^3fx-26\,a^3\right)\cos\left(fx+e\right)-\left(30\,a^3fx-\cdots\right.}{3\left(c^2f\cos\left(fx+e\right)^2-c^2f\cos\left(fx+e\right)-2\,c^2f+\left(c^2f\cos\left(fx+e\right)\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -1/3*(3*a^3*cos(f*x + e)^3 + 30*a^3*f*x + 8*a^3 - (15*a^3*f*x + 31*a^3)*cos
(f*x + e)^2 + (15*a^3*f*x - 26*a^3)*cos(f*x + e) - (30*a^3*f*x - 3*a^3*cos(
f*x + e)^2 - 8*a^3 + (15*a^3*f*x - 34*a^3)*cos(f*x + e))*sin(f*x + e))/(c^2
*f*cos(f*x + e)^2 - c^2*f*cos(f*x + e) - 2*c^2*f + (c^2*f*cos(f*x + e) + 2*
c^2*f)*sin(f*x + e))
```

Sympy [A] time = 49.1151, size = 1282, normalized size = 13.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((15*a**3*f*x*tan(e/2 + f*x/2)**5/(3*c**2*f*tan(e/2 + f*x/2)**5 -
9*c**2*f*tan(e/2 + f*x/2)**4 + 12*c**2*f*tan(e/2 + f*x/2)**3 - 12*c**2*f*ta
n(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) - 45*a**3*f*x*tan
(e/2 + f*x/2)**4/(3*c**2*f*tan(e/2 + f*x/2)**5 - 9*c**2*f*tan(e/2 + f*x/2)*
*4 + 12*c**2*f*tan(e/2 + f*x/2)**3 - 12*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2
*f*tan(e/2 + f*x/2) - 3*c**2*f) + 60*a**3*f*x*tan(e/2 + f*x/2)**3/(3*c**2*f
*tan(e/2 + f*x/2)**5 - 9*c**2*f*tan(e/2 + f*x/2)**4 + 12*c**2*f*tan(e/2 + f
*x/2)**3 - 12*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c*
*2*f) - 60*a**3*f*x*tan(e/2 + f*x/2)**2/(3*c**2*f*tan(e/2 + f*x/2)**5 - 9*c
**2*f*tan(e/2 + f*x/2)**4 + 12*c**2*f*tan(e/2 + f*x/2)**3 - 12*c**2*f*tan(e
/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) + 45*a**3*f*x*tan(e/
2 + f*x/2)/(3*c**2*f*tan(e/2 + f*x/2)**5 - 9*c**2*f*tan(e/2 + f*x/2)**4 + 1
2*c**2*f*tan(e/2 + f*x/2)**3 - 12*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan
(e/2 + f*x/2) - 3*c**2*f) - 15*a**3*f*x/(3*c**2*f*tan(e/2 + f*x/2)**5 - 9*c
**2*f*tan(e/2 + f*x/2)**4 + 12*c**2*f*tan(e/2 + f*x/2)**3 - 12*c**2*f*tan(e
/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) + 8*a**3*tan(e/2 + f
```

```
*x/2)**5/(3*c**2*f*tan(e/2 + f*x/2)**5 - 9*c**2*f*tan(e/2 + f*x/2)**4 + 12*
c**2*f*tan(e/2 + f*x/2)**3 - 12*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e
/2 + f*x/2) - 3*c**2*f) - 70*a**3*tan(e/2 + f*x/2)**3/(3*c**2*f*tan(e/2 + f
*x/2)**5 - 9*c**2*f*tan(e/2 + f*x/2)**4 + 12*c**2*f*tan(e/2 + f*x/2)**3 - 1
2*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) + 50*a
**3*tan(e/2 + f*x/2)**2/(3*c**2*f*tan(e/2 + f*x/2)**5 - 9*c**2*f*tan(e/2 +
f*x/2)**4 + 12*c**2*f*tan(e/2 + f*x/2)**3 - 12*c**2*f*tan(e/2 + f*x/2)**2 +
 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) - 90*a**3*tan(e/2 + f*x/2)/(3*c**2*f
*tan(e/2 + f*x/2)**5 - 9*c**2*f*tan(e/2 + f*x/2)**4 + 12*c**2*f*tan(e/2 + f
*x/2)**3 - 12*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c*
*2*f) + 38*a**3/(3*c**2*f*tan(e/2 + f*x/2)**5 - 9*c**2*f*tan(e/2 + f*x/2)**
4 + 12*c**2*f*tan(e/2 + f*x/2)**3 - 12*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*
f*tan(e/2 + f*x/2) - 3*c**2*f), Ne(f, 0)), (x*(a*sin(e) + a)**3/(-c*sin(e)
+ c)**2, True))
```

Giac [A] time = 1.75864, size = 136, normalized size = 1.48

$$\frac{\frac{15\left(fx+e\right)a^{3}}{c^{2}}-\frac{6\,a^{3}}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+1\right)c^{2}}+\frac{8\left(3\,a^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-12\,a^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+5\,a^{3}\right)}{c^{2}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^{3}}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/3*(15*(f*x + e)*a^3/c^2 - 6*a^3/((tan(1/2*f*x + 1/2*e)^2 + 1)*c^2) + 8*(3
*a^3*tan(1/2*f*x + 1/2*e)^2 - 12*a^3*tan(1/2*f*x + 1/2*e) + 5*a^3)/(c^2*(ta
n(1/2*f*x + 1/2*e) - 1)^3))/f
```

3.255 $\int \frac{(a+a\sin(e+fx))^3}{(c-c\sin(e+fx))^3} \, dx$

Optimal. Leaf size=106

$$\frac{2a^3c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^5}+\frac{2a^3\cos(e+fx)}{f\left(c^3-c^3\sin(e+fx)\right)}-\frac{a^3x}{c^3}-\frac{2a^3\cos^3(e+fx)}{3f(c-c\sin(e+fx))^3}$$

```
[Out] -((a^3*x)/c^3) + (2*a^3*c^2*Cos[e + f*x]^5)/(5*f*(c - c*Sin[e + f*x])^5) -
(2*a^3*Cos[e + f*x]^3)/(3*f*(c - c*Sin[e + f*x])^3) + (2*a^3*Cos[e + f*x])/
(f*(c^3 - c^3*Sin[e + f*x]))
```

Rubi [A] time = 0.188967, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2736, 2680, 8}

$$\frac{2a^3c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^5}+\frac{2a^3\cos(e+fx)}{f\left(c^3-c^3\sin(e+fx)\right)}-\frac{a^3x}{c^3}-\frac{2a^3\cos^3(e+fx)}{3f(c-c\sin(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3/(c - c*Sin[e + f*x])^3,x]
```

```
[Out] -((a^3*x)/c^3) + (2*a^3*c^2*Cos[e + f*x]^5)/(5*f*(c - c*Sin[e + f*x])^5) -
(2*a^3*Cos[e + f*x]^3)/(3*f*(c - c*Sin[e + f*x])^3) + (2*a^3*Cos[e + f*x])/
(f*(c^3 - c^3*Sin[e + f*x]))
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^3}{(c-c\sin(e+fx))^3}\,dx = \left(a^3c^3\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^6}\,dx$$
$$= \frac{2a^3c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^5} - \left(a^3c\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^4}\,dx$$
$$= \frac{2a^3c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^5} - \frac{2a^3\cos^3(e+fx)}{3f(c-c\sin(e+fx))^3} + \frac{a^3\int \frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^2}\,dx}{c}$$
$$= \frac{2a^3c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^5} - \frac{2a^3\cos^3(e+fx)}{3f(c-c\sin(e+fx))^3} + \frac{2a^3\cos(e+fx)}{f\left(c^3-c^3\sin(e+fx)\right)} - \frac{a^3\int 1\,dx}{c^3}$$
$$= -\frac{a^3x}{c^3} + \frac{2a^3c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^5} - \frac{2a^3\cos^3(e+fx)}{3f(c-c\sin(e+fx))^3} + \frac{2a^3\cos(e+fx)}{f\left(c^3-c^3\sin(e+fx)\right)}$$

Mathematica [B] time = 0.455322, size = 249, normalized size = 2.35

$$\frac{(a\sin(e+fx)+a)^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(48\sin\left(\frac{1}{2}(e+fx)\right)-15(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+f\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3/(c - c*Sin[e + f*x])^3,x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(24*(Cos[(e + f*x)/2] - Sin[(e + f*x
)/2]) - 44*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3 - 15*(e + f*x)*(Cos[(e +
 f*x)/2] - Sin[(e + f*x)/2])^5 + 48*Sin[(e + f*x)/2] - 88*(Cos[(e + f*x)/2]
 - Sin[(e + f*x)/2])^2*Sin[(e + f*x)/2] + 92*(Cos[(e + f*x)/2] - Sin[(e + f
*x)/2])^4*Sin[(e + f*x)/2])*(a + a*Sin[e + f*x])^3)/(15*f*(Cos[(e + f*x)/2]
 + Sin[(e + f*x)/2])^6*(c - c*Sin[e + f*x])^3)
```

Maple [A] time = 0.092, size = 143, normalized size = 1.4

$$-\frac{64\,a^3}{5\,fc^3}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{-5} - 32\,\frac{a^3}{fc^3\left(\tan\left(1/2\,fx+e/2\right)-1\right)^4} - \frac{80\,a^3}{3\,fc^3}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{-3} - 8\,\frac{a^3}{fc^3\left(\tan\left(1/2\,fx+e/\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^3,x)
```

```
[Out] -64/5/f*a^3/c^3/(tan(1/2*f*x+1/2*e)-1)^5-32/f*a^3/c^3/(tan(1/2*f*x+1/2*e)-1
)^4-80/3/f*a^3/c^3/(tan(1/2*f*x+1/2*e)-1)^3-8/f*a^3/c^3/(tan(1/2*f*x+1/2*e)
-1)^2-4/f*a^3/c^3/(tan(1/2*f*x+1/2*e)-1)-2/f*a^3/c^3*arctan(tan(1/2*f*x+1/2
*e))
```

Maxima [B] time = 2.66346, size = 1060, normalized size = 10.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] -2/15*(a^3*((95*sin(f*x + e)/(cos(f*x + e) + 1) - 145*sin(f*x + e)^2/(cos(f
*x + e) + 1)^2 + 75*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 15*sin(f*x + e)^4
/(cos(f*x + e) + 1)^4 - 22)/(c^3 - 5*c^3*sin(f*x + e)/(cos(f*x + e) + 1) +
10*c^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 10*c^3*sin(f*x + e)^3/(cos(f*x
 + e) + 1)^3 + 5*c^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - c^3*sin(f*x + e)
^5/(cos(f*x + e) + 1)^5) + 15*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c^3)
+ a^3*(20*sin(f*x + e)/(cos(f*x + e) + 1) - 40*sin(f*x + e)^2/(cos(f*x + e)
 + 1)^2 + 30*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 15*sin(f*x + e)^4/(cos(f
*x + e) + 1)^4 - 7)/(c^3 - 5*c^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*c^3*s
in(f*x + e)^2/(cos(f*x + e) + 1)^2 - 10*c^3*sin(f*x + e)^3/(cos(f*x + e) +
1)^3 + 5*c^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - c^3*sin(f*x + e)^5/(cos(
f*x + e) + 1)^5) - 9*a^3*(5*sin(f*x + e)/(cos(f*x + e) + 1) - 5*sin(f*x + e
)^2/(cos(f*x + e) + 1)^2 + 5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 1)/(c^3
- 5*c^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*c^3*sin(f*x + e)^2/(cos(f*x +
e) + 1)^2 - 10*c^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*c^3*sin(f*x + e)
^4/(cos(f*x + e) + 1)^4 - c^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 6*a^3*
(5*sin(f*x + e)/(cos(f*x + e) + 1) - 10*sin(f*x + e)^2/(cos(f*x + e) + 1)^2
 - 1)/(c^3 - 5*c^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*c^3*sin(f*x + e)^2/
(cos(f*x + e) + 1)^2 - 10*c^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*c^3*s
in(f*x + e)^4/(cos(f*x + e) + 1)^4 - c^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^
5))/f
```

Fricas [B] time = 1.35291, size = 551, normalized size = 5.2

$$\frac{60\,a^3fx-\left(15\,a^3fx-46\,a^3\right)\cos\left(fx+e\right)^3-24\,a^3-\left(45\,a^3fx+2\,a^3\right)\cos\left(fx+e\right)^2+6\left(5\,a^3fx-12\,a^3\right)\cos\left(fx+e\right)-}{15\left(c^3f\cos\left(fx+e\right)^3+3\,c^3f\cos\left(fx+e\right)^2-2\,c^3f\cos\left(fx+e\right)-4\,c^3f-\left(c^3f\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/15*(60*a^3*f*x - (15*a^3*f*x - 46*a^3)*cos(f*x + e)^3 - 24*a^3 - (45*a^3*
f*x + 2*a^3)*cos(f*x + e)^2 + 6*(5*a^3*f*x - 12*a^3)*cos(f*x + e) - (60*a^3
*f*x + 24*a^3 - (15*a^3*f*x + 46*a^3)*cos(f*x + e)^2 + 6*(5*a^3*f*x - 8*a^3
)*cos(f*x + e))*sin(f*x + e))/(c^3*f*cos(f*x + e)^3 + 3*c^3*f*cos(f*x + e)^
2 - 2*c^3*f*cos(f*x + e) - 4*c^3*f - (c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*
x + e) - 4*c^3*f)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 2.35508, size = 150, normalized size = 1.42

$$-\frac{\frac{15\,(fx+e)a^3}{c^3}+\frac{4\left(15\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-30\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+100\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-50\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+13\,a^3\right)}{c^3\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^5}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -1/15*(15*(f*x + e)*a^3/c^3 + 4*(15*a^3*tan(1/2*f*x + 1/2*e)^4 - 30*a^3*tan
(1/2*f*x + 1/2*e)^3 + 100*a^3*tan(1/2*f*x + 1/2*e)^2 - 50*a^3*tan(1/2*f*x +
 1/2*e) + 13*a^3)/(c^3*(tan(1/2*f*x + 1/2*e) - 1)^5))/f
```

3.256 $\int \frac{(a+a\sin(e+fx))^3}{(c-c\sin(e+fx))^4}\,dx$

Optimal. Leaf size=34

$$\frac{a^3c^3\cos^7(e+fx)}{7f(c-c\sin(e+fx))^7}$$

```
[Out] (a^3*c^3*Cos[e + f*x]^7)/(7*f*(c - c*Sin[e + f*x])^7)
```

Rubi [A] time = 0.0902245, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2736, 2671}

$$\frac{a^3c^3\cos^7(e+fx)}{7f(c-c\sin(e+fx))^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3/(c - c*Sin[e + f*x])^4,x]
```

```
[Out] (a^3*c^3*Cos[e + f*x]^7)/(7*f*(c - c*Sin[e + f*x])^7)
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+a\sin(e+fx))^3}{(c-c\sin(e+fx))^4}\,dx &= \left(a^3c^3\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^7}\,dx\\ &= \frac{a^3c^3\cos^7(e+fx)}{7f(c-c\sin(e+fx))^7}\end{aligned}$$

Mathematica [B] time = 0.776164, size = 93, normalized size = 2.74

$$\frac{a^3\left(35\cos\left(\frac{1}{2}(e+fx)\right)-21\cos\left(\frac{3}{2}(e+fx)\right)-7\cos\left(\frac{5}{2}(e+fx)\right)+\cos\left(\frac{7}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}{28c^4f(\sin(e+fx)-1)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3/(c - c*Sin[e + f*x])^4,x]
```

```
[Out] (a^3*(35*Cos[(e + f*x)/2] - 21*Cos[(3*(e + f*x))/2] - 7*Cos[(5*(e + f*x))/2
] + Cos[(7*(e + f*x))/2])*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))/(28*c^4*f*
(-1 + Sin[e + f*x])^4)
```

Maple [B] time = 0.099, size = 118, normalized size = 3.5

$$2\,{\frac {{a}^{3}}{f{c}^{4}} \left(-{\frac {64}{7\, \left(\tan \left(1/2\,fx+e/2 \right) -1 \right) ^{7}}}-48\, \left(\tan \left(1/2\,fx+e/2 \right) -1 \right) ^{-5}-6\, \left(\tan \left(1/2\,fx+e/2 \right) -1 \right) ^{-2}-20\, \left(\tan \left(1/2\,fx+e/ \right. \right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^4,x)
```

```
[Out] 2/f*a^3/c^4*(-64/7/(tan(1/2*f*x+1/2*e)-1)^7-48/(tan(1/2*f*x+1/2*e)-1)^5-6/(
tan(1/2*f*x+1/2*e)-1)^2-20/(tan(1/2*f*x+1/2*e)-1)^3-40/(tan(1/2*f*x+1/2*e)-
1)^4-1/(tan(1/2*f*x+1/2*e)-1)-32/(tan(1/2*f*x+1/2*e)-1)^6)
```

Maxima [B] time = 1.54743, size = 1411, normalized size = 41.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^4,x, algorithm="maxima")
```

```
[Out] 2/35*(a^3*(91*sin(f*x + e)/(cos(f*x + e) + 1) - 168*sin(f*x + e)^2/(cos(f*x
 + e) + 1)^2 + 280*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 175*sin(f*x + e)^4
/(cos(f*x + e) + 1)^4 + 105*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 13)/(c^4
- 7*c^4*sin(f*x + e)/(cos(f*x + e) + 1) + 21*c^4*sin(f*x + e)^2/(cos(f*x +
e) + 1)^2 - 35*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 35*c^4*sin(f*x + e
)^4/(cos(f*x + e) + 1)^4 - 21*c^4*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 7*c
^4*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - c^4*sin(f*x + e)^7/(cos(f*x + e) +
 1)^7) - a^3*(49*sin(f*x + e)/(cos(f*x + e) + 1) - 147*sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 + 210*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 210*sin(f*x + e
)^4/(cos(f*x + e) + 1)^4 + 105*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 35*sin
(f*x + e)^6/(cos(f*x + e) + 1)^6 - 12)/(c^4 - 7*c^4*sin(f*x + e)/(cos(f*x +
 e) + 1) + 21*c^4*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 35*c^4*sin(f*x + e)
^3/(cos(f*x + e) + 1)^3 + 35*c^4*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 21*c
^4*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 7*c^4*sin(f*x + e)^6/(cos(f*x + e)
 + 1)^6 - c^4*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) - 4*a^3*(14*sin(f*x + e)
/(cos(f*x + e) + 1) - 42*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 35*sin(f*x +
 e)^3/(cos(f*x + e) + 1)^3 - 35*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 2)/(c
^4 - 7*c^4*sin(f*x + e)/(cos(f*x + e) + 1) + 21*c^4*sin(f*x + e)^2/(cos(f*x
 + e) + 1)^2 - 35*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 35*c^4*sin(f*x
+ e)^4/(cos(f*x + e) + 1)^4 - 21*c^4*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 +
7*c^4*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - c^4*sin(f*x + e)^7/(cos(f*x + e
) + 1)^7) + 2*a^3*(7*sin(f*x + e)/(cos(f*x + e) + 1) - 21*sin(f*x + e)^2/(c
os(f*x + e) + 1)^2 + 35*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 1)/(c^4 - 7*c
^4*sin(f*x + e)/(cos(f*x + e) + 1) + 21*c^4*sin(f*x + e)^2/(cos(f*x + e) +
1)^2 - 35*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 35*c^4*sin(f*x + e)^4/(
cos(f*x + e) + 1)^4 - 21*c^4*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 7*c^4*si
n(f*x + e)^6/(cos(f*x + e) + 1)^6 - c^4*sin(f*x + e)^7/(cos(f*x + e) + 1)^7
))/f
```

Fricas [B] time = 1.36482, size = 528, normalized size = 15.53

$$\frac{a^3\cos\left(fx+e\right)^4-3\,a^3\cos\left(fx+e\right)^3-8\,a^3\cos\left(fx+e\right)^2+4\,a^3\cos\left(fx+e\right)+8\,a^3-\left(a^3\cos\left(fx+e\right)^3+4\,a^3\cos\ldots\right.}{7\left(c^4f\cos\left(fx+e\right)^4-3\,c^4f\cos\left(fx+e\right)^3-8\,c^4f\cos\left(fx+e\right)^2+4\,c^4f\cos\left(fx+e\right)+8\,c^4f+\left(c^4f\cos\left(fx+e\right)^3+4\,c\ldots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^4,x, algorithm="fricas")
```

```
[Out] 1/7*(a^3*cos(f*x + e)^4 - 3*a^3*cos(f*x + e)^3 - 8*a^3*cos(f*x + e)^2 + 4*a
^3*cos(f*x + e) + 8*a^3 - (a^3*cos(f*x + e)^3 + 4*a^3*cos(f*x + e)^2 - 4*a^
3*cos(f*x + e) - 8*a^3)*sin(f*x + e))/(c^4*f*cos(f*x + e)^4 - 3*c^4*f*cos(f
*x + e)^3 - 8*c^4*f*cos(f*x + e)^2 + 4*c^4*f*cos(f*x + e) + 8*c^4*f + (c^4*
f*cos(f*x + e)^3 + 4*c^4*f*cos(f*x + e)^2 - 4*c^4*f*cos(f*x + e) - 8*c^4*f)
*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 2.04922, size = 104, normalized size = 3.06

$$-\frac{2\left(7\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6+35\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+21\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+a^3\right)}{7\,c^4f\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^4,x, algorithm="giac")
```

```
[Out] -2/7*(7*a^3*tan(1/2*f*x + 1/2*e)^6 + 35*a^3*tan(1/2*f*x + 1/2*e)^4 + 21*a^3
*tan(1/2*f*x + 1/2*e)^2 + a^3)/(c^4*f*(tan(1/2*f*x + 1/2*e) - 1)^7)
```

3.257 $\int \frac{(a+a\sin(e+fx))^3}{(c-c\sin(e+fx))^5}\,dx$

Optimal. Leaf size=69

$$\frac{a^3c^2\cos^7(e+fx)}{63f(c-c\sin(e+fx))^7}+\frac{a^3c^3\cos^7(e+fx)}{9f(c-c\sin(e+fx))^8}$$

```
[Out] (a^3*c^3*Cos[e + f*x]^7)/(9*f*(c - c*Sin[e + f*x])^8) + (a^3*c^2*Cos[e + f*
x]^7)/(63*f*(c - c*Sin[e + f*x])^7)
```

Rubi [A] time = 0.139355, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2736, 2672, 2671}

$$\frac{a^3c^2\cos^7(e+fx)}{63f(c-c\sin(e+fx))^7}+\frac{a^3c^3\cos^7(e+fx)}{9f(c-c\sin(e+fx))^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3/(c - c*Sin[e + f*x])^5,x]
```

```
[Out] (a^3*c^3*Cos[e + f*x]^7)/(9*f*(c - c*Sin[e + f*x])^8) + (a^3*c^2*Cos[e + f*
x]^7)/(63*f*(c - c*Sin[e + f*x])^7)
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^3}{(c-c\sin(e+fx))^5}\,dx = \left(a^3c^3\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^8}\,dx$$
$$= \frac{a^3c^3\cos^7(e+fx)}{9f(c-c\sin(e+fx))^8} + \frac{1}{9}\left(a^3c^2\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^7}\,dx$$
$$= \frac{a^3c^3\cos^7(e+fx)}{9f(c-c\sin(e+fx))^8} + \frac{a^3c^2\cos^7(e+fx)}{63f(c-c\sin(e+fx))^7}$$

Mathematica [A] time = 0.720795, size = 135, normalized size = 1.96

$$\frac{a^3\left(189\sin\left(\frac{1}{2}(e+fx)\right)+105\sin\left(\frac{3}{2}(e+fx)\right)-27\sin\left(\frac{5}{2}(e+fx)\right)-\sin\left(\frac{9}{2}(e+fx)\right)+315\cos\left(\frac{1}{2}(e+fx)\right)-189\cos\left(\right.\right.}{504c^5f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3/(c - c*Sin[e + f*x])^5,x]
```

```
[Out] (a^3*(315*Cos[(e + f*x)/2] - 189*Cos[(3*(e + f*x))/2] - 63*Cos[(5*(e + f*x)
)/2] + 9*Cos[(7*(e + f*x))/2] + 189*Sin[(e + f*x)/2] + 105*Sin[(3*(e + f*x)
)/2] - 27*Sin[(5*(e + f*x))/2] - Sin[(9*(e + f*x))/2]))/(504*c^5*f*(Cos[(e
+ f*x)/2] - Sin[(e + f*x)/2])^9)
```

Maple [B] time = 0.109, size = 148, normalized size = 2.1

$$2\,\frac{a^3}{fc^5}\left(-\frac{928}{7\left(\tan\left(1/2\,fx+e/2\right)-1\right)^7}-76\left(\tan\left(1/2\,fx+e/2\right)-1\right)^{-4}-\frac{496}{3\left(\tan\left(1/2\,fx+e/2\right)-1\right)^6}-\frac{86}{3\left(\tan\left(1/2\,fx+\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^5,x)
```

```
[Out] 2/f*a^3/c^5*(-928/7/(tan(1/2*f*x+1/2*e)-1)^7-76/(tan(1/2*f*x+1/2*e)-1)^4-49
6/3/(tan(1/2*f*x+1/2*e)-1)^6-86/3/(tan(1/2*f*x+1/2*e)-1)^3-1/(tan(1/2*f*x+1
/2*e)-1)-64/(tan(1/2*f*x+1/2*e)-1)^8-136/(tan(1/2*f*x+1/2*e)-1)^5-128/9/(ta
n(1/2*f*x+1/2*e)-1)^9-7/(tan(1/2*f*x+1/2*e)-1)^2)
```

Maxima [B] time = 2.19098, size = 1875, normalized size = 27.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^5,x, algorithm="maxima")
```

```
[Out] -2/315*(a^3*(432*sin(f*x + e)/(cos(f*x + e) + 1) - 1728*sin(f*x + e)^2/(cos
(f*x + e) + 1)^2 + 3612*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 5418*sin(f*x
+ e)^4/(cos(f*x + e) + 1)^4 + 5040*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 33
60*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 1260*sin(f*x + e)^7/(cos(f*x + e)
+ 1)^7 - 315*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 83)/(c^5 - 9*c^5*sin(f*x
```

```
 + e)/(cos(f*x + e) + 1) + 36*c^5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 84*
c^5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 126*c^5*sin(f*x + e)^4/(cos(f*x +
 e) + 1)^4 - 126*c^5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 84*c^5*sin(f*x +
 e)^6/(cos(f*x + e) + 1)^6 - 36*c^5*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 9
*c^5*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - c^5*sin(f*x + e)^9/(cos(f*x + e)
 + 1)^9) - 15*a^3*(45*sin(f*x + e)/(cos(f*x + e) + 1) - 117*sin(f*x + e)^2/
(cos(f*x + e) + 1)^2 + 273*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 315*sin(f*
x + e)^4/(cos(f*x + e) + 1)^4 + 315*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 1
47*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 63*sin(f*x + e)^7/(cos(f*x + e) +
1)^7 - 5)/(c^5 - 9*c^5*sin(f*x + e)/(cos(f*x + e) + 1) + 36*c^5*sin(f*x + e
)^2/(cos(f*x + e) + 1)^2 - 84*c^5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 126
*c^5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 126*c^5*sin(f*x + e)^5/(cos(f*x
+ e) + 1)^5 + 84*c^5*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 36*c^5*sin(f*x +
 e)^7/(cos(f*x + e) + 1)^7 + 9*c^5*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - c^
5*sin(f*x + e)^9/(cos(f*x + e) + 1)^9) - 10*a^3*(9*sin(f*x + e)/(cos(f*x +
e) + 1) - 36*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 84*sin(f*x + e)^3/(cos(f
*x + e) + 1)^3 - 63*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 63*sin(f*x + e)^5
/(cos(f*x + e) + 1)^5 - 1)/(c^5 - 9*c^5*sin(f*x + e)/(cos(f*x + e) + 1) + 3
6*c^5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 84*c^5*sin(f*x + e)^3/(cos(f*x
+ e) + 1)^3 + 126*c^5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 126*c^5*sin(f*x
 + e)^5/(cos(f*x + e) + 1)^5 + 84*c^5*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 -
 36*c^5*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 9*c^5*sin(f*x + e)^8/(cos(f*x
 + e) + 1)^8 - c^5*sin(f*x + e)^9/(cos(f*x + e) + 1)^9) + 42*a^3*(9*sin(f*x
 + e)/(cos(f*x + e) + 1) - 36*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 54*sin(
f*x + e)^3/(cos(f*x + e) + 1)^3 - 81*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 +
45*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 30*sin(f*x + e)^6/(cos(f*x + e) +
1)^6 - 1)/(c^5 - 9*c^5*sin(f*x + e)/(cos(f*x + e) + 1) + 36*c^5*sin(f*x + e
)^2/(cos(f*x + e) + 1)^2 - 84*c^5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 126
*c^5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 126*c^5*sin(f*x + e)^5/(cos(f*x
+ e) + 1)^5 + 84*c^5*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 36*c^5*sin(f*x +
 e)^7/(cos(f*x + e) + 1)^7 + 9*c^5*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - c^
5*sin(f*x + e)^9/(cos(f*x + e) + 1)^9))/f
```

Fricas [B] time = 1.38983, size = 675, normalized size = 9.78

$$-\frac{a^3\cos\left(fx+e\right)^5 - 4\,a^3\cos\left(fx+e\right)^4 + 19\,a^3\cos\left(fx+e\right)^3 + 52\,a^3\cos\left(fx+e\right)^2 - 28\,a^3\cos\left(fx+e\right) - 56\,a^3 + \left(\cdots\right.}{63\left(c^5 f\cos\left(fx+e\right)^5 + 5\,c^5 f\cos\left(fx+e\right)^4 - 8\,c^5 f\cos\left(fx+e\right)^3 - 20\,c^5 f\cos\left(fx+e\right)^2 + 8\,c^5 f\cos\left(fx+e\right) + 16\,c^5 f - \cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^5,x, algorithm="fricas")
```

```
[Out] -1/63*(a^3*cos(f*x + e)^5 - 4*a^3*cos(f*x + e)^4 + 19*a^3*cos(f*x + e)^3 +
52*a^3*cos(f*x + e)^2 - 28*a^3*cos(f*x + e) - 56*a^3 + (a^3*cos(f*x + e)^4
+ 5*a^3*cos(f*x + e)^3 + 24*a^3*cos(f*x + e)^2 - 28*a^3*cos(f*x + e) - 56*a
^3)*sin(f*x + e))/(c^5*f*cos(f*x + e)^5 + 5*c^5*f*cos(f*x + e)^4 - 8*c^5*f*
cos(f*x + e)^3 - 20*c^5*f*cos(f*x + e)^2 + 8*c^5*f*cos(f*x + e) + 16*c^5*f
- (c^5*f*cos(f*x + e)^4 - 4*c^5*f*cos(f*x + e)^3 - 12*c^5*f*cos(f*x + e)^2
+ 8*c^5*f*cos(f*x + e) + 16*c^5*f)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**5,x)
```

```
[Out] Timed out
```

Giac [B] time = 2.08094, size = 219, normalized size = 3.17

$$-\frac{2\left(63\,a^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{8}-63\,a^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{7}+483\,a^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{6}-315\,a^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{5}+693\,a^{3}\,\mathrm{t}}{63\,c^{5}f\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^5,x, algorithm="giac")
```

```
[Out] -2/63*(63*a^3*tan(1/2*f*x + 1/2*e)^8 - 63*a^3*tan(1/2*f*x + 1/2*e)^7 + 483*
a^3*tan(1/2*f*x + 1/2*e)^6 - 315*a^3*tan(1/2*f*x + 1/2*e)^5 + 693*a^3*tan(1
/2*f*x + 1/2*e)^4 - 189*a^3*tan(1/2*f*x + 1/2*e)^3 + 225*a^3*tan(1/2*f*x +
1/2*e)^2 - 9*a^3*tan(1/2*f*x + 1/2*e) + 8*a^3)/(c^5*f*(tan(1/2*f*x + 1/2*e)
 - 1)^9)
```

3.258 $\int \frac{(a+a\sin(e+fx))^3}{(c-c\sin(e+fx))^6}\,dx$

Optimal. Leaf size=101

$$\frac{2a^3c^2\cos^7(e+fx)}{99f(c-c\sin(e+fx))^8}+\frac{a^3c^3\cos^7(e+fx)}{11f(c-c\sin(e+fx))^9}+\frac{2a^3c\cos^7(e+fx)}{693f(c-c\sin(e+fx))^7}$$

```
[Out] (a^3*c^3*Cos[e + f*x]^7)/(11*f*(c - c*Sin[e + f*x])^9) + (2*a^3*c^2*Cos[e +
 f*x]^7)/(99*f*(c - c*Sin[e + f*x])^8) + (2*a^3*c*Cos[e + f*x]^7)/(693*f*(c
 - c*Sin[e + f*x])^7)
```

Rubi [A] time = 0.182151, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2736, 2672, 2671}

$$\frac{2a^3c^2\cos^7(e+fx)}{99f(c-c\sin(e+fx))^8}+\frac{a^3c^3\cos^7(e+fx)}{11f(c-c\sin(e+fx))^9}+\frac{2a^3c\cos^7(e+fx)}{693f(c-c\sin(e+fx))^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3/(c - c*Sin[e + f*x])^6,x]
```

```
[Out] (a^3*c^3*Cos[e + f*x]^7)/(11*f*(c - c*Sin[e + f*x])^9) + (2*a^3*c^2*Cos[e +
 f*x]^7)/(99*f*(c - c*Sin[e + f*x])^8) + (2*a^3*c*Cos[e + f*x]^7)/(693*f*(c
 - c*Sin[e + f*x])^7)
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] && !IGtQ[m, 0]
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] && !ILtQ[p, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^3}{(c-c\sin(e+fx))^6}\,dx = \left(a^3c^3\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^9}\,dx$$
$$= \frac{a^3c^3\cos^7(e+fx)}{11f(c-c\sin(e+fx))^9} + \frac{1}{11}\left(2a^3c^2\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^8}\,dx$$
$$= \frac{a^3c^3\cos^7(e+fx)}{11f(c-c\sin(e+fx))^9} + \frac{2a^3c^2\cos^7(e+fx)}{99f(c-c\sin(e+fx))^8} + \frac{1}{99}\left(2a^3c\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^7}\,dx$$
$$= \frac{a^3c^3\cos^7(e+fx)}{11f(c-c\sin(e+fx))^9} + \frac{2a^3c^2\cos^7(e+fx)}{99f(c-c\sin(e+fx))^8} + \frac{2a^3c\cos^7(e+fx)}{693f(c-c\sin(e+fx))^7}$$

Mathematica [A] time = 0.925097, size = 145, normalized size = 1.44

$$-\frac{a^3\left(-2079\sin\left(\frac{1}{2}(e+fx)\right)-1155\sin\left(\frac{3}{2}(e+fx)\right)+297\sin\left(\frac{5}{2}(e+fx)\right)+11\sin\left(\frac{9}{2}(e+fx)\right)-2541\cos\left(\frac{1}{2}(e+fx)\right)\right.}{5544c^6f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3/(c - c*Sin[e + f*x])^6,x]
```

```
[Out] -(a^3*(-2541*Cos[(e + f*x)/2] + 1485*Cos[(3*(e + f*x))/2] + 462*Cos[(5*(e +
 f*x))/2] - 55*Cos[(7*(e + f*x))/2] + Cos[(11*(e + f*x))/2] - 2079*Sin[(e +
 f*x)/2] - 1155*Sin[(3*(e + f*x))/2] + 297*Sin[(5*(e + f*x))/2] + 11*Sin[(9
*(e + f*x))/2]))/(5544*c^6*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^11)
```

Maple [A] time = 0.119, size = 178, normalized size = 1.8

$$2\,\frac{a^3}{fc^6}\left(-8\left(\tan\left(1/2\,fx+e/2\right)-1\right)^{-2}-\frac{4272}{7\left(\tan\left(1/2\,fx+e/2\right)-1\right)^7}-128\left(\tan\left(1/2\,fx+e/2\right)-1\right)^{-10}-292\left(\tan\left(1/2\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^6,x)
```

```
[Out] 2/f*a^3/c^6*(-8/(tan(1/2*f*x+1/2*e)-1)^2-4272/7/(tan(1/2*f*x+1/2*e)-1)^7-12
8/(tan(1/2*f*x+1/2*e)-1)^10-292/(tan(1/2*f*x+1/2*e)-1)^5-544/(tan(1/2*f*x+1
/2*e)-1)^8-256/11/(tan(1/2*f*x+1/2*e)-1)^11-126/(tan(1/2*f*x+1/2*e)-1)^4-11
6/3/(tan(1/2*f*x+1/2*e)-1)^3-1/(tan(1/2*f*x+1/2*e)-1)-3008/9/(tan(1/2*f*x+1
/2*e)-1)^9-1480/3/(tan(1/2*f*x+1/2*e)-1)^6)
```

Maxima [B] time = 1.89596, size = 2341, normalized size = 23.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^6,x, algorithm="maxima")
```

```
[Out] -2/3465*(5*a^3*(913*sin(f*x + e)/(cos(f*x + e) + 1) - 4565*sin(f*x + e)^2/(
cos(f*x + e) + 1)^2 + 12540*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 25080*sin
```

```
(f*x + e)^4/(cos(f*x + e) + 1)^4 + 33726*sin(f*x + e)^5/(cos(f*x + e) + 1)^
5 - 33726*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 23100*sin(f*x + e)^7/(cos(f
*x + e) + 1)^7 - 11550*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 3465*sin(f*x +
 e)^9/(cos(f*x + e) + 1)^9 - 693*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 14
6)/(c^6 - 11*c^6*sin(f*x + e)/(cos(f*x + e) + 1) + 55*c^6*sin(f*x + e)^2/(c
os(f*x + e) + 1)^2 - 165*c^6*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 330*c^6*
sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 462*c^6*sin(f*x + e)^5/(cos(f*x + e)
+ 1)^5 + 462*c^6*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 330*c^6*sin(f*x + e)
^7/(cos(f*x + e) + 1)^7 + 165*c^6*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 55*
c^6*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 11*c^6*sin(f*x + e)^10/(cos(f*x +
 e) + 1)^10 - c^6*sin(f*x + e)^11/(cos(f*x + e) + 1)^11) - 9*a^3*(671*sin(f
*x + e)/(cos(f*x + e) + 1) - 2200*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 660
0*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 10890*sin(f*x + e)^4/(cos(f*x + e)
+ 1)^4 + 15246*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 12936*sin(f*x + e)^6/(
cos(f*x + e) + 1)^6 + 9240*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 3465*sin(f
*x + e)^8/(cos(f*x + e) + 1)^8 + 1155*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 -
 61)/(c^6 - 11*c^6*sin(f*x + e)/(cos(f*x + e) + 1) + 55*c^6*sin(f*x + e)^2/
(cos(f*x + e) + 1)^2 - 165*c^6*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 330*c^
6*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 462*c^6*sin(f*x + e)^5/(cos(f*x + e
) + 1)^5 + 462*c^6*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 330*c^6*sin(f*x +
e)^7/(cos(f*x + e) + 1)^7 + 165*c^6*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 5
5*c^6*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 11*c^6*sin(f*x + e)^10/(cos(f*x
 + e) + 1)^10 - c^6*sin(f*x + e)^11/(cos(f*x + e) + 1)^11) - 2*a^3*(341*sin
(f*x + e)/(cos(f*x + e) + 1) - 1705*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 5
115*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 6765*sin(f*x + e)^4/(cos(f*x + e)
 + 1)^4 + 9471*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 4851*sin(f*x + e)^6/(c
os(f*x + e) + 1)^6 + 3465*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 31)/(c^6 -
11*c^6*sin(f*x + e)/(cos(f*x + e) + 1) + 55*c^6*sin(f*x + e)^2/(cos(f*x + e
) + 1)^2 - 165*c^6*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 330*c^6*sin(f*x +
e)^4/(cos(f*x + e) + 1)^4 - 462*c^6*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 4
62*c^6*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 330*c^6*sin(f*x + e)^7/(cos(f*
x + e) + 1)^7 + 165*c^6*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 55*c^6*sin(f*
x + e)^9/(cos(f*x + e) + 1)^9 + 11*c^6*sin(f*x + e)^10/(cos(f*x + e) + 1)^1
0 - c^6*sin(f*x + e)^11/(cos(f*x + e) + 1)^11) + 12*a^3*(253*sin(f*x + e)/(
cos(f*x + e) + 1) - 1265*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2640*sin(f*x
 + e)^3/(cos(f*x + e) + 1)^3 - 5280*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 5
313*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 5313*sin(f*x + e)^6/(cos(f*x + e)
 + 1)^6 + 2310*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 1155*sin(f*x + e)^8/(c
os(f*x + e) + 1)^8 - 23)/(c^6 - 11*c^6*sin(f*x + e)/(cos(f*x + e) + 1) + 55
*c^6*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 165*c^6*sin(f*x + e)^3/(cos(f*x
+ e) + 1)^3 + 330*c^6*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 462*c^6*sin(f*x
 + e)^5/(cos(f*x + e) + 1)^5 + 462*c^6*sin(f*x + e)^6/(cos(f*x + e) + 1)^6
- 330*c^6*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 165*c^6*sin(f*x + e)^8/(cos
(f*x + e) + 1)^8 - 55*c^6*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 11*c^6*sin(
f*x + e)^10/(cos(f*x + e) + 1)^10 - c^6*sin(f*x + e)^11/(cos(f*x + e) + 1)^
11))/f
```

Fricas [B] time = 1.34543, size = 832, normalized size = 8.24

$$\frac{2\,a^3\cos\left(fx+e\right)^6+12\,a^3\cos\left(fx+e\right)^5-25\,a^3\cos\left(fx+e\right)^4+161\,a^3\cos\left(fx+e\right)^3+448\,a^3\cos\left(fx+e\right)^2-252\,a^3\,\mathrm{c}}{693\left(c^6f\cos\left(fx+e\right)^6-5\,c^6f\cos\left(fx+e\right)^5-18\,c^6f\cos\left(fx+e\right)^4+20\,c^6f\cos\left(fx+e\right)^3+48\,c^6f\cos\left(fx+e\right)^2-16\,c^6f\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^6,x, algorithm="fricas")
```

```
[Out] 1/693*(2*a^3*cos(f*x + e)^6 + 12*a^3*cos(f*x + e)^5 - 25*a^3*cos(f*x + e)^4
 + 161*a^3*cos(f*x + e)^3 + 448*a^3*cos(f*x + e)^2 - 252*a^3*cos(f*x + e) -
 504*a^3 - (2*a^3*cos(f*x + e)^5 - 10*a^3*cos(f*x + e)^4 - 35*a^3*cos(f*x +
 e)^3 - 196*a^3*cos(f*x + e)^2 + 252*a^3*cos(f*x + e) + 504*a^3)*sin(f*x +
e))/(c^6*f*cos(f*x + e)^6 - 5*c^6*f*cos(f*x + e)^5 - 18*c^6*f*cos(f*x + e)^
4 + 20*c^6*f*cos(f*x + e)^3 + 48*c^6*f*cos(f*x + e)^2 - 16*c^6*f*cos(f*x +
e) - 32*c^6*f + (c^6*f*cos(f*x + e)^5 + 6*c^6*f*cos(f*x + e)^4 - 12*c^6*f*c
os(f*x + e)^3 - 32*c^6*f*cos(f*x + e)^2 + 16*c^6*f*cos(f*x + e) + 32*c^6*f)
*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**6,x)
```

```
[Out] Timed out
```

Giac [B] time = 2.22028, size = 265, normalized size = 2.62

$$-\frac{2\left(693\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{10}-1386\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^9+8085\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^8-10626\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^7+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^6,x, algorithm="giac")
```

```
[Out] -2/693*(693*a^3*tan(1/2*f*x + 1/2*e)^10 - 1386*a^3*tan(1/2*f*x + 1/2*e)^9 +
 8085*a^3*tan(1/2*f*x + 1/2*e)^8 - 10626*a^3*tan(1/2*f*x + 1/2*e)^7 + 21252
*a^3*tan(1/2*f*x + 1/2*e)^6 - 15246*a^3*tan(1/2*f*x + 1/2*e)^5 + 15444*a^3*
tan(1/2*f*x + 1/2*e)^4 - 4950*a^3*tan(1/2*f*x + 1/2*e)^3 + 2959*a^3*tan(1/2
*f*x + 1/2*e)^2 - 176*a^3*tan(1/2*f*x + 1/2*e) + 79*a^3)/(c^6*f*(tan(1/2*f*
x + 1/2*e) - 1)^11)
```

3.259 $\int \frac{(a+a\sin(e+fx))^3}{(c-c\sin(e+fx))^7}\,dx$

Optimal. Leaf size=132

$$\frac{3a^3c^2\cos^7(e+fx)}{143f(c-c\sin(e+fx))^9}+\frac{a^3c^3\cos^7(e+fx)}{13f(c-c\sin(e+fx))^{10}}+\frac{2a^3\cos^7(e+fx)}{3003f(c-c\sin(e+fx))^7}+\frac{2a^3c\cos^7(e+fx)}{429f(c-c\sin(e+fx))^8}$$

```
[Out] (a^3*c^3*Cos[e + f*x]^7)/(13*f*(c - c*Sin[e + f*x])^10) + (3*a^3*c^2*Cos[e
+ f*x]^7)/(143*f*(c - c*Sin[e + f*x])^9) + (2*a^3*c*Cos[e + f*x]^7)/(429*f*
(c - c*Sin[e + f*x])^8) + (2*a^3*Cos[e + f*x]^7)/(3003*f*(c - c*Sin[e + f*x
])^7)
```

Rubi [A] time = 0.229953, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2736, 2672, 2671}

$$\frac{3a^3c^2\cos^7(e+fx)}{143f(c-c\sin(e+fx))^9}+\frac{a^3c^3\cos^7(e+fx)}{13f(c-c\sin(e+fx))^{10}}+\frac{2a^3\cos^7(e+fx)}{3003f(c-c\sin(e+fx))^7}+\frac{2a^3c\cos^7(e+fx)}{429f(c-c\sin(e+fx))^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3/(c - c*Sin[e + f*x])^7,x]
```


```
[Out] (a^3*c^3*Cos[e + f*x]^7)/(13*f*(c - c*Sin[e + f*x])^10) + (3*a^3*c^2*Cos[e
+ f*x]^7)/(143*f*(c - c*Sin[e + f*x])^9) + (2*a^3*c*Cos[e + f*x]^7)/(429*f*
(c - c*Sin[e + f*x])^8) + (2*a^3*Cos[e + f*x]^7)/(3003*f*(c - c*Sin[e + f*x
])^7)
```


Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^3}{(c-c\sin(e+fx))^7}dx = \left(a^3c^3\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^{10}}dx$$
$$= \frac{a^3c^3\cos^7(e+fx)}{13f(c-c\sin(e+fx))^{10}} + \frac{1}{13}\left(3a^3c^2\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^9}dx$$
$$= \frac{a^3c^3\cos^7(e+fx)}{13f(c-c\sin(e+fx))^{10}} + \frac{3a^3c^2\cos^7(e+fx)}{143f(c-c\sin(e+fx))^9} + \frac{1}{143}\left(6a^3c\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^8}$$
$$= \frac{a^3c^3\cos^7(e+fx)}{13f(c-c\sin(e+fx))^{10}} + \frac{3a^3c^2\cos^7(e+fx)}{143f(c-c\sin(e+fx))^9} + \frac{2a^3c\cos^7(e+fx)}{429f(c-c\sin(e+fx))^8} + \frac{1}{429}(2$$
$$= \frac{a^3c^3\cos^7(e+fx)}{13f(c-c\sin(e+fx))^{10}} + \frac{3a^3c^2\cos^7(e+fx)}{143f(c-c\sin(e+fx))^9} + \frac{2a^3c\cos^7(e+fx)}{429f(c-c\sin(e+fx))^8} + \frac{2a}{3003f}$$

Mathematica [A] time = 1.8649, size = 157, normalized size = 1.19

$$\frac{a^3\left(16302\sin\left(\frac{1}{2}(e+fx)\right)+9009\sin\left(\frac{3}{2}(e+fx)\right)-2288\sin\left(\frac{5}{2}(e+fx)\right)-78\sin\left(\frac{9}{2}(e+fx)\right)+\sin\left(\frac{13}{2}(e+fx)\right)+18\right.}{48048c^7f\left(\cos\left(\frac{1}{2}(e+fx)\right)\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3/(c - c*Sin[e + f*x])^7,x]
```

```
[Out] (a^3*(18018*Cos[(e + f*x)/2] - 10296*Cos[(3*(e + f*x))/2] - 3003*Cos[(5*(e
+ f*x))/2] + 286*Cos[(7*(e + f*x))/2] - 13*Cos[(11*(e + f*x))/2] + 16302*Si
n[(e + f*x)/2] + 9009*Sin[(3*(e + f*x))/2] - 2288*Sin[(5*(e + f*x))/2] - 78
*Sin[(9*(e + f*x))/2] + Sin[(13*(e + f*x))/2]))/(48048*c^7*f*(Cos[(e + f*x)
/2] - Sin[(e + f*x)/2])^13)
```

Maple [A] time = 0.129, size = 208, normalized size = 1.6

$$2\frac{a^3}{fc^7}\left(-\frac{13112}{7\left(\tan\left(1/2\,fx+e/2\right)-1\right)^7}-\frac{512}{13\left(\tan\left(1/2\,fx+e/2\right)-1\right)^{13}}-540\left(\tan\left(1/2\,fx+e/2\right)-1\right)^{-5}-9\left(\tan\left(1/2\,fx\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^7,x)
```

```
[Out] 2/f*a^3/c^7*(-13112/7/(tan(1/2*f*x+1/2*e)-1)^7-512/13/(tan(1/2*f*x+1/2*e)-1
)^13-540/(tan(1/2*f*x+1/2*e)-1)^5-9/(tan(1/2*f*x+1/2*e)-1)^2-256/(tan(1/2*f
*x+1/2*e)-1)^12-1148/(tan(1/2*f*x+1/2*e)-1)^6-8832/11/(tan(1/2*f*x+1/2*e)-1
)^11-50/(tan(1/2*f*x+1/2*e)-1)^3-192/(tan(1/2*f*x+1/2*e)-1)^4-1600/(tan(1/2
*f*x+1/2*e)-1)^10-2352/(tan(1/2*f*x+1/2*e)-1)^8-1/(tan(1/2*f*x+1/2*e)-1)-67
52/3/(tan(1/2*f*x+1/2*e)-1)^9)
```

Maxima [B] time = 2.04692, size = 2805, normalized size = 21.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^7,x, algorithm="maxima")
```

```
[Out] -2/15015*(2*a^3*(4771*sin(f*x + e)/(cos(f*x + e) + 1) - 28626*sin(f*x + e)^
2/(cos(f*x + e) + 1)^2 + 74932*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 187330
*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 265122*sin(f*x + e)^5/(cos(f*x + e)
+ 1)^5 - 353496*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 276276*sin(f*x + e)^7
/(cos(f*x + e) + 1)^7 - 207207*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 75075*
sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - 30030*sin(f*x + e)^10/(cos(f*x + e) +
 1)^10 - 367)/(c^7 - 13*c^7*sin(f*x + e)/(cos(f*x + e) + 1) + 78*c^7*sin(f*
x + e)^2/(cos(f*x + e) + 1)^2 - 286*c^7*sin(f*x + e)^3/(cos(f*x + e) + 1)^3
 + 715*c^7*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 1287*c^7*sin(f*x + e)^5/(c
os(f*x + e) + 1)^5 + 1716*c^7*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 1716*c^
7*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 1287*c^7*sin(f*x + e)^8/(cos(f*x +
e) + 1)^8 - 715*c^7*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 286*c^7*sin(f*x +
 e)^10/(cos(f*x + e) + 1)^10 - 78*c^7*sin(f*x + e)^11/(cos(f*x + e) + 1)^11
 + 13*c^7*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - c^7*sin(f*x + e)^13/(cos(
f*x + e) + 1)^13) + 5*a^3*(3796*sin(f*x + e)/(cos(f*x + e) + 1) - 22776*sin
(f*x + e)^2/(cos(f*x + e) + 1)^2 + 77506*sin(f*x + e)^3/(cos(f*x + e) + 1)^
3 - 193765*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 339768*sin(f*x + e)^5/(cos
(f*x + e) + 1)^5 - 453024*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 444444*sin(
f*x + e)^7/(cos(f*x + e) + 1)^7 - 333333*sin(f*x + e)^8/(cos(f*x + e) + 1)^
8 + 180180*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - 72072*sin(f*x + e)^10/(cos
(f*x + e) + 1)^10 + 18018*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 - 3003*sin(
f*x + e)^12/(cos(f*x + e) + 1)^12 - 523)/(c^7 - 13*c^7*sin(f*x + e)/(cos(f*
x + e) + 1) + 78*c^7*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 286*c^7*sin(f*x
+ e)^3/(cos(f*x + e) + 1)^3 + 715*c^7*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 -
 1287*c^7*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 1716*c^7*sin(f*x + e)^6/(co
s(f*x + e) + 1)^6 - 1716*c^7*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 1287*c^7
*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 715*c^7*sin(f*x + e)^9/(cos(f*x + e)
 + 1)^9 + 286*c^7*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 78*c^7*sin(f*x +
e)^11/(cos(f*x + e) + 1)^11 + 13*c^7*sin(f*x + e)^12/(cos(f*x + e) + 1)^12
- c^7*sin(f*x + e)^13/(cos(f*x + e) + 1)^13) - 35*a^3*(611*sin(f*x + e)/(co
s(f*x + e) + 1) - 2379*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 8723*sin(f*x +
 e)^3/(cos(f*x + e) + 1)^3 - 18590*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 33
462*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 40326*sin(f*x + e)^6/(cos(f*x + e
) + 1)^6 + 40326*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 27027*sin(f*x + e)^8
/(cos(f*x + e) + 1)^8 + 15015*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - 4719*si
n(f*x + e)^10/(cos(f*x + e) + 1)^10 + 1287*sin(f*x + e)^11/(cos(f*x + e) +
1)^11 - 47)/(c^7 - 13*c^7*sin(f*x + e)/(cos(f*x + e) + 1) + 78*c^7*sin(f*x
+ e)^2/(cos(f*x + e) + 1)^2 - 286*c^7*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 +
 715*c^7*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 1287*c^7*sin(f*x + e)^5/(cos
(f*x + e) + 1)^5 + 1716*c^7*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 1716*c^7*
sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 1287*c^7*sin(f*x + e)^8/(cos(f*x + e)
 + 1)^8 - 715*c^7*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 286*c^7*sin(f*x + e
)^10/(cos(f*x + e) + 1)^10 - 78*c^7*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 +
 13*c^7*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - c^7*sin(f*x + e)^13/(cos(f*
x + e) + 1)^13) - 154*a^3*(13*sin(f*x + e)/(cos(f*x + e) + 1) - 78*sin(f*x
+ e)^2/(cos(f*x + e) + 1)^2 + 286*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 520
*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 936*sin(f*x + e)^5/(cos(f*x + e) + 1
)^5 - 858*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 858*sin(f*x + e)^7/(cos(f*x
 + e) + 1)^7 - 351*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 195*sin(f*x + e)^9
/(cos(f*x + e) + 1)^9 - 1)/(c^7 - 13*c^7*sin(f*x + e)/(cos(f*x + e) + 1) +
78*c^7*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 286*c^7*sin(f*x + e)^3/(cos(f*
x + e) + 1)^3 + 715*c^7*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 1287*c^7*sin(
f*x + e)^5/(cos(f*x + e) + 1)^5 + 1716*c^7*sin(f*x + e)^6/(cos(f*x + e) + 1
)^6 - 1716*c^7*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 1287*c^7*sin(f*x + e)^
8/(cos(f*x + e) + 1)^8 - 715*c^7*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 286*
c^7*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 78*c^7*sin(f*x + e)^11/(cos(f*x
 + e) + 1)^11 + 13*c^7*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - c^7*sin(f*x
```

```
+ e)^13/(cos(f*x + e) + 1)^13))/f
```

Fricas [B] time = 1.46854, size = 976, normalized size = 7.39

$$-\frac{2\,a^3\cos\left(fx+e\right)^7-12\,a^3\cos\left(fx+e\right)^6-49\,a^3\cos\left(fx+e\right)^5+70\,a^3\cos\left(fx+e\right)^4-567\,a^3\cos\left(fx+e\right)^3-1596}{3003\left(c^7f\cos\left(fx+e\right)^7+7\,c^7f\cos\left(fx+e\right)^6-18\,c^7f\cos\left(fx+e\right)^5-56\,c^7f\cos\left(fx+e\right)^4+48\,c^7f\cos\left(fx+e\right)^3+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^7,x, algorithm="fricas")
```

```
[Out] -1/3003*(2*a^3*cos(f*x + e)^7 - 12*a^3*cos(f*x + e)^6 - 49*a^3*cos(f*x + e)
^5 + 70*a^3*cos(f*x + e)^4 - 567*a^3*cos(f*x + e)^3 - 1596*a^3*cos(f*x + e)
^2 + 924*a^3*cos(f*x + e) + 1848*a^3 + (2*a^3*cos(f*x + e)^6 + 14*a^3*cos(f
*x + e)^5 - 35*a^3*cos(f*x + e)^4 - 105*a^3*cos(f*x + e)^3 - 672*a^3*cos(f*
x + e)^2 + 924*a^3*cos(f*x + e) + 1848*a^3)*sin(f*x + e))/(c^7*f*cos(f*x +
e)^7 + 7*c^7*f*cos(f*x + e)^6 - 18*c^7*f*cos(f*x + e)^5 - 56*c^7*f*cos(f*x
+ e)^4 + 48*c^7*f*cos(f*x + e)^3 + 112*c^7*f*cos(f*x + e)^2 - 32*c^7*f*cos(
f*x + e) - 64*c^7*f - (c^7*f*cos(f*x + e)^6 - 6*c^7*f*cos(f*x + e)^5 - 24*c
^7*f*cos(f*x + e)^4 + 32*c^7*f*cos(f*x + e)^3 + 80*c^7*f*cos(f*x + e)^2 - 3
2*c^7*f*cos(f*x + e) - 64*c^7*f)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**7,x)
```

```
[Out] Timed out
```

Giac [A] time = 2.29861, size = 311, normalized size = 2.36

$$-\frac{2\left(3003\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{12}-9009\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{11}+51051\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{10}-99099\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^7,x, algorithm="giac")
```

```
[Out] -2/3003*(3003*a^3*tan(1/2*f*x + 1/2*e)^12 - 9009*a^3*tan(1/2*f*x + 1/2*e)^1
1 + 51051*a^3*tan(1/2*f*x + 1/2*e)^10 - 99099*a^3*tan(1/2*f*x + 1/2*e)^9 +
216216*a^3*tan(1/2*f*x + 1/2*e)^8 - 246246*a^3*tan(1/2*f*x + 1/2*e)^7 + 285
714*a^3*tan(1/2*f*x + 1/2*e)^6 - 182754*a^3*tan(1/2*f*x + 1/2*e)^5 + 122551
*a^3*tan(1/2*f*x + 1/2*e)^4 - 37609*a^3*tan(1/2*f*x + 1/2*e)^3 + 15171*a^3*
tan(1/2*f*x + 1/2*e)^2 - 1027*a^3*tan(1/2*f*x + 1/2*e) + 310*a^3)/(c^7*f*(t
an(1/2*f*x + 1/2*e) - 1)^13)
```

3.260 $\int \frac{(a+a\sin(e+fx))^3}{(c-c\sin(e+fx))^8}\,dx$

Optimal. Leaf size=166

$$\frac{4a^3c^2\cos^7(e+fx)}{195f(c-c\sin(e+fx))^{10}}+\frac{a^3c^3\cos^7(e+fx)}{15f(c-c\sin(e+fx))^{11}}+\frac{8a^3\cos^7(e+fx)}{45045cf(c-c\sin(e+fx))^7}+\frac{8a^3\cos^7(e+fx)}{6435f(c-c\sin(e+fx))^8}+\frac{4a}{715f(}$$

```
[Out] (a^3*c^3*Cos[e + f*x]^7)/(15*f*(c - c*Sin[e + f*x])^11) + (4*a^3*c^2*Cos[e
+ f*x]^7)/(195*f*(c - c*Sin[e + f*x])^10) + (4*a^3*c*Cos[e + f*x]^7)/(715*f
*(c - c*Sin[e + f*x])^9) + (8*a^3*Cos[e + f*x]^7)/(6435*f*(c - c*Sin[e + f*
x])^8) + (8*a^3*Cos[e + f*x]^7)/(45045*c*f*(c - c*Sin[e + f*x])^7)
```

Rubi [A] time = 0.287297, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2736, 2672, 2671}

$$\frac{4a^3c^2\cos^7(e+fx)}{195f(c-c\sin(e+fx))^{10}}+\frac{a^3c^3\cos^7(e+fx)}{15f(c-c\sin(e+fx))^{11}}+\frac{8a^3\cos^7(e+fx)}{45045cf(c-c\sin(e+fx))^7}+\frac{8a^3\cos^7(e+fx)}{6435f(c-c\sin(e+fx))^8}+\frac{4a}{715f(}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3/(c - c*Sin[e + f*x])^8,x]
```

```
[Out] (a^3*c^3*Cos[e + f*x]^7)/(15*f*(c - c*Sin[e + f*x])^11) + (4*a^3*c^2*Cos[e
+ f*x]^7)/(195*f*(c - c*Sin[e + f*x])^10) + (4*a^3*c*Cos[e + f*x]^7)/(715*f
*(c - c*Sin[e + f*x])^9) + (8*a^3*Cos[e + f*x]^7)/(6435*f*(c - c*Sin[e + f*
x])^8) + (8*a^3*Cos[e + f*x]^7)/(45045*c*f*(c - c*Sin[e + f*x])^7)
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] && !IGtQ[m, 0]
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] && !ILtQ[p, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^3}{(c-c\sin(e+fx))^8}\,dx = \left(a^3c^3\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^{11}}\,dx$$
$$= \frac{a^3c^3\cos^7(e+fx)}{15f(c-c\sin(e+fx))^{11}} + \frac{1}{15}\left(4a^3c^2\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^{10}}\,dx$$
$$= \frac{a^3c^3\cos^7(e+fx)}{15f(c-c\sin(e+fx))^{11}} + \frac{4a^3c^2\cos^7(e+fx)}{195f(c-c\sin(e+fx))^{10}} + \frac{1}{65}\left(4a^3c\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^{9}}$$
$$= \frac{a^3c^3\cos^7(e+fx)}{15f(c-c\sin(e+fx))^{11}} + \frac{4a^3c^2\cos^7(e+fx)}{195f(c-c\sin(e+fx))^{10}} + \frac{4a^3c\cos^7(e+fx)}{715f(c-c\sin(e+fx))^{9}} + \frac{1}{715}($$
$$= \frac{a^3c^3\cos^7(e+fx)}{15f(c-c\sin(e+fx))^{11}} + \frac{4a^3c^2\cos^7(e+fx)}{195f(c-c\sin(e+fx))^{10}} + \frac{4a^3c\cos^7(e+fx)}{715f(c-c\sin(e+fx))^{9}} + \frac{8}{6435f}$$
$$= \frac{a^3c^3\cos^7(e+fx)}{15f(c-c\sin(e+fx))^{11}} + \frac{4a^3c^2\cos^7(e+fx)}{195f(c-c\sin(e+fx))^{10}} + \frac{4a^3c\cos^7(e+fx)}{715f(c-c\sin(e+fx))^{9}} + \frac{8}{6435f}$$

Mathematica [A] time = 2.11265, size = 209, normalized size = 1.26

$$(a\sin(e+fx)+a)^3\left(\cos\left(\tfrac{1}{2}(e+fx)\right)-\sin\left(\tfrac{1}{2}(e+fx)\right)\right)\left(109395\sin\left(\tfrac{1}{2}(e+fx)\right)+60060\sin\left(\tfrac{3}{2}(e+fx)\right)-15015\sin\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3/(c - c*Sin[e + f*x])^8,x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a + a*Sin[e + f*x])^3*(115830*Cos[(
e + f*x)/2] - 65065*Cos[(3*(e + f*x))/2] - 18018*Cos[(5*(e + f*x))/2] + 136
5*Cos[(7*(e + f*x))/2] - 105*Cos[(11*(e + f*x))/2] + Cos[(15*(e + f*x))/2]
+ 109395*Sin[(e + f*x)/2] + 60060*Sin[(3*(e + f*x))/2] - 15015*Sin[(5*(e +
f*x))/2] - 455*Sin[(9*(e + f*x))/2] + 15*Sin[(13*(e + f*x))/2]))/(360360*f*
(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6*(c - c*Sin[e + f*x])^8)
```

Maple [A] time = 0.139, size = 238, normalized size = 1.4

$$2\,\frac{a^3}{fc^8}\left(-276\,\left(\tan\left(1/2\,fx+e/2\right)-1\right)^{-4}-\frac{24320}{13\,\left(\tan\left(1/2\,fx+e/2\right)-1\right)^{13}}-\frac{32288}{7\,\left(\tan\left(1/2\,fx+e/2\right)-1\right)^{7}}-512\,\left(\tan\left(1/2\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^8,x)
```

```
[Out] 2/f*a^3/c^8*(-276/(tan(1/2*f*x+1/2*e)-1)^4-24320/13/(tan(1/2*f*x+1/2*e)-1)^
13-32288/7/(tan(1/2*f*x+1/2*e)-1)^7-512/(tan(1/2*f*x+1/2*e)-1)^14-188/3/(ta
n(1/2*f*x+1/2*e)-1)^3-13184/3/(tan(1/2*f*x+1/2*e)-1)^12-47072/5/(tan(1/2*f*
x+1/2*e)-1)^10-81344/11/(tan(1/2*f*x+1/2*e)-1)^11-4536/5/(tan(1/2*f*x+1/2*e
)-1)^5-2304/(tan(1/2*f*x+1/2*e)-1)^6-10/(tan(1/2*f*x+1/2*e)-1)^2-7352/(tan(
1/2*f*x+1/2*e)-1)^8-1/(tan(1/2*f*x+1/2*e)-1)-84112/9/(tan(1/2*f*x+1/2*e)-1)
^9-1024/15/(tan(1/2*f*x+1/2*e)-1)^15)
```

Maxima [B] time = 1.94779, size = 3270, normalized size = 19.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^8,x, algorithm="maxima")
```

```
[Out] 2/45045*(3*a^3*(17715*sin(f*x + e)/(cos(f*x + e) + 1) - 78960*sin(f*x + e)^
2/(cos(f*x + e) + 1)^2 + 342160*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 89134
5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 1960959*sin(f*x + e)^5/(cos(f*x + e
) + 1)^5 - 3043040*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 3912480*sin(f*x +
e)^7/(cos(f*x + e) + 1)^7 - 3687255*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 2
867865*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - 1585584*sin(f*x + e)^10/(cos(f
*x + e) + 1)^10 + 720720*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 - 195195*sin
(f*x + e)^12/(cos(f*x + e) + 1)^12 + 45045*sin(f*x + e)^13/(cos(f*x + e) +
1)^13 - 1181)/(c^8 - 15*c^8*sin(f*x + e)/(cos(f*x + e) + 1) + 105*c^8*sin(f
*x + e)^2/(cos(f*x + e) + 1)^2 - 455*c^8*sin(f*x + e)^3/(cos(f*x + e) + 1)^
3 + 1365*c^8*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 3003*c^8*sin(f*x + e)^5/
(cos(f*x + e) + 1)^5 + 5005*c^8*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 6435*
c^8*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 6435*c^8*sin(f*x + e)^8/(cos(f*x
+ e) + 1)^8 - 5005*c^8*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 3003*c^8*sin(f
*x + e)^10/(cos(f*x + e) + 1)^10 - 1365*c^8*sin(f*x + e)^11/(cos(f*x + e) +
 1)^11 + 455*c^8*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - 105*c^8*sin(f*x +
e)^13/(cos(f*x + e) + 1)^13 + 15*c^8*sin(f*x + e)^14/(cos(f*x + e) + 1)^14
- c^8*sin(f*x + e)^15/(cos(f*x + e) + 1)^15) - 7*a^3*(7845*sin(f*x + e)/(co
s(f*x + e) + 1) - 54915*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 222950*sin(f*
x + e)^3/(cos(f*x + e) + 1)^3 - 668850*sin(f*x + e)^4/(cos(f*x + e) + 1)^4
+ 1444443*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 2407405*sin(f*x + e)^6/(cos
(f*x + e) + 1)^6 + 3063060*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 3063060*si
n(f*x + e)^8/(cos(f*x + e) + 1)^8 + 2357355*sin(f*x + e)^9/(cos(f*x + e) +
1)^9 - 1414413*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 + 630630*sin(f*x + e)^
11/(cos(f*x + e) + 1)^11 - 210210*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 + 4
5045*sin(f*x + e)^13/(cos(f*x + e) + 1)^13 - 6435*sin(f*x + e)^14/(cos(f*x
+ e) + 1)^14 - 952)/(c^8 - 15*c^8*sin(f*x + e)/(cos(f*x + e) + 1) + 105*c^8
*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 455*c^8*sin(f*x + e)^3/(cos(f*x + e)
 + 1)^3 + 1365*c^8*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 3003*c^8*sin(f*x +
 e)^5/(cos(f*x + e) + 1)^5 + 5005*c^8*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 -
 6435*c^8*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 6435*c^8*sin(f*x + e)^8/(co
s(f*x + e) + 1)^8 - 5005*c^8*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 3003*c^8
*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 1365*c^8*sin(f*x + e)^11/(cos(f*x
+ e) + 1)^11 + 455*c^8*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - 105*c^8*sin(
f*x + e)^13/(cos(f*x + e) + 1)^13 + 15*c^8*sin(f*x + e)^14/(cos(f*x + e) +
1)^14 - c^8*sin(f*x + e)^15/(cos(f*x + e) + 1)^15) - 12*a^3*(1740*sin(f*x +
 e)/(cos(f*x + e) + 1) - 12180*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 37765*
sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 113295*sin(f*x + e)^4/(cos(f*x + e) +
 1)^4 + 204204*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 340340*sin(f*x + e)^6/
(cos(f*x + e) + 1)^6 + 373230*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 373230*
sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 240240*sin(f*x + e)^9/(cos(f*x + e) +
 1)^9 - 144144*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 + 45045*sin(f*x + e)^1
1/(cos(f*x + e) + 1)^11 - 15015*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - 116
)/(c^8 - 15*c^8*sin(f*x + e)/(cos(f*x + e) + 1) + 105*c^8*sin(f*x + e)^2/(c
os(f*x + e) + 1)^2 - 455*c^8*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 1365*c^8
*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 3003*c^8*sin(f*x + e)^5/(cos(f*x + e
) + 1)^5 + 5005*c^8*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 6435*c^8*sin(f*x
+ e)^7/(cos(f*x + e) + 1)^7 + 6435*c^8*sin(f*x + e)^8/(cos(f*x + e) + 1)^8
- 5005*c^8*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 3003*c^8*sin(f*x + e)^10/(
cos(f*x + e) + 1)^10 - 1365*c^8*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 + 455
*c^8*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - 105*c^8*sin(f*x + e)^13/(cos(f
```

```
*x + e) + 1)^13 + 15*c^8*sin(f*x + e)^14/(cos(f*x + e) + 1)^14 - c^8*sin(f*
x + e)^15/(cos(f*x + e) + 1)^15) + 6*a^3*(675*sin(f*x + e)/(cos(f*x + e) +
1) - 4725*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 20475*sin(f*x + e)^3/(cos(f
*x + e) + 1)^3 - 46410*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 102102*sin(f*x
 + e)^5/(cos(f*x + e) + 1)^5 - 130130*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 +
 167310*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 122265*sin(f*x + e)^8/(cos(f*
x + e) + 1)^8 + 95095*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - 33033*sin(f*x +
 e)^10/(cos(f*x + e) + 1)^10 + 15015*sin(f*x + e)^11/(cos(f*x + e) + 1)^11
- 45)/(c^8 - 15*c^8*sin(f*x + e)/(cos(f*x + e) + 1) + 105*c^8*sin(f*x + e)^
2/(cos(f*x + e) + 1)^2 - 455*c^8*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 1365
*c^8*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 3003*c^8*sin(f*x + e)^5/(cos(f*x
 + e) + 1)^5 + 5005*c^8*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 6435*c^8*sin(
f*x + e)^7/(cos(f*x + e) + 1)^7 + 6435*c^8*sin(f*x + e)^8/(cos(f*x + e) + 1
)^8 - 5005*c^8*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 3003*c^8*sin(f*x + e)^
10/(cos(f*x + e) + 1)^10 - 1365*c^8*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 +
 455*c^8*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - 105*c^8*sin(f*x + e)^13/(c
os(f*x + e) + 1)^13 + 15*c^8*sin(f*x + e)^14/(cos(f*x + e) + 1)^14 - c^8*si
n(f*x + e)^15/(cos(f*x + e) + 1)^15))/f
```

Fricas [B] time = 1.44161, size = 1138, normalized size = 6.86

$$\frac{8\,a^3\cos\left(fx+e\right)^8+64\,a^3\cos\left(fx+e\right)^7-196\,a^3\cos\left(fx+e\right)^6-672\,a^3\cos\left(fx+e\right)^5+735\,a^3\cos\left(fx+e\right)^4-7161\,a^3}{45045\left(c^8f\cos\left(fx+e\right)^8-7\,c^8f\cos\left(fx+e\right)^7-32\,c^8f\cos\left(fx+e\right)^6+56\,c^8f\cos\left(fx+e\right)^5+160\,c^8f\cos\left(fx+e\right)^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^8,x, algorithm="fricas")
```

```
[Out] 1/45045*(8*a^3*cos(f*x + e)^8 + 64*a^3*cos(f*x + e)^7 - 196*a^3*cos(f*x + e
)^6 - 672*a^3*cos(f*x + e)^5 + 735*a^3*cos(f*x + e)^4 - 7161*a^3*cos(f*x +
e)^3 - 20328*a^3*cos(f*x + e)^2 + 12012*a^3*cos(f*x + e) + 24024*a^3 - (8*a
^3*cos(f*x + e)^7 - 56*a^3*cos(f*x + e)^6 - 252*a^3*cos(f*x + e)^5 + 420*a^
3*cos(f*x + e)^4 + 1155*a^3*cos(f*x + e)^3 + 8316*a^3*cos(f*x + e)^2 - 1201
2*a^3*cos(f*x + e) - 24024*a^3)*sin(f*x + e))/(c^8*f*cos(f*x + e)^8 - 7*c^8
*f*cos(f*x + e)^7 - 32*c^8*f*cos(f*x + e)^6 + 56*c^8*f*cos(f*x + e)^5 + 160
*c^8*f*cos(f*x + e)^4 - 112*c^8*f*cos(f*x + e)^3 - 256*c^8*f*cos(f*x + e)^2
 + 64*c^8*f*cos(f*x + e) + 128*c^8*f + (c^8*f*cos(f*x + e)^7 + 8*c^8*f*cos(
f*x + e)^6 - 24*c^8*f*cos(f*x + e)^5 - 80*c^8*f*cos(f*x + e)^4 + 80*c^8*f*c
os(f*x + e)^3 + 192*c^8*f*cos(f*x + e)^2 - 64*c^8*f*cos(f*x + e) - 128*c^8*
f)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**8,x)
```

```
[Out] Timed out
```

Giac [A] time = 2.20153, size = 356, normalized size = 2.14

$$-\frac{2\left(45045\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{14}-180180\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{13}+1066065\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{12}-2702700\,a^3\tan\left(\frac{1}{2}\,fx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^8,x, algorithm="giac")
```

```
[Out] -2/45045*(45045*a^3*tan(1/2*f*x + 1/2*e)^14 - 180180*a^3*tan(1/2*f*x + 1/2*
e)^13 + 1066065*a^3*tan(1/2*f*x + 1/2*e)^12 - 2702700*a^3*tan(1/2*f*x + 1/2
*e)^11 + 6675669*a^3*tan(1/2*f*x + 1/2*e)^10 - 10210200*a^3*tan(1/2*f*x + 1
/2*e)^9 + 14124825*a^3*tan(1/2*f*x + 1/2*e)^8 - 13178880*a^3*tan(1/2*f*x +
1/2*e)^7 + 11026015*a^3*tan(1/2*f*x + 1/2*e)^6 - 6066060*a^3*tan(1/2*f*x +
1/2*e)^5 + 3088995*a^3*tan(1/2*f*x + 1/2*e)^4 - 864500*a^3*tan(1/2*f*x + 1/
2*e)^3 + 265335*a^3*tan(1/2*f*x + 1/2*e)^2 - 18600*a^3*tan(1/2*f*x + 1/2*e)
 + 4243*a^3)/(c^8*f*(tan(1/2*f*x + 1/2*e) - 1)^15)
```

3.261 $\int \frac{(c-c\sin(e+fx))^4}{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=118

$$-\frac{2a^3c^4\cos^7(e+fx)}{f(a\sin(e+fx)+a)^4}-\frac{35c^4\cos^3(e+fx)}{3af}-\frac{14ac^4\cos^5(e+fx)}{f(a\sin(e+fx)+a)^2}-\frac{35c^4\sin(e+fx)\cos(e+fx)}{2af}-\frac{35c^4x}{2a}$$

```
[Out] (-35*c^4*x)/(2*a) - (35*c^4*Cos[e + f*x]^3)/(3*a*f) - (35*c^4*Cos[e + f*x]*
Sin[e + f*x])/(2*a*f) - (2*a^3*c^4*Cos[e + f*x]^7)/(f*(a + a*Sin[e + f*x])^
4) - (14*a*c^4*Cos[e + f*x]^5)/(f*(a + a*Sin[e + f*x])^2)
```

Rubi [A] time = 0.195605, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {2736, 2680, 2682, 2635, 8}

$$-\frac{2a^3c^4\cos^7(e+fx)}{f(a\sin(e+fx)+a)^4}-\frac{35c^4\cos^3(e+fx)}{3af}-\frac{14ac^4\cos^5(e+fx)}{f(a\sin(e+fx)+a)^2}-\frac{35c^4\sin(e+fx)\cos(e+fx)}{2af}-\frac{35c^4x}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sin[e + f*x])^4/(a + a*Sin[e + f*x]),x]
```


```
[Out] (-35*c^4*x)/(2*a) - (35*c^4*Cos[e + f*x]^3)/(3*a*f) - (35*c^4*Cos[e + f*x]*
Sin[e + f*x])/(2*a*f) - (2*a^3*c^4*Cos[e + f*x]^7)/(f*(a + a*Sin[e + f*x])^
4) - (14*a*c^4*Cos[e + f*x]^5)/(f*(a + a*Sin[e + f*x])^2)
```


Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
```

]

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{(c-c\sin(e+fx))^4}{a+a\sin(e+fx)}\,dx = \left(a^4c^4\right)\int \frac{\cos^8(e+fx)}{(a+a\sin(e+fx))^5}\,dx$$
$$= -\frac{2a^3c^4\cos^7(e+fx)}{f(a+a\sin(e+fx))^4} - \left(7a^2c^4\right)\int \frac{\cos^6(e+fx)}{(a+a\sin(e+fx))^3}\,dx$$
$$= -\frac{2a^3c^4\cos^7(e+fx)}{f(a+a\sin(e+fx))^4} - \frac{14ac^4\cos^5(e+fx)}{f(a+a\sin(e+fx))^2} - \left(35c^4\right)\int \frac{\cos^4(e+fx)}{a+a\sin(e+fx)}\,dx$$
$$= -\frac{35c^4\cos^3(e+fx)}{3af} - \frac{2a^3c^4\cos^7(e+fx)}{f(a+a\sin(e+fx))^4} - \frac{14ac^4\cos^5(e+fx)}{f(a+a\sin(e+fx))^2} - \frac{\left(35c^4\right)\int \cos^2(e+fx)}{a}$$
$$= -\frac{35c^4\cos^3(e+fx)}{3af} - \frac{35c^4\cos(e+fx)\sin(e+fx)}{2af} - \frac{2a^3c^4\cos^7(e+fx)}{f(a+a\sin(e+fx))^4} - \frac{14ac^4\cos^5(e}{f(a+a\sin(e+}$$
$$= -\frac{35c^4x}{2a} - \frac{35c^4\cos^3(e+fx)}{3af} - \frac{35c^4\cos(e+fx)\sin(e+fx)}{2af} - \frac{2a^3c^4\cos^7(e+fx)}{f(a+a\sin(e+fx))^4} - \frac{14ac}{f(a+}$$

Mathematica [A] time = 1.3807, size = 175, normalized size = 1.48

$$-\frac{c^4(\sin(e+fx)-1)^4\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)(-15\sin(2(e+fx))+141\cos(e+fx)-\cos(3}{12af(\sin(e+fx)+1)\left(\cos\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sin[e + f*x])^4/(a + a*Sin[e + f*x]),x]
```

```
[Out] -(c^4*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])^4*(Sin[(e +
 f*x)/2]*(-384 + 210*e + 210*f*x + 141*Cos[e + f*x] - Cos[3*(e + f*x)] - 15
*Sin[2*(e + f*x)]) + Cos[(e + f*x)/2]*(210*e + 210*f*x + 141*Cos[e + f*x] -
 Cos[3*(e + f*x)] - 15*Sin[2*(e + f*x)])))/(12*a*f*(Cos[(e + f*x)/2] - Sin[
(e + f*x)/2])^8*(1 + Sin[e + f*x]))
```

Maple [A] time = 0.092, size = 219, normalized size = 1.9

$$-5\,\frac{c^4\left(\tan\left(1/2\,fx+e/2\right)\right)^5}{af\left(1+\left(\tan\left(1/2\,fx+e/2\right)\right)^2\right)^3} - 22\,\frac{c^4\left(\tan\left(1/2\,fx+e/2\right)\right)^4}{af\left(1+\left(\tan\left(1/2\,fx+e/2\right)\right)^2\right)^3} - 48\,\frac{c^4\left(\tan\left(1/2\,fx+e/2\right)\right)^2}{af\left(1+\left(\tan\left(1/2\,fx+e/2\right)\right)^2\right)^3} + 5\,\frac{c^4}{af\left(1+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))^4/(a+a*sin(f*x+e)),x)
```

```
[Out] -5/f*c^4/a/(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)^5-22/f*c^4/a/(1+ta
n(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)^4-48/f*c^4/a/(1+tan(1/2*f*x+1/2*e)
^2)^3*tan(1/2*f*x+1/2*e)^2+5/f*c^4/a/(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x
+1/2*e)-70/3/f*c^4/a/(1+tan(1/2*f*x+1/2*e)^2)^3-35/f*c^4/a*arctan(tan(1/2*f
```

```
*x+1/2*e))-32/f*c^4/a/(tan(1/2*f*x+1/2*e)+1)
```

Maxima [B] time = 2.30217, size = 972, normalized size = 8.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^4/(a+a*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] -1/3*(c^4*((7*sin(f*x + e)/(cos(f*x + e) + 1) + 39*sin(f*x + e)^2/(cos(f*x
+ e) + 1)^2 + 24*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 24*sin(f*x + e)^4/(c
os(f*x + e) + 1)^4 + 9*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 9*sin(f*x + e)
^6/(cos(f*x + e) + 1)^6 + 16)/(a + a*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a*
sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 3*a*sin(f*x + e)^3/(cos(f*x + e) + 1)
^3 + 3*a*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 3*a*sin(f*x + e)^5/(cos(f*x
+ e) + 1)^5 + a*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + a*sin(f*x + e)^7/(cos
(f*x + e) + 1)^7) + 9*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a) + 12*c^4*(
(sin(f*x + e)/(cos(f*x + e) + 1) + 5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 +
3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^4/(cos(f*x + e) + 1)
^4 + 4)/(a + a*sin(f*x + e)/(cos(f*x + e) + 1) + 2*a*sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 + 2*a*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + a*sin(f*x + e)^4/
(cos(f*x + e) + 1)^4 + a*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 3*arctan(si
n(f*x + e)/(cos(f*x + e) + 1))/a) + 36*c^4*((sin(f*x + e)/(cos(f*x + e) + 1
) + sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2)/(a + a*sin(f*x + e)/(cos(f*x +
 e) + 1) + a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a*sin(f*x + e)^3/(cos(f*
x + e) + 1)^3) + arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a) + 24*c^4*(arcta
n(sin(f*x + e)/(cos(f*x + e) + 1))/a + 1/(a + a*sin(f*x + e)/(cos(f*x + e)
+ 1))) + 6*c^4/(a + a*sin(f*x + e)/(cos(f*x + e) + 1)))/f
```

Fricas [A] time = 1.31893, size = 390, normalized size = 3.31

$$\frac{2\,c^4\cos\left(fx+e\right)^4-13\,c^4\cos\left(fx+e\right)^3-105\,c^4fx-72\,c^4\cos\left(fx+e\right)^2-96\,c^4-3\left(35\,c^4fx+51\,c^4\right)\cos\left(fx+e\right)+\left(2\right.}{6\left(af\cos\left(fx+e\right)+af\sin\left(fx+e\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^4/(a+a*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*c^4*cos(f*x + e)^4 - 13*c^4*cos(f*x + e)^3 - 105*c^4*f*x - 72*c^4*co
s(f*x + e)^2 - 96*c^4 - 3*(35*c^4*f*x + 51*c^4)*cos(f*x + e) + (2*c^4*cos(f
*x + e)^3 - 105*c^4*f*x + 15*c^4*cos(f*x + e)^2 - 57*c^4*cos(f*x + e) + 96*
c^4)*sin(f*x + e))/(a*f*cos(f*x + e) + a*f*sin(f*x + e) + a*f)
```

Sympy [A] time = 42.4116, size = 2106, normalized size = 17.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))**4/(a+a*sin(f*x+e)),x)
```

```
[Out] Piecewise((-105*c**4*f*x*tan(e/2 + f*x/2)**7/(6*a*f*tan(e/2 + f*x/2)**7 + 6
*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*
x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*t
an(e/2 + f*x/2) + 6*a*f) - 105*c**4*f*x*tan(e/2 + f*x/2)**6/(6*a*f*tan(e/2
+ f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a
*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/
2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 315*c**4*f*x*tan(e/2 + f*x/2)**5/
(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f
*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f
*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 315*c**4*f*x*tan(e
/2 + f*x/2)**4/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*
a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x
/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 315
*c**4*f*x*tan(e/2 + f*x/2)**3/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 +
f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*
f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2)
 + 6*a*f) - 315*c**4*f*x*tan(e/2 + f*x/2)**2/(6*a*f*tan(e/2 + f*x/2)**7 + 6
*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*
x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*t
an(e/2 + f*x/2) + 6*a*f) - 105*c**4*f*x*tan(e/2 + f*x/2)/(6*a*f*tan(e/2 + f
*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*
tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)*
*2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 105*c**4*f*x/(6*a*f*tan(e/2 + f*x/2)
**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e
/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 +
6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 222*c**4*tan(e/2 + f*x/2)**7/(6*a*f*tan(e
/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 1
8*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f
*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 504*c**4*tan(e/2 + f*x/2)**5/(
6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*
x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*
tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 42*c**4*tan(e/2 + f
*x/2)**4/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*ta
n(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3
 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 378*c**4*
tan(e/2 + f*x/2)**3/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6
+ 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2
+ f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f)
- 168*c**4*tan(e/2 + f*x/2)**2/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 +
 f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a
*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2
) + 6*a*f) + 112*c**4*tan(e/2 + f*x/2)/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*t
an(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**
4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2
 + f*x/2) + 6*a*f) - 110*c**4/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 +
f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*
f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2)
 + 6*a*f), Ne(f, 0)), (x*(-c*sin(e) + c)**4/(a*sin(e) + a), True))
```

Giac [A] time = 2.27692, size = 182, normalized size = 1.54

$$-\frac{\frac{105\left(fx+e\right)c^{4}}{a}+\frac{192\,c^{4}}{a\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)}+\frac{2\left(15\,c^{4}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{5}+66\,c^{4}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}+144\,c^{4}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-15\,c^{4}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+70\,c^{4}\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+1\right)^{3}a}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^4/(a+a*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] -1/6*(105*(f*x + e)*c^4/a + 192*c^4/(a*(tan(1/2*f*x + 1/2*e) + 1)) + 2*(15*
c^4*tan(1/2*f*x + 1/2*e)^5 + 66*c^4*tan(1/2*f*x + 1/2*e)^4 + 144*c^4*tan(1/
2*f*x + 1/2*e)^2 - 15*c^4*tan(1/2*f*x + 1/2*e) + 70*c^4)/((tan(1/2*f*x + 1/
2*e)^2 + 1)^3*a))/f
```

3.262 $\int \frac{(c-c\sin(e+fx))^3}{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=92

$$-\frac{2a^2c^3\cos^5(e+fx)}{f(a\sin(e+fx)+a)^3}-\frac{15c^3\cos(e+fx)}{2af}-\frac{5c^3\cos^3(e+fx)}{2f(a\sin(e+fx)+a)}-\frac{15c^3x}{2a}$$

```
[Out] (-15*c^3*x)/(2*a) - (15*c^3*Cos[e + f*x])/(2*a*f) - (2*a^2*c^3*Cos[e + f*x]
^5)/(f*(a + a*Sin[e + f*x])^3) - (5*c^3*Cos[e + f*x]^3)/(2*f*(a + a*Sin[e +
 f*x]))
```

Rubi [A] time = 0.17626, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {2736, 2680, 2679, 2682, 8}

$$-\frac{2a^2c^3\cos^5(e+fx)}{f(a\sin(e+fx)+a)^3}-\frac{15c^3\cos(e+fx)}{2af}-\frac{5c^3\cos^3(e+fx)}{2f(a\sin(e+fx)+a)}-\frac{15c^3x}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sin[e + f*x])^3/(a + a*Sin[e + f*x]),x]
```

```
[Out] (-15*c^3*x)/(2*a) - (15*c^3*Cos[e + f*x])/(2*a*f) - (2*a^2*c^3*Cos[e + f*x]
^5)/(f*(a + a*Sin[e + f*x])^3) - (5*c^3*Cos[e + f*x]^3)/(2*f*(a + a*Sin[e +
 f*x]))
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2679

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(a*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(c-c\sin(e+fx))^3}{a+a\sin(e+fx)}\,dx &= \left(a^3c^3\right)\int \frac{\cos^6(e+fx)}{(a+a\sin(e+fx))^4}\,dx\\ &= -\frac{2a^2c^3\cos^5(e+fx)}{f(a+a\sin(e+fx))^3}-\left(5ac^3\right)\int \frac{\cos^4(e+fx)}{(a+a\sin(e+fx))^2}\,dx\\ &= -\frac{2a^2c^3\cos^5(e+fx)}{f(a+a\sin(e+fx))^3}-\frac{5c^3\cos^3(e+fx)}{2f(a+a\sin(e+fx))}-\frac{1}{2}\left(15c^3\right)\int \frac{\cos^2(e+fx)}{a+a\sin(e+fx)}\,dx\\ &= -\frac{15c^3\cos(e+fx)}{2af}-\frac{2a^2c^3\cos^5(e+fx)}{f(a+a\sin(e+fx))^3}-\frac{5c^3\cos^3(e+fx)}{2f(a+a\sin(e+fx))}-\frac{\left(15c^3\right)\int 1\,dx}{2a}\\ &= -\frac{15c^3x}{2a}-\frac{15c^3\cos(e+fx)}{2af}-\frac{2a^2c^3\cos^5(e+fx)}{f(a+a\sin(e+fx))^3}-\frac{5c^3\cos^3(e+fx)}{2f(a+a\sin(e+fx))}\end{aligned}$$

Mathematica [A] time = 0.507868, size = 155, normalized size = 1.68

$$\frac{c^3(\sin(e+fx)-1)^3\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)(-\sin(2(e+fx))+16\cos(e+fx)+30e+3\right.}{4af(\sin(e+fx)+1)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e-\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sin[e + f*x])^3/(a + a*Sin[e + f*x]),x]
```

```
[Out] (c^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])^3*(Sin[(e +
f*x)/2]*(-64 + 30*e + 30*f*x + 16*Cos[e + f*x] - Sin[2*(e + f*x)]) + Cos[(e
 + f*x)/2]*(30*(e + f*x) + 16*Cos[e + f*x] - Sin[2*(e + f*x)])))/(4*a*f*(Co
s[(e + f*x)/2] - Sin[(e + f*x)/2])^6*(1 + Sin[e + f*x]))
```

Maple [B] time = 0.078, size = 181, normalized size = 2.

$$-\frac{c^3}{af}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3\left(1+\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2\right)^{-2}-8\,\frac{c^3\left(\tan\left(1/2\,fx+e/2\right)\right)^2}{af\left(1+\left(\tan\left(1/2\,fx+e/2\right)\right)^2\right)^2}+\frac{c^3}{af}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\left(1+\left(\tan\left(\frac{f}{}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))^3/(a+a*sin(f*x+e)),x)
```

```
[Out] -1/f*c^3/a/(1+tan(1/2*f*x+1/2*e)^2)^2*tan(1/2*f*x+1/2*e)^3-8/f*c^3/a/(1+tan
(1/2*f*x+1/2*e)^2)^2*tan(1/2*f*x+1/2*e)^2+1/f*c^3/a/(1+tan(1/2*f*x+1/2*e)^2
)^2*tan(1/2*f*x+1/2*e)-8/f*c^3/a/(1+tan(1/2*f*x+1/2*e)^2)^2-15/f*c^3/a*arct
an(tan(1/2*f*x+1/2*e))-16/f*c^3/a/(tan(1/2*f*x+1/2*e)+1)
```

Maxima [B] time = 2.02787, size = 572, normalized size = 6.22

$$-\frac{c^3\left(\frac{\frac{\sin(fx+e)}{\cos(fx+e)+1}+\frac{5\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+4}{a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}+\frac{2a\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{2a\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{a\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{a\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}+\frac{3\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a}\right)+6c^3\left(\frac{\frac{\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)}{(\cos(fx+e)+}}{a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}+\frac{a\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+}\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^3/(a+a*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] -(c^3*((sin(f*x + e)/(cos(f*x + e) + 1) + 5*sin(f*x + e)^2/(cos(f*x + e) +
1)^2 + 3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^4/(cos(f*x +
e) + 1)^4 + 4)/(a + a*sin(f*x + e)/(cos(f*x + e) + 1) + 2*a*sin(f*x + e)^2/
(cos(f*x + e) + 1)^2 + 2*a*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + a*sin(f*x
+ e)^4/(cos(f*x + e) + 1)^4 + a*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 3*ar
ctan(sin(f*x + e)/(cos(f*x + e) + 1))/a) + 6*c^3*((sin(f*x + e)/(cos(f*x +
e) + 1) + sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2)/(a + a*sin(f*x + e)/(cos
(f*x + e) + 1) + a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a*sin(f*x + e)^3/(
cos(f*x + e) + 1)^3) + arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a) + 6*c^3*(
arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a + 1/(a + a*sin(f*x + e)/(cos(f*x
+ e) + 1))) + 2*c^3/(a + a*sin(f*x + e)/(cos(f*x + e) + 1)))/f
```

Fricas [A] time = 1.39535, size = 313, normalized size = 3.4

$$-\frac{c^3\cos\left(fx+e\right)^3+15\,c^3fx+8\,c^3\cos\left(fx+e\right)^2+16\,c^3+\left(15\,c^3fx+23\,c^3\right)\cos\left(fx+e\right)+\left(15\,c^3fx-c^3\cos\left(fx+e\right)^2+\right.}{2\left(af\cos\left(fx+e\right)+af\sin\left(fx+e\right)+af\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^3/(a+a*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] -1/2*(c^3*cos(f*x + e)^3 + 15*c^3*f*x + 8*c^3*cos(f*x + e)^2 + 16*c^3 + (15
*c^3*f*x + 23*c^3)*cos(f*x + e) + (15*c^3*f*x - c^3*cos(f*x + e)^2 + 7*c^3*
cos(f*x + e) - 16*c^3)*sin(f*x + e))/(a*f*cos(f*x + e) + a*f*sin(f*x + e) +
 a*f)
```

Sympy [A] time = 16.8178, size = 1170, normalized size = 12.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))**3/(a+a*sin(f*x+e)),x)
```

```
[Out] Piecewise((-15*c**3*f*x*tan(e/2 + f*x/2)**5/(2*a*f*tan(e/2 + f*x/2)**5 + 2*
a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2
)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 15*c**3*f*x*tan(e/2 + f*x/2)**4/(2
*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/
2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 30*c*
```

```
*3*f*x*tan(e/2 + f*x/2)**3/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x
/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(
e/2 + f*x/2) + 2*a*f) - 30*c**3*f*x*tan(e/2 + f*x/2)**2/(2*a*f*tan(e/2 + f*
x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan
(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 15*c**3*f*x*tan(e/2 +
f*x/2)/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e
/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f
) - 15*c**3*f*x/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*
a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2
) + 2*a*f) + 14*c**3*tan(e/2 + f*x/2)**5/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f
*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**
2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 20*c**3*tan(e/2 + f*x/2)**4/(2*a*f*ta
n(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 +
 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 10*c**3*tan(
e/2 + f*x/2)**3/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*
a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2
) + 2*a*f) - 50*c**3*tan(e/2 + f*x/2)**2/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f
*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**
2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 34*c**3/(2*a*f*tan(e/2 + f*x/2)**5 +
2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x
/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f), Ne(f, 0)), (x*(-c*sin(e) + c)**3/
(a*sin(e) + a), True))
```

Giac [A] time = 2.07312, size = 158, normalized size = 1.72

$$-\frac{\frac{15\,(fx+e)c^3}{a} + \frac{32\,c^3}{a\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)} + \frac{2\left(c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+8\,c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+8\,c^3\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)^2 a}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^3/(a+a*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] -1/2*(15*(f*x + e)*c^3/a + 32*c^3/(a*(tan(1/2*f*x + 1/2*e) + 1)) + 2*(c^3*t
an(1/2*f*x + 1/2*e)^3 + 8*c^3*tan(1/2*f*x + 1/2*e)^2 - c^3*tan(1/2*f*x + 1/
2*e) + 8*c^3)/((tan(1/2*f*x + 1/2*e)^2 + 1)^2*a))/f
```

3.263 $\int \frac{(c-c\sin(e+fx))^2}{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=56

$$-\frac{3c^2\cos(e+fx)}{af}-\frac{2ac^2\cos^3(e+fx)}{f(a\sin(e+fx)+a)^2}-\frac{3c^2x}{a}$$

```
[Out] (-3*c^2*x)/a - (3*c^2*Cos[e + f*x])/(a*f) - (2*a*c^2*Cos[e + f*x]^3)/(f*(a
+ a*Sin[e + f*x])^2)
```

Rubi [A] time = 0.135655, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2736, 2680, 2682, 8}

$$-\frac{3c^2\cos(e+fx)}{af}-\frac{2ac^2\cos^3(e+fx)}{f(a\sin(e+fx)+a)^2}-\frac{3c^2x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sin[e + f*x])^2/(a + a*Sin[e + f*x]),x]
```

```
[Out] (-3*c^2*x)/a - (3*c^2*Cos[e + f*x])/(a*f) - (2*a*c^2*Cos[e + f*x]^3)/(f*(a
+ a*Sin[e + f*x])^2)
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{(c - c\sin(e + fx))^2}{a + a\sin(e + fx)}\,dx = \left(a^2c^2\right)\int \frac{\cos^4(e + fx)}{(a + a\sin(e + fx))^3}\,dx$$
$$= -\frac{2ac^2\cos^3(e + fx)}{f(a + a\sin(e + fx))^2} - \left(3c^2\right)\int \frac{\cos^2(e + fx)}{a + a\sin(e + fx)}\,dx$$
$$= -\frac{3c^2\cos(e + fx)}{af} - \frac{2ac^2\cos^3(e + fx)}{f(a + a\sin(e + fx))^2} - \frac{\left(3c^2\right)\int 1\,dx}{a}$$
$$= -\frac{3c^2x}{a} - \frac{3c^2\cos(e + fx)}{af} - \frac{2ac^2\cos^3(e + fx)}{f(a + a\sin(e + fx))^2}$$

Mathematica [B] time = 0.366895, size = 129, normalized size = 2.3

$$-\frac{c^2(\sin(e + fx) - 1)^2\left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right)\right)\left(\cos\left(\frac{1}{2}(e + fx)\right)(3(e + fx) + \cos(e + fx)) + \sin\left(\frac{1}{2}(e + fx)\right.\right.}{af(\sin(e + fx) + 1)\left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right)\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sin[e + f*x])^2/(a + a*Sin[e + f*x]),x]
```

```
[Out] -((c^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(Cos[(e + f*x)/2]*(3*(e + f*x)
 + Cos[e + f*x]) + (-8 + 3*e + 3*f*x + Cos[e + f*x])*Sin[(e + f*x)/2])*(-1
+ Sin[e + f*x])^2)/(a*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*(1 + Sin[e
+ f*x])))
```

Maple [A] time = 0.071, size = 73, normalized size = 1.3

$$-2\,\frac{c^2}{af\left(1 + \left(\tan\left(1/2\,fx + e/2\right)\right)^2\right)} - 6\,\frac{c^2\arctan\left(\tan\left(1/2\,fx + e/2\right)\right)}{af} - 8\,\frac{c^2}{af\left(\tan\left(1/2\,fx + e/2\right) + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))^2/(a+a*sin(f*x+e)),x)
```

```
[Out] -2/f*c^2/a/(1+tan(1/2*f*x+1/2*e)^2)-6/f*c^2/a*arctan(tan(1/2*f*x+1/2*e))-8/
f*c^2/a/(tan(1/2*f*x+1/2*e)+1)
```

Maxima [B] time = 2.14845, size = 284, normalized size = 5.07

$$-\frac{2\left(c^2\left(\frac{\frac{\sin(fx+e)}{\cos(fx+e)+1} + \frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2} + 2}{a + \frac{a\sin(fx+e)}{\cos(fx+e)+1} + \frac{a\sin(fx+e)^2}{(\cos(fx+e)+1)^2} + \frac{a\sin(fx+e)^3}{(\cos(fx+e)+1)^3}} + \frac{\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a}\right) + 2\,c^2\left(\frac{\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a} + \frac{1}{a + \frac{a\sin(fx+e)}{\cos(fx+e)+1}}\right) + \frac{c^2}{a + \frac{a\sin(fx+e}{\cos(fx+e)+}}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^2/(a+a*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] -2*(c^2*((sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^2/(cos(f*x + e) +
1)^2 + 2)/(a + a*sin(f*x + e)/(cos(f*x + e) + 1) + a*sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 + a*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + arctan(sin(f*x + e
)/(cos(f*x + e) + 1))/a) + 2*c^2*(arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a
 + 1/(a + a*sin(f*x + e)/(cos(f*x + e) + 1))) + c^2/(a + a*sin(f*x + e)/(co
s(f*x + e) + 1)))/f
```

Fricas [A] time = 1.36464, size = 238, normalized size = 4.25

$$-\frac{3\,c^2fx+c^2\cos\left(fx+e\right)^2+4\,c^2+\left(3\,c^2fx+5\,c^2\right)\cos\left(fx+e\right)+\left(3\,c^2fx+c^2\cos\left(fx+e\right)-4\,c^2\right)\sin\left(fx+e\right)}{af\cos\left(fx+e\right)+af\sin\left(fx+e\right)+af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^2/(a+a*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] -(3*c^2*f*x + c^2*cos(f*x + e)^2 + 4*c^2 + (3*c^2*f*x + 5*c^2)*cos(f*x + e)
 + (3*c^2*f*x + c^2*cos(f*x + e) - 4*c^2)*sin(f*x + e))/(a*f*cos(f*x + e) +
 a*f*sin(f*x + e) + a*f)
```

Sympy [A] time = 7.78863, size = 456, normalized size = 8.14

$$\begin{cases}-\frac{3c^2fx\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)}{af\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+af\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+af\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+af}-\frac{3c^2fx\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{af\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+af\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+af\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+af}-\frac{3c^2fx\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{af\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+af\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+\cdots} \\ \frac{x(-c\sin(e)+c)^2}{a\sin(e)+a}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))**2/(a+a*sin(f*x+e)),x)
```

```
[Out] Piecewise((-3*c**2*f*x*tan(e/2 + f*x/2)**3/(a*f*tan(e/2 + f*x/2)**3 + a*f*t
an(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) - 3*c**2*f*x*tan(e/2 + f*x
/2)**2/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f
*x/2) + a*f) - 3*c**2*f*x*tan(e/2 + f*x/2)/(a*f*tan(e/2 + f*x/2)**3 + a*f*t
an(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) - 3*c**2*f*x/(a*f*tan(e/2
+ f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) + 2*c**
2*tan(e/2 + f*x/2)**3/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 +
a*f*tan(e/2 + f*x/2) + a*f) - 6*c**2*tan(e/2 + f*x/2)**2/(a*f*tan(e/2 + f*x
/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) - 8*c**2/(a*
f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*
f), Ne(f, 0)), (x*(-c*sin(e) + c)**2/(a*sin(e) + a), True))
```

Giac [A] time = 1.93332, size = 135, normalized size = 2.41

$$-\frac{\frac{3\left(fx+e\right)c^2}{a}+\frac{2\left(4\,c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+5\,c^2\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)a}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^2/(a+a*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] -(3*(f*x + e)*c^2/a + 2*(4*c^2*tan(1/2*f*x + 1/2*e)^2 + c^2*tan(1/2*f*x + 1
/2*e) + 5*c^2)/((tan(1/2*f*x + 1/2*e)^3 + tan(1/2*f*x + 1/2*e)^2 + tan(1/2*
f*x + 1/2*e) + 1)*a))/f
```

3.264 $\int \frac{c-c\sin(e+fx)}{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=32

$$-\frac{2c\cos(e+fx)}{f(a\sin(e+fx)+a)}-\frac{cx}{a}$$

```
[Out] -((c*x)/a) - (2*c*Cos[e + f*x])/(f*(a + a*Sin[e + f*x]))
```

Rubi [A] time = 0.0439464, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2735, 2648}

$$-\frac{2c\cos(e+fx)}{f(a\sin(e+fx)+a)}-\frac{cx}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sin[e + f*x])/(a + a*Sin[e + f*x]),x]
```

```
[Out] -((c*x)/a) - (2*c*Cos[e + f*x])/(f*(a + a*Sin[e + f*x]))
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{c-c\sin(e+fx)}{a+a\sin(e+fx)}\,dx &= -\frac{cx}{a}+(2c)\int \frac{1}{a+a\sin(e+fx)}\,dx\\ &= -\frac{cx}{a}-\frac{2c\cos(e+fx)}{f(a+a\sin(e+fx))}\end{aligned}$$

Mathematica [B] time = 0.180096, size = 79, normalized size = 2.47

$$-\frac{c\left(fx\sin\left(e+\frac{fx}{2}\right)-4\sin\left(\frac{fx}{2}\right)+fx\cos\left(\frac{fx}{2}\right)\right)}{af\left(\sin\left(\frac{e}{2}\right)+\cos\left(\frac{e}{2}\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sin[e + f*x])/(a + a*Sin[e + f*x]),x]
```

```
[Out] -((c*(f*x*Cos[(f*x)/2] - 4*Sin[(f*x)/2] + f*x*Sin[e + (f*x)/2]))/(a*f*(Cos[
e/2] + Sin[e/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])))
```

Maple [A] time = 0.056, size = 43, normalized size = 1.3

$$-2\,{\frac {c\arctan \left(\tan \left(1/2\,fx+e/2 \right) \right) }{af}}-4\,{\frac {c}{af \left(\tan \left(1/2\,fx+e/2 \right) +1 \right) }}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))/(a+a*sin(f*x+e)),x)
```

```
[Out] -2/f*c/a*arctan(tan(1/2*f*x+1/2*e))-4/f*c/a/(tan(1/2*f*x+1/2*e)+1)
```

Maxima [B] time = 1.88392, size = 104, normalized size = 3.25

$$-\frac{2\left(c\left(\frac{\arctan\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}\right)}{a}+\frac{1}{a+\frac{a\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}}\right)+\frac{c}{a+\frac{a\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))/(a+a*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] -2*(c*(arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a + 1/(a + a*sin(f*x + e)/(c
os(f*x + e) + 1))) + c/(a + a*sin(f*x + e)/(cos(f*x + e) + 1)))/f
```

Fricas [A] time = 1.31992, size = 159, normalized size = 4.97

$$-\frac{cfx + \left(cfx + 2\,c\right)\cos\left(fx + e\right) + \left(cfx - 2\,c\right)\sin\left(fx + e\right) + 2\,c}{af\cos\left(fx + e\right) + af\sin\left(fx + e\right) + af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))/(a+a*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] -(c*f*x + (c*f*x + 2*c)*cos(f*x + e) + (c*f*x - 2*c)*sin(f*x + e) + 2*c)/(a
*f*cos(f*x + e) + a*f*sin(f*x + e) + a*f)
```

Sympy [A] time = 3.36027, size = 90, normalized size = 2.81

$$\begin{cases} -\frac{cfx\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{af\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+af} - \frac{cfx}{af\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+af} - \frac{4c}{af\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+af} & \text{for}\: f \neq 0 \\ \frac{x\left(-c\sin\left(e\right)+c\right)}{a\sin\left(e\right)+a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))/(a+a*sin(f*x+e)),x)
```

```
[Out] Piecewise((-c*f*x*tan(e/2 + f*x/2)/(a*f*tan(e/2 + f*x/2) + a*f) - c*f*x/(a*
f*tan(e/2 + f*x/2) + a*f) - 4*c/(a*f*tan(e/2 + f*x/2) + a*f), Ne(f, 0)), (x
*(-c*sin(e) + c)/(a*sin(e) + a), True))
```

Giac [A] time = 2.05113, size = 50, normalized size = 1.56

$$-\frac{\frac{(fx+e)c}{a}+\frac{4c}{a\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))/(a+a*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] -((f*x + e)*c/a + 4*c/(a*(tan(1/2*f*x + 1/2*e) + 1)))/f
```

3.265 $\int \frac{1}{(a+a\sin(e+fx))(c-c\sin(e+fx))}\,dx$

Optimal. Leaf size=16

$$\frac{\tan(e+fx)}{acf}$$

```
[Out] Tan[e + f*x]/(a*c*f)
```

Rubi [A] time = 0.0674709, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2736, 3767, 8}

$$\frac{\tan(e+fx)}{acf}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])),x]
```

```
[Out] Tan[e + f*x]/(a*c*f)
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+a\sin(e+fx))(c-c\sin(e+fx))}\,dx &= \frac{\int \sec^2(e+fx)\,dx}{ac}\\
&= -\frac{\text{Subst}\left(\int 1\,dx, x, -\tan(e+fx)\right)}{acf}\\
&= \frac{\tan(e+fx)}{acf}
\end{aligned}$$

Mathematica [A] time = 0.0123707, size = 16, normalized size = 1.

$$\frac{\tan(e+fx)}{acf}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])),x]
```

```
[Out] Tan[e + f*x]/(a*c*f)
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a + a\sin\left(fx + e\right)\right)\left(c - c\sin\left(fx + e\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))/(c-c*sin(f*x+e)),x)
```

```
[Out] int(1/(a+a*sin(f*x+e))/(c-c*sin(f*x+e)),x)
```

Maxima [A] time = 1.75087, size = 22, normalized size = 1.38

$$\frac{\tan\left(fx + e\right)}{acf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] tan(f*x + e)/(a*c*f)
```

Fricas [A] time = 1.27716, size = 47, normalized size = 2.94

$$\frac{\sin\left(fx + e\right)}{acf\cos\left(fx + e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] sin(f*x + e)/(a*c*f*cos(f*x + e))
```

Sympy [A] time = 2.81456, size = 49, normalized size = 3.06

$$\begin{cases} -\frac{2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{acf\tan^{2}\left(\frac{e}{2}+\frac{fx}{2}\right)-acf} & \text{for}\: f \neq 0 \\ \frac{x}{\left(a\sin\left(e\right)+a\right)\left(-c\sin\left(e\right)+c\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c-c*sin(f*x+e)),x)
```

```
[Out] Piecewise((-2*tan(e/2 + f*x/2)/(a*c*f*tan(e/2 + f*x/2)**2 - a*c*f), Ne(f, 0
)), (x/((a*sin(e) + a)*(-c*sin(e) + c)), True))
```

Giac [A] time = 2.03161, size = 23, normalized size = 1.44

$$\frac{\tan\left(fx+e\right)}{acf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] tan(f*x + e)/(a*c*f)
```

3.266 $\int \frac{1}{(a+a\sin(e+fx))(c-c\sin(e+fx))^2}\,dx$

Optimal. Leaf size=53

$$\frac{2\tan(e+fx)}{3ac^2f}+\frac{\sec(e+fx)}{3af\left(c^2-c^2\sin(e+fx)\right)}$$

```
[Out] Sec[e + f*x]/(3*a*f*(c^2 - c^2*Sin[e + f*x])) + (2*Tan[e + f*x])/(3*a*c^2*f
)
```

Rubi [A] time = 0.11045, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2736, 2672, 3767, 8}

$$\frac{2\tan(e+fx)}{3ac^2f}+\frac{\sec(e+fx)}{3af\left(c^2-c^2\sin(e+fx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^2),x]
```

```
[Out] Sec[e + f*x]/(3*a*f*(c^2 - c^2*Sin[e + f*x])) + (2*Tan[e + f*x])/(3*a*c^2*f
)
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))(c-c\sin(e+fx))^2}\,dx = \frac{\int \frac{\sec^2(e+fx)}{c-c\sin(e+fx)}\,dx}{ac}$$
$$= \frac{\sec(e+fx)}{3af\left(c^2-c^2\sin(e+fx)\right)} + \frac{2\int \sec^2(e+fx)\,dx}{3ac^2}$$
$$= \frac{\sec(e+fx)}{3af\left(c^2-c^2\sin(e+fx)\right)} - \frac{2\,\mathrm{Subst}(\int 1\,dx, x, -\tan(e+fx))}{3ac^2f}$$
$$= \frac{\sec(e+fx)}{3af\left(c^2-c^2\sin(e+fx)\right)} + \frac{2\tan(e+fx)}{3ac^2f}$$

Mathematica [A] time = 0.425996, size = 87, normalized size = 1.64

$$\frac{\sin(e+fx)+8\sin(2(e+fx))+\sin(3(e+fx))+4\cos(e+fx)-2\cos(2(e+fx))+4\cos(3(e+fx))-2}{24ac^2f(\sin(e+fx)-1)^2(\sin(e+fx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^2),x]
```

```
[Out] (-2 + 4*Cos[e + f*x] - 2*Cos[2*(e + f*x)] + 4*Cos[3*(e + f*x)] + Sin[e + f*
x] + 8*Sin[2*(e + f*x)] + Sin[3*(e + f*x)])/(24*a*c^2*f*(-1 + Sin[e + f*x])
^2*(1 + Sin[e + f*x]))
```

Maple [A] time = 0.049, size = 73, normalized size = 1.4

$$2\,\frac{1}{afc^2}\left(-1/3\,\left(\tan\left(1/2\,fx+e/2\right)-1\right)^{-3}-1/2\,\left(\tan\left(1/2\,fx+e/2\right)-1\right)^{-2}-3/4\,\left(\tan\left(1/2\,fx+e/2\right)-1\right)^{-1}-1/4\,\left(\tan\left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^2,x)
```

```
[Out] 2/f/a/c^2*(-1/3/(tan(1/2*f*x+1/2*e)-1)^3-1/2/(tan(1/2*f*x+1/2*e)-1)^2-3/4/(
tan(1/2*f*x+1/2*e)-1)-1/4/(tan(1/2*f*x+1/2*e)+1))
```

Maxima [B] time = 1.41166, size = 192, normalized size = 3.62

$$\frac{2\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-\frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+1\right)}{3\left(ac^2-\frac{2ac^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{2ac^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}-\frac{ac^2\sin(fx+e)^4}{(\cos(fx+e)+1)^4}\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 2/3*(sin(f*x + e)/(cos(f*x + e) + 1) - 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^
2 + 3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 1)/((a*c^2 - 2*a*c^2*sin(f*x +
e)/(cos(f*x + e) + 1) + 2*a*c^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - a*c^2
```

```
*sin(f*x + e)^4/(cos(f*x + e) + 1)^4)*f)
```

Fricas [A] time = 1.24241, size = 142, normalized size = 2.68

$$-\frac{2\cos\left(fx+e\right)^{2}+2\sin\left(fx+e\right)-1}{3\left(ac^{2}f\cos\left(fx+e\right)\sin\left(fx+e\right)-ac^{2}f\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -1/3*(2*cos(f*x + e)^2 + 2*sin(f*x + e) - 1)/(a*c^2*f*cos(f*x + e)*sin(f*x
+ e) - a*c^2*f*cos(f*x + e))
```

Sympy [A] time = 8.5386, size = 328, normalized size = 6.19

$$\begin{cases} \frac{\tan^{4}\left(\frac{e}{2}+\frac{fx}{2}\right)}{3ac^{2}f\tan^{4}\left(\frac{e}{2}+\frac{fx}{2}\right)-6ac^{2}f\tan^{3}\left(\frac{e}{2}+\frac{fx}{2}\right)+6ac^{2}f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-3ac^{2}f}-\frac{8\tan^{3}\left(\frac{e}{2}+\frac{fx}{2}\right)}{3ac^{2}f\tan^{4}\left(\frac{e}{2}+\frac{fx}{2}\right)-6ac^{2}f\tan^{3}\left(\frac{e}{2}+\frac{fx}{2}\right)+6ac^{2}f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-3ac^{2}f}+\frac{\quad}{3ac^{2}f\tan^{4}(\quad} \\ \frac{x}{(a\sin(e)+a)(-c\sin(e)+c)^{2}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((tan(e/2 + f*x/2)**4/(3*a*c**2*f*tan(e/2 + f*x/2)**4 - 6*a*c**2*f
*tan(e/2 + f*x/2)**3 + 6*a*c**2*f*tan(e/2 + f*x/2) - 3*a*c**2*f) - 8*tan(e/
2 + f*x/2)**3/(3*a*c**2*f*tan(e/2 + f*x/2)**4 - 6*a*c**2*f*tan(e/2 + f*x/2)
**3 + 6*a*c**2*f*tan(e/2 + f*x/2) - 3*a*c**2*f) + 6*tan(e/2 + f*x/2)**2/(3*
a*c**2*f*tan(e/2 + f*x/2)**4 - 6*a*c**2*f*tan(e/2 + f*x/2)**3 + 6*a*c**2*f*
tan(e/2 + f*x/2) - 3*a*c**2*f) - 3/(3*a*c**2*f*tan(e/2 + f*x/2)**4 - 6*a*c*
*2*f*tan(e/2 + f*x/2)**3 + 6*a*c**2*f*tan(e/2 + f*x/2) - 3*a*c**2*f), Ne(f,
 0)), (x/((a*sin(e) + a)*(-c*sin(e) + c)**2), True))
```

Giac [A] time = 2.04361, size = 104, normalized size = 1.96

$$-\frac{\frac{3}{ac^{2}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}+\frac{9\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-12\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+7}{ac^{2}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^{3}}}{6f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] -1/6*(3/(a*c^2*(tan(1/2*f*x + 1/2*e) + 1)) + (9*tan(1/2*f*x + 1/2*e)^2 - 12
*tan(1/2*f*x + 1/2*e) + 7)/(a*c^2*(tan(1/2*f*x + 1/2*e) - 1)^3))/f
```

3.267 $\int \frac{1}{(a+a\sin(e+fx))(c-c\sin(e+fx))^3}\,dx$

Optimal. Leaf size=85

$$\frac{2\tan(e+fx)}{5ac^3f}+\frac{\sec(e+fx)}{5af\left(c^3-c^3\sin(e+fx)\right)}+\frac{\sec(e+fx)}{5acf(c-c\sin(e+fx))^2}$$

```
[Out] Sec[e + f*x]/(5*a*c*f*(c - c*Sin[e + f*x])^2) + Sec[e + f*x]/(5*a*f*(c^3 -
c^3*Sin[e + f*x])) + (2*Tan[e + f*x])/(5*a*c^3*f)
```

Rubi [A] time = 0.15787, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2736, 2672, 3767, 8}

$$\frac{2\tan(e+fx)}{5ac^3f}+\frac{\sec(e+fx)}{5af\left(c^3-c^3\sin(e+fx)\right)}+\frac{\sec(e+fx)}{5acf(c-c\sin(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^3),x]
```

```
[Out] Sec[e + f*x]/(5*a*c*f*(c - c*Sin[e + f*x])^2) + Sec[e + f*x]/(5*a*f*(c^3 -
c^3*Sin[e + f*x])) + (2*Tan[e + f*x])/(5*a*c^3*f)
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))(c-c\sin(e+fx))^3}\,dx = \frac{\int \frac{\sec^2(e+fx)}{(c-c\sin(e+fx))^2}\,dx}{ac}$$
$$= \frac{\sec(e+fx)}{5acf(c-c\sin(e+fx))^2} + \frac{3\int \frac{\sec^2(e+fx)}{c-c\sin(e+fx)}\,dx}{5ac^2}$$
$$= \frac{\sec(e+fx)}{5acf(c-c\sin(e+fx))^2} + \frac{\sec(e+fx)}{5af\left(c^3-c^3\sin(e+fx)\right)} + \frac{2\int \sec^2(e+fx)\,dx}{5ac^3}$$
$$= \frac{\sec(e+fx)}{5acf(c-c\sin(e+fx))^2} + \frac{\sec(e+fx)}{5af\left(c^3-c^3\sin(e+fx)\right)} - \frac{2\,\text{Subst}(\int 1\,dx, x, \ldots}{5ac^3 \ldots}$$
$$= \frac{\sec(e+fx)}{5acf(c-c\sin(e+fx))^2} + \frac{\sec(e+fx)}{5af\left(c^3-c^3\sin(e+fx)\right)} + \frac{2\tan(e+fx)}{5ac^3f}$$

Mathematica [A] time = 0.669206, size = 111, normalized size = 1.31

$$-\frac{12\sin(e+fx)+32\sin(2(e+fx))+12\sin(3(e+fx))-8\sin(4(e+fx))+32\cos(e+fx)-12\cos(2(e+fx))+32\cos(\ldots}{160ac^3f(\sin(e+fx)-1)^3(\sin(e+fx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^3),x]
```

```
[Out] -(-15 + 32*Cos[e + f*x] - 12*Cos[2*(e + f*x)] + 32*Cos[3*(e + f*x)] + 3*Cos
[4*(e + f*x)] + 12*Sin[e + f*x] + 32*Sin[2*(e + f*x)] + 12*Sin[3*(e + f*x)]
 - 8*Sin[4*(e + f*x)])/(160*a*c^3*f*(-1 + Sin[e + f*x])^3*(1 + Sin[e + f*x]
))
```

Maple [A] time = 0.063, size = 103, normalized size = 1.2

$$2\,\frac{1}{afc^3}\left(-2/5\,\left(\tan\left(1/2\,fx+e/2\right)-1\right)^{-5}-\left(\tan\left(1/2\,fx+e/2\right)-1\right)^{-4}-3/2\,\left(\tan\left(1/2\,fx+e/2\right)-1\right)^{-3}-5/4\,\left(\tan\left(1/2\,fx \ldots\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^3,x)
```

```
[Out] 2/f/a/c^3*(-2/5/(tan(1/2*f*x+1/2*e)-1)^5-1/(tan(1/2*f*x+1/2*e)-1)^4-3/2/(ta
n(1/2*f*x+1/2*e)-1)^3-5/4/(tan(1/2*f*x+1/2*e)-1)^2-7/8/(tan(1/2*f*x+1/2*e)-
1)-1/8/(tan(1/2*f*x+1/2*e)+1))
```

Maxima [B] time = 1.46496, size = 285, normalized size = 3.35

$$-\frac{2\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}-\frac{10\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{10\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{5\sin(fx+e)^5}{(\cos(fx+e)+1)^5}-2\right)}{5\left(ac^3-\frac{4ac^3\sin(fx+e)}{\cos(fx+e)+1}+\frac{5ac^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{5ac^3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{4ac^3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}-\frac{ac^3\sin(fx+e)^6}{(\cos(fx+e)+1)^6}\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] -2/5*(3*sin(f*x + e)/(cos(f*x + e) + 1) - 10*sin(f*x + e)^3/(cos(f*x + e) +
 1)^3 + 10*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 5*sin(f*x + e)^5/(cos(f*x
+ e) + 1)^5 - 2)/((a*c^3 - 4*a*c^3*sin(f*x + e)/(cos(f*x + e) + 1) + 5*a*c^
3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 5*a*c^3*sin(f*x + e)^4/(cos(f*x + e
) + 1)^4 + 4*a*c^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - a*c^3*sin(f*x + e)
^6/(cos(f*x + e) + 1)^6)*f)
```

Fricas [A] time = 1.30729, size = 209, normalized size = 2.46

$$-\frac{4 \cos\left(fx+e\right)^{2}-\left(2 \cos\left(fx+e\right)^{2}-3\right) \sin\left(fx+e\right)-2}{5 \left(ac^{3} f \cos\left(fx+e\right)^{3}+2 \, ac^{3} f \cos\left(fx+e\right) \sin\left(fx+e\right)-2 \, ac^{3} f \cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -1/5*(4*cos(f*x + e)^2 - (2*cos(f*x + e)^2 - 3)*sin(f*x + e) - 2)/(a*c^3*f*
cos(f*x + e)^3 + 2*a*c^3*f*cos(f*x + e)*sin(f*x + e) - 2*a*c^3*f*cos(f*x +
e))
```

Sympy [A] time = 17.9411, size = 738, normalized size = 8.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((-3*tan(e/2 + f*x/2)**6/(10*a*c**3*f*tan(e/2 + f*x/2)**6 - 40*a*c
**3*f*tan(e/2 + f*x/2)**5 + 50*a*c**3*f*tan(e/2 + f*x/2)**4 - 50*a*c**3*f*t
an(e/2 + f*x/2)**2 + 40*a*c**3*f*tan(e/2 + f*x/2) - 10*a*c**3*f) - 8*tan(e/
2 + f*x/2)**5/(10*a*c**3*f*tan(e/2 + f*x/2)**6 - 40*a*c**3*f*tan(e/2 + f*x/
2)**5 + 50*a*c**3*f*tan(e/2 + f*x/2)**4 - 50*a*c**3*f*tan(e/2 + f*x/2)**2 +
 40*a*c**3*f*tan(e/2 + f*x/2) - 10*a*c**3*f) + 25*tan(e/2 + f*x/2)**4/(10*a
*c**3*f*tan(e/2 + f*x/2)**6 - 40*a*c**3*f*tan(e/2 + f*x/2)**5 + 50*a*c**3*f
*tan(e/2 + f*x/2)**4 - 50*a*c**3*f*tan(e/2 + f*x/2)**2 + 40*a*c**3*f*tan(e/
2 + f*x/2) - 10*a*c**3*f) - 40*tan(e/2 + f*x/2)**3/(10*a*c**3*f*tan(e/2 + f
*x/2)**6 - 40*a*c**3*f*tan(e/2 + f*x/2)**5 + 50*a*c**3*f*tan(e/2 + f*x/2)**
4 - 50*a*c**3*f*tan(e/2 + f*x/2)**2 + 40*a*c**3*f*tan(e/2 + f*x/2) - 10*a*c
**3*f) + 15*tan(e/2 + f*x/2)**2/(10*a*c**3*f*tan(e/2 + f*x/2)**6 - 40*a*c**
3*f*tan(e/2 + f*x/2)**5 + 50*a*c**3*f*tan(e/2 + f*x/2)**4 - 50*a*c**3*f*tan
(e/2 + f*x/2)**2 + 40*a*c**3*f*tan(e/2 + f*x/2) - 10*a*c**3*f) - 5/(10*a*c*
*3*f*tan(e/2 + f*x/2)**6 - 40*a*c**3*f*tan(e/2 + f*x/2)**5 + 50*a*c**3*f*ta
n(e/2 + f*x/2)**4 - 50*a*c**3*f*tan(e/2 + f*x/2)**2 + 40*a*c**3*f*tan(e/2 +
 f*x/2) - 10*a*c**3*f), Ne(f, 0)), (x/((a*sin(e) + a)*(-c*sin(e) + c)**3),
True))
```

Giac [A] time = 2.01864, size = 142, normalized size = 1.67

$$-\frac{\frac{5}{ac^3\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}+\frac{35\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-90\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+120\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-70\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+21}{ac^3\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^5}}{20f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -1/20*(5/(a*c^3*(tan(1/2*f*x + 1/2*e) + 1)) + (35*tan(1/2*f*x + 1/2*e)^4 -
90*tan(1/2*f*x + 1/2*e)^3 + 120*tan(1/2*f*x + 1/2*e)^2 - 70*tan(1/2*f*x + 1
/2*e) + 21)/(a*c^3*(tan(1/2*f*x + 1/2*e) - 1)^5))/f
```

3.268 $\int \frac{1}{(a+a\sin(e+fx))(c-c\sin(e+fx))^4}\,dx$

Optimal. Leaf size=118

$$\frac{8\tan(e+fx)}{35ac^4f}+\frac{4\sec(e+fx)}{35af\left(c^4-c^4\sin(e+fx)\right)}+\frac{4\sec(e+fx)}{35af\left(c^2-c^2\sin(e+fx)\right)^2}+\frac{\sec(e+fx)}{7acf(c-c\sin(e+fx))^3}$$

```
[Out] Sec[e + f*x]/(7*a*c*f*(c - c*Sin[e + f*x])^3) + (4*Sec[e + f*x])/(35*a*f*(c
^2 - c^2*Sin[e + f*x])^2) + (4*Sec[e + f*x])/(35*a*f*(c^4 - c^4*Sin[e + f*x
])) + (8*Tan[e + f*x])/(35*a*c^4*f)
```

Rubi [A] time = 0.206891, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2736, 2672, 3767, 8}

$$\frac{8\tan(e+fx)}{35ac^4f}+\frac{4\sec(e+fx)}{35af\left(c^4-c^4\sin(e+fx)\right)}+\frac{4\sec(e+fx)}{35af\left(c^2-c^2\sin(e+fx)\right)^2}+\frac{\sec(e+fx)}{7acf(c-c\sin(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^4),x]
```

```
[Out] Sec[e + f*x]/(7*a*c*f*(c - c*Sin[e + f*x])^3) + (4*Sec[e + f*x])/(35*a*f*(c
^2 - c^2*Sin[e + f*x])^2) + (4*Sec[e + f*x])/(35*a*f*(c^4 - c^4*Sin[e + f*x
])) + (8*Tan[e + f*x])/(35*a*c^4*f)
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))(c-c\sin(e+fx))^4}\,dx = \frac{\int \frac{\sec^2(e+fx)}{(c-c\sin(e+fx))^3}\,dx}{ac}$$
$$= \frac{\sec(e+fx)}{7acf(c-c\sin(e+fx))^3} + \frac{4\int \frac{\sec^2(e+fx)}{(c-c\sin(e+fx))^2}\,dx}{7ac^2}$$
$$= \frac{\sec(e+fx)}{7acf(c-c\sin(e+fx))^3} + \frac{4\sec(e+fx)}{35af\left(c^2-c^2\sin(e+fx)\right)^2} + \frac{12\int \frac{\sec^2(e+fx)}{c-c\sin(e+fx)}}{35ac^3}$$
$$= \frac{\sec(e+fx)}{7acf(c-c\sin(e+fx))^3} + \frac{4\sec(e+fx)}{35af\left(c^2-c^2\sin(e+fx)\right)^2} + \frac{4\sec(e+}{35af\left(c^4-c^4\sin\right.}$$
$$= \frac{\sec(e+fx)}{7acf(c-c\sin(e+fx))^3} + \frac{4\sec(e+fx)}{35af\left(c^2-c^2\sin(e+fx)\right)^2} + \frac{4\sec(e+}{35af\left(c^4-c^4\sin\right.}$$
$$= \frac{\sec(e+fx)}{7acf(c-c\sin(e+fx))^3} + \frac{4\sec(e+fx)}{35af\left(c^2-c^2\sin(e+fx)\right)^2} + \frac{4\sec(e+}{35af\left(c^4-c^4\sin\right.}$$

Mathematica [A] time = 0.740573, size = 131, normalized size = 1.11

$$\frac{406\sin(e+fx)+512\sin(2(e+fx))+377\sin(3(e+fx))-384\sin(4(e+fx))-29\sin(5(e+fx))+896\cos(e+fx)-2}{4480ac^4f(\sin(e+fx)-1)^4(\sin(e+fx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^4),x]
```

```
[Out] (-406 + 896*Cos[e + f*x] - 232*Cos[2*(e + f*x)] + 832*Cos[3*(e + f*x)] + 17
4*Cos[4*(e + f*x)] - 64*Cos[5*(e + f*x)] + 406*Sin[e + f*x] + 512*Sin[2*(e
+ f*x)] + 377*Sin[3*(e + f*x)] - 384*Sin[4*(e + f*x)] - 29*Sin[5*(e + f*x)]
)/(4480*a*c^4*f*(-1 + Sin[e + f*x])^4*(1 + Sin[e + f*x]))
```

Maple [A] time = 0.059, size = 133, normalized size = 1.1

$$2\,\frac{1}{afc^4}\left(-4/7\,\left(\tan\left(1/2\,fx+e/2\right)-1\right)^{-7}-2\,\left(\tan\left(1/2\,fx+e/2\right)-1\right)^{-6}-\frac{19}{5\,\left(\tan\left(1/2\,fx+e/2\right)-1\right)^5}-9/2\,\left(\tan\left(1/2\,fx+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^4,x)
```

```
[Out] 2/f/a/c^4*(-4/7/(tan(1/2*f*x+1/2*e)-1)^7-2/(tan(1/2*f*x+1/2*e)-1)^6-19/5/(t
an(1/2*f*x+1/2*e)-1)^5-9/2/(tan(1/2*f*x+1/2*e)-1)^4-15/4/(tan(1/2*f*x+1/2*e
)-1)^3-17/8/(tan(1/2*f*x+1/2*e)-1)^2-15/16/(tan(1/2*f*x+1/2*e)-1)-1/16/(tan
(1/2*f*x+1/2*e)+1))
```

Maxima [B] time = 1.56919, size = 431, normalized size = 3.65

$$-\frac{2\left(\frac{43\sin(fx+e)}{\cos(fx+e)+1}-\frac{77\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{7\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{105\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{175\sin(fx+e)^5}{(\cos(fx+e)+1)^5}+\frac{105\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-\frac{35\sin(fx+e)^7}{(\cos(fx+e)+1)^7}-13\right)}{35\left(ac^4-\frac{6ac^4\sin(fx+e)}{\cos(fx+e)+1}+\frac{14ac^4\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{14ac^4\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{14ac^4\sin(fx+e)^5}{(\cos(fx+e)+1)^5}-\frac{14ac^4\sin(fx+e)^6}{(\cos(fx+e)+1)^6}+\frac{6ac^4\sin(fx+e)^7}{(\cos(fx+e)+1)^7}-\frac{ac^4\sin(fx+e)^8}{(\cos(fx+e)+1)}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^4,x, algorithm="maxima")
```

```
[Out] -2/35*(43*sin(f*x + e)/(cos(f*x + e) + 1) - 77*sin(f*x + e)^2/(cos(f*x + e)
 + 1)^2 + 7*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 105*sin(f*x + e)^4/(cos(f
*x + e) + 1)^4 - 175*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 105*sin(f*x + e)
^6/(cos(f*x + e) + 1)^6 - 35*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 13)/((a*
c^4 - 6*a*c^4*sin(f*x + e)/(cos(f*x + e) + 1) + 14*a*c^4*sin(f*x + e)^2/(co
s(f*x + e) + 1)^2 - 14*a*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 14*a*c^4
*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 14*a*c^4*sin(f*x + e)^6/(cos(f*x + e
) + 1)^6 + 6*a*c^4*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - a*c^4*sin(f*x + e)
^8/(cos(f*x + e) + 1)^8)*f)
```

Fricas [A] time = 1.3308, size = 279, normalized size = 2.36

$$\frac{8\cos\left(fx+e\right)^4-36\cos\left(fx+e\right)^2+4\left(6\cos\left(fx+e\right)^2-5\right)\sin\left(fx+e\right)+15}{35\left(3\,ac^4f\cos\left(fx+e\right)^3-4\,ac^4f\cos\left(fx+e\right)-\left(ac^4f\cos\left(fx+e\right)^3-4\,ac^4f\cos\left(fx+e\right)\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^4,x, algorithm="fricas")
```

```
[Out] 1/35*(8*cos(f*x + e)^4 - 36*cos(f*x + e)^2 + 4*(6*cos(f*x + e)^2 - 5)*sin(f
*x + e) + 15)/(3*a*c^4*f*cos(f*x + e)^3 - 4*a*c^4*f*cos(f*x + e) - (a*c^4*f
*cos(f*x + e)^3 - 4*a*c^4*f*cos(f*x + e))*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 2.17976, size = 180, normalized size = 1.53

$$-\frac{\frac{35}{ac^4\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}+\frac{525\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^6-1960\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5+4025\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-4480\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+3143\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-1176\tan}{ac^4\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^7}}{280\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^4,x, algorithm="giac")
```

```
[Out] -1/280*(35/(a*c^4*(tan(1/2*f*x + 1/2*e) + 1)) + (525*tan(1/2*f*x + 1/2*e)^6
 - 1960*tan(1/2*f*x + 1/2*e)^5 + 4025*tan(1/2*f*x + 1/2*e)^4 - 4480*tan(1/2
```

```
*f*x + 1/2*e)^3 + 3143*tan(1/2*f*x + 1/2*e)^2 - 1176*tan(1/2*f*x + 1/2*e) +
 243)/(a*c^4*(tan(1/2*f*x + 1/2*e) - 1)^7))/f
```

3.269 $\int \frac{(c-c\sin(e+fx))^5}{(a+a\sin(e+fx))^2}\,dx$

Optimal. Leaf size=148

$$\frac{35c^5\cos^3(e+fx)}{a^2f}-\frac{2a^4c^5\cos^9(e+fx)}{3f(a\sin(e+fx)+a)^6}+\frac{6a^2c^5\cos^7(e+fx)}{f(a\sin(e+fx)+a)^4}+\frac{105c^5\sin(e+fx)\cos(e+fx)}{2a^2f}+\frac{105c^5x}{2a^2}+\frac{42}{f(a}$$

```
[Out] (105*c^5*x)/(2*a^2) + (35*c^5*Cos[e + f*x]^3)/(a^2*f) + (105*c^5*Cos[e + f*
x]*Sin[e + f*x])/(2*a^2*f) - (2*a^4*c^5*Cos[e + f*x]^9)/(3*f*(a + a*Sin[e +
 f*x])^6) + (6*a^2*c^5*Cos[e + f*x]^7)/(f*(a + a*Sin[e + f*x])^4) + (42*c^5
*Cos[e + f*x]^5)/(f*(a + a*Sin[e + f*x])^2)
```

Rubi [A] time = 0.239855, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {2736, 2680, 2682, 2635, 8}

$$\frac{35c^5\cos^3(e+fx)}{a^2f}-\frac{2a^4c^5\cos^9(e+fx)}{3f(a\sin(e+fx)+a)^6}+\frac{6a^2c^5\cos^7(e+fx)}{f(a\sin(e+fx)+a)^4}+\frac{105c^5\sin(e+fx)\cos(e+fx)}{2a^2f}+\frac{105c^5x}{2a^2}+\frac{42}{f(a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sin[e + f*x])^5/(a + a*Sin[e + f*x])^2,x]
```


```
[Out] (105*c^5*x)/(2*a^2) + (35*c^5*Cos[e + f*x]^3)/(a^2*f) + (105*c^5*Cos[e + f*
x]*Sin[e + f*x])/(2*a^2*f) - (2*a^4*c^5*Cos[e + f*x]^9)/(3*f*(a + a*Sin[e +
 f*x])^6) + (6*a^2*c^5*Cos[e + f*x]^7)/(f*(a + a*Sin[e + f*x])^4) + (42*c^5
*Cos[e + f*x]^5)/(f*(a + a*Sin[e + f*x])^2)
```


Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
```

```
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{(c-c\sin(e+fx))^5}{(a+a\sin(e+fx))^2}\,dx = \left(a^5c^5\right)\int \frac{\cos^{10}(e+fx)}{(a+a\sin(e+fx))^7}\,dx$$
$$= -\frac{2a^4c^5\cos^9(e+fx)}{3f(a+a\sin(e+fx))^6} - \left(3a^3c^5\right)\int \frac{\cos^8(e+fx)}{(a+a\sin(e+fx))^5}\,dx$$
$$= -\frac{2a^4c^5\cos^9(e+fx)}{3f(a+a\sin(e+fx))^6} + \frac{6a^2c^5\cos^7(e+fx)}{f(a+a\sin(e+fx))^4} + \left(21ac^5\right)\int \frac{\cos^6(e+fx)}{(a+a\sin(e+fx))^3}\,dx$$
$$= -\frac{2a^4c^5\cos^9(e+fx)}{3f(a+a\sin(e+fx))^6} + \frac{6a^2c^5\cos^7(e+fx)}{f(a+a\sin(e+fx))^4} + \frac{42c^5\cos^5(e+fx)}{f(a+a\sin(e+fx))^2} + \frac{\left(105c^5\right)\int \frac{\cos^4(\cdots}{a+a\sin\cdots}}{a}$$
$$= \frac{35c^5\cos^3(e+fx)}{a^2f} - \frac{2a^4c^5\cos^9(e+fx)}{3f(a+a\sin(e+fx))^6} + \frac{6a^2c^5\cos^7(e+fx)}{f(a+a\sin(e+fx))^4} + \frac{42c^5\cos^5(e+fx)}{f(a+a\sin(e+fx))^2}$$
$$= \frac{35c^5\cos^3(e+fx)}{a^2f} + \frac{105c^5\cos(e+fx)\sin(e+fx)}{2a^2f} - \frac{2a^4c^5\cos^9(e+fx)}{3f(a+a\sin(e+fx))^6} + \frac{6a^2c^5\cos^7(\cdots}{f(a+a\sin(\cdots}$$
$$= \frac{105c^5x}{2a^2} + \frac{35c^5\cos^3(e+fx)}{a^2f} + \frac{105c^5\cos(e+fx)\sin(e+fx)}{2a^2f} - \frac{2a^4c^5\cos^9(e+fx)}{3f(a+a\sin(e+fx))^6} + \frac{6\cdots}{f(\cdots}$$

Mathematica [A] time = 0.726095, size = 276, normalized size = 1.86

$$(c-c\sin(e+fx))^5\left(\sin\left(\tfrac{1}{2}(e+fx)\right)+\cos\left(\tfrac{1}{2}(e+fx)\right)\right)\left(256\sin\left(\tfrac{1}{2}(e+fx)\right)+630(e+fx)\left(\sin\left(\tfrac{1}{2}(e+fx)\right)+\cos\left(\tfrac{1}{2}(e+\cdots\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sin[e + f*x])^5/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(c - c*Sin[e + f*x])^5*(256*Sin[(e +
 f*x)/2] - 128*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) - 1664*Sin[(e + f*x)/2
]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + 630*(e + f*x)*(Cos[(e + f*x)/2]
 + Sin[(e + f*x)/2])^3 + 285*Cos[e + f*x]*(Cos[(e + f*x)/2] + Sin[(e + f*x)
/2])^3 - Cos[3*(e + f*x)]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 - 21*(Cos
[(e + f*x)/2] + Sin[(e + f*x)/2])^3*Sin[2*(e + f*x)]))/(12*f*(Cos[(e + f*x)
/2] - Sin[(e + f*x)/2])^10*(a + a*Sin[e + f*x])^2)
```

Maple [A] time = 0.108, size = 267, normalized size = 1.8

$$7\,\frac{c^5\left(\tan\left(1/2\,fx+e/2\right)\right)^5}{a^2f\left(1+\left(\tan\left(1/2\,fx+e/2\right)\right)^2\right)^3}+46\,\frac{c^5\left(\tan\left(1/2\,fx+e/2\right)\right)^4}{a^2f\left(1+\left(\tan\left(1/2\,fx+e/2\right)\right)^2\right)^3}+96\,\frac{c^5\left(\tan\left(1/2\,fx+e/2\right)\right)^2}{a^2f\left(1+\left(\tan\left(1/2\,fx+e/2\right)\right)^2\right)^3}-7\,\frac{\cdots}{a^2f\left(\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))^5/(a+a*sin(f*x+e))^2,x)
```

```
[Out] 7/f*c^5/a^2/(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)^5+46/f*c^5/a^2/(1
+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)^4+96/f*c^5/a^2/(1+tan(1/2*f*x+1
/2*e)^2)^3*tan(1/2*f*x+1/2*e)^2-7/f*c^5/a^2/(1+tan(1/2*f*x+1/2*e)^2)^3*tan(
1/2*f*x+1/2*e)+142/3/f*c^5/a^2/(1+tan(1/2*f*x+1/2*e)^2)^3+105/f*c^5/a^2*arc
tan(tan(1/2*f*x+1/2*e))-128/3/f*c^5/a^2/(tan(1/2*f*x+1/2*e)+1)^3+64/f*c^5/a
^2/(tan(1/2*f*x+1/2*e)+1)^2+96/f*c^5/a^2/(tan(1/2*f*x+1/2*e)+1)
```

Maxima [B] time = 3.30545, size = 1760, normalized size = 11.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^5/(a+a*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/3*(5*c^5*((75*sin(f*x + e)/(cos(f*x + e) + 1) + 97*sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 + 126*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 98*sin(f*x + e)^4
/(cos(f*x + e) + 1)^4 + 63*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 21*sin(f*x
 + e)^6/(cos(f*x + e) + 1)^6 + 32)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e)
+ 1) + 5*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 7*a^2*sin(f*x + e)^3/(co
s(f*x + e) + 1)^3 + 7*a^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 5*a^2*sin(f
*x + e)^5/(cos(f*x + e) + 1)^5 + 3*a^2*sin(f*x + e)^6/(cos(f*x + e) + 1)^6
+ a^2*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) + 21*arctan(sin(f*x + e)/(cos(f*
x + e) + 1))/a^2) + 2*c^5*((57*sin(f*x + e)/(cos(f*x + e) + 1) + 99*sin(f*x
 + e)^2/(cos(f*x + e) + 1)^2 + 155*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15
3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 135*sin(f*x + e)^5/(cos(f*x + e) +
1)^5 + 85*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 45*sin(f*x + e)^7/(cos(f*x
+ e) + 1)^7 + 15*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 24)/(a^2 + 3*a^2*sin
(f*x + e)/(cos(f*x + e) + 1) + 6*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 +
10*a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 12*a^2*sin(f*x + e)^4/(cos(f*x
 + e) + 1)^4 + 12*a^2*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 10*a^2*sin(f*x
+ e)^6/(cos(f*x + e) + 1)^6 + 6*a^2*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 3
*a^2*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + a^2*sin(f*x + e)^9/(cos(f*x + e)
 + 1)^9) + 15*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) + 40*c^5*((12*si
n(f*x + e)/(cos(f*x + e) + 1) + 11*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 9*
sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4
 + 5)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 4*a^2*sin(f*x + e)^2/(
cos(f*x + e) + 1)^2 + 4*a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*a^2*sin
(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^2*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)
 + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) + 20*c^5*((9*sin(f*x + e)
/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4)/(a^2 + 3*a
^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1
)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 3*arctan(sin(f*x + e)/(cos
(f*x + e) + 1))/a^2) - 2*c^5*(3*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x
 + e)^2/(cos(f*x + e) + 1)^2 + 2)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) +
 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f
*x + e) + 1)^3) + 10*c^5*(3*sin(f*x + e)/(cos(f*x + e) + 1) + 1)/(a^2 + 3*a
^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1
)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3))/f
```

Fricas [A] time = 1.44187, size = 581, normalized size = 3.93

$$-\frac{2 c^5 \cos\left(fx+e\right)^5 + 19 c^5 \cos\left(fx+e\right)^4 - 106 c^5 \cos\left(fx+e\right)^3 + 630 c^5 fx - 64 c^5 - 7\left(45 c^5 fx - 77 c^5\right)\cos\left(fx+e\right)^2 + (}{6\left(a^2 f \cos\left(fx+e\right)^2 - \right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^5/(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -1/6*(2*c^5*cos(f*x + e)^5 + 19*c^5*cos(f*x + e)^4 - 106*c^5*cos(f*x + e)^3
 + 630*c^5*f*x - 64*c^5 - 7*(45*c^5*f*x - 77*c^5)*cos(f*x + e)^2 + (315*c^5
*f*x + 598*c^5)*cos(f*x + e) - (2*c^5*cos(f*x + e)^4 - 17*c^5*cos(f*x + e)^
3 - 630*c^5*f*x - 123*c^5*cos(f*x + e)^2 - 64*c^5 - (315*c^5*f*x + 662*c^5)
*cos(f*x + e))*sin(f*x + e))/(a^2*f*cos(f*x + e)^2 - a^2*f*cos(f*x + e) - 2
*a^2*f - (a^2*f*cos(f*x + e) + 2*a^2*f)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))**5/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 2.15119, size = 274, normalized size = 1.85

$$\frac{\frac{315\left(fx+e\right)c^5}{a^2} + \frac{2\left(309 c^5 \tan\left(\frac{1}{2} fx+\frac{1}{2} e\right)^8 + 969 c^5 \tan\left(\frac{1}{2} fx+\frac{1}{2} e\right)^7 + 1693 c^5 \tan\left(\frac{1}{2} fx+\frac{1}{2} e\right)^6 + 3027 c^5 \tan\left(\frac{1}{2} fx+\frac{1}{2} e\right)^5 + 2901 c^5 \tan\left(\frac{1}{2} fx+\frac{1}{2} e\right)^4 + 3247 c^5 \tan\left(\frac{1}{2}\right.\right.}{\left(\tan\left(\frac{1}{2} fx+\frac{1}{2} e\right)^3 + \tan\left(\frac{1}{2} fx+\frac{1}{2} e\right)^2 + \tan\left(\frac{1}{2} fx+\frac{1}{2} e\right)+1\right)^3 a^2}}{6 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^5/(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/6*(315*(f*x + e)*c^5/a^2 + 2*(309*c^5*tan(1/2*f*x + 1/2*e)^8 + 969*c^5*ta
n(1/2*f*x + 1/2*e)^7 + 1693*c^5*tan(1/2*f*x + 1/2*e)^6 + 3027*c^5*tan(1/2*f
*x + 1/2*e)^5 + 2901*c^5*tan(1/2*f*x + 1/2*e)^4 + 3247*c^5*tan(1/2*f*x + 1/
2*e)^3 + 1995*c^5*tan(1/2*f*x + 1/2*e)^2 + 1173*c^5*tan(1/2*f*x + 1/2*e) +
494*c^5)/((tan(1/2*f*x + 1/2*e)^3 + tan(1/2*f*x + 1/2*e)^2 + tan(1/2*f*x +
1/2*e) + 1)^3*a^2))/f
```

3.270 $\int \frac{(c-c\sin(e+fx))^4}{(a+a\sin(e+fx))^2}\,dx$

Optimal. Leaf size=135

$$\frac{35c^4\cos(e+fx)}{2a^2f}-\frac{2a^3c^4\cos^7(e+fx)}{3f(a\sin(e+fx)+a)^5}+\frac{14a^4c^4\cos^5(e+fx)}{3f\left(a^2\sin(e+fx)+a^2\right)^3}+\frac{35c^4\cos^3(e+fx)}{6f\left(a^2\sin(e+fx)+a^2\right)}+\frac{35c^4x}{2a^2}$$

```
[Out] (35*c^4*x)/(2*a^2) + (35*c^4*Cos[e + f*x])/(2*a^2*f) - (2*a^3*c^4*Cos[e + f
*x]^7)/(3*f*(a + a*Sin[e + f*x])^5) + (14*a^4*c^4*Cos[e + f*x]^5)/(3*f*(a^2
 + a^2*Sin[e + f*x])^3) + (35*c^4*Cos[e + f*x]^3)/(6*f*(a^2 + a^2*Sin[e + f
*x]))
```

Rubi [A] time = 0.224106, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {2736, 2680, 2679, 2682, 8}

$$\frac{35c^4\cos(e+fx)}{2a^2f}-\frac{2a^3c^4\cos^7(e+fx)}{3f(a\sin(e+fx)+a)^5}+\frac{14a^4c^4\cos^5(e+fx)}{3f\left(a^2\sin(e+fx)+a^2\right)^3}+\frac{35c^4\cos^3(e+fx)}{6f\left(a^2\sin(e+fx)+a^2\right)}+\frac{35c^4x}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sin[e + f*x])^4/(a + a*Sin[e + f*x])^2,x]
```


```
[Out] (35*c^4*x)/(2*a^2) + (35*c^4*Cos[e + f*x])/(2*a^2*f) - (2*a^3*c^4*Cos[e + f
*x]^7)/(3*f*(a + a*Sin[e + f*x])^5) + (14*a^4*c^4*Cos[e + f*x]^5)/(3*f*(a^2
 + a^2*Sin[e + f*x])^3) + (35*c^4*Cos[e + f*x]^3)/(6*f*(a^2 + a^2*Sin[e + f
*x]))
```


Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2679

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(a*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{(c-c\sin(e+fx))^4}{(a+a\sin(e+fx))^2}\,dx = \left(a^4c^4\right)\int \frac{\cos^8(e+fx)}{(a+a\sin(e+fx))^6}\,dx$$
$$= -\frac{2a^3c^4\cos^7(e+fx)}{3f(a+a\sin(e+fx))^5} - \frac{1}{3}\left(7a^2c^4\right)\int \frac{\cos^6(e+fx)}{(a+a\sin(e+fx))^4}\,dx$$
$$= -\frac{2a^3c^4\cos^7(e+fx)}{3f(a+a\sin(e+fx))^5} + \frac{14ac^4\cos^5(e+fx)}{3f(a+a\sin(e+fx))^3} + \frac{1}{3}\left(35c^4\right)\int \frac{\cos^4(e+fx)}{(a+a\sin(e+fx))^2}\,dx$$
$$= -\frac{2a^3c^4\cos^7(e+fx)}{3f(a+a\sin(e+fx))^5} + \frac{14ac^4\cos^5(e+fx)}{3f(a+a\sin(e+fx))^3} + \frac{35c^4\cos^3(e+fx)}{6f\left(a^2+a^2\sin(e+fx)\right)} + \frac{\left(35c^4\right)\int \frac{\text{co}}{a+a}}{2}$$
$$= \frac{35c^4\cos(e+fx)}{2a^2f} - \frac{2a^3c^4\cos^7(e+fx)}{3f(a+a\sin(e+fx))^5} + \frac{14ac^4\cos^5(e+fx)}{3f(a+a\sin(e+fx))^3} + \frac{35c^4\cos^3(e+fx)}{6f\left(a^2+a^2\sin(e+f\right.}$$
$$= \frac{35c^4x}{2a^2} + \frac{35c^4\cos(e+fx)}{2a^2f} - \frac{2a^3c^4\cos^7(e+fx)}{3f(a+a\sin(e+fx))^5} + \frac{14ac^4\cos^5(e+fx)}{3f(a+a\sin(e+fx))^3} + \frac{35c^4\cos}{6f\left(a^2+a^2\right.}$$

Mathematica [A] time = 0.503876, size = 243, normalized size = 1.8

$$(c-c\sin(e+fx))^4\left(\sin\left(\tfrac{1}{2}(e+fx)\right)+\cos\left(\tfrac{1}{2}(e+fx)\right)\right)\left(128\sin\left(\tfrac{1}{2}(e+fx)\right)+210(e+fx)\left(\sin\left(\tfrac{1}{2}(e+fx)\right)+\cos\left(\tfrac{1}{2}(e+\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sin[e + f*x])^4/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(c - c*Sin[e + f*x])^4*(128*Sin[(e +
 f*x)/2] - 64*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) - 640*Sin[(e + f*x)/2]*
(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + 210*(e + f*x)*(Cos[(e + f*x)/2] +
 Sin[(e + f*x)/2])^3 + 72*Cos[e + f*x]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)^3 - 3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*Sin[2*(e + f*x)]))/(12*f*(C
os[(e + f*x)/2] - Sin[(e + f*x)/2])^8*(a + a*Sin[e + f*x])^2)
```

Maple [A] time = 0.093, size = 229, normalized size = 1.7

$$\frac{c^4}{a^2f}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3\left(1+\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2\right)^{-2}+12\,\frac{c^4\left(\tan\left(1/2\,fx+e/2\right)\right)^2}{a^2f\left(1+\left(\tan\left(1/2\,fx+e/2\right)\right)^2\right)^2}-\frac{c^4}{a^2f}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\left(1+\left(\tan\left(\frac{f}{2}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))^4/(a+a*sin(f*x+e))^2,x)
```

```
[Out] 1/f*c^4/a^2/(1+tan(1/2*f*x+1/2*e)^2)^2*tan(1/2*f*x+1/2*e)^3+12/f*c^4/a^2/(1
+tan(1/2*f*x+1/2*e)^2)^2*tan(1/2*f*x+1/2*e)^2-1/f*c^4/a^2/(1+tan(1/2*f*x+1/
2*e)^2)^2*tan(1/2*f*x+1/2*e)+12/f*c^4/a^2/(1+tan(1/2*f*x+1/2*e)^2)^2+35/f*c
^4/a^2*arctan(tan(1/2*f*x+1/2*e))-64/3/f*c^4/a^2/(tan(1/2*f*x+1/2*e)+1)^3+3
2/f*c^4/a^2/(tan(1/2*f*x+1/2*e)+1)^2+32/f*c^4/a^2/(tan(1/2*f*x+1/2*e)+1)
```

Maxima [B] time = 2.35734, size = 1219, normalized size = 9.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^4/(a+a*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/3*(c^4*((75*sin(f*x + e)/(cos(f*x + e) + 1) + 97*sin(f*x + e)^2/(cos(f*x
+ e) + 1)^2 + 126*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 98*sin(f*x + e)^4/(
cos(f*x + e) + 1)^4 + 63*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 21*sin(f*x +
 e)^6/(cos(f*x + e) + 1)^6 + 32)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) +
1) + 5*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 7*a^2*sin(f*x + e)^3/(cos(
f*x + e) + 1)^3 + 7*a^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 5*a^2*sin(f*x
 + e)^5/(cos(f*x + e) + 1)^5 + 3*a^2*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 +
a^2*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) + 21*arctan(sin(f*x + e)/(cos(f*x
+ e) + 1))/a^2) + 16*c^4*((12*sin(f*x + e)/(cos(f*x + e) + 1) + 11*sin(f*x
+ e)^2/(cos(f*x + e) + 1)^2 + 9*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin
(f*x + e)^4/(cos(f*x + e) + 1)^4 + 5)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x +
e) + 1) + 4*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4*a^2*sin(f*x + e)^3/
(cos(f*x + e) + 1)^3 + 3*a^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^2*sin(
f*x + e)^5/(cos(f*x + e) + 1)^5) + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1)
)/a^2) + 12*c^4*((9*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos
(f*x + e) + 1)^2 + 4)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*
sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)
^3) + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) - 2*c^4*(3*sin(f*x + e
)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2)/(a^2 + 3*
a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) +
1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 8*c^4*(3*sin(f*x + e)/(co
s(f*x + e) + 1) + 1)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*s
in(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^
3))/f
```

Fricas [A] time = 1.43884, size = 513, normalized size = 3.8

$$-\frac{3\,c^4\cos\left(fx+e\right)^4-30\,c^4\cos\left(fx+e\right)^3+210\,c^4fx-32\,c^4-\left(105\,c^4fx-193\,c^4\right)\cos\left(fx+e\right)^2+\left(105\,c^4fx+194\,c^4\right)}{6\left(a^2f\cos\left(fx+e\right)^2-a^2f\cos\left(fx+e\right)-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^4/(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -1/6*(3*c^4*cos(f*x + e)^4 - 30*c^4*cos(f*x + e)^3 + 210*c^4*f*x - 32*c^4 -
 (105*c^4*f*x - 193*c^4)*cos(f*x + e)^2 + (105*c^4*f*x + 194*c^4)*cos(f*x +
 e) + (3*c^4*cos(f*x + e)^3 + 210*c^4*f*x + 33*c^4*cos(f*x + e)^2 + 32*c^4
```

```
+ (105*c^4*f*x + 226*c^4)*cos(f*x + e))*sin(f*x + e))/(a^2*f*cos(f*x + e)^2
 - a^2*f*cos(f*x + e) - 2*a^2*f - (a^2*f*cos(f*x + e) + 2*a^2*f)*sin(f*x +
e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))**4/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 2.22172, size = 205, normalized size = 1.52

$$\frac{\frac{105\,(fx+e)c^4}{a^2} + \frac{6\left(c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+12\,c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+12\,c^4\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)^2a^2} + \frac{64\left(3\,c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+9\,c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+4\,c^4\right)}{a^2\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^3}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^4/(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/6*(105*(f*x + e)*c^4/a^2 + 6*(c^4*tan(1/2*f*x + 1/2*e)^3 + 12*c^4*tan(1/2
*f*x + 1/2*e)^2 - c^4*tan(1/2*f*x + 1/2*e) + 12*c^4)/((tan(1/2*f*x + 1/2*e)
^2 + 1)^2*a^2) + 64*(3*c^4*tan(1/2*f*x + 1/2*e)^2 + 9*c^4*tan(1/2*f*x + 1/2
*e) + 4*c^4)/(a^2*(tan(1/2*f*x + 1/2*e) + 1)^3))/f
```

3.271 $\int \frac{(c-c\sin(e+fx))^3}{(a+a\sin(e+fx))^2}\,dx$

Optimal. Leaf size=90

$$\frac{5c^3\cos(e+fx)}{a^2f}-\frac{2a^2c^3\cos^5(e+fx)}{3f(a\sin(e+fx)+a)^4}+\frac{5c^3x}{a^2}+\frac{10c^3\cos^3(e+fx)}{3f(a\sin(e+fx)+a)^2}$$

```
[Out] (5*c^3*x)/a^2 + (5*c^3*Cos[e + f*x])/(a^2*f) - (2*a^2*c^3*Cos[e + f*x]^5)/(
3*f*(a + a*Sin[e + f*x])^4) + (10*c^3*Cos[e + f*x]^3)/(3*f*(a + a*Sin[e + f
*x])^2)
```

Rubi [A] time = 0.175377, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2736, 2680, 2682, 8}

$$\frac{5c^3\cos(e+fx)}{a^2f}-\frac{2a^2c^3\cos^5(e+fx)}{3f(a\sin(e+fx)+a)^4}+\frac{5c^3x}{a^2}+\frac{10c^3\cos^3(e+fx)}{3f(a\sin(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sin[e + f*x])^3/(a + a*Sin[e + f*x])^2,x]
```


```
[Out] (5*c^3*x)/a^2 + (5*c^3*Cos[e + f*x])/(a^2*f) - (2*a^2*c^3*Cos[e + f*x]^5)/(
3*f*(a + a*Sin[e + f*x])^4) + (10*c^3*Cos[e + f*x]^3)/(3*f*(a + a*Sin[e + f
*x])^2)
```


Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{(c-c\sin(e+fx))^3}{(a+a\sin(e+fx))^2}\,dx = \left(a^3c^3\right)\int \frac{\cos^6(e+fx)}{(a+a\sin(e+fx))^5}\,dx$$
$$= -\frac{2a^2c^3\cos^5(e+fx)}{3f(a+a\sin(e+fx))^4} - \frac{1}{3}\left(5ac^3\right)\int \frac{\cos^4(e+fx)}{(a+a\sin(e+fx))^3}\,dx$$
$$= -\frac{2a^2c^3\cos^5(e+fx)}{3f(a+a\sin(e+fx))^4} + \frac{10c^3\cos^3(e+fx)}{3f(a+a\sin(e+fx))^2} + \frac{\left(5c^3\right)\int \frac{\cos^2(e+fx)}{a+a\sin(e+fx)}\,dx}{a}$$
$$= \frac{5c^3\cos(e+fx)}{a^2f} - \frac{2a^2c^3\cos^5(e+fx)}{3f(a+a\sin(e+fx))^4} + \frac{10c^3\cos^3(e+fx)}{3f(a+a\sin(e+fx))^2} + \frac{\left(5c^3\right)\int 1\,dx}{a^2}$$
$$= \frac{5c^3x}{a^2} + \frac{5c^3\cos(e+fx)}{a^2f} - \frac{2a^2c^3\cos^5(e+fx)}{3f(a+a\sin(e+fx))^4} + \frac{10c^3\cos^3(e+fx)}{3f(a+a\sin(e+fx))^2}$$

Mathematica [B] time = 0.360541, size = 210, normalized size = 2.33

$$\frac{(c-c\sin(e+fx))^3\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(16\sin\left(\frac{1}{2}(e+fx)\right)+15(e+fx)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx\right.\right.\right.}{3f(a\sin(e\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sin[e + f*x])^3/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(16*Sin[(e + f*x)/2] - 8*(Cos[(e + f
*x)/2] + Sin[(e + f*x)/2]) - 56*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e
 + f*x)/2])^2 + 15*(e + f*x)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 + 3*Co
s[e + f*x]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3)*(c - c*Sin[e + f*x])^3)
/(3*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^6*(a + a*Sin[e + f*x])^2)
```

Maple [A] time = 0.089, size = 121, normalized size = 1.3

$$2\,\frac{c^3}{a^2f\left(1+\left(\tan\left(1/2\,fx+e/2\right)\right)^2\right)}+10\,\frac{c^3\arctan\left(\tan\left(1/2\,fx+e/2\right)\right)}{a^2f}-\frac{32\,c^3}{3\,a^2f}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{-3}+16\,\frac{c}{a^2f\left(\tan\left(1/2\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))^3/(a+a*sin(f*x+e))^2,x)
```

```
[Out] 2/f*c^3/a^2/(1+tan(1/2*f*x+1/2*e)^2)+10/f*c^3/a^2*arctan(tan(1/2*f*x+1/2*e)
)-32/3/f*c^3/a^2/(tan(1/2*f*x+1/2*e)+1)^3+16/f*c^3/a^2/(tan(1/2*f*x+1/2*e)+
1)^2+8/f*c^3/a^2/(tan(1/2*f*x+1/2*e)+1)
```

Maxima [B] time = 3.91568, size = 797, normalized size = 8.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^3/(a+a*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 2/3*(2*c^3*((12*sin(f*x + e)/(cos(f*x + e) + 1) + 11*sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 + 9*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^4/(c
os(f*x + e) + 1)^4 + 5)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 4*a^
2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4*a^2*sin(f*x + e)^3/(cos(f*x + e)
+ 1)^3 + 3*a^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^2*sin(f*x + e)^5/(co
s(f*x + e) + 1)^5) + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) + 3*c^3
*((9*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^
2 + 4)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/
(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 3*arctan(
sin(f*x + e)/(cos(f*x + e) + 1))/a^2) - c^3*(3*sin(f*x + e)/(cos(f*x + e) +
 1) + 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2)/(a^2 + 3*a^2*sin(f*x + e)/
(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*
x + e)^3/(cos(f*x + e) + 1)^3) + 3*c^3*(3*sin(f*x + e)/(cos(f*x + e) + 1) +
 1)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(co
s(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3))/f
```

Fricas [B] time = 1.37893, size = 433, normalized size = 4.81

$$\frac{3\,c^3\cos\left(fx+e\right)^3-30\,c^3fx+8\,c^3+\left(15\,c^3fx-31\,c^3\right)\cos\left(fx+e\right)^2-\left(15\,c^3fx+26\,c^3\right)\cos\left(fx+e\right)-\left(30\,c^3fx+3\,c^3\right.}{3\left(a^2f\cos\left(fx+e\right)^2-a^2f\cos\left(fx+e\right)-2\,a^2f-\left(a^2f\cos\left(fx+e\right)+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^3/(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(3*c^3*cos(f*x + e)^3 - 30*c^3*f*x + 8*c^3 + (15*c^3*f*x - 31*c^3)*cos(
f*x + e)^2 - (15*c^3*f*x + 26*c^3)*cos(f*x + e) - (30*c^3*f*x + 3*c^3*cos(f
*x + e)^2 + 8*c^3 + (15*c^3*f*x + 34*c^3)*cos(f*x + e))*sin(f*x + e))/(a^2*
f*cos(f*x + e)^2 - a^2*f*cos(f*x + e) - 2*a^2*f - (a^2*f*cos(f*x + e) + 2*a
^2*f)*sin(f*x + e))
```

Sympy [A] time = 51.7506, size = 1282, normalized size = 14.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))**3/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((15*c**3*f*x*tan(e/2 + f*x/2)**5/(3*a**2*f*tan(e/2 + f*x/2)**5 +
9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*ta
n(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 45*c**3*f*x*tan
(e/2 + f*x/2)**4/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)*
*4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2
*f*tan(e/2 + f*x/2) + 3*a**2*f) + 60*c**3*f*x*tan(e/2 + f*x/2)**3/(3*a**2*f
*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f
*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a*
*2*f) + 60*c**3*f*x*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a
**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e
/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 45*c**3*f*x*tan(e/
2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 1
2*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan
(e/2 + f*x/2) + 3*a**2*f) + 15*c**3*f*x/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a
```

```
**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e
/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 8*c**3*tan(e/2 + f
*x/2)**5/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*
a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e
/2 + f*x/2) + 3*a**2*f) + 70*c**3*tan(e/2 + f*x/2)**3/(3*a**2*f*tan(e/2 + f
*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 1
2*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 50*c
**3*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 +
f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 +
 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 90*c**3*tan(e/2 + f*x/2)/(3*a**2*f
*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f
*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a*
*2*f) + 38*c**3/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**
4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*
f*tan(e/2 + f*x/2) + 3*a**2*f), Ne(f, 0)), (x*(-c*sin(e) + c)**3/(a*sin(e)
+ a)**2, True))
```

Giac [A] time = 2.16426, size = 136, normalized size = 1.51

$$\frac{\frac{15\,(fx+e)c^3}{a^2} + \frac{6\,c^3}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)a^2} + \frac{8\left(3\,c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+12\,c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+5\,c^3\right)}{a^2\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^3}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^3/(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/3*(15*(f*x + e)*c^3/a^2 + 6*c^3/((tan(1/2*f*x + 1/2*e)^2 + 1)*a^2) + 8*(3
*c^3*tan(1/2*f*x + 1/2*e)^2 + 12*c^3*tan(1/2*f*x + 1/2*e) + 5*c^3)/(a^2*(ta
n(1/2*f*x + 1/2*e) + 1)^3))/f
```

3.272 $\int \frac{(c-c\sin(e+fx))^2}{(a+a\sin(e+fx))^2}\,dx$

Optimal. Leaf size=70

$$\frac{2c^2\cos(e+fx)}{f\left(a^2\sin(e+fx)+a^2\right)}+\frac{c^2x}{a^2}-\frac{2ac^2\cos^3(e+fx)}{3f(a\sin(e+fx)+a)^3}$$

```
[Out] (c^2*x)/a^2 - (2*a*c^2*Cos[e + f*x]^3)/(3*f*(a + a*Sin[e + f*x])^3) + (2*c^
2*Cos[e + f*x])/(f*(a^2 + a^2*Sin[e + f*x]))
```

Rubi [A] time = 0.131786, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2736, 2680, 8}

$$\frac{2c^2\cos(e+fx)}{f\left(a^2\sin(e+fx)+a^2\right)}+\frac{c^2x}{a^2}-\frac{2ac^2\cos^3(e+fx)}{3f(a\sin(e+fx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sin[e + f*x])^2/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] (c^2*x)/a^2 - (2*a*c^2*Cos[e + f*x]^3)/(3*f*(a + a*Sin[e + f*x])^3) + (2*c^
2*Cos[e + f*x])/(f*(a^2 + a^2*Sin[e + f*x]))
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{(c-c\sin(e+fx))^2}{(a+a\sin(e+fx))^2}\,dx = \left(a^2c^2\right)\int \frac{\cos^4(e+fx)}{(a+a\sin(e+fx))^4}\,dx$$
$$= -\frac{2ac^2\cos^3(e+fx)}{3f(a+a\sin(e+fx))^3} - c^2\int \frac{\cos^2(e+fx)}{(a+a\sin(e+fx))^2}\,dx$$
$$= -\frac{2ac^2\cos^3(e+fx)}{3f(a+a\sin(e+fx))^3} + \frac{2c^2\cos(e+fx)}{f\left(a^2+a^2\sin(e+fx)\right)} + \frac{c^2\int 1\,dx}{a^2}$$
$$= \frac{c^2x}{a^2} - \frac{2ac^2\cos^3(e+fx)}{3f(a+a\sin(e+fx))^3} + \frac{2c^2\cos(e+fx)}{f\left(a^2+a^2\sin(e+fx)\right)}$$

Mathematica [A] time = 0.585637, size = 119, normalized size = 1.7

$$\frac{c^2\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(3(3e+3fx-8)\cos\left(\frac{1}{2}(e+fx)\right)+(-3e-3fx+16)\cos\left(\frac{3}{2}(e+fx)\right)+6\sin\left(\frac{1}{2}(e\right.\right.}{6a^2f(\sin(e+fx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sin[e + f*x])^2/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] (c^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(3*(-8 + 3*e + 3*f*x)*Cos[(e + f
*x)/2] + (16 - 3*e - 3*f*x)*Cos[(3*(e + f*x))/2] + 6*(2*(-2 + e + f*x) + (e
 + f*x)*Cos[e + f*x])*Sin[(e + f*x)/2]))/(6*a^2*f*(1 + Sin[e + f*x])^2)
```

Maple [A] time = 0.078, size = 71, normalized size = 1.

$$2\,\frac{c^2\arctan\left(\tan\left(1/2\,fx+e/2\right)\right)}{a^2f} - \frac{16\,c^2}{3\,a^2f}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{-3} + 8\,\frac{c^2}{a^2f\left(\tan\left(1/2\,fx+e/2\right)+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))^2/(a+a*sin(f*x+e))^2,x)
```

```
[Out] 2/f*c^2/a^2*arctan(tan(1/2*f*x+1/2*e))-16/3/f*c^2/a^2/(tan(1/2*f*x+1/2*e)+1
)^3+8/f*c^2/a^2/(tan(1/2*f*x+1/2*e)+1)^2
```

Maxima [B] time = 2.15464, size = 487, normalized size = 6.96

$$\frac{2\left(c^2\left(\frac{\frac{9\sin(fx+e)}{\cos(fx+e)+1}+\frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+4}{a^2+\frac{3a^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{3a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}+\frac{3\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^2}\right)-\frac{c^2\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}+\frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+2\right)}{a^2+\frac{3a^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{3a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}+\frac{\ldots}{a^2+\frac{3a^2\sin(fx\ldots}{\cos(fx+e)\ldots}}\right.}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^2/(a+a*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 2/3*(c^2*((9*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x +
e) + 1)^2 + 4)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x
```

```
 + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 3
*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) - c^2*(3*sin(f*x + e)/(cos(f*
x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2)/(a^2 + 3*a^2*sin(f
*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^
2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 2*c^2*(3*sin(f*x + e)/(cos(f*x + e
) + 1) + 1)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x +
e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3))/f
```

Fricas [B] time = 1.39614, size = 362, normalized size = 5.17

$$-\frac{6\,c^2fx-\left(3\,c^2fx-8\,c^2\right)\cos\left(fx+e\right)^2-4\,c^2+\left(3\,c^2fx+4\,c^2\right)\cos\left(fx+e\right)+\left(6\,c^2fx+4\,c^2+\left(3\,c^2fx+8\,c^2\right)\cos\left(fx\right.\right.}{3\left(a^2f\cos\left(fx+e\right)^2-a^2f\cos\left(fx+e\right)-2\,a^2f-\left(a^2f\cos\left(fx+e\right)+2\,a^2f\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^2/(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -1/3*(6*c^2*f*x - (3*c^2*f*x - 8*c^2)*cos(f*x + e)^2 - 4*c^2 + (3*c^2*f*x +
 4*c^2)*cos(f*x + e) + (6*c^2*f*x + 4*c^2 + (3*c^2*f*x + 8*c^2)*cos(f*x + e
))*sin(f*x + e))/(a^2*f*cos(f*x + e)^2 - a^2*f*cos(f*x + e) - 2*a^2*f - (a^
2*f*cos(f*x + e) + 2*a^2*f)*sin(f*x + e))
```

Sympy [A] time = 12.4763, size = 486, normalized size = 6.94

$$\begin{cases} \frac{3c^2fx\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+3a^2f} + \frac{9c^2fx\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+3a^2f} + \frac{}{3a^2f\tan^3\left(\frac{e}{2}+\right.} \\ \frac{x(-c\sin(e)+c)^2}{(a\sin(e)+a)^2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))**2/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((3*c**2*f*x*tan(e/2 + f*x/2)**3/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9
*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 9*c**
2*f*x*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2
+ f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 9*c**2*f*x*tan(e/2 +
f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**
2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 3*c**2*f*x/(3*a**2*f*tan(e/2 + f*x/2)**3
 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 8
*c**2*tan(e/2 + f*x/2)**3/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2
+ f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 24*c**2*tan(e/2 + f*x
/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**
2*f*tan(e/2 + f*x/2) + 3*a**2*f), Ne(f, 0)), (x*(-c*sin(e) + c)**2/(a*sin(e
) + a)**2, True))
```

Giac [A] time = 2.13887, size = 78, normalized size = 1.11

$$\frac{\frac{3\left(fx+e\right)c^2}{a^2}+\frac{8\left(3\,c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+c^2\right)}{a^2\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^3}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^2/(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/3*(3*(f*x + e)*c^2/a^2 + 8*(3*c^2*tan(1/2*f*x + 1/2*e) + c^2)/(a^2*(tan(1
/2*f*x + 1/2*e) + 1)^3))/f
```

3.273 $\int \frac{c-c\sin(e+fx)}{(a+a\sin(e+fx))^2}\,dx$

Optimal. Leaf size=29

$$-\frac{ac\cos^3(e+fx)}{3f(a\sin(e+fx)+a)^3}$$

```
[Out] -(a*c*Cos[e + f*x]^3)/(3*f*(a + a*Sin[e + f*x])^3)
```

Rubi [A] time = 0.0668245, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2736, 2671}

$$-\frac{ac\cos^3(e+fx)}{3f(a\sin(e+fx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sin[e + f*x])/(a + a*Sin[e + f*x])^2,x]

[Out] -(a*c*Cos[e + f*x]^3)/(3*f*(a + a*Sin[e + f*x])^3)
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{c-c\sin(e+fx)}{(a+a\sin(e+fx))^2}\,dx &= (ac)\int \frac{\cos^2(e+fx)}{(a+a\sin(e+fx))^3}\,dx\\ &= -\frac{ac\cos^3(e+fx)}{3f(a+a\sin(e+fx))^3}\end{aligned}$$

Mathematica [B] time = 0.264063, size = 70, normalized size = 2.41

$$\frac{c\left(\cos\left(e+\frac{3fx}{2}\right)-3\cos\left(e+\frac{fx}{2}\right)\right)}{3a^2f\left(\sin\left(\frac{e}{2}\right)+\cos\left(\frac{e}{2}\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sin[e + f*x])/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] (c*(-3*Cos[e + (f*x)/2] + Cos[e + (3*f*x)/2]))/(3*a^2*f*(Cos[e/2] + Sin[e/2
])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3)
```

Maple [B] time = 0.069, size = 56, normalized size = 1.9

$$2\,{\frac {c}{{a}^{2}f} \left(-4/3\, \left(\tan \left(1/2\,fx+e/2 \right) +1 \right) ^{-3}+2\, \left(\tan \left(1/2\,fx+e/2 \right) +1 \right) ^{-2}- \left(\tan \left(1/2\,fx+e/2 \right) +1 \right) ^{-1} \right) }$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))/(a+a*sin(f*x+e))^2,x)
```

```
[Out] 2/f*c/a^2*(-4/3/(tan(1/2*f*x+1/2*e)+1)^3+2/(tan(1/2*f*x+1/2*e)+1)^2-1/(tan(
1/2*f*x+1/2*e)+1))
```

Maxima [B] time = 1.77858, size = 290, normalized size = 10.

$$-\frac{2\left(\frac{c\left(\frac{3\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{3\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+2\right)}{a^{2}+\frac{3a^{2}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{3a^{2}\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{a^{2}\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}}-\frac{c\left(\frac{3\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+1\right)}{a^{2}+\frac{3a^{2}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{3a^{2}\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{a^{2}\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}}\right)}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))/(a+a*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] -2/3*(c*(3*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e)
 + 1)^2 + 2)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x +
 e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) - c*(
3*sin(f*x + e)/(cos(f*x + e) + 1) + 1)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x +
 e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(
cos(f*x + e) + 1)^3))/f
```

Fricas [B] time = 1.29729, size = 251, normalized size = 8.66

$$-\frac{c\cos\left(fx+e\right)^{2}-c\cos\left(fx+e\right)+\left(c\cos\left(fx+e\right)+2\,c\right)\sin\left(fx+e\right)-2\,c}{3\left(a^{2}f\cos\left(fx+e\right)^{2}-a^{2}f\cos\left(fx+e\right)-2\,a^{2}f-\left(a^{2}f\cos\left(fx+e\right)+2\,a^{2}f\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))/(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -1/3*(c*cos(f*x + e)^2 - c*cos(f*x + e) + (c*cos(f*x + e) + 2*c)*sin(f*x +
e) - 2*c)/(a^2*f*cos(f*x + e)^2 - a^2*f*cos(f*x + e) - 2*a^2*f - (a^2*f*cos
(f*x + e) + 2*a^2*f)*sin(f*x + e))
```

Sympy [A] time = 6.88263, size = 158, normalized size = 5.45

$$\begin{cases} -\frac{6c\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+3a^2f} - \frac{2c}{3a^2f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+3a^2f} & \text{for } f \neq 0 \\ \frac{x(-c\sin(e)+c)}{(a\sin(e)+a)^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((-6*c*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*
f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 2*c/(3*a**2
*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 +
f*x/2) + 3*a**2*f), Ne(f, 0)), (x*(-c*sin(e) + c)/(a*sin(e) + a)**2, True))
```

Giac [A] time = 2.04768, size = 53, normalized size = 1.83

$$-\frac{2\left(3\,c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+c\right)}{3\,a^2f\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))/(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] -2/3*(3*c*tan(1/2*f*x + 1/2*e)^2 + c)/(a^2*f*(tan(1/2*f*x + 1/2*e) + 1)^3)
```

3.274 $\int \frac{1}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))}\,dx$

Optimal. Leaf size=52

$$\frac{2\tan(e+fx)}{3a^2cf}-\frac{\sec(e+fx)}{3cf\left(a^2\sin(e+fx)+a^2\right)}$$

```
[Out] -Sec[e + f*x]/(3*c*f*(a^2 + a^2*Sin[e + f*x])) + (2*Tan[e + f*x])/(3*a^2*c*
f)
```

Rubi [A] time = 0.10668, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2736, 2672, 3767, 8}

$$\frac{2\tan(e+fx)}{3a^2cf}-\frac{\sec(e+fx)}{3cf\left(a^2\sin(e+fx)+a^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])),x]
```

```
[Out] -Sec[e + f*x]/(3*c*f*(a^2 + a^2*Sin[e + f*x])) + (2*Tan[e + f*x])/(3*a^2*c*
f)
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] && !IGtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))}\,dx = \frac{\int \frac{\sec^2(e+fx)}{a+a\sin(e+fx)}\,dx}{ac}$$
$$= -\frac{\sec(e+fx)}{3cf\left(a^2+a^2\sin(e+fx)\right)} + \frac{2\int \sec^2(e+fx)\,dx}{3a^2c}$$
$$= -\frac{\sec(e+fx)}{3cf\left(a^2+a^2\sin(e+fx)\right)} - \frac{2\,\text{Subst}(\int 1\,dx, x, -\tan(e+fx))}{3a^2cf}$$
$$= -\frac{\sec(e+fx)}{3cf\left(a^2+a^2\sin(e+fx)\right)} + \frac{2\tan(e+fx)}{3a^2cf}$$

Mathematica [A] time = 0.461606, size = 87, normalized size = 1.67

$$-\frac{\sin(e+fx)+8\sin(2(e+fx))+\sin(3(e+fx))-4\cos(e+fx)+2\cos(2(e+fx))-4\cos(3(e+fx))+2}{24a^2cf(\sin(e+fx)-1)(\sin(e+fx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])),x]
```

```
[Out] -(2 - 4*Cos[e + f*x] + 2*Cos[2*(e + f*x)] - 4*Cos[3*(e + f*x)] + Sin[e + f*
x] + 8*Sin[2*(e + f*x)] + Sin[3*(e + f*x)])/(24*a^2*c*f*(-1 + Sin[e + f*x])
*(1 + Sin[e + f*x])^2)
```

Maple [A] time = 0.046, size = 73, normalized size = 1.4

$$2\,\frac{1}{a^2cf}\left(-1/4\,\left(\tan\left(1/2\,fx+e/2\right)-1\right)^{-1}-1/3\,\left(\tan\left(1/2\,fx+e/2\right)+1\right)^{-3}+1/2\,\left(\tan\left(1/2\,fx+e/2\right)+1\right)^{-2}-3/4\,\left(\tan\left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e)),x)
```

```
[Out] 2/f/a^2/c*(-1/4/(tan(1/2*f*x+1/2*e)-1)-1/3/(tan(1/2*f*x+1/2*e)+1)^3+1/2/(ta
n(1/2*f*x+1/2*e)+1)^2-3/4/(tan(1/2*f*x+1/2*e)+1))
```

Maxima [B] time = 1.55873, size = 192, normalized size = 3.69

$$\frac{2\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+\frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}-1\right)}{3\left(a^2c+\frac{2a^2c\sin(fx+e)}{\cos(fx+e)+1}-\frac{2a^2c\sin(fx+e)^3}{(\cos(fx+e)+1)^3}-\frac{a^2c\sin(fx+e)^4}{(\cos(fx+e)+1)^4}\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] 2/3*(sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^
2 + 3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 1)/((a^2*c + 2*a^2*c*sin(f*x +
e)/(cos(f*x + e) + 1) - 2*a^2*c*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - a^2*c
```

```
*sin(f*x + e)^4/(cos(f*x + e) + 1)^4)*f)
```

Fricas [A] time = 1.37038, size = 142, normalized size = 2.73

$$-\frac{2 \cos\left(fx+e\right)^{2} - 2 \sin\left(fx+e\right) - 1}{3\left(a^{2}cf \cos\left(fx+e\right)\sin\left(fx+e\right) + a^{2}cf \cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] -1/3*(2*cos(f*x + e)^2 - 2*sin(f*x + e) - 1)/(a^2*c*f*cos(f*x + e)*sin(f*x
+ e) + a^2*c*f*cos(f*x + e))
```

Sympy [A] time = 9.1402, size = 328, normalized size = 6.31

$$\begin{cases} -\frac{\tan^{4}\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^{2}cf\tan^{4}\left(\frac{e}{2}+\frac{fx}{2}\right)+6a^{2}cf\tan^{3}\left(\frac{e}{2}+\frac{fx}{2}\right)-6a^{2}cf\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-3a^{2}cf} - \frac{8\tan^{3}\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^{2}cf\tan^{4}\left(\frac{e}{2}+\frac{fx}{2}\right)+6a^{2}cf\tan^{3}\left(\frac{e}{2}+\frac{fx}{2}\right)-6a^{2}cf\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-3a^{2}cf} - \frac{}{3a^{2}cf\tan^{4}} \\ \frac{x}{\left(a\sin\left(e\right)+a\right)^{2}\left(-c\sin\left(e\right)+c\right)} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**2/(c-c*sin(f*x+e)),x)
```

```
[Out] Piecewise((-tan(e/2 + f*x/2)**4/(3*a**2*c*f*tan(e/2 + f*x/2)**4 + 6*a**2*c*
f*tan(e/2 + f*x/2)**3 - 6*a**2*c*f*tan(e/2 + f*x/2) - 3*a**2*c*f) - 8*tan(e
/2 + f*x/2)**3/(3*a**2*c*f*tan(e/2 + f*x/2)**4 + 6*a**2*c*f*tan(e/2 + f*x/2
)**3 - 6*a**2*c*f*tan(e/2 + f*x/2) - 3*a**2*c*f) - 6*tan(e/2 + f*x/2)**2/(3
*a**2*c*f*tan(e/2 + f*x/2)**4 + 6*a**2*c*f*tan(e/2 + f*x/2)**3 - 6*a**2*c*f
*tan(e/2 + f*x/2) - 3*a**2*c*f) + 3/(3*a**2*c*f*tan(e/2 + f*x/2)**4 + 6*a**
2*c*f*tan(e/2 + f*x/2)**3 - 6*a**2*c*f*tan(e/2 + f*x/2) - 3*a**2*c*f), Ne(f
, 0)), (x/((a*sin(e) + a)**2*(-c*sin(e) + c)), True))
```

Giac [A] time = 1.98881, size = 104, normalized size = 2.

$$-\frac{\frac{3}{a^{2}c\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)} + \frac{9\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+12\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+7}{a^{2}c\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^{3}}}{6f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] -1/6*(3/(a^2*c*(tan(1/2*f*x + 1/2*e) - 1)) + (9*tan(1/2*f*x + 1/2*e)^2 + 12
*tan(1/2*f*x + 1/2*e) + 7)/(a^2*c*(tan(1/2*f*x + 1/2*e) + 1)^3))/f
```

3.275 $\int \frac{1}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^2}\,dx$

Optimal. Leaf size=38

$$\frac{\tan^3(e+fx)}{3a^2c^2f}+\frac{\tan(e+fx)}{a^2c^2f}$$

```
[Out] Tan[e + f*x]/(a^2*c^2*f) + Tan[e + f*x]^3/(3*a^2*c^2*f)
```

Rubi [A] time = 0.0632528, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2736, 3767}

$$\frac{\tan^3(e+fx)}{3a^2c^2f}+\frac{\tan(e+fx)}{a^2c^2f}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^2),x]
```

```
[Out] Tan[e + f*x]/(a^2*c^2*f) + Tan[e + f*x]^3/(3*a^2*c^2*f)
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^2}\,dx &= \frac{\int \sec^4(e+fx)\,dx}{a^2c^2}\\
&= -\frac{\text{Subst}\left(\int \left(1+x^2\right)dx, x, -\tan(e+fx)\right)}{a^2c^2f}\\
&= \frac{\tan(e+fx)}{a^2c^2f}+\frac{\tan^3(e+fx)}{3a^2c^2f}
\end{aligned}$$

Mathematica [A] time = 0.0537988, size = 29, normalized size = 0.76

$$\frac{\frac{1}{3}\tan^3(e+fx)+\tan(e+fx)}{a^2c^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^2),x]
```

```
[Out] (Tan[e + f*x] + Tan[e + f*x]^3/3)/(a^2*c^2*f)
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a + a\sin\left(fx + e\right)\right)^{2} \left(c - c\sin\left(fx + e\right)\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^2,x)
```

```
[Out] int(1/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^2,x)
```

Maxima [A] time = 1.43591, size = 38, normalized size = 1.

$$\frac{\tan\left(fx + e\right)^{3} + 3\,\tan\left(fx + e\right)}{3\,a^{2}c^{2}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/3*(tan(f*x + e)^3 + 3*tan(f*x + e))/(a^2*c^2*f)
```

Fricas [A] time = 1.20854, size = 92, normalized size = 2.42

$$\frac{\left(2\,\cos\left(fx + e\right)^{2} + 1\right)\sin\left(fx + e\right)}{3\,a^{2}c^{2}f\cos\left(fx + e\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(2*cos(f*x + e)^2 + 1)*sin(f*x + e)/(a^2*c^2*f*cos(f*x + e)^3)
```

Sympy [A] time = 13.2746, size = 286, normalized size = 7.53

$$\begin{cases} -\frac{6\tan^{5}\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^{2}c^{2}f\tan^{6}\left(\frac{e}{2}+\frac{fx}{2}\right)-9a^{2}c^{2}f\tan^{4}\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^{2}c^{2}f\tan^{2}\left(\frac{e}{2}+\frac{fx}{2}\right)-3a^{2}c^{2}f} + \frac{4\tan^{3}\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^{2}c^{2}f\tan^{6}\left(\frac{e}{2}+\frac{fx}{2}\right)-9a^{2}c^{2}f\tan^{4}\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^{2}c^{2}f\tan^{2}\left(\frac{e}{2}+\frac{fx}{2}\right)-3a^{2}c^{2}f} - \\ \frac{x}{(a\sin(e)+a)^{2}(-c\sin(e)+c)^{2}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**2/(c-c*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((-6*tan(e/2 + f*x/2)**5/(3*a**2*c**2*f*tan(e/2 + f*x/2)**6 - 9*a*
*2*c**2*f*tan(e/2 + f*x/2)**4 + 9*a**2*c**2*f*tan(e/2 + f*x/2)**2 - 3*a**2*
c**2*f) + 4*tan(e/2 + f*x/2)**3/(3*a**2*c**2*f*tan(e/2 + f*x/2)**6 - 9*a**2
*c**2*f*tan(e/2 + f*x/2)**4 + 9*a**2*c**2*f*tan(e/2 + f*x/2)**2 - 3*a**2*c*
*2*f) - 6*tan(e/2 + f*x/2)/(3*a**2*c**2*f*tan(e/2 + f*x/2)**6 - 9*a**2*c**2
*f*tan(e/2 + f*x/2)**4 + 9*a**2*c**2*f*tan(e/2 + f*x/2)**2 - 3*a**2*c**2*f)
, Ne(f, 0)), (x/((a*sin(e) + a)**2*(-c*sin(e) + c)**2), True))
```

Giac [A] time = 2.23937, size = 41, normalized size = 1.08

$$\frac{\tan\left(fx+e\right)^3+3\,\tan\left(fx+e\right)}{3\,a^2c^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/3*(tan(f*x + e)^3 + 3*tan(f*x + e))/(a^2*c^2*f)
```

3.276 $\int \frac{1}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^3}\,dx$

Optimal. Leaf size=76

$$\frac{4\tan^3(e+fx)}{15a^2c^3f}+\frac{4\tan(e+fx)}{5a^2c^3f}+\frac{\sec^3(e+fx)}{5a^2f\left(c^3-c^3\sin(e+fx)\right)}$$

```
[Out] Sec[e + f*x]^3/(5*a^2*f*(c^3 - c^3*Sin[e + f*x])) + (4*Tan[e + f*x])/(5*a^2
*c^3*f) + (4*Tan[e + f*x]^3)/(15*a^2*c^3*f)
```

Rubi [A] time = 0.113286, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2736, 2672, 3767}

$$\frac{4\tan^3(e+fx)}{15a^2c^3f}+\frac{4\tan(e+fx)}{5a^2c^3f}+\frac{\sec^3(e+fx)}{5a^2f\left(c^3-c^3\sin(e+fx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^3),x]
```

```
[Out] Sec[e + f*x]^3/(5*a^2*f*(c^3 - c^3*Sin[e + f*x])) + (4*Tan[e + f*x])/(5*a^2
*c^3*f) + (4*Tan[e + f*x]^3)/(15*a^2*c^3*f)
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^3}\,dx = \frac{\int \frac{\sec^4(e+fx)}{c-c\sin(e+fx)}\,dx}{a^2c^2}$$
$$= \frac{\sec^3(e+fx)}{5a^2f\left(c^3-c^3\sin(e+fx)\right)} + \frac{4\int \sec^4(e+fx)\,dx}{5a^2c^3}$$
$$= \frac{\sec^3(e+fx)}{5a^2f\left(c^3-c^3\sin(e+fx)\right)} - \frac{4\,\text{Subst}\left(\int \left(1+x^2\right)dx, x, -\tan(e+fx)\right.}{5a^2c^3f}$$
$$= \frac{\sec^3(e+fx)}{5a^2f\left(c^3-c^3\sin(e+fx)\right)} + \frac{4\tan(e+fx)}{5a^2c^3f} + \frac{4\tan^3(e+fx)}{15a^2c^3f}$$

Mathematica [A] time = 0.82248, size = 131, normalized size = 1.72

$$-\frac{18\sin(e+fx)+512\sin(2(e+fx))+27\sin(3(e+fx))+128\sin(4(e+fx))+9\sin(5(e+fx))+128\cos(e+fx)-}{1920a^2c^3f(\sin(e+fx)-1)^3(\sin(e+fx}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^3),x]
```

```
[Out] -(-54 + 128*Cos[e + f*x] - 72*Cos[2*(e + f*x)] + 192*Cos[3*(e + f*x)] - 18*
Cos[4*(e + f*x)] + 64*Cos[5*(e + f*x)] + 18*Sin[e + f*x] + 512*Sin[2*(e + f
*x)] + 27*Sin[3*(e + f*x)] + 128*Sin[4*(e + f*x)] + 9*Sin[5*(e + f*x)])/(19
20*a^2*c^3*f*(-1 + Sin[e + f*x])^3*(1 + Sin[e + f*x])^2)
```

Maple [A] time = 0.059, size = 133, normalized size = 1.8

$$2\,\frac{1}{fc^3a^2}\left(-1/5\,\left(\tan\left(1/2\,fx+e/2\right)-1\right)^{-5}-1/2\,\left(\tan\left(1/2\,fx+e/2\right)-1\right)^{-4}-5/6\,\left(\tan\left(1/2\,fx+e/2\right)-1\right)^{-3}-3/4\,\left(\tan\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^3,x)
```

```
[Out] 2/f/c^3/a^2*(-1/5/(tan(1/2*f*x+1/2*e)-1)^5-1/2/(tan(1/2*f*x+1/2*e)-1)^4-5/6
/(tan(1/2*f*x+1/2*e)-1)^3-3/4/(tan(1/2*f*x+1/2*e)-1)^2-11/16/(tan(1/2*f*x+1
/2*e)-1)-1/12/(tan(1/2*f*x+1/2*e)+1)^3+1/8/(tan(1/2*f*x+1/2*e)+1)^2-5/16/(t
an(1/2*f*x+1/2*e)+1))
```

Maxima [B] time = 1.37926, size = 452, normalized size = 5.95

$$\frac{2\left(\frac{9\sin(fx+e)}{\cos(fx+e)+1}-\frac{21\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{13\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{25\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{5\sin(fx+e)^5}{(\cos(fx+e)+1)^5}-\frac{15\sin(fx+e)^6}{(\cos(fx+e)+1)^6}+\frac{15\sin(fx+e)^7}{(\cos(fx+e)+1)^7}+3\right.}{15\left(a^2c^3-\frac{2a^2c^3\sin(fx+e)}{\cos(fx+e)+1}-\frac{2a^2c^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{6a^2c^3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}-\frac{6a^2c^3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}+\frac{2a^2c^3\sin(fx+e)^6}{(\cos(fx+e)+1)^6}+\frac{2a^2c^3\sin(fx+e)^7}{(\cos(fx+e)+1)^7}-\frac{a^2c^3\sin(}{(\cos(fx+e}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 2/15*(9*sin(f*x + e)/(cos(f*x + e) + 1) - 21*sin(f*x + e)^2/(cos(f*x + e) +
 1)^2 + 13*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 25*sin(f*x + e)^4/(cos(f*x
 + e) + 1)^4 - 5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 15*sin(f*x + e)^6/(c
os(f*x + e) + 1)^6 + 15*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 3)/((a^2*c^3
- 2*a^2*c^3*sin(f*x + e)/(cos(f*x + e) + 1) - 2*a^2*c^3*sin(f*x + e)^2/(cos
(f*x + e) + 1)^2 + 6*a^2*c^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 6*a^2*c^
3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 2*a^2*c^3*sin(f*x + e)^6/(cos(f*x +
 e) + 1)^6 + 2*a^2*c^3*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - a^2*c^3*sin(f*
x + e)^8/(cos(f*x + e) + 1)^8)*f)
```

Fricas [A] time = 1.29684, size = 211, normalized size = 2.78

$$-\frac{8\,\cos\left(fx+e\right)^{4}-4\,\cos\left(fx+e\right)^{2}+4\left(2\,\cos\left(fx+e\right)^{2}+1\right)\sin\left(fx+e\right)-1}{15\left(a^{2}c^{3}f\cos\left(fx+e\right)^{3}\sin\left(fx+e\right)-a^{2}c^{3}f\cos\left(fx+e\right)^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -1/15*(8*cos(f*x + e)^4 - 4*cos(f*x + e)^2 + 4*(2*cos(f*x + e)^2 + 1)*sin(f
*x + e) - 1)/(a^2*c^3*f*cos(f*x + e)^3*sin(f*x + e) - a^2*c^3*f*cos(f*x + e
)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**2/(c-c*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 2.02454, size = 180, normalized size = 2.37

$$-\frac{\frac{5\left(15\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+24\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+13\right)}{a^{2}c^{3}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^{3}}+\frac{165\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-480\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+650\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-400\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+113}{a^{2}c^{3}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^{5}}}{120\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -1/120*(5*(15*tan(1/2*f*x + 1/2*e)^2 + 24*tan(1/2*f*x + 1/2*e) + 13)/(a^2*c
^3*(tan(1/2*f*x + 1/2*e) + 1)^3) + (165*tan(1/2*f*x + 1/2*e)^4 - 480*tan(1/
2*f*x + 1/2*e)^3 + 650*tan(1/2*f*x + 1/2*e)^2 - 400*tan(1/2*f*x + 1/2*e) +
113)/(a^2*c^3*(tan(1/2*f*x + 1/2*e) - 1)^5))/f
```

3.277 $\int \frac{1}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^4}\,dx$

Optimal. Leaf size=111

$$\frac{4\tan^3(e+fx)}{21a^2c^4f}+\frac{4\tan(e+fx)}{7a^2c^4f}+\frac{\sec^3(e+fx)}{7a^2f\left(c^4-c^4\sin(e+fx)\right)}+\frac{\sec^3(e+fx)}{7a^2f\left(c^2-c^2\sin(e+fx)\right)^2}$$

```
[Out] Sec[e + f*x]^3/(7*a^2*f*(c^2 - c^2*Sin[e + f*x])^2) + Sec[e + f*x]^3/(7*a^2
*f*(c^4 - c^4*Sin[e + f*x])) + (4*Tan[e + f*x])/(7*a^2*c^4*f) + (4*Tan[e +
f*x]^3)/(21*a^2*c^4*f)
```

Rubi [A] time = 0.159483, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2736, 2672, 3767}

$$\frac{4\tan^3(e+fx)}{21a^2c^4f}+\frac{4\tan(e+fx)}{7a^2c^4f}+\frac{\sec^3(e+fx)}{7a^2f\left(c^4-c^4\sin(e+fx)\right)}+\frac{\sec^3(e+fx)}{7a^2f\left(c^2-c^2\sin(e+fx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^4),x]
```

```
[Out] Sec[e + f*x]^3/(7*a^2*f*(c^2 - c^2*Sin[e + f*x])^2) + Sec[e + f*x]^3/(7*a^2
*f*(c^4 - c^4*Sin[e + f*x])) + (4*Tan[e + f*x])/(7*a^2*c^4*f) + (4*Tan[e +
f*x]^3)/(21*a^2*c^4*f)
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^4}\,dx = \frac{\int \frac{\sec^4(e+fx)}{(c-c\sin(e+fx))^2}\,dx}{a^2c^2}$$
$$= \frac{\sec^3(e+fx)}{7a^2f\left(c^2-c^2\sin(e+fx)\right)^2} + \frac{5\int \frac{\sec^4(e+fx)}{c-c\sin(e+fx)}\,dx}{7a^2c^3}$$
$$= \frac{\sec^3(e+fx)}{7a^2f\left(c^2-c^2\sin(e+fx)\right)^2} + \frac{\sec^3(e+fx)}{7a^2f\left(c^4-c^4\sin(e+fx)\right)} + \frac{4\int \sec^4(e+f}{7a^2c^4}$$
$$= \frac{\sec^3(e+fx)}{7a^2f\left(c^2-c^2\sin(e+fx)\right)^2} + \frac{\sec^3(e+fx)}{7a^2f\left(c^4-c^4\sin(e+fx)\right)} - \frac{4\,\text{Subst}\left(\int (1\right.}{}$$
$$= \frac{\sec^3(e+fx)}{7a^2f\left(c^2-c^2\sin(e+fx)\right)^2} + \frac{\sec^3(e+fx)}{7a^2f\left(c^4-c^4\sin(e+fx)\right)} + \frac{4\tan(e+fx)}{7a^2c^4f}$$

Mathematica [A] time = 0.934362, size = 151, normalized size = 1.36

$$\frac{120\sin(e+fx)+1088\sin(2(e+fx))+180\sin(3(e+fx))+128\sin(4(e+fx))+60\sin(5(e+fx))-64\sin(6(e+fx))+}{5376a^2c^4f(\sin(e+}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^4),x]
```

```
[Out] (-210 + 512*Cos[e + f*x] - 255*Cos[2*(e + f*x)] + 768*Cos[3*(e + f*x)] - 30
*Cos[4*(e + f*x)] + 256*Cos[5*(e + f*x)] + 15*Cos[6*(e + f*x)] + 120*Sin[e
+ f*x] + 1088*Sin[2*(e + f*x)] + 180*Sin[3*(e + f*x)] + 128*Sin[4*(e + f*x)
] + 60*Sin[5*(e + f*x)] - 64*Sin[6*(e + f*x)])/(5376*a^2*c^4*f*(-1 + Sin[e
+ f*x])^4*(1 + Sin[e + f*x])^2)
```

Maple [A] time = 0.066, size = 163, normalized size = 1.5

$$2\frac{1}{a^2fc^4}\left(-2/7\left(\tan\left(1/2\,fx+e/2\right)-1\right)^{-7}-\left(\tan\left(1/2\,fx+e/2\right)-1\right)^{-6}-2\left(\tan\left(1/2\,fx+e/2\right)-1\right)^{-5}-5/2\left(\tan\left(1/2\,fx+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^4,x)
```

```
[Out] 2/f/a^2/c^4*(-2/7/(tan(1/2*f*x+1/2*e)-1)^7-1/(tan(1/2*f*x+1/2*e)-1)^6-2/(ta
n(1/2*f*x+1/2*e)-1)^5-5/2/(tan(1/2*f*x+1/2*e)-1)^4-55/24/(tan(1/2*f*x+1/2*e
)-1)^3-23/16/(tan(1/2*f*x+1/2*e)-1)^2-13/16/(tan(1/2*f*x+1/2*e)-1)-1/24/(ta
n(1/2*f*x+1/2*e)+1)^3+1/16/(tan(1/2*f*x+1/2*e)+1)^2-3/16/(tan(1/2*f*x+1/2*e
)+1))
```

Maxima [B] time = 1.61514, size = 576, normalized size = 5.19

$$-\frac{2\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}+\frac{24\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{76\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{28\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{42\sin(fx+e)^5}{(\cos(fx+e)+1)^5}-\frac{56\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-\frac{28\sin(fx+e)^7}{(\cos(fx+e)+1)^7}+\right.}{21\left(a^2c^4-\frac{4a^2c^4\sin(fx+e)}{\cos(fx+e)+1}+\frac{3a^2c^4\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{8a^2c^4\sin(fx+e)^3}{(\cos(fx+e)+1)^3}-\frac{14a^2c^4\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{14a^2c^4\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-\frac{8a^2c^4\sin(fx+e)^7}{(\cos(fx+e)+1)^7}-\frac{3a^2c^4\sin}{(\cos(fx+}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^4,x, algorithm="maxima")
```

```
[Out] -2/21*(3*sin(f*x + e)/(cos(f*x + e) + 1) + 24*sin(f*x + e)^2/(cos(f*x + e)
+ 1)^2 - 76*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 28*sin(f*x + e)^4/(cos(f*
x + e) + 1)^4 + 42*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 56*sin(f*x + e)^6/
(cos(f*x + e) + 1)^6 - 28*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 42*sin(f*x
+ e)^8/(cos(f*x + e) + 1)^8 - 21*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - 6)/(
(a^2*c^4 - 4*a^2*c^4*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*c^4*sin(f*x +
e)^2/(cos(f*x + e) + 1)^2 + 8*a^2*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 -
 14*a^2*c^4*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 14*a^2*c^4*sin(f*x + e)^6
/(cos(f*x + e) + 1)^6 - 8*a^2*c^4*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 3*a
^2*c^4*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 4*a^2*c^4*sin(f*x + e)^9/(cos(
f*x + e) + 1)^9 - a^2*c^4*sin(f*x + e)^10/(cos(f*x + e) + 1)^10)*f)
```

Fricas [A] time = 1.38188, size = 278, normalized size = 2.5

$$-\frac{16 \cos\left(fx+e\right)^{4} - 8 \cos\left(fx+e\right)^{2} - \left(8 \cos\left(fx+e\right)^{4} - 12 \cos\left(fx+e\right)^{2} - 5\right) \sin\left(fx+e\right) - 2}{21 \left(a^{2} c^{4} f \cos\left(fx+e\right)^{5} + 2 a^{2} c^{4} f \cos\left(fx+e\right)^{3} \sin\left(fx+e\right) - 2 a^{2} c^{4} f \cos\left(fx+e\right)^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^4,x, algorithm="fricas")
```

```
[Out] -1/21*(16*cos(f*x + e)^4 - 8*cos(f*x + e)^2 - (8*cos(f*x + e)^4 - 12*cos(f*
x + e)^2 - 5)*sin(f*x + e) - 2)/(a^2*c^4*f*cos(f*x + e)^5 + 2*a^2*c^4*f*cos
(f*x + e)^3*sin(f*x + e) - 2*a^2*c^4*f*cos(f*x + e)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**2/(c-c*sin(f*x+e))**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 2.18122, size = 217, normalized size = 1.95

$$-\frac{\frac{7\left(9 \tan\left(\frac{1}{2} fx+\frac{1}{2} e\right)^{2}+15 \tan\left(\frac{1}{2} fx+\frac{1}{2} e\right)+8\right)}{a^{2} c^{4}\left(\tan\left(\frac{1}{2} fx+\frac{1}{2} e\right)+1\right)^{3}} + \frac{273 \tan\left(\frac{1}{2} fx+\frac{1}{2} e\right)^{6}-1155 \tan\left(\frac{1}{2} fx+\frac{1}{2} e\right)^{5}+2450 \tan\left(\frac{1}{2} fx+\frac{1}{2} e\right)^{4}-2870 \tan\left(\frac{1}{2} fx+\frac{1}{2} e\right)^{3}+2037 \tan(}{a^{2} c^{4}\left(\tan\left(\frac{1}{2} fx+\frac{1}{2} e\right)-1\right)^{7}}}{168 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^4,x, algorithm="giac")
```

```
[Out] -1/168*(7*(9*tan(1/2*f*x + 1/2*e)^2 + 15*tan(1/2*f*x + 1/2*e) + 8)/(a^2*c^4
*(tan(1/2*f*x + 1/2*e) + 1)^3) + (273*tan(1/2*f*x + 1/2*e)^6 - 1155*tan(1/2
*f*x + 1/2*e)^5 + 2450*tan(1/2*f*x + 1/2*e)^4 - 2870*tan(1/2*f*x + 1/2*e)^3
 + 2037*tan(1/2*f*x + 1/2*e)^2 - 791*tan(1/2*f*x + 1/2*e) + 152)/(a^2*c^4*(
tan(1/2*f*x + 1/2*e) - 1)^7))/f
```

3.278 $\int \frac{1}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^5}\,dx$

Optimal. Leaf size=144

$$\frac{8\tan^3(e+fx)}{63a^2c^5f}+\frac{8\tan(e+fx)}{21a^2c^5f}+\frac{2\sec^3(e+fx)}{21a^2f\left(c^5-c^5\sin(e+fx)\right)}+\frac{2\sec^3(e+fx)}{21a^2c^3f(c-c\sin(e+fx))^2}+\frac{\sec^3(e+fx)}{9a^2c^2f(c-c\sin(e+f$$

```
[Out] Sec[e + f*x]^3/(9*a^2*c^2*f*(c - c*Sin[e + f*x])^3) + (2*Sec[e + f*x]^3)/(2
1*a^2*c^3*f*(c - c*Sin[e + f*x])^2) + (2*Sec[e + f*x]^3)/(21*a^2*f*(c^5 - c
^5*Sin[e + f*x])) + (8*Tan[e + f*x])/(21*a^2*c^5*f) + (8*Tan[e + f*x]^3)/(6
3*a^2*c^5*f)
```

Rubi [A] time = 0.215191, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2736, 2672, 3767}

$$\frac{8\tan^3(e+fx)}{63a^2c^5f}+\frac{8\tan(e+fx)}{21a^2c^5f}+\frac{2\sec^3(e+fx)}{21a^2f\left(c^5-c^5\sin(e+fx)\right)}+\frac{2\sec^3(e+fx)}{21a^2c^3f(c-c\sin(e+fx))^2}+\frac{\sec^3(e+fx)}{9a^2c^2f(c-c\sin(e+f$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^5),x]
```



```
[Out] Sec[e + f*x]^3/(9*a^2*c^2*f*(c - c*Sin[e + f*x])^3) + (2*Sec[e + f*x]^3)/(2
1*a^2*c^3*f*(c - c*Sin[e + f*x])^2) + (2*Sec[e + f*x]^3)/(21*a^2*f*(c^5 - c
^5*Sin[e + f*x])) + (8*Tan[e + f*x])/(21*a^2*c^5*f) + (8*Tan[e + f*x]^3)/(6
3*a^2*c^5*f)
```


Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] && !IGtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^5}\,dx = \frac{\int \frac{\sec^4(e+fx)}{(c-c\sin(e+fx))^3}\,dx}{a^2c^2}$$

$$= \frac{\sec^3(e+fx)}{9a^2c^2f(c-c\sin(e+fx))^3} + \frac{2\int \frac{\sec^4(e+fx)}{(c-c\sin(e+fx))^2}\,dx}{3a^2c^3}$$

$$= \frac{\sec^3(e+fx)}{9a^2c^2f(c-c\sin(e+fx))^3} + \frac{2\sec^3(e+fx)}{21a^2c^3f(c-c\sin(e+fx))^2} + \frac{10\int \frac{\sec^4(e+f}{c-c\sin(e+}}{21a^2c^4}$$

$$= \frac{\sec^3(e+fx)}{9a^2c^2f(c-c\sin(e+fx))^3} + \frac{2\sec^3(e+fx)}{21a^2c^3f(c-c\sin(e+fx))^2} + \frac{2\sec^3(}{21a^2f\left(c^5-c\right.}$$

$$= \frac{\sec^3(e+fx)}{9a^2c^2f(c-c\sin(e+fx))^3} + \frac{2\sec^3(e+fx)}{21a^2c^3f(c-c\sin(e+fx))^2} + \frac{2\sec^3(}{21a^2f\left(c^5-c\right.}$$

$$= \frac{\sec^3(e+fx)}{9a^2c^2f(c-c\sin(e+fx))^3} + \frac{2\sec^3(e+fx)}{21a^2c^3f(c-c\sin(e+fx))^2} + \frac{2\sec^3(}{21a^2f\left(c^5-c\right.}$$

Mathematica [A] time = 1.13137, size = 193, normalized size = 1.34

$$\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)(18432\sin(e+fx)+4185\sin(2(e+fx))+1024$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^5),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)*(-5580*Cos[e + f*x] + 13824*Cos[2*(e + f*x)] - 310*Cos[3*(e + f*x)] + 614
4*Cos[4*(e + f*x)] + 930*Cos[5*(e + f*x)] - 512*Cos[6*(e + f*x)] + 18432*Si
n[e + f*x] + 4185*Sin[2*(e + f*x)] + 1024*Sin[3*(e + f*x)] + 1860*Sin[4*(e
+ f*x)] - 3072*Sin[5*(e + f*x)] - 155*Sin[6*(e + f*x)]))/(64512*f*(a + a*Si
n[e + f*x])^2*(c - c*Sin[e + f*x])^5)
```

Maple [A] time = 0.067, size = 193, normalized size = 1.3

$$2\frac{1}{a^2fc^5}\left(-4/9\left(\tan\left(1/2\,fx+e/2\right)-1\right)^{-9}-2\left(\tan\left(1/2\,fx+e/2\right)-1\right)^{-8}-\frac{34}{7\left(\tan\left(1/2\,fx+e/2\right)-1\right)^{7}}-\frac{23}{3\left(\tan\left(1/2\,fx+\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^5,x)
```

```
[Out] 2/f/a^2/c^5*(-4/9/(tan(1/2*f*x+1/2*e)-1)^9-2/(tan(1/2*f*x+1/2*e)-1)^8-34/7/
(tan(1/2*f*x+1/2*e)-1)^7-23/3/(tan(1/2*f*x+1/2*e)-1)^6-35/4/(tan(1/2*f*x+1/
2*e)-1)^5-59/8/(tan(1/2*f*x+1/2*e)-1)^4-19/4/(tan(1/2*f*x+1/2*e)-1)^3-9/4/(
tan(1/2*f*x+1/2*e)-1)^2-57/64/(tan(1/2*f*x+1/2*e)-1)-1/48/(tan(1/2*f*x+1/2*
e)+1)^3+1/32/(tan(1/2*f*x+1/2*e)+1)^2-7/64/(tan(1/2*f*x+1/2*e)+1))
```

Maxima [B] time = 1.73964, size = 701, normalized size = 4.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^5,x, algorithm="maxima")
```

```
[Out] -2/63*(51*sin(f*x + e)/(cos(f*x + e) + 1) - 39*sin(f*x + e)^2/(cos(f*x + e)
 + 1)^2 - 235*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 450*sin(f*x + e)^4/(cos
(f*x + e) + 1)^4 - 306*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 294*sin(f*x +
e)^6/(cos(f*x + e) + 1)^6 + 378*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 63*si
n(f*x + e)^8/(cos(f*x + e) + 1)^8 - 273*sin(f*x + e)^9/(cos(f*x + e) + 1)^9
 + 189*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 63*sin(f*x + e)^11/(cos(f*x
+ e) + 1)^11 - 19)/((a^2*c^5 - 6*a^2*c^5*sin(f*x + e)/(cos(f*x + e) + 1) +
12*a^2*c^5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 2*a^2*c^5*sin(f*x + e)^3/(
cos(f*x + e) + 1)^3 - 27*a^2*c^5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 36*a
^2*c^5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 36*a^2*c^5*sin(f*x + e)^7/(cos
(f*x + e) + 1)^7 + 27*a^2*c^5*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 2*a^2*c
^5*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - 12*a^2*c^5*sin(f*x + e)^10/(cos(f*
x + e) + 1)^10 + 6*a^2*c^5*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 - a^2*c^5*
sin(f*x + e)^12/(cos(f*x + e) + 1)^12)*f)
```

Fricas [A] time = 1.38318, size = 351, normalized size = 2.44

$$\frac{16\cos\left(fx+e\right)^{6}-72\cos\left(fx+e\right)^{4}+30\cos\left(fx+e\right)^{2}+2\left(24\cos\left(fx+e\right)^{4}-20\cos\left(fx+e\right)^{2}-7\right)\sin\left(fx+e\right)+7}{63\left(3a^{2}c^{5}f\cos\left(fx+e\right)^{5}-4a^{2}c^{5}f\cos\left(fx+e\right)^{3}-\left(a^{2}c^{5}f\cos\left(fx+e\right)^{5}-4a^{2}c^{5}f\cos\left(fx+e\right)^{3}\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^5,x, algorithm="fricas")
```

```
[Out] 1/63*(16*cos(f*x + e)^6 - 72*cos(f*x + e)^4 + 30*cos(f*x + e)^2 + 2*(24*cos
(f*x + e)^4 - 20*cos(f*x + e)^2 - 7)*sin(f*x + e) + 7)/(3*a^2*c^5*f*cos(f*x
 + e)^5 - 4*a^2*c^5*f*cos(f*x + e)^3 - (a^2*c^5*f*cos(f*x + e)^5 - 4*a^2*c^
5*f*cos(f*x + e)^3)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**2/(c-c*sin(f*x+e))**5,x)
```

```
[Out] Timed out
```

Giac [A] time = 2.04002, size = 255, normalized size = 1.77

$$-\frac{\frac{21\left(21\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+36\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+19\right)}{a^{2}c^{5}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^{3}}+\frac{3591\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{8}-19656\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{7}+56196\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{6}-95760\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{5}+10\cdots}{a^{2}c^{5}\left(\tan\left(\frac{1}{2}\cdots\right.\right.}}{2016\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^5,x, algorithm="giac")
```

```
[Out] -1/2016*(21*(21*tan(1/2*f*x + 1/2*e)^2 + 36*tan(1/2*f*x + 1/2*e) + 19)/(a^2
*c^5*(tan(1/2*f*x + 1/2*e) + 1)^3) + (3591*tan(1/2*f*x + 1/2*e)^8 - 19656*t
an(1/2*f*x + 1/2*e)^7 + 56196*tan(1/2*f*x + 1/2*e)^6 - 95760*tan(1/2*f*x +
1/2*e)^5 + 107730*tan(1/2*f*x + 1/2*e)^4 - 79464*tan(1/2*f*x + 1/2*e)^3 + 3
8484*tan(1/2*f*x + 1/2*e)^2 - 10944*tan(1/2*f*x + 1/2*e) + 1615)/(a^2*c^5*(
tan(1/2*f*x + 1/2*e) - 1)^9))/f
```

3.279 $\int \frac{(c-c\sin(e+fx))^5}{(a+a\sin(e+fx))^3}\,dx$

Optimal. Leaf size=161

$$-\frac{63c^5\cos(e+fx)}{2a^3f}-\frac{2a^4c^5\cos^9(e+fx)}{5f(a\sin(e+fx)+a)^7}+\frac{6a^2c^5\cos^7(e+fx)}{5f(a\sin(e+fx)+a)^5}-\frac{21c^5\cos^3(e+fx)}{2f\left(a^3\sin(e+fx)+a^3\right)}-\frac{63c^5x}{2a^3}-\frac{42c^5\mathrm{co}}{5f(a\sin(}$$

```
[Out] (-63*c^5*x)/(2*a^3) - (63*c^5*Cos[e + f*x])/(2*a^3*f) - (2*a^4*c^5*Cos[e +
f*x]^9)/(5*f*(a + a*Sin[e + f*x])^7) + (6*a^2*c^5*Cos[e + f*x]^7)/(5*f*(a +
 a*Sin[e + f*x])^5) - (42*c^5*Cos[e + f*x]^5)/(5*f*(a + a*Sin[e + f*x])^3)
- (21*c^5*Cos[e + f*x]^3)/(2*f*(a^3 + a^3*Sin[e + f*x]))
```

Rubi [A] time = 0.27595, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {2736, 2680, 2679, 2682, 8}

$$-\frac{63c^5\cos(e+fx)}{2a^3f}-\frac{2a^4c^5\cos^9(e+fx)}{5f(a\sin(e+fx)+a)^7}+\frac{6a^2c^5\cos^7(e+fx)}{5f(a\sin(e+fx)+a)^5}-\frac{21c^5\cos^3(e+fx)}{2f\left(a^3\sin(e+fx)+a^3\right)}-\frac{63c^5x}{2a^3}-\frac{42c^5\mathrm{co}}{5f(a\sin(}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sin[e + f*x])^5/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] (-63*c^5*x)/(2*a^3) - (63*c^5*Cos[e + f*x])/(2*a^3*f) - (2*a^4*c^5*Cos[e +
f*x]^9)/(5*f*(a + a*Sin[e + f*x])^7) + (6*a^2*c^5*Cos[e + f*x]^7)/(5*f*(a +
 a*Sin[e + f*x])^5) - (42*c^5*Cos[e + f*x]^5)/(5*f*(a + a*Sin[e + f*x])^3)
- (21*c^5*Cos[e + f*x]^3)/(2*f*(a^3 + a^3*Sin[e + f*x]))
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2679

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(a*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{(c-c\sin(e+fx))^5}{(a+a\sin(e+fx))^3}\,dx = \left(a^5c^5\right)\int \frac{\cos^{10}(e+fx)}{(a+a\sin(e+fx))^8}\,dx$$
$$= -\frac{2a^4c^5\cos^9(e+fx)}{5f(a+a\sin(e+fx))^7} - \frac{1}{5}\left(9a^3c^5\right)\int \frac{\cos^8(e+fx)}{(a+a\sin(e+fx))^6}\,dx$$
$$= -\frac{2a^4c^5\cos^9(e+fx)}{5f(a+a\sin(e+fx))^7} + \frac{6a^2c^5\cos^7(e+fx)}{5f(a+a\sin(e+fx))^5} + \frac{1}{5}\left(21ac^5\right)\int \frac{\cos^6(e+fx)}{(a+a\sin(e+fx))^4}\,dx$$
$$= -\frac{2a^4c^5\cos^9(e+fx)}{5f(a+a\sin(e+fx))^7} + \frac{6a^2c^5\cos^7(e+fx)}{5f(a+a\sin(e+fx))^5} - \frac{42c^5\cos^5(e+fx)}{5f(a+a\sin(e+fx))^3} - \frac{\left(21c^5\right)\int \frac{\cos}{(a+a\,\mathrm{s}}}{a}$$
$$= -\frac{2a^4c^5\cos^9(e+fx)}{5f(a+a\sin(e+fx))^7} + \frac{6a^2c^5\cos^7(e+fx)}{5f(a+a\sin(e+fx))^5} - \frac{42c^5\cos^5(e+fx)}{5f(a+a\sin(e+fx))^3} - \frac{21c^5\cos^3(}{2f\left(a^3+a^3\sin\right.}$$
$$= -\frac{63c^5\cos(e+fx)}{2a^3f} - \frac{2a^4c^5\cos^9(e+fx)}{5f(a+a\sin(e+fx))^7} + \frac{6a^2c^5\cos^7(e+fx)}{5f(a+a\sin(e+fx))^5} - \frac{42c^5\cos^5(e+fx)}{5f(a+a\sin(e+fx}$$
$$= -\frac{63c^5x}{2a^3} - \frac{63c^5\cos(e+fx)}{2a^3f} - \frac{2a^4c^5\cos^9(e+fx)}{5f(a+a\sin(e+fx))^7} + \frac{6a^2c^5\cos^7(e+fx)}{5f(a+a\sin(e+fx))^5} - \frac{42c^5\cos}{5f(a+a\sin}$$

Mathematica [A] time = 0.849494, size = 303, normalized size = 1.88

$$(c-c\sin(e+fx))^5\left(\sin\left(\tfrac{1}{2}(e+fx)\right)+\cos\left(\tfrac{1}{2}(e+fx)\right)\right)\left(256\sin\left(\tfrac{1}{2}(e+fx)\right)-630(e+fx)\left(\sin\left(\tfrac{1}{2}(e+fx)\right)+\cos\left(\tfrac{1}{2}(e+\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sin[e + f*x])^5/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(c - c*Sin[e + f*x])^5*(256*Sin[(e +
 f*x)/2] - 128*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) - 896*Sin[(e + f*x)/2]
*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + 448*(Cos[(e + f*x)/2] + Sin[(e +
 f*x)/2])^3 + 2304*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4
 - 630*(e + f*x)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5 - 160*Cos[e + f*x]
*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5 + 5*(Cos[(e + f*x)/2] + Sin[(e + f
*x)/2])^5*Sin[2*(e + f*x)]))/(20*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^10
*(a + a*Sin[e + f*x])^3)
```

Maple [A] time = 0.108, size = 277, normalized size = 1.7

$$-\frac{c^5}{fa^3}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3\left(1+\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2\right)^{-2}-16\,\frac{c^5\left(\tan\left(1/2\,fx+e/2\right)\right)^2}{fa^3\left(1+\left(\tan\left(1/2\,fx+e/2\right)\right)^2\right)^2}+\frac{c^5}{fa^3}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\left(1+\left(\mathrm{ta}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))^5/(a+a*sin(f*x+e))^3,x)
```

```
[Out] -1/f*c^5/a^3/(1+tan(1/2*f*x+1/2*e)^2)^2*tan(1/2*f*x+1/2*e)^3-16/f*c^5/a^3/(
1+tan(1/2*f*x+1/2*e)^2)^2*tan(1/2*f*x+1/2*e)^2+1/f*c^5/a^3/(1+tan(1/2*f*x+1
/2*e)^2)^2*tan(1/2*f*x+1/2*e)-16/f*c^5/a^3/(1+tan(1/2*f*x+1/2*e)^2)^2-63/f*
c^5/a^3*arctan(tan(1/2*f*x+1/2*e))-256/5/f*c^5/a^3/(tan(1/2*f*x+1/2*e)+1)^5
+128/f*c^5/a^3/(tan(1/2*f*x+1/2*e)+1)^4-64/f*c^5/a^3/(tan(1/2*f*x+1/2*e)+1)
^3-32/f*c^5/a^3/(tan(1/2*f*x+1/2*e)+1)^2-64/f*c^5/a^3/(tan(1/2*f*x+1/2*e)+1
)
```

Maxima [B] time = 3.68809, size = 2020, normalized size = 12.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^5/(a+a*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] -1/15*(c^5*((1325*sin(f*x + e)/(cos(f*x + e) + 1) + 2673*sin(f*x + e)^2/(co
s(f*x + e) + 1)^2 + 3805*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 4329*sin(f*x
 + e)^4/(cos(f*x + e) + 1)^4 + 3575*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 2
275*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 975*sin(f*x + e)^7/(cos(f*x + e)
+ 1)^7 + 195*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 304)/(a^3 + 5*a^3*sin(f*
x + e)/(cos(f*x + e) + 1) + 12*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 20
*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 26*a^3*sin(f*x + e)^4/(cos(f*x +
 e) + 1)^4 + 26*a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 20*a^3*sin(f*x +
e)^6/(cos(f*x + e) + 1)^6 + 12*a^3*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 5*
a^3*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + a^3*sin(f*x + e)^9/(cos(f*x + e)
+ 1)^9) + 195*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^3) + 30*c^5*((105*s
in(f*x + e)/(cos(f*x + e) + 1) + 189*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 +
200*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 160*sin(f*x + e)^4/(cos(f*x + e)
+ 1)^4 + 75*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 15*sin(f*x + e)^6/(cos(f*
x + e) + 1)^6 + 24)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 11*a^3*s
in(f*x + e)^2/(cos(f*x + e) + 1)^2 + 15*a^3*sin(f*x + e)^3/(cos(f*x + e) +
1)^3 + 15*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 11*a^3*sin(f*x + e)^5/(
cos(f*x + e) + 1)^5 + 5*a^3*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + a^3*sin(f
*x + e)^7/(cos(f*x + e) + 1)^7) + 15*arctan(sin(f*x + e)/(cos(f*x + e) + 1)
)/a^3) + 20*c^5*((95*sin(f*x + e)/(cos(f*x + e) + 1) + 145*sin(f*x + e)^2/(
cos(f*x + e) + 1)^2 + 75*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*sin(f*x +
 e)^4/(cos(f*x + e) + 1)^4 + 22)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) +
1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(co
s(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x
 + e)^5/(cos(f*x + e) + 1)^5) + 15*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/
a^3) + 2*c^5*(20*sin(f*x + e)/(cos(f*x + e) + 1) + 40*sin(f*x + e)^2/(cos(f
*x + e) + 1)^2 + 30*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*sin(f*x + e)^4
/(cos(f*x + e) + 1)^4 + 7)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 1
0*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x
+ e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^
```

```
5/(cos(f*x + e) + 1)^5) + 40*c^5*(5*sin(f*x + e)/(cos(f*x + e) + 1) + 10*si
n(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x +
 e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)
^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*s
in(f*x + e)^5/(cos(f*x + e) + 1)^5) - 30*c^5*(5*sin(f*x + e)/(cos(f*x + e)
+ 1) + 5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 5*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*
x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3
+ 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x +
 e) + 1)^5))/f
```

Fricas [A] time = 1.38678, size = 707, normalized size = 4.39

$$-\frac{5\,c^5\cos\left(fx+e\right)^5+70\,c^5\cos\left(fx+e\right)^4-1260\,c^5fx-64\,c^5+7\left(45\,c^5fx+113\,c^5\right)\cos\left(fx+e\right)^3+\left(945\,c^5fx-502\,c^5\right)\cos\cdots}{10\left(a^3f\cos\left(fx+e\right)^3+3\,a^3\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^5/(a+a*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -1/10*(5*c^5*cos(f*x + e)^5 + 70*c^5*cos(f*x + e)^4 - 1260*c^5*f*x - 64*c^5
 + 7*(45*c^5*f*x + 113*c^5)*cos(f*x + e)^3 + (945*c^5*f*x - 502*c^5)*cos(f*
x + e)^2 - 2*(315*c^5*f*x + 646*c^5)*cos(f*x + e) - (5*c^5*cos(f*x + e)^4 -
 65*c^5*cos(f*x + e)^3 + 1260*c^5*f*x - 64*c^5 - 3*(105*c^5*f*x - 242*c^5)*
cos(f*x + e)^2 + 2*(315*c^5*f*x + 614*c^5)*cos(f*x + e))*sin(f*x + e))/(a^3
*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f
 + (a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))**5/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 2.13942, size = 251, normalized size = 1.56

$$-\frac{\frac{315\left(fx+e\right)c^5}{a^3}+\frac{10\left(c^5\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+16\,c^5\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c^5\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+16\,c^5\right)}{\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+1\right)^2a^3}+\frac{64\left(10\,c^5\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+45\,c^5\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+85\,c^5\tan\left(\frac{1}{2}\cdots\right.\right.}{a^3\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^5}}{10\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^5/(a+a*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -1/10*(315*(f*x + e)*c^5/a^3 + 10*(c^5*tan(1/2*f*x + 1/2*e)^3 + 16*c^5*tan(
1/2*f*x + 1/2*e)^2 - c^5*tan(1/2*f*x + 1/2*e) + 16*c^5)/((tan(1/2*f*x + 1/2
```

```
*e)^2 + 1)^2*a^3) + 64*(10*c^5*tan(1/2*f*x + 1/2*e)^4 + 45*c^5*tan(1/2*f*x
+ 1/2*e)^3 + 85*c^5*tan(1/2*f*x + 1/2*e)^2 + 55*c^5*tan(1/2*f*x + 1/2*e) +
13*c^5)/(a^3*(tan(1/2*f*x + 1/2*e) + 1)^5))/f
```

3.280 $\int \frac{(c-c\sin(e+fx))^4}{(a+a\sin(e+fx))^3}\,dx$

Optimal. Leaf size=124

$$-\frac{7c^4\cos(e+fx)}{a^3f}-\frac{2a^3c^4\cos^7(e+fx)}{5f(a\sin(e+fx)+a)^6}-\frac{7c^4x}{a^3}+\frac{14ac^4\cos^5(e+fx)}{15f(a\sin(e+fx)+a)^4}-\frac{14c^4\cos^3(e+fx)}{3af(a\sin(e+fx)+a)^2}$$

```
[Out] (-7*c^4*x)/a^3 - (7*c^4*Cos[e + f*x])/(a^3*f) - (2*a^3*c^4*Cos[e + f*x]^7)/
(5*f*(a + a*Sin[e + f*x])^6) + (14*a*c^4*Cos[e + f*x]^5)/(15*f*(a + a*Sin[e
 + f*x])^4) - (14*c^4*Cos[e + f*x]^3)/(3*a*f*(a + a*Sin[e + f*x])^2)
```

Rubi [A] time = 0.221114, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2736, 2680, 2682, 8}

$$-\frac{7c^4\cos(e+fx)}{a^3f}-\frac{2a^3c^4\cos^7(e+fx)}{5f(a\sin(e+fx)+a)^6}-\frac{7c^4x}{a^3}+\frac{14ac^4\cos^5(e+fx)}{15f(a\sin(e+fx)+a)^4}-\frac{14c^4\cos^3(e+fx)}{3af(a\sin(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sin[e + f*x])^4/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] (-7*c^4*x)/a^3 - (7*c^4*Cos[e + f*x])/(a^3*f) - (2*a^3*c^4*Cos[e + f*x]^7)/
(5*f*(a + a*Sin[e + f*x])^6) + (14*a*c^4*Cos[e + f*x]^5)/(15*f*(a + a*Sin[e
 + f*x])^4) - (14*c^4*Cos[e + f*x]^3)/(3*a*f*(a + a*Sin[e + f*x])^2)
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{(c-c\sin(e+fx))^4}{(a+a\sin(e+fx))^3}\,dx = \left(a^4c^4\right)\int \frac{\cos^8(e+fx)}{(a+a\sin(e+fx))^7}\,dx$$
$$= -\frac{2a^3c^4\cos^7(e+fx)}{5f(a+a\sin(e+fx))^6} - \frac{1}{5}\left(7a^2c^4\right)\int \frac{\cos^6(e+fx)}{(a+a\sin(e+fx))^5}\,dx$$
$$= -\frac{2a^3c^4\cos^7(e+fx)}{5f(a+a\sin(e+fx))^6} + \frac{14ac^4\cos^5(e+fx)}{15f(a+a\sin(e+fx))^4} + \frac{1}{3}\left(7c^4\right)\int \frac{\cos^4(e+fx)}{(a+a\sin(e+fx))^3}\,dx$$
$$= -\frac{2a^3c^4\cos^7(e+fx)}{5f(a+a\sin(e+fx))^6} + \frac{14ac^4\cos^5(e+fx)}{15f(a+a\sin(e+fx))^4} - \frac{14c^4\cos^3(e+fx)}{3af(a+a\sin(e+fx))^2} - \frac{\left(7c^4\right)\int}{}$$
$$= -\frac{7c^4\cos(e+fx)}{a^3f} - \frac{2a^3c^4\cos^7(e+fx)}{5f(a+a\sin(e+fx))^6} + \frac{14ac^4\cos^5(e+fx)}{15f(a+a\sin(e+fx))^4} - \frac{14c^4\cos^3(e+}{3af(a+a\sin(e}$$
$$= -\frac{7c^4x}{a^3} - \frac{7c^4\cos(e+fx)}{a^3f} - \frac{2a^3c^4\cos^7(e+fx)}{5f(a+a\sin(e+fx))^6} + \frac{14ac^4\cos^5(e+fx)}{15f(a+a\sin(e+fx))^4} - \frac{14c^4\,c}{3af(a+}$$

Mathematica [B] time = 0.601386, size = 270, normalized size = 2.18

$$\frac{(c-c\sin(e+fx))^4\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(96\sin\left(\frac{1}{2}(e+fx)\right)-105(e+fx)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sin[e + f*x])^4/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(96*Sin[(e + f*x)/2] - 48*(Cos[(e +
f*x)/2] + Sin[(e + f*x)/2]) - 256*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[
(e + f*x)/2])^2 + 128*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 + 464*Sin[(e
+ f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4 - 105*(e + f*x)*(Cos[(e +
 f*x)/2] + Sin[(e + f*x)/2])^5 - 15*Cos[e + f*x]*(Cos[(e + f*x)/2] + Sin[(e
 + f*x)/2])^5)*(c - c*Sin[e + f*x])^4)/(15*f*(Cos[(e + f*x)/2] - Sin[(e + f
*x)/2])^8*(a + a*Sin[e + f*x])^3)
```

Maple [A] time = 0.099, size = 145, normalized size = 1.2

$$-2\frac{c^4}{fa^3\left(1+\left(\tan\left(1/2\,fx+e/2\right)\right)^2\right)} - 14\frac{c^4\arctan\left(\tan\left(1/2\,fx+e/2\right)\right)}{fa^3} - \frac{128\,c^4}{5\,fa^3}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{-5} + 64\frac{}{fa^3\left(\tan\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))^4/(a+a*sin(f*x+e))^3,x)
```

```
[Out] -2/f*c^4/a^3/(1+tan(1/2*f*x+1/2*e)^2)-14/f*c^4/a^3*arctan(tan(1/2*f*x+1/2*e
))-128/5/f*c^4/a^3/(tan(1/2*f*x+1/2*e)+1)^5+64/f*c^4/a^3/(tan(1/2*f*x+1/2*e
)+1)^4-128/3/f*c^4/a^3/(tan(1/2*f*x+1/2*e)+1)^3-16/f*c^4/a^3/(tan(1/2*f*x+1
/2*e)+1)
```

Maxima [B] time = 1.96228, size = 1480, normalized size = 11.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^4/(a+a*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] -2/15*(3*c^4*((105*sin(f*x + e)/(cos(f*x + e) + 1) + 189*sin(f*x + e)^2/(co
s(f*x + e) + 1)^2 + 200*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 160*sin(f*x +
 e)^4/(cos(f*x + e) + 1)^4 + 75*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 15*si
n(f*x + e)^6/(cos(f*x + e) + 1)^6 + 24)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x
+ e) + 1) + 11*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 15*a^3*sin(f*x + e
)^3/(cos(f*x + e) + 1)^3 + 15*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 11*
a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 5*a^3*sin(f*x + e)^6/(cos(f*x + e
) + 1)^6 + a^3*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) + 15*arctan(sin(f*x + e
)/(cos(f*x + e) + 1))/a^3) + 4*c^4*((95*sin(f*x + e)/(cos(f*x + e) + 1) + 1
45*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 75*sin(f*x + e)^3/(cos(f*x + e) +
1)^3 + 15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 22)/(a^3 + 5*a^3*sin(f*x +
e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3
*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) +
 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 15*arctan(sin(f*x + e)/(
cos(f*x + e) + 1))/a^3) + c^4*(20*sin(f*x + e)/(cos(f*x + e) + 1) + 40*sin(
f*x + e)^2/(cos(f*x + e) + 1)^2 + 30*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 +
15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 7)/(a^3 + 5*a^3*sin(f*x + e)/(cos(
f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x
 + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 +
a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 12*c^4*(5*sin(f*x + e)/(cos(f*x
+ e) + 1) + 10*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/(a^3 + 5*a^3*sin(f*
x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10
*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x +
e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) - 12*c^4*(5*sin(f*x +
e)/(cos(f*x + e) + 1) + 5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 5*sin(f*x +
 e)^3/(cos(f*x + e) + 1)^3 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1
) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos
(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x
+ e)^5/(cos(f*x + e) + 1)^5))/f
```

Fricas [B] time = 1.3706, size = 633, normalized size = 5.1

$$-\frac{15\,c^4\cos\left(fx+e\right)^4-420\,c^4fx-48\,c^4+\left(105\,c^4fx+277\,c^4\right)\cos\left(fx+e\right)^3+\left(315\,c^4fx-134\,c^4\right)\cos\left(fx+e\right)^2-6\left(35\,c\right.}{15\left(a^3f\cos\left(fx+e\right)^3+3\,a^3f\cos\left(fx+e\right)^2-2\,a^3f\,c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^4/(a+a*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -1/15*(15*c^4*cos(f*x + e)^4 - 420*c^4*f*x - 48*c^4 + (105*c^4*f*x + 277*c^
4)*cos(f*x + e)^3 + (315*c^4*f*x - 134*c^4)*cos(f*x + e)^2 - 6*(35*c^4*f*x
+ 74*c^4)*cos(f*x + e) + (15*c^4*cos(f*x + e)^3 - 420*c^4*f*x + 48*c^4 + (1
05*c^4*f*x - 262*c^4)*cos(f*x + e)^2 - 6*(35*c^4*f*x + 66*c^4)*cos(f*x + e)
)*sin(f*x + e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 - 2*a^3*f*co
s(f*x + e) - 4*a^3*f + (a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3
*f)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))**4/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 2.05805, size = 182, normalized size = 1.47

$$-\frac{\frac{105\,(fx+e)c^4}{a^3}+\frac{30\,c^4}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)a^3}+\frac{16\left(15\,c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+60\,c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+130\,c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+80\,c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+19\,c^4\right)}{a^3\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^5}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^4/(a+a*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -1/15*(105*(f*x + e)*c^4/a^3 + 30*c^4/((tan(1/2*f*x + 1/2*e)^2 + 1)*a^3) +
16*(15*c^4*tan(1/2*f*x + 1/2*e)^4 + 60*c^4*tan(1/2*f*x + 1/2*e)^3 + 130*c^4
*tan(1/2*f*x + 1/2*e)^2 + 80*c^4*tan(1/2*f*x + 1/2*e) + 19*c^4)/(a^3*(tan(1
/2*f*x + 1/2*e) + 1)^5))/f
```

3.281 $\int \frac{(c-c\sin(e+fx))^3}{(a+a\sin(e+fx))^3} dx$

Optimal. Leaf size=103

$$-\frac{2a^2c^3\cos^5(e+fx)}{5f(a\sin(e+fx)+a)^5}-\frac{2c^3\cos(e+fx)}{f\left(a^3\sin(e+fx)+a^3\right)}-\frac{c^3x}{a^3}+\frac{2c^3\cos^3(e+fx)}{3f(a\sin(e+fx)+a)^3}$$

```
[Out] -((c^3*x)/a^3) - (2*a^2*c^3*Cos[e + f*x]^5)/(5*f*(a + a*Sin[e + f*x])^5) +
(2*c^3*Cos[e + f*x]^3)/(3*f*(a + a*Sin[e + f*x])^3) - (2*c^3*Cos[e + f*x])/
(f*(a^3 + a^3*Sin[e + f*x]))
```

Rubi [A] time = 0.179353, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2736, 2680, 8}

$$-\frac{2a^2c^3\cos^5(e+fx)}{5f(a\sin(e+fx)+a)^5}-\frac{2c^3\cos(e+fx)}{f\left(a^3\sin(e+fx)+a^3\right)}-\frac{c^3x}{a^3}+\frac{2c^3\cos^3(e+fx)}{3f(a\sin(e+fx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sin[e + f*x])^3/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] -((c^3*x)/a^3) - (2*a^2*c^3*Cos[e + f*x]^5)/(5*f*(a + a*Sin[e + f*x])^5) +
(2*c^3*Cos[e + f*x]^3)/(3*f*(a + a*Sin[e + f*x])^3) - (2*c^3*Cos[e + f*x])/
(f*(a^3 + a^3*Sin[e + f*x]))
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{(c-c\sin(e+fx))^3}{(a+a\sin(e+fx))^3}\,dx = \left(a^3c^3\right)\int \frac{\cos^6(e+fx)}{(a+a\sin(e+fx))^6}\,dx$$
$$= -\frac{2a^2c^3\cos^5(e+fx)}{5f(a+a\sin(e+fx))^5} - \left(ac^3\right)\int \frac{\cos^4(e+fx)}{(a+a\sin(e+fx))^4}\,dx$$
$$= -\frac{2a^2c^3\cos^5(e+fx)}{5f(a+a\sin(e+fx))^5} + \frac{2c^3\cos^3(e+fx)}{3f(a+a\sin(e+fx))^3} + \frac{c^3\int \frac{\cos^2(e+fx)}{(a+a\sin(e+fx))^2}\,dx}{a}$$
$$= -\frac{2a^2c^3\cos^5(e+fx)}{5f(a+a\sin(e+fx))^5} + \frac{2c^3\cos^3(e+fx)}{3f(a+a\sin(e+fx))^3} - \frac{2c^3\cos(e+fx)}{f\left(a^3+a^3\sin(e+fx)\right)} - \frac{c^3\int 1\,dx}{a^3}$$
$$= -\frac{c^3x}{a^3} - \frac{2a^2c^3\cos^5(e+fx)}{5f(a+a\sin(e+fx))^5} + \frac{2c^3\cos^3(e+fx)}{3f(a+a\sin(e+fx))^3} - \frac{2c^3\cos(e+fx)}{f\left(a^3+a^3\sin(e+fx)\right)}$$

Mathematica [B] time = 0.427202, size = 239, normalized size = 2.32

$$(c-c\sin(e+fx))^3\left(\sin\left(\tfrac{1}{2}(e+fx)\right)+\cos\left(\tfrac{1}{2}(e+fx)\right)\right)\left(48\sin\left(\tfrac{1}{2}(e+fx)\right)-15(e+fx)\left(\sin\left(\tfrac{1}{2}(e+fx)\right)+\cos\left(\tfrac{1}{2}(e+\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sin[e + f*x])^3/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(48*Sin[(e + f*x)/2] - 24*(Cos[(e +
f*x)/2] + Sin[(e + f*x)/2]) - 88*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(
e + f*x)/2])^2 + 44*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 + 92*Sin[(e + f
*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4 - 15*(e + f*x)*(Cos[(e + f*x
)/2] + Sin[(e + f*x)/2])^5)*(c - c*Sin[e + f*x])^3)/(15*f*(Cos[(e + f*x)/2]
 - Sin[(e + f*x)/2])^6*(a + a*Sin[e + f*x])^3)
```

Maple [A] time = 0.092, size = 143, normalized size = 1.4

$$-2\,\frac{c^3\arctan\left(\tan\left(1/2\,fx+e/2\right)\right)}{fa^3} - \frac{64\,c^3}{5\,fa^3}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{-5} + 32\,\frac{c^3}{fa^3\left(\tan\left(1/2\,fx+e/2\right)+1\right)^4} - \frac{80\,c^3}{3\,fa^3}\left(\tan\left(\frac{f}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))^3/(a+a*sin(f*x+e))^3,x)
```

```
[Out] -2/f*c^3/a^3*arctan(tan(1/2*f*x+1/2*e))-64/5/f*c^3/a^3/(tan(1/2*f*x+1/2*e)+
1)^5+32/f*c^3/a^3/(tan(1/2*f*x+1/2*e)+1)^4-80/3/f*c^3/a^3/(tan(1/2*f*x+1/2*
e)+1)^3+8/f*c^3/a^3/(tan(1/2*f*x+1/2*e)+1)^2-4/f*c^3/a^3/(tan(1/2*f*x+1/2*e
)+1)
```

Maxima [B] time = 2.06637, size = 1054, normalized size = 10.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^3/(a+a*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] -2/15*(c^3*((95*sin(f*x + e)/(cos(f*x + e) + 1) + 145*sin(f*x + e)^2/(cos(f
*x + e) + 1)^2 + 75*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*sin(f*x + e)^4
/(cos(f*x + e) + 1)^4 + 22)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) +
10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x
 + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)
^5/(cos(f*x + e) + 1)^5) + 15*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^3)
+ c^3*(20*sin(f*x + e)/(cos(f*x + e) + 1) + 40*sin(f*x + e)^2/(cos(f*x + e)
 + 1)^2 + 30*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*sin(f*x + e)^4/(cos(f
*x + e) + 1)^4 + 7)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*s
in(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) +
1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(
f*x + e) + 1)^5) + 6*c^3*(5*sin(f*x + e)/(cos(f*x + e) + 1) + 10*sin(f*x +
e)^2/(cos(f*x + e) + 1)^2 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1)
 + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(
f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x +
 e)^5/(cos(f*x + e) + 1)^5) - 9*c^3*(5*sin(f*x + e)/(cos(f*x + e) + 1) + 5*
sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3
 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/
(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*s
in(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^
5))/f
```

Fricas [B] time = 1.3382, size = 551, normalized size = 5.35

$$\frac{60\,c^3fx - \left(15\,c^3fx + 46\,c^3\right)\cos\left(fx+e\right)^3 + 24\,c^3 - \left(45\,c^3fx - 2\,c^3\right)\cos\left(fx+e\right)^2 + 6\left(5\,c^3fx + 12\,c^3\right)\cos\left(fx+e\right) + \left(60\right.}{15\left(a^3f\cos\left(fx+e\right)^3 + 3\,a^3f\cos\left(fx+e\right)^2 - 2\,a^3f\cos\left(fx+e\right) - 4\,a^3f + \left(a^3f\cos\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^3/(a+a*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/15*(60*c^3*f*x - (15*c^3*f*x + 46*c^3)*cos(f*x + e)^3 + 24*c^3 - (45*c^3*
f*x - 2*c^3)*cos(f*x + e)^2 + 6*(5*c^3*f*x + 12*c^3)*cos(f*x + e) + (60*c^3
*f*x - 24*c^3 - (15*c^3*f*x - 46*c^3)*cos(f*x + e)^2 + 6*(5*c^3*f*x + 8*c^3
)*cos(f*x + e))*sin(f*x + e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^
2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f + (a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*
x + e) - 4*a^3*f)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))**3/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.89325, size = 150, normalized size = 1.46

$$-\frac{\frac{15\,(fx+e)c^3}{a^3}+\frac{4\left(15\,c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+30\,c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+100\,c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+50\,c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+13\,c^3\right)}{a^3\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^5}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^3/(a+a*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -1/15*(15*(f*x + e)*c^3/a^3 + 4*(15*c^3*tan(1/2*f*x + 1/2*e)^4 + 30*c^3*tan
(1/2*f*x + 1/2*e)^3 + 100*c^3*tan(1/2*f*x + 1/2*e)^2 + 50*c^3*tan(1/2*f*x +
 1/2*e) + 13*c^3)/(a^3*(tan(1/2*f*x + 1/2*e) + 1)^5))/f
```

3.282 $\int \frac{(c-c\sin(e+fx))^2}{(a+a\sin(e+fx))^3}\,dx$

Optimal. Leaf size=33

$$-\frac{a^2c^2\cos^5(e+fx)}{5f(a\sin(e+fx)+a)^5}$$

```
[Out] -(a^2*c^2*Cos[e + f*x]^5)/(5*f*(a + a*Sin[e + f*x])^5)
```

Rubi [A] time = 0.0858378, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2736, 2671}

$$-\frac{a^2c^2\cos^5(e+fx)}{5f(a\sin(e+fx)+a)^5}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sin[e + f*x])^2/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] -(a^2*c^2*Cos[e + f*x]^5)/(5*f*(a + a*Sin[e + f*x])^5)
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(c-c\sin(e+fx))^2}{(a+a\sin(e+fx))^3}\,dx &= \left(a^2c^2\right)\int \frac{\cos^4(e+fx)}{(a+a\sin(e+fx))^5}\,dx\\ &= -\frac{a^2c^2\cos^5(e+fx)}{5f(a+a\sin(e+fx))^5}\end{aligned}$$

Mathematica [B] time = 0.388886, size = 81, normalized size = 2.45

$$\frac{c^2\left(10\sin\left(\frac{1}{2}(e+fx)\right)+5\sin\left(\frac{3}{2}(e+fx)\right)-\sin\left(\frac{5}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}{10a^3f(\sin(e+fx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sin[e + f*x])^2/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] (c^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(10*Sin[(e + f*x)/2] + 5*Sin[(3*
(e + f*x))/2] - Sin[(5*(e + f*x))/2]))/(10*a^3*f*(1 + Sin[e + f*x])^3)
```

Maple [B] time = 0.083, size = 88, normalized size = 2.7

$$2\,\frac{c^2}{f a^3}\left(-8\,\left(\tan\left(1/2\,fx+e/2\right)+1\right)^{-3}-\frac{16}{5\,\left(\tan\left(1/2\,fx+e/2\right)+1\right)^{5}}-\left(\tan\left(1/2\,fx+e/2\right)+1\right)^{-1}+8\,\left(\tan\left(1/2\,fx+e/\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))^2/(a+a*sin(f*x+e))^3,x)
```

```
[Out] 2/f*c^2/a^3*(-8/(tan(1/2*f*x+1/2*e)+1)^3-16/5/(tan(1/2*f*x+1/2*e)+1)^5-1/(t
an(1/2*f*x+1/2*e)+1)+8/(tan(1/2*f*x+1/2*e)+1)^4+4/(tan(1/2*f*x+1/2*e)+1)^2)
```

Maxima [B] time = 1.19826, size = 748, normalized size = 22.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^2/(a+a*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] -2/15*(c^2*(20*sin(f*x + e)/(cos(f*x + e) + 1) + 40*sin(f*x + e)^2/(cos(f*x
 + e) + 1)^2 + 30*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*sin(f*x + e)^4/(
cos(f*x + e) + 1)^4 + 7)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*
a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/
(cos(f*x + e) + 1)^5) + 2*c^2*(5*sin(f*x + e)/(cos(f*x + e) + 1) + 10*sin(f
*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e)
 + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/
(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(
f*x + e)^5/(cos(f*x + e) + 1)^5) - 6*c^2*(5*sin(f*x + e)/(cos(f*x + e) + 1)
 + 5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 5*sin(f*x + e)^3/(cos(f*x + e) +
 1)^3 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x +
e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*
a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e)
+ 1)^5))/f
```

Fricas [B] time = 1.28623, size = 400, normalized size = 12.12

$$-\frac{c^2\cos\left(fx+e\right)^3+3\,c^2\cos\left(fx+e\right)^2-2\,c^2\cos\left(fx+e\right)-4\,c^2-\left(c^2\cos\left(fx+e\right)^2-2\,c^2\cos\left(fx+e\right)-4\,c^2\right)\mathrm{s}}{5\left(a^3f\cos\left(fx+e\right)^3+3\,a^3f\cos\left(fx+e\right)^2-2\,a^3f\cos\left(fx+e\right)-4\,a^3f+\left(a^3f\cos\left(fx+e\right)^2-2\,a^3f\cos\left(fx+e\right)-4\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^2/(a+a*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -1/5*(c^2*cos(f*x + e)^3 + 3*c^2*cos(f*x + e)^2 - 2*c^2*cos(f*x + e) - 4*c^
2 - (c^2*cos(f*x + e)^2 - 2*c^2*cos(f*x + e) - 4*c^2)*sin(f*x + e))/(a^3*f*
```

```
cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f +
(a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f)*sin(f*x + e))
```

Sympy [A] time = 49.998, size = 362, normalized size = 10.97

$$\begin{cases} \frac{2c^2\tan^5\left(\frac{e}{2}+\frac{fx}{2}\right)}{5a^3f\tan^5\left(\frac{e}{2}+\frac{fx}{2}\right)+25a^3f\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)+50a^3f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+50a^3f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+25a^3f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+5a^3f} + \frac{}{5a^3f\tan^5\left(\frac{e}{2}+\frac{fx}{2}\right)+25a^3f\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)+5} \\ \frac{x(-c\sin(e)+c)^2}{(a\sin(e)+a)^3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))**2/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((2*c**2*tan(e/2 + f*x/2)**5/(5*a**3*f*tan(e/2 + f*x/2)**5 + 25*a*
*3*f*tan(e/2 + f*x/2)**4 + 50*a**3*f*tan(e/2 + f*x/2)**3 + 50*a**3*f*tan(e/
2 + f*x/2)**2 + 25*a**3*f*tan(e/2 + f*x/2) + 5*a**3*f) + 20*c**2*tan(e/2 +
f*x/2)**3/(5*a**3*f*tan(e/2 + f*x/2)**5 + 25*a**3*f*tan(e/2 + f*x/2)**4 + 5
0*a**3*f*tan(e/2 + f*x/2)**3 + 50*a**3*f*tan(e/2 + f*x/2)**2 + 25*a**3*f*ta
n(e/2 + f*x/2) + 5*a**3*f) + 10*c**2*tan(e/2 + f*x/2)/(5*a**3*f*tan(e/2 + f
*x/2)**5 + 25*a**3*f*tan(e/2 + f*x/2)**4 + 50*a**3*f*tan(e/2 + f*x/2)**3 +
50*a**3*f*tan(e/2 + f*x/2)**2 + 25*a**3*f*tan(e/2 + f*x/2) + 5*a**3*f), Ne(
f, 0)), (x*(-c*sin(e) + c)**2/(a*sin(e) + a)**3, True))
```

Giac [A] time = 1.89197, size = 81, normalized size = 2.45

$$-\frac{2\left(5\,c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+10\,c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+c^2\right)}{5\,a^3f\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^2/(a+a*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -2/5*(5*c^2*tan(1/2*f*x + 1/2*e)^4 + 10*c^2*tan(1/2*f*x + 1/2*e)^2 + c^2)/(
a^3*f*(tan(1/2*f*x + 1/2*e) + 1)^5)
```

3.283 $\int \frac{c-c\sin(e+fx)}{(a+a\sin(e+fx))^3}\,dx$

Optimal. Leaf size=58

$$-\frac{c\cos^3(e+fx)}{15f(a\sin(e+fx)+a)^3}-\frac{ac\cos^3(e+fx)}{5f(a\sin(e+fx)+a)^4}$$

```
[Out] -(a*c*Cos[e + f*x]^3)/(5*f*(a + a*Sin[e + f*x])^4) - (c*Cos[e + f*x]^3)/(15
*f*(a + a*Sin[e + f*x])^3)
```

Rubi [A] time = 0.106667, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2736, 2672, 2671}

$$-\frac{c\cos^3(e+fx)}{15f(a\sin(e+fx)+a)^3}-\frac{ac\cos^3(e+fx)}{5f(a\sin(e+fx)+a)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sin[e + f*x])/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] -(a*c*Cos[e + f*x]^3)/(5*f*(a + a*Sin[e + f*x])^4) - (c*Cos[e + f*x]^3)/(15
*f*(a + a*Sin[e + f*x])^3)
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rubi steps

$$\int \frac{c-c\sin(e+fx)}{(a+a\sin(e+fx))^3}\,dx=(ac)\int \frac{\cos^2(e+fx)}{(a+a\sin(e+fx))^4}\,dx$$
$$=-\frac{ac\cos^3(e+fx)}{5f(a+a\sin(e+fx))^4}+\frac{1}{5}c\int \frac{\cos^2(e+fx)}{(a+a\sin(e+fx))^3}\,dx$$
$$=-\frac{ac\cos^3(e+fx)}{5f(a+a\sin(e+fx))^4}-\frac{c\cos^3(e+fx)}{15f(a+a\sin(e+fx))^3}$$

Mathematica [A] time = 0.329819, size = 92, normalized size = 1.59

$$\frac{c\left(\sin\left(2e+\frac{5fx}{2}\right)-15\cos\left(e+\frac{fx}{2}\right)+5\cos\left(e+\frac{3fx}{2}\right)+5\sin\left(\frac{fx}{2}\right)\right)}{30a^3f\left(\sin\left(\frac{e}{2}\right)+\cos\left(\frac{e}{2}\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sin[e + f*x])/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] (c*(-15*Cos[e + (f*x)/2] + 5*Cos[e + (3*f*x)/2] + 5*Sin[(f*x)/2] + Sin[2*e
+ (5*f*x)/2]))/(30*a^3*f*(Cos[e/2] + Sin[e/2])*(Cos[(e + f*x)/2] + Sin[(e +
 f*x)/2])^5)
```

Maple [A] time = 0.074, size = 86, normalized size = 1.5

$$2\,{\frac{c}{f{a}^{3}}}\left(-14/3\,\left(\tan\left(1/2\,fx+e/2\right)+1\right)^{-3}-8/5\,\left(\tan\left(1/2\,fx+e/2\right)+1\right)^{-5}+4\,\left(\tan\left(1/2\,fx+e/2\right)+1\right)^{-4}-\left(\tan\left(1/2\,fx+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))/(a+a*sin(f*x+e))^3,x)
```

```
[Out] 2/f*c/a^3*(-14/3/(tan(1/2*f*x+1/2*e)+1)^3-8/5/(tan(1/2*f*x+1/2*e)+1)^5+4/(t
an(1/2*f*x+1/2*e)+1)^4-1/(tan(1/2*f*x+1/2*e)+1)+3/(tan(1/2*f*x+1/2*e)+1)^2)
```

Maxima [B] time = 1.20779, size = 522, normalized size = 9.

$$-\frac{2\left(\frac{c\left(\frac{20\sin(fx+e)}{\cos(fx+e)+1}+\frac{40\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{30\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{15\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+7\right)}{a^3+\frac{5a^3\sin(fx+e)}{\cos(fx+e)+1}+\frac{10a^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{10a^3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{5a^3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{a^3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}-\frac{3c\left(\frac{5\sin(fx+e)}{\cos(fx+e)+1}+\frac{5\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{5\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+1\right.}{a^3+\frac{5a^3\sin(fx+e)}{\cos(fx+e)+1}+\frac{10a^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{10a^3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{5a^3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}}\right.}{15f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))/(a+a*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] -2/15*(c*(20*sin(f*x + e)/(cos(f*x + e) + 1) + 40*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 + 30*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*sin(f*x + e)^4/(co
s(f*x + e) + 1)^4 + 7)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^
3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e)
 + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(c
```

```
os(f*x + e) + 1)^5) - 3*c*(5*sin(f*x + e)/(cos(f*x + e) + 1) + 5*sin(f*x +
e)^2/(cos(f*x + e) + 1)^2 + 5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 1)/(a^3
 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e
)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5))/f
```

Fricas [B] time = 1.28202, size = 381, normalized size = 6.57

$$\frac{c\cos\left(fx+e\right)^{3}-2\,c\cos\left(fx+e\right)^{2}+3\,c\cos\left(fx+e\right)-\left(c\cos\left(fx+e\right)^{2}+3\,c\cos\left(fx+e\right)+6\,c\right)\sin\left(fx+\cdots\right.}{15\left(a^{3}f\cos\left(fx+e\right)^{3}+3\,a^{3}f\cos\left(fx+e\right)^{2}-2\,a^{3}f\cos\left(fx+e\right)-4\,a^{3}f+\left(a^{3}f\cos\left(fx+e\right)^{2}-2\,a^{3}f\cos\left(fx+e\right)-4\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))/(a+a*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/15*(c*cos(f*x + e)^3 - 2*c*cos(f*x + e)^2 + 3*c*cos(f*x + e) - (c*cos(f*x
 + e)^2 + 3*c*cos(f*x + e) + 6*c)*sin(f*x + e) + 6*c)/(a^3*f*cos(f*x + e)^3
 + 3*a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f + (a^3*f*cos(f*x
 + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f)*sin(f*x + e))
```

Sympy [A] time = 19.1246, size = 573, normalized size = 9.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((2*c*tan(e/2 + f*x/2)**5/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3
*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/
2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 20*c*tan(e/2 + f*
x/2)**4/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 15
0*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*t
an(e/2 + f*x/2) + 15*a**3*f) - 10*c*tan(e/2 + f*x/2)**3/(15*a**3*f*tan(e/2
+ f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**
3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f
) - 30*c*tan(e/2 + f*x/2)**2/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan
(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*
x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 6*c/(15*a**3*f*tan(e/2
+ f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**
3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f
), Ne(f, 0)), (x*(-c*sin(e) + c)/(a*sin(e) + a)**3, True))
```

Giac [A] time = 2.04911, size = 113, normalized size = 1.95

$$-\frac{2\left(15\,c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}+15\,c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+25\,c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+5\,c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+4\,c\right)}{15\,a^{3}f\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))/(a+a*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -2/15*(15*c*tan(1/2*f*x + 1/2*e)^4 + 15*c*tan(1/2*f*x + 1/2*e)^3 + 25*c*tan
(1/2*f*x + 1/2*e)^2 + 5*c*tan(1/2*f*x + 1/2*e) + 4*c)/(a^3*f*(tan(1/2*f*x +
 1/2*e) + 1)^5)
```

3.284 $\int \frac{1}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))}\,dx$

Optimal. Leaf size=83

$$\frac{2\tan(e+fx)}{5a^3cf}-\frac{\sec(e+fx)}{5cf\left(a^3\sin(e+fx)+a^3\right)}-\frac{\sec(e+fx)}{5acf(a\sin(e+fx)+a)^2}$$

```
[Out] -Sec[e + f*x]/(5*a*c*f*(a + a*Sin[e + f*x])^2) - Sec[e + f*x]/(5*c*f*(a^3 +
 a^3*Sin[e + f*x])) + (2*Tan[e + f*x])/(5*a^3*c*f)
```

Rubi [A] time = 0.147634, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2736, 2672, 3767, 8}

$$\frac{2\tan(e+fx)}{5a^3cf}-\frac{\sec(e+fx)}{5cf\left(a^3\sin(e+fx)+a^3\right)}-\frac{\sec(e+fx)}{5acf(a\sin(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])),x]
```

```
[Out] -Sec[e + f*x]/(5*a*c*f*(a + a*Sin[e + f*x])^2) - Sec[e + f*x]/(5*c*f*(a^3 +
 a^3*Sin[e + f*x])) + (2*Tan[e + f*x])/(5*a^3*c*f)
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))}dx = \frac{\int \frac{\sec^2(e+fx)}{(a+a\sin(e+fx))^2}dx}{ac}$$
$$= -\frac{\sec(e+fx)}{5acf(a+a\sin(e+fx))^2} + \frac{3\int \frac{\sec^2(e+fx)}{a+a\sin(e+fx)}dx}{5a^2c}$$
$$= -\frac{\sec(e+fx)}{5acf(a+a\sin(e+fx))^2} - \frac{\sec(e+fx)}{5cf\left(a^3+a^3\sin(e+fx)\right)} + \frac{2\int \sec^2(e+fx)}{5a^3c}$$
$$= -\frac{\sec(e+fx)}{5acf(a+a\sin(e+fx))^2} - \frac{\sec(e+fx)}{5cf\left(a^3+a^3\sin(e+fx)\right)} - \frac{2\,\mathrm{Subst}(\int 1\,dx,}{5a}$$
$$= -\frac{\sec(e+fx)}{5acf(a+a\sin(e+fx))^2} - \frac{\sec(e+fx)}{5cf\left(a^3+a^3\sin(e+fx)\right)} + \frac{2\tan(e+fx)}{5a^3cf}$$

Mathematica [A] time = 0.58773, size = 111, normalized size = 1.34

$$\frac{-12\sin(e+fx)-32\sin(2(e+fx))-12\sin(3(e+fx))+8\sin(4(e+fx))+32\cos(e+fx)-12\cos(2(e+fx))+32\cos(}{160a^3cf(\sin(e+fx)-1)(\sin(e+fx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])),x]
```

```
[Out] (-15 + 32*Cos[e + f*x] - 12*Cos[2*(e + f*x)] + 32*Cos[3*(e + f*x)] + 3*Cos[
4*(e + f*x)] - 12*Sin[e + f*x] - 32*Sin[2*(e + f*x)] - 12*Sin[3*(e + f*x)]
+ 8*Sin[4*(e + f*x)])/(160*a^3*c*f*(-1 + Sin[e + f*x])*(1 + Sin[e + f*x])^3
)
```

Maple [A] time = 0.06, size = 101, normalized size = 1.2

$$2\frac{1}{a^3cf}\left(-1/8\left(\tan\left(1/2\,fx+e/2\right)-1\right)^{-1}-2/5\left(\tan\left(1/2\,fx+e/2\right)+1\right)^{-5}+\left(\tan\left(1/2\,fx+e/2\right)+1\right)^{-4}-3/2\left(\tan\left(1/2\,fx\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e)),x)
```

```
[Out] 2/f/a^3/c*(-1/8/(tan(1/2*f*x+1/2*e)-1)-2/5/(tan(1/2*f*x+1/2*e)+1)^5+1/(tan(
1/2*f*x+1/2*e)+1)^4-3/2/(tan(1/2*f*x+1/2*e)+1)^3+5/4/(tan(1/2*f*x+1/2*e)+1)
^2-7/8/(tan(1/2*f*x+1/2*e)+1))
```

Maxima [B] time = 1.13727, size = 285, normalized size = 3.43

$$-\frac{2\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}-\frac{10\sin(fx+e)^3}{(\cos(fx+e)+1)^3}-\frac{10\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{5\sin(fx+e)^5}{(\cos(fx+e)+1)^5}+2\right)}{5\left(a^3c+\frac{4a^3c\sin(fx+e)}{\cos(fx+e)+1}+\frac{5a^3c\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{5a^3c\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{4a^3c\sin(fx+e)^5}{(\cos(fx+e)+1)^5}-\frac{a^3c\sin(fx+e)^6}{(\cos(fx+e)+1)^6}\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] -2/5*(3*sin(f*x + e)/(cos(f*x + e) + 1) - 10*sin(f*x + e)^3/(cos(f*x + e) +
 1)^3 - 10*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 5*sin(f*x + e)^5/(cos(f*x
+ e) + 1)^5 + 2)/((a^3*c + 4*a^3*c*sin(f*x + e)/(cos(f*x + e) + 1) + 5*a^3*
c*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 5*a^3*c*sin(f*x + e)^4/(cos(f*x + e
) + 1)^4 - 4*a^3*c*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - a^3*c*sin(f*x + e)
^6/(cos(f*x + e) + 1)^6)*f)
```

Fricas [A] time = 1.37743, size = 208, normalized size = 2.51

$$\frac{4\cos\left(fx+e\right)^2+\left(2\cos\left(fx+e\right)^2-3\right)\sin\left(fx+e\right)-2}{5\left(a^3cf\cos\left(fx+e\right)^3-2\,a^3cf\cos\left(fx+e\right)\sin\left(fx+e\right)-2\,a^3cf\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/5*(4*cos(f*x + e)^2 + (2*cos(f*x + e)^2 - 3)*sin(f*x + e) - 2)/(a^3*c*f*c
os(f*x + e)^3 - 2*a^3*c*f*cos(f*x + e)*sin(f*x + e) - 2*a^3*c*f*cos(f*x + e
))
```

Sympy [A] time = 17.9375, size = 738, normalized size = 8.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**3/(c-c*sin(f*x+e)),x)
```

```
[Out] Piecewise((3*tan(e/2 + f*x/2)**6/(10*a**3*c*f*tan(e/2 + f*x/2)**6 + 40*a**3
*c*f*tan(e/2 + f*x/2)**5 + 50*a**3*c*f*tan(e/2 + f*x/2)**4 - 50*a**3*c*f*ta
n(e/2 + f*x/2)**2 - 40*a**3*c*f*tan(e/2 + f*x/2) - 10*a**3*c*f) - 8*tan(e/2
 + f*x/2)**5/(10*a**3*c*f*tan(e/2 + f*x/2)**6 + 40*a**3*c*f*tan(e/2 + f*x/2
)**5 + 50*a**3*c*f*tan(e/2 + f*x/2)**4 - 50*a**3*c*f*tan(e/2 + f*x/2)**2 -
40*a**3*c*f*tan(e/2 + f*x/2) - 10*a**3*c*f) - 25*tan(e/2 + f*x/2)**4/(10*a*
*3*c*f*tan(e/2 + f*x/2)**6 + 40*a**3*c*f*tan(e/2 + f*x/2)**5 + 50*a**3*c*f*
tan(e/2 + f*x/2)**4 - 50*a**3*c*f*tan(e/2 + f*x/2)**2 - 40*a**3*c*f*tan(e/2
 + f*x/2) - 10*a**3*c*f) - 40*tan(e/2 + f*x/2)**3/(10*a**3*c*f*tan(e/2 + f*
x/2)**6 + 40*a**3*c*f*tan(e/2 + f*x/2)**5 + 50*a**3*c*f*tan(e/2 + f*x/2)**4
 - 50*a**3*c*f*tan(e/2 + f*x/2)**2 - 40*a**3*c*f*tan(e/2 + f*x/2) - 10*a**3
*c*f) - 15*tan(e/2 + f*x/2)**2/(10*a**3*c*f*tan(e/2 + f*x/2)**6 + 40*a**3*c
*f*tan(e/2 + f*x/2)**5 + 50*a**3*c*f*tan(e/2 + f*x/2)**4 - 50*a**3*c*f*tan(
e/2 + f*x/2)**2 - 40*a**3*c*f*tan(e/2 + f*x/2) - 10*a**3*c*f) + 5/(10*a**3*
c*f*tan(e/2 + f*x/2)**6 + 40*a**3*c*f*tan(e/2 + f*x/2)**5 + 50*a**3*c*f*tan
(e/2 + f*x/2)**4 - 50*a**3*c*f*tan(e/2 + f*x/2)**2 - 40*a**3*c*f*tan(e/2 +
f*x/2) - 10*a**3*c*f), Ne(f, 0)), (x/((a*sin(e) + a)**3*(-c*sin(e) + c)), T
rue))
```

<u>Giac</u> [A] time = 2.1182, size = 142, normalized size = 1.71

$$-\frac{\frac{5}{a^{3}c\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}+\frac{35\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}+90\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{3}+120\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+70\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+21}{a^{3}c\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^{5}}}{20f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] -1/20*(5/(a^3*c*(tan(1/2*f*x + 1/2*e) - 1)) + (35*tan(1/2*f*x + 1/2*e)^4 +
90*tan(1/2*f*x + 1/2*e)^3 + 120*tan(1/2*f*x + 1/2*e)^2 + 70*tan(1/2*f*x + 1
/2*e) + 21)/(a^3*c*(tan(1/2*f*x + 1/2*e) + 1)^5))/f
```

3.285 $\int \frac{1}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^2}\,dx$

Optimal. Leaf size=75

$$\frac{4\tan^3(e+fx)}{15a^3c^2f}+\frac{4\tan(e+fx)}{5a^3c^2f}-\frac{\sec^3(e+fx)}{5c^2f\left(a^3\sin(e+fx)+a^3\right)}$$

```
[Out] -Sec[e + f*x]^3/(5*c^2*f*(a^3 + a^3*Sin[e + f*x])) + (4*Tan[e + f*x])/(5*a^
3*c^2*f) + (4*Tan[e + f*x]^3)/(15*a^3*c^2*f)
```

Rubi [A] time = 0.110116, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2736, 2672, 3767}

$$\frac{4\tan^3(e+fx)}{15a^3c^2f}+\frac{4\tan(e+fx)}{5a^3c^2f}-\frac{\sec^3(e+fx)}{5c^2f\left(a^3\sin(e+fx)+a^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^2),x]
```

```
[Out] -Sec[e + f*x]^3/(5*c^2*f*(a^3 + a^3*Sin[e + f*x])) + (4*Tan[e + f*x])/(5*a^
3*c^2*f) + (4*Tan[e + f*x]^3)/(15*a^3*c^2*f)
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^2}\,dx = \frac{\int \frac{\sec^4(e+fx)}{a+a\sin(e+fx)}\,dx}{a^2c^2}$$
$$= -\frac{\sec^3(e+fx)}{5c^2f\left(a^3+a^3\sin(e+fx)\right)} + \frac{4\int \sec^4(e+fx)\,dx}{5a^3c^2}$$
$$= -\frac{\sec^3(e+fx)}{5c^2f\left(a^3+a^3\sin(e+fx)\right)} - \frac{4\,\text{Subst}\left(\int \left(1+x^2\right)\,dx, x, -\tan(e+fx)\right)}{5a^3c^2f}$$
$$= -\frac{\sec^3(e+fx)}{5c^2f\left(a^3+a^3\sin(e+fx)\right)} + \frac{4\tan(e+fx)}{5a^3c^2f} + \frac{4\tan^3(e+fx)}{15a^3c^2f}$$

Mathematica [A] time = 0.743909, size = 131, normalized size = 1.75

$$\frac{18\sin(e+fx)+512\sin(2(e+fx))+27\sin(3(e+fx))+128\sin(4(e+fx))+9\sin(5(e+fx))-128\cos(e+fx)+72\,\text{cc}}{1920a^3c^2f(\sin(e+fx)-1)^2(\sin(e+fx)+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^2),x]
```

```
[Out] (54 - 128*Cos[e + f*x] + 72*Cos[2*(e + f*x)] - 192*Cos[3*(e + f*x)] + 18*Co
s[4*(e + f*x)] - 64*Cos[5*(e + f*x)] + 18*Sin[e + f*x] + 512*Sin[2*(e + f*x
)] + 27*Sin[3*(e + f*x)] + 128*Sin[4*(e + f*x)] + 9*Sin[5*(e + f*x)])/(1920
*a^3*c^2*f*(-1 + Sin[e + f*x])^2*(1 + Sin[e + f*x])^3)
```

Maple [A] time = 0.055, size = 133, normalized size = 1.8

$$2\frac{1}{fa^3c^2}\left(-1/12\left(\tan\left(1/2\,fx+e/2\right)-1\right)^{-3}-1/8\left(\tan\left(1/2\,fx+e/2\right)-1\right)^{-2}-\frac{5}{16\tan\left(1/2\,fx+e/2\right)-16}-1/5\left(\tan\left(1/2\,f\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^2,x)
```

```
[Out] 2/f/a^3/c^2*(-1/12/(tan(1/2*f*x+1/2*e)-1)^3-1/8/(tan(1/2*f*x+1/2*e)-1)^2-5/
16/(tan(1/2*f*x+1/2*e)-1)-1/5/(tan(1/2*f*x+1/2*e)+1)^5+1/2/(tan(1/2*f*x+1/2
*e)+1)^4-5/6/(tan(1/2*f*x+1/2*e)+1)^3+3/4/(tan(1/2*f*x+1/2*e)+1)^2-11/16/(t
an(1/2*f*x+1/2*e)+1))
```

Maxima [B] time = 1.10226, size = 452, normalized size = 6.03

$$\frac{2\left(\frac{9\sin(fx+e)}{\cos(fx+e)+1}+\frac{21\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{13\sin(fx+e)^3}{(\cos(fx+e)+1)^3}-\frac{25\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{5\sin(fx+e)^5}{(\cos(fx+e)+1)^5}+\frac{15\sin(fx+e)^6}{(\cos(fx+e)+1)^6}+\frac{15\sin(fx+e)^7}{(\cos(fx+e)+1)^7}-3\right)}{15\left(a^3c^2+\frac{2a^3c^2\sin(fx+e)}{\cos(fx+e)+1}-\frac{2a^3c^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{6a^3c^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{6a^3c^2\sin(fx+e)^5}{(\cos(fx+e)+1)^5}+\frac{2a^3c^2\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-\frac{2a^3c^2\sin(fx+e)^7}{(\cos(fx+e)+1)^7}-\frac{a^3c^2\sin(fx+e}{(\cos(fx+e)+1}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 2/15*(9*sin(f*x + e)/(cos(f*x + e) + 1) + 21*sin(f*x + e)^2/(cos(f*x + e) +
 1)^2 + 13*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 25*sin(f*x + e)^4/(cos(f*x
 + e) + 1)^4 - 5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 15*sin(f*x + e)^6/(c
os(f*x + e) + 1)^6 + 15*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 3)/((a^3*c^2
+ 2*a^3*c^2*sin(f*x + e)/(cos(f*x + e) + 1) - 2*a^3*c^2*sin(f*x + e)^2/(cos
(f*x + e) + 1)^2 - 6*a^3*c^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 6*a^3*c^
2*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 2*a^3*c^2*sin(f*x + e)^6/(cos(f*x +
 e) + 1)^6 - 2*a^3*c^2*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - a^3*c^2*sin(f*
x + e)^8/(cos(f*x + e) + 1)^8)*f)
```

Fricas [A] time = 1.53963, size = 211, normalized size = 2.81

$$-\frac{8\cos\left(fx+e\right)^{4}-4\cos\left(fx+e\right)^{2}-4\left(2\cos\left(fx+e\right)^{2}+1\right)\sin\left(fx+e\right)-1}{15\left(a^{3}c^{2}f\cos\left(fx+e\right)^{3}\sin\left(fx+e\right)+a^{3}c^{2}f\cos\left(fx+e\right)^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -1/15*(8*cos(f*x + e)^4 - 4*cos(f*x + e)^2 - 4*(2*cos(f*x + e)^2 + 1)*sin(f
*x + e) - 1)/(a^3*c^2*f*cos(f*x + e)^3*sin(f*x + e) + a^3*c^2*f*cos(f*x + e
)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 2.09315, size = 180, normalized size = 2.4

$$-\frac{\frac{5\left(15\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-24\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+13\right)}{a^{3}c^{2}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^{3}}+\frac{165\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}+480\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{3}+650\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+400\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+113}{a^{3}c^{2}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^{5}}}{120f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] -1/120*(5*(15*tan(1/2*f*x + 1/2*e)^2 - 24*tan(1/2*f*x + 1/2*e) + 13)/(a^3*c
^2*(tan(1/2*f*x + 1/2*e) - 1)^3) + (165*tan(1/2*f*x + 1/2*e)^4 + 480*tan(1/
2*f*x + 1/2*e)^3 + 650*tan(1/2*f*x + 1/2*e)^2 + 400*tan(1/2*f*x + 1/2*e) +
113)/(a^3*c^2*(tan(1/2*f*x + 1/2*e) + 1)^5))/f
```

3.286 $\int \frac{1}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^3}\,dx$

Optimal. Leaf size=59

$$\frac{\tan^5(e+fx)}{5a^3c^3f}+\frac{2\tan^3(e+fx)}{3a^3c^3f}+\frac{\tan(e+fx)}{a^3c^3f}$$

```
[Out] Tan[e + f*x]/(a^3*c^3*f) + (2*Tan[e + f*x]^3)/(3*a^3*c^3*f) + Tan[e + f*x]^
5/(5*a^3*c^3*f)
```

Rubi [A] time = 0.0725032, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2736, 3767}

$$\frac{\tan^5(e+fx)}{5a^3c^3f}+\frac{2\tan^3(e+fx)}{3a^3c^3f}+\frac{\tan(e+fx)}{a^3c^3f}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^3),x]
```

```
[Out] Tan[e + f*x]/(a^3*c^3*f) + (2*Tan[e + f*x]^3)/(3*a^3*c^3*f) + Tan[e + f*x]^
5/(5*a^3*c^3*f)
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^3}\,dx &= \frac{\int \sec^6(e+fx)\,dx}{a^3c^3}\\ &= -\frac{\operatorname{Subst}\left(\int \left(1+2x^2+x^4\right)\,dx,x,-\tan(e+fx)\right)}{a^3c^3f}\\ &= \frac{\tan(e+fx)}{a^3c^3f}+\frac{2\tan^3(e+fx)}{3a^3c^3f}+\frac{\tan^5(e+fx)}{5a^3c^3f}\end{aligned}$$

Mathematica [A] time = 0.12801, size = 41, normalized size = 0.69

$$\frac{\frac{1}{5}\tan^5(e+fx)+\frac{2}{3}\tan^3(e+fx)+\tan(e+fx)}{a^3c^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^3),x]
```

```
[Out] (Tan[e + f*x] + (2*Tan[e + f*x]^3)/3 + Tan[e + f*x]^5/5)/(a^3*c^3*f)
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a + a \sin \left(fx + e\right)\right)^3 \left(c - c \sin \left(fx + e\right)\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^3,x)
```

```
[Out] int(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^3,x)
```

Maxima [A] time = 1.13831, size = 54, normalized size = 0.92

$$\frac{3 \tan \left(fx + e\right)^5 + 10 \tan \left(fx + e\right)^3 + 15 \tan \left(fx + e\right)}{15\, a^3 c^3 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 1/15*(3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 + 15*tan(f*x + e))/(a^3*c^3*f)
```

Fricas [A] time = 1.56539, size = 119, normalized size = 2.02

$$\frac{\left(8 \cos \left(fx + e\right)^4 + 4 \cos \left(fx + e\right)^2 + 3\right) \sin \left(fx + e\right)}{15\, a^3 c^3 f \cos \left(fx + e\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/15*(8*cos(f*x + e)^4 + 4*cos(f*x + e)^2 + 3)*sin(f*x + e)/(a^3*c^3*f*cos(
f*x + e)^5)
```

Sympy [A] time = 61.7444, size = 687, normalized size = 11.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((-30*tan(e/2 + f*x/2)**9/(15*a**3*c**3*f*tan(e/2 + f*x/2)**10 - 7
5*a**3*c**3*f*tan(e/2 + f*x/2)**8 + 150*a**3*c**3*f*tan(e/2 + f*x/2)**6 - 1
50*a**3*c**3*f*tan(e/2 + f*x/2)**4 + 75*a**3*c**3*f*tan(e/2 + f*x/2)**2 - 1
5*a**3*c**3*f) + 40*tan(e/2 + f*x/2)**7/(15*a**3*c**3*f*tan(e/2 + f*x/2)**1
0 - 75*a**3*c**3*f*tan(e/2 + f*x/2)**8 + 150*a**3*c**3*f*tan(e/2 + f*x/2)**
6 - 150*a**3*c**3*f*tan(e/2 + f*x/2)**4 + 75*a**3*c**3*f*tan(e/2 + f*x/2)**
2 - 15*a**3*c**3*f) - 116*tan(e/2 + f*x/2)**5/(15*a**3*c**3*f*tan(e/2 + f*x
/2)**10 - 75*a**3*c**3*f*tan(e/2 + f*x/2)**8 + 150*a**3*c**3*f*tan(e/2 + f*
x/2)**6 - 150*a**3*c**3*f*tan(e/2 + f*x/2)**4 + 75*a**3*c**3*f*tan(e/2 + f*
x/2)**2 - 15*a**3*c**3*f) + 40*tan(e/2 + f*x/2)**3/(15*a**3*c**3*f*tan(e/2
+ f*x/2)**10 - 75*a**3*c**3*f*tan(e/2 + f*x/2)**8 + 150*a**3*c**3*f*tan(e/2
 + f*x/2)**6 - 150*a**3*c**3*f*tan(e/2 + f*x/2)**4 + 75*a**3*c**3*f*tan(e/2
 + f*x/2)**2 - 15*a**3*c**3*f) - 30*tan(e/2 + f*x/2)/(15*a**3*c**3*f*tan(e/
2 + f*x/2)**10 - 75*a**3*c**3*f*tan(e/2 + f*x/2)**8 + 150*a**3*c**3*f*tan(e
/2 + f*x/2)**6 - 150*a**3*c**3*f*tan(e/2 + f*x/2)**4 + 75*a**3*c**3*f*tan(e
/2 + f*x/2)**2 - 15*a**3*c**3*f), Ne(f, 0)), (x/((a*sin(e) + a)**3*(-c*sin(
e) + c)**3), True))
```

Giac [A] time = 2.01524, size = 58, normalized size = 0.98

$$\frac{3 \tan\left(fx+e\right)^5 + 10 \tan\left(fx+e\right)^3 + 15 \tan\left(fx+e\right)}{15 a^3 c^3 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/15*(3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 + 15*tan(f*x + e))/(a^3*c^3*f)
```

3.287 $\int \frac{1}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^4} dx$

Optimal. Leaf size=97

$$\frac{6\tan^5(e+fx)}{35a^3c^4f}+\frac{4\tan^3(e+fx)}{7a^3c^4f}+\frac{6\tan(e+fx)}{7a^3c^4f}+\frac{\sec^5(e+fx)}{7a^3f\left(c^4-c^4\sin(e+fx)\right)}$$

```
[Out] Sec[e + f*x]^5/(7*a^3*f*(c^4 - c^4*Sin[e + f*x])) + (6*Tan[e + f*x])/(7*a^3
*c^4*f) + (4*Tan[e + f*x]^3)/(7*a^3*c^4*f) + (6*Tan[e + f*x]^5)/(35*a^3*c^4
*f)
```

Rubi [A] time = 0.119633, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2736, 2672, 3767}

$$\frac{6\tan^5(e+fx)}{35a^3c^4f}+\frac{4\tan^3(e+fx)}{7a^3c^4f}+\frac{6\tan(e+fx)}{7a^3c^4f}+\frac{\sec^5(e+fx)}{7a^3f\left(c^4-c^4\sin(e+fx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^4),x]
```

```
[Out] Sec[e + f*x]^5/(7*a^3*f*(c^4 - c^4*Sin[e + f*x])) + (6*Tan[e + f*x])/(7*a^3
*c^4*f) + (4*Tan[e + f*x]^3)/(7*a^3*c^4*f) + (6*Tan[e + f*x]^5)/(35*a^3*c^4
*f)
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^4}\,dx = \frac{\int \frac{\sec^6(e+fx)}{c-c\sin(e+fx)}\,dx}{a^3c^3}$$
$$= \frac{\sec^5(e+fx)}{7a^3f\left(c^4-c^4\sin(e+fx)\right)} + \frac{6\int \sec^6(e+fx)\,dx}{7a^3c^4}$$
$$= \frac{\sec^5(e+fx)}{7a^3f\left(c^4-c^4\sin(e+fx)\right)} - \frac{6\,\text{Subst}\left(\int \left(1+2x^2+x^4\right)dx, x, -\tan(e+f$$
$$= \frac{\sec^5(e+fx)}{7a^3f\left(c^4-c^4\sin(e+fx)\right)} + \frac{6\tan(e+fx)}{7a^3c^4f} + \frac{4\tan^3(e+fx)}{7a^3c^4f} + \frac{6\tan^5(e$$

Mathematica [A] time = 1.07088, size = 193, normalized size = 1.99

$$\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)(5120\sin(e+fx)+125\sin(2(e+fx))+2560\,\text{si}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^4),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)*(-500*Cos[e + f*x] + 1280*Cos[2*(e + f*x)] - 250*Cos[3*(e + f*x)] + 1024*
Cos[4*(e + f*x)] - 50*Cos[5*(e + f*x)] + 256*Cos[6*(e + f*x)] + 5120*Sin[e
+ f*x] + 125*Sin[2*(e + f*x)] + 2560*Sin[3*(e + f*x)] + 100*Sin[4*(e + f*x)
] + 512*Sin[5*(e + f*x)] + 25*Sin[6*(e + f*x)]))/(17920*f*(a + a*Sin[e + f*
x])^3*(c - c*Sin[e + f*x])^4)
```

Maple [B] time = 0.069, size = 193, normalized size = 2.

$$2\,\frac{1}{fa^3c^4}\left(-1/7\left(\tan\left(1/2\,fx+e/2\right)-1\right)^{-7}-1/2\left(\tan\left(1/2\,fx+e/2\right)-1\right)^{-6}-\frac{21}{20\left(\tan\left(1/2\,fx+e/2\right)-1\right)^5}-\frac{1}{8\left(\tan\left(1/2\,f\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^4,x)
```

```
[Out] 2/f/a^3/c^4*(-1/7/(tan(1/2*f*x+1/2*e)-1)^7-1/2/(tan(1/2*f*x+1/2*e)-1)^6-21/
20/(tan(1/2*f*x+1/2*e)-1)^5-11/8/(tan(1/2*f*x+1/2*e)-1)^4-11/8/(tan(1/2*f*x
+1/2*e)-1)^3-15/16/(tan(1/2*f*x+1/2*e)-1)^2-21/32/(tan(1/2*f*x+1/2*e)-1)-1/
20/(tan(1/2*f*x+1/2*e)+1)^5+1/8/(tan(1/2*f*x+1/2*e)+1)^4-1/4/(tan(1/2*f*x+1
/2*e)+1)^3+1/4/(tan(1/2*f*x+1/2*e)+1)^2-11/32/(tan(1/2*f*x+1/2*e)+1))
```

Maxima [B] time = 1.29869, size = 701, normalized size = 7.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^4,x, algorithm="maxima")
```

```
[Out] 2/35*(25*sin(f*x + e)/(cos(f*x + e) + 1) - 55*sin(f*x + e)^2/(cos(f*x + e)
+ 1)^2 + 15*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 130*sin(f*x + e)^4/(cos(f
*x + e) + 1)^4 + 26*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 182*sin(f*x + e)^
6/(cos(f*x + e) + 1)^6 + 126*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 105*sin(
f*x + e)^8/(cos(f*x + e) + 1)^8 - 35*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 -
35*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 + 35*sin(f*x + e)^11/(cos(f*x + e)
 + 1)^11 + 5)/((a^3*c^4 - 2*a^3*c^4*sin(f*x + e)/(cos(f*x + e) + 1) - 4*a^3
*c^4*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*c^4*sin(f*x + e)^3/(cos(f
*x + e) + 1)^3 + 5*a^3*c^4*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 20*a^3*c^4
*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 20*a^3*c^4*sin(f*x + e)^7/(cos(f*x +
 e) + 1)^7 - 5*a^3*c^4*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 10*a^3*c^4*sin
(f*x + e)^9/(cos(f*x + e) + 1)^9 + 4*a^3*c^4*sin(f*x + e)^10/(cos(f*x + e)
+ 1)^10 + 2*a^3*c^4*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 - a^3*c^4*sin(f*x
 + e)^12/(cos(f*x + e) + 1)^12)*f)
```

Fricas [A] time = 1.66689, size = 263, normalized size = 2.71

$$-\frac{16 \cos\left(fx+e\right)^{6}-8 \cos\left(fx+e\right)^{4}-2 \cos\left(fx+e\right)^{2}+2\left(8 \cos\left(fx+e\right)^{4}+4 \cos\left(fx+e\right)^{2}+3\right)\sin\left(fx+e\right)-1}{35\left(a^{3}c^{4}f\cos\left(fx+e\right)^{5}\sin\left(fx+e\right)-a^{3}c^{4}f\cos\left(fx+e\right)^{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^4,x, algorithm="fricas")
```

```
[Out] -1/35*(16*cos(f*x + e)^6 - 8*cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 2*(8*cos(f
*x + e)^4 + 4*cos(f*x + e)^2 + 3)*sin(f*x + e) - 1)/(a^3*c^4*f*cos(f*x + e)
^5*sin(f*x + e) - a^3*c^4*f*cos(f*x + e)^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 2.27087, size = 255, normalized size = 2.63

$$-\frac{\frac{7\left(55 \tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}+180 \tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{3}+250 \tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+160 \tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+43\right)}{a^{3}c^{4}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^{5}}+\frac{735 \tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{6}-3360 \tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{5}+7315 \tan}{}}{560\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^4,x, algorithm="giac")
```

```
[Out] -1/560*(7*(55*tan(1/2*f*x + 1/2*e)^4 + 180*tan(1/2*f*x + 1/2*e)^3 + 250*tan
(1/2*f*x + 1/2*e)^2 + 160*tan(1/2*f*x + 1/2*e) + 43)/(a^3*c^4*(tan(1/2*f*x
```

```
+ 1/2*e) + 1)^5) + (735*tan(1/2*f*x + 1/2*e)^6 - 3360*tan(1/2*f*x + 1/2*e)^
5 + 7315*tan(1/2*f*x + 1/2*e)^4 - 8820*tan(1/2*f*x + 1/2*e)^3 + 6321*tan(1/
2*f*x + 1/2*e)^2 - 2492*tan(1/2*f*x + 1/2*e) + 461)/(a^3*c^4*(tan(1/2*f*x +
 1/2*e) - 1)^7))/f
```

3.288 $\int \frac{1}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^5}\,dx$

Optimal. Leaf size=131

$$\frac{2\tan^5(e+fx)}{15a^3c^5f}+\frac{4\tan^3(e+fx)}{9a^3c^5f}+\frac{2\tan(e+fx)}{3a^3c^5f}+\frac{\sec^5(e+fx)}{9a^3f\left(c^5-c^5\sin(e+fx)\right)}+\frac{\sec^5(e+fx)}{9a^3c^3f(c-c\sin(e+fx))^2}$$

```
[Out] Sec[e + f*x]^5/(9*a^3*c^3*f*(c - c*Sin[e + f*x])^2) + Sec[e + f*x]^5/(9*a^3
*f*(c^5 - c^5*Sin[e + f*x])) + (2*Tan[e + f*x])/(3*a^3*c^5*f) + (4*Tan[e +
f*x]^3)/(9*a^3*c^5*f) + (2*Tan[e + f*x]^5)/(15*a^3*c^5*f)
```

Rubi [A] time = 0.170823, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2736, 2672, 3767}

$$\frac{2\tan^5(e+fx)}{15a^3c^5f}+\frac{4\tan^3(e+fx)}{9a^3c^5f}+\frac{2\tan(e+fx)}{3a^3c^5f}+\frac{\sec^5(e+fx)}{9a^3f\left(c^5-c^5\sin(e+fx)\right)}+\frac{\sec^5(e+fx)}{9a^3c^3f(c-c\sin(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^5),x]
```

```
[Out] Sec[e + f*x]^5/(9*a^3*c^3*f*(c - c*Sin[e + f*x])^2) + Sec[e + f*x]^5/(9*a^3
*f*(c^5 - c^5*Sin[e + f*x])) + (2*Tan[e + f*x])/(3*a^3*c^5*f) + (4*Tan[e +
f*x]^3)/(9*a^3*c^5*f) + (2*Tan[e + f*x]^5)/(15*a^3*c^5*f)
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^5}\,dx = \frac{\int \frac{\sec^6(e+fx)}{(c-c\sin(e+fx))^2}\,dx}{a^3c^3}$$
$$= \frac{\sec^5(e+fx)}{9a^3c^3f(c-c\sin(e+fx))^2} + \frac{7\int \frac{\sec^6(e+fx)}{c-c\sin(e+fx)}\,dx}{9a^3c^4}$$
$$= \frac{\sec^5(e+fx)}{9a^3c^3f(c-c\sin(e+fx))^2} + \frac{\sec^5(e+fx)}{9a^3f\left(c^5-c^5\sin(e+fx)\right)} + \frac{2\int \sec^6(e+f$$
$$= \frac{\sec^5(e+fx)}{9a^3c^3f(c-c\sin(e+fx))^2} + \frac{\sec^5(e+fx)}{9a^3f\left(c^5-c^5\sin(e+fx)\right)} - \frac{2\,\mathrm{Subst}\left(\int (1$$
$$= \frac{\sec^5(e+fx)}{9a^3c^3f(c-c\sin(e+fx))^2} + \frac{\sec^5(e+fx)}{9a^3f\left(c^5-c^5\sin(e+fx)\right)} + \frac{2\tan(e+fx)}{3a^3c^5f}$$

Mathematica [A] time = 1.32107, size = 213, normalized size = 1.63

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)(46080\sin(e+fx)+3500\sin(2(e+fx))+1945$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^5),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)*(-7875*Cos[e + f*x] + 20480*Cos[2*(e + f*x)] - 3325*Cos[3*(e + f*x)] + 16
384*Cos[4*(e + f*x)] - 175*Cos[5*(e + f*x)] + 4096*Cos[6*(e + f*x)] + 175*C
os[7*(e + f*x)] + 46080*Sin[e + f*x] + 3500*Sin[2*(e + f*x)] + 19456*Sin[3*
(e + f*x)] + 2800*Sin[4*(e + f*x)] + 1024*Sin[5*(e + f*x)] + 700*Sin[6*(e +
 f*x)] - 1024*Sin[7*(e + f*x)]))/(184320*f*(a + a*Sin[e + f*x])^3*(c - c*Si
n[e + f*x])^5)
```

Maple [A] time = 0.081, size = 223, normalized size = 1.7

$$2\,\frac{1}{fc^5a^3}\left(-2/9\left(\tan\left(1/2\,fx+e/2\right)-1\right)^{-9}-\left(\tan\left(1/2\,fx+e/2\right)-1\right)^{-8}-5/2\left(\tan\left(1/2\,fx+e/2\right)-1\right)^{-7}-\frac{49}{12\left(\tan\left(1/2\,fx\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^5,x)
```

```
[Out] 2/f/c^5/a^3*(-2/9/(tan(1/2*f*x+1/2*e)-1)^9-1/(tan(1/2*f*x+1/2*e)-1)^8-5/2/(
tan(1/2*f*x+1/2*e)-1)^7-49/12/(tan(1/2*f*x+1/2*e)-1)^6-49/10/(tan(1/2*f*x+1
/2*e)-1)^5-35/8/(tan(1/2*f*x+1/2*e)-1)^4-49/16/(tan(1/2*f*x+1/2*e)-1)^3-51/
32/(tan(1/2*f*x+1/2*e)-1)^2-99/128/(tan(1/2*f*x+1/2*e)-1)-1/40/(tan(1/2*f*x
+1/2*e)+1)^5+1/16/(tan(1/2*f*x+1/2*e)+1)^4-13/96/(tan(1/2*f*x+1/2*e)+1)^3+9
/64/(tan(1/2*f*x+1/2*e)+1)^2-29/128/(tan(1/2*f*x+1/2*e)+1))
```

Maxima [B] time = 1.28406, size = 824, normalized size = 6.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^5,x, algorithm="maxima")
```

```
[Out] 2/45*(5*sin(f*x + e)/(cos(f*x + e) + 1) - 80*sin(f*x + e)^2/(cos(f*x + e) +
 1)^2 + 190*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 50*sin(f*x + e)^4/(cos(f*
x + e) + 1)^4 - 269*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 96*sin(f*x + e)^6
/(cos(f*x + e) + 1)^6 + 516*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 354*sin(f
*x + e)^8/(cos(f*x + e) + 1)^8 - 69*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 2
40*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 + 30*sin(f*x + e)^11/(cos(f*x + e)
 + 1)^11 - 90*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 + 45*sin(f*x + e)^13/(c
os(f*x + e) + 1)^13 + 10)/((a^3*c^5 - 4*a^3*c^5*sin(f*x + e)/(cos(f*x + e)
+ 1) + a^3*c^5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 16*a^3*c^5*sin(f*x + e
)^3/(cos(f*x + e) + 1)^3 - 19*a^3*c^5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 -
 20*a^3*c^5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 45*a^3*c^5*sin(f*x + e)^6
/(cos(f*x + e) + 1)^6 - 45*a^3*c^5*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 20
*a^3*c^5*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 19*a^3*c^5*sin(f*x + e)^10/(
cos(f*x + e) + 1)^10 - 16*a^3*c^5*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 - a
^3*c^5*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 + 4*a^3*c^5*sin(f*x + e)^13/(c
os(f*x + e) + 1)^13 - a^3*c^5*sin(f*x + e)^14/(cos(f*x + e) + 1)^14)*f)
```

Fricas [A] time = 1.61598, size = 333, normalized size = 2.54

$$-\frac{32 \cos\left(fx+e\right)^{6} - 16 \cos\left(fx+e\right)^{4} - 4 \cos\left(fx+e\right)^{2} - \left(16 \cos\left(fx+e\right)^{6} - 24 \cos\left(fx+e\right)^{4} - 10 \cos\left(fx+e\right)^{2} - 7\right.}{45\left(a^{3}c^{5}f\cos\left(fx+e\right)^{7} + 2\,a^{3}c^{5}f\cos\left(fx+e\right)^{5}\sin\left(fx+e\right) - 2\,a^{3}c^{5}f\cos\left(fx+e\right)^{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^5,x, algorithm="fricas")
```

```
[Out] -1/45*(32*cos(f*x + e)^6 - 16*cos(f*x + e)^4 - 4*cos(f*x + e)^2 - (16*cos(f
*x + e)^6 - 24*cos(f*x + e)^4 - 10*cos(f*x + e)^2 - 7)*sin(f*x + e) - 2)/(a
^3*c^5*f*cos(f*x + e)^7 + 2*a^3*c^5*f*cos(f*x + e)^5*sin(f*x + e) - 2*a^3*c
^5*f*cos(f*x + e)^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**5,x)
```

```
[Out] Timed out
```

Giac [A] time = 2.02414, size = 293, normalized size = 2.24

$$-\frac{\frac{3\left(435\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}+1470\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{3}+2060\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+1330\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+353\right)}{a^{3}c^{5}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^{5}}+\frac{4455\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{8}-26460\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{7}+7}{}}{288}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^5,x, algorithm="giac")
```

```
[Out] -1/2880*(3*(435*tan(1/2*f*x + 1/2*e)^4 + 1470*tan(1/2*f*x + 1/2*e)^3 + 2060
*tan(1/2*f*x + 1/2*e)^2 + 1330*tan(1/2*f*x + 1/2*e) + 353)/(a^3*c^5*(tan(1/
2*f*x + 1/2*e) + 1)^5) + (4455*tan(1/2*f*x + 1/2*e)^8 - 26460*tan(1/2*f*x +
 1/2*e)^7 + 78120*tan(1/2*f*x + 1/2*e)^6 - 137340*tan(1/2*f*x + 1/2*e)^5 +
157374*tan(1/2*f*x + 1/2*e)^4 - 118356*tan(1/2*f*x + 1/2*e)^3 + 57744*tan(1
/2*f*x + 1/2*e)^2 - 16596*tan(1/2*f*x + 1/2*e) + 2339)/(a^3*c^5*(tan(1/2*f*
x + 1/2*e) - 1)^9))/f
```

3.289 $\int \frac{1}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^6}\,dx$

Optimal. Leaf size=167

$$\frac{16\tan^5(e+fx)}{165a^3c^6f}+\frac{32\tan^3(e+fx)}{99a^3c^6f}+\frac{16\tan(e+fx)}{33a^3c^6f}+\frac{8\sec^5(e+fx)}{99a^3f\left(c^6-c^6\sin(e+fx)\right)}+\frac{8\sec^5(e+fx)}{99a^3f\left(c^3-c^3\sin(e+fx)\right)^2}+$$

```
[Out] Sec[e + f*x]^5/(11*a^3*f*(c^2 - c^2*Sin[e + f*x])^3) + (8*Sec[e + f*x]^5)/(
99*a^3*f*(c^3 - c^3*Sin[e + f*x])^2) + (8*Sec[e + f*x]^5)/(99*a^3*f*(c^6 -
c^6*Sin[e + f*x])) + (16*Tan[e + f*x])/(33*a^3*c^6*f) + (32*Tan[e + f*x]^3)
/(99*a^3*c^6*f) + (16*Tan[e + f*x]^5)/(165*a^3*c^6*f)
```

Rubi [A] time = 0.217262, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2736, 2672, 3767}

$$\frac{16\tan^5(e+fx)}{165a^3c^6f}+\frac{32\tan^3(e+fx)}{99a^3c^6f}+\frac{16\tan(e+fx)}{33a^3c^6f}+\frac{8\sec^5(e+fx)}{99a^3f\left(c^6-c^6\sin(e+fx)\right)}+\frac{8\sec^5(e+fx)}{99a^3f\left(c^3-c^3\sin(e+fx)\right)^2}+$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^6),x]
```


```
[Out] Sec[e + f*x]^5/(11*a^3*f*(c^2 - c^2*Sin[e + f*x])^3) + (8*Sec[e + f*x]^5)/(
99*a^3*f*(c^3 - c^3*Sin[e + f*x])^2) + (8*Sec[e + f*x]^5)/(99*a^3*f*(c^6 -
c^6*Sin[e + f*x])) + (16*Tan[e + f*x])/(33*a^3*c^6*f) + (32*Tan[e + f*x]^3)
/(99*a^3*c^6*f) + (16*Tan[e + f*x]^5)/(165*a^3*c^6*f)
```


Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] && !IGtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^6}\,dx = \frac{\int \frac{\sec^6(e+fx)}{(c-c\sin(e+fx))^3}\,dx}{a^3c^3}$$

$$= \frac{\sec^5(e+fx)}{11a^3f\left(c^2-c^2\sin(e+fx)\right)^3} + \frac{8\int \frac{\sec^6(e+fx)}{(c-c\sin(e+fx))^2}\,dx}{11a^3c^4}$$

$$= \frac{\sec^5(e+fx)}{11a^3f\left(c^2-c^2\sin(e+fx)\right)^3} + \frac{8\sec^5(e+fx)}{99a^3f\left(c^3-c^3\sin(e+fx)\right)^2} + \frac{56\int \frac{\sec^6(}{c-c\sin}}{99a^3}$$

$$= \frac{\sec^5(e+fx)}{11a^3f\left(c^2-c^2\sin(e+fx)\right)^3} + \frac{8\sec^5(e+fx)}{99a^3f\left(c^3-c^3\sin(e+fx)\right)^2} + \frac{8\sec}{99a^3f\left(c^6-\right.}$$

$$= \frac{\sec^5(e+fx)}{11a^3f\left(c^2-c^2\sin(e+fx)\right)^3} + \frac{8\sec^5(e+fx)}{99a^3f\left(c^3-c^3\sin(e+fx)\right)^2} + \frac{8\sec}{99a^3f\left(c^6-\right.}$$

$$= \frac{\sec^5(e+fx)}{11a^3f\left(c^2-c^2\sin(e+fx)\right)^3} + \frac{8\sec^5(e+fx)}{99a^3f\left(c^3-c^3\sin(e+fx)\right)^2} + \frac{8\sec}{99a^3f\left(c^6-\right.}$$

Mathematica [A] time = 1.56477, size = 233, normalized size = 1.4

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)(1802240\sin(e+fx)+247170\sin(2(e+fx))+}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^6),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)*(-411950*Cos[e + f*x] + 1081344*Cos[2*(e + f*x)] - 127330*Cos[3*(e + f*x)
] + 819200*Cos[4*(e + f*x)] + 37450*Cos[5*(e + f*x)] + 163840*Cos[6*(e + f*
x)] + 22470*Cos[7*(e + f*x)] - 16384*Cos[8*(e + f*x)] + 1802240*Sin[e + f*x
] + 247170*Sin[2*(e + f*x)] + 557056*Sin[3*(e + f*x)] + 187250*Sin[4*(e + f
*x)] - 163840*Sin[5*(e + f*x)] + 37450*Sin[6*(e + f*x)] - 98304*Sin[7*(e +
f*x)] - 3745*Sin[8*(e + f*x)]))/(8110080*f*(a + a*Sin[e + f*x])^3*(c - c*Si
n[e + f*x])^6)
```

Maple [A] time = 0.076, size = 253, normalized size = 1.5

$$2\,\frac{1}{fc^6a^3}\left(-4/11\,\left(\tan\left(1/2\,fx+e/2\right)-1\right)^{-11}-2\,\left(\tan\left(1/2\,fx+e/2\right)-1\right)^{-10}-\frac{53}{9\,\left(\tan\left(1/2\,fx+e/2\right)-1\right)^{9}}-23/2\,\left(\tan\left(1/2\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^6,x)
```

```
[Out] 2/f/c^6/a^3*(-4/11/(tan(1/2*f*x+1/2*e)-1)^11-2/(tan(1/2*f*x+1/2*e)-1)^10-53
/9/(tan(1/2*f*x+1/2*e)-1)^9-23/2/(tan(1/2*f*x+1/2*e)-1)^8-33/2/(tan(1/2*f*x
+1/2*e)-1)^7-217/12/(tan(1/2*f*x+1/2*e)-1)^6-623/40/(tan(1/2*f*x+1/2*e)-1)^
5-169/16/(tan(1/2*f*x+1/2*e)-1)^4-365/64/(tan(1/2*f*x+1/2*e)-1)^3-303/128/(
tan(1/2*f*x+1/2*e)-1)^2-219/256/(tan(1/2*f*x+1/2*e)-1)-1/80/(tan(1/2*f*x+1/
2*e)+1)^5+1/32/(tan(1/2*f*x+1/2*e)+1)^4-7/96/(tan(1/2*f*x+1/2*e)+1)^3+5/64/
```

```
(tan(1/2*f*x+1/2*e)+1)^2-37/256/(tan(1/2*f*x+1/2*e)+1))
```

Maxima [B] time = 1.35087, size = 949, normalized size = 5.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^6,x, algorithm="maxima")
```

```
[Out] -2/495*(255*sin(f*x + e)/(cos(f*x + e) + 1) + 235*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 - 3065*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3775*sin(f*x + e)^4
/(cos(f*x + e) + 1)^4 + 667*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 8217*sin(
f*x + e)^6/(cos(f*x + e) + 1)^6 + 2035*sin(f*x + e)^7/(cos(f*x + e) + 1)^7
+ 8745*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 11715*sin(f*x + e)^9/(cos(f*x
+ e) + 1)^9 + 33*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 + 4917*sin(f*x + e)^
11/(cos(f*x + e) + 1)^11 - 2475*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - 181
5*sin(f*x + e)^13/(cos(f*x + e) + 1)^13 + 1485*sin(f*x + e)^14/(cos(f*x + e
) + 1)^14 - 495*sin(f*x + e)^15/(cos(f*x + e) + 1)^15 - 125)/((a^3*c^6 - 6*
a^3*c^6*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*c^6*sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 + 10*a^3*c^6*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 50*a^3*c^6
*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 34*a^3*c^6*sin(f*x + e)^5/(cos(f*x +
 e) + 1)^5 + 66*a^3*c^6*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 110*a^3*c^6*s
in(f*x + e)^7/(cos(f*x + e) + 1)^7 + 110*a^3*c^6*sin(f*x + e)^9/(cos(f*x +
e) + 1)^9 - 66*a^3*c^6*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 34*a^3*c^6*s
in(f*x + e)^11/(cos(f*x + e) + 1)^11 + 50*a^3*c^6*sin(f*x + e)^12/(cos(f*x
+ e) + 1)^12 - 10*a^3*c^6*sin(f*x + e)^13/(cos(f*x + e) + 1)^13 - 10*a^3*c^
6*sin(f*x + e)^14/(cos(f*x + e) + 1)^14 + 6*a^3*c^6*sin(f*x + e)^15/(cos(f*
x + e) + 1)^15 - a^3*c^6*sin(f*x + e)^16/(cos(f*x + e) + 1)^16)*f)
```

Fricas [A] time = 1.64909, size = 412, normalized size = 2.47

$$\frac{128 \cos\left(fx+e\right)^{8} - 576 \cos\left(fx+e\right)^{6} + 240 \cos\left(fx+e\right)^{4} + 56 \cos\left(fx+e\right)^{2} + 8\left(48 \cos\left(fx+e\right)^{6} - 40 \cos\left(fx+e\right.\right.}{495\left(3\,a^{3}c^{6}f \cos\left(fx+e\right)^{7} - 4\,a^{3}c^{6}f \cos\left(fx+e\right)^{5} - \left(a^{3}c^{6}f \cos\left(fx+e\right)^{7} - 4\,a^{3}c^{6}f \cos\left(fx-\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^6,x, algorithm="fricas")
```

```
[Out] 1/495*(128*cos(f*x + e)^8 - 576*cos(f*x + e)^6 + 240*cos(f*x + e)^4 + 56*co
s(f*x + e)^2 + 8*(48*cos(f*x + e)^6 - 40*cos(f*x + e)^4 - 14*cos(f*x + e)^2
 - 9)*sin(f*x + e) + 27)/(3*a^3*c^6*f*cos(f*x + e)^7 - 4*a^3*c^6*f*cos(f*x
+ e)^5 - (a^3*c^6*f*cos(f*x + e)^7 - 4*a^3*c^6*f*cos(f*x + e)^5)*sin(f*x +
e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**6,x)
```

```
[Out] Timed out
```

Giac [A] time = 2.10262, size = 331, normalized size = 1.98

$$-\frac{\frac{33\left(555\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+1920\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+2710\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+1760\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+463\right)}{a^3c^6\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^5}+\frac{108405\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{10}-784080\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^9+}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^6,x, algorithm="giac")
```

```
[Out] -1/63360*(33*(555*tan(1/2*f*x + 1/2*e)^4 + 1920*tan(1/2*f*x + 1/2*e)^3 + 27
10*tan(1/2*f*x + 1/2*e)^2 + 1760*tan(1/2*f*x + 1/2*e) + 463)/(a^3*c^6*(tan(
1/2*f*x + 1/2*e) + 1)^5) + (108405*tan(1/2*f*x + 1/2*e)^10 - 784080*tan(1/2
*f*x + 1/2*e)^9 + 2901195*tan(1/2*f*x + 1/2*e)^8 - 6652800*tan(1/2*f*x + 1/
2*e)^7 + 10407474*tan(1/2*f*x + 1/2*e)^6 - 11435424*tan(1/2*f*x + 1/2*e)^5
+ 8949270*tan(1/2*f*x + 1/2*e)^4 - 4899840*tan(1/2*f*x + 1/2*e)^3 + 1816265
*tan(1/2*f*x + 1/2*e)^2 - 411664*tan(1/2*f*x + 1/2*e) + 47279)/(a^3*c^6*(ta
n(1/2*f*x + 1/2*e) - 1)^11))/f
```

3.290 $\int (a + a\sin(e + fx))(c - c\sin(e + fx))^{7/2}\,dx$

Optimal. Leaf size=137

$$\frac{256ac^5\cos^3(e+fx)}{315f(c-c\sin(e+fx))^{3/2}}+\frac{64ac^4\cos^3(e+fx)}{105f\sqrt{c-c\sin(e+fx)}}+\frac{8ac^3\cos^3(e+fx)\sqrt{c-c\sin(e+fx)}}{21f}+\frac{2ac^2\cos^3(e+fx)(c-c}{9f}$$

```
[Out] (256*a*c^5*Cos[e + f*x]^3)/(315*f*(c - c*Sin[e + f*x])^(3/2)) + (64*a*c^4*C
os[e + f*x]^3)/(105*f*Sqrt[c - c*Sin[e + f*x]]) + (8*a*c^3*Cos[e + f*x]^3*S
qrt[c - c*Sin[e + f*x]])/(21*f) + (2*a*c^2*Cos[e + f*x]^3*(c - c*Sin[e + f*
x])^(3/2))/(9*f)
```

Rubi [A] time = 0.294873, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2736, 2674, 2673}

$$\frac{256ac^5\cos^3(e+fx)}{315f(c-c\sin(e+fx))^{3/2}}+\frac{64ac^4\cos^3(e+fx)}{105f\sqrt{c-c\sin(e+fx)}}+\frac{8ac^3\cos^3(e+fx)\sqrt{c-c\sin(e+fx)}}{21f}+\frac{2ac^2\cos^3(e+fx)(c-c}{9f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7/2),x]
```

```
[Out] (256*a*c^5*Cos[e + f*x]^3)/(315*f*(c - c*Sin[e + f*x])^(3/2)) + (64*a*c^4*C
os[e + f*x]^3)/(105*f*Sqrt[c - c*Sin[e + f*x]]) + (8*a*c^3*Cos[e + f*x]^3*S
qrt[c - c*Sin[e + f*x]])/(21*f) + (2*a*c^2*Cos[e + f*x]^3*(c - c*Sin[e + f*
x])^(3/2))/(9*f)
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rubi steps

$$\int (a + a\sin(e + fx))(c - c\sin(e + fx))^{7/2}\,dx = (ac)\int \cos^2(e + fx)(c - c\sin(e + fx))^{5/2}\,dx$$
$$= \frac{2ac^2\cos^3(e + fx)(c - c\sin(e + fx))^{3/2}}{9f} + \frac{1}{3}\left(4ac^2\right)\int \cos^2(e + fx)(c - c\,\mathrm{si}$$
$$= \frac{8ac^3\cos^3(e + fx)\sqrt{c - c\sin(e + fx)}}{21f} + \frac{2ac^2\cos^3(e + fx)(c - c\sin(e + fx)}{9f}$$
$$= \frac{64ac^4\cos^3(e + fx)}{105f\sqrt{c - c\sin(e + fx)}} + \frac{8ac^3\cos^3(e + fx)\sqrt{c - c\sin(e + fx)}}{21f} + \frac{2ac^2\,\mathrm{co}}{}$$
$$= \frac{256ac^5\cos^3(e + fx)}{315f(c - c\sin(e + fx))^{3/2}} + \frac{64ac^4\cos^3(e + fx)}{105f\sqrt{c - c\sin(e + fx)}} + \frac{8ac^3\cos^3(e + fx)}{21}$$

Mathematica [A] time = 0.840671, size = 104, normalized size = 0.76

$$\frac{ac^3\sqrt{c - c\sin(e + fx)}\left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right)\right)^3(-1389\sin(e + fx) + 35\sin(3(e + fx)) - 330\cos(2(e + fx)) +}{630f\left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7/2),x]
```

```
[Out] (a*c^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*Sqrt[c - c*Sin[e + f*x]]*(16
06 - 330*Cos[2*(e + f*x)] - 1389*Sin[e + f*x] + 35*Sin[3*(e + f*x)]))/(630*
f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))
```

Maple [A] time = 0.573, size = 79, normalized size = 0.6

$$\frac{\left(-2 + 2\sin\left(fx + e\right)\right)c^4\left(1 + \sin\left(fx + e\right)\right)^2 a\left(35\left(\sin\left(fx + e\right)\right)^3 - 165\left(\sin\left(fx + e\right)\right)^2 + 321\sin\left(fx + e\right) - 319\right)}{315\,f\cos\left(fx + e\right)}\frac{}{\sqrt{c - c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x)
```

```
[Out] 2/315*(-1+sin(f*x+e))*c^4*(1+sin(f*x+e))^2*a*(35*sin(f*x+e)^3-165*sin(f*x+e
)^2+321*sin(f*x+e)-319)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx + e\right) + a\right)\left(-c\sin\left(fx + e\right) + c\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)*(-c*sin(f*x + e) + c)^(7/2), x)
```

Fricas [A] time = 1.1593, size = 466, normalized size = 3.4

$$-\frac{2\left(35\,ac^3\cos\left(fx+e\right)^5-95\,ac^3\cos\left(fx+e\right)^4-226\,ac^3\cos\left(fx+e\right)^3+32\,ac^3\cos\left(fx+e\right)^2-128\,ac^3\cos\left(fx+e\right)-\right.}{315\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x, algorithm="fricas")
```

```
[Out] -2/315*(35*a*c^3*cos(f*x + e)^5 - 95*a*c^3*cos(f*x + e)^4 - 226*a*c^3*cos(f
*x + e)^3 + 32*a*c^3*cos(f*x + e)^2 - 128*a*c^3*cos(f*x + e) - 256*a*c^3 +
(35*a*c^3*cos(f*x + e)^4 + 130*a*c^3*cos(f*x + e)^3 - 96*a*c^3*cos(f*x + e)
^2 - 128*a*c^3*cos(f*x + e) - 256*a*c^3)*sin(f*x + e))*sqrt(-c*sin(f*x + e)
 + c)/(f*cos(f*x + e) - f*sin(f*x + e) + f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c-c*sin(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a\sin\left(fx+e\right)+a\right)\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)*(-c*sin(f*x + e) + c)^(7/2), x)
```

3.291 $\int (a + a\sin(e + fx))(c - c\sin(e + fx))^{5/2}\,dx$

Optimal. Leaf size=103

$$\frac{64ac^4\cos^3(e+fx)}{105f(c-c\sin(e+fx))^{3/2}}+\frac{16ac^3\cos^3(e+fx)}{35f\sqrt{c-c\sin(e+fx)}}+\frac{2ac^2\cos^3(e+fx)\sqrt{c-c\sin(e+fx)}}{7f}$$

```
[Out] (64*a*c^4*Cos[e + f*x]^3)/(105*f*(c - c*Sin[e + f*x])^(3/2)) + (16*a*c^3*Co
s[e + f*x]^3)/(35*f*Sqrt[c - c*Sin[e + f*x]]) + (2*a*c^2*Cos[e + f*x]^3*Sqr
t[c - c*Sin[e + f*x]])/(7*f)
```

Rubi [A] time = 0.222452, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2736, 2674, 2673}

$$\frac{64ac^4\cos^3(e+fx)}{105f(c-c\sin(e+fx))^{3/2}}+\frac{16ac^3\cos^3(e+fx)}{35f\sqrt{c-c\sin(e+fx)}}+\frac{2ac^2\cos^3(e+fx)\sqrt{c-c\sin(e+fx)}}{7f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] (64*a*c^4*Cos[e + f*x]^3)/(105*f*(c - c*Sin[e + f*x])^(3/2)) + (16*a*c^3*Co
s[e + f*x]^3)/(35*f*Sqrt[c - c*Sin[e + f*x]]) + (2*a*c^2*Cos[e + f*x]^3*Sqr
t[c - c*Sin[e + f*x]])/(7*f)
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rubi steps

$$\int (a+a\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx = (ac)\int \cos^2(e+fx)(c-c\sin(e+fx))^{3/2}\,dx$$
$$= \frac{2ac^2\cos^3(e+fx)\sqrt{c-c\sin(e+fx)}}{7f} + \frac{1}{7}\left(8ac^2\right)\int \cos^2(e+fx)\sqrt{c-c$$
$$= \frac{16ac^3\cos^3(e+fx)}{35f\sqrt{c-c\sin(e+fx)}} + \frac{2ac^2\cos^3(e+fx)\sqrt{c-c\sin(e+fx)}}{7f} + \frac{1}{35}(3$$
$$= \frac{64ac^4\cos^3(e+fx)}{105f(c-c\sin(e+fx))^{3/2}} + \frac{16ac^3\cos^3(e+fx)}{35f\sqrt{c-c\sin(e+fx)}} + \frac{2ac^2\cos^3(e+f$$

Mathematica [A] time = 0.532364, size = 94, normalized size = 0.91

$$-\frac{ac^2\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3(108\sin(e+fx)+15\cos(2(e+fx))-157)}{105f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] -(a*c^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*(-157 + 15*Cos[2*(e + f*x)]
 + 108*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(105*f*(Cos[(e + f*x)/2] - S
in[(e + f*x)/2]))
```

Maple [A] time = 0.52, size = 69, normalized size = 0.7

$$-\frac{\left(-2+2\sin\left(fx+e\right)\right)c^3\left(1+\sin\left(fx+e\right)\right)^2 a\left(15\left(\sin\left(fx+e\right)\right)^2-54\sin\left(fx+e\right)+71\right)}{105\,f\cos\left(fx+e\right)}\frac{1}{\sqrt{c-c\sin\left(fx+e\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] -2/105*(-1+sin(f*x+e))*c^3*(1+sin(f*x+e))^2*a*(15*sin(f*x+e)^2-54*sin(f*x+e
)+71)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)*(-c*sin(f*x + e) + c)^(5/2), x)
```

Fricas [A] time = 1.09818, size = 385, normalized size = 3.74

$$\frac{2\left(15\,ac^{2}\cos\left(fx+e\right)^{4}+39\,ac^{2}\cos\left(fx+e\right)^{3}-8\,ac^{2}\cos\left(fx+e\right)^{2}+32\,ac^{2}\cos\left(fx+e\right)+64\,ac^{2}-\left(15\,ac^{2}\cos\left(fx+e\right)^{3}\right.\right.}{105\left(f\cos\left(fx+e\right)-f\sin\left(fx+e\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/105*(15*a*c^2*cos(f*x + e)^4 + 39*a*c^2*cos(f*x + e)^3 - 8*a*c^2*cos(f*x
+ e)^2 + 32*a*c^2*cos(f*x + e) + 64*a*c^2 - (15*a*c^2*cos(f*x + e)^3 - 24*a
*c^2*cos(f*x + e)^2 - 32*a*c^2*cos(f*x + e) - 64*a*c^2)*sin(f*x + e))*sqrt(
-c*sin(f*x + e) + c)/(f*cos(f*x + e) - f*sin(f*x + e) + f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.292 $\int (a+a\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=69

$$\frac{2ac^2\cos^3(e+fx)}{5f\sqrt{c-c\sin(e+fx)}}+\frac{8ac^3\cos^3(e+fx)}{15f(c-c\sin(e+fx))^{3/2}}$$

```
[Out] (8*a*c^3*Cos[e + f*x]^3)/(15*f*(c - c*Sin[e + f*x])^(3/2)) + (2*a*c^2*Cos[e
 + f*x]^3)/(5*f*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 0.157268, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2736, 2674, 2673}

$$\frac{2ac^2\cos^3(e+fx)}{5f\sqrt{c-c\sin(e+fx)}}+\frac{8ac^3\cos^3(e+fx)}{15f(c-c\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2),x]
```


```
[Out] (8*a*c^3*Cos[e + f*x]^3)/(15*f*(c - c*Sin[e + f*x])^(3/2)) + (2*a*c^2*Cos[e
 + f*x]^3)/(5*f*Sqrt[c - c*Sin[e + f*x]])
```


Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rubi steps

$$\int (a+a\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx=(ac)\int \cos^2(e+fx)\sqrt{c-c\sin(e+fx)}\,dx$$
$$=\frac{2ac^2\cos^3(e+fx)}{5f\sqrt{c-c\sin(e+fx)}}+\frac{1}{5}\left(4ac^2\right)\int \frac{\cos^2(e+fx)}{\sqrt{c-c\sin(e+fx)}}\,dx$$
$$=\frac{8ac^3\cos^3(e+fx)}{15f(c-c\sin(e+fx))^{3/2}}+\frac{2ac^2\cos^3(e+fx)}{5f\sqrt{c-c\sin(e+fx)}}$$

Mathematica [A] time = 0.336405, size = 82, normalized size = 1.19

$$-\frac{2ac(3\sin(e+fx)-7)\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3}{15f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-2*a*c*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*(-7 + 3*Sin[e + f*x])*Sqrt[
c - c*Sin[e + f*x]])/(15*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))
```

Maple [A] time = 0.506, size = 59, normalized size = 0.9

$$\frac{\left(-2+2\sin\left(fx+e\right)\right)c^2\left(1+\sin\left(fx+e\right)\right)^2a\left(3\sin\left(fx+e\right)-7\right)}{15f\cos\left(fx+e\right)}\frac{1}{\sqrt{c-c\sin\left(fx+e\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] 2/15*(-1+sin(f*x+e))*c^2*(1+sin(f*x+e))^2*a*(3*sin(f*x+e)-7)/cos(f*x+e)/(c-
c*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a\sin\left(fx+e\right)+a\right)\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)*(-c*sin(f*x + e) + c)^(3/2), x)
```

Fricas [A] time = 1.07368, size = 284, normalized size = 4.12

$$\frac{2\left(3ac\cos\left(fx+e\right)^3-ac\cos\left(fx+e\right)^2+4ac\cos\left(fx+e\right)+8ac+\left(3ac\cos\left(fx+e\right)^2+4ac\cos\left(fx+e\right)+8ac\right)\sin\left(f\right.\right.}{15\left(f\cos\left(fx+e\right)-f\sin\left(fx+e\right)+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*a*c*cos(f*x + e)^3 - a*c*cos(f*x + e)^2 + 4*a*c*cos(f*x + e) + 8*a*
c + (3*a*c*cos(f*x + e)^2 + 4*a*c*cos(f*x + e) + 8*a*c)*sin(f*x + e))*sqrt(
-c*sin(f*x + e) + c)/(f*cos(f*x + e) - f*sin(f*x + e) + f)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int c\sqrt{-c\sin\left(e+fx\right)+c}\,dx+\int -c\sqrt{-c\sin\left(e+fx\right)+c}\sin^{2}\left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] a*(Integral(c*sqrt(-c*sin(e + f*x) + c), x) + Integral(-c*sqrt(-c*sin(e + f
*x) + c)*sin(e + f*x)**2, x))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.293 $\int (a+a\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx$

Optimal. Leaf size=34

$$\frac{2ac^2\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{3/2}}$$

```
[Out] (2*a*c^2*Cos[e + f*x]^3)/(3*f*(c - c*Sin[e + f*x])^(3/2))
```

Rubi [A] time = 0.0918206, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2736, 2673}

$$\frac{2ac^2\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (2*a*c^2*Cos[e + f*x]^3)/(3*f*(c - c*Sin[e + f*x])^(3/2))
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rubi steps

$$\begin{aligned}\int (a+a\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx &= (ac)\int \frac{\cos^2(e+fx)}{\sqrt{c-c\sin(e+fx)}}\,dx\\ &= \frac{2ac^2\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{3/2}}\end{aligned}$$

Mathematica [B] time = 0.117942, size = 71, normalized size = 2.09

$$\frac{2a\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3}{3f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (2*a*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*Sqrt[c - c*Sin[e + f*x]])/(3*f
*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))
```

Maple [A] time = 0.375, size = 47, normalized size = 1.4

$$-\frac{\left(-2+2\sin\left(fx+e\right)\right)c\left(1+\sin\left(fx+e\right)\right)^{2}a}{3f\cos\left(fx+e\right)}\frac{1}{\sqrt{c-c\sin\left(fx+e\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] -2/3*(-1+sin(f*x+e))*c*(1+sin(f*x+e))^2*a/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)
/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a\sin\left(fx+e\right)+a\right)\sqrt{-c\sin\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c), x)
```

Fricas [B] time = 1.04304, size = 203, normalized size = 5.97

$$-\frac{2\left(a\cos\left(fx+e\right)^{2}-a\cos\left(fx+e\right)-\left(a\cos\left(fx+e\right)+2a\right)\sin\left(fx+e\right)-2a\right)\sqrt{-c\sin\left(fx+e\right)+c}}{3\left(f\cos\left(fx+e\right)-f\sin\left(fx+e\right)+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*(a*cos(f*x + e)^2 - a*cos(f*x + e) - (a*cos(f*x + e) + 2*a)*sin(f*x +
e) - 2*a)*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e) - f*sin(f*x + e) + f)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int\sqrt{-c\sin\left(e+fx\right)+c}\sin\left(e+fx\right)dx+\int\sqrt{-c\sin\left(e+fx\right)+c}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] a*(Integral(sqrt(-c*sin(e + f*x) + c)*sin(e + f*x), x) + Integral(sqrt(-c*s
in(e + f*x) + c), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)\sqrt{-c\sin\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c), x)
```

3.294 $\int \frac{a+a\sin(e+fx)}{\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=77

$$\frac{2\sqrt{2}a\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{c}f}-\frac{2a\cos(e+fx)}{f\sqrt{c-c\sin(e+fx)}}$$

```
[Out] (2*Sqrt[2]*a*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x
]])])/(Sqrt[c]*f) - (2*a*Cos[e + f*x])/(f*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 0.138476, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2736, 2679, 2649, 206}

$$\frac{2\sqrt{2}a\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{c}f}-\frac{2a\cos(e+fx)}{f\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])/Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (2*Sqrt[2]*a*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x
]])])/(Sqrt[c]*f) - (2*a*Cos[e + f*x])/(f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2679

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(a*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+a\sin(e+fx)}{\sqrt{c-c\sin(e+fx)}}\,dx &= (ac)\int \frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^{3/2}}\,dx\\
&= -\frac{2a\cos(e+fx)}{f\sqrt{c-c\sin(e+fx)}}+(2a)\int \frac{1}{\sqrt{c-c\sin(e+fx)}}\,dx\\
&= -\frac{2a\cos(e+fx)}{f\sqrt{c-c\sin(e+fx)}}-\frac{(4a)\operatorname{Subst}\left(\int \frac{1}{2c-x^2}\,dx,x,-\frac{c\cos(e+fx)}{\sqrt{c-c\sin(e+fx)}}\right)}{f}\\
&= \frac{2\sqrt{2}a\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{c}f}-\frac{2a\cos(e+fx)}{f\sqrt{c-c\sin(e+fx)}}
\end{aligned}
$$

Mathematica [A] time = 0.585694, size = 135, normalized size = 1.75

$$
-\frac{2a\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sqrt{c}(\sin(e+fx)+1)+\sqrt{2}\sqrt{-c(\sin(e+fx)+1)}\tan^{-1}\left(\frac{\sqrt{-c(\sin(e+fx)+1)}}{\sqrt{2}\sqrt{c}}\right)\right)}{\sqrt{c}f\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])/Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (-2*a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Sqrt[c]*(1 + Sin[e + f*x]) + S
qrt[2]*ArcTan[Sqrt[-(c*(1 + Sin[e + f*x]))]/(Sqrt[2]*Sqrt[c])]*Sqrt[-(c*(1
+ Sin[e + f*x]))]))/(Sqrt[c]*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c
 - c*Sin[e + f*x]])
```

Maple [A] time = 0.576, size = 94, normalized size = 1.2

$$
-2\,\frac{\left(-1+\sin\left(fx+e\right)\right)\sqrt{c\left(1+\sin\left(fx+e\right)\right)}a}{c\cos\left(fx+e\right)\sqrt{c-c\sin\left(fx+e\right)}f}\left(\sqrt{c}\sqrt{2}\operatorname{Artanh}\left(1/2\,\frac{\sqrt{c\left(1+\sin\left(fx+e\right)\right)}\sqrt{2}}{\sqrt{c}}\right)-\sqrt{c\left(1+\sin\left(fx+e\right)\right)}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] -2*(-1+sin(f*x+e))*(c*(1+sin(f*x+e)))^(1/2)*a*(c^(1/2)*2^(1/2)*arctanh(1/2*
(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))-(c*(1+sin(f*x+e)))^(1/2))/c/cos(f
*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$
\int \frac{a\sin\left(fx+e\right)+a}{\sqrt{-c\sin\left(fx+e\right)+c}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)/sqrt(-c*sin(f*x + e) + c), x)
```

Fricas [B] time = 1.0731, size = 544, normalized size = 7.06

$$\frac{\frac{\sqrt{2}\left(ac\cos\left(fx+e\right)-ac\sin\left(fx+e\right)+ac\right)\log\left(-\frac{\cos\left(fx+e\right)^{2}+\left(\cos\left(fx+e\right)-2\right)\sin\left(fx+e\right)+\frac{2\sqrt{2}\sqrt{-c\sin\left(fx+e\right)+c}\left(\cos\left(fx+e\right)+\sin\left(fx+e\right)+1\right)}{\sqrt{c}}+3\cos\left(fx+e\right)+2}{\cos\left(fx+e\right)^{2}+\left(\cos\left(fx+e\right)+2\right)\sin\left(fx+e\right)-\cos\left(fx+e\right)-2}\right)}{\sqrt{c}}-2\left(a\cos\left(fx\cdots\right.\right.}{cf\cos\left(fx+e\right)-cf\sin\left(fx+e\right)+cf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] (sqrt(2)*(a*c*cos(f*x + e) - a*c*sin(f*x + e) + a*c)*log(-(cos(f*x + e)^2 +
 (cos(f*x + e) - 2)*sin(f*x + e) + 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*(cos
(f*x + e) + sin(f*x + e) + 1)/sqrt(c) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2
 + (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2))/sqrt(c) - 2*(a*cos(
f*x + e) + a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c))/(c*f*cos(f*x + e)
 - c*f*sin(f*x + e) + c*f)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int\frac{\sin\left(e+fx\right)}{\sqrt{-c\sin\left(e+fx\right)+c}}\,dx+\int\frac{1}{\sqrt{-c\sin\left(e+fx\right)+c}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] a*(Integral(sin(e + f*x)/sqrt(-c*sin(e + f*x) + c), x) + Integral(1/sqrt(-c
*sin(e + f*x) + c), x))
```

Giac [B] time = 2.39374, size = 267, normalized size = 3.47

$$\frac{2\left(\frac{2\sqrt{2}a\arctan\left(-\frac{\sqrt{2}\left(\sqrt{c}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+c}-\sqrt{c}\right)}{2\sqrt{-c}}\right)}{\sqrt{-c}\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}-\frac{\sqrt{2}\left(2ac\arctan\left(\frac{\sqrt{c}}{\sqrt{-c}}\right)+a\sqrt{-c}\sqrt{c}\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{\sqrt{-c}c}+\frac{\frac{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}+\cdots}{\sqrt{c\tan\left(\frac{1}{2}fx\cdots\right.}}\cdots\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] 2*(2*sqrt(2)*a*arctan(-1/2*sqrt(2)*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*t
an(1/2*f*x + 1/2*e)^2 + c) - sqrt(c))/sqrt(-c))/(sqrt(-c)*sgn(tan(1/2*f*x +
 1/2*e) - 1)) - sqrt(2)*(2*a*c*arctan(sqrt(c)/sqrt(-c)) + a*sqrt(-c)*sqrt(c
))*sgn(tan(1/2*f*x + 1/2*e) - 1)/(sqrt(-c)*c) + (a*tan(1/2*f*x + 1/2*e)/sgn
(tan(1/2*f*x + 1/2*e) - 1) + a/sgn(tan(1/2*f*x + 1/2*e) - 1))/sqrt(c*tan(1/
2*f*x + 1/2*e)^2 + c))/f
```

3.295 $\int \frac{a+a\sin(e+fx)}{(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=76

$$\frac{a\cos(e+fx)}{f(c-c\sin(e+fx))^{3/2}}-\frac{a\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{2}c^{3/2}f}$$

```
[Out] -((a*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])])/(S
qrt[2]*c^(3/2)*f)) + (a*Cos[e + f*x])/(f*(c - c*Sin[e + f*x])^(3/2))
```

Rubi [A] time = 0.140707, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2736, 2680, 2649, 206}

$$\frac{a\cos(e+fx)}{f(c-c\sin(e+fx))^{3/2}}-\frac{a\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{2}c^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])/(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] -((a*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])])/(S
qrt[2]*c^(3/2)*f)) + (a*Cos[e + f*x])/(f*(c - c*Sin[e + f*x])^(3/2))
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+a\sin(e+fx)}{(c-c\sin(e+fx))^{3/2}}\,dx &= (ac)\int \frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^{5/2}}\,dx\\ &= \frac{a\cos(e+fx)}{f(c-c\sin(e+fx))^{3/2}}-\frac{a\int \frac{1}{\sqrt{c-c\sin(e+fx)}}\,dx}{2c}\\ &= \frac{a\cos(e+fx)}{f(c-c\sin(e+fx))^{3/2}}+\frac{a\,\mathrm{Subst}\left(\int \frac{1}{2c-x^2}\,dx,x,-\frac{c\cos(e+fx)}{\sqrt{c-c\sin(e+fx)}}\right)}{cf}\\ &= -\frac{a\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{2}c^{3/2}f}+\frac{a\cos(e+fx)}{f(c-c\sin(e+fx))^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.622697, size = 107, normalized size = 1.41

$$\frac{a\sec(e+fx)\left(2\sqrt{c}(\sin(e+fx)+1)-\sqrt{2}(\sin(e+fx)-1)\sqrt{-c(\sin(e+fx)+1)}\tan^{-1}\left(\frac{\sqrt{-c(\sin(e+fx)+1)}}{\sqrt{2}\sqrt{c}}\right)\right)}{2c^{3/2}f\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])/(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] (a*Sec[e + f*x]*(2*Sqrt[c]*(1 + Sin[e + f*x]) - Sqrt[2]*ArcTan[Sqrt[-(c*(1
+ Sin[e + f*x]))]/(Sqrt[2]*Sqrt[c])]*(-1 + Sin[e + f*x])*Sqrt[-(c*(1 + Sin[
e + f*x]))]))/(2*c^(3/2)*f*Sqrt[c - c*Sin[e + f*x]])
```

Maple [A] time = 0.425, size = 120, normalized size = 1.6

$$\frac{a}{2f\cos\left(fx+e\right)}\left(\sqrt{2}\mathrm{Artanh}\left(\frac{\sqrt{2}}{2}\sqrt{c\left(1+\sin\left(fx+e\right)\right)}\frac{1}{\sqrt{c}}\right)c\sin\left(fx+e\right)-\sqrt{2}\mathrm{Artanh}\left(\frac{\sqrt{2}}{2}\sqrt{c\left(1+\sin\left(fx+e\right)\right)}\frac{1}{\sqrt{c}}\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] 1/2/c^(5/2)*a*(2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2)
)*c*sin(f*x+e)-2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2)
)*c+2*(c*(1+sin(f*x+e)))^(1/2)*c^(1/2))*(c*(1+sin(f*x+e)))^(1/2)/cos(f*x+e)
/(c-c*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a\sin\left(fx+e\right)+a}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)/(-c*sin(f*x + e) + c)^(3/2), x)
```

Fricas [B] time = 1.06673, size = 687, normalized size = 9.04

$$\frac{\frac{\sqrt{2}\left(ac\cos\left(fx+e\right)^{2}-ac\cos\left(fx+e\right)-2\,ac+\left(ac\cos\left(fx+e\right)+2\,ac\right)\sin\left(fx+e\right)\right)\log\left(-\frac{\cos\left(fx+e\right)^{2}+\left(\cos\left(fx+e\right)-2\right)\sin\left(fx+e\right)-\frac{2\sqrt{2}\sqrt{-c\sin\left(fx+e\right)+c}\left(\cos\left(fx+e\right)+\sin\left(fx+e\right)+1\right)}{\sqrt{c}}}{\cos\left(fx+e\right)^{2}+\left(\cos\left(fx+e\right)+2\right)\sin\left(fx+e\right)-\cos\left(fx+e\right)-2}\right)}{\sqrt{c}}}{4\left(c^{2}f\cos\left(fx+e\right)^{2}-c^{2}f\cos\left(fx+e\right)-2\,c^{2}f+\left(c^{2}f\cos\left(fx+e\right)\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(2)*(a*c*cos(f*x + e)^2 - a*c*cos(f*x + e) - 2*a*c + (a*c*cos(f*x
+ e) + 2*a*c)*sin(f*x + e))*log(-(cos(f*x + e)^2 + (cos(f*x + e) - 2)*sin(f
*x + e) - 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*(cos(f*x + e) + sin(f*x + e)
+ 1)/sqrt(c) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin
(f*x + e) - cos(f*x + e) - 2))/sqrt(c) - 4*(a*cos(f*x + e) + a*sin(f*x + e)
 + a)*sqrt(-c*sin(f*x + e) + c))/(c^2*f*cos(f*x + e)^2 - c^2*f*cos(f*x + e)
 - 2*c^2*f + (c^2*f*cos(f*x + e) + 2*c^2*f)*sin(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int\frac{\sin\left(e+fx\right)}{-c\sqrt{-c\sin\left(e+fx\right)+c}\sin\left(e+fx\right)+c\sqrt{-c\sin\left(e+fx\right)+c}}\,dx+\int\frac{1}{-c\sqrt{-c\sin\left(e+fx\right)+c}\sin\left(e+fx\right)+c\sqrt{\cdots}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] a*(Integral(sin(e + f*x)/(-c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x) + c*sqr
t(-c*sin(e + f*x) + c)), x) + Integral(1/(-c*sqrt(-c*sin(e + f*x) + c)*sin(
e + f*x) + c*sqrt(-c*sin(e + f*x) + c)), x))
```

Giac [B] time = 2.38424, size = 428, normalized size = 5.63

$$-\frac{\frac{\sqrt{2}a\arctan\left(-\frac{\sqrt{2}\left(\sqrt{c}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+c}-\sqrt{c}\right)}{2\sqrt{-c}}\right)}{\sqrt{-c}c\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}-\frac{2\left(3\left(\sqrt{c}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+c}\right)^{3}a-\left(\sqrt{c}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{c\tan\left(\frac{1}{2}fx+\cdots\right.}\right.\right.}{\left(\left(\sqrt{c}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+c}\right)^{2}-2\left(\sqrt{c}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{\cdots}\right.\right.}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] -(sqrt(2)*a*arctan(-1/2*sqrt(2)*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(
1/2*f*x + 1/2*e)^2 + c) - sqrt(c))/sqrt(-c))/(sqrt(-c)*c*sgn(tan(1/2*f*x +
1/2*e) - 1)) - 2*(3*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/
2*e)^2 + c))^3*a - (sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2
*e)^2 + c))^2*a*sqrt(c) - (sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*
x + 1/2*e)^2 + c))*a*c - a*c^(3/2))/(((sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(
c*tan(1/2*f*x + 1/2*e)^2 + c))^2 - 2*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c
*tan(1/2*f*x + 1/2*e)^2 + c))*sqrt(c) - c)^2*c*sgn(tan(1/2*f*x + 1/2*e) - 1
)))/f
```

3.296 $\int \frac{a+a\sin(e+fx)}{(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=113

$$-\frac{a\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{8\sqrt{2}c^{5/2}f}-\frac{a\cos(e+fx)}{8cf(c-c\sin(e+fx))^{3/2}}+\frac{a\cos(e+fx)}{2f(c-c\sin(e+fx))^{5/2}}$$

```
[Out] -(a*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])])/(8*
Sqrt[2]*c^(5/2)*f) + (a*Cos[e + f*x])/(2*f*(c - c*Sin[e + f*x])^(5/2)) - (a
*Cos[e + f*x])/(8*c*f*(c - c*Sin[e + f*x])^(3/2))
```

Rubi [A] time = 0.163481, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {2736, 2680, 2650, 2649, 206}

$$-\frac{a\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{8\sqrt{2}c^{5/2}f}-\frac{a\cos(e+fx)}{8cf(c-c\sin(e+fx))^{3/2}}+\frac{a\cos(e+fx)}{2f(c-c\sin(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])/(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] -(a*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])])/(8*
Sqrt[2]*c^(5/2)*f) + (a*Cos[e + f*x])/(2*f*(c - c*Sin[e + f*x])^(5/2)) - (a
*Cos[e + f*x])/(8*c*f*(c - c*Sin[e + f*x])^(3/2))
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
```

```
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{a+a\sin(e+fx)}{(c-c\sin(e+fx))^{5/2}}\,dx = (ac)\int \frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^{7/2}}\,dx$$

$$= \frac{a\cos(e+fx)}{2f(c-c\sin(e+fx))^{5/2}} - \frac{a\int \frac{1}{(c-c\sin(e+fx))^{3/2}}\,dx}{4c}$$

$$= \frac{a\cos(e+fx)}{2f(c-c\sin(e+fx))^{5/2}} - \frac{a\cos(e+fx)}{8cf(c-c\sin(e+fx))^{3/2}} - \frac{a\int \frac{1}{\sqrt{c-c\sin(e+fx)}}\,dx}{16c^2}$$

$$= \frac{a\cos(e+fx)}{2f(c-c\sin(e+fx))^{5/2}} - \frac{a\cos(e+fx)}{8cf(c-c\sin(e+fx))^{3/2}} + \frac{a\,\mathrm{Subst}\left(\int \frac{1}{2c-x^2}\,dx, x, -\frac{c\cos(e+fx)}{\sqrt{c-c\sin(e+fx)}}\right.}{8c^2f}$$

$$= -\frac{a\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{8\sqrt{2}c^{5/2}f} + \frac{a\cos(e+fx)}{2f(c-c\sin(e+fx))^{5/2}} - \frac{a\cos(e+fx)}{8cf(c-c\sin(e+fx))^{3/2}}$$

Mathematica [A] time = 0.911654, size = 176, normalized size = 1.56

$$\frac{a\left(2\sqrt{2}\sqrt{-c(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^4\tan^{-1}\left(\frac{\sqrt{-c(\sin(e+fx)+1)}}{\sqrt{2}\sqrt{c}}\right)-2\sqrt{c}(-8\sin(e+fx)+\cos(2(\ldots}{32c^{5/2}f\sqrt{c-c\sin(e+fx)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])/(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] (a*(-2*Sqrt[c]*(-7 + Cos[2*(e + f*x)] - 8*Sin[e + f*x]) + 2*Sqrt[2]*ArcTan[
Sqrt[-(c*(1 + Sin[e + f*x]))]/(Sqrt[2]*Sqrt[c])]*(Cos[(e + f*x)/2] - Sin[(e
 + f*x)/2])^4*Sqrt[-(c*(1 + Sin[e + f*x]))]))/(32*c^(5/2)*f*(Cos[(e + f*x)/
2] - Sin[(e + f*x)/2])^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*S
in[e + f*x]])
```

Maple [A] time = 0.714, size = 189, normalized size = 1.7

$$-\frac{a}{\left(-16+16\sin\left(fx+e\right)\right)\cos\left(fx+e\right)f}\left(-\sqrt{2}\mathrm{Artanh}\left(\frac{\sqrt{2}}{2}\sqrt{c\left(1+\sin\left(fx+e\right)\right)}\frac{1}{\sqrt{c}}\right)\left(\sin\left(fx+e\right)\right)^2c^3+2\left(c\left(1+\sin(\ldots\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] -1/16/c^(11/2)*a*(-2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(
1/2))*sin(f*x+e)^2*c^3+2*(c*(1+sin(f*x+e)))^(3/2)*c^(3/2)+2*2^(1/2)*arctanh
```

```
(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)*c^3+4*(c*(1+sin(f
*x+e)))^(1/2)*c^(5/2)-2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/
c^(1/2))*c^3)*(c*(1+sin(f*x+e)))^(1/2)/(-1+sin(f*x+e))/cos(f*x+e)/(c-c*sin(
f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a \sin\left(fx+e\right)+a}{\left(-c \sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)/(-c*sin(f*x + e) + c)^(5/2), x)
```

Fricas [B] time = 1.14269, size = 878, normalized size = 7.77

$$\frac{\sqrt{2}\left(a \cos\left(fx+e\right)^{3}+3\,a \cos\left(fx+e\right)^{2}-2\,a \cos\left(fx+e\right)-\left(a \cos\left(fx+e\right)^{2}-2\,a \cos\left(fx+e\right)-4\,a\right) \sin\left(fx+e\right)-4\,a \cdots}{32\left(c^{3} f \cos\left(fx+e\right)^{3}+\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/32*(sqrt(2)*(a*cos(f*x + e)^3 + 3*a*cos(f*x + e)^2 - 2*a*cos(f*x + e) - (
a*cos(f*x + e)^2 - 2*a*cos(f*x + e) - 4*a)*sin(f*x + e) - 4*a)*sqrt(c)*log(
-(c*cos(f*x + e)^2 - 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*sqrt(c)*(cos(f*x +
 e) + sin(f*x + e) + 1) + 3*c*cos(f*x + e) + (c*cos(f*x + e) - 2*c)*sin(f*x
 + e) + 2*c)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x +
e) - 2)) + 4*(a*cos(f*x + e)^2 - 3*a*cos(f*x + e) - (a*cos(f*x + e) + 4*a)*
sin(f*x + e) - 4*a)*sqrt(-c*sin(f*x + e) + c))/(c^3*f*cos(f*x + e)^3 + 3*c^
3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x + e) - 4*c^3*f - (c^3*f*cos(f*x + e)^2
 - 2*c^3*f*cos(f*x + e) - 4*c^3*f)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] sage2
```

3.297 $\int \frac{a+a\sin(e+fx)}{(c-c\sin(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=145

$$-\frac{a\cos(e+fx)}{32c^2f(c-c\sin(e+fx))^{3/2}}-\frac{a\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{32\sqrt{2}c^{7/2}f}-\frac{a\cos(e+fx)}{24cf(c-c\sin(e+fx))^{5/2}}+\frac{a\cos(e+fx)}{3f(c-c\sin(e+fx))^{7/2}}$$

```
[Out] -(a*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])])/(32
*Sqrt[2]*c^(7/2)*f) + (a*Cos[e + f*x])/(3*f*(c - c*Sin[e + f*x])^(7/2)) - (
a*Cos[e + f*x])/(24*c*f*(c - c*Sin[e + f*x])^(5/2)) - (a*Cos[e + f*x])/(32*
c^2*f*(c - c*Sin[e + f*x])^(3/2))
```

Rubi [A] time = 0.193465, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {2736, 2680, 2650, 2649, 206}

$$-\frac{a\cos(e+fx)}{32c^2f(c-c\sin(e+fx))^{3/2}}-\frac{a\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{32\sqrt{2}c^{7/2}f}-\frac{a\cos(e+fx)}{24cf(c-c\sin(e+fx))^{5/2}}+\frac{a\cos(e+fx)}{3f(c-c\sin(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])/(c - c*Sin[e + f*x])^(7/2),x]
```


```
[Out] -(a*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])])/(32
*Sqrt[2]*c^(7/2)*f) + (a*Cos[e + f*x])/(3*f*(c - c*Sin[e + f*x])^(7/2)) - (
a*Cos[e + f*x])/(24*c*f*(c - c*Sin[e + f*x])^(5/2)) - (a*Cos[e + f*x])/(32*
c^2*f*(c - c*Sin[e + f*x])^(3/2))
```


Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{a+a\sin(e+fx)}{(c-c\sin(e+fx))^{7/2}}\,dx = (ac)\int \frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^{9/2}}\,dx$$

$$= \frac{a\cos(e+fx)}{3f(c-c\sin(e+fx))^{7/2}} - \frac{a\int \frac{1}{(c-c\sin(e+fx))^{5/2}}\,dx}{6c}$$

$$= \frac{a\cos(e+fx)}{3f(c-c\sin(e+fx))^{7/2}} - \frac{a\cos(e+fx)}{24cf(c-c\sin(e+fx))^{5/2}} - \frac{a\int \frac{1}{(c-c\sin(e+fx))^{3/2}}\,dx}{16c^2}$$

$$= \frac{a\cos(e+fx)}{3f(c-c\sin(e+fx))^{7/2}} - \frac{a\cos(e+fx)}{24cf(c-c\sin(e+fx))^{5/2}} - \frac{a\cos(e+fx)}{32c^2f(c-c\sin(e+fx))^{3/2}} - \frac{a\int \frac{}{\sqrt{c}}$$

$$= \frac{a\cos(e+fx)}{3f(c-c\sin(e+fx))^{7/2}} - \frac{a\cos(e+fx)}{24cf(c-c\sin(e+fx))^{5/2}} - \frac{a\cos(e+fx)}{32c^2f(c-c\sin(e+fx))^{3/2}} + \frac{a\,\mathrm{Sub}}{}$$

$$= -\frac{a\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{32\sqrt{2}c^{7/2}f} + \frac{a\cos(e+fx)}{3f(c-c\sin(e+fx))^{7/2}} - \frac{a\cos(e+fx)}{24cf(c-c\sin(e+fx))^{5/2}} - \frac{}{32c^2}$$

Mathematica [A] time = 1.1603, size = 189, normalized size = 1.3

$$\frac{a\left(2\sqrt{c}(131\sin(e+fx)+3(\sin(3(e+fx))+38)-14\cos(2(e+fx)))+12\sqrt{2}\sqrt{-c(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\right.\right.}{768c^{7/2}f\sqrt{c-c\sin(e+fx)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^5\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])/(c - c*Sin[e + f*x])^(7/2),x]
```

```
[Out] (a*(12*Sqrt[2]*ArcTan[Sqrt[-(c*(1 + Sin[e + f*x]))]/(Sqrt[2]*Sqrt[c])]*(Cos
[(e + f*x)/2] - Sin[(e + f*x)/2])^6*Sqrt[-(c*(1 + Sin[e + f*x]))] + 2*Sqrt[
c]*(-14*Cos[2*(e + f*x)] + 131*Sin[e + f*x] + 3*(38 + Sin[3*(e + f*x)])))))/
(768*c^(7/2)*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] +
Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]])
```

Maple [A] time = 0.719, size = 243, normalized size = 1.7

$$\frac{a}{192\left(-1+\sin\left(fx+e\right)\right)^2\cos\left(fx+e\right)f}\left(24\sqrt{c\left(1+\sin\left(fx+e\right)\right)}c^{9/2}+32\left(c\left(1+\sin\left(fx+e\right)\right)\right)^{3/2}c^{7/2}-6\left(c\left(1+\sin\left(f\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x)
```

```
[Out] 1/192*a*(24*(c*(1+sin(f*x+e)))^(1/2)*c^(9/2)+32*(c*(1+sin(f*x+e)))^(3/2)*c^
(7/2)-6*(c*(1+sin(f*x+e)))^(5/2)*c^(5/2)+3*2^(1/2)*arctanh(1/2*(c*(1+sin(f*
x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)^3*c^5-9*2^(1/2)*arctanh(1/2*(c*(1+
sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)^2*c^5+9*2^(1/2)*arctanh(1/2*
(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)*c^5-3*2^(1/2)*arctanh(
1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*c^5)*(c*(1+sin(f*x+e)))^(1/2)
/c^(17/2)/(-1+sin(f*x+e))^2/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a\sin\left(fx+e\right)+a}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)/(-c*sin(f*x + e) + c)^(7/2), x)
```

Fricas [B] time = 1.19135, size = 1073, normalized size = 7.4

$$\frac{3\sqrt{2}\left(a\cos\left(fx+e\right)^{4}-3\,a\cos\left(fx+e\right)^{3}-8\,a\cos\left(fx+e\right)^{2}+4\,a\cos\left(fx+e\right)+\left(a\cos\left(fx+e\right)^{3}+4\,a\cos\left(fx+e\right)^{2}-\right.\right.}{38}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/384*(3*sqrt(2)*(a*cos(f*x + e)^4 - 3*a*cos(f*x + e)^3 - 8*a*cos(f*x + e)^
2 + 4*a*cos(f*x + e) + (a*cos(f*x + e)^3 + 4*a*cos(f*x + e)^2 - 4*a*cos(f*x
 + e) - 8*a)*sin(f*x + e) + 8*a)*sqrt(c)*log(-(c*cos(f*x + e)^2 - 2*sqrt(2)
*sqrt(-c*sin(f*x + e) + c)*sqrt(c)*(cos(f*x + e) + sin(f*x + e) + 1) + 3*c*
cos(f*x + e) + (c*cos(f*x + e) - 2*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2 +
 (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) + 4*(3*a*cos(f*x + e)
^3 - 7*a*cos(f*x + e)^2 + 22*a*cos(f*x + e) + (3*a*cos(f*x + e)^2 + 10*a*co
s(f*x + e) + 32*a)*sin(f*x + e) + 32*a)*sqrt(-c*sin(f*x + e) + c))/(c^4*f*c
os(f*x + e)^4 - 3*c^4*f*cos(f*x + e)^3 - 8*c^4*f*cos(f*x + e)^2 + 4*c^4*f*c
os(f*x + e) + 8*c^4*f + (c^4*f*cos(f*x + e)^3 + 4*c^4*f*cos(f*x + e)^2 - 4*
c^4*f*cos(f*x + e) - 8*c^4*f)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x, algorithm="giac")
```

```
[Out] sage2
```

3.298 $\int (a+a\sin(e+fx))^2(c-c\sin(e+fx))^{7/2}\,dx$

Optimal. Leaf size=145

$$\frac{256a^2c^6\cos^5(e+fx)}{1155f(c-c\sin(e+fx))^{5/2}}+\frac{64a^2c^5\cos^5(e+fx)}{231f(c-c\sin(e+fx))^{3/2}}+\frac{8a^2c^4\cos^5(e+fx)}{33f\sqrt{c-c\sin(e+fx)}}+\frac{2a^2c^3\cos^5(e+fx)\sqrt{c-c\sin(e+fx)}}{11f}$$

```
[Out] (256*a^2*c^6*Cos[e + f*x]^5)/(1155*f*(c - c*Sin[e + f*x])^(5/2)) + (64*a^2*
c^5*Cos[e + f*x]^5)/(231*f*(c - c*Sin[e + f*x])^(3/2)) + (8*a^2*c^4*Cos[e +
 f*x]^5)/(33*f*Sqrt[c - c*Sin[e + f*x]]) + (2*a^2*c^3*Cos[e + f*x]^5*Sqrt[c
 - c*Sin[e + f*x]])/(11*f)
```

Rubi [A] time = 0.327226, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {2736, 2674, 2673}

$$\frac{256a^2c^6\cos^5(e+fx)}{1155f(c-c\sin(e+fx))^{5/2}}+\frac{64a^2c^5\cos^5(e+fx)}{231f(c-c\sin(e+fx))^{3/2}}+\frac{8a^2c^4\cos^5(e+fx)}{33f\sqrt{c-c\sin(e+fx)}}+\frac{2a^2c^3\cos^5(e+fx)\sqrt{c-c\sin(e+fx)}}{11f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(7/2),x]
```


```
[Out] (256*a^2*c^6*Cos[e + f*x]^5)/(1155*f*(c - c*Sin[e + f*x])^(5/2)) + (64*a^2*
c^5*Cos[e + f*x]^5)/(231*f*(c - c*Sin[e + f*x])^(3/2)) + (8*a^2*c^4*Cos[e +
 f*x]^5)/(33*f*Sqrt[c - c*Sin[e + f*x]]) + (2*a^2*c^3*Cos[e + f*x]^5*Sqrt[c
 - c*Sin[e + f*x]])/(11*f)
```


Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rubi steps

$$\int (a+a\sin(e+fx))^2(c-c\sin(e+fx))^{7/2}\,dx = \left(a^2c^2\right)\int \cos^4(e+fx)(c-c\sin(e+fx))^{3/2}\,dx$$
$$= \frac{2a^2c^3\cos^5(e+fx)\sqrt{c-c\sin(e+fx)}}{11f} + \frac{1}{11}\left(12a^2c^3\right)\int \cos^4(e+fx)\sqrt{c-$$
$$= \frac{8a^2c^4\cos^5(e+fx)}{33f\sqrt{c-c\sin(e+fx)}} + \frac{2a^2c^3\cos^5(e+fx)\sqrt{c-c\sin(e+fx)}}{11f} + \frac{1}{33}(32$$
$$= \frac{64a^2c^5\cos^5(e+fx)}{231f(c-c\sin(e+fx))^{3/2}} + \frac{8a^2c^4\cos^5(e+fx)}{33f\sqrt{c-c\sin(e+fx)}} + \frac{2a^2c^3\cos^5(e+fx}{11$$
$$= \frac{256a^2c^6\cos^5(e+fx)}{1155f(c-c\sin(e+fx))^{5/2}} + \frac{64a^2c^5\cos^5(e+fx)}{231f(c-c\sin(e+fx))^{3/2}} + \frac{8a^2c^4\cos^5(}{33f\sqrt{c-c\sin}$$

Mathematica [B] time = 6.43329, size = 1105, normalized size = 7.62

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(7/2),x]
```

```
[Out] (7*Cos[(e + f*x)/2]*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(7/2))/(8*f
*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/
2])^4) - (Cos[(3*(e + f*x))/2]*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^
(7/2))/(8*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin
[(e + f*x)/2])^4) + (11*Cos[(5*(e + f*x))/2]*(a + a*Sin[e + f*x])^2*(c - c*
Sin[e + f*x])^(7/2))/(80*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e
+ f*x)/2] + Sin[(e + f*x)/2])^4) + (Cos[(7*(e + f*x))/2]*(a + a*Sin[e + f*x
])^2*(c - c*Sin[e + f*x])^(7/2))/(112*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2
])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4) + (Cos[(9*(e + f*x))/2]*(a +
a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(7/2))/(48*f*(Cos[(e + f*x)/2] - Sin
[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4) + (Cos[(11*(e + f
*x))/2]*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(7/2))/(176*f*(Cos[(e +
 f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4) + (
7*Sin[(e + f*x)/2]*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(7/2))/(8*f*
(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2
])^4) + ((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(7/2)*Sin[(3*(e + f*x)
)/2])/(8*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[
(e + f*x)/2])^4) + (11*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(7/2)*Si
n[(5*(e + f*x))/2])/(80*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e +
 f*x)/2] + Sin[(e + f*x)/2])^4) - ((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*
x])^(7/2)*Sin[(7*(e + f*x))/2])/(112*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]
)^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4) + ((a + a*Sin[e + f*x])^2*(c -
 c*Sin[e + f*x])^(7/2)*Sin[(9*(e + f*x))/2])/(48*f*(Cos[(e + f*x)/2] - Sin[
(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4) - ((a + a*Sin[e +
f*x])^2*(c - c*Sin[e + f*x])^(7/2)*Sin[(11*(e + f*x))/2])/(176*f*(Cos[(e +
f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4)
```

Maple [A] time = 0.632, size = 81, normalized size = 0.6

$$\frac{\left(-2+2\sin\left(fx+e\right)\right)c^4\left(1+\sin\left(fx+e\right)\right)^3a^2\left(105\left(\sin\left(fx+e\right)\right)^3-455\left(\sin\left(fx+e\right)\right)^2+755\sin\left(fx+e\right)-533\right)}{1155\,f\cos\left(fx+e\right)}\frac{}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e))^(7/2),x)
```

```
[Out] 2/1155*(-1+sin(f*x+e))*c^4*(1+sin(f*x+e))^3*a^2*(105*sin(f*x+e)^3-455*sin(f
*x+e)^2+755*sin(f*x+e)-533)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a \sin\left(fx+e\right)+a\right)^{2} \left(-c \sin\left(fx+e\right)+c\right)^{\frac{7}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^2*(-c*sin(f*x + e) + c)^(7/2), x)
```

Fricas [A] time = 1.09928, size = 576, normalized size = 3.97

$$\frac{2\left(105\,a^{2}c^{3}\cos\left(fx+e\right)^{6}+245\,a^{2}c^{3}\cos\left(fx+e\right)^{5}-20\,a^{2}c^{3}\cos\left(fx+e\right)^{4}+32\,a^{2}c^{3}\cos\left(fx+e\right)^{3}-64\,a^{2}c^{3}\cos\left(fx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e))^(7/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(105*a^2*c^3*cos(f*x + e)^6 + 245*a^2*c^3*cos(f*x + e)^5 - 20*a^2*c^
3*cos(f*x + e)^4 + 32*a^2*c^3*cos(f*x + e)^3 - 64*a^2*c^3*cos(f*x + e)^2 +
256*a^2*c^3*cos(f*x + e) + 512*a^2*c^3 - (105*a^2*c^3*cos(f*x + e)^5 - 140*
a^2*c^3*cos(f*x + e)^4 - 160*a^2*c^3*cos(f*x + e)^3 - 192*a^2*c^3*cos(f*x +
 e)^2 - 256*a^2*c^3*cos(f*x + e) - 512*a^2*c^3)*sin(f*x + e))*sqrt(-c*sin(f
*x + e) + c)/(f*cos(f*x + e) - f*sin(f*x + e) + f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(c-c*sin(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a \sin\left(fx+e\right)+a\right)^{2} \left(-c \sin\left(fx+e\right)+c\right)^{\frac{7}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^2*(-c*sin(f*x + e) + c)^(7/2), x)
```

3.299 $\int (a + a\sin(e + fx))^2(c - c\sin(e + fx))^{5/2}\,dx$

Optimal. Leaf size=109

$$\frac{2a^2c^3\cos^5(e+fx)}{9f\sqrt{c-c\sin(e+fx)}}+\frac{16a^2c^4\cos^5(e+fx)}{63f(c-c\sin(e+fx))^{3/2}}+\frac{64a^2c^5\cos^5(e+fx)}{315f(c-c\sin(e+fx))^{5/2}}$$

```
[Out] (64*a^2*c^5*Cos[e + f*x]^5)/(315*f*(c - c*Sin[e + f*x])^(5/2)) + (16*a^2*c^
4*Cos[e + f*x]^5)/(63*f*(c - c*Sin[e + f*x])^(3/2)) + (2*a^2*c^3*Cos[e + f*
x]^5)/(9*f*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 0.259677, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {2736, 2674, 2673}

$$\frac{2a^2c^3\cos^5(e+fx)}{9f\sqrt{c-c\sin(e+fx)}}+\frac{16a^2c^4\cos^5(e+fx)}{63f(c-c\sin(e+fx))^{3/2}}+\frac{64a^2c^5\cos^5(e+fx)}{315f(c-c\sin(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] (64*a^2*c^5*Cos[e + f*x]^5)/(315*f*(c - c*Sin[e + f*x])^(5/2)) + (16*a^2*c^
4*Cos[e + f*x]^5)/(63*f*(c - c*Sin[e + f*x])^(3/2)) + (2*a^2*c^3*Cos[e + f*
x]^5)/(9*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rubi steps

$$\int (a+a\sin(e+fx))^2(c-c\sin(e+fx))^{5/2}\,dx = \left(a^2c^2\right)\int \cos^4(e+fx)\sqrt{c-c\sin(e+fx)}\,dx$$
$$= \frac{2a^2c^3\cos^5(e+fx)}{9f\sqrt{c-c\sin(e+fx)}} + \frac{1}{9}\left(8a^2c^3\right)\int \frac{\cos^4(e+fx)}{\sqrt{c-c\sin(e+fx)}}\,dx$$
$$= \frac{16a^2c^4\cos^5(e+fx)}{63f(c-c\sin(e+fx))^{3/2}} + \frac{2a^2c^3\cos^5(e+fx)}{9f\sqrt{c-c\sin(e+fx)}} + \frac{1}{63}\left(32a^2c^4\right)\int \frac{\mathrm{c}}{(c-}$$
$$= \frac{64a^2c^5\cos^5(e+fx)}{315f(c-c\sin(e+fx))^{5/2}} + \frac{16a^2c^4\cos^5(e+fx)}{63f(c-c\sin(e+fx))^{3/2}} + \frac{2a^2c^3\cos^5(e+}{9f\sqrt{c-c\sin(e}}$$

Mathematica [A] time = 5.38052, size = 96, normalized size = 0.88

$$-\frac{a^2c^2\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^5(220\sin(e+fx)+35\cos(2(e+fx))-249)}{315f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] -(a^2*c^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5*(-249 + 35*Cos[2*(e + f*x
)] + 220*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(315*f*(Cos[(e + f*x)/2] -
 Sin[(e + f*x)/2]))
```

Maple [A] time = 0.478, size = 71, normalized size = 0.7

$$-\frac{\left(-2+2\sin\left(fx+e\right)\right)c^3\left(1+\sin\left(fx+e\right)\right)^3a^2\left(35\left(\sin\left(fx+e\right)\right)^2-110\sin\left(fx+e\right)+107\right)}{315f\cos\left(fx+e\right)}\frac{1}{\sqrt{c-c\sin\left(fx+e\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] -2/315*(-1+sin(f*x+e))*c^3*(1+sin(f*x+e))^3*a^2*(35*sin(f*x+e)^2-110*sin(f*
x+e)+107)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^2\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^2*(-c*sin(f*x + e) + c)^(5/2), x)
```

Fricas [B] time = 1.07699, size = 486, normalized size = 4.46

$$\frac{2\left(35\,a^2c^2\cos\left(fx+e\right)^5-5\,a^2c^2\cos\left(fx+e\right)^4+8\,a^2c^2\cos\left(fx+e\right)^3-16\,a^2c^2\cos\left(fx+e\right)^2+64\,a^2c^2\cos\left(fx+e\right)+\cdots\right.}{315\,(\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*a^2*c^2*cos(f*x + e)^5 - 5*a^2*c^2*cos(f*x + e)^4 + 8*a^2*c^2*cos
(f*x + e)^3 - 16*a^2*c^2*cos(f*x + e)^2 + 64*a^2*c^2*cos(f*x + e) + 128*a^2
*c^2 + (35*a^2*c^2*cos(f*x + e)^4 + 40*a^2*c^2*cos(f*x + e)^3 + 48*a^2*c^2*
cos(f*x + e)^2 + 64*a^2*c^2*cos(f*x + e) + 128*a^2*c^2)*sin(f*x + e))*sqrt(
-c*sin(f*x + e) + c)/(f*cos(f*x + e) - f*sin(f*x + e) + f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^2\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^2*(-c*sin(f*x + e) + c)^(5/2), x)
```

3.300 $\int (a+a\sin(e+fx))^2(c-c\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=73

$$\frac{2a^2c^3\cos^5(e+fx)}{7f(c-c\sin(e+fx))^{3/2}}+\frac{8a^2c^4\cos^5(e+fx)}{35f(c-c\sin(e+fx))^{5/2}}$$

```
[Out] (8*a^2*c^4*Cos[e + f*x]^5)/(35*f*(c - c*Sin[e + f*x])^(5/2)) + (2*a^2*c^3*C
os[e + f*x]^5)/(7*f*(c - c*Sin[e + f*x])^(3/2))
```

Rubi [A] time = 0.195061, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {2736, 2674, 2673}

$$\frac{2a^2c^3\cos^5(e+fx)}{7f(c-c\sin(e+fx))^{3/2}}+\frac{8a^2c^4\cos^5(e+fx)}{35f(c-c\sin(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] (8*a^2*c^4*Cos[e + f*x]^5)/(35*f*(c - c*Sin[e + f*x])^(5/2)) + (2*a^2*c^3*C
os[e + f*x]^5)/(7*f*(c - c*Sin[e + f*x])^(3/2))
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rubi steps

$$\int (a+a\sin(e+fx))^2(c-c\sin(e+fx))^{3/2}\,dx = \left(a^2c^2\right)\int \frac{\cos^4(e+fx)}{\sqrt{c-c\sin(e+fx)}}\,dx$$
$$= \frac{2a^2c^3\cos^5(e+fx)}{7f(c-c\sin(e+fx))^{3/2}} + \frac{1}{7}\left(4a^2c^3\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^{3/2}}\,dx$$
$$= \frac{8a^2c^4\cos^5(e+fx)}{35f(c-c\sin(e+fx))^{5/2}} + \frac{2a^2c^3\cos^5(e+fx)}{7f(c-c\sin(e+fx))^{3/2}}$$

Mathematica [A] time = 1.36272, size = 84, normalized size = 1.15

$$-\frac{2a^2c(5\sin(e+fx)-9)\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^5}{35f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-2*a^2*c*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5*(-9 + 5*Sin[e + f*x])*Sqr
t[c - c*Sin[e + f*x]])/(35*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))
```

Maple [A] time = 0.67, size = 61, normalized size = 0.8

$$\frac{\left(-2+2\sin\left(fx+e\right)\right)c^2\left(1+\sin\left(fx+e\right)\right)^3a^2\left(5\sin\left(fx+e\right)-9\right)}{35f\cos\left(fx+e\right)}\frac{1}{\sqrt{c-c\sin\left(fx+e\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] 2/35*(-1+sin(f*x+e))*c^2*(1+sin(f*x+e))^3*a^2*(5*sin(f*x+e)-9)/cos(f*x+e)/(
c-c*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^2\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^2*(-c*sin(f*x + e) + c)^(3/2), x)
```

Fricas [B] time = 1.02344, size = 374, normalized size = 5.12

$$-\frac{2\left(5a^2c\cos\left(fx+e\right)^4-a^2c\cos\left(fx+e\right)^3+2a^2c\cos\left(fx+e\right)^2-8a^2c\cos\left(fx+e\right)-16a^2c-\left(5a^2c\cos\left(fx+e\right)^3+\right.}{35\left(f\cos\left(fx+e\right)-f\sin\left(fx+e\right)+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] -2/35*(5*a^2*c*cos(f*x + e)^4 - a^2*c*cos(f*x + e)^3 + 2*a^2*c*cos(f*x + e)
^2 - 8*a^2*c*cos(f*x + e) - 16*a^2*c - (5*a^2*c*cos(f*x + e)^3 + 6*a^2*c*co
s(f*x + e)^2 + 8*a^2*c*cos(f*x + e) + 16*a^2*c)*sin(f*x + e))*sqrt(-c*sin(f
*x + e) + c)/(f*cos(f*x + e) - f*sin(f*x + e) + f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.301 $\int (a+a\sin(e+fx))^2\sqrt{c-c\sin(e+fx)}\,dx$

Optimal. Leaf size=36

$$\frac{2a^2c^3\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{5/2}}$$

```
[Out] (2*a^2*c^3*Cos[e + f*x]^5)/(5*f*(c - c*Sin[e + f*x])^(5/2))
```

Rubi [A] time = 0.125124, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2736, 2673}

$$\frac{2a^2c^3\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2*Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (2*a^2*c^3*Cos[e + f*x]^5)/(5*f*(c - c*Sin[e + f*x])^(5/2))
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rubi steps

$$\begin{aligned}\int (a+a\sin(e+fx))^2\sqrt{c-c\sin(e+fx)}\,dx &= \left(a^2c^2\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^{3/2}}\,dx\\ &= \frac{2a^2c^3\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{5/2}}\end{aligned}$$

Mathematica [B] time = 0.236341, size = 73, normalized size = 2.03

$$\frac{2a^2\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^5}{5f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2*Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (2*a^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5*Sqrt[c - c*Sin[e + f*x]])/(5
*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))
```

Maple [A] time = 0.333, size = 49, normalized size = 1.4

$$-\frac{\left(-2+2\sin\left(fx+e\right)\right)c\left(1+\sin\left(fx+e\right)\right)^{3}a^{2}}{5f\cos\left(fx+e\right)}\frac{1}{\sqrt{c-c\sin\left(fx+e\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] -2/5*(-1+sin(f*x+e))*c*(1+sin(f*x+e))^3*a^2/cos(f*x+e)/(c-c*sin(f*x+e))^(1/
2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a\sin\left(fx+e\right)+a\right)^{2}\sqrt{-c\sin\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^2*sqrt(-c*sin(f*x + e) + c), x)
```

Fricas [B] time = 1.02983, size = 281, normalized size = 7.81

$$-\frac{2\left(a^{2}\cos\left(fx+e\right)^{3}+3a^{2}\cos\left(fx+e\right)^{2}-2a^{2}\cos\left(fx+e\right)-4a^{2}+\left(a^{2}\cos\left(fx+e\right)^{2}-2a^{2}\cos\left(fx+e\right)-4a^{2}\right)\sin\left(fx\right.}{5\left(f\cos\left(fx+e\right)-f\sin\left(fx+e\right)+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] -2/5*(a^2*cos(f*x + e)^3 + 3*a^2*cos(f*x + e)^2 - 2*a^2*cos(f*x + e) - 4*a^
2 + (a^2*cos(f*x + e)^2 - 2*a^2*cos(f*x + e) - 4*a^2)*sin(f*x + e))*sqrt(-c
*sin(f*x + e) + c)/(f*cos(f*x + e) - f*sin(f*x + e) + f)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^{2}\left(\int 2\sqrt{-c\sin\left(e+fx\right)+c}\sin\left(e+fx\right)dx+\int\sqrt{-c\sin\left(e+fx\right)+c}\sin^{2}\left(e+fx\right)dx+\int\sqrt{-c\sin\left(e+fx\right)+c}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] a**2*(Integral(2*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x), x) + Integral(sqrt
(-c*sin(e + f*x) + c)*sin(e + f*x)**2, x) + Integral(sqrt(-c*sin(e + f*x) +
 c), x))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.302 $\int \frac{(a+a\sin(e+fx))^2}{\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=115

$$-\frac{2a^2c\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{3/2}}-\frac{4a^2\cos(e+fx)}{f\sqrt{c-c\sin(e+fx)}}+\frac{4\sqrt{2}a^2\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{c}f}$$

```
[Out] (4*Sqrt[2]*a^2*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f
*x]])])/(Sqrt[c]*f) - (2*a^2*c*Cos[e + f*x]^3)/(3*f*(c - c*Sin[e + f*x])^(3
/2)) - (4*a^2*Cos[e + f*x])/(f*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 0.244165, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2736, 2679, 2649, 206}

$$-\frac{2a^2c\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{3/2}}-\frac{4a^2\cos(e+fx)}{f\sqrt{c-c\sin(e+fx)}}+\frac{4\sqrt{2}a^2\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{c}f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2/Sqrt[c - c*Sin[e + f*x]],x]
```


```
[Out] (4*Sqrt[2]*a^2*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f
*x]])])/(Sqrt[c]*f) - (2*a^2*c*Cos[e + f*x]^3)/(3*f*(c - c*Sin[e + f*x])^(3
/2)) - (4*a^2*Cos[e + f*x])/(f*Sqrt[c - c*Sin[e + f*x]])
```


Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2679

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(a*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^2}{\sqrt{c-c\sin(e+fx)}}\,dx = \left(a^2c^2\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^{5/2}}\,dx$$
$$= -\frac{2a^2c\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{3/2}} + \left(2a^2c\right)\int \frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^{3/2}}\,dx$$
$$= -\frac{2a^2c\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{3/2}} - \frac{4a^2\cos(e+fx)}{f\sqrt{c-c\sin(e+fx)}} + \left(4a^2\right)\int \frac{1}{\sqrt{c-c\sin(e+fx)}}\,dx$$
$$= -\frac{2a^2c\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{3/2}} - \frac{4a^2\cos(e+fx)}{f\sqrt{c-c\sin(e+fx)}} - \frac{\left(8a^2\right)\mathrm{Subst}\left(\int \frac{1}{2c-x^2}\,dx, x, -\frac{c\cos(e+f}{\sqrt{c-c\sin(e+}}\right.}{f}$$
$$= \frac{4\sqrt{2}a^2\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{c}f} - \frac{2a^2c\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{3/2}} - \frac{4a^2\cos(e+fx)}{f\sqrt{c-c\sin(e+fx)}}$$

Mathematica [C] time = 0.443477, size = 130, normalized size = 1.13

$$-\frac{a^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(15\sin\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{3}{2}(e+fx)\right)+15\cos\left(\frac{1}{2}(e+fx)\right)-\cos\left(\frac{3}{2}(e+fx)\right)\right.}{3f\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2/Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] -(a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*((24 + 24*I)*(-1)^(1/4)*ArcTan[
(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/4])] + 15*Cos[(e + f*x)/2] - Cos[
(3*(e + f*x))/2] + 15*Sin[(e + f*x)/2] + Sin[(3*(e + f*x))/2]))/(3*f*Sqrt[c
 - c*Sin[e + f*x]])
```

Maple [A] time = 0.674, size = 112, normalized size = 1.

$$-\frac{\left(-2+2\sin\left(fx+e\right)\right)a^2}{3c^2\cos\left(fx+e\right)f}\sqrt{c\left(1+\sin\left(fx+e\right)\right)}\left(6c^{3/2}\sqrt{2}\mathrm{Artanh}\left(1/2\,\frac{\sqrt{c\left(1+\sin\left(fx+e\right)\right)}\sqrt{2}}{\sqrt{c}}\right)-\left(c\left(1+\sin\left(fx+e\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] -2/3*(-1+sin(f*x+e))*(c*(1+sin(f*x+e)))^(1/2)*a^2*(6*c^(3/2)*2^(1/2)*arctan
h(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))-(c*(1+sin(f*x+e)))^(3/2)-6*
(c*(1+sin(f*x+e)))^(1/2)*c)/c^2/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^2}{\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^2/sqrt(-c*sin(f*x + e) + c), x)
```

Fricas [B] time = 1.09188, size = 630, normalized size = 5.48

$$\frac{2\left(\frac{3\sqrt{2}\left(a^{2}c\cos\left(fx+e\right)-a^{2}c\sin\left(fx+e\right)+a^{2}c\right)\log\left(-\frac{\cos\left(fx+e\right)^{2}+\left(\cos\left(fx+e\right)-2\right)\sin\left(fx+e\right)+\frac{2\sqrt{2}\sqrt{-c\sin\left(fx+e\right)+c}\left(\cos\left(fx+e\right)+\sin\left(fx+e\right)+1\right)}{\sqrt{c}}+3\cos\left(fx+e\right)+2}{\cos\left(fx+e\right)^{2}+\left(\cos\left(fx+e\right)+2\right)\sin\left(fx+e\right)-\cos\left(fx+e\right)-2}\right)}{\sqrt{c}}+\left(a^{2}\cos\left(\right.\right.}{3\left(cf\cos\left(fx+e\right)-cf\sin\left(fx+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(3*sqrt(2)*(a^2*c*cos(f*x + e) - a^2*c*sin(f*x + e) + a^2*c)*log(-(cos(
f*x + e)^2 + (cos(f*x + e) - 2)*sin(f*x + e) + 2*sqrt(2)*sqrt(-c*sin(f*x +
e) + c)*(cos(f*x + e) + sin(f*x + e) + 1)/sqrt(c) + 3*cos(f*x + e) + 2)/(co
s(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2))/sqrt(c)
 + (a^2*cos(f*x + e)^2 - 7*a^2*cos(f*x + e) - 8*a^2 - (a^2*cos(f*x + e) + 8
*a^2)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c))/(c*f*cos(f*x + e) - c*f*sin(
f*x + e) + c*f)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^{2}\left(\int\frac{2\sin\left(e+fx\right)}{\sqrt{-c\sin\left(e+fx\right)+c}}\,dx+\int\frac{\sin^{2}\left(e+fx\right)}{\sqrt{-c\sin\left(e+fx\right)+c}}\,dx+\int\frac{1}{\sqrt{-c\sin\left(e+fx\right)+c}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2/(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] a**2*(Integral(2*sin(e + f*x)/sqrt(-c*sin(e + f*x) + c), x) + Integral(sin(
e + f*x)**2/sqrt(-c*sin(e + f*x) + c), x) + Integral(1/sqrt(-c*sin(e + f*x)
 + c), x))
```

Giac [B] time = 3.50761, size = 375, normalized size = 3.26

$$\frac{\frac{24\sqrt{2}a^{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{c}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+c}-\sqrt{c}\right)}{2\sqrt{-c}}\right)}{\sqrt{-c}\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}+\frac{\left(\left(\frac{7a^{2}\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{c^{5}}+\frac{9a^{2}\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{c^{5}}\right)\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+\frac{9a^{2}s}{}\right.}{\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+c\right)^{\frac{3}{2}}}}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*(24*sqrt(2)*a^2*arctan(-1/2*sqrt(2)*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqr
t(c*tan(1/2*f*x + 1/2*e)^2 + c) - sqrt(c))/sqrt(-c))/(sqrt(-c)*sgn(tan(1/2*
f*x + 1/2*e) - 1)) + (((7*a^2*sgn(tan(1/2*f*x + 1/2*e) - 1)*tan(1/2*f*x + 1
/2*e)/c^5 + 9*a^2*sgn(tan(1/2*f*x + 1/2*e) - 1)/c^5)*tan(1/2*f*x + 1/2*e) +
 9*a^2*sgn(tan(1/2*f*x + 1/2*e) - 1)/c^5)*tan(1/2*f*x + 1/2*e) + 7*a^2*sgn(
tan(1/2*f*x + 1/2*e) - 1)/c^5)/(c*tan(1/2*f*x + 1/2*e)^2 + c)^(3/2) - 8*sqr
t(2)*(3*a^2*c^(15/2)*arctan(sqrt(c)/sqrt(-c)) + a^2*sqrt(-c)*c)*sgn(tan(1/2
*f*x + 1/2*e) - 1)/(sqrt(-c)*c^(15/2)))/f
```

3.303 $\int \frac{(a+a\sin(e+fx))^2}{(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=115

$$-\frac{3\sqrt{2}a^2\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{c^{3/2}f}+\frac{a^2c\cos^3(e+fx)}{f(c-c\sin(e+fx))^{5/2}}+\frac{3a^2\cos(e+fx)}{cf\sqrt{c-c\sin(e+fx)}}$$

```
[Out] (-3*Sqrt[2]*a^2*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e +
f*x]])])/(c^(3/2)*f) + (a^2*c*Cos[e + f*x]^3)/(f*(c - c*Sin[e + f*x])^(5/2)
) + (3*a^2*Cos[e + f*x])/(c*f*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 0.244084, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {2736, 2680, 2679, 2649, 206}

$$-\frac{3\sqrt{2}a^2\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{c^{3/2}f}+\frac{a^2c\cos^3(e+fx)}{f(c-c\sin(e+fx))^{5/2}}+\frac{3a^2\cos(e+fx)}{cf\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2/(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-3*Sqrt[2]*a^2*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e +
f*x]])])/(c^(3/2)*f) + (a^2*c*Cos[e + f*x]^3)/(f*(c - c*Sin[e + f*x])^(5/2)
) + (3*a^2*Cos[e + f*x])/(c*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2679

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(a*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^2}{(c-c\sin(e+fx))^{3/2}}\,dx = \left(a^2c^2\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^{7/2}}\,dx$$
$$= \frac{a^2c\cos^3(e+fx)}{f(c-c\sin(e+fx))^{5/2}} - \frac{1}{2}\left(3a^2\right)\int \frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^{3/2}}\,dx$$
$$= \frac{a^2c\cos^3(e+fx)}{f(c-c\sin(e+fx))^{5/2}} + \frac{3a^2\cos(e+fx)}{cf\sqrt{c-c\sin(e+fx)}} - \frac{\left(3a^2\right)\int \frac{1}{\sqrt{c-c\sin(e+fx)}}\,dx}{c}$$
$$= \frac{a^2c\cos^3(e+fx)}{f(c-c\sin(e+fx))^{5/2}} + \frac{3a^2\cos(e+fx)}{cf\sqrt{c-c\sin(e+fx)}} + \frac{\left(6a^2\right)\text{Subst}\left(\int \frac{1}{2c-x^2}\,dx, x, -\frac{c\cos(e+fx)}{\sqrt{c-c\sin(e+fx)}}\right)}{cf}$$
$$= -\frac{3\sqrt{2}a^2\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{c^{3/2}f} + \frac{a^2c\cos^3(e+fx)}{f(c-c\sin(e+fx))^{5/2}} + \frac{3a^2\cos(e+fx)}{cf\sqrt{c-c\sin(e+fx)}}$$

Mathematica [C] time = 0.65125, size = 149, normalized size = 1.3

$$-\frac{a^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(3\sin\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{3}{2}(e+fx)\right)+3\cos\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{3}{2}(e+fx)\right)-\left(\ldots\right.}{cf(\sin(e+fx)-1)\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2/(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] -((a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(3*Cos[(e + f*x)/2] + Cos[(3*(
e + f*x))/2] + 3*Sin[(e + f*x)/2] - (6 + 6*I)*(-1)^(1/4)*ArcTan[(1/2 + I/2)
*(-1)^(1/4)*(1 + Tan[(e + f*x)/4])]*(-1 + Sin[e + f*x]) - Sin[(3*(e + f*x))
/2]))/(c*f*(-1 + Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]]))
```

Maple [A] time = 0.622, size = 145, normalized size = 1.3

$$\frac{a^2}{f\cos(fx+e)}\left(3\sqrt{2}\text{Artanh}\left(1/2\frac{\sqrt{c(1+\sin(fx+e))}\sqrt{2}}{\sqrt{c}}\right)c\sin(fx+e)-3\sqrt{2}\text{Artanh}\left(1/2\frac{\sqrt{c(1+\sin(fx+e))}\sqrt{\ldots}}{\sqrt{c}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] a^2*(3*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*c*sin(
f*x+e)-3*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*c-2*
(c*(1+sin(f*x+e)))^(1/2)*c^(1/2)*sin(f*x+e)+4*(c*(1+sin(f*x+e)))^(1/2)*c^(1
/2))*(c*(1+sin(f*x+e)))^(1/2)/c^(5/2)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^2}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^2/(-c*sin(f*x + e) + c)^(3/2), x)
```

Fricas [B] time = 1.14709, size = 771, normalized size = 6.7

$$\frac{\frac{3\sqrt{2}\left(a^2c\cos\left(fx+e\right)^2-a^2c\cos\left(fx+e\right)-2a^2c+\left(a^2c\cos\left(fx+e\right)+2a^2c\right)\sin\left(fx+e\right)\right)\log\left(-\frac{\cos\left(fx+e\right)^2+\left(\cos\left(fx+e\right)-2\right)\sin\left(fx+e\right)-\frac{2\sqrt{2}\sqrt{-c\sin\left(fx+e\right)+c}\left(\cos\left(fx+e\right)+\sin\left(fx+e\right)\right.}{\sqrt{c}}}{\cos\left(fx+e\right)^2+\left(\cos\left(fx+e\right)+2\right)\sin\left(fx+e\right)-\cos\left(fx+e\right)-2}\right.}{\sqrt{c}}}{2\left(c^2f\cos\left(fx+e\right)^2-c^2f\cos\left(fx+e\right)-2c^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(3*sqrt(2)*(a^2*c*cos(f*x + e)^2 - a^2*c*cos(f*x + e) - 2*a^2*c + (a^2*
c*cos(f*x + e) + 2*a^2*c)*sin(f*x + e))*log(-(cos(f*x + e)^2 + (cos(f*x + e
) - 2)*sin(f*x + e) - 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*(cos(f*x + e) + s
in(f*x + e) + 1)/sqrt(c) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2 + (cos(f*x +
 e) + 2)*sin(f*x + e) - cos(f*x + e) - 2))/sqrt(c) - 4*(a^2*cos(f*x + e)^2
+ 2*a^2*cos(f*x + e) + a^2 - (a^2*cos(f*x + e) - a^2)*sin(f*x + e))*sqrt(-c
*sin(f*x + e) + c))/(c^2*f*cos(f*x + e)^2 - c^2*f*cos(f*x + e) - 2*c^2*f +
(c^2*f*cos(f*x + e) + 2*c^2*f)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2/(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] sage2
```

3.304 $\int \frac{(a+a\sin(e+fx))^2}{(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=122

$$\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{4\sqrt{2}c^{5/2}f}+\frac{a^2c\cos^3(e+fx)}{2f(c-c\sin(e+fx))^{7/2}}-\frac{3a^2\cos(e+fx)}{4cf(c-c\sin(e+fx))^{3/2}}$$

```
[Out] (3*a^2*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])])/
(4*Sqrt[2]*c^(5/2)*f) + (a^2*c*Cos[e + f*x]^3)/(2*f*(c - c*Sin[e + f*x])^(7
/2)) - (3*a^2*Cos[e + f*x])/(4*c*f*(c - c*Sin[e + f*x])^(3/2))
```

Rubi [A] time = 0.240452, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2736, 2680, 2649, 206}

$$\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{4\sqrt{2}c^{5/2}f}+\frac{a^2c\cos^3(e+fx)}{2f(c-c\sin(e+fx))^{7/2}}-\frac{3a^2\cos(e+fx)}{4cf(c-c\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2/(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] (3*a^2*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])])/
(4*Sqrt[2]*c^(5/2)*f) + (a^2*c*Cos[e + f*x]^3)/(2*f*(c - c*Sin[e + f*x])^(7
/2)) - (3*a^2*Cos[e + f*x])/(4*c*f*(c - c*Sin[e + f*x])^(3/2))
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+a\sin(e+fx))^2}{(c-c\sin(e+fx))^{5/2}}\,dx &= \left(a^2c^2\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^{9/2}}\,dx\\ &= \frac{a^2c\cos^3(e+fx)}{2f(c-c\sin(e+fx))^{7/2}}-\frac{1}{4}\left(3a^2\right)\int \frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^{5/2}}\,dx\\ &= \frac{a^2c\cos^3(e+fx)}{2f(c-c\sin(e+fx))^{7/2}}-\frac{3a^2\cos(e+fx)}{4cf(c-c\sin(e+fx))^{3/2}}+\frac{\left(3a^2\right)\int \frac{1}{\sqrt{c-c\sin(e+fx)}}\,dx}{8c^2}\\ &= \frac{a^2c\cos^3(e+fx)}{2f(c-c\sin(e+fx))^{7/2}}-\frac{3a^2\cos(e+fx)}{4cf(c-c\sin(e+fx))^{3/2}}-\frac{\left(3a^2\right)\text{Subst}\left(\int \frac{1}{2c-x^2}\,dx,x,-\frac{c\cos}{\sqrt{c-c\sin}}\right.}{4c^2f}\\ &= \frac{3a^2\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{4\sqrt{2}c^{5/2}f}+\frac{a^2c\cos^3(e+fx)}{2f(c-c\sin(e+fx))^{7/2}}-\frac{3a^2\cos(e+fx)}{4cf(c-c\sin(e+fx))^{3/2}}\end{aligned}$$

Mathematica [C] time = 0.960644, size = 163, normalized size = 1.34

$$\frac{a^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(3\sin\left(\frac{1}{2}(e+fx)\right)+5\sin\left(\frac{3}{2}(e+fx)\right)+3\cos\left(\frac{1}{2}(e+fx)\right)-5\cos\left(\frac{3}{2}(e+fx)\right)\right.}{8c^2f(\sin(e+fx)-1)^2\sqrt{c-c\sin}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2/(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] (a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(3*Cos[(e + f*x)/2] - 5*Cos[(3*(
e + f*x))/2] + 3*Sin[(e + f*x)/2] + (3 + 3*I)*(-1)^(1/4)*ArcTan[(1/2 + I/2)
*(-1)^(1/4)*(1 + Tan[(e + f*x)/4])]*(-3 + Cos[2*(e + f*x)] + 4*Sin[e + f*x]
) + 5*Sin[(3*(e + f*x))/2]))/(8*c^2*f*(-1 + Sin[e + f*x])^2*Sqrt[c - c*Sin[
e + f*x]])
```

Maple [A] time = 0.735, size = 191, normalized size = 1.6

$$-\frac{a^2}{\left(-8+8\sin\left(fx+e\right)\right)\cos\left(fx+e\right)f}\left(3\sqrt{2}\text{Artanh}\left(1/2\,\frac{\sqrt{c\left(1+\sin\left(fx+e\right)\right)}\sqrt{2}}{\sqrt{c}}\right)\left(\sin\left(fx+e\right)\right)^2c^2-6\sqrt{2}\text{Artanh}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] -1/8/c^(9/2)*a^2*(3*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^
(1/2))*sin(f*x+e)^2*c^2-6*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1
/2)/c^(1/2))*sin(f*x+e)*c^2+10*(c*(1+sin(f*x+e)))^(3/2)*c^(1/2)+3*2^(1/2)*a
rctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*c^2-12*(c*(1+sin(f*x+e
)))^(1/2)*c^(3/2))*(c*(1+sin(f*x+e)))^(1/2)/(-1+sin(f*x+e))/cos(f*x+e)/(c-c
*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^2}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^2/(-c*sin(f*x + e) + c)^(5/2), x)
```

Fricas [B] time = 1.13205, size = 915, normalized size = 7.5

$$\frac{3\sqrt{2}\left(a^2\cos\left(fx+e\right)^3+3\,a^2\cos\left(fx+e\right)^2-2\,a^2\cos\left(fx+e\right)-4\,a^2-\left(a^2\cos\left(fx+e\right)^2-2\,a^2\cos\left(fx+e\right)-4\,a^2\right)\sin\left(f\right.\right.}{16\left(c^3f\cos\left(fx+e\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/16*(3*sqrt(2)*(a^2*cos(f*x + e)^3 + 3*a^2*cos(f*x + e)^2 - 2*a^2*cos(f*x
+ e) - 4*a^2 - (a^2*cos(f*x + e)^2 - 2*a^2*cos(f*x + e) - 4*a^2)*sin(f*x +
e))*sqrt(c)*log(-(c*cos(f*x + e)^2 + 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*sq
rt(c)*(cos(f*x + e) + sin(f*x + e) + 1) + 3*c*cos(f*x + e) + (c*cos(f*x + e
) - 2*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x +
 e) - cos(f*x + e) - 2)) + 4*(5*a^2*cos(f*x + e)^2 + a^2*cos(f*x + e) - 4*a
^2 - (5*a^2*cos(f*x + e) + 4*a^2)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c))/
(c^3*f*cos(f*x + e)^3 + 3*c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x + e) - 4*c
^3*f - (c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x + e) - 4*c^3*f)*sin(f*x + e)
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2/(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] sage2
```

3.305 $\int \frac{(a+a\sin(e+fx))^2}{(c-c\sin(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=156

$$\frac{a^2\cos(e+fx)}{16c^2f(c-c\sin(e+fx))^{3/2}}+\frac{a^2\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{16\sqrt{2}c^{7/2}f}+\frac{a^2c\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{9/2}}-\frac{a^2\cos(e+fx)}{4cf(c-c\sin(e+fx))^{5/2}}$$

```
[Out] (a^2*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])])/(1
6*Sqrt[2]*c^(7/2)*f) + (a^2*c*Cos[e + f*x]^3)/(3*f*(c - c*Sin[e + f*x])^(9/
2)) - (a^2*Cos[e + f*x])/(4*c*f*(c - c*Sin[e + f*x])^(5/2)) + (a^2*Cos[e +
f*x])/(16*c^2*f*(c - c*Sin[e + f*x])^(3/2))
```

Rubi [A] time = 0.272922, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {2736, 2680, 2650, 2649, 206}

$$\frac{a^2\cos(e+fx)}{16c^2f(c-c\sin(e+fx))^{3/2}}+\frac{a^2\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{16\sqrt{2}c^{7/2}f}+\frac{a^2c\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{9/2}}-\frac{a^2\cos(e+fx)}{4cf(c-c\sin(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2/(c - c*Sin[e + f*x])^(7/2),x]
```


```
[Out] (a^2*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])])/(1
6*Sqrt[2]*c^(7/2)*f) + (a^2*c*Cos[e + f*x]^3)/(3*f*(c - c*Sin[e + f*x])^(9/
2)) - (a^2*Cos[e + f*x])/(4*c*f*(c - c*Sin[e + f*x])^(5/2)) + (a^2*Cos[e +
f*x])/(16*c^2*f*(c - c*Sin[e + f*x])^(3/2))
```


Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^2}{(c-c\sin(e+fx))^{7/2}}\,dx = \left(a^2c^2\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^{11/2}}\,dx$$
$$= \frac{a^2c\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{9/2}} - \frac{1}{2}a^2\int \frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^{7/2}}\,dx$$
$$= \frac{a^2c\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{9/2}} - \frac{a^2\cos(e+fx)}{4cf(c-c\sin(e+fx))^{5/2}} + \frac{a^2\int \frac{1}{(c-c\sin(e+fx))^{3/2}}\,dx}{8c^2}$$
$$= \frac{a^2c\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{9/2}} - \frac{a^2\cos(e+fx)}{4cf(c-c\sin(e+fx))^{5/2}} + \frac{a^2\cos(e+fx)}{16c^2f(c-c\sin(e+fx))^{3/2}} + \frac{a^2\int \frac{}{\sqrt{c}}}{}$$
$$= \frac{a^2c\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{9/2}} - \frac{a^2\cos(e+fx)}{4cf(c-c\sin(e+fx))^{5/2}} + \frac{a^2\cos(e+fx)}{16c^2f(c-c\sin(e+fx))^{3/2}} - \frac{a^2\,\mathrm{Subs}}{}$$
$$= \frac{a^2\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{16\sqrt{2}c^{7/2}f} + \frac{a^2c\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{9/2}} - \frac{a^2\cos(e+fx)}{4cf(c-c\sin(e+fx))^{5/2}} + \frac{}{16c^2f(}$$

Mathematica [C] time = 0.97242, size = 307, normalized size = 1.97

$$\frac{a^2(\sin(e+fx)+1)^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(64\sin\left(\frac{1}{2}(e+fx)\right)+3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^5+6}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2/(c - c*Sin[e + f*x])^(7/2),x]
```

```
[Out] (a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(32*(Cos[(e + f*x)/2] - Sin[(e +
 f*x)/2]) - 28*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3 + 3*(Cos[(e + f*x)/2
] - Sin[(e + f*x)/2])^5 - (3 + 3*I)*(-1)^(1/4)*ArcTan[(1/2 + I/2)*(-1)^(1/4
)*(1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^6 + 64*Sin[
(e + f*x)/2] - 56*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*Sin[(e + f*x)/2]
+ 6*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*Sin[(e + f*x)/2])*(1 + Sin[e +
f*x])^2)/(48*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*(c - c*Sin[e + f*x])
^(7/2))
```

Maple [A] time = 0.802, size = 245, normalized size = 1.6

$$-\frac{a^2}{96\left(-1+\sin\left(fx+e\right)\right)^2\cos\left(fx+e\right)f}\left(3\sqrt{2}\mathrm{Artanh}\left(1/2\,\frac{\sqrt{c\left(1+\sin\left(fx+e\right)\right)}\sqrt{2}}{\sqrt{c}}\right)\left(\sin\left(fx+e\right)\right)^3c^4+24\sqrt{c\left(1+\sin\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(7/2),x)
```

```
[Out] -1/96*a^2*(3*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*
sin(f*x+e)^3*c^4+24*(c*(1+sin(f*x+e)))^(1/2)*c^(7/2)-32*(c*(1+sin(f*x+e)))^
(3/2)*c^(5/2)-6*(c*(1+sin(f*x+e)))^(5/2)*c^(3/2)-9*2^(1/2)*arctanh(1/2*(c*(
1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)^2*c^4+9*2^(1/2)*arctanh(1/
2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)*c^4-3*2^(1/2)*arctan
h(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*c^4)*(c*(1+sin(f*x+e)))^(1/
2)/c^(15/2)/(-1+sin(f*x+e))^2/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^2}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^2/(-c*sin(f*x + e) + c)^(7/2), x)
```

Fricas [B] time = 1.15374, size = 1118, normalized size = 7.17

$$3\sqrt{2}\left(a^2\cos\left(fx+e\right)^4-3\,a^2\cos\left(fx+e\right)^3-8\,a^2\cos\left(fx+e\right)^2+4\,a^2\cos\left(fx+e\right)+8\,a^2+\left(a^2\cos\left(fx+e\right)^3+4\,a^2\cos\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/192*(3*sqrt(2)*(a^2*cos(f*x + e)^4 - 3*a^2*cos(f*x + e)^3 - 8*a^2*cos(f*x
 + e)^2 + 4*a^2*cos(f*x + e) + 8*a^2 + (a^2*cos(f*x + e)^3 + 4*a^2*cos(f*x
+ e)^2 - 4*a^2*cos(f*x + e) - 8*a^2)*sin(f*x + e))*sqrt(c)*log(-(c*cos(f*x
+ e)^2 + 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*sqrt(c)*(cos(f*x + e) + sin(f*
x + e) + 1) + 3*c*cos(f*x + e) + (c*cos(f*x + e) - 2*c)*sin(f*x + e) + 2*c)
/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) - 4
*(3*a^2*cos(f*x + e)^3 + 25*a^2*cos(f*x + e)^2 - 10*a^2*cos(f*x + e) - 32*a
^2 + (3*a^2*cos(f*x + e)^2 - 22*a^2*cos(f*x + e) - 32*a^2)*sin(f*x + e))*sq
rt(-c*sin(f*x + e) + c))/(c^4*f*cos(f*x + e)^4 - 3*c^4*f*cos(f*x + e)^3 - 8
*c^4*f*cos(f*x + e)^2 + 4*c^4*f*cos(f*x + e) + 8*c^4*f + (c^4*f*cos(f*x + e
)^3 + 4*c^4*f*cos(f*x + e)^2 - 4*c^4*f*cos(f*x + e) - 8*c^4*f)*sin(f*x + e)
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2/(c-c*sin(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(7/2),x, algorithm="giac")
```

```
[Out] sage2
```

3.306 $\int \frac{(a+a\sin(e+fx))^2}{(c-c\sin(e+fx))^{9/2}}\,dx$

Optimal. Leaf size=190

$$\frac{3a^2\cos(e+fx)}{256c^3f(c-c\sin(e+fx))^{3/2}}+\frac{a^2\cos(e+fx)}{64c^2f(c-c\sin(e+fx))^{5/2}}+\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{256\sqrt{2}c^{9/2}f}+\frac{a^2c\cos^3(e+fx)}{4f(c-c\sin(e+fx))^{11/2}}$$

```
[Out] (3*a^2*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])])/
(256*Sqrt[2]*c^(9/2)*f) + (a^2*c*Cos[e + f*x]^3)/(4*f*(c - c*Sin[e + f*x])^
(11/2)) - (a^2*Cos[e + f*x])/(8*c*f*(c - c*Sin[e + f*x])^(7/2)) + (a^2*Cos[
e + f*x])/(64*c^2*f*(c - c*Sin[e + f*x])^(5/2)) + (3*a^2*Cos[e + f*x])/(256
*c^3*f*(c - c*Sin[e + f*x])^(3/2))
```

Rubi [A] time = 0.303295, antiderivative size = 190, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {2736, 2680, 2650, 2649, 206}

$$\frac{3a^2\cos(e+fx)}{256c^3f(c-c\sin(e+fx))^{3/2}}+\frac{a^2\cos(e+fx)}{64c^2f(c-c\sin(e+fx))^{5/2}}+\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{256\sqrt{2}c^{9/2}f}+\frac{a^2c\cos^3(e+fx)}{4f(c-c\sin(e+fx))^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2/(c - c*Sin[e + f*x])^(9/2),x]
```

```
[Out] (3*a^2*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])])/
(256*Sqrt[2]*c^(9/2)*f) + (a^2*c*Cos[e + f*x]^3)/(4*f*(c - c*Sin[e + f*x])^
(11/2)) - (a^2*Cos[e + f*x])/(8*c*f*(c - c*Sin[e + f*x])^(7/2)) + (a^2*Cos[
e + f*x])/(64*c^2*f*(c - c*Sin[e + f*x])^(5/2)) + (3*a^2*Cos[e + f*x])/(256
*c^3*f*(c - c*Sin[e + f*x])^(3/2))
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^2}{(c-c\sin(e+fx))^{9/2}}\,dx = \left(a^2c^2\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^{13/2}}\,dx$$
$$= \frac{a^2c\cos^3(e+fx)}{4f(c-c\sin(e+fx))^{11/2}} - \frac{1}{8}\left(3a^2\right)\int \frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^{9/2}}\,dx$$
$$= \frac{a^2c\cos^3(e+fx)}{4f(c-c\sin(e+fx))^{11/2}} - \frac{a^2\cos(e+fx)}{8cf(c-c\sin(e+fx))^{7/2}} + \frac{a^2\int \frac{1}{(c-c\sin(e+fx))^{5/2}}\,dx}{16c^2}$$
$$= \frac{a^2c\cos^3(e+fx)}{4f(c-c\sin(e+fx))^{11/2}} - \frac{a^2\cos(e+fx)}{8cf(c-c\sin(e+fx))^{7/2}} + \frac{a^2\cos(e+fx)}{64c^2f(c-c\sin(e+fx))^{5/2}} + \frac{\left(3a^2\right)\dots}{\dots}$$
$$= \frac{a^2c\cos^3(e+fx)}{4f(c-c\sin(e+fx))^{11/2}} - \frac{a^2\cos(e+fx)}{8cf(c-c\sin(e+fx))^{7/2}} + \frac{a^2\cos(e+fx)}{64c^2f(c-c\sin(e+fx))^{5/2}} + \frac{\dots}{256c^3\dots}$$
$$= \frac{a^2c\cos^3(e+fx)}{4f(c-c\sin(e+fx))^{11/2}} - \frac{a^2\cos(e+fx)}{8cf(c-c\sin(e+fx))^{7/2}} + \frac{a^2\cos(e+fx)}{64c^2f(c-c\sin(e+fx))^{5/2}} + \frac{\dots}{256c^3\dots}$$
$$= \frac{3a^2\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{256\sqrt{2}c^{9/2}f} + \frac{a^2c\cos^3(e+fx)}{4f(c-c\sin(e+fx))^{11/2}} - \frac{a^2\cos(e+fx)}{8cf(c-c\sin(e+fx))^{7/2}} + \frac{\dots}{64c^2\dots}$$

Mathematica [C] time = 1.4723, size = 371, normalized size = 1.95

$$\frac{a^2(\sin(e+fx)+1)^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(256\sin\left(\frac{1}{2}(e+fx)\right)+3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7+\dots\right)}{\dots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2/(c - c*Sin[e + f*x])^(9/2),x]
```

```
[Out] (a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(128*(Cos[(e + f*x)/2] - Sin[(e
+ f*x)/2]) - 96*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3 + 4*(Cos[(e + f*x)/
2] - Sin[(e + f*x)/2])^5 + 3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7 - (3 +
 3*I)*(-1)^(1/4)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/4])]*(Cos
[(e + f*x)/2] - Sin[(e + f*x)/2])^8 + 256*Sin[(e + f*x)/2] - 192*(Cos[(e +
f*x)/2] - Sin[(e + f*x)/2])^2*Sin[(e + f*x)/2] + 8*(Cos[(e + f*x)/2] - Sin[
(e + f*x)/2])^4*Sin[(e + f*x)/2] + 6*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^
6*Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^2)/(256*f*(Cos[(e + f*x)/2] + Sin[(e
 + f*x)/2])^4*(c - c*Sin[e + f*x])^(9/2))
```

Maple [A] time = 0.812, size = 299, normalized size = 1.6

$$\frac{a^2}{512\left(-1+\sin\left(fx+e\right)\right)^3\cos\left(fx+e\right)f}\left(6\left(c\left(1+\sin\left(fx+e\right)\right)\right)^{7/2}c^{5/2}-3\sqrt{2}\mathrm{Artanh}\left(1/2\,\frac{\sqrt{c\left(1+\sin\left(fx+e\right)\right)}\sqrt{2}}{\sqrt{c}}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(9/2),x)
```

```
[Out] 1/512/c^(21/2)*a^2*(6*(c*(1+sin(f*x+e)))^(7/2)*c^(5/2)-3*2^(1/2)*arctanh(1/
2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)^4*c^6-44*(c*(1+sin(f
*x+e)))^(5/2)*c^(7/2)+12*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/
2)/c^(1/2))*sin(f*x+e)^3*c^6-88*(c*(1+sin(f*x+e)))^(3/2)*c^(9/2)-18*2^(1/2)
*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)^2*c^6+48*
(c*(1+sin(f*x+e)))^(1/2)*c^(11/2)+12*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))
^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)*c^6-3*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+
e)))^(1/2)*2^(1/2)/c^(1/2))*c^6)*(c*(1+sin(f*x+e)))^(1/2)/(-1+sin(f*x+e))^3
/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^2}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^2/(-c*sin(f*x + e) + c)^(9/2), x)
```

Fricas [B] time = 1.20554, size = 1327, normalized size = 6.98

$$\frac{3\sqrt{2}\left(a^2\cos\left(fx+e\right)^5+5\,a^2\cos\left(fx+e\right)^4-8\,a^2\cos\left(fx+e\right)^3-20\,a^2\cos\left(fx+e\right)^2+8\,a^2\cos\left(fx+e\right)+16\,a^2-\left(a^2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(9/2),x, algorithm="fricas")
```

```
[Out] 1/1024*(3*sqrt(2)*(a^2*cos(f*x + e)^5 + 5*a^2*cos(f*x + e)^4 - 8*a^2*cos(f*
x + e)^3 - 20*a^2*cos(f*x + e)^2 + 8*a^2*cos(f*x + e) + 16*a^2 - (a^2*cos(f
*x + e)^4 - 4*a^2*cos(f*x + e)^3 - 12*a^2*cos(f*x + e)^2 + 8*a^2*cos(f*x +
e) + 16*a^2)*sin(f*x + e))*sqrt(c)*log(-(c*cos(f*x + e)^2 + 2*sqrt(2)*sqrt(
-c*sin(f*x + e) + c)*sqrt(c)*(cos(f*x + e) + sin(f*x + e) + 1) + 3*c*cos(f*
x + e) + (c*cos(f*x + e) - 2*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2 + (cos(
f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) - 4*(3*a^2*cos(f*x + e)^4 +
 13*a^2*cos(f*x + e)^3 + 86*a^2*cos(f*x + e)^2 - 52*a^2*cos(f*x + e) - 128*
a^2 - (3*a^2*cos(f*x + e)^3 - 10*a^2*cos(f*x + e)^2 + 76*a^2*cos(f*x + e) +
 128*a^2)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c))/(c^5*f*cos(f*x + e)^5 +
```

```
5*c^5*f*cos(f*x + e)^4 - 8*c^5*f*cos(f*x + e)^3 - 20*c^5*f*cos(f*x + e)^2 +
 8*c^5*f*cos(f*x + e) + 16*c^5*f - (c^5*f*cos(f*x + e)^4 - 4*c^5*f*cos(f*x
+ e)^3 - 12*c^5*f*cos(f*x + e)^2 + 8*c^5*f*cos(f*x + e) + 16*c^5*f)*sin(f*x
 + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2/(c-c*sin(f*x+e))**(9/2),x)

[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(9/2),x, algorithm="giac")

[Out] sage2
```

3.307 $\int (a+a\sin(e+fx))^3(c-c\sin(e+fx))^{7/2}\,dx$

Optimal. Leaf size=145

$$\frac{2a^3c^4\cos^7(e+fx)}{13f\sqrt{c-c\sin(e+fx)}}+\frac{24a^3c^5\cos^7(e+fx)}{143f(c-c\sin(e+fx))^{3/2}}+\frac{64a^3c^6\cos^7(e+fx)}{429f(c-c\sin(e+fx))^{5/2}}+\frac{256a^3c^7\cos^7(e+fx)}{3003f(c-c\sin(e+fx))^{7/2}}$$

```
[Out] (256*a^3*c^7*Cos[e + f*x]^7)/(3003*f*(c - c*Sin[e + f*x])^(7/2)) + (64*a^3*
c^6*Cos[e + f*x]^7)/(429*f*(c - c*Sin[e + f*x])^(5/2)) + (24*a^3*c^5*Cos[e
+ f*x]^7)/(143*f*(c - c*Sin[e + f*x])^(3/2)) + (2*a^3*c^4*Cos[e + f*x]^7)/(
13*f*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 0.330216, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {2736, 2674, 2673}

$$\frac{2a^3c^4\cos^7(e+fx)}{13f\sqrt{c-c\sin(e+fx)}}+\frac{24a^3c^5\cos^7(e+fx)}{143f(c-c\sin(e+fx))^{3/2}}+\frac{64a^3c^6\cos^7(e+fx)}{429f(c-c\sin(e+fx))^{5/2}}+\frac{256a^3c^7\cos^7(e+fx)}{3003f(c-c\sin(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(7/2),x]
```

```
[Out] (256*a^3*c^7*Cos[e + f*x]^7)/(3003*f*(c - c*Sin[e + f*x])^(7/2)) + (64*a^3*
c^6*Cos[e + f*x]^7)/(429*f*(c - c*Sin[e + f*x])^(5/2)) + (24*a^3*c^5*Cos[e
+ f*x]^7)/(143*f*(c - c*Sin[e + f*x])^(3/2)) + (2*a^3*c^4*Cos[e + f*x]^7)/(
13*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rubi steps

$$\int (a+a\sin(e+fx))^3(c-c\sin(e+fx))^{7/2}\,dx = \left(a^3c^3\right)\int \cos^6(e+fx)\sqrt{c-c\sin(e+fx)}\,dx$$
$$= \frac{2a^3c^4\cos^7(e+fx)}{13f\sqrt{c-c\sin(e+fx)}} + \frac{1}{13}\left(12a^3c^4\right)\int \frac{\cos^6(e+fx)}{\sqrt{c-c\sin(e+fx)}}\,dx$$
$$= \frac{24a^3c^5\cos^7(e+fx)}{143f(c-c\sin(e+fx))^{3/2}} + \frac{2a^3c^4\cos^7(e+fx)}{13f\sqrt{c-c\sin(e+fx)}} + \frac{1}{143}\left(96a^3c^5\right)\int \frac{}{(c}$$
$$= \frac{64a^3c^6\cos^7(e+fx)}{429f(c-c\sin(e+fx))^{5/2}} + \frac{24a^3c^5\cos^7(e+fx)}{143f(c-c\sin(e+fx))^{3/2}} + \frac{2a^3c^4\cos^7(e}{13f\sqrt{c-c\sin(}}$$
$$= \frac{256a^3c^7\cos^7(e+fx)}{3003f(c-c\sin(e+fx))^{7/2}} + \frac{64a^3c^6\cos^7(e+fx)}{429f(c-c\sin(e+fx))^{5/2}} + \frac{24a^3c^5\cos}{143f(c-c\sin}$$

Mathematica [A] time = 6.18494, size = 112, normalized size = 0.77

$$\frac{a^3c^3\cos^6(e+fx)\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)(-6377\sin(e+fx)+231\sin(3(e+fx))-1890\,c}{6006f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(7/2),x]
```

```
[Out] (a^3*c^3*Cos[e + f*x]^6*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Si
n[e + f*x]]*(5230 - 1890*Cos[2*(e + f*x)] - 6377*Sin[e + f*x] + 231*Sin[3*(
e + f*x)]))/(6006*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7)
```

Maple [A] time = 0.545, size = 81, normalized size = 0.6

$$\frac{\left(-2+2\sin\left(fx+e\right)\right)c^4\left(1+\sin\left(fx+e\right)\right)^4a^3\left(231\left(\sin\left(fx+e\right)\right)^3-945\left(\sin\left(fx+e\right)\right)^2+1421\sin\left(fx+e\right)-835\right)}{3003\,f\cos\left(fx+e\right)}\frac{}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^(7/2),x)
```

```
[Out] 2/3003*(-1+sin(f*x+e))*c^4*(1+sin(f*x+e))^4*a^3*(231*sin(f*x+e)^3-945*sin(f
*x+e)^2+1421*sin(f*x+e)-835)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^3\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^3*(-c*sin(f*x + e) + c)^(7/2), x)
```

Fricas [B] time = 1.09878, size = 656, normalized size = 4.52

$$\frac{2\left(231\,a^3c^3\cos\left(fx+e\right)^7-21\,a^3c^3\cos\left(fx+e\right)^6+28\,a^3c^3\cos\left(fx+e\right)^5-40\,a^3c^3\cos\left(fx+e\right)^4+64\,a^3c^3\cos\left(fx+e\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^(7/2),x, algorithm="fricas")
```

```
[Out] 2/3003*(231*a^3*c^3*cos(f*x + e)^7 - 21*a^3*c^3*cos(f*x + e)^6 + 28*a^3*c^3
*cos(f*x + e)^5 - 40*a^3*c^3*cos(f*x + e)^4 + 64*a^3*c^3*cos(f*x + e)^3 - 1
28*a^3*c^3*cos(f*x + e)^2 + 512*a^3*c^3*cos(f*x + e) + 1024*a^3*c^3 + (231*
a^3*c^3*cos(f*x + e)^6 + 252*a^3*c^3*cos(f*x + e)^5 + 280*a^3*c^3*cos(f*x +
 e)^4 + 320*a^3*c^3*cos(f*x + e)^3 + 384*a^3*c^3*cos(f*x + e)^2 + 512*a^3*c
^3*cos(f*x + e) + 1024*a^3*c^3)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c)/(f*
cos(f*x + e) - f*sin(f*x + e) + f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(c-c*sin(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a\sin\left(fx+e\right)+a\right)^3\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^3*(-c*sin(f*x + e) + c)^(7/2), x)
```

3.308 $\int (a+a\sin(e+fx))^3(c-c\sin(e+fx))^{5/2}\,dx$

Optimal. Leaf size=109

$$\frac{2a^3c^4\cos^7(e+fx)}{11f(c-c\sin(e+fx))^{3/2}}+\frac{16a^3c^5\cos^7(e+fx)}{99f(c-c\sin(e+fx))^{5/2}}+\frac{64a^3c^6\cos^7(e+fx)}{693f(c-c\sin(e+fx))^{7/2}}$$

```
[Out] (64*a^3*c^6*Cos[e + f*x]^7)/(693*f*(c - c*Sin[e + f*x])^(7/2)) + (16*a^3*c^
5*Cos[e + f*x]^7)/(99*f*(c - c*Sin[e + f*x])^(5/2)) + (2*a^3*c^4*Cos[e + f*
x]^7)/(11*f*(c - c*Sin[e + f*x])^(3/2))
```

Rubi [A] time = 0.268288, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {2736, 2674, 2673}

$$\frac{2a^3c^4\cos^7(e+fx)}{11f(c-c\sin(e+fx))^{3/2}}+\frac{16a^3c^5\cos^7(e+fx)}{99f(c-c\sin(e+fx))^{5/2}}+\frac{64a^3c^6\cos^7(e+fx)}{693f(c-c\sin(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] (64*a^3*c^6*Cos[e + f*x]^7)/(693*f*(c - c*Sin[e + f*x])^(7/2)) + (16*a^3*c^
5*Cos[e + f*x]^7)/(99*f*(c - c*Sin[e + f*x])^(5/2)) + (2*a^3*c^4*Cos[e + f*
x]^7)/(11*f*(c - c*Sin[e + f*x])^(3/2))
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rubi steps

$$\int (a+a\sin(e+fx))^3(c-c\sin(e+fx))^{5/2}\,dx = \left(a^3c^3\right)\int \frac{\cos^6(e+fx)}{\sqrt{c-c\sin(e+fx)}}\,dx$$
$$= \frac{2a^3c^4\cos^7(e+fx)}{11f(c-c\sin(e+fx))^{3/2}} + \frac{1}{11}\left(8a^3c^4\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^{3/2}}\,dx$$
$$= \frac{16a^3c^5\cos^7(e+fx)}{99f(c-c\sin(e+fx))^{5/2}} + \frac{2a^3c^4\cos^7(e+fx)}{11f(c-c\sin(e+fx))^{3/2}} + \frac{1}{99}\left(32a^3c^5\right)\int$$
$$= \frac{64a^3c^6\cos^7(e+fx)}{693f(c-c\sin(e+fx))^{7/2}} + \frac{16a^3c^5\cos^7(e+fx)}{99f(c-c\sin(e+fx))^{5/2}} + \frac{2a^3c^4\cos}{11f(c-c\sin}$$

Mathematica [B] time = 6.4651, size = 1105, normalized size = 10.14

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] (5*Cos[(e + f*x)/2]*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(5/2))/(8*f
*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] + Sin[(e + f*x)/
2])^6) - (5*Cos[(3*(e + f*x))/2]*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x]
)^(5/2))/(24*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] +
Sin[(e + f*x)/2])^6) + (Cos[(5*(e + f*x))/2]*(a + a*Sin[e + f*x])^3*(c - c*
Sin[e + f*x])^(5/2))/(16*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e
+ f*x)/2] + Sin[(e + f*x)/2])^6) - (5*Cos[(7*(e + f*x))/2]*(a + a*Sin[e + f
*x])^3*(c - c*Sin[e + f*x])^(5/2))/(112*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)
/2])^5*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6) + (Cos[(9*(e + f*x))/2]*(a
+ a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(5/2))/(144*f*(Cos[(e + f*x)/2] -
Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6) - (Cos[(11*(e
+ f*x))/2]*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(5/2))/(176*f*(Cos[(
e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6)
+ (5*Sin[(e + f*x)/2]*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(5/2))/(8
*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] + Sin[(e + f*x
)/2])^6) + (5*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(5/2)*Sin[(3*(e +
 f*x))/2])/(24*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2]
+ Sin[(e + f*x)/2])^6) + ((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(5/2)
*Sin[(5*(e + f*x))/2])/(16*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(
e + f*x)/2] + Sin[(e + f*x)/2])^6) + (5*(a + a*Sin[e + f*x])^3*(c - c*Sin[e
 + f*x])^(5/2)*Sin[(7*(e + f*x))/2])/(112*f*(Cos[(e + f*x)/2] - Sin[(e + f*
x)/2])^5*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6) + ((a + a*Sin[e + f*x])^3
*(c - c*Sin[e + f*x])^(5/2)*Sin[(9*(e + f*x))/2])/(144*f*(Cos[(e + f*x)/2]
- Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6) + ((a + a*Si
n[e + f*x])^3*(c - c*Sin[e + f*x])^(5/2)*Sin[(11*(e + f*x))/2])/(176*f*(Cos
[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6
)
```

Maple [A] time = 0.525, size = 71, normalized size = 0.7

$$-\frac{\left(-2+2\sin\left(fx+e\right)\right)c^3\left(1+\sin\left(fx+e\right)\right)^4a^3\left(63\left(\sin\left(fx+e\right)\right)^2-182\sin\left(fx+e\right)+151\right)}{693f\cos\left(fx+e\right)}\frac{1}{\sqrt{c-c\sin\left(fx+e\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] -2/693*(-1+sin(f*x+e))*c^3*(1+sin(f*x+e))^4*a^3*(63*sin(f*x+e)^2-182*sin(f*
x+e)+151)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^{3}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^3*(-c*sin(f*x + e) + c)^(5/2), x)
```

Fricas [B] time = 1.09341, size = 567, normalized size = 5.2

$$-\frac{2\left(63\,a^{3}c^{2}\cos\left(fx+e\right)^{6}-7\,a^{3}c^{2}\cos\left(fx+e\right)^{5}+10\,a^{3}c^{2}\cos\left(fx+e\right)^{4}-16\,a^{3}c^{2}\cos\left(fx+e\right)^{3}+32\,a^{3}c^{2}\cos\left(fx+e\right)^{2}-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/693*(63*a^3*c^2*cos(f*x + e)^6 - 7*a^3*c^2*cos(f*x + e)^5 + 10*a^3*c^2*c
os(f*x + e)^4 - 16*a^3*c^2*cos(f*x + e)^3 + 32*a^3*c^2*cos(f*x + e)^2 - 128
*a^3*c^2*cos(f*x + e) - 256*a^3*c^2 - (63*a^3*c^2*cos(f*x + e)^5 + 70*a^3*c
^2*cos(f*x + e)^4 + 80*a^3*c^2*cos(f*x + e)^3 + 96*a^3*c^2*cos(f*x + e)^2 +
 128*a^3*c^2*cos(f*x + e) + 256*a^3*c^2)*sin(f*x + e))*sqrt(-c*sin(f*x + e)
 + c)/(f*cos(f*x + e) - f*sin(f*x + e) + f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^{3}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^3*(-c*sin(f*x + e) + c)^(5/2), x)
```

3.309 $\int (a+a\sin(e+fx))^3(c-c\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=73

$$\frac{2a^3c^4\cos^7(e+fx)}{9f(c-c\sin(e+fx))^{5/2}}+\frac{8a^3c^5\cos^7(e+fx)}{63f(c-c\sin(e+fx))^{7/2}}$$

```
[Out] (8*a^3*c^5*Cos[e + f*x]^7)/(63*f*(c - c*Sin[e + f*x])^(7/2)) + (2*a^3*c^4*C
os[e + f*x]^7)/(9*f*(c - c*Sin[e + f*x])^(5/2))
```

Rubi [A] time = 0.202857, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {2736, 2674, 2673}

$$\frac{2a^3c^4\cos^7(e+fx)}{9f(c-c\sin(e+fx))^{5/2}}+\frac{8a^3c^5\cos^7(e+fx)}{63f(c-c\sin(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] (8*a^3*c^5*Cos[e + f*x]^7)/(63*f*(c - c*Sin[e + f*x])^(7/2)) + (2*a^3*c^4*C
os[e + f*x]^7)/(9*f*(c - c*Sin[e + f*x])^(5/2))
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rubi steps

$$\int (a+a\sin(e+fx))^3(c-c\sin(e+fx))^{3/2}\,dx = \left(a^3c^3\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^{3/2}}\,dx$$
$$= \frac{2a^3c^4\cos^7(e+fx)}{9f(c-c\sin(e+fx))^{5/2}} + \frac{1}{9}\left(4a^3c^4\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^{5/2}}\,dx$$
$$= \frac{8a^3c^5\cos^7(e+fx)}{63f(c-c\sin(e+fx))^{7/2}} + \frac{2a^3c^4\cos^7(e+fx)}{9f(c-c\sin(e+fx))^{5/2}}$$

Mathematica [A] time = 2.98018, size = 84, normalized size = 1.15

$$-\frac{2a^3c(7\sin(e+fx)-11)\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^7}{63f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-2*a^3*c*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(-11 + 7*Sin[e + f*x])*Sq
rt[c - c*Sin[e + f*x]])/(63*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))
```

Maple [A] time = 0.536, size = 61, normalized size = 0.8

$$\frac{\left(-2+2\sin\left(fx+e\right)\right)c^2\left(1+\sin\left(fx+e\right)\right)^4a^3\left(7\sin\left(fx+e\right)-11\right)}{63f\cos\left(fx+e\right)}\frac{1}{\sqrt{c-c\sin\left(fx+e\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] 2/63*(-1+sin(f*x+e))*c^2*(1+sin(f*x+e))^4*a^3*(7*sin(f*x+e)-11)/cos(f*x+e)/
(c-c*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^3\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^3*(-c*sin(f*x + e) + c)^(3/2), x)
```

Fricas [B] time = 1.04366, size = 451, normalized size = 6.18

$$-\frac{2\left(7a^3c\cos\left(fx+e\right)^5+17a^3c\cos\left(fx+e\right)^4-2a^3c\cos\left(fx+e\right)^3+4a^3c\cos\left(fx+e\right)^2-16a^3c\cos\left(fx+e\right)-32a^3 \ldots\right.}{63\left(f\cos\left(\ldots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] -2/63*(7*a^3*c*cos(f*x + e)^5 + 17*a^3*c*cos(f*x + e)^4 - 2*a^3*c*cos(f*x +
 e)^3 + 4*a^3*c*cos(f*x + e)^2 - 16*a^3*c*cos(f*x + e) - 32*a^3*c + (7*a^3*
c*cos(f*x + e)^4 - 10*a^3*c*cos(f*x + e)^3 - 12*a^3*c*cos(f*x + e)^2 - 16*a
^3*c*cos(f*x + e) - 32*a^3*c)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c)/(f*co
s(f*x + e) - f*sin(f*x + e) + f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(fx+e)+a)^3(-c\sin(fx+e)+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^3*(-c*sin(f*x + e) + c)^(3/2), x)
```

3.310 $\int (a+a\sin(e+fx))^3\sqrt{c-c\sin(e+fx)}\,dx$

Optimal. Leaf size=36

$$\frac{2a^3c^4\cos^7(e+fx)}{7f(c-c\sin(e+fx))^{7/2}}$$

```
[Out] (2*a^3*c^4*Cos[e + f*x]^7)/(7*f*(c - c*Sin[e + f*x])^(7/2))
```

Rubi [A] time = 0.127129, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2736, 2673}

$$\frac{2a^3c^4\cos^7(e+fx)}{7f(c-c\sin(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3*Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (2*a^3*c^4*Cos[e + f*x]^7)/(7*f*(c - c*Sin[e + f*x])^(7/2))
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rubi steps

$$\begin{aligned}\int (a+a\sin(e+fx))^3\sqrt{c-c\sin(e+fx)}\,dx &= \left(a^3c^3\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^{5/2}}\,dx\\ &= \frac{2a^3c^4\cos^7(e+fx)}{7f(c-c\sin(e+fx))^{7/2}}\end{aligned}$$

Mathematica [B] time = 0.373951, size = 73, normalized size = 2.03

$$\frac{2a^3\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^7}{7f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3*Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (2*a^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*Sqrt[c - c*Sin[e + f*x]])/(7
*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))
```

Maple [A] time = 0.383, size = 49, normalized size = 1.4

$$-\frac{\left(-2+2\sin\left(fx+e\right)\right)c\left(1+\sin\left(fx+e\right)\right)^{4}a^{3}}{7f\cos\left(fx+e\right)}\frac{1}{\sqrt{c-c\sin\left(fx+e\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] -2/7*(-1+sin(f*x+e))*c*(1+sin(f*x+e))^4*a^3/cos(f*x+e)/(c-c*sin(f*x+e))^(1/
2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a\sin\left(fx+e\right)+a\right)^{3}\sqrt{-c\sin\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^3*sqrt(-c*sin(f*x + e) + c), x)
```

Fricas [B] time = 1.05076, size = 342, normalized size = 9.5

$$\frac{2\left(a^{3}\cos\left(fx+e\right)^{4}-3a^{3}\cos\left(fx+e\right)^{3}-8a^{3}\cos\left(fx+e\right)^{2}+4a^{3}\cos\left(fx+e\right)+8a^{3}-\left(a^{3}\cos\left(fx+e\right)^{3}+4a^{3}\cos\left(fx\right.\right.\right.}{7\left(f\cos\left(fx+e\right)-f\sin\left(fx+e\right)+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/7*(a^3*cos(f*x + e)^4 - 3*a^3*cos(f*x + e)^3 - 8*a^3*cos(f*x + e)^2 + 4*a
^3*cos(f*x + e) + 8*a^3 - (a^3*cos(f*x + e)^3 + 4*a^3*cos(f*x + e)^2 - 4*a^
3*cos(f*x + e) - 8*a^3)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x
+ e) - f*sin(f*x + e) + f)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^{3}\left(\int 3\sqrt{-c\sin\left(e+fx\right)+c}\sin\left(e+fx\right)dx+\int 3\sqrt{-c\sin\left(e+fx\right)+c}\sin^{2}\left(e+fx\right)dx+\int\sqrt{-c\sin\left(e+fx\right)+c}\sin^{3}\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] a**3*(Integral(3*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x), x) + Integral(3*sq
rt(-c*sin(e + f*x) + c)*sin(e + f*x)**2, x) + Integral(sqrt(-c*sin(e + f*x)
 + c)*sin(e + f*x)**3, x) + Integral(sqrt(-c*sin(e + f*x) + c), x))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.311 $\int \frac{(a+a\sin(e+fx))^3}{\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=151

$$-\frac{2a^3c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{5/2}}-\frac{4a^3c\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{3/2}}-\frac{8a^3\cos(e+fx)}{f\sqrt{c-c\sin(e+fx)}}+\frac{8\sqrt{2}a^3\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{c}f}$$

```
[Out] (8*Sqrt[2]*a^3*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f
*x]])])/(Sqrt[c]*f) - (2*a^3*c^2*Cos[e + f*x]^5)/(5*f*(c - c*Sin[e + f*x])^
(5/2)) - (4*a^3*c*Cos[e + f*x]^3)/(3*f*(c - c*Sin[e + f*x])^(3/2)) - (8*a^3
*Cos[e + f*x])/(f*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 0.315871, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2736, 2679, 2649, 206}

$$-\frac{2a^3c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{5/2}}-\frac{4a^3c\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{3/2}}-\frac{8a^3\cos(e+fx)}{f\sqrt{c-c\sin(e+fx)}}+\frac{8\sqrt{2}a^3\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{c}f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3/Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (8*Sqrt[2]*a^3*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f
*x]])])/(Sqrt[c]*f) - (2*a^3*c^2*Cos[e + f*x]^5)/(5*f*(c - c*Sin[e + f*x])^
(5/2)) - (4*a^3*c*Cos[e + f*x]^3)/(3*f*(c - c*Sin[e + f*x])^(3/2)) - (8*a^3
*Cos[e + f*x])/(f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2679

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(a*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^3}{\sqrt{c-c\sin(e+fx)}}\,dx = \left(a^3c^3\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^{7/2}}\,dx$$
$$= -\frac{2a^3c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{5/2}} + \left(2a^3c^2\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^{5/2}}\,dx$$
$$= -\frac{2a^3c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{5/2}} - \frac{4a^3c\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{3/2}} + \left(4a^3c\right)\int \frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^{3/2}}\,dx$$
$$= -\frac{2a^3c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{5/2}} - \frac{4a^3c\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{3/2}} - \frac{8a^3\cos(e+fx)}{f\sqrt{c-c\sin(e+fx)}} + \left(8a^3\right)\int \frac{}{\sqrt{c}}$$
$$= -\frac{2a^3c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{5/2}} - \frac{4a^3c\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{3/2}} - \frac{8a^3\cos(e+fx)}{f\sqrt{c-c\sin(e+fx)}} - \frac{\left(16a^3\right)\text{Subs}}{}$$
$$= \frac{8\sqrt{2}a^3\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{c}f} - \frac{2a^3c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{5/2}} - \frac{4a^3c\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{3/2}} - \frac{}{f}$$

Mathematica [C] time = 0.723938, size = 156, normalized size = 1.03

$$-\frac{a^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(330\sin\left(\frac{1}{2}(e+fx)\right)+35\sin\left(\frac{3}{2}(e+fx)\right)-3\sin\left(\frac{5}{2}(e+fx)\right)+330\cos\left(\frac{1}{2}(e\right.\right.}{30f\sqrt{c-c\sin}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3/Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] -(a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*((480 + 480*I)*(-1)^(1/4)*ArcTa
n[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/4])] + 330*Cos[(e + f*x)/2] - 3
5*Cos[(3*(e + f*x))/2] - 3*Cos[(5*(e + f*x))/2] + 330*Sin[(e + f*x)/2] + 35
*Sin[(3*(e + f*x))/2] - 3*Sin[(5*(e + f*x))/2]))/(30*f*Sqrt[c - c*Sin[e + f
*x]])
```

Maple [A] time = 0.767, size = 129, normalized size = 0.9

$$-\frac{\left(-2+2\sin\left(fx+e\right)\right)a^3}{15c^3\cos\left(fx+e\right)f}\sqrt{c\left(1+\sin\left(fx+e\right)\right)}\left(60c^{5/2}\sqrt{2}\text{Artanh}\left(1/2\,\frac{\sqrt{c\left(1+\sin\left(fx+e\right)\right)}\sqrt{2}}{\sqrt{c}}\right)-3\left(c\left(1+\sin\left(fx\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] -2/15*(-1+sin(f*x+e))*(c*(1+sin(f*x+e)))^(1/2)*a^3*(60*c^(5/2)*2^(1/2)*arct
anh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))-3*(c*(1+sin(f*x+e)))^(5/2
)-10*c*(c*(1+sin(f*x+e)))^(3/2)-60*c^2*(c*(1+sin(f*x+e)))^(1/2))/c^3/cos(f*
x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{3}}{\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^3/sqrt(-c*sin(f*x + e) + c), x)
```

Fricas [B] time = 1.15413, size = 707, normalized size = 4.68

$$\frac{2\left(\frac{30\sqrt{2}\left(a^{3}c\cos\left(fx+e\right)-a^{3}c\sin\left(fx+e\right)+a^{3}c\right)\log\left(-\frac{\cos\left(fx+e\right)^{2}+\left(\cos\left(fx+e\right)-2\right)\sin\left(fx+e\right)+\frac{2\sqrt{2}\sqrt{-c\sin\left(fx+e\right)+c}\left(\cos\left(fx+e\right)+\sin\left(fx+e\right)+1\right)}{\sqrt{c}}+3\cos\left(fx+e\right)+2}{\cos\left(fx+e\right)^{2}+\left(\cos\left(fx+e\right)+2\right)\sin\left(fx+e\right)-\cos\left(fx+e\right)-2}\right)}{\sqrt{c}}+\left(3a^{3}\mathrm{co}\cdots\right.}{15\left(cf\,\mathrm{c}\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15*(30*sqrt(2)*(a^3*c*cos(f*x + e) - a^3*c*sin(f*x + e) + a^3*c)*log(-(co
s(f*x + e)^2 + (cos(f*x + e) - 2)*sin(f*x + e) + 2*sqrt(2)*sqrt(-c*sin(f*x
+ e) + c)*(cos(f*x + e) + sin(f*x + e) + 1)/sqrt(c) + 3*cos(f*x + e) + 2)/(
cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2))/sqrt(
c) + (3*a^3*cos(f*x + e)^3 + 19*a^3*cos(f*x + e)^2 - 76*a^3*cos(f*x + e) -
92*a^3 + (3*a^3*cos(f*x + e)^2 - 16*a^3*cos(f*x + e) - 92*a^3)*sin(f*x + e)
)*sqrt(-c*sin(f*x + e) + c))/(c*f*cos(f*x + e) - c*f*sin(f*x + e) + c*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 2.52584, size = 466, normalized size = 3.09

$$\frac{960\sqrt{2}a^{3}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{c}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+c}-\sqrt{c}\right)}{2\sqrt{-c}}\right)}{\sqrt{-c}\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}+\frac{\left(\left(\left(\left(\left(\frac{73a^{3}\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{c^{7}}+\frac{105a^{3}\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{c^{7}}\right)\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\cdots\right.\right.\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] 1/60*(960*sqrt(2)*a^3*arctan(-1/2*sqrt(2)*(sqrt(c)*tan(1/2*f*x + 1/2*e) - s
qrt(c*tan(1/2*f*x + 1/2*e)^2 + c) - sqrt(c))/sqrt(-c))/(sqrt(-c)*sgn(tan(1/
2*f*x + 1/2*e) - 1)) + (((((73*a^3*sgn(tan(1/2*f*x + 1/2*e) - 1)*tan(1/2*f*
x + 1/2*e)/c^7 + 105*a^3*sgn(tan(1/2*f*x + 1/2*e) - 1)/c^7)*tan(1/2*f*x + 1
/2*e) + 190*a^3*sgn(tan(1/2*f*x + 1/2*e) - 1)/c^7)*tan(1/2*f*x + 1/2*e) + 1
90*a^3*sgn(tan(1/2*f*x + 1/2*e) - 1)/c^7)*tan(1/2*f*x + 1/2*e) + 105*a^3*sg
n(tan(1/2*f*x + 1/2*e) - 1)/c^7)*tan(1/2*f*x + 1/2*e) + 73*a^3*sgn(tan(1/2*
f*x + 1/2*e) - 1)/c^7)/(c*tan(1/2*f*x + 1/2*e)^2 + c)^(5/2) - 4*sqrt(2)*(24
0*a^3*c^(21/2)*arctan(sqrt(c)/sqrt(-c)) + 23*a^3*sqrt(-c)*c)*sgn(tan(1/2*f*
x + 1/2*e) - 1)/(sqrt(-c)*c^(21/2)))/f
```

3.312 $\int \frac{(a+a\sin(e+fx))^3}{(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=150

$$\frac{a^3c^2\cos^5(e+fx)}{f(c-c\sin(e+fx))^{7/2}}-\frac{10\sqrt{2}a^3\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{c^{3/2}f}+\frac{5a^3\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{3/2}}+\frac{10a^3\cos(e+fx)}{cf\sqrt{c-c\sin(e+fx)}}$$

```
[Out] (-10*Sqrt[2]*a^3*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e +
 f*x]])])/(c^(3/2)*f) + (a^3*c^2*Cos[e + f*x]^5)/(f*(c - c*Sin[e + f*x])^(7
/2)) + (5*a^3*Cos[e + f*x]^3)/(3*f*(c - c*Sin[e + f*x])^(3/2)) + (10*a^3*Co
s[e + f*x])/(c*f*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 0.320458, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {2736, 2680, 2679, 2649, 206}

$$\frac{a^3c^2\cos^5(e+fx)}{f(c-c\sin(e+fx))^{7/2}}-\frac{10\sqrt{2}a^3\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{c^{3/2}f}+\frac{5a^3\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{3/2}}+\frac{10a^3\cos(e+fx)}{cf\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3/(c - c*Sin[e + f*x])^(3/2),x]
```


```
[Out] (-10*Sqrt[2]*a^3*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e +
 f*x]])])/(c^(3/2)*f) + (a^3*c^2*Cos[e + f*x]^5)/(f*(c - c*Sin[e + f*x])^(7
/2)) + (5*a^3*Cos[e + f*x]^3)/(3*f*(c - c*Sin[e + f*x])^(3/2)) + (10*a^3*Co
s[e + f*x])/(c*f*Sqrt[c - c*Sin[e + f*x]])
```


Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2679

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(a*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
```

```
egersQ[2*m, 2*p]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^3}{(c-c\sin(e+fx))^{3/2}}\,dx = \left(a^3c^3\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^{9/2}}\,dx$$
$$= \frac{a^3c^2\cos^5(e+fx)}{f(c-c\sin(e+fx))^{7/2}} - \frac{1}{2}\left(5a^3c\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^{5/2}}\,dx$$
$$= \frac{a^3c^2\cos^5(e+fx)}{f(c-c\sin(e+fx))^{7/2}} + \frac{5a^3\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{3/2}} - \left(5a^3\right)\int \frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^{3/2}}\,dx$$
$$= \frac{a^3c^2\cos^5(e+fx)}{f(c-c\sin(e+fx))^{7/2}} + \frac{5a^3\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{3/2}} + \frac{10a^3\cos(e+fx)}{cf\sqrt{c-c\sin(e+fx)}} - \frac{\left(10a^3\right)\int \frac{}{\sqrt{c-}}}{c}$$
$$= \frac{a^3c^2\cos^5(e+fx)}{f(c-c\sin(e+fx))^{7/2}} + \frac{5a^3\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{3/2}} + \frac{10a^3\cos(e+fx)}{cf\sqrt{c-c\sin(e+fx)}} + \frac{\left(20a^3\right)\text{Subs}}{}$$
$$= -\frac{10\sqrt{2}a^3\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{c^{3/2}f} + \frac{a^3c^2\cos^5(e+fx)}{f(c-c\sin(e+fx))^{7/2}} + \frac{5a^3\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{3/2}}$$

Mathematica [C] time = 0.798989, size = 173, normalized size = 1.15

$$-\frac{a^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(50\sin\left(\frac{1}{2}(e+fx)\right)-25\sin\left(\frac{3}{2}(e+fx)\right)+\sin\left(\frac{5}{2}(e+fx)\right)+50\cos\left(\frac{1}{2}(e+fx\right.\right.}{6cf(\sin(e+fx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3/(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] -(a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(50*Cos[(e + f*x)/2] + 25*Cos[(
3*(e + f*x))/2] + Cos[(5*(e + f*x))/2] + 50*Sin[(e + f*x)/2] - (120 + 120*I
)*(-1)^(1/4)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/4])]*(-1 + Si
n[e + f*x]) - 25*Sin[(3*(e + f*x))/2] + Sin[(5*(e + f*x))/2]))/(6*c*f*(-1 +
 Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])
```

Maple [A] time = 0.599, size = 189, normalized size = 1.3

$$\frac{2a^3}{3f\cos\left(fx+e\right)}\left(15\sqrt{2}\text{Artanh}\left(1/2\,\frac{\sqrt{c\left(1+\sin\left(fx+e\right)\right)}\sqrt{2}}{\sqrt{c}}\right)\sin\left(fx+e\right)c^2-12\sqrt{c\left(1+\sin\left(fx+e\right)\right)}c^{3/2}\sin\left(fx+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] 2/3*a^3*(15*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*s
in(f*x+e)*c^2-12*(c*(1+sin(f*x+e)))^(1/2)*c^(3/2)*sin(f*x+e)-(c*(1+sin(f*x+
e)))^(3/2)*c^(1/2)*sin(f*x+e)-15*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/
2)*2^(1/2)/c^(1/2))*c^2+18*(c*(1+sin(f*x+e)))^(1/2)*c^(3/2)+(c*(1+sin(f*x+e
)))^(3/2)*c^(1/2))*(c*(1+sin(f*x+e)))^(1/2)/c^(7/2)/cos(f*x+e)/(c-c*sin(f*x
+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^3}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^3/(-c*sin(f*x + e) + c)^(3/2), x)
```

Fricas [B] time = 1.12119, size = 844, normalized size = 5.63

$$\frac{\frac{15\sqrt{2}\left(a^3c\cos\left(fx+e\right)^2-a^3c\cos\left(fx+e\right)-2a^3c+\left(a^3c\cos\left(fx+e\right)+2a^3c\right)\sin\left(fx+e\right)\right)\log\left(-\frac{\cos\left(fx+e\right)^2+\left(\cos\left(fx+e\right)-2\right)\sin\left(fx+e\right)-\frac{2\sqrt{2}\sqrt{-c\sin\left(fx+e\right)+c}\left(\cos\left(fx+e\right)+\sin\left(fx+e\right)\cdots\right.}{\sqrt{c}}}{\cos\left(fx+e\right)^2+\left(\cos\left(fx+e\right)+2\right)\sin\left(fx+e\right)-\cos\left(fx+e\right)-2}\right.\cdots}{\sqrt{c}}\cdots}{3\left(c^2f\cos\left(fx+e\right)^2\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(15*sqrt(2)*(a^3*c*cos(f*x + e)^2 - a^3*c*cos(f*x + e) - 2*a^3*c + (a^3
*c*cos(f*x + e) + 2*a^3*c)*sin(f*x + e))*log(-(cos(f*x + e)^2 + (cos(f*x +
e) - 2)*sin(f*x + e) - 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*(cos(f*x + e) +
sin(f*x + e) + 1)/sqrt(c) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2 + (cos(f*x
+ e) + 2)*sin(f*x + e) - cos(f*x + e) - 2))/sqrt(c) - 2*(a^3*cos(f*x + e)^3
 + 13*a^3*cos(f*x + e)^2 + 18*a^3*cos(f*x + e) + 6*a^3 + (a^3*cos(f*x + e)^
2 - 12*a^3*cos(f*x + e) + 6*a^3)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c))/(
c^2*f*cos(f*x + e)^2 - c^2*f*cos(f*x + e) - 2*c^2*f + (c^2*f*cos(f*x + e) +
 2*c^2*f)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] sage2
```

3.313 $\int \frac{(a+a\sin(e+fx))^3}{(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=157

$$\frac{a^3c^2\cos^5(e+fx)}{2f(c-c\sin(e+fx))^{9/2}}-\frac{15a^3\cos(e+fx)}{4c^2f\sqrt{c-c\sin(e+fx)}}+\frac{15a^3\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{2\sqrt{2}c^{5/2}f}-\frac{5a^3\cos^3(e+fx)}{4f(c-c\sin(e+fx))^{5/2}}$$

```
[Out] (15*a^3*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])])
/(2*Sqrt[2]*c^(5/2)*f) + (a^3*c^2*Cos[e + f*x]^5)/(2*f*(c - c*Sin[e + f*x])
^(9/2)) - (5*a^3*Cos[e + f*x]^3)/(4*f*(c - c*Sin[e + f*x])^(5/2)) - (15*a^3
*Cos[e + f*x])/(4*c^2*f*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 0.325292, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {2736, 2680, 2679, 2649, 206}

$$\frac{a^3c^2\cos^5(e+fx)}{2f(c-c\sin(e+fx))^{9/2}}-\frac{15a^3\cos(e+fx)}{4c^2f\sqrt{c-c\sin(e+fx)}}+\frac{15a^3\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{2\sqrt{2}c^{5/2}f}-\frac{5a^3\cos^3(e+fx)}{4f(c-c\sin(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3/(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] (15*a^3*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])])
/(2*Sqrt[2]*c^(5/2)*f) + (a^3*c^2*Cos[e + f*x]^5)/(2*f*(c - c*Sin[e + f*x])
^(9/2)) - (5*a^3*Cos[e + f*x]^3)/(4*f*(c - c*Sin[e + f*x])^(5/2)) - (15*a^3
*Cos[e + f*x])/(4*c^2*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2679

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(a*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
```

```
egersQ[2*m, 2*p]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^3}{(c-c\sin(e+fx))^{5/2}}\,dx = \left(a^3c^3\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^{11/2}}\,dx$$

$$= \frac{a^3c^2\cos^5(e+fx)}{2f(c-c\sin(e+fx))^{9/2}} - \frac{1}{4}\left(5a^3c\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^{7/2}}\,dx$$

$$= \frac{a^3c^2\cos^5(e+fx)}{2f(c-c\sin(e+fx))^{9/2}} - \frac{5a^3\cos^3(e+fx)}{4f(c-c\sin(e+fx))^{5/2}} + \frac{\left(15a^3\right)\int \frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^{3/2}}\,dx}{8c}$$

$$= \frac{a^3c^2\cos^5(e+fx)}{2f(c-c\sin(e+fx))^{9/2}} - \frac{5a^3\cos^3(e+fx)}{4f(c-c\sin(e+fx))^{5/2}} - \frac{15a^3\cos(e+fx)}{4c^2f\sqrt{c-c\sin(e+fx)}} + \frac{\left(15a^3\right)\int}{}$$

$$= \frac{a^3c^2\cos^5(e+fx)}{2f(c-c\sin(e+fx))^{9/2}} - \frac{5a^3\cos^3(e+fx)}{4f(c-c\sin(e+fx))^{5/2}} - \frac{15a^3\cos(e+fx)}{4c^2f\sqrt{c-c\sin(e+fx)}} - \frac{\left(15a^3\right)\mathrm{Su}}{}$$

$$= \frac{15a^3\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{2\sqrt{2}c^{5/2}f} + \frac{a^3c^2\cos^5(e+fx)}{2f(c-c\sin(e+fx))^{9/2}} - \frac{5a^3\cos^3(e+fx)}{4f(c-c\sin(e+fx))^{5/2}} - \frac{}{4c}$$

Mathematica [C] time = 0.998543, size = 187, normalized size = 1.19

$$\frac{a^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(-5\sin\left(\frac{1}{2}(e+fx)\right)+15\sin\left(\frac{3}{2}(e+fx)\right)+2\sin\left(\frac{5}{2}(e+fx)\right)-5\cos\left(\frac{1}{2}(e+fx\right)}{4c^2f(\sin}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3/(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] (a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(-5*Cos[(e + f*x)/2] - 15*Cos[(3
*(e + f*x))/2] + 2*Cos[(5*(e + f*x))/2] - 5*Sin[(e + f*x)/2] + (15 + 15*I)*
(-1)^(1/4)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/4])]*(-3 + Cos[
2*(e + f*x)] + 4*Sin[e + f*x]) + 15*Sin[(3*(e + f*x))/2] + 2*Sin[(5*(e + f*
x))/2]))/(4*c^2*f*(-1 + Sin[e + f*x])^2*Sqrt[c - c*Sin[e + f*x]])
```

Maple [A] time = 0.744, size = 239, normalized size = 1.5

$$-\frac{a^3}{\left(-4+4\sin\left(fx+e\right)\right)\cos\left(fx+e\right)f}\left(15\sqrt{2}\mathrm{Artanh}\left(1/2\,\frac{\sqrt{c\left(1+\sin\left(fx+e\right)\right)}\sqrt{2}}{\sqrt{c}}\right)\left(\sin\left(fx+e\right)\right)^2c^2-8\sqrt{c\left(1+\mathrm{si}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] -1/4/c^(9/2)*a^3*(15*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c
^(1/2))*sin(f*x+e)^2*c^2-8*(c*(1+sin(f*x+e)))^(1/2)*c^(3/2)*sin(f*x+e)^2-30
*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)*c
^2+18*(c*(1+sin(f*x+e)))^(3/2)*c^(1/2)+16*(c*(1+sin(f*x+e)))^(1/2)*c^(3/2)*
sin(f*x+e)+15*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))
*c^2-36*(c*(1+sin(f*x+e)))^(1/2)*c^(3/2))*(c*(1+sin(f*x+e)))^(1/2)/(-1+sin(
f*x+e))/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{3}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^3/(-c*sin(f*x + e) + c)^(5/2), x)
```

Fricas [B] time = 1.19308, size = 984, normalized size = 6.27

$$\frac{15\sqrt{2}\left(a^{3}\cos\left(fx+e\right)^{3}+3\,a^{3}\cos\left(fx+e\right)^{2}-2\,a^{3}\cos\left(fx+e\right)-4\,a^{3}-\left(a^{3}\cos\left(fx+e\right)^{2}-2\,a^{3}\cos\left(fx+e\right)-4\,a^{3}\right)\sin\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/8*(15*sqrt(2)*(a^3*cos(f*x + e)^3 + 3*a^3*cos(f*x + e)^2 - 2*a^3*cos(f*x
+ e) - 4*a^3 - (a^3*cos(f*x + e)^2 - 2*a^3*cos(f*x + e) - 4*a^3)*sin(f*x +
e))*sqrt(c)*log(-(c*cos(f*x + e)^2 + 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*sq
rt(c)*(cos(f*x + e) + sin(f*x + e) + 1) + 3*c*cos(f*x + e) + (c*cos(f*x + e
) - 2*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x +
 e) - cos(f*x + e) - 2)) - 4*(4*a^3*cos(f*x + e)^3 - 13*a^3*cos(f*x + e)^2
- 13*a^3*cos(f*x + e) + 4*a^3 + (4*a^3*cos(f*x + e)^2 + 17*a^3*cos(f*x + e)
 + 4*a^3)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c))/(c^3*f*cos(f*x + e)^3 +
3*c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x + e) - 4*c^3*f - (c^3*f*cos(f*x +
e)^2 - 2*c^3*f*cos(f*x + e) - 4*c^3*f)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] sage2
```

3.314 $\int \frac{(a+a\sin(e+fx))^3}{(c-c\sin(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=157

$$\frac{a^3c^2\cos^5(e+fx)}{3f(c-c\sin(e+fx))^{11/2}}+\frac{5a^3\cos(e+fx)}{8c^2f(c-c\sin(e+fx))^{3/2}}-\frac{5a^3\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{8\sqrt{2}c^{7/2}f}-\frac{5a^3\cos^3(e+fx)}{12f(c-c\sin(e+fx))^{7/2}}$$

```
[Out] (-5*a^3*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])])
/(8*Sqrt[2]*c^(7/2)*f) + (a^3*c^2*Cos[e + f*x]^5)/(3*f*(c - c*Sin[e + f*x])
^(11/2)) - (5*a^3*Cos[e + f*x]^3)/(12*f*(c - c*Sin[e + f*x])^(7/2)) + (5*a^
3*Cos[e + f*x])/(8*c^2*f*(c - c*Sin[e + f*x])^(3/2))
```

Rubi [A] time = 0.323023, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2736, 2680, 2649, 206}

$$\frac{a^3c^2\cos^5(e+fx)}{3f(c-c\sin(e+fx))^{11/2}}+\frac{5a^3\cos(e+fx)}{8c^2f(c-c\sin(e+fx))^{3/2}}-\frac{5a^3\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{8\sqrt{2}c^{7/2}f}-\frac{5a^3\cos^3(e+fx)}{12f(c-c\sin(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3/(c - c*Sin[e + f*x])^(7/2),x]
```

```
[Out] (-5*a^3*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])])
/(8*Sqrt[2]*c^(7/2)*f) + (a^3*c^2*Cos[e + f*x]^5)/(3*f*(c - c*Sin[e + f*x])
^(11/2)) - (5*a^3*Cos[e + f*x]^3)/(12*f*(c - c*Sin[e + f*x])^(7/2)) + (5*a^
3*Cos[e + f*x])/(8*c^2*f*(c - c*Sin[e + f*x])^(3/2))
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^3}{(c-c\sin(e+fx))^{7/2}}\,dx = \left(a^3c^3\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^{13/2}}\,dx$$
$$= \frac{a^3c^2\cos^5(e+fx)}{3f(c-c\sin(e+fx))^{11/2}} - \frac{1}{6}\left(5a^3c\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^{9/2}}\,dx$$
$$= \frac{a^3c^2\cos^5(e+fx)}{3f(c-c\sin(e+fx))^{11/2}} - \frac{5a^3\cos^3(e+fx)}{12f(c-c\sin(e+fx))^{7/2}} + \frac{\left(5a^3\right)\int \frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^{5/2}}\,dx}{8c}$$
$$= \frac{a^3c^2\cos^5(e+fx)}{3f(c-c\sin(e+fx))^{11/2}} - \frac{5a^3\cos^3(e+fx)}{12f(c-c\sin(e+fx))^{7/2}} + \frac{5a^3\cos(e+fx)}{8c^2f(c-c\sin(e+fx))^{3/2}} - \frac{(5a^3}{}$$
$$= \frac{a^3c^2\cos^5(e+fx)}{3f(c-c\sin(e+fx))^{11/2}} - \frac{5a^3\cos^3(e+fx)}{12f(c-c\sin(e+fx))^{7/2}} + \frac{5a^3\cos(e+fx)}{8c^2f(c-c\sin(e+fx))^{3/2}} + \frac{(5a^3}{}$$
$$= -\frac{5a^3\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{8\sqrt{2}c^{7/2}f} + \frac{a^3c^2\cos^5(e+fx)}{3f(c-c\sin(e+fx))^{11/2}} - \frac{5a^3\cos^3(e+fx)}{12f(c-c\sin(e+fx))^{7/2}} +$$

Mathematica [C] time = 1.56934, size = 307, normalized size = 1.96

$$\frac{a^3(\sin(e+fx)+1)^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(64\sin\left(\frac{1}{2}(e+fx)\right)+33\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^5\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3/(c - c*Sin[e + f*x])^(7/2),x]
```

```
[Out] (a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(32*(Cos[(e + f*x)/2] - Sin[(e +
 f*x)/2]) - 52*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3 + 33*(Cos[(e + f*x)/
2] - Sin[(e + f*x)/2])^5 + (15 + 15*I)*(-1)^(1/4)*ArcTan[(1/2 + I/2)*(-1)^(
1/4)*(1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^6 + 64*S
in[(e + f*x)/2] - 104*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*Sin[(e + f*x)
/2] + 66*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*Sin[(e + f*x)/2])*(1 + Sin
[e + f*x])^3)/(24*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6*(c - c*Sin[e +
f*x])^(7/2))
```

Maple [A] time = 0.645, size = 245, normalized size = 1.6

$$\frac{a^3}{48\left(-1+\sin\left(fx+e\right)\right)^2\cos\left(fx+e\right)f}\left(15\sqrt{2}\mathrm{Artanh}\left(1/2\,\frac{\sqrt{c\left(1+\sin\left(fx+e\right)\right)}\sqrt{2}}{\sqrt{c}}\right)\left(\sin\left(fx+e\right)\right)^3c^3-45\sqrt{2}\mathrm{Arta}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(7/2),x)
```

```
[Out] 1/48/c^(13/2)*a^3*(15*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/
c^(1/2))*sin(f*x+e)^3*c^3-45*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2
^(1/2)/c^(1/2))*sin(f*x+e)^2*c^3+66*(c*(1+sin(f*x+e)))^(5/2)*c^(1/2)+45*2^(
1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)*c^3-1
60*(c*(1+sin(f*x+e)))^(3/2)*c^(3/2)-15*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)
))^(1/2)*2^(1/2)/c^(1/2))*c^3+120*(c*(1+sin(f*x+e)))^(1/2)*c^(5/2))*(c*(1+s
in(f*x+e)))^(1/2)/(-1+sin(f*x+e))^2/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^3}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^3/(-c*sin(f*x + e) + c)^(7/2), x)
```

Fricas [B] time = 1.12637, size = 1121, normalized size = 7.14

$$15\sqrt{2}\left(a^3\cos\left(fx+e\right)^4-3\,a^3\cos\left(fx+e\right)^3-8\,a^3\cos\left(fx+e\right)^2+4\,a^3\cos\left(fx+e\right)+8\,a^3+\left(a^3\cos\left(fx+e\right)^3+4\,a^3\cos\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/96*(15*sqrt(2)*(a^3*cos(f*x + e)^4 - 3*a^3*cos(f*x + e)^3 - 8*a^3*cos(f*x
 + e)^2 + 4*a^3*cos(f*x + e) + 8*a^3 + (a^3*cos(f*x + e)^3 + 4*a^3*cos(f*x
+ e)^2 - 4*a^3*cos(f*x + e) - 8*a^3)*sin(f*x + e))*sqrt(c)*log(-(c*cos(f*x
+ e)^2 - 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*sqrt(c)*(cos(f*x + e) + sin(f*
x + e) + 1) + 3*c*cos(f*x + e) + (c*cos(f*x + e) - 2*c)*sin(f*x + e) + 2*c)
/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) - 4
*(33*a^3*cos(f*x + e)^3 + 19*a^3*cos(f*x + e)^2 - 46*a^3*cos(f*x + e) - 32*
a^3 + (33*a^3*cos(f*x + e)^2 + 14*a^3*cos(f*x + e) - 32*a^3)*sin(f*x + e))*
sqrt(-c*sin(f*x + e) + c))/(c^4*f*cos(f*x + e)^4 - 3*c^4*f*cos(f*x + e)^3 -
 8*c^4*f*cos(f*x + e)^2 + 4*c^4*f*cos(f*x + e) + 8*c^4*f + (c^4*f*cos(f*x +
 e)^3 + 4*c^4*f*cos(f*x + e)^2 - 4*c^4*f*cos(f*x + e) - 8*c^4*f)*sin(f*x +
e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(7/2),x, algorithm="giac")
```

```
[Out] sage2
```

3.315 $\int \frac{(a+a\sin(e+fx))^3}{(c-c\sin(e+fx))^{9/2}}\,dx$

Optimal. Leaf size=191

$$\frac{a^3c^2\cos^5(e+fx)}{4f(c-c\sin(e+fx))^{13/2}}-\frac{5a^3\cos(e+fx)}{128c^3f(c-c\sin(e+fx))^{3/2}}+\frac{5a^3\cos(e+fx)}{32c^2f(c-c\sin(e+fx))^{5/2}}-\frac{5a^3\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{128\sqrt{2}c^{9/2}f}-$$

```
[Out] (-5*a^3*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])])
/(128*Sqrt[2]*c^(9/2)*f) + (a^3*c^2*Cos[e + f*x]^5)/(4*f*(c - c*Sin[e + f*x
])^(13/2)) - (5*a^3*Cos[e + f*x]^3)/(24*f*(c - c*Sin[e + f*x])^(9/2)) + (5*
a^3*Cos[e + f*x])/(32*c^2*f*(c - c*Sin[e + f*x])^(5/2)) - (5*a^3*Cos[e + f*
x])/(128*c^3*f*(c - c*Sin[e + f*x])^(3/2))
```

Rubi [A] time = 0.353228, antiderivative size = 191, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {2736, 2680, 2650, 2649, 206}

$$\frac{a^3c^2\cos^5(e+fx)}{4f(c-c\sin(e+fx))^{13/2}}-\frac{5a^3\cos(e+fx)}{128c^3f(c-c\sin(e+fx))^{3/2}}+\frac{5a^3\cos(e+fx)}{32c^2f(c-c\sin(e+fx))^{5/2}}-\frac{5a^3\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{128\sqrt{2}c^{9/2}f}-$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3/(c - c*Sin[e + f*x])^(9/2),x]
```

```
[Out] (-5*a^3*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])])
/(128*Sqrt[2]*c^(9/2)*f) + (a^3*c^2*Cos[e + f*x]^5)/(4*f*(c - c*Sin[e + f*x
])^(13/2)) - (5*a^3*Cos[e + f*x]^3)/(24*f*(c - c*Sin[e + f*x])^(9/2)) + (5*
a^3*Cos[e + f*x])/(32*c^2*f*(c - c*Sin[e + f*x])^(5/2)) - (5*a^3*Cos[e + f*
x])/(128*c^3*f*(c - c*Sin[e + f*x])^(3/2))
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^3}{(c-c\sin(e+fx))^{9/2}}\,dx = \left(a^3c^3\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^{15/2}}\,dx$$

$$= \frac{a^3c^2\cos^5(e+fx)}{4f(c-c\sin(e+fx))^{13/2}} - \frac{1}{8}\left(5a^3c\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^{11/2}}\,dx$$

$$= \frac{a^3c^2\cos^5(e+fx)}{4f(c-c\sin(e+fx))^{13/2}} - \frac{5a^3\cos^3(e+fx)}{24f(c-c\sin(e+fx))^{9/2}} + \frac{\left(5a^3\right)\int \frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^{7/2}}\,dx}{16c}$$

$$= \frac{a^3c^2\cos^5(e+fx)}{4f(c-c\sin(e+fx))^{13/2}} - \frac{5a^3\cos^3(e+fx)}{24f(c-c\sin(e+fx))^{9/2}} + \frac{5a^3\cos(e+fx)}{32c^2f(c-c\sin(e+fx))^{5/2}} - \frac{(5a}{}$$

$$= \frac{a^3c^2\cos^5(e+fx)}{4f(c-c\sin(e+fx))^{13/2}} - \frac{5a^3\cos^3(e+fx)}{24f(c-c\sin(e+fx))^{9/2}} + \frac{5a^3\cos(e+fx)}{32c^2f(c-c\sin(e+fx))^{5/2}} - \frac{}{128}$$

$$= \frac{a^3c^2\cos^5(e+fx)}{4f(c-c\sin(e+fx))^{13/2}} - \frac{5a^3\cos^3(e+fx)}{24f(c-c\sin(e+fx))^{9/2}} + \frac{5a^3\cos(e+fx)}{32c^2f(c-c\sin(e+fx))^{5/2}} - \frac{}{128}$$

$$= -\frac{5a^3\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{128\sqrt{2}c^{9/2}f} + \frac{a^3c^2\cos^5(e+fx)}{4f(c-c\sin(e+fx))^{13/2}} - \frac{5a^3\cos^3(e+fx)}{24f(c-c\sin(e+fx))^{9/2}} +$$

Mathematica [C] time = 2.57538, size = 371, normalized size = 1.94

$$\frac{a^3(\sin(e+fx)+1)^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(768\sin\left(\frac{1}{2}(e+fx)\right)-15\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3/(c - c*Sin[e + f*x])^(9/2),x]
```

```
[Out] (a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(384*(Cos[(e + f*x)/2] - Sin[(e
+ f*x)/2]) - 544*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3 + 236*(Cos[(e + f*
x)/2] - Sin[(e + f*x)/2])^5 - 15*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7 +
(15 + 15*I)*(-1)^(1/4)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/4])
]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^8 + 768*Sin[(e + f*x)/2] - 1088*(Co
s[(e + f*x)/2] - Sin[(e + f*x)/2])^2*Sin[(e + f*x)/2] + 472*(Cos[(e + f*x)/
2] - Sin[(e + f*x)/2])^4*Sin[(e + f*x)/2] - 30*(Cos[(e + f*x)/2] - Sin[(e +
 f*x)/2])^6*Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^3)/(384*f*(Cos[(e + f*x)/2
] + Sin[(e + f*x)/2])^6*(c - c*Sin[e + f*x])^(9/2))
```

Maple [A] time = 0.991, size = 299, normalized size = 1.6

$$-\frac{a^{3}}{768\left(-1+\sin\left(fx+e\right)\right)^{3}\cos\left(fx+e\right)f}\left(-15\sqrt{2}\mathrm{Artanh}\left(1/2\,\frac{\sqrt{c\left(1+\sin\left(fx+e\right)\right)}\sqrt{2}}{\sqrt{c}}\right)\left(\sin\left(fx+e\right)\right)^{4}c^{5}+30\,\left(c\left(1+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(9/2),x)
```

```
[Out] -1/768/c^(19/2)*a^3*(-15*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/
2)/c^(1/2))*sin(f*x+e)^4*c^5+30*(c*(1+sin(f*x+e)))^(7/2)*c^(3/2)+60*2^(1/2)
*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)^3*c^5+292
*(c*(1+sin(f*x+e)))^(5/2)*c^(5/2)-90*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))
^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)^2*c^5-440*(c*(1+sin(f*x+e)))^(3/2)*c^(7/
2)+60*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin(f*x
+e)*c^5+240*(c*(1+sin(f*x+e)))^(1/2)*c^(9/2)-15*2^(1/2)*arctanh(1/2*(c*(1+s
in(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*c^5)*(c*(1+sin(f*x+e)))^(1/2)/(-1+sin(f*
x+e))^3/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a\sin\left(fx+e\right)+a\right)^{3}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^3/(-c*sin(f*x + e) + c)^(9/2), x)
```

Fricas [B] time = 1.23502, size = 1338, normalized size = 7.01

$$\frac{15\sqrt{2}\left(a^{3}\cos\left(fx+e\right)^{5}+5\,a^{3}\cos\left(fx+e\right)^{4}-8\,a^{3}\cos\left(fx+e\right)^{3}-20\,a^{3}\cos\left(fx+e\right)^{2}+8\,a^{3}\cos\left(fx+e\right)+16\,a^{3}-\left(a^{3}\cos\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(9/2),x, algorithm="fricas")
```

```
[Out] 1/1536*(15*sqrt(2)*(a^3*cos(f*x + e)^5 + 5*a^3*cos(f*x + e)^4 - 8*a^3*cos(f
*x + e)^3 - 20*a^3*cos(f*x + e)^2 + 8*a^3*cos(f*x + e) + 16*a^3 - (a^3*cos(
f*x + e)^4 - 4*a^3*cos(f*x + e)^3 - 12*a^3*cos(f*x + e)^2 + 8*a^3*cos(f*x +
 e) + 16*a^3)*sin(f*x + e))*sqrt(c)*log(-(c*cos(f*x + e)^2 - 2*sqrt(2)*sqrt
(-c*sin(f*x + e) + c)*sqrt(c)*(cos(f*x + e) + sin(f*x + e) + 1) + 3*c*cos(f
*x + e) + (c*cos(f*x + e) - 2*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2 + (cos
(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) + 4*(15*a^3*cos(f*x + e)^4
 - 191*a^3*cos(f*x + e)^3 - 338*a^3*cos(f*x + e)^2 + 252*a^3*cos(f*x + e) +
 384*a^3 - (15*a^3*cos(f*x + e)^3 + 206*a^3*cos(f*x + e)^2 - 132*a^3*cos(f*
x + e) - 384*a^3)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c))/(c^5*f*cos(f*x +
```

```
e)^5 + 5*c^5*f*cos(f*x + e)^4 - 8*c^5*f*cos(f*x + e)^3 - 20*c^5*f*cos(f*x
+ e)^2 + 8*c^5*f*cos(f*x + e) + 16*c^5*f - (c^5*f*cos(f*x + e)^4 - 4*c^5*f*
cos(f*x + e)^3 - 12*c^5*f*cos(f*x + e)^2 + 8*c^5*f*cos(f*x + e) + 16*c^5*f)
*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(9/2),x, algorithm="giac")
```

```
[Out] sage2
```

3.316 $\int \frac{(a+a\sin(e+fx))^3}{(c-c\sin(e+fx))^{11/2}}\,dx$

Optimal. Leaf size=225

$$\frac{a^3c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{15/2}}-\frac{3a^3\cos(e+fx)}{512c^4f(c-c\sin(e+fx))^{3/2}}-\frac{a^3\cos(e+fx)}{128c^3f(c-c\sin(e+fx))^{5/2}}+\frac{a^3\cos(e+fx)}{16c^2f(c-c\sin(e+fx))^{7/2}}-\frac{3a}{}$$

```
[Out] (-3*a^3*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])])
/(512*Sqrt[2]*c^(11/2)*f) + (a^3*c^2*Cos[e + f*x]^5)/(5*f*(c - c*Sin[e + f*
x])^(15/2)) - (a^3*Cos[e + f*x]^3)/(8*f*(c - c*Sin[e + f*x])^(11/2)) + (a^3
*Cos[e + f*x])/(16*c^2*f*(c - c*Sin[e + f*x])^(7/2)) - (a^3*Cos[e + f*x])/(
128*c^3*f*(c - c*Sin[e + f*x])^(5/2)) - (3*a^3*Cos[e + f*x])/(512*c^4*f*(c
- c*Sin[e + f*x])^(3/2))
```

Rubi [A] time = 0.387612, antiderivative size = 225, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {2736, 2680, 2650, 2649, 206}

$$\frac{a^3c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{15/2}}-\frac{3a^3\cos(e+fx)}{512c^4f(c-c\sin(e+fx))^{3/2}}-\frac{a^3\cos(e+fx)}{128c^3f(c-c\sin(e+fx))^{5/2}}+\frac{a^3\cos(e+fx)}{16c^2f(c-c\sin(e+fx))^{7/2}}-\frac{3a}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3/(c - c*Sin[e + f*x])^(11/2),x]
```


```
[Out] (-3*a^3*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])])
/(512*Sqrt[2]*c^(11/2)*f) + (a^3*c^2*Cos[e + f*x]^5)/(5*f*(c - c*Sin[e + f*
x])^(15/2)) - (a^3*Cos[e + f*x]^3)/(8*f*(c - c*Sin[e + f*x])^(11/2)) + (a^3
*Cos[e + f*x])/(16*c^2*f*(c - c*Sin[e + f*x])^(7/2)) - (a^3*Cos[e + f*x])/(
128*c^3*f*(c - c*Sin[e + f*x])^(5/2)) - (3*a^3*Cos[e + f*x])/(512*c^4*f*(c
- c*Sin[e + f*x])^(3/2))
```


Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] && !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
```

```
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^3}{(c-c\sin(e+fx))^{11/2}}\,dx = \left(a^3c^3\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^{17/2}}\,dx$$
$$= \frac{a^3c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{15/2}} - \frac{1}{2}\left(a^3c\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^{13/2}}\,dx$$
$$= \frac{a^3c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{15/2}} - \frac{a^3\cos^3(e+fx)}{8f(c-c\sin(e+fx))^{11/2}} + \frac{\left(3a^3\right)\int \frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^{9/2}}\,dx}{16c}$$
$$= \frac{a^3c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{15/2}} - \frac{a^3\cos^3(e+fx)}{8f(c-c\sin(e+fx))^{11/2}} + \frac{a^3\cos(e+fx)}{16c^2f(c-c\sin(e+fx))^{7/2}} - \frac{a^3}{}$$
$$= \frac{a^3c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{15/2}} - \frac{a^3\cos^3(e+fx)}{8f(c-c\sin(e+fx))^{11/2}} + \frac{a^3\cos(e+fx)}{16c^2f(c-c\sin(e+fx))^{7/2}} - \frac{}{12}$$
$$= \frac{a^3c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{15/2}} - \frac{a^3\cos^3(e+fx)}{8f(c-c\sin(e+fx))^{11/2}} + \frac{a^3\cos(e+fx)}{16c^2f(c-c\sin(e+fx))^{7/2}} - \frac{}{12}$$
$$= \frac{a^3c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{15/2}} - \frac{a^3\cos^3(e+fx)}{8f(c-c\sin(e+fx))^{11/2}} + \frac{a^3\cos(e+fx)}{16c^2f(c-c\sin(e+fx))^{7/2}} - \frac{}{12}$$
$$= -\frac{3a^3\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{512\sqrt{2}c^{11/2}f} + \frac{a^3c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{15/2}} - \frac{a^3\cos^3(e+fx)}{8f(c-c\sin(e+fx))^{11/2}} +$$

Mathematica [C] time = 4.14668, size = 435, normalized size = 1.93

$$\frac{a^3(\sin(e+fx)+1)^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(4096\sin\left(\frac{1}{2}(e+fx)\right)-15\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3/(c - c*Sin[e + f*x])^(11/2),x]
```

```
[Out] (a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(2048*(Cos[(e + f*x)/2] - Sin[(e
 + f*x)/2]) - 2688*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3 + 992*(Cos[(e +
f*x)/2] - Sin[(e + f*x)/2])^5 - 20*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7
- 15*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9 + (15 + 15*I)*(-1)^(1/4)*ArcTa
n[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] - Sin[(e
```

```
+ f*x)/2])^10 + 4096*Sin[(e + f*x)/2] - 5376*(Cos[(e + f*x)/2] - Sin[(e +
f*x)/2])^2*Sin[(e + f*x)/2] + 1984*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*
Sin[(e + f*x)/2] - 40*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^6*Sin[(e + f*x)
/2] - 30*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^8*Sin[(e + f*x)/2])*(1 + Sin
[e + f*x])^3)/(2560*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6*(c - c*Sin[e
+ f*x])^(11/2))
```

Maple [A] time = 0.837, size = 353, normalized size = 1.6

$$\frac{a^3}{5120\left(-1+\sin\left(fx+e\right)\right)^4\cos\left(fx+e\right)f}\left(480\sqrt{c\left(1+\sin\left(fx+e\right)\right)}c^{13/2}-1120\left(c\left(1+\sin\left(fx+e\right)\right)\right)^{3/2}c^{11/2}+1024\left(c\left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(11/2),x)
```

```
[Out] 1/5120*a^3*(480*(c*(1+sin(f*x+e)))^(1/2)*c^(13/2)-1120*(c*(1+sin(f*x+e)))^(
3/2)*c^(11/2)+1024*(c*(1+sin(f*x+e)))^(5/2)*c^(9/2)+280*(c*(1+sin(f*x+e)))^
(7/2)*c^(7/2)-30*(c*(1+sin(f*x+e)))^(9/2)*c^(5/2)+15*2^(1/2)*arctanh(1/2*(c
*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)^5*c^7-75*2^(1/2)*arctanh
(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)^4*c^7+150*2^(1/2)
*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)^3*c^7-150
*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)^2
*c^7+75*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin(f
*x+e)*c^7-15*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*
c^7)*(c*(1+sin(f*x+e)))^(1/2)/c^(25/2)/(-1+sin(f*x+e))^4/cos(f*x+e)/(c-c*si
n(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^3}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(11/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^3/(-c*sin(f*x + e) + c)^(11/2), x)
```

Fricas [B] time = 1.25892, size = 1553, normalized size = 6.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(11/2),x, algorithm="fricas")
```

```
[Out] 1/10240*(15*sqrt(2)*(a^3*cos(f*x + e)^6 - 5*a^3*cos(f*x + e)^5 - 18*a^3*cos
(f*x + e)^4 + 20*a^3*cos(f*x + e)^3 + 48*a^3*cos(f*x + e)^2 - 16*a^3*cos(f*
```

```
x + e) - 32*a^3 + (a^3*cos(f*x + e)^5 + 6*a^3*cos(f*x + e)^4 - 12*a^3*cos(f
*x + e)^3 - 32*a^3*cos(f*x + e)^2 + 16*a^3*cos(f*x + e) + 32*a^3)*sin(f*x +
 e))*sqrt(c)*log(-(c*cos(f*x + e)^2 - 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*s
qrt(c)*(cos(f*x + e) + sin(f*x + e) + 1) + 3*c*cos(f*x + e) + (c*cos(f*x +
e) - 2*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x
+ e) - cos(f*x + e) - 2)) + 4*(15*a^3*cos(f*x + e)^5 - 65*a^3*cos(f*x + e)^
4 + 812*a^3*cos(f*x + e)^3 + 1796*a^3*cos(f*x + e)^2 - 1144*a^3*cos(f*x + e
) - 2048*a^3 + (15*a^3*cos(f*x + e)^4 + 80*a^3*cos(f*x + e)^3 + 892*a^3*cos
(f*x + e)^2 - 904*a^3*cos(f*x + e) - 2048*a^3)*sin(f*x + e))*sqrt(-c*sin(f*
x + e) + c))/(c^6*f*cos(f*x + e)^6 - 5*c^6*f*cos(f*x + e)^5 - 18*c^6*f*cos(
f*x + e)^4 + 20*c^6*f*cos(f*x + e)^3 + 48*c^6*f*cos(f*x + e)^2 - 16*c^6*f*c
os(f*x + e) - 32*c^6*f + (c^6*f*cos(f*x + e)^5 + 6*c^6*f*cos(f*x + e)^4 - 1
2*c^6*f*cos(f*x + e)^3 - 32*c^6*f*cos(f*x + e)^2 + 16*c^6*f*cos(f*x + e) +
32*c^6*f)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**(11/2),x)

[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$sage_2$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(11/2),x, algorithm="giac")

[Out] sage2
```

3.317 $\int \frac{(c-c\sin(e+fx))^{7/2}}{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=132

$$\frac{64c^2\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{5af}-\frac{256c^3\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{5af}+\frac{2\sec(e+fx)(c-c\sin(e+fx))^{7/2}}{5af}+\frac{8c\,\mathrm{se}}{}$$

```
[Out] (-256*c^3*Sec[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(5*a*f) + (64*c^2*Sec[e +
f*x]*(c - c*Sin[e + f*x])^(3/2))/(5*a*f) + (8*c*Sec[e + f*x]*(c - c*Sin[e +
 f*x])^(5/2))/(5*a*f) + (2*Sec[e + f*x]*(c - c*Sin[e + f*x])^(7/2))/(5*a*f)
```

Rubi [A] time = 0.344804, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {2736, 2674, 2673}

$$\frac{64c^2\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{5af}-\frac{256c^3\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{5af}+\frac{2\sec(e+fx)(c-c\sin(e+fx))^{7/2}}{5af}+\frac{8c\,\mathrm{se}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sin[e + f*x])^(7/2)/(a + a*Sin[e + f*x]),x]
```


```
[Out] (-256*c^3*Sec[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(5*a*f) + (64*c^2*Sec[e +
f*x]*(c - c*Sin[e + f*x])^(3/2))/(5*a*f) + (8*c*Sec[e + f*x]*(c - c*Sin[e +
 f*x])^(5/2))/(5*a*f) + (2*Sec[e + f*x]*(c - c*Sin[e + f*x])^(7/2))/(5*a*f)
```


Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rubi steps

$$\int \frac{(c-c\sin(e+fx))^{7/2}}{a+a\sin(e+fx)}\,dx = \frac{\int \sec^2(e+fx)(c-c\sin(e+fx))^{9/2}\,dx}{ac}$$
$$= \frac{2\sec(e+fx)(c-c\sin(e+fx))^{7/2}}{5af} + \frac{12\int \sec^2(e+fx)(c-c\sin(e+fx))^{7/2}\,dx}{5a}$$
$$= \frac{8c\sec(e+fx)(c-c\sin(e+fx))^{5/2}}{5af} + \frac{2\sec(e+fx)(c-c\sin(e+fx))^{7/2}}{5af} + \frac{(32c)\int \sec^2(}{}$$
$$= \frac{64c^2\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{5af} + \frac{8c\sec(e+fx)(c-c\sin(e+fx))^{5/2}}{5af} + \frac{2\sec(e+}{}$$
$$= -\frac{256c^3\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{5af} + \frac{64c^2\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{5af} + \frac{8c\sec(}{}$$

Mathematica [A] time = 2.12381, size = 112, normalized size = 0.85

$$\frac{c^3\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)(-175\sin(e+fx)+\sin(3(e+fx))-14\cos(2(e+fx))-350}{10af(\sin(e+fx)+1)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sin[e + f*x])^(7/2)/(a + a*Sin[e + f*x]),x]
```

```
[Out] (c^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]*(-350 -
 14*Cos[2*(e + f*x)] - 175*Sin[e + f*x] + Sin[3*(e + f*x)]))/(10*a*f*(Cos[(
e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x]))
```

Maple [A] time = 0.479, size = 69, normalized size = 0.5

$$\frac{2c^4\left(-1+\sin\left(fx+e\right)\right)\left(\left(\sin\left(fx+e\right)\right)^3-7\left(\sin\left(fx+e\right)\right)^2+43\sin\left(fx+e\right)+91\right)}{5af\cos\left(fx+e\right)}\frac{1}{\sqrt{c-c\sin\left(fx+e\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e)),x)
```

```
[Out] 2/5*c^4/a*(-1+sin(f*x+e))*(sin(f*x+e)^3-7*sin(f*x+e)^2+43*sin(f*x+e)+91)/co
s(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Maxima [B] time = 1.89519, size = 321, normalized size = 2.43

$$\frac{2\left(91c^{\frac{7}{2}}+\frac{86c^{\frac{7}{2}}\sin(fx+e)}{\cos(fx+e)+1}+\frac{336c^{\frac{7}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{266c^{\frac{7}{2}}\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{490c^{\frac{7}{2}}\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{266c^{\frac{7}{2}}\sin(fx+e)^5}{(\cos(fx+e)+1)^5}+\frac{336c^{\frac{7}{2}}\sin(fx+e)^6}{(\cos(fx+e)+1)^6}+\frac{86c^{\frac{7}{2}}}{(\cos}\right.}{5\left(a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}\right)f\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] 2/5*(91*c^(7/2) + 86*c^(7/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 336*c^(7/2)*
sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 266*c^(7/2)*sin(f*x + e)^3/(cos(f*x +
 e) + 1)^3 + 490*c^(7/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 266*c^(7/2)*
sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 336*c^(7/2)*sin(f*x + e)^6/(cos(f*x +
 e) + 1)^6 + 86*c^(7/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 91*c^(7/2)*si
n(f*x + e)^8/(cos(f*x + e) + 1)^8)/((a + a*sin(f*x + e)/(cos(f*x + e) + 1))
*f*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(7/2))
```

Fricas [A] time = 1.03254, size = 173, normalized size = 1.31

$$-\frac{2\left(7\,c^3\cos\left(fx+e\right)^2+84\,c^3-\left(c^3\cos\left(fx+e\right)^2-44\,c^3\right)\sin\left(fx+e\right)\right)\sqrt{-c\sin\left(fx+e\right)+c}}{5\,af\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] -2/5*(7*c^3*cos(f*x + e)^2 + 84*c^3 - (c^3*cos(f*x + e)^2 - 44*c^3)*sin(f*x
 + e))*sqrt(-c*sin(f*x + e) + c)/(a*f*cos(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))**(7/2)/(a+a*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.78945, size = 575, normalized size = 4.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] -1/20*(4*(40*sqrt(2)*c^10 - 11*sqrt(2)*a^6*c + 22*a^6*c)*sgn(tan(1/2*f*x +
1/2*e) - 1)/(sqrt(2)*a*c^(13/2) - a*c^(13/2)) - 640*((sqrt(c)*tan(1/2*f*x +
 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))*c^4*sgn(tan(1/2*f*x + 1/2*e)
- 1) - c^(9/2)*sgn(tan(1/2*f*x + 1/2*e) - 1))/(((sqrt(c)*tan(1/2*f*x + 1/2*
e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^2 + 2*(sqrt(c)*tan(1/2*f*x + 1/2*e
) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))*sqrt(c) - c)*a) - (51*a^5*sgn(tan(1
/2*f*x + 1/2*e) - 1)/c^3 + (35*a^5*sgn(tan(1/2*f*x + 1/2*e) - 1)/c^3 + (90*
a^5*sgn(tan(1/2*f*x + 1/2*e) - 1)/c^3 + (90*a^5*sgn(tan(1/2*f*x + 1/2*e) -
1)/c^3 + (51*a^5*sgn(tan(1/2*f*x + 1/2*e) - 1)*tan(1/2*f*x + 1/2*e)/c^3 + 3
5*a^5*sgn(tan(1/2*f*x + 1/2*e) - 1)/c^3)*tan(1/2*f*x + 1/2*e))*tan(1/2*f*x
+ 1/2*e))*tan(1/2*f*x + 1/2*e))*tan(1/2*f*x + 1/2*e))/(c*tan(1/2*f*x + 1/2*
e)^2 + c)^(5/2))/f
```

3.318 $\int \frac{(c-c\sin(e+fx))^{5/2}}{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=98

$$-\frac{64c^2\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{3af}+\frac{2\sec(e+fx)(c-c\sin(e+fx))^{5/2}}{3af}+\frac{16c\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{3af}$$

```
[Out] (-64*c^2*Sec[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(3*a*f) + (16*c*Sec[e + f*x
]*(c - c*Sin[e + f*x])^(3/2))/(3*a*f) + (2*Sec[e + f*x]*(c - c*Sin[e + f*x]
)^(5/2))/(3*a*f)
```

Rubi [A] time = 0.269496, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {2736, 2674, 2673}

$$-\frac{64c^2\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{3af}+\frac{2\sec(e+fx)(c-c\sin(e+fx))^{5/2}}{3af}+\frac{16c\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{3af}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sin[e + f*x])^(5/2)/(a + a*Sin[e + f*x]),x]
```

```
[Out] (-64*c^2*Sec[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(3*a*f) + (16*c*Sec[e + f*x
]*(c - c*Sin[e + f*x])^(3/2))/(3*a*f) + (2*Sec[e + f*x]*(c - c*Sin[e + f*x]
)^(5/2))/(3*a*f)
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rubi steps

$$\int \frac{(c-c\sin(e+fx))^{5/2}}{a+a\sin(e+fx)}\,dx = \frac{\int \sec^2(e+fx)(c-c\sin(e+fx))^{7/2}\,dx}{ac}$$
$$= \frac{2\sec(e+fx)(c-c\sin(e+fx))^{5/2}}{3af} + \frac{8\int \sec^2(e+fx)(c-c\sin(e+fx))^{5/2}\,dx}{3a}$$
$$= \frac{16c\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{3af} + \frac{2\sec(e+fx)(c-c\sin(e+fx))^{5/2}}{3af} + \frac{(32c)\int \sec^2(e\cdot}{}$$
$$= -\frac{64c^2\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{3af} + \frac{16c\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{3af} + \frac{2\sec(e+fx}{}$$

Mathematica [A] time = 0.704548, size = 102, normalized size = 1.04

$$-\frac{c^2\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)(20\sin(e+fx)+\cos(2(e+fx))+45)}{3af(\sin(e+fx)+1)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sin[e + f*x])^(5/2)/(a + a*Sin[e + f*x]),x]
```

```
[Out] -(c^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(45 + Cos[2*(e + f*x)] + 20*Sin
[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(3*a*f*(Cos[(e + f*x)/2] - Sin[(e + f*
x)/2])*(1 + Sin[e + f*x]))
```

Maple [A] time = 0.403, size = 59, normalized size = 0.6

$$-\frac{2c^3\left(-1+\sin\left(fx+e\right)\right)\left(\left(\sin\left(fx+e\right)\right)^2-10\sin\left(fx+e\right)-23\right)}{3af\cos\left(fx+e\right)}\frac{1}{\sqrt{c-c\sin\left(fx+e\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e)),x)
```

```
[Out] -2/3*c^3/a*(-1+sin(f*x+e))*(sin(f*x+e)^2-10*sin(f*x+e)-23)/cos(f*x+e)/(c-c*
sin(f*x+e))^(1/2)/f
```

Maxima [B] time = 1.79514, size = 259, normalized size = 2.64

$$\frac{2\left(23c^{\frac{5}{2}}+\frac{20c^{\frac{5}{2}}\sin(fx+e)}{\cos(fx+e)+1}+\frac{65c^{\frac{5}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{40c^{\frac{5}{2}}\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{65c^{\frac{5}{2}}\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{20c^{\frac{5}{2}}\sin(fx+e)^5}{(\cos(fx+e)+1)^5}+\frac{23c^{\frac{5}{2}}\sin(fx+e)^6}{(\cos(fx+e)+1)^6}\right)}{3\left(a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}\right)f\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] 2/3*(23*c^(5/2) + 20*c^(5/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 65*c^(5/2)*s
in(f*x + e)^2/(cos(f*x + e) + 1)^2 + 40*c^(5/2)*sin(f*x + e)^3/(cos(f*x + e
) + 1)^3 + 65*c^(5/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 20*c^(5/2)*sin(
f*x + e)^5/(cos(f*x + e) + 1)^5 + 23*c^(5/2)*sin(f*x + e)^6/(cos(f*x + e) +
 1)^6)/((a + a*sin(f*x + e)/(cos(f*x + e) + 1))*f*(sin(f*x + e)^2/(cos(f*x
+ e) + 1)^2 + 1)^(5/2))
```

Fricas [A] time = 1.07028, size = 139, normalized size = 1.42

$$-\frac{2\left(c^2\cos\left(fx+e\right)^2+10\,c^2\sin\left(fx+e\right)+22\,c^2\right)\sqrt{-c\sin\left(fx+e\right)+c}}{3\,af\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] -2/3*(c^2*cos(f*x + e)^2 + 10*c^2*sin(f*x + e) + 22*c^2)*sqrt(-c*sin(f*x +
e) + c)/(a*f*cos(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))**(5/2)/(a+a*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.6757, size = 486, normalized size = 4.96

$$-\frac{\frac{2\left(6\sqrt{2}c^7-5\sqrt{2}a^4c+10\,a^4c\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)}{\sqrt{2}ac^{\frac{9}{2}}-ac^{\frac{9}{2}}}-\frac{\frac{11\,a^3\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)}{c^2}+\left(\frac{9\,a^3\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)}{c^2}+\left(\frac{11\,a^3\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{c^2}+\cdots}{\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+c\right)^{\frac{3}{2}}}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] -1/3*(2*(6*sqrt(2)*c^7 - 5*sqrt(2)*a^4*c + 10*a^4*c)*sgn(tan(1/2*f*x + 1/2*
e) - 1)/(sqrt(2)*a*c^(9/2) - a*c^(9/2)) - (11*a^3*sgn(tan(1/2*f*x + 1/2*e)
- 1)/c^2 + (9*a^3*sgn(tan(1/2*f*x + 1/2*e) - 1)/c^2 + (11*a^3*sgn(tan(1/2*f
*x + 1/2*e) - 1)*tan(1/2*f*x + 1/2*e)/c^2 + 9*a^3*sgn(tan(1/2*f*x + 1/2*e)
- 1)/c^2)*tan(1/2*f*x + 1/2*e))*tan(1/2*f*x + 1/2*e))/(c*tan(1/2*f*x + 1/2*
e)^2 + c)^(3/2) - 48*((sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x +
1/2*e)^2 + c))*c^3*sgn(tan(1/2*f*x + 1/2*e) - 1) - c^(7/2)*sgn(tan(1/2*f*x
+ 1/2*e) - 1))/(((sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e
)^2 + c))^2 + 2*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)
^2 + c))*sqrt(c) - c)*a))/f
```

3.319 $\int \frac{(c-c\sin(e+fx))^{3/2}}{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=60

$$\frac{2\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{af}-\frac{8c\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{af}$$

```
[Out] (-8*c*Sec[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(a*f) + (2*Sec[e + f*x]*(c - c
*Sin[e + f*x])^(3/2))/(a*f)
```

Rubi [A] time = 0.203895, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {2736, 2674, 2673}

$$\frac{2\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{af}-\frac{8c\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{af}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sin[e + f*x])^(3/2)/(a + a*Sin[e + f*x]),x]
```

```
[Out] (-8*c*Sec[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(a*f) + (2*Sec[e + f*x]*(c - c
*Sin[e + f*x])^(3/2))/(a*f)
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rubi steps

$$\int \frac{(c-c\sin(e+fx))^{3/2}}{a+a\sin(e+fx)}\,dx = \frac{\int \sec^2(e+fx)(c-c\sin(e+fx))^{5/2}\,dx}{ac}$$
$$= \frac{2\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{af} + \frac{4\int \sec^2(e+fx)(c-c\sin(e+fx))^{3/2}\,dx}{a}$$
$$= -\frac{8c\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{af} + \frac{2\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{af}$$

Mathematica [A] time = 0.29723, size = 88, normalized size = 1.47

$$-\frac{2c(\sin(e+fx)+3)\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}{af(\sin(e+fx)+1)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sin[e + f*x])^(3/2)/(a + a*Sin[e + f*x]),x]
```

```
[Out] (-2*c*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(3 + Sin[e + f*x])*Sqrt[c - c*S
in[e + f*x]])/(a*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])
)
```

Maple [A] time = 0.454, size = 49, normalized size = 0.8

$$2\,\frac{c^2\left(-1+\sin\left(fx+e\right)\right)\left(3+\sin\left(fx+e\right)\right)}{\cos\left(fx+e\right)a\sqrt{c-c\sin\left(fx+e\right)}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e)),x)
```

```
[Out] 2*c^2/a*(-1+sin(f*x+e))*(3+sin(f*x+e))/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Maxima [B] time = 2.43061, size = 197, normalized size = 3.28

$$\frac{2\left(3\,c^{\frac{3}{2}}+\frac{2\,c^{\frac{3}{2}}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{6\,c^{\frac{3}{2}}\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}+\frac{2\,c^{\frac{3}{2}}\sin\left(fx+e\right)^3}{\left(\cos\left(fx+e\right)+1\right)^3}+\frac{3\,c^{\frac{3}{2}}\sin\left(fx+e\right)^4}{\left(\cos\left(fx+e\right)+1\right)^4}\right)}{\left(a+\frac{a\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}\right)f\left(\frac{\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}+1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] 2*(3*c^(3/2) + 2*c^(3/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 6*c^(3/2)*sin(f*
x + e)^2/(cos(f*x + e) + 1)^2 + 2*c^(3/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)
^3 + 3*c^(3/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4)/((a + a*sin(f*x + e)/(c
os(f*x + e) + 1))*f*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(3/2))
```

Fricas [A] time = 1.00194, size = 97, normalized size = 1.62

$$-\frac{2\left(c\sin\left(fx+e\right)+3\,c\right)\sqrt{-c\sin\left(fx+e\right)+c}}{af\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] -2*(c*sin(f*x + e) + 3*c)*sqrt(-c*sin(f*x + e) + c)/(a*f*cos(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))**(3/2)/(a+a*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.57878, size = 358, normalized size = 5.97

$$-\frac{2\left(\frac{2\,c^{\frac{3}{2}}\operatorname{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)}{\sqrt{2}a-a}-\frac{\frac{c^{2}\operatorname{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{a}+\frac{c^{2}\operatorname{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)}{a}}{\sqrt{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+c}}-\frac{4\left(\left(\sqrt{c}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-\sqrt{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+c}\right)c^{2}\operatorname{sgn}(\mathrm{t}\cdots}{\left(\left(\sqrt{c}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-\sqrt{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+c}\right)^{2}+2\left(\sqrt{c}\,\mathrm{t}\cdots\right.}\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] -2*(2*c^(3/2)*sgn(tan(1/2*f*x + 1/2*e) - 1)/(sqrt(2)*a - a) - (c^2*sgn(tan(
1/2*f*x + 1/2*e) - 1)*tan(1/2*f*x + 1/2*e)/a + c^2*sgn(tan(1/2*f*x + 1/2*e)
 - 1)/a)/sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c) - 4*((sqrt(c)*tan(1/2*f*x + 1/2
*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))*c^2*sgn(tan(1/2*f*x + 1/2*e) - 1)
 - c^(5/2)*sgn(tan(1/2*f*x + 1/2*e) - 1))/(((sqrt(c)*tan(1/2*f*x + 1/2*e) -
 sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^2 + 2*(sqrt(c)*tan(1/2*f*x + 1/2*e) -
sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))*sqrt(c) - c)*a))/f
```

3.320 $\int \frac{\sqrt{c-c\sin(e+fx)}}{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=29

$$-\frac{2\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{af}$$

```
[Out] (-2*Sec[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(a*f)
```

Rubi [A] time = 0.12751, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2736, 2673}

$$-\frac{2\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{af}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c*Sin[e + f*x]]/(a + a*Sin[e + f*x]),x]
```

```
[Out] (-2*Sec[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(a*f)
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{c-c\sin(e+fx)}}{a+a\sin(e+fx)}\,dx &= \frac{\int \sec^2(e+fx)(c-c\sin(e+fx))^{3/2}\,dx}{ac}\\ &= -\frac{2\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{af}\end{aligned}$$

Mathematica [A] time = 0.0964544, size = 29, normalized size = 1.

$$-\frac{2\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{af}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c*Sin[e + f*x]]/(a + a*Sin[e + f*x]),x]
```

```
[Out] (-2*Sec[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(a*f)
```

Maple [A] time = 0.408, size = 39, normalized size = 1.3

$$2\,{\frac {c \left(-1+\sin \left(fx+e \right) \right) }{\cos \left(fx+e \right) a\sqrt {c-c\sin \left(fx+e \right) }f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e)),x)
```

```
[Out] 2*c/a*(-1+sin(f*x+e))/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Maxima [B] time = 2.24396, size = 103, normalized size = 3.55

$$\frac{2 \left(\sqrt{c} + \frac{\sqrt{c} \sin\left(f x + e\right)^{2}}{{\left(\cos\left(f x + e\right) + 1\right)}^{2}}\right)}{{\left(a + \frac{a \sin\left(f x + e\right)}{\cos\left(f x + e\right) + 1}\right)} f \sqrt{\frac{\sin\left(f x + e\right)^{2}}{{\left(\cos\left(f x + e\right) + 1\right)}^{2}} + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] 2*(sqrt(c) + sqrt(c)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2)/((a + a*sin(f*x +
 e)/(cos(f*x + e) + 1))*f*sqrt(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1))
```

Fricas [A] time = 1.00828, size = 66, normalized size = 2.28

$$-\frac{2 \sqrt{-c \sin\left(f x + e\right) + c}}{a f \cos\left(f x + e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] -2*sqrt(-c*sin(f*x + e) + c)/(a*f*cos(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sqrt{- c \sin{\left(e + f x \right)} + c}}{\sin{\left(e + f x \right)} + 1}\, dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))**(1/2)/(a+a*sin(f*x+e)),x)
```

```
[Out] Integral(sqrt(-c*sin(e + f*x) + c)/(sin(e + f*x) + 1), x)/a
```

Giac [B] time = 1.50303, size = 262, normalized size = 9.03

$$-\frac{\frac{\sqrt{2}\sqrt{c}\operatorname{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{\sqrt{2}a-a}-\frac{4\left(\left(\sqrt{c}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+c}\right)c\operatorname{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)-c^{\frac{3}{2}}\operatorname{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)\right)}{\left(\left(\sqrt{c}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+c}\right)^2+2\left(\sqrt{c}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+c}\right)\sqrt{c}-c\right)a}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] -(sqrt(2)*sqrt(c)*sgn(tan(1/2*f*x + 1/2*e) - 1)/(sqrt(2)*a - a) - 4*((sqrt(
c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))*c*sgn(tan(1/2
*f*x + 1/2*e) - 1) - c^(3/2)*sgn(tan(1/2*f*x + 1/2*e) - 1))/(((sqrt(c)*tan(
1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^2 + 2*(sqrt(c)*tan(1
/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))*sqrt(c) - c)*a))/f
```

3.321 $\int \frac{1}{(a+a\sin(e+fx))\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=83

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{2}a\sqrt{c}f}-\frac{\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{acf}$$

```
[Out] ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])]/(Sqrt[2]
*a*Sqrt[c]*f) - (Sec[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(a*c*f)
```

Rubi [A] time = 0.158731, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2736, 2675, 2649, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{2}a\sqrt{c}f}-\frac{\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{acf}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]]),x]
```

```
[Out] ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])]/(Sqrt[2]
*a*Sqrt[c]*f) - (Sec[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(a*c*f)
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2675

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^m)/(a*f*g*(p + 1)), x] + Dist[(a*(m + p + 1))/(g^2*(p + 1)), Int[(g*Cos
[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[p, -2*m] && IntegersQ[m
+ 1/2, 2*p]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))\sqrt{c-c\sin(e+fx)}}\,dx=\frac{\int \sec^2(e+fx)\sqrt{c-c\sin(e+fx)}\,dx}{ac}$$
$$=-\frac{\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{acf}+\frac{\int \frac{1}{\sqrt{c-c\sin(e+fx)}}\,dx}{2a}$$
$$=-\frac{\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{acf}-\frac{\text{Subst}\left(\int \frac{1}{2c-x^2}\,dx,x,-\frac{c\cos(e+fx)}{\sqrt{c-c\sin(e+fx)}}\right)}{af}$$
$$=\frac{\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{2}a\sqrt{c}f}-\frac{\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{acf}$$

Mathematica [C] time = 0.31445, size = 97, normalized size = 1.17

$$-\frac{\cos(e+fx)\left(1+(1+i)\sqrt[4]{-1}\tan^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt[4]{-1}\left(\tan\left(\frac{1}{4}(e+fx)\right)+1\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\right)}{af(\sin(e+fx)+1)\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]]),x]
```

```
[Out] -((Cos[e + f*x]*(1 + (1 + I)*(-1)^(1/4)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 +
Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])))/(a*f*(1 + Sin[e
+ f*x])*Sqrt[c - c*Sin[e + f*x]]))
```

Maple [A] time = 0.625, size = 85, normalized size = 1.

$$-\frac{-1+\sin\left(fx+e\right)}{2\,af\cos\left(fx+e\right)}\left(\sqrt{2}\text{Artanh}\left(\frac{\sqrt{2}}{2}\sqrt{c\left(1+\sin\left(fx+e\right)\right)}\frac{1}{\sqrt{c}}\right)c\sqrt{c\left(1+\sin\left(fx+e\right)\right)}-2\,c^{3/2}\right)c^{-\frac{3}{2}}\frac{1}{\sqrt{c-c\sin\left(fx+e\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] -1/2/a*(-1+sin(f*x+e))*(2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2
)/c^(1/2))*c*(c*(1+sin(f*x+e)))^(1/2)-2*c^(3/2))/c^(3/2)/cos(f*x+e)/(c-c*si
n(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\sin\left(fx+e\right)+a\right)\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)), x)
```

Fricas [B] time = 1.09838, size = 425, normalized size = 5.12

$$\frac{\sqrt{2}\sqrt{c}\cos\left(fx+e\right)\log\left(-\frac{\cos\left(fx+e\right)^2+\left(\cos\left(fx+e\right)-2\right)\sin\left(fx+e\right)+\frac{2\sqrt{2}\sqrt{-c\sin\left(fx+e\right)+c}\left(\cos\left(fx+e\right)+\sin\left(fx+e\right)+1\right)}{\sqrt{c}}+3\cos\left(fx+e\right)+2}{\cos\left(fx+e\right)^2+\left(\cos\left(fx+e\right)+2\right)\sin\left(fx+e\right)-\cos\left(fx+e\right)-2}\right)-4\sqrt{-c\sin\left(fx\right.}}{4\,acf\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(2)*sqrt(c)*cos(f*x + e)*log(-(cos(f*x + e)^2 + (cos(f*x + e) - 2)
*sin(f*x + e) + 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*(cos(f*x + e) + sin(f*x
 + e) + 1)/sqrt(c) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2 + (cos(f*x + e) +
2)*sin(f*x + e) - cos(f*x + e) - 2)) - 4*sqrt(-c*sin(f*x + e) + c))/(a*c*f*
cos(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{\sqrt{-c\sin\left(e+fx\right)+c}\sin\left(e+fx\right)+\sqrt{-c\sin\left(e+fx\right)+c}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-c*sin(e + f*x) + c)*sin(e + f*x) + sqrt(-c*sin(e + f*x) +
 c)), x)/a
```

Giac [B] time = 1.6202, size = 408, normalized size = 4.92

$$\frac{\frac{\left(2\sqrt{2}c\arctan\left(\frac{\sqrt{c}}{\sqrt{-c}}\right)-2c\arctan\left(\frac{\sqrt{c}}{\sqrt{-c}}\right)+\sqrt{-c}\sqrt{c}\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{\sqrt{2}a\sqrt{-c}c-2a\sqrt{-c}c}+\frac{\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{c}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+c}-\sqrt{c}\right)}{2\sqrt{-c}}\right)}{a\sqrt{-c}\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}+\frac{\ldots}{\left(\left(\sqrt{c}\tan\left(\frac{1}{2}fx+\frac{1}{2}\right.\right.\right.}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] ((2*sqrt(2)*c*arctan(sqrt(c)/sqrt(-c)) - 2*c*arctan(sqrt(c)/sqrt(-c)) + sqr
t(-c)*sqrt(c))*sgn(tan(1/2*f*x + 1/2*e) - 1)/(sqrt(2)*a*sqrt(-c)*c - 2*a*sq
rt(-c)*c) + sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqr
t(c*tan(1/2*f*x + 1/2*e)^2 + c) - sqrt(c))/sqrt(-c))/(a*sqrt(-c)*sgn(tan(1/
2*f*x + 1/2*e) - 1)) + 2*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x
```

```
 + 1/2*e)^2 + c) - sqrt(c))/(((sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/
2*f*x + 1/2*e)^2 + c))^2 + 2*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2
*f*x + 1/2*e)^2 + c))*sqrt(c) - c)*a*sgn(tan(1/2*f*x + 1/2*e) - 1)))/f
```

3.322 $\int \frac{1}{(a+a\sin(e+fx))(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=117

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{4\sqrt{2}ac^{3/2}f}+\frac{3\cos(e+fx)}{4af(c-c\sin(e+fx))^{3/2}}-\frac{\sec(e+fx)}{acf\sqrt{c-c\sin(e+fx)}}$$

```
[Out] (3*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])])/(4*S
qrt[2]*a*c^(3/2)*f) + (3*Cos[e + f*x])/(4*a*f*(c - c*Sin[e + f*x])^(3/2)) -
 Sec[e + f*x]/(a*c*f*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 0.192404, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {2736, 2687, 2650, 2649, 206}

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{4\sqrt{2}ac^{3/2}f}+\frac{3\cos(e+fx)}{4af(c-c\sin(e+fx))^{3/2}}-\frac{\sec(e+fx)}{acf\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2)),x]
```

```
[Out] (3*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])])/(4*S
qrt[2]*a*c^(3/2)*f) + (3*Cos[e + f*x])/(4*a*f*(c - c*Sin[e + f*x])^(3/2)) -
 Sec[e + f*x]/(a*c*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2687

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p + 1)*Sq
rt[a + b*Sin[e + f*x]]), x] + Dist[(a*(2*p + 1))/(2*g^2*(p + 1)), Int[(g*Co
s[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
```

```
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))(c-c\sin(e+fx))^{3/2}}\,dx = \frac{\int \frac{\sec^2(e+fx)}{\sqrt{c-c\sin(e+fx)}}\,dx}{ac}$$
$$= -\frac{\sec(e+fx)}{acf\sqrt{c-c\sin(e+fx)}} + \frac{3\int \frac{1}{(c-c\sin(e+fx))^{3/2}}\,dx}{2a}$$
$$= \frac{3\cos(e+fx)}{4af(c-c\sin(e+fx))^{3/2}} - \frac{\sec(e+fx)}{acf\sqrt{c-c\sin(e+fx)}} + \frac{3\int \frac{1}{\sqrt{c-c\sin(e+fx)}}}{8ac}$$
$$= \frac{3\cos(e+fx)}{4af(c-c\sin(e+fx))^{3/2}} - \frac{\sec(e+fx)}{acf\sqrt{c-c\sin(e+fx)}} - \frac{3\,\mathrm{Subst}\left(\int \frac{1}{2c-x^2}\right.}{}$$
$$= \frac{3\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{4\sqrt{2}ac^{3/2}f} + \frac{3\cos(e+fx)}{4af(c-c\sin(e+fx))^{3/2}} - \frac{\sec(e+}{acf\sqrt{c-c\,\mathrm{s}}}$$

Mathematica [C] time = 0.603284, size = 125, normalized size = 1.07

$$-\frac{\sec(e+fx)\left(-3\sin(e+fx)+(3+3i)\sqrt[4]{-1}\tan^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt[4]{-1}\left(\tan\left(\frac{1}{4}(e+fx)\right)+1\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+\right.\right.}{4acf\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2)),x]
```

```
[Out] -(Sec[e + f*x]*(1 + (3 + 3*I)*(-1)^(1/4)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 +
 Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*(Cos[(e + f*x)/
2] + Sin[(e + f*x)/2]) - 3*Sin[e + f*x]))/(4*a*c*f*Sqrt[c - c*Sin[e + f*x]]
)
```

Maple [A] time = 0.638, size = 134, normalized size = 1.2

$$-\frac{1}{8af\cos(fx+e)}\left(3\sqrt{c\left(1+\sin(fx+e)\right)}\sqrt{2}\mathrm{Artanh}\left(1/2\,\frac{\sqrt{c\left(1+\sin(fx+e)\right)}\sqrt{2}}{\sqrt{c}}\right)\sin(fx+e)\,c-3\sqrt{2}\mathrm{Artanh}\left(1\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] -1/8/c^(5/2)/a*(3*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)*arctanh(1/2*(c*(1+sin(f*
x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)*c-3*2^(1/2)*arctanh(1/2*(c*(1+sin(
```

```
f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*c*(c*(1+sin(f*x+e)))^(1/2)-6*c^(3/2)*sin(f*
x+e)+2*c^(3/2))/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\sin\left(fx+e\right)+a\right)\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)*(-c*sin(f*x + e) + c)^(3/2)), x)
```

Fricas [B] time = 1.13358, size = 563, normalized size = 4.81

$$\frac{3\sqrt{2}\left(\cos\left(fx+e\right)\sin\left(fx+e\right)-\cos\left(fx+e\right)\right)\sqrt{c}\log\left(-\frac{c\cos\left(fx+e\right)^2+2\sqrt{2}\sqrt{-c\sin\left(fx+e\right)+c}\sqrt{c}\left(\cos\left(fx+e\right)+\sin\left(fx+e\right)+1\right)+3c\cos\left(fx+e\right)+(}{\cos\left(fx+e\right)^2+\left(\cos\left(fx+e\right)+2\right)\sin\left(fx+e\right)-\cos\left(fx+e\right)-2}\right.}{16\left(ac^2f\cos\left(fx+e\right)\sin\left(fx+e\right)-ac^2f\cos\left(fx+e\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/16*(3*sqrt(2)*(cos(f*x + e)*sin(f*x + e) - cos(f*x + e))*sqrt(c)*log(-(c*
cos(f*x + e)^2 + 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*sqrt(c)*(cos(f*x + e)
+ sin(f*x + e) + 1) + 3*c*cos(f*x + e) + (c*cos(f*x + e) - 2*c)*sin(f*x + e
) + 2*c)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) -
 2)) - 4*sqrt(-c*sin(f*x + e) + c)*(3*sin(f*x + e) - 1))/(a*c^2*f*cos(f*x +
 e)*sin(f*x + e) - a*c^2*f*cos(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{-c\sqrt{-c\sin\left(e+fx\right)+c}\sin^2\left(e+fx\right)+c\sqrt{-c\sin\left(e+fx\right)+c}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral(1/(-c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)**2 + c*sqrt(-c*sin(e
+ f*x) + c)), x)/a
```

Giac [B] time = 1.99257, size = 628, normalized size = 5.37

$$\frac{3\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{c}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+c}-\sqrt{c}\right)}{2\sqrt{-c}}\right)}{a\sqrt{-c}c\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}+\frac{4\left(\sqrt{c}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+c}-\sqrt{c}\right)}{\left(\left(\sqrt{c}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+c}\right)^2+2\left(\sqrt{c}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right.}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*(3*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*t
an(1/2*f*x + 1/2*e)^2 + c) - sqrt(c))/sqrt(-c))/(a*sqrt(-c)*c*sgn(tan(1/2*f
*x + 1/2*e) - 1)) + 4*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x +
1/2*e)^2 + c) - sqrt(c))/(((sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f
*x + 1/2*e)^2 + c))^2 + 2*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*
x + 1/2*e)^2 + c))*sqrt(c) - c)*a*c*sgn(tan(1/2*f*x + 1/2*e) - 1)) + 2*(3*(
sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^3 - (sqr
t(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^2*sqrt(c) -
 (sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))*c - c^
(3/2))/(((sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c)
)^2 - 2*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))
*sqrt(c) - c)^2*a*c*sgn(tan(1/2*f*x + 1/2*e) - 1)))/f
```

3.323 $\int \frac{1}{(a+a\sin(e+fx))(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=156

$$-\frac{5\sec(e+fx)}{8ac^2f\sqrt{c-c\sin(e+fx)}}+\frac{15\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{32\sqrt{2}ac^{5/2}f}+\frac{15\cos(e+fx)}{32acf(c-c\sin(e+fx))^{3/2}}+\frac{\sec(e+fx)}{4acf(c-c\sin(e+fx))^{3/2}}$$

```
[Out] (15*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])])/(32
*Sqrt[2]*a*c^(5/2)*f) + (15*Cos[e + f*x])/(32*a*c*f*(c - c*Sin[e + f*x])^(3
/2)) + Sec[e + f*x]/(4*a*c*f*(c - c*Sin[e + f*x])^(3/2)) - (5*Sec[e + f*x])
/(8*a*c^2*f*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 0.266892, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2736, 2681, 2687, 2650, 2649, 206}

$$-\frac{5\sec(e+fx)}{8ac^2f\sqrt{c-c\sin(e+fx)}}+\frac{15\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{32\sqrt{2}ac^{5/2}f}+\frac{15\cos(e+fx)}{32acf(c-c\sin(e+fx))^{3/2}}+\frac{\sec(e+fx)}{4acf(c-c\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2)),x]
```


```
[Out] (15*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])])/(32
*Sqrt[2]*a*c^(5/2)*f) + (15*Cos[e + f*x])/(32*a*c*f*(c - c*Sin[e + f*x])^(3
/2)) + Sec[e + f*x]/(4*a*c*f*(c - c*Sin[e + f*x])^(3/2)) - (5*Sec[e + f*x])
/(8*a*c^2*f*Sqrt[c - c*Sin[e + f*x]])
```


Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2681

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*(2*m + p + 1)), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
 IntegersQ[2*m, 2*p]
```

Rule 2687

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p + 1)*Sq
rt[a + b*Sin[e + f*x]]), x] + Dist[(a*(2*p + 1))/(2*g^2*(p + 1)), Int[(g*Co
s[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))(c-c\sin(e+fx))^{5/2}}\,dx = \frac{\int \frac{\sec^2(e+fx)}{(c-c\sin(e+fx))^{3/2}}\,dx}{ac}$$

$$= \frac{\sec(e+fx)}{4acf(c-c\sin(e+fx))^{3/2}} + \frac{5\int \frac{\sec^2(e+fx)}{\sqrt{c-c\sin(e+fx)}}\,dx}{8ac^2}$$

$$= \frac{\sec(e+fx)}{4acf(c-c\sin(e+fx))^{3/2}} - \frac{5\sec(e+fx)}{8ac^2f\sqrt{c-c\sin(e+fx)}} + \frac{15\int \frac{1}{(c-c\sin(e+}}{16ac}$$

$$= \frac{15\cos(e+fx)}{32acf(c-c\sin(e+fx))^{3/2}} + \frac{\sec(e+fx)}{4acf(c-c\sin(e+fx))^{3/2}} - \frac{5\sec}{8ac^2f\sqrt{c-}}$$

$$= \frac{15\cos(e+fx)}{32acf(c-c\sin(e+fx))^{3/2}} + \frac{\sec(e+fx)}{4acf(c-c\sin(e+fx))^{3/2}} - \frac{5\sec}{8ac^2f\sqrt{c-}}$$

$$= \frac{15\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{32\sqrt{2}ac^{5/2}f} + \frac{15\cos(e+fx)}{32acf(c-c\sin(e+fx))^{3/2}} + \frac{\mathrm{se}}{4acf(c-}$$

Mathematica [C] time = 0.778693, size = 162, normalized size = 1.04

$$\frac{\left(\frac{1}{128}+\frac{i}{128}\right)\cos(e+fx)\left((1-i)(40\sin(e+fx)+15\cos(2(e+fx))-9)-60\sqrt[4]{-1}\tan^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt[4]{-1}\left(\tan\left(\frac{1}{4}(e+fx)\right.\right.\right.\right.}{ac^2f(\sin(e+fx)-1)^2(\sin(e+fx)+1)\sqrt{c-c\mathrm{s}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2)),x]
```

```
[Out] ((1/128 + I/128)*Cos[e + f*x]*(-60*(-1)^(1/4)*ArcTan[(1/2 + I/2)*(-1)^(1/4)
*(1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*(Cos[(e +
f*x)/2] + Sin[(e + f*x)/2]) + (1 - I)*(-9 + 15*Cos[2*(e + f*x)] + 40*Sin[e
+ f*x])))/(a*c^2*f*(-1 + Sin[e + f*x])^2*(1 + Sin[e + f*x])*Sqrt[c - c*Sin[
e + f*x]])
```

Maple [A] time = 0.703, size = 210, normalized size = 1.4

$$-\frac{1}{64\,a\left(-1+\sin\left(fx+e\right)\right)\cos\left(fx+e\right)f}\left(15\sqrt{c\left(1+\sin\left(fx+e\right)\right)}\sqrt{2}\mathrm{Artanh}\left(1/2\,\frac{\sqrt{c\left(1+\sin\left(fx+e\right)\right)}\sqrt{2}}{\sqrt{c}}\right)\left(\sin\left(fx+e\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] -1/64/c^(9/2)/a*(15*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)*arctanh(1/2*(c*(1+sin(
f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)^2*c^2-30*c^(5/2)*sin(f*x+e)^2-30
*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1
/2)/c^(1/2))*sin(f*x+e)*c^2+40*c^(5/2)*sin(f*x+e)+15*(c*(1+sin(f*x+e)))^(1/
2)*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*c^2+6*c^(5
/2))/(-1+sin(f*x+e))/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\sin\left(fx+e\right)+a\right)\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)*(-c*sin(f*x + e) + c)^(5/2)), x)
```

Fricas [A] time = 1.13242, size = 663, normalized size = 4.25

$$\frac{15\sqrt{2}\left(\cos\left(fx+e\right)^{3}+2\cos\left(fx+e\right)\sin\left(fx+e\right)-2\cos\left(fx+e\right)\right)\sqrt{c}\log\left(-\frac{c\cos\left(fx+e\right)^{2}+2\sqrt{2}\sqrt{-c\sin\left(fx+e\right)+c}\sqrt{c}\left(\cos\left(fx+e\right)+\sin\right.}{\cos\left(fx+e\right)^{2}+\left(\cos\left(fx+e\right)\right.}\right.}{128\left(ac^{3}f\cos\left(fx+e\right)^{3}+2\,ac^{3}f\cos\left(fx+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/128*(15*sqrt(2)*(cos(f*x + e)^3 + 2*cos(f*x + e)*sin(f*x + e) - 2*cos(f*x
 + e))*sqrt(c)*log(-(c*cos(f*x + e)^2 + 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)
*sqrt(c)*(cos(f*x + e) + sin(f*x + e) + 1) + 3*c*cos(f*x + e) + (c*cos(f*x
+ e) - 2*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*
x + e) - cos(f*x + e) - 2)) - 4*(15*cos(f*x + e)^2 + 20*sin(f*x + e) - 12)*
sqrt(-c*sin(f*x + e) + c))/(a*c^3*f*cos(f*x + e)^3 + 2*a*c^3*f*cos(f*x + e)
*sin(f*x + e) - 2*a*c^3*f*cos(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] sage2
```

3.324 $\int \frac{(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^2}\,dx$

Optimal. Leaf size=176

$$-\frac{1024c^2\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{5a^2f}+\frac{4096c^3\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{15a^2f}+\frac{2\sec^3(e+fx)(c-c\sin(e+fx))}{5a^2cf}$$

```
[Out] (4096*c^3*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(3/2))/(15*a^2*f) - (1024*c^2
*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(5/2))/(5*a^2*f) + (128*c*Sec[e + f*x]
^3*(c - c*Sin[e + f*x])^(7/2))/(5*a^2*f) + (32*Sec[e + f*x]^3*(c - c*Sin[e
+ f*x])^(9/2))/(15*a^2*f) + (2*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(11/2))/
(5*a^2*c*f)
```

Rubi [A] time = 0.419948, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {2736, 2674, 2673}

$$-\frac{1024c^2\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{5a^2f}+\frac{4096c^3\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{15a^2f}+\frac{2\sec^3(e+fx)(c-c\sin(e+fx))}{5a^2cf}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sin[e + f*x])^(9/2)/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] (4096*c^3*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(3/2))/(15*a^2*f) - (1024*c^2
*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(5/2))/(5*a^2*f) + (128*c*Sec[e + f*x]
^3*(c - c*Sin[e + f*x])^(7/2))/(5*a^2*f) + (32*Sec[e + f*x]^3*(c - c*Sin[e
+ f*x])^(9/2))/(15*a^2*f) + (2*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(11/2))/
(5*a^2*c*f)
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rubi steps

$$\int \frac{(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^2}\,dx = \frac{\int \sec^4(e+fx)(c-c\sin(e+fx))^{13/2}\,dx}{a^2c^2}$$
$$= \frac{2\sec^3(e+fx)(c-c\sin(e+fx))^{11/2}}{5a^2cf} + \frac{16\int \sec^4(e+fx)(c-c\sin(e+fx))^{11/2}\,dx}{5a^2c}$$
$$= \frac{32\sec^3(e+fx)(c-c\sin(e+fx))^{9/2}}{15a^2f} + \frac{2\sec^3(e+fx)(c-c\sin(e+fx))^{11/2}}{5a^2cf} + \frac{64\int \sec^4}{}$$
$$= \frac{128c\sec^3(e+fx)(c-c\sin(e+fx))^{7/2}}{5a^2f} + \frac{32\sec^3(e+fx)(c-c\sin(e+fx))^{9/2}}{15a^2f} + \frac{2\sec^3(}{}$$
$$= -\frac{1024c^2\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{5a^2f} + \frac{128c\sec^3(e+fx)(c-c\sin(e+fx))^{7/2}}{5a^2f} + \frac{3}{}$$
$$= \frac{4096c^3\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{15a^2f} - \frac{1024c^2\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{5a^2f} + \frac{1}{}$$

Mathematica [A] time = 3.07125, size = 124, normalized size = 0.7

$$\frac{c^4\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)(8568\sin(e+fx)+56\sin(3(e+fx))-1044\cos(2(e+fx))}{60a^2f(\sin(e+fx)+1)^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sin[e + f*x])^(9/2)/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] (c^4*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]*(6825 -
 1044*Cos[2*(e + f*x)] + 3*Cos[4*(e + f*x)] + 8568*Sin[e + f*x] + 56*Sin[3*
(e + f*x)]))/(60*a^2*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f
*x])^2)
```

Maple [A] time = 0.638, size = 91, normalized size = 0.5

$$-\frac{2c^5\left(-1+\sin\left(fx+e\right)\right)\left(3\left(\sin\left(fx+e\right)\right)^4-28\left(\sin\left(fx+e\right)\right)^3+258\left(\sin\left(fx+e\right)\right)^2+1092\sin\left(fx+e\right)+723\right)}{15a^2\left(1+\sin\left(fx+e\right)\right)\cos\left(fx+e\right)f}\frac{}{\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^2,x)
```

```
[Out] -2/15*c^5/a^2*(-1+sin(f*x+e))/(1+sin(f*x+e))*(3*sin(f*x+e)^4-28*sin(f*x+e)^
3+258*sin(f*x+e)^2+1092*sin(f*x+e)+723)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Maxima [B] time = 1.73688, size = 513, normalized size = 2.91

$$-\frac{2\left(723c^{\frac{9}{2}}+\frac{2184c^{\frac{9}{2}}\sin(fx+e)}{\cos(fx+e)+1}+\frac{5370c^{\frac{9}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{10696c^{\frac{9}{2}}\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{15021c^{\frac{9}{2}}\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{21168c^{\frac{9}{2}}\sin(fx+e)^5}{(\cos(fx+e)+1)^5}+\frac{20748c^{\frac{9}{2}}\sin(}{(\cos(fx+e)}\right.}{15\left(a^2+\frac{3a^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{3a^2\sin(fx+e)^2}{(\cos(fx+e)+1)}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] -2/15*(723*c^(9/2) + 2184*c^(9/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 5370*c^
(9/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10696*c^(9/2)*sin(f*x + e)^3/(c
os(f*x + e) + 1)^3 + 15021*c^(9/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 21
168*c^(9/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 20748*c^(9/2)*sin(f*x + e
)^6/(cos(f*x + e) + 1)^6 + 21168*c^(9/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^
7 + 15021*c^(9/2)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 10696*c^(9/2)*sin(f
*x + e)^9/(cos(f*x + e) + 1)^9 + 5370*c^(9/2)*sin(f*x + e)^10/(cos(f*x + e)
 + 1)^10 + 2184*c^(9/2)*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 + 723*c^(9/2)
*sin(f*x + e)^12/(cos(f*x + e) + 1)^12)/((a^2 + 3*a^2*sin(f*x + e)/(cos(f*x
 + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3
/(cos(f*x + e) + 1)^3)*f*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(9/2))
```

Fricas [A] time = 1.06822, size = 261, normalized size = 1.48

$$\frac{2\left(3c^4\cos\left(fx+e\right)^4-264c^4\cos\left(fx+e\right)^2+984c^4+28\left(c^4\cos\left(fx+e\right)^2+38c^4\right)\sin\left(fx+e\right)\right)\sqrt{-c\sin\left(fx+e\right)+c}}{15\left(a^2f\cos\left(fx+e\right)\sin\left(fx+e\right)+a^2f\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 2/15*(3*c^4*cos(f*x + e)^4 - 264*c^4*cos(f*x + e)^2 + 984*c^4 + 28*(c^4*cos
(f*x + e)^2 + 38*c^4)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c)/(a^2*f*cos(f*
x + e)*sin(f*x + e) + a^2*f*cos(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))**(9/2)/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 2.09839, size = 887, normalized size = 5.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] -1/60*(4*(511*sqrt(2)*a^12*c - 730*a^12*c - 920*sqrt(2)*c^10 + 1200*c^10)*s
gn(tan(1/2*f*x + 1/2*e) - 1)/(5*sqrt(2)*a^2*c^(11/2) - 7*a^2*c^(11/2)) + (3
23*a^10*sgn(tan(1/2*f*x + 1/2*e) - 1)/c^2 + (255*a^10*sgn(tan(1/2*f*x + 1/2
```

```
*e) - 1)/c^2 + (590*a^10*sgn(tan(1/2*f*x + 1/2*e) - 1)/c^2 + (590*a^10*sgn(
tan(1/2*f*x + 1/2*e) - 1)/c^2 + 17*(19*a^10*sgn(tan(1/2*f*x + 1/2*e) - 1)*t
an(1/2*f*x + 1/2*e)/c^2 + 15*a^10*sgn(tan(1/2*f*x + 1/2*e) - 1)/c^2)*tan(1/
2*f*x + 1/2*e))*tan(1/2*f*x + 1/2*e))*tan(1/2*f*x + 1/2*e))*tan(1/2*f*x + 1
/2*e))/(c*tan(1/2*f*x + 1/2*e)^2 + c)^(5/2) + 1280*(3*(sqrt(c)*tan(1/2*f*x
+ 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^5*c^5*sgn(tan(1/2*f*x + 1/2*
e) - 1) + 15*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2
+ c))^4*c^(11/2)*sgn(tan(1/2*f*x + 1/2*e) - 1) - 10*(sqrt(c)*tan(1/2*f*x +
1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^3*c^6*sgn(tan(1/2*f*x + 1/2*e)
 - 1) - 30*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 +
c))^2*c^(13/2)*sgn(tan(1/2*f*x + 1/2*e) - 1) + 27*(sqrt(c)*tan(1/2*f*x + 1/
2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))*c^7*sgn(tan(1/2*f*x + 1/2*e) - 1
) - 5*c^(15/2)*sgn(tan(1/2*f*x + 1/2*e) - 1))/(((sqrt(c)*tan(1/2*f*x + 1/2*
e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^2 + 2*(sqrt(c)*tan(1/2*f*x + 1/2*e
) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))*sqrt(c) - c)^3*a^2))/f
```

3.325 $\int \frac{(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^2}\,dx$

Optimal. Leaf size=136

$$\frac{256c^2\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{3a^2f}+\frac{2\sec^3(e+fx)(c-c\sin(e+fx))^{9/2}}{3a^2cf}+\frac{8\sec^3(e+fx)(c-c\sin(e+fx))^{7/2}}{a^2f}-64$$

```
[Out] (256*c^2*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(3/2))/(3*a^2*f) - (64*c*Sec[e
 + f*x]^3*(c - c*Sin[e + f*x])^(5/2))/(a^2*f) + (8*Sec[e + f*x]^3*(c - c*Si
n[e + f*x])^(7/2))/(a^2*f) + (2*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(9/2))/
(3*a^2*c*f)
```

Rubi [A] time = 0.332457, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {2736, 2674, 2673}

$$\frac{256c^2\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{3a^2f}+\frac{2\sec^3(e+fx)(c-c\sin(e+fx))^{9/2}}{3a^2cf}+\frac{8\sec^3(e+fx)(c-c\sin(e+fx))^{7/2}}{a^2f}-64$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sin[e + f*x])^(7/2)/(a + a*Sin[e + f*x])^2,x]
```



```
[Out] (256*c^2*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(3/2))/(3*a^2*f) - (64*c*Sec[e
 + f*x]^3*(c - c*Sin[e + f*x])^(5/2))/(a^2*f) + (8*Sec[e + f*x]^3*(c - c*Si
n[e + f*x])^(7/2))/(a^2*f) + (2*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(9/2))/
(3*a^2*c*f)
```


Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rubi steps

$$\int \frac{(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^2}\,dx = \frac{\int \sec^4(e+fx)(c-c\sin(e+fx))^{11/2}\,dx}{a^2c^2}$$
$$= \frac{2\sec^3(e+fx)(c-c\sin(e+fx))^{9/2}}{3a^2cf} + \frac{4\int \sec^4(e+fx)(c-c\sin(e+fx))^{9/2}\,dx}{a^2c}$$
$$= \frac{8\sec^3(e+fx)(c-c\sin(e+fx))^{7/2}}{a^2f} + \frac{2\sec^3(e+fx)(c-c\sin(e+fx))^{9/2}}{3a^2cf} + \frac{32\int \sec^4(e$$
$$= -\frac{64c\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{a^2f} + \frac{8\sec^3(e+fx)(c-c\sin(e+fx))^{7/2}}{a^2f} + \frac{2\sec^3(e$$
$$= \frac{256c^2\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{3a^2f} - \frac{64c\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{a^2f} + \frac{8\sec$$

Mathematica [A] time = 1.18221, size = 112, normalized size = 0.82

$$\frac{c^3\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)(273\sin(e+fx)+\sin(3(e+fx))-30\cos(2(e+fx))+210)}{6a^2f(\sin(e+fx)+1)^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sin[e + f*x])^(7/2)/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] (c^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]*(210 -
30*Cos[2*(e + f*x)] + 273*Sin[e + f*x] + Sin[3*(e + f*x)]))/(6*a^2*f*(Cos[(
e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^2)
```

Maple [A] time = 0.581, size = 79, normalized size = 0.6

$$\frac{2c^4(-1+\sin(fx+e))\left((\sin(fx+e))^3-15(\sin(fx+e))^2-69\sin(fx+e)-45\right)}{3a^2(1+\sin(fx+e))\cos(fx+e)f}\frac{1}{\sqrt{c-c\sin(fx+e)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^2,x)
```

```
[Out] 2/3*c^4/a^2*(-1+sin(f*x+e))/(1+sin(f*x+e))*(sin(f*x+e)^3-15*sin(f*x+e)^2-69
*sin(f*x+e)-45)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Maxima [B] time = 1.86055, size = 451, normalized size = 3.32

$$-\frac{2\left(45c^{\frac{7}{2}}+\frac{138c^{\frac{7}{2}}\sin(fx+e)}{\cos(fx+e)+1}+\frac{285c^{\frac{7}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{544c^{\frac{7}{2}}\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{630c^{\frac{7}{2}}\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{812c^{\frac{7}{2}}\sin(fx+e)^5}{(\cos(fx+e)+1)^5}+\frac{630c^{\frac{7}{2}}\sin(fx+e)^6}{(\cos(fx+e)+1)^6}+\frac{54}{(}\right.}{3\left(a^2+\frac{3a^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{3a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)f\left(\frac{\sin(fx}{(\cos(fx+}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] -2/3*(45*c^(7/2) + 138*c^(7/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 285*c^(7/2
)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 544*c^(7/2)*sin(f*x + e)^3/(cos(f*x
 + e) + 1)^3 + 630*c^(7/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 812*c^(7/2
)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 630*c^(7/2)*sin(f*x + e)^6/(cos(f*x
 + e) + 1)^6 + 544*c^(7/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 285*c^(7/2
)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 138*c^(7/2)*sin(f*x + e)^9/(cos(f*x
 + e) + 1)^9 + 45*c^(7/2)*sin(f*x + e)^10/(cos(f*x + e) + 1)^10)/((a^2 + 3*
a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) +
1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3)*f*(sin(f*x + e)^2/(cos(f*x
+ e) + 1)^2 + 1)^(7/2))
```

Fricas [A] time = 1.06585, size = 223, normalized size = 1.64

$$-\frac{2\left(15\,c^3\cos\left(fx+e\right)^2-60\,c^3-\left(c^3\cos\left(fx+e\right)^2+68\,c^3\right)\sin\left(fx+e\right)\right)\sqrt{-c\sin\left(fx+e\right)+c}}{3\left(a^2f\cos\left(fx+e\right)\sin\left(fx+e\right)+a^2f\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -2/3*(15*c^3*cos(f*x + e)^2 - 60*c^3 - (c^3*cos(f*x + e)^2 + 68*c^3)*sin(f*
x + e))*sqrt(-c*sin(f*x + e) + c)/(a^2*f*cos(f*x + e)*sin(f*x + e) + a^2*f*
cos(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))**(7/2)/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 2.67561, size = 802, normalized size = 5.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] -1/3*(8*(14*sqrt(2)*a^8*sqrt(c) - 20*a^8*sqrt(c) - 7*sqrt(2)*c^(13/2) + 9*c
^(13/2))*sgn(tan(1/2*f*x + 1/2*e) - 1)/(5*sqrt(2)*a^2*c^3 - 7*a^2*c^3) + (1
7*a^6*sgn(tan(1/2*f*x + 1/2*e) - 1)/c + (15*a^6*sgn(tan(1/2*f*x + 1/2*e) -
1)/c + (17*a^6*sgn(tan(1/2*f*x + 1/2*e) - 1)*tan(1/2*f*x + 1/2*e)/c + 15*a^
6*sgn(tan(1/2*f*x + 1/2*e) - 1)/c)*tan(1/2*f*x + 1/2*e))*tan(1/2*f*x + 1/2*
e))/(c*tan(1/2*f*x + 1/2*e)^2 + c)^(3/2) + 16*(3*(sqrt(c)*tan(1/2*f*x + 1/2
```

```
*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^5*c^4*sgn(tan(1/2*f*x + 1/2*e) -
1) + 21*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))
^4*c^(9/2)*sgn(tan(1/2*f*x + 1/2*e) - 1) - 14*(sqrt(c)*tan(1/2*f*x + 1/2*e)
 - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^3*c^5*sgn(tan(1/2*f*x + 1/2*e) - 1)
- 42*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^2*
c^(11/2)*sgn(tan(1/2*f*x + 1/2*e) - 1) + 39*(sqrt(c)*tan(1/2*f*x + 1/2*e) -
 sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))*c^6*sgn(tan(1/2*f*x + 1/2*e) - 1) - 7*
c^(13/2)*sgn(tan(1/2*f*x + 1/2*e) - 1))/(((sqrt(c)*tan(1/2*f*x + 1/2*e) - s
qrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^2 + 2*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sq
rt(c*tan(1/2*f*x + 1/2*e)^2 + c))*sqrt(c) - c)^3*a^2))/f
```

3.326 $\int \frac{(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^2}\,dx$

Optimal. Leaf size=100

$$\frac{2\sec^3(e+fx)(c-c\sin(e+fx))^{7/2}}{a^2cf}-\frac{16\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{a^2f}+\frac{64c\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{3a^2f}$$

```
[Out] (64*c*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(3/2))/(3*a^2*f) - (16*Sec[e + f*
x]^3*(c - c*Sin[e + f*x])^(5/2))/(a^2*f) + (2*Sec[e + f*x]^3*(c - c*Sin[e +
 f*x])^(7/2))/(a^2*c*f)
```

Rubi [A] time = 0.26357, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {2736, 2674, 2673}

$$\frac{2\sec^3(e+fx)(c-c\sin(e+fx))^{7/2}}{a^2cf}-\frac{16\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{a^2f}+\frac{64c\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{3a^2f}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sin[e + f*x])^(5/2)/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] (64*c*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(3/2))/(3*a^2*f) - (16*Sec[e + f*
x]^3*(c - c*Sin[e + f*x])^(5/2))/(a^2*f) + (2*Sec[e + f*x]^3*(c - c*Sin[e +
 f*x])^(7/2))/(a^2*c*f)
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rubi steps

$$\int \frac{(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^2}\,dx = \frac{\int \sec^4(e+fx)(c-c\sin(e+fx))^{9/2}\,dx}{a^2c^2}$$
$$= \frac{2\sec^3(e+fx)(c-c\sin(e+fx))^{7/2}}{a^2cf} + \frac{8\int \sec^4(e+fx)(c-c\sin(e+fx))^{7/2}\,dx}{a^2c}$$
$$= -\frac{16\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{a^2f} + \frac{2\sec^3(e+fx)(c-c\sin(e+fx))^{7/2}}{a^2cf} - \frac{32\int \sec^4}{}$$
$$= \frac{64c\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{3a^2f} - \frac{16\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{a^2f} + \frac{2\sec^3(e}{}$$

Mathematica [A] time = 0.780483, size = 104, normalized size = 1.04

$$\frac{c^2\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)(36\sin(e+fx)-3\cos(2(e+fx))+25)}{3a^2f(\sin(e+fx)+1)^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sin[e + f*x])^(5/2)/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] (c^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(25 - 3*Cos[2*(e + f*x)] + 36*Si
n[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(3*a^2*f*(Cos[(e + f*x)/2] - Sin[(e +
 f*x)/2])*(1 + Sin[e + f*x])^2)
```

Maple [A] time = 0.601, size = 71, normalized size = 0.7

$$-\frac{2\,c^3\left(-1+\sin\left(fx+e\right)\right)\left(3\left(\sin\left(fx+e\right)\right)^2+18\sin\left(fx+e\right)+11\right)}{3\,a^2\left(1+\sin\left(fx+e\right)\right)\cos\left(fx+e\right)f}\frac{1}{\sqrt{c-c\sin\left(fx+e\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^2,x)
```

```
[Out] -2/3*c^3/a^2*(-1+sin(f*x+e))/(1+sin(f*x+e))*(3*sin(f*x+e)^2+18*sin(f*x+e)+1
1)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Maxima [B] time = 1.77455, size = 389, normalized size = 3.89

$$-\frac{2\left(11\,c^{\frac{5}{2}}+\frac{36\,c^{\frac{5}{2}}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{56\,c^{\frac{5}{2}}\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}+\frac{108\,c^{\frac{5}{2}}\sin\left(fx+e\right)^3}{\left(\cos\left(fx+e\right)+1\right)^3}+\frac{90\,c^{\frac{5}{2}}\sin\left(fx+e\right)^4}{\left(\cos\left(fx+e\right)+1\right)^4}+\frac{108\,c^{\frac{5}{2}}\sin\left(fx+e\right)^5}{\left(\cos\left(fx+e\right)+1\right)^5}+\frac{56\,c^{\frac{5}{2}}\sin\left(fx+e\right)^6}{\left(\cos\left(fx+e\right)+1\right)^6}+\frac{36\,c^{\frac{5}{2}}\,\mathrm{s}}{(\cos(f}\right.}{3\left(a^2+\frac{3\,a^2\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{3\,a^2\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}+\frac{a^2\sin\left(fx+e\right)^3}{\left(\cos\left(fx+e\right)+1\right)^3}\right)f\left(\frac{\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}+1\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] -2/3*(11*c^(5/2) + 36*c^(5/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 56*c^(5/2)*
sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 108*c^(5/2)*sin(f*x + e)^3/(cos(f*x +
 e) + 1)^3 + 90*c^(5/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 108*c^(5/2)*s
in(f*x + e)^5/(cos(f*x + e) + 1)^5 + 56*c^(5/2)*sin(f*x + e)^6/(cos(f*x + e
) + 1)^6 + 36*c^(5/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 11*c^(5/2)*sin(
f*x + e)^8/(cos(f*x + e) + 1)^8)/((a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) +
 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f
*x + e) + 1)^3)*f*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(5/2))
```

Fricas [A] time = 1.07377, size = 190, normalized size = 1.9

$$-\frac{2\left(3\,c^{2}\cos\left(fx+e\right)^{2}-18\,c^{2}\sin\left(fx+e\right)-14\,c^{2}\right)\sqrt{-c\sin\left(fx+e\right)+c}}{3\left(a^{2}f\cos\left(fx+e\right)\sin\left(fx+e\right)+a^{2}f\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -2/3*(3*c^2*cos(f*x + e)^2 - 18*c^2*sin(f*x + e) - 14*c^2)*sqrt(-c*sin(f*x
+ e) + c)/(a^2*f*cos(f*x + e)*sin(f*x + e) + a^2*f*cos(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))**(5/2)/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 2.49808, size = 603, normalized size = 6.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] -2/3*(8*(2*sqrt(2)*c^(5/2) - 3*c^(5/2))*sgn(tan(1/2*f*x + 1/2*e) - 1)/(5*sq
rt(2)*a^2 - 7*a^2) + 3*(c^3*sgn(tan(1/2*f*x + 1/2*e) - 1)*tan(1/2*f*x + 1/2
*e)/a^2 + c^3*sgn(tan(1/2*f*x + 1/2*e) - 1)/a^2)/sqrt(c*tan(1/2*f*x + 1/2*e
)^2 + c) + 16*(3*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e
)^2 + c))^4*c^(7/2)*sgn(tan(1/2*f*x + 1/2*e) - 1) - 2*(sqrt(c)*tan(1/2*f*x
+ 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^3*c^4*sgn(tan(1/2*f*x + 1/2*
e) - 1) - 6*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 +
 c))^2*c^(9/2)*sgn(tan(1/2*f*x + 1/2*e) - 1) + 6*(sqrt(c)*tan(1/2*f*x + 1/2
*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))*c^5*sgn(tan(1/2*f*x + 1/2*e) - 1)
 - c^(11/2)*sgn(tan(1/2*f*x + 1/2*e) - 1))/(((sqrt(c)*tan(1/2*f*x + 1/2*e)
- sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^2 + 2*(sqrt(c)*tan(1/2*f*x + 1/2*e) -
 sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))*sqrt(c) - c)^3*a^2))/f
```

3.327 $\int \frac{(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^2}\,dx$

Optimal. Leaf size=68

$$\frac{8\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{3a^2f}-\frac{2\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{a^2cf}$$

```
[Out] (8*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(3/2))/(3*a^2*f) - (2*Sec[e + f*x]^3
*(c - c*Sin[e + f*x])^(5/2))/(a^2*c*f)
```

Rubi [A] time = 0.195654, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {2736, 2674, 2673}

$$\frac{8\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{3a^2f}-\frac{2\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{a^2cf}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sin[e + f*x])^(3/2)/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] (8*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(3/2))/(3*a^2*f) - (2*Sec[e + f*x]^3
*(c - c*Sin[e + f*x])^(5/2))/(a^2*c*f)
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rubi steps

$$\int \frac{(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^2}\,dx = \frac{\int \sec^4(e+fx)(c-c\sin(e+fx))^{7/2}\,dx}{a^2c^2}$$
$$= -\frac{2\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{a^2cf} - \frac{4\int \sec^4(e+fx)(c-c\sin(e+fx))^{5/2}\,dx}{a^2c}$$
$$= \frac{8\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{3a^2f} - \frac{2\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{a^2cf}$$

Mathematica [A] time = 0.351781, size = 92, normalized size = 1.35

$$\frac{2c(3\sin(e+fx)+1)\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}{3a^2f(\sin(e+fx)+1)^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sin[e + f*x])^(3/2)/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] (2*c*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(1 + 3*Sin[e + f*x])*Sqrt[c - c*
Sin[e + f*x]])/(3*a^2*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e +
f*x])^2)
```

Maple [A] time = 0.582, size = 61, normalized size = 0.9

$$-\frac{2c^2(-1+\sin(fx+e))(3\sin(fx+e)+1)}{3a^2(1+\sin(fx+e))\cos(fx+e)f}\frac{1}{\sqrt{c-c\sin(fx+e)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^2,x)
```

```
[Out] -2/3*c^2/a^2*(-1+sin(f*x+e))/(1+sin(f*x+e))*(3*sin(f*x+e)+1)/cos(f*x+e)/(c-
c*sin(f*x+e))^(1/2)/f
```

Maxima [B] time = 1.87996, size = 323, normalized size = 4.75

$$-\frac{2\left(c^{\frac{3}{2}}+\frac{6c^{\frac{3}{2}}\sin(fx+e)}{\cos(fx+e)+1}+\frac{3c^{\frac{3}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{12c^{\frac{3}{2}}\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3c^{\frac{3}{2}}\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{6c^{\frac{3}{2}}\sin(fx+e)^5}{(\cos(fx+e)+1)^5}+\frac{c^{\frac{3}{2}}\sin(fx+e)^6}{(\cos(fx+e)+1)^6}\right)}{3\left(a^2+\frac{3a^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{3a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)f\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] -2/3*(c^(3/2) + 6*c^(3/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 3*c^(3/2)*sin(f
*x + e)^2/(cos(f*x + e) + 1)^2 + 12*c^(3/2)*sin(f*x + e)^3/(cos(f*x + e) +
1)^3 + 3*c^(3/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 6*c^(3/2)*sin(f*x +
```

```
e)^5/(cos(f*x + e) + 1)^5 + c^(3/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6)/((
a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x
 + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3)*f*(sin(f*x + e)^2/(
cos(f*x + e) + 1)^2 + 1)^(3/2))
```

Fricas [A] time = 1.03494, size = 147, normalized size = 2.16

$$\frac{2\left(3 c \sin\left(fx+e\right)+c\right)\sqrt{-c \sin\left(fx+e\right)+c}}{3\left(a^{2} f \cos\left(fx+e\right)\sin\left(fx+e\right)+a^{2} f \cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 2/3*(3*c*sin(f*x + e) + c)*sqrt(-c*sin(f*x + e) + c)/(a^2*f*cos(f*x + e)*si
n(f*x + e) + a^2*f*cos(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))**(3/2)/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.79874, size = 579, normalized size = 8.51

$$-\frac{2\left(\frac{\left(2\sqrt{2}c^{\frac{3}{2}}-3c^{\frac{3}{2}}\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{5\sqrt{2}a^{2}-7a^{2}}-\frac{2\left(3\left(\sqrt{c}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+c}\right)^{5}c^{2}\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)-3\left(\sqrt{c}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{c\tan\cdots}\right.\right.}{\cdots}\right)}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] -2/3*((2*sqrt(2)*c^(3/2) - 3*c^(3/2))*sgn(tan(1/2*f*x + 1/2*e) - 1)/(5*sqrt
(2)*a^2 - 7*a^2) - 2*(3*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x
+ 1/2*e)^2 + c))^5*c^2*sgn(tan(1/2*f*x + 1/2*e) - 1) - 3*(sqrt(c)*tan(1/2*f
*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^4*c^(5/2)*sgn(tan(1/2*f*x
 + 1/2*e) - 1) + 2*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2
*e)^2 + c))^3*c^3*sgn(tan(1/2*f*x + 1/2*e) - 1) + 6*(sqrt(c)*tan(1/2*f*x +
1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^2*c^(7/2)*sgn(tan(1/2*f*x + 1/
2*e) - 1) - 9*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2
 + c))*c^4*sgn(tan(1/2*f*x + 1/2*e) - 1) + c^(9/2)*sgn(tan(1/2*f*x + 1/2*e)
 - 1))/(((sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c)
)^2 + 2*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))
*sqrt(c) - c)^3*a^2))/f
```

3.328 $\int \frac{\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^2}\,dx$

Optimal. Leaf size=36

$$-\frac{2\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{3a^2cf}$$

```
[Out] (-2*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(3/2))/(3*a^2*c*f)
```

Rubi [A] time = 0.136258, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2736, 2673}

$$-\frac{2\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{3a^2cf}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c*Sin[e + f*x]]/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] (-2*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(3/2))/(3*a^2*c*f)
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^2}\,dx &= \frac{\int \sec^4(e+fx)(c-c\sin(e+fx))^{5/2}\,dx}{a^2c^2}\\ &= -\frac{2\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{3a^2cf}\end{aligned}$$

Mathematica [B] time = 0.118548, size = 73, normalized size = 2.03

$$-\frac{2\sqrt{c-c\sin(e+fx)}}{3a^2f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c*Sin[e + f*x]]/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] (-2*Sqrt[c - c*Sin[e + f*x]])/(3*a^2*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]
)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3)
```

Maple [A] time = 0.404, size = 49, normalized size = 1.4

$$\frac{2c\left(-1+\sin\left(fx+e\right)\right)}{3a^{2}\left(1+\sin\left(fx+e\right)\right)\cos\left(fx+e\right)f}\frac{1}{\sqrt{c-c\sin\left(fx+e\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^2,x)
```

```
[Out] 2/3*c/a^2*(-1+sin(f*x+e))/(1+sin(f*x+e))/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/
f
```

Maxima [B] time = 1.56676, size = 201, normalized size = 5.58

$$\frac{2\left(\sqrt{c}+\frac{2\sqrt{c}\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{\sqrt{c}\sin\left(fx+e\right)^{4}}{\left(\cos\left(fx+e\right)+1\right)^{4}}\right)}{3\left(a^{2}+\frac{3a^{2}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{3a^{2}\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{a^{2}\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}\right)f\sqrt{\frac{\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 2/3*(sqrt(c) + 2*sqrt(c)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + sqrt(c)*sin(
f*x + e)^4/(cos(f*x + e) + 1)^4)/((a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) +
 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f
*x + e) + 1)^3)*f*sqrt(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1))
```

Fricas [A] time = 1.05771, size = 117, normalized size = 3.25

$$-\frac{2\sqrt{-c\sin\left(fx+e\right)+c}}{3\left(a^{2}f\cos\left(fx+e\right)\sin\left(fx+e\right)+a^{2}f\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -2/3*sqrt(-c*sin(f*x + e) + c)/(a^2*f*cos(f*x + e)*sin(f*x + e) + a^2*f*cos
(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sqrt{-c\sin\left(e+fx\right)+c}}{\sin^{2}\left(e+fx\right)+2\sin\left(e+fx\right)+1}\,dx}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))**(1/2)/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Integral(sqrt(-c*sin(e + f*x) + c)/(sin(e + f*x)**2 + 2*sin(e + f*x) + 1),
x)/a**2
```

Giac [B] time = 1.51499, size = 578, normalized size = 16.06

$$-\frac{\frac{\left(13\sqrt{2}\sqrt{c}-18\sqrt{c}\right)\operatorname{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{5\sqrt{2}a^{2}-7a^{2}}-\frac{8\left(3\left(\sqrt{c}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+c}\right)^{5}c\operatorname{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)+3\left(\sqrt{c}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{c\tan\left(\frac{1}{2}fx\right.}\right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] -1/6*((13*sqrt(2)*sqrt(c) - 18*sqrt(c))*sgn(tan(1/2*f*x + 1/2*e) - 1)/(5*sq
rt(2)*a^2 - 7*a^2) - 8*(3*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*
x + 1/2*e)^2 + c))^5*c*sgn(tan(1/2*f*x + 1/2*e) - 1) + 3*(sqrt(c)*tan(1/2*f
*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^4*c^(3/2)*sgn(tan(1/2*f*x
 + 1/2*e) - 1) - 2*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2
*e)^2 + c))^3*c^2*sgn(tan(1/2*f*x + 1/2*e) - 1) - 6*(sqrt(c)*tan(1/2*f*x +
1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^2*c^(5/2)*sgn(tan(1/2*f*x + 1/
2*e) - 1) + 3*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2
 + c))*c^3*sgn(tan(1/2*f*x + 1/2*e) - 1) - c^(7/2)*sgn(tan(1/2*f*x + 1/2*e)
 - 1))/(((sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c)
)^2 + 2*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))
*sqrt(c) - c)^3*a^2))/f
```

3.329 $\int \frac{1}{(a+a\sin(e+fx))^2\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=124

$$-\frac{\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{3a^2c^2f}-\frac{\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{2a^2cf}+\frac{\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{2\sqrt{2}a^2\sqrt{c}f}$$

```
[Out] ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])]/(2*Sqrt[
2]*a^2*Sqrt[c]*f) - (Sec[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(2*a^2*c*f) - (
Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(3/2))/(3*a^2*c^2*f)
```

Rubi [A] time = 0.224182, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2736, 2675, 2649, 206}

$$-\frac{\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{3a^2c^2f}-\frac{\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{2a^2cf}+\frac{\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{2\sqrt{2}a^2\sqrt{c}f}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^2*Sqrt[c - c*Sin[e + f*x]]),x]
```

```
[Out] ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])]/(2*Sqrt[
2]*a^2*Sqrt[c]*f) - (Sec[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(2*a^2*c*f) - (
Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(3/2))/(3*a^2*c^2*f)
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2675

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^m)/(a*f*g*(p + 1)), x] + Dist[(a*(m + p + 1))/(g^2*(p + 1)), Int[(g*Cos
[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[p, -2*m] && IntegersQ[m
+ 1/2, 2*p]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^2\sqrt{c-c\sin(e+fx)}}\,dx = \frac{\int \sec^4(e+fx)(c-c\sin(e+fx))^{3/2}\,dx}{a^2c^2}$$
$$= -\frac{\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{3a^2c^2f} + \frac{\int \sec^2(e+fx)\sqrt{c-c\sin(e+fx)}\,dx}{2a^2c}$$
$$= -\frac{\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{2a^2cf} - \frac{\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{3a^2c^2f} + \frac{\int}{}$$
$$= -\frac{\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{2a^2cf} - \frac{\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{3a^2c^2f} - \frac{\mathrm{Su}}{}$$
$$= \frac{\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{2\sqrt{2}a^2\sqrt{c}f} - \frac{\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{2a^2cf} - \frac{\sec^3(e+fx)}{}$$

Mathematica [C] time = 0.481597, size = 109, normalized size = 0.88

$$\frac{\cos(e+fx)\left(-3\sin(e+fx)+(-3-3i)\sqrt[4]{-1}\tan^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt[4]{-1}\left(\tan\left(\frac{1}{4}(e+fx)\right)+1\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx\right.\right.\right.}{6a^2f(\sin(e+fx)+1)^2\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^2*Sqrt[c - c*Sin[e + f*x]]),x]
```

```
[Out] (Cos[e + f*x]*(-5 - (3 + 3*I)*(-1)^(1/4)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 +
 Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 - 3*Sin[e + f*x
]))/(6*a^2*f*(1 + Sin[e + f*x])^2*Sqrt[c - c*Sin[e + f*x]])
```

Maple [A] time = 0.681, size = 109, normalized size = 0.9

$$-\frac{-1+\sin(fx+e)}{12a^2(1+\sin(fx+e))\cos(fx+e)f}\left(-6c^{7/2}\sin(fx+e)+3\sqrt{2}\mathrm{Artanh}\left(1/2\frac{\sqrt{c(1+\sin(fx+e))}\sqrt{2}}{\sqrt{c}}\right)c^2\left(c\left(1+\sin\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] -1/12*(-1+sin(f*x+e))*(-6*c^(7/2)*sin(f*x+e)+3*2^(1/2)*arctanh(1/2*(c*(1+si
n(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*c^2*(c*(1+sin(f*x+e)))^(3/2)-10*c^(7/2))/
a^2/c^(7/2)/(1+sin(f*x+e))/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\sin(fx+e)+a)^2\sqrt{-c\sin(fx+e)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)^2*sqrt(-c*sin(f*x + e) + c)), x)
```

Fricas [A] time = 1.15062, size = 563, normalized size = 4.54

$$\frac{3\sqrt{2}\left(\cos\left(fx+e\right)\sin\left(fx+e\right)+\cos\left(fx+e\right)\right)\sqrt{c}\log\left(-\frac{c\cos\left(fx+e\right)^2+2\sqrt{2}\sqrt{-c\sin\left(fx+e\right)+c}\sqrt{c}\left(\cos\left(fx+e\right)+\sin\left(fx+e\right)+1\right)+3c\cos\left(fx+\right.}{\cos\left(fx+e\right)^2+\left(\cos\left(fx+e\right)+2\right)\sin\left(fx+e\right)-\cos\left(fx+\right.}\right.}{24\left(a^2cf\cos\left(fx+e\right)\sin\left(fx+e\right)+a^2cf\cos\left(fx\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(1/2),x, algorithm="fricas"
)
```

```
[Out] 1/24*(3*sqrt(2)*(cos(f*x + e)*sin(f*x + e) + cos(f*x + e))*sqrt(c)*log(-(c*
cos(f*x + e)^2 + 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*sqrt(c)*(cos(f*x + e)
+ sin(f*x + e) + 1) + 3*c*cos(f*x + e) + (c*cos(f*x + e) - 2*c)*sin(f*x + e
) + 2*c)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) -
 2)) - 4*sqrt(-c*sin(f*x + e) + c)*(3*sin(f*x + e) + 5))/(a^2*c*f*cos(f*x +
 e)*sin(f*x + e) + a^2*c*f*cos(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{\sqrt{-c\sin\left(e+fx\right)+c}\sin^2\left(e+fx\right)+2\sqrt{-c\sin\left(e+fx\right)+c}\sin\left(e+fx\right)+\sqrt{-c\sin\left(e+fx\right)+c}}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**2/(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)**2 + 2*sqrt(-c*sin(e + f
*x) + c)*sin(e + f*x) + sqrt(-c*sin(e + f*x) + c)), x)/a**2
```

Giac [B] time = 1.70229, size = 664, normalized size = 5.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*((30*sqrt(2)*c*arctan(sqrt(c)/sqrt(-c)) - 42*c*arctan(sqrt(c)/sqrt(-c))
 - 15*sqrt(2)*sqrt(-c)*sqrt(c) + 22*sqrt(-c)*sqrt(c))*sgn(tan(1/2*f*x + 1/2
*e) - 1)/(7*sqrt(2)*a^2*sqrt(-c)*c - 10*a^2*sqrt(-c)*c) + 3*sqrt(2)*arctan(
-1/2*sqrt(2)*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2
+ c) - sqrt(c))/sqrt(-c))/(a^2*sqrt(-c)*sgn(tan(1/2*f*x + 1/2*e) - 1)) + 2*
```

```
(9*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^5 +
15*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^4*sq
rt(c) - 10*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 +
c))^3*c - 30*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2
+ c))^2*c^(3/2) + 21*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1
/2*e)^2 + c))*c^2 - 5*c^(5/2))/(((sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan
(1/2*f*x + 1/2*e)^2 + c))^2 + 2*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(
1/2*f*x + 1/2*e)^2 + c))*sqrt(c) - c)^3*a^2*sgn(tan(1/2*f*x + 1/2*e) - 1)))
/f
```

3.330 $\int \frac{1}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=155

$$-\frac{\sec^3(e+fx)\sqrt{c-c\sin(e+fx)}}{3a^2c^2f}+\frac{5\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{8\sqrt{2}a^2c^{3/2}f}+\frac{5\cos(e+fx)}{8a^2f(c-c\sin(e+fx))^{3/2}}-\frac{5\sec(e+fx)}{6a^2cf\sqrt{c-c\sin(e+fx)}}$$

```
[Out] (5*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])])/(8*S
qrt[2]*a^2*c^(3/2)*f) + (5*Cos[e + f*x])/(8*a^2*f*(c - c*Sin[e + f*x])^(3/2
)) - (5*Sec[e + f*x])/(6*a^2*c*f*Sqrt[c - c*Sin[e + f*x]]) - (Sec[e + f*x]^
3*Sqrt[c - c*Sin[e + f*x]])/(3*a^2*c^2*f)
```

Rubi [A] time = 0.249886, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2736, 2675, 2687, 2650, 2649, 206}

$$-\frac{\sec^3(e+fx)\sqrt{c-c\sin(e+fx)}}{3a^2c^2f}+\frac{5\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{8\sqrt{2}a^2c^{3/2}f}+\frac{5\cos(e+fx)}{8a^2f(c-c\sin(e+fx))^{3/2}}-\frac{5\sec(e+fx)}{6a^2cf\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(3/2)),x]
```

```
[Out] (5*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])])/(8*S
qrt[2]*a^2*c^(3/2)*f) + (5*Cos[e + f*x])/(8*a^2*f*(c - c*Sin[e + f*x])^(3/2
)) - (5*Sec[e + f*x])/(6*a^2*c*f*Sqrt[c - c*Sin[e + f*x]]) - (Sec[e + f*x]^
3*Sqrt[c - c*Sin[e + f*x]])/(3*a^2*c^2*f)
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2675

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^m)/(a*f*g*(p + 1)), x] + Dist[(a*(m + p + 1))/(g^2*(p + 1)), Int[(g*Cos
[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[p, -2*m] && IntegersQ[m
+ 1/2, 2*p]
```

Rule 2687

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p + 1)*Sq
rt[a + b*Sin[e + f*x]]), x] + Dist[(a*(2*p + 1))/(2*g^2*(p + 1)), Int[(g*Co
s[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^{3/2}}\,dx = \frac{\int \sec^4(e+fx)\sqrt{c-c\sin(e+fx)}\,dx}{a^2c^2}$$

$$= -\frac{\sec^3(e+fx)\sqrt{c-c\sin(e+fx)}}{3a^2c^2f} + \frac{5\int \frac{\sec^2(e+fx)}{\sqrt{c-c\sin(e+fx)}}\,dx}{6a^2c}$$

$$= -\frac{5\sec(e+fx)}{6a^2cf\sqrt{c-c\sin(e+fx)}} - \frac{\sec^3(e+fx)\sqrt{c-c\sin(e+fx)}}{3a^2c^2f} + \frac{5\int \frac{1}{(c-c\,\text{s}}$$

$$= \frac{5\cos(e+fx)}{8a^2f(c-c\sin(e+fx))^{3/2}} - \frac{5\sec(e+fx)}{6a^2cf\sqrt{c-c\sin(e+fx)}} - \frac{\sec^3(e+fx)\sqrt{c}}{3a^2}$$

$$= \frac{5\cos(e+fx)}{8a^2f(c-c\sin(e+fx))^{3/2}} - \frac{5\sec(e+fx)}{6a^2cf\sqrt{c-c\sin(e+fx)}} - \frac{\sec^3(e+fx)\sqrt{c}}{3a^2}$$

$$= \frac{5\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{8\sqrt{2}a^2c^{3/2}f} + \frac{5\cos(e+fx)}{8a^2f(c-c\sin(e+fx))^{3/2}} - \frac{5\sec(e}{6a^2cf\sqrt{c-c\,\text{s}}}$$

Mathematica [C] time = 0.785651, size = 164, normalized size = 1.06

$$\frac{\left(\frac{1}{96}+\frac{i}{96}\right)\cos(e+fx)\left((1-i)(-20\sin(e+fx)+15\cos(2(e+fx))+11)+60\sqrt[4]{-1}\tan^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt[4]{-1}\left(\tan\left(\frac{1}{4}(e+fx)\right)\right.\right.\right.}{a^2cf(\sin(e+fx)-1)(\sin(e+fx)+1)^2\sqrt{c-c\sin}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(3/2)),x]
```

```
[Out] ((1/96 + I/96)*Cos[e + f*x]*(60*(-1)^(1/4)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1
 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*(Cos[(e + f*x
)/2] + Sin[(e + f*x)/2])^3 + (1 - I)*(11 + 15*Cos[2*(e + f*x)] - 20*Sin[e +
 f*x])))/(a^2*c*f*(-1 + Sin[e + f*x])*(1 + Sin[e + f*x])^2*Sqrt[c - c*Sin[e
 + f*x]])
```

Maple [A] time = 0.569, size = 157, normalized size = 1.

$$-\frac{1}{48 a^2 \left(1+\sin \left(f x+e\right)\right) \cos \left(f x+e\right) f}\left(15 \left(c \left(1+\sin \left(f x+e\right)\right)\right)^{3/2} \sqrt{2}\mathrm{Artanh}\left(1/2\,\frac{\sqrt{c \left(1+\sin \left(f x+e\right)\right)}\sqrt{2}}{\sqrt{c}}\right)\sin \left(f\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] -1/48/c^(7/2)/a^2*(15*(c*(1+sin(f*x+e)))^(3/2)*2^(1/2)*arctanh(1/2*(c*(1+si
n(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)*c-15*(c*(1+sin(f*x+e)))^(3/2)*
2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*c-20*c^(5/2)*
sin(f*x+e)-30*c^(5/2)*sin(f*x+e)^2+26*c^(5/2))/(1+sin(f*x+e))/cos(f*x+e)/(c
-c*sin(f*x+e))^(1/2)/f
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(3/2),x, algorithm="maxima"
)
```

```
[Out] Timed out
```

Fricas [A] time = 1.1494, size = 512, normalized size = 3.3

$$\frac{15 \sqrt{2}\sqrt{c}\cos \left(f x+e\right)^3 \log \left(-\frac{c \cos \left(f x+e\right)^2+2 \sqrt{2}\sqrt{-c \sin \left(f x+e\right)+c}\sqrt{c}\left(\cos \left(f x+e\right)+\sin \left(f x+e\right)+1\right)+3 c \cos \left(f x+e\right)+\left(c \cos \left(f x+e\right)-2 c\right)\sin \left(f x+e\right)+2\,c}{\cos \left(f x+e\right)^2+\left(\cos \left(f x+e\right)+2\right)\sin \left(f x+e\right)-\cos \left(f x+e\right)-2}\right.}{96\, a^2 c^2 f \cos \left(f x+e\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(3/2),x, algorithm="fricas"
)
```

```
[Out] 1/96*(15*sqrt(2)*sqrt(c)*cos(f*x + e)^3*log(-(c*cos(f*x + e)^2 + 2*sqrt(2)*
sqrt(-c*sin(f*x + e) + c)*sqrt(c)*(cos(f*x + e) + sin(f*x + e) + 1) + 3*c*c
os(f*x + e) + (c*cos(f*x + e) - 2*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2 +
(cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) - 4*(15*cos(f*x + e)^2
 - 10*sin(f*x + e) - 2)*sqrt(-c*sin(f*x + e) + c))/(a^2*c^2*f*cos(f*x + e)^
3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**2/(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] sage2
```

3.331 $\int \frac{1}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=192

$$-\frac{\sec^3(e+fx)}{3a^2c^2f\sqrt{c-c\sin(e+fx)}}-\frac{35\sec(e+fx)}{48a^2c^2f\sqrt{c-c\sin(e+fx)}}+\frac{35\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{64\sqrt{2}a^2c^{5/2}f}+\frac{35\cos(e+fx)}{64a^2cf(c-c\sin(e+fx))^{3/2}}$$

```
[Out] (35*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])])/(64
*Sqrt[2]*a^2*c^(5/2)*f) + (35*Cos[e + f*x])/(64*a^2*c*f*(c - c*Sin[e + f*x]
)^(3/2)) + (7*Sec[e + f*x])/(24*a^2*c*f*(c - c*Sin[e + f*x])^(3/2)) - (35*S
ec[e + f*x])/(48*a^2*c^2*f*Sqrt[c - c*Sin[e + f*x]]) - Sec[e + f*x]^3/(3*a^
2*c^2*f*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 0.330746, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2736, 2687, 2681, 2650, 2649, 206}

$$-\frac{\sec^3(e+fx)}{3a^2c^2f\sqrt{c-c\sin(e+fx)}}-\frac{35\sec(e+fx)}{48a^2c^2f\sqrt{c-c\sin(e+fx)}}+\frac{35\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{64\sqrt{2}a^2c^{5/2}f}+\frac{35\cos(e+fx)}{64a^2cf(c-c\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(5/2)),x]
```


```
[Out] (35*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])])/(64
*Sqrt[2]*a^2*c^(5/2)*f) + (35*Cos[e + f*x])/(64*a^2*c*f*(c - c*Sin[e + f*x]
)^(3/2)) + (7*Sec[e + f*x])/(24*a^2*c*f*(c - c*Sin[e + f*x])^(3/2)) - (35*S
ec[e + f*x])/(48*a^2*c^2*f*Sqrt[c - c*Sin[e + f*x]]) - Sec[e + f*x]^3/(3*a^
2*c^2*f*Sqrt[c - c*Sin[e + f*x]])
```


Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2687

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p + 1)*Sq
rt[a + b*Sin[e + f*x]]), x] + Dist[(a*(2*p + 1))/(2*g^2*(p + 1)), Int[(g*Co
s[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 2681

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*(2*m + p + 1)), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
```

```
IntegersQ[2*m, 2*p]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^{5/2}}\,dx = \frac{\int \frac{\sec^4(e+fx)}{\sqrt{c-c\sin(e+fx)}}\,dx}{a^2c^2}$$

$$= -\frac{\sec^3(e+fx)}{3a^2c^2f\sqrt{c-c\sin(e+fx)}} + \frac{7\int \frac{\sec^2(e+fx)}{(c-c\sin(e+fx))^{3/2}}\,dx}{6a^2c}$$

$$= \frac{7\sec(e+fx)}{24a^2cf(c-c\sin(e+fx))^{3/2}} - \frac{\sec^3(e+fx)}{3a^2c^2f\sqrt{c-c\sin(e+fx)}} + \frac{35\int \frac{\sec^2(e\cdots}{\sqrt{c-c\sin\cdots}}}{48a^2c\cdots}$$

$$= \frac{7\sec(e+fx)}{24a^2cf(c-c\sin(e+fx))^{3/2}} - \frac{35\sec(e+fx)}{48a^2c^2f\sqrt{c-c\sin(e+fx)}} - \frac{\sec^3\cdots}{3a^2c^2f\sqrt{c-\cdots}}$$

$$= \frac{35\cos(e+fx)}{64a^2cf(c-c\sin(e+fx))^{3/2}} + \frac{7\sec(e+fx)}{24a^2cf(c-c\sin(e+fx))^{3/2}} - \frac{35\,\mathrm{s}\cdots}{48a^2c^2f\sqrt{\cdots}}$$

$$= \frac{35\cos(e+fx)}{64a^2cf(c-c\sin(e+fx))^{3/2}} + \frac{7\sec(e+fx)}{24a^2cf(c-c\sin(e+fx))^{3/2}} - \frac{35\,\mathrm{s}\cdots}{48a^2c^2f\sqrt{\cdots}}$$

$$= \frac{35\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{64\sqrt{2}a^2c^{5/2}f} + \frac{35\cos(e+fx)}{64a^2cf(c-c\sin(e+fx))^{3/2}} + \frac{7\,\mathrm{s}\cdots}{24a^2cf(c\cdots}$$

Mathematica [C] time = 1.1363, size = 156, normalized size = 0.81

$$-\frac{\left(\frac{1}{1536}+\frac{i}{1536}\right)\sec^3(e+fx)\left((1-i)(-329\sin(e+fx)-105\sin(3(e+fx))+70\cos(2(e+fx))+102)+840\sqrt[4]{-1}\tan^{-1}\left(\left(\cdots\right.\right.\right.}{a^2c^2f\sqrt{c-c\sin\cdots}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(5/2)),x]
```

```
[Out] ((-1/1536 - I/1536)*Sec[e + f*x]^3*(840*(-1)^(1/4)*ArcTan[(1/2 + I/2)*(-1)^
(1/4)*(1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*(Cos[
(e + f*x)/2] + Sin[(e + f*x)/2])^3 + (1 - I)*(102 + 70*Cos[2*(e + f*x)] - 3
29*Sin[e + f*x] - 105*Sin[3*(e + f*x)])))/(a^2*c^2*f*Sqrt[c - c*Sin[e + f*x
]])
```

Maple [A] time = 0.728, size = 233, normalized size = 1.2

$$-\frac{1}{384\,a^2\left(1+\sin\left(fx+e\right)\right)\left(-1+\sin\left(fx+e\right)\right)\cos\left(fx+e\right)f}\left(105\left(c\left(1+\sin\left(fx+e\right)\right)\right)^{3/2}\sqrt{2}\mathrm{Artanh}\left(1/2\frac{\sqrt{c\left(1+\sin\right.}}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] -1/384/c^(11/2)/a^2*(105*(c*(1+sin(f*x+e)))^(3/2)*2^(1/2)*arctanh(1/2*(c*(1
+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)^2*c^2+70*c^(7/2)*sin(f*x+e)
^2-210*c^(7/2)*sin(f*x+e)^3-210*(c*(1+sin(f*x+e)))^(3/2)*2^(1/2)*arctanh(1/
2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)*c^2+322*c^(7/2)*sin(
f*x+e)+105*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*c^
2*(c*(1+sin(f*x+e)))^(3/2)-86*c^(7/2))/(1+sin(f*x+e))/(-1+sin(f*x+e))/cos(f
*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(5/2),x, algorithm="maxima"
)
```

```
[Out] Timed out
```

Fricas [A] time = 1.24951, size = 643, normalized size = 3.35

$$\frac{105\sqrt{2}\left(\cos\left(fx+e\right)^3\sin\left(fx+e\right)-\cos\left(fx+e\right)^3\right)\sqrt{c}\log\left(-\frac{c\cos\left(fx+e\right)^2+2\sqrt{2}\sqrt{-c\sin\left(fx+e\right)+c}\sqrt{c}\left(\cos\left(fx+e\right)+\sin\left(fx+e\right)+1\right)+3\,c\cos}{\cos\left(fx+e\right)^2+\left(\cos\left(fx+e\right)+2\right)\sin\left(fx+e\right)-\cos}\right.}{768\left(a^2c^3f\cos\left(fx+e\right)^3\sin\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(5/2),x, algorithm="fricas"
)
```

```
[Out] 1/768*(105*sqrt(2)*(cos(f*x + e)^3*sin(f*x + e) - cos(f*x + e)^3)*sqrt(c)*l
og(-(c*cos(f*x + e)^2 + 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*sqrt(c)*(cos(f*
x + e) + sin(f*x + e) + 1) + 3*c*cos(f*x + e) + (c*cos(f*x + e) - 2*c)*sin(
f*x + e) + 2*c)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x
```

```
 + e) - 2)) + 4*(35*cos(f*x + e)^2 - 7*(15*cos(f*x + e)^2 + 8)*sin(f*x + e)
 + 8)*sqrt(-c*sin(f*x + e) + c))/(a^2*c^3*f*cos(f*x + e)^3*sin(f*x + e) - a
^2*c^3*f*cos(f*x + e)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**2/(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 3.08634, size = 1083, normalized size = 5.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] 1/192*(105*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt
(c*tan(1/2*f*x + 1/2*e)^2 + c) - sqrt(c))/sqrt(-c))/(a^2*sqrt(-c)*c^2*sgn(t
an(1/2*f*x + 1/2*e) - 1)) + 16*(15*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*t
an(1/2*f*x + 1/2*e)^2 + c))^5 + 33*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*t
an(1/2*f*x + 1/2*e)^2 + c))^4*sqrt(c) - 22*(sqrt(c)*tan(1/2*f*x + 1/2*e) -
sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^3*c - 66*(sqrt(c)*tan(1/2*f*x + 1/2*e)
- sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^2*c^(3/2) + 51*(sqrt(c)*tan(1/2*f*x +
 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))*c^2 - 11*c^(5/2))/(((sqrt(c)*
tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^2 + 2*(sqrt(c)*t
an(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))*sqrt(c) - c)^3*a^
2*c^2*sgn(tan(1/2*f*x + 1/2*e) - 1)) + 6*(53*(sqrt(c)*tan(1/2*f*x + 1/2*e)
- sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^7 - 179*(sqrt(c)*tan(1/2*f*x + 1/2*e)
 - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^6*sqrt(c) + 127*(sqrt(c)*tan(1/2*f*x
 + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^5*c + 195*(sqrt(c)*tan(1/2*
f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^4*c^(3/2) + 7*(sqrt(c)*t
an(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^3*c^2 - 121*(sqrt
(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^2*c^(5/2) -
67*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))*c^3
- 15*c^(7/2))/(((sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)
^2 + c))^2 - 2*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^
2 + c))*sqrt(c) - c)^4*a^2*c^2*sgn(tan(1/2*f*x + 1/2*e) - 1)))/f
```

3.332 $\int \frac{(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^3}\,dx$

Optimal. Leaf size=174

$$\frac{2\sec^5(e+fx)(c-c\sin(e+fx))^{13/2}}{3a^3c^2f}-\frac{4096c^2\sec^5(e+fx)(c-c\sin(e+fx))^{5/2}}{15a^3f}+\frac{32\sec^5(e+fx)(c-c\sin(e+fx))^{11}}{3a^3cf}$$

```
[Out] (-4096*c^2*Sec[e + f*x]^5*(c - c*Sin[e + f*x])^(5/2))/(15*a^3*f) + (1024*c*
Sec[e + f*x]^5*(c - c*Sin[e + f*x])^(7/2))/(3*a^3*f) - (128*Sec[e + f*x]^5*
(c - c*Sin[e + f*x])^(9/2))/(a^3*f) + (32*Sec[e + f*x]^5*(c - c*Sin[e + f*x
])^(11/2))/(3*a^3*c*f) + (2*Sec[e + f*x]^5*(c - c*Sin[e + f*x])^(13/2))/(3*
a^3*c^2*f)
```

Rubi [A] time = 0.403117, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {2736, 2674, 2673}

$$\frac{2\sec^5(e+fx)(c-c\sin(e+fx))^{13/2}}{3a^3c^2f}-\frac{4096c^2\sec^5(e+fx)(c-c\sin(e+fx))^{5/2}}{15a^3f}+\frac{32\sec^5(e+fx)(c-c\sin(e+fx))^{11}}{3a^3cf}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sin[e + f*x])^(9/2)/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] (-4096*c^2*Sec[e + f*x]^5*(c - c*Sin[e + f*x])^(5/2))/(15*a^3*f) + (1024*c*
Sec[e + f*x]^5*(c - c*Sin[e + f*x])^(7/2))/(3*a^3*f) - (128*Sec[e + f*x]^5*
(c - c*Sin[e + f*x])^(9/2))/(a^3*f) + (32*Sec[e + f*x]^5*(c - c*Sin[e + f*x
])^(11/2))/(3*a^3*c*f) + (2*Sec[e + f*x]^5*(c - c*Sin[e + f*x])^(13/2))/(3*
a^3*c^2*f)
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rubi steps

$$\int \frac{(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^3}\,dx = \frac{\int \sec^6(e+fx)(c-c\sin(e+fx))^{15/2}\,dx}{a^3c^3}$$
$$= \frac{2\sec^5(e+fx)(c-c\sin(e+fx))^{13/2}}{3a^3c^2f} + \frac{16\int \sec^6(e+fx)(c-c\sin(e+fx))^{13/2}\,dx}{3a^3c^2}$$
$$= \frac{32\sec^5(e+fx)(c-c\sin(e+fx))^{11/2}}{3a^3cf} + \frac{2\sec^5(e+fx)(c-c\sin(e+fx))^{13/2}}{3a^3c^2f} + \frac{64\int \sec^6(e$$
$$= -\frac{128\sec^5(e+fx)(c-c\sin(e+fx))^{9/2}}{a^3f} + \frac{32\sec^5(e+fx)(c-c\sin(e+fx))^{11/2}}{3a^3cf} + \frac{2\sec^5(e$$
$$= \frac{1024c\sec^5(e+fx)(c-c\sin(e+fx))^{7/2}}{3a^3f} - \frac{128\sec^5(e+fx)(c-c\sin(e+fx))^{9/2}}{a^3f} + \frac{32\sec^5($$
$$= -\frac{4096c^2\sec^5(e+fx)(c-c\sin(e+fx))^{5/2}}{15a^3f} + \frac{1024c\sec^5(e+fx)(c-c\sin(e+fx))^{7/2}}{3a^3f} - 128$$

Mathematica [A] time = 3.01368, size = 124, normalized size = 0.71

$$\frac{c^4\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)(-7800\sin(e+fx)+200\sin(3(e+fx))+2740\cos(2(e+fx))}{60a^3f(\sin(e+fx)+1)^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sin[e + f*x])^(9/2)/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] (c^4*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]*(-5649
+ 2740*Cos[2*(e + f*x)] + 5*Cos[4*(e + f*x)] - 7800*Sin[e + f*x] + 200*Sin[
3*(e + f*x)]))/(60*a^3*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e +
 f*x])^3)
```

Maple [A] time = 0.761, size = 91, normalized size = 0.5

$$-\frac{2c^5\left(-1+\sin(fx+e)\right)\left(5(\sin(fx+e))^4-100(\sin(fx+e))^3-690(\sin(fx+e))^2-900\sin(fx+e)-363\right)}{15a^3\left(1+\sin(fx+e)\right)^2\cos(fx+e)f}\frac{}{\sqrt{c-}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^3,x)
```

```
[Out] -2/15*c^5/a^3*(-1+sin(f*x+e))/(1+sin(f*x+e))^2*(5*sin(f*x+e)^4-100*sin(f*x+
e)^3-690*sin(f*x+e)^2-900*sin(f*x+e)-363)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)
/f
```

Maxima [B] time = 1.82271, size = 637, normalized size = 3.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 2/15*(363*c^(9/2) + 1800*c^(9/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 5301*c^(
9/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 11600*c^(9/2)*sin(f*x + e)^3/(co
s(f*x + e) + 1)^3 + 21343*c^(9/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 302
00*c^(9/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 40065*c^(9/2)*sin(f*x + e)
^6/(cos(f*x + e) + 1)^6 + 40800*c^(9/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7
 + 40065*c^(9/2)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 30200*c^(9/2)*sin(f*
x + e)^9/(cos(f*x + e) + 1)^9 + 21343*c^(9/2)*sin(f*x + e)^10/(cos(f*x + e)
 + 1)^10 + 11600*c^(9/2)*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 + 5301*c^(9/
2)*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 + 1800*c^(9/2)*sin(f*x + e)^13/(co
s(f*x + e) + 1)^13 + 363*c^(9/2)*sin(f*x + e)^14/(cos(f*x + e) + 1)^14)/((a
^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x
 + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x +
 e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)*f*(si
n(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(9/2))
```

Fricas [A] time = 1.14366, size = 301, normalized size = 1.73

$$-\frac{2\left(5\,c^4\cos\left(fx+e\right)^4+680\,c^4\cos\left(fx+e\right)^2-1048\,c^4+100\left(c^4\cos\left(fx+e\right)^2-10\,c^4\right)\sin\left(fx+e\right)\right)\sqrt{-c\sin\left(fx+e\right)+c}}{15\left(a^3f\cos\left(fx+e\right)^3-2\,a^3f\cos\left(fx+e\right)\sin\left(fx+e\right)-2\,a^3f\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -2/15*(5*c^4*cos(f*x + e)^4 + 680*c^4*cos(f*x + e)^2 - 1048*c^4 + 100*(c^4*
cos(f*x + e)^2 - 10*c^4)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c)/(a^3*f*cos
(f*x + e)^3 - 2*a^3*f*cos(f*x + e)*sin(f*x + e) - 2*a^3*f*cos(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))**(9/2)/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 2.34077, size = 1084, normalized size = 6.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/15*(2*(2255*sqrt(2)*a^12*sqrt(c) - 3190*a^12*sqrt(c) - 1206*sqrt(2)*c^(13
/2) + 1700*c^(13/2))*sgn(tan(1/2*f*x + 1/2*e) - 1)/(29*sqrt(2)*a^3*c^2 - 41
*a^3*c^2) + 5*(23*a^9*sgn(tan(1/2*f*x + 1/2*e) - 1) + (21*a^9*sgn(tan(1/2*f
*x + 1/2*e) - 1) + (23*a^9*sgn(tan(1/2*f*x + 1/2*e) - 1)*tan(1/2*f*x + 1/2*
```

```
e) + 21*a^9*sgn(tan(1/2*f*x + 1/2*e) - 1))*tan(1/2*f*x + 1/2*e))*tan(1/2*f*
x + 1/2*e))/(c*tan(1/2*f*x + 1/2*e)^2 + c)^(3/2) + 32*(15*(sqrt(c)*tan(1/2*
f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^9*c^5*sgn(tan(1/2*f*x +
1/2*e) - 1) + 105*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*
e)^2 + c))^8*c^(11/2)*sgn(tan(1/2*f*x + 1/2*e) - 1) + 340*(sqrt(c)*tan(1/2*
f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^7*c^6*sgn(tan(1/2*f*x +
1/2*e) - 1) - 260*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*
e)^2 + c))^6*c^(13/2)*sgn(tan(1/2*f*x + 1/2*e) - 1) - 1054*(sqrt(c)*tan(1/2
*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^5*c^7*sgn(tan(1/2*f*x +
 1/2*e) - 1) + 670*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2
*e)^2 + c))^4*c^(15/2)*sgn(tan(1/2*f*x + 1/2*e) - 1) + 900*(sqrt(c)*tan(1/2
*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^3*c^8*sgn(tan(1/2*f*x +
 1/2*e) - 1) - 980*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2
*e)^2 + c))^2*c^(17/2)*sgn(tan(1/2*f*x + 1/2*e) - 1) + 295*(sqrt(c)*tan(1/2
*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))*c^9*sgn(tan(1/2*f*x + 1
/2*e) - 1) - 31*c^(19/2)*sgn(tan(1/2*f*x + 1/2*e) - 1))/(((sqrt(c)*tan(1/2*
f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^2 + 2*(sqrt(c)*tan(1/2*f
*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))*sqrt(c) - c)^5*a^3))/f
```

3.333 $\int \frac{(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^3}\,dx$

Optimal. Leaf size=134

$$\frac{2\sec^5(e+fx)(c-c\sin(e+fx))^{11/2}}{a^3c^2f}-\frac{24\sec^5(e+fx)(c-c\sin(e+fx))^{9/2}}{a^3cf}+\frac{64\sec^5(e+fx)(c-c\sin(e+fx))^{7/2}}{a^3f}-$$

```
[Out] (-256*c*Sec[e + f*x]^5*(c - c*Sin[e + f*x])^(5/2))/(5*a^3*f) + (64*Sec[e +
f*x]^5*(c - c*Sin[e + f*x])^(7/2))/(a^3*f) - (24*Sec[e + f*x]^5*(c - c*Sin[
e + f*x])^(9/2))/(a^3*c*f) + (2*Sec[e + f*x]^5*(c - c*Sin[e + f*x])^(11/2))
/(a^3*c^2*f)
```

Rubi [A] time = 0.330242, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {2736, 2674, 2673}

$$\frac{2\sec^5(e+fx)(c-c\sin(e+fx))^{11/2}}{a^3c^2f}-\frac{24\sec^5(e+fx)(c-c\sin(e+fx))^{9/2}}{a^3cf}+\frac{64\sec^5(e+fx)(c-c\sin(e+fx))^{7/2}}{a^3f}-$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sin[e + f*x])^(7/2)/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] (-256*c*Sec[e + f*x]^5*(c - c*Sin[e + f*x])^(5/2))/(5*a^3*f) + (64*Sec[e +
f*x]^5*(c - c*Sin[e + f*x])^(7/2))/(a^3*f) - (24*Sec[e + f*x]^5*(c - c*Sin[
e + f*x])^(9/2))/(a^3*c*f) + (2*Sec[e + f*x]^5*(c - c*Sin[e + f*x])^(11/2))
/(a^3*c^2*f)
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rubi steps

$$\int \frac{(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^3}\,dx = \frac{\int \sec^6(e+fx)(c-c\sin(e+fx))^{13/2}\,dx}{a^3c^3}$$
$$= \frac{2\sec^5(e+fx)(c-c\sin(e+fx))^{11/2}}{a^3c^2f} + \frac{12\int \sec^6(e+fx)(c-c\sin(e+fx))^{11/2}\,dx}{a^3c^2}$$
$$= -\frac{24\sec^5(e+fx)(c-c\sin(e+fx))^{9/2}}{a^3cf} + \frac{2\sec^5(e+fx)(c-c\sin(e+fx))^{11/2}}{a^3c^2f} - \frac{96\int \sec^6(e$$
$$= \frac{64\sec^5(e+fx)(c-c\sin(e+fx))^{7/2}}{a^3f} - \frac{24\sec^5(e+fx)(c-c\sin(e+fx))^{9/2}}{a^3cf} + \frac{2\sec^5(e+fx$$
$$= -\frac{256c\sec^5(e+fx)(c-c\sin(e+fx))^{5/2}}{5a^3f} + \frac{64\sec^5(e+fx)(c-c\sin(e+fx))^{7/2}}{a^3f} - \frac{24\sec^5(e$$

Mathematica [A] time = 1.24318, size = 114, normalized size = 0.85

$$\frac{c^3\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)(-235\sin(e+fx)+5\sin(3(e+fx))+90\cos(2(e+fx))-182)}{10a^3f(\sin(e+fx)+1)^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sin[e + f*x])^(7/2)/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] (c^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]*(-182 +
 90*Cos[2*(e + f*x)] - 235*Sin[e + f*x] + 5*Sin[3*(e + f*x)]))/(10*a^3*f*(C
os[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^3)
```

Maple [A] time = 0.622, size = 81, normalized size = 0.6

$$\frac{2c^4(-1+\sin(fx+e))\left(5(\sin(fx+e))^3+45(\sin(fx+e))^2+55\sin(fx+e)+23\right)}{5a^3(1+\sin(fx+e))^2\cos(fx+e)f}\frac{1}{\sqrt{c-c\sin(fx+e)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^3,x)
```

```
[Out] 2/5*c^4/a^3*(-1+sin(f*x+e))/(1+sin(f*x+e))^2*(5*sin(f*x+e)^3+45*sin(f*x+e)^
2+55*sin(f*x+e)+23)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Maxima [B] time = 1.83459, size = 575, normalized size = 4.29

$$\frac{2\left(23c^{\frac{7}{2}}+\frac{110c^{\frac{7}{2}}\sin(fx+e)}{\cos(fx+e)+1}+\frac{318c^{\frac{7}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{590c^{\frac{7}{2}}\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{1065c^{\frac{7}{2}}\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{1220c^{\frac{7}{2}}\sin(fx+e)^5}{(\cos(fx+e)+1)^5}+\frac{1540c^{\frac{7}{2}}\sin(fx+e)^6}{(\cos(fx+e)+1)^6}+\frac{1220}{(\text{co}}\right.}{5\left(a^3+\frac{5a^3\sin(fx+e)}{\cos(fx+e)+1}+\frac{10a^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{10a^3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{5a^3\sin(f}{(\cos(fx+e}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 2/5*(23*c^(7/2) + 110*c^(7/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 318*c^(7/2)
*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 590*c^(7/2)*sin(f*x + e)^3/(cos(f*x
+ e) + 1)^3 + 1065*c^(7/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 1220*c^(7/
2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 1540*c^(7/2)*sin(f*x + e)^6/(cos(f
*x + e) + 1)^6 + 1220*c^(7/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 1065*c^
(7/2)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 590*c^(7/2)*sin(f*x + e)^9/(cos
(f*x + e) + 1)^9 + 318*c^(7/2)*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 + 110*
c^(7/2)*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 + 23*c^(7/2)*sin(f*x + e)^12/
(cos(f*x + e) + 1)^12)/((a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a
^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e
) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(
cos(f*x + e) + 1)^5)*f*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(7/2))
```

Fricas [A] time = 1.1173, size = 262, normalized size = 1.96

$$-\frac{2\left(45\,c^{3}\cos\left(fx+e\right)^{2}-68\,c^{3}+5\left(c^{3}\cos\left(fx+e\right)^{2}-12\,c^{3}\right)\sin\left(fx+e\right)\right)\sqrt{-c\sin\left(fx+e\right)+c}}{5\left(a^{3}f\cos\left(fx+e\right)^{3}-2\,a^{3}f\cos\left(fx+e\right)\sin\left(fx+e\right)-2\,a^{3}f\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -2/5*(45*c^3*cos(f*x + e)^2 - 68*c^3 + 5*(c^3*cos(f*x + e)^2 - 12*c^3)*sin(
f*x + e))*sqrt(-c*sin(f*x + e) + c)/(a^3*f*cos(f*x + e)^3 - 2*a^3*f*cos(f*x
 + e)*sin(f*x + e) - 2*a^3*f*cos(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))**(7/2)/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 2.17722, size = 975, normalized size = 7.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 2/5*(2*(67*sqrt(2)*c^(7/2) - 95*c^(7/2))*sgn(tan(1/2*f*x + 1/2*e) - 1)/(29*
sqrt(2)*a^3 - 41*a^3) + 5*(c^4*sgn(tan(1/2*f*x + 1/2*e) - 1)*tan(1/2*f*x +
1/2*e)/a^3 + c^4*sgn(tan(1/2*f*x + 1/2*e) - 1)/a^3)/sqrt(c*tan(1/2*f*x + 1/
2*e)^2 + c) + 4*(5*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2
```

```
*e)^2 + c))^9*c^4*sgn(tan(1/2*f*x + 1/2*e) - 1) + 15*(sqrt(c)*tan(1/2*f*x +
 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^8*c^(9/2)*sgn(tan(1/2*f*x + 1
/2*e) - 1) + 100*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e
)^2 + c))^7*c^5*sgn(tan(1/2*f*x + 1/2*e) - 1) - 60*(sqrt(c)*tan(1/2*f*x + 1
/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^6*c^(11/2)*sgn(tan(1/2*f*x + 1/
2*e) - 1) - 306*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)
^2 + c))^5*c^6*sgn(tan(1/2*f*x + 1/2*e) - 1) + 210*(sqrt(c)*tan(1/2*f*x + 1
/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^4*c^(13/2)*sgn(tan(1/2*f*x + 1/
2*e) - 1) + 260*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)
^2 + c))^3*c^7*sgn(tan(1/2*f*x + 1/2*e) - 1) - 300*(sqrt(c)*tan(1/2*f*x + 1
/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^2*c^(15/2)*sgn(tan(1/2*f*x + 1/
2*e) - 1) + 85*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^
2 + c))*c^8*sgn(tan(1/2*f*x + 1/2*e) - 1) - 9*c^(17/2)*sgn(tan(1/2*f*x + 1/
2*e) - 1))/(((sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2
+ c))^2 + 2*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 +
 c))*sqrt(c) - c)^5*a^3))/f
```

3.334 $\int \frac{(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^3} dx$

Optimal. Leaf size=104

$$-\frac{2\sec^5(e+fx)(c-c\sin(e+fx))^{9/2}}{a^3c^2f}+\frac{16\sec^5(e+fx)(c-c\sin(e+fx))^{7/2}}{3a^3cf}-\frac{64\sec^5(e+fx)(c-c\sin(e+fx))^{5/2}}{15a^3f}$$

```
[Out] (-64*Sec[e + f*x]^5*(c - c*Sin[e + f*x])^(5/2))/(15*a^3*f) + (16*Sec[e + f*
x]^5*(c - c*Sin[e + f*x])^(7/2))/(3*a^3*c*f) - (2*Sec[e + f*x]^5*(c - c*Sin
[e + f*x])^(9/2))/(a^3*c^2*f)
```

Rubi [A] time = 0.269207, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {2736, 2674, 2673}

$$-\frac{2\sec^5(e+fx)(c-c\sin(e+fx))^{9/2}}{a^3c^2f}+\frac{16\sec^5(e+fx)(c-c\sin(e+fx))^{7/2}}{3a^3cf}-\frac{64\sec^5(e+fx)(c-c\sin(e+fx))^{5/2}}{15a^3f}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sin[e + f*x])^(5/2)/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] (-64*Sec[e + f*x]^5*(c - c*Sin[e + f*x])^(5/2))/(15*a^3*f) + (16*Sec[e + f*
x]^5*(c - c*Sin[e + f*x])^(7/2))/(3*a^3*c*f) - (2*Sec[e + f*x]^5*(c - c*Sin
[e + f*x])^(9/2))/(a^3*c^2*f)
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rubi steps

$$\int \frac{(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^3}\,dx = \frac{\int \sec^6(e+fx)(c-c\sin(e+fx))^{11/2}\,dx}{a^3c^3}$$
$$= -\frac{2\sec^5(e+fx)(c-c\sin(e+fx))^{9/2}}{a^3c^2f} - \frac{8\int \sec^6(e+fx)(c-c\sin(e+fx))^{9/2}\,dx}{a^3c^2}$$
$$= \frac{16\sec^5(e+fx)(c-c\sin(e+fx))^{7/2}}{3a^3cf} - \frac{2\sec^5(e+fx)(c-c\sin(e+fx))^{9/2}}{a^3c^2f} + \frac{32\int \sec^6(e+$$
$$= -\frac{64\sec^5(e+fx)(c-c\sin(e+fx))^{5/2}}{15a^3f} + \frac{16\sec^5(e+fx)(c-c\sin(e+fx))^{7/2}}{3a^3cf} - \frac{2\sec^5(e+$$

Mathematica [A] time = 0.841201, size = 104, normalized size = 1.

$$\frac{c^2\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)(-20\sin(e+fx)+15\cos(2(e+fx))-29)}{15a^3f(\sin(e+fx)+1)^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sin[e + f*x])^(5/2)/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] (c^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-29 + 15*Cos[2*(e + f*x)] - 20*
Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(15*a^3*f*(Cos[(e + f*x)/2] - Sin[(
e + f*x)/2])*(1 + Sin[e + f*x])^3)
```

Maple [A] time = 0.504, size = 71, normalized size = 0.7

$$\frac{2c^3\left(-1+\sin\left(fx+e\right)\right)\left(15\left(\sin\left(fx+e\right)\right)^2+10\sin\left(fx+e\right)+7\right)}{15a^3\left(1+\sin\left(fx+e\right)\right)^2\cos\left(fx+e\right)f}\frac{1}{\sqrt{c-c\sin\left(fx+e\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^3,x)
```

```
[Out] 2/15*c^3/a^3*(-1+sin(f*x+e))/(1+sin(f*x+e))^2*(15*sin(f*x+e)^2+10*sin(f*x+e
)+7)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Maxima [B] time = 1.82795, size = 513, normalized size = 4.93

$$\frac{2\left(7c^{\frac{5}{2}}+\frac{20c^{\frac{5}{2}}\sin(fx+e)}{\cos(fx+e)+1}+\frac{95c^{\frac{5}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{80c^{\frac{5}{2}}\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{250c^{\frac{5}{2}}\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{120c^{\frac{5}{2}}\sin(fx+e)^5}{(\cos(fx+e)+1)^5}+\frac{250c^{\frac{5}{2}}\sin(fx+e)^6}{(\cos(fx+e)+1)^6}+\frac{80c^{\frac{5}{2}}\sin(fx-}{(\cos(fx+e)+}\right.}{15\left(a^3+\frac{5a^3\sin(fx+e)}{\cos(fx+e)+1}+\frac{10a^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{10a^3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{5a^3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{a^3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 2/15*(7*c^(5/2) + 20*c^(5/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 95*c^(5/2)*s
in(f*x + e)^2/(cos(f*x + e) + 1)^2 + 80*c^(5/2)*sin(f*x + e)^3/(cos(f*x + e
) + 1)^3 + 250*c^(5/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 120*c^(5/2)*si
n(f*x + e)^5/(cos(f*x + e) + 1)^5 + 250*c^(5/2)*sin(f*x + e)^6/(cos(f*x + e
) + 1)^6 + 80*c^(5/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 95*c^(5/2)*sin(
f*x + e)^8/(cos(f*x + e) + 1)^8 + 20*c^(5/2)*sin(f*x + e)^9/(cos(f*x + e) +
 1)^9 + 7*c^(5/2)*sin(f*x + e)^10/(cos(f*x + e) + 1)^10)/((a^3 + 5*a^3*sin(
f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 +
10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x
+ e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)*f*(sin(f*x + e)^2/(c
os(f*x + e) + 1)^2 + 1)^(5/2))
```

Fricas [A] time = 1.12084, size = 230, normalized size = 2.21

$$-\frac{2\left(15\,c^{2}\cos\left(fx+e\right)^{2}-10\,c^{2}\sin\left(fx+e\right)-22\,c^{2}\right)\sqrt{-c\sin\left(fx+e\right)+c}}{15\left(a^{3}f\cos\left(fx+e\right)^{3}-2\,a^{3}f\cos\left(fx+e\right)\sin\left(fx+e\right)-2\,a^{3}f\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -2/15*(15*c^2*cos(f*x + e)^2 - 10*c^2*sin(f*x + e) - 22*c^2)*sqrt(-c*sin(f*
x + e) + c)/(a^3*f*cos(f*x + e)^3 - 2*a^3*f*cos(f*x + e)*sin(f*x + e) - 2*a
^3*f*cos(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))**(5/2)/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.94364, size = 878, normalized size = 8.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -2/15*((39*sqrt(2)*c^(5/2) - 55*c^(5/2))*sgn(tan(1/2*f*x + 1/2*e) - 1)/(29*
sqrt(2)*a^3 - 41*a^3) - 2*(15*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/
2*f*x + 1/2*e)^2 + c))^9*c^3*sgn(tan(1/2*f*x + 1/2*e) - 1) - 15*(sqrt(c)*ta
n(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^8*c^(7/2)*sgn(tan(
1/2*f*x + 1/2*e) - 1) + 100*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*
f*x + 1/2*e)^2 + c))^7*c^4*sgn(tan(1/2*f*x + 1/2*e) - 1) - 20*(sqrt(c)*tan(
1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^6*c^(9/2)*sgn(tan(1/
```

```
2*f*x + 1/2*e) - 1) - 238*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*
x + 1/2*e)^2 + c))^5*c^5*sgn(tan(1/2*f*x + 1/2*e) - 1) + 190*(sqrt(c)*tan(1
/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^4*c^(11/2)*sgn(tan(1/
2*f*x + 1/2*e) - 1) + 180*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*
x + 1/2*e)^2 + c))^3*c^6*sgn(tan(1/2*f*x + 1/2*e) - 1) - 260*(sqrt(c)*tan(1
/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^2*c^(13/2)*sgn(tan(1/
2*f*x + 1/2*e) - 1) + 55*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x
 + 1/2*e)^2 + c))*c^7*sgn(tan(1/2*f*x + 1/2*e) - 1) - 7*c^(15/2)*sgn(tan(1/
2*f*x + 1/2*e) - 1))/(((sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x +
 1/2*e)^2 + c))^2 + 2*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x +
1/2*e)^2 + c))*sqrt(c) - c)^5*a^3))/f
```

3.335 $\int \frac{(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^3}\,dx$

Optimal. Leaf size=73

$$\frac{8\sec^5(e+fx)(c-c\sin(e+fx))^{5/2}}{15a^3cf}-\frac{2\sec^5(e+fx)(c-c\sin(e+fx))^{7/2}}{3a^3c^2f}$$

```
[Out] (8*Sec[e + f*x]^5*(c - c*Sin[e + f*x])^(5/2))/(15*a^3*c*f) - (2*Sec[e + f*x
]^5*(c - c*Sin[e + f*x])^(7/2))/(3*a^3*c^2*f)
```

Rubi [A] time = 0.195981, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {2736, 2674, 2673}

$$\frac{8\sec^5(e+fx)(c-c\sin(e+fx))^{5/2}}{15a^3cf}-\frac{2\sec^5(e+fx)(c-c\sin(e+fx))^{7/2}}{3a^3c^2f}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sin[e + f*x])^(3/2)/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] (8*Sec[e + f*x]^5*(c - c*Sin[e + f*x])^(5/2))/(15*a^3*c*f) - (2*Sec[e + f*x
]^5*(c - c*Sin[e + f*x])^(7/2))/(3*a^3*c^2*f)
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rubi steps

$$\int \frac{(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^3}\,dx = \frac{\int \sec^6(e+fx)(c-c\sin(e+fx))^{9/2}\,dx}{a^3c^3}$$
$$= -\frac{2\sec^5(e+fx)(c-c\sin(e+fx))^{7/2}}{3a^3c^2f} - \frac{4\int \sec^6(e+fx)(c-c\sin(e+fx))^{7/2}\,dx}{3a^3c^2}$$
$$= \frac{8\sec^5(e+fx)(c-c\sin(e+fx))^{5/2}}{15a^3cf} - \frac{2\sec^5(e+fx)(c-c\sin(e+fx))^{7/2}}{3a^3c^2f}$$

Mathematica [A] time = 0.376438, size = 92, normalized size = 1.26

$$\frac{2c(5\sin(e+fx)-1)\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}{15a^3f(\sin(e+fx)+1)^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sin[e + f*x])^(3/2)/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] (2*c*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + 5*Sin[e + f*x])*Sqrt[c - c
*Sin[e + f*x]])/(15*a^3*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e
+ f*x])^3)
```

Maple [A] time = 0.651, size = 61, normalized size = 0.8

$$-\frac{2c^2(-1+\sin(fx+e))(5\sin(fx+e)-1)}{15a^3(1+\sin(fx+e))^2\cos(fx+e)f}\frac{1}{\sqrt{c-c\sin(fx+e)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^3,x)
```

```
[Out] -2/15*c^2/a^3*(-1+sin(f*x+e))/(1+sin(f*x+e))^2*(5*sin(f*x+e)-1)/cos(f*x+e)/
(c-c*sin(f*x+e))^(1/2)/f
```

Maxima [B] time = 1.83898, size = 447, normalized size = 6.12

$$\frac{2\left(c^{\frac{3}{2}} - \frac{10c^{\frac{3}{2}}\sin(fx+e)}{\cos(fx+e)+1} + \frac{4c^{\frac{3}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2} - \frac{30c^{\frac{3}{2}}\sin(fx+e)^3}{(\cos(fx+e)+1)^3} + \frac{6c^{\frac{3}{2}}\sin(fx+e)^4}{(\cos(fx+e)+1)^4} - \frac{30c^{\frac{3}{2}}\sin(fx+e)^5}{(\cos(fx+e)+1)^5} + \frac{4c^{\frac{3}{2}}\sin(fx+e)^6}{(\cos(fx+e)+1)^6} - \frac{10c^{\frac{3}{2}}\sin(fx+e)^7}{(\cos(fx+e)+1)^7} + \cdots\right.}{15\left(a^3 + \frac{5a^3\sin(fx+e)}{\cos(fx+e)+1} + \frac{10a^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2} + \frac{10a^3\sin(fx+e)^3}{(\cos(fx+e)+1)^3} + \frac{5a^3\sin(fx+e)^4}{(\cos(fx+e)+1)^4} + \frac{a^3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)f\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 2/15*(c^(3/2) - 10*c^(3/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 4*c^(3/2)*sin(
f*x + e)^2/(cos(f*x + e) + 1)^2 - 30*c^(3/2)*sin(f*x + e)^3/(cos(f*x + e) +
 1)^3 + 6*c^(3/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 30*c^(3/2)*sin(f*x
```

```
+ e)^5/(cos(f*x + e) + 1)^5 + 4*c^(3/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6
 - 10*c^(3/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + c^(3/2)*sin(f*x + e)^8/
(cos(f*x + e) + 1)^8)/((a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^
3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e)
 + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(c
os(f*x + e) + 1)^5)*f*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(3/2))
```

Fricas [A] time = 1.05636, size = 186, normalized size = 2.55

$$-\frac{2\left(5\,c\sin\left(fx+e\right)-c\right)\sqrt{-c\sin\left(fx+e\right)+c}}{15\left(a^{3}f\cos\left(fx+e\right)^{3}-2\,a^{3}f\cos\left(fx+e\right)\sin\left(fx+e\right)-2\,a^{3}f\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -2/15*(5*c*sin(f*x + e) - c)*sqrt(-c*sin(f*x + e) + c)/(a^3*f*cos(f*x + e)^
3 - 2*a^3*f*cos(f*x + e)*sin(f*x + e) - 2*a^3*f*cos(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))**(3/2)/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.85955, size = 803, normalized size = 11.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -1/15*((99*sqrt(2)*c^(3/2) - 140*c^(3/2))*sgn(tan(1/2*f*x + 1/2*e) - 1)/(29
*sqrt(2)*a^3 - 41*a^3) - 4*(15*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1
/2*f*x + 1/2*e)^2 + c))^9*c^2*sgn(tan(1/2*f*x + 1/2*e) - 1) + 15*(sqrt(c)*t
an(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^8*c^(5/2)*sgn(tan
(1/2*f*x + 1/2*e) - 1) + 40*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*
f*x + 1/2*e)^2 + c))^7*c^3*sgn(tan(1/2*f*x + 1/2*e) - 1) - 20*(sqrt(c)*tan(
1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^6*c^(7/2)*sgn(tan(1/
2*f*x + 1/2*e) - 1) - 34*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x
 + 1/2*e)^2 + c))^5*c^4*sgn(tan(1/2*f*x + 1/2*e) - 1) + 10*(sqrt(c)*tan(1/2
*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^4*c^(9/2)*sgn(tan(1/2*f
*x + 1/2*e) - 1) - 20*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x +
1/2*e)^2 + c))^2*c^(11/2)*sgn(tan(1/2*f*x + 1/2*e) - 1) - 5*(sqrt(c)*tan(1/
2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))*c^6*sgn(tan(1/2*f*x +
1/2*e) - 1) - c^(13/2)*sgn(tan(1/2*f*x + 1/2*e) - 1))/(((sqrt(c)*tan(1/2*f*
```

```
x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^2 + 2*(sqrt(c)*tan(1/2*f*x
 + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))*sqrt(c) - c)^5*a^3))/f
```

3.336 $\int \frac{\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^3}\,dx$

Optimal. Leaf size=36

$$-\frac{2\sec^5(e+fx)(c-c\sin(e+fx))^{5/2}}{5a^3c^2f}$$

```
[Out] (-2*Sec[e + f*x]^5*(c - c*Sin[e + f*x])^(5/2))/(5*a^3*c^2*f)
```

Rubi [A] time = 0.122419, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2736, 2673}

$$-\frac{2\sec^5(e+fx)(c-c\sin(e+fx))^{5/2}}{5a^3c^2f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c*Sin[e + f*x]]/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] (-2*Sec[e + f*x]^5*(c - c*Sin[e + f*x])^(5/2))/(5*a^3*c^2*f)
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^3}\,dx &= \frac{\int \sec^6(e+fx)(c-c\sin(e+fx))^{7/2}\,dx}{a^3c^3}\\ &= -\frac{2\sec^5(e+fx)(c-c\sin(e+fx))^{5/2}}{5a^3c^2f}\end{aligned}$$

Mathematica [B] time = 0.1418, size = 73, normalized size = 2.03

$$-\frac{2\sqrt{c-c\sin(e+fx)}}{5a^3f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c*Sin[e + f*x]]/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] (-2*Sqrt[c - c*Sin[e + f*x]])/(5*a^3*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]
)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)
```

Maple [A] time = 0.425, size = 49, normalized size = 1.4

$$\frac{2c\left(-1+\sin\left(fx+e\right)\right)}{5a^3\left(1+\sin\left(fx+e\right)\right)^2\cos\left(fx+e\right)f}\frac{1}{\sqrt{c-c\sin\left(fx+e\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^3,x)
```

```
[Out] 2/5*c/a^3*(-1+sin(f*x+e))/(1+sin(f*x+e))^2/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2
)/f
```

Maxima [B] time = 1.74812, size = 294, normalized size = 8.17

$$\frac{2\left(\sqrt{c}+\frac{3\sqrt{c}\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}+\frac{3\sqrt{c}\sin\left(fx+e\right)^4}{\left(\cos\left(fx+e\right)+1\right)^4}+\frac{\sqrt{c}\sin\left(fx+e\right)^6}{\left(\cos\left(fx+e\right)+1\right)^6}\right)}{5\left(a^3+\frac{5a^3\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{10a^3\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}+\frac{10a^3\sin\left(fx+e\right)^3}{\left(\cos\left(fx+e\right)+1\right)^3}+\frac{5a^3\sin\left(fx+e\right)^4}{\left(\cos\left(fx+e\right)+1\right)^4}+\frac{a^3\sin\left(fx+e\right)^5}{\left(\cos\left(fx+e\right)+1\right)^5}\right)f\sqrt{\frac{\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 2/5*(sqrt(c) + 3*sqrt(c)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 3*sqrt(c)*si
n(f*x + e)^4/(cos(f*x + e) + 1)^4 + sqrt(c)*sin(f*x + e)^6/(cos(f*x + e) +
1)^6)/((a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2
/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*
sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)
^5)*f*sqrt(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1))
```

Fricas [A] time = 1.07758, size = 153, normalized size = 4.25

$$\frac{2\sqrt{-c\sin\left(fx+e\right)+c}}{5\left(a^3f\cos\left(fx+e\right)^3-2a^3f\cos\left(fx+e\right)\sin\left(fx+e\right)-2a^3f\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 2/5*sqrt(-c*sin(f*x + e) + c)/(a^3*f*cos(f*x + e)^3 - 2*a^3*f*cos(f*x + e)*
sin(f*x + e) - 2*a^3*f*cos(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))**(1/2)/(a+a*sin(f*x+e))**3,x)

[Out] Timed out
```

Giac [B] time = 1.70931, size = 875, normalized size = 24.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^3,x, algorithm="giac")

[Out] -1/20*((191*sqrt(2)*sqrt(c) - 270*sqrt(c))*sgn(tan(1/2*f*x + 1/2*e) - 1)/(2
9*sqrt(2)*a^3 - 41*a^3) - 16*(5*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(
1/2*f*x + 1/2*e)^2 + c))^9*c*sgn(tan(1/2*f*x + 1/2*e) - 1) + 15*(sqrt(c)*ta
n(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^8*c^(3/2)*sgn(tan(
1/2*f*x + 1/2*e) - 1) + 20*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f
*x + 1/2*e)^2 + c))^7*c^2*sgn(tan(1/2*f*x + 1/2*e) - 1) - 20*(sqrt(c)*tan(1
/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^6*c^(5/2)*sgn(tan(1/2
*f*x + 1/2*e) - 1) - 34*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x
+ 1/2*e)^2 + c))^5*c^3*sgn(tan(1/2*f*x + 1/2*e) - 1) + 10*(sqrt(c)*tan(1/2*
f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^4*c^(7/2)*sgn(tan(1/2*f*
x + 1/2*e) - 1) + 20*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1
/2*e)^2 + c))^3*c^4*sgn(tan(1/2*f*x + 1/2*e) - 1) - 20*(sqrt(c)*tan(1/2*f*x
 + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^2*c^(9/2)*sgn(tan(1/2*f*x +
 1/2*e) - 1) + 5*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e
)^2 + c))*c^5*sgn(tan(1/2*f*x + 1/2*e) - 1) - c^(11/2)*sgn(tan(1/2*f*x + 1/
2*e) - 1))/(((sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2
+ c))^2 + 2*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 +
 c))*sqrt(c) - c)^5*a^3))/f
```

3.337 $\int \frac{1}{(a+a\sin(e+fx))^3\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=160

$$-\frac{\sec^5(e+fx)(c-c\sin(e+fx))^{5/2}}{5a^3c^3f}-\frac{\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{6a^3c^2f}-\frac{\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{4a^3cf}+\frac{\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{4\sqrt{2}a^3\sqrt{c}f}$$

```
[Out] ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])]/(4*Sqrt[
2]*a^3*Sqrt[c]*f) - (Sec[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(4*a^3*c*f) - (
Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(3/2))/(6*a^3*c^2*f) - (Sec[e + f*x]^5*
(c - c*Sin[e + f*x])^(5/2))/(5*a^3*c^3*f)
```

Rubi [A] time = 0.294272, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2736, 2675, 2649, 206}

$$-\frac{\sec^5(e+fx)(c-c\sin(e+fx))^{5/2}}{5a^3c^3f}-\frac{\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{6a^3c^2f}-\frac{\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{4a^3cf}+\frac{\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{4\sqrt{2}a^3\sqrt{c}f}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^3*Sqrt[c - c*Sin[e + f*x]]),x]
```

```
[Out] ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])]/(4*Sqrt[
2]*a^3*Sqrt[c]*f) - (Sec[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(4*a^3*c*f) - (
Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(3/2))/(6*a^3*c^2*f) - (Sec[e + f*x]^5*
(c - c*Sin[e + f*x])^(5/2))/(5*a^3*c^3*f)
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2675

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^m)/(a*f*g*(p + 1)), x] + Dist[(a*(m + p + 1))/(g^2*(p + 1)), Int[(g*Cos
[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[p, -2*m] && IntegersQ[m
+ 1/2, 2*p]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^3\sqrt{c-c\sin(e+fx)}}\,dx = \frac{\int \sec^6(e+fx)(c-c\sin(e+fx))^{5/2}\,dx}{a^3c^3}$$
$$= -\frac{\sec^5(e+fx)(c-c\sin(e+fx))^{5/2}}{5a^3c^3f} + \frac{\int \sec^4(e+fx)(c-c\sin(e+fx))}{2a^3c^2}$$
$$= -\frac{\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{6a^3c^2f} - \frac{\sec^5(e+fx)(c-c\sin(e+fx))^{5/2}}{5a^3c^3f}$$
$$= -\frac{\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{4a^3cf} - \frac{\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{6a^3c^2f} -$$
$$= -\frac{\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{4a^3cf} - \frac{\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{6a^3c^2f} -$$
$$= \frac{\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{4\sqrt{2}a^3\sqrt{c}f} - \frac{\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{4a^3cf} - \sec^3(e+$$

Mathematica [C] time = 0.598332, size = 189, normalized size = 1.18

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(-15\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^4-\right.}{60a^3f(}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^3*Sqrt[c - c*Sin[e + f*x]]),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)*(-12 - 10*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 - 15*(Cos[(e + f*x)/2]
+ Sin[(e + f*x)/2])^4 - (15 + 15*I)*(-1)^(1/4)*ArcTan[(1/2 + I/2)*(-1)^(1/4
)*(1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5))/(60*a^3
*f*(1 + Sin[e + f*x])^3*Sqrt[c - c*Sin[e + f*x]])
```

Maple [A] time = 0.618, size = 122, normalized size = 0.8

$$\frac{-1+\sin(fx+e)}{120\,a^3\left(1+\sin(fx+e)\right)^2\cos(fx+e)\,f}\left(74\,c^{11/2}+80\,c^{11/2}\sin(fx+e)+30\,c^{11/2}\left(\sin(fx+e)\right)^2-15\sqrt{2}\mathrm{Artanh}\left(1/\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] 1/120/a^3*(-1+sin(f*x+e))/c^(11/2)/(1+sin(f*x+e))^2*(74*c^(11/2)+80*c^(11/2
)*sin(f*x+e)+30*c^(11/2)*sin(f*x+e)^2-15*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+
e)))^(1/2)*2^(1/2)/c^(1/2))*c^3*(c*(1+sin(f*x+e)))^(5/2))/cos(f*x+e)/(c-c*s
```

```
in(f*x+e))^(1/2)/f
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(1/2),x, algorithm="maxima"
)
```

```
[Out] Timed out
```

Fricas [A] time = 1.16261, size = 663, normalized size = 4.14

$$\frac{15\sqrt{2}\left(\cos\left(fx+e\right)^{3}-2\cos\left(fx+e\right)\sin\left(fx+e\right)-2\cos\left(fx+e\right)\right)\sqrt{c}\log\left(-\frac{c\cos\left(fx+e\right)^{2}+2\sqrt{2}\sqrt{-c\sin\left(fx+e\right)+c}\sqrt{c}\left(\cos\left(fx+e\right)+\sin\right.}{\cos\left(fx+e\right)^{2}+\left(\cos\left(fx+e\right)\right.}\right.}{240\left(a^{3}cf\cos\left(fx+e\right)^{3}-2a^{3}cf\cos\left(fx+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(1/2),x, algorithm="fricas"
)
```

```
[Out] 1/240*(15*sqrt(2)*(cos(f*x + e)^3 - 2*cos(f*x + e)*sin(f*x + e) - 2*cos(f*x
 + e))*sqrt(c)*log(-(c*cos(f*x + e)^2 + 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)
*sqrt(c)*(cos(f*x + e) + sin(f*x + e) + 1) + 3*c*cos(f*x + e) + (c*cos(f*x
+ e) - 2*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*
x + e) - cos(f*x + e) - 2)) - 4*(15*cos(f*x + e)^2 - 40*sin(f*x + e) - 52)*
sqrt(-c*sin(f*x + e) + c))/(a^3*c*f*cos(f*x + e)^3 - 2*a^3*c*f*cos(f*x + e)
*sin(f*x + e) - 2*a^3*c*f*cos(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.77975, size = 891, normalized size = 5.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(1/2),x, algorithm="giac")

[Out] 1/60*((870*sqrt(2)*c*arctan(sqrt(c)/sqrt(-c)) - 1230*c*arctan(sqrt(c)/sqrt(
-c)) - 850*sqrt(2)*sqrt(-c)*sqrt(c) + 1203*sqrt(-c)*sqrt(c))*sgn(tan(1/2*f*
x + 1/2*e) - 1)/(41*sqrt(2)*a^3*sqrt(-c)*c - 58*a^3*sqrt(-c)*c) + 15*sqrt(2
)*arctan(-1/2*sqrt(2)*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x +
1/2*e)^2 + c) - sqrt(c))/sqrt(-c))/(a^3*sqrt(-c)*sgn(tan(1/2*f*x + 1/2*e) -
 1)) + 2*(105*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2
 + c))^9 + 435*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^
2 + c))^8*sqrt(c) + 580*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x
+ 1/2*e)^2 + c))^7*c - 620*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f
*x + 1/2*e)^2 + c))^6*c^(3/2) - 1258*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c
*tan(1/2*f*x + 1/2*e)^2 + c))^5*c^2 + 490*(sqrt(c)*tan(1/2*f*x + 1/2*e) - s
qrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^4*c^(5/2) + 900*(sqrt(c)*tan(1/2*f*x + 1
/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^3*c^3 - 860*(sqrt(c)*tan(1/2*f*
x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^2*c^(7/2) + 265*(sqrt(c)*t
an(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))*c^4 - 37*c^(9/2))
/(((sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^2 +
2*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))*sqrt(
c) - c)^5*a^3*sgn(tan(1/2*f*x + 1/2*e) - 1)))/f
```

3.338 $\int \frac{1}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=191

$$-\frac{\sec^5(e+fx)(c-c\sin(e+fx))^{3/2}}{5a^3c^3f}-\frac{7\sec^3(e+fx)\sqrt{c-c\sin(e+fx)}}{30a^3c^2f}+\frac{7\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{16\sqrt{2}a^3c^{3/2}f}+\frac{7\cos(e+f}{16a^3f(c-c\sin(e}$$

```
[Out] (7*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])])/(16*
Sqrt[2]*a^3*c^(3/2)*f) + (7*Cos[e + f*x])/(16*a^3*f*(c - c*Sin[e + f*x])^(3
/2)) - (7*Sec[e + f*x])/(12*a^3*c*f*Sqrt[c - c*Sin[e + f*x]]) - (7*Sec[e +
f*x]^3*Sqrt[c - c*Sin[e + f*x]])/(30*a^3*c^2*f) - (Sec[e + f*x]^5*(c - c*Si
n[e + f*x])^(3/2))/(5*a^3*c^3*f)
```

Rubi [A] time = 0.328142, antiderivative size = 191, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2736, 2675, 2687, 2650, 2649, 206}

$$-\frac{\sec^5(e+fx)(c-c\sin(e+fx))^{3/2}}{5a^3c^3f}-\frac{7\sec^3(e+fx)\sqrt{c-c\sin(e+fx)}}{30a^3c^2f}+\frac{7\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{16\sqrt{2}a^3c^{3/2}f}+\frac{7\cos(e+f}{16a^3f(c-c\sin(e}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(3/2)),x]
```

```
[Out] (7*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])])/(16*
Sqrt[2]*a^3*c^(3/2)*f) + (7*Cos[e + f*x])/(16*a^3*f*(c - c*Sin[e + f*x])^(3
/2)) - (7*Sec[e + f*x])/(12*a^3*c*f*Sqrt[c - c*Sin[e + f*x]]) - (7*Sec[e +
f*x]^3*Sqrt[c - c*Sin[e + f*x]])/(30*a^3*c^2*f) - (Sec[e + f*x]^5*(c - c*Si
n[e + f*x])^(3/2))/(5*a^3*c^3*f)
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2675

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^m)/(a*f*g*(p + 1)), x] + Dist[(a*(m + p + 1))/(g^2*(p + 1)), Int[(g*Cos
[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[p, -2*m] && IntegersQ[m
+ 1/2, 2*p]
```

Rule 2687

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p + 1)*Sq
rt[a + b*Sin[e + f*x]]), x] + Dist[(a*(2*p + 1))/(2*g^2*(p + 1)), Int[(g*Co
s[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e, f
```

```
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^{3/2}}\,dx = \frac{\int \sec^6(e+fx)(c-c\sin(e+fx))^{3/2}\,dx}{a^3c^3}$$
$$= -\frac{\sec^5(e+fx)(c-c\sin(e+fx))^{3/2}}{5a^3c^3f} + \frac{7\int \sec^4(e+fx)\sqrt{c-c\sin(e+}}{10a^3c^2}$$
$$= -\frac{7\sec^3(e+fx)\sqrt{c-c\sin(e+fx)}}{30a^3c^2f} - \frac{\sec^5(e+fx)(c-c\sin(e+fx))^{3}}{5a^3c^3f}$$
$$= -\frac{7\sec(e+fx)}{12a^3cf\sqrt{c-c\sin(e+fx)}} - \frac{7\sec^3(e+fx)\sqrt{c-c\sin(e+fx)}}{30a^3c^2f} - \text{se}$$
$$= \frac{7\cos(e+fx)}{16a^3f(c-c\sin(e+fx))^{3/2}} - \frac{7\sec(e+fx)}{12a^3cf\sqrt{c-c\sin(e+fx)}} - \frac{7\sec^3(e+}{}$$
$$= \frac{7\cos(e+fx)}{16a^3f(c-c\sin(e+fx))^{3/2}} - \frac{7\sec(e+fx)}{12a^3cf\sqrt{c-c\sin(e+fx)}} - \frac{7\sec^3(e+}{}$$
$$= \frac{7\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{16\sqrt{2}a^3c^{3/2}f} + \frac{7\cos(e+fx)}{16a^3f(c-c\sin(e+fx))^{3/2}} - \frac{7\text{s}}{12a^3cf\sqrt{}}$$

Mathematica [C] time = 1.2306, size = 174, normalized size = 0.91

$$\frac{\left(\frac{1}{1920}+\frac{i}{1920}\right)\cos(e+fx)\left((1-i)(-231\sin(e+fx)+105\sin(3(e+fx))+350\cos(2(e+fx))+206)+840\sqrt[4]{-1}\tan^{-1}\right.}{a^3cf(\sin(e+fx)-1)(\sin(e+f}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(3/2)),x]
```

```
[Out] ((1/1920 + I/1920)*Cos[e + f*x]*(840*(-1)^(1/4)*ArcTan[(1/2 + I/2)*(-1)^(1/
4)*(1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*(Cos[(e
```

```
+ f*x)/2] + Sin[(e + f*x)/2])^5 + (1 - I)*(206 + 350*Cos[2*(e + f*x)] - 231
*Sin[e + f*x] + 105*Sin[3*(e + f*x)])))/(a^3*c*f*(-1 + Sin[e + f*x])*(1 + S
in[e + f*x])^3*Sqrt[c - c*Sin[e + f*x]])
```

Maple [A] time = 0.603, size = 170, normalized size = 0.9

$$-\frac{1}{480\,a^{3}\left(1+\sin\left(fx+e\right)\right)^{2}\cos\left(fx+e\right)f}\left(105\left(c\left(1+\sin\left(fx+e\right)\right)\right)^{5/2}\sqrt{2}\mathrm{Artanh}\left(1/2\,\frac{\sqrt{c\left(1+\sin\left(fx+e\right)\right)}\sqrt{2}}{\sqrt{c}}\right)\sin\left(f\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] -1/480/c^(9/2)/a^3*(105*(c*(1+sin(f*x+e)))^(5/2)*2^(1/2)*arctanh(1/2*(c*(1+
sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)*c-105*(c*(1+sin(f*x+e)))^(5/
2)*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*c+42*c^(7/
2)*sin(f*x+e)-350*c^(7/2)*sin(f*x+e)^2-210*c^(7/2)*sin(f*x+e)^3+278*c^(7/2)
)/(1+sin(f*x+e))^2/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(3/2),x, algorithm="maxima"
)
```

```
[Out] Timed out
```

Fricas [A] time = 1.21606, size = 645, normalized size = 3.38

$$\frac{105\,\sqrt{2}\left(\cos\left(fx+e\right)^{3}\sin\left(fx+e\right)+\cos\left(fx+e\right)^{3}\right)\sqrt{c}\log\left(-\frac{c\cos\left(fx+e\right)^{2}+2\,\sqrt{2}\sqrt{-c\sin\left(fx+e\right)+c}\sqrt{c}\left(\cos\left(fx+e\right)+\sin\left(fx+e\right)+1\right)+3\,c\cos\left(fx\right.}{\cos\left(fx+e\right)^{2}+\left(\cos\left(fx+e\right)+2\right)\sin\left(fx+e\right)-\cos\left(f\right.}\right.}{960\left(a^{3}c^{2}f\cos\left(fx+e\right)^{3}\sin\left(f\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(3/2),x, algorithm="fricas"
)
```

```
[Out] 1/960*(105*sqrt(2)*(cos(f*x + e)^3*sin(f*x + e) + cos(f*x + e)^3)*sqrt(c)*l
og(-(c*cos(f*x + e)^2 + 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*sqrt(c)*(cos(f*
x + e) + sin(f*x + e) + 1) + 3*c*cos(f*x + e) + (c*cos(f*x + e) - 2*c)*sin(
f*x + e) + 2*c)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x
 + e) - 2)) - 4*(175*cos(f*x + e)^2 + 21*(5*cos(f*x + e)^2 - 4)*sin(f*x + e
) - 36)*sqrt(-c*sin(f*x + e) + c))/(a^3*c^2*f*cos(f*x + e)^3*sin(f*x + e) +
 a^3*c^2*f*cos(f*x + e)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] sage2
```

3.339 $\int \frac{1}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=228

$$-\frac{\sec^5(e+fx)\sqrt{c-c\sin(e+fx)}}{5a^3c^3f}-\frac{3\sec^3(e+fx)}{10a^3c^2f\sqrt{c-c\sin(e+fx)}}-\frac{21\sec(e+fx)}{32a^3c^2f\sqrt{c-c\sin(e+fx)}}+\frac{63\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{128\sqrt{2}a^3c^{5/2}f}$$

```
[Out] (63*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])])/(12
8*Sqrt[2]*a^3*c^(5/2)*f) + (63*Cos[e + f*x])/(128*a^3*c*f*(c - c*Sin[e + f*
x])^(3/2)) + (21*Sec[e + f*x])/(80*a^3*c*f*(c - c*Sin[e + f*x])^(3/2)) - (2
1*Sec[e + f*x])/(32*a^3*c^2*f*Sqrt[c - c*Sin[e + f*x]]) - (3*Sec[e + f*x]^3
)/(10*a^3*c^2*f*Sqrt[c - c*Sin[e + f*x]]) - (Sec[e + f*x]^5*Sqrt[c - c*Sin[
e + f*x]])/(5*a^3*c^3*f)
```

Rubi [A] time = 0.407421, antiderivative size = 228, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2736, 2675, 2687, 2681, 2650, 2649, 206}

$$-\frac{\sec^5(e+fx)\sqrt{c-c\sin(e+fx)}}{5a^3c^3f}-\frac{3\sec^3(e+fx)}{10a^3c^2f\sqrt{c-c\sin(e+fx)}}-\frac{21\sec(e+fx)}{32a^3c^2f\sqrt{c-c\sin(e+fx)}}+\frac{63\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{128\sqrt{2}a^3c^{5/2}f}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(5/2)),x]
```


```
[Out] (63*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])])/(12
8*Sqrt[2]*a^3*c^(5/2)*f) + (63*Cos[e + f*x])/(128*a^3*c*f*(c - c*Sin[e + f*
x])^(3/2)) + (21*Sec[e + f*x])/(80*a^3*c*f*(c - c*Sin[e + f*x])^(3/2)) - (2
1*Sec[e + f*x])/(32*a^3*c^2*f*Sqrt[c - c*Sin[e + f*x]]) - (3*Sec[e + f*x]^3
)/(10*a^3*c^2*f*Sqrt[c - c*Sin[e + f*x]]) - (Sec[e + f*x]^5*Sqrt[c - c*Sin[
e + f*x]])/(5*a^3*c^3*f)
```


Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2675

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^m)/(a*f*g*(p + 1)), x] + Dist[(a*(m + p + 1))/(g^2*(p + 1)), Int[(g*Cos
[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[p, -2*m] && IntegersQ[m
+ 1/2, 2*p]
```

Rule 2687

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p + 1)*Sq
rt[a + b*Sin[e + f*x]]), x] + Dist[(a*(2*p + 1))/(2*g^2*(p + 1)), Int[(g*Co
```

```
s[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 2681

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*(2*m + p + 1)), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
 IntegersQ[2*m, 2*p]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^{5/2}}\,dx = \frac{\int \sec^6(e+fx)\sqrt{c-c\sin(e+fx)}\,dx}{a^3c^3}$$

$$= -\frac{\sec^5(e+fx)\sqrt{c-c\sin(e+fx)}}{5a^3c^3f} + \frac{9\int \frac{\sec^4(e+fx)}{\sqrt{c-c\sin(e+fx)}}\,dx}{10a^3c^2}$$

$$= -\frac{3\sec^3(e+fx)}{10a^3c^2f\sqrt{c-c\sin(e+fx)}} - \frac{\sec^5(e+fx)\sqrt{c-c\sin(e+fx)}}{5a^3c^3f} + \frac{21}{}$$

$$= \frac{21\sec(e+fx)}{80a^3cf(c-c\sin(e+fx))^{3/2}} - \frac{3\sec^3(e+fx)}{10a^3c^2f\sqrt{c-c\sin(e+fx)}} - \frac{\sec^5(e+}{}$$

$$= \frac{21\sec(e+fx)}{80a^3cf(c-c\sin(e+fx))^{3/2}} - \frac{21\sec(e+fx)}{32a^3c^2f\sqrt{c-c\sin(e+fx)}} - \frac{3}{10a^3c^2f}$$

$$= \frac{63\cos(e+fx)}{128a^3cf(c-c\sin(e+fx))^{3/2}} + \frac{21\sec(e+fx)}{80a^3cf(c-c\sin(e+fx))^{3/2}} - \frac{}{32a^3c}$$

$$= \frac{63\cos(e+fx)}{128a^3cf(c-c\sin(e+fx))^{3/2}} + \frac{21\sec(e+fx)}{80a^3cf(c-c\sin(e+fx))^{3/2}} - \frac{}{32a^3c}$$

$$= \frac{63\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{128\sqrt{2}a^3c^{5/2}f} + \frac{63\cos(e+fx)}{128a^3cf(c-c\sin(e+fx))^{3/2}} + \frac{}{80a^3c}$$

Mathematica [C] time = 1.50582, size = 443, normalized size = 1.94

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(-240\cos^4(e+fx)+75\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(5/2)),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)*(-240*Cos[e + f*x]^4 - 32*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4 - 80*(C
os[(e + f*x)/2] - Sin[(e + f*x)/2])^4*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])
^2 + 20*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e +
f*x)/2])^5 + 75*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(Cos[(e + f*x)/2] +
 Sin[(e + f*x)/2])^5 - (315 + 315*I)*(-1)^(1/4)*ArcTan[(1/2 + I/2)*(-1)^(1/
4)*(1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*(Cos[(e
+ f*x)/2] + Sin[(e + f*x)/2])^5 + 40*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + S
in[(e + f*x)/2])^5 + 150*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*Sin[(e + f
*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5))/(640*a^3*f*(1 + Sin[e + f*
x])^3*(c - c*Sin[e + f*x])^(5/2))
```

Maple [A] time = 0.767, size = 246, normalized size = 1.1

$$-\frac{1}{1280\,a^3\left(1+\sin\left(fx+e\right)\right)^2\left(-1+\sin\left(fx+e\right)\right)\cos\left(fx+e\right)f}\left(315\,\left(c\left(1+\sin\left(fx+e\right)\right)\right)^{5/2}\sqrt{2}\mathrm{Artanh}\left(1/2\,\frac{\sqrt{c\left(1+\sin\right.}}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] -1/1280/c^(13/2)/a^3*(315*(c*(1+sin(f*x+e)))^(5/2)*2^(1/2)*arctanh(1/2*(c*(
1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)^2*c^2+1176*c^(9/2)*sin(f*x
+e)^2-420*c^(9/2)*sin(f*x+e)^3-630*c^(9/2)*sin(f*x+e)^4-630*(c*(1+sin(f*x+e
)))^(5/2)*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin
(f*x+e)*c^2+708*c^(9/2)*sin(f*x+e)+315*(c*(1+sin(f*x+e)))^(5/2)*2^(1/2)*arc
tanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*c^2-514*c^(9/2))/(1+sin(
f*x+e))^2/(-1+sin(f*x+e))/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(5/2),x, algorithm="maxima"
)
```

```
[Out] Timed out
```

Fricas [A] time = 1.20115, size = 578, normalized size = 2.54

$$\frac{315\sqrt{2}\sqrt{c}\cos\left(fx+e\right)^5\log\left(-\frac{c\cos\left(fx+e\right)^2+2\sqrt{2}\sqrt{-c\sin\left(fx+e\right)+c}\sqrt{c}\left(\cos\left(fx+e\right)+\sin\left(fx+e\right)+1\right)+3c\cos\left(fx+e\right)+\left(c\cos\left(fx+e\right)-2c\right)\sin\left(fx+e\right)+}{\cos\left(fx+e\right)^2+\left(\cos\left(fx+e\right)+2\right)\sin\left(fx+e\right)-\cos\left(fx+e\right)-2}\right.}{2560\,a^3c^3f\cos}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(5/2),x, algorithm="fricas"
)
```

```
[Out] 1/2560*(315*sqrt(2)*sqrt(c)*cos(f*x + e)^5*log(-(c*cos(f*x + e)^2 + 2*sqrt(
2)*sqrt(-c*sin(f*x + e) + c)*sqrt(c)*(cos(f*x + e) + sin(f*x + e) + 1) + 3*
c*cos(f*x + e) + (c*cos(f*x + e) - 2*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2
 + (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) - 4*(315*cos(f*x +
e)^4 - 42*cos(f*x + e)^2 - 6*(35*cos(f*x + e)^2 + 24)*sin(f*x + e) - 16)*sq
rt(-c*sin(f*x + e) + c))/(a^3*c^3*f*cos(f*x + e)^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] sage2
```

3.340 $\int \sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{7/2}\,dx$

Optimal. Leaf size=43

$$-\frac{a\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4f\sqrt{a\sin(e+fx)+a}}$$

```
[Out] -(a*Cos[e + f*x]*(c - c*Sin[e + f*x])^(7/2))/(4*f*Sqrt[a + a*Sin[e + f*x]])
```

Rubi [A] time = 0.0829605, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {2738}

$$-\frac{a\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4f\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(7/2),x]
```

```
[Out] -(a*Cos[e + f*x]*(c - c*Sin[e + f*x])^(7/2))/(4*f*Sqrt[a + a*Sin[e + f*x]])
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\int \sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{7/2}\,dx=-\frac{a\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4f\sqrt{a+a\sin(e+fx)}}$$

Mathematica [A] time = 0.395553, size = 83, normalized size = 1.93

$$-\frac{c^3\sec(e+fx)\sqrt{a(\sin(e+fx)+1)}\sqrt{c-c\sin(e+fx)}(8(\sin(3(e+fx))-7\sin(e+fx))-28\cos(2(e+fx))+\cos(4(e+}{32f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(7/2),x]
```

```
[Out] -(c^3*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]]*(-28
*Cos[2*(e + f*x)] + Cos[4*(e + f*x)] + 8*(-7*Sin[e + f*x] + Sin[3*(e + f*x)
])))/(32*f)
```

Maple [B] time = 0.211, size = 103, normalized size = 2.4

$$\frac{\sin(fx+e)\left((\cos(fx+e))^6+\sin(fx+e)(\cos(fx+e))^4+(\cos(fx+e))^2\sin(fx+e)-(\cos(fx+e))^2+4\sin(fx}{4f(\cos(fx+e))^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(1/2)*(c-c*sin(f*x+e))^(7/2),x)
```

```
[Out] 1/4/f*(-c*(-1+sin(f*x+e)))^(7/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(1/2)*(cos(f
*x+e)^6+sin(f*x+e)*cos(f*x+e)^4+cos(f*x+e)^2*sin(f*x+e)-cos(f*x+e)^2+4*sin(
f*x+e)+4)/cos(f*x+e)^7
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a \sin\left(fx+e\right)+a}\left(-c \sin\left(fx+e\right)+c\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(c-c*sin(f*x+e))^(7/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e) + c)^(7/2), x)
```

Fricas [B] time = 1.13083, size = 232, normalized size = 5.4

$$-\frac{\left(c^{3} \cos\left(fx+e\right)^{4}-8 c^{3} \cos\left(fx+e\right)^{2}+7 c^{3}+4 \left(c^{3} \cos\left(fx+e\right)^{2}-2 c^{3}\right) \sin\left(fx+e\right)\right) \sqrt{a \sin\left(fx+e\right)+a} \sqrt{-c \sin\left(f\right.}}{4 f \cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(c-c*sin(f*x+e))^(7/2),x, algorithm="frica
s")
```

```
[Out] -1/4*(c^3*cos(f*x + e)^4 - 8*c^3*cos(f*x + e)^2 + 7*c^3 + 4*(c^3*cos(f*x +
e)^2 - 2*c^3)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) +
 c)/(f*cos(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(1/2)*(c-c*sin(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(c-c*sin(f*x+e))^(7/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError
```

3.341 $\int \sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}\,dx$

Optimal. Leaf size=43

$$-\frac{a\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3f\sqrt{a\sin(e+fx)+a}}$$

```
[Out] -(a*Cos[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(3*f*Sqrt[a + a*Sin[e + f*x]])
```

Rubi [A] time = 0.0820688, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {2738}

$$-\frac{a\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3f\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] -(a*Cos[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(3*f*Sqrt[a + a*Sin[e + f*x]])
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\int \sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}\,dx=-\frac{a\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3f\sqrt{a+a\sin(e+fx)}}$$

Mathematica [A] time = 0.286207, size = 74, normalized size = 1.72

$$\frac{c^2\sec(e+fx)\sqrt{a(\sin(e+fx)+1)}\sqrt{c-c\sin(e+fx)}(15\sin(e+fx)-\sin(3(e+fx))+6\cos(2(e+fx)))}{12f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] (c^2*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]]*(6*Co
s[2*(e + f*x)] + 15*Sin[e + f*x] - Sin[3*(e + f*x)]))/(12*f)
```

Maple [B] time = 0.179, size = 78, normalized size = 1.8

$$\frac{\sin\left(fx+e\right)\left(\left(\cos\left(fx+e\right)\right)^{4}+\left(\cos\left(fx+e\right)\right)^{2}\sin\left(fx+e\right)+2\sin\left(fx+e\right)+2\right)}{3f\left(\cos\left(fx+e\right)\right)^{5}}\left(-c\left(-1+\sin\left(fx+e\right)\right)\right)^{\frac{5}{2}}\sqrt{a\left(1+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(1/2)*(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] 1/3/f*(-c*(-1+sin(f*x+e)))^(5/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(1/2)*(cos(f
*x+e)^4+cos(f*x+e)^2*sin(f*x+e)+2*sin(f*x+e)+2)/cos(f*x+e)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a \sin \left(fx+e\right)+a}\left(-c \sin \left(fx+e\right)+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(c-c*sin(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e) + c)^(5/2), x)
```

Fricas [B] time = 1.10867, size = 200, normalized size = 4.65

$$\frac{\left(3\,c^{2}\cos \left(fx+e\right)^{2}-3\,c^{2}-\left(c^{2}\cos \left(fx+e\right)^{2}-4\,c^{2}\right)\sin \left(fx+e\right)\right)\sqrt{a \sin \left(fx+e\right)+a}\sqrt{-c \sin \left(fx+e\right)+c}}{3\,f \cos \left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(c-c*sin(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] 1/3*(3*c^2*cos(f*x + e)^2 - 3*c^2 - (c^2*cos(f*x + e)^2 - 4*c^2)*sin(f*x +
e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(1/2)*(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(c-c*sin(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError
```

3.342 $\int \sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=43

$$-\frac{a\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}$$

```
[Out] -(a*Cos[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(2*f*Sqrt[a + a*Sin[e + f*x]])
```

Rubi [A] time = 0.0805129, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {2738}

$$-\frac{a\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] -(a*Cos[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(2*f*Sqrt[a + a*Sin[e + f*x]])
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\int \sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}\,dx=-\frac{a\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a+a\sin(e+fx)}}$$

Mathematica [A] time = 0.21214, size = 60, normalized size = 1.4

$$\frac{c\sec(e+fx)\sqrt{a(\sin(e+fx)+1)}\sqrt{c-c\sin(e+fx)}(4\sin(e+fx)+\cos(2(e+fx)))}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] (c*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*(Cos[2*(e + f*x)] + 4*Sin[e + f*
x])*Sqrt[c - c*Sin[e + f*x]])/(4*f)
```

Maple [A] time = 0.176, size = 61, normalized size = 1.4

$$\frac{\sin(fx+e)\left(\left(\cos(fx+e)\right)^2+\sin(fx+e)+1\right)}{2f\left(\cos(fx+e)\right)^3}\left(-c\left(-1+\sin(fx+e)\right)\right)^{\frac{3}{2}}\sqrt{a\left(1+\sin(fx+e)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(1/2)*(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] 1/2/f*(-c*(-1+sin(f*x+e)))^(3/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(1/2)*(cos(f
*x+e)^2+sin(f*x+e)+1)/cos(f*x+e)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\sin\left(fx+e\right)+a}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(c-c*sin(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e) + c)^(3/2), x)
```

Fricas [A] time = 1.09567, size = 155, normalized size = 3.6

$$\frac{\left(c\cos\left(fx+e\right)^{2}+2\,c\sin\left(fx+e\right)-c\right)\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}}{2\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(c-c*sin(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] 1/2*(c*cos(f*x + e)^2 + 2*c*sin(f*x + e) - c)*sqrt(a*sin(f*x + e) + a)*sqrt
(-c*sin(f*x + e) + c)/(f*cos(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(1/2)*(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(c-c*sin(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: AttributeError
```

3.343 $\int \sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}\,dx$

Optimal. Leaf size=41

$$-\frac{a\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}$$

```
[Out] -((a*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f*x]]))
```

Rubi [A] time = 0.0730127, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {2738}

$$-\frac{a\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] -((a*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f*x]]))
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\int \sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}\,dx=-\frac{a\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a+a\sin(e+fx)}}$$

Mathematica [A] time = 0.0869807, size = 39, normalized size = 0.95

$$\frac{\tan(e+fx)\sqrt{a(\sin(e+fx)+1)}\sqrt{c-c\sin(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]]*Tan[e + f*x])/f
```

Maple [A] time = 0.178, size = 44, normalized size = 1.1

$$\frac{\sin\left(fx+e\right)}{f\cos\left(fx+e\right)}\sqrt{-c\left(-1+\sin\left(fx+e\right)\right)}\sqrt{a\left(1+\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(1/2)*(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] 1/f*(-c*(-1+sin(f*x+e)))^(1/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(1/2)/cos(f*x+
e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(c-c*sin(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c), x)
```

Fricas [A] time = 1.02725, size = 111, normalized size = 2.71

$$\frac{\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}\sin\left(fx+e\right)}{f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(c-c*sin(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*sin(f*x + e)/(f*cos(f*x
+ e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\left(\sin\left(e+fx\right)+1\right)}\sqrt{-c\left(\sin\left(e+fx\right)-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(1/2)*(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))*sqrt(-c*(sin(e + f*x) - 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(c-c*sin(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c), x)
```

3.344 $\int \frac{\sqrt{a+a\sin(e+fx)}}{\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=52

$$-\frac{a\cos(e+fx)\log(1-\sin(e+fx))}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

```
[Out] -((a*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c
 - c*Sin[e + f*x]]))
```

Rubi [A] time = 0.100986, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {2737, 2667, 31}

$$-\frac{a\cos(e+fx)\log(1-\sin(e+fx))}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]/Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] -((a*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c
 - c*Sin[e + f*x]]))
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+a\sin(e+fx)}}{\sqrt{c-c\sin(e+fx)}}\,dx &= \frac{(ac\cos(e+fx))\int \frac{\cos(e+fx)}{c-c\sin(e+fx)}\,dx}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}\\
&= -\frac{(a\cos(e+fx))\,\text{Subst}\left(\int \frac{1}{c+x}\,dx, x, -c\sin(e+fx)\right)}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}\\
&= -\frac{a\cos(e+fx)\log(1-\sin(e+fx))}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}
\end{aligned}$$

Mathematica [C] time = 0.919851, size = 119, normalized size = 2.29

$$-\frac{\sqrt{2}\left(e^{i(e+fx)}-i\right)\left(fx+2i\log\left(i-e^{i(e+fx)}\right)\right)\sqrt{a(\sin(e+fx)+1)}}{f\left(e^{i(e+fx)}+i\right)\sqrt{ice^{-i(e+fx)}\left(e^{i(e+fx)}-i\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]/Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] -((Sqrt[2]*(-I + E^(I*(e + f*x)))*(f*x + (2*I)*Log[I - E^(I*(e + f*x))])*Sq
rt[a*(1 + Sin[e + f*x])])/(Sqrt[(I*c*(-I + E^(I*(e + f*x)))^2)/E^(I*(e + f*
x))]*(I + E^(I*(e + f*x)))*f))
```

Maple [B] time = 0.161, size = 106, normalized size = 2.

$$-\frac{-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)}{f\left(1-\cos\left(fx+e\right)+\sin\left(fx+e\right)\right)}\sqrt{a\left(1+\sin\left(fx+e\right)\right)}\left(2\ln\left(-\frac{-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)-\ln\left(2\left(\cos\left(fx\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] -1/f*(a*(1+sin(f*x+e)))^(1/2)*(-1+cos(f*x+e)+sin(f*x+e))*(2*ln(-(-1+cos(f*x
+e)+sin(f*x+e))/sin(f*x+e))-ln(2/(cos(f*x+e)+1)))/(1-cos(f*x+e)+sin(f*x+e))
/(-c*(-1+sin(f*x+e)))^(1/2)
```

Maxima [A] time = 1.82271, size = 85, normalized size = 1.63

$$\frac{\frac{2\sqrt{a}\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{\sqrt{c}}-\frac{\sqrt{a}\log\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)}{\sqrt{c}}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] (2*sqrt(a)*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/sqrt(c) - sqrt(a)*log(s
in(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/sqrt(c))/f
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}}{c\sin\left(fx+e\right)-c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] integral(-sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c*sin(f*x + e
) - c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}}{\sqrt{-c\left(\sin\left(e+fx\right)-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(1/2)/(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))/sqrt(-c*(sin(e + f*x) - 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a\sin\left(fx+e\right)+a}}{\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)/sqrt(-c*sin(f*x + e) + c), x)
```

3.345 $\int \frac{\sqrt{a+a\sin(e+fx)}}{(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=40

$$\frac{a\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}$$

```
[Out] (a*Cos[e + f*x])/(f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2))
```

Rubi [A] time = 0.0823106, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {2738}

$$\frac{a\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]/(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] (a*Cos[e + f*x])/(f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2))
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sin(e+fx)}}{(c-c\sin(e+fx))^{3/2}}\,dx = \frac{a\cos(e+fx)}{f\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}}$$

Mathematica [B] time = 0.208228, size = 84, normalized size = 2.1

$$\frac{\sqrt{a(\sin(e+fx)+1)}\sqrt{c-c\sin(e+fx)}}{c^2f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]/(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] (Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]])/(c^2*f*(Cos[(e + f*x)
/2] - Sin[(e + f*x)/2])^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))
```

Maple [A] time = 0.171, size = 68, normalized size = 1.7

$$\frac{\left(-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)\right)\sin\left(fx+e\right)}{f\left(1-\cos\left(fx+e\right)+\sin\left(fx+e\right)\right)}\sqrt{a\left(1+\sin\left(fx+e\right)\right)}\left(-c\left(-1+\sin\left(fx+e\right)\right)\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] 1/f*(-1+cos(f*x+e)+sin(f*x+e))*sin(f*x+e)*(a*(1+sin(f*x+e)))^(1/2)/(1-cos(f
*x+e)+sin(f*x+e))/(-c*(-1+sin(f*x+e)))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a \sin \left(fx+e\right)+a}}{\left(-c \sin \left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)/(-c*sin(f*x + e) + c)^(3/2), x)
```

Fricas [A] time = 1.04962, size = 146, normalized size = 3.65

$$-\frac{\sqrt{a \sin \left(fx+e\right)+a}\sqrt{-c \sin \left(fx+e\right)+c}}{c^2 f \cos \left(fx+e\right) \sin \left(fx+e\right)-c^2 f \cos \left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] -sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c^2*f*cos(f*x + e)*sin
(f*x + e) - c^2*f*cos(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a \left(\sin \left(e+fx\right)+1\right)}}{\left(-c \left(\sin \left(e+fx\right)-1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(1/2)/(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))/(-c*(sin(e + f*x) - 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a \sin \left(fx+e\right)+a}}{\left(-c \sin \left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)/(-c*sin(f*x + e) + c)^(3/2), x)
```

3.346 $\int \frac{\sqrt{a+a\sin(e+fx)}}{(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=43

$$\frac{a\cos(e+fx)}{2f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/2}}$$

```
[Out] (a*Cos[e + f*x])/(2*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(5/2))
```

Rubi [A] time = 0.0849277, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {2738}

$$\frac{a\cos(e+fx)}{2f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]/(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] (a*Cos[e + f*x])/(2*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(5/2))
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sin(e+fx)}}{(c-c\sin(e+fx))^{5/2}}\,dx=\frac{a\cos(e+fx)}{2f\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}}$$

Mathematica [B] time = 0.206599, size = 87, normalized size = 2.02

$$\frac{\sqrt{a(\sin(e+fx)+1)}\sqrt{c-c\sin(e+fx)}}{2c^3f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^5\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]/(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] (Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]])/(2*c^3*f*(Cos[(e + f*
x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))
```

Maple [B] time = 0.178, size = 96, normalized size = 2.2

$$-\frac{\left(\sin\left(fx+e\right)\cos\left(fx+e\right)-\left(\cos\left(fx+e\right)\right)^2-3\sin\left(fx+e\right)-2\cos\left(fx+e\right)+3\right)\sin\left(fx+e\right)}{2f\left(1-\cos\left(fx+e\right)+\sin\left(fx+e\right)\right)}\sqrt{a\left(1+\sin\left(fx+e\right)\right)}\,(\cdot$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] -1/2/f*(sin(f*x+e)*cos(f*x+e)-cos(f*x+e)^2-3*sin(f*x+e)-2*cos(f*x+e)+3)*sin
(f*x+e)*(a*(1+sin(f*x+e)))^(1/2)/(1-cos(f*x+e)+sin(f*x+e))/(-c*(-1+sin(f*x+
e)))^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a \sin\left(fx+e\right)+a}}{\left(-c \sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)/(-c*sin(f*x + e) + c)^(5/2), x)
```

Fricas [A] time = 1.09644, size = 188, normalized size = 4.37

$$-\frac{\sqrt{a \sin\left(fx+e\right)+a}\sqrt{-c \sin\left(fx+e\right)+c}}{2\left(c^{3} f \cos\left(fx+e\right)^{3}+2\,c^{3} f \cos\left(fx+e\right)\sin\left(fx+e\right)-2\,c^{3} f \cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] -1/2*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c^3*f*cos(f*x + e)
^3 + 2*c^3*f*cos(f*x + e)*sin(f*x + e) - 2*c^3*f*cos(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(1/2)/(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a \sin\left(fx+e\right)+a}}{\left(-c \sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)/(-c*sin(f*x + e) + c)^(5/2), x)
```

3.347 $\int \frac{\sqrt{a+a\sin(e+fx)}}{(c-c\sin(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=43

$$\frac{a\cos(e+fx)}{3f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{7/2}}$$

```
[Out] (a*Cos[e + f*x])/(3*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(7/2))
```

Rubi [A] time = 0.08369, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {2738}

$$\frac{a\cos(e+fx)}{3f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]/(c - c*Sin[e + f*x])^(7/2),x]
```

```
[Out] (a*Cos[e + f*x])/(3*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(7/2))
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sin(e+fx)}}{(c-c\sin(e+fx))^{7/2}}\,dx = \frac{a\cos(e+fx)}{3f\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{7/2}}$$

Mathematica [B] time = 0.274498, size = 87, normalized size = 2.02

$$\frac{\sqrt{a(\sin(e+fx)+1)}\sqrt{c-c\sin(e+fx)}}{3c^4f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]/(c - c*Sin[e + f*x])^(7/2),x]
```

```
[Out] (Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]])/(3*c^4*f*(Cos[(e + f*
x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))
```

Maple [B] time = 0.18, size = 120, normalized size = 2.8

$$-\frac{\left(\left(\cos\left(fx+e\right)\right)^2\sin\left(fx+e\right)+\left(\cos\left(fx+e\right)\right)^3+3\sin\left(fx+e\right)\cos\left(fx+e\right)-4\left(\cos\left(fx+e\right)\right)^2-7\sin\left(fx+e\right)-\right.}{3f\left(1-\cos\left(fx+e\right)+\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(7/2),x)
```

```
[Out] -1/3/f*(cos(f*x+e)^2*sin(f*x+e)+cos(f*x+e)^3+3*sin(f*x+e)*cos(f*x+e)-4*cos(
f*x+e)^2-7*sin(f*x+e)-4*cos(f*x+e)+7)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(1/2)/(
1-cos(f*x+e)+sin(f*x+e))/(-c*(-1+sin(f*x+e)))^(7/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a\sin\left(fx+e\right)+a}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(7/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)/(-c*sin(f*x + e) + c)^(7/2), x)
```

Fricas [B] time = 1.16651, size = 224, normalized size = 5.21

$$-\frac{\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}}{3\left(3\,c^{4}f\cos\left(fx+e\right)^{3}-4\,c^{4}f\cos\left(fx+e\right)-\left(c^{4}f\cos\left(fx+e\right)^{3}-4\,c^{4}f\cos\left(fx+e\right)\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(7/2),x, algorithm="frica
s")
```

```
[Out] -1/3*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(3*c^4*f*cos(f*x +
e)^3 - 4*c^4*f*cos(f*x + e) - (c^4*f*cos(f*x + e)^3 - 4*c^4*f*cos(f*x + e))
*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(1/2)/(c-c*sin(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a\sin\left(fx+e\right)+a}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(7/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)/(-c*sin(f*x + e) + c)^(7/2), x)
```

3.348 $\int (a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{7/2}\,dx$

Optimal. Leaf size=89

$$-\frac{a^2\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{10f\sqrt{a\sin(e+fx)+a}}-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{7/2}}{5f}$$

```
[Out] -(a^2*Cos[e + f*x]*(c - c*Sin[e + f*x])^(7/2))/(10*f*Sqrt[a + a*Sin[e + f*x
]]) - (a*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(7/2))/
(5*f)
```

Rubi [A] time = 0.187908, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2740, 2738}

$$-\frac{a^2\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{10f\sqrt{a\sin(e+fx)+a}}-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{7/2}}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(7/2),x]
```

```
[Out] -(a^2*Cos[e + f*x]*(c - c*Sin[e + f*x])^(7/2))/(10*f*Sqrt[a + a*Sin[e + f*x
]]) - (a*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(7/2))/
(5*f)
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] && !LtQ[n, -1] && !(IGtQ[n - 1/2, 0] && LtQ[n, m]) && !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{7/2}\,dx=-\frac{a\cos(e+fx)\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{7/2}}{5f}+\frac{1}{5}(2a)\int \sqrt{a+$$

$$=-\frac{a^2\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{10f\sqrt{a+a\sin(e+fx)}}-\frac{a\cos(e+fx)\sqrt{a+a\sin(e+fx}}{5f}$$

Mathematica [A] time = 1.02234, size = 146, normalized size = 1.64

$$-\frac{c^3(\sin(e+fx)-1)^3(a(\sin(e+fx)+1))^{3/2}\sqrt{c-c\sin(e+fx)}(70\sin(e+fx)+5\sin(3(e+fx))-\sin(5(e+fx))+20\cos}{80f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(7/2),x]
```

```
[Out] -(c^3*(-1 + Sin[e + f*x])^3*(a*(1 + Sin[e + f*x]))^(3/2)*Sqrt[c - c*Sin[e +
 f*x]]*(20*Cos[2*(e + f*x)] + 5*Cos[4*(e + f*x)] + 70*Sin[e + f*x] + 5*Sin[
3*(e + f*x)] - Sin[5*(e + f*x)]))/(80*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2
])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3)
```

Maple [A] time = 0.168, size = 106, normalized size = 1.2

$$\frac{\sin\left(fx+e\right)\left(2\left(\cos\left(fx+e\right)\right)^{6}+\sin\left(fx+e\right)\left(\cos\left(fx+e\right)\right)^{4}+2\left(\cos\left(fx+e\right)\right)^{4}+3\left(\cos\left(fx+e\right)\right)^{2}\sin\left(fx+e\right)+\right.}{10f\left(\cos\left(fx+e\right)\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(7/2),x)
```

```
[Out] 1/10/f*(-c*(-1+sin(f*x+e)))^(7/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(3/2)*(2*co
s(f*x+e)^6+sin(f*x+e)*cos(f*x+e)^4+2*cos(f*x+e)^4+3*cos(f*x+e)^2*sin(f*x+e)
+6*sin(f*x+e)+6)/cos(f*x+e)^7
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(7/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)*(-c*sin(f*x + e) + c)^(7/2), x)
```

Fricas [A] time = 1.46939, size = 248, normalized size = 2.79

$$\frac{\left(5\,ac^{3}\cos\left(fx+e\right)^{4}-5\,ac^{3}-2\left(ac^{3}\cos\left(fx+e\right)^{4}-2\,ac^{3}\cos\left(fx+e\right)^{2}-4\,ac^{3}\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c}}{10\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(7/2),x, algorithm="frica
s")
```

```
[Out] 1/10*(5*a*c^3*cos(f*x + e)^4 - 5*a*c^3 - 2*(a*c^3*cos(f*x + e)^4 - 2*a*c^3*
cos(f*x + e)^2 - 4*a*c^3)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*si
n(f*x + e) + c)/(f*cos(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*(c-c*sin(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(7/2),x, algorithm="giac"
)
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)*(-c*sin(f*x + e) + c)^(7/2), x)
```

3.349 $\int (a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}\,dx$

Optimal. Leaf size=89

$$-\frac{a^2\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{6f\sqrt{a\sin(e+fx)+a}}-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/2}}{4f}$$

```
[Out] -(a^2*Cos[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(6*f*Sqrt[a + a*Sin[e + f*x]
]) - (a*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(5/2))/(
4*f)
```

Rubi [A] time = 0.17939, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2740, 2738}

$$-\frac{a^2\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{6f\sqrt{a\sin(e+fx)+a}}-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/2}}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] -(a^2*Cos[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(6*f*Sqrt[a + a*Sin[e + f*x]
]) - (a*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(5/2))/(
4*f)
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] && !LtQ[n, -1] && !(IGtQ[n - 1/2, 0] && LtQ[n, m]) && !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}\,dx=-\frac{a\cos(e+fx)\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}}{4f}+\frac{1}{2}a\int\sqrt{a\cdots}$$
$$=-\frac{a^2\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{6f\sqrt{a+a\sin(e+fx)}}-\frac{a\cos(e+fx)\sqrt{a+a\sin(e+\cdots}}{4f}$$

Mathematica [A] time = 0.631901, size = 137, normalized size = 1.54

$$\frac{c^2(\sin(e+fx)-1)^2(a(\sin(e+fx)+1))^{3/2}\sqrt{c-c\sin(e+fx)}(8(9\sin(e+fx)+\sin(3(e+fx)))+12\cos(2(e+fx))+3\cdots}{96f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^5\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] (c^2*(-1 + Sin[e + f*x])^2*(a*(1 + Sin[e + f*x]))^(3/2)*Sqrt[c - c*Sin[e +
f*x]]*(12*Cos[2*(e + f*x)] + 3*Cos[4*(e + f*x)] + 8*(9*Sin[e + f*x] + Sin[3
*(e + f*x)])))/(96*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)
/2] + Sin[(e + f*x)/2])^3)
```

Maple [A] time = 0.148, size = 90, normalized size = 1.

$$\frac{\sin\left(fx+e\right)\left(3\left(\cos\left(fx+e\right)\right)^{4}+\left(\cos\left(fx+e\right)\right)^{2}\sin\left(fx+e\right)+4\left(\cos\left(fx+e\right)\right)^{2}+5\sin\left(fx+e\right)+5\right)}{12f\left(\cos\left(fx+e\right)\right)^{5}}\left(-c\left(-1+\sin\left(f\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] 1/12/f*(-c*(-1+sin(f*x+e)))^(5/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(3/2)*(3*co
s(f*x+e)^4+cos(f*x+e)^2*sin(f*x+e)+4*cos(f*x+e)^2+5*sin(f*x+e)+5)/cos(f*x+e
)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)*(-c*sin(f*x + e) + c)^(5/2), x)
```

Fricas [A] time = 1.42356, size = 215, normalized size = 2.42

$$\frac{\left(3\,ac^{2}\cos\left(fx+e\right)^{4}-3\,ac^{2}+4\left(ac^{2}\cos\left(fx+e\right)^{2}+2\,ac^{2}\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}}{12\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] 1/12*(3*a*c^2*cos(f*x + e)^4 - 3*a*c^2 + 4*(a*c^2*cos(f*x + e)^2 + 2*a*c^2)
*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*
x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError
```

3.350 $\int (a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=89

$$-\frac{a^2\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{3f\sqrt{a\sin(e+fx)+a}}-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}{3f}$$

```
[Out] -(a^2*Cos[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(3*f*Sqrt[a + a*Sin[e + f*x]
]) - (a*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2))/(
3*f)
```

Rubi [A] time = 0.178479, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2740, 2738}

$$-\frac{a^2\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{3f\sqrt{a\sin(e+fx)+a}}-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] -(a^2*Cos[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(3*f*Sqrt[a + a*Sin[e + f*x]
]) - (a*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2))/(
3*f)
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] && !LtQ[n, -1] && !(IGtQ[n - 1/2, 0] && LtQ[n, m]) && !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}\,dx=-\frac{a\cos(e+fx)\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}}{3f}+\frac{1}{3}(2a)\int \sqrt{a+$$

$$=-\frac{a^2\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{3f\sqrt{a+a\sin(e+fx)}}-\frac{a\cos(e+fx)\sqrt{a+a\sin(e+fx}}{3f}$$

Mathematica [A] time = 0.429396, size = 70, normalized size = 0.79

$$-\frac{c(\sin(e+fx)-1)(9\sin(e+fx)+\sin(3(e+fx)))\sec^3(e+fx)(a(\sin(e+fx)+1))^{3/2}\sqrt{c-c\sin(e+fx)}}{12f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] -(c*Sec[e + f*x]^3*(-1 + Sin[e + f*x])*(a*(1 + Sin[e + f*x]))^(3/2)*Sqrt[c
- c*Sin[e + f*x]]*(9*Sin[e + f*x] + Sin[3*(e + f*x)]))/(12*f)
```

Maple [A] time = 0.141, size = 55, normalized size = 0.6

$$\frac{\left(\left(\cos\left(fx+e\right)\right)^{2}+2\right)\sin\left(fx+e\right)}{3f\left(\cos\left(fx+e\right)\right)^{3}}\left(-c\left(-1+\sin\left(fx+e\right)\right)\right)^{\frac{3}{2}}\left(a\left(1+\sin\left(fx+e\right)\right)\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] 1/3/f*(cos(f*x+e)^2+2)*(-c*(-1+sin(f*x+e)))^(3/2)*(a*(1+sin(f*x+e)))^(3/2)*
sin(f*x+e)/cos(f*x+e)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)*(-c*sin(f*x + e) + c)^(3/2), x)
```

Fricas [A] time = 1.44806, size = 155, normalized size = 1.74

$$\frac{\left(ac\cos\left(fx+e\right)^{2}+2\,ac\right)\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}\sin\left(fx+e\right)}{3\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] 1/3*(a*c*cos(f*x + e)^2 + 2*a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x +
 e) + c)*sin(f*x + e)/(f*cos(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: AttributeError
```

3.351 $\int (a + a\sin(e + fx))^{3/2}\sqrt{c - c\sin(e + fx)}\,dx$

Optimal. Leaf size=43

$$\frac{c\cos(e + fx)(a\sin(e + fx) + a)^{3/2}}{2f\sqrt{c - c\sin(e + fx)}}$$

```
[Out] (c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(2*f*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 0.0818158, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {2738}

$$\frac{c\cos(e + fx)(a\sin(e + fx) + a)^{3/2}}{2f\sqrt{c - c\sin(e + fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)*Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(2*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\int (a + a\sin(e + fx))^{3/2}\sqrt{c - c\sin(e + fx)}\,dx = \frac{c\cos(e + fx)(a + a\sin(e + fx))^{3/2}}{2f\sqrt{c - c\sin(e + fx)}}$$

Mathematica [A] time = 0.203004, size = 60, normalized size = 1.4

$$-\frac{a\sec(e + fx)\sqrt{a(\sin(e + fx) + 1)}\sqrt{c - c\sin(e + fx)}(\cos(2(e + fx)) - 4\sin(e + fx))}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)*Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] -(a*Sec[e + f*x]*(Cos[2*(e + f*x)] - 4*Sin[e + f*x])*Sqrt[a*(1 + Sin[e + f*
x])]*Sqrt[c - c*Sin[e + f*x]])/(4*f)
```

Maple [A] time = 0.158, size = 63, normalized size = 1.5

$$-\frac{\sin\left(fx+e\right)\left(-1-\left(\cos\left(fx+e\right)\right)^{2}+\sin\left(fx+e\right)\right)}{2f\left(\cos\left(fx+e\right)\right)^{3}}\sqrt{-c\left(-1+\sin\left(fx+e\right)\right)}\left(a\left(1+\sin\left(fx+e\right)\right)\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] -1/2/f*(-c*(-1+sin(f*x+e)))^(1/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(3/2)*(-1-c
os(f*x+e)^2+sin(f*x+e))/cos(f*x+e)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\sqrt{-c\sin\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)*sqrt(-c*sin(f*x + e) + c), x)
```

Fricas [A] time = 1.35472, size = 157, normalized size = 3.65

$$-\frac{\left(a\cos\left(fx+e\right)^{2}-2\,a\sin\left(fx+e\right)-a\right)\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}}{2\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] -1/2*(a*cos(f*x + e)^2 - 2*a*sin(f*x + e) - a)*sqrt(a*sin(f*x + e) + a)*sqr
t(-c*sin(f*x + e) + c)/(f*cos(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\sqrt{-c\sin\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)*sqrt(-c*sin(f*x + e) + c), x)
```

3.352 $\int \frac{(a+a\sin(e+fx))^{3/2}}{\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=96

$$-\frac{2a^2\cos(e+fx)\log(1-\sin(e+fx))}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}$$

```
[Out] (-2*a^2*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f*x]]*Sqr
t[c - c*Sin[e + f*x]]) - (a*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(f*Sqrt[
c - c*Sin[e + f*x]])
```

Rubi [A] time = 0.191691, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2740, 2737, 2667, 31}

$$-\frac{2a^2\cos(e+fx)\log(1-\sin(e+fx))}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)/Sqrt[c - c*Sin[e + f*x]],x]
```


```
[Out] (-2*a^2*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f*x]]*Sqr
t[c - c*Sin[e + f*x]]) - (a*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(f*Sqrt[
c - c*Sin[e + f*x]])
```


Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] && !LtQ[n, -1] && !(IGtQ[n - 1/2, 0] && LtQ[n, m]) && !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] || !IntegerQ[m + 1/2
])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+a\sin(e+fx))^{3/2}}{\sqrt{c-c\sin(e+fx)}}\,dx &= -\frac{a\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{f\sqrt{c-c\sin(e+fx)}}+(2a)\int \frac{\sqrt{a+a\sin(e+fx)}}{\sqrt{c-c\sin(e+fx)}}\,dx\\
&= -\frac{a\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{f\sqrt{c-c\sin(e+fx)}}+\frac{\left(2a^2c\cos(e+fx)\right)\int \frac{\cos(e+fx)}{c-c\sin(e+fx)}\,dx}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}\\
&= -\frac{a\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{f\sqrt{c-c\sin(e+fx)}}-\frac{\left(2a^2\cos(e+fx)\right)\operatorname{Subst}\left(\int \frac{1}{c+x}\,dx,x,-c\sin(e+fx)\right)}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}\\
&= -\frac{2a^2\cos(e+fx)\log(1-\sin(e+fx))}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}-\frac{a\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{f\sqrt{c-c\sin(e+fx)}}
\end{aligned}$$

Mathematica [A] time = 0.317503, size = 113, normalized size = 1.18

$$-\frac{(a(\sin(e+fx)+1))^{3/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin(e+fx)+4\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)}{f\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)/Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] -((((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(3/2)*(4*Lo
g[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + Sin[e + f*x]))/(f*(Cos[(e + f*x)/2
] + Sin[(e + f*x)/2])^3*Sqrt[c - c*Sin[e + f*x]]))
```

Maple [B] time = 0.17, size = 252, normalized size = 2.6

$$\frac{1}{f\left(\sin\left(fx+e\right)\cos\left(fx+e\right)+\left(\cos\left(fx+e\right)\right)^2-2\sin\left(fx+e\right)+\cos\left(fx+e\right)-2\right)}\left(\sin\left(fx+e\right)\cos\left(fx+e\right)-2\sin\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] 1/f*(sin(f*x+e)*cos(f*x+e)-2*sin(f*x+e)*ln(2/(cos(f*x+e)+1))+4*sin(f*x+e)*l
n(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-cos(f*x+e)^2-2*cos(f*x+e)*ln(2/(c
os(f*x+e)+1))+4*cos(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-sin(f
*x+e)+2*ln(2/(cos(f*x+e)+1))-4*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+1
)*(a*(1+sin(f*x+e)))^(3/2)/(sin(f*x+e)*cos(f*x+e)+cos(f*x+e)^2-2*sin(f*x+e)
+cos(f*x+e)-2)/(-c*(-1+sin(f*x+e)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}{\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)/sqrt(-c*sin(f*x + e) + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-\frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\sqrt{-c\sin\left(fx+e\right)+c}}{c\sin\left(fx+e\right)-c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] integral(-(a*sin(f*x + e) + a)^(3/2)*sqrt(-c*sin(f*x + e) + c)/(c*sin(f*x +
 e) - c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}{\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)/sqrt(-c*sin(f*x + e) + c), x)
```

3.353 $\int \frac{(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=97

$$\frac{a^2\cos(e+fx)\log(1-\sin(e+fx))}{cf\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f(c-c\sin(e+fx))^{3/2}}$$

```
[Out] (a*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(f*(c - c*Sin[e + f*x])^(3/2)) +
(a^2*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt
[c - c*Sin[e + f*x]])
```

Rubi [A] time = 0.195201, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2739, 2737, 2667, 31}

$$\frac{a^2\cos(e+fx)\log(1-\sin(e+fx))}{cf\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f(c-c\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)/(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] (a*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(f*(c - c*Sin[e + f*x])^(3/2)) +
(a^2*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt
[c - c*Sin[e + f*x]])
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(2*n + 1)), x] - Dist[(b*(2*m - 1))/(d*
(2*n + 1)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{3/2}}\,dx = \frac{a\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{f(c-c\sin(e+fx))^{3/2}} - \frac{a\int \frac{\sqrt{a+a\sin(e+fx)}}{\sqrt{c-c\sin(e+fx)}}\,dx}{c}$$
$$= \frac{a\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{f(c-c\sin(e+fx))^{3/2}} - \frac{\left(a^2\cos(e+fx)\right)\int \frac{\cos(e+fx)}{c-c\sin(e+fx)}\,dx}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$
$$= \frac{a\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{f(c-c\sin(e+fx))^{3/2}} + \frac{\left(a^2\cos(e+fx)\right)\text{Subst}\left(\int \frac{1}{c+x}\,dx, x, -c\sin(e+fx)\right)}{cf\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$
$$= \frac{a\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{f(c-c\sin(e+fx))^{3/2}} + \frac{a^2\cos(e+fx)\log(1-\sin(e+fx))}{cf\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

Mathematica [A] time = 0.454203, size = 153, normalized size = 1.58

$$\frac{2a\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin(e+fx)\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-\log\left(\text{cc}\right.\right.}{cf(\sin(e+fx)-1)\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)/(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] (2*a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(-1 -
 Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + Log[Cos[(e + f*x)/2] - Sin[(e +
 f*x)/2]]*Sin[e + f*x]))/(c*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + S
in[e + f*x])*Sqrt[c - c*Sin[e + f*x]])
```

Maple [B] time = 0.14, size = 377, normalized size = 3.9

$$-\frac{1}{f\left(\sin(fx+e)\cos(fx+e)+(\cos(fx+e))^2-2\sin(fx+e)+\cos(fx+e)-2\right)}\left(\ln\left(2\left(\cos(fx+e)+1\right)^{-1}\right)\sin\left(fx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] -1/f*(ln(2/(cos(f*x+e)+1))*sin(f*x+e)*cos(f*x+e)-2*ln(-(-1+cos(f*x+e)+sin(f
*x+e))/sin(f*x+e))*sin(f*x+e)*cos(f*x+e)-cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))+
2*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+2*sin(f*x+e)*cos(
f*x+e)-2*sin(f*x+e)*ln(2/(cos(f*x+e)+1))+4*sin(f*x+e)*ln(-(-1+cos(f*x+e)+si
n(f*x+e))/sin(f*x+e))-2*cos(f*x+e)^2-cos(f*x+e)*ln(2/(cos(f*x+e)+1))+2*cos(
f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-2*sin(f*x+e)+2*ln(2/(cos(
f*x+e)+1))-4*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+2)*(a*(1+sin(f*x+e)
))^(3/2)/(sin(f*x+e)*cos(f*x+e)+cos(f*x+e)^2-2*sin(f*x+e)+cos(f*x+e)-2)/(-c
*(-1+sin(f*x+e)))^(3/2)
```

Maxima [A] time = 1.72081, size = 185, normalized size = 1.91

$$-\frac{\frac{2\,a^{\frac{3}{2}}\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{c^{\frac{3}{2}}}-\frac{a^{\frac{3}{2}}\log\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)}{c^{\frac{3}{2}}}+\frac{4\,a^{\frac{3}{2}}\sqrt{c}\sin(fx+e)}{\left(c^2-\frac{2\,c^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{c^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}\right)(\cos(fx+e)+1)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] -(2*a^(3/2)*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/c^(3/2) - a^(3/2)*log(
sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/c^(3/2) + 4*a^(3/2)*sqrt(c)*sin(f*
x + e)/((c^2 - 2*c^2*sin(f*x + e)/(cos(f*x + e) + 1) + c^2*sin(f*x + e)^2/(
cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1)))/f
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{(a\sin(fx+e)+a)^{\frac{3}{2}}\sqrt{-c\sin(fx+e)+c}}{c^2\cos(fx+e)^2+2\,c^2\sin(fx+e)-2\,c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] integral(-(a*sin(f*x + e) + a)^(3/2)*sqrt(-c*sin(f*x + e) + c)/(c^2*cos(f*x
 + e)^2 + 2*c^2*sin(f*x + e) - 2*c^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\sin(fx+e)+a)^{\frac{3}{2}}}{(-c\sin(fx+e)+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)/(-c*sin(f*x + e) + c)^(3/2), x)
```

3.354 $\int \frac{(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=42

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{4f(c-c\sin(e+fx))^{5/2}}$$

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(4*f*(c - c*Sin[e + f*x])^(5/2))
```

Rubi [A] time = 0.0909264, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {2742}

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{4f(c-c\sin(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)/(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(4*f*(c - c*Sin[e + f*x])^(5/2))
```

Rule 2742

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{5/2}}\,dx=\frac{\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{4f(c-c\sin(e+fx))^{5/2}}$$

Mathematica [B] time = 0.469106, size = 99, normalized size = 2.36

$$\frac{a\sin(e+fx)\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}{c^2f(\sin(e+fx)-1)^2\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)/(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] (a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sin[e + f*x]*Sqrt[a*(1 + Sin[e + f
*x])])/(c^2*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])^2*S
qrt[c - c*Sin[e + f*x]])
```

Maple [B] time = 0.132, size = 90, normalized size = 2.1

$$-\frac{\left(-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)\right)\sin\left(fx+e\right)}{f\left(\sin\left(fx+e\right)\cos\left(fx+e\right)+\left(\cos\left(fx+e\right)\right)^{2}-2\sin\left(fx+e\right)+\cos\left(fx+e\right)-2\right)}\left(a\left(1+\sin\left(fx+e\right)\right)\right)^{\frac{3}{2}}\left(-c\left(-1+\sin\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] -1/f*(-1+cos(f*x+e)+sin(f*x+e))*sin(f*x+e)*(a*(1+sin(f*x+e)))^(3/2)/(sin(f*
x+e)*cos(f*x+e)+cos(f*x+e)^2-2*sin(f*x+e)+cos(f*x+e)-2)/(-c*(-1+sin(f*x+e))
)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)/(-c*sin(f*x + e) + c)^(5/2), x)
```

Fricas [B] time = 1.06562, size = 203, normalized size = 4.83

$$-\frac{\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}a\sin\left(fx+e\right)}{c^{3}f\cos\left(fx+e\right)^{3}+2\,c^{3}f\cos\left(fx+e\right)\sin\left(fx+e\right)-2\,c^{3}f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] -sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*a*sin(f*x + e)/(c^3*f*c
os(f*x + e)^3 + 2*c^3*f*cos(f*x + e)*sin(f*x + e) - 2*c^3*f*cos(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)/(-c*sin(f*x + e) + c)^(5/2), x)
```

3.355 $\int \frac{(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=88

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{24cf(c-c\sin(e+fx))^{5/2}}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{6f(c-c\sin(e+fx))^{7/2}}$$

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(6*f*(c - c*Sin[e + f*x])^(7/2))
+ (Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(24*c*f*(c - c*Sin[e + f*x])^(5
/2))
```

Rubi [A] time = 0.184431, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2743, 2742}

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{24cf(c-c\sin(e+fx))^{5/2}}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{6f(c-c\sin(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)/(c - c*Sin[e + f*x])^(7/2),x]
```

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(6*f*(c - c*Sin[e + f*x])^(7/2))
+ (Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(24*c*f*(c - c*Sin[e + f*x])^(5
/2))
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rule 2742

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{7/2}}\,dx &= \frac{\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{6f(c-c\sin(e+fx))^{7/2}}+\frac{\int \frac{(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{5/2}}\,dx}{6c}\\ &= \frac{\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{6f(c-c\sin(e+fx))^{7/2}}+\frac{\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{24cf(c-c\sin(e+fx))^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.576118, size = 106, normalized size = 1.2

$$-\frac{a(3\sin(e+fx)+1)\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}{6c^3f(\sin(e+fx)-1)^3\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)/(c - c*Sin[e + f*x])^(7/2),x]
```

```
[Out] -(a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(1 + 3
*Sin[e + f*x]))/(6*c^3*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e
+ f*x])^3*Sqrt[c - c*Sin[e + f*x]])
```

Maple [A] time = 0.141, size = 141, normalized size = 1.6

$$\frac{\left(\left(\cos\left(fx+e\right)\right)^{2}\sin\left(fx+e\right)+\left(\cos\left(fx+e\right)\right)^{3}+3\sin\left(fx+e\right)\cos\left(fx+e\right)-4\left(\cos\left(fx+e\right)\right)^{2}-10\sin\left(fx+e\right)-\cdots}{6f\left(\sin\left(fx+e\right)\cos\left(fx+e\right)+\left(\cos\left(fx+e\right)\right)^{2}-2\sin\left(fx+e\right)+\cos\left(fx+e\right)-2\right)\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(7/2),x)
```

```
[Out] 1/6/f*(cos(f*x+e)^2*sin(f*x+e)+cos(f*x+e)^3+3*sin(f*x+e)*cos(f*x+e)-4*cos(f
*x+e)^2-10*sin(f*x+e)-7*cos(f*x+e)+10)*(a*(1+sin(f*x+e)))^(3/2)*sin(f*x+e)/
(sin(f*x+e)*cos(f*x+e)+cos(f*x+e)^2-2*sin(f*x+e)+cos(f*x+e)-2)/(-c*(-1+sin(
f*x+e)))^(7/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(7/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)/(-c*sin(f*x + e) + c)^(7/2), x)
```

Fricas [A] time = 1.10811, size = 255, normalized size = 2.9

$$-\frac{\left(3\,a\sin\left(fx+e\right)+a\right)\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}}{6\left(3\,c^{4}f\cos\left(fx+e\right)^{3}-4\,c^{4}f\cos\left(fx+e\right)-\left(c^{4}f\cos\left(fx+e\right)^{3}-4\,c^{4}f\cos\left(fx+e\right)\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(7/2),x, algorithm="frica
s")
```

```
[Out] -1/6*(3*a*sin(f*x + e) + a)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) +
 c)/(3*c^4*f*cos(f*x + e)^3 - 4*c^4*f*cos(f*x + e) - (c^4*f*cos(f*x + e)^3
- 4*c^4*f*cos(f*x + e))*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(7/2),x, algorithm="giac"
)
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)/(-c*sin(f*x + e) + c)^(7/2), x)
```

3.356 $\int \frac{(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{9/2}}\,dx$

Optimal. Leaf size=92

$$\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{4f(c-c\sin(e+fx))^{9/2}}-\frac{a^2\cos(e+fx)}{12cf\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{7/2}}$$

```
[Out] (a*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(4*f*(c - c*Sin[e + f*x])^(9/2))
- (a^2*Cos[e + f*x])/(12*c*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^
(7/2))
```

Rubi [A] time = 0.177274, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2739, 2738}

$$\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{4f(c-c\sin(e+fx))^{9/2}}-\frac{a^2\cos(e+fx)}{12cf\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)/(c - c*Sin[e + f*x])^(9/2),x]
```

```
[Out] (a*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(4*f*(c - c*Sin[e + f*x])^(9/2))
- (a^2*Cos[e + f*x])/(12*c*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^
(7/2))
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(2*n + 1)), x] - Dist[(b*(2*m - 1))/(d*
(2*n + 1)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{9/2}}\,dx &= \frac{a\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{4f(c-c\sin(e+fx))^{9/2}}-\frac{a\int \frac{\sqrt{a+a\sin(e+fx)}}{(c-c\sin(e+fx))^{7/2}}\,dx}{4c}\\ &= \frac{a\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{4f(c-c\sin(e+fx))^{9/2}}-\frac{a^2\cos(e+fx)}{12cf\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{7/2}}\end{aligned}$$

Mathematica [A] time = 1.0532, size = 106, normalized size = 1.15

$$\frac{a(2\sin(e+fx)+1)\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}{6c^4f(\sin(e+fx)-1)^4\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)/(c - c*Sin[e + f*x])^(9/2),x]
```

```
[Out] (a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(1 + 2*
Sin[e + f*x]))/(6*c^4*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e +
 f*x])^4*Sqrt[c - c*Sin[e + f*x]])
```

Maple [B] time = 0.15, size = 169, normalized size = 1.8

$$-\frac{\left(\sin\left(fx+e\right)\left(\cos\left(fx+e\right)\right)^{3}-\left(\cos\left(fx+e\right)\right)^{4}-5\left(\cos\left(fx+e\right)\right)^{2}\sin\left(fx+e\right)-4\left(\cos\left(fx+e\right)\right)^{3}-7\sin\left(fx+e\right)\mathrm{co}}{6f\left(\sin\left(fx+e\right)\cos\left(fx+e\right)+\left(\cos\left(fx+e\right)\right)^{2}-2\sin\left(f\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(9/2),x)
```

```
[Out] -1/6/f*(sin(f*x+e)*cos(f*x+e)^3-cos(f*x+e)^4-5*cos(f*x+e)^2*sin(f*x+e)-4*co
s(f*x+e)^3-7*sin(f*x+e)*cos(f*x+e)+12*cos(f*x+e)^2+17*sin(f*x+e)+10*cos(f*x
+e)-17)*(a*(1+sin(f*x+e)))^(3/2)*sin(f*x+e)/(sin(f*x+e)*cos(f*x+e)+cos(f*x+
e)^2-2*sin(f*x+e)+cos(f*x+e)-2)/(-c*(-1+sin(f*x+e)))^(9/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(9/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)/(-c*sin(f*x + e) + c)^(9/2), x)
```

Fricas [A] time = 1.13219, size = 288, normalized size = 3.13

$$\frac{\left(2\,a\sin\left(fx+e\right)+a\right)\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}}{6\left(c^5f\cos\left(fx+e\right)^5-8\,c^5f\cos\left(fx+e\right)^3+8\,c^5f\cos\left(fx+e\right)+4\left(c^5f\cos\left(fx+e\right)^3-2\,c^5f\cos\left(fx+e\right)\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(9/2),x, algorithm="frica
s")
```

```
[Out] 1/6*(2*a*sin(f*x + e) + a)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) +
c)/(c^5*f*cos(f*x + e)^5 - 8*c^5*f*cos(f*x + e)^3 + 8*c^5*f*cos(f*x + e) +
4*(c^5*f*cos(f*x + e)^3 - 2*c^5*f*cos(f*x + e))*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(9/2),x, algorithm="giac"
)
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)/(-c*sin(f*x + e) + c)^(9/2), x)
```

3.357 $\int \frac{(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{11/2}}\,dx$

Optimal. Leaf size=92

$$\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{5f(c-c\sin(e+fx))^{11/2}}-\frac{a^2\cos(e+fx)}{20cf\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{9/2}}$$

```
[Out] (a*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(5*f*(c - c*Sin[e + f*x])^(11/2))
 - (a^2*Cos[e + f*x])/(20*c*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])
^(9/2))
```

Rubi [A] time = 0.176202, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2739, 2738}

$$\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{5f(c-c\sin(e+fx))^{11/2}}-\frac{a^2\cos(e+fx)}{20cf\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)/(c - c*Sin[e + f*x])^(11/2),x]
```

```
[Out] (a*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(5*f*(c - c*Sin[e + f*x])^(11/2))
 - (a^2*Cos[e + f*x])/(20*c*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])
^(9/2))
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(2*n + 1)), x] - Dist[(b*(2*m - 1))/(d*
(2*n + 1)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{11/2}}\,dx &= \frac{a\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{5f(c-c\sin(e+fx))^{11/2}}-\frac{a\int \frac{\sqrt{a+a\sin(e+fx)}}{(c-c\sin(e+fx))^{9/2}}\,dx}{5c}\\ &= \frac{a\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{5f(c-c\sin(e+fx))^{11/2}}-\frac{a^2\cos(e+fx)}{20cf\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{9/2}}\end{aligned}$$

Mathematica [A] time = 1.50409, size = 106, normalized size = 1.15

$$-\frac{a(5\sin(e+fx)+3)\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}{20c^5f(\sin(e+fx)-1)^5\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)/(c - c*Sin[e + f*x])^(11/2),x]
```

```
[Out] -(a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(3 + 5
*Sin[e + f*x]))/(20*c^5*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e
 + f*x])^5*Sqrt[c - c*Sin[e + f*x]])
```

Maple [B] time = 0.163, size = 196, normalized size = 2.1

$$-\frac{\left(3\sin\left(fx+e\right)\left(\cos\left(fx+e\right)\right)^{4}+3\left(\cos\left(fx+e\right)\right)^{5}+15\sin\left(fx+e\right)\left(\cos\left(fx+e\right)\right)^{3}-18\left(\cos\left(fx+e\right)\right)^{4}-51\left(\cos\right.\right.}{20f\left(\sin\left(fx+e\right)\cos\left(f\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(11/2),x)
```

```
[Out] -1/20/f*(3*sin(f*x+e)*cos(f*x+e)^4+3*cos(f*x+e)^5+15*sin(f*x+e)*cos(f*x+e)^
3-18*cos(f*x+e)^4-51*cos(f*x+e)^2*sin(f*x+e)-36*cos(f*x+e)^3-45*sin(f*x+e)*
cos(f*x+e)+96*cos(f*x+e)^2+98*sin(f*x+e)+53*cos(f*x+e)-98)*(a*(1+sin(f*x+e)
))^(3/2)*sin(f*x+e)/(sin(f*x+e)*cos(f*x+e)+cos(f*x+e)^2-2*sin(f*x+e)+cos(f*
x+e)-2)/(-c*(-1+sin(f*x+e)))^(11/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(11/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)/(-c*sin(f*x + e) + c)^(11/2), x)
```

Fricas [A] time = 1.19594, size = 331, normalized size = 3.6

$$\frac{\left(5\,a\sin\left(fx+e\right)+3\,a\right)\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}}{20\left(5\,c^{6}f\cos\left(fx+e\right)^{5}-20\,c^{6}f\cos\left(fx+e\right)^{3}+16\,c^{6}f\cos\left(fx+e\right)-\left(c^{6}f\cos\left(fx+e\right)^{5}-12\,c^{6}f\cos\left(fx+e\right)^{3}+16\,c^{6}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(11/2),x, algorithm="fric
as")
```

```
[Out] 1/20*(5*a*sin(f*x + e) + 3*a)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e)
 + c)/(5*c^6*f*cos(f*x + e)^5 - 20*c^6*f*cos(f*x + e)^3 + 16*c^6*f*cos(f*x
+ e) - (c^6*f*cos(f*x + e)^5 - 12*c^6*f*cos(f*x + e)^3 + 16*c^6*f*cos(f*x +
 e))*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(11/2),x, algorithm="giac
")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)/(-c*sin(f*x + e) + c)^(11/2), x)
```

3.358 $\int (a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{7/2}\,dx$

Optimal. Leaf size=134

$$-\frac{2a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{7/2}}{15f}-\frac{a^3\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{15f\sqrt{a\sin(e+fx)+a}}-\frac{a\cos(e+fx)(a\sin(e$$

```
[Out] -(a^3*Cos[e + f*x]*(c - c*Sin[e + f*x])^(7/2))/(15*f*Sqrt[a + a*Sin[e + f*x
]]) - (2*a^2*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(7/
2))/(15*f) - (a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x]
)^(7/2))/(6*f)
```

Rubi [A] time = 0.270132, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2740, 2738}

$$-\frac{2a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{7/2}}{15f}-\frac{a^3\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{15f\sqrt{a\sin(e+fx)+a}}-\frac{a\cos(e+fx)(a\sin(e$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(7/2),x]
```


```
[Out] -(a^3*Cos[e + f*x]*(c - c*Sin[e + f*x])^(7/2))/(15*f*Sqrt[a + a*Sin[e + f*x
]]) - (2*a^2*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(7/
2))/(15*f) - (a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x]
)^(7/2))/(6*f)
```


Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] && !LtQ[n, -1] && !(IGtQ[n - 1/2, 0] && LtQ[n, m]) && !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int (a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{7/2}\,dx &= -\frac{a\cos(e+fx)(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{7/2}}{6f}+\frac{1}{3}(2a)\int\\ &= -\frac{2a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{7/2}}{15f}-\frac{a\cos(e\\ &= -\frac{a^3\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{15f\sqrt{a+a\sin(e+fx)}}-\frac{2a^2\cos(e+fx)\sqrt{a+a\sin(e}{15}\end{aligned}$$

Mathematica [A] time = 1.37587, size = 156, normalized size = 1.16

$$-\frac{c^3(\sin(e+fx)-1)^3(a(\sin(e+fx)+1))^{5/2}\sqrt{c-c\sin(e+fx)}(600\sin(e+fx)+100\sin(3(e+fx))+12\sin(5(e+fx))+}{960f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(7/2),x]
```

```
[Out] -(c^3*(-1 + Sin[e + f*x])^3*(a*(1 + Sin[e + f*x]))^(5/2)*Sqrt[c - c*Sin[e +
 f*x]]*(75*Cos[2*(e + f*x)] + 30*Cos[4*(e + f*x)] + 5*Cos[6*(e + f*x)] + 60
0*Sin[e + f*x] + 100*Sin[3*(e + f*x)] + 12*Sin[5*(e + f*x)]))/(960*f*(Cos[(
e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)
```

Maple [A] time = 0.165, size = 116, normalized size = 0.9

$$\frac{\sin\left(fx+e\right)\left(5\left(\cos\left(fx+e\right)\right)^6+\sin\left(fx+e\right)\left(\cos\left(fx+e\right)\right)^4+6\left(\cos\left(fx+e\right)\right)^4+3\left(\cos\left(fx+e\right)\right)^2\sin\left(fx+e\right)+8\left(\right.\right.}{30f\left(\cos\left(fx+e\right)\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(7/2),x)
```

```
[Out] 1/30/f*(-c*(-1+sin(f*x+e)))^(7/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(5/2)*(5*co
s(f*x+e)^6+sin(f*x+e)*cos(f*x+e)^4+6*cos(f*x+e)^4+3*cos(f*x+e)^2*sin(f*x+e)
+8*cos(f*x+e)^2+11*sin(f*x+e)+11)/cos(f*x+e)^7
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(7/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)*(-c*sin(f*x + e) + c)^(7/2), x)
```

Fricas [A] time = 1.20894, size = 265, normalized size = 1.98

$$\frac{\left(5a^2c^3\cos\left(fx+e\right)^6-5a^2c^3+2\left(3a^2c^3\cos\left(fx+e\right)^4+4a^2c^3\cos\left(fx+e\right)^2+8a^2c^3\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}}{30f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(7/2),x, algorithm="frica
s")
```

```
[Out] 1/30*(5*a^2*c^3*cos(f*x + e)^6 - 5*a^2*c^3 + 2*(3*a^2*c^3*cos(f*x + e)^4 +
4*a^2*c^3*cos(f*x + e)^2 + 8*a^2*c^3)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a
)*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*(c-c*sin(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a \sin\left(fx+e\right)+a\right)^{\frac{5}{2}} \left(-c \sin\left(fx+e\right)+c\right)^{\frac{7}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(7/2),x, algorithm="giac"
)
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)*(-c*sin(f*x + e) + c)^(7/2), x)
```

3.359 $\int (a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}\,dx$

Optimal. Leaf size=134

$$-\frac{2a^3\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{15f\sqrt{a\sin(e+fx)+a}}-\frac{a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/2}}{5f}-\frac{a\cos(e+fx)(a\sin(e+\ldots}{}$$

```
[Out] (-2*a^3*Cos[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(15*f*Sqrt[a + a*Sin[e + f
*x]]) - (a^2*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(5/
2))/(5*f) - (a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])
^(5/2))/(5*f)
```

Rubi [A] time = 0.271218, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2740, 2738}

$$-\frac{2a^3\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{15f\sqrt{a\sin(e+fx)+a}}-\frac{a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/2}}{5f}-\frac{a\cos(e+fx)(a\sin(e+\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] (-2*a^3*Cos[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(15*f*Sqrt[a + a*Sin[e + f
*x]]) - (a^2*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(5/
2))/(5*f) - (a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])
^(5/2))/(5*f)
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int (a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}\,dx &= -\frac{a\cos(e+fx)(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}}{5f}+\frac{1}{5}(4a)\int (a\ldots\\ &= -\frac{a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}}{5f}-\frac{a\cos(e+fx\ldots}{}\\ &= -\frac{2a^3\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{15f\sqrt{a+a\sin(e+fx)}}-\frac{a^2\cos(e+fx)\sqrt{a+a\sin(e+\ldots}}{5f}\end{aligned}$$

Mathematica [A] time = 0.537671, size = 77, normalized size = 0.57

$$\frac{a^2c^2(150\sin(e+fx)+25\sin(3(e+fx))+3\sin(5(e+fx)))\sec(e+fx)\sqrt{a(\sin(e+fx)+1)}\sqrt{c-c\sin(e+fx)}}{240f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] (a^2*c^2*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]]*(
150*Sin[e + f*x] + 25*Sin[3*(e + f*x)] + 3*Sin[5*(e + f*x)]))/(240*f)
```

Maple [A] time = 0.145, size = 67, normalized size = 0.5

$$\frac{\left(3\left(\cos\left(fx+e\right)\right)^4+4\left(\cos\left(fx+e\right)\right)^2+8\right)\sin\left(fx+e\right)}{15f\left(\cos\left(fx+e\right)\right)^5}\left(-c\left(-1+\sin\left(fx+e\right)\right)\right)^{\frac{5}{2}}\left(a\left(1+\sin\left(fx+e\right)\right)\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] 1/15/f*(3*cos(f*x+e)^4+4*cos(f*x+e)^2+8)*(-c*(-1+sin(f*x+e)))^(5/2)*sin(f*x
+e)*(a*(1+sin(f*x+e)))^(5/2)/cos(f*x+e)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)*(-c*sin(f*x + e) + c)^(5/2), x)
```

Fricas [A] time = 1.09305, size = 207, normalized size = 1.54

$$\frac{\left(3a^2c^2\cos\left(fx+e\right)^4+4a^2c^2\cos\left(fx+e\right)^2+8a^2c^2\right)\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}\sin\left(fx+e\right)}{15f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] 1/15*(3*a^2*c^2*cos(f*x + e)^4 + 4*a^2*c^2*cos(f*x + e)^2 + 8*a^2*c^2)*sqrt
(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*sin(f*x + e)/(f*cos(f*x + e)
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError
```

3.360 $\int (a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=89

$$\frac{c^2\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{6f\sqrt{c-c\sin(e+fx)}}+\frac{c\cos(e+fx)(a\sin(e+fx)+a)^{5/2}\sqrt{c-c\sin(e+fx)}}{4f}$$

```
[Out] (c^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(6*f*Sqrt[c - c*Sin[e + f*x]]
) + (c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x]])/(4
*f)
```

Rubi [A] time = 0.17159, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2740, 2738}

$$\frac{c^2\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{6f\sqrt{c-c\sin(e+fx)}}+\frac{c\cos(e+fx)(a\sin(e+fx)+a)^{5/2}\sqrt{c-c\sin(e+fx)}}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] (c^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(6*f*Sqrt[c - c*Sin[e + f*x]]
) + (c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x]])/(4
*f)
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] && !LtQ[n, -1] && !(IGtQ[n - 1/2, 0] && LtQ[n, m]) && !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}\,dx=\frac{c\cos(e+fx)(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}{4f}+\frac{1}{2}c\int (a+a$$
$$=\frac{c^2\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{6f\sqrt{c-c\sin(e+fx)}}+\frac{c\cos(e+fx)(a+a\sin(e+f}{4f}$$

Mathematica [A] time = 0.664354, size = 133, normalized size = 1.49

$$-\frac{c(\sin(e+fx)-1)(a(\sin(e+fx)+1))^{5/2}\sqrt{c-c\sin(e+fx)}(8(9\sin(e+fx)+\sin(3(e+fx)))-12\cos(2(e+fx))-3}{96f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] -(c*(-1 + Sin[e + f*x])*(a*(1 + Sin[e + f*x]))^(5/2)*Sqrt[c - c*Sin[e + f*x
]]*(-12*Cos[2*(e + f*x)] - 3*Cos[4*(e + f*x)] + 8*(9*Sin[e + f*x] + Sin[3*(
e + f*x)])))/(96*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(Cos[(e + f*x)/2
] + Sin[(e + f*x)/2])^5)
```

Maple [A] time = 0.141, size = 90, normalized size = 1.

$$-\frac{\sin\left(fx+e\right)\left(-3\left(\cos\left(fx+e\right)\right)^{4}+\left(\cos\left(fx+e\right)\right)^{2}\sin\left(fx+e\right)-4\left(\cos\left(fx+e\right)\right)^{2}+5\sin\left(fx+e\right)-5\right)}{12f\left(\cos\left(fx+e\right)\right)^{5}}\left(-c\left(-1+\sin\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] -1/12/f*(-c*(-1+sin(f*x+e)))^(3/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(5/2)*(-3*
cos(f*x+e)^4+cos(f*x+e)^2*sin(f*x+e)-4*cos(f*x+e)^2+5*sin(f*x+e)-5)/cos(f*x
+e)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)*(-c*sin(f*x + e) + c)^(3/2), x)
```

Fricas [A] time = 1.088, size = 216, normalized size = 2.43

$$-\frac{\left(3a^{2}c\cos\left(fx+e\right)^{4}-3a^{2}c-4\left(a^{2}c\cos\left(fx+e\right)^{2}+2a^{2}c\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}}{12f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] -1/12*(3*a^2*c*cos(f*x + e)^4 - 3*a^2*c - 4*(a^2*c*cos(f*x + e)^2 + 2*a^2*c
)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(f*cos(f
*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError
```

3.361 $\int (a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}\,dx$

Optimal. Leaf size=43

$$\frac{c\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{3f\sqrt{c-c\sin(e+fx)}}$$

```
[Out] (c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(3*f*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 0.0802235, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {2738}

$$\frac{c\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{3f\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(3*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}\,dx=\frac{c\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3f\sqrt{c-c\sin(e+fx)}}$$

Mathematica [A] time = 0.266344, size = 72, normalized size = 1.67

$$-\frac{a^2\sec(e+fx)\sqrt{a(\sin(e+fx)+1)}\sqrt{c-c\sin(e+fx)}(-15\sin(e+fx)+\sin(3(e+fx))+6\cos(2(e+fx)))}{12f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] -(a^2*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]]*(6*C
os[2*(e + f*x)] - 15*Sin[e + f*x] + Sin[3*(e + f*x)]))/(12*f)
```

Maple [B] time = 0.183, size = 80, normalized size = 1.9

$$-\frac{\sin(fx+e)\left(-(\cos(fx+e))^4+(\cos(fx+e))^2\sin(fx+e)-2+2\sin(fx+e)\right)}{3f(\cos(fx+e))^5}\sqrt{-c(-1+\sin(fx+e))}\,(a(1+\sin$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] -1/3/f*(-c*(-1+sin(f*x+e)))^(1/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(5/2)*(-cos
(f*x+e)^4+cos(f*x+e)^2*sin(f*x+e)-2+2*sin(f*x+e))/cos(f*x+e)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\sqrt{-c\sin\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)*sqrt(-c*sin(f*x + e) + c), x)
```

Fricas [B] time = 1.05643, size = 201, normalized size = 4.67

$$-\frac{\left(3\,a^2\cos\left(fx+e\right)^2-3\,a^2+\left(a^2\cos\left(fx+e\right)^2-4\,a^2\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}}{3\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] -1/3*(3*a^2*cos(f*x + e)^2 - 3*a^2 + (a^2*cos(f*x + e)^2 - 4*a^2)*sin(f*x +
 e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError
```

3.362 $\int \frac{(a+a\sin(e+fx))^{5/2}}{\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=141

$$-\frac{2a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}-\frac{4a^3\cos(e+fx)\log(1-\sin(e+fx))}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sin(e+fx)}}$$

```
[Out] (-4*a^3*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f*x]]*Sqr
t[c - c*Sin[e + f*x]]) - (2*a^2*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(f*S
qrt[c - c*Sin[e + f*x]]) - (a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(2*f
*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 0.279105, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2740, 2737, 2667, 31}

$$-\frac{2a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}-\frac{4a^3\cos(e+fx)\log(1-\sin(e+fx))}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)/Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (-4*a^3*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f*x]]*Sqr
t[c - c*Sin[e + f*x]]) - (2*a^2*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(f*S
qrt[c - c*Sin[e + f*x]]) - (a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(2*f
*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{5/2}}{\sqrt{c-c\sin(e+fx)}}\,dx = -\frac{a\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2f\sqrt{c-c\sin(e+fx)}} + (2a)\int \frac{(a+a\sin(e+fx))^{3/2}}{\sqrt{c-c\sin(e+fx)}}\,dx$$

$$= -\frac{2a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{f\sqrt{c-c\sin(e+fx)}} - \frac{a\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2f\sqrt{c-c\sin(e+fx)}} + \left(4a^2\right)\int \frac{\sqrt{a+}}{\sqrt{c-}}$$

$$= -\frac{2a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{f\sqrt{c-c\sin(e+fx)}} - \frac{a\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2f\sqrt{c-c\sin(e+fx)}} + \frac{\left(4a^3c\cos(e+\right.}{\sqrt{a+a\sin(e-}}$$

$$= -\frac{2a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{f\sqrt{c-c\sin(e+fx)}} - \frac{a\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2f\sqrt{c-c\sin(e+fx)}} - \frac{\left(4a^3\cos(e+f\right.}{f\sqrt{a+}}$$

$$= -\frac{4a^3\cos(e+fx)\log(1-\sin(e+fx))}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{2a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{f\sqrt{c-c\sin(e+fx)}} - \frac{a\cos(e+}{2f}$$

Mathematica [A] time = 0.577391, size = 127, normalized size = 0.9

$$-\frac{(a(\sin(e+fx)+1))^{5/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(12\sin(e+fx)-\cos(2(e+fx))+32\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\right.\right.}{4f\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)/Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] -((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(5/2)*(-Cos[
2*(e + f*x)] + 32*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + 12*Sin[e + f*x
]))/(4*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5*Sqrt[c - c*Sin[e + f*x]])
```

Maple [B] time = 0.184, size = 315, normalized size = 2.2

$$-\frac{1}{2f\left(\left(\cos\left(fx+e\right)\right)^2\sin\left(fx+e\right)-\left(\cos\left(fx+e\right)\right)^3+2\sin\left(fx+e\right)\cos\left(fx+e\right)+3\left(\cos\left(fx+e\right)\right)^2-4\sin\left(fx+e\right)+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] -1/2/f*(cos(f*x+e)^2*sin(f*x+e)+cos(f*x+e)^3-6*sin(f*x+e)*cos(f*x+e)-16*sin
(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+8*sin(f*x+e)*ln(2/(cos(f
*x+e)+1))+5*cos(f*x+e)^2-16*cos(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f
*x+e))+8*cos(f*x+e)*ln(2/(cos(f*x+e)+1))+5*sin(f*x+e)-cos(f*x+e)+16*ln(-(-1
+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-8*ln(2/(cos(f*x+e)+1))-5)*(a*(1+sin(f*x
+e)))^(5/2)/(cos(f*x+e)^2*sin(f*x+e)-cos(f*x+e)^3+2*sin(f*x+e)*cos(f*x+e)+3
*cos(f*x+e)^2-4*sin(f*x+e)+2*cos(f*x+e)-4)/(-c*(-1+sin(f*x+e)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}{\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)/sqrt(-c*sin(f*x + e) + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^{2}\cos\left(fx+e\right)^{2}-2\,a^{2}\sin\left(fx+e\right)-2\,a^{2}\right)\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}}{c\sin\left(fx+e\right)-c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] integral((a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x + e) - 2*a^2)*sqrt(a*sin(f*x +
 e) + a)*sqrt(-c*sin(f*x + e) + c)/(c*sin(f*x + e) - c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}{\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)/sqrt(-c*sin(f*x + e) + c), x)
```

3.363 $\int \frac{(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=144

$$\frac{2a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{cf\sqrt{c-c\sin(e+fx)}}+\frac{4a^3\cos(e+fx)\log(1-\sin(e+fx))}{cf\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{f(c-c\sin(e+fx))^{3/2}}$$

```
[Out] (a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(f*(c - c*Sin[e + f*x])^(3/2))
+ (4*a^3*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c*f*Sqrt[a + a*Sin[e + f*x]]*
Sqrt[c - c*Sin[e + f*x]]) + (2*a^2*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(
c*f*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 0.29033, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2739, 2740, 2737, 2667, 31}

$$\frac{2a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{cf\sqrt{c-c\sin(e+fx)}}+\frac{4a^3\cos(e+fx)\log(1-\sin(e+fx))}{cf\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{f(c-c\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)/(c - c*Sin[e + f*x])^(3/2),x]
```


```
[Out] (a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(f*(c - c*Sin[e + f*x])^(3/2))
+ (4*a^3*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c*f*Sqrt[a + a*Sin[e + f*x]]*
Sqrt[c - c*Sin[e + f*x]]) + (2*a^2*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(
c*f*Sqrt[c - c*Sin[e + f*x]])
```


Rule 2739

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(2*n + 1)), x] - Dist[(b*(2*m - 1))/(d*
(2*n + 1)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{3/2}}\,dx = \frac{a\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{f(c-c\sin(e+fx))^{3/2}} - \frac{(2a)\int\frac{(a+a\sin(e+fx))^{3/2}}{\sqrt{c-c\sin(e+fx)}}\,dx}{c}$$

$$= \frac{a\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{f(c-c\sin(e+fx))^{3/2}} + \frac{2a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{cf\sqrt{c-c\sin(e+fx)}} - \frac{(4a^2)\int\frac{\sqrt{a+}}{\sqrt{c-}}}{c}$$

$$= \frac{a\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{f(c-c\sin(e+fx))^{3/2}} + \frac{2a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{cf\sqrt{c-c\sin(e+fx)}} - \frac{(4a^3\cos(e}{\sqrt{a+a\sin(e}}$$

$$= \frac{a\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{f(c-c\sin(e+fx))^{3/2}} + \frac{2a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{cf\sqrt{c-c\sin(e+fx)}} + \frac{(4a^3\cos(e+}{cf\sqrt{a}}$$

$$= \frac{a\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{f(c-c\sin(e+fx))^{3/2}} + \frac{4a^3\cos(e+fx)\log(1-\sin(e+fx))}{cf\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{2a^2\cos}{}$$

Mathematica [A] time = 0.805996, size = 169, normalized size = 1.17

$$-\frac{a^2\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos(2(e+fx))+16\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx\right.\right.\right.}{2cf(\sin(e+fx)-1)\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\mathrm{co}\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)/(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] -(a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(7 +
 Cos[2*(e + f*x)] + 16*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + (2 - 16*L
og[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]])*Sin[e + f*x]))/(2*c*f*(Cos[(e + f*
x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])
```

Maple [B] time = 0.154, size = 439, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] -1/f*(cos(f*x+e)^2*sin(f*x+e)-8*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*
sin(f*x+e)*cos(f*x+e)+4*ln(2/(cos(f*x+e)+1))*sin(f*x+e)*cos(f*x+e)+cos(f*x+
e)^3+8*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-4*cos(f*x+e)
^2*ln(2/(cos(f*x+e)+1))+5*sin(f*x+e)*cos(f*x+e)+16*sin(f*x+e)*ln(-(-1+cos(f
*x+e)+sin(f*x+e))/sin(f*x+e))-8*sin(f*x+e)*ln(2/(cos(f*x+e)+1))-6*cos(f*x+e
)^2+8*cos(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-4*cos(f*x+e)*ln
(2/(cos(f*x+e)+1))-6*sin(f*x+e)-cos(f*x+e)-16*ln(-(-1+cos(f*x+e)+sin(f*x+e)
)/sin(f*x+e))+8*ln(2/(cos(f*x+e)+1))+6)*(a*(1+sin(f*x+e)))^(5/2)/(cos(f*x+e
)^2*sin(f*x+e)-cos(f*x+e)^3+2*sin(f*x+e)*cos(f*x+e)+3*cos(f*x+e)^2-4*sin(f*
x+e)+2*cos(f*x+e)-4)/(-c*(-1+sin(f*x+e)))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)/(-c*sin(f*x + e) + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(a^{2}\cos\left(fx+e\right)^{2}-2\,a^{2}\sin\left(fx+e\right)-2\,a^{2}\right)\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}}{c^{2}\cos\left(fx+e\right)^{2}+2\,c^{2}\sin\left(fx+e\right)-2\,c^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] integral((a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x + e) - 2*a^2)*sqrt(a*sin(f*x +
 e) + a)*sqrt(-c*sin(f*x + e) + c)/(c^2*cos(f*x + e)^2 + 2*c^2*sin(f*x + e)
 - 2*c^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)/(-c*sin(f*x + e) + c)^(3/2), x)
```

3.364 $\int \frac{(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=147

$$-\frac{a^3\cos(e+fx)\log(1-\sin(e+fx))}{c^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{cf(c-c\sin(e+fx))^{3/2}}+\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f(c-c\sin(e+fx))^{5/2}}$$

```
[Out] (a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(2*f*(c - c*Sin[e + f*x])^(5/2)
) - (a^2*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(c*f*(c - c*Sin[e + f*x])^(
3/2)) - (a^3*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c^2*f*Sqrt[a + a*Sin[e +
f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 0.292987, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2739, 2737, 2667, 31}

$$-\frac{a^3\cos(e+fx)\log(1-\sin(e+fx))}{c^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{cf(c-c\sin(e+fx))^{3/2}}+\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f(c-c\sin(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)/(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] (a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(2*f*(c - c*Sin[e + f*x])^(5/2)
) - (a^2*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(c*f*(c - c*Sin[e + f*x])^(
3/2)) - (a^3*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c^2*f*Sqrt[a + a*Sin[e +
f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(2*n + 1)), x] - Dist[(b*(2*m - 1))/(d*
(2*n + 1)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{5/2}}\,dx = \frac{a\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2f(c-c\sin(e+fx))^{5/2}} - \frac{a\int \frac{(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{3/2}}\,dx}{c}$$

$$= \frac{a\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2f(c-c\sin(e+fx))^{5/2}} - \frac{a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{cf(c-c\sin(e+fx))^{3/2}} + \frac{a^2\int \frac{\sqrt{a+a\sin(e}}{\sqrt{c-c\sin(e}}}{c^2}$$

$$= \frac{a\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2f(c-c\sin(e+fx))^{5/2}} - \frac{a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{cf(c-c\sin(e+fx))^{3/2}} + \frac{\left(a^3\cos(e+\right.}{c\sqrt{a+a\sin(e}}$$

$$= \frac{a\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2f(c-c\sin(e+fx))^{5/2}} - \frac{a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{cf(c-c\sin(e+fx))^{3/2}} - \frac{\left(a^3\cos(e+f\right.}{c^2f\sqrt{a+}}$$

$$= \frac{a\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2f(c-c\sin(e+fx))^{5/2}} - \frac{a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{cf(c-c\sin(e+fx))^{3/2}} - \frac{a^3\cos(e+}{c^2f\sqrt{a+a\sin}}$$

Mathematica [A] time = 1.15252, size = 190, normalized size = 1.29

$$\frac{a^2\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos(2(e+fx))\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-3\right.}{c^2f(\sin(e+fx)-1)^2\sqrt{c-c\sin(e+fx)}\left(\sin\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)/(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] (a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(-2 -
 3*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + Cos[2*(e + f*x)]*Log[Cos[(e +
 f*x)/2] - Sin[(e + f*x)/2]] + 4*(1 + Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/
2]])*Sin[e + f*x]))/(c^2*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[
e + f*x])^2*Sqrt[c - c*Sin[e + f*x]])
```

Maple [B] time = 0.151, size = 556, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] -1/f*(2*sin(f*x+e)*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-
sin(f*x+e)*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))+2*cos(f*x+e)^3*ln(-(-1+cos(f*x
+e)+sin(f*x+e))/sin(f*x+e))-cos(f*x+e)^3*ln(2/(cos(f*x+e)+1))-2*cos(f*x+e)^
2*sin(f*x+e)+4*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*sin(f*x+e)*cos(f*
x+e)-2*ln(2/(cos(f*x+e)+1))*sin(f*x+e)*cos(f*x+e)-2*cos(f*x+e)^3-6*cos(f*x+
e)^2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+3*cos(f*x+e)^2*ln(2/(cos(f*
x+e)+1))-8*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+4*sin(f*x+
e)*ln(2/(cos(f*x+e)+1))+2*cos(f*x+e)^2-4*cos(f*x+e)*ln(-(-1+cos(f*x+e)+sin(
```

```
f*x+e))/sin(f*x+e))+2*cos(f*x+e)*ln(2/(cos(f*x+e)+1))+2*sin(f*x+e)+2*cos(f*
x+e)+8*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-4*ln(2/(cos(f*x+e)+1))-2)
*(a*(1+sin(f*x+e)))^(5/2)/(cos(f*x+e)^2*sin(f*x+e)-cos(f*x+e)^3+2*sin(f*x+e
)*cos(f*x+e)+3*cos(f*x+e)^2-4*sin(f*x+e)+2*cos(f*x+e)-4)/(-c*(-1+sin(f*x+e)
))^(5/2)
```

Maxima [A] time = 2.49532, size = 248, normalized size = 1.69

$$-\frac{\frac{8 a^{\frac{5}{2}} \sqrt{c} \sin\left(fx+e\right)^2}{\left(c^3 - \frac{4 c^3 \sin\left(fx+e\right)}{\cos\left(fx+e\right)+1} + \frac{6 c^3 \sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2} - \frac{4 c^3 \sin\left(fx+e\right)^3}{\left(\cos\left(fx+e\right)+1\right)^3} + \frac{c^3 \sin\left(fx+e\right)^4}{\left(\cos\left(fx+e\right)+1\right)^4}\right)\left(\cos\left(fx+e\right)+1\right)^2} - \frac{2 a^{\frac{5}{2}} \log\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1} - 1\right)}{c^{\frac{5}{2}}} + \frac{a^{\frac{5}{2}} \log\left(\frac{\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2} + 1\right)}{c^{\frac{5}{2}}}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] -(8*a^(5/2)*sqrt(c)*sin(f*x + e)^2/((c^3 - 4*c^3*sin(f*x + e)/(cos(f*x + e)
 + 1) + 6*c^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 4*c^3*sin(f*x + e)^3/(c
os(f*x + e) + 1)^3 + c^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4)*(cos(f*x + e)
 + 1)^2) - 2*a^(5/2)*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/c^(5/2) + a^(
5/2)*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/c^(5/2))/f
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^2 \cos\left(fx+e\right)^2 - 2 a^2 \sin\left(fx+e\right) - 2 a^2\right)\sqrt{a \sin\left(fx+e\right)+a}\sqrt{-c \sin\left(fx+e\right)+c}}{3 c^3 \cos\left(fx+e\right)^2 - 4 c^3 - \left(c^3 \cos\left(fx+e\right)^2 - 4 c^3\right)\sin\left(fx+e\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] integral((a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x + e) - 2*a^2)*sqrt(a*sin(f*x +
 e) + a)*sqrt(-c*sin(f*x + e) + c)/(3*c^3*cos(f*x + e)^2 - 4*c^3 - (c^3*cos
(f*x + e)^2 - 4*c^3)*sin(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)/(-c*sin(f*x + e) + c)^(5/2), x)
```

3.365 $\int \frac{(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=42

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{6f(c-c\sin(e+fx))^{7/2}}$$

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(6*f*(c - c*Sin[e + f*x])^(7/2))
```

Rubi [A] time = 0.0920967, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {2742}

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{6f(c-c\sin(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)/(c - c*Sin[e + f*x])^(7/2),x]
```

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(6*f*(c - c*Sin[e + f*x])^(7/2))
```

Rule 2742

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{7/2}}\,dx=\frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{6f(c-c\sin(e+fx))^{7/2}}$$

Mathematica [B] time = 0.98255, size = 110, normalized size = 2.62

$$\frac{a^2(3\cos(2(e+fx))-5)\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}{6c^3f(\sin(e+fx)-1)^3\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)/(c - c*Sin[e + f*x])^(7/2),x]
```

```
[Out] (a^2*(-5 + 3*Cos[2*(e + f*x)])*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a
*(1 + Sin[e + f*x])])/(6*c^3*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 +
Sin[e + f*x])^3*Sqrt[c - c*Sin[e + f*x]])
```

Maple [B] time = 0.142, size = 136, normalized size = 3.2

$$\frac{\left(\cos\left(fx+e\right)-2\right)\left(\cos\left(fx+e\right)+2\right)\left(-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)\right)\sin\left(fx+e\right)}{3f\left(\left(\cos\left(fx+e\right)\right)^{2}\sin\left(fx+e\right)-\left(\cos\left(fx+e\right)\right)^{3}+2\sin\left(fx+e\right)\cos\left(fx+e\right)+3\left(\cos\left(fx+e\right)\right)^{2}-4\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(7/2),x)
```

```
[Out] 1/3/f*(cos(f*x+e)-2)*(cos(f*x+e)+2)*(-1+cos(f*x+e)+sin(f*x+e))*(a*(1+sin(f*
x+e)))^(5/2)*sin(f*x+e)/(cos(f*x+e)^2*sin(f*x+e)-cos(f*x+e)^3+2*sin(f*x+e)*
cos(f*x+e)+3*cos(f*x+e)^2-4*sin(f*x+e)+2*cos(f*x+e)-4)/(-c*(-1+sin(f*x+e)))
^(7/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(7/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)/(-c*sin(f*x + e) + c)^(7/2), x)
```

Fricas [B] time = 1.0975, size = 265, normalized size = 6.31

$$\frac{\left(3a^{2}\cos\left(fx+e\right)^{2}-4a^{2}\right)\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}}{3\left(3c^{4}f\cos\left(fx+e\right)^{3}-4c^{4}f\cos\left(fx+e\right)-\left(c^{4}f\cos\left(fx+e\right)^{3}-4c^{4}f\cos\left(fx+e\right)\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(7/2),x, algorithm="frica
s")
```

```
[Out] 1/3*(3*a^2*cos(f*x + e)^2 - 4*a^2)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x
 + e) + c)/(3*c^4*f*cos(f*x + e)^3 - 4*c^4*f*cos(f*x + e) - (c^4*f*cos(f*x
+ e)^3 - 4*c^4*f*cos(f*x + e))*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(7/2),x, algorithm="giac"
)
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)/(-c*sin(f*x + e) + c)^(7/2), x)
```

3.366 $\int \frac{(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{9/2}}\,dx$

Optimal. Leaf size=88

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{48cf(c-c\sin(e+fx))^{7/2}}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{8f(c-c\sin(e+fx))^{9/2}}$$

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(8*f*(c - c*Sin[e + f*x])^(9/2))
+ (Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(48*c*f*(c - c*Sin[e + f*x])^(7
/2))
```

Rubi [A] time = 0.192558, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2743, 2742}

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{48cf(c-c\sin(e+fx))^{7/2}}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{8f(c-c\sin(e+fx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)/(c - c*Sin[e + f*x])^(9/2),x]
```

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(8*f*(c - c*Sin[e + f*x])^(9/2))
+ (Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(48*c*f*(c - c*Sin[e + f*x])^(7
/2))
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rule 2742

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{9/2}}\,dx &= \frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{8f(c-c\sin(e+fx))^{9/2}}+\frac{\int \frac{(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{7/2}}\,dx}{8c}\\ &= \frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{8f(c-c\sin(e+fx))^{9/2}}+\frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{48cf(c-c\sin(e+fx))^{7/2}}\end{aligned}$$

Mathematica [A] time = 2.18865, size = 118, normalized size = 1.34

$$\frac{a^2\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(4\sin(e+fx)-3\cos(2(e+fx))+5)}{12c^4f(\sin(e+fx)-1)^4\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)/(c - c*Sin[e + f*x])^(9/2),x]
```

```
[Out] (a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(5 -
3*Cos[2*(e + f*x)] + 4*Sin[e + f*x]))/(12*c^4*f*(Cos[(e + f*x)/2] + Sin[(e
+ f*x)/2])*(-1 + Sin[e + f*x])^4*Sqrt[c - c*Sin[e + f*x]])
```

Maple [B] time = 0.155, size = 199, normalized size = 2.3

$$-\frac{\left(\sin\left(fx+e\right)\left(\cos\left(fx+e\right)\right)^3-\left(\cos\left(fx+e\right)\right)^4-5\left(\cos\left(fx+e\right)\right)^2\sin\left(fx+e\right)-4\left(\cos\left(fx+e\right)\right)^3-4\sin\left(fx+e\right)\mathrm{co}\cdots\right.}{6f\left(\left(\cos\left(fx+e\right)\right)^2\sin\left(fx+e\right)-\left(\cos\left(fx+e\right)\right)^3+2\sin\left(fx+e\right)\cos\left(fx+e\right)+3\left(\mathrm{c}\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(9/2),x)
```

```
[Out] -1/6/f*(sin(f*x+e)*cos(f*x+e)^3-cos(f*x+e)^4-5*cos(f*x+e)^2*sin(f*x+e)-4*co
s(f*x+e)^3-4*sin(f*x+e)*cos(f*x+e)+9*cos(f*x+e)^2+14*sin(f*x+e)+10*cos(f*x+
e)-14)*(a*(1+sin(f*x+e)))^(5/2)*sin(f*x+e)/(cos(f*x+e)^2*sin(f*x+e)-cos(f*x
+e)^3+2*sin(f*x+e)*cos(f*x+e)+3*cos(f*x+e)^2-4*sin(f*x+e)+2*cos(f*x+e)-4)/(
-c*(-1+sin(f*x+e)))^(9/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(9/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)/(-c*sin(f*x + e) + c)^(9/2), x)
```

Fricas [A] time = 1.12148, size = 328, normalized size = 3.73

$$-\frac{\left(3\,a^2\cos\left(fx+e\right)^2-2\,a^2\sin\left(fx+e\right)-4\,a^2\right)\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}}{6\left(c^5f\cos\left(fx+e\right)^5-8\,c^5f\cos\left(fx+e\right)^3+8\,c^5f\cos\left(fx+e\right)+4\left(c^5f\cos\left(fx+e\right)^3-2\,c^5f\cos\left(fx+e\right)\right)\sin\left(fx+e\right)\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(9/2),x, algorithm="frica
s")
```

```
[Out] -1/6*(3*a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x + e) - 4*a^2)*sqrt(a*sin(f*x + e
) + a)*sqrt(-c*sin(f*x + e) + c)/(c^5*f*cos(f*x + e)^5 - 8*c^5*f*cos(f*x +
e)^3 + 8*c^5*f*cos(f*x + e) + 4*(c^5*f*cos(f*x + e)^3 - 2*c^5*f*cos(f*x + e
))*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(9/2),x, algorithm="giac"
)
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)/(-c*sin(f*x + e) + c)^(9/2), x)
```

3.367 $\int \frac{(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{11/2}}\,dx$

Optimal. Leaf size=133

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{240c^2f(c-c\sin(e+fx))^{7/2}}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{40cf(c-c\sin(e+fx))^{9/2}}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{10f(c-c\sin(e+fx))^{11/2}}$$

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(10*f*(c - c*Sin[e + f*x])^(11/2)
) + (Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(40*c*f*(c - c*Sin[e + f*x])^
(9/2)) + (Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(240*c^2*f*(c - c*Sin[e
+ f*x])^(7/2))
```

Rubi [A] time = 0.282607, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2743, 2742}

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{240c^2f(c-c\sin(e+fx))^{7/2}}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{40cf(c-c\sin(e+fx))^{9/2}}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{10f(c-c\sin(e+fx))^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)/(c - c*Sin[e + f*x])^(11/2),x]
```

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(10*f*(c - c*Sin[e + f*x])^(11/2)
) + (Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(40*c*f*(c - c*Sin[e + f*x])^
(9/2)) + (Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(240*c^2*f*(c - c*Sin[e
+ f*x])^(7/2))
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rule 2742

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{11/2}}\,dx = \frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{10f(c-c\sin(e+fx))^{11/2}} + \frac{\int \frac{(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{9/2}}\,dx}{5c}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{10f(c-c\sin(e+fx))^{11/2}} + \frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{40cf(c-c\sin(e+fx))^{9/2}} + \frac{\int \frac{(a+a\sin(e+fx))^{5}}{(c-c\sin(e+fx))^{7}}}{40c^2}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{10f(c-c\sin(e+fx))^{11/2}} + \frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{40cf(c-c\sin(e+fx))^{9/2}} + \frac{\cos(e+fx)(a-}{240c^2f(c-c}$$

Mathematica [A] time = 3.35421, size = 118, normalized size = 0.89

$$-\frac{a^2\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(10\sin(e+fx)-5\cos(2(e+fx))+9)}{30c^5f(\sin(e+fx)-1)^5\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)/(c - c*Sin[e + f*x])^(11/2),x]
```

```
[Out] -(a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(9 -
 5*Cos[2*(e + f*x)] + 10*Sin[e + f*x]))/(30*c^5*f*(Cos[(e + f*x)/2] + Sin[(
e + f*x)/2])*(-1 + Sin[e + f*x])^5*Sqrt[c - c*Sin[e + f*x]])
```

Maple [A] time = 0.197, size = 226, normalized size = 1.7

$$-\frac{\left(2\sin\left(fx+e\right)\left(\cos\left(fx+e\right)\right)^4+2\left(\cos\left(fx+e\right)\right)^5+10\sin\left(fx+e\right)\left(\cos\left(fx+e\right)\right)^3-12\left(\cos\left(fx+e\right)\right)^4-34\left(\cos\right.\right.}{15f\left(\left(\cos\left(fx+e\right)\right)^2\sin\left(fx+e\right)-\left(\cos\left(fx+e\right)\right)^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(11/2),x)
```

```
[Out] -1/15/f*(2*sin(f*x+e)*cos(f*x+e)^4+2*cos(f*x+e)^5+10*sin(f*x+e)*cos(f*x+e)^
3-12*cos(f*x+e)^4-34*cos(f*x+e)^2*sin(f*x+e)-24*cos(f*x+e)^3-25*sin(f*x+e)*
cos(f*x+e)+59*cos(f*x+e)^2+62*sin(f*x+e)+37*cos(f*x+e)-62)*(a*(1+sin(f*x+e)
))^(5/2)*sin(f*x+e)/(cos(f*x+e)^2*sin(f*x+e)-cos(f*x+e)^3+2*sin(f*x+e)*cos(
f*x+e)+3*cos(f*x+e)^2-4*sin(f*x+e)+2*cos(f*x+e)-4)/(-c*(-1+sin(f*x+e)))^(11
/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(11/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)/(-c*sin(f*x + e) + c)^(11/2), x)
```

Fricas [A] time = 1.15467, size = 369, normalized size = 2.77

$$-\frac{\left(5\,a^{2}\cos\left(fx+e\right)^{2}-5\,a^{2}\sin\left(fx+e\right)-7\,a^{2}\right)\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}}{15\left(5\,c^{6}f\cos\left(fx+e\right)^{5}-20\,c^{6}f\cos\left(fx+e\right)^{3}+16\,c^{6}f\cos\left(fx+e\right)-\left(c^{6}f\cos\left(fx+e\right)^{5}-12\,c^{6}f\cos\left(fx+e\right)^{3}+16\,c^{6}f\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(11/2),x, algorithm="fric
as")
```

```
[Out] -1/15*(5*a^2*cos(f*x + e)^2 - 5*a^2*sin(f*x + e) - 7*a^2)*sqrt(a*sin(f*x +
e) + a)*sqrt(-c*sin(f*x + e) + c)/(5*c^6*f*cos(f*x + e)^5 - 20*c^6*f*cos(f*
x + e)^3 + 16*c^6*f*cos(f*x + e) - (c^6*f*cos(f*x + e)^5 - 12*c^6*f*cos(f*x
 + e)^3 + 16*c^6*f*cos(f*x + e))*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(11/2),x, algorithm="giac
")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)/(-c*sin(f*x + e) + c)^(11/2), x)
```

3.368 $\int \frac{(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{13/2}} dx$

Optimal. Leaf size=140

$$\frac{a^3\cos(e+fx)}{60c^2f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{9/2}}-\frac{a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{15cf(c-c\sin(e+fx))^{11/2}}+\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{6f(c-c\sin(e+fx))^{13/2}}$$

```
[Out] (a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(6*f*(c - c*Sin[e + f*x])^(13/2
)) - (a^2*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(15*c*f*(c - c*Sin[e + f*x
])^(11/2)) + (a^3*Cos[e + f*x])/(60*c^2*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*S
in[e + f*x])^(9/2))
```

Rubi [A] time = 0.272628, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2739, 2738}

$$\frac{a^3\cos(e+fx)}{60c^2f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{9/2}}-\frac{a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{15cf(c-c\sin(e+fx))^{11/2}}+\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{6f(c-c\sin(e+fx))^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)/(c - c*Sin[e + f*x])^(13/2),x]
```

```
[Out] (a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(6*f*(c - c*Sin[e + f*x])^(13/2
)) - (a^2*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(15*c*f*(c - c*Sin[e + f*x
])^(11/2)) + (a^3*Cos[e + f*x])/(60*c^2*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*S
in[e + f*x])^(9/2))
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(2*n + 1)), x] - Dist[(b*(2*m - 1))/(d*
(2*n + 1)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{13/2}}\,dx = \frac{a\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{6f(c-c\sin(e+fx))^{13/2}} - \frac{a\int \frac{(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{11/2}}\,dx}{3c}$$

$$= \frac{a\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{6f(c-c\sin(e+fx))^{13/2}} - \frac{a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{15cf(c-c\sin(e+fx))^{11/2}} + \frac{a^2\int \frac{\sqrt{a+a\sin(e+f}}{(c-c\sin(e+fx)}}{15c^2}$$

$$= \frac{a\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{6f(c-c\sin(e+fx))^{13/2}} - \frac{a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{15cf(c-c\sin(e+fx))^{11/2}} + \frac{a}{60c^2f\sqrt{a+a\sin}}$$

Mathematica [A] time = 4.74052, size = 118, normalized size = 0.84

$$\frac{a^2\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(36\sin(e+fx)-15\cos(2(e+fx))+29)}{120c^6f(\sin(e+fx)-1)^6\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)/(c - c*Sin[e + f*x])^(13/2),x]
```

```
[Out] (a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(29 -
 15*Cos[2*(e + f*x)] + 36*Sin[e + f*x]))/(120*c^6*f*(Cos[(e + f*x)/2] + Sin
[(e + f*x)/2])*(-1 + Sin[e + f*x])^6*Sqrt[c - c*Sin[e + f*x]])
```

Maple [B] time = 0.187, size = 252, normalized size = 1.8

$$\frac{\left(7\sin(fx+e)(\cos(fx+e))^5-7(\cos(fx+e))^6-49\sin(fx+e)(\cos(fx+e))^4-42(\cos(fx+e))^5-119\sin(fx}{60f\left((\cos(fx+e))^2\sin\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(13/2),x)
```

```
[Out] 1/60/f*(7*sin(f*x+e)*cos(f*x+e)^5-7*cos(f*x+e)^6-49*sin(f*x+e)*cos(f*x+e)^4
-42*cos(f*x+e)^5-119*sin(f*x+e)*cos(f*x+e)^3+168*cos(f*x+e)^4+343*cos(f*x+e
)^2*sin(f*x+e)+224*cos(f*x+e)^3+202*sin(f*x+e)*cos(f*x+e)-545*cos(f*x+e)^2-
444*sin(f*x+e)-242*cos(f*x+e)+444)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(5/2)/(cos
(f*x+e)^2*sin(f*x+e)-cos(f*x+e)^3+2*sin(f*x+e)*cos(f*x+e)+3*cos(f*x+e)^2-4*
sin(f*x+e)+2*cos(f*x+e)-4)/(-c*(-1+sin(f*x+e)))^(13/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(13/2),x, algorithm="maxi
ma")
```

```
[Out] Timed out
```

Fricas [A] time = 1.1942, size = 409, normalized size = 2.92

$$\frac{\left(15\,a^2\cos\left(fx+e\right)^2-18\,a^2\sin\left(fx+e\right)-22\,a^2\right)\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin}}{60\left(c^7f\cos\left(fx+e\right)^7-18\,c^7f\cos\left(fx+e\right)^5+48\,c^7f\cos\left(fx+e\right)^3-32\,c^7f\cos\left(fx+e\right)+2\left(3\,c^7f\cos\left(fx+e\right)^5-16\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(13/2),x, algorithm="fric
as")
```

```
[Out] 1/60*(15*a^2*cos(f*x + e)^2 - 18*a^2*sin(f*x + e) - 22*a^2)*sqrt(a*sin(f*x
+ e) + a)*sqrt(-c*sin(f*x + e) + c)/(c^7*f*cos(f*x + e)^7 - 18*c^7*f*cos(f*
x + e)^5 + 48*c^7*f*cos(f*x + e)^3 - 32*c^7*f*cos(f*x + e) + 2*(3*c^7*f*cos
(f*x + e)^5 - 16*c^7*f*cos(f*x + e)^3 + 16*c^7*f*cos(f*x + e))*sin(f*x + e)
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(13/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(13/2),x, algorithm="giac
")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)/(-c*sin(f*x + e) + c)^(13/2), x)
```

3.369 $\int (a + a\sin(e + fx))^{7/2}(c - c\sin(e + fx))^{9/2}\,dx$

Optimal. Leaf size=179

$$-\frac{3a^2\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{9/2}}{28f}-\frac{a^3\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{9/2}}{14f}-\frac{a^4}{}$$

```
[Out] -(a^4*Cos[e + f*x]*(c - c*Sin[e + f*x])^(9/2))/(35*f*Sqrt[a + a*Sin[e + f*x
]]) - (a^3*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(9/2)
)/(14*f) - (3*a^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*
x])^(9/2))/(28*f) - (a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e
 + f*x])^(9/2))/(8*f)
```

Rubi [A] time = 0.36556, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2740, 2738}

$$-\frac{3a^2\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{9/2}}{28f}-\frac{a^3\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{9/2}}{14f}-\frac{a^4}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(7/2)*(c - c*Sin[e + f*x])^(9/2),x]
```


```
[Out] -(a^4*Cos[e + f*x]*(c - c*Sin[e + f*x])^(9/2))/(35*f*Sqrt[a + a*Sin[e + f*x
]]) - (a^3*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(9/2)
)/(14*f) - (3*a^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*
x])^(9/2))/(28*f) - (a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e
 + f*x])^(9/2))/(8*f)
```


Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{9/2}\,dx = -\frac{a\cos(e+fx)(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{9/2}}{8f} + \frac{1}{4}(3a)\int$$

$$= -\frac{3a^2\cos(e+fx)(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{9/2}}{28f} - \frac{a\cos(}{}$$

$$= -\frac{a^3\cos(e+fx)\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{9/2}}{14f} - \frac{3a^2\cos(e}{}$$

$$= -\frac{a^4\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{35f\sqrt{a+a\sin(e+fx)}} - \frac{a^3\cos(e+fx)\sqrt{a+a\sin(e+}}{14f}$$

Mathematica [A] time = 5.62273, size = 127, normalized size = 0.71

$$\frac{a^3c^4\sec(e+fx)\sqrt{a(\sin(e+fx)+1)}\sqrt{c-c\sin(e+fx)}(19600\sin(e+fx)+3920\sin(3(e+fx))+784\sin(5(e+fx))}{35840f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(7/2)*(c - c*Sin[e + f*x])^(9/2),x]
```

```
[Out] (a^3*c^4*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]]*(
1960*Cos[2*(e + f*x)] + 980*Cos[4*(e + f*x)] + 280*Cos[6*(e + f*x)] + 35*Co
s[8*(e + f*x)] + 19600*Sin[e + f*x] + 3920*Sin[3*(e + f*x)] + 784*Sin[5*(e
+ f*x)] + 80*Sin[7*(e + f*x)]))/(35840*f)
```

Maple [A] time = 0.203, size = 143, normalized size = 0.8

$$\frac{\sin(fx+e)\left(35\left(\cos(fx+e)\right)^8+5\left(\cos(fx+e)\right)^6\sin(fx+e)+40\left(\cos(fx+e)\right)^6+13\sin(fx+e)\left(\cos(fx+e\right.\right.}{280f\left(\cos(fx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(9/2),x)
```

```
[Out] 1/280/f*(-c*(-1+sin(f*x+e)))^(9/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(7/2)*(35*
cos(f*x+e)^8+5*cos(f*x+e)^6*sin(f*x+e)+40*cos(f*x+e)^6+13*sin(f*x+e)*cos(f*
x+e)^4+48*cos(f*x+e)^4+29*cos(f*x+e)^2*sin(f*x+e)+64*cos(f*x+e)^2+93*sin(f*
x+e)+93)/cos(f*x+e)^9
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin(fx+e)+a\right)^{\frac{7}{2}}\left(-c\sin(fx+e)+c\right)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(9/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(7/2)*(-c*sin(f*x + e) + c)^(9/2), x)
```

Fricas [A] time = 1.20983, size = 306, normalized size = 1.71

$$\frac{\left(35\,a^3c^4\cos\left(fx+e\right)^8-35\,a^3c^4+8\left(5\,a^3c^4\cos\left(fx+e\right)^6+6\,a^3c^4\cos\left(fx+e\right)^4+8\,a^3c^4\cos\left(fx+e\right)^2+16\,a^3c^4\right)\sin\left(fx\right.}{280\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(9/2),x, algorithm="frica
s")
```

```
[Out] 1/280*(35*a^3*c^4*cos(f*x + e)^8 - 35*a^3*c^4 + 8*(5*a^3*c^4*cos(f*x + e)^6
 + 6*a^3*c^4*cos(f*x + e)^4 + 8*a^3*c^4*cos(f*x + e)^2 + 16*a^3*c^4)*sin(f*
x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(7/2)*(c-c*sin(f*x+e))**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

sage_2

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(9/2),x, algorithm="giac"
)
```

```
[Out] sage2
```

3.370 $\int (a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{7/2}\,dx$

Optimal. Leaf size=179

$$-\frac{2a^4\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{35f\sqrt{a\sin(e+fx)+a}}-\frac{4a^3\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{7/2}}{35f}-\frac{a^2\cos(e+fx)(a\sin$$

```
[Out] (-2*a^4*Cos[e + f*x]*(c - c*Sin[e + f*x])^(7/2))/(35*f*Sqrt[a + a*Sin[e + f
*x]]) - (4*a^3*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(
7/2))/(35*f) - (a^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e +
f*x])^(7/2))/(7*f) - (a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[
e + f*x])^(7/2))/(7*f)
```

Rubi [A] time = 0.366398, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2740, 2738}

$$-\frac{2a^4\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{35f\sqrt{a\sin(e+fx)+a}}-\frac{4a^3\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{7/2}}{35f}-\frac{a^2\cos(e+fx)(a\sin$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(7/2)*(c - c*Sin[e + f*x])^(7/2),x]
```


```
[Out] (-2*a^4*Cos[e + f*x]*(c - c*Sin[e + f*x])^(7/2))/(35*f*Sqrt[a + a*Sin[e + f
*x]]) - (4*a^3*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(
7/2))/(35*f) - (a^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e +
f*x])^(7/2))/(7*f) - (a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[
e + f*x])^(7/2))/(7*f)
```


Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] && !LtQ[n, -1] && !(IGtQ[n - 1/2, 0] && LtQ[n, m]) && !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{7/2}\,dx = -\frac{a\cos(e+fx)(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{7/2}}{7f}+\frac{1}{7}(6a)\int (a$$
$$= -\frac{a^2\cos(e+fx)(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{7/2}}{7f}-\frac{a\cos(e+}{}$$
$$= -\frac{4a^3\cos(e+fx)\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{7/2}}{35f}-\frac{a^2\cos(e+}{}$$
$$= -\frac{2a^4\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{35f\sqrt{a+a\sin(e+fx)}}-\frac{4a^3\cos(e+fx)\sqrt{a+a\sin(e+}}{35f}$$

Mathematica [A] time = 1.01169, size = 87, normalized size = 0.49

$$\frac{a^3c^3(1225\sin(e+fx)+245\sin(3(e+fx))+49\sin(5(e+fx))+5\sin(7(e+fx)))\sec(e+fx)\sqrt{a(\sin(e+fx)+1)}\sqrt{c-c}}{2240f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(7/2)*(c - c*Sin[e + f*x])^(7/2),x]
```

```
[Out] (a^3*c^3*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]]*(
1225*Sin[e + f*x] + 245*Sin[3*(e + f*x)] + 49*Sin[5*(e + f*x)] + 5*Sin[7*(e
 + f*x)]))/(2240*f)
```

Maple [A] time = 0.156, size = 77, normalized size = 0.4

$$\frac{\left(5\left(\cos\left(fx+e\right)\right)^6+6\left(\cos\left(fx+e\right)\right)^4+8\left(\cos\left(fx+e\right)\right)^2+16\right)\sin\left(fx+e\right)}{35f\left(\cos\left(fx+e\right)\right)^7}\left(-c\left(-1+\sin\left(fx+e\right)\right)\right)^{\frac{7}{2}}\left(a\left(1+\sin\left(fx+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(7/2),x)
```

```
[Out] 1/35/f*(5*cos(f*x+e)^6+6*cos(f*x+e)^4+8*cos(f*x+e)^2+16)*(-c*(-1+sin(f*x+e)
))^(7/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(7/2)/cos(f*x+e)^7
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a\sin\left(fx+e\right)+a\right)^{\frac{7}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(7/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(7/2)*(-c*sin(f*x + e) + c)^(7/2), x)
```

Fricas [A] time = 1.1831, size = 244, normalized size = 1.36

$$\frac{\left(5\,a^{3}c^{3}\cos\left(fx+e\right)^{6}+6\,a^{3}c^{3}\cos\left(fx+e\right)^{4}+8\,a^{3}c^{3}\cos\left(fx+e\right)^{2}+16\,a^{3}c^{3}\right)\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}}{35\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(7/2),x, algorithm="frica
s")
```

```
[Out] 1/35*(5*a^3*c^3*cos(f*x + e)^6 + 6*a^3*c^3*cos(f*x + e)^4 + 8*a^3*c^3*cos(f
*x + e)^2 + 16*a^3*c^3)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*
sin(f*x + e)/(f*cos(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(7/2)*(c-c*sin(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(7/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError
```

3.371 $\int (a + a\sin(e + fx))^{7/2}(c - c\sin(e + fx))^{5/2}\,dx$

Optimal. Leaf size=134

$$\frac{2c^2\cos(e+fx)(a\sin(e+fx)+a)^{7/2}\sqrt{c-c\sin(e+fx)}}{15f}+\frac{c^3\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{15f\sqrt{c-c\sin(e+fx)}}+\frac{c\cos(e+fx)(a\sin(e+f$$

```
[Out] (c^3*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(15*f*Sqrt[c - c*Sin[e + f*x]
]) + (2*c^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2)*Sqrt[c - c*Sin[e + f*x]
])/(15*f) + (c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2)*(c - c*Sin[e + f*x])
^(3/2))/(6*f)
```

Rubi [A] time = 0.263819, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2740, 2738}

$$\frac{2c^2\cos(e+fx)(a\sin(e+fx)+a)^{7/2}\sqrt{c-c\sin(e+fx)}}{15f}+\frac{c^3\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{15f\sqrt{c-c\sin(e+fx)}}+\frac{c\cos(e+fx)(a\sin(e+f$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(7/2)*(c - c*Sin[e + f*x])^(5/2),x]
```


```
[Out] (c^3*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(15*f*Sqrt[c - c*Sin[e + f*x]
]) + (2*c^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2)*Sqrt[c - c*Sin[e + f*x]
])/(15*f) + (c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2)*(c - c*Sin[e + f*x])
^(3/2))/(6*f)
```


Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{5/2}\,dx=\frac{c\cos(e+fx)(a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{3/2}}{6f}+\frac{1}{3}(2c)\int (a+$$

$$=\frac{2c^2\cos(e+fx)(a+a\sin(e+fx))^{7/2}\sqrt{c-c\sin(e+fx)}}{15f}+\frac{c\cos(e+fx)}{}$$

$$=\frac{c^3\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{15f\sqrt{c-c\sin(e+fx)}}+\frac{2c^2\cos(e+fx)(a+a\sin(e+fx}{15f}$$

Mathematica [A] time = 1.22532, size = 107, normalized size = 0.8

$$\frac{a^3c^2\sec(e+fx)\sqrt{a(\sin(e+fx)+1)}\sqrt{c-c\sin(e+fx)}(600\sin(e+fx)+100\sin(3(e+fx))+12\sin(5(e+fx))-75\cos}{960f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(7/2)*(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] (a^3*c^2*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]]*(
-75*Cos[2*(e + f*x)] - 30*Cos[4*(e + f*x)] - 5*Cos[6*(e + f*x)] + 600*Sin[e
 + f*x] + 100*Sin[3*(e + f*x)] + 12*Sin[5*(e + f*x)]))/(960*f)
```

Maple [A] time = 0.167, size = 116, normalized size = 0.9

$$-\frac{\sin\left(fx+e\right)\left(-5\left(\cos\left(fx+e\right)\right)^{6}+\sin\left(fx+e\right)\left(\cos\left(fx+e\right)\right)^{4}-6\left(\cos\left(fx+e\right)\right)^{4}+3\left(\cos\left(fx+e\right)\right)^{2}\sin\left(fx+e\right)}{30f\left(\cos\left(fx+e\right)\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] -1/30/f*(-c*(-1+sin(f*x+e)))^(5/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(7/2)*(-5*
cos(f*x+e)^6+sin(f*x+e)*cos(f*x+e)^4-6*cos(f*x+e)^4+3*cos(f*x+e)^2*sin(f*x+
e)-8*cos(f*x+e)^2+11*sin(f*x+e)-11)/cos(f*x+e)^7
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a\sin\left(fx+e\right)+a\right)^{\frac{7}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(7/2)*(-c*sin(f*x + e) + c)^(5/2), x)
```

Fricas [A] time = 1.1617, size = 266, normalized size = 1.99

$$-\frac{\left(5\,a^3c^2\cos\left(fx+e\right)^6-5\,a^3c^2-2\left(3\,a^3c^2\cos\left(fx+e\right)^4+4\,a^3c^2\cos\left(fx+e\right)^2+8\,a^3c^2\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)}}{30\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] -1/30*(5*a^3*c^2*cos(f*x + e)^6 - 5*a^3*c^2 - 2*(3*a^3*c^2*cos(f*x + e)^4 +
 4*a^3*c^2*cos(f*x + e)^2 + 8*a^3*c^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) +
```

```
a)*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(7/2)*(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] sage2
```

3.372 $\int (a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=89

$$\frac{c^2\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{10f\sqrt{c-c\sin(e+fx)}}+\frac{c\cos(e+fx)(a\sin(e+fx)+a)^{7/2}\sqrt{c-c\sin(e+fx)}}{5f}$$

```
[Out] (c^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(10*f*Sqrt[c - c*Sin[e + f*x]
]) + (c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2)*Sqrt[c - c*Sin[e + f*x]])/(
5*f)
```

Rubi [A] time = 0.169982, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2740, 2738}

$$\frac{c^2\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{10f\sqrt{c-c\sin(e+fx)}}+\frac{c\cos(e+fx)(a\sin(e+fx)+a)^{7/2}\sqrt{c-c\sin(e+fx)}}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(7/2)*(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] (c^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(10*f*Sqrt[c - c*Sin[e + f*x]
]) + (c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2)*Sqrt[c - c*Sin[e + f*x]])/(
5*f)
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] && !LtQ[n, -1] && !(IGtQ[n - 1/2, 0] && LtQ[n, m]) && !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{3/2}\,dx=\frac{c\cos(e+fx)(a+a\sin(e+fx))^{7/2}\sqrt{c-c\sin(e+fx)}}{5f}+\frac{1}{5}(2c)\int (a+$$

$$=\frac{c^2\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{10f\sqrt{c-c\sin(e+fx)}}+\frac{c\cos(e+fx)(a+a\sin(e+f}{5f}$$

Mathematica [A] time = 0.963196, size = 93, normalized size = 1.04

$$-\frac{a^3c\sec(e+fx)\sqrt{a(\sin(e+fx)+1)}\sqrt{c-c\sin(e+fx)}(-70\sin(e+fx)-5\sin(3(e+fx))+\sin(5(e+fx))+20\cos(2}{80f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(7/2)*(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] -(a^3*c*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]]*(2
0*Cos[2*(e + f*x)] + 5*Cos[4*(e + f*x)] - 70*Sin[e + f*x] - 5*Sin[3*(e + f*
x)] + Sin[5*(e + f*x)]))/(80*f)
```

Maple [A] time = 0.161, size = 106, normalized size = 1.2

$$-\frac{\sin\left(fx+e\right)\left(-2\left(\cos\left(fx+e\right)\right)^{6}+\sin\left(fx+e\right)\left(\cos\left(fx+e\right)\right)^{4}-2\left(\cos\left(fx+e\right)\right)^{4}+3\left(\cos\left(fx+e\right)\right)^{2}\sin\left(fx+e\right)+6}{10f\left(\cos\left(fx+e\right)\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] -1/10/f*(-c*(-1+sin(f*x+e)))^(3/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(7/2)*(-2*
cos(f*x+e)^6+sin(f*x+e)*cos(f*x+e)^4-2*cos(f*x+e)^4+3*cos(f*x+e)^2*sin(f*x+
e)+6*sin(f*x+e)-6)/cos(f*x+e)^7
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^{\frac{7}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(7/2)*(-c*sin(f*x + e) + c)^(3/2), x)
```

Fricas [A] time = 1.11897, size = 250, normalized size = 2.81

$$-\frac{\left(5\,a^{3}c\cos\left(fx+e\right)^{4}-5\,a^{3}c+2\left(a^{3}c\cos\left(fx+e\right)^{4}-2\,a^{3}c\cos\left(fx+e\right)^{2}-4\,a^{3}c\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin}}{10f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] -1/10*(5*a^3*c*cos(f*x + e)^4 - 5*a^3*c + 2*(a^3*c*cos(f*x + e)^4 - 2*a^3*c
*cos(f*x + e)^2 - 4*a^3*c)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*s
in(f*x + e) + c)/(f*cos(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(7/2)*(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] sage2
```

3.373 $\int (a+a\sin(e+fx))^{7/2}\sqrt{c-c\sin(e+fx)}\,dx$

Optimal. Leaf size=43

$$\frac{c\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{4f\sqrt{c-c\sin(e+fx)}}$$

```
[Out] (c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(4*f*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 0.0815774, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {2738}

$$\frac{c\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{4f\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(7/2)*Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(4*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{7/2}\sqrt{c-c\sin(e+fx)}\,dx=\frac{c\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{4f\sqrt{c-c\sin(e+fx)}}$$

Mathematica [A] time = 0.340935, size = 82, normalized size = 1.91

$$\frac{a^3\sec(e+fx)\sqrt{a(\sin(e+fx)+1)}\sqrt{c-c\sin(e+fx)}(56\sin(e+fx)-8\sin(3(e+fx))-28\cos(2(e+fx))+\cos(4(e+f}{32f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(7/2)*Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (a^3*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]]*(-28*
Cos[2*(e + f*x)] + Cos[4*(e + f*x)] + 56*Sin[e + f*x] - 8*Sin[3*(e + f*x)])
)/(32*f)
```

Maple [B] time = 0.17, size = 103, normalized size = 2.4

$$-\frac{\sin\left(fx+e\right)\left(-\left(\cos\left(fx+e\right)\right)^{6}+\sin\left(fx+e\right)\left(\cos\left(fx+e\right)\right)^{4}+\left(\cos\left(fx+e\right)\right)^{2}\sin\left(fx+e\right)+\left(\cos\left(fx+e\right)\right)^{2}+4\sin\left(\right.}{4f\left(\cos\left(fx+e\right)\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] -1/4/f*(-c*(-1+sin(f*x+e)))^(1/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(7/2)*(-cos
(f*x+e)^6+sin(f*x+e)*cos(f*x+e)^4+cos(f*x+e)^2*sin(f*x+e)+cos(f*x+e)^2+4*si
n(f*x+e)-4)/cos(f*x+e)^7
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^{\frac{7}{2}}\sqrt{-c\sin\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(7/2)*sqrt(-c*sin(f*x + e) + c), x)
```

Fricas [B] time = 1.1097, size = 231, normalized size = 5.37

$$\frac{\left(a^3\cos\left(fx+e\right)^4-8\,a^3\cos\left(fx+e\right)^2+7\,a^3-4\left(a^3\cos\left(fx+e\right)^2-2\,a^3\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(f\right.}}{4\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] 1/4*(a^3*cos(f*x + e)^4 - 8*a^3*cos(f*x + e)^2 + 7*a^3 - 4*(a^3*cos(f*x + e
)^2 - 2*a^3)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) +
c)/(f*cos(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(7/2)*(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError
```

3.374 $\int \frac{(a+a\sin(e+fx))^{7/2}}{\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=184

$$-\frac{4a^3\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}-\frac{a^2\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{f\sqrt{c-c\sin(e+fx)}}-\frac{8a^4\cos(e+fx)\log(1-\sin(e+fx))}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-$$

```
[Out] (-8*a^4*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f*x]]*Sqr
t[c - c*Sin[e + f*x]]) - (4*a^3*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(f*S
qrt[c - c*Sin[e + f*x]]) - (a^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(f
*Sqrt[c - c*Sin[e + f*x]]) - (a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(3
*f*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 0.376197, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2740, 2737, 2667, 31}

$$-\frac{4a^3\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}-\frac{a^2\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{f\sqrt{c-c\sin(e+fx)}}-\frac{8a^4\cos(e+fx)\log(1-\sin(e+fx))}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(7/2)/Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (-8*a^4*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f*x]]*Sqr
t[c - c*Sin[e + f*x]]) - (4*a^3*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(f*S
qrt[c - c*Sin[e + f*x]]) - (a^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(f
*Sqrt[c - c*Sin[e + f*x]]) - (a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(3
*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{7/2}}{\sqrt{c-c\sin(e+fx)}}\,dx = -\frac{a\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3f\sqrt{c-c\sin(e+fx)}} + (2a)\int \frac{(a+a\sin(e+fx))^{5/2}}{\sqrt{c-c\sin(e+fx)}}\,dx$$
$$= -\frac{a^2\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{f\sqrt{c-c\sin(e+fx)}} - \frac{a\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3f\sqrt{c-c\sin(e+fx)}} + \left(4a^2\right)\int \frac{(a+}{\sqrt{c}}$$
$$= -\frac{4a^3\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{f\sqrt{c-c\sin(e+fx)}} - \frac{a^2\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{f\sqrt{c-c\sin(e+fx)}} - \frac{a\cos(e+fx)}{3f\sqrt{c}}$$
$$= -\frac{4a^3\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{f\sqrt{c-c\sin(e+fx)}} - \frac{a^2\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{f\sqrt{c-c\sin(e+fx)}} - \frac{a\cos(e+fx)}{3f\sqrt{c}}$$
$$= -\frac{4a^3\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{f\sqrt{c-c\sin(e+fx)}} - \frac{a^2\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{f\sqrt{c-c\sin(e+fx)}} - \frac{a\cos(e+fx)}{3f\sqrt{c}}$$
$$= -\frac{8a^4\cos(e+fx)\log(1-\sin(e+fx))}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{4a^3\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{f\sqrt{c-c\sin(e+fx)}} - \frac{a^2\cos(e+}{f}$$

Mathematica [A] time = 1.02392, size = 150, normalized size = 0.82

$$-\frac{a^3(\sin(e+fx)+1)^3\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(87\sin(e+fx)-\sin(3(e+fx))-12\cos(2\right.}{12f\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(7/2)/Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] -(a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^3*Sqrt[a*(1
+ Sin[e + f*x])]*(-12*Cos[2*(e + f*x)] + 192*Log[Cos[(e + f*x)/2] - Sin[(e
+ f*x)/2]] + 87*Sin[e + f*x] - Sin[3*(e + f*x)]))/(12*f*(Cos[(e + f*x)/2] +
 Sin[(e + f*x)/2])^7*Sqrt[c - c*Sin[e + f*x]])
```

Maple [B] time = 0.197, size = 367, normalized size = 2.

$$\frac{1}{3f\left(\sin\left(fx+e\right)\left(\cos\left(fx+e\right)\right)^3+\left(\cos\left(fx+e\right)\right)^4-4\left(\cos\left(fx+e\right)\right)^2\sin\left(fx+e\right)+3\left(\cos\left(fx+e\right)\right)^3-4\sin\left(fx+e\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] 1/3/f*(sin(f*x+e)*cos(f*x+e)^3-cos(f*x+e)^4+5*cos(f*x+e)^2*sin(f*x+e)+6*cos
(f*x+e)^3-22*sin(f*x+e)*cos(f*x+e)-48*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x
+e))/sin(f*x+e))+24*sin(f*x+e)*ln(2/(cos(f*x+e)+1))+17*cos(f*x+e)^2-48*cos(
f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+24*cos(f*x+e)*ln(2/(cos(f
```

```
*x+e)+1))+16*sin(f*x+e)-6*cos(f*x+e)+48*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(
f*x+e))-24*ln(2/(cos(f*x+e)+1))-16)*(a*(1+sin(f*x+e)))^(7/2)/(sin(f*x+e)*co
s(f*x+e)^3+cos(f*x+e)^4-4*cos(f*x+e)^2*sin(f*x+e)+3*cos(f*x+e)^3-4*sin(f*x+
e)*cos(f*x+e)-8*cos(f*x+e)^2+8*sin(f*x+e)-4*cos(f*x+e)+8)/(-c*(-1+sin(f*x+e
)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\sin(fx+e)+a)^{\frac{7}{2}}}{\sqrt{-c\sin(fx+e)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(7/2)/sqrt(-c*sin(f*x + e) + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(3\,a^3\cos\left(fx+e\right)^2-4\,a^3+\left(a^3\cos\left(fx+e\right)^2-4\,a^3\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}}{c\sin\left(fx+e\right)-c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] integral((3*a^3*cos(f*x + e)^2 - 4*a^3 + (a^3*cos(f*x + e)^2 - 4*a^3)*sin(f
*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c*sin(f*x + e)
 - c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(7/2)/(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\sin(fx+e)+a)^{\frac{7}{2}}}{\sqrt{-c\sin(fx+e)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate((a*sin(f*x + e) + a)^(7/2)/sqrt(-c*sin(f*x + e) + c), x)
```

3.375 $\int \frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=192

$$\frac{6a^3\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{cf\sqrt{c-c\sin(e+fx)}}+\frac{3a^2\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2cf\sqrt{c-c\sin(e+fx)}}+\frac{12a^4\cos(e+fx)\log(1-\sin(e+fx))}{cf\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

```
[Out] (a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(f*(c - c*Sin[e + f*x])^(3/2))
+ (12*a^4*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c*f*Sqrt[a + a*Sin[e + f*x]]
*Sqrt[c - c*Sin[e + f*x]]) + (6*a^3*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/
(c*f*Sqrt[c - c*Sin[e + f*x]]) + (3*a^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(
3/2))/(2*c*f*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 0.395516, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2739, 2740, 2737, 2667, 31}

$$\frac{6a^3\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{cf\sqrt{c-c\sin(e+fx)}}+\frac{3a^2\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2cf\sqrt{c-c\sin(e+fx)}}+\frac{12a^4\cos(e+fx)\log(1-\sin(e+fx))}{cf\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(7/2)/(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] (a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(f*(c - c*Sin[e + f*x])^(3/2))
+ (12*a^4*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c*f*Sqrt[a + a*Sin[e + f*x]]
*Sqrt[c - c*Sin[e + f*x]]) + (6*a^3*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/
(c*f*Sqrt[c - c*Sin[e + f*x]]) + (3*a^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(
3/2))/(2*c*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(2*n + 1)), x] - Dist[(b*(2*m - 1))/(d*
(2*n + 1)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
```

```
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{3/2}}\,dx = \frac{a\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{f(c-c\sin(e+fx))^{3/2}} - \frac{(3a)\int \frac{(a+a\sin(e+fx))^{5/2}}{\sqrt{c-c\sin(e+fx)}}\,dx}{c}$$

$$= \frac{a\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{f(c-c\sin(e+fx))^{3/2}} + \frac{3a^2\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2cf\sqrt{c-c\sin(e+fx)}} - \frac{\left(6a^2\right)\int \frac{(a+a\sin}{\sqrt{c-c}}}{c}$$

$$= \frac{a\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{f(c-c\sin(e+fx))^{3/2}} + \frac{6a^3\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{cf\sqrt{c-c\sin(e+fx)}} + \frac{3a^2\cos(e+fx)}{2cf\sqrt{c-}}$$

$$= \frac{a\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{f(c-c\sin(e+fx))^{3/2}} + \frac{6a^3\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{cf\sqrt{c-c\sin(e+fx)}} + \frac{3a^2\cos(e+fx)}{2cf\sqrt{c-}}$$

$$= \frac{a\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{f(c-c\sin(e+fx))^{3/2}} + \frac{6a^3\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{cf\sqrt{c-c\sin(e+fx)}} + \frac{3a^2\cos(e+fx)}{2cf\sqrt{c-}}$$

$$= \frac{a\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{f(c-c\sin(e+fx))^{3/2}} + \frac{12a^4\cos(e+fx)\log(1-\sin(e+fx))}{cf\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{6a^3\cos(e}{cf}$$

Mathematica [A] time = 1.59808, size = 179, normalized size = 0.93

$$-\frac{a^3\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin(3(e+fx))+18\cos(2(e+fx))+192\log\left(\cos\left(\frac{1}{2}(e+fx\right.\right.\right.}{8cf(\sin(e+fx)-1)\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(7/2)/(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] -(a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(44
+ 18*Cos[2*(e + f*x)] + 192*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + (39
- 192*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]])*Sin[e + f*x] + Sin[3*(e + f
*x)]))/(8*c*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])*Sqr
t[c - c*Sin[e + f*x]])
```

Maple [B] time = 0.176, size = 491, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] 1/2/f*(sin(f*x+e)*cos(f*x+e)^3-cos(f*x+e)^4+8*cos(f*x+e)^2*sin(f*x+e)+24*ln
(2/(cos(f*x+e)+1))*sin(f*x+e)*cos(f*x+e)-48*ln(-(-1+cos(f*x+e)+sin(f*x+e))/
sin(f*x+e))*sin(f*x+e)*cos(f*x+e)+9*cos(f*x+e)^3-24*cos(f*x+e)^2*ln(2/(cos(
f*x+e)+1))+48*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+25*si
n(f*x+e)*cos(f*x+e)-48*sin(f*x+e)*ln(2/(cos(f*x+e)+1))+96*sin(f*x+e)*ln(-(-
1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-33*cos(f*x+e)^2-24*cos(f*x+e)*ln(2/(co
s(f*x+e)+1))+48*cos(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-34*si
n(f*x+e)-9*cos(f*x+e)+48*ln(2/(cos(f*x+e)+1))-96*ln(-(-1+cos(f*x+e)+sin(f*x
+e))/sin(f*x+e))+34)*(a*(1+sin(f*x+e)))^(7/2)/(sin(f*x+e)*cos(f*x+e)^3+cos(
f*x+e)^4-4*cos(f*x+e)^2*sin(f*x+e)+3*cos(f*x+e)^3-4*sin(f*x+e)*cos(f*x+e)-8
*cos(f*x+e)^2+8*sin(f*x+e)-4*cos(f*x+e)+8)/(-c*(-1+sin(f*x+e)))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{7}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(7/2)/(-c*sin(f*x + e) + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(3\,a^3\cos\left(fx+e\right)^2-4\,a^3+\left(a^3\cos\left(fx+e\right)^2-4\,a^3\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}}{c^2\cos\left(fx+e\right)^2+2\,c^2\sin\left(fx+e\right)-2\,c^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] integral((3*a^3*cos(f*x + e)^2 - 4*a^3 + (a^3*cos(f*x + e)^2 - 4*a^3)*sin(f
*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c^2*cos(f*x +
e)^2 + 2*c^2*sin(f*x + e) - 2*c^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(7/2)/(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{7}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate((a*sin(f*x + e) + a)^(7/2)/(-c*sin(f*x + e) + c)^(3/2), x)
```

3.376 $\int \frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=195

$$-\frac{3a^3\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{c^2f\sqrt{c-c\sin(e+fx)}}-\frac{6a^4\cos(e+fx)\log(1-\sin(e+fx))}{c^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{3a^2\cos(e+fx)(a\sin(e+fx)+a)^3}{2cf(c-c\sin(e+fx))^{3/2}}$$

```
[Out] (a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(2*f*(c - c*Sin[e + f*x])^(5/2)
) - (3*a^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(2*c*f*(c - c*Sin[e + f
*x])^(3/2)) - (6*a^4*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c^2*f*Sqrt[a + a*
Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) - (3*a^3*Cos[e + f*x]*Sqrt[a + a*Si
n[e + f*x]])/(c^2*f*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 0.401665, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2739, 2740, 2737, 2667, 31}

$$-\frac{3a^3\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{c^2f\sqrt{c-c\sin(e+fx)}}-\frac{6a^4\cos(e+fx)\log(1-\sin(e+fx))}{c^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{3a^2\cos(e+fx)(a\sin(e+fx)+a)^3}{2cf(c-c\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(7/2)/(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] (a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(2*f*(c - c*Sin[e + f*x])^(5/2)
) - (3*a^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(2*c*f*(c - c*Sin[e + f
*x])^(3/2)) - (6*a^4*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c^2*f*Sqrt[a + a*
Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) - (3*a^3*Cos[e + f*x]*Sqrt[a + a*Si
n[e + f*x]])/(c^2*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(2*n + 1)), x] - Dist[(b*(2*m - 1))/(d*
(2*n + 1)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
```

```
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{5/2}}\,dx = \frac{a\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{2f(c-c\sin(e+fx))^{5/2}} - \frac{(3a)\int \frac{(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{3/2}}\,dx}{2c}$$

$$= \frac{a\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{2f(c-c\sin(e+fx))^{5/2}} - \frac{3a^2\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2cf(c-c\sin(e+fx))^{3/2}} + \frac{\left(3a^2\right)\int \frac{(a+a\sin}{\sqrt{c-c}}}{c^2}$$

$$= \frac{a\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{2f(c-c\sin(e+fx))^{5/2}} - \frac{3a^2\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2cf(c-c\sin(e+fx))^{3/2}} - \frac{3a^3\cos(e+f}{c^2f\sqrt{c}}$$

$$= \frac{a\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{2f(c-c\sin(e+fx))^{5/2}} - \frac{3a^2\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2cf(c-c\sin(e+fx))^{3/2}} - \frac{3a^3\cos(e+f}{c^2f\sqrt{c}}$$

$$= \frac{a\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{2f(c-c\sin(e+fx))^{5/2}} - \frac{3a^2\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2cf(c-c\sin(e+fx))^{3/2}} - \frac{3a^3\cos(e+f}{c^2f\sqrt{c}}$$

$$= \frac{a\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{2f(c-c\sin(e+fx))^{5/2}} - \frac{3a^2\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2cf(c-c\sin(e+fx))^{3/2}} - \frac{6a^4\cos(e+}{c^2f\sqrt{a+a\sin}}$$

Mathematica [A] time = 1.90439, size = 207, normalized size = 1.06

$$\frac{a^3\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin(3(e+fx))-72\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)+\right.}{4c^2f(\sin(e+fx)-1)^2\sqrt{c-c\sin}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(7/2)/(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] (a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(-28
- 72*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + 4*Cos[2*(e + f*x)]*(-1 + 6*
Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]) + (41 + 96*Log[Cos[(e + f*x)/2] -
 Sin[(e + f*x)/2]])*Sin[e + f*x] + Sin[3*(e + f*x)]))/(4*c^2*f*(Cos[(e + f*
x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])^2*Sqrt[c - c*Sin[e + f*x]])
```

Maple [B] time = 0.173, size = 618, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] -1/f*(6*sin(f*x+e)*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))+6*cos(f*x+e)^3*ln(2/(c
os(f*x+e)+1))-sin(f*x+e)*cos(f*x+e)^3-12*sin(f*x+e)*cos(f*x+e)^2*ln(-(-1+co
s(f*x+e)+sin(f*x+e))/sin(f*x+e))+cos(f*x+e)^4-12*cos(f*x+e)^3*ln(-(-1+cos(f
*x+e)+sin(f*x+e))/sin(f*x+e))+12*ln(2/(cos(f*x+e)+1))*sin(f*x+e)*cos(f*x+e)
-18*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))+11*cos(f*x+e)^2*sin(f*x+e)-24*ln(-(-1
+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*sin(f*x+e)*cos(f*x+e)+10*cos(f*x+e)^3+3
6*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-24*sin(f*x+e)*ln(
2/(cos(f*x+e)+1))-12*cos(f*x+e)*ln(2/(cos(f*x+e)+1))+6*sin(f*x+e)*cos(f*x+e
)+48*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-17*cos(f*x+e)^2+
24*cos(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+24*ln(2/(cos(f*x+e
)+1))-16*sin(f*x+e)-10*cos(f*x+e)-48*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x
+e))+16)*(a*(1+sin(f*x+e)))^(7/2)/(sin(f*x+e)*cos(f*x+e)^3+cos(f*x+e)^4-4*c
os(f*x+e)^2*sin(f*x+e)+3*cos(f*x+e)^3-4*sin(f*x+e)*cos(f*x+e)-8*cos(f*x+e)^
2+8*sin(f*x+e)-4*cos(f*x+e)+8)/(-c*(-1+sin(f*x+e)))^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{7}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(7/2)/(-c*sin(f*x + e) + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(3\,a^3\cos\left(fx+e\right)^2-4\,a^3+\left(a^3\cos\left(fx+e\right)^2-4\,a^3\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}}{3\,c^3\cos\left(fx+e\right)^2-4\,c^3-\left(c^3\cos\left(fx+e\right)^2-4\,c^3\right)\sin\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] integral((3*a^3*cos(f*x + e)^2 - 4*a^3 + (a^3*cos(f*x + e)^2 - 4*a^3)*sin(f
*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(3*c^3*cos(f*x
+ e)^2 - 4*c^3 - (c^3*cos(f*x + e)^2 - 4*c^3)*sin(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(7/2)/(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{7}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate((a*sin(f*x + e) + a)^(7/2)/(-c*sin(f*x + e) + c)^(5/2), x)
```

3.377 $\int \frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=193

$$\frac{a^3\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{c^2f(c-c\sin(e+fx))^{3/2}}+\frac{a^4\cos(e+fx)\log(1-\sin(e+fx))}{c^3f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{a^2\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2cf(c-c\sin(e+fx))^{5/2}}+$$

```
[Out] (a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(3*f*(c - c*Sin[e + f*x])^(7/2)
) - (a^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(2*c*f*(c - c*Sin[e + f*x
])^(5/2)) + (a^3*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(c^2*f*(c - c*Sin[e
 + f*x])^(3/2)) + (a^4*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c^3*f*Sqrt[a +
a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 0.40755, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2739, 2737, 2667, 31}

$$\frac{a^3\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{c^2f(c-c\sin(e+fx))^{3/2}}+\frac{a^4\cos(e+fx)\log(1-\sin(e+fx))}{c^3f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{a^2\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2cf(c-c\sin(e+fx))^{5/2}}+$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(7/2)/(c - c*Sin[e + f*x])^(7/2),x]
```


```
[Out] (a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(3*f*(c - c*Sin[e + f*x])^(7/2)
) - (a^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(2*c*f*(c - c*Sin[e + f*x
])^(5/2)) + (a^3*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(c^2*f*(c - c*Sin[e
 + f*x])^(3/2)) + (a^4*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c^3*f*Sqrt[a +
a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```


Rule 2739

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(2*n + 1)), x] - Dist[(b*(2*m - 1))/(d*
(2*n + 1)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{7/2}}\,dx = \frac{a\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3f(c-c\sin(e+fx))^{7/2}} - \frac{a\int \frac{(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{5/2}}\,dx}{c}$$

$$= \frac{a\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3f(c-c\sin(e+fx))^{7/2}} - \frac{a^2\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2cf(c-c\sin(e+fx))^{5/2}} + \frac{a^2\int \frac{(a+a\sin(e+}{(c-c\sin(e+}}{c^2}$$

$$= \frac{a\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3f(c-c\sin(e+fx))^{7/2}} - \frac{a^2\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2cf(c-c\sin(e+fx))^{5/2}} + \frac{a^3\cos(e+fx)}{c^2f(c-c\sin}$$

$$= \frac{a\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3f(c-c\sin(e+fx))^{7/2}} - \frac{a^2\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2cf(c-c\sin(e+fx))^{5/2}} + \frac{a^3\cos(e+fx)}{c^2f(c-c\sin}$$

$$= \frac{a\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3f(c-c\sin(e+fx))^{7/2}} - \frac{a^2\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2cf(c-c\sin(e+fx))^{5/2}} + \frac{a^3\cos(e+fx)}{c^2f(c-c\sin}$$

$$= \frac{a\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3f(c-c\sin(e+fx))^{7/2}} - \frac{a^2\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2cf(c-c\sin(e+fx))^{5/2}} + \frac{a^3\cos(e+fx)}{c^2f(c-c\sin}$$

Mathematica [A] time = 2.39388, size = 232, normalized size = 1.2

$$\frac{a^3\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(-3\sin(3(e+fx))\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-3}{6c^3f(\sin(e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(7/2)/(c - c*Sin[e + f*x])^(7/2),x]
```

```
[Out] (a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(-34
- 30*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + 18*Cos[2*(e + f*x)]*(1 + Lo
g[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]) + 9*(4 + 5*Log[Cos[(e + f*x)/2] - S
in[(e + f*x)/2]])*Sin[e + f*x] - 3*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]
*Sin[3*(e + f*x)]))/(6*c^3*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Si
n[e + f*x])^3*Sqrt[c - c*Sin[e + f*x]])
```

Maple [B] time = 0.168, size = 748, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(7/2),x)
```

```
[Out] -1/3/f*(-20+3*sin(f*x+e)*cos(f*x+e)^3*ln(2/(cos(f*x+e)+1))-6*sin(f*x+e)*cos
(f*x+e)^3*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+24*sin(f*x+e)*cos(f*x+
e)^2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-12*ln(2/(cos(f*x+e)+1))*sin
```

```
(f*x+e)*cos(f*x+e)+20*sin(f*x+e)+6*cos(f*x+e)+48*ln(-(-1+cos(f*x+e)+sin(f*x
+e))/sin(f*x+e))-24*ln(2/(cos(f*x+e)+1))-14*sin(f*x+e)*cos(f*x+e)+28*cos(f*
x+e)^2-3*cos(f*x+e)^4*ln(2/(cos(f*x+e)+1))+6*cos(f*x+e)^4*ln(-(-1+cos(f*x+e
)+sin(f*x+e))/sin(f*x+e))-12*sin(f*x+e)*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))-6
*cos(f*x+e)^3+8*sin(f*x+e)*cos(f*x+e)^3+24*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1)
)-48*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+24*ln(-(-1+cos
(f*x+e)+sin(f*x+e))/sin(f*x+e))*sin(f*x+e)*cos(f*x+e)-14*cos(f*x+e)^2*sin(f
*x+e)-8*cos(f*x+e)^4+18*cos(f*x+e)^3*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x
+e))-9*cos(f*x+e)^3*ln(2/(cos(f*x+e)+1))+24*sin(f*x+e)*ln(2/(cos(f*x+e)+1))
-48*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+12*cos(f*x+e)*ln(
2/(cos(f*x+e)+1))-24*cos(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e)))
*(a*(1+sin(f*x+e)))^(7/2)/(sin(f*x+e)*cos(f*x+e)^3+cos(f*x+e)^4-4*cos(f*x+e
)^2*sin(f*x+e)+3*cos(f*x+e)^3-4*sin(f*x+e)*cos(f*x+e)-8*cos(f*x+e)^2+8*sin(
f*x+e)-4*cos(f*x+e)+8)/(-c*(-1+sin(f*x+e)))^(7/2)
```

Maxima [A] time = 1.80198, size = 454, normalized size = 2.35

$$-\frac{\frac{6a^{\frac{7}{2}}\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{c^{\frac{7}{2}}}-\frac{3a^{\frac{7}{2}}\log\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)}{c^{\frac{7}{2}}}+\frac{4\left(\frac{3a^{\frac{7}{2}}\sqrt{c}\sin(fx+e)}{\cos(fx+e)+1}-\frac{6a^{\frac{7}{2}}\sqrt{c}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{22a^{\frac{7}{2}}\sqrt{c}\sin(fx+e)^3}{(\cos(fx+e)+1)^3}-\frac{6a^{\frac{7}{2}}\sqrt{c}\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{3a^{\frac{7}{2}}\sqrt{c}\sin(f\cdots}{(\cos(fx+e)\cdots}\right.}{c^4-\frac{6c^4\sin(fx+e)}{\cos(fx+e)+1}+\frac{15c^4\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{20c^4\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{15c^4\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{6c^4\sin(fx+e)^5}{(\cos(fx+e)+1)^5}+\frac{c^4\sin(\cdots}{(\cos(fx\cdots}}}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(7/2),x, algorithm="maxim
a")
```

```
[Out] -1/3*(6*a^(7/2)*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/c^(7/2) - 3*a^(7/2
)*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/c^(7/2) + 4*(3*a^(7/2)*sqrt(
c)*sin(f*x + e)/(cos(f*x + e) + 1) - 6*a^(7/2)*sqrt(c)*sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 + 22*a^(7/2)*sqrt(c)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 -
6*a^(7/2)*sqrt(c)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 3*a^(7/2)*sqrt(c)*s
in(f*x + e)^5/(cos(f*x + e) + 1)^5)/(c^4 - 6*c^4*sin(f*x + e)/(cos(f*x + e)
 + 1) + 15*c^4*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 20*c^4*sin(f*x + e)^3/
(cos(f*x + e) + 1)^3 + 15*c^4*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 6*c^4*s
in(f*x + e)^5/(cos(f*x + e) + 1)^5 + c^4*sin(f*x + e)^6/(cos(f*x + e) + 1)^
6))/f
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(3a^3\cos\left(fx+e\right)^2-4a^3+\left(a^3\cos\left(fx+e\right)^2-4a^3\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+\cdots}}{c^4\cos\left(fx+e\right)^4-8c^4\cos\left(fx+e\right)^2+8c^4+4\left(c^4\cos\left(fx+e\right)^2-2c^4\right)\sin\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(7/2),x, algorithm="frica
s")
```

```
[Out] integral(-(3*a^3*cos(f*x + e)^2 - 4*a^3 + (a^3*cos(f*x + e)^2 - 4*a^3)*sin(
f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c^4*cos(f*x +
 e)^4 - 8*c^4*cos(f*x + e)^2 + 8*c^4 + 4*(c^4*cos(f*x + e)^2 - 2*c^4)*sin(f
```

```
*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(7/2)/(c-c*sin(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{7}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(7/2),x, algorithm="giac"
)
```

```
[Out] integrate((a*sin(f*x + e) + a)^(7/2)/(-c*sin(f*x + e) + c)^(7/2), x)
```

3.378 $\int \frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{9/2}}\,dx$

Optimal. Leaf size=42

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{8f(c-c\sin(e+fx))^{9/2}}$$

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(8*f*(c - c*Sin[e + f*x])^(9/2))
```

Rubi [A] time = 0.093497, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {2742}

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{8f(c-c\sin(e+fx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(7/2)/(c - c*Sin[e + f*x])^(9/2),x]
```

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(8*f*(c - c*Sin[e + f*x])^(9/2))
```

Rule 2742

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{9/2}}\,dx = \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{8f(c-c\sin(e+fx))^{9/2}}$$

Mathematica [B] time = 4.42188, size = 115, normalized size = 2.74

$$-\frac{a^3(\sin(3(e+fx))-7\sin(e+fx))\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}{4c^4f(\sin(e+fx)-1)^4\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(7/2)/(c - c*Sin[e + f*x])^(9/2),x]
```

```
[Out] -(a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(-7*
Sin[e + f*x] + Sin[3*(e + f*x)]))/(4*c^4*f*(Cos[(e + f*x)/2] + Sin[(e + f*x
)/2])*(-1 + Sin[e + f*x])^4*Sqrt[c - c*Sin[e + f*x]])
```

Maple [B] time = 0.139, size = 154, normalized size = 3.7

$$-\frac{\left(\left(\cos \left(f x+e\right)\right)^{2}-2\right) \left(-1+\cos \left(f x+e\right)+\sin \left(f x+e\right)\right) \mathrm{si}}{f \left(\sin \left(f x+e\right) \left(\cos \left(f x+e\right)\right)^{3}+\left(\cos \left(f x+e\right)\right)^{4}-4 \left(\cos \left(f x+e\right)\right)^{2} \sin \left(f x+e\right)+3 \left(\cos \left(f x+e\right)\right)^{3}-4 \sin \left(f x+e\right) \mathrm{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(9/2),x)
```

```
[Out] -1/f*(cos(f*x+e)^2-2)*(-1+cos(f*x+e)+sin(f*x+e))*(a*(1+sin(f*x+e)))^(7/2)*s
in(f*x+e)/(sin(f*x+e)*cos(f*x+e)^3+cos(f*x+e)^4-4*cos(f*x+e)^2*sin(f*x+e)+3
*cos(f*x+e)^3-4*sin(f*x+e)*cos(f*x+e)-8*cos(f*x+e)^2+8*sin(f*x+e)-4*cos(f*x
+e)+8)/(-c*(-1+sin(f*x+e)))^(9/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a \sin \left(f x+e\right)+a\right)^{\frac{7}{2}}}{\left(-c \sin \left(f x+e\right)+c\right)^{\frac{9}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(9/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(7/2)/(-c*sin(f*x + e) + c)^(9/2), x)
```

Fricas [B] time = 1.13011, size = 309, normalized size = 7.36

$$-\frac{\left(a^{3} \cos \left(f x+e\right)^{2}-2 a^{3}\right) \sqrt{a \sin \left(f x+e\right)+a} \sqrt{-c \sin \left(f x+e\right)+c} \sin \left(f x+e\right)}{c^{5} f \cos \left(f x+e\right)^{5}-8 c^{5} f \cos \left(f x+e\right)^{3}+8 c^{5} f \cos \left(f x+e\right)+4 \left(c^{5} f \cos \left(f x+e\right)^{3}-2 c^{5} f \cos \left(f x+e\right)\right) \sin \left(f x+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(9/2),x, algorithm="frica
s")
```

```
[Out] -(a^3*cos(f*x + e)^2 - 2*a^3)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e)
 + c)*sin(f*x + e)/(c^5*f*cos(f*x + e)^5 - 8*c^5*f*cos(f*x + e)^3 + 8*c^5*f
*cos(f*x + e) + 4*(c^5*f*cos(f*x + e)^3 - 2*c^5*f*cos(f*x + e))*sin(f*x + e
))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(7/2)/(c-c*sin(f*x+e))**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{7}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(9/2),x, algorithm="giac"
)
```

```
[Out] integrate((a*sin(f*x + e) + a)^(7/2)/(-c*sin(f*x + e) + c)^(9/2), x)
```

3.379 $\int \frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{11/2}}\,dx$

Optimal. Leaf size=88

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{80cf(c-c\sin(e+fx))^{9/2}}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{10f(c-c\sin(e+fx))^{11/2}}$$

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(10*f*(c - c*Sin[e + f*x])^(11/2)
) + (Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(80*c*f*(c - c*Sin[e + f*x])^
(9/2))
```

Rubi [A] time = 0.186293, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2743, 2742}

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{80cf(c-c\sin(e+fx))^{9/2}}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{10f(c-c\sin(e+fx))^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(7/2)/(c - c*Sin[e + f*x])^(11/2),x]
```

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(10*f*(c - c*Sin[e + f*x])^(11/2)
) + (Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(80*c*f*(c - c*Sin[e + f*x])^
(9/2))
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rule 2742

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{11/2}}\,dx &= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{10f(c-c\sin(e+fx))^{11/2}}+\frac{\int \frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{9/2}}\,dx}{10c}\\ &= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{10f(c-c\sin(e+fx))^{11/2}}+\frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{80cf(c-c\sin(e+fx))^{9/2}}\end{aligned}$$

Mathematica [B] time = 6.59981, size = 331, normalized size = 3.76

$$-\frac{(a(\sin(e+fx)+1))^{7/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7}{2f(c-c\sin(e+fx))^{11/2}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^7}+\frac{2(a(\sin(e+fx)+1))^{7/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e\right.\right.}{f(c-c\sin(e+fx))^{11/2}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(7/2)/(c - c*Sin[e + f*x])^(11/2),x]
```

```
[Out] (8*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(7/2))/(5*f
*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(11/2)) - (3*
(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(a*(1 + Sin[e + f*x]))^(7/2))/(f*(C
os[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(11/2)) + (2*(Co
s[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(a*(1 + Sin[e + f*x]))^(7/2))/(f*(Cos[
(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(11/2)) - ((Cos[(e
+ f*x)/2] - Sin[(e + f*x)/2])^7*(a*(1 + Sin[e + f*x]))^(7/2))/(2*f*(Cos[(e
+ f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(11/2))
```

Maple [B] time = 0.165, size = 247, normalized size = 2.8

$$\frac{\left(\sin\left(fx+e\right)\left(\cos\left(fx+e\right)\right)^4+\left(\cos\left(fx+e\right)\right)^5+5\sin\left(fx+e\right)\left(\cos\left(fx+e\right)\right)^3-6\left(\cos\left(fx+e\right)\right)^4-22\left(\cos\left(fx+\right.\right.\right.}{10f\left(\sin\left(fx+e\right)\left(\cos\left(fx+e\right)\right)^3+\left(\cos\left(fx+e\right)\right)^4-4\left(\cos\left(fx+e\right)\right)^2\sin\left(fx+e\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(11/2),x)
```

```
[Out] 1/10/f*(sin(f*x+e)*cos(f*x+e)^4+cos(f*x+e)^5+5*sin(f*x+e)*cos(f*x+e)^3-6*co
s(f*x+e)^4-22*cos(f*x+e)^2*sin(f*x+e)-17*cos(f*x+e)^3-10*sin(f*x+e)*cos(f*x
+e)+32*cos(f*x+e)^2+36*sin(f*x+e)+26*cos(f*x+e)-36)*sin(f*x+e)*(a*(1+sin(f*
x+e)))^(7/2)/(sin(f*x+e)*cos(f*x+e)^3+cos(f*x+e)^4-4*cos(f*x+e)^2*sin(f*x+e
)+3*cos(f*x+e)^3-4*sin(f*x+e)*cos(f*x+e)-8*cos(f*x+e)^2+8*sin(f*x+e)-4*cos(
f*x+e)+8)/(-c*(-1+sin(f*x+e)))^(11/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{7}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(11/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(7/2)/(-c*sin(f*x + e) + c)^(11/2), x)
```

Fricas [B] time = 1.21122, size = 402, normalized size = 4.57

$$-\frac{\left(5\,a^{3}\cos\left(fx+e\right)^{2}-6\,a^{3}+5\left(a^{3}\cos\left(fx+e\right)^{2}-2\,a^{3}\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx\right.}}{10\left(5\,c^{6}f\cos\left(fx+e\right)^{5}-20\,c^{6}f\cos\left(fx+e\right)^{3}+16\,c^{6}f\cos\left(fx+e\right)-\left(c^{6}f\cos\left(fx+e\right)^{5}-12\,c^{6}f\cos\left(fx+e\right)^{3}+16\,c^{6}f\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(11/2),x, algorithm="fric
as")
```

```
[Out] -1/10*(5*a^3*cos(f*x + e)^2 - 6*a^3 + 5*(a^3*cos(f*x + e)^2 - 2*a^3)*sin(f*
x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(5*c^6*f*cos(f*x
 + e)^5 - 20*c^6*f*cos(f*x + e)^3 + 16*c^6*f*cos(f*x + e) - (c^6*f*cos(f*x
+ e)^5 - 12*c^6*f*cos(f*x + e)^3 + 16*c^6*f*cos(f*x + e))*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(7/2)/(c-c*sin(f*x+e))**(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{7}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(11/2),x, algorithm="giac
")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(7/2)/(-c*sin(f*x + e) + c)^(11/2), x)
```

3.380 $\int \frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{13/2}}\,dx$

Optimal. Leaf size=133

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{480c^2f(c-c\sin(e+fx))^{9/2}}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{60cf(c-c\sin(e+fx))^{11/2}}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{12f(c-c\sin(e+fx))^{13/2}}$$

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(12*f*(c - c*Sin[e + f*x])^(13/2)
) + (Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(60*c*f*(c - c*Sin[e + f*x])^
(11/2)) + (Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(480*c^2*f*(c - c*Sin[e
 + f*x])^(9/2))
```

Rubi [A] time = 0.294136, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2743, 2742}

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{480c^2f(c-c\sin(e+fx))^{9/2}}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{60cf(c-c\sin(e+fx))^{11/2}}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{12f(c-c\sin(e+fx))^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(7/2)/(c - c*Sin[e + f*x])^(13/2),x]
```

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(12*f*(c - c*Sin[e + f*x])^(13/2)
) + (Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(60*c*f*(c - c*Sin[e + f*x])^
(11/2)) + (Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(480*c^2*f*(c - c*Sin[e
 + f*x])^(9/2))
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rule 2742

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{13/2}}\,dx = \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{12f(c-c\sin(e+fx))^{13/2}} + \frac{\int \frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{11/2}}dx}{6c}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{12f(c-c\sin(e+fx))^{13/2}} + \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{60cf(c-c\sin(e+fx))^{11/2}} + \frac{\int \frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{9/2}} d}{60c^2}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{12f(c-c\sin(e+fx))^{13/2}} + \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{60cf(c-c\sin(e+fx))^{11/2}} + \frac{\cos(e+fx)(a+a}{480c^2f(c-c\sin}$$

Mathematica [B] time = 6.64031, size = 335, normalized size = 2.52

$$-\frac{(a(\sin(e+fx)+1))^{7/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7}{3f(c-c\sin(e+fx))^{13/2}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^7} + \frac{3(a(\sin(e+fx)+1))^{7/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+\right.\right.}{2f(c-c\sin(e+fx))^{13/2}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(7/2)/(c - c*Sin[e + f*x])^(13/2),x]
```

```
[Out] (4*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(7/2))/(3*f
*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(13/2)) - (12
*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(a*(1 + Sin[e + f*x]))^(7/2))/(5*f
*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(13/2)) + (3*
(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(a*(1 + Sin[e + f*x]))^(7/2))/(2*f*
(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(13/2)) - ((Co
s[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(a*(1 + Sin[e + f*x]))^(7/2))/(3*f*(Co
s[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(13/2))
```

Maple [B] time = 0.179, size = 276, normalized size = 2.1

$$-\frac{\left(3\sin\left(fx+e\right)\left(\cos\left(fx+e\right)\right)^5-3\left(\cos\left(fx+e\right)\right)^6-21\sin\left(fx+e\right)\left(\cos\left(fx+e\right)\right)^4-18\left(\cos\left(fx+e\right)\right)^5-51\sin\left(fx\right.\right.}{30f\left(\sin\left(fx+e\right)\left(\cos\left(fx+e\right)\right)^3+\left(\cos\left(fx+e\right)\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(13/2),x)
```

```
[Out] -1/30/f*(3*sin(f*x+e)*cos(f*x+e)^5-3*cos(f*x+e)^6-21*sin(f*x+e)*cos(f*x+e)^
4-18*cos(f*x+e)^5-51*sin(f*x+e)*cos(f*x+e)^3+72*cos(f*x+e)^4+157*cos(f*x+e)
^2*sin(f*x+e)+106*cos(f*x+e)^3+78*sin(f*x+e)*cos(f*x+e)-235*cos(f*x+e)^2-19
6*sin(f*x+e)-118*cos(f*x+e)+196)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(7/2)/(sin(f
*x+e)*cos(f*x+e)^3+cos(f*x+e)^4-4*cos(f*x+e)^2*sin(f*x+e)+3*cos(f*x+e)^3-4*
sin(f*x+e)*cos(f*x+e)-8*cos(f*x+e)^2+8*sin(f*x+e)-4*cos(f*x+e)+8)/(-c*(-1+s
in(f*x+e)))^(13/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(13/2),x, algorithm="maxi
ma")
```

```
[Out] Timed out
```

Fricas [A] time = 1.22279, size = 446, normalized size = 3.35

$$\frac{\left(15\,a^3\cos\left(fx+e\right)^2-18\,a^3+2\left(5\,a^3\cos\left(fx+e\right)^2-11\,a^3\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)}}{30\left(c^7f\cos\left(fx+e\right)^7-18\,c^7f\cos\left(fx+e\right)^5+48\,c^7f\cos\left(fx+e\right)^3-32\,c^7f\cos\left(fx+e\right)+2\left(3\,c^7f\cos\left(fx+e\right)^5-16\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(13/2),x, algorithm="fric
as")
```

```
[Out] 1/30*(15*a^3*cos(f*x + e)^2 - 18*a^3 + 2*(5*a^3*cos(f*x + e)^2 - 11*a^3)*si
n(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c^7*f*cos(f
*x + e)^7 - 18*c^7*f*cos(f*x + e)^5 + 48*c^7*f*cos(f*x + e)^3 - 32*c^7*f*co
s(f*x + e) + 2*(3*c^7*f*cos(f*x + e)^5 - 16*c^7*f*cos(f*x + e)^3 + 16*c^7*f
*cos(f*x + e))*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(7/2)/(c-c*sin(f*x+e))**(13/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{7}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(13/2),x, algorithm="giac
")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(7/2)/(-c*sin(f*x + e) + c)^(13/2), x)
```

3.381 $\int \frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{15/2}}\,dx$

Optimal. Leaf size=178

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{2240c^3f(c-c\sin(e+fx))^{9/2}}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{280c^2f(c-c\sin(e+fx))^{11/2}}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{56cf(c-c\sin(e+fx))^{13/2}}+\frac{\cos(e+fx)(a}{14f(c-c}$$

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(14*f*(c - c*Sin[e + f*x])^(15/2)
) + (Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(56*c*f*(c - c*Sin[e + f*x])^
(13/2)) + (Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(280*c^2*f*(c - c*Sin[e
 + f*x])^(11/2)) + (Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(2240*c^3*f*(c
 - c*Sin[e + f*x])^(9/2))
```

Rubi [A] time = 0.380435, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2743, 2742}

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{2240c^3f(c-c\sin(e+fx))^{9/2}}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{280c^2f(c-c\sin(e+fx))^{11/2}}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{56cf(c-c\sin(e+fx))^{13/2}}+\frac{\cos(e+fx)(a}{14f(c-c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(7/2)/(c - c*Sin[e + f*x])^(15/2),x]
```


```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(14*f*(c - c*Sin[e + f*x])^(15/2)
) + (Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(56*c*f*(c - c*Sin[e + f*x])^
(13/2)) + (Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(280*c^2*f*(c - c*Sin[e
 + f*x])^(11/2)) + (Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(2240*c^3*f*(c
 - c*Sin[e + f*x])^(9/2))
```


Rule 2743

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rule 2742

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{15/2}}dx = \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{14f(c-c\sin(e+fx))^{15/2}} + \frac{3\int \frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{13/2}}dx}{14c}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{14f(c-c\sin(e+fx))^{15/2}} + \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{56cf(c-c\sin(e+fx))^{13/2}} + \frac{\int \frac{(a+a\sin(e+fx))^{\cdots}}{(c-c\sin(e+fx))^{1\cdots}}}{28c^2}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{14f(c-c\sin(e+fx))^{15/2}} + \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{56cf(c-c\sin(e+fx))^{13/2}} + \frac{\cos(e+fx)(a\cdots}{280c^2f(c-c\cdots}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{14f(c-c\sin(e+fx))^{15/2}} + \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{56cf(c-c\sin(e+fx))^{13/2}} + \frac{\cos(e+fx)(a\cdots}{280c^2f(c-c\cdots}$$

Mathematica [A] time = 6.68938, size = 333, normalized size = 1.87

$$-\frac{(a(\sin(e+fx)+1))^{7/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7}{4f(c-c\sin(e+fx))^{15/2}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^7} + \frac{6(a(\sin(e+fx)+1))^{7/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e\cdots\right.\right.}{5f(c-c\sin(e+fx))^{15/2}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(\cdots\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(7/2)/(c - c*Sin[e + f*x])^(15/2),x]
```

```
[Out] (8*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(7/2))/(7*f
*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(15/2)) - (2*
(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(a*(1 + Sin[e + f*x]))^(7/2))/(f*(C
os[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(15/2)) + (6*(Co
s[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(a*(1 + Sin[e + f*x]))^(7/2))/(5*f*(Co
s[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(15/2)) - ((Cos[(
e + f*x)/2] - Sin[(e + f*x)/2])^7*(a*(1 + Sin[e + f*x]))^(7/2))/(4*f*(Cos[(
e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(15/2))
```

Maple [A] time = 0.207, size = 302, normalized size = 1.7

$$-\frac{\left(13\left(\cos\left(fx+e\right)\right)^6\sin\left(fx+e\right)+13\left(\cos\left(fx+e\right)\right)^7+91\sin\left(fx+e\right)\left(\cos\left(fx+e\right)\right)^5-104\left(\cos\left(fx+e\right)\right)^6-403\cdots\right.}{140f\left(\sin\left(\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(15/2),x)
```

```
[Out] -1/140/f*(13*cos(f*x+e)^6*sin(f*x+e)+13*cos(f*x+e)^7+91*sin(f*x+e)*cos(f*x+
e)^5-104*cos(f*x+e)^6-403*sin(f*x+e)*cos(f*x+e)^4-312*cos(f*x+e)^5-637*sin(
f*x+e)*cos(f*x+e)^3+1040*cos(f*x+e)^4+1712*cos(f*x+e)^2*sin(f*x+e)+1075*cos
(f*x+e)^3+756*sin(f*x+e)*cos(f*x+e)-2468*cos(f*x+e)^2-1672*sin(f*x+e)-916*c
os(f*x+e)+1672)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(7/2)/(sin(f*x+e)*cos(f*x+e)^
3+cos(f*x+e)^4-4*cos(f*x+e)^2*sin(f*x+e)+3*cos(f*x+e)^3-4*sin(f*x+e)*cos(f*
x+e)-8*cos(f*x+e)^2+8*sin(f*x+e)-4*cos(f*x+e)+8)/(-c*(-1+sin(f*x+e)))^(15/2
)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(15/2),x, algorithm="maxi
ma")
```

```
[Out] Timed out
```

Fricas [A] time = 1.27381, size = 481, normalized size = 2.7

$$\frac{\left(63\,a^3\cos\left(fx+e\right)^2-76\,a^3+7\left(5\,a^3\cos\left(fx+e\right)^2-12\,a^3\right)\sin\left(fx+e\right)\right)\sqrt{a\sin}}{140\left(7\,c^8f\cos\left(fx+e\right)^7-56\,c^8f\cos\left(fx+e\right)^5+112\,c^8f\cos\left(fx+e\right)^3-64\,c^8f\cos\left(fx+e\right)-\left(c^8f\cos\left(fx+e\right)^7-24\,c^8\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(15/2),x, algorithm="fric
as")
```

```
[Out] 1/140*(63*a^3*cos(f*x + e)^2 - 76*a^3 + 7*(5*a^3*cos(f*x + e)^2 - 12*a^3)*s
in(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(7*c^8*f*co
s(f*x + e)^7 - 56*c^8*f*cos(f*x + e)^5 + 112*c^8*f*cos(f*x + e)^3 - 64*c^8*
f*cos(f*x + e) - (c^8*f*cos(f*x + e)^7 - 24*c^8*f*cos(f*x + e)^5 + 80*c^8*f
*cos(f*x + e)^3 - 64*c^8*f*cos(f*x + e))*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(7/2)/(c-c*sin(f*x+e))**(15/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{7}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{15}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(15/2),x, algorithm="giac
")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(7/2)/(-c*sin(f*x + e) + c)^(15/2), x)
```

3.382 $\int \frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{17/2}}\,dx$

Optimal. Leaf size=188

$$-\frac{a^4\cos(e+fx)}{280c^3f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{11/2}}+\frac{a^3\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{56c^2f(c-c\sin(e+fx))^{13/2}}-\frac{3a^2\cos(e+fx)(a\sin(e+fx)}{56cf(c-c\sin(e+fx))^{1}}$$

```
[Out] (a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(8*f*(c - c*Sin[e + f*x])^(17/2
)) - (3*a^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(56*c*f*(c - c*Sin[e +
 f*x])^(15/2)) + (a^3*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(56*c^2*f*(c -
 c*Sin[e + f*x])^(13/2)) - (a^4*Cos[e + f*x])/(280*c^3*f*Sqrt[a + a*Sin[e +
 f*x]]*(c - c*Sin[e + f*x])^(11/2))
```

Rubi [A] time = 0.384105, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2739, 2738}

$$-\frac{a^4\cos(e+fx)}{280c^3f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{11/2}}+\frac{a^3\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{56c^2f(c-c\sin(e+fx))^{13/2}}-\frac{3a^2\cos(e+fx)(a\sin(e+fx)}{56cf(c-c\sin(e+fx))^{1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(7/2)/(c - c*Sin[e + f*x])^(17/2),x]
```

```
[Out] (a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(8*f*(c - c*Sin[e + f*x])^(17/2
)) - (3*a^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(56*c*f*(c - c*Sin[e +
 f*x])^(15/2)) + (a^3*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(56*c^2*f*(c -
 c*Sin[e + f*x])^(13/2)) - (a^4*Cos[e + f*x])/(280*c^3*f*Sqrt[a + a*Sin[e +
 f*x]]*(c - c*Sin[e + f*x])^(11/2))
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(2*n + 1)), x] - Dist[(b*(2*m - 1))/(d*
(2*n + 1)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{17/2}}\,dx = \frac{a\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{8f(c-c\sin(e+fx))^{17/2}} - \frac{(3a)\int \frac{(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{15/2}}dx}{8c}$$

$$= \frac{a\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{8f(c-c\sin(e+fx))^{17/2}} - \frac{3a^2\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{56cf(c-c\sin(e+fx))^{15/2}} + \frac{(3a^2)\int \frac{(a+a\,\mathrm{s}}{(c-c\,\mathrm{s}}}{28}$$

$$= \frac{a\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{8f(c-c\sin(e+fx))^{17/2}} - \frac{3a^2\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{56cf(c-c\sin(e+fx))^{15/2}} + \frac{a^3\cos(e+fx}{56c^2f(c-}$$

$$= \frac{a\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{8f(c-c\sin(e+fx))^{17/2}} - \frac{3a^2\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{56cf(c-c\sin(e+fx))^{15/2}} + \frac{a^3\cos(e+fx}{56c^2f(c-}$$

Mathematica [A] time = 6.02393, size = 128, normalized size = 0.68

$$\frac{a^3\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(65\sin(e+fx)-7\sin(3(e+fx))-28\cos(2(e+fx))+40)}{140c^8f(\sin(e+fx)-1)^8\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(7/2)/(c - c*Sin[e + f*x])^(17/2),x]
```

```
[Out] (a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(40 -
 28*Cos[2*(e + f*x)] + 65*Sin[e + f*x] - 7*Sin[3*(e + f*x)]))/(140*c^8*f*(C
os[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])^8*Sqrt[c - c*Sin[e
+ f*x]])
```

Maple [A] time = 0.228, size = 328, normalized size = 1.7

$$\left(3\sin(fx+e)(\cos(fx+e))^7 - 3(\cos(fx+e))^8 - 27(\cos(fx+e))^6\sin(fx+e) - 24(\cos(fx+e))^7 - 93\sin(fx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(17/2),x)
```

```
[Out] 1/35/f*(3*sin(f*x+e)*cos(f*x+e)^7-3*cos(f*x+e)^8-27*cos(f*x+e)^6*sin(f*x+e)
-24*cos(f*x+e)^7-93*sin(f*x+e)*cos(f*x+e)^5+120*cos(f*x+e)^6+333*sin(f*x+e)
*cos(f*x+e)^4+240*cos(f*x+e)^5+387*sin(f*x+e)*cos(f*x+e)^3-720*cos(f*x+e)^4
-970*cos(f*x+e)^2*sin(f*x+e)-583*cos(f*x+e)^3-367*sin(f*x+e)*cos(f*x+e)+133
7*cos(f*x+e)^2+769*sin(f*x+e)+402*cos(f*x+e)-769)*sin(f*x+e)*(a*(1+sin(f*x+
e)))^(7/2)/(sin(f*x+e)*cos(f*x+e)^3+cos(f*x+e)^4-4*cos(f*x+e)^2*sin(f*x+e)+
3*cos(f*x+e)^3-4*sin(f*x+e)*cos(f*x+e)-8*cos(f*x+e)^2+8*sin(f*x+e)-4*cos(f*
x+e)+8)/(-c*(-1+sin(f*x+e)))^(17/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(17/2),x, algorithm="maxi
ma")
```

```
[Out] Timed out
```

Fricas [A] time = 1.34539, size = 516, normalized size = 2.74

$$-\frac{\left(14\,a^{3}\cos\left(fx+e\right)^{2}-17\,a^{3}+\left(7\,a^{3}\cos\left(fx+e\right)^{2}-18\,a^{3}\right)\sin\left(\right.}{35\left(c^{9}f\cos\left(fx+e\right)^{9}-32\,c^{9}f\cos\left(fx+e\right)^{7}+160\,c^{9}f\cos\left(fx+e\right)^{5}-256\,c^{9}f\cos\left(fx+e\right)^{3}+128\,c^{9}f\cos\left(fx+e\right)-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(17/2),x, algorithm="fric
as")
```

```
[Out] -1/35*(14*a^3*cos(f*x + e)^2 - 17*a^3 + (7*a^3*cos(f*x + e)^2 - 18*a^3)*sin
(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c^9*f*cos(f*
x + e)^9 - 32*c^9*f*cos(f*x + e)^7 + 160*c^9*f*cos(f*x + e)^5 - 256*c^9*f*c
os(f*x + e)^3 + 128*c^9*f*cos(f*x + e) + 8*(c^9*f*cos(f*x + e)^7 - 10*c^9*f
*cos(f*x + e)^5 + 24*c^9*f*cos(f*x + e)^3 - 16*c^9*f*cos(f*x + e))*sin(f*x
+ e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(7/2)/(c-c*sin(f*x+e))**(17/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{7}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{17}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(17/2),x, algorithm="giac
")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(7/2)/(-c*sin(f*x + e) + c)^(17/2), x)
```

3.383 $\int \frac{(c-c\sin(e+fx))^{5/2}}{\sqrt{a+a\sin(e+fx)}}\,dx$

Optimal. Leaf size=139

$$\frac{2c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}+\frac{4c^3\cos(e+fx)\log(\sin(e+fx)+1)}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}$$

```
[Out] (4*c^3*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f*x]]*Sqrt
[c - c*Sin[e + f*x]]) + (2*c^2*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(f*Sq
rt[a + a*Sin[e + f*x]]) + (c*Cos[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(2*f*
Sqrt[a + a*Sin[e + f*x]])
```

Rubi [A] time = 0.28084, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2740, 2737, 2667, 31}

$$\frac{2c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}+\frac{4c^3\cos(e+fx)\log(\sin(e+fx)+1)}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sin[e + f*x])^(5/2)/Sqrt[a + a*Sin[e + f*x]],x]
```


```
[Out] (4*c^3*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f*x]]*Sqrt
[c - c*Sin[e + f*x]]) + (2*c^2*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(f*Sq
rt[a + a*Sin[e + f*x]]) + (c*Cos[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(2*f*
Sqrt[a + a*Sin[e + f*x]])
```


Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] && !LtQ[n, -1] && !(IGtQ[n - 1/2, 0] && LtQ[n, m]) && !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] || !IntegerQ[m + 1/2
])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{(c-c\sin(e+fx))^{5/2}}{\sqrt{a+a\sin(e+fx)}}\,dx = \frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a+a\sin(e+fx)}} + (2c)\int \frac{(c-c\sin(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}}\,dx$$

$$= \frac{2c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a+a\sin(e+fx)}} + \frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a+a\sin(e+fx)}} + \left(4c^2\right)\int \frac{\sqrt{c-\ldots}}{\sqrt{a+\ldots}}$$

$$= \frac{2c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a+a\sin(e+fx)}} + \frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a+a\sin(e+fx)}} + \frac{\left(4ac^3\cos(e+\ldots\right.}{\sqrt{a+a\sin(e+\ldots}}$$

$$= \frac{2c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a+a\sin(e+fx)}} + \frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a+a\sin(e+fx)}} + \frac{\left(4c^3\cos(e+f\ldots\right.}{f\sqrt{a+\ldots}}$$

$$= \frac{4c^3\cos(e+fx)\log(1+\sin(e+fx))}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{2c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a+a\sin(e+fx)}} + \frac{c\cos(e+\ldots}{2f\ldots}$$

Mathematica [A] time = 0.561192, size = 136, normalized size = 0.98

$$-\frac{c^2(\sin(e+fx)-1)^2\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(12\sin(e+fx)+\cos(2(e+fx))-32\log\ldots\right.}{4f\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sin[e + f*x])^(5/2)/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] -(c^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])^2*(Cos[2*(e
 + f*x)] - 32*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + 12*Sin[e + f*x])*S
qrt[c - c*Sin[e + f*x]])/(4*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*Sqrt[
a*(1 + Sin[e + f*x])])
```

Maple [B] time = 0.198, size = 320, normalized size = 2.3

$$-\frac{1}{2f\left(\left(\cos\left(fx+e\right)\right)^3+\left(\cos\left(fx+e\right)\right)^2\sin\left(fx+e\right)-3\left(\cos\left(fx+e\right)\right)^2+2\sin\left(fx+e\right)\cos\left(fx+e\right)-2\cos\left(fx+e\ldots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] -1/2/f*(cos(f*x+e)^3-cos(f*x+e)^2*sin(f*x+e)+5*cos(f*x+e)^2+8*cos(f*x+e)*ln
(2/(cos(f*x+e)+1))+6*sin(f*x+e)*cos(f*x+e)-16*cos(f*x+e)*ln(-(-1+cos(f*x+e)
-sin(f*x+e))/sin(f*x+e))-8*sin(f*x+e)*ln(2/(cos(f*x+e)+1))+16*sin(f*x+e)*ln
(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))-cos(f*x+e)-8*ln(2/(cos(f*x+e)+1))-
5*sin(f*x+e)+16*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))-5)*(-c*(-1+sin(f
*x+e)))^(5/2)/(cos(f*x+e)^3+cos(f*x+e)^2*sin(f*x+e)-3*cos(f*x+e)^2+2*sin(f*
x+e)*cos(f*x+e)-2*cos(f*x+e)-4*sin(f*x+e)+4)/(a*(1+sin(f*x+e)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}{\sqrt{a\sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate((-c*sin(f*x + e) + c)^(5/2)/sqrt(a*sin(f*x + e) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(c^{2}\cos\left(fx+e\right)^{2}+2\,c^{2}\sin\left(fx+e\right)-2\,c^{2}\right)\sqrt{-c\sin\left(fx+e\right)+c}}{\sqrt{a\sin\left(fx+e\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] integral(-(c^2*cos(f*x + e)^2 + 2*c^2*sin(f*x + e) - 2*c^2)*sqrt(-c*sin(f*x
 + e) + c)/sqrt(a*sin(f*x + e) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))**(5/2)/(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}{\sqrt{a\sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate((-c*sin(f*x + e) + c)^(5/2)/sqrt(a*sin(f*x + e) + a), x)
```

3.384 $\int \frac{(c-c\sin(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}}\,dx$

Optimal. Leaf size=93

$$\frac{2c^2\cos(e+fx)\log(\sin(e+fx)+1)}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}$$

```
[Out] (2*c^2*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f*x]]*Sqrt
[c - c*Sin[e + f*x]]) + (c*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(f*Sqrt[a
 + a*Sin[e + f*x]])
```

Rubi [A] time = 0.187189, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2740, 2737, 2667, 31}

$$\frac{2c^2\cos(e+fx)\log(\sin(e+fx)+1)}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sin[e + f*x])^(3/2)/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] (2*c^2*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f*x]]*Sqrt
[c - c*Sin[e + f*x]]) + (c*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(f*Sqrt[a
 + a*Sin[e + f*x]])
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{(c-c\sin(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}}\,dx=\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a+a\sin(e+fx)}}+(2c)\int \frac{\sqrt{c-c\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}}\,dx$$
$$=\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a+a\sin(e+fx)}}+\frac{\left(2ac^2\cos(e+fx)\right)\int \frac{\cos(e+fx)}{a+a\sin(e+fx)}\,dx}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$
$$=\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a+a\sin(e+fx)}}+\frac{\left(2c^2\cos(e+fx)\right)\text{Subst}\left(\int \frac{1}{a+x}\,dx,x,a\sin(e+fx)\right)}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$
$$=\frac{2c^2\cos(e+fx)\log(1+\sin(e+fx))}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}+\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a+a\sin(e+fx)}}$$

Mathematica [A] time = 0.292943, size = 119, normalized size = 1.28

$$\frac{c(\sin(e+fx)-1)\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin(e+fx)-4\log\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e\right.\right.\right.}{f\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sin[e + f*x])^(3/2)/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] (c*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])*(-4*Log[Cos[(e
 + f*x)/2] + Sin[(e + f*x)/2]] + Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(f
*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*Sqrt[a*(1 + Sin[e + f*x])])
```

Maple [B] time = 0.174, size = 261, normalized size = 2.8

$$-\frac{1}{f\left(\sin\left(fx+e\right)\cos\left(fx+e\right)-\left(\cos\left(fx+e\right)\right)^2-2\sin\left(fx+e\right)-\cos\left(fx+e\right)+2\right)}\left(\sin\left(fx+e\right)\cos\left(fx+e\right)+4\sin\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] -1/f*(sin(f*x+e)*cos(f*x+e)+4*sin(f*x+e)*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin
(f*x+e))-2*sin(f*x+e)*ln(2/(cos(f*x+e)+1))+cos(f*x+e)^2-4*cos(f*x+e)*ln(-(-
1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))+2*cos(f*x+e)*ln(2/(cos(f*x+e)+1))-sin(
f*x+e)+4*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))-2*ln(2/(cos(f*x+e)+1))-
1)*(-c*(-1+sin(f*x+e)))^(3/2)/(sin(f*x+e)*cos(f*x+e)-cos(f*x+e)^2-2*sin(f*x
+e)-cos(f*x+e)+2)/(a*(1+sin(f*x+e)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}{\sqrt{a\sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate((-c*sin(f*x + e) + c)^(3/2)/sqrt(a*sin(f*x + e) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}{\sqrt{a\sin\left(fx+e\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] integral((-c*sin(f*x + e) + c)^(3/2)/sqrt(a*sin(f*x + e) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(-c\left(\sin\left(e+fx\right)-1\right)\right)^{\frac{3}{2}}}{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))**(3/2)/(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((-c*(sin(e + f*x) - 1))**(3/2)/sqrt(a*(sin(e + f*x) + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}{\sqrt{a\sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate((-c*sin(f*x + e) + c)^(3/2)/sqrt(a*sin(f*x + e) + a), x)
```

3.385 $\int \frac{\sqrt{c-c\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}}\,dx$

Optimal. Leaf size=49

$$\frac{c\cos(e+fx)\log(\sin(e+fx)+1)}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

```
[Out] (c*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c -
 c*Sin[e + f*x]])
```

Rubi [A] time = 0.100471, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {2737, 2667, 31}

$$\frac{c\cos(e+fx)\log(\sin(e+fx)+1)}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c*Sin[e + f*x]]/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] (c*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c -
 c*Sin[e + f*x]])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{c-c\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}}\,dx &= \frac{(ac\cos(e+fx))\int \frac{\cos(e+fx)}{a+a\sin(e+fx)}\,dx}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}\\ &= \frac{(c\cos(e+fx))\operatorname{Subst}\left(\int \frac{1}{a+x}\,dx,x,a\sin(e+fx)\right)}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}\\ &= \frac{c\cos(e+fx)\log(1+\sin(e+fx))}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}\end{aligned}$$

Mathematica [C] time = 0.976221, size = 118, normalized size = 2.41

$$-\frac{\sqrt{2}\left(e^{i(e+fx)}+i\right)\left(fx+2i\log\left(e^{i(e+fx)}+i\right)\right)\sqrt{c-c\sin(e+fx)}}{f\left(e^{i(e+fx)}-i\right)\sqrt{-iae^{-i(e+fx)}\left(e^{i(e+fx)}+i\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c*Sin[e + f*x]]/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] -((Sqrt[2]*(I + E^(I*(e + f*x)))*(f*x + (2*I)*Log[I + E^(I*(e + f*x))])*Sqr
t[c - c*Sin[e + f*x]])/((-I + E^(I*(e + f*x)))*Sqrt[((-I)*a*(I + E^(I*(e +
f*x)))^2)/E^(I*(e + f*x))]*f))
```

Maple [B] time = 0.127, size = 106, normalized size = 2.2

$$-\frac{1-\cos\left(fx+e\right)+\sin\left(fx+e\right)}{f\left(-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)\right)}\sqrt{-c\left(-1+\sin\left(fx+e\right)\right)}\left(\ln\left(2\left(\cos\left(fx+e\right)+1\right)^{-1}\right)-2\ln\left(-\frac{-1+\cos\left(fx+e\right.}{\sin\left(f\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] -1/f*(1-cos(f*x+e)+sin(f*x+e))*(-c*(-1+sin(f*x+e)))^(1/2)*(ln(2/(cos(f*x+e)
+1))-2*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e)))/(a*(1+sin(f*x+e)))^(1/2)
/(-1+cos(f*x+e)+sin(f*x+e))
```

Maxima [A] time = 1.81834, size = 86, normalized size = 1.76

$$-\frac{\frac{2\sqrt{c}\log\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+1\right)}{\sqrt{a}}-\frac{\sqrt{c}\log\left(\frac{\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}+1\right)}{\sqrt{a}}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] -(2*sqrt(c)*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/sqrt(a) - sqrt(c)*log(
sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/sqrt(a))/f
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-c\sin\left(fx+e\right)+c}}{\sqrt{a\sin\left(fx+e\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] integral(sqrt(-c*sin(f*x + e) + c)/sqrt(a*sin(f*x + e) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c\left(\sin\left(e+fx\right)-1\right)}}{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))**(1/2)/(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(sin(e + f*x) - 1))/sqrt(a*(sin(e + f*x) + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c\sin\left(fx+e\right)+c}}{\sqrt{a\sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(-c*sin(f*x + e) + c)/sqrt(a*sin(f*x + e) + a), x)
```

3.386 $\int \frac{1}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=46

$$\frac{\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

```
[Out] (ArcTanh[Sin[e + f*x]]*Cos[e + f*x])/(f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c
*Sin[e + f*x]])
```

Rubi [A] time = 0.086145, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2741, 3770}

$$\frac{\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]),x]
```

```
[Out] (ArcTanh[Sin[e + f*x]]*Cos[e + f*x])/(f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c
*Sin[e + f*x]])
```

Rule 2741

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Dist[Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]
]*Sqrt[c + d*Sin[e + f*x]]), Int[1/Cos[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}\,dx &= \frac{\cos(e+fx)\int \sec(e+fx)\,dx}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}\\ &= \frac{\tanh^{-1}(\sin(e+fx))\cos(e+fx)}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}\end{aligned}$$

Mathematica [A] time = 0.243021, size = 89, normalized size = 1.93

$$-\frac{\cos(e+fx)\left(\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\right)}{f\sqrt{a(\sin(e+fx)+1)}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]),x]
```

```
[Out] -((Cos[e + f*x]*(Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - Log[Cos[(e + f*
x)/2] + Sin[(e + f*x)/2]]))/(f*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e
+ f*x]]))
```

Maple [B] time = 0.164, size = 92, normalized size = 2.

$$-\frac{\cos\left(fx+e\right)}{f}\left(\ln\left(-\frac{-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)-\ln\left(-\frac{-1+\cos\left(fx+e\right)-\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)\right)\frac{1}{\sqrt{a\left(1+\sin\left(fx+e\right)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] -1/f*(ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-ln(-(-1+cos(f*x+e)-sin(f*x
+e))/sin(f*x+e)))*cos(f*x+e)/(a*(1+sin(f*x+e)))^(1/2)/(-c*(-1+sin(f*x+e)))^
(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate(1/(sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)), x)
```

Fricas [A] time = 1.219, size = 393, normalized size = 8.54

$$\left[\frac{\sqrt{ac}\log\left(-\frac{ac\cos\left(fx+e\right)^{3}-2\,ac\cos\left(fx+e\right)-2\,\sqrt{ac}\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}\sin\left(fx+e\right)}{\cos\left(fx+e\right)^{3}}\right)}{2\,acf},-\frac{\sqrt{-ac}\arctan\left(\frac{\sqrt{-ac}\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+\right.}}{ac\cos\left(fx+e\right)\sin\left(fx+e\right)}\right.}{acf}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2),x, algorithm="fri
cas")
```

```
[Out] [1/2*sqrt(a*c)*log(-(a*c*cos(f*x + e)^3 - 2*a*c*cos(f*x + e) - 2*sqrt(a*c)*
sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*sin(f*x + e))/cos(f*x +
e)^3)/(a*c*f), -sqrt(-a*c)*arctan(sqrt(-a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(
-c*sin(f*x + e) + c)/(a*c*cos(f*x + e)*sin(f*x + e)))/(a*c*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}\sqrt{-c\left(\sin\left(e+fx\right)-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**(1/2)/(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a*(sin(e + f*x) + 1))*sqrt(-c*(sin(e + f*x) - 1))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate(1/(sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)), x)
```

3.387 $\int \frac{1}{\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=95

$$\frac{\cos(e+fx)}{2f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}+\frac{\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{2cf\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

```
[Out] Cos[e + f*x]/(2*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2)) + (A
rcTanh[Sin[e + f*x]]*Cos[e + f*x])/(2*c*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c -
 c*Sin[e + f*x]])
```

Rubi [A] time = 0.175376, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {2743, 2741, 3770}

$$\frac{\cos(e+fx)}{2f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}+\frac{\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{2cf\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2)),x]
```

```
[Out] Cos[e + f*x]/(2*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2)) + (A
rcTanh[Sin[e + f*x]]*Cos[e + f*x])/(2*c*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c -
 c*Sin[e + f*x]])
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rule 2741

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Dist[Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]
]*Sqrt[c + d*Sin[e + f*x]]), Int[1/Cos[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}}\,dx = \frac{\cos(e+fx)}{2f\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}} + \frac{\int \frac{1}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+}}}{2c}$$

$$= \frac{\cos(e+fx)}{2f\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}} + \frac{\cos(e+fx)\int \sec(}{2c\sqrt{a+a\sin(e+fx)}\sqrt{c}}$$

$$= \frac{\cos(e+fx)}{2f\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}} + \frac{\tanh^{-1}(\sin(e+fx)}{2cf\sqrt{a+a\sin(e+fx)}\sqrt{}}$$

Mathematica [A] time = 0.408667, size = 161, normalized size = 1.69

$$-\frac{\cos(e+fx)\left(-\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)+\log\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)+\sin(e+fx)\left(\log\right.\right.}{2cf(\sin(e+fx)-1)\sqrt{a(\sin(e+fx)+1)}\sqrt{c-}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2)),x]
```

```
[Out] -(Cos[e + f*x]*(1 - Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + Log[Cos[(e +
 f*x)/2] + Sin[(e + f*x)/2]] + (Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] -
Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]])*Sin[e + f*x]))/(2*c*f*(-1 + Sin[e
 + f*x])*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]])
```

Maple [A] time = 0.177, size = 165, normalized size = 1.7

$$\frac{\cos(fx+e)}{2f}\left(\sin(fx+e)\ln\left(-\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right)-\sin(fx+e)\ln\left(-\frac{-1+\cos(fx+e)-\sin(fx+e}{\sin(fx+e)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] 1/2/f*(sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-sin(f*x+e)*ln(
-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))+sin(f*x+e)-ln(-(-1+cos(f*x+e)+sin(f
*x+e))/sin(f*x+e))+ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e)))*cos(f*x+e)/(
a*(1+sin(f*x+e)))^(1/2)/(-c*(-1+sin(f*x+e)))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\sin(fx+e)+a}\,(-c\sin(fx+e)+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(3/2),x, algorithm="max
ima")
```

```
[Out] integrate(1/(sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e) + c)^(3/2)), x)
```

Fricas [A] time = 1.29885, size = 803, normalized size = 8.45

$$\left[\frac{\sqrt{ac}\left(\cos\left(fx+e\right)\sin\left(fx+e\right)-\cos\left(fx+e\right)\right)\log\left(-\frac{ac\cos\left(fx+e\right)^{3}-2\,ac\cos\left(fx+e\right)-2\,\sqrt{ac}\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}\sin\left(fx+e\right)}{\cos\left(fx+e\right)^{3}}\right)-2}{4\left(ac^{2}f\cos\left(fx+e\right)\sin\left(fx+e\right)-ac^{2}f\cos\left(fx+e\right)\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(3/2),x, algorithm="fri
cas")
```

```
[Out] [1/4*(sqrt(a*c)*(cos(f*x + e)*sin(f*x + e) - cos(f*x + e))*log(-(a*c*cos(f*
x + e)^3 - 2*a*c*cos(f*x + e) - 2*sqrt(a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-
c*sin(f*x + e) + c)*sin(f*x + e))/cos(f*x + e)^3) - 2*sqrt(a*sin(f*x + e) +
 a)*sqrt(-c*sin(f*x + e) + c))/(a*c^2*f*cos(f*x + e)*sin(f*x + e) - a*c^2*f
*cos(f*x + e)), -1/2*(sqrt(-a*c)*(cos(f*x + e)*sin(f*x + e) - cos(f*x + e))
*arctan(sqrt(-a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(a*c*
cos(f*x + e)*sin(f*x + e))) + sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e)
 + c))/(a*c^2*f*cos(f*x + e)*sin(f*x + e) - a*c^2*f*cos(f*x + e))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}\left(-c\left(\sin\left(e+fx\right)-1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**(1/2)/(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral(1/(sqrt(a*(sin(e + f*x) + 1))*(-c*(sin(e + f*x) - 1))**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\sin\left(fx+e\right)+a}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(3/2),x, algorithm="gia
c")
```

```
[Out] integrate(1/(sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e) + c)^(3/2)), x)
```

3.388 $\int \frac{1}{\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=140

$$\frac{\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{4c^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{\cos(e+fx)}{4cf\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}+\frac{\cos(e+fx}{4f\sqrt{a\sin(e+fx)+a}(c-}\ldots$$

```
[Out] Cos[e + f*x]/(4*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(5/2)) + Co
s[e + f*x]/(4*c*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2)) + (A
rcTanh[Sin[e + f*x]]*Cos[e + f*x])/(4*c^2*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c
 - c*Sin[e + f*x]])
```

Rubi [A] time = 0.268799, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {2743, 2741, 3770}

$$\frac{\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{4c^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{\cos(e+fx)}{4cf\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}+\frac{\cos(e+fx}{4f\sqrt{a\sin(e+fx)+a}(c-}\ldots$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(5/2)),x]
```

```
[Out] Cos[e + f*x]/(4*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(5/2)) + Co
s[e + f*x]/(4*c*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2)) + (A
rcTanh[Sin[e + f*x]]*Cos[e + f*x])/(4*c^2*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c
 - c*Sin[e + f*x]])
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rule 2741

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Dist[Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]
]*Sqrt[c + d*Sin[e + f*x]]), Int[1/Cos[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}}\,dx = \frac{\cos(e+fx)}{4f\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}} + \frac{\int \frac{1}{\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3}}}{2c}$$

$$= \frac{\cos(e+fx)}{4f\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}} + \frac{\cos(e+fx)}{4cf\sqrt{a+a\sin(e+fx)}(c-}$$

$$= \frac{\cos(e+fx)}{4f\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}} + \frac{\cos(e+fx)}{4cf\sqrt{a+a\sin(e+fx)}(c-}$$

$$= \frac{\cos(e+fx)}{4f\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}} + \frac{\cos(e+fx)}{4cf\sqrt{a+a\sin(e+fx)}(c-}$$

Mathematica [A] time = 0.617281, size = 224, normalized size = 1.6

$$\frac{\cos(e+fx)\left(-3\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)+\cos(2(e+fx))\left(\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-\log\left(\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(5/2)),x]
```

```
[Out] (Cos[e + f*x]*(4 - 3*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + Cos[2*(e +
f*x)]*(Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - Log[Cos[(e + f*x)/2] + Si
n[(e + f*x)/2]]) + 3*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + (-2 + 4*Log
[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - 4*Log[Cos[(e + f*x)/2] + Sin[(e + f
*x)/2]])*Sin[e + f*x]))/(8*c^2*f*(-1 + Sin[e + f*x])^2*Sqrt[a*(1 + Sin[e +
f*x])]*Sqrt[c - c*Sin[e + f*x]])
```

Maple [B] time = 0.188, size = 252, normalized size = 1.8

$$\frac{\cos\left(fx+e\right)}{4f}\left(\left(\cos\left(fx+e\right)\right)^{2}\ln\left(-\frac{-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)-\left(\cos\left(fx+e\right)\right)^{2}\ln\left(-\frac{-1+\cos\left(fx+e\right)-\sin\left(fx\right.}{\sin\left(fx+e\right)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] 1/4/f*(cos(f*x+e)^2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-cos(f*x+e)^2
*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))+2*sin(f*x+e)*ln(-(-1+cos(f*x+e)
+sin(f*x+e))/sin(f*x+e))-2*sin(f*x+e)*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*
x+e))+2*cos(f*x+e)^2+3*sin(f*x+e)-2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+
e))+2*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))-2)*cos(f*x+e)/(a*(1+sin(f*
x+e)))^(1/2)/(-c*(-1+sin(f*x+e)))^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\sin\left(fx+e\right)+a}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(5/2),x, algorithm="max
ima")
```

```
[Out] integrate(1/(sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e) + c)^(5/2)), x)
```

Fricas [A] time = 1.35324, size = 990, normalized size = 7.07

$$\left[\frac{\left(\cos\left(fx+e\right)^{3}+2\,\cos\left(fx+e\right)\sin\left(fx+e\right)-2\,\cos\left(fx+e\right)\right)\sqrt{ac}\log\left(-\frac{ac\cos\left(fx+e\right)^{3}-2\,ac\cos\left(fx+e\right)-2\,\sqrt{ac}\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-}}{\cos\left(fx+e\right)^{3}}\right.}{8\left(ac^{3}f\cos\left(fx+e\right)^{3}+2\,ac^{3}f\cos\left(fx+e\right)\sin\left(fx+e\right)\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(5/2),x, algorithm="fri
cas")
```

```
[Out] [1/8*((cos(f*x + e)^3 + 2*cos(f*x + e)*sin(f*x + e) - 2*cos(f*x + e))*sqrt(
a*c)*log(-(a*c*cos(f*x + e)^3 - 2*a*c*cos(f*x + e) - 2*sqrt(a*c)*sqrt(a*sin
(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*sin(f*x + e))/cos(f*x + e)^3) + 2*
sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*(sin(f*x + e) - 2))/(a*c
^3*f*cos(f*x + e)^3 + 2*a*c^3*f*cos(f*x + e)*sin(f*x + e) - 2*a*c^3*f*cos(f
*x + e)), -1/4*((cos(f*x + e)^3 + 2*cos(f*x + e)*sin(f*x + e) - 2*cos(f*x +
 e))*sqrt(-a*c)*arctan(sqrt(-a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x
+ e) + c)/(a*c*cos(f*x + e)*sin(f*x + e))) - sqrt(a*sin(f*x + e) + a)*sqrt(
-c*sin(f*x + e) + c)*(sin(f*x + e) - 2))/(a*c^3*f*cos(f*x + e)^3 + 2*a*c^3*
f*cos(f*x + e)*sin(f*x + e) - 2*a*c^3*f*cos(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**(1/2)/(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\sin\left(fx+e\right)+a}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(5/2),x, algorithm="gia
c")
```

```
[Out] integrate(1/(sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e) + c)^(5/2)), x)
```

3.389 $\int \frac{(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=191

$$-\frac{6c^3\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a\sin(e+fx)+a}}-\frac{3c^2\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2af\sqrt{a\sin(e+fx)+a}}-\frac{12c^4\cos(e+fx)\log(\sin(e+fx)+1)}{af\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{c}{}$$

```
[Out] (-12*c^4*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(a*f*Sqrt[a + a*Sin[e + f*x]]*
Sqrt[c - c*Sin[e + f*x]]) - (6*c^3*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(
a*f*Sqrt[a + a*Sin[e + f*x]]) - (3*c^2*Cos[e + f*x]*(c - c*Sin[e + f*x])^(3
/2))/(2*a*f*Sqrt[a + a*Sin[e + f*x]]) - (c*Cos[e + f*x]*(c - c*Sin[e + f*x]
)^(5/2))/(f*(a + a*Sin[e + f*x])^(3/2))
```

Rubi [A] time = 0.376912, antiderivative size = 191, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2739, 2740, 2737, 2667, 31}

$$-\frac{6c^3\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a\sin(e+fx)+a}}-\frac{3c^2\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2af\sqrt{a\sin(e+fx)+a}}-\frac{12c^4\cos(e+fx)\log(\sin(e+fx)+1)}{af\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{c}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sin[e + f*x])^(7/2)/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-12*c^4*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(a*f*Sqrt[a + a*Sin[e + f*x]]*
Sqrt[c - c*Sin[e + f*x]]) - (6*c^3*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(
a*f*Sqrt[a + a*Sin[e + f*x]]) - (3*c^2*Cos[e + f*x]*(c - c*Sin[e + f*x])^(3
/2))/(2*a*f*Sqrt[a + a*Sin[e + f*x]]) - (c*Cos[e + f*x]*(c - c*Sin[e + f*x]
)^(5/2))/(f*(a + a*Sin[e + f*x])^(3/2))
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(2*n + 1)), x] - Dist[(b*(2*m - 1))/(d*
(2*n + 1)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
```

```
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{3/2}}\,dx = -\frac{c\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{f(a+a\sin(e+fx))^{3/2}} - \frac{(3c)\int \frac{(c-c\sin(e+fx))^{5/2}}{\sqrt{a+a\sin(e+fx)}}\,dx}{a}$$

$$= -\frac{3c^2\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2af\sqrt{a+a\sin(e+fx)}} - \frac{c\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{f(a+a\sin(e+fx))^{3/2}} - \frac{\left(6c^2\right)\int \frac{(c-}{\sqrt{}}}{}$$

$$= -\frac{6c^3\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a+a\sin(e+fx)}} - \frac{3c^2\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2af\sqrt{a+a\sin(e+fx)}} - \frac{c\cos(e+}{f(a}$$

$$= -\frac{6c^3\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a+a\sin(e+fx)}} - \frac{3c^2\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2af\sqrt{a+a\sin(e+fx)}} - \frac{c\cos(e+}{f(a}$$

$$= -\frac{6c^3\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a+a\sin(e+fx)}} - \frac{3c^2\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2af\sqrt{a+a\sin(e+fx)}} - \frac{c\cos(e+}{f(a}$$

$$= -\frac{12c^4\cos(e+fx)\log(1+\sin(e+fx))}{af\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{6c^3\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a+a\sin(e+fx)}} - \frac{3c^2\,\mathrm{co}}{}$$

Mathematica [A] time = 1.71082, size = 162, normalized size = 0.85

$$\frac{c^3\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin(3(e+fx))-18\cos(2(e+fx))-192\log\left(\sin\left(\frac{1}{2}(e+fx\right.\right.\right.}{8f(a(\sin(e+fx)+1))^{3/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sin[e + f*x])^(7/2)/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] (c^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]*(-44 -
18*Cos[2*(e + f*x)] - 192*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + (39 -
192*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]])*Sin[e + f*x] + Sin[3*(e + f*x
)]))/(8*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(3/2
))
```

Maple [B] time = 0.174, size = 501, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(3/2),x)
```

```
[Out] -1/2/f*(sin(f*x+e)*cos(f*x+e)^3+cos(f*x+e)^4+8*cos(f*x+e)^2*sin(f*x+e)+24*l
n(2/(cos(f*x+e)+1))*sin(f*x+e)*cos(f*x+e)-48*sin(f*x+e)*cos(f*x+e)*ln(-(-1+
cos(f*x+e)-sin(f*x+e))/sin(f*x+e))-9*cos(f*x+e)^3+24*cos(f*x+e)^2*ln(2/(cos
(f*x+e)+1))-48*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))+25*s
in(f*x+e)*cos(f*x+e)-48*sin(f*x+e)*ln(2/(cos(f*x+e)+1))+96*sin(f*x+e)*ln(-(
-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))+33*cos(f*x+e)^2+24*cos(f*x+e)*ln(2/(c
os(f*x+e)+1))-48*cos(f*x+e)*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))-34*s
in(f*x+e)+9*cos(f*x+e)-48*ln(2/(cos(f*x+e)+1))+96*ln(-(-1+cos(f*x+e)-sin(f*
x+e))/sin(f*x+e))-34)*(-c*(-1+sin(f*x+e)))^(7/2)/(sin(f*x+e)*cos(f*x+e)^3-c
os(f*x+e)^4-4*cos(f*x+e)^2*sin(f*x+e)-3*cos(f*x+e)^3-4*sin(f*x+e)*cos(f*x+e
)+8*cos(f*x+e)^2+8*sin(f*x+e)+4*cos(f*x+e)-8)/(a*(1+sin(f*x+e)))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate((-c*sin(f*x + e) + c)^(7/2)/(a*sin(f*x + e) + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(3\,c^3\cos\left(fx+e\right)^2-4\,c^3-\left(c^3\cos\left(fx+e\right)^2-4\,c^3\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}}{a^2\cos\left(fx+e\right)^2-2\,a^2\sin\left(fx+e\right)-2\,a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] integral((3*c^3*cos(f*x + e)^2 - 4*c^3 - (c^3*cos(f*x + e)^2 - 4*c^3)*sin(f
*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(a^2*cos(f*x +
e)^2 - 2*a^2*sin(f*x + e) - 2*a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))**(7/2)/(a+a*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate((-c*sin(f*x + e) + c)^(7/2)/(a*sin(f*x + e) + a)^(3/2), x)
```

3.390 $\int \frac{(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=143

$$-\frac{2c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a\sin(e+fx)+a}}-\frac{4c^3\cos(e+fx)\log(\sin(e+fx)+1)}{af\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{f(a\sin(e+fx)+a)^{3/2}}$$

```
[Out] (-4*c^3*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(a*f*Sqrt[a + a*Sin[e + f*x]]*S
qrt[c - c*Sin[e + f*x]]) - (2*c^2*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(a
*f*Sqrt[a + a*Sin[e + f*x]]) - (c*Cos[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/
(f*(a + a*Sin[e + f*x])^(3/2))
```

Rubi [A] time = 0.28775, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2739, 2740, 2737, 2667, 31}

$$-\frac{2c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a\sin(e+fx)+a}}-\frac{4c^3\cos(e+fx)\log(\sin(e+fx)+1)}{af\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{f(a\sin(e+fx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sin[e + f*x])^(5/2)/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-4*c^3*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(a*f*Sqrt[a + a*Sin[e + f*x]]*S
qrt[c - c*Sin[e + f*x]]) - (2*c^2*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(a
*f*Sqrt[a + a*Sin[e + f*x]]) - (c*Cos[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/
(f*(a + a*Sin[e + f*x])^(3/2))
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(2*n + 1)), x] - Dist[(b*(2*m - 1))/(d*
(2*n + 1)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{3/2}}\,dx = -\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{f(a+a\sin(e+fx))^{3/2}} - \frac{(2c)\int \frac{(c-c\sin(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}}\,dx}{a}$$

$$= -\frac{2c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a+a\sin(e+fx)}} - \frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{f(a+a\sin(e+fx))^{3/2}} - \frac{\left(4c^2\right)\int \frac{\sqrt{c-c}}{\sqrt{a+a}}}{a}$$

$$= -\frac{2c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a+a\sin(e+fx)}} - \frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{f(a+a\sin(e+fx))^{3/2}} - \frac{\left(4c^3\cos(e\right.}{\sqrt{a+a\sin(e}}$$

$$= -\frac{2c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a+a\sin(e+fx)}} - \frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{f(a+a\sin(e+fx))^{3/2}} - \frac{\left(4c^3\cos(e+\right.}{af\sqrt{a}}$$

$$= -\frac{4c^3\cos(e+fx)\log(1+\sin(e+fx))}{af\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{2c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a+a\sin(e+fx)}} - \frac{c\cos(}{}$$

Mathematica [A] time = 0.780627, size = 153, normalized size = 1.07

$$-\frac{c^2\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos(2(e+fx))+16\log\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right.\right.\right.}{2f(a(\sin(e+fx)+1))^{3/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sin[e + f*x])^(5/2)/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] -(c^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]*(7 + C
os[2*(e + f*x)] + 16*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + 2*(-1 + 8*L
og[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]])*Sin[e + f*x]))/(2*f*(Cos[(e + f*x)
/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(3/2))
```

Maple [B] time = 0.157, size = 446, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(3/2),x)
```

```
[Out] 1/f*(cos(f*x+e)^2*sin(f*x+e)-8*sin(f*x+e)*cos(f*x+e)*ln(-(-1+cos(f*x+e)-sin
(f*x+e))/sin(f*x+e))+4*ln(2/(cos(f*x+e)+1))*sin(f*x+e)*cos(f*x+e)-cos(f*x+e
)^3-8*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))+4*cos(f*x+e)^
2*ln(2/(cos(f*x+e)+1))+5*sin(f*x+e)*cos(f*x+e)+16*sin(f*x+e)*ln(-(-1+cos(f*
x+e)-sin(f*x+e))/sin(f*x+e))-8*sin(f*x+e)*ln(2/(cos(f*x+e)+1))+6*cos(f*x+e)
^2-8*cos(f*x+e)*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))+4*cos(f*x+e)*ln(
2/(cos(f*x+e)+1))-6*sin(f*x+e)+cos(f*x+e)+16*ln(-(-1+cos(f*x+e)-sin(f*x+e))
/sin(f*x+e))-8*ln(2/(cos(f*x+e)+1))-6)*(-c*(-1+sin(f*x+e)))^(5/2)/(cos(f*x+
e)^3+cos(f*x+e)^2*sin(f*x+e)-3*cos(f*x+e)^2+2*sin(f*x+e)*cos(f*x+e)-2*cos(f
*x+e)-4*sin(f*x+e)+4)/(a*(1+sin(f*x+e)))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate((-c*sin(f*x + e) + c)^(5/2)/(a*sin(f*x + e) + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(c^2\cos\left(fx+e\right)^2+2\,c^2\sin\left(fx+e\right)-2\,c^2\right)\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}}{a^2\cos\left(fx+e\right)^2-2\,a^2\sin\left(fx+e\right)-2\,a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] integral((c^2*cos(f*x + e)^2 + 2*c^2*sin(f*x + e) - 2*c^2)*sqrt(a*sin(f*x +
 e) + a)*sqrt(-c*sin(f*x + e) + c)/(a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x + e)
 - 2*a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))**(5/2)/(a+a*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate((-c*sin(f*x + e) + c)^(5/2)/(a*sin(f*x + e) + a)^(3/2), x)
```

3.391 $\int \frac{(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=97

$$-\frac{c^2\cos(e+fx)\log(\sin(e+fx)+1)}{af\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f(a\sin(e+fx)+a)^{3/2}}$$

```
[Out] -((c^2*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(a*f*Sqrt[a + a*Sin[e + f*x]]*Sq
rt[c - c*Sin[e + f*x]])) - (c*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(f*(a
+ a*Sin[e + f*x])^(3/2))
```

Rubi [A] time = 0.197259, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2739, 2737, 2667, 31}

$$-\frac{c^2\cos(e+fx)\log(\sin(e+fx)+1)}{af\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f(a\sin(e+fx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sin[e + f*x])^(3/2)/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] -((c^2*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(a*f*Sqrt[a + a*Sin[e + f*x]]*Sq
rt[c - c*Sin[e + f*x]])) - (c*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(f*(a
+ a*Sin[e + f*x])^(3/2))
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(2*n + 1)), x] - Dist[(b*(2*m - 1))/(d*
(2*n + 1)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] && !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] || !IntegerQ[m + 1/2
])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{3/2}}\,dx &= -\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f(a+a\sin(e+fx))^{3/2}}-\frac{c\int\frac{\sqrt{c-c\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}}\,dx}{a}\\
&= -\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f(a+a\sin(e+fx))^{3/2}}-\frac{\left(c^2\cos(e+fx)\right)\int\frac{\cos(e+fx)}{a+a\sin(e+fx)}\,dx}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}\\
&= -\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f(a+a\sin(e+fx))^{3/2}}-\frac{\left(c^2\cos(e+fx)\right)\operatorname{Subst}\left(\int\frac{1}{a+x}\,dx,x,a\sin(e+fx)\right)}{af\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}\\
&= -\frac{c^2\cos(e+fx)\log(1+\sin(e+fx))}{af\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}-\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f(a+a\sin(e+fx))^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.453933, size = 134, normalized size = 1.38

$$-\frac{2c\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin(e+fx)\log\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)+\log\right.}{f(a(\sin(e+fx)+1))^{3/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sin[e + f*x])^(3/2)/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-2*c*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]*(1 + L
og[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + Log[Cos[(e + f*x)/2] + Sin[(e + f
*x)/2]]*Sin[e + f*x]))/(f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin
[e + f*x]))^(3/2))
```

Maple [B] time = 0.141, size = 388, normalized size = 4.

$$\frac{1}{f\left(\sin\left(fx+e\right)\cos\left(fx+e\right)-\left(\cos\left(fx+e\right)\right)^{2}-2\sin\left(fx+e\right)-\cos\left(fx+e\right)+2\right)}\left(\ln\left(2\left(\cos\left(fx+e\right)+1\right)^{-1}\right)\sin\left(f\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2),x)
```

```
[Out] 1/f*(ln(2/(cos(f*x+e)+1))*sin(f*x+e)*cos(f*x+e)+cos(f*x+e)^2*ln(2/(cos(f*x+
e)+1))-2*sin(f*x+e)*cos(f*x+e)*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))-2
*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))-2*sin(f*x+e)*ln(2/
(cos(f*x+e)+1))+cos(f*x+e)*ln(2/(cos(f*x+e)+1))+2*sin(f*x+e)*cos(f*x+e)+4*s
in(f*x+e)*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))+2*cos(f*x+e)^2-2*cos(f
*x+e)*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))-2*ln(2/(cos(f*x+e)+1))-2*s
in(f*x+e)+4*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))-2)*(-c*(-1+sin(f*x+e
)))^(3/2)/(sin(f*x+e)*cos(f*x+e)-cos(f*x+e)^2-2*sin(f*x+e)-cos(f*x+e)+2)/(a
*(1+sin(f*x+e)))^(3/2)
```

Maxima [A] time = 1.77917, size = 184, normalized size = 1.9

$$\frac{\frac{2c^{\frac{3}{2}}\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^{\frac{3}{2}}}-\frac{c^{\frac{3}{2}}\log\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)}{a^{\frac{3}{2}}}-\frac{4\sqrt{a}c^{\frac{3}{2}}\sin(fx+e)}{\left(a^2+\frac{2a^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}\right)(\cos(fx+e)+1)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] (2*c^(3/2)*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^(3/2) - c^(3/2)*log(s
in(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/a^(3/2) - 4*sqrt(a)*c^(3/2)*sin(f*x
 + e)/((a^2 + 2*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + a^2*sin(f*x + e)^2/(c
os(f*x + e) + 1)^2)*(cos(f*x + e) + 1)))/f
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{a\sin\left(fx+e\right)+a}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}{a^2\cos\left(fx+e\right)^2-2a^2\sin\left(fx+e\right)-2a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] integral(-sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e) + c)^(3/2)/(a^2*cos(f*x
 + e)^2 - 2*a^2*sin(f*x + e) - 2*a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(-c\left(\sin\left(e+fx\right)-1\right)\right)^{\frac{3}{2}}}{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))**(3/2)/(a+a*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral((-c*(sin(e + f*x) - 1))**(3/2)/(a*(sin(e + f*x) + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate((-c*sin(f*x + e) + c)^(3/2)/(a*sin(f*x + e) + a)^(3/2), x)
```

3.392 $\int \frac{\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=41

$$-\frac{c\cos(e+fx)}{f(a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}}$$

```
[Out] -((c*Cos[e + f*x])/(f*(a + a*Sin[e + f*x])^(3/2)*Sqrt[c - c*Sin[e + f*x]]))
```

Rubi [A] time = 0.0854564, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {2738}

$$-\frac{c\cos(e+fx)}{f(a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c*Sin[e + f*x]]/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] -((c*Cos[e + f*x])/(f*(a + a*Sin[e + f*x])^(3/2)*Sqrt[c - c*Sin[e + f*x]]))
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\int \frac{\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{3/2}}\,dx = -\frac{c\cos(e+fx)}{f(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}}$$

Mathematica [B] time = 0.19293, size = 85, normalized size = 2.07

$$-\frac{\sqrt{a(\sin(e+fx)+1)}\sqrt{c-c\sin(e+fx)}}{a^2 f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c*Sin[e + f*x]]/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] -((Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]])/(a^2*f*(Cos[(e + f*
x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3))
```

Maple [A] time = 0.161, size = 69, normalized size = 1.7

$$-\frac{\sin(fx+e)\left(-1+\cos(fx+e)-\sin(fx+e)\right)}{f\left(-1+\cos(fx+e)+\sin(fx+e)\right)}\sqrt{-c\left(-1+\sin(fx+e)\right)}\left(a\left(1+\sin(fx+e)\right)\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(3/2),x)
```

```
[Out] -1/f*(-c*(-1+sin(f*x+e)))^(1/2)*sin(f*x+e)*(-1+cos(f*x+e)-sin(f*x+e))/(-1+c
os(f*x+e)+sin(f*x+e))/(a*(1+sin(f*x+e)))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c\sin\left(fx+e\right)+c}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(-c*sin(f*x + e) + c)/(a*sin(f*x + e) + a)^(3/2), x)
```

Fricas [A] time = 1.03902, size = 146, normalized size = 3.56

$$-\frac{\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}}{a^2f\cos\left(fx+e\right)\sin\left(fx+e\right)+a^2f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] -sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(a^2*f*cos(f*x + e)*sin
(f*x + e) + a^2*f*cos(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c\left(\sin\left(e+fx\right)-1\right)}}{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))**(1/2)/(a+a*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral(sqrt(-c*(sin(e + f*x) - 1))/(a*(sin(e + f*x) + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c\sin\left(fx+e\right)+c}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(-c*sin(f*x + e) + c)/(a*sin(f*x + e) + a)^(3/2), x)
```

3.393 $\int \frac{1}{(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=95

$$\frac{\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{2af\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}}$$

```
[Out] -Cos[e + f*x]/(2*f*(a + a*Sin[e + f*x])^(3/2)*Sqrt[c - c*Sin[e + f*x]]) + (
ArcTanh[Sin[e + f*x]]*Cos[e + f*x])/(2*a*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c
- c*Sin[e + f*x]])
```

Rubi [A] time = 0.176238, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {2743, 2741, 3770}

$$\frac{\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{2af\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^(3/2)*Sqrt[c - c*Sin[e + f*x]]),x]
```


```
[Out] -Cos[e + f*x]/(2*f*(a + a*Sin[e + f*x])^(3/2)*Sqrt[c - c*Sin[e + f*x]]) + (
ArcTanh[Sin[e + f*x]]*Cos[e + f*x])/(2*a*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c
- c*Sin[e + f*x]])
```


Rule 2743

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rule 2741

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Dist[Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]
]*Sqrt[c + d*Sin[e + f*x]]), Int[1/Cos[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}}\,dx = -\frac{\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}} + \frac{\int \frac{1}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+f}}}{2a}$$
$$= -\frac{\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}} + \frac{\cos(e+fx)\int \sec(e}{2a\sqrt{a+a\sin(e+fx)}\sqrt{c-}}$$
$$= -\frac{\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}} + \frac{\tanh^{-1}(\sin(e+fx))}{2af\sqrt{a+a\sin(e+fx)}\sqrt{c}}$$

Mathematica [A] time = 0.397532, size = 148, normalized size = 1.56

$$-\frac{\cos(e+fx)\left(\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)+\sin(e+fx)\left(\log\left(\cos\right.\right.\right.}{2f(a(\sin(e+fx)+1))^{3/2}\sqrt{c-c\sin(e+fx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^(3/2)*Sqrt[c - c*Sin[e + f*x]]),x]
```

```
[Out] -(Cos[e + f*x]*(1 + Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - Log[Cos[(e +
 f*x)/2] + Sin[(e + f*x)/2]] + (Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] -
Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]])*Sin[e + f*x]))/(2*f*(a*(1 + Sin[e
 + f*x]))^(3/2)*Sqrt[c - c*Sin[e + f*x]])
```

Maple [A] time = 0.178, size = 167, normalized size = 1.8

$$-\frac{\cos(fx+e)}{2f}\left(\sin(fx+e)\ln\left(-\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right)-\sin(fx+e)\ln\left(-\frac{-1+\cos(fx+e)-\sin(fx+e)}{\sin(fx+e)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] -1/2/f*(sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-sin(f*x+e)*ln
(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))-sin(f*x+e)+ln(-(-1+cos(f*x+e)+sin(
f*x+e))/sin(f*x+e))-ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e)))*cos(f*x+e)/
(a*(1+sin(f*x+e)))^(3/2)/(-c*(-1+sin(f*x+e)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\sin(fx+e)+a\right)^{\frac{3}{2}}\sqrt{-c\sin(fx+e)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)^(3/2)*sqrt(-c*sin(f*x + e) + c)), x)
```

Fricas [A] time = 1.32618, size = 803, normalized size = 8.45

$$\left[\frac{\sqrt{ac}\left(\cos\left(fx+e\right)\sin\left(fx+e\right)+\cos\left(fx+e\right)\right)\log\left(-\frac{ac\cos\left(fx+e\right)^3-2\,ac\cos\left(fx+e\right)-2\,\sqrt{ac}\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}\sin\left(fx+e\right)}{\cos\left(fx+e\right)^3}\right.}{4\left(a^2cf\cos\left(fx+e\right)\sin\left(fx+e\right)+a^2cf\cos\left(fx+e\right)\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2),x, algorithm="fri
cas")
```

```
[Out] [1/4*(sqrt(a*c)*(cos(f*x + e)*sin(f*x + e) + cos(f*x + e))*log(-(a*c*cos(f*
x + e)^3 - 2*a*c*cos(f*x + e) - 2*sqrt(a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-
c*sin(f*x + e) + c)*sin(f*x + e))/cos(f*x + e)^3) - 2*sqrt(a*sin(f*x + e) +
 a)*sqrt(-c*sin(f*x + e) + c))/(a^2*c*f*cos(f*x + e)*sin(f*x + e) + a^2*c*f
*cos(f*x + e)), -1/2*(sqrt(-a*c)*(cos(f*x + e)*sin(f*x + e) + cos(f*x + e))
*arctan(sqrt(-a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(a*c*
cos(f*x + e)*sin(f*x + e))) + sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e)
 + c))/(a^2*c*f*cos(f*x + e)*sin(f*x + e) + a^2*c*f*cos(f*x + e))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}\sqrt{-c\left(\sin\left(e+fx\right)-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/((a*(sin(e + f*x) + 1))**(3/2)*sqrt(-c*(sin(e + f*x) - 1))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)^(3/2)*sqrt(-c*sin(f*x + e) + c)), x)
```

3.394 $\int \frac{1}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=143

$$\frac{\cos(e+fx)}{2af\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}-\frac{\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{3/2}}+\frac{\cos(e+fx)\tanh^{-1}(\mathrm{si}}{2acf\sqrt{a\sin(e+fx)+a}\sqrt{c}}$$

```
[Out] -Cos[e + f*x]/(2*f*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(3/2)) +
 Cos[e + f*x]/(2*a*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2)) +
 (ArcTanh[Sin[e + f*x]]*Cos[e + f*x])/(2*a*c*f*Sqrt[a + a*Sin[e + f*x]]*Sqr
t[c - c*Sin[e + f*x]])
```

Rubi [A] time = 0.278176, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {2743, 2741, 3770}

$$\frac{\cos(e+fx)}{2af\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}-\frac{\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{3/2}}+\frac{\cos(e+fx)\tanh^{-1}(\mathrm{si}}{2acf\sqrt{a\sin(e+fx)+a}\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(3/2)),x]
```

```
[Out] -Cos[e + f*x]/(2*f*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(3/2)) +
 Cos[e + f*x]/(2*a*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2)) +
 (ArcTanh[Sin[e + f*x]]*Cos[e + f*x])/(2*a*c*f*Sqrt[a + a*Sin[e + f*x]]*Sqr
t[c - c*Sin[e + f*x]])
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rule 2741

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Dist[Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]
]*Sqrt[c + d*Sin[e + f*x]]), Int[1/Cos[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}}\,dx = -\frac{\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}} + \frac{\int \frac{1}{\sqrt{a+a\sin(e+fx)}(c-c}}{a}$$

$$= -\frac{\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}} + \frac{\mathrm{cc}}{2af\sqrt{a+a\sin(e-}}$$

$$= -\frac{\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}} + \frac{\mathrm{cc}}{2af\sqrt{a+a\sin(e-}}$$

$$= -\frac{\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}} + \frac{\mathrm{cc}}{2af\sqrt{a+a\sin(e-}}$$

Mathematica [A] time = 0.608316, size = 170, normalized size = 1.19

$$\frac{\cos(e+fx)\left(-2\sin(e+fx)+\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)+\cos(2(e+fx))\left(\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}\right.\right.\right.}{4cf(\sin(e+fx)-1)(a(\sin(e+fx)+1))^{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(3/2)),x]
```

```
[Out] (Cos[e + f*x]*(Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + Cos[2*(e + f*x)]*
(Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - Log[Cos[(e + f*x)/2] + Sin[(e +
 f*x)/2]]) - Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] - 2*Sin[e + f*x]))/(4
*c*f*(-1 + Sin[e + f*x])*(a*(1 + Sin[e + f*x]))^(3/2)*Sqrt[c - c*Sin[e + f*
x]])
```

Maple [A] time = 0.15, size = 117, normalized size = 0.8

$$-\frac{\cos(fx+e)}{2f}\left(\left(\cos(fx+e)\right)^{2}\ln\left(-\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right)-\left(\cos(fx+e)\right)^{2}\ln\left(-\frac{-1+\cos(fx+e)-\sin}{\sin(fx+e)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] -1/2/f*(cos(f*x+e)^2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-cos(f*x+e)^
2*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))-sin(f*x+e))*cos(f*x+e)/(a*(1+s
in(f*x+e)))^(3/2)/(-c*(-1+sin(f*x+e)))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\sin(fx+e)+a\right)^{\frac{3}{2}}\left(-c\sin(fx+e)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2),x, algorithm="max
ima")
```

[Out] integrate(1/((a*sin(f*x + e) + a)^(3/2)*(-c*sin(f*x + e) + c)^(3/2)), x)

Fricas [A] time = 1.29499, size = 676, normalized size = 4.73

$$\left[\frac{\sqrt{ac}\cos\left(fx+e\right)^{3}\log\left(-\frac{ac\cos\left(fx+e\right)^{3}-2\,ac\cos\left(fx+e\right)-2\,\sqrt{ac}\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}\sin\left(fx+e\right)}{\cos\left(fx+e\right)^{3}}\right)+2\,\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin}}{4\,a^{2}c^{2}f\cos\left(fx+e\right)^{3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2),x, algorithm="fricas")

[Out] [1/4*(sqrt(a*c)*cos(f*x + e)^3*log(-(a*c*cos(f*x + e)^3 - 2*a*c*cos(f*x + e) - 2*sqrt(a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*sin(f*x + e))/cos(f*x + e)^3) + 2*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*sin(f*x + e))/(a^2*c^2*f*cos(f*x + e)^3), -1/2*(sqrt(-a*c)*arctan(sqrt(-a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(a*c*cos(f*x + e)*sin(f*x + e)))*cos(f*x + e)^3 - sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*sin(f*x + e))/(a^2*c^2*f*cos(f*x + e)^3)]

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sin(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(3/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2),x, algorithm="giac")

[Out] integrate(1/((a*sin(f*x + e) + a)^(3/2)*(-c*sin(f*x + e) + c)^(3/2)), x)

3.395 $\int \frac{1}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=191

$$\frac{3\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{8ac^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{3\cos(e+fx)}{8acf\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}+\frac{3\cos(e+}{8af\sqrt{a\sin(e+fx)+a}(}$$

```
[Out] -Cos[e + f*x]/(2*f*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(5/2)) +
 (3*Cos[e + f*x])/(8*a*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(5/2
)) + (3*Cos[e + f*x])/(8*a*c*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x]
)^(3/2)) + (3*ArcTanh[Sin[e + f*x]]*Cos[e + f*x])/(8*a*c^2*f*Sqrt[a + a*Sin
[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 0.374454, antiderivative size = 191, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {2743, 2741, 3770}

$$\frac{3\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{8ac^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{3\cos(e+fx)}{8acf\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}+\frac{3\cos(e+}{8af\sqrt{a\sin(e+fx)+a}(}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(5/2)),x]
```

```
[Out] -Cos[e + f*x]/(2*f*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(5/2)) +
 (3*Cos[e + f*x])/(8*a*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(5/2
)) + (3*Cos[e + f*x])/(8*a*c*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x]
)^(3/2)) + (3*ArcTanh[Sin[e + f*x]]*Cos[e + f*x])/(8*a*c^2*f*Sqrt[a + a*Sin
[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rule 2741

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Dist[Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]
]*Sqrt[c + d*Sin[e + f*x]]), Int[1/Cos[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}}\,dx = -\frac{\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}} + \frac{3\int \frac{1}{\sqrt{a+a\sin(e+fx)}(c-c\,\mathrm{si}}}{2a}$$

$$= -\frac{\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}} + \frac{3\cos(}{8af\sqrt{a+a\sin(e+f}}$$

$$= -\frac{\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}} + \frac{3\cos(}{8af\sqrt{a+a\sin(e+f}}$$

$$= -\frac{\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}} + \frac{3\cos(}{8af\sqrt{a+a\sin(e+f}}$$

$$= -\frac{\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}} + \frac{3\cos(}{8af\sqrt{a+a\sin(e+f}}$$

Mathematica [A] time = 0.77338, size = 287, normalized size = 1.5

$$\left(\cos\left(\tfrac{1}{2}(e+fx)\right)-\sin\left(\tfrac{1}{2}(e+fx)\right)\right)\left(\sin\left(\tfrac{1}{2}(e+fx)\right)+\cos\left(\tfrac{1}{2}(e+fx)\right)\right)\left(2\cos^2(e+fx)-\left(\cos\left(\tfrac{1}{2}(e+fx)\right)-\sin\left(\tfrac{1}{2}(e+\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(5/2)),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)*(2*Cos[e + f*x]^2 - (Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4 + (Cos[(e + f
*x)/2] + Sin[(e + f*x)/2])^2 - 3*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]*(
Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)^2 + 3*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]*(Cos[(e + f*x)/2] - Sin[(e
 + f*x)/2])^4*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2))/(8*f*(a*(1 + Sin[e
+ f*x]))^(3/2)*(c - c*Sin[e + f*x])^(5/2))
```

Maple [A] time = 0.167, size = 227, normalized size = 1.2

$$\frac{\cos(fx+e)}{8f}\left(3\sin(fx+e)(\cos(fx+e))^2\ln\left(-\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right)-3\ln\left(-\frac{-1+\cos(fx+e)-\sin(f}{\sin(fx+e)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] 1/8/f*(3*sin(f*x+e)*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))
-3*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))*sin(f*x+e)*cos(f*x+e)^2+2*cos
(f*x+e)^2*sin(f*x+e)-3*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+
e))+3*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))+cos(f*x+e)^2+
3*sin(f*x+e)-1)*cos(f*x+e)/(a*(1+sin(f*x+e)))^(3/2)/(-c*(-1+sin(f*x+e)))^(5
/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\sin(fx+e)+a\right)^{\frac{3}{2}}\left(-c\sin(fx+e)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2),x, algorithm="max
ima")
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)^(3/2)*(-c*sin(f*x + e) + c)^(5/2)), x)
```

Fricas [A] time = 1.38152, size = 950, normalized size = 4.97

$$\left[\frac{3\left(\cos\left(fx+e\right)^{3}\sin\left(fx+e\right)-\cos\left(fx+e\right)^{3}\right)\sqrt{ac}\log\left(-\frac{ac\cos\left(fx+e\right)^{3}-2\,ac\cos\left(fx+e\right)-2\,\sqrt{ac}\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}\sin\left(fx\right.}{\cos\left(fx+e\right)^{3}}\right.}{16\left(a^{2}c^{3}f\cos\left(fx+e\right)^{3}\sin\left(fx+e\right)-a^{2}c^{3}f\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2),x, algorithm="fri
cas")
```

```
[Out] [1/16*(3*(cos(f*x + e)^3*sin(f*x + e) - cos(f*x + e)^3)*sqrt(a*c)*log(-(a*c
*cos(f*x + e)^3 - 2*a*c*cos(f*x + e) - 2*sqrt(a*c)*sqrt(a*sin(f*x + e) + a)
*sqrt(-c*sin(f*x + e) + c)*sin(f*x + e))/cos(f*x + e)^3) - 2*(3*cos(f*x + e
)^2 + 3*sin(f*x + e) - 1)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c
))/(a^2*c^3*f*cos(f*x + e)^3*sin(f*x + e) - a^2*c^3*f*cos(f*x + e)^3), -1/8
*(3*(cos(f*x + e)^3*sin(f*x + e) - cos(f*x + e)^3)*sqrt(-a*c)*arctan(sqrt(-
a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(a*c*cos(f*x + e)*s
in(f*x + e))) + (3*cos(f*x + e)^2 + 3*sin(f*x + e) - 1)*sqrt(a*sin(f*x + e)
 + a)*sqrt(-c*sin(f*x + e) + c))/(a^2*c^3*f*cos(f*x + e)^3*sin(f*x + e) - a
^2*c^3*f*cos(f*x + e)^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2),x, algorithm="gia
c")
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)^(3/2)*(-c*sin(f*x + e) + c)^(5/2)), x)
```

3.396 $\int \frac{(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^{5/2}} \, dx$

Optimal. Leaf size=237

$$\frac{12c^4\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a\sin(e+fx)+a}}+\frac{3c^3\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{a^2f\sqrt{a\sin(e+fx)+a}}+\frac{24c^5\cos(e+fx)\log(\sin(e+fx)+1)}{a^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+$$

```
[Out] (24*c^5*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(a^2*f*Sqrt[a + a*Sin[e + f*x]]
*Sqrt[c - c*Sin[e + f*x]]) + (12*c^4*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])
/(a^2*f*Sqrt[a + a*Sin[e + f*x]]) + (3*c^3*Cos[e + f*x]*(c - c*Sin[e + f*x]
)^(3/2))/(a^2*f*Sqrt[a + a*Sin[e + f*x]]) + (2*c^2*Cos[e + f*x]*(c - c*Sin[
e + f*x])^(5/2))/(a*f*(a + a*Sin[e + f*x])^(3/2)) - (c*Cos[e + f*x]*(c - c*
Sin[e + f*x])^(7/2))/(2*f*(a + a*Sin[e + f*x])^(5/2))
```

Rubi [A] time = 0.492747, antiderivative size = 237, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2739, 2740, 2737, 2667, 31}

$$\frac{12c^4\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a\sin(e+fx)+a}}+\frac{3c^3\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{a^2f\sqrt{a\sin(e+fx)+a}}+\frac{24c^5\cos(e+fx)\log(\sin(e+fx)+1)}{a^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sin[e + f*x])^(9/2)/(a + a*Sin[e + f*x])^(5/2),x]
```


```
[Out] (24*c^5*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(a^2*f*Sqrt[a + a*Sin[e + f*x]]
*Sqrt[c - c*Sin[e + f*x]]) + (12*c^4*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])
/(a^2*f*Sqrt[a + a*Sin[e + f*x]]) + (3*c^3*Cos[e + f*x]*(c - c*Sin[e + f*x]
)^(3/2))/(a^2*f*Sqrt[a + a*Sin[e + f*x]]) + (2*c^2*Cos[e + f*x]*(c - c*Sin[
e + f*x])^(5/2))/(a*f*(a + a*Sin[e + f*x])^(3/2)) - (c*Cos[e + f*x]*(c - c*
Sin[e + f*x])^(7/2))/(2*f*(a + a*Sin[e + f*x])^(5/2))
```


Rule 2739

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(2*n + 1)), x] - Dist[(b*(2*m - 1))/(d*
(2*n + 1)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
```

```
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = -\frac{c\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{2f(a+a\sin(e+fx))^{5/2}} - \frac{(2c)\int \frac{(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{3/2}}\,dx}{a}$$

$$= \frac{2c^2\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{af(a+a\sin(e+fx))^{3/2}} - \frac{c\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{2f(a+a\sin(e+fx))^{5/2}} + \frac{\left(6c^2\right)\int \frac{(c-c\cdots}{\sqrt{a\cdots}}}{\cdots}$$

$$= \frac{3c^3\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{a^2f\sqrt{a+a\sin(e+fx)}} + \frac{2c^2\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{af(a+a\sin(e+fx))^{3/2}} - \frac{c\cos(e+\cdots}{2f(\cdots}$$

$$= \frac{12c^4\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a+a\sin(e+fx)}} + \frac{3c^3\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{a^2f\sqrt{a+a\sin(e+fx)}} + \frac{2c^2\cos(e\cdots}{af(\cdots}$$

$$= \frac{12c^4\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a+a\sin(e+fx)}} + \frac{3c^3\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{a^2f\sqrt{a+a\sin(e+fx)}} + \frac{2c^2\cos(e\cdots}{af(\cdots}$$

$$= \frac{12c^4\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a+a\sin(e+fx)}} + \frac{3c^3\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{a^2f\sqrt{a+a\sin(e+fx)}} + \frac{2c^2\cos(e\cdots}{af(\cdots}$$

$$= \frac{24c^5\cos(e+fx)\log(1+\sin(e+fx))}{a^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{12c^4\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a+a\sin(e+fx)}} + \frac{3c^3\,\mathrm{c}\cdots}{\cdots}$$

Mathematica [A] time = 5.14368, size = 202, normalized size = 0.85

$$\frac{c^4\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(320\sin(e+fx)+24\sin(3(e+fx))+\cos(4(e+fx))+\cos(\cdots\right.}{1\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sin[e + f*x])^(9/2)/(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] (c^4*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]*(273 +
Cos[4*(e + f*x)] + Cos[2*(e + f*x)]*(106 - 384*Log[Cos[(e + f*x)/2] + Sin[(
e + f*x)/2]]) + 1152*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + 320*Sin[e +
 f*x] + 1536*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]*Sin[e + f*x] + 24*Sin
[3*(e + f*x)]))/(16*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e +
```

```
f*x]))^(5/2))
```

Maple [B] time = 0.196, size = 685, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^(5/2),x)
```

```
[Out] 1/2/f*(-132+96*ln(2/(cos(f*x+e)+1))*sin(f*x+e)*cos(f*x+e)-132*sin(f*x+e)+74
*cos(f*x+e)-192*ln(2/(cos(f*x+e)+1))+58*sin(f*x+e)*cos(f*x+e)+143*cos(f*x+e
)^2+48*sin(f*x+e)*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))-cos(f*x+e)^5-73*cos(f*x
+e)^3-12*sin(f*x+e)*cos(f*x+e)^3+144*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))-192*
sin(f*x+e)*cos(f*x+e)*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))-288*cos(f*
x+e)^2*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))+96*cos(f*x+e)^3*ln(-(-1+c
os(f*x+e)-sin(f*x+e))/sin(f*x+e))-96*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x
+e))*sin(f*x+e)*cos(f*x+e)^2+sin(f*x+e)*cos(f*x+e)^4-192*cos(f*x+e)*ln(-(-1
+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))+384*sin(f*x+e)*ln(-(-1+cos(f*x+e)-sin(f
*x+e))/sin(f*x+e))+384*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))+85*cos(f*
x+e)^2*sin(f*x+e)-11*cos(f*x+e)^4-48*cos(f*x+e)^3*ln(2/(cos(f*x+e)+1))-192*
sin(f*x+e)*ln(2/(cos(f*x+e)+1))+96*cos(f*x+e)*ln(2/(cos(f*x+e)+1)))*(-c*(-1
+sin(f*x+e)))^(9/2)/(sin(f*x+e)*cos(f*x+e)^4+cos(f*x+e)^5+4*sin(f*x+e)*cos(
f*x+e)^3-5*cos(f*x+e)^4-12*cos(f*x+e)^2*sin(f*x+e)-8*cos(f*x+e)^3-8*sin(f*x
+e)*cos(f*x+e)+20*cos(f*x+e)^2+16*sin(f*x+e)+8*cos(f*x+e)-16)/(a*(1+sin(f*x
+e)))^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{9}{2}}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] integrate((-c*sin(f*x + e) + c)^(9/2)/(a*sin(f*x + e) + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(c^4\cos\left(fx+e\right)^4-8\,c^4\cos\left(fx+e\right)^2+8\,c^4+4\left(c^4\cos\left(fx+e\right)^2-2\,c^4\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-}}{3\,a^3\cos\left(fx+e\right)^2-4\,a^3+\left(a^3\cos\left(fx+e\right)^2-4\,a^3\right)\sin\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] integral(-(c^4*cos(f*x + e)^4 - 8*c^4*cos(f*x + e)^2 + 8*c^4 + 4*(c^4*cos(f
*x + e)^2 - 2*c^4)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x +
 e) + c)/(3*a^3*cos(f*x + e)^2 - 4*a^3 + (a^3*cos(f*x + e)^2 - 4*a^3)*sin(f
*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))**(9/2)/(a+a*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{9}{2}}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate((-c*sin(f*x + e) + c)^(9/2)/(a*sin(f*x + e) + a)^(5/2), x)
```

3.397 $\int \frac{(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=193

$$\frac{3c^3\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a\sin(e+fx)+a}}+\frac{6c^4\cos(e+fx)\log(\sin(e+fx)+1)}{a^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{3c^2\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2af(a\sin(e+fx)+a)^{3/2}} - c$$

```
[Out] (6*c^4*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(a^2*f*Sqrt[a + a*Sin[e + f*x]]*
Sqrt[c - c*Sin[e + f*x]]) + (3*c^3*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(
a^2*f*Sqrt[a + a*Sin[e + f*x]]) + (3*c^2*Cos[e + f*x]*(c - c*Sin[e + f*x])^
(3/2))/(2*a*f*(a + a*Sin[e + f*x])^(3/2)) - (c*Cos[e + f*x]*(c - c*Sin[e +
f*x])^(5/2))/(2*f*(a + a*Sin[e + f*x])^(5/2))
```

Rubi [A] time = 0.38682, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2739, 2740, 2737, 2667, 31}

$$\frac{3c^3\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a\sin(e+fx)+a}}+\frac{6c^4\cos(e+fx)\log(\sin(e+fx)+1)}{a^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{3c^2\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2af(a\sin(e+fx)+a)^{3/2}} - c$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sin[e + f*x])^(7/2)/(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] (6*c^4*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(a^2*f*Sqrt[a + a*Sin[e + f*x]]*
Sqrt[c - c*Sin[e + f*x]]) + (3*c^3*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(
a^2*f*Sqrt[a + a*Sin[e + f*x]]) + (3*c^2*Cos[e + f*x]*(c - c*Sin[e + f*x])^
(3/2))/(2*a*f*(a + a*Sin[e + f*x])^(3/2)) - (c*Cos[e + f*x]*(c - c*Sin[e +
f*x])^(5/2))/(2*f*(a + a*Sin[e + f*x])^(5/2))
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(2*n + 1)), x] - Dist[(b*(2*m - 1))/(d*
(2*n + 1)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
```

```
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = -\frac{c\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{2f(a+a\sin(e+fx))^{5/2}} - \frac{(3c)\int \frac{(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{3/2}}\,dx}{2a}$$

$$= \frac{3c^2\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2af(a+a\sin(e+fx))^{3/2}} - \frac{c\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{2f(a+a\sin(e+fx))^{5/2}} + \frac{\left(3c^2\right)\int \frac{(c-c \cdots}{\sqrt{a \cdots}}}{\cdots}$$

$$= \frac{3c^3\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a+a\sin(e+fx)}} + \frac{3c^2\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2af(a+a\sin(e+fx))^{3/2}} - \frac{c\cos(e+f \cdots}{2f(a+\cdots}$$

$$= \frac{3c^3\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a+a\sin(e+fx)}} + \frac{3c^2\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2af(a+a\sin(e+fx))^{3/2}} - \frac{c\cos(e+f \cdots}{2f(a+\cdots}$$

$$= \frac{3c^3\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a+a\sin(e+fx)}} + \frac{3c^2\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2af(a+a\sin(e+fx))^{3/2}} - \frac{c\cos(e+f \cdots}{2f(a+\cdots}$$

$$= \frac{6c^4\cos(e+fx)\log(1+\sin(e+fx))}{a^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{3c^3\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a+a\sin(e+fx)}} + \frac{3c^2\text{co}\cdots}{2\cdots}$$

Mathematica [A] time = 2.02332, size = 187, normalized size = 0.97

$$\frac{c^3\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin(3(e+fx))+\cos(2(e+fx))\left(4-24\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\cdots\right.\right.\right.}{4f(a(\sin(e+fx)+1\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sin[e + f*x])^(7/2)/(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] (c^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]*(28 + C
os[2*(e + f*x)]*(4 - 24*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]) + 72*Log[
Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + (41 + 96*Log[Cos[(e + f*x)/2] + Sin[
(e + f*x)/2]])*Sin[e + f*x] + Sin[3*(e + f*x)]))/(4*f*(Cos[(e + f*x)/2] - S
in[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(5/2))
```

Maple [B] time = 0.168, size = 633, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(5/2),x)
```

```
[Out] -1/f*(sin(f*x+e)*cos(f*x+e)^3-6*sin(f*x+e)*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1)
)+12*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))*sin(f*x+e)*cos(f*x+e)^2+cos
(f*x+e)^4+6*cos(f*x+e)^3*ln(2/(cos(f*x+e)+1))-12*cos(f*x+e)^3*ln(-(-1+cos(f
*x+e)-sin(f*x+e))/sin(f*x+e))-11*cos(f*x+e)^2*sin(f*x+e)-12*ln(2/(cos(f*x+e
)+1))*sin(f*x+e)*cos(f*x+e)+24*sin(f*x+e)*cos(f*x+e)*ln(-(-1+cos(f*x+e)-sin
(f*x+e))/sin(f*x+e))+10*cos(f*x+e)^3-18*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))+3
6*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))-6*sin(f*x+e)*cos(
f*x+e)+24*sin(f*x+e)*ln(2/(cos(f*x+e)+1))-48*sin(f*x+e)*ln(-(-1+cos(f*x+e)-
sin(f*x+e))/sin(f*x+e))-17*cos(f*x+e)^2-12*cos(f*x+e)*ln(2/(cos(f*x+e)+1))+
24*cos(f*x+e)*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))+16*sin(f*x+e)-10*c
os(f*x+e)+24*ln(2/(cos(f*x+e)+1))-48*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x
+e))+16)*(-c*(-1+sin(f*x+e)))^(7/2)/(sin(f*x+e)*cos(f*x+e)^3-cos(f*x+e)^4-4
*cos(f*x+e)^2*sin(f*x+e)-3*cos(f*x+e)^3-4*sin(f*x+e)*cos(f*x+e)+8*cos(f*x+e
)^2+8*sin(f*x+e)+4*cos(f*x+e)-8)/(a*(1+sin(f*x+e)))^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] integrate((-c*sin(f*x + e) + c)^(7/2)/(a*sin(f*x + e) + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(3\,c^3\cos\left(fx+e\right)^2-4\,c^3-\left(c^3\cos\left(fx+e\right)^2-4\,c^3\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}}{3\,a^3\cos\left(fx+e\right)^2-4\,a^3+\left(a^3\cos\left(fx+e\right)^2-4\,a^3\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] integral((3*c^3*cos(f*x + e)^2 - 4*c^3 - (c^3*cos(f*x + e)^2 - 4*c^3)*sin(f
*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(3*a^3*cos(f*x
+ e)^2 - 4*a^3 + (a^3*cos(f*x + e)^2 - 4*a^3)*sin(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))**(7/2)/(a+a*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate((-c*sin(f*x + e) + c)^(7/2)/(a*sin(f*x + e) + a)^(5/2), x)
```

3.398 $\int \frac{(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=143

$$\frac{c^3\cos(e+fx)\log(\sin(e+fx)+1)}{a^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af(a\sin(e+fx)+a)^{3/2}}-\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f(a\sin(e+fx)+a)^{5/2}}$$

```
[Out] (c^3*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(a^2*f*Sqrt[a + a*Sin[e + f*x]]*Sq
rt[c - c*Sin[e + f*x]]) + (c^2*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(a*f*
(a + a*Sin[e + f*x])^(3/2)) - (c*Cos[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(
2*f*(a + a*Sin[e + f*x])^(5/2))
```

Rubi [A] time = 0.302824, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2739, 2737, 2667, 31}

$$\frac{c^3\cos(e+fx)\log(\sin(e+fx)+1)}{a^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af(a\sin(e+fx)+a)^{3/2}}-\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f(a\sin(e+fx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sin[e + f*x])^(5/2)/(a + a*Sin[e + f*x])^(5/2),x]
```



```
[Out] (c^3*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(a^2*f*Sqrt[a + a*Sin[e + f*x]]*Sq
rt[c - c*Sin[e + f*x]]) + (c^2*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(a*f*
(a + a*Sin[e + f*x])^(3/2)) - (c*Cos[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(
2*f*(a + a*Sin[e + f*x])^(5/2))
```


Rule 2739

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(2*n + 1)), x] - Dist[(b*(2*m - 1))/(d*
(2*n + 1)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = -\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f(a+a\sin(e+fx))^{5/2}} - \frac{c\int \frac{(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{3/2}}\,dx}{a}$$

$$= \frac{c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af(a+a\sin(e+fx))^{3/2}} - \frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f(a+a\sin(e+fx))^{5/2}} + \frac{c^2\int \frac{\sqrt{c-c\sin(e+}}{\sqrt{a+a\sin(e+}}}{a^2}$$

$$= \frac{c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af(a+a\sin(e+fx))^{3/2}} - \frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f(a+a\sin(e+fx))^{5/2}} + \frac{(c^3\cos(e+}{a\sqrt{a+a\sin(e}}$$

$$= \frac{c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af(a+a\sin(e+fx))^{3/2}} - \frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f(a+a\sin(e+fx))^{5/2}} + \frac{(c^3\cos(e+fx)}{a^2f\sqrt{a+a}}$$

$$= \frac{c^3\cos(e+fx)\log(1+\sin(e+fx))}{a^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af(a+a\sin(e+fx))^{3/2}} - \frac{c\cos(e}{2f}$$

Mathematica [A] time = 1.18108, size = 172, normalized size = 1.2

$$\frac{c^2\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(-\cos(2(e+fx))\log\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)+3}{f(a(\sin(e+fx)+1))^{5/2}\left(\cos\left(\frac{1}{2}(e+\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sin[e + f*x])^(5/2)/(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] (c^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]*(2 + 3*
Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] - Cos[2*(e + f*x)]*Log[Cos[(e + f*
x)/2] + Sin[(e + f*x)/2]] + 4*(1 + Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]
)*Sin[e + f*x]))/(f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f
*x]))^(5/2))
```

Maple [B] time = 0.148, size = 566, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(5/2),x)
```

```
[Out] 1/f*(2*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))*sin(f*x+e)*cos(f*x+e)^2-2
*cos(f*x+e)^3*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))-sin(f*x+e)*cos(f*x
+e)^2*ln(2/(cos(f*x+e)+1))+cos(f*x+e)^3*ln(2/(cos(f*x+e)+1))+4*sin(f*x+e)*c
os(f*x+e)*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))+6*cos(f*x+e)^2*ln(-(-1
+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))-2*cos(f*x+e)^2*sin(f*x+e)-2*ln(2/(cos(f
*x+e)+1))*sin(f*x+e)*cos(f*x+e)+2*cos(f*x+e)^3-3*cos(f*x+e)^2*ln(2/(cos(f*x
+e)+1))-8*sin(f*x+e)*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))+4*cos(f*x+e
)*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))+4*sin(f*x+e)*ln(2/(cos(f*x+e)+
```

```
1))-2*cos(f*x+e)^2-2*cos(f*x+e)*ln(2/(cos(f*x+e)+1))-8*ln(-(-1+cos(f*x+e)-s
in(f*x+e))/sin(f*x+e))+2*sin(f*x+e)-2*cos(f*x+e)+4*ln(2/(cos(f*x+e)+1))+2)*
(-c*(-1+sin(f*x+e)))^(5/2)/(cos(f*x+e)^3+cos(f*x+e)^2*sin(f*x+e)-3*cos(f*x+
e)^2+2*sin(f*x+e)*cos(f*x+e)-2*cos(f*x+e)-4*sin(f*x+e)+4)/(a*(1+sin(f*x+e))
)^(5/2)
```

Maxima [A] time = 1.81496, size = 247, normalized size = 1.73

$$\frac{\frac{8\sqrt{a}c^{\frac{5}{2}}\sin\left(fx+e\right)^2}{\left(a^3+\frac{4a^3\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{6a^3\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}+\frac{4a^3\sin\left(fx+e\right)^3}{\left(\cos\left(fx+e\right)+1\right)^3}+\frac{a^3\sin\left(fx+e\right)^4}{\left(\cos\left(fx+e\right)+1\right)^4}\right)\left(\cos\left(fx+e\right)+1\right)^2}-\frac{2c^{\frac{5}{2}}\log\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+1\right)}{a^{\frac{5}{2}}}+\frac{c^{\frac{5}{2}}\log\left(\frac{\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}+1\right)}{a^{\frac{5}{2}}}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] (8*sqrt(a)*c^(5/2)*sin(f*x + e)^2/((a^3 + 4*a^3*sin(f*x + e)/(cos(f*x + e)
+ 1) + 6*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4*a^3*sin(f*x + e)^3/(co
s(f*x + e) + 1)^3 + a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4)*(cos(f*x + e)
+ 1)^2) - 2*c^(5/2)*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^(5/2) + c^(5
/2)*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/a^(5/2))/f
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(c^2\cos\left(fx+e\right)^2+2c^2\sin\left(fx+e\right)-2c^2\right)\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}}{3a^3\cos\left(fx+e\right)^2-4a^3+\left(a^3\cos\left(fx+e\right)^2-4a^3\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] integral((c^2*cos(f*x + e)^2 + 2*c^2*sin(f*x + e) - 2*c^2)*sqrt(a*sin(f*x +
 e) + a)*sqrt(-c*sin(f*x + e) + c)/(3*a^3*cos(f*x + e)^2 - 4*a^3 + (a^3*cos
(f*x + e)^2 - 4*a^3)*sin(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))**(5/2)/(a+a*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate((-c*sin(f*x + e) + c)^(5/2)/(a*sin(f*x + e) + a)^(5/2), x)
```

3.399 $\int \frac{(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=42

$$-\frac{\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{4f(a\sin(e+fx)+a)^{5/2}}$$

```
[Out] -(Cos[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(4*f*(a + a*Sin[e + f*x])^(5/2))
```

Rubi [A] time = 0.0924086, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {2742}

$$-\frac{\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{4f(a\sin(e+fx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sin[e + f*x])^(3/2)/(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] -(Cos[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(4*f*(a + a*Sin[e + f*x])^(5/2))
```

Rule 2742

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rubi steps

$$\int \frac{(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = -\frac{\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{4f(a+a\sin(e+fx))^{5/2}}$$

Mathematica [B] time = 0.467379, size = 86, normalized size = 2.05

$$\frac{c\sin(e+fx)\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}{f(a(\sin(e+fx)+1))^{5/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sin[e + f*x])^(3/2)/(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] (c*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sin[e + f*x]*Sqrt[c - c*Sin[e + f*
x]])/(f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(5/2))
```

Maple [B] time = 0.132, size = 96, normalized size = 2.3

$$-\frac{\sin\left(fx+e\right)\left(1-\cos\left(fx+e\right)+\sin\left(fx+e\right)\right)}{f\left(\sin\left(fx+e\right)\cos\left(fx+e\right)-\left(\cos\left(fx+e\right)\right)^{2}-2\sin\left(fx+e\right)-\cos\left(fx+e\right)+2\right)}\left(-c\left(-1+\sin\left(fx+e\right)\right)\right)^{\frac{3}{2}}\left(a\left(1+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2),x)
```

```
[Out] -1/f*sin(f*x+e)*(-c*(-1+sin(f*x+e)))^(3/2)*(1-cos(f*x+e)+sin(f*x+e))/(sin(f
*x+e)*cos(f*x+e)-cos(f*x+e)^2-2*sin(f*x+e)-cos(f*x+e)+2)/(a*(1+sin(f*x+e)))
^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] integrate((-c*sin(f*x + e) + c)^(3/2)/(a*sin(f*x + e) + a)^(5/2), x)
```

Fricas [B] time = 1.39044, size = 203, normalized size = 4.83

$$-\frac{\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}c\sin\left(fx+e\right)}{a^{3}f\cos\left(fx+e\right)^{3}-2\,a^{3}f\cos\left(fx+e\right)\sin\left(fx+e\right)-2\,a^{3}f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] -sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*c*sin(f*x + e)/(a^3*f*c
os(f*x + e)^3 - 2*a^3*f*cos(f*x + e)*sin(f*x + e) - 2*a^3*f*cos(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))**(3/2)/(a+a*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate((-c*sin(f*x + e) + c)^(3/2)/(a*sin(f*x + e) + a)^(5/2), x)
```

3.400 $\int \frac{\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=43

$$-\frac{c\cos(e+fx)}{2f(a\sin(e+fx)+a)^{5/2}\sqrt{c-c\sin(e+fx)}}$$

```
[Out] -(c*Cos[e + f*x])/(2*f*(a + a*Sin[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 0.0850884, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {2738}

$$-\frac{c\cos(e+fx)}{2f(a\sin(e+fx)+a)^{5/2}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c*Sin[e + f*x]]/(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] -(c*Cos[e + f*x])/(2*f*(a + a*Sin[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\int \frac{\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{5/2}}\,dx = -\frac{c\cos(e+fx)}{2f(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}$$

Mathematica [B] time = 0.213874, size = 87, normalized size = 2.02

$$-\frac{\sqrt{a(\sin(e+fx)+1)}\sqrt{c-c\sin(e+fx)}}{2a^3f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c*Sin[e + f*x]]/(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] -(Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]])/(2*a^3*f*(Cos[(e + f
*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)
```

Maple [B] time = 0.164, size = 92, normalized size = 2.1

$$-\frac{\left(\left(\cos\left(fx+e\right)\right)^2+\sin\left(fx+e\right)\cos\left(fx+e\right)+2\cos\left(fx+e\right)-3\sin\left(fx+e\right)-3\right)\sin\left(fx+e\right)}{2f\left(-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)\right)}\sqrt{-c\left(-1+\sin\left(fx+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(5/2),x)
```

```
[Out] -1/2/f*(cos(f*x+e)^2+sin(f*x+e)*cos(f*x+e)+2*cos(f*x+e)-3*sin(f*x+e)-3)*sin
(f*x+e)*(-c*(-1+sin(f*x+e)))^(1/2)/(-1+cos(f*x+e)+sin(f*x+e))/(a*(1+sin(f*x
+e)))^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c\sin\left(fx+e\right)+c}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(-c*sin(f*x + e) + c)/(a*sin(f*x + e) + a)^(5/2), x)
```

Fricas [A] time = 1.14289, size = 186, normalized size = 4.33

$$\frac{\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}}{2\left(a^{3}f\cos\left(fx+e\right)^{3}-2\,a^{3}f\cos\left(fx+e\right)\sin\left(fx+e\right)-2\,a^{3}f\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] 1/2*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(a^3*f*cos(f*x + e)^
3 - 2*a^3*f*cos(f*x + e)*sin(f*x + e) - 2*a^3*f*cos(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))**(1/2)/(a+a*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c\sin\left(fx+e\right)+c}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(-c*sin(f*x + e) + c)/(a*sin(f*x + e) + a)^(5/2), x)
```

3.401 $\int \frac{1}{(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=140

$$\frac{\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{4a^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{\cos(e+fx)}{4af(a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}}-\frac{\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}\sqrt{c-}}$$

```
[Out] -Cos[e + f*x]/(4*f*(a + a*Sin[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x]]) - C
os[e + f*x]/(4*a*f*(a + a*Sin[e + f*x])^(3/2)*Sqrt[c - c*Sin[e + f*x]]) + (
ArcTanh[Sin[e + f*x]]*Cos[e + f*x])/(4*a^2*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[
c - c*Sin[e + f*x]])
```

Rubi [A] time = 0.270781, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {2743, 2741, 3770}

$$\frac{\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{4a^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{\cos(e+fx)}{4af(a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}}-\frac{\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}\sqrt{c-}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x]]),x]
```

```
[Out] -Cos[e + f*x]/(4*f*(a + a*Sin[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x]]) - C
os[e + f*x]/(4*a*f*(a + a*Sin[e + f*x])^(3/2)*Sqrt[c - c*Sin[e + f*x]]) + (
ArcTanh[Sin[e + f*x]]*Cos[e + f*x])/(4*a^2*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[
c - c*Sin[e + f*x]])
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rule 2741

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Dist[Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]
]*Sqrt[c + d*Sin[e + f*x]]), Int[1/Cos[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}\,dx = -\frac{\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}} + \frac{\int \frac{1}{(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin}}}{2a}$$
$$= -\frac{\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}} - \frac{\cos(e}{4af(a+a\sin(e+fx))}$$
$$= -\frac{\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}} - \frac{\cos(e}{4af(a+a\sin(e+fx))}$$
$$= -\frac{\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}} - \frac{\cos(e}{4af(a+a\sin(e+fx))}$$

Mathematica [A] time = 0.628072, size = 211, normalized size = 1.51

$$\frac{\cos(e+fx)\left(-3\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)+\cos(2(e+fx))\left(\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-\mathrm{l}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x]]),x]
```

```
[Out] (Cos[e + f*x]*(-4 - 3*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + Cos[2*(e +
 f*x)]*(Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - Log[Cos[(e + f*x)/2] + S
in[(e + f*x)/2]]) + 3*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + (-2 - 4*Lo
g[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + 4*Log[Cos[(e + f*x)/2] + Sin[(e +
f*x)/2]])*Sin[e + f*x]))/(8*f*(a*(1 + Sin[e + f*x]))^(5/2)*Sqrt[c - c*Sin[e
 + f*x]])
```

Maple [B] time = 0.181, size = 252, normalized size = 1.8

$$-\frac{\cos(fx+e)}{4f}\left(-\left(\cos(fx+e)\right)^2\ln\left(-\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right)+\left(\cos(fx+e)\right)^2\ln\left(-\frac{-1+\cos(fx+e)-\mathrm{s}}{\sin(fx+e}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] -1/4/f*(-cos(f*x+e)^2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+cos(f*x+e)
^2*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))+2*sin(f*x+e)*ln(-(-1+cos(f*x+
e)+sin(f*x+e))/sin(f*x+e))-2*sin(f*x+e)*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(
f*x+e))+2*cos(f*x+e)^2-3*sin(f*x+e)+2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*
x+e))-2*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))-2)*cos(f*x+e)/(a*(1+sin(
f*x+e)))^(5/2)/(-c*(-1+sin(f*x+e)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\sin(fx+e)+a\right)^{\frac{5}{2}}\sqrt{-c\sin(fx+e)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)^(5/2)*sqrt(-c*sin(f*x + e) + c)), x)
```

Fricas [A] time = 1.32909, size = 990, normalized size = 7.07

$$\left[\frac{\left(\cos\left(fx+e\right)^{3} - 2\cos\left(fx+e\right)\sin\left(fx+e\right) - 2\cos\left(fx+e\right)\right)\sqrt{ac}\log\left(-\frac{ac\cos\left(fx+e\right)^{3} - 2\,ac\cos\left(fx+e\right) - 2\sqrt{ac}\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin}}{\cos\left(fx+e\right)^{3}}\right.}{8\left(a^{3}cf\cos\left(fx+e\right)^{3} - 2\,a^{3}cf\cos\left(fx+e\right)\sin\left(fx+e\right) - 2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2),x, algorithm="fri
cas")
```

```
[Out] [1/8*((cos(f*x + e)^3 - 2*cos(f*x + e)*sin(f*x + e) - 2*cos(f*x + e))*sqrt(
a*c)*log(-(a*c*cos(f*x + e)^3 - 2*a*c*cos(f*x + e) - 2*sqrt(a*c)*sqrt(a*sin
(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*sin(f*x + e))/cos(f*x + e)^3) + 2*
sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*(sin(f*x + e) + 2))/(a^3
*c*f*cos(f*x + e)^3 - 2*a^3*c*f*cos(f*x + e)*sin(f*x + e) - 2*a^3*c*f*cos(f
*x + e)), -1/4*((cos(f*x + e)^3 - 2*cos(f*x + e)*sin(f*x + e) - 2*cos(f*x +
 e))*sqrt(-a*c)*arctan(sqrt(-a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x
+ e) + c)/(a*c*cos(f*x + e)*sin(f*x + e))) - sqrt(a*sin(f*x + e) + a)*sqrt(
-c*sin(f*x + e) + c)*(sin(f*x + e) + 2))/(a^3*c*f*cos(f*x + e)^3 - 2*a^3*c*
f*cos(f*x + e)*sin(f*x + e) - 2*a^3*c*f*cos(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)^(5/2)*sqrt(-c*sin(f*x + e) + c)), x)
```

3.402 $\int \frac{1}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=188

$$\frac{3\cos(e+fx)}{8a^2f\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}+\frac{3\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{8a^2cf\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{3\cos(e\cdots}{8af(a\sin(e+fx)+a)^{3/\cdots}}$$

```
[Out] -Cos[e + f*x]/(4*f*(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(3/2)) -
 (3*Cos[e + f*x])/(8*a*f*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(3
/2)) + (3*Cos[e + f*x])/(8*a^2*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*
x])^(3/2)) + (3*ArcTanh[Sin[e + f*x]]*Cos[e + f*x])/(8*a^2*c*f*Sqrt[a + a*S
in[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 0.383201, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {2743, 2741, 3770}

$$\frac{3\cos(e+fx)}{8a^2f\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}+\frac{3\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{8a^2cf\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{3\cos(e\cdots}{8af(a\sin(e+fx)+a)^{3/\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(3/2)),x]
```

```
[Out] -Cos[e + f*x]/(4*f*(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(3/2)) -
 (3*Cos[e + f*x])/(8*a*f*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(3
/2)) + (3*Cos[e + f*x])/(8*a^2*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*
x])^(3/2)) + (3*ArcTanh[Sin[e + f*x]]*Cos[e + f*x])/(8*a^2*c*f*Sqrt[a + a*S
in[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rule 2741

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Dist[Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]
]*Sqrt[c + d*Sin[e + f*x]]), Int[1/Cos[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}}\,dx = -\frac{\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}} + \frac{3\int \frac{1}{(a+a\sin(e+fx))^{3/2}(c-c}}{4a}$$

$$= -\frac{\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}} - \frac{3\cos}{8af(a+a\sin(e+fx}$$

$$= -\frac{\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}} - \frac{3\cos}{8af(a+a\sin(e+fx}$$

$$= -\frac{\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}} - \frac{3\cos}{8af(a+a\sin(e+fx}$$

$$= -\frac{\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}} - \frac{3\cos}{8af(a+a\sin(e+fx}$$

Mathematica [A] time = 0.744613, size = 287, normalized size = 1.53

$$\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(-2\cos^2(e+fx)+\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(3/2)),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)*(-2*Cos[e + f*x]^2 - (Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2 + (Cos[(e +
f*x)/2] + Sin[(e + f*x)/2])^4 - 3*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]*
(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2
])^4 + 3*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]*(Cos[(e + f*x)/2] - Sin[(
e + f*x)/2])^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4))/(8*f*(a*(1 + Sin[e
 + f*x]))^(5/2)*(c - c*Sin[e + f*x])^(3/2))
```

Maple [A] time = 0.17, size = 227, normalized size = 1.2

$$-\frac{\cos(fx+e)}{8f}\left(3\sin(fx+e)(\cos(fx+e))^2\ln\left(-\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right)-3\ln\left(-\frac{-1+\cos(fx+e)-\sin(}{\sin(fx+e)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] -1/8/f*(3*sin(f*x+e)*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e)
)-3*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))*sin(f*x+e)*cos(f*x+e)^2-2*co
s(f*x+e)^2*sin(f*x+e)+3*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x
+e))-3*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))+cos(f*x+e)^2
-3*sin(f*x+e)-1)*cos(f*x+e)/(a*(1+sin(f*x+e)))^(5/2)/(-c*(-1+sin(f*x+e)))^(
3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\sin(fx+e)+a)^{\frac{5}{2}}(-c\sin(fx+e)+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2),x, algorithm="max
ima")
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)^(5/2)*(-c*sin(f*x + e) + c)^(3/2)), x)
```

Fricas [A] time = 1.35597, size = 950, normalized size = 5.05

$$\left[\frac{3\left(\cos\left(fx+e\right)^{3}\sin\left(fx+e\right)+\cos\left(fx+e\right)^{3}\right)\sqrt{ac}\log\left(-\frac{ac\cos\left(fx+e\right)^{3}-2\,ac\cos\left(fx+e\right)-2\,\sqrt{ac}\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}\sin\left(fx\right.}{\cos\left(fx+e\right)^{3}}\right.}{16\left(a^{3}c^{2}f\cos\left(fx+e\right)^{3}\sin\left(fx+e\right)+a^{3}c^{2}f\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2),x, algorithm="fri
cas")
```

```
[Out] [1/16*(3*(cos(f*x + e)^3*sin(f*x + e) + cos(f*x + e)^3)*sqrt(a*c)*log(-(a*c
*cos(f*x + e)^3 - 2*a*c*cos(f*x + e) - 2*sqrt(a*c)*sqrt(a*sin(f*x + e) + a)
*sqrt(-c*sin(f*x + e) + c)*sin(f*x + e))/cos(f*x + e)^3) - 2*(3*cos(f*x + e
)^2 - 3*sin(f*x + e) - 1)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c
))/(a^3*c^2*f*cos(f*x + e)^3*sin(f*x + e) + a^3*c^2*f*cos(f*x + e)^3), -1/8
*(3*(cos(f*x + e)^3*sin(f*x + e) + cos(f*x + e)^3)*sqrt(-a*c)*arctan(sqrt(-
a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(a*c*cos(f*x + e)*s
in(f*x + e))) + (3*cos(f*x + e)^2 - 3*sin(f*x + e) - 1)*sqrt(a*sin(f*x + e)
 + a)*sqrt(-c*sin(f*x + e) + c))/(a^3*c^2*f*cos(f*x + e)^3*sin(f*x + e) + a
^3*c^2*f*cos(f*x + e)^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2),x, algorithm="gia
c")
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)^(5/2)*(-c*sin(f*x + e) + c)^(3/2)), x)
```

3.403 $\int \frac{1}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=236

$$\frac{3\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{8a^2c^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{3\cos(e+fx)}{8a^2cf\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}+\frac{3\cos(e+}{8a^2f\sqrt{a\sin(e+fx)+a}(c}$$

```
[Out] -Cos[e + f*x]/(4*f*(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(5/2)) -
 Cos[e + f*x]/(2*a*f*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(5/2))
 + (3*Cos[e + f*x])/(8*a^2*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^
(5/2)) + (3*Cos[e + f*x])/(8*a^2*c*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e
+ f*x])^(3/2)) + (3*ArcTanh[Sin[e + f*x]]*Cos[e + f*x])/(8*a^2*c^2*f*Sqrt[a
 + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 0.478639, antiderivative size = 236, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {2743, 2741, 3770}

$$\frac{3\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{8a^2c^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{3\cos(e+fx)}{8a^2cf\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}+\frac{3\cos(e+}{8a^2f\sqrt{a\sin(e+fx)+a}(c}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(5/2)),x]
```


```
[Out] -Cos[e + f*x]/(4*f*(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(5/2)) -
 Cos[e + f*x]/(2*a*f*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(5/2))
 + (3*Cos[e + f*x])/(8*a^2*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^
(5/2)) + (3*Cos[e + f*x])/(8*a^2*c*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e
+ f*x])^(3/2)) + (3*ArcTanh[Sin[e + f*x]]*Cos[e + f*x])/(8*a^2*c^2*f*Sqrt[a
 + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```


Rule 2743

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rule 2741

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Dist[Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]
]*Sqrt[c + d*Sin[e + f*x]]), Int[1/Cos[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}}\,dx = -\frac{\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}} + \frac{\int \frac{1}{(a+a\sin(e+fx))^{3/2}(c}}{a}$$

$$= -\frac{\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}} - \frac{c}{2af(a+a\sin(e+}$$

$$= -\frac{\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}} - \frac{c}{2af(a+a\sin(e+}$$

$$= -\frac{\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}} - \frac{c}{2af(a+a\sin(e+}$$

$$= -\frac{\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}} - \frac{c}{2af(a+a\sin(e+}$$

$$= -\frac{\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}} - \frac{c}{2af(a+a\sin(e+}$$

Mathematica [A] time = 0.999232, size = 237, normalized size = 1.

$$\sec^3(e+fx)\left(22\sin(e+fx)+6\sin(3(e+fx))-9\log\left(\cos\left(\tfrac{1}{2}(e+fx)\right)-\sin\left(\tfrac{1}{2}(e+fx)\right)\right)-12\cos(2(e+fx))\left(\log\left(c\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(5/2)),x]
```

```
[Out] (Sec[e + f*x]^3*(-9*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - 12*Cos[2*(e
+ f*x)]*(Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - Log[Cos[(e + f*x)/2] +
Sin[(e + f*x)/2]]) - 3*Cos[4*(e + f*x)]*(Log[Cos[(e + f*x)/2] - Sin[(e + f*
x)/2]] - Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]) + 9*Log[Cos[(e + f*x)/2]
 + Sin[(e + f*x)/2]] + 22*Sin[e + f*x] + 6*Sin[3*(e + f*x)]))/(64*a^2*c^2*f
*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]])
```

Maple [A] time = 0.171, size = 134, normalized size = 0.6

$$\frac{\cos(fx+e)}{8f}\left(-3\left(\cos(fx+e)\right)^4\ln\left(-\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right)+3\left(\cos(fx+e)\right)^4\ln\left(-\frac{-1+\cos(fx+e)}{\sin(fx}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] 1/8/f*(-3*cos(f*x+e)^4*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+3*cos(f*x
+e)^4*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))+3*cos(f*x+e)^2*sin(f*x+e)+
2*sin(f*x+e))*cos(f*x+e)/(a*(1+sin(f*x+e)))^(5/2)/(-c*(-1+sin(f*x+e)))^(5/2
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\sin(fx+e)+a\right)^{\frac{5}{2}}\left(-c\sin(fx+e)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2),x, algorithm="max
ima")
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)^(5/2)*(-c*sin(f*x + e) + c)^(5/2)), x)
```

Fricas [A] time = 1.32375, size = 745, normalized size = 3.16

$$\left[\frac{3\sqrt{ac}\cos\left(fx+e\right)^{5}\log\left(-\frac{ac\cos\left(fx+e\right)^{3}-2\,ac\cos\left(fx+e\right)-2\,\sqrt{ac}\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}\sin\left(fx+e\right)}{\cos\left(fx+e\right)^{3}}\right)+2\left(3\,\cos\left(fx+e\right)^{2}+2\right)\sqrt{a}}{16\,a^{3}c^{3}f\cos\left(fx+e\right)^{5}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2),x, algorithm="fri
cas")
```

```
[Out] [1/16*(3*sqrt(a*c)*cos(f*x + e)^5*log(-(a*c*cos(f*x + e)^3 - 2*a*c*cos(f*x
+ e) - 2*sqrt(a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*sin(f
*x + e))/cos(f*x + e)^3) + 2*(3*cos(f*x + e)^2 + 2)*sqrt(a*sin(f*x + e) + a
)*sqrt(-c*sin(f*x + e) + c)*sin(f*x + e))/(a^3*c^3*f*cos(f*x + e)^5), -1/8*
(3*sqrt(-a*c)*arctan(sqrt(-a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x +
e) + c)/(a*c*cos(f*x + e)*sin(f*x + e)))*cos(f*x + e)^5 - (3*cos(f*x + e)^2
 + 2)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*sin(f*x + e))/(a^3
*c^3*f*cos(f*x + e)^5)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2),x, algorithm="gia
c")
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)^(5/2)*(-c*sin(f*x + e) + c)^(5/2)), x)
```

3.404 $\int (a + a\sin(e + fx))^m (c - c\sin(e + fx))^n \, dx$

Optimal. Leaf size=110

$$\frac{c2^{n+\frac{1}{2}}\cos(e+fx)(1-\sin(e+fx))^{\frac{1}{2}-n}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{n-1}\,{}_2F_1\left(\frac{1}{2}(2m+1),\frac{1}{2}(1-2n);\frac{1}{2}(2m+3);\right.}{f(2m+1)}$$

```
[Out] (2^(1/2 + n)*c*Cos[e + f*x]*Hypergeometric2F1[(1 + 2*m)/2, (1 - 2*n)/2, (3
+ 2*m)/2, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*x])^(1/2 - n)*(a + a*Sin[e +
 f*x])^m*(c - c*Sin[e + f*x])^(-1 + n))/(f*(1 + 2*m))
```

Rubi [A] time = 0.154717, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2745, 2689, 70, 69}

$$\frac{c2^{n+\frac{1}{2}}\cos(e+fx)(1-\sin(e+fx))^{\frac{1}{2}-n}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{n-1}\,{}_2F_1\left(\frac{1}{2}(2m+1),\frac{1}{2}(1-2n);\frac{1}{2}(2m+3);\right.}{f(2m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^n,x]
```

```
[Out] (2^(1/2 + n)*c*Cos[e + f*x]*Hypergeometric2F1[(1 + 2*m)/2, (1 - 2*n)/2, (3
+ 2*m)/2, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*x])^(1/2 - n)*(a + a*Sin[e +
 f*x])^m*(c - c*Sin[e + f*x])^(-1 + n))/(f*(1 + 2*m))
```

Rule 2745

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e
 + f*x])^FracPart[m]*(c + d*Sin[e + f*x])^FracPart[m])/Cos[e + f*x]^(2*Frac
Part[m]), Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; Fr
eeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
&& (FractionQ[m] ||  !FractionQ[n])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
```

```
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (a + a\sin(e+fx))^m (c - c\sin(e+fx))^n \, dx = \left(\cos^{-2m}(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^m\right)\int \cos^{2m}(e+fx$$

$$= \frac{\left(c^2\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{\frac{1}{2}(-1-2m)+m}(c+c\sin\right.}{}$$

$$= \frac{\left(2^{-\frac{1}{2}+n}c^2\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-\frac{1}{2}+\frac{1}{2}(-1-2m)+m}\right.}{}$$

$$= \frac{2^{\frac{1}{2}+n}c\cos(e+fx)\,{}_2F_1\left(\frac{1}{2}(1+2m),\frac{1}{2}(1-2n);\frac{1}{2}(3+2m);\frac{1}{2}(1+\sin(e+fx\right.}{f(1+2m}$$

Mathematica [C] time = 3.07898, size = 365, normalized size = 3.32

$$\frac{4(2n+3)\sin\left(\frac{1}{8}(2e+2fx-\pi\right.}{f(2n+1)\left((2n+3)\cos^2\left(\frac{1}{8}(2e+2fx-\pi)\right)F_1\left(n+\frac{1}{2};-2m,2(m+n)+1;n+\frac{3}{2};\tan^2\left(\frac{1}{8}(-2e-2fx+\pi)\right),-\tan^2\left(\frac{1}{8}(2e\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^n,x]
```

```
[Out] (4*(3 + 2*n)*AppellF1[1/2 + n, -2*m, 1 + 2*(m + n), 3/2 + n, Tan[(-2*e + Pi
 - 2*f*x)/8]^2, -Tan[(2*e - Pi + 2*f*x)/8]^2]*Cos[(2*e - Pi + 2*f*x)/8]^3*(
a*(1 + Sin[e + f*x]))^m*(c - c*Sin[e + f*x])^n*Sin[(2*e - Pi + 2*f*x)/8])/(
f*(1 + 2*n)*((3 + 2*n)*AppellF1[1/2 + n, -2*m, 1 + 2*(m + n), 3/2 + n, Tan[
(-2*e + Pi - 2*f*x)/8]^2, -Tan[(2*e - Pi + 2*f*x)/8]^2]*Cos[(2*e - Pi + 2*f
*x)/8]^2 - 2*(2*m*AppellF1[3/2 + n, 1 - 2*m, 1 + 2*(m + n), 5/2 + n, Tan[(-
2*e + Pi - 2*f*x)/8]^2, -Tan[(2*e - Pi + 2*f*x)/8]^2] + (1 + 2*m + 2*n)*App
ellF1[3/2 + n, -2*m, 2*(1 + m + n), 5/2 + n, Tan[(-2*e + Pi - 2*f*x)/8]^2,
-Tan[(2*e - Pi + 2*f*x)/8]^2])*Sin[(2*e - Pi + 2*f*x)/8]^2))
```

Maple [F] time = 0.95, size = 0, normalized size = 0.

$$\int \left(a + a\sin\left(fx+e\right)\right)^m \left(c - c\sin\left(fx+e\right)\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right) + a\right)^m \left(-c\sin\left(fx+e\right) + c\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\left(\sin\left(e+fx\right)+1\right)\right)^{m}\left(-c\left(\sin\left(e+fx\right)-1\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**n,x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**m*(-c*(sin(e + f*x) - 1))**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^n, x)
```

3.405 $\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^3\,dx$

Optimal. Leaf size=86

$$-\frac{a^4c^3 2^{m+\frac{1}{2}}\cos^7(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^{m-4}\,{}_2F_1\left(\frac{7}{2},\frac{1}{2}-m;\frac{9}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{7f}$$

```
[Out] -(2^(1/2 + m)*a^4*c^3*Cos[e + f*x]^7*Hypergeometric2F1[7/2, 1/2 - m, 9/2, (
1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(1/2 - m)*(a + a*Sin[e + f*x])^(-4
+ m))/(7*f)
```

Rubi [A] time = 0.142781, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2736, 2689, 70, 69}

$$-\frac{a^4c^3 2^{m+\frac{1}{2}}\cos^7(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^{m-4}\,{}_2F_1\left(\frac{7}{2},\frac{1}{2}-m;\frac{9}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{7f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^3,x]
```

```
[Out] -(2^(1/2 + m)*a^4*c^3*Cos[e + f*x]^7*Hypergeometric2F1[7/2, 1/2 - m, 9/2, (
1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(1/2 - m)*(a + a*Sin[e + f*x])^(-4
+ m))/(7*f)
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
```

```
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^3\,dx = \left(a^3c^3\right)\int \cos^6(e+fx)(a+a\sin(e+fx))^{-3+m}\,dx$$
$$= \frac{\left(a^5c^3\cos^7(e+fx)\right)\text{Subst}\left(\int (a-ax)^{5/2}(a+ax)^{-\frac{1}{2}+m}\,dx,x,\sin(e+fx\right.}{f(a-a\sin(e+fx))^{7/2}(a+a\sin(e+fx))^{7/2}}$$
$$= \frac{\left(2^{-\frac{1}{2}+m}a^5c^3\cos^7(e+fx)(a+a\sin(e+fx))^{-4+m}\left(\frac{a+a\sin(e+fx)}{a}\right)^{\frac{1}{2}-m}\right)\text{Su}}{f(a-a\sin(e+fx))^{7/}}$$
$$= -\frac{2^{\frac{1}{2}+m}a^4c^3\cos^7(e+fx)\,{}_2F_1\left(\frac{7}{2},\frac{1}{2}-m;\frac{9}{2};\frac{1}{2}(1-\sin(e+fx))\right)(1+\sin(}{7f}$$

Mathematica [F] time = 180.047, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^3,x]
```

```
[Out] $Aborted
```

Maple [F] time = 2.334, size = 0, normalized size = 0.

$$\int \left(a+a\sin\left(fx+e\right)\right)^{m}\left(c-c\sin\left(fx+e\right)\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^3,x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \left(c\sin\left(fx+e\right)-c\right)^{3}\left(a\sin\left(fx+e\right)+a\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] -integrate((c*sin(f*x + e) - c)^3*(a*sin(f*x + e) + a)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(3\,c^3\cos\left(fx+e\right)^2-4\,c^3-\left(c^3\cos\left(fx+e\right)^2-4\,c^3\right)\sin\left(fx+e\right)\right)\left(a\sin\left(fx+e\right)+a\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] integral(-(3*c^3*cos(f*x + e)^2 - 4*c^3 - (c^3*cos(f*x + e)^2 - 4*c^3)*sin(
f*x + e))*(a*sin(f*x + e) + a)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\left(c\sin\left(fx+e\right)-c\right)^3\left(a\sin\left(fx+e\right)+a\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate(-(c*sin(f*x + e) - c)^3*(a*sin(f*x + e) + a)^m, x)
```

3.406 $\int (a + a\sin(e + fx))^m (c - c\sin(e + fx))^2 \, dx$

Optimal. Leaf size=86

$$-\frac{a^3 c^2 2^{m+\frac{1}{2}} \cos^5(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^{m-3} {}_2F_1\left(\frac{5}{2},\frac{1}{2}-m;\frac{7}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{5f}$$

```
[Out] -(2^(1/2 + m)*a^3*c^2*Cos[e + f*x]^5*Hypergeometric2F1[5/2, 1/2 - m, 7/2, (
1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(1/2 - m)*(a + a*Sin[e + f*x])^(-3
+ m))/(5*f)
```

Rubi [A] time = 0.139956, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2736, 2689, 70, 69}

$$-\frac{a^3 c^2 2^{m+\frac{1}{2}} \cos^5(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^{m-3} {}_2F_1\left(\frac{5}{2},\frac{1}{2}-m;\frac{7}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^2,x]
```

```
[Out] -(2^(1/2 + m)*a^3*c^2*Cos[e + f*x]^5*Hypergeometric2F1[5/2, 1/2 - m, 7/2, (
1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(1/2 - m)*(a + a*Sin[e + f*x])^(-3
+ m))/(5*f)
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
```

```
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^2\,dx = \left(a^2c^2\right)\int \cos^4(e+fx)(a+a\sin(e+fx))^{-2+m}\,dx$$

$$= \frac{\left(a^4c^2\cos^5(e+fx)\right)\text{Subst}\left(\int (a-ax)^{3/2}(a+ax)^{-\frac{1}{2}+m}\,dx,x,\sin(e+fx)\right)}{f(a-a\sin(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}$$

$$= \frac{\left(2^{-\frac{1}{2}+m}a^4c^2\cos^5(e+fx)(a+a\sin(e+fx))^{-3+m}\left(\frac{a+a\sin(e+fx)}{a}\right)^{\frac{1}{2}-m}\right)\text{Subst}}{f(a-a\sin(e+fx))^{5/2}}$$

$$= -\frac{2^{\frac{1}{2}+m}a^3c^2\cos^5(e+fx)\,{}_2F_1\left(\frac{5}{2},\frac{1}{2}-m;\frac{7}{2};\frac{1}{2}(1-\sin(e+fx))\right)(1+\sin(e+}{5f}$$

Mathematica [C] time = 155.36, size = 88512, normalized size = 1029.21

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^2,x]
```

```
[Out] Result too large to show
```

Maple [F] time = 2.758, size = 0, normalized size = 0.

$$\int \left(a+a\sin\left(fx+e\right)\right)^m\left(c-c\sin\left(fx+e\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^2,x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c\sin\left(fx+e\right)-c\right)^2\left(a\sin\left(fx+e\right)+a\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate((c*sin(f*x + e) - c)^2*(a*sin(f*x + e) + a)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(c^2\cos\left(fx+e\right)^2+2\,c^2\sin\left(fx+e\right)-2\,c^2\right)\left(a\sin\left(fx+e\right)+a\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral(-(c^2*cos(f*x + e)^2 + 2*c^2*sin(f*x + e) - 2*c^2)*(a*sin(f*x + e)
 + a)^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$c^2\left(\int -2\left(a\sin\left(e+fx\right)+a\right)^m\sin\left(e+fx\right)dx+\int\left(a\sin\left(e+fx\right)+a\right)^m\sin^2\left(e+fx\right)dx+\int\left(a\sin\left(e+fx\right)+a\right)^m d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**2,x)
```

```
[Out] c**2*(Integral(-2*(a*sin(e + f*x) + a)**m*sin(e + f*x), x) + Integral((a*si
n(e + f*x) + a)**m*sin(e + f*x)**2, x) + Integral((a*sin(e + f*x) + a)**m,
x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(c\sin\left(fx+e\right)-c\right)^2\left(a\sin\left(fx+e\right)+a\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((c*sin(f*x + e) - c)^2*(a*sin(f*x + e) + a)^m, x)
```

3.407 $\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))\,dx$

Optimal. Leaf size=84

$$-\frac{a^2c2^{m+\frac{1}{2}}\cos^3(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^{m-2}\,{}_2F_1\left(\frac{3}{2},\frac{1}{2}-m;\frac{5}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{3f}$$

```
[Out] -(2^(1/2 + m)*a^2*c*Cos[e + f*x]^3*Hypergeometric2F1[3/2, 1/2 - m, 5/2, (1
- Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(1/2 - m)*(a + a*Sin[e + f*x])^(-2 +
m))/(3*f)
```

Rubi [A] time = 0.112976, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2736, 2689, 70, 69}

$$-\frac{a^2c2^{m+\frac{1}{2}}\cos^3(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^{m-2}\,{}_2F_1\left(\frac{3}{2},\frac{1}{2}-m;\frac{5}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x]),x]
```

```
[Out] -(2^(1/2 + m)*a^2*c*Cos[e + f*x]^3*Hypergeometric2F1[3/2, 1/2 - m, 5/2, (1
- Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(1/2 - m)*(a + a*Sin[e + f*x])^(-2 +
m))/(3*f)
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && !In
tegerQ[m] && !IntegerQ[n] && (RationalQ[m] || !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
```

```
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (a + a\sin(e + fx))^m (c - c\sin(e + fx))\,dx = (ac)\int \cos^2(e + fx)(a + a\sin(e + fx))^{-1+m}\,dx$$
$$= \frac{\left(a^3 c\cos^3(e + fx)\right)\operatorname{Subst}\left(\int \sqrt{a - ax}(a + ax)^{-\frac{1}{2}+m}\,dx, x, \sin(e + fx)\right)}{f(a - a\sin(e + fx))^{3/2}(a + a\sin(e + fx))^{3/2}}$$
$$= \frac{\left(2^{-\frac{1}{2}+m} a^3 c\cos^3(e + fx)(a + a\sin(e + fx))^{-2+m}\left(\frac{a+a\sin(e+fx)}{a}\right)^{\frac{1}{2}-m}\operatorname{Subs}\right.}{f(a - a\sin(e + fx))^{3/2}}$$
$$= -\frac{2^{\frac{1}{2}+m} a^2 c\cos^3(e + fx)\,{}_2F_1\left(\frac{3}{2}, \frac{1}{2} - m; \frac{5}{2}; \frac{1}{2}(1 - \sin(e + fx))\right)(1 + \sin(e}{3f}$$

Mathematica [C] time = 1.65244, size = 261, normalized size = 3.11

$$-\frac{(-1)^{3/4} c 2^{-2m-1} e^{-\frac{3}{2}i(e+fx)}\left(-(-1)^{3/4} e^{-\frac{1}{2}i(e+fx)}\left(e^{i(e+fx)} + i\right)\right)^{2m+1}(\sin(e + fx) - 1)\left((m - 1)m e^{2i(e+fx)}\,{}_2F_1\left(1, m; -m; -ie^{-i(}\right.\right.}{f(m - 1)n}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x]),x]
```

```
[Out] -((((-1)^(3/4)*2^(-1 - 2*m)*c*(-(((-1)^(3/4)*(I + E^(I*(e + f*x))))/E^((I/2)
*(e + f*x))))^(1 + 2*m)*(E^((2*I)*(e + f*x))*(-1 + m)*m*Hypergeometric2F1[1
, m, -m, (-I)/E^(I*(e + f*x))] - (1 + m)*(2*E^(I*(e + f*x))*(-1 + m)*Hyperg
eometric2F1[1, 1 + m, 1 - m, (-I)/E^(I*(e + f*x))] - m*Hypergeometric2F1[1,
 2 + m, 2 - m, (-I)/E^(I*(e + f*x))]))*(-1 + Sin[e + f*x])*(a*(1 + Sin[e +
f*x]))^m)/(E^(((3*I)/2)*(e + f*x))*f*(-1 + m)*m*(1 + m)*(Cos[(e + f*x)/2] -
 Sin[(e + f*x)/2])^2*Sin[(2*e + Pi + 2*f*x)/4]^(2*m)))
```

Maple [F] time = 0.977, size = 0, normalized size = 0.

$$\int \left(a + a\sin\left(fx + e\right)\right)^m \left(c - c\sin\left(fx + e\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e)),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \left(c\sin\left(fx + e\right) - c\right)\left(a\sin\left(fx + e\right) + a\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] -integrate((c*sin(f*x + e) - c)*(a*sin(f*x + e) + a)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(c\sin\left(fx+e\right)-c\right)\left(a\sin\left(fx+e\right)+a\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral(-(c*sin(f*x + e) - c)*(a*sin(f*x + e) + a)^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-c\left(\int\left(a\sin\left(e+fx\right)+a\right)^{m}\sin\left(e+fx\right)dx+\int-\left(a\sin\left(e+fx\right)+a\right)^{m}dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(c-c*sin(f*x+e)),x)
```

```
[Out] -c*(Integral((a*sin(e + f*x) + a)**m*sin(e + f*x), x) + Integral(-(a*sin(e
+ f*x) + a)**m, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int-\left(c\sin\left(fx+e\right)-c\right)\left(a\sin\left(fx+e\right)+a\right)^{m}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate(-(c*sin(f*x + e) - c)*(a*sin(f*x + e) + a)^m, x)
```

3.408 $\int \frac{(a+a\sin(e+fx))^m}{c-c\sin(e+fx)}\,dx$

Optimal. Leaf size=76

$$\frac{2^{m+\frac{1}{2}}\sec(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^m\,{}_2F_1\left(-\frac{1}{2},\frac{1}{2}-m;\frac{1}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{cf}$$

```
[Out] (2^(1/2 + m)*Hypergeometric2F1[-1/2, 1/2 - m, 1/2, (1 - Sin[e + f*x])/2]*Se
c[e + f*x]*(1 + Sin[e + f*x])^(1/2 - m)*(a + a*Sin[e + f*x])^m)/(c*f)
```

Rubi [A] time = 0.132706, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2736, 2689, 70, 69}

$$\frac{2^{m+\frac{1}{2}}\sec(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^m\,{}_2F_1\left(-\frac{1}{2},\frac{1}{2}-m;\frac{1}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{cf}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m/(c - c*Sin[e + f*x]),x]
```

```
[Out] (2^(1/2 + m)*Hypergeometric2F1[-1/2, 1/2 - m, 1/2, (1 - Sin[e + f*x])/2]*Se
c[e + f*x]*(1 + Sin[e + f*x])^(1/2 - m)*(a + a*Sin[e + f*x])^m)/(c*f)
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && !In
tegerQ[m] && !IntegerQ[n] && (RationalQ[m] || !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
```

```
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^m}{c-c\sin(e+fx)}\,dx = \frac{\int \sec^2(e+fx)(a+a\sin(e+fx))^{1+m}\,dx}{ac}$$

$$= \frac{\left(a\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}\right)\operatorname{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}}{(a-ax)^{3/2}}\,dx, x, \sin(e+fx)\right.}{cf}$$

$$= \frac{\left(2^{-\frac{1}{2}+m}a\sec(e+fx)\sqrt{a-a\sin(e+fx)}(a+a\sin(e+fx))^m\left(\frac{a+a\sin(e+fx)}{a}\right)^{\frac{1}{2}-m}\right)\operatorname{Subst}\left(\int \frac{\left(\frac{1}{2}\right.}{(a}\right.}{cf}$$

$$= \frac{2^{\frac{1}{2}+m}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{2}-m;\frac{1}{2};\frac{1}{2}(1-\sin(e+fx))\right)\sec(e+fx)(1+\sin(e+fx))^{\frac{1}{2}-m}(a+a\sin(e+f}{cf}$$

Mathematica [C] time = 15.8283, size = 3844, normalized size = 50.58

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m/(c - c*Sin[e + f*x]),x]
```

```
[Out] -((Cos[(-e + Pi/2 - f*x)/4]^2)^(2*m)*Cot[(-e + Pi/2 - f*x)/4]*(Cos[(e + f*x
)/2] - Sin[(e + f*x)/2])^2*(a + a*Sin[e + f*x])^m*(-(AppellF1[-1/2, -2*m, 2
*m, 1/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*(Sec[(-e
+ Pi/2 - f*x)/4]^2)^(2*m)) + (3*AppellF1[1/2, -2*m, 2*m, 3/2, Tan[(-e + Pi/
2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Tan[(-e + Pi/2 - f*x)/4]^2*(1 -
 Tan[(-e + Pi/2 - f*x)/4]^2)^(2*m))/(3*AppellF1[1/2, -2*m, 2*m, 3/2, Tan[(-
e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - 4*m*(AppellF1[3/2, 1 -
 2*m, 2*m, 5/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] +
AppellF1[3/2, -2*m, 1 + 2*m, 5/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi
/2 - f*x)/4]^2])*Tan[(-e + Pi/2 - f*x)/4]^2)))/(2*f*(c - c*Sin[e + f*x])*(C
os[Pi/4 + (e - Pi/2 + f*x)/2] - Sin[Pi/4 + (e - Pi/2 + f*x)/2])^2*(-(m*(Cos
[(-e + Pi/2 - f*x)/4]^2)^(2*m)*(-(AppellF1[-1/2, -2*m, 2*m, 1/2, Tan[(-e +
Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*(Sec[(-e + Pi/2 - f*x)/4]^2)
^(2*m)) + (3*AppellF1[1/2, -2*m, 2*m, 3/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan
[(-e + Pi/2 - f*x)/4]^2]*Tan[(-e + Pi/2 - f*x)/4]^2*(1 - Tan[(-e + Pi/2 - f
*x)/4]^2)^(2*m))/(3*AppellF1[1/2, -2*m, 2*m, 3/2, Tan[(-e + Pi/2 - f*x)/4]^
2, -Tan[(-e + Pi/2 - f*x)/4]^2] - 4*m*(AppellF1[3/2, 1 - 2*m, 2*m, 5/2, Tan
[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + AppellF1[3/2, -2*m,
 1 + 2*m, 5/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2])*Ta
n[(-e + Pi/2 - f*x)/4]^2)))/2 - ((Cos[(-e + Pi/2 - f*x)/4]^2)^(2*m)*Csc[(-e
 + Pi/2 - f*x)/4]^2*(-(AppellF1[-1/2, -2*m, 2*m, 1/2, Tan[(-e + Pi/2 - f*x)
/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*(Sec[(-e + Pi/2 - f*x)/4]^2)^(2*m)) + (
3*AppellF1[1/2, -2*m, 2*m, 3/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2
 - f*x)/4]^2]*Tan[(-e + Pi/2 - f*x)/4]^2*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(
2*m))/(3*AppellF1[1/2, -2*m, 2*m, 3/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e
 + Pi/2 - f*x)/4]^2] - 4*m*(AppellF1[3/2, 1 - 2*m, 2*m, 5/2, Tan[(-e + Pi/2
 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + AppellF1[3/2, -2*m, 1 + 2*m, 5
/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2])*Tan[(-e + Pi/
2 - f*x)/4]^2)))/8 + ((Cos[(-e + Pi/2 - f*x)/4]^2)^(2*m)*Cot[(-e + Pi/2 - f
*x)/4]*(-(m*AppellF1[-1/2, -2*m, 2*m, 1/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan
```

```
[(-e + Pi/2 - f*x)/4]^2]*(Sec[(-e + Pi/2 - f*x)/4]^2)^(2*m)*Tan[(-e + Pi/2
- f*x)/4]) - (Sec[(-e + Pi/2 - f*x)/4]^2)^(2*m)*(m*AppellF1[1/2, 1 - 2*m, 2
*m, 3/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e +
 Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4] + m*AppellF1[1/2, -2*m, 1 + 2*m,
 3/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi
/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4]) + (3*AppellF1[1/2, -2*m, 2*m, 3/2,
 Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 -
f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4]*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(2*m))/
(2*(3*AppellF1[1/2, -2*m, 2*m, 3/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e +
Pi/2 - f*x)/4]^2] - 4*m*(AppellF1[3/2, 1 - 2*m, 2*m, 5/2, Tan[(-e + Pi/2 -
f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + AppellF1[3/2, -2*m, 1 + 2*m, 5/2,
 Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2])*Tan[(-e + Pi/2 -
 f*x)/4]^2)) + (3*Tan[(-e + Pi/2 - f*x)/4]^2*(-(m*AppellF1[3/2, 1 - 2*m, 2*
m, 5/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e +
Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4])/3 - (m*AppellF1[3/2, -2*m, 1 + 2
*m, 5/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e +
 Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4])/3)*(1 - Tan[(-e + Pi/2 - f*x)/4
]^2)^(2*m))/(3*AppellF1[1/2, -2*m, 2*m, 3/2, Tan[(-e + Pi/2 - f*x)/4]^2, -T
an[(-e + Pi/2 - f*x)/4]^2] - 4*m*(AppellF1[3/2, 1 - 2*m, 2*m, 5/2, Tan[(-e
+ Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + AppellF1[3/2, -2*m, 1 +
2*m, 5/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2])*Tan[(-e
 + Pi/2 - f*x)/4]^2) - (3*m*AppellF1[1/2, -2*m, 2*m, 3/2, Tan[(-e + Pi/2 -
f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e
+ Pi/2 - f*x)/4]^3*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(-1 + 2*m))/(3*AppellF1
[1/2, -2*m, 2*m, 3/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]
^2] - 4*m*(AppellF1[3/2, 1 - 2*m, 2*m, 5/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Ta
n[(-e + Pi/2 - f*x)/4]^2] + AppellF1[3/2, -2*m, 1 + 2*m, 5/2, Tan[(-e + Pi/
2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2])*Tan[(-e + Pi/2 - f*x)/4]^2) -
(3*AppellF1[1/2, -2*m, 2*m, 3/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/
2 - f*x)/4]^2]*Tan[(-e + Pi/2 - f*x)/4]^2*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^
(2*m)*(-2*m*(AppellF1[3/2, 1 - 2*m, 2*m, 5/2, Tan[(-e + Pi/2 - f*x)/4]^2, -
Tan[(-e + Pi/2 - f*x)/4]^2] + AppellF1[3/2, -2*m, 1 + 2*m, 5/2, Tan[(-e + P
i/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2])*Sec[(-e + Pi/2 - f*x)/4]^2*T
an[(-e + Pi/2 - f*x)/4] + 3*(-(m*AppellF1[3/2, 1 - 2*m, 2*m, 5/2, Tan[(-e +
 Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*
Tan[(-e + Pi/2 - f*x)/4])/3 - (m*AppellF1[3/2, -2*m, 1 + 2*m, 5/2, Tan[(-e
+ Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2
*Tan[(-e + Pi/2 - f*x)/4])/3) - 4*m*Tan[(-e + Pi/2 - f*x)/4]^2*((-6*m*Appel
lF1[5/2, 1 - 2*m, 1 + 2*m, 7/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2
 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4])/5 + (3*(
1 - 2*m)*AppellF1[5/2, 2 - 2*m, 2*m, 7/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[
(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4]
)/10 - (3*(1 + 2*m)*AppellF1[5/2, -2*m, 2 + 2*m, 7/2, Tan[(-e + Pi/2 - f*x)
/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi
/2 - f*x)/4])/10)))/(3*AppellF1[1/2, -2*m, 2*m, 3/2, Tan[(-e + Pi/2 - f*x)/
4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - 4*m*(AppellF1[3/2, 1 - 2*m, 2*m, 5/2,
Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + AppellF1[3/2, -2
*m, 1 + 2*m, 5/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2])
*Tan[(-e + Pi/2 - f*x)/4]^2)^2))/2))
```

Maple [F] time = 0.181, size = 0, normalized size = 0.

$$\int \frac{\left(a+a\sin\left(fx+e\right)\right)^{m}}{c-c\sin\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m/(c-c*sin(f*x+e)),x)
```

```
[Out] int((a+a*sin(f*x+e))^m/(c-c*sin(f*x+e)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{m}}{c\sin\left(fx+e\right)-c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c-c*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] -integrate((a*sin(f*x + e) + a)^m/(c*sin(f*x + e) - c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(a\sin\left(fx+e\right)+a\right)^{m}}{c\sin\left(fx+e\right)-c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c-c*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral(-(a*sin(f*x + e) + a)^m/(c*sin(f*x + e) - c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{\left(a\sin\left(e+fx\right)+a\right)^{m}}{\sin\left(e+fx\right)-1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m/(c-c*sin(f*x+e)),x)
```

```
[Out] -Integral((a*sin(e + f*x) + a)**m/(sin(e + f*x) - 1), x)/c
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(a\sin\left(fx+e\right)+a\right)^{m}}{c\sin\left(fx+e\right)-c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c-c*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate(-(a*sin(f*x + e) + a)^m/(c*sin(f*x + e) - c), x)
```

3.409 $\int \frac{(a+a\sin(e+fx))^m}{(c-c\sin(e+fx))^2}\,dx$

Optimal. Leaf size=86

$$\frac{2^{m+\frac{1}{2}}\sec^3(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^{m+1}\,{}_2F_1\left(-\frac{3}{2},\frac{1}{2}-m;-\frac{1}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{3ac^2f}$$

```
[Out] (2^(1/2 + m)*Hypergeometric2F1[-3/2, 1/2 - m, -1/2, (1 - Sin[e + f*x])/2]*S
ec[e + f*x]^3*(1 + Sin[e + f*x])^(1/2 - m)*(a + a*Sin[e + f*x])^(1 + m))/(3
*a*c^2*f)
```

Rubi [A] time = 0.131682, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2736, 2689, 70, 69}

$$\frac{2^{m+\frac{1}{2}}\sec^3(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^{m+1}\,{}_2F_1\left(-\frac{3}{2},\frac{1}{2}-m;-\frac{1}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{3ac^2f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m/(c - c*Sin[e + f*x])^2,x]
```

```
[Out] (2^(1/2 + m)*Hypergeometric2F1[-3/2, 1/2 - m, -1/2, (1 - Sin[e + f*x])/2]*S
ec[e + f*x]^3*(1 + Sin[e + f*x])^(1/2 - m)*(a + a*Sin[e + f*x])^(1 + m))/(3
*a*c^2*f)
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && !In
tegerQ[m] && !IntegerQ[n] && (RationalQ[m] || !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
```

```
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^m}{(c-c\sin(e+fx))^2}\,dx = \frac{\int \sec^4(e+fx)(a+a\sin(e+fx))^{2+m}\,dx}{a^2c^2}$$

$$= \frac{\left(\sec^3(e+fx)(a-a\sin(e+fx))^{3/2}(a+a\sin(e+fx))^{3/2}\right)\text{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}}{(a-ax)^{5/2}}\,dx,x,\sin(e+\ldots$$

$$= \frac{\left(2^{-\frac{1}{2}+m}\sec^3(e+fx)(a-a\sin(e+fx))^{3/2}(a+a\sin(e+fx))^{1+m}\left(\frac{a+a\sin(e+fx)}{a}\right)^{\frac{1}{2}-m}\right)\text{Subst}\left(\ldots\right.}{c^2f}$$

$$= \frac{2^{\frac{1}{2}+m}\,{}_2F_1\left(-\frac{3}{2},\frac{1}{2}-m;-\frac{1}{2};\frac{1}{2}(1-\sin(e+fx))\right)\sec^3(e+fx)(1+\sin(e+fx))^{\frac{1}{2}-m}(a+a\sin(e+\ldots}{3ac^2f}$$

Mathematica [C] time = 20.8011, size = 5391, normalized size = 62.69

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m/(c - c*Sin[e + f*x])^2,x]
```

```
[Out] Result too large to show
```

Maple [F] time = 0.312, size = 0, normalized size = 0.

$$\int \frac{\left(a+a\sin\left(fx+e\right)\right)^m}{\left(c-c\sin\left(fx+e\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^2,x)
```

```
[Out] int((a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^m}{\left(c\sin\left(fx+e\right)-c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m/(c*sin(f*x + e) - c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(a\sin\left(fx+e\right)+a\right)^{m}}{c^{2}\cos\left(fx+e\right)^{2}+2\,c^{2}\sin\left(fx+e\right)-2\,c^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral(-(a*sin(f*x + e) + a)^m/(c^2*cos(f*x + e)^2 + 2*c^2*sin(f*x + e) -
 2*c^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\left(a\sin\left(e+fx\right)+a\right)^{m}}{\sin^{2}\left(e+fx\right)-2\sin\left(e+fx\right)+1}\,dx}{c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m/(c-c*sin(f*x+e))**2,x)
```

```
[Out] Integral((a*sin(e + f*x) + a)**m/(sin(e + f*x)**2 - 2*sin(e + f*x) + 1), x)
/c**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{m}}{\left(c\sin\left(fx+e\right)-c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m/(c*sin(f*x + e) - c)^2, x)
```

3.410 $\int \frac{(a+a\sin(e+fx))^m}{(c-c\sin(e+fx))^3} dx$

Optimal. Leaf size=86

$$\frac{2^{m+\frac{1}{2}} \sec^5(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^{m+2}\,{}_2F_1\left(-\frac{5}{2},\frac{1}{2}-m;-\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{5a^2c^3f}$$

```
[Out] (2^(1/2 + m)*Hypergeometric2F1[-5/2, 1/2 - m, -3/2, (1 - Sin[e + f*x])/2]*S
ec[e + f*x]^5*(1 + Sin[e + f*x])^(1/2 - m)*(a + a*Sin[e + f*x])^(2 + m))/(5
*a^2*c^3*f)
```

Rubi [A] time = 0.133367, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2736, 2689, 70, 69}

$$\frac{2^{m+\frac{1}{2}} \sec^5(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^{m+2}\,{}_2F_1\left(-\frac{5}{2},\frac{1}{2}-m;-\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{5a^2c^3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m/(c - c*Sin[e + f*x])^3,x]
```

```
[Out] (2^(1/2 + m)*Hypergeometric2F1[-5/2, 1/2 - m, -3/2, (1 - Sin[e + f*x])/2]*S
ec[e + f*x]^5*(1 + Sin[e + f*x])^(1/2 - m)*(a + a*Sin[e + f*x])^(2 + m))/(5
*a^2*c^3*f)
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
```

```
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^m}{(c-c\sin(e+fx))^3}\,dx = \frac{\int \sec^6(e+fx)(a+a\sin(e+fx))^{3+m}\,dx}{a^3c^3}$$

$$= \frac{\left(\sec^5(e+fx)(a-a\sin(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}\right)\text{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}}{(a-ax)^{7/2}}\,dx,x,\sin(e\right.}{ac^3f}$$

$$= \frac{\left(2^{-\frac{1}{2}+m}\sec^5(e+fx)(a-a\sin(e+fx))^{5/2}(a+a\sin(e+fx))^{2+m}\left(\frac{a+a\sin(e+fx)}{a}\right)^{\frac{1}{2}-m}\right)\text{Subs}}{ac^3f}$$

$$= \frac{2^{\frac{1}{2}+m}\,{}_2F_1\left(-\frac{5}{2},\frac{1}{2}-m;-\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)\sec^5(e+fx)(1+\sin(e+fx))^{\frac{1}{2}-m}(a+a\sin}{5a^2c^3f}$$

Mathematica [C] time = 23.3967, size = 7184, normalized size = 83.53

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m/(c - c*Sin[e + f*x])^3,x]
```

```
[Out] Result too large to show
```

Maple [F] time = 0.368, size = 0, normalized size = 0.

$$\int \frac{\left(a+a\sin\left(fx+e\right)\right)^m}{\left(c-c\sin\left(fx+e\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^3,x)
```

```
[Out] int((a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(a\sin\left(fx+e\right)+a\right)^m}{\left(c\sin\left(fx+e\right)-c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] -integrate((a*sin(f*x + e) + a)^m/(c*sin(f*x + e) - c)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(a\sin\left(fx+e\right)+a\right)^m}{3\,c^3\cos\left(fx+e\right)^2-4\,c^3-\left(c^3\cos\left(fx+e\right)^2-4\,c^3\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] integral(-(a*sin(f*x + e) + a)^m/(3*c^3*cos(f*x + e)^2 - 4*c^3 - (c^3*cos(f
*x + e)^2 - 4*c^3)*sin(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m/(c-c*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(a\sin\left(fx+e\right)+a\right)^m}{\left(c\sin\left(fx+e\right)-c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate(-(a*sin(f*x + e) + a)^m/(c*sin(f*x + e) - c)^3, x)
```

3.411 $\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^{5/2}\,dx$

Optimal. Leaf size=160

$$\frac{16c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}(a\sin(e+fx)+a)^m}{f\left(4m^2+16m+15\right)}+\frac{64c^3\cos(e+fx)(a\sin(e+fx)+a)^m}{f(2m+5)\left(4m^2+8m+3\right)\sqrt{c-c\sin(e+fx)}}+\frac{2c\cos(e+$$

```
[Out] (64*c^3*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(f*(5 + 2*m)*(3 + 8*m + 4*m^2)
*Sqrt[c - c*Sin[e + f*x]]) + (16*c^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*Sq
rt[c - c*Sin[e + f*x]])/(f*(15 + 16*m + 4*m^2)) + (2*c*Cos[e + f*x]*(a + a*
Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(3/2))/(f*(5 + 2*m))
```

Rubi [A] time = 0.251838, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2740, 2738}

$$\frac{16c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}(a\sin(e+fx)+a)^m}{f\left(4m^2+16m+15\right)}+\frac{64c^3\cos(e+fx)(a\sin(e+fx)+a)^m}{f(2m+5)\left(4m^2+8m+3\right)\sqrt{c-c\sin(e+fx)}}+\frac{2c\cos(e+$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] (64*c^3*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(f*(5 + 2*m)*(3 + 8*m + 4*m^2)
*Sqrt[c - c*Sin[e + f*x]]) + (16*c^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*Sq
rt[c - c*Sin[e + f*x]])/(f*(15 + 16*m + 4*m^2)) + (2*c*Cos[e + f*x]*(a + a*
Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(3/2))/(f*(5 + 2*m))
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^{5/2}\,dx=\frac{2c\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{3/2}}{f(5+2m)}+\frac{(8c)\int (a+$$

$$=\frac{16c^2\cos(e+fx)(a+a\sin(e+fx))^m\sqrt{c-c\sin(e+fx)}}{f\left(15+16m+4m^2\right)}+\frac{2c\cos(e+$$

$$=\frac{64c^3\cos(e+fx)(a+a\sin(e+fx))^m}{f\left(15+46m+36m^2+8m^3\right)\sqrt{c-c\sin(e+fx)}}+\frac{16c^2\cos(e+fx)(a$$

Mathematica [A] time = 2.54694, size = 149, normalized size = 0.93

$$-\frac{c^2\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)(a(\sin(e+fx)+1))^m\left(4\left(4m^2+16m+7\right)\sin(e+fx)+\left(4m^2\right.\right.}{f(2m+1)(2m+3)(2m+5)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] -((c^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^m*Sqrt[
c - c*Sin[e + f*x]]*(-89 - 56*m - 12*m^2 + (3 + 8*m + 4*m^2)*Cos[2*(e + f*x
)] + 4*(7 + 16*m + 4*m^2)*Sin[e + f*x]))/(f*(1 + 2*m)*(3 + 2*m)*(5 + 2*m)*(
Cos[(e + f*x)/2] - Sin[(e + f*x)/2])))
```

Maple [F] time = 4.983, size = 0, normalized size = 0.

$$\int \left(a+a\sin\left(fx+e\right)\right)^{m}\left(c-c\sin\left(fx+e\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(5/2),x)
```

Maxima [A] time = 1.98713, size = 392, normalized size = 2.45

$$-\frac{2\left(\left(4\,m^2+24\,m+43\right)a^m c^{\frac{5}{2}}-\frac{\left(12\,m^2+40\,m-15\right)a^m c^{\frac{5}{2}}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{2\left(4\,m^2+8\,m+35\right)a^m c^{\frac{5}{2}}\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}+\frac{2\left(4\,m^2+8\,m+35\right)a^m c^{\frac{5}{2}}\sin\left(fx+e\right)^3}{\left(\cos\left(fx+e\right)+1\right)^3}-\cdots\right.}{\left(8\,m^3+36\,m^2+46\,m+15\right)f\left(\frac{\sin(\cdots}{(\cos(f\cdots}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] -2*((4*m^2 + 24*m + 43)*a^m*c^(5/2) - (12*m^2 + 40*m - 15)*a^m*c^(5/2)*sin(
f*x + e)/(cos(f*x + e) + 1) + 2*(4*m^2 + 8*m + 35)*a^m*c^(5/2)*sin(f*x + e)
^2/(cos(f*x + e) + 1)^2 + 2*(4*m^2 + 8*m + 35)*a^m*c^(5/2)*sin(f*x + e)^3/(
cos(f*x + e) + 1)^3 - (12*m^2 + 40*m - 15)*a^m*c^(5/2)*sin(f*x + e)^4/(cos(
f*x + e) + 1)^4 + (4*m^2 + 24*m + 43)*a^m*c^(5/2)*sin(f*x + e)^5/(cos(f*x +
 e) + 1)^5)*e^(2*m*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1) - m*log(sin(f*x
 + e)^2/(cos(f*x + e) + 1)^2 + 1))/((8*m^3 + 36*m^2 + 46*m + 15)*f*(sin(f*x
 + e)^2/(cos(f*x + e) + 1)^2 + 1)^(5/2))
```

Fricas [A] time = 1.15751, size = 630, normalized size = 3.94

$$-\frac{2\left(\left(4\,c^2m^2+8\,c^2m+3\,c^2\right)\cos\left(fx+e\right)^3-\left(4\,c^2m^2+24\,c^2m+11\,c^2\right)\cos\left(fx+e\right)^2-32\,c^2-2\left(4\,c^2m^2+16\,c^2m+23\,c^2\right.\right.}{8\,fm^3+36\,fm^2+46\,fm+\left(8\,fm^3+36\,fm^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] -2*((4*c^2*m^2 + 8*c^2*m + 3*c^2)*cos(f*x + e)^3 - (4*c^2*m^2 + 24*c^2*m +
11*c^2)*cos(f*x + e)^2 - 32*c^2 - 2*(4*c^2*m^2 + 16*c^2*m + 23*c^2)*cos(f*x
 + e) + ((4*c^2*m^2 + 8*c^2*m + 3*c^2)*cos(f*x + e)^2 - 32*c^2 + 2*(4*c^2*m
^2 + 16*c^2*m + 7*c^2)*cos(f*x + e))*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c
)*(a*sin(f*x + e) + a)^m/(8*f*m^3 + 36*f*m^2 + 46*f*m + (8*f*m^3 + 36*f*m^2
 + 46*f*m + 15*f)*cos(f*x + e) - (8*f*m^3 + 36*f*m^2 + 46*f*m + 15*f)*sin(f
*x + e) + 15*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.412 $\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=100

$$\frac{8c^2\cos(e+fx)(a\sin(e+fx)+a)^m}{f\left(4m^2+8m+3\right)\sqrt{c-c\sin(e+fx)}}+\frac{2c\cos(e+fx)\sqrt{c-c\sin(e+fx)}(a\sin(e+fx)+a)^m}{f(2m+3)}$$

```
[Out] (8*c^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(f*(3 + 8*m + 4*m^2)*Sqrt[c - c
*Sin[e + f*x]]) + (2*c*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*Sqrt[c - c*Sin[e
 + f*x]])/(f*(3 + 2*m))
```

Rubi [A] time = 0.149104, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2740, 2738}

$$\frac{8c^2\cos(e+fx)(a\sin(e+fx)+a)^m}{f\left(4m^2+8m+3\right)\sqrt{c-c\sin(e+fx)}}+\frac{2c\cos(e+fx)\sqrt{c-c\sin(e+fx)}(a\sin(e+fx)+a)^m}{f(2m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] (8*c^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(f*(3 + 8*m + 4*m^2)*Sqrt[c - c
*Sin[e + f*x]]) + (2*c*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*Sqrt[c - c*Sin[e
 + f*x]])/(f*(3 + 2*m))
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^{3/2}\,dx &= \frac{2c\cos(e+fx)(a+a\sin(e+fx))^m\sqrt{c-c\sin(e+fx)}}{f(3+2m)}+\frac{(4c)\int (a+a\sin\\ &= \frac{8c^2\cos(e+fx)(a+a\sin(e+fx))^m}{f\left(3+8m+4m^2\right)\sqrt{c-c\sin(e+fx)}}+\frac{2c\cos(e+fx)(a+a\sin(e+fx)}{f(3+2m)}\end{aligned}$$

Mathematica [A] time = 0.497649, size = 110, normalized size = 1.1

$$-\frac{2c\sqrt{c-c\sin(e+fx)}((2m+1)\sin(e+fx)-2m-5)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)(a(\sin(e+fx)+1))^m}{f(2m+1)(2m+3)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-2*c*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^m*Sqrt[c
 - c*Sin[e + f*x]]*(-5 - 2*m + (1 + 2*m)*Sin[e + f*x]))/(f*(1 + 2*m)*(3 + 2
*m)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))
```

Maple [F] time = 0.148, size = 0, normalized size = 0.

$$\int \left(a+a\sin\left(fx+e\right)\right)^{m}\left(c-c\sin\left(fx+e\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(3/2),x)
```

Maxima [B] time = 1.87242, size = 261, normalized size = 2.61

$$-\frac{2\left(a^{m}c^{\frac{3}{2}}(2m+5)-\frac{a^{m}c^{\frac{3}{2}}(2m-3)\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-\frac{a^{m}c^{\frac{3}{2}}(2m-3)\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{a^{m}c^{\frac{3}{2}}(2m+5)\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}\right)e^{\left(2m\log\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+1\right)-m\log\left(\frac{\sin(f}{(\cos(fx}\right.\right.}}{\left(4m^{2}+8m+3\right)f\left(\frac{\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] -2*(a^m*c^(3/2)*(2*m + 5) - a^m*c^(3/2)*(2*m - 3)*sin(f*x + e)/(cos(f*x + e
) + 1) - a^m*c^(3/2)*(2*m - 3)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^m*c^
(3/2)*(2*m + 5)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3)*e^(2*m*log(sin(f*x + e
)/(cos(f*x + e) + 1) + 1) - m*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1))
/((4*m^2 + 8*m + 3)*f*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(3/2))
```

Fricas [A] time = 1.14181, size = 359, normalized size = 3.59

$$\frac{2\left((2cm+c)\cos\left(fx+e\right)^{2}+(2cm+5c)\cos\left(fx+e\right)-\left((2cm+c)\cos\left(fx+e\right)-4c\right)\sin\left(fx+e\right)+4c\right)\sqrt{-c\sin\left(fx\right.}}{4fm^{2}+8fm+\left(4fm^{2}+8fm+3f\right)\cos\left(fx+e\right)-\left(4fm^{2}+8fm+3f\right)\sin\left(fx+e\right)+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] 2*((2*c*m + c)*cos(f*x + e)^2 + (2*c*m + 5*c)*cos(f*x + e) - ((2*c*m + c)*c
os(f*x + e) - 4*c)*sin(f*x + e) + 4*c)*sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x
 + e) + a)^m/(4*f*m^2 + 8*f*m + (4*f*m^2 + 8*f*m + 3*f)*cos(f*x + e) - (4*f
```

```
*m^2 + 8*f*m + 3*f)*sin(f*x + e) + 3*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**(3/2),x)

[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(3/2),x, algorithm="giac")

[Out] sage2
```

3.413 $\int (a+a\sin (e+fx))^m\sqrt {c-c\sin (e+fx)}\,dx$

Optimal. Leaf size=46

$$\frac{2c\cos(e+fx)(a\sin(e+fx)+a)^m}{f(2m+1)\sqrt{c-c\sin(e+fx)}}$$

```
[Out] (2*c*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(f*(1 + 2*m)*Sqrt[c - c*Sin[e + f
*x]])
```

Rubi [A] time = 0.0677496, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {2738}

$$\frac{2c\cos(e+fx)(a\sin(e+fx)+a)^m}{f(2m+1)\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (2*c*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(f*(1 + 2*m)*Sqrt[c - c*Sin[e + f
*x]])
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\int (a+a\sin(e+fx))^m\sqrt{c-c\sin(e+fx)}\,dx=\frac{2c\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+2m)\sqrt{c-c\sin(e+fx)}}$$

Mathematica [A] time = 0.172022, size = 85, normalized size = 1.85

$$\frac{2\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)(a(\sin(e+fx)+1))^m}{f(2m+1)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^m*Sqrt[c -
c*Sin[e + f*x]])/(f*(1 + 2*m)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))
```

Maple [F] time = 0.149, size = 0, normalized size = 0.

$$\int \left(a + a\sin\left(fx + e\right)\right)^{m} \sqrt{c - c\sin\left(fx + e\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1/2),x)
```

Maxima [B] time = 1.81542, size = 157, normalized size = 3.41

$$-\frac{2\left(a^{m}\sqrt{c} + \frac{a^{m}\sqrt{c}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}\right)e^{\left(2\,m\log\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+1\right)-m\log\left(\frac{\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+1\right)\right)}}{f(2\,m+1)\sqrt{\frac{\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] -2*(a^m*sqrt(c) + a^m*sqrt(c)*sin(f*x + e)/(cos(f*x + e) + 1))*e^(2*m*log(s
in(f*x + e)/(cos(f*x + e) + 1) + 1) - m*log(sin(f*x + e)^2/(cos(f*x + e) +
1)^2 + 1))/(f*(2*m + 1)*sqrt(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1))
```

Fricas [A] time = 1.1034, size = 205, normalized size = 4.46

$$\frac{2\sqrt{-c\sin\left(fx+e\right)+c}\left(a\sin\left(fx+e\right)+a\right)^{m}\left(\cos\left(fx+e\right)+\sin\left(fx+e\right)+1\right)}{2\,fm+\left(2\,fm+f\right)\cos\left(fx+e\right)-\left(2\,fm+f\right)\sin\left(fx+e\right)+f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x + e) + a)^m*(cos(f*x + e) + sin(f*x
+ e) + 1)/(2*f*m + (2*f*m + f)*cos(f*x + e) - (2*f*m + f)*sin(f*x + e) + f)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\left(\sin\left(e+fx\right)+1\right)\right)^{m} \sqrt{-c\left(\sin\left(e+fx\right)-1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**m*sqrt(-c*(sin(e + f*x) - 1)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.414 $\int \frac{(a+a\sin(e+fx))^m}{\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=68

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^m\,{}_2F_1\left(1,m+\frac{1}{2};m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1)\right)}{f(2m+1)\sqrt{c-c\sin(e+fx)}}$$

```
[Out] (Cos[e + f*x]*Hypergeometric2F1[1, 1/2 + m, 3/2 + m, (1 + Sin[e + f*x])/2]*
(a + a*Sin[e + f*x])^m)/(f*(1 + 2*m)*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 0.133801, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {2745, 2667, 68}

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^m\,{}_2F_1\left(1,m+\frac{1}{2};m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1)\right)}{f(2m+1)\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m/Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (Cos[e + f*x]*Hypergeometric2F1[1, 1/2 + m, 3/2 + m, (1 + Sin[e + f*x])/2]*
(a + a*Sin[e + f*x])^m)/(f*(1 + 2*m)*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2745

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e
 + f*x])^FracPart[m]*(c + d*Sin[e + f*x])^FracPart[m])/Cos[e + f*x]^(2*Frac
Part[m]), Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; Fr
eeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
&& (FractionQ[m] ||  !FractionQ[n])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^m}{\sqrt{c-c\sin(e+fx)}}\,dx = \frac{\cos(e+fx)\int \sec(e+fx)(a+a\sin(e+fx))^{\frac{1}{2}+m}\,dx}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

$$= \frac{(a\cos(e+fx))\,\mathrm{Subst}\left(\int \frac{(a+x)^{-\frac{1}{2}+m}}{a-x}\,dx, x, a\sin(e+fx)\right)}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

$$= \frac{\cos(e+fx)\,{}_2F_1\left(1,\frac{1}{2}+m;\frac{3}{2}+m;\frac{1}{2}(1+\sin(e+fx))\right)(a+a\sin(e+fx))^m}{f(1+2m)\sqrt{c-c\sin(e+fx)}}$$

Mathematica [B] time = 1.43693, size = 157, normalized size = 2.31

$$\frac{2^{-2m-\frac{3}{2}}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(a(\sin(e+fx)+1))^m\left(4^m\,{}_2F_1\left(1,2m;2m+1;\sin\left(\frac{1}{4}(2e+2fx+\pi)\right)\right)-\mathrm{sec}\right.}{fm\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^m/Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (2^(-3/2 - 2*m)*(4^m*Hypergeometric2F1[1, 2*m, 1 + 2*m, Sin[(2*e + Pi + 2*f
*x)/4]] - Hypergeometric2F1[2*m, 2*m, 1 + 2*m, (1 - Tan[(2*e - Pi + 2*f*x)/
8]^2)/2]*(Sec[(2*e - Pi + 2*f*x)/8]^2)^(2*m))*(Cos[(e + f*x)/2] - Sin[(e +
f*x)/2])*(a*(1 + Sin[e + f*x]))^m)/(f*m*Sqrt[c - c*Sin[e + f*x]])
```

Maple [F] time = 0.148, size = 0, normalized size = 0.

$$\int (a+a\sin(fx+e))^m \frac{1}{\sqrt{c-c\sin(fx+e)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] int((a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\sin(fx+e)+a)^m}{\sqrt{-c\sin(fx+e)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m/sqrt(-c*sin(f*x + e) + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-c\sin\left(fx+e\right)+c}\left(a\sin\left(fx+e\right)+a\right)^m}{c\sin\left(fx+e\right)-c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x + e) + a)^m/(c*sin(f*x + e)
- c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^m}{\sqrt{-c\left(\sin\left(e+fx\right)-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m/(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**m/sqrt(-c*(sin(e + f*x) - 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a\sin\left(fx+e\right)+a\right)^m}{\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m/sqrt(-c*sin(f*x + e) + c), x)
```

3.415 $\int \frac{(a+a\sin(e+fx))^m}{(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=74

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^m\,{}_2F_1\left(2,m+\frac{1}{2};m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1)\right)}{2cf(2m+1)\sqrt{c-c\sin(e+fx)}}$$

```
[Out] (Cos[e + f*x]*Hypergeometric2F1[2, 1/2 + m, 3/2 + m, (1 + Sin[e + f*x])/2]*
(a + a*Sin[e + f*x])^m)/(2*c*f*(1 + 2*m)*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 0.157211, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {2745, 2667, 68}

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^m\,{}_2F_1\left(2,m+\frac{1}{2};m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1)\right)}{2cf(2m+1)\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m/(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] (Cos[e + f*x]*Hypergeometric2F1[2, 1/2 + m, 3/2 + m, (1 + Sin[e + f*x])/2]*
(a + a*Sin[e + f*x])^m)/(2*c*f*(1 + 2*m)*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2745

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e
 + f*x])^FracPart[m]*(c + d*Sin[e + f*x])^FracPart[m])/Cos[e + f*x]^(2*Frac
Part[m]), Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; Fr
eeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
&& (FractionQ[m] ||  !FractionQ[n])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^m}{(c-c\sin(e+fx))^{3/2}}\,dx = \frac{\cos(e+fx)\int \sec^3(e+fx)(a+a\sin(e+fx))^{\frac{3}{2}+m}\,dx}{ac\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

$$= \frac{\left(a^2\cos(e+fx)\right)\text{Subst}\left(\int \frac{(a+x)^{-\frac{1}{2}+m}}{(a-x)^2}\,dx, x, a\sin(e+fx)\right)}{cf\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

$$= \frac{\cos(e+fx)\,{}_2F_1\left(2,\frac{1}{2}+m;\frac{3}{2}+m;\frac{1}{2}(1+\sin(e+fx))\right)(a+a\sin(e+fx))^m}{2cf(1+2m)\sqrt{c-c\sin(e+fx)}}$$

Mathematica [C] time = 17.3428, size = 3006, normalized size = 40.62

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m/(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] -((Cos[(-e + Pi/2 - f*x)/4]^2)^(2*m)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^
3*(a + a*Sin[e + f*x])^m*(AppellF1[1, -2*m, 2*m, 2, Tan[(-e + Pi/2 - f*x)/4
]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*(Sec[(-e + Pi/2 - f*x)/4]^2)^(2*m)*Tan[(-
e + Pi/2 - f*x)/4]^2 - (AppellF1[1, -2*m, 2*m, 2, Cot[(-e + Pi/2 - f*x)/4]^
2, -Cot[(-e + Pi/2 - f*x)/4]^2]*Cot[(-e + Pi/2 - f*x)/4]^2*(Csc[(-e + Pi/2
- f*x)/4]^2)^(2*m)*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(2*m))/(1 - Cot[(-e + P
i/2 - f*x)/4]^2)^(2*m) + (2^(1 - 2*m)*AppellF1[1 + 2*m, 2*m, 1, 2 + 2*m, (1
 - Tan[(-e + Pi/2 - f*x)/4]^2)/2, 1 - Tan[(-e + Pi/2 - f*x)/4]^2]*(-1 + Tan
[(-e + Pi/2 - f*x)/4]^2)*(1 - Tan[(-e + Pi/2 - f*x)/4]^4)^(2*m))/(1 + 2*m))
)/(8*Sqrt[2]*f*(c - c*Sin[e + f*x])^(3/2)*(Cos[Pi/4 + (e - Pi/2 + f*x)/2] -
 Sin[Pi/4 + (e - Pi/2 + f*x)/2])^3*(-(m*Cos[(-e + Pi/2 - f*x)/4]*(Cos[(-e +
 Pi/2 - f*x)/4]^2)^(-1 + 2*m)*Sin[(-e + Pi/2 - f*x)/4]*(AppellF1[1, -2*m, 2
*m, 2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*(Sec[(-e +
Pi/2 - f*x)/4]^2)^(2*m)*Tan[(-e + Pi/2 - f*x)/4]^2 - (AppellF1[1, -2*m, 2*m
, 2, Cot[(-e + Pi/2 - f*x)/4]^2, -Cot[(-e + Pi/2 - f*x)/4]^2]*Cot[(-e + Pi/
2 - f*x)/4]^2*(Csc[(-e + Pi/2 - f*x)/4]^2)^(2*m)*(1 - Tan[(-e + Pi/2 - f*x)
/4]^2)^(2*m))/(1 - Cot[(-e + Pi/2 - f*x)/4]^2)^(2*m) + (2^(1 - 2*m)*AppellF
1[1 + 2*m, 2*m, 1, 2 + 2*m, (1 - Tan[(-e + Pi/2 - f*x)/4]^2)/2, 1 - Tan[(-e
 + Pi/2 - f*x)/4]^2]*(-1 + Tan[(-e + Pi/2 - f*x)/4]^2)*(1 - Tan[(-e + Pi/2
- f*x)/4]^4)^(2*m))/(1 + 2*m)))/(8*Sqrt[2]) + ((Cos[(-e + Pi/2 - f*x)/4]^2)
^(2*m)*((AppellF1[1, -2*m, 2*m, 2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + P
i/2 - f*x)/4]^2]*(Sec[(-e + Pi/2 - f*x)/4]^2)^(1 + 2*m)*Tan[(-e + Pi/2 - f*
x)/4])/2 + m*AppellF1[1, -2*m, 2*m, 2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e
 + Pi/2 - f*x)/4]^2]*(Sec[(-e + Pi/2 - f*x)/4]^2)^(2*m)*Tan[(-e + Pi/2 - f*
x)/4]^3 + (Sec[(-e + Pi/2 - f*x)/4]^2)^(2*m)*Tan[(-e + Pi/2 - f*x)/4]^2*(-(
m*AppellF1[2, 1 - 2*m, 2*m, 3, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2
- f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4])/2 - (m*Ap
pellF1[2, -2*m, 1 + 2*m, 3, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f
*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4])/2) + (m*Appe
llF1[1, -2*m, 2*m, 2, Cot[(-e + Pi/2 - f*x)/4]^2, -Cot[(-e + Pi/2 - f*x)/4]
^2]*Cot[(-e + Pi/2 - f*x)/4]^3*(Csc[(-e + Pi/2 - f*x)/4]^2)^(2*m)*(1 - Tan[
(-e + Pi/2 - f*x)/4]^2)^(2*m))/(1 - Cot[(-e + Pi/2 - f*x)/4]^2)^(2*m) + m*A
ppellF1[1, -2*m, 2*m, 2, Cot[(-e + Pi/2 - f*x)/4]^2, -Cot[(-e + Pi/2 - f*x)
/4]^2]*Cot[(-e + Pi/2 - f*x)/4]^3*(1 - Cot[(-e + Pi/2 - f*x)/4]^2)^(-1 - 2*
m)*(Csc[(-e + Pi/2 - f*x)/4]^2)^(1 + 2*m)*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^
(2*m) + (AppellF1[1, -2*m, 2*m, 2, Cot[(-e + Pi/2 - f*x)/4]^2, -Cot[(-e + P
i/2 - f*x)/4]^2]*Cot[(-e + Pi/2 - f*x)/4]*(Csc[(-e + Pi/2 - f*x)/4]^2)^(1 +
 2*m)*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(2*m))/(2*(1 - Cot[(-e + Pi/2 - f*x)
```

```
/4]^2)^(2*m)) - (Cot[(-e + Pi/2 - f*x)/4]^2*(Csc[(-e + Pi/2 - f*x)/4]^2)^(2
*m)*((m*AppellF1[2, 1 - 2*m, 2*m, 3, Cot[(-e + Pi/2 - f*x)/4]^2, -Cot[(-e +
 Pi/2 - f*x)/4]^2]*Cot[(-e + Pi/2 - f*x)/4]*Csc[(-e + Pi/2 - f*x)/4]^2)/2 +
 (m*AppellF1[2, -2*m, 1 + 2*m, 3, Cot[(-e + Pi/2 - f*x)/4]^2, -Cot[(-e + Pi
/2 - f*x)/4]^2]*Cot[(-e + Pi/2 - f*x)/4]*Csc[(-e + Pi/2 - f*x)/4]^2)/2)*(1
- Tan[(-e + Pi/2 - f*x)/4]^2)^(2*m))/(1 - Cot[(-e + Pi/2 - f*x)/4]^2)^(2*m)
 + (m*AppellF1[1, -2*m, 2*m, 2, Cot[(-e + Pi/2 - f*x)/4]^2, -Cot[(-e + Pi/2
 - f*x)/4]^2]*Csc[(-e + Pi/2 - f*x)/4]*(Csc[(-e + Pi/2 - f*x)/4]^2)^(2*m)*S
ec[(-e + Pi/2 - f*x)/4]*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(-1 + 2*m))/(1 - C
ot[(-e + Pi/2 - f*x)/4]^2)^(2*m) + (AppellF1[1 + 2*m, 2*m, 1, 2 + 2*m, (1 -
 Tan[(-e + Pi/2 - f*x)/4]^2)/2, 1 - Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + P
i/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4]*(1 - Tan[(-e + Pi/2 - f*x)/4]^4)^(
2*m))/(2^(2*m)*(1 + 2*m)) + (2^(1 - 2*m)*(-((1 + 2*m)*AppellF1[2 + 2*m, 2*m
, 2, 3 + 2*m, (1 - Tan[(-e + Pi/2 - f*x)/4]^2)/2, 1 - Tan[(-e + Pi/2 - f*x)
/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4])/(2*(2 + 2*m)) -
 (m*(1 + 2*m)*AppellF1[2 + 2*m, 1 + 2*m, 1, 3 + 2*m, (1 - Tan[(-e + Pi/2 -
f*x)/4]^2)/2, 1 - Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Ta
n[(-e + Pi/2 - f*x)/4])/(2*(2 + 2*m)))*(-1 + Tan[(-e + Pi/2 - f*x)/4]^2)*(1
 - Tan[(-e + Pi/2 - f*x)/4]^4)^(2*m))/(1 + 2*m) - (2^(2 - 2*m)*m*AppellF1[1
 + 2*m, 2*m, 1, 2 + 2*m, (1 - Tan[(-e + Pi/2 - f*x)/4]^2)/2, 1 - Tan[(-e +
Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4]^3*(-1
 + Tan[(-e + Pi/2 - f*x)/4]^2)*(1 - Tan[(-e + Pi/2 - f*x)/4]^4)^(-1 + 2*m))
/(1 + 2*m)))/(8*Sqrt[2])))
```

Maple [F] time = 0.148, size = 0, normalized size = 0.

$$\int \left(a+a\sin\left(fx+e\right)\right)^{m}\left(c-c\sin\left(fx+e\right)\right)^{-\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] int((a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{m}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m/(-c*sin(f*x + e) + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-c\sin\left(fx+e\right)+c}\left(a\sin\left(fx+e\right)+a\right)^{m}}{c^{2}\cos\left(fx+e\right)^{2}+2\,c^{2}\sin\left(fx+e\right)-2\,c^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x + e) + a)^m/(c^2*cos(f*x + e
)^2 + 2*c^2*sin(f*x + e) - 2*c^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m/(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{m}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m/(-c*sin(f*x + e) + c)^(3/2), x)
```

3.416 $\int \frac{(a+a\sin(e+fx))^m}{(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=74

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^m\,{}_2F_1\left(3,m+\frac{1}{2};m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1)\right)}{4c^2f(2m+1)\sqrt{c-c\sin(e+fx)}}$$

```
[Out] (Cos[e + f*x]*Hypergeometric2F1[3, 1/2 + m, 3/2 + m, (1 + Sin[e + f*x])/2]*
(a + a*Sin[e + f*x])^m)/(4*c^2*f*(1 + 2*m)*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 0.159911, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {2745, 2667, 68}

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^m\,{}_2F_1\left(3,m+\frac{1}{2};m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1)\right)}{4c^2f(2m+1)\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m/(c - c*Sin[e + f*x])^(5/2),x]
```


```
[Out] (Cos[e + f*x]*Hypergeometric2F1[3, 1/2 + m, 3/2 + m, (1 + Sin[e + f*x])/2]*
(a + a*Sin[e + f*x])^m)/(4*c^2*f*(1 + 2*m)*Sqrt[c - c*Sin[e + f*x]])
```


Rule 2745

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e
 + f*x])^FracPart[m]*(c + d*Sin[e + f*x])^FracPart[m])/Cos[e + f*x]^(2*Frac
Part[m]), Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; Fr
eeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
&& (FractionQ[m] ||  !FractionQ[n])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^m}{(c-c\sin(e+fx))^{5/2}}\,dx = \frac{\cos(e+fx)\int \sec^5(e+fx)(a+a\sin(e+fx))^{\frac{5}{2}+m}\,dx}{a^2c^2\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

$$= \frac{\left(a^3\cos(e+fx)\right)\text{Subst}\left(\int \frac{(a+x)^{-\frac{1}{2}+m}}{(a-x)^3}\,dx, x, a\sin(e+fx)\right)}{c^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

$$= \frac{\cos(e+fx)\,{}_2F_1\left(3,\frac{1}{2}+m;\frac{3}{2}+m;\frac{1}{2}(1+\sin(e+fx))\right)(a+a\sin(e+fx))^m}{4c^2f(1+2m)\sqrt{c-c\sin(e+fx)}}$$

Mathematica [C] time = 22.0685, size = 5136, normalized size = 69.41

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m/(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] Result too large to show
```

Maple [F] time = 0.144, size = 0, normalized size = 0.

$$\int \left(a+a\sin\left(fx+e\right)\right)^m\left(c-c\sin\left(fx+e\right)\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] int((a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^m}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m/(-c*sin(f*x + e) + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-c\sin\left(fx+e\right)+c}\left(a\sin\left(fx+e\right)+a\right)^m}{3\,c^3\cos\left(fx+e\right)^2-4\,c^3-\left(c^3\cos\left(fx+e\right)^2-4\,c^3\right)\sin\left(fx+e\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x + e) + a)^m/(3*c^3*cos(f*x +
 e)^2 - 4*c^3 - (c^3*cos(f*x + e)^2 - 4*c^3)*sin(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m/(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a \sin \left(fx + e\right) + a\right)^{m}}{\left(-c \sin \left(fx + e\right) + c\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m/(-c*sin(f*x + e) + c)^(5/2), x)
```

3.417 $\int \frac{(a+a\sin(e+fx))^m}{\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=68

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^m\,{}_2F_1\left(1,m+\frac{1}{2};m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1)\right)}{f(2m+1)\sqrt{c-c\sin(e+fx)}}$$

```
[Out] (Cos[e + f*x]*Hypergeometric2F1[1, 1/2 + m, 3/2 + m, (1 + Sin[e + f*x])/2]*
(a + a*Sin[e + f*x])^m)/(f*(1 + 2*m)*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 0.128541, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {2745, 2667, 68}

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^m\,{}_2F_1\left(1,m+\frac{1}{2};m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1)\right)}{f(2m+1)\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m/Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (Cos[e + f*x]*Hypergeometric2F1[1, 1/2 + m, 3/2 + m, (1 + Sin[e + f*x])/2]*
(a + a*Sin[e + f*x])^m)/(f*(1 + 2*m)*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2745

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e
 + f*x])^FracPart[m]*(c + d*Sin[e + f*x])^FracPart[m])/Cos[e + f*x]^(2*Frac
Part[m]), Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; Fr
eeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
&& (FractionQ[m] ||  !FractionQ[n])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^m}{\sqrt{c-c\sin(e+fx)}}\,dx = \frac{\cos(e+fx)\int \sec(e+fx)(a+a\sin(e+fx))^{\frac{1}{2}+m}\,dx}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

$$= \frac{(a\cos(e+fx))\,\text{Subst}\left(\int \frac{(a+x)^{-\frac{1}{2}+m}}{a-x}\,dx, x, a\sin(e+fx)\right)}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

$$= \frac{\cos(e+fx)\,{}_2F_1\left(1,\frac{1}{2}+m;\frac{3}{2}+m;\frac{1}{2}(1+\sin(e+fx))\right)(a+a\sin(e+fx))^m}{f(1+2m)\sqrt{c-c\sin(e+fx)}}$$

Mathematica [B] time = 0.491723, size = 157, normalized size = 2.31

$$\frac{2^{-2m-\frac{3}{2}}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(a(\sin(e+fx)+1))^m\left(4^m\,{}_2F_1\left(1,2m;2m+1;\sin\left(\frac{1}{4}(2e+2fx+\pi)\right)\right)-\text{se}\right.}{fm\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^m/Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (2^(-3/2 - 2*m)*(4^m*Hypergeometric2F1[1, 2*m, 1 + 2*m, Sin[(2*e + Pi + 2*f
*x)/4]] - Hypergeometric2F1[2*m, 2*m, 1 + 2*m, (1 - Tan[(2*e - Pi + 2*f*x)/
8]^2)/2]*(Sec[(2*e - Pi + 2*f*x)/8]^2)^(2*m))*(Cos[(e + f*x)/2] - Sin[(e +
f*x)/2])*(a*(1 + Sin[e + f*x]))^m)/(f*m*Sqrt[c - c*Sin[e + f*x]])
```

Maple [F] time = 0.006, size = 0, normalized size = 0.

$$\int (a+a\sin(fx+e))^m \frac{1}{\sqrt{c-c\sin(fx+e)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] int((a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\sin(fx+e)+a)^m}{\sqrt{-c\sin(fx+e)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m/sqrt(-c*sin(f*x + e) + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-c\sin\left(fx+e\right)+c}\left(a\sin\left(fx+e\right)+a\right)^{m}}{c\sin\left(fx+e\right)-c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x + e) + a)^m/(c*sin(f*x + e)
- c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{m}}{\sqrt{-c\left(\sin\left(e+fx\right)-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m/(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**m/sqrt(-c*(sin(e + f*x) - 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a\sin\left(fx+e\right)+a\right)^{m}}{\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m/sqrt(-c*sin(f*x + e) + c), x)
```

3.418 $\int \frac{(c+c\sin(e+fx))^m}{\sqrt{a-a\sin(e+fx)}}\,dx$

Optimal. Leaf size=68

$$\frac{\cos(e+fx)(c\sin(e+fx)+c)^m\,{}_2F_1\left(1,m+\frac{1}{2};m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1)\right)}{f(2m+1)\sqrt{a-a\sin(e+fx)}}$$

```
[Out] (Cos[e + f*x]*Hypergeometric2F1[1, 1/2 + m, 3/2 + m, (1 + Sin[e + f*x])/2]*
(c + c*Sin[e + f*x])^m)/(f*(1 + 2*m)*Sqrt[a - a*Sin[e + f*x]])
```

Rubi [A] time = 0.137928, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {2745, 2667, 68}

$$\frac{\cos(e+fx)(c\sin(e+fx)+c)^m\,{}_2F_1\left(1,m+\frac{1}{2};m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1)\right)}{f(2m+1)\sqrt{a-a\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + c*Sin[e + f*x])^m/Sqrt[a - a*Sin[e + f*x]],x]
```

```
[Out] (Cos[e + f*x]*Hypergeometric2F1[1, 1/2 + m, 3/2 + m, (1 + Sin[e + f*x])/2]*
(c + c*Sin[e + f*x])^m)/(f*(1 + 2*m)*Sqrt[a - a*Sin[e + f*x]])
```

Rule 2745

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e
 + f*x])^FracPart[m]*(c + d*Sin[e + f*x])^FracPart[m])/Cos[e + f*x]^(2*Frac
Part[m]), Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; Fr
eeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
&& (FractionQ[m] ||  !FractionQ[n])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{(c+c\sin(e+fx))^m}{\sqrt{a-a\sin(e+fx)}}\,dx = \frac{\cos(e+fx)\int \sec(e+fx)(c+c\sin(e+fx))^{\frac{1}{2}+m}\,dx}{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}$$
$$= \frac{(c\cos(e+fx))\,\text{Subst}\left(\int \frac{(c+x)^{-\frac{1}{2}+m}}{c-x}\,dx, x, c\sin(e+fx)\right)}{f\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}$$
$$= \frac{\cos(e+fx)\,{}_2F_1\left(1,\frac{1}{2}+m;\frac{3}{2}+m;\frac{1}{2}(1+\sin(e+fx))\right)(c+c\sin(e+fx))^m}{f(1+2m)\sqrt{a-a\sin(e+fx)}}$$

Mathematica [B] time = 0.426783, size = 157, normalized size = 2.31

$$\frac{2^{-2m-\frac{3}{2}}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(c(\sin(e+fx)+1))^m\left(4^m\,{}_2F_1\left(1,2m;2m+1;\sin\left(\frac{1}{4}(2e+2fx+\pi)\right)\right)-\sec^2\left(\right.\right.}{fm\sqrt{a-a\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + c*Sin[e + f*x])^m/Sqrt[a - a*Sin[e + f*x]],x]
```

```
[Out] (2^(-3/2 - 2*m)*(4^m*Hypergeometric2F1[1, 2*m, 1 + 2*m, Sin[(2*e + Pi + 2*f
*x)/4]] - Hypergeometric2F1[2*m, 2*m, 1 + 2*m, (1 - Tan[(2*e - Pi + 2*f*x)/
8]^2)/2]*(Sec[(2*e - Pi + 2*f*x)/8]^2)^(2*m))*(Cos[(e + f*x)/2] - Sin[(e +
f*x)/2])*(c*(1 + Sin[e + f*x]))^m)/(f*m*Sqrt[a - a*Sin[e + f*x]])
```

Maple [F] time = 0.175, size = 0, normalized size = 0.

$$\int \left(c+c\sin\left(fx+e\right)\right)^m \frac{1}{\sqrt{a-a\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+c*sin(f*x+e))^m/(a-a*sin(f*x+e))^(1/2),x)
```

```
[Out] int((c+c*sin(f*x+e))^m/(a-a*sin(f*x+e))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c\sin\left(fx+e\right)+c\right)^m}{\sqrt{-a\sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+c*sin(f*x+e))^m/(a-a*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((c*sin(f*x + e) + c)^m/sqrt(-a*sin(f*x + e) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-a\sin\left(fx+e\right)+a}\left(c\sin\left(fx+e\right)+c\right)^m}{a\sin\left(fx+e\right)-a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+c*sin(f*x+e))^m/(a-a*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a*sin(f*x + e) + a)*(c*sin(f*x + e) + c)^m/(a*sin(f*x + e)
- a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(c\left(\sin\left(e+fx\right)+1\right)\right)^m}{\sqrt{-a\left(\sin\left(e+fx\right)-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+c*sin(f*x+e))**m/(a-a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((c*(sin(e + f*x) + 1))**m/sqrt(-a*(sin(e + f*x) - 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(c\sin\left(fx+e\right)+c\right)^m}{\sqrt{-a\sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+c*sin(f*x+e))^m/(a-a*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((c*sin(f*x + e) + c)^m/sqrt(-a*sin(f*x + e) + a), x)
```

3.419 $\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-3-m}\,dx$

Optimal. Leaf size=164

$$\frac{2\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}}{c^2f(2m+5)\left(4m^2+8m+3\right)}+\frac{2\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-2}}{cf\left(4m^2+16m+15\right)}+\text{co}$$

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-3 - m))/(f*(5 +
 2*m)) + (2*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-2 -
m))/(c*f*(15 + 16*m + 4*m^2)) + (2*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c -
 c*Sin[e + f*x])^(-1 - m))/(c^2*f*(5 + 2*m)*(3 + 8*m + 4*m^2))
```

Rubi [A] time = 0.223402, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2743, 2742}

$$\frac{2\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}}{c^2f(2m+5)\left(4m^2+8m+3\right)}+\frac{2\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-2}}{cf\left(4m^2+16m+15\right)}+\text{co}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-3 - m),x]
```


```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-3 - m))/(f*(5 +
 2*m)) + (2*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-2 -
m))/(c*f*(15 + 16*m + 4*m^2)) + (2*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c -
 c*Sin[e + f*x])^(-1 - m))/(c^2*f*(5 + 2*m)*(3 + 8*m + 4*m^2))
```


Rule 2743

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rule 2742

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rubi steps

$$\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-3-m}\,dx=\frac{\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-3-m}}{f(5+2m)}+\frac{2\int(a+a\,\text{si}}{}$$

$$=\frac{\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-3-m}}{f(5+2m)}+\frac{2\cos(e+f\text{x}}{}$$

$$=\frac{\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-3-m}}{f(5+2m)}+\frac{2\cos(e+f\text{x}}{}$$

Mathematica [A] time = 8.60579, size = 174, normalized size = 1.06

$$\frac{2^{-m-2}\cos\left(\frac{1}{2}\left(-e-fx+\frac{\pi}{2}\right)\right)\sin^{-2m-5}\left(\frac{1}{2}\left(-e-fx+\frac{\pi}{2}\right)\right)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-3}\left(\cos\left(\frac{1}{2}(e+fx)\right)\right.}{f(2m+1)(2m+3)(2m+}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-3 - m),x]
```

```
[Out] (2^(-2 - m)*Cos[(-e + Pi/2 - f*x)/2]*Sin[(-e + Pi/2 - f*x)/2]^(-5 - 2*m)*(a
 + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-3 - m)*(4*(2 + 3*m + m^2) + Cos
[2*(-e + Pi/2 - f*x)] - 2*(3 + 2*m)*Sin[e + f*x]))/(f*(1 + 2*m)*(3 + 2*m)*(
5 + 2*m)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^(2*(-3 - m)))
```

Maple [F] time = 0.342, size = 0, normalized size = 0.

$$\int\left(a+a\sin\left(fx+e\right)\right)^{m}\left(c-c\sin\left(fx+e\right)\right)^{-3-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-3-m),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-3-m),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{-m-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-3-m),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(-m - 3), x)
```

Fricas [A] time = 1.08885, size = 251, normalized size = 1.53

$$-\frac{\left(2\cos\left(fx+e\right)^{3}+2\left(2m+3\right)\cos\left(fx+e\right)\sin\left(fx+e\right)-\left(4m^{2}+12m+9\right)\cos\left(fx+e\right)\right)\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\,\mathrm{s}\right.}{8fm^{3}+36fm^{2}+46fm+15f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-3-m),x, algorithm="fricas")
```

```
[Out] -(2*cos(f*x + e)^3 + 2*(2*m + 3)*cos(f*x + e)*sin(f*x + e) - (4*m^2 + 12*m
+ 9)*cos(f*x + e))*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(-m - 3)/(8
*f*m^3 + 36*f*m^2 + 46*f*m + 15*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**(-3-m),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(fx+e)+a)^m(-c\sin(fx+e)+c)^{-m-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-3-m),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(-m - 3), x)
```

3.420 $\int (a+a\sin(e+fx))^m (c-c\sin(e+fx))^{-2-m}\,dx$

Optimal. Leaf size=101

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}}{cf\left(4m^2+8m+3\right)}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-2}}{f(2m+3)}$$

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-2 - m))/(f*(3 +
 2*m)) + (Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 - m)
)/(c*f*(3 + 8*m + 4*m^2))
```

Rubi [A] time = 0.134702, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2743, 2742}

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}}{cf\left(4m^2+8m+3\right)}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-2}}{f(2m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-2 - m),x]
```

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-2 - m))/(f*(3 +
 2*m)) + (Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 - m)
)/(c*f*(3 + 8*m + 4*m^2))
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rule 2742

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rubi steps

$$\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-2-m}\,dx = \frac{\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-2-m}}{f(3+2m)}+\int (a+a\,\mathrm{s}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-2-m}}{f(3+2m)}+\cos(e+$$

Mathematica [A] time = 3.12659, size = 136, normalized size = 1.35

$$-\frac{2^{-m}\cos\left(\frac{1}{2}\left(-e-fx+\frac{\pi}{2}\right)\right)\sin^{-2m-3}\left(\frac{1}{2}\left(-e-fx+\frac{\pi}{2}\right)\right)(\sin(e+fx)-2(m+1))(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m}}{f\left(8m^2+16m+6\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-2 - m),x]
```

```
[Out] -((Cos[(-e + Pi/2 - f*x)/2]*Sin[(-e + Pi/2 - f*x)/2]^(-3 - 2*m)*(-2*(1 + m)
 + Sin[e + f*x])*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-2 - m))/(2^m
*f*(6 + 16*m + 8*m^2)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^(2*(-2 - m))))
```

Maple [F] time = 0.298, size = 0, normalized size = 0.

$$\int\left(a+a\sin\left(fx+e\right)\right)^{m}\left(c-c\sin\left(fx+e\right)\right)^{-2-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-2-m),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-2-m),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{-m-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-2-m),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(-m - 2), x)
```

Fricas [A] time = 1.14477, size = 178, normalized size = 1.76

$$\frac{\left(2\,(m+1)\cos\left(fx+e\right)-\cos\left(fx+e\right)\sin\left(fx+e\right)\right)\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{-m-2}}{4\,fm^2+8\,fm+3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-2-m),x, algorithm="fricas")
```

```
[Out] (2*(m + 1)*cos(f*x + e) - cos(f*x + e)*sin(f*x + e))*(a*sin(f*x + e) + a)^m
*(-c*sin(f*x + e) + c)^(-m - 2)/(4*f*m^2 + 8*f*m + 3*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**(-2-m),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{-m-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-2-m),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(-m - 2), x)
```

3.421 $\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-1-m}\,dx$

Optimal. Leaf size=46

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}}{f(2m+1)}$$

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 - m))/(f*(1 +
 2*m))
```

Rubi [A] time = 0.0669966, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {2742}

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}}{f(2m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 - m),x]
```

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 - m))/(f*(1 +
 2*m))
```

Rule 2742

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rubi steps

$$\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-1-m}\,dx=\frac{\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-1-m}}{f(1+2m)}$$

Mathematica [B] time = 1.54917, size = 107, normalized size = 2.33

$$\frac{2^{-m}\sin\left(\frac{1}{4}(2e+2fx+\pi)\right)\cos^{-2m-1}\left(\frac{1}{4}(2e+2fx+\pi)\right)(a(\sin(e+fx)+1))^m(c-c\sin(e+fx))^{-m}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\right.}{cf(2m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 - m),x]
```

```
[Out] (Cos[(2*e + Pi + 2*f*x)/4]^(-1 - 2*m)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])
^(2*m)*(a*(1 + Sin[e + f*x]))^m*Sin[(2*e + Pi + 2*f*x)/4])/(2^m*c*f*(1 + 2*
m)*(c - c*Sin[e + f*x])^m)
```

Maple [F] time = 0.263, size = 0, normalized size = 0.

$$\int \left(a+a\sin\left(fx+e\right)\right)^{m}\left(c-c\sin\left(fx+e\right)\right)^{-1-m}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1-m),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1-m),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{-m-1}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1-m),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(-m - 1), x)
```

Fricas [A] time = 1.09797, size = 108, normalized size = 2.35

$$\frac{\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{-m-1}\cos\left(fx+e\right)}{2\,fm+f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1-m),x, algorithm="fricas")
```

```
[Out] (a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(-m - 1)*cos(f*x + e)/(2*f*m +
 f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**(-1-m),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{-m-1}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1-m),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(-m - 1), x)
```

3.422 $\int (a + a\sin(e + fx))^m (c - c\sin(e + fx))^{-m}\,dx$

Optimal. Leaf size=112

$$\frac{c2^{\frac{1}{2}-m}\cos(e+fx)(1-\sin(e+fx))^{m+\frac{1}{2}}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}\,{}_2F_1\left(\frac{1}{2}(2m+1),\frac{1}{2}(2m+1);\frac{1}{2}(2m+\right.}{f(2m+1)}$$

```
[Out] (2^(1/2 - m)*c*Cos[e + f*x]*Hypergeometric2F1[(1 + 2*m)/2, (1 + 2*m)/2, (3
+ 2*m)/2, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*x])^(1/2 + m)*(a + a*Sin[e +
 f*x])^m*(c - c*Sin[e + f*x])^(-1 - m))/(f*(1 + 2*m))
```

Rubi [A] time = 0.157636, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2745, 2689, 70, 69}

$$\frac{c2^{\frac{1}{2}-m}\cos(e+fx)(1-\sin(e+fx))^{m+\frac{1}{2}}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}\,{}_2F_1\left(\frac{1}{2}(2m+1),\frac{1}{2}(2m+1);\frac{1}{2}(2m+\right.}{f(2m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m/(c - c*Sin[e + f*x])^m,x]
```

```
[Out] (2^(1/2 - m)*c*Cos[e + f*x]*Hypergeometric2F1[(1 + 2*m)/2, (1 + 2*m)/2, (3
+ 2*m)/2, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*x])^(1/2 + m)*(a + a*Sin[e +
 f*x])^m*(c - c*Sin[e + f*x])^(-1 - m))/(f*(1 + 2*m))
```

Rule 2745

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e
 + f*x])^FracPart[m]*(c + d*Sin[e + f*x])^FracPart[m])/Cos[e + f*x]^(2*Frac
Part[m]), Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; Fr
eeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
&& (FractionQ[m] ||  !FractionQ[n])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
```

```
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-m}\,dx = \left(\cos^{-2m}(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^m\right)\int \cos^{2m}(e+f$$

$$= \frac{\left(c^2\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{\frac{1}{2}(-1-2m)+m}(c+c\,\mathrm{si}\right.}{}$$

$$= \frac{\left(2^{-\frac{1}{2}-m}c^2\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-\frac{1}{2}+\frac{1}{2}(-1-2m)}\right.}{}$$

$$= \frac{2^{\frac{1}{2}-m}c\cos(e+fx)\,{}_2F_1\left(\frac{1}{2}(1+2m),\frac{1}{2}(1+2m);\frac{1}{2}(3+2m);\frac{1}{2}(1+\sin(e+\right.}{f(1+}$$

Mathematica [C] time = 2.93898, size = 388, normalized size = 3.46

$$\frac{2^{1-m}(2m-3)\sin^2\left(\frac{1}{8}(2e+2fx+3\pi)\right)\cos^{1-2m}\left(\frac{1}{4}(2e+2fx+\pi)\right)\left(a(\sin(\right.}{f(2m-1)\left(2\sin^2\left(\frac{1}{8}(2e+2fx-\pi)\right)\left(2mF_1\left(\frac{3}{2}-m;1-2m,1;\frac{5}{2}-m;\tan^2\left(\frac{1}{8}(-2e-2fx+\pi)\right),-\tan^2\left(\frac{1}{8}(2e+2fx-\pi)\right)\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m/(c - c*Sin[e + f*x])^m,x]
```

```
[Out] (2^(1 - m)*(-3 + 2*m)*AppellF1[1/2 - m, -2*m, 1, 3/2 - m, Tan[(-2*e + Pi -
2*f*x)/8]^2, -Tan[(2*e - Pi + 2*f*x)/8]^2]*Cos[(2*e + Pi + 2*f*x)/4]^(1 - 2
*m)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^(2*m)*(a*(1 + Sin[e + f*x]))^m*Si
n[(2*e + 3*Pi + 2*f*x)/8]^2)/(f*(-1 + 2*m)*(c - c*Sin[e + f*x])^m*((-3 + 2*
m)*AppellF1[1/2 - m, -2*m, 1, 3/2 - m, Tan[(-2*e + Pi - 2*f*x)/8]^2, -Tan[(
2*e - Pi + 2*f*x)/8]^2]*Cos[(2*e - Pi + 2*f*x)/8]^2 + 2*(2*m*AppellF1[3/2 -
 m, 1 - 2*m, 1, 5/2 - m, Tan[(-2*e + Pi - 2*f*x)/8]^2, -Tan[(2*e - Pi + 2*f
*x)/8]^2] + AppellF1[3/2 - m, -2*m, 2, 5/2 - m, Tan[(-2*e + Pi - 2*f*x)/8]^
2, -Tan[(2*e - Pi + 2*f*x)/8]^2])*Sin[(2*e - Pi + 2*f*x)/8]^2))
```

Maple [F] time = 0.463, size = 0, normalized size = 0.

$$\int \frac{\left(a+a\sin\left(fx+e\right)\right)^m}{\left(c-c\sin\left(fx+e\right)\right)^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m/((c-c*sin(f*x+e))^m),x)
```

```
[Out] int((a+a*sin(f*x+e))^m/((c-c*sin(f*x+e))^m),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^m}{\left(-c\sin\left(fx+e\right)+c\right)^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/((c-c*sin(f*x+e))^m),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m/(-c*sin(f*x + e) + c)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a\sin\left(fx+e\right)+a\right)^m}{\left(-c\sin\left(fx+e\right)+c\right)^m}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/((c-c*sin(f*x+e))^m),x, algorithm="fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)^m/(-c*sin(f*x + e) + c)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m/((c-c*sin(f*x+e))**m),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^m}{\left(-c\sin\left(fx+e\right)+c\right)^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/((c-c*sin(f*x+e))^m),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m/(-c*sin(f*x + e) + c)^m, x)
```

3.423 $\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^{1-m}\,dx$

Optimal. Leaf size=114

$$\frac{c^2 2^{\frac{3}{2}-m}\cos(e+fx)(1-\sin(e+fx))^{m+\frac{1}{2}}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}\,{}_2F_1\left(\frac{1}{2}(2m-1),\frac{1}{2}(2m+1);\frac{1}{2}(2m+3)\right.}{f(2m+1)}$$

```
[Out] (2^(3/2 - m)*c^2*Cos[e + f*x]*Hypergeometric2F1[(-1 + 2*m)/2, (1 + 2*m)/2,
(3 + 2*m)/2, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*x])^(1/2 + m)*(a + a*Sin[
e + f*x])^m*(c - c*Sin[e + f*x])^(-1 - m))/(f*(1 + 2*m))
```

Rubi [A] time = 0.186005, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2745, 2689, 70, 69}

$$\frac{c^2 2^{\frac{3}{2}-m}\cos(e+fx)(1-\sin(e+fx))^{m+\frac{1}{2}}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}\,{}_2F_1\left(\frac{1}{2}(2m-1),\frac{1}{2}(2m+1);\frac{1}{2}(2m+3)\right.}{f(2m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(1 - m),x]
```

```
[Out] (2^(3/2 - m)*c^2*Cos[e + f*x]*Hypergeometric2F1[(-1 + 2*m)/2, (1 + 2*m)/2,
(3 + 2*m)/2, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*x])^(1/2 + m)*(a + a*Sin[
e + f*x])^m*(c - c*Sin[e + f*x])^(-1 - m))/(f*(1 + 2*m))
```

Rule 2745

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e
 + f*x])^FracPart[m]*(c + d*Sin[e + f*x])^FracPart[m])/Cos[e + f*x]^(2*Frac
Part[m]), Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; Fr
eeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
&& (FractionQ[m] ||  !FractionQ[n])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
```

```
a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^{1-m}\,dx = \left(\cos^{-2m}(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^m\right)\int \cos^{2m}(e$$

$$= \frac{\left(c^2\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{\frac{1}{2}(-1-2m)+m}(c+\right.}{}$$

$$= \frac{\left(2^{\frac{1}{2}-m}c^3\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-\frac{1}{2}+\frac{1}{2}(-1-2\right.}{}$$

$$= \frac{2^{\frac{3}{2}-m}c^2\cos(e+fx)\,{}_2F_1\left(\frac{1}{2}(-1+2m),\frac{1}{2}(1+2m);\frac{1}{2}(3+2m);\frac{1}{2}(1+\sin\right.}{f}$$

Mathematica [C] time = 7.55476, size = 602, normalized size = 5.28

$$-\frac{}{f(2m-1)\left(2\tan^2\left(\frac{1}{8}(2e+2fx-\pi)\right)\left(2mF_1\left(\frac{3}{2}-m;1-2m,2;\frac{5}{2}-m;\tan^2\left(\frac{1}{8}(-2e-2fx+\pi)\right),-\tan^2\left(\frac{1}{8}(2e+2fx\right.\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(1 - m),x]
```

```
[Out] -((2^(2 - m)*c*(-3 + 2*m)*(AppellF1[1/2 - m, -2*m, 2, 3/2 - m, Tan[(-2*e +
Pi - 2*f*x)/8]^2, -Tan[(2*e - Pi + 2*f*x)/8]^2] - AppellF1[1/2 - m, -2*m, 3
, 3/2 - m, Tan[(-2*e + Pi - 2*f*x)/8]^2, -Tan[(2*e - Pi + 2*f*x)/8]^2])*Cos
[(2*e + Pi + 2*f*x)/4]^(3 - 2*m)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^(2*(
-1 + m))*(-1 + Sin[e + f*x])*(a*(1 + Sin[e + f*x]))^m)/(f*(-1 + 2*m)*(c - c
*Sin[e + f*x])^m*((-3 + 2*m)*AppellF1[1/2 - m, -2*m, 2, 3/2 - m, Tan[(-2*e
+ Pi - 2*f*x)/8]^2, -Tan[(2*e - Pi + 2*f*x)/8]^2] + (3 - 2*m)*AppellF1[1/2
- m, -2*m, 3, 3/2 - m, Tan[(-2*e + Pi - 2*f*x)/8]^2, -Tan[(2*e - Pi + 2*f*x
)/8]^2] + 2*(2*m*AppellF1[3/2 - m, 1 - 2*m, 2, 5/2 - m, Tan[(-2*e + Pi - 2*
f*x)/8]^2, -Tan[(2*e - Pi + 2*f*x)/8]^2] - 2*m*AppellF1[3/2 - m, 1 - 2*m, 3
, 5/2 - m, Tan[(-2*e + Pi - 2*f*x)/8]^2, -Tan[(2*e - Pi + 2*f*x)/8]^2] + 2*
AppellF1[3/2 - m, -2*m, 3, 5/2 - m, Tan[(-2*e + Pi - 2*f*x)/8]^2, -Tan[(2*e
 - Pi + 2*f*x)/8]^2] - 3*AppellF1[3/2 - m, -2*m, 4, 5/2 - m, Tan[(-2*e + Pi
 - 2*f*x)/8]^2, -Tan[(2*e - Pi + 2*f*x)/8]^2])*Tan[(2*e - Pi + 2*f*x)/8]^2)
))
```

Maple [F] time = 0.256, size = 0, normalized size = 0.

$$\int \left(a+a\sin\left(fx+e\right)\right)^m\left(c-c\sin\left(fx+e\right)\right)^{1-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1-m),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1-m),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(fx+e)+a)^m(-c\sin(fx+e)+c)^{-m+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1-m),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(-m + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((a\sin(fx+e)+a)^m(-c\sin(fx+e)+c)^{-m+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1-m),x, algorithm="fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(-m + 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**(1-m),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(fx+e)+a)^m(-c\sin(fx+e)+c)^{-m+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1-m),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(-m + 1), x)
```

3.424 $\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^{2-m}\,dx$

Optimal. Leaf size=114

$$\frac{c^3 2^{\frac{5}{2}-m}\cos(e+fx)(1-\sin(e+fx))^{m+\frac{1}{2}}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}\,{}_2F_1\left(\frac{1}{2}(2m-3),\frac{1}{2}(2m+1);\frac{1}{2}(2m+\right.}{f(2m+1)}$$

```
[Out] (2^(5/2 - m)*c^3*Cos[e + f*x]*Hypergeometric2F1[(-3 + 2*m)/2, (1 + 2*m)/2,
(3 + 2*m)/2, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*x])^(1/2 + m)*(a + a*Sin[
e + f*x])^m*(c - c*Sin[e + f*x])^(-1 - m))/(f*(1 + 2*m))
```

Rubi [A] time = 0.182276, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2745, 2689, 70, 69}

$$\frac{c^3 2^{\frac{5}{2}-m}\cos(e+fx)(1-\sin(e+fx))^{m+\frac{1}{2}}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}\,{}_2F_1\left(\frac{1}{2}(2m-3),\frac{1}{2}(2m+1);\frac{1}{2}(2m+\right.}{f(2m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(2 - m),x]
```

```
[Out] (2^(5/2 - m)*c^3*Cos[e + f*x]*Hypergeometric2F1[(-3 + 2*m)/2, (1 + 2*m)/2,
(3 + 2*m)/2, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*x])^(1/2 + m)*(a + a*Sin[
e + f*x])^m*(c - c*Sin[e + f*x])^(-1 - m))/(f*(1 + 2*m))
```

Rule 2745

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e
 + f*x])^FracPart[m]*(c + d*Sin[e + f*x])^FracPart[m])/Cos[e + f*x]^(2*Frac
Part[m]), Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; Fr
eeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
&& (FractionQ[m] ||  !FractionQ[n])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
```

```
a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^{2-m}\,dx = \left(\cos^{-2m}(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^m\right)\int \cos^{2m}(e+$$

$$= \frac{\left(c^2\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{\frac{1}{2}(-1-2m)+m}(c+c\sin\right.}{}$$

$$= \frac{\left(2^{\frac{3}{2}-m}c^4\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-\frac{1}{2}+\frac{1}{2}(-1-2m)}\right.}{}$$

$$= \frac{2^{\frac{5}{2}-m}c^3\cos(e+fx)\,{}_2F_1\left(\frac{1}{2}(-3+2m),\frac{1}{2}(1+2m);\frac{1}{2}(3+2m);\frac{1}{2}(1+\sin(\right.}{f(1}$$

Mathematica [C] time = 12.7401, size = 1201, normalized size = 10.54

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(2 - m),x]
```

```
[Out] (2^(4 - m)*(-3 + 2*m)*(AppellF1[1/2 - m, -2*m, 3, 3/2 - m, Tan[(-e + Pi/2 -
 f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - 2*AppellF1[1/2 - m, -2*m, 4, 3/2
 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + AppellF1[1
/2 - m, -2*m, 5, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x
)/4]^2])*Cos[(-e + Pi/2 - f*x)/4]*Sin[(-e + Pi/2 - f*x)/4]*Sin[(-e + Pi/2 -
 f*x)/2]^(4 - 2*m)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(2 - m))/(f*
(-1 + 2*m)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^(2*(2 - m))*((-3 + 2*m)*Ap
pellF1[1/2 - m, -2*m, 3, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi
/2 - f*x)/4]^2] + (6 - 4*m)*AppellF1[1/2 - m, -2*m, 4, 3/2 - m, Tan[(-e + P
i/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - 3*AppellF1[1/2 - m, -2*m, 5
, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + 2*m*A
ppellF1[1/2 - m, -2*m, 5, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + P
i/2 - f*x)/4]^2] - 8*AppellF1[3/2 - m, -2*m, 5, 5/2 - m, Tan[(-e + Pi/2 - f
*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2 + 8*Appel
lF1[3/2 - m, -2*m, 5, 5/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2
- f*x)/4]^2]*Cos[(-e + Pi/2 - f*x)/2]*Sec[(-e + Pi/2 - f*x)/4]^2 + 4*m*Appe
llF1[3/2 - m, 1 - 2*m, 3, 5/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + P
i/2 - f*x)/4]^2]*Tan[(-e + Pi/2 - f*x)/4]^2 - 8*m*AppellF1[3/2 - m, 1 - 2*m
, 4, 5/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Tan[
(-e + Pi/2 - f*x)/4]^2 + 4*m*AppellF1[3/2 - m, 1 - 2*m, 5, 5/2 - m, Tan[(-e
 + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Tan[(-e + Pi/2 - f*x)/4]^
2 + 6*AppellF1[3/2 - m, -2*m, 4, 5/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[
(-e + Pi/2 - f*x)/4]^2]*Tan[(-e + Pi/2 - f*x)/4]^2 + 10*AppellF1[3/2 - m, -
2*m, 6, 5/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*T
an[(-e + Pi/2 - f*x)/4]^2))
```

Maple [F] time = 0.289, size = 0, normalized size = 0.

$$\int \left(a+a\sin\left(fx+e\right)\right)^m\left(c-c\sin\left(fx+e\right)\right)^{2-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(2-m),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(2-m),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(fx+e)+a)^m(-c\sin(fx+e)+c)^{-m+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(2-m),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(-m + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((a\sin(fx+e)+a)^m(-c\sin(fx+e)+c)^{-m+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(2-m),x, algorithm="fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(-m + 2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**(2-m),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(fx+e)+a)^m(-c\sin(fx+e)+c)^{-m+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(2-m),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(-m + 2), x)
```

3.425 $\int (a+a\sin(e+fx))(c+d\sin(e+fx))^4\,dx$

Optimal. Leaf size=227

$$-\frac{a\left(112c^2d^2+95c^3d+12c^4+80cd^3+16d^4\right)\cos(e+fx)}{30f}-\frac{a\left(12c^2+35cd+16d^2\right)\cos(e+fx)(c+d\sin(e+fx))^2}{60f}-\frac{ad\,(}{}$$

```
[Out] (a*(8*c^4 + 16*c^3*d + 24*c^2*d^2 + 12*c*d^3 + 3*d^4)*x)/8 - (a*(12*c^4 + 9
5*c^3*d + 112*c^2*d^2 + 80*c*d^3 + 16*d^4)*Cos[e + f*x])/(30*f) - (a*d*(24*
c^3 + 130*c^2*d + 116*c*d^2 + 45*d^3)*Cos[e + f*x]*Sin[e + f*x])/(120*f) -
(a*(12*c^2 + 35*c*d + 16*d^2)*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(60*f) -
 (a*(4*c + 5*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(20*f) - (a*Cos[e + f*
x]*(c + d*Sin[e + f*x])^4)/(5*f)
```

Rubi [A] time = 0.281751, antiderivative size = 227, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2753, 2734}

$$-\frac{a\left(112c^2d^2+95c^3d+12c^4+80cd^3+16d^4\right)\cos(e+fx)}{30f}-\frac{a\left(12c^2+35cd+16d^2\right)\cos(e+fx)(c+d\sin(e+fx))^2}{60f}-\frac{ad\,(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^4,x]
```


```
[Out] (a*(8*c^4 + 16*c^3*d + 24*c^2*d^2 + 12*c*d^3 + 3*d^4)*x)/8 - (a*(12*c^4 + 9
5*c^3*d + 112*c^2*d^2 + 80*c*d^3 + 16*d^4)*Cos[e + f*x])/(30*f) - (a*d*(24*
c^3 + 130*c^2*d + 116*c*d^2 + 45*d^3)*Cos[e + f*x]*Sin[e + f*x])/(120*f) -
(a*(12*c^2 + 35*c*d + 16*d^2)*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(60*f) -
 (a*(4*c + 5*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(20*f) - (a*Cos[e + f*
x]*(c + d*Sin[e + f*x])^4)/(5*f)
```


Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))(c+d\sin(e+fx))^4\,dx = -\frac{a\cos(e+fx)(c+d\sin(e+fx))^4}{5f}+\frac{1}{5}\int (c+d\sin(e+fx))^3(a(5c+4d$$
$$= -\frac{a(4c+5d)\cos(e+fx)(c+d\sin(e+fx))^3}{20f}-\frac{a\cos(e+fx)(c+d\sin(e$$
$$= -\frac{a\left(12c^2+35cd+16d^2\right)\cos(e+fx)(c+d\sin(e+fx))^2}{60f}-\frac{a(4c+5d)\,c}{}$$
$$= \frac{1}{8}a\left(8c^4+16c^3d+24c^2d^2+12cd^3+3d^4\right)x-\frac{a\left(12c^4+95c^3d+112c^2d^2\right.}{}$$

Mathematica [A] time = 1.39053, size = 207, normalized size = 0.91

$$\frac{a(\sin(e+fx)+1)\left(15\left(-8d\left(6c^2d+4c^3+4cd^2+d^3\right)\sin(2(e+fx))+4fx\left(24c^2d^2+16c^3d+8c^4+12cd^3+3d^4\right)+d^3\right.\right.}{480f\left(\mathrm{s}\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^4,x]
```

```
[Out] (a*(1 + Sin[e + f*x])*(-60*(8*c^4 + 32*c^3*d + 36*c^2*d^2 + 24*c*d^3 + 5*d^
4)*Cos[e + f*x] + 10*d^2*(24*c^2 + 16*c*d + 5*d^2)*Cos[3*(e + f*x)] - 6*d^4
*Cos[5*(e + f*x)] + 15*(4*(8*c^4 + 16*c^3*d + 24*c^2*d^2 + 12*c*d^3 + 3*d^4
)*f*x - 8*d*(4*c^3 + 6*c^2*d + 4*c*d^2 + d^3)*Sin[2*(e + f*x)] + d^3*(4*c +
 d)*Sin[4*(e + f*x)])))/(480*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2)
```

Maple [A] time = 0.046, size = 259, normalized size = 1.1

$$\frac{1}{f}\left(-ac^4\cos\left(fx+e\right)+4\,ac^3d\left(-1/2\,\sin\left(fx+e\right)\cos\left(fx+e\right)+1/2\,fx+e/2\right)-2\,ac^2d^2\left(2+\left(\sin\left(fx+e\right)\right)^2\right)\cos\left(fx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(c+d*sin(f*x+e))^4,x)
```

```
[Out] 1/f*(-a*c^4*cos(f*x+e)+4*a*c^3*d*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)
-2*a*c^2*d^2*(2+sin(f*x+e)^2)*cos(f*x+e)+4*a*c*d^3*(-1/4*(sin(f*x+e)^3+3/2*
sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-1/5*a*d^4*(8/3+sin(f*x+e)^4+4/3*sin(f
*x+e)^2)*cos(f*x+e)+a*c^4*(f*x+e)-4*a*c^3*d*cos(f*x+e)+6*a*c^2*d^2*(-1/2*si
n(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-4/3*a*c*d^3*(2+sin(f*x+e)^2)*cos(f*x+e)+
a*d^4*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e))
```

Maxima [A] time = 1.05633, size = 338, normalized size = 1.49

$$\frac{480\left(fx+e\right)ac^4+480\left(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right)\right)ac^3d+960\left(\cos\left(fx+e\right)^3-3\,\cos\left(fx+e\right)\right)ac^2d^2+720\left(2\,fx+2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e))^4,x, algorithm="maxima")
```

```
[Out] 1/480*(480*(f*x + e)*a*c^4 + 480*(2*f*x + 2*e - sin(2*f*x + 2*e))*a*c^3*d +
 960*(cos(f*x + e)^3 - 3*cos(f*x + e))*a*c^2*d^2 + 720*(2*f*x + 2*e - sin(2
*f*x + 2*e))*a*c^2*d^2 + 640*(cos(f*x + e)^3 - 3*cos(f*x + e))*a*c*d^3 + 60
*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*a*c*d^3 - 32*(3*co
s(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*a*d^4 + 15*(12*f*x + 12
*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*a*d^4 - 480*a*c^4*cos(f*x + e)
- 1920*a*c^3*d*cos(f*x + e))/f
```

Fricas [A] time = 1.1881, size = 481, normalized size = 2.12

$$-\frac{24\,ad^4\cos\left(fx+e\right)^5-80\left(3\,ac^2d^2+2\,acd^3+ad^4\right)\cos\left(fx+e\right)^3-15\left(8\,ac^4+16\,ac^3d+24\,ac^2d^2+12\,acd^3+3\,ad^4\right)fx+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e))^4,x, algorithm="fricas")
```

```
[Out] -1/120*(24*a*d^4*cos(f*x + e)^5 - 80*(3*a*c^2*d^2 + 2*a*c*d^3 + a*d^4)*cos(
f*x + e)^3 - 15*(8*a*c^4 + 16*a*c^3*d + 24*a*c^2*d^2 + 12*a*c*d^3 + 3*a*d^4
)*f*x + 120*(a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 + a*d^4)*cos(f*x +
 e) - 15*(2*(4*a*c*d^3 + a*d^4)*cos(f*x + e)^3 - (16*a*c^3*d + 24*a*c^2*d^2
 + 20*a*c*d^3 + 5*a*d^4)*cos(f*x + e))*sin(f*x + e))/f
```

Sympy [A] time = 4.01411, size = 580, normalized size = 2.56

$$\begin{cases} ac^4x-\frac{ac^4\cos\left(e+fx\right)}{f}+2ac^3dx\sin^2\left(e+fx\right)+2ac^3dx\cos^2\left(e+fx\right)-\frac{2ac^3d\sin\left(e+fx\right)\cos\left(e+fx\right)}{f}-\frac{4ac^3d\cos\left(e+fx\right)}{f}+3ac^2d^2x\sin \\ x\left(c+d\sin\left(e\right)\right)^4\left(a\sin\left(e\right)+a\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e))**4,x)
```

```
[Out] Piecewise((a*c**4*x - a*c**4*cos(e + f*x)/f + 2*a*c**3*d*x*sin(e + f*x)**2
+ 2*a*c**3*d*x*cos(e + f*x)**2 - 2*a*c**3*d*sin(e + f*x)*cos(e + f*x)/f - 4
*a*c**3*d*cos(e + f*x)/f + 3*a*c**2*d**2*x*sin(e + f*x)**2 + 3*a*c**2*d**2*
x*cos(e + f*x)**2 - 6*a*c**2*d**2*sin(e + f*x)**2*cos(e + f*x)/f - 3*a*c**2
*d**2*sin(e + f*x)*cos(e + f*x)/f - 4*a*c**2*d**2*cos(e + f*x)**3/f + 3*a*c
*d**3*x*sin(e + f*x)**4/2 + 3*a*c*d**3*x*sin(e + f*x)**2*cos(e + f*x)**2 +
3*a*c*d**3*x*cos(e + f*x)**4/2 - 5*a*c*d**3*sin(e + f*x)**3*cos(e + f*x)/(2
*f) - 4*a*c*d**3*sin(e + f*x)**2*cos(e + f*x)/f - 3*a*c*d**3*sin(e + f*x)*c
os(e + f*x)**3/(2*f) - 8*a*c*d**3*cos(e + f*x)**3/(3*f) + 3*a*d**4*x*sin(e
+ f*x)**4/8 + 3*a*d**4*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 3*a*d**4*x*cos
(e + f*x)**4/8 - a*d**4*sin(e + f*x)**4*cos(e + f*x)/f - 5*a*d**4*sin(e + f
*x)**3*cos(e + f*x)/(8*f) - 4*a*d**4*sin(e + f*x)**2*cos(e + f*x)**3/(3*f)
- 3*a*d**4*sin(e + f*x)*cos(e + f*x)**3/(8*f) - 8*a*d**4*cos(e + f*x)**5/(1
5*f), Ne(f, 0)), (x*(c + d*sin(e))**4*(a*sin(e) + a), True))
```

Giac [A] time = 1.29283, size = 367, normalized size = 1.62

$$-\frac{ad^4\cos\left(5\,fx+5\,e\right)}{80\,f}+\frac{acd^3\cos\left(3\,fx+3\,e\right)}{3\,f}+\frac{acd^3\sin\left(4\,fx+4\,e\right)}{8\,f}+\frac{ad^4\sin\left(4\,fx+4\,e\right)}{32\,f}+\frac{1}{8}\left(8\,ac^4+24\,ac^2d^2+3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e))^4,x, algorithm="giac")
```

```
[Out] -1/80*a*d^4*cos(5*f*x + 5*e)/f + 1/3*a*c*d^3*cos(3*f*x + 3*e)/f + 1/8*a*c*d
^3*sin(4*f*x + 4*e)/f + 1/32*a*d^4*sin(4*f*x + 4*e)/f + 1/8*(8*a*c^4 + 24*a
*c^2*d^2 + 3*a*d^4)*x + 1/2*(4*a*c^3*d + 3*a*c*d^3)*x + 1/48*(24*a*c^2*d^2
+ 5*a*d^4)*cos(3*f*x + 3*e)/f - 1/8*(8*a*c^4 + 36*a*c^2*d^2 + 5*a*d^4)*cos(
f*x + e)/f - (4*a*c^3*d + 3*a*c*d^3)*cos(f*x + e)/f - (a*c^3*d + a*c*d^3)*s
in(2*f*x + 2*e)/f - 1/4*(6*a*c^2*d^2 + a*d^4)*sin(2*f*x + 2*e)/f
```

3.426 $\int (a+a\sin(e+fx))(c+d\sin(e+fx))^3\,dx$

Optimal. Leaf size=162

$$-\frac{a\left(16c^2d+3c^3+12cd^2+4d^3\right)\cos(e+fx)}{6f}-\frac{ad\left(6c^2+20cd+9d^2\right)\sin(e+fx)\cos(e+fx)}{24f}+\frac{1}{8}ax\left(12c^2d+8c^3+12cd^2\right.$$

```
[Out] (a*(8*c^3 + 12*c^2*d + 12*c*d^2 + 3*d^3)*x)/8 - (a*(3*c^3 + 16*c^2*d + 12*c
*d^2 + 4*d^3)*Cos[e + f*x])/(6*f) - (a*d*(6*c^2 + 20*c*d + 9*d^2)*Cos[e + f
*x]*Sin[e + f*x])/(24*f) - (a*(3*c + 4*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])
^2)/(12*f) - (a*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(4*f)
```

Rubi [A] time = 0.189431, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2753, 2734}

$$-\frac{a\left(16c^2d+3c^3+12cd^2+4d^3\right)\cos(e+fx)}{6f}-\frac{ad\left(6c^2+20cd+9d^2\right)\sin(e+fx)\cos(e+fx)}{24f}+\frac{1}{8}ax\left(12c^2d+8c^3+12cd^2\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^3,x]
```

```
[Out] (a*(8*c^3 + 12*c^2*d + 12*c*d^2 + 3*d^3)*x)/8 - (a*(3*c^3 + 16*c^2*d + 12*c
*d^2 + 4*d^3)*Cos[e + f*x])/(6*f) - (a*d*(6*c^2 + 20*c*d + 9*d^2)*Cos[e + f
*x]*Sin[e + f*x])/(24*f) - (a*(3*c + 4*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])
^2)/(12*f) - (a*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(4*f)
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int (a+a\sin(e+fx))(c+d\sin(e+fx))^3\,dx &= -\frac{a\cos(e+fx)(c+d\sin(e+fx))^3}{4f}+\frac{1}{4}\int (c+d\sin(e+fx))^2(a(4c+3d)+\\ &= -\frac{a(3c+4d)\cos(e+fx)(c+d\sin(e+fx))^2}{12f}-\frac{a\cos(e+fx)(c+d\sin(e+\\ &= \frac{1}{8}a\left(8c^3+12c^2d+12cd^2+3d^3\right)x-\frac{a\left(3c^3+16c^2d+12cd^2+4d^3\right)\cos(e+\end{aligned}$$

Mathematica [A] time = 0.805007, size = 124, normalized size = 0.77

$$\frac{a\left(3\left(-8d\left(3c^2+3cd+d^2\right)\sin(2(e+fx))+4fx\left(12c^2d+8c^3+12cd^2+3d^3\right)+d^3\sin(4(e+fx))\right)-24\left(12c^2d+4c^3+\right.}{96f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^3,x]
```

```
[Out] (a*(-24*(4*c^3 + 12*c^2*d + 9*c*d^2 + 3*d^3)*Cos[e + f*x] + 8*d^2*(3*c + d)
*Cos[3*(e + f*x)] + 3*(4*(8*c^3 + 12*c^2*d + 12*c*d^2 + 3*d^3)*f*x - 8*d*(3
*c^2 + 3*c*d + d^2)*Sin[2*(e + f*x)] + d^3*Sin[4*(e + f*x)])))/(96*f)
```

Maple [A] time = 0.039, size = 182, normalized size = 1.1

$$\frac{1}{f}\left(-ac^3\cos\left(fx+e\right)+3\,ac^2d\left(-1/2\,\sin\left(fx+e\right)\cos\left(fx+e\right)+1/2\,fx+e/2\right)-acd^2\left(2+\left(\sin\left(fx+e\right)\right)^2\right)\cos\left(fx+e\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(c+d*sin(f*x+e))^3,x)
```

```
[Out] 1/f*(-a*c^3*cos(f*x+e)+3*a*c^2*d*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)
-a*c*d^2*(2+sin(f*x+e)^2)*cos(f*x+e)+a*d^3*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+
e))*cos(f*x+e)+3/8*f*x+3/8*e)+a*c^3*(f*x+e)-3*a*c^2*d*cos(f*x+e)+3*a*c*d^2*
(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-1/3*a*d^3*(2+sin(f*x+e)^2)*cos(f
*x+e))
```

Maxima [A] time = 1.17298, size = 236, normalized size = 1.46

$$\frac{96\left(fx+e\right)ac^3+72\left(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right)\right)ac^2d+96\left(\cos\left(fx+e\right)^3-3\,\cos\left(fx+e\right)\right)acd^2+72\left(2\,fx+2\,e-\mathrm{s}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 1/96*(96*(f*x + e)*a*c^3 + 72*(2*f*x + 2*e - sin(2*f*x + 2*e))*a*c^2*d + 96
*(cos(f*x + e)^3 - 3*cos(f*x + e))*a*c*d^2 + 72*(2*f*x + 2*e - sin(2*f*x +
2*e))*a*c*d^2 + 32*(cos(f*x + e)^3 - 3*cos(f*x + e))*a*d^3 + 3*(12*f*x + 12
*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*a*d^3 - 96*a*c^3*cos(f*x + e) -
 288*a*c^2*d*cos(f*x + e))/f
```

Fricas [A] time = 1.13413, size = 340, normalized size = 2.1

$$\frac{8\left(3\,acd^2+ad^3\right)\cos\left(fx+e\right)^3+3\left(8\,ac^3+12\,ac^2d+12\,acd^2+3\,ad^3\right)fx-24\left(ac^3+3\,ac^2d+3\,acd^2+ad^3\right)\cos\left(fx+\right.}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/24*(8*(3*a*c*d^2 + a*d^3)*cos(f*x + e)^3 + 3*(8*a*c^3 + 12*a*c^2*d + 12*a
*c*d^2 + 3*a*d^3)*f*x - 24*(a*c^3 + 3*a*c^2*d + 3*a*c*d^2 + a*d^3)*cos(f*x
+ e) + 3*(2*a*d^3*cos(f*x + e)^3 - (12*a*c^2*d + 12*a*c*d^2 + 5*a*d^3)*cos(
f*x + e))*sin(f*x + e))/f
```

Sympy [A] time = 1.81782, size = 386, normalized size = 2.38

$$\begin{cases} ac^3x - \frac{ac^3\cos(e+fx)}{f} + \frac{3ac^2dx\sin^2(e+fx)}{2} + \frac{3ac^2dx\cos^2(e+fx)}{2} - \frac{3ac^2d\sin(e+fx)\cos(e+fx)}{2f} - \frac{3ac^2d\cos(e+fx)}{f} + \frac{3acd^2x\sin^2(e+fx)}{2} + \frac{3acd^2 \cdots}{} \\ x(c+d\sin(e))^3(a\sin(e)+a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((a*c**3*x - a*c**3*cos(e + f*x)/f + 3*a*c**2*d*x*sin(e + f*x)**2/
2 + 3*a*c**2*d*x*cos(e + f*x)**2/2 - 3*a*c**2*d*sin(e + f*x)*cos(e + f*x)/(
2*f) - 3*a*c**2*d*cos(e + f*x)/f + 3*a*c*d**2*x*sin(e + f*x)**2/2 + 3*a*c*d
**2*x*cos(e + f*x)**2/2 - 3*a*c*d**2*sin(e + f*x)**2*cos(e + f*x)/f - 3*a*c
*d**2*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*a*c*d**2*cos(e + f*x)**3/f + 3*a*
d**3*x*sin(e + f*x)**4/8 + 3*a*d**3*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 3
*a*d**3*x*cos(e + f*x)**4/8 - 5*a*d**3*sin(e + f*x)**3*cos(e + f*x)/(8*f) -
 a*d**3*sin(e + f*x)**2*cos(e + f*x)/f - 3*a*d**3*sin(e + f*x)*cos(e + f*x)
**3/(8*f) - 2*a*d**3*cos(e + f*x)**3/(3*f), Ne(f, 0)), (x*(c + d*sin(e))**3
*(a*sin(e) + a), True))
```

Giac [A] time = 1.23924, size = 258, normalized size = 1.59

$$\frac{acd^2\cos(3fx+3e)}{4f} + \frac{ad^3\cos(3fx+3e)}{12f} + \frac{ad^3\sin(4fx+4e)}{32f} - \frac{3acd^2\sin(2fx+2e)}{4f} + \frac{1}{2}\left(2ac^3+3acd^2\right)x + \frac{3}{8}\left(4 \cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/4*a*c*d^2*cos(3*f*x + 3*e)/f + 1/12*a*d^3*cos(3*f*x + 3*e)/f + 1/32*a*d^3
*sin(4*f*x + 4*e)/f - 3/4*a*c*d^2*sin(2*f*x + 2*e)/f + 1/2*(2*a*c^3 + 3*a*c
*d^2)*x + 3/8*(4*a*c^2*d + a*d^3)*x - 1/4*(4*a*c^3 + 9*a*c*d^2)*cos(f*x + e
)/f - 3/4*(4*a*c^2*d + a*d^3)*cos(f*x + e)/f - 1/4*(3*a*c^2*d + a*d^3)*sin(
2*f*x + 2*e)/f
```

3.427 $\int (a+a\sin(e+fx))(c+d\sin(e+fx))^2\,dx$

Optimal. Leaf size=99

$$-\frac{2a\left(c^2+3cd+d^2\right)\cos(e+fx)}{3f}+\frac{1}{2}ax\left(2c^2+2cd+d^2\right)-\frac{a\cos(e+fx)(c+d\sin(e+fx))^2}{3f}-\frac{ad(2c+3d)\sin(e+fx)}{6f}$$

```
[Out] (a*(2*c^2 + 2*c*d + d^2)*x)/2 - (2*a*(c^2 + 3*c*d + d^2)*Cos[e + f*x])/(3*f
) - (a*d*(2*c + 3*d)*Cos[e + f*x]*Sin[e + f*x])/(6*f) - (a*Cos[e + f*x]*(c
+ d*Sin[e + f*x])^2)/(3*f)
```

Rubi [A] time = 0.0926097, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2753, 2734}

$$-\frac{2a\left(c^2+3cd+d^2\right)\cos(e+fx)}{3f}+\frac{1}{2}ax\left(2c^2+2cd+d^2\right)-\frac{a\cos(e+fx)(c+d\sin(e+fx))^2}{3f}-\frac{ad(2c+3d)\sin(e+fx)}{6f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^2,x]
```

```
[Out] (a*(2*c^2 + 2*c*d + d^2)*x)/2 - (2*a*(c^2 + 3*c*d + d^2)*Cos[e + f*x])/(3*f
) - (a*d*(2*c + 3*d)*Cos[e + f*x]*Sin[e + f*x])/(6*f) - (a*Cos[e + f*x]*(c
+ d*Sin[e + f*x])^2)/(3*f)
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int (a+a\sin(e+fx))(c+d\sin(e+fx))^2\,dx &= -\frac{a\cos(e+fx)(c+d\sin(e+fx))^2}{3f}+\frac{1}{3}\int (c+d\sin(e+fx))(a(3c+2d\\ &= \frac{1}{2}a\left(2c^2+2cd+d^2\right)x-\frac{2a\left(c^2+3cd+d^2\right)\cos(e+fx)}{3f}-\frac{ad(2c+3d)\,c\end{aligned}$$

Mathematica [A] time = 0.399076, size = 89, normalized size = 0.9

$$\frac{a\left(-3\left(4c^2+8cd+3d^2\right)\cos(e+fx)+12c^2fx-6cd\sin(2(e+fx))+12cdfx-3d^2\sin(2(e+fx))+d^2\cos(3(e+fx))\right.}{12f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^2,x]
```

```
[Out] (a*(12*c^2*f*x + 12*c*d*f*x + 6*d^2*f*x - 3*(4*c^2 + 8*c*d + 3*d^2)*Cos[e +
 f*x] + d^2*Cos[3*(e + f*x)] - 6*c*d*Sin[2*(e + f*x)] - 3*d^2*Sin[2*(e + f*
x)]))/(12*f)
```

Maple [A] time = 0.033, size = 115, normalized size = 1.2

$$\frac{1}{f}\left(-ac^2\cos\left(fx+e\right)+2\,acd\left(-1/2\,\sin\left(fx+e\right)\cos\left(fx+e\right)+1/2\,fx+e/2\right)-\frac{ad^2\left(2+\left(\sin\left(fx+e\right)\right)^2\right)\cos\left(fx+e\right)}{3}+a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(c+d*sin(f*x+e))^2,x)
```

```
[Out] 1/f*(-a*c^2*cos(f*x+e)+2*a*c*d*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-1
/3*a*d^2*(2+sin(f*x+e)^2)*cos(f*x+e)+a*c^2*(f*x+e)-2*a*c*d*cos(f*x+e)+a*d^2
*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e))
```

Maxima [A] time = 0.986632, size = 151, normalized size = 1.53

$$\frac{12\left(fx+e\right)ac^2+6\left(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right)\right)acd+4\left(\cos\left(fx+e\right)^3-3\,\cos\left(fx+e\right)\right)ad^2+3\left(2\,fx+2\,e-\sin\left(2\,fx+\right.\right.}{12\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/12*(12*(f*x + e)*a*c^2 + 6*(2*f*x + 2*e - sin(2*f*x + 2*e))*a*c*d + 4*(co
s(f*x + e)^3 - 3*cos(f*x + e))*a*d^2 + 3*(2*f*x + 2*e - sin(2*f*x + 2*e))*a
*d^2 - 12*a*c^2*cos(f*x + e) - 24*a*c*d*cos(f*x + e))/f
```

Fricas [A] time = 1.09289, size = 215, normalized size = 2.17

$$\frac{2\,ad^2\cos\left(fx+e\right)^3+3\left(2\,ac^2+2\,acd+ad^2\right)fx-3\left(2\,acd+ad^2\right)\cos\left(fx+e\right)\sin\left(fx+e\right)-6\left(ac^2+2\,acd+ad^2\right)\cos\left(f\right.}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/6*(2*a*d^2*cos(f*x + e)^3 + 3*(2*a*c^2 + 2*a*c*d + a*d^2)*f*x - 3*(2*a*c*
d + a*d^2)*cos(f*x + e)*sin(f*x + e) - 6*(a*c^2 + 2*a*c*d + a*d^2)*cos(f*x
+ e))/f
```

Sympy [A] time = 0.802562, size = 199, normalized size = 2.01

$$\begin{cases} ac^2x - \frac{ac^2\cos(e+fx)}{f} + acdx\sin^2(e+fx) + acdx\cos^2(e+fx) - \frac{acd\sin(e+fx)\cos(e+fx)}{f} - \frac{2acd\cos(e+fx)}{f} + \frac{ad^2x\sin^2(e+fx)}{2} + \\ x(c+d\sin(e))^2(a\sin(e)+a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((a*c**2*x - a*c**2*cos(e + f*x)/f + a*c*d*x*sin(e + f*x)**2 + a*c
*d*x*cos(e + f*x)**2 - a*c*d*sin(e + f*x)*cos(e + f*x)/f - 2*a*c*d*cos(e +
f*x)/f + a*d**2*x*sin(e + f*x)**2/2 + a*d**2*x*cos(e + f*x)**2/2 - a*d**2*s
in(e + f*x)**2*cos(e + f*x)/f - a*d**2*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*
a*d**2*cos(e + f*x)**3/(3*f), Ne(f, 0)), (x*(c + d*sin(e))**2*(a*sin(e) + a
), True))
```

Giac [A] time = 1.27741, size = 158, normalized size = 1.6

$$acdx + \frac{ad^2\cos(3fx+3e)}{12f} - \frac{2acd\cos(fx+e)}{f} - \frac{acd\sin(2fx+2e)}{2f} - \frac{ad^2\sin(2fx+2e)}{4f} + \frac{1}{2}(2ac^2+ad^2)x - \frac{(4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] a*c*d*x + 1/12*a*d^2*cos(3*f*x + 3*e)/f - 2*a*c*d*cos(f*x + e)/f - 1/2*a*c*
d*sin(2*f*x + 2*e)/f - 1/4*a*d^2*sin(2*f*x + 2*e)/f + 1/2*(2*a*c^2 + a*d^2)
*x - 1/4*(4*a*c^2 + 3*a*d^2)*cos(f*x + e)/f
```

3.428 $\int (a+a\sin(e+fx))(c+d\sin(e+fx))\,dx$

Optimal. Leaf size=48

$$-\frac{a(c+d)\cos(e+fx)}{f}+\frac{1}{2}ax(2c+d)-\frac{ad\sin(e+fx)\cos(e+fx)}{2f}$$

```
[Out] (a*(2*c + d)*x)/2 - (a*(c + d)*Cos[e + f*x])/f - (a*d*Cos[e + f*x]*Sin[e +
f*x])/(2*f)
```

Rubi [A] time = 0.023605, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2734}

$$-\frac{a(c+d)\cos(e+fx)}{f}+\frac{1}{2}ax(2c+d)-\frac{ad\sin(e+fx)\cos(e+fx)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])*(c + d*Sin[e + f*x]),x]
```

```
[Out] (a*(2*c + d)*x)/2 - (a*(c + d)*Cos[e + f*x])/f - (a*d*Cos[e + f*x]*Sin[e +
f*x])/(2*f)
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))(c+d\sin(e+fx))\,dx=\frac{1}{2}a(2c+d)x-\frac{a(c+d)\cos(e+fx)}{f}-\frac{ad\cos(e+fx)\sin(e+fx)}{2f}$$

Mathematica [A] time = 0.112512, size = 45, normalized size = 0.94

$$\frac{a(-4(c+d)\cos(e+fx)+4cfx-d\sin(2(e+fx))+2de+2dfx)}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])*(c + d*Sin[e + f*x]),x]
```

```
[Out] (a*(2*d*e + 4*c*f*x + 2*d*f*x - 4*(c + d)*Cos[e + f*x] - d*Sin[2*(e + f*x)]
))/(4*f)
```

Maple [A] time = 0.029, size = 59, normalized size = 1.2

$$\frac{1}{f}\left(da\left(-\frac{\sin\left(fx+e\right)\cos\left(fx+e\right)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-ca\cos\left(fx+e\right)-da\cos\left(fx+e\right)+ca\left(fx+e\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(c+d*sin(f*x+e)),x)
```

```
[Out] 1/f*(d*a*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-c*a*cos(f*x+e)-d*a*cos(
f*x+e)+c*a*(f*x+e))
```

Maxima [A] time = 1.10094, size = 77, normalized size = 1.6

$$\frac{4\left(fx+e\right)ac+\left(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right)\right)ad-4\,ac\cos\left(fx+e\right)-4\,ad\cos\left(fx+e\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/4*(4*(f*x + e)*a*c + (2*f*x + 2*e - sin(2*f*x + 2*e))*a*d - 4*a*c*cos(f*x
 + e) - 4*a*d*cos(f*x + e))/f
```

Fricas [A] time = 1.06934, size = 120, normalized size = 2.5

$$-\frac{ad\cos\left(fx+e\right)\sin\left(fx+e\right)-(2\,ac+ad)fx+2\,(ac+ad)\cos\left(fx+e\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] -1/2*(a*d*cos(f*x + e)*sin(f*x + e) - (2*a*c + a*d)*f*x + 2*(a*c + a*d)*cos
(f*x + e))/f
```

Sympy [A] time = 0.331355, size = 94, normalized size = 1.96

$$\begin{cases} acx-\frac{ac\cos\left(e+fx\right)}{f}+\frac{adx\sin^{2}\left(e+fx\right)}{2}+\frac{adx\cos^{2}\left(e+fx\right)}{2}-\frac{ad\sin\left(e+fx\right)\cos\left(e+fx\right)}{2f}-\frac{ad\cos\left(e+fx\right)}{f} & \text{for}\ f\neq 0\\ x\left(c+d\sin\left(e\right)\right)\left(a\sin\left(e\right)+a\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e)),x)
```

```
[Out] Piecewise((a*c*x - a*c*cos(e + f*x)/f + a*d*x*sin(e + f*x)**2/2 + a*d*x*cos
(e + f*x)**2/2 - a*d*sin(e + f*x)*cos(e + f*x)/(2*f) - a*d*cos(e + f*x)/f,
Ne(f, 0)), (x*(c + d*sin(e))*(a*sin(e) + a), True))
```

Giac [A] time = 1.25065, size = 74, normalized size = 1.54

$$acx+\frac{1}{2}\,adx-\frac{ac\cos\left(fx+e\right)}{f}-\frac{ad\cos\left(fx+e\right)}{f}-\frac{ad\sin\left(2\,fx+2\,e\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] a*c*x + 1/2*a*d*x - a*c*cos(f*x + e)/f - a*d*cos(f*x + e)/f - 1/4*a*d*sin(2
*f*x + 2*e)/f
```

3.429 $\int (a + a\sin(e + fx))\,dx$

Optimal. Leaf size=16

$$ax - \frac{a\cos(e + fx)}{f}$$

```
[Out] a*x - (a*Cos[e + f*x])/f
```

Rubi [A] time = 0.0082288, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {2638}

$$ax - \frac{a\cos(e + fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[a + a*Sin[e + f*x],x]

[Out] a*x - (a*Cos[e + f*x])/f
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a + a\sin(e + fx))\,dx &= ax + a\int \sin(e + fx)\,dx\\ &= ax - \frac{a\cos(e + fx)}{f}\end{aligned}$$

Mathematica [A] time = 0.0064691, size = 27, normalized size = 1.69

$$\frac{a\sin(e)\sin(fx)}{f} - \frac{a\cos(e)\cos(fx)}{f} + ax$$

Antiderivative was successfully verified.

```
[In] Integrate[a + a*Sin[e + f*x],x]

[Out] a*x - (a*Cos[e]*Cos[f*x])/f + (a*Sin[e]*Sin[f*x])/f
```

Maple [A] time = 0.007, size = 17, normalized size = 1.1

$$ax - \frac{\cos\left(fx + e\right)a}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+a*sin(f*x+e),x)
```

```
[Out] a*x-a*cos(f*x+e)/f
```

Maxima [A] time = 1.12515, size = 22, normalized size = 1.38

$$ax - \frac{a\cos\left(fx + e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+a*sin(f*x+e),x, algorithm="maxima")
```

```
[Out] a*x - a*cos(f*x + e)/f
```

Fricas [A] time = 1.02352, size = 38, normalized size = 2.38

$$\frac{afx - a\cos\left(fx + e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+a*sin(f*x+e),x, algorithm="fricas")
```

```
[Out] (a*f*x - a*cos(f*x + e))/f
```

Sympy [A] time = 0.158875, size = 19, normalized size = 1.19

$$ax + a\left(\begin{cases} -\frac{\cos\left(e+fx\right)}{f} & \text{for}\ f \neq 0 \\ x\sin\left(e\right) & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+a*sin(f*x+e),x)
```

```
[Out] a*x + a*Piecewise((-cos(e + f*x)/f, Ne(f, 0)), (x*sin(e), True))
```

Giac [A] time = 1.29333, size = 23, normalized size = 1.44

$$ax - \frac{a\cos\left(fx + e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+a*sin(f*x+e),x, algorithm="giac")
```

```
[Out] a*x - a*cos(f*x + e)/f
```

3.430 $\int \frac{a+a\sin(e+fx)}{c+d\sin(e+fx)}\,dx$

Optimal. Leaf size=63

$$\frac{ax}{d}-\frac{2a(c-d)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{df\sqrt{c^2-d^2}}$$

```
[Out] (a*x)/d - (2*a*(c - d)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(d
*Sqrt[c^2 - d^2]*f)
```

Rubi [A] time = 0.0911328, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2735, 2660, 618, 204}

$$\frac{ax}{d}-\frac{2a(c-d)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{df\sqrt{c^2-d^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])/(c + d*Sin[e + f*x]),x]
```

```
[Out] (a*x)/d - (2*a*(c - d)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(d
*Sqrt[c^2 - d^2]*f)
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{a+a\sin(e+fx)}{c+d\sin(e+fx)}\,dx = \frac{ax}{d} - \frac{(a(c-d))\int \frac{1}{c+d\sin(e+fx)}\,dx}{d}$$

$$= \frac{ax}{d} - \frac{(2a(c-d))\,\text{Subst}\left(\int \frac{1}{c+2dx+cx^2}\,dx, x, \tan\left(\frac{1}{2}(e+fx)\right)\right)}{df}$$

$$= \frac{ax}{d} + \frac{(4a(c-d))\,\text{Subst}\left(\int \frac{1}{-4(c^2-d^2)-x^2}\,dx, x, 2d+2c\tan\left(\frac{1}{2}(e+fx)\right)\right)}{df}$$

$$= \frac{ax}{d} - \frac{2a(c-d)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{d\sqrt{c^2-d^2}f}$$

Mathematica [C] time = 0.321298, size = 182, normalized size = 2.89

$$\frac{a(\sin(e+fx)+1)\left(fx\sqrt{c^2-d^2}\sqrt{(\cos(e)-i\sin(e))^2}-2(c-d)(\cos(e)-i\sin(e))\tan^{-1}\left(\frac{(\cos(e)-i\sin(e))\sec\left(\frac{fx}{2}\right)\left(c\sin\left(\frac{fx}{2}\right)+d\cos\left(e\right.\right.}{\sqrt{c^2-d^2}\sqrt{(\cos(e)-i\sin(e))^2}}\right.\right.}{df\sqrt{c^2-d^2}\sqrt{(\cos(e)-i\sin(e))^2}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])/(c + d*Sin[e + f*x]),x]
```

```
[Out] (a*(-2*(c - d)*ArcTan[(Sec[(f*x)/2]*(Cos[e] - I*Sin[e])*(d*Cos[e + (f*x)/2]
 + c*Sin[(f*x)/2]))/(Sqrt[c^2 - d^2]*Sqrt[(Cos[e] - I*Sin[e])^2])]*(Cos[e]
- I*Sin[e]) + Sqrt[c^2 - d^2]*f*x*Sqrt[(Cos[e] - I*Sin[e])^2])*(1 + Sin[e +
 f*x]))/(d*Sqrt[c^2 - d^2]*f*Sqrt[(Cos[e] - I*Sin[e])^2]*(Cos[(e + f*x)/2]
+ Sin[(e + f*x)/2])^2)
```

Maple [B] time = 0.083, size = 119, normalized size = 1.9

$$-2\,\frac{ca}{df\sqrt{c^2-d^2}}\arctan\left(1/2\,\frac{2\,c\tan\left(1/2\,fx+e/2\right)+2\,d}{\sqrt{c^2-d^2}}\right)+2\,\frac{a}{f\sqrt{c^2-d^2}}\arctan\left(1/2\,\frac{2\,c\tan\left(1/2\,fx+e/2\right)+2\,d}{\sqrt{c^2-d^2}}\right)+2\,\frac{a\,\text{ar}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

```
[Out] -2/f*a/d/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^
(1/2))*c+2/f*a/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2
-d^2)^(1/2))+2/f*a/d*arctan(tan(1/2*f*x+1/2*e))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm="maxima")
```

[Out] Exception raised: ValueError

Fricas [A] time = 1.18265, size = 520, normalized size = 8.25

$$\left[\frac{2\,afx + a\sqrt{-\frac{c-d}{c+d}}\log\left(\frac{\left(2\,c^2-d^2\right)\cos\left(fx+e\right)^2-2\,cd\sin\left(fx+e\right)-c^2-d^2+2\left(\left(c^2+cd\right)\cos\left(fx+e\right)\sin\left(fx+e\right)+\left(cd+d^2\right)\cos\left(fx+e\right)\right)\sqrt{-\frac{c-d}{c+d}}}{d^2\cos\left(fx+e\right)^2-2\,cd\sin\left(fx+e\right)-c^2-d^2}\right)}{2\,df}, \frac{afx + a\sqrt{\cdots}}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*f*x + a*sqrt(-(c - d)/(c + d))*log(((2*c^2 - d^2)*cos(f*x + e)^2
- 2*c*d*sin(f*x + e) - c^2 - d^2 + 2*((c^2 + c*d)*cos(f*x + e)*sin(f*x + e)
 + (c*d + d^2)*cos(f*x + e))*sqrt(-(c - d)/(c + d)))/(d^2*cos(f*x + e)^2 -
2*c*d*sin(f*x + e) - c^2 - d^2)))/(d*f), (a*f*x + a*sqrt((c - d)/(c + d))*a
rctan(-(c*sin(f*x + e) + d)*sqrt((c - d)/(c + d))/((c - d)*cos(f*x + e))))/
(d*f)]
```

Sympy [A] time = 162.134, size = 269, normalized size = 4.27

$$\begin{cases} \frac{\tilde{\infty}x(a\sin(e)+a)}{\sin(e)} & \text{for}\ c = 0 \wedge d = 0 \wedge f = 0 \\ \frac{afx\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{df\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-df} - \frac{afx}{df\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-df} + \frac{4a}{df\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-df} & \text{for}\ c = -d \\ \frac{ax-\frac{a\cos(e+fx)}{f}}{c} & \text{for}\ d = 0 \\ \frac{x(a\sin(e)+a)}{c+d\sin(e)} & \text{for}\ f = 0 \\ \frac{ax+\frac{a\log\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{f}}{d} & \text{for}\ c = 0 \\ \frac{acfx}{cdf+d^2f} + \frac{adfx}{cdf+d^2f} + \frac{a\sqrt{-c^2+d^2}\log\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+\frac{d}{c}-\frac{\sqrt{-c^2+d^2}}{c}\right)}{cdf+d^2f} - \frac{a\sqrt{-c^2+d^2}\log\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+\frac{d}{c}+\frac{\sqrt{-c^2+d^2}}{c}\right)}{cdf+d^2f} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

```
[Out] Piecewise((zoo*x*(a*sin(e) + a)/sin(e), Eq(c, 0) & Eq(d, 0) & Eq(f, 0)), (a
*f*x*tan(e/2 + f*x/2)/(d*f*tan(e/2 + f*x/2) - d*f) - a*f*x/(d*f*tan(e/2 + f
*x/2) - d*f) + 4*a/(d*f*tan(e/2 + f*x/2) - d*f), Eq(c, -d)), ((a*x - a*cos(
e + f*x)/f)/c, Eq(d, 0)), (x*(a*sin(e) + a)/(c + d*sin(e)), Eq(f, 0)), ((a*
x + a*log(tan(e/2 + f*x/2))/f)/d, Eq(c, 0)), (a*c*f*x/(c*d*f + d**2*f) + a*
d*f*x/(c*d*f + d**2*f) + a*sqrt(-c**2 + d**2)*log(tan(e/2 + f*x/2) + d/c -
sqrt(-c**2 + d**2)/c)/(c*d*f + d**2*f) - a*sqrt(-c**2 + d**2)*log(tan(e/2 +
 f*x/2) + d/c + sqrt(-c**2 + d**2)/c)/(c*d*f + d**2*f), True))
```

Giac [A] time = 1.32485, size = 116, normalized size = 1.84

$$\frac{\frac{(fx+e)a}{d} - \frac{2\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+d}{\sqrt{c^2-d^2}}\right)\right)(ac-ad)}{\sqrt{c^2-d^2}d}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] ((f*x + e)*a/d - 2*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan
(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))*(a*c - a*d)/(sqrt(c^2 - d^2)*d))/f
```

3.431 $\int \frac{a+a\sin(e+fx)}{(c+d\sin(e+fx))^2}\,dx$

Optimal. Leaf size=83

$$\frac{2a\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{f(c+d)\sqrt{c^2-d^2}}-\frac{a\cos(e+fx)}{f(c+d)(c+d\sin(e+fx))}$$

```
[Out] (2*a*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/((c + d)*Sqrt[c^2 -
d^2]*f) - (a*Cos[e + f*x])/((c + d)*f*(c + d*Sin[e + f*x]))
```

Rubi [A] time = 0.091673, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2754, 12, 2660, 618, 204}

$$\frac{2a\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{f(c+d)\sqrt{c^2-d^2}}-\frac{a\cos(e+fx)}{f(c+d)(c+d\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])/(c + d*Sin[e + f*x])^2,x]
```

```
[Out] (2*a*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/((c + d)*Sqrt[c^2 -
d^2]*f) - (a*Cos[e + f*x])/((c + d)*f*(c + d*Sin[e + f*x]))
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+a\sin(e+fx)}{(c+d\sin(e+fx))^2}\,dx &= -\frac{a\cos(e+fx)}{(c+d)f(c+d\sin(e+fx))}-\frac{\int \frac{a(c-d)}{c+d\sin(e+fx)}\,dx}{-c^2+d^2}\\ &= -\frac{a\cos(e+fx)}{(c+d)f(c+d\sin(e+fx))}+\frac{a\int \frac{1}{c+d\sin(e+fx)}\,dx}{c+d}\\ &= -\frac{a\cos(e+fx)}{(c+d)f(c+d\sin(e+fx))}+\frac{(2a)\,\mathrm{Subst}\left(\int \frac{1}{c+2dx+cx^2}\,dx,x,\tan\left(\frac{1}{2}(e+fx)\right)\right)}{(c+d)f}\\ &= -\frac{a\cos(e+fx)}{(c+d)f(c+d\sin(e+fx))}-\frac{(4a)\,\mathrm{Subst}\left(\int \frac{1}{-4(c^2-d^2)-x^2}\,dx,x,2d+2c\tan\left(\frac{1}{2}(e+fx)\right)\right)}{(c+d)f}\\ &= \frac{2a\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{(c+d)\sqrt{c^2-d^2}f}-\frac{a\cos(e+fx)}{(c+d)f(c+d\sin(e+fx))}\end{aligned}$$

Mathematica [C] time = 0.556515, size = 220, normalized size = 2.65

$$\frac{a(\sin(e+fx)+1)\left(2\sqrt{c^2-d^2}\csc(e)\sqrt{(\cos(e)-i\sin(e))^2}(c\cos(e)+d\sin(fx))+4d(\cos(e)-i\sin(e))(c+d\sin(e+fx))\,\mathrm{ta}\right.}{2df(c+d)\sqrt{c^2-d^2}\sqrt{(\cos(e)-i\sin(e))^2}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^2(c+d\,\mathrm{si}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])/(c + d*Sin[e + f*x])^2,x]
```

```
[Out] (a*(1 + Sin[e + f*x])*(2*Sqrt[c^2 - d^2]*Csc[e]*Sqrt[(Cos[e] - I*Sin[e])^2]
*(c*Cos[e] + d*Sin[f*x]) + 4*d*ArcTan[(Sec[(f*x)/2]*(Cos[e] - I*Sin[e])*(d*
Cos[e + (f*x)/2] + c*Sin[(f*x)/2]))/(Sqrt[c^2 - d^2]*Sqrt[(Cos[e] - I*Sin[e
])^2])]*(Cos[e] - I*Sin[e])*(c + d*Sin[e + f*x])))/(2*d*(c + d)*Sqrt[c^2 -
d^2]*f*Sqrt[(Cos[e] - I*Sin[e])^2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2*
(c + d*Sin[e + f*x]))
```

Maple [A] time = 0.102, size = 147, normalized size = 1.8

$$-2\,\frac{da\tan\left(1/2\,fx+e/2\right)}{f\left(c\left(\tan\left(1/2\,fx+e/2\right)\right)^2+2\,\tan\left(1/2\,fx+e/2\right)d+c\right)(c+d)\,c}-2\,\frac{a}{f\left(c\left(\tan\left(1/2\,fx+e/2\right)\right)^2+2\,\tan\left(1/2\,fx+e/2\right)c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^2,x)
```

```
[Out] -2/f*a/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)*d/(c+d)/c*tan(1/2*
f*x+1/2*e)-2/f*a/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)/(c+d)+2/
```

```
f*a/(c+d)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)
^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.42825, size = 790, normalized size = 9.52

$$\left[-\frac{\left(ad\sin\left(fx+e\right)+ac\right)\sqrt{-c^2+d^2}\log\left(\frac{\left(2c^2-d^2\right)\cos\left(fx+e\right)^2-2cd\sin\left(fx+e\right)-c^2-d^2+2\left(c\cos\left(fx+e\right)\sin\left(fx+e\right)+d\cos\left(fx+e\right)\right)\sqrt{-c^2+d^2}}{d^2\cos\left(fx+e\right)^2-2cd\sin\left(fx+e\right)-c^2-d^2}\right)+2\cdots}{2\left(\left(c^3d+c^2d^2-cd^3-d^4\right)f\sin\left(fx+e\right)+\left(c^4+c^3d-c^2d^2-cd^3\right)f\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] [-1/2*((a*d*sin(f*x + e) + a*c)*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(f*x
 + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x + e) +
 d*cos(f*x + e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e)
 - c^2 - d^2)) + 2*(a*c^2 - a*d^2)*cos(f*x + e))/((c^3*d + c^2*d^2 - c*d^3
- d^4)*f*sin(f*x + e) + (c^4 + c^3*d - c^2*d^2 - c*d^3)*f), -((a*d*sin(f*x
+ e) + a*c)*sqrt(c^2 - d^2)*arctan(-(c*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*c
os(f*x + e))) + (a*c^2 - a*d^2)*cos(f*x + e))/((c^3*d + c^2*d^2 - c*d^3 - d
^4)*f*sin(f*x + e) + (c^4 + c^3*d - c^2*d^2 - c*d^3)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.30054, size = 174, normalized size = 2.1

$$\frac{2\left(\frac{\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+d}{\sqrt{c^2-d^2}}\right)\right)a}{\sqrt{c^2-d^2}(c+d)}-\frac{ad\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+ac}{\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+c\right)\left(c^2+cd\right)}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 2*((pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e
) + d)/sqrt(c^2 - d^2)))*a/(sqrt(c^2 - d^2)*(c + d)) - (a*d*tan(1/2*f*x + 1
/2*e) + a*c)/((c*tan(1/2*f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e) + c)*(c^
2 + c*d)))/f
```

3.432 $\int \frac{a+a\sin(e+fx)}{(c+d\sin(e+fx))^3}\,dx$

Optimal. Leaf size=134

$$\frac{a(2c-d)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{f(c+d)\left(c^2-d^2\right)^{3/2}}-\frac{a(c-2d)\cos(e+fx)}{2f(c-d)(c+d)^2(c+d\sin(e+fx))}-\frac{a\cos(e+fx)}{2f(c+d)(c+d\sin(e+fx))^2}$$

```
[Out] (a*(2*c - d)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/((c + d)*(c^
2 - d^2)^(3/2)*f) - (a*Cos[e + f*x])/(2*(c + d)*f*(c + d*Sin[e + f*x])^2) -
 (a*(c - 2*d)*Cos[e + f*x])/(2*(c - d)*(c + d)^2*f*(c + d*Sin[e + f*x]))
```

Rubi [A] time = 0.184297, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2754, 12, 2660, 618, 204}

$$\frac{a(2c-d)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{f(c+d)\left(c^2-d^2\right)^{3/2}}-\frac{a(c-2d)\cos(e+fx)}{2f(c-d)(c+d)^2(c+d\sin(e+fx))}-\frac{a\cos(e+fx)}{2f(c+d)(c+d\sin(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])/(c + d*Sin[e + f*x])^3,x]
```



```
[Out] (a*(2*c - d)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/((c + d)*(c^
2 - d^2)^(3/2)*f) - (a*Cos[e + f*x])/(2*(c + d)*f*(c + d*Sin[e + f*x])^2) -
 (a*(c - 2*d)*Cos[e + f*x])/(2*(c - d)*(c + d)^2*f*(c + d*Sin[e + f*x]))
```


Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{a+a\sin(e+fx)}{(c+d\sin(e+fx))^3}\,dx = -\frac{a\cos(e+fx)}{2(c+d)f(c+d\sin(e+fx))^2} - \frac{\int \frac{-2a(c-d)-a(c-d)\sin(e+fx)}{(c+d\sin(e+fx))^2}\,dx}{2\left(c^2-d^2\right)}$$

$$= -\frac{a\cos(e+fx)}{2(c+d)f(c+d\sin(e+fx))^2} - \frac{a(c-2d)\cos(e+fx)}{2(c-d)(c+d)^2f(c+d\sin(e+fx))} + \frac{\int \frac{a(c-d)(2c-d)}{c+d\sin(e+fx)}\,dx}{2\left(c^2-d^2\right)^2}$$

$$= -\frac{a\cos(e+fx)}{2(c+d)f(c+d\sin(e+fx))^2} - \frac{a(c-2d)\cos(e+fx)}{2(c-d)(c+d)^2f(c+d\sin(e+fx))} + \frac{(a(2c-d))\int \frac{1}{c+d\sin(}}{2(c-d)(c+d}$$

$$= -\frac{a\cos(e+fx)}{2(c+d)f(c+d\sin(e+fx))^2} - \frac{a(c-2d)\cos(e+fx)}{2(c-d)(c+d)^2f(c+d\sin(e+fx))} + \frac{(a(2c-d))\,\text{Subst}\big(}{(}$$

$$= -\frac{a\cos(e+fx)}{2(c+d)f(c+d\sin(e+fx))^2} - \frac{a(c-2d)\cos(e+fx)}{2(c-d)(c+d)^2f(c+d\sin(e+fx))} - \frac{(2a(2c-d))\,\text{Subst}\big(}{}$$

$$= \frac{a(2c-d)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{(c-d)(c+d)^2\sqrt{c^2-d^2}\,f} - \frac{a\cos(e+fx)}{2(c+d)f(c+d\sin(e+fx))^2} - \frac{a(c-2d)\cos(e+}{2(c-d)(c+d)^2f(c+d\,\text{s}}$$

Mathematica [C] time = 1.14404, size = 242, normalized size = 1.81

$$\frac{a(\sin(e+fx)+1)\left(\frac{4(2c-d)(\cos(e)-i\sin(e))\tan^{-1}\left(\frac{(\cos(e)-i\sin(e))\sec\left(\frac{fx}{2}\right)\left(c\sin\left(\frac{fx}{2}\right)+d\cos\left(e+\frac{fx}{2}\right)\right)}{\sqrt{c^2-d^2}\sqrt{(\cos(e)-i\sin(e))^2}}\right)}{(c-d)\sqrt{c^2-d^2}\sqrt{(\cos(e)-i\sin(e))^2}} + \frac{2(c+d)\csc(e)(c\cos(e)+d\sin(fx))}{d(c+d\sin(e+fx))^2} + \frac{2(c-2d)\csc(e)}{(c-d)(}\right.}{4f(c+d)^2\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])/(c + d*Sin[e + f*x])^3,x]
```

```
[Out] (a*(1 + Sin[e + f*x])*((4*(2*c - d)*ArcTan[(Sec[(f*x)/2]*(Cos[e] - I*Sin[e]
)*(d*Cos[e + (f*x)/2] + c*Sin[(f*x)/2]))/(Sqrt[c^2 - d^2]*Sqrt[(Cos[e] - I*
Sin[e])^2])]*(Cos[e] - I*Sin[e]))/((c - d)*Sqrt[c^2 - d^2]*Sqrt[(Cos[e] - I
*Sin[e])^2]) + (2*(c + d)*Csc[e]*(c*Cos[e] + d*Sin[f*x]))/(d*(c + d*Sin[e +
 f*x])^2) + ((-4*c + 2*d)*Cot[e] + 2*(c - 2*d)*Csc[e]*Sin[f*x])/((c - d)*(c
 + d*Sin[e + f*x]))))/(4*(c + d)^2*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^
2)
```

Maple [B] time = 0.122, size = 1104, normalized size = 8.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^3,x)
```

```
[Out] -3/f*a/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2*d/(c^3+c^2*d-c*d
^2-d^3)*c*tan(1/2*f*x+1/2*e)^3+2/f*a/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+
1/2*e)*d+c)^2*d^2/(c^3+c^2*d-c*d^2-d^3)*tan(1/2*f*x+1/2*e)^3+2/f*a/(c*tan(1
/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2*d^3/(c^3+c^2*d-c*d^2-d^3)/c*tan
(1/2*f*x+1/2*e)^3-2/f*a/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2
/(c^3+c^2*d-c*d^2-d^3)*c^2*tan(1/2*f*x+1/2*e)^2+2/f*a/(c*tan(1/2*f*x+1/2*e)
^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^3+c^2*d-c*d^2-d^3)*c*tan(1/2*f*x+1/2*e)^2
*d-3/f*a/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^3+c^2*d-c*d
^2-d^3)*tan(1/2*f*x+1/2*e)^2*d^2+4/f*a/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*
x+1/2*e)*d+c)^2/(c^3+c^2*d-c*d^2-d^3)/c*tan(1/2*f*x+1/2*e)^2*d^3+2/f*a/(c*t
an(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^3+c^2*d-c*d^2-d^3)/c^2*t
an(1/2*f*x+1/2*e)^2*d^4-5/f*a/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*
d+c)^2*d*c/(c^3+c^2*d-c*d^2-d^3)*tan(1/2*f*x+1/2*e)+6/f*a/(c*tan(1/2*f*x+1/
2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2*d^2/(c^3+c^2*d-c*d^2-d^3)*tan(1/2*f*x+1/
2*e)+2/f*a/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2*d^3/c/(c^3+c
^2*d-c*d^2-d^3)*tan(1/2*f*x+1/2*e)-2/f*a/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*
f*x+1/2*e)*d+c)^2/(c^3+c^2*d-c*d^2-d^3)*c^2+2/f*a/(c*tan(1/2*f*x+1/2*e)^2+2
*tan(1/2*f*x+1/2*e)*d+c)^2/(c^3+c^2*d-c*d^2-d^3)*c*d+1/f*a/(c*tan(1/2*f*x+1
/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^3+c^2*d-c*d^2-d^3)*d^2+2/f*a/(c^3+c^
2*d-c*d^2-d^3)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2
-d^2)^(1/2))*c-1/f*a/(c^3+c^2*d-c*d^2-d^3)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*
tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.71909, size = 1729, normalized size = 12.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] [1/4*(2*(a*c^3*d - 2*a*c^2*d^2 - a*c*d^3 + 2*a*d^4)*cos(f*x + e)*sin(f*x +
e) + (2*a*c^3 - a*c^2*d + 2*a*c*d^2 - a*d^3 - (2*a*c*d^2 - a*d^3)*cos(f*x +
 e)^2 + 2*(2*a*c^2*d - a*c*d^2)*sin(f*x + e))*sqrt(-c^2 + d^2)*log(((2*c^2
- d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 + 2*(c*cos(f*x + e)*
sin(f*x + e) + d*cos(f*x + e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*
d*sin(f*x + e) - c^2 - d^2)) + 2*(2*a*c^4 - 2*a*c^3*d - 3*a*c^2*d^2 + 2*a*c
*d^3 + a*d^4)*cos(f*x + e))/((c^5*d^2 + c^4*d^3 - 2*c^3*d^4 - 2*c^2*d^5 + c
*d^6 + d^7)*f*cos(f*x + e)^2 - 2*(c^6*d + c^5*d^2 - 2*c^4*d^3 - 2*c^3*d^4 +
 c^2*d^5 + c*d^6)*f*sin(f*x + e) - (c^7 + c^6*d - c^5*d^2 - c^4*d^3 - c^3*d
```

```
^4 - c^2*d^5 + c*d^6 + d^7)*f), 1/2*((a*c^3*d - 2*a*c^2*d^2 - a*c*d^3 + 2*a
*d^4)*cos(f*x + e)*sin(f*x + e) + (2*a*c^3 - a*c^2*d + 2*a*c*d^2 - a*d^3 -
(2*a*c*d^2 - a*d^3)*cos(f*x + e)^2 + 2*(2*a*c^2*d - a*c*d^2)*sin(f*x + e))*
sqrt(c^2 - d^2)*arctan(-(c*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))
) + (2*a*c^4 - 2*a*c^3*d - 3*a*c^2*d^2 + 2*a*c*d^3 + a*d^4)*cos(f*x + e))/(
(c^5*d^2 + c^4*d^3 - 2*c^3*d^4 - 2*c^2*d^5 + c*d^6 + d^7)*f*cos(f*x + e)^2
- 2*(c^6*d + c^5*d^2 - 2*c^4*d^3 - 2*c^3*d^4 + c^2*d^5 + c*d^6)*f*sin(f*x +
 e) - (c^7 + c^6*d - c^5*d^2 - c^4*d^3 - c^3*d^4 - c^2*d^5 + c*d^6 + d^7)*f
)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.4063, size = 518, normalized size = 3.87

$$\frac{\frac{\left(\pi \left\lfloor \frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor \operatorname{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+d}{\sqrt{c^2-d^2}}\right)\right)(2ac-ad)}{\left(c^3+c^2d-cd^2-d^3\right)\sqrt{c^2-d^2}} - \frac{3ac^3d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-2ac^2d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-2acd^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+2ac^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2 - \cdots}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] ((pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e)
+ d)/sqrt(c^2 - d^2)))*(2*a*c - a*d)/((c^3 + c^2*d - c*d^2 - d^3)*sqrt(c^2
- d^2)) - (3*a*c^3*d*tan(1/2*f*x + 1/2*e)^3 - 2*a*c^2*d^2*tan(1/2*f*x + 1/2
*e)^3 - 2*a*c*d^3*tan(1/2*f*x + 1/2*e)^3 + 2*a*c^4*tan(1/2*f*x + 1/2*e)^2 -
 2*a*c^3*d*tan(1/2*f*x + 1/2*e)^2 + 3*a*c^2*d^2*tan(1/2*f*x + 1/2*e)^2 - 4*
a*c*d^3*tan(1/2*f*x + 1/2*e)^2 - 2*a*d^4*tan(1/2*f*x + 1/2*e)^2 + 5*a*c^3*d
*tan(1/2*f*x + 1/2*e) - 6*a*c^2*d^2*tan(1/2*f*x + 1/2*e) - 2*a*c*d^3*tan(1/
2*f*x + 1/2*e) + 2*a*c^4 - 2*a*c^3*d - a*c^2*d^2)/((c^5 + c^4*d - c^3*d^2 -
 c^2*d^3)*(c*tan(1/2*f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e) + c)^2))/f
```

3.433 $\int \frac{a+a\sin(e+fx)}{(c+d\sin(e+fx))^4}\,dx$

Optimal. Leaf size=192

$$\frac{a\left(2c^2-2cd+d^2\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{f(c+d)\left(c^2-d^2\right)^{5/2}}-\frac{a(c-4d)(2c-d)\cos(e+fx)}{6f(c-d)^2(c+d)^3(c+d\sin(e+fx))}-\frac{a(2c-3d)\cos(e+fx)}{6f(c-d)(c+d)^2(c+d\sin(e+fx}$$

```
[Out] (a*(2*c^2 - 2*c*d + d^2)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/
((c + d)*(c^2 - d^2)^(5/2)*f) - (a*Cos[e + f*x])/(3*(c + d)*f*(c + d*Sin[e
+ f*x])^3) - (a*(2*c - 3*d)*Cos[e + f*x])/(6*(c - d)*(c + d)^2*f*(c + d*Sin
[e + f*x])^2) - (a*(c - 4*d)*(2*c - d)*Cos[e + f*x])/(6*(c - d)^2*(c + d)^3
*f*(c + d*Sin[e + f*x]))
```

Rubi [A] time = 0.334907, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2754, 12, 2660, 618, 204}

$$\frac{a\left(2c^2-2cd+d^2\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{f(c+d)\left(c^2-d^2\right)^{5/2}}-\frac{a(c-4d)(2c-d)\cos(e+fx)}{6f(c-d)^2(c+d)^3(c+d\sin(e+fx))}-\frac{a(2c-3d)\cos(e+fx)}{6f(c-d)(c+d)^2(c+d\sin(e+fx}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])/(c + d*Sin[e + f*x])^4,x]
```

```
[Out] (a*(2*c^2 - 2*c*d + d^2)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/
((c + d)*(c^2 - d^2)^(5/2)*f) - (a*Cos[e + f*x])/(3*(c + d)*f*(c + d*Sin[e
+ f*x])^3) - (a*(2*c - 3*d)*Cos[e + f*x])/(6*(c - d)*(c + d)^2*f*(c + d*Sin
[e + f*x])^2) - (a*(c - 4*d)*(2*c - d)*Cos[e + f*x])/(6*(c - d)^2*(c + d)^3
*f*(c + d*Sin[e + f*x]))
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{a+a\sin(e+fx)}{(c+d\sin(e+fx))^4}\,dx = -\frac{a\cos(e+fx)}{3(c+d)f(c+d\sin(e+fx))^3} - \frac{\int \frac{-3a(c-d)-2a(c-d)\sin(e+fx)}{(c+d\sin(e+fx))^3}\,dx}{3\left(c^2-d^2\right)}$$

$$= -\frac{a\cos(e+fx)}{3(c+d)f(c+d\sin(e+fx))^3} - \frac{a(2c-3d)\cos(e+fx)}{6(c-d)(c+d)^2f(c+d\sin(e+fx))^2} + \frac{\int \frac{2a(3c-2d)(c-d)+a(2c\ldots}{(c+d\sin(\ldots}}{6\left(c^2-\ldots\right.}$$

$$= -\frac{a\cos(e+fx)}{3(c+d)f(c+d\sin(e+fx))^3} - \frac{a(2c-3d)\cos(e+fx)}{6(c-d)(c+d)^2f(c+d\sin(e+fx))^2} - \frac{a(c-4d)(2c-\ldots}{6(c-d)^2(c+d)^3f(\ldots}$$

$$= -\frac{a\cos(e+fx)}{3(c+d)f(c+d\sin(e+fx))^3} - \frac{a(2c-3d)\cos(e+fx)}{6(c-d)(c+d)^2f(c+d\sin(e+fx))^2} - \frac{a(c-4d)(2c-\ldots}{6(c-d)^2(c+d)^3f(\ldots}$$

$$= -\frac{a\cos(e+fx)}{3(c+d)f(c+d\sin(e+fx))^3} - \frac{a(2c-3d)\cos(e+fx)}{6(c-d)(c+d)^2f(c+d\sin(e+fx))^2} - \frac{a(c-4d)(2c-\ldots}{6(c-d)^2(c+d)^3f(\ldots}$$

$$= -\frac{a\cos(e+fx)}{3(c+d)f(c+d\sin(e+fx))^3} - \frac{a(2c-3d)\cos(e+fx)}{6(c-d)(c+d)^2f(c+d\sin(e+fx))^2} - \frac{a(c-4d)(2c-\ldots}{6(c-d)^2(c+d)^3f(\ldots}$$

$$= \frac{a\left(2c^2-2cd+d^2\right)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{(c-d)^2(c+d)^3\sqrt{c^2-d^2}\,f} - \frac{a\cos(e+fx)}{3(c+d)f(c+d\sin(e+fx))^3} - \frac{a(2c-3\ldots}{6(c-d)(c+d)^2\ldots}$$

Mathematica [C] time = 2.69252, size = 428, normalized size = 2.23

$$a(\sin(e+fx)+1)\left(\frac{2c\left(14c^2d^2-18c^3d+4c^4-27cd^3+12d^4\right)\cot(e)-d\csc(e)\left(-30c^2d^2\sin(2e+fx)+2c^2d^2\sin(2e+3fx)+3d\left(-16c^2d+4c^3+6cd^2+d^3\right)\cos(e+2fx)-\ldots\right.}{\ldots}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])/(c + d*Sin[e + f*x])^4,x]
```

```
[Out] (a*(1 + Sin[e + f*x])*((24*(2*c^2 - 2*c*d + d^2)*ArcTan[(Sec[(f*x)/2]*(Cos[
e] - I*Sin[e])*(d*Cos[e + (f*x)/2] + c*Sin[(f*x)/2]))/(Sqrt[c^2 - d^2]*Sqrt
[(Cos[e] - I*Sin[e])^2])]*(Cos[e] - I*Sin[e]))/(Sqrt[c^2 - d^2]*Sqrt[(Cos[e
] - I*Sin[e])^2]) + (2*c*(4*c^4 - 18*c^3*d + 14*c^2*d^2 - 27*c*d^3 + 12*d^4
)*Cot[e] - d*Csc[e]*(3*d*(4*c^3 - 16*c^2*d + 6*c*d^2 + d^3)*Cos[e + 2*f*x]
- 3*d^2*(2*c^2 - 2*c*d + d^2)*Cos[3*e + 2*f*x] - 24*c^4*Sin[f*x] + 78*c^3*d
```

```
*Sin[f*x] - 24*c^2*d^2*Sin[f*x] + 12*c*d^3*Sin[f*x] - 12*d^4*Sin[f*x] + 30*
c^3*d*Sin[2*e + f*x] - 30*c^2*d^2*Sin[2*e + f*x] + 15*c*d^3*Sin[2*e + f*x]
+ 2*c^2*d^2*Sin[2*e + 3*f*x] - 9*c*d^3*Sin[2*e + 3*f*x] + 4*d^4*Sin[2*e + 3
*f*x]))/(d*(c + d*Sin[e + f*x])^3)))/(24*(c - d)^2*(c + d)^3*f*(Cos[(e + f*
x)/2] + Sin[(e + f*x)/2])^2)
```

Maple [B] time = 0.145, size = 3104, normalized size = 16.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^4,x)
```

```
[Out] -2/3/f*a/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^3/(c^5+c^4*d-2*c
^3*d^2-2*c^2*d^3+c*d^4+d^5)*d^4+10/f*a/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*
x+1/2*e)*d+c)^3*d^4/(c^5+c^4*d-2*c^3*d^2-2*c^2*d^3+c*d^4+d^5)*tan(1/2*f*x+1
/2*e)^3-4/f*a/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^3*d^4/(c^5+
c^4*d-2*c^3*d^2-2*c^2*d^3+c*d^4+d^5)*tan(1/2*f*x+1/2*e)-2/f*a/(c*tan(1/2*f*
x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^3*d^4/(c^5+c^4*d-2*c^3*d^2-2*c^2*d^3+c
*d^4+d^5)*tan(1/2*f*x+1/2*e)^5-1/f*a/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+
1/2*e)*d+c)^3/(c^5+c^4*d-2*c^3*d^2-2*c^2*d^3+c*d^4+d^5)*c*d^3+2/f*a/(c^5+c^
4*d-2*c^3*d^2-2*c^2*d^3+c*d^4+d^5)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*
f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*c^2-4/f*a/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/
2*f*x+1/2*e)*d+c)^3/(c^5+c^4*d-2*c^3*d^2-2*c^2*d^3+c*d^4+d^5)/c^2*tan(1/2*f
*x+1/2*e)^4*d^6-12/f*a/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^3*
c^3*d/(c^5+c^4*d-2*c^3*d^2-2*c^2*d^3+c*d^4+d^5)*tan(1/2*f*x+1/2*e)^3+24/f*a
/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^3*c^2*d^2/(c^5+c^4*d-2*c
^3*d^2-2*c^2*d^3+c*d^4+d^5)*tan(1/2*f*x+1/2*e)^3-4/f*a/(c*tan(1/2*f*x+1/2*e
)^2+2*tan(1/2*f*x+1/2*e)*d+c)^3*c*d^3/(c^5+c^4*d-2*c^3*d^2-2*c^2*d^3+c*d^4+
d^5)*tan(1/2*f*x+1/2*e)^3-4/3/f*a/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2
*e)*d+c)^3/c*d^5/(c^5+c^4*d-2*c^3*d^2-2*c^2*d^3+c*d^4+d^5)*tan(1/2*f*x+1/2*
e)^3-2/f*a/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^3/(c^5+c^4*d-2
*c^3*d^2-2*c^2*d^3+c*d^4+d^5)*c^4+1/f*a/(c^5+c^4*d-2*c^3*d^2-2*c^2*d^3+c*d^
4+d^5)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1
/2))*d^2-2/f*a/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^3/(c^5+c^4
*d-2*c^3*d^2-2*c^2*d^3+c*d^4+d^5)*c^4*tan(1/2*f*x+1/2*e)^4-4/f*a/(c*tan(1/2
*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^3/(c^5+c^4*d-2*c^3*d^2-2*c^2*d^3+c*
d^4+d^5)*c^4*tan(1/2*f*x+1/2*e)^2+4/f*a/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f
*x+1/2*e)*d+c)^3/(c^5+c^4*d-2*c^3*d^2-2*c^2*d^3+c*d^4+d^5)*c^3*d+2/3/f*a/(c
*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^3/(c^5+c^4*d-2*c^3*d^2-2*c^
2*d^3+c*d^4+d^5)*c^2*d^2+6/f*a/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)
*d+c)^3/(c^5+c^4*d-2*c^3*d^2-2*c^2*d^3+c*d^4+d^5)*tan(1/2*f*x+1/2*e)^4*d^4-
4/f*a/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^3/c^2*d^6/(c^5+c^4*
d-2*c^3*d^2-2*c^2*d^3+c*d^4+d^5)*tan(1/2*f*x+1/2*e)^3-8/3/f*a/(c*tan(1/2*f*
x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^3/c^3*d^7/(c^5+c^4*d-2*c^3*d^2-2*c^2*d
^3+c*d^4+d^5)*tan(1/2*f*x+1/2*e)^3+8/f*a/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*
f*x+1/2*e)*d+c)^3/(c^5+c^4*d-2*c^3*d^2-2*c^2*d^3+c*d^4+d^5)*c^3*tan(1/2*f*x
+1/2*e)^2*d-12/f*a/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^3/(c^5
+c^4*d-2*c^3*d^2-2*c^2*d^3+c*d^4+d^5)*c^2*tan(1/2*f*x+1/2*e)^2*d^2+28/f*a/(
c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^3/(c^5+c^4*d-2*c^3*d^2-2*c
^2*d^3+c*d^4+d^5)*c*tan(1/2*f*x+1/2*e)^2*d^3-6/f*a/(c*tan(1/2*f*x+1/2*e)^2+
2*tan(1/2*f*x+1/2*e)*d+c)^3/(c^5+c^4*d-2*c^3*d^2-2*c^2*d^3+c*d^4+d^5)/c*tan
(1/2*f*x+1/2*e)^2*d^5-4/f*a/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+
c)^3/(c^5+c^4*d-2*c^3*d^2-2*c^2*d^3+c*d^4+d^5)/c^2*tan(1/2*f*x+1/2*e)^2*d^6
-8/f*a/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^3*d*c^3/(c^5+c^4*d
-2*c^3*d^2-2*c^2*d^3+c*d^4+d^5)*tan(1/2*f*x+1/2*e)+19/f*a/(c*tan(1/2*f*x+1/
```

```
2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^3*d^2*c^2/(c^5+c^4*d-2*c^3*d^2-2*c^2*d^3+c
*d^4+d^5)*tan(1/2*f*x+1/2*e)-2/f*a/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/
2*e)*d+c)^3*d^5/c/(c^5+c^4*d-2*c^3*d^2-2*c^2*d^3+c*d^4+d^5)*tan(1/2*f*x+1/2
*e)-4/f*a/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^3*d*c^3/(c^5+c^
4*d-2*c^3*d^2-2*c^2*d^3+c*d^4+d^5)*tan(1/2*f*x+1/2*e)^5+5/f*a/(c*tan(1/2*f*
x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^3*d^2*c^2/(c^5+c^4*d-2*c^3*d^2-2*c^2*d
^3+c*d^4+d^5)*tan(1/2*f*x+1/2*e)^5+4/f*a/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*
f*x+1/2*e)*d+c)^3*d^3*c/(c^5+c^4*d-2*c^3*d^2-2*c^2*d^3+c*d^4+d^5)*tan(1/2*f
*x+1/2*e)^5-2/f*a/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^3*d^5/c
/(c^5+c^4*d-2*c^3*d^2-2*c^2*d^3+c*d^4+d^5)*tan(1/2*f*x+1/2*e)^5+4/f*a/(c*ta
n(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^3/(c^5+c^4*d-2*c^3*d^2-2*c^2*d
^3+c*d^4+d^5)*c^3*tan(1/2*f*x+1/2*e)^4*d-10/f*a/(c*tan(1/2*f*x+1/2*e)^2+2*t
an(1/2*f*x+1/2*e)*d+c)^3/(c^5+c^4*d-2*c^3*d^2-2*c^2*d^3+c*d^4+d^5)*c^2*tan(
1/2*f*x+1/2*e)^4*d^2+17/f*a/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+
c)^3/(c^5+c^4*d-2*c^3*d^2-2*c^2*d^3+c*d^4+d^5)*c*tan(1/2*f*x+1/2*e)^4*d^3-2
/f*a/(c^5+c^4*d-2*c^3*d^2-2*c^2*d^3+c*d^4+d^5)/(c^2-d^2)^(1/2)*arctan(1/2*(
2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*c*d-6/f*a/(c*tan(1/2*f*x+1/2*e
)^2+2*tan(1/2*f*x+1/2*e)*d+c)^3/(c^5+c^4*d-2*c^3*d^2-2*c^2*d^3+c*d^4+d^5)/c
*tan(1/2*f*x+1/2*e)^4*d^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.28681, size = 2877, normalized size = 14.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^4,x, algorithm="fricas")
```

```
[Out] [-1/12*(2*(2*a*c^4*d^2 - 9*a*c^3*d^3 + 2*a*c^2*d^4 + 9*a*c*d^5 - 4*a*d^6)*c
os(f*x + e)^3 - 6*(2*a*c^5*d - 7*a*c^4*d^2 + 8*a*c^2*d^4 - 2*a*c*d^5 - a*d^
6)*cos(f*x + e)*sin(f*x + e) - 3*(2*a*c^5 - 2*a*c^4*d + 7*a*c^3*d^2 - 6*a*c
^2*d^3 + 3*a*c*d^4 - 3*(2*a*c^3*d^2 - 2*a*c^2*d^3 + a*c*d^4)*cos(f*x + e)^2
 + (6*a*c^4*d - 6*a*c^3*d^2 + 5*a*c^2*d^3 - 2*a*c*d^4 + a*d^5 - (2*a*c^2*d^
3 - 2*a*c*d^4 + a*d^5)*cos(f*x + e)^2)*sin(f*x + e))*sqrt(-c^2 + d^2)*log((
(2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 + 2*(c*cos(f*
x + e)*sin(f*x + e) + d*cos(f*x + e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2
 - 2*c*d*sin(f*x + e) - c^2 - d^2)) - 12*(a*c^6 - 2*a*c^5*d - a*c^4*d^2 + a
*c^3*d^3 + a*c^2*d^4 + a*c*d^5 - a*d^6)*cos(f*x + e))/(3*(c^8*d^2 + c^7*d^3
 - 3*c^6*d^4 - 3*c^5*d^5 + 3*c^4*d^6 + 3*c^3*d^7 - c^2*d^8 - c*d^9)*f*cos(f
*x + e)^2 - (c^10 + c^9*d - 6*c^6*d^4 - 6*c^5*d^5 + 8*c^4*d^6 + 8*c^3*d^7 -
 3*c^2*d^8 - 3*c*d^9)*f + ((c^7*d^3 + c^6*d^4 - 3*c^5*d^5 - 3*c^4*d^6 + 3*c
^3*d^7 + 3*c^2*d^8 - c*d^9 - d^10)*f*cos(f*x + e)^2 - (3*c^9*d + 3*c^8*d^2
- 8*c^7*d^3 - 8*c^6*d^4 + 6*c^5*d^5 + 6*c^4*d^6 - c*d^9 - d^10)*f)*sin(f*x
+ e)), -1/6*((2*a*c^4*d^2 - 9*a*c^3*d^3 + 2*a*c^2*d^4 + 9*a*c*d^5 - 4*a*d^6
)*cos(f*x + e)^3 - 3*(2*a*c^5*d - 7*a*c^4*d^2 + 8*a*c^2*d^4 - 2*a*c*d^5 - a
```

```
*d^6)*cos(f*x + e)*sin(f*x + e) - 3*(2*a*c^5 - 2*a*c^4*d + 7*a*c^3*d^2 - 6*
a*c^2*d^3 + 3*a*c*d^4 - 3*(2*a*c^3*d^2 - 2*a*c^2*d^3 + a*c*d^4)*cos(f*x + e
)^2 + (6*a*c^4*d - 6*a*c^3*d^2 + 5*a*c^2*d^3 - 2*a*c*d^4 + a*d^5 - (2*a*c^2
*d^3 - 2*a*c*d^4 + a*d^5)*cos(f*x + e)^2)*sin(f*x + e))*sqrt(c^2 - d^2)*arc
tan(-(c*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) - 6*(a*c^6 - 2*a*
c^5*d - a*c^4*d^2 + a*c^3*d^3 + a*c^2*d^4 + a*c*d^5 - a*d^6)*cos(f*x + e))/
(3*(c^8*d^2 + c^7*d^3 - 3*c^6*d^4 - 3*c^5*d^5 + 3*c^4*d^6 + 3*c^3*d^7 - c^2
*d^8 - c*d^9)*f*cos(f*x + e)^2 - (c^10 + c^9*d - 6*c^6*d^4 - 6*c^5*d^5 + 8*
c^4*d^6 + 8*c^3*d^7 - 3*c^2*d^8 - 3*c*d^9)*f + ((c^7*d^3 + c^6*d^4 - 3*c^5*
d^5 - 3*c^4*d^6 + 3*c^3*d^7 + 3*c^2*d^8 - c*d^9 - d^10)*f*cos(f*x + e)^2 -
(3*c^9*d + 3*c^8*d^2 - 8*c^7*d^3 - 8*c^6*d^4 + 6*c^5*d^5 + 6*c^4*d^6 - c*d^
9 - d^10)*f)*sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))**4,x)

[Out] Timed out
```

Giac [B] time = 1.43983, size = 1091, normalized size = 5.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^4,x, algorithm="giac")

[Out] 1/3*(3*(2*a*c^2 - 2*a*c*d + a*d^2)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c)
 + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/((c^5 + c^4*d - 2*
c^3*d^2 - 2*c^2*d^3 + c*d^4 + d^5)*sqrt(c^2 - d^2)) - (12*a*c^6*d*tan(1/2*f
*x + 1/2*e)^5 - 15*a*c^5*d^2*tan(1/2*f*x + 1/2*e)^5 - 12*a*c^4*d^3*tan(1/2*
f*x + 1/2*e)^5 + 6*a*c^3*d^4*tan(1/2*f*x + 1/2*e)^5 + 6*a*c^2*d^5*tan(1/2*f
*x + 1/2*e)^5 + 6*a*c^7*tan(1/2*f*x + 1/2*e)^4 - 12*a*c^6*d*tan(1/2*f*x + 1
/2*e)^4 + 30*a*c^5*d^2*tan(1/2*f*x + 1/2*e)^4 - 51*a*c^4*d^3*tan(1/2*f*x +
1/2*e)^4 - 18*a*c^3*d^4*tan(1/2*f*x + 1/2*e)^4 + 18*a*c^2*d^5*tan(1/2*f*x +
 1/2*e)^4 + 12*a*c*d^6*tan(1/2*f*x + 1/2*e)^4 + 36*a*c^6*d*tan(1/2*f*x + 1/
2*e)^3 - 72*a*c^5*d^2*tan(1/2*f*x + 1/2*e)^3 + 12*a*c^4*d^3*tan(1/2*f*x + 1
/2*e)^3 - 30*a*c^3*d^4*tan(1/2*f*x + 1/2*e)^3 + 4*a*c^2*d^5*tan(1/2*f*x + 1
/2*e)^3 + 12*a*c*d^6*tan(1/2*f*x + 1/2*e)^3 + 8*a*d^7*tan(1/2*f*x + 1/2*e)^
3 + 12*a*c^7*tan(1/2*f*x + 1/2*e)^2 - 24*a*c^6*d*tan(1/2*f*x + 1/2*e)^2 + 3
6*a*c^5*d^2*tan(1/2*f*x + 1/2*e)^2 - 84*a*c^4*d^3*tan(1/2*f*x + 1/2*e)^2 +
18*a*c^2*d^5*tan(1/2*f*x + 1/2*e)^2 + 12*a*c*d^6*tan(1/2*f*x + 1/2*e)^2 + 2
4*a*c^6*d*tan(1/2*f*x + 1/2*e) - 57*a*c^5*d^2*tan(1/2*f*x + 1/2*e) + 12*a*c
^3*d^4*tan(1/2*f*x + 1/2*e) + 6*a*c^2*d^5*tan(1/2*f*x + 1/2*e) + 6*a*c^7 -
12*a*c^6*d - 2*a*c^5*d^2 + 3*a*c^4*d^3 + 2*a*c^3*d^4)/((c^8 + c^7*d - 2*c^6
*d^2 - 2*c^5*d^3 + c^4*d^4 + c^3*d^5)*(c*tan(1/2*f*x + 1/2*e)^2 + 2*d*tan(1
/2*f*x + 1/2*e) + c)^3))/f
```

3.434 $\int (a+a\sin(e+fx))^2(c+d\sin(e+fx))^4\,dx$

Optimal. Leaf size=318

$$\frac{a^2\left(-311c^3d^2-448c^2d^3-48c^4d+4c^5-288cd^4-64d^5\right)\cos(e+fx)}{60df}+\frac{a^2\left(4c^2-48cd-55d^2\right)\cos(e+fx)(c+d\sin(e+fx))^3}{120df}$$

```
[Out] (a^2*(24*c^4 + 64*c^3*d + 84*c^2*d^2 + 48*c*d^3 + 11*d^4)*x)/16 + (a^2*(4*c
^5 - 48*c^4*d - 311*c^3*d^2 - 448*c^2*d^3 - 288*c*d^4 - 64*d^5)*Cos[e + f*x
])/(60*d*f) + (a^2*(8*c^4 - 96*c^3*d - 438*c^2*d^2 - 464*c*d^3 - 165*d^4)*C
os[e + f*x]*Sin[e + f*x])/(240*f) + (a^2*(4*c^3 - 48*c^2*d - 123*c*d^2 - 64
*d^3)*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(120*d*f) + (a^2*(4*c^2 - 48*c*d
 - 55*d^2)*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(120*d*f) + (a^2*(c - 12*d)
*Cos[e + f*x]*(c + d*Sin[e + f*x])^4)/(30*d*f) - (a^2*Cos[e + f*x]*(c + d*S
in[e + f*x])^5)/(6*d*f)
```

Rubi [A] time = 0.462589, antiderivative size = 318, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2763, 2753, 2734}

$$\frac{a^2\left(-311c^3d^2-448c^2d^3-48c^4d+4c^5-288cd^4-64d^5\right)\cos(e+fx)}{60df}+\frac{a^2\left(4c^2-48cd-55d^2\right)\cos(e+fx)(c+d\sin(e+fx))^3}{120df}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^4,x]
```



```
[Out] (a^2*(24*c^4 + 64*c^3*d + 84*c^2*d^2 + 48*c*d^3 + 11*d^4)*x)/16 + (a^2*(4*c
^5 - 48*c^4*d - 311*c^3*d^2 - 448*c^2*d^3 - 288*c*d^4 - 64*d^5)*Cos[e + f*x
])/(60*d*f) + (a^2*(8*c^4 - 96*c^3*d - 438*c^2*d^2 - 464*c*d^3 - 165*d^4)*C
os[e + f*x]*Sin[e + f*x])/(240*f) + (a^2*(4*c^3 - 48*c^2*d - 123*c*d^2 - 64
*d^3)*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(120*d*f) + (a^2*(4*c^2 - 48*c*d
 - 55*d^2)*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(120*d*f) + (a^2*(c - 12*d)
*Cos[e + f*x]*(c + d*Sin[e + f*x])^4)/(30*d*f) - (a^2*Cos[e + f*x]*(c + d*S
in[e + f*x])^5)/(6*d*f)
```


Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))^2(c+d\sin(e+fx))^4\,dx = -\frac{a^2\cos(e+fx)(c+d\sin(e+fx))^5}{6df}+\frac{\int \left(11a^2d-a^2(c-12d)\sin(e+f\right.}{6d}$$
$$=\frac{a^2(c-12d)\cos(e+fx)(c+d\sin(e+fx))^4}{30df}-\frac{a^2\cos(e+fx)(c+d\sin(}{6df}$$
$$=\frac{a^2\left(4c^2-48cd-55d^2\right)\cos(e+fx)(c+d\sin(e+fx))^3}{120df}+\frac{a^2(c-12d)\,c}{}$$
$$=\frac{a^2\left(4c^3-48c^2d-123cd^2-64d^3\right)\cos(e+fx)(c+d\sin(e+fx))^2}{120df}+\frac{a^2}{}$$
$$=\frac{1}{16}a^2\left(24c^4+64c^3d+84c^2d^2+48cd^3+11d^4\right)x+\frac{a^2\left(4c^5-48c^4d-31\right.}{}$$

Mathematica [A] time = 1.38604, size = 262, normalized size = 0.82

$$-\frac{a^2\cos(e+fx)\left(30\left(84c^2d^2+64c^3d+24c^4+48cd^3+11d^4\right)\sin^{-1}\left(\frac{\sqrt{1-\sin(e+fx)}}{\sqrt{2}}\right)+\sqrt{\cos^2(e+fx)}\left(10d^2\left(36c^2+48cd\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^4,x]
```

```
[Out] -(a^2*Cos[e + f*x]*(30*(24*c^4 + 64*c^3*d + 84*c^2*d^2 + 48*c*d^3 + 11*d^4)
*ArcSin[Sqrt[1 - Sin[e + f*x]]/Sqrt[2]] + Sqrt[Cos[e + f*x]^2]*(32*(15*c^4
+ 50*c^3*d + 60*c^2*d^2 + 36*c*d^3 + 8*d^4) + 15*(8*c^4 + 64*c^3*d + 84*c^2
*d^2 + 48*c*d^3 + 11*d^4)*Sin[e + f*x] + 64*d*(5*c^3 + 15*c^2*d + 9*c*d^2 +
 2*d^3)*Sin[e + f*x]^2 + 10*d^2*(36*c^2 + 48*c*d + 11*d^2)*Sin[e + f*x]^3 +
 96*d^3*(2*c + d)*Sin[e + f*x]^4 + 40*d^4*Sin[e + f*x]^5)))/(240*f*Sqrt[Cos
[e + f*x]^2])
```

Maple [A] time = 0.056, size = 462, normalized size = 1.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^4,x)
```

```
[Out] 1/f*(a^2*c^4*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-4/3*a^2*c^3*d*(2+si
n(f*x+e)^2)*cos(f*x+e)+6*a^2*c^2*d^2*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*co
s(f*x+e)+3/8*f*x+3/8*e)-4/5*a^2*c*d^3*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*c
os(f*x+e)+a^2*d^4*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos
(f*x+e)+5/16*f*x+5/16*e)-2*a^2*c^4*cos(f*x+e)+8*a^2*c^3*d*(-1/2*sin(f*x+e)*
cos(f*x+e)+1/2*f*x+1/2*e)-4*a^2*c^2*d^2*(2+sin(f*x+e)^2)*cos(f*x+e)+8*a^2*c
```

```
*d^3*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-2/5*a^2*
d^4*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)+a^2*c^4*(f*x+e)-4*a^2*c^
3*d*cos(f*x+e)+6*a^2*c^2*d^2*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-4/3
*a^2*c*d^3*(2+sin(f*x+e)^2)*cos(f*x+e)+a^2*d^4*(-1/4*(sin(f*x+e)^3+3/2*sin(
f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e))
```

Maxima [A] time = 1.22161, size = 609, normalized size = 1.92

$$\frac{240\left(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right)\right)a^2c^4+960\left(fx+e\right)a^2c^4+1280\left(\cos\left(fx+e\right)^3-3\,\cos\left(fx+e\right)\right)a^2c^3d+1920\left(2\,fx+2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^4,x, algorithm="maxima")
```

```
[Out] 1/960*(240*(2*f*x + 2*e - sin(2*f*x + 2*e))*a^2*c^4 + 960*(f*x + e)*a^2*c^4
 + 1280*(cos(f*x + e)^3 - 3*cos(f*x + e))*a^2*c^3*d + 1920*(2*f*x + 2*e - s
in(2*f*x + 2*e))*a^2*c^3*d + 3840*(cos(f*x + e)^3 - 3*cos(f*x + e))*a^2*c^2
*d^2 + 180*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*a^2*c^2*
d^2 + 1440*(2*f*x + 2*e - sin(2*f*x + 2*e))*a^2*c^2*d^2 - 256*(3*cos(f*x +
e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*a^2*c*d^3 + 1280*(cos(f*x + e)^
3 - 3*cos(f*x + e))*a^2*c*d^3 + 240*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*s
in(2*f*x + 2*e))*a^2*c*d^3 - 128*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15
*cos(f*x + e))*a^2*d^4 + 5*(4*sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*sin(4*
f*x + 4*e) - 48*sin(2*f*x + 2*e))*a^2*d^4 + 30*(12*f*x + 12*e + sin(4*f*x +
 4*e) - 8*sin(2*f*x + 2*e))*a^2*d^4 - 1920*a^2*c^4*cos(f*x + e) - 3840*a^2*
c^3*d*cos(f*x + e))/f
```

Fricas [A] time = 1.89266, size = 662, normalized size = 2.08

$$-\frac{96\left(2\,a^2cd^3+a^2d^4\right)\cos\left(fx+e\right)^5-320\left(a^2c^3d+3\,a^2c^2d^2+3\,a^2cd^3+a^2d^4\right)\cos\left(fx+e\right)^3-15\left(24\,a^2c^4+64\,a^2c^3d+84\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^4,x, algorithm="fricas")
```

```
[Out] -1/240*(96*(2*a^2*c*d^3 + a^2*d^4)*cos(f*x + e)^5 - 320*(a^2*c^3*d + 3*a^2*
c^2*d^2 + 3*a^2*c*d^3 + a^2*d^4)*cos(f*x + e)^3 - 15*(24*a^2*c^4 + 64*a^2*c
^3*d + 84*a^2*c^2*d^2 + 48*a^2*c*d^3 + 11*a^2*d^4)*f*x + 480*(a^2*c^4 + 4*a
^2*c^3*d + 6*a^2*c^2*d^2 + 4*a^2*c*d^3 + a^2*d^4)*cos(f*x + e) + 5*(8*a^2*d
^4*cos(f*x + e)^5 - 2*(36*a^2*c^2*d^2 + 48*a^2*c*d^3 + 19*a^2*d^4)*cos(f*x
+ e)^3 + 3*(8*a^2*c^4 + 64*a^2*c^3*d + 108*a^2*c^2*d^2 + 80*a^2*c*d^3 + 21*
a^2*d^4)*cos(f*x + e))*sin(f*x + e))/f
```

Sympy [A] time = 8.36999, size = 1136, normalized size = 3.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(c+d*sin(f*x+e))**4,x)
```

```
[Out] Piecewise((a**2*c**4*x*sin(e + f*x)**2/2 + a**2*c**4*x*cos(e + f*x)**2/2 +
a**2*c**4*x - a**2*c**4*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*a**2*c**4*cos(e
 + f*x)/f + 4*a**2*c**3*d*x*sin(e + f*x)**2 + 4*a**2*c**3*d*x*cos(e + f*x)*
*2 - 4*a**2*c**3*d*sin(e + f*x)**2*cos(e + f*x)/f - 4*a**2*c**3*d*sin(e + f
*x)*cos(e + f*x)/f - 8*a**2*c**3*d*cos(e + f*x)**3/(3*f) - 4*a**2*c**3*d*co
s(e + f*x)/f + 9*a**2*c**2*d**2*x*sin(e + f*x)**4/4 + 9*a**2*c**2*d**2*x*si
n(e + f*x)**2*cos(e + f*x)**2/2 + 3*a**2*c**2*d**2*x*sin(e + f*x)**2 + 9*a*
*2*c**2*d**2*x*cos(e + f*x)**4/4 + 3*a**2*c**2*d**2*x*cos(e + f*x)**2 - 15*
a**2*c**2*d**2*sin(e + f*x)**3*cos(e + f*x)/(4*f) - 12*a**2*c**2*d**2*sin(e
 + f*x)**2*cos(e + f*x)/f - 9*a**2*c**2*d**2*sin(e + f*x)*cos(e + f*x)**3/(
4*f) - 3*a**2*c**2*d**2*sin(e + f*x)*cos(e + f*x)/f - 8*a**2*c**2*d**2*cos(
e + f*x)**3/f + 3*a**2*c*d**3*x*sin(e + f*x)**4 + 6*a**2*c*d**3*x*sin(e + f
*x)**2*cos(e + f*x)**2 + 3*a**2*c*d**3*x*cos(e + f*x)**4 - 4*a**2*c*d**3*si
n(e + f*x)**4*cos(e + f*x)/f - 5*a**2*c*d**3*sin(e + f*x)**3*cos(e + f*x)/f
 - 16*a**2*c*d**3*sin(e + f*x)**2*cos(e + f*x)**3/(3*f) - 4*a**2*c*d**3*sin
(e + f*x)**2*cos(e + f*x)/f - 3*a**2*c*d**3*sin(e + f*x)*cos(e + f*x)**3/f
- 32*a**2*c*d**3*cos(e + f*x)**5/(15*f) - 8*a**2*c*d**3*cos(e + f*x)**3/(3*
f) + 5*a**2*d**4*x*sin(e + f*x)**6/16 + 15*a**2*d**4*x*sin(e + f*x)**4*cos(
e + f*x)**2/16 + 3*a**2*d**4*x*sin(e + f*x)**4/8 + 15*a**2*d**4*x*sin(e + f
*x)**2*cos(e + f*x)**4/16 + 3*a**2*d**4*x*sin(e + f*x)**2*cos(e + f*x)**2/4
 + 5*a**2*d**4*x*cos(e + f*x)**6/16 + 3*a**2*d**4*x*cos(e + f*x)**4/8 - 11*
a**2*d**4*sin(e + f*x)**5*cos(e + f*x)/(16*f) - 2*a**2*d**4*sin(e + f*x)**4
*cos(e + f*x)/f - 5*a**2*d**4*sin(e + f*x)**3*cos(e + f*x)**3/(6*f) - 5*a**
2*d**4*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 8*a**2*d**4*sin(e + f*x)**2*cos
(e + f*x)**3/(3*f) - 5*a**2*d**4*sin(e + f*x)*cos(e + f*x)**5/(16*f) - 3*a*
*2*d**4*sin(e + f*x)*cos(e + f*x)**3/(8*f) - 16*a**2*d**4*cos(e + f*x)**5/(
15*f), Ne(f, 0)), (x*(c + d*sin(e))**4*(a*sin(e) + a)**2, True))
```

Giac [A] time = 1.39704, size = 618, normalized size = 1.94

$$\frac{a^2cd^3\cos\left(3\,fx+3\,e\right)}{3\,f}-\frac{a^2d^4\sin\left(6\,fx+6\,e\right)}{192\,f}+\frac{a^2d^4\sin\left(4\,fx+4\,e\right)}{32\,f}+\frac{1}{16}\left(8\,a^2c^4+64\,a^2c^3d+36\,a^2c^2d^2+48\,a^2cd^3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^4,x, algorithm="giac")
```

```
[Out] 1/3*a^2*c*d^3*cos(3*f*x + 3*e)/f - 1/192*a^2*d^4*sin(6*f*x + 6*e)/f + 1/32*
a^2*d^4*sin(4*f*x + 4*e)/f + 1/16*(8*a^2*c^4 + 64*a^2*c^3*d + 36*a^2*c^2*d^
2 + 48*a^2*c*d^3 + 5*a^2*d^4)*x + 1/8*(8*a^2*c^4 + 24*a^2*c^2*d^2 + 3*a^2*d
^4)*x - 1/40*(2*a^2*c*d^3 + a^2*d^4)*cos(5*f*x + 5*e)/f + 1/24*(8*a^2*c^3*d
 + 24*a^2*c^2*d^2 + 10*a^2*c*d^3 + 5*a^2*d^4)*cos(3*f*x + 3*e)/f - 1/4*(8*a
^2*c^4 + 12*a^2*c^3*d + 36*a^2*c^2*d^2 + 10*a^2*c*d^3 + 5*a^2*d^4)*cos(f*x
+ e)/f - (4*a^2*c^3*d + 3*a^2*c*d^3)*cos(f*x + e)/f + 1/64*(12*a^2*c^2*d^2
+ 16*a^2*c*d^3 + 3*a^2*d^4)*sin(4*f*x + 4*e)/f - 1/64*(16*a^2*c^4 + 128*a^2
*c^3*d + 96*a^2*c^2*d^2 + 128*a^2*c*d^3 + 15*a^2*d^4)*sin(2*f*x + 2*e)/f -
1/4*(6*a^2*c^2*d^2 + a^2*d^4)*sin(2*f*x + 2*e)/f
```

3.435 $\int (a + a\sin(e + fx))^2 (c + d\sin(e + fx))^3 \, dx$

Optimal. Leaf size=233

$$\frac{a^2 \left(-44c^2d^2 - 10c^3d + c^4 - 40cd^3 - 12d^4\right) \cos(e + fx)}{10df} + \frac{a^2 \left(c^2 - 10cd - 12d^2\right) \cos(e + fx)(c + d\sin(e + fx))^2}{20df} + \frac{a^2 \left(-20\right.}{}$$

```
[Out] (3*a^2*(2*c + d)*(2*c^2 + 3*c*d + 2*d^2)*x)/8 + (a^2*(c^4 - 10*c^3*d - 44*c
^2*d^2 - 40*c*d^3 - 12*d^4)*Cos[e + f*x])/(10*d*f) + (a^2*(2*c^3 - 20*c^2*d
 - 57*c*d^2 - 30*d^3)*Cos[e + f*x]*Sin[e + f*x])/(40*f) + (a^2*(c^2 - 10*c*
d - 12*d^2)*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(20*d*f) + (a^2*(c - 10*d)
*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(20*d*f) - (a^2*Cos[e + f*x]*(c + d*S
in[e + f*x])^4)/(5*d*f)
```

Rubi [A] time = 0.308547, antiderivative size = 233, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2763, 2753, 2734}

$$\frac{a^2 \left(-44c^2d^2 - 10c^3d + c^4 - 40cd^3 - 12d^4\right) \cos(e + fx)}{10df} + \frac{a^2 \left(c^2 - 10cd - 12d^2\right) \cos(e + fx)(c + d\sin(e + fx))^2}{20df} + \frac{a^2 \left(-20\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^3,x]
```


```
[Out] (3*a^2*(2*c + d)*(2*c^2 + 3*c*d + 2*d^2)*x)/8 + (a^2*(c^4 - 10*c^3*d - 44*c
^2*d^2 - 40*c*d^3 - 12*d^4)*Cos[e + f*x])/(10*d*f) + (a^2*(2*c^3 - 20*c^2*d
 - 57*c*d^2 - 30*d^3)*Cos[e + f*x]*Sin[e + f*x])/(40*f) + (a^2*(c^2 - 10*c*
d - 12*d^2)*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(20*d*f) + (a^2*(c - 10*d)
*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(20*d*f) - (a^2*Cos[e + f*x]*(c + d*S
in[e + f*x])^4)/(5*d*f)
```


Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
```

```
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int (a + a\sin(e + fx))^2(c + d\sin(e + fx))^3\,dx = -\frac{a^2\cos(e + fx)(c + d\sin(e + fx))^4}{5df} + \frac{\int \left(9a^2d - a^2(c - 10d)\sin(e + f\right.}{5d}$$

$$= \frac{a^2(c - 10d)\cos(e + fx)(c + d\sin(e + fx))^3}{20df} - \frac{a^2\cos(e + fx)(c + d\sin(}{5df}$$

$$= \frac{a^2\left(c^2 - 10cd - 12d^2\right)\cos(e + fx)(c + d\sin(e + fx))^2}{20df} + \frac{a^2(c - 10d)\cos}{}$$

$$= \frac{3}{8}a^2(2c + d)\left(2c^2 + 3cd + 2d^2\right)x + \frac{a^2\left(c^4 - 10c^3d - 44c^2d^2 - 40cd^3 - 1\right.}{10df}$$

Mathematica [A] time = 0.909665, size = 204, normalized size = 0.88

$$-\frac{a^2\cos(e + fx)\left(30\left(8c^2d + 4c^3 + 7cd^2 + 2d^3\right)\sin^{-1}\left(\frac{\sqrt{1-\sin(e+fx)}}{\sqrt{2}}\right) + \sqrt{\cos^2(e + fx)}\left(8d\left(5c^2 + 10cd + 3d^2\right)\sin^2(e + f\right.\right.}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^3,x]
```

```
[Out] -(a^2*Cos[e + f*x]*(30*(4*c^3 + 8*c^2*d + 7*c*d^2 + 2*d^3)*ArcSin[Sqrt[1 -
Sin[e + f*x]]/Sqrt[2]] + Sqrt[Cos[e + f*x]^2]*(8*(10*c^3 + 25*c^2*d + 20*c*
d^2 + 6*d^3) + 5*(4*c^3 + 24*c^2*d + 21*c*d^2 + 6*d^3)*Sin[e + f*x] + 8*d*(
5*c^2 + 10*c*d + 3*d^2)*Sin[e + f*x]^2 + 10*d^2*(3*c + 2*d)*Sin[e + f*x]^3
+ 8*d^3*Sin[e + f*x]^4)))/(40*f*Sqrt[Cos[e + f*x]^2])
```

Maple [A] time = 0.046, size = 329, normalized size = 1.4

$$\frac{1}{f}\left(a^2c^3\left(-\frac{\sin\left(fx+e\right)\cos\left(fx+e\right)}{2} + \frac{fx}{2} + \frac{e}{2}\right) - a^2c^2d\left(2 + \left(\sin\left(fx+e\right)\right)^2\right)\cos\left(fx+e\right) + 3\,a^2cd^2\left(-1/4\left(\left(\sin\left(fx+\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^3,x)
```

```
[Out] 1/f*(a^2*c^3*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-a^2*c^2*d*(2+sin(f*
x+e)^2)*cos(f*x+e)+3*a^2*c*d^2*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+
e)+3/8*f*x+3/8*e)-1/5*a^2*d^3*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e
)-2*a^2*c^3*cos(f*x+e)+6*a^2*c^2*d*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*
e)-2*a^2*c*d^2*(2+sin(f*x+e)^2)*cos(f*x+e)+2*a^2*d^3*(-1/4*(sin(f*x+e)^3+3/
2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)+a^2*c^3*(f*x+e)-3*a^2*c^2*d*cos(f*x
+e)+3*a^2*c*d^2*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-1/3*a^2*d^3*(2+s
in(f*x+e)^2)*cos(f*x+e))
```

Maxima [A] time = 1.17888, size = 429, normalized size = 1.84

$$\frac{120\left(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right)\right)a^2c^3+480\left(fx+e\right)a^2c^3+480\left(\cos\left(fx+e\right)^3-3\,\cos\left(fx+e\right)\right)a^2c^2d+720\left(2\,fx+2\,e\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 1/480*(120*(2*f*x + 2*e - sin(2*f*x + 2*e))*a^2*c^3 + 480*(f*x + e)*a^2*c^3
 + 480*(cos(f*x + e)^3 - 3*cos(f*x + e))*a^2*c^2*d + 720*(2*f*x + 2*e - sin
(2*f*x + 2*e))*a^2*c^2*d + 960*(cos(f*x + e)^3 - 3*cos(f*x + e))*a^2*c*d^2
+ 45*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*a^2*c*d^2 + 36
0*(2*f*x + 2*e - sin(2*f*x + 2*e))*a^2*c*d^2 - 32*(3*cos(f*x + e)^5 - 10*co
s(f*x + e)^3 + 15*cos(f*x + e))*a^2*d^3 + 160*(cos(f*x + e)^3 - 3*cos(f*x +
 e))*a^2*d^3 + 30*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*a
^2*d^3 - 960*a^2*c^3*cos(f*x + e) - 1440*a^2*c^2*d*cos(f*x + e))/f
```

Fricas [A] time = 1.79178, size = 474, normalized size = 2.03

$$-\frac{8\,a^2d^3\cos\left(fx+e\right)^5-40\left(a^2c^2d+2\,a^2cd^2+a^2d^3\right)\cos\left(fx+e\right)^3-15\left(4\,a^2c^3+8\,a^2c^2d+7\,a^2cd^2+2\,a^2d^3\right)fx+80\left(a^2c^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -1/40*(8*a^2*d^3*cos(f*x + e)^5 - 40*(a^2*c^2*d + 2*a^2*c*d^2 + a^2*d^3)*co
s(f*x + e)^3 - 15*(4*a^2*c^3 + 8*a^2*c^2*d + 7*a^2*c*d^2 + 2*a^2*d^3)*f*x +
 80*(a^2*c^3 + 3*a^2*c^2*d + 3*a^2*c*d^2 + a^2*d^3)*cos(f*x + e) - 5*(2*(3*
a^2*c*d^2 + 2*a^2*d^3)*cos(f*x + e)^3 - (4*a^2*c^3 + 24*a^2*c^2*d + 27*a^2*
c*d^2 + 10*a^2*d^3)*cos(f*x + e))*sin(f*x + e))/f
```

Sympy [A] time = 4.25343, size = 729, normalized size = 3.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(c+d*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((a**2*c**3*x*sin(e + f*x)**2/2 + a**2*c**3*x*cos(e + f*x)**2/2 +
a**2*c**3*x - a**2*c**3*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*a**2*c**3*cos(e
 + f*x)/f + 3*a**2*c**2*d*x*sin(e + f*x)**2 + 3*a**2*c**2*d*x*cos(e + f*x)*
*2 - 3*a**2*c**2*d*sin(e + f*x)**2*cos(e + f*x)/f - 3*a**2*c**2*d*sin(e + f
*x)*cos(e + f*x)/f - 2*a**2*c**2*d*cos(e + f*x)**3/f - 3*a**2*c**2*d*cos(e
+ f*x)/f + 9*a**2*c*d**2*x*sin(e + f*x)**4/8 + 9*a**2*c*d**2*x*sin(e + f*x)
**2*cos(e + f*x)**2/4 + 3*a**2*c*d**2*x*sin(e + f*x)**2/2 + 9*a**2*c*d**2*x
*cos(e + f*x)**4/8 + 3*a**2*c*d**2*x*cos(e + f*x)**2/2 - 15*a**2*c*d**2*sin
(e + f*x)**3*cos(e + f*x)/(8*f) - 6*a**2*c*d**2*sin(e + f*x)**2*cos(e + f*x
)/f - 9*a**2*c*d**2*sin(e + f*x)*cos(e + f*x)**3/(8*f) - 3*a**2*c*d**2*sin(
e + f*x)*cos(e + f*x)/(2*f) - 4*a**2*c*d**2*cos(e + f*x)**3/f + 3*a**2*d**3
*x*sin(e + f*x)**4/4 + 3*a**2*d**3*x*sin(e + f*x)**2*cos(e + f*x)**2/2 + 3*
```

```
a**2*d**3*x*cos(e + f*x)**4/4 - a**2*d**3*sin(e + f*x)**4*cos(e + f*x)/f -
5*a**2*d**3*sin(e + f*x)**3*cos(e + f*x)/(4*f) - 4*a**2*d**3*sin(e + f*x)**
2*cos(e + f*x)**3/(3*f) - a**2*d**3*sin(e + f*x)**2*cos(e + f*x)/f - 3*a**2
*d**3*sin(e + f*x)*cos(e + f*x)**3/(4*f) - 8*a**2*d**3*cos(e + f*x)**5/(15*
f) - 2*a**2*d**3*cos(e + f*x)**3/(3*f), Ne(f, 0)), (x*(c + d*sin(e))**3*(a*
sin(e) + a)**2, True))
```

Giac [A] time = 1.38968, size = 437, normalized size = 1.88

$$-\frac{a^2d^3\cos\left(5\,fx+5\,e\right)}{80\,f}+\frac{a^2d^3\cos\left(3\,fx+3\,e\right)}{12\,f}-\frac{3\,a^2cd^2\sin\left(2\,fx+2\,e\right)}{4\,f}+\frac{1}{8}\left(4\,a^2c^3+24\,a^2c^2d+9\,a^2cd^2+6\,a^2d^3\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -1/80*a^2*d^3*cos(5*f*x + 5*e)/f + 1/12*a^2*d^3*cos(3*f*x + 3*e)/f - 3/4*a^
2*c*d^2*sin(2*f*x + 2*e)/f + 1/8*(4*a^2*c^3 + 24*a^2*c^2*d + 9*a^2*c*d^2 +
6*a^2*d^3)*x + 1/2*(2*a^2*c^3 + 3*a^2*c*d^2)*x + 1/48*(12*a^2*c^2*d + 24*a^
2*c*d^2 + 5*a^2*d^3)*cos(3*f*x + 3*e)/f - 1/8*(16*a^2*c^3 + 18*a^2*c^2*d +
36*a^2*c*d^2 + 5*a^2*d^3)*cos(f*x + e)/f - 3/4*(4*a^2*c^2*d + a^2*d^3)*cos(
f*x + e)/f + 1/32*(3*a^2*c*d^2 + 2*a^2*d^3)*sin(4*f*x + 4*e)/f - 1/4*(a^2*c
^3 + 6*a^2*c^2*d + 3*a^2*c*d^2 + 2*a^2*d^3)*sin(2*f*x + 2*e)/f
```

3.436 $\int (a+a\sin(e+fx))^2(c+d\sin(e+fx))^2\,dx$

Optimal. Leaf size=156

$$-\frac{a^2\left(12c^2+16cd+7d^2\right)\cos(e+fx)}{6f}-\frac{a^2\left(12c^2+16cd+7d^2\right)\sin(e+fx)\cos(e+fx)}{24f}+\frac{1}{8}a^2x\left(12c^2+16cd+7d^2\right)-\frac{d(8$$

```
[Out] (a^2*(12*c^2 + 16*c*d + 7*d^2)*x)/8 - (a^2*(12*c^2 + 16*c*d + 7*d^2)*Cos[e
+ f*x])/(6*f) - (a^2*(12*c^2 + 16*c*d + 7*d^2)*Cos[e + f*x]*Sin[e + f*x])/(
24*f) - ((8*c - d)*d*Cos[e + f*x]*(a + a*Sin[e + f*x])^2)/(12*f) - (d^2*Cos
[e + f*x]*(a + a*Sin[e + f*x])^3)/(4*a*f)
```

Rubi [A] time = 0.201897, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2761, 2751, 2644}

$$-\frac{a^2\left(12c^2+16cd+7d^2\right)\cos(e+fx)}{6f}-\frac{a^2\left(12c^2+16cd+7d^2\right)\sin(e+fx)\cos(e+fx)}{24f}+\frac{1}{8}a^2x\left(12c^2+16cd+7d^2\right)-\frac{d(8$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^2,x]
```

```
[Out] (a^2*(12*c^2 + 16*c*d + 7*d^2)*x)/8 - (a^2*(12*c^2 + 16*c*d + 7*d^2)*Cos[e
+ f*x])/(6*f) - (a^2*(12*c^2 + 16*c*d + 7*d^2)*Cos[e + f*x]*Sin[e + f*x])/(
24*f) - ((8*c - d)*d*Cos[e + f*x]*(a + a*Sin[e + f*x])^2)/(12*f) - (d^2*Cos
[e + f*x]*(a + a*Sin[e + f*x])^3)/(4*a*f)
```

Rule 2761

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^2, x_Symbol] :> -Simp[(d^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m
 + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*S
imp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2644

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[((2*a^2 + b
^2)*x)/2, x] + (-Simp[(2*a*b*Cos[c + d*x])/d, x] - Simp[(b^2*Cos[c + d*x]*S
in[c + d*x])/(2*d), x]) /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$\int (a+a\sin(e+fx))^2(c+d\sin(e+fx))^2\,dx = -\frac{d^2\cos(e+fx)(a+a\sin(e+fx))^3}{4af}+\frac{\int (a+a\sin(e+fx))^2\left(a\left(4c^2+\right.\right.}{}$$
$$= -\frac{(8c-d)d\cos(e+fx)(a+a\sin(e+fx))^2}{12f}-\frac{d^2\cos(e+fx)(a+a\sin(}{4af}$$
$$= \frac{1}{8}a^2\left(12c^2+16cd+7d^2\right)x-\frac{a^2\left(12c^2+16cd+7d^2\right)\cos(e+fx)}{6f}-\frac{a^2\left(\right.}{}$$

Mathematica [A] time = 0.524634, size = 148, normalized size = 0.95

$$-\frac{a^2\cos(e+fx)\left(6\left(12c^2+16cd+7d^2\right)\sin^{-1}\left(\frac{\sqrt{1-\sin(e+fx)}}{\sqrt{2}}\right)+\sqrt{\cos^2(e+fx)}\left(3\left(4c^2+16cd+7d^2\right)\sin(e+fx)+16\left(3\right.\right.\right.}{24f\sqrt{\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^2,x]
```

```
[Out] -(a^2*Cos[e + f*x]*(6*(12*c^2 + 16*c*d + 7*d^2)*ArcSin[Sqrt[1 - Sin[e + f*x
]]/Sqrt[2]] + Sqrt[Cos[e + f*x]^2]*(16*(3*c^2 + 5*c*d + 2*d^2) + 3*(4*c^2 +
 16*c*d + 7*d^2)*Sin[e + f*x] + 16*d*(c + d)*Sin[e + f*x]^2 + 6*d^2*Sin[e +
 f*x]^3)))/(24*f*Sqrt[Cos[e + f*x]^2])
```

Maple [A] time = 0.039, size = 219, normalized size = 1.4

$$\frac{1}{f}\left(a^2c^2\left(-\frac{\sin(fx+e)\cos(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-\frac{2a^2cd\left(2+(\sin(fx+e))^2\right)\cos(fx+e)}{3}+a^2d^2\left(-\frac{\cos(fx+e)}{4}\left((\mathrm{s}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^2,x)
```

```
[Out] 1/f*(a^2*c^2*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-2/3*a^2*c*d*(2+sin(
f*x+e)^2)*cos(f*x+e)+a^2*d^2*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)
+3/8*f*x+3/8*e)-2*a^2*c^2*cos(f*x+e)+4*a^2*c*d*(-1/2*sin(f*x+e)*cos(f*x+e)+
1/2*f*x+1/2*e)-2/3*a^2*d^2*(2+sin(f*x+e)^2)*cos(f*x+e)+a^2*c^2*(f*x+e)-2*a^
2*c*d*cos(f*x+e)+a^2*d^2*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e))
```

Maxima [A] time = 1.29266, size = 285, normalized size = 1.83

$$\frac{24\left(2fx+2e-\sin(2fx+2e)\right)a^2c^2+96(fx+e)a^2c^2+64\left(\cos(fx+e)^3-3\cos(fx+e)\right)a^2cd+96\left(2fx+2e-\mathrm{s}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/96*(24*(2*f*x + 2*e - sin(2*f*x + 2*e))*a^2*c^2 + 96*(f*x + e)*a^2*c^2 +
64*(cos(f*x + e)^3 - 3*cos(f*x + e))*a^2*c*d + 96*(2*f*x + 2*e - sin(2*f*x
```

```
+ 2*e))*a^2*c*d + 64*(cos(f*x + e)^3 - 3*cos(f*x + e))*a^2*d^2 + 3*(12*f*x
+ 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*a^2*d^2 + 24*(2*f*x + 2*e -
 sin(2*f*x + 2*e))*a^2*d^2 - 192*a^2*c^2*cos(f*x + e) - 192*a^2*c*d*cos(f*x
 + e))/f
```

Fricas [A] time = 1.68755, size = 324, normalized size = 2.08

$$\frac{16\left(a^2cd+a^2d^2\right)\cos\left(fx+e\right)^3+3\left(12\,a^2c^2+16\,a^2cd+7\,a^2d^2\right)fx-48\left(a^2c^2+2\,a^2cd+a^2d^2\right)\cos\left(fx+e\right)+3\left(2\,a^2d^2\cos\right.}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/24*(16*(a^2*c*d + a^2*d^2)*cos(f*x + e)^3 + 3*(12*a^2*c^2 + 16*a^2*c*d +
7*a^2*d^2)*f*x - 48*(a^2*c^2 + 2*a^2*c*d + a^2*d^2)*cos(f*x + e) + 3*(2*a^2
*d^2*cos(f*x + e)^3 - (4*a^2*c^2 + 16*a^2*c*d + 9*a^2*d^2)*cos(f*x + e))*si
n(f*x + e))/f
```

Sympy [A] time = 1.88581, size = 459, normalized size = 2.94

$$\begin{cases} \frac{a^2c^2x\sin^2\left(e+fx\right)}{2}+\frac{a^2c^2x\cos^2\left(e+fx\right)}{2}+a^2c^2x-\frac{a^2c^2\sin\left(e+fx\right)\cos\left(e+fx\right)}{2f}-\frac{2a^2c^2\cos\left(e+fx\right)}{f}+2a^2cdx\sin^2\left(e+fx\right)+2a^2cdx\cos^2\left(e\right. \\ x\left(c+d\sin\left(e\right)\right)^2\left(a\sin\left(e\right)+a\right)^2 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(c+d*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((a**2*c**2*x*sin(e + f*x)**2/2 + a**2*c**2*x*cos(e + f*x)**2/2 +
a**2*c**2*x - a**2*c**2*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*a**2*c**2*cos(e
 + f*x)/f + 2*a**2*c*d*x*sin(e + f*x)**2 + 2*a**2*c*d*x*cos(e + f*x)**2 - 2
*a**2*c*d*sin(e + f*x)**2*cos(e + f*x)/f - 2*a**2*c*d*sin(e + f*x)*cos(e +
f*x)/f - 4*a**2*c*d*cos(e + f*x)**3/(3*f) - 2*a**2*c*d*cos(e + f*x)/f + 3*a
**2*d**2*x*sin(e + f*x)**4/8 + 3*a**2*d**2*x*sin(e + f*x)**2*cos(e + f*x)**
2/4 + a**2*d**2*x*sin(e + f*x)**2/2 + 3*a**2*d**2*x*cos(e + f*x)**4/8 + a**
2*d**2*x*cos(e + f*x)**2/2 - 5*a**2*d**2*sin(e + f*x)**3*cos(e + f*x)/(8*f)
 - 2*a**2*d**2*sin(e + f*x)**2*cos(e + f*x)/f - 3*a**2*d**2*sin(e + f*x)*co
s(e + f*x)**3/(8*f) - a**2*d**2*sin(e + f*x)*cos(e + f*x)/(2*f) - 4*a**2*d*
*2*cos(e + f*x)**3/(3*f), Ne(f, 0)), (x*(c + d*sin(e))**2*(a*sin(e) + a)**2
, True))
```

Giac [A] time = 1.34071, size = 281, normalized size = 1.8

$$-\frac{2\,a^2cd\cos\left(fx+e\right)}{f}+\frac{a^2d^2\sin\left(4\,fx+4\,e\right)}{32\,f}-\frac{a^2d^2\sin\left(2\,fx+2\,e\right)}{4\,f}+\frac{1}{8}\left(4\,a^2c^2+16\,a^2cd+3\,a^2d^2\right)x+\frac{1}{2}\left(2\,a^2c^2+a^2d^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] -2*a^2*c*d*cos(f*x + e)/f + 1/32*a^2*d^2*sin(4*f*x + 4*e)/f - 1/4*a^2*d^2*s
in(2*f*x + 2*e)/f + 1/8*(4*a^2*c^2 + 16*a^2*c*d + 3*a^2*d^2)*x + 1/2*(2*a^2
*c^2 + a^2*d^2)*x + 1/6*(a^2*c*d + a^2*d^2)*cos(3*f*x + 3*e)/f - 1/2*(4*a^2
*c^2 + 3*a^2*c*d + 3*a^2*d^2)*cos(f*x + e)/f - 1/4*(a^2*c^2 + 4*a^2*c*d + a
^2*d^2)*sin(2*f*x + 2*e)/f
```

3.437 $\int (a+a\sin(e+fx))^2(c+d\sin(e+fx))\,dx$

Optimal. Leaf size=94

$$-\frac{2a^2(3c+2d)\cos(e+fx)}{3f}-\frac{a^2(3c+2d)\sin(e+fx)\cos(e+fx)}{6f}+\frac{1}{2}a^2x(3c+2d)-\frac{d\cos(e+fx)(a\sin(e+fx)+a)^2}{3f}$$

```
[Out] (a^2*(3*c + 2*d)*x)/2 - (2*a^2*(3*c + 2*d)*Cos[e + f*x])/(3*f) - (a^2*(3*c
+ 2*d)*Cos[e + f*x]*Sin[e + f*x])/(6*f) - (d*Cos[e + f*x]*(a + a*Sin[e + f*
x])^2)/(3*f)
```

Rubi [A] time = 0.0623795, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2751, 2644}

$$-\frac{2a^2(3c+2d)\cos(e+fx)}{3f}-\frac{a^2(3c+2d)\sin(e+fx)\cos(e+fx)}{6f}+\frac{1}{2}a^2x(3c+2d)-\frac{d\cos(e+fx)(a\sin(e+fx)+a)^2}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x]),x]
```

```
[Out] (a^2*(3*c + 2*d)*x)/2 - (2*a^2*(3*c + 2*d)*Cos[e + f*x])/(3*f) - (a^2*(3*c
+ 2*d)*Cos[e + f*x]*Sin[e + f*x])/(6*f) - (d*Cos[e + f*x]*(a + a*Sin[e + f*
x])^2)/(3*f)
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2644

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[((2*a^2 + b
^2)*x)/2, x] + (-Simp[(2*a*b*Cos[c + d*x])/d, x] - Simp[(b^2*Cos[c + d*x]*S
in[c + d*x])/(2*d), x]) /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a+a\sin(e+fx))^2(c+d\sin(e+fx))\,dx &= -\frac{d\cos(e+fx)(a+a\sin(e+fx))^2}{3f}+\frac{1}{3}(3c+2d)\int (a+a\sin(e+fx))^2\,dx\\ &= \frac{1}{2}a^2(3c+2d)x-\frac{2a^2(3c+2d)\cos(e+fx)}{3f}-\frac{a^2(3c+2d)\cos(e+fx)\sin(e\end{aligned}$$
$$\frac{}{6f}$$

Mathematica [A] time = 0.331744, size = 106, normalized size = 1.13

$$-\frac{a^2\cos(e+fx)\left(6(3c+2d)\sin^{-1}\left(\frac{\sqrt{1-\sin(e+fx)}}{\sqrt{2}}\right)+\sqrt{\cos^2(e+fx)}\left(3(c+2d)\sin(e+fx)+2(6c+5d)+2d\sin^2(e+fx)\right)\right)}{6f\sqrt{\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x]),x]
```

```
[Out] -(a^2*Cos[e + f*x]*(6*(3*c + 2*d)*ArcSin[Sqrt[1 - Sin[e + f*x]]/Sqrt[2]] +
Sqrt[Cos[e + f*x]^2]*(2*(6*c + 5*d) + 3*(c + 2*d)*Sin[e + f*x] + 2*d*Sin[e
+ f*x]^2)))/(6*f*Sqrt[Cos[e + f*x]^2])
```

Maple [A] time = 0.033, size = 117, normalized size = 1.2

$$\frac{1}{f}\left(a^2c\left(-\frac{\sin(fx+e)\cos(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-\frac{a^2d\left(2+(\sin(fx+e))^2\right)\cos(fx+e)}{3}-2a^2c\cos(fx+e)+2a^2d(-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(c+d*sin(f*x+e)),x)
```

```
[Out] 1/f*(a^2*c*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-1/3*a^2*d*(2+sin(f*x+
e)^2)*cos(f*x+e)-2*a^2*c*cos(f*x+e)+2*a^2*d*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2
*f*x+1/2*e)+a^2*c*(f*x+e)-a^2*d*cos(f*x+e))
```

Maxima [A] time = 1.13387, size = 154, normalized size = 1.64

$$\frac{3(2fx+2e-\sin(2fx+2e))a^2c+12(fx+e)a^2c+4\left(\cos(fx+e)^3-3\cos(fx+e)\right)a^2d+6(2fx+2e-\sin(2f}{12f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c+d*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/12*(3*(2*f*x + 2*e - sin(2*f*x + 2*e))*a^2*c + 12*(f*x + e)*a^2*c + 4*(co
s(f*x + e)^3 - 3*cos(f*x + e))*a^2*d + 6*(2*f*x + 2*e - sin(2*f*x + 2*e))*a
^2*d - 24*a^2*c*cos(f*x + e) - 12*a^2*d*cos(f*x + e))/f
```

Fricas [A] time = 1.59518, size = 192, normalized size = 2.04

$$\frac{2a^2d\cos(fx+e)^3+3\left(3a^2c+2a^2d\right)fx-3\left(a^2c+2a^2d\right)\cos(fx+e)\sin(fx+e)-12\left(a^2c+a^2d\right)\cos(fx+e)}{6f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c+d*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*a^2*d*cos(f*x + e)^3 + 3*(3*a^2*c + 2*a^2*d)*f*x - 3*(a^2*c + 2*a^2*
d)*cos(f*x + e)*sin(f*x + e) - 12*(a^2*c + a^2*d)*cos(f*x + e))/f
```

Sympy [A] time = 0.800184, size = 199, normalized size = 2.12

$$\begin{cases}\frac{a^2cx\sin^2(e+fx)}{2}+\frac{a^2cx\cos^2(e+fx)}{2}+a^2cx-\frac{a^2c\sin(e+fx)\cos(e+fx)}{2f}-\frac{2a^2c\cos(e+fx)}{f}+a^2dx\sin^2(e+fx)+a^2dx\cos^2(e+fx)\\ x(c+d\sin(e))(a\sin(e)+a)^2\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(c+d*sin(f*x+e)),x)
```

```
[Out] Piecewise((a**2*c*x*sin(e + f*x)**2/2 + a**2*c*x*cos(e + f*x)**2/2 + a**2*c
*x - a**2*c*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*a**2*c*cos(e + f*x)/f + a**
2*d*x*sin(e + f*x)**2 + a**2*d*x*cos(e + f*x)**2 - a**2*d*sin(e + f*x)**2*c
os(e + f*x)/f - a**2*d*sin(e + f*x)*cos(e + f*x)/f - 2*a**2*d*cos(e + f*x)*
*3/(3*f) - a**2*d*cos(e + f*x)/f, Ne(f, 0)), (x*(c + d*sin(e))*(a*sin(e) +
a)**2, True))
```

Giac [A] time = 1.3898, size = 147, normalized size = 1.56

$$a^2cx + \frac{a^2d\cos\left(3fx+3e\right)}{12f} - \frac{a^2d\cos\left(fx+e\right)}{f} + \frac{1}{2}\left(a^2c+2a^2d\right)x - \frac{\left(8a^2c+3a^2d\right)\cos\left(fx+e\right)}{4f} - \frac{\left(a^2c+2a^2d\right)\sin\left(2f\right.}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c+d*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] a^2*c*x + 1/12*a^2*d*cos(3*f*x + 3*e)/f - a^2*d*cos(f*x + e)/f + 1/2*(a^2*c
 + 2*a^2*d)*x - 1/4*(8*a^2*c + 3*a^2*d)*cos(f*x + e)/f - 1/4*(a^2*c + 2*a^2
*d)*sin(2*f*x + 2*e)/f
```

3.438 $\int (a+a\sin(e+fx))^2\,dx$

Optimal. Leaf size=45

$$-\frac{2a^2\cos(e+fx)}{f}-\frac{a^2\sin(e+fx)\cos(e+fx)}{2f}+\frac{3a^2x}{2}$$

```
[Out] (3*a^2*x)/2 - (2*a^2*Cos[e + f*x])/f - (a^2*Cos[e + f*x]*Sin[e + f*x])/(2*f
)
```

Rubi [A] time = 0.0148743, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2644}

$$-\frac{2a^2\cos(e+fx)}{f}-\frac{a^2\sin(e+fx)\cos(e+fx)}{2f}+\frac{3a^2x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2,x]
```

```
[Out] (3*a^2*x)/2 - (2*a^2*Cos[e + f*x])/f - (a^2*Cos[e + f*x]*Sin[e + f*x])/(2*f
)
```

Rule 2644

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[((2*a^2 + b
^2)*x)/2, x] + (-Simp[(2*a*b*Cos[c + d*x])/d, x] - Simp[(b^2*Cos[c + d*x]*S
in[c + d*x])/(2*d), x]) /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$\int (a+a\sin(e+fx))^2\,dx=\frac{3a^2x}{2}-\frac{2a^2\cos(e+fx)}{f}-\frac{a^2\cos(e+fx)\sin(e+fx)}{2f}$$

Mathematica [A] time = 0.188047, size = 34, normalized size = 0.76

$$-\frac{a^2(-6(e+fx)+\sin(2(e+fx))+8\cos(e+fx))}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2,x]
```

```
[Out] -(a^2*(-6*(e + f*x) + 8*Cos[e + f*x] + Sin[2*(e + f*x)]))/(4*f)
```

Maple [A] time = 0.023, size = 52, normalized size = 1.2

$$\frac{1}{f}\left(a^2\left(-\frac{\sin(fx+e)\cos(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-2\cos(fx+e)a^2+a^2(fx+e)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2,x)
```

```
[Out] 1/f*(a^2*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-2*cos(f*x+e)*a^2+a^2*(f
*x+e))
```

Maxima [A] time = 1.10774, size = 63, normalized size = 1.4

$$a^2x + \frac{(2\,fx + 2\,e - \sin(2\,fx + 2\,e))a^2}{4\,f} - \frac{2\,a^2\cos(fx + e)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] a^2*x + 1/4*(2*f*x + 2*e - sin(2*f*x + 2*e))*a^2/f - 2*a^2*cos(f*x + e)/f
```

Fricas [A] time = 1.58226, size = 97, normalized size = 2.16

$$\frac{3\,a^2fx - a^2\cos(fx + e)\sin(fx + e) - 4\,a^2\cos(fx + e)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(3*a^2*f*x - a^2*cos(f*x + e)*sin(f*x + e) - 4*a^2*cos(f*x + e))/f
```

Sympy [A] time = 0.296199, size = 78, normalized size = 1.73

$$\begin{cases} \frac{a^2x\sin^2(e+fx)}{2} + \frac{a^2x\cos^2(e+fx)}{2} + a^2x - \frac{a^2\sin(e+fx)\cos(e+fx)}{2f} - \frac{2a^2\cos(e+fx)}{f} & \text{for } f \neq 0 \\ x\,(a\sin(e) + a)^2 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((a**2*x*sin(e + f*x)**2/2 + a**2*x*cos(e + f*x)**2/2 + a**2*x - a
**2*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*a**2*cos(e + f*x)/f, Ne(f, 0)), (x*
(a*sin(e) + a)**2, True))
```

Giac [A] time = 1.32693, size = 54, normalized size = 1.2

$$\frac{3}{2}\,a^2x - \frac{2\,a^2\cos(fx + e)}{f} - \frac{a^2\sin(2\,fx + 2\,e)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 3/2*a^2*x - 2*a^2*cos(f*x + e)/f - 1/4*a^2*sin(2*f*x + 2*e)/f
```

3.439 $\int \frac{(a+a\sin(e+fx))^2}{c+d\sin(e+fx)}\,dx$

Optimal. Leaf size=92

$$\frac{2a^2(c-d)^2\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{d^2f\sqrt{c^2-d^2}}-\frac{a^2x(c-2d)}{d^2}-\frac{a^2\cos(e+fx)}{df}$$

```
[Out] -((a^2*(c - 2*d)*x)/d^2) + (2*a^2*(c - d)^2*ArcTan[(d + c*Tan[(e + f*x)/2])
/Sqrt[c^2 - d^2]])/(d^2*Sqrt[c^2 - d^2]*f) - (a^2*Cos[e + f*x])/(d*f)
```

Rubi [A] time = 0.202015, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2746, 2735, 2660, 618, 204}

$$\frac{2a^2(c-d)^2\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{d^2f\sqrt{c^2-d^2}}-\frac{a^2x(c-2d)}{d^2}-\frac{a^2\cos(e+fx)}{df}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2/(c + d*Sin[e + f*x]),x]
```

```
[Out] -((a^2*(c - 2*d)*x)/d^2) + (2*a^2*(c - d)^2*ArcTan[(d + c*Tan[(e + f*x)/2])
/Sqrt[c^2 - d^2]])/(d^2*Sqrt[c^2 - d^2]*f) - (a^2*Cos[e + f*x])/(d*f)
```

Rule 2746

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2/((c_.) + (d_.)*sin[(e_.) + (f
_.)*(x_)]), x_Symbol] :> -Simp[(b^2*Cos[e + f*x])/(d*f), x] + Dist[1/d, Int
[Simp[a^2*d - b*(b*c - 2*a*d)*Sin[e + f*x], x]/(c + d*Sin[e + f*x]), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^2}{c+d\sin(e+fx)}\,dx = -\frac{a^2\cos(e+fx)}{df}+\frac{\int \frac{a^2d-a^2(c-2d)\sin(e+fx)}{c+d\sin(e+fx)}\,dx}{d}$$

$$= -\frac{a^2(c-2d)x}{d^2}-\frac{a^2\cos(e+fx)}{df}+\frac{\left(a^2(c-d)^2\right)\int \frac{1}{c+d\sin(e+fx)}\,dx}{d^2}$$

$$= -\frac{a^2(c-2d)x}{d^2}-\frac{a^2\cos(e+fx)}{df}+\frac{\left(2a^2(c-d)^2\right)\text{Subst}\left(\int \frac{1}{c+2dx+cx^2}\,dx,x,\tan\left(\frac{1}{2}(e+fx)\right)\right)}{d^2f}$$

$$= -\frac{a^2(c-2d)x}{d^2}-\frac{a^2\cos(e+fx)}{df}-\frac{\left(4a^2(c-d)^2\right)\text{Subst}\left(\int \frac{1}{-4(c^2-d^2)-x^2}\,dx,x,2d+2c\tan\left(\frac{1}{2}(e+fx)\right)\right)}{d^2f}$$

$$= -\frac{a^2(c-2d)x}{d^2}+\frac{2a^2(c-d)^2\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{d^2\sqrt{c^2-d^2}\,f}-\frac{a^2\cos(e+fx)}{df}$$

Mathematica [A] time = 0.405786, size = 130, normalized size = 1.41

$$-\frac{a^2(\sin(e+fx)+1)^2\left(\sqrt{c^2-d^2}((c-2d)(e+fx)+d\cos(e+fx))-2(c-d)^2\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{d^2f\sqrt{c^2-d^2}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2/(c + d*Sin[e + f*x]),x]
```

```
[Out] -((a^2*(-2*(c - d)^2*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]] + Sqr
t[c^2 - d^2]*((c - 2*d)*(e + f*x) + d*Cos[e + f*x]))*(1 + Sin[e + f*x])^2)/
(d^2*Sqrt[c^2 - d^2]*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4))
```

Maple [B] time = 0.095, size = 228, normalized size = 2.5

$$2\,\frac{a^2c^2}{fd^2\sqrt{c^2-d^2}}\arctan\left(1/2\,\frac{2\,c\tan\left(1/2\,fx+e/2\right)+2\,d}{\sqrt{c^2-d^2}}\right)-4\,\frac{a^2c}{df\sqrt{c^2-d^2}}\arctan\left(1/2\,\frac{2\,c\tan\left(1/2\,fx+e/2\right)+2\,d}{\sqrt{c^2-d^2}}\right)+2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e)),x)
```

```
[Out] 2/f*a^2/d^2/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^
2)^(1/2))*c^2-4/f*a^2/d/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+
2*d)/(c^2-d^2)^(1/2))*c+2/f*a^2/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x
+1/2*e)+2*d)/(c^2-d^2)^(1/2))-2/f*a^2/d/(1+tan(1/2*f*x+1/2*e)^2)-2/f*a^2/d^
2*arctan(tan(1/2*f*x+1/2*e))*c+4/f*a^2/d*arctan(tan(1/2*f*x+1/2*e))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.72513, size = 668, normalized size = 7.26

$$\left[-\frac{2\,a^2d\cos\left(fx+e\right)+2\left(a^2c-2\,a^2d\right)fx+\left(a^2c-a^2d\right)\sqrt{-\frac{c-d}{c+d}}\log\left(\frac{\left(2\,c^2-d^2\right)\cos\left(fx+e\right)^2-2\,cd\sin\left(fx+e\right)-c^2-d^2+2\left(\left(c^2+cd\right)\cos\left(fx+e\right)\sin\ldots\right.}{d^2\cos\left(fx+e\right)^2-2\,cd\sin\left(fx+e\right)-c^2-\ldots}\right.}{2\,d^2f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*a^2*d*cos(f*x + e) + 2*(a^2*c - 2*a^2*d)*f*x + (a^2*c - a^2*d)*sqr
t(-(c - d)/(c + d))*log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e)
- c^2 - d^2 + 2*((c^2 + c*d)*cos(f*x + e)*sin(f*x + e) + (c*d + d^2)*cos(f*
x + e))*sqrt(-(c - d)/(c + d)))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) -
c^2 - d^2)))/(d^2*f), -(a^2*d*cos(f*x + e) + (a^2*c - 2*a^2*d)*f*x + (a^2*c
 - a^2*d)*sqrt((c - d)/(c + d))*arctan(-(c*sin(f*x + e) + d)*sqrt((c - d)/(
c + d))/((c - d)*cos(f*x + e))))/(d^2*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2/(c+d*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.41076, size = 184, normalized size = 2.

$$-\frac{\frac{\left(a^2c-2\,a^2d\right)\left(fx+e\right)}{d^2}+\frac{2\,a^2}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)d}-\frac{2\left(a^2c^2-2\,a^2cd+a^2d^2\right)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{\sqrt{c^2-d^2}\,d^2}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] -((a^2*c - 2*a^2*d)*(f*x + e)/d^2 + 2*a^2/((tan(1/2*f*x + 1/2*e)^2 + 1)*d)
- 2*(a^2*c^2 - 2*a^2*c*d + a^2*d^2)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c
) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/(sqrt(c^2 - d^2)*
d^2))/f
```

3.440 $\int \frac{(a+a\sin(e+fx))^2}{(c+d\sin(e+fx))^2}\,dx$

Optimal. Leaf size=112

$$-\frac{2a^2(c-d)^2(c+2d)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{d^2f\left(c^2-d^2\right)^{3/2}}+\frac{a^2(c-d)\cos(e+fx)}{df(c+d)(c+d\sin(e+fx))}+\frac{a^2x}{d^2}$$

```
[Out] (a^2*x)/d^2 - (2*a^2*(c - d)^2*(c + 2*d)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sq
rt[c^2 - d^2]])/(d^2*(c^2 - d^2)^(3/2)*f) + (a^2*(c - d)*Cos[e + f*x])/(d*(
c + d)*f*(c + d*Sin[e + f*x]))
```

Rubi [A] time = 0.182345, antiderivative size = 115, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2762, 2735, 2660, 618, 204}

$$-\frac{2a^2(c-d)(c+2d)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{d^2f(c+d)\sqrt{c^2-d^2}}+\frac{a^2(c-d)\cos(e+fx)}{df(c+d)(c+d\sin(e+fx))}+\frac{a^2x}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2/(c + d*Sin[e + f*x])^2,x]
```

```
[Out] (a^2*x)/d^2 - (2*a^2*(c - d)*(c + 2*d)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt
[c^2 - d^2]])/(d^2*(c + d)*Sqrt[c^2 - d^2]*f) + (a^2*(c - d)*Cos[e + f*x])/
(d*(c + d)*f*(c + d*Sin[e + f*x]))
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+a\sin(e+fx))^2}{(c+d\sin(e+fx))^2}\,dx &= \frac{a^2(c-d)\cos(e+fx)}{d(c+d)f(c+d\sin(e+fx))} - \frac{a\int \frac{-2ad-a(c+d)\sin(e+fx)}{c+d\sin(e+fx)}\,dx}{d(c+d)} \\
&= \frac{a^2x}{d^2} + \frac{a^2(c-d)\cos(e+fx)}{d(c+d)f(c+d\sin(e+fx))} - \frac{\left(a^2(c-d)(c+2d)\right)\int \frac{1}{c+d\sin(e+fx)}\,dx}{d^2(c+d)} \\
&= \frac{a^2x}{d^2} + \frac{a^2(c-d)\cos(e+fx)}{d(c+d)f(c+d\sin(e+fx))} - \frac{\left(2a^2(c-d)(c+2d)\right)\mathrm{Subst}\left(\int \frac{1}{c+2dx+cx^2}\,dx, x, \tan\left(\right.\right.}{d^2(c+d)f} \\
&= \frac{a^2x}{d^2} + \frac{a^2(c-d)\cos(e+fx)}{d(c+d)f(c+d\sin(e+fx))} + \frac{\left(4a^2(c-d)(c+2d)\right)\mathrm{Subst}\left(\int \frac{1}{-4(c^2-d^2)-x^2}\,dx, x, 2d\right.}{d^2(c+d)f} \\
&= \frac{a^2x}{d^2} - \frac{2a^2(c-d)(c+2d)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{d^2(c+d)\sqrt{c^2-d^2}\,f} + \frac{a^2(c-d)\cos(e+fx)}{d(c+d)f(c+d\sin(e+fx))}
\end{aligned}$$

Mathematica [A] time = 0.458566, size = 139, normalized size = 1.24

$$\frac{a^2(\sin(e+fx)+1)^2\left(-\frac{2(c^2+cd-2d^2)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{(c+d)\sqrt{c^2-d^2}} + \frac{d(c-d)\cos(e+fx)}{(c+d)(c+d\sin(e+fx))} + e + fx\right)}{d^2f\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2/(c + d*Sin[e + f*x])^2,x]
```

```
[Out] (a^2*(1 + Sin[e + f*x])^2*(e + f*x - (2*(c^2 + c*d - 2*d^2)*ArcTan[(d + c*T
an[(e + f*x)/2])/Sqrt[c^2 - d^2]])/((c + d)*Sqrt[c^2 - d^2]) + ((c - d)*d*C
os[e + f*x])/((c + d)*(c + d*Sin[e + f*x]))))/(d^2*f*(Cos[(e + f*x)/2] + Si
n[(e + f*x)/2])^4)
```

Maple [B] time = 0.112, size = 389, normalized size = 3.5

$$2\,\frac{a^2\tan\left(1/2\,fx+e/2\right)}{f\left(c\left(\tan\left(1/2\,fx+e/2\right)\right)^2+2\,\tan\left(1/2\,fx+e/2\right)d+c\right)(c+d)} - 2\,\frac{a^2d\tan\left(1/2\,fx+e/2\right)}{f\left(c\left(\tan\left(1/2\,fx+e/2\right)\right)^2+2\,\tan\left(1/2\,fx+e/2\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^2,x)
```

```
[Out] 2/f*a^2/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)/(c+d)*tan(1/2*f*x
+1/2*e)-2/f*a^2*d/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)/(c+d)/c
*tan(1/2*f*x+1/2*e)+2/f*a^2/d/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*
d+c)/(c+d)*c-2/f*a^2/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)/(c+d
)-2/f*a^2/d^2/(c+d)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)
/(c^2-d^2)^(1/2))*c^2-2/f*a^2/d/(c+d)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1
/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*c+4/f*a^2/(c+d)/(c^2-d^2)^(1/2)*arctan(
1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))+2/f*a^2/d^2*arctan(tan(1/
2*f*x+1/2*e))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.837, size = 1054, normalized size = 9.41

$$\left[\frac{2\left(a^2cd + a^2d^2\right)fx\sin\left(fx+e\right) + 2\left(a^2c^2 + a^2cd\right)fx + \left(a^2c^2 + 2\,a^2cd + \left(a^2cd + 2\,a^2d^2\right)\sin\left(fx+e\right)\right)\sqrt{-\frac{c-d}{c+d}}\log\left(\frac{\left(2\,c^2-d^2\right)\cdots}{\cdots}\right.}{2\left(\left(cd^3 + d^4\right)f\sin\left(fx+e\right) + \cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*(a^2*c*d + a^2*d^2)*f*x*sin(f*x + e) + 2*(a^2*c^2 + a^2*c*d)*f*x +
(a^2*c^2 + 2*a^2*c*d + (a^2*c*d + 2*a^2*d^2)*sin(f*x + e))*sqrt(-(c - d)/(c
 + d))*log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 +
 2*((c^2 + c*d)*cos(f*x + e)*sin(f*x + e) + (c*d + d^2)*cos(f*x + e))*sqrt(
-(c - d)/(c + d)))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) +
 2*(a^2*c*d - a^2*d^2)*cos(f*x + e))/((c*d^3 + d^4)*f*sin(f*x + e) + (c^2*d
^2 + c*d^3)*f), ((a^2*c*d + a^2*d^2)*f*x*sin(f*x + e) + (a^2*c^2 + a^2*c*d)
*f*x + (a^2*c^2 + 2*a^2*c*d + (a^2*c*d + 2*a^2*d^2)*sin(f*x + e))*sqrt((c -
 d)/(c + d))*arctan(-(c*sin(f*x + e) + d)*sqrt((c - d)/(c + d))/((c - d)*co
s(f*x + e))) + (a^2*c*d - a^2*d^2)*cos(f*x + e))/((c*d^3 + d^4)*f*sin(f*x +
 e) + (c^2*d^2 + c*d^3)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2/(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.38669, size = 277, normalized size = 2.47

$$\frac{\frac{(fx+e)a^2}{d^2} - \frac{2\left(a^2c^2+a^2cd-2\,a^2d^2\right)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{\left(cd^2+d^3\right)\sqrt{c^2-d^2}} + \frac{2\left(a^2cd\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-a^2d^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+a^2c^2-a^2cd\right)}{\left(c^2d+cd^2\right)\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+2\,d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+c\right)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] ((f*x + e)*a^2/d^2 - 2*(a^2*c^2 + a^2*c*d - 2*a^2*d^2)*(pi*floor(1/2*(f*x +
 e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2))
)/((c*d^2 + d^3)*sqrt(c^2 - d^2)) + 2*(a^2*c*d*tan(1/2*f*x + 1/2*e) - a^2*d
^2*tan(1/2*f*x + 1/2*e) + a^2*c^2 - a^2*c*d)/((c^2*d + c*d^2)*(c*tan(1/2*f*
x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e) + c)))/f
```

3.441 $\int \frac{(a+a\sin(e+fx))^2}{(c+d\sin(e+fx))^3}\,dx$

Optimal. Leaf size=138

$$\frac{3a^2\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{f(c+d)^2\sqrt{c^2-d^2}}-\frac{a^2(c+4d)\cos(e+fx)}{2df(c+d)^2(c+d\sin(e+fx))}+\frac{a^2(c-d)\cos(e+fx)}{2df(c+d)(c+d\sin(e+fx))^2}$$

```
[Out] (3*a^2*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/((c + d)^2*Sqrt[c^
2 - d^2]*f) + (a^2*(c - d)*Cos[e + f*x])/(2*d*(c + d)*f*(c + d*Sin[e + f*x]
)^2) - (a^2*(c + 4*d)*Cos[e + f*x])/(2*d*(c + d)^2*f*(c + d*Sin[e + f*x]))
```

Rubi [A] time = 0.181949, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {2762, 2754, 12, 2660, 618, 204}

$$\frac{3a^2\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{f(c+d)^2\sqrt{c^2-d^2}}-\frac{a^2(c+4d)\cos(e+fx)}{2df(c+d)^2(c+d\sin(e+fx))}+\frac{a^2(c-d)\cos(e+fx)}{2df(c+d)(c+d\sin(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2/(c + d*Sin[e + f*x])^3,x]
```

```
[Out] (3*a^2*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/((c + d)^2*Sqrt[c^
2 - d^2]*f) + (a^2*(c - d)*Cos[e + f*x])/(2*d*(c + d)*f*(c + d*Sin[e + f*x]
)^2) - (a^2*(c + 4*d)*Cos[e + f*x])/(2*d*(c + d)^2*f*(c + d*Sin[e + f*x]))
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^2}{(c+d\sin(e+fx))^3}\,dx = \frac{a^2(c-d)\cos(e+fx)}{2d(c+d)f(c+d\sin(e+fx))^2} - \frac{a\int \frac{-4ad-a(c+3d)\sin(e+fx)}{(c+d\sin(e+fx))^2}\,dx}{2d(c+d)}$$

$$= \frac{a^2(c-d)\cos(e+fx)}{2d(c+d)f(c+d\sin(e+fx))^2} - \frac{a^2(c+4d)\cos(e+fx)}{2d(c+d)^2f(c+d\sin(e+fx))} + \frac{a\int \frac{3a(c-d)d}{c+d\sin(e+fx)}\,dx}{2(c-d)d(c+d)^2}$$

$$= \frac{a^2(c-d)\cos(e+fx)}{2d(c+d)f(c+d\sin(e+fx))^2} - \frac{a^2(c+4d)\cos(e+fx)}{2d(c+d)^2f(c+d\sin(e+fx))} + \frac{\left(3a^2\right)\int \frac{1}{c+d\sin(e+fx)}\,dx}{2(c+d)^2}$$

$$= \frac{a^2(c-d)\cos(e+fx)}{2d(c+d)f(c+d\sin(e+fx))^2} - \frac{a^2(c+4d)\cos(e+fx)}{2d(c+d)^2f(c+d\sin(e+fx))} + \frac{\left(3a^2\right)\text{Subst}\left(\int \frac{1}{c+2dx+\cdots}\right.}{(c\cdots}$$

$$= \frac{a^2(c-d)\cos(e+fx)}{2d(c+d)f(c+d\sin(e+fx))^2} - \frac{a^2(c+4d)\cos(e+fx)}{2d(c+d)^2f(c+d\sin(e+fx))} - \frac{\left(6a^2\right)\text{Subst}\left(\int \frac{1}{-4(c^2-\cdots}\right.}{\cdots}$$

$$= \frac{3a^2\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{(c+d)^2\sqrt{c^2-d^2}\,f} + \frac{a^2(c-d)\cos(e+fx)}{2d(c+d)f(c+d\sin(e+fx))^2} - \frac{a^2(c+4d)\cos(e+fx}{2d(c+d)^2f(c+d\sin(e+\cdots}$$

Mathematica [A] time = 0.663653, size = 140, normalized size = 1.01

$$\frac{a^2\cos(e+fx)\left(-\frac{(c+4d)\sin(e+fx)+4c+d}{(c+d)(c+d\sin(e+fx))^2} - \frac{6\tan^{-1}\left(\frac{\sqrt{d-c}\sqrt{1-\sin(e+fx)}}{\sqrt{-c-d}\sqrt{\sin(e+fx)+1}}\right)}{(-c-d)^{3/2}\sqrt{d-c}\sqrt{\cos^2(e+fx)}}\right)}{2f(c+d)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2/(c + d*Sin[e + f*x])^3,x]
```

```
[Out] (a^2*Cos[e + f*x]*((-6*ArcTan[(Sqrt[-c + d]*Sqrt[1 - Sin[e + f*x]])/(Sqrt[-
c - d]*Sqrt[1 + Sin[e + f*x]])])/((-c - d)^(3/2)*Sqrt[-c + d]*Sqrt[Cos[e +
f*x]^2]) - (4*c + d + (c + 4*d)*Sin[e + f*x])/((c + d)*(c + d*Sin[e + f*x])
^2)))/(2*(c + d)*f)
```

Maple [B] time = 0.132, size = 799, normalized size = 5.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^3,x)
```

```
[Out] 1/f*a^2/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)
*c*tan(1/2*f*x+1/2*e)^3-4/f*a^2/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e
)*d+c)^2/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3*d-2/f*a^2/(c*tan(1/2*f*x+1/2*
e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)/c*tan(1/2*f*x+1/2*e)^3*d^2
-4/f*a^2/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2
)*c*tan(1/2*f*x+1/2*e)^2-1/f*a^2/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*
e)*d+c)^2/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^2*d-8/f*a^2/(c*tan(1/2*f*x+1/2
*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)/c*tan(1/2*f*x+1/2*e)^2*d^
2-2/f*a^2/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^
2)/c^2*tan(1/2*f*x+1/2*e)^2*d^3-1/f*a^2/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f
*x+1/2*e)*d+c)^2*c/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)-12/f*a^2/(c*tan(1/2*f
*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)*
d-2/f*a^2/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/c/(c^2+2*c*d+
d^2)*tan(1/2*f*x+1/2*e)*d^2-4/f*a^2/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1
/2*e)*d+c)^2/(c^2+2*c*d+d^2)*c-1/f*a^2/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*
x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*d+3/f*a^2/(c^2+2*c*d+d^2)/(c^2-d^2)^(1/2)*a
rctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.84876, size = 1449, normalized size = 10.5

$$\left[\frac{2\left(a^2c^3 + 4\,a^2c^2d - a^2cd^2 - 4\,a^2d^3\right)\cos\left(fx+e\right)\sin\left(fx+e\right) - 3\left(a^2d^2\cos\left(fx+e\right)^2 - 2\,a^2cd\sin\left(fx+e\right) - a^2c^2 - a^2d^2\right)}{4\left(\left(c^4d^2 + 2\,c^3d^3 - 2\,cd^5 - d^6\right)f\cos\left(fx+e\right)^2 - 2\left(c^5d + 2\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] [1/4*(2*(a^2*c^3 + 4*a^2*c^2*d - a^2*c*d^2 - 4*a^2*d^3)*cos(f*x + e)*sin(f*
x + e) - 3*(a^2*d^2*cos(f*x + e)^2 - 2*a^2*c*d*sin(f*x + e) - a^2*c^2 - a^2
*d^2)*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x +
e) - c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x + e) + d*cos(f*x + e))*sqrt(-c^2
 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) + 2*(4*a^2*
```

```
c^3 + a^2*c^2*d - 4*a^2*c*d^2 - a^2*d^3)*cos(f*x + e))/((c^4*d^2 + 2*c^3*d^
3 - 2*c*d^5 - d^6)*f*cos(f*x + e)^2 - 2*(c^5*d + 2*c^4*d^2 - 2*c^2*d^4 - c*
d^5)*f*sin(f*x + e) - (c^6 + 2*c^5*d + c^4*d^2 - c^2*d^4 - 2*c*d^5 - d^6)*f
), 1/2*((a^2*c^3 + 4*a^2*c^2*d - a^2*c*d^2 - 4*a^2*d^3)*cos(f*x + e)*sin(f*
x + e) - 3*(a^2*d^2*cos(f*x + e)^2 - 2*a^2*c*d*sin(f*x + e) - a^2*c^2 - a^2
*d^2)*sqrt(c^2 - d^2)*arctan(-(c*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x
 + e))) + (4*a^2*c^3 + a^2*c^2*d - 4*a^2*c*d^2 - a^2*d^3)*cos(f*x + e))/((c
^4*d^2 + 2*c^3*d^3 - 2*c*d^5 - d^6)*f*cos(f*x + e)^2 - 2*(c^5*d + 2*c^4*d^2
 - 2*c^2*d^4 - c*d^5)*f*sin(f*x + e) - (c^6 + 2*c^5*d + c^4*d^2 - c^2*d^4 -
 2*c*d^5 - d^6)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2/(c+d*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.45219, size = 470, normalized size = 3.41

$$\frac{\frac{3\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+d}{\sqrt{c^2-d^2}}\right)\right)a^2}{\left(c^2+2cd+d^2\right)\sqrt{c^2-d^2}}+\frac{a^2c^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-4a^2c^2d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-2a^2cd^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-4a^2c^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-\cdots}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] (3*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e
) + d)/sqrt(c^2 - d^2)))*a^2/((c^2 + 2*c*d + d^2)*sqrt(c^2 - d^2)) + (a^2*c
^3*tan(1/2*f*x + 1/2*e)^3 - 4*a^2*c^2*d*tan(1/2*f*x + 1/2*e)^3 - 2*a^2*c*d^
2*tan(1/2*f*x + 1/2*e)^3 - 4*a^2*c^3*tan(1/2*f*x + 1/2*e)^2 - a^2*c^2*d*tan
(1/2*f*x + 1/2*e)^2 - 8*a^2*c*d^2*tan(1/2*f*x + 1/2*e)^2 - 2*a^2*d^3*tan(1/
2*f*x + 1/2*e)^2 - a^2*c^3*tan(1/2*f*x + 1/2*e) - 12*a^2*c^2*d*tan(1/2*f*x
+ 1/2*e) - 2*a^2*c*d^2*tan(1/2*f*x + 1/2*e) - 4*a^2*c^3 - a^2*c^2*d)/((c^4
+ 2*c^3*d + c^2*d^2)*(c*tan(1/2*f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e) +
 c)^2))/f
```

3.442 $\int \frac{(a+a\sin(e+fx))^2}{(c+d\sin(e+fx))^4}\,dx$

Optimal. Leaf size=207

$$\frac{a^2(3c-2d)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{f(c-d)(c+d)^3\sqrt{c^2-d^2}}-\frac{a^2\left(c^2+6cd-10d^2\right)\cos(e+fx)}{6df(c-d)(c+d)^3(c+d\sin(e+fx))}-\frac{a^2(c+6d)\cos(e+fx)}{6df(c+d)^2(c+d\sin(e+fx))^2}+\frac{a^2(c}{3df(c+}$$

```
[Out] (a^2*(3*c - 2*d)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/((c - d)
*(c + d)^3*Sqrt[c^2 - d^2]*f) + (a^2*(c - d)*Cos[e + f*x])/(3*d*(c + d)*f*(
c + d*Sin[e + f*x])^3) - (a^2*(c + 6*d)*Cos[e + f*x])/(6*d*(c + d)^2*f*(c +
 d*Sin[e + f*x])^2) - (a^2*(c^2 + 6*c*d - 10*d^2)*Cos[e + f*x])/(6*(c - d)*
d*(c + d)^3*f*(c + d*Sin[e + f*x]))
```

Rubi [A] time = 0.314724, antiderivative size = 207, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {2762, 2754, 12, 2660, 618, 204}

$$\frac{a^2(3c-2d)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{f(c-d)(c+d)^3\sqrt{c^2-d^2}}-\frac{a^2\left(c^2+6cd-10d^2\right)\cos(e+fx)}{6df(c-d)(c+d)^3(c+d\sin(e+fx))}-\frac{a^2(c+6d)\cos(e+fx)}{6df(c+d)^2(c+d\sin(e+fx))^2}+\frac{a^2(c}{3df(c+}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2/(c + d*Sin[e + f*x])^4,x]
```

```
[Out] (a^2*(3*c - 2*d)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/((c - d)
*(c + d)^3*Sqrt[c^2 - d^2]*f) + (a^2*(c - d)*Cos[e + f*x])/(3*d*(c + d)*f*(
c + d*Sin[e + f*x])^3) - (a^2*(c + 6*d)*Cos[e + f*x])/(6*d*(c + d)^2*f*(c +
 d*Sin[e + f*x])^2) - (a^2*(c^2 + 6*c*d - 10*d^2)*Cos[e + f*x])/(6*(c - d)*
d*(c + d)^3*f*(c + d*Sin[e + f*x]))
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+a\sin(e+fx))^2}{(c+d\sin(e+fx))^4}\,dx &= \frac{a^2(c-d)\cos(e+fx)}{3d(c+d)f(c+d\sin(e+fx))^3} - \frac{a\int \frac{-6ad-a(c+5d)\sin(e+fx)}{(c+d\sin(e+fx))^3}\,dx}{3d(c+d)}\\
&= \frac{a^2(c-d)\cos(e+fx)}{3d(c+d)f(c+d\sin(e+fx))^3} - \frac{a^2(c+6d)\cos(e+fx)}{6d(c+d)^2f(c+d\sin(e+fx))^2} + \frac{a\int \frac{10a(c-d)d+a(c-d)(c+6}{(c+d\sin(e+f}}{6(c-d)d(c+}\\
&= \frac{a^2(c-d)\cos(e+fx)}{3d(c+d)f(c+d\sin(e+fx))^3} - \frac{a^2(c+6d)\cos(e+fx)}{6d(c+d)^2f(c+d\sin(e+fx))^2} - \frac{a^2\left(c^2+6cd-10d^2\right.}{6(c-d)d(c+d)^3f(c}\\
&= \frac{a^2(c-d)\cos(e+fx)}{3d(c+d)f(c+d\sin(e+fx))^3} - \frac{a^2(c+6d)\cos(e+fx)}{6d(c+d)^2f(c+d\sin(e+fx))^2} - \frac{a^2\left(c^2+6cd-10d^2\right.}{6(c-d)d(c+d)^3f(c}\\
&= \frac{a^2(c-d)\cos(e+fx)}{3d(c+d)f(c+d\sin(e+fx))^3} - \frac{a^2(c+6d)\cos(e+fx)}{6d(c+d)^2f(c+d\sin(e+fx))^2} - \frac{a^2\left(c^2+6cd-10d^2\right.}{6(c-d)d(c+d)^3f(c}\\
&= \frac{a^2(c-d)\cos(e+fx)}{3d(c+d)f(c+d\sin(e+fx))^3} - \frac{a^2(c+6d)\cos(e+fx)}{6d(c+d)^2f(c+d\sin(e+fx))^2} - \frac{a^2\left(c^2+6cd-10d^2\right.}{6(c-d)d(c+d)^3f(c}\\
&= \frac{a^2(3c-2d)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{(c-d)(c+d)^3\sqrt{c^2-d^2}f} + \frac{a^2(c-d)\cos(e+fx)}{3d(c+d)f(c+d\sin(e+fx))^3} - \frac{a^2(c+6d)\,\mathrm{co}}{6d(c+d)^2f(c+d}
\end{aligned}$$

Mathematica [A] time = 2.43868, size = 196, normalized size = 0.95

$$\frac{a^2\cos(e+fx)\left(-\frac{d(\sin(e+fx)+1)^2}{(c+d\sin(e+fx))^3}-\frac{(3c-2d)\left(\frac{6\tan^{-1}\left(\frac{\sqrt{d-c}\sqrt{1-\sin(e+fx)}}{\sqrt{-c-d}\sqrt{\sin(e+fx)+1}}\right)}{\sqrt{-c-d}\sqrt{d-c}}-\frac{\sqrt{\cos^2(e+fx)}((c+4d)\sin(e+fx)+4c+d)}{(c+d\sin(e+fx))^2}\right)}{2(c+d)^2\sqrt{\cos^2(e+fx)}}\right)}{3f(d-c)(c+d)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2/(c + d*Sin[e + f*x])^4,x]
```

```
[Out] (a^2*Cos[e + f*x]*(-((d*(1 + Sin[e + f*x])^2)/(c + d*Sin[e + f*x])^3) - ((3
*c - 2*d)*((6*ArcTan[(Sqrt[-c + d]*Sqrt[1 - Sin[e + f*x]])/(Sqrt[-c - d]*Sq
rt[1 + Sin[e + f*x]])])/(Sqrt[-c - d]*Sqrt[-c + d]) - (Sqrt[Cos[e + f*x]^2]
*(4*c + d + (c + 4*d)*Sin[e + f*x]))/(c + d*Sin[e + f*x])^2))/(2*(c + d)^2*
Sqrt[Cos[e + f*x]^2])))/(3*(-c + d)*(c + d)*f)
```

Maple [B] time = 0.155, size = 2425, normalized size = 11.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^4,x)
```

```
[Out] 1/f*a^2/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^3*c^3/(c^4+2*c^3*
d-2*c*d^3-d^4)*tan(1/2*f*x+1/2*e)^5-4/f*a^2/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1
/2*f*x+1/2*e)*d+c)^3*c^3/(c^4+2*c^3*d-2*c*d^3-d^4)*tan(1/2*f*x+1/2*e)^4-4/f
*a^2/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^3/(c^4+2*c^3*d-2*c*d
^3-d^4)*c^3+2/3/f*a^2/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^3/(
c^4+2*c^3*d-2*c*d^3-d^4)*d^3-18/f*a^2/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x
+1/2*e)*d+c)^3*c^2/(c^4+2*c^3*d-2*c*d^3-d^4)*tan(1/2*f*x+1/2*e)*d+14/f*a^2/
(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^3*c/(c^4+2*c^3*d-2*c*d^3-
d^4)*tan(1/2*f*x+1/2*e)*d^2+2/f*a^2/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1
/2*e)*d+c)^3/c/(c^4+2*c^3*d-2*c*d^3-d^4)*tan(1/2*f*x+1/2*e)*d^4-6/f*a^2/(c*
tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^3*c^2/(c^4+2*c^3*d-2*c*d^3-d
^4)*tan(1/2*f*x+1/2*e)^5*d+2/f*a^2/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/
2*e)*d+c)^3/c/(c^4+2*c^3*d-2*c*d^3-d^4)*tan(1/2*f*x+1/2*e)^5*d^4+3/f*a^2/(c
*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^3*c^2/(c^4+2*c^3*d-2*c*d^3-
d^4)*tan(1/2*f*x+1/2*e)^4*d-18/f*a^2/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+
1/2*e)*d+c)^3*c/(c^4+2*c^3*d-2*c*d^3-d^4)*tan(1/2*f*x+1/2*e)^4*d^2+12/f*a^2
/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^3/c/(c^4+2*c^3*d-2*c*d^3
-d^4)*tan(1/2*f*x+1/2*e)^4*d^4+4/f*a^2/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*
x+1/2*e)*d+c)^3/c^2/(c^4+2*c^3*d-2*c*d^3-d^4)*tan(1/2*f*x+1/2*e)^4*d^5-24/f
*a^2/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^3*c^2*d/(c^4+2*c^3*d
-2*c*d^3-d^4)*tan(1/2*f*x+1/2*e)^3+14/f*a^2/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1
/2*f*x+1/2*e)*d+c)^3*c*d^2/(c^4+2*c^3*d-2*c*d^3-d^4)*tan(1/2*f*x+1/2*e)^3+4
0/3/f*a^2/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^3/c*d^4/(c^4+2*
c^3*d-2*c*d^3-d^4)*tan(1/2*f*x+1/2*e)^3+8/f*a^2/(c*tan(1/2*f*x+1/2*e)^2+2*t
an(1/2*f*x+1/2*e)*d+c)^3/c^2*d^5/(c^4+2*c^3*d-2*c*d^3-d^4)*tan(1/2*f*x+1/2*
e)^3+8/3/f*a^2/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^3/c^3*d^6/
(c^4+2*c^3*d-2*c*d^3-d^4)*tan(1/2*f*x+1/2*e)^3+4/f*a^2/(c*tan(1/2*f*x+1/2*e
)^2+2*tan(1/2*f*x+1/2*e)*d+c)^3*c^2/(c^4+2*c^3*d-2*c*d^3-d^4)*tan(1/2*f*x+1
/2*e)^2*d-8/f*a^2/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^3*c^3/(
c^4+2*c^3*d-2*c*d^3-d^4)*tan(1/2*f*x+1/2*e)^2+7/3/f*a^2/(c*tan(1/2*f*x+1/2*
e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^3/(c^4+2*c^3*d-2*c*d^3-d^4)*c^2*d+2/f*a^2/(c
*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^3/(c^4+2*c^3*d-2*c*d^3-d^4)
*c*d^2+4/f*a^2/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^3/(c^4+2*c
^3*d-2*c*d^3-d^4)*tan(1/2*f*x+1/2*e)^5*d^3+8/f*a^2/(c*tan(1/2*f*x+1/2*e)^2+
2*tan(1/2*f*x+1/2*e)*d+c)^3/(c^4+2*c^3*d-2*c*d^3-d^4)*tan(1/2*f*x+1/2*e)^4*
d^3-4/f*a^2/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^3*d^3/(c^4+2*
c^3*d-2*c*d^3-d^4)*tan(1/2*f*x+1/2*e)^3+22/f*a^2/(c*tan(1/2*f*x+1/2*e)^2+2*
tan(1/2*f*x+1/2*e)*d+c)^3/(c^4+2*c^3*d-2*c*d^3-d^4)*tan(1/2*f*x+1/2*e)^2*d^
3+8/f*a^2/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^3/(c^4+2*c^3*d-
2*c*d^3-d^4)*tan(1/2*f*x+1/2*e)*d^3+3/f*a^2/(c^4+2*c^3*d-2*c*d^3-d^4)/(c^2-
d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*c-2/f*a
```

```
^2/(c^4+2*c^3*d-2*c*d^3-d^4)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/
2*e)+2*d)/(c^2-d^2)^(1/2))*d-1/f*a^2/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+
1/2*e)*d+c)^3*c^3/(c^4+2*c^3*d-2*c*d^3-d^4)*tan(1/2*f*x+1/2*e)-24/f*a^2/(c*
tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^3*c/(c^4+2*c^3*d-2*c*d^3-d^4
)*tan(1/2*f*x+1/2*e)^2*d^2+12/f*a^2/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1
/2*e)*d+c)^3/c/(c^4+2*c^3*d-2*c*d^3-d^4)*tan(1/2*f*x+1/2*e)^2*d^4+4/f*a^2/(
c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^3/c^2/(c^4+2*c^3*d-2*c*d^3
-d^4)*tan(1/2*f*x+1/2*e)^2*d^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.43455, size = 2874, normalized size = 13.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^4,x, algorithm="fricas")
```

```
[Out] [-1/12*(2*(a^2*c^4*d + 6*a^2*c^3*d^2 - 11*a^2*c^2*d^3 - 6*a^2*c*d^4 + 10*a^
2*d^5)*cos(f*x + e)^3 - 6*(a^2*c^5 + 6*a^2*c^4*d - 8*a^2*c^3*d^2 - 8*a^2*c^
2*d^3 + 7*a^2*c*d^4 + 2*a^2*d^5)*cos(f*x + e)*sin(f*x + e) - 3*(3*a^2*c^4 -
 2*a^2*c^3*d + 9*a^2*c^2*d^2 - 6*a^2*c*d^3 - 3*(3*a^2*c^2*d^2 - 2*a^2*c*d^3
)*cos(f*x + e)^2 + (9*a^2*c^3*d - 6*a^2*c^2*d^2 + 3*a^2*c*d^3 - 2*a^2*d^4 -
 (3*a^2*c*d^3 - 2*a^2*d^4)*cos(f*x + e)^2)*sin(f*x + e))*sqrt(-c^2 + d^2)*l
og(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 + 2*(c*co
s(f*x + e)*sin(f*x + e) + d*cos(f*x + e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x +
e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) - 12*(2*a^2*c^5 - a^2*c^4*d - 2*a^2
*c^3*d^2 - a^2*c^2*d^3 + 2*a^2*d^5)*cos(f*x + e))/(3*(c^7*d^2 + 2*c^6*d^3 -
 c^5*d^4 - 4*c^4*d^5 - c^3*d^6 + 2*c^2*d^7 + c*d^8)*f*cos(f*x + e)^2 - (c^9
 + 2*c^8*d + 2*c^7*d^2 + 2*c^6*d^3 - 4*c^5*d^4 - 10*c^4*d^5 - 2*c^3*d^6 + 6
*c^2*d^7 + 3*c*d^8)*f + ((c^6*d^3 + 2*c^5*d^4 - c^4*d^5 - 4*c^3*d^6 - c^2*d
^7 + 2*c*d^8 + d^9)*f*cos(f*x + e)^2 - (3*c^8*d + 6*c^7*d^2 - 2*c^6*d^3 - 1
0*c^5*d^4 - 4*c^4*d^5 + 2*c^3*d^6 + 2*c^2*d^7 + 2*c*d^8 + d^9)*f)*sin(f*x +
 e)), -1/6*((a^2*c^4*d + 6*a^2*c^3*d^2 - 11*a^2*c^2*d^3 - 6*a^2*c*d^4 + 10*
a^2*d^5)*cos(f*x + e)^3 - 3*(a^2*c^5 + 6*a^2*c^4*d - 8*a^2*c^3*d^2 - 8*a^2*
c^2*d^3 + 7*a^2*c*d^4 + 2*a^2*d^5)*cos(f*x + e)*sin(f*x + e) - 3*(3*a^2*c^4
 - 2*a^2*c^3*d + 9*a^2*c^2*d^2 - 6*a^2*c*d^3 - 3*(3*a^2*c^2*d^2 - 2*a^2*c*d
^3)*cos(f*x + e)^2 + (9*a^2*c^3*d - 6*a^2*c^2*d^2 + 3*a^2*c*d^3 - 2*a^2*d^4
 - (3*a^2*c*d^3 - 2*a^2*d^4)*cos(f*x + e)^2)*sin(f*x + e))*sqrt(c^2 - d^2)*
arctan(-(c*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) - 6*(2*a^2*c^5
 - a^2*c^4*d - 2*a^2*c^3*d^2 - a^2*c^2*d^3 + 2*a^2*d^5)*cos(f*x + e))/(3*(c
^7*d^2 + 2*c^6*d^3 - c^5*d^4 - 4*c^4*d^5 - c^3*d^6 + 2*c^2*d^7 + c*d^8)*f*c
os(f*x + e)^2 - (c^9 + 2*c^8*d + 2*c^7*d^2 + 2*c^6*d^3 - 4*c^5*d^4 - 10*c^4
*d^5 - 2*c^3*d^6 + 6*c^2*d^7 + 3*c*d^8)*f + ((c^6*d^3 + 2*c^5*d^4 - c^4*d^5
 - 4*c^3*d^6 - c^2*d^7 + 2*c*d^8 + d^9)*f*cos(f*x + e)^2 - (3*c^8*d + 6*c^7
*d^2 - 2*c^6*d^3 - 10*c^5*d^4 - 4*c^4*d^5 + 2*c^3*d^6 + 2*c^2*d^7 + 2*c*d^8
```

```
+ d^9)*f)*sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2/(c+d*sin(f*x+e))**4,x)

[Out] Timed out
```

Giac [B] time = 1.76368, size = 1054, normalized size = 5.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^4,x, algorithm="giac")

[Out] 1/3*(3*(3*a^2*c - 2*a^2*d)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arcta
n((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/((c^4 + 2*c^3*d - 2*c*d^3
- d^4)*sqrt(c^2 - d^2)) + (3*a^2*c^6*tan(1/2*f*x + 1/2*e)^5 - 18*a^2*c^5*d*
tan(1/2*f*x + 1/2*e)^5 + 12*a^2*c^3*d^3*tan(1/2*f*x + 1/2*e)^5 + 6*a^2*c^2*
d^4*tan(1/2*f*x + 1/2*e)^5 - 12*a^2*c^6*tan(1/2*f*x + 1/2*e)^4 + 9*a^2*c^5*
d*tan(1/2*f*x + 1/2*e)^4 - 54*a^2*c^4*d^2*tan(1/2*f*x + 1/2*e)^4 + 24*a^2*c
^3*d^3*tan(1/2*f*x + 1/2*e)^4 + 36*a^2*c^2*d^4*tan(1/2*f*x + 1/2*e)^4 + 12*
a^2*c*d^5*tan(1/2*f*x + 1/2*e)^4 - 72*a^2*c^5*d*tan(1/2*f*x + 1/2*e)^3 + 42
*a^2*c^4*d^2*tan(1/2*f*x + 1/2*e)^3 - 12*a^2*c^3*d^3*tan(1/2*f*x + 1/2*e)^3
 + 40*a^2*c^2*d^4*tan(1/2*f*x + 1/2*e)^3 + 24*a^2*c*d^5*tan(1/2*f*x + 1/2*e
)^3 + 8*a^2*d^6*tan(1/2*f*x + 1/2*e)^3 - 24*a^2*c^6*tan(1/2*f*x + 1/2*e)^2
+ 12*a^2*c^5*d*tan(1/2*f*x + 1/2*e)^2 - 72*a^2*c^4*d^2*tan(1/2*f*x + 1/2*e)
^2 + 66*a^2*c^3*d^3*tan(1/2*f*x + 1/2*e)^2 + 36*a^2*c^2*d^4*tan(1/2*f*x + 1
/2*e)^2 + 12*a^2*c*d^5*tan(1/2*f*x + 1/2*e)^2 - 3*a^2*c^6*tan(1/2*f*x + 1/2
*e) - 54*a^2*c^5*d*tan(1/2*f*x + 1/2*e) + 42*a^2*c^4*d^2*tan(1/2*f*x + 1/2*
e) + 24*a^2*c^3*d^3*tan(1/2*f*x + 1/2*e) + 6*a^2*c^2*d^4*tan(1/2*f*x + 1/2*
e) - 12*a^2*c^6 + 7*a^2*c^5*d + 6*a^2*c^4*d^2 + 2*a^2*c^3*d^3)/((c^7 + 2*c^
6*d - 2*c^4*d^3 - c^3*d^4)*(c*tan(1/2*f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/
2*e) + c)^3))/f
```

3.443 $\int \frac{(a+a\sin(e+fx))^2}{(c+d\sin(e+fx))^5}\,dx$

Optimal. Leaf size=286

$$\frac{a^2\left(12c^2-16cd+7d^2\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{4f(c-d)^2(c+d)^4\sqrt{c^2-d^2}}-\frac{a^2\left(16c^2d+2c^3-59cd^2+32d^3\right)\cos(e+fx)}{24df(c-d)^2(c+d)^4(c+d\sin(e+fx))}-\frac{a^2\left(2c^2+16cd-21\right.}{24df(c-d)(c+d)^3(c}$$

```
[Out] (a^2*(12*c^2 - 16*c*d + 7*d^2)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d
^2]])/(4*(c - d)^2*(c + d)^4*Sqrt[c^2 - d^2]*f) + (a^2*(c - d)*Cos[e + f*x]
)/(4*d*(c + d)*f*(c + d*Sin[e + f*x])^4) - (a^2*(c + 8*d)*Cos[e + f*x])/(12
*d*(c + d)^2*f*(c + d*Sin[e + f*x])^3) - (a^2*(2*c^2 + 16*c*d - 21*d^2)*Cos
[e + f*x])/(24*(c - d)*d*(c + d)^3*f*(c + d*Sin[e + f*x])^2) - (a^2*(2*c^3
+ 16*c^2*d - 59*c*d^2 + 32*d^3)*Cos[e + f*x])/(24*(c - d)^2*d*(c + d)^4*f*(
c + d*Sin[e + f*x]))
```

Rubi [A] time = 0.506972, antiderivative size = 286, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {2762, 2754, 12, 2660, 618, 204}

$$\frac{a^2\left(12c^2-16cd+7d^2\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{4f(c-d)^2(c+d)^4\sqrt{c^2-d^2}}-\frac{a^2\left(16c^2d+2c^3-59cd^2+32d^3\right)\cos(e+fx)}{24df(c-d)^2(c+d)^4(c+d\sin(e+fx))}-\frac{a^2\left(2c^2+16cd-21\right.}{24df(c-d)(c+d)^3(c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2/(c + d*Sin[e + f*x])^5,x]
```

```
[Out] (a^2*(12*c^2 - 16*c*d + 7*d^2)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d
^2]])/(4*(c - d)^2*(c + d)^4*Sqrt[c^2 - d^2]*f) + (a^2*(c - d)*Cos[e + f*x]
)/(4*d*(c + d)*f*(c + d*Sin[e + f*x])^4) - (a^2*(c + 8*d)*Cos[e + f*x])/(12
*d*(c + d)^2*f*(c + d*Sin[e + f*x])^3) - (a^2*(2*c^2 + 16*c*d - 21*d^2)*Cos
[e + f*x])/(24*(c - d)*d*(c + d)^3*f*(c + d*Sin[e + f*x])^2) - (a^2*(2*c^3
+ 16*c^2*d - 59*c*d^2 + 32*d^3)*Cos[e + f*x])/(24*(c - d)^2*d*(c + d)^4*f*(
c + d*Sin[e + f*x]))
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
```

```
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^2}{(c+d\sin(e+fx))^5}\,dx = \frac{a^2(c-d)\cos(e+fx)}{4d(c+d)f(c+d\sin(e+fx))^4} - \frac{a\int \frac{-8ad-a(c+7d)\sin(e+fx)}{(c+d\sin(e+fx))^4}\,dx}{4d(c+d)}$$

$$= \frac{a^2(c-d)\cos(e+fx)}{4d(c+d)f(c+d\sin(e+fx))^4} - \frac{a^2(c+8d)\cos(e+fx)}{12d(c+d)^2f(c+d\sin(e+fx))^3} + \frac{a\int \frac{21a(c-d)d+2a(c-d)(c+8d \ldots}{(c+d\sin(e+fx)) \ldots}}{12(c-d)d(c+ \ldots}$$

$$= \frac{a^2(c-d)\cos(e+fx)}{4d(c+d)f(c+d\sin(e+fx))^4} - \frac{a^2(c+8d)\cos(e+fx)}{12d(c+d)^2f(c+d\sin(e+fx))^3} - \frac{a^2\left(2c^2+16cd-21d^2 \ldots\right.}{24(c-d)d(c+d)^3f(c+ \ldots}$$

$$= \frac{a^2(c-d)\cos(e+fx)}{4d(c+d)f(c+d\sin(e+fx))^4} - \frac{a^2(c+8d)\cos(e+fx)}{12d(c+d)^2f(c+d\sin(e+fx))^3} - \frac{a^2\left(2c^2+16cd-21d^2 \ldots\right.}{24(c-d)d(c+d)^3f(c+ \ldots}$$

$$= \frac{a^2(c-d)\cos(e+fx)}{4d(c+d)f(c+d\sin(e+fx))^4} - \frac{a^2(c+8d)\cos(e+fx)}{12d(c+d)^2f(c+d\sin(e+fx))^3} - \frac{a^2\left(2c^2+16cd-21d^2 \ldots\right.}{24(c-d)d(c+d)^3f(c+ \ldots}$$

$$= \frac{a^2(c-d)\cos(e+fx)}{4d(c+d)f(c+d\sin(e+fx))^4} - \frac{a^2(c+8d)\cos(e+fx)}{12d(c+d)^2f(c+d\sin(e+fx))^3} - \frac{a^2\left(2c^2+16cd-21d^2 \ldots\right.}{24(c-d)d(c+d)^3f(c+ \ldots}$$

$$= \frac{a^2(c-d)\cos(e+fx)}{4d(c+d)f(c+d\sin(e+fx))^4} - \frac{a^2(c+8d)\cos(e+fx)}{12d(c+d)^2f(c+d\sin(e+fx))^3} - \frac{a^2\left(2c^2+16cd-21d^2 \ldots\right.}{24(c-d)d(c+d)^3f(c+ \ldots}$$

$$= \frac{a^2\left(12c^2-16cd+7d^2\right)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{4(c-d)^2(c+d)^4\sqrt{c^2-d^2}\,f} + \frac{a^2(c-d)\cos(e+fx)}{4d(c+d)f(c+d\sin(e+fx))^4} - \frac{a^2(c \ldots}{12d(c+d \ldots}$$

Mathematica [A] time = 3.90137, size = 269, normalized size = 0.94

$$\frac{a^2\cos(e+fx)\left(\frac{\left(12c^2-16cd+7d^2\right)(c+d\sin(e+fx))^2\left(\sqrt{-c-d}\sqrt{d-c}\sqrt{\cos^2(e+fx)}((c+4d)\sin(e+fx)+4c+d)-6(c+d\sin(e+fx))^2\tan^{-1}\left(\frac{\sqrt{d-c}\sqrt{1-\sin(e+fx)}}{\sqrt{-c-d}\sqrt{\sin(e+fx)+1}}\right)\right)}{(-c-d)^{7/2}(d-c)^{3/2}\sqrt{\cos^2(e+fx)}}\right.}{24f(d-c)(c+d)(c+d\sin(e+fx))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2/(c + d*Sin[e + f*x])^5,x]
```

```
[Out] (a^2*Cos[e + f*x]*(-6*d*(1 + Sin[e + f*x])^2 - (2*(5*c - 2*d)*d*(1 + Sin[e
+ f*x])^2*(c + d*Sin[e + f*x]))/((c - d)*(c + d)) + ((12*c^2 - 16*c*d + 7*d
^2)*(c + d*Sin[e + f*x])^2*(-6*ArcTan[(Sqrt[-c + d]*Sqrt[1 - Sin[e + f*x]])
/(Sqrt[-c - d]*Sqrt[1 + Sin[e + f*x]])]*(c + d*Sin[e + f*x])^2 + Sqrt[-c -
d]*Sqrt[-c + d]*Sqrt[Cos[e + f*x]^2]*(4*c + d + (c + 4*d)*Sin[e + f*x])))/(
(-c - d)^(7/2)*(-c + d)^(3/2)*Sqrt[Cos[e + f*x]^2])))/(24*(-c + d)*(c + d)*
f*(c + d*Sin[e + f*x])^4)
```

Maple [B] time = 0.192, size = 6466, normalized size = 22.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^5,x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.12127, size = 4629, normalized size = 16.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^5,x, algorithm="fricas")
```

```
[Out] [1/48*(2*(8*a^2*c^6*d + 64*a^2*c^5*d^2 - 208*a^2*c^4*d^3 + 16*a^2*c^3*d^4 +
 221*a^2*c^2*d^5 - 80*a^2*c*d^6 - 21*a^2*d^7)*cos(f*x + e)^3 - 3*(12*a^2*c^
6 - 16*a^2*c^5*d + 79*a^2*c^4*d^2 - 96*a^2*c^3*d^3 + 54*a^2*c^2*d^4 - 16*a^
2*c*d^5 + 7*a^2*d^6 + (12*a^2*c^2*d^4 - 16*a^2*c*d^5 + 7*a^2*d^6)*cos(f*x +
```

```
 e)^4 - 2*(36*a^2*c^4*d^2 - 48*a^2*c^3*d^3 + 33*a^2*c^2*d^4 - 16*a^2*c*d^5
+ 7*a^2*d^6)*cos(f*x + e)^2 + 4*(12*a^2*c^5*d - 16*a^2*c^4*d^2 + 19*a^2*c^3
*d^3 - 16*a^2*c^2*d^4 + 7*a^2*c*d^5 - (12*a^2*c^3*d^3 - 16*a^2*c^2*d^4 + 7*
a^2*c*d^5)*cos(f*x + e)^2)*sin(f*x + e))*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2
)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f
*x + e) + d*cos(f*x + e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin
(f*x + e) - c^2 - d^2)) - 6*(16*a^2*c^7 - 20*a^2*c^6*d - 45*a^2*c^4*d^3 + 1
6*a^2*c^3*d^4 + 74*a^2*c^2*d^5 - 32*a^2*c*d^6 - 9*a^2*d^7)*cos(f*x + e) + 2
*((2*a^2*c^5*d^2 + 16*a^2*c^4*d^3 - 61*a^2*c^3*d^4 + 16*a^2*c^2*d^5 + 59*a^
2*c*d^6 - 32*a^2*d^7)*cos(f*x + e)^3 - 3*(4*a^2*c^7 + 32*a^2*c^6*d - 79*a^2
*c^5*d^2 - 16*a^2*c^4*d^3 + 70*a^2*c^3*d^4 + 5*a^2*c*d^6 - 16*a^2*d^7)*cos(
f*x + e))*sin(f*x + e))/((c^8*d^4 + 2*c^7*d^5 - 2*c^6*d^6 - 6*c^5*d^7 + 6*c
^3*d^9 + 2*c^2*d^10 - 2*c*d^11 - d^12)*f*cos(f*x + e)^4 - 2*(3*c^10*d^2 + 6
*c^9*d^3 - 5*c^8*d^4 - 16*c^7*d^5 - 2*c^6*d^6 + 12*c^5*d^7 + 6*c^4*d^8 - c^
2*d^10 - 2*c*d^11 - d^12)*f*cos(f*x + e)^2 + (c^12 + 2*c^11*d + 4*c^10*d^2
+ 6*c^9*d^3 - 11*c^8*d^4 - 28*c^7*d^5 + 28*c^5*d^7 + 11*c^4*d^8 - 6*c^3*d^9
 - 4*c^2*d^10 - 2*c*d^11 - d^12)*f - 4*((c^9*d^3 + 2*c^8*d^4 - 2*c^7*d^5 -
6*c^6*d^6 + 6*c^4*d^8 + 2*c^3*d^9 - 2*c^2*d^10 - c*d^11)*f*cos(f*x + e)^2 -
 (c^11*d + 2*c^10*d^2 - c^9*d^3 - 4*c^8*d^4 - 2*c^7*d^5 + 2*c^5*d^7 + 4*c^4
*d^8 + c^3*d^9 - 2*c^2*d^10 - c*d^11)*f)*sin(f*x + e)), 1/24*((8*a^2*c^6*d
+ 64*a^2*c^5*d^2 - 208*a^2*c^4*d^3 + 16*a^2*c^3*d^4 + 221*a^2*c^2*d^5 - 80*
a^2*c*d^6 - 21*a^2*d^7)*cos(f*x + e)^3 - 3*(12*a^2*c^6 - 16*a^2*c^5*d + 79*
a^2*c^4*d^2 - 96*a^2*c^3*d^3 + 54*a^2*c^2*d^4 - 16*a^2*c*d^5 + 7*a^2*d^6 +
(12*a^2*c^2*d^4 - 16*a^2*c*d^5 + 7*a^2*d^6)*cos(f*x + e)^4 - 2*(36*a^2*c^4*
d^2 - 48*a^2*c^3*d^3 + 33*a^2*c^2*d^4 - 16*a^2*c*d^5 + 7*a^2*d^6)*cos(f*x +
 e)^2 + 4*(12*a^2*c^5*d - 16*a^2*c^4*d^2 + 19*a^2*c^3*d^3 - 16*a^2*c^2*d^4
+ 7*a^2*c*d^5 - (12*a^2*c^3*d^3 - 16*a^2*c^2*d^4 + 7*a^2*c*d^5)*cos(f*x + e
)^2)*sin(f*x + e))*sqrt(c^2 - d^2)*arctan(-(c*sin(f*x + e) + d)/(sqrt(c^2 -
 d^2)*cos(f*x + e))) - 3*(16*a^2*c^7 - 20*a^2*c^6*d - 45*a^2*c^4*d^3 + 16*a
^2*c^3*d^4 + 74*a^2*c^2*d^5 - 32*a^2*c*d^6 - 9*a^2*d^7)*cos(f*x + e) + ((2*
a^2*c^5*d^2 + 16*a^2*c^4*d^3 - 61*a^2*c^3*d^4 + 16*a^2*c^2*d^5 + 59*a^2*c*d
^6 - 32*a^2*d^7)*cos(f*x + e)^3 - 3*(4*a^2*c^7 + 32*a^2*c^6*d - 79*a^2*c^5*
d^2 - 16*a^2*c^4*d^3 + 70*a^2*c^3*d^4 + 5*a^2*c*d^6 - 16*a^2*d^7)*cos(f*x +
 e))*sin(f*x + e))/((c^8*d^4 + 2*c^7*d^5 - 2*c^6*d^6 - 6*c^5*d^7 + 6*c^3*d^
9 + 2*c^2*d^10 - 2*c*d^11 - d^12)*f*cos(f*x + e)^4 - 2*(3*c^10*d^2 + 6*c^9*
d^3 - 5*c^8*d^4 - 16*c^7*d^5 - 2*c^6*d^6 + 12*c^5*d^7 + 6*c^4*d^8 - c^2*d^1
0 - 2*c*d^11 - d^12)*f*cos(f*x + e)^2 + (c^12 + 2*c^11*d + 4*c^10*d^2 + 6*c
^9*d^3 - 11*c^8*d^4 - 28*c^7*d^5 + 28*c^5*d^7 + 11*c^4*d^8 - 6*c^3*d^9 - 4*
c^2*d^10 - 2*c*d^11 - d^12)*f - 4*((c^9*d^3 + 2*c^8*d^4 - 2*c^7*d^5 - 6*c^6
*d^6 + 6*c^4*d^8 + 2*c^3*d^9 - 2*c^2*d^10 - c*d^11)*f*cos(f*x + e)^2 - (c^1
1*d + 2*c^10*d^2 - c^9*d^3 - 4*c^8*d^4 - 2*c^7*d^5 + 2*c^5*d^7 + 4*c^4*d^8
+ c^3*d^9 - 2*c^2*d^10 - c*d^11)*f)*sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2/(c+d*sin(f*x+e))**5,x)

[Out] Timed out
```

Giac [B] time = 1.6169, size = 2099, normalized size = 7.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^5,x, algorithm="giac")
```

```
[Out] 1/12*(3*(12*a^2*c^2 - 16*a^2*c*d + 7*a^2*d^2)*(pi*floor(1/2*(f*x + e)/pi +
1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/((c^6 +
 2*c^5*d - c^4*d^2 - 4*c^3*d^3 - c^2*d^4 + 2*c*d^5 + d^6)*sqrt(c^2 - d^2))
+ (12*a^2*c^9*tan(1/2*f*x + 1/2*e)^7 - 96*a^2*c^8*d*tan(1/2*f*x + 1/2*e)^7
+ 45*a^2*c^7*d^2*tan(1/2*f*x + 1/2*e)^7 + 96*a^2*c^6*d^3*tan(1/2*f*x + 1/2*
e)^7 + 24*a^2*c^5*d^4*tan(1/2*f*x + 1/2*e)^7 - 48*a^2*c^4*d^5*tan(1/2*f*x +
 1/2*e)^7 - 24*a^2*c^3*d^6*tan(1/2*f*x + 1/2*e)^7 - 48*a^2*c^9*tan(1/2*f*x
+ 1/2*e)^6 + 84*a^2*c^8*d*tan(1/2*f*x + 1/2*e)^6 - 432*a^2*c^7*d^2*tan(1/2*
f*x + 1/2*e)^6 + 411*a^2*c^6*d^3*tan(1/2*f*x + 1/2*e)^6 + 336*a^2*c^5*d^4*t
an(1/2*f*x + 1/2*e)^6 - 24*a^2*c^4*d^5*tan(1/2*f*x + 1/2*e)^6 - 192*a^2*c^3
*d^6*tan(1/2*f*x + 1/2*e)^6 - 72*a^2*c^2*d^7*tan(1/2*f*x + 1/2*e)^6 + 12*a^
2*c^9*tan(1/2*f*x + 1/2*e)^5 - 480*a^2*c^8*d*tan(1/2*f*x + 1/2*e)^5 + 597*a
^2*c^7*d^2*tan(1/2*f*x + 1/2*e)^5 - 480*a^2*c^6*d^3*tan(1/2*f*x + 1/2*e)^5
+ 836*a^2*c^5*d^4*tan(1/2*f*x + 1/2*e)^5 + 208*a^2*c^4*d^5*tan(1/2*f*x + 1/
2*e)^5 - 152*a^2*c^3*d^6*tan(1/2*f*x + 1/2*e)^5 - 256*a^2*c^2*d^7*tan(1/2*f
*x + 1/2*e)^5 - 96*a^2*c*d^8*tan(1/2*f*x + 1/2*e)^5 - 144*a^2*c^9*tan(1/2*f
*x + 1/2*e)^4 + 204*a^2*c^8*d*tan(1/2*f*x + 1/2*e)^4 - 1104*a^2*c^7*d^2*tan
(1/2*f*x + 1/2*e)^4 + 1617*a^2*c^6*d^3*tan(1/2*f*x + 1/2*e)^4 - 48*a^2*c^5*
d^4*tan(1/2*f*x + 1/2*e)^4 + 406*a^2*c^4*d^5*tan(1/2*f*x + 1/2*e)^4 - 256*a
^2*c^3*d^6*tan(1/2*f*x + 1/2*e)^4 - 184*a^2*c^2*d^7*tan(1/2*f*x + 1/2*e)^4
- 128*a^2*c*d^8*tan(1/2*f*x + 1/2*e)^4 - 48*a^2*d^9*tan(1/2*f*x + 1/2*e)^4
- 12*a^2*c^9*tan(1/2*f*x + 1/2*e)^3 - 672*a^2*c^8*d*tan(1/2*f*x + 1/2*e)^3
+ 1035*a^2*c^7*d^2*tan(1/2*f*x + 1/2*e)^3 - 672*a^2*c^6*d^3*tan(1/2*f*x + 1
/2*e)^3 + 1220*a^2*c^5*d^4*tan(1/2*f*x + 1/2*e)^3 - 80*a^2*c^4*d^5*tan(1/2*
f*x + 1/2*e)^3 - 152*a^2*c^3*d^6*tan(1/2*f*x + 1/2*e)^3 - 256*a^2*c^2*d^7*t
an(1/2*f*x + 1/2*e)^3 - 96*a^2*c*d^8*tan(1/2*f*x + 1/2*e)^3 - 144*a^2*c^9*t
an(1/2*f*x + 1/2*e)^2 + 188*a^2*c^8*d*tan(1/2*f*x + 1/2*e)^2 - 656*a^2*c^7*
d^2*tan(1/2*f*x + 1/2*e)^2 + 1201*a^2*c^6*d^3*tan(1/2*f*x + 1/2*e)^2 - 16*a
^2*c^5*d^4*tan(1/2*f*x + 1/2*e)^2 - 120*a^2*c^4*d^5*tan(1/2*f*x + 1/2*e)^2
- 192*a^2*c^3*d^6*tan(1/2*f*x + 1/2*e)^2 - 72*a^2*c^2*d^7*tan(1/2*f*x + 1/2
*e)^2 - 12*a^2*c^9*tan(1/2*f*x + 1/2*e) - 288*a^2*c^8*d*tan(1/2*f*x + 1/2*e
) + 499*a^2*c^7*d^2*tan(1/2*f*x + 1/2*e) + 32*a^2*c^6*d^3*tan(1/2*f*x + 1/2
*e) - 64*a^2*c^5*d^4*tan(1/2*f*x + 1/2*e) - 80*a^2*c^4*d^5*tan(1/2*f*x + 1/
2*e) - 24*a^2*c^3*d^6*tan(1/2*f*x + 1/2*e) - 48*a^2*c^9 + 68*a^2*c^8*d + 16
*a^2*c^7*d^2 - 5*a^2*c^6*d^3 - 16*a^2*c^5*d^4 - 6*a^2*c^4*d^5)/((c^10 + 2*c
^9*d - c^8*d^2 - 4*c^7*d^3 - c^6*d^4 + 2*c^5*d^5 + c^4*d^6)*(c*tan(1/2*f*x
+ 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e) + c)^4))/f
```

3.444 $\int (a+a\sin(e+fx))^3(c+d\sin(e+fx))^3\,dx$

Optimal. Leaf size=215

$$-\frac{a^3d\left(18c^2+54cd+23d^2\right)\sin^3(e+fx)\cos(e+fx)}{24f}-\frac{a^3\left(90c^2d+24c^3+78cd^2+23d^3\right)\sin(e+fx)\cos(e+fx)}{16f}+\frac{1}{16}a^3$$

```
[Out] (a^3*(40*c^3 + 90*c^2*d + 78*c*d^2 + 23*d^3)*x)/16 - (4*a^3*(c + d)^3*Cos[e
 + f*x])/f + (a^3*(c + d)^2*(c + 7*d)*Cos[e + f*x]^3)/(3*f) - (3*a^3*d^2*(c
 + d)*Cos[e + f*x]^5)/(5*f) - (a^3*(24*c^3 + 90*c^2*d + 78*c*d^2 + 23*d^3)*
Cos[e + f*x]*Sin[e + f*x])/(16*f) - (a^3*d*(18*c^2 + 54*c*d + 23*d^2)*Cos[e
 + f*x]*Sin[e + f*x]^3)/(24*f) - (a^3*d^3*Cos[e + f*x]*Sin[e + f*x]^5)/(6*f
)
```

Rubi [A] time = 0.544649, antiderivative size = 326, normalized size of antiderivative = 1.52, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2763, 2968, 3023, 2753, 2734}

$$-\frac{a^3\left(107c^3d^2+472c^2d^3-18c^4d+2c^5+456cd^4+136d^5\right)\cos(e+fx)}{60d^2f}-\frac{a^3\left(2c^2-18cd+115d^2\right)\cos(e+fx)(c+d\sin(e+fx))}{120d^2f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^3,x]
```

```
[Out] (a^3*(40*c^3 + 90*c^2*d + 78*c*d^2 + 23*d^3)*x)/16 - (a^3*(2*c^5 - 18*c^4*d
 + 107*c^3*d^2 + 472*c^2*d^3 + 456*c*d^4 + 136*d^5)*Cos[e + f*x])/(60*d^2*f
) - (a^3*(4*c^4 - 36*c^3*d + 216*c^2*d^2 + 626*c*d^3 + 345*d^4)*Cos[e + f*x
]*Sin[e + f*x])/(240*d*f) - (a^3*(2*c^3 - 18*c^2*d + 111*c*d^2 + 136*d^3)*C
os[e + f*x]*(c + d*Sin[e + f*x])^2)/(120*d^2*f) - (a^3*(2*c^2 - 18*c*d + 11
5*d^2)*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(120*d^2*f) + (a^3*(2*c - 13*d)
*Cos[e + f*x]*(c + d*Sin[e + f*x])^4)/(30*d^2*f) - (Cos[e + f*x]*(a^3 + a^3
*Sin[e + f*x])*(c + d*Sin[e + f*x])^4)/(6*d*f)
```

Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
```

```
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int (a + a\sin(e+fx))^3(c+d\sin(e+fx))^3\,dx = -\frac{\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)(c+d\sin(e+fx))^4}{6df} + \frac{\int (a+a\sin(\cdots}{\cdots}$$

$$= -\frac{\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)(c+d\sin(e+fx))^4}{6df} + \frac{\int (c+d\sin(\cdots}{\cdots}$$

$$= \frac{a^3(2c-13d)\cos(e+fx)(c+d\sin(e+fx))^4}{30d^2f} - \frac{\cos(e+fx)\left(a^3+a^3\sin\cdots\right.}{\cdots}$$

$$= -\frac{a^3\left(2c^2-18cd+115d^2\right)\cos(e+fx)(c+d\sin(e+fx))^3}{120d^2f} + \frac{a^3(2c-13\cdots}{\cdots}$$

$$= -\frac{a^3\left(2c^3-18c^2d+111cd^2+136d^3\right)\cos(e+fx)(c+d\sin(e+fx))^2}{120d^2f} - \cdots$$

$$= \frac{1}{16}a^3\left(40c^3+90c^2d+78cd^2+23d^3\right)x - \frac{a^3\left(2c^5-18c^4d+107c^3d^2+\cdots\right.}{\cdots}$$

Mathematica [A] time = 1.41372, size = 233, normalized size = 1.08

$$-\frac{a^3\cos(e+fx)\left(30\left(90c^2d+40c^3+78cd^2+23d^3\right)\sin^{-1}\left(\frac{\sqrt{1-\sin(e+fx)}}{\sqrt{2}}\right)+\sqrt{\cos^2(e+fx)}\left(10d\left(18c^2+54cd+23d^2\right)\mathrm{si}\cdots\right.\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^3,x]
```

```
[Out] -(a^3*Cos[e + f*x]*(30*(40*c^3 + 90*c^2*d + 78*c*d^2 + 23*d^3)*ArcSin[Sqrt[
1 - Sin[e + f*x]]/Sqrt[2]] + Sqrt[Cos[e + f*x]^2]*(16*(55*c^3 + 135*c^2*d +
 114*c*d^2 + 34*d^3) + 15*(24*c^3 + 90*c^2*d + 78*c*d^2 + 23*d^3)*Sin[e + f
*x] + 16*(5*c^3 + 45*c^2*d + 57*c*d^2 + 17*d^3)*Sin[e + f*x]^2 + 10*d*(18*c
^2 + 54*c*d + 23*d^2)*Sin[e + f*x]^3 + 144*d^2*(c + d)*Sin[e + f*x]^4 + 40*
d^3*Sin[e + f*x]^5)))/(240*f*Sqrt[Cos[e + f*x]^2])
```

Maple [B] time = 0.051, size = 481, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(c+d*sin(f*x+e))^3,x)
```

```
[Out] 1/f*(-1/3*a^3*c^3*(2+sin(f*x+e)^2)*cos(f*x+e)+3*a^3*c^2*d*(-1/4*(sin(f*x+e)
^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-3/5*a^3*c*d^2*(8/3+sin(f*x+e)^
4+4/3*sin(f*x+e)^2)*cos(f*x+e)+a^3*d^3*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3
+15/8*sin(f*x+e))*cos(f*x+e)+5/16*f*x+5/16*e)+3*a^3*c^3*(-1/2*sin(f*x+e)*co
s(f*x+e)+1/2*f*x+1/2*e)-3*a^3*c^2*d*(2+sin(f*x+e)^2)*cos(f*x+e)+9*a^3*c*d^2
*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-3/5*a^3*d^3*
(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)-3*a^3*c^3*cos(f*x+e)+9*a^3*c
^2*d*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-3*a^3*c*d^2*(2+sin(f*x+e)^2
)*cos(f*x+e)+3*a^3*d^3*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f
*x+3/8*e)+a^3*c^3*(f*x+e)-3*a^3*c^2*d*cos(f*x+e)+3*a^3*c*d^2*(-1/2*sin(f*x+
e)*cos(f*x+e)+1/2*f*x+1/2*e)-1/3*a^3*d^3*(2+sin(f*x+e)^2)*cos(f*x+e))
```

Maxima [B] time = 1.19856, size = 633, normalized size = 2.94

$$\frac{320\left(\cos\left(fx+e\right)^{3}-3\cos\left(fx+e\right)\right)a^{3}c^{3}+720\left(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right)\right)a^{3}c^{3}+960\left(fx+e\right)a^{3}c^{3}+2880\left(\cos\left(fx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c+d*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 1/960*(320*(cos(f*x + e)^3 - 3*cos(f*x + e))*a^3*c^3 + 720*(2*f*x + 2*e - s
in(2*f*x + 2*e))*a^3*c^3 + 960*(f*x + e)*a^3*c^3 + 2880*(cos(f*x + e)^3 - 3
*cos(f*x + e))*a^3*c^2*d + 90*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f
*x + 2*e))*a^3*c^2*d + 2160*(2*f*x + 2*e - sin(2*f*x + 2*e))*a^3*c^2*d - 19
2*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*a^3*c*d^2 + 2880
*(cos(f*x + e)^3 - 3*cos(f*x + e))*a^3*c*d^2 + 270*(12*f*x + 12*e + sin(4*f
*x + 4*e) - 8*sin(2*f*x + 2*e))*a^3*c*d^2 + 720*(2*f*x + 2*e - sin(2*f*x +
2*e))*a^3*c*d^2 - 192*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x +
e))*a^3*d^3 + 320*(cos(f*x + e)^3 - 3*cos(f*x + e))*a^3*d^3 + 5*(4*sin(2*f*
x + 2*e)^3 + 60*f*x + 60*e + 9*sin(4*f*x + 4*e) - 48*sin(2*f*x + 2*e))*a^3*
d^3 + 90*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*a^3*d^3 -
2880*a^3*c^3*cos(f*x + e) - 2880*a^3*c^2*d*cos(f*x + e))/f
```

Fricas [A] time = 2.14481, size = 590, normalized size = 2.74

$$-\frac{144\left(a^{3}cd^{2}+a^{3}d^{3}\right)\cos\left(fx+e\right)^{5}-80\left(a^{3}c^{3}+9\,a^{3}c^{2}d+15\,a^{3}cd^{2}+7\,a^{3}d^{3}\right)\cos\left(fx+e\right)^{3}-15\left(40\,a^{3}c^{3}+90\,a^{3}c^{2}d+78\,a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c+d*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -1/240*(144*(a^3*c*d^2 + a^3*d^3)*cos(f*x + e)^5 - 80*(a^3*c^3 + 9*a^3*c^2*
d + 15*a^3*c*d^2 + 7*a^3*d^3)*cos(f*x + e)^3 - 15*(40*a^3*c^3 + 90*a^3*c^2*
d + 78*a^3*c*d^2 + 23*a^3*d^3)*f*x + 960*(a^3*c^3 + 3*a^3*c^2*d + 3*a^3*c*d
^2 + a^3*d^3)*cos(f*x + e) + 5*(8*a^3*d^3*cos(f*x + e)^5 - 2*(18*a^3*c^2*d
+ 54*a^3*c*d^2 + 31*a^3*d^3)*cos(f*x + e)^3 + 3*(24*a^3*c^3 + 102*a^3*c^2*d
 + 114*a^3*c*d^2 + 41*a^3*d^3)*cos(f*x + e))*sin(f*x + e))/f
```

Sympy [A] time = 8.79437, size = 1176, normalized size = 5.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(c+d*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((3*a**3*c**3*x*sin(e + f*x)**2/2 + 3*a**3*c**3*x*cos(e + f*x)**2/
2 + a**3*c**3*x - a**3*c**3*sin(e + f*x)**2*cos(e + f*x)/f - 3*a**3*c**3*si
n(e + f*x)*cos(e + f*x)/(2*f) - 2*a**3*c**3*cos(e + f*x)**3/(3*f) - 3*a**3*
c**3*cos(e + f*x)/f + 9*a**3*c**2*d*x*sin(e + f*x)**4/8 + 9*a**3*c**2*d*x*s
in(e + f*x)**2*cos(e + f*x)**2/4 + 9*a**3*c**2*d*x*sin(e + f*x)**2/2 + 9*a*
*3*c**2*d*x*cos(e + f*x)**4/8 + 9*a**3*c**2*d*x*cos(e + f*x)**2/2 - 15*a**3
*c**2*d*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 9*a**3*c**2*d*sin(e + f*x)**2*
cos(e + f*x)/f - 9*a**3*c**2*d*sin(e + f*x)*cos(e + f*x)**3/(8*f) - 9*a**3*
c**2*d*sin(e + f*x)*cos(e + f*x)/(2*f) - 6*a**3*c**2*d*cos(e + f*x)**3/f -
3*a**3*c**2*d*cos(e + f*x)/f + 27*a**3*c*d**2*x*sin(e + f*x)**4/8 + 27*a**3
*c*d**2*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 3*a**3*c*d**2*x*sin(e + f*x)*
*2/2 + 27*a**3*c*d**2*x*cos(e + f*x)**4/8 + 3*a**3*c*d**2*x*cos(e + f*x)**2
/2 - 3*a**3*c*d**2*sin(e + f*x)**4*cos(e + f*x)/f - 45*a**3*c*d**2*sin(e +
f*x)**3*cos(e + f*x)/(8*f) - 4*a**3*c*d**2*sin(e + f*x)**2*cos(e + f*x)**3/
f - 9*a**3*c*d**2*sin(e + f*x)**2*cos(e + f*x)/f - 27*a**3*c*d**2*sin(e + f
*x)*cos(e + f*x)**3/(8*f) - 3*a**3*c*d**2*sin(e + f*x)*cos(e + f*x)/(2*f) -
 8*a**3*c*d**2*cos(e + f*x)**5/(5*f) - 6*a**3*c*d**2*cos(e + f*x)**3/f + 5*
a**3*d**3*x*sin(e + f*x)**6/16 + 15*a**3*d**3*x*sin(e + f*x)**4*cos(e + f*x
)**2/16 + 9*a**3*d**3*x*sin(e + f*x)**4/8 + 15*a**3*d**3*x*sin(e + f*x)**2*
cos(e + f*x)**4/16 + 9*a**3*d**3*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 5*a*
*3*d**3*x*cos(e + f*x)**6/16 + 9*a**3*d**3*x*cos(e + f*x)**4/8 - 11*a**3*d*
*3*sin(e + f*x)**5*cos(e + f*x)/(16*f) - 3*a**3*d**3*sin(e + f*x)**4*cos(e
+ f*x)/f - 5*a**3*d**3*sin(e + f*x)**3*cos(e + f*x)**3/(6*f) - 15*a**3*d**3
*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 4*a**3*d**3*sin(e + f*x)**2*cos(e + f
*x)**3/f - a**3*d**3*sin(e + f*x)**2*cos(e + f*x)/f - 5*a**3*d**3*sin(e + f
*x)*cos(e + f*x)**5/(16*f) - 9*a**3*d**3*sin(e + f*x)*cos(e + f*x)**3/(8*f)
 - 8*a**3*d**3*cos(e + f*x)**5/(5*f) - 2*a**3*d**3*cos(e + f*x)**3/(3*f), N
e(f, 0)), (x*(c + d*sin(e))**3*(a*sin(e) + a)**3, True))
```

Giac [A] time = 1.41731, size = 504, normalized size = 2.34

$$\frac{a^3d^3\cos\left(3fx+3e\right)}{12f}-\frac{a^3d^3\sin\left(6fx+6e\right)}{192f}-\frac{3a^3cd^2\sin\left(2fx+2e\right)}{4f}+\frac{1}{16}\left(24a^3c^3+90a^3c^2d+54a^3cd^2+23a^3d^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c+d*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/12*a^3*d^3*cos(3*f*x + 3*e)/f - 1/192*a^3*d^3*sin(6*f*x + 6*e)/f - 3/4*a^
3*c*d^2*sin(2*f*x + 2*e)/f + 1/16*(24*a^3*c^3 + 90*a^3*c^2*d + 54*a^3*c*d^2
```

```
 + 23*a^3*d^3)*x + 1/2*(2*a^3*c^3 + 3*a^3*c*d^2)*x - 3/80*(a^3*c*d^2 + a^3*
d^3)*cos(5*f*x + 5*e)/f + 1/48*(4*a^3*c^3 + 36*a^3*c^2*d + 51*a^3*c*d^2 + 1
5*a^3*d^3)*cos(3*f*x + 3*e)/f - 3/8*(10*a^3*c^3 + 18*a^3*c^2*d + 23*a^3*c*d
^2 + 5*a^3*d^3)*cos(f*x + e)/f - 3/4*(4*a^3*c^2*d + a^3*d^3)*cos(f*x + e)/f
 + 3/64*(2*a^3*c^2*d + 6*a^3*c*d^2 + 3*a^3*d^3)*sin(4*f*x + 4*e)/f - 3/64*(
16*a^3*c^3 + 64*a^3*c^2*d + 48*a^3*c*d^2 + 21*a^3*d^3)*sin(2*f*x + 2*e)/f
```

3.445 $\int (a + a\sin(e + fx))^3 (c + d\sin(e + fx))^2 \, dx$

Optimal. Leaf size=164

$$\frac{a^3 \left(c^2 + 6cd + 5d^2\right) \cos^3(e + fx)}{3f} - \frac{a^3 \left(12c^2 + 30cd + 13d^2\right) \sin(e + fx) \cos(e + fx)}{8f} + \frac{1}{8} a^3 x \left(20c^2 + 30cd + 13d^2\right) - \ldots$$

```
[Out] (a^3*(20*c^2 + 30*c*d + 13*d^2)*x)/8 - (4*a^3*(c + d)^2*Cos[e + f*x])/f + (
a^3*(c^2 + 6*c*d + 5*d^2)*Cos[e + f*x]^3)/(3*f) - (a^3*d^2*Cos[e + f*x]^5)/
(5*f) - (a^3*(12*c^2 + 30*c*d + 13*d^2)*Cos[e + f*x]*Sin[e + f*x])/(8*f) -
(a^3*d*(2*c + 3*d)*Cos[e + f*x]*Sin[e + f*x]^3)/(4*f)
```

Rubi [A] time = 0.259766, antiderivative size = 189, normalized size of antiderivative = 1.15, number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {2761, 2751, 2645, 2638, 2635, 8, 2633}

$$\frac{a^3 \left(20c^2 + 30cd + 13d^2\right) \cos^3(e + fx)}{60f} - \frac{a^3 \left(20c^2 + 30cd + 13d^2\right) \cos(e + fx)}{5f} - \frac{3a^3 \left(20c^2 + 30cd + 13d^2\right) \sin(e + fx) \ldots}{40f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^2,x]
```

```
[Out] (a^3*(20*c^2 + 30*c*d + 13*d^2)*x)/8 - (a^3*(20*c^2 + 30*c*d + 13*d^2)*Cos[
e + f*x])/(5*f) + (a^3*(20*c^2 + 30*c*d + 13*d^2)*Cos[e + f*x]^3)/(60*f) -
(3*a^3*(20*c^2 + 30*c*d + 13*d^2)*Cos[e + f*x]*Sin[e + f*x])/(40*f) - ((10*
c - d)*d*Cos[e + f*x]*(a + a*Sin[e + f*x])^3)/(20*f) - (d^2*Cos[e + f*x]*(a
 + a*Sin[e + f*x])^4)/(5*a*f)
```

Rule 2761

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^2, x_Symbol] :> -Simp[(d^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m
 + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*S
imp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2645

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Int[ExpandTri
g[(a + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 -
b^2, 0] && IGtQ[n, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))^3(c+d\sin(e+fx))^2\,dx = -\frac{d^2\cos(e+fx)(a+a\sin(e+fx))^4}{5af}+\frac{\int (a+a\sin(e+fx))^3\left(a\left(5c^2+4d^2\right.\right.}{5a}$$

$$= -\frac{(10c-d)d\cos(e+fx)(a+a\sin(e+fx))^3}{20f}-\frac{d^2\cos(e+fx)(a+a\sin(e}{5af}$$

$$= -\frac{(10c-d)d\cos(e+fx)(a+a\sin(e+fx))^3}{20f}-\frac{d^2\cos(e+fx)(a+a\sin(e}{5af}$$

$$= \frac{1}{20}a^3\left(20c^2+30cd+13d^2\right)x-\frac{(10c-d)d\cos(e+fx)(a+a\sin(e+fx))^3}{20f}$$

$$= \frac{1}{20}a^3\left(20c^2+30cd+13d^2\right)x-\frac{3a^3\left(20c^2+30cd+13d^2\right)\cos(e+fx)}{20f}-$$

$$= \frac{1}{8}a^3\left(20c^2+30cd+13d^2\right)x-\frac{a^3\left(20c^2+30cd+13d^2\right)\cos(e+fx)}{5f}+$$

Mathematica [A] time = 0.729981, size = 177, normalized size = 1.08

$$-\frac{a^3\cos(e+fx)\left(30\left(20c^2+30cd+13d^2\right)\sin^{-1}\left(\frac{\sqrt{1-\sin(e+fx)}}{\sqrt{2}}\right)+\sqrt{\cos^2(e+fx)}\left(8\left(5c^2+30cd+19d^2\right)\sin^2(e+fx)+15\right.\right.}{120f\sqrt{\cos^2(e+}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^2,x]
```

```
[Out] -(a^3*Cos[e + f*x]*(30*(20*c^2 + 30*c*d + 13*d^2)*ArcSin[Sqrt[1 - Sin[e + f
*x]]/Sqrt[2]] + Sqrt[Cos[e + f*x]^2]*(8*(55*c^2 + 90*c*d + 38*d^2) + 15*(12
*c^2 + 30*c*d + 13*d^2)*Sin[e + f*x] + 8*(5*c^2 + 30*c*d + 19*d^2)*Sin[e +
f*x]^2 + 30*d*(2*c + 3*d)*Sin[e + f*x]^3 + 24*d^2*Sin[e + f*x]^4)))/(120*f*
Sqrt[Cos[e + f*x]^2])
```

Maple [B] time = 0.045, size = 319, normalized size = 2.

$$\frac{1}{f}\left(-\frac{a^3c^2\left(2+\left(\sin\left(fx+e\right)\right)^2\right)\cos\left(fx+e\right)}{3}+2\,a^3cd\left(-1/4\,\left(\left(\sin\left(fx+e\right)\right)^3+3/2\,\sin\left(fx+e\right)\right)\cos\left(fx+e\right)+3/8\,fx+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(c+d*sin(f*x+e))^2,x)
```

```
[Out] 1/f*(-1/3*a^3*c^2*(2+sin(f*x+e)^2)*cos(f*x+e)+2*a^3*c*d*(-1/4*(sin(f*x+e)^3
+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-1/5*a^3*d^2*(8/3+sin(f*x+e)^4+4/
3*sin(f*x+e)^2)*cos(f*x+e)+3*a^3*c^2*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/
2*e)-2*a^3*c*d*(2+sin(f*x+e)^2)*cos(f*x+e)+3*a^3*d^2*(-1/4*(sin(f*x+e)^3+3/
2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-3*a^3*c^2*cos(f*x+e)+6*a^3*c*d*(-1/
2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-a^3*d^2*(2+sin(f*x+e)^2)*cos(f*x+e)+
a^3*c^2*(f*x+e)-2*a^3*c*d*cos(f*x+e)+a^3*d^2*(-1/2*sin(f*x+e)*cos(f*x+e)+1/
2*f*x+1/2*e))
```

Maxima [A] time = 1.16491, size = 416, normalized size = 2.54

$$\frac{160\left(\cos\left(fx+e\right)^{3}-3\cos\left(fx+e\right)\right)a^{3}c^{2}+360\left(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right)\right)a^{3}c^{2}+480\left(fx+e\right)a^{3}c^{2}+960\left(\cos\left(fx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c+d*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/480*(160*(cos(f*x + e)^3 - 3*cos(f*x + e))*a^3*c^2 + 360*(2*f*x + 2*e - s
in(2*f*x + 2*e))*a^3*c^2 + 480*(f*x + e)*a^3*c^2 + 960*(cos(f*x + e)^3 - 3*
cos(f*x + e))*a^3*c*d + 30*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x
+ 2*e))*a^3*c*d + 720*(2*f*x + 2*e - sin(2*f*x + 2*e))*a^3*c*d - 32*(3*cos(
f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*a^3*d^2 + 480*(cos(f*x +
e)^3 - 3*cos(f*x + e))*a^3*d^2 + 45*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*s
in(2*f*x + 2*e))*a^3*d^2 + 120*(2*f*x + 2*e - sin(2*f*x + 2*e))*a^3*d^2 - 1
440*a^3*c^2*cos(f*x + e) - 960*a^3*c*d*cos(f*x + e))/f
```

Fricas [A] time = 2.03207, size = 413, normalized size = 2.52

$$-\frac{24\,a^{3}d^{2}\cos\left(fx+e\right)^{5}-40\left(a^{3}c^{2}+6\,a^{3}cd+5\,a^{3}d^{2}\right)\cos\left(fx+e\right)^{3}-15\left(20\,a^{3}c^{2}+30\,a^{3}cd+13\,a^{3}d^{2}\right)fx+480\left(a^{3}c^{2}+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c+d*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -1/120*(24*a^3*d^2*cos(f*x + e)^5 - 40*(a^3*c^2 + 6*a^3*c*d + 5*a^3*d^2)*co
s(f*x + e)^3 - 15*(20*a^3*c^2 + 30*a^3*c*d + 13*a^3*d^2)*f*x + 480*(a^3*c^2
 + 2*a^3*c*d + a^3*d^2)*cos(f*x + e) - 15*(2*(2*a^3*c*d + 3*a^3*d^2)*cos(f*
x + e)^3 - (12*a^3*c^2 + 34*a^3*c*d + 19*a^3*d^2)*cos(f*x + e))*sin(f*x + e
))/f
```

Sympy [A] time = 4.03135, size = 702, normalized size = 4.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(c+d*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((3*a**3*c**2*x*sin(e + f*x)**2/2 + 3*a**3*c**2*x*cos(e + f*x)**2/
2 + a**3*c**2*x - a**3*c**2*sin(e + f*x)**2*cos(e + f*x)/f - 3*a**3*c**2*si
n(e + f*x)*cos(e + f*x)/(2*f) - 2*a**3*c**2*cos(e + f*x)**3/(3*f) - 3*a**3*
c**2*cos(e + f*x)/f + 3*a**3*c*d*x*sin(e + f*x)**4/4 + 3*a**3*c*d*x*sin(e +
 f*x)**2*cos(e + f*x)**2/2 + 3*a**3*c*d*x*sin(e + f*x)**2 + 3*a**3*c*d*x*co
s(e + f*x)**4/4 + 3*a**3*c*d*x*cos(e + f*x)**2 - 5*a**3*c*d*sin(e + f*x)**3
*cos(e + f*x)/(4*f) - 6*a**3*c*d*sin(e + f*x)**2*cos(e + f*x)/f - 3*a**3*c*
d*sin(e + f*x)*cos(e + f*x)**3/(4*f) - 3*a**3*c*d*sin(e + f*x)*cos(e + f*x)
/f - 4*a**3*c*d*cos(e + f*x)**3/f - 2*a**3*c*d*cos(e + f*x)/f + 9*a**3*d**2
*x*sin(e + f*x)**4/8 + 9*a**3*d**2*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + a*
*3*d**2*x*sin(e + f*x)**2/2 + 9*a**3*d**2*x*cos(e + f*x)**4/8 + a**3*d**2*x
*cos(e + f*x)**2/2 - a**3*d**2*sin(e + f*x)**4*cos(e + f*x)/f - 15*a**3*d**
2*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 4*a**3*d**2*sin(e + f*x)**2*cos(e +
f*x)**3/(3*f) - 3*a**3*d**2*sin(e + f*x)**2*cos(e + f*x)/f - 9*a**3*d**2*si
n(e + f*x)*cos(e + f*x)**3/(8*f) - a**3*d**2*sin(e + f*x)*cos(e + f*x)/(2*f
) - 8*a**3*d**2*cos(e + f*x)**5/(15*f) - 2*a**3*d**2*cos(e + f*x)**3/f, Ne(
f, 0)), (x*(c + d*sin(e))**2*(a*sin(e) + a)**3, True))
```

Giac [A] time = 1.31324, size = 339, normalized size = 2.07

$$-\frac{a^3d^2\cos\left(5\,fx+5\,e\right)}{80\,f}-\frac{2\,a^3cd\cos\left(fx+e\right)}{f}-\frac{a^3d^2\sin\left(2\,fx+2\,e\right)}{4\,f}+\frac{3}{8}\left(4\,a^3c^2+10\,a^3cd+3\,a^3d^2\right)x+\frac{1}{2}\left(2\,a^3c^2+a^3d^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c+d*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] -1/80*a^3*d^2*cos(5*f*x + 5*e)/f - 2*a^3*c*d*cos(f*x + e)/f - 1/4*a^3*d^2*s
in(2*f*x + 2*e)/f + 3/8*(4*a^3*c^2 + 10*a^3*c*d + 3*a^3*d^2)*x + 1/2*(2*a^3
*c^2 + a^3*d^2)*x + 1/48*(4*a^3*c^2 + 24*a^3*c*d + 17*a^3*d^2)*cos(3*f*x +
3*e)/f - 1/8*(30*a^3*c^2 + 36*a^3*c*d + 23*a^3*d^2)*cos(f*x + e)/f + 1/32*(
2*a^3*c*d + 3*a^3*d^2)*sin(4*f*x + 4*e)/f - 1/4*(3*a^3*c^2 + 8*a^3*c*d + 3*
a^3*d^2)*sin(2*f*x + 2*e)/f
```

3.446 $\int (a+a\sin(e+fx))^3(c+d\sin(e+fx))\,dx$

Optimal. Leaf size=110

$$\frac{a^3(c+3d)\cos^3(e+fx)}{3f}-\frac{4a^3(c+d)\cos(e+fx)}{f}-\frac{3a^3(4c+5d)\sin(e+fx)\cos(e+fx)}{8f}+\frac{5}{8}a^3x(4c+3d)-\frac{a^3d\sin^3(\ldots}{\ldots}$$

```
[Out] (5*a^3*(4*c + 3*d)*x)/8 - (4*a^3*(c + d)*Cos[e + f*x])/f + (a^3*(c + 3*d)*C
os[e + f*x]^3)/(3*f) - (3*a^3*(4*c + 5*d)*Cos[e + f*x]*Sin[e + f*x])/(8*f)
- (a^3*d*Cos[e + f*x]*Sin[e + f*x]^3)/(4*f)
```

Rubi [A] time = 0.0967658, antiderivative size = 117, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2751, 2645, 2638, 2635, 8, 2633}

$$\frac{a^3(4c+3d)\cos^3(e+fx)}{12f}-\frac{a^3(4c+3d)\cos(e+fx)}{f}-\frac{3a^3(4c+3d)\sin(e+fx)\cos(e+fx)}{8f}+\frac{5}{8}a^3x(4c+3d)-\frac{d\cos(\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x]),x]
```

```
[Out] (5*a^3*(4*c + 3*d)*x)/8 - (a^3*(4*c + 3*d)*Cos[e + f*x])/f + (a^3*(4*c + 3*
d)*Cos[e + f*x]^3)/(12*f) - (3*a^3*(4*c + 3*d)*Cos[e + f*x]*Sin[e + f*x])/(
8*f) - (d*Cos[e + f*x]*(a + a*Sin[e + f*x])^3)/(4*f)
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2645

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Int[ExpandTri
g[(a + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 -
b^2, 0] && IGtQ[n, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))^3(c+d\sin(e+fx))\,dx = -\frac{d\cos(e+fx)(a+a\sin(e+fx))^3}{4f}+\frac{1}{4}(4c+3d)\int (a+a\sin(e+fx))^3\,dx$$
$$= -\frac{d\cos(e+fx)(a+a\sin(e+fx))^3}{4f}+\frac{1}{4}(4c+3d)\int \left(a^3+3a^3\sin(e+fx)+\right.$$
$$= \frac{1}{4}a^3(4c+3d)x-\frac{d\cos(e+fx)(a+a\sin(e+fx))^3}{4f}+\frac{1}{4}\left(a^3(4c+3d)\right)\int \sin$$
$$= \frac{1}{4}a^3(4c+3d)x-\frac{3a^3(4c+3d)\cos(e+fx)}{4f}-\frac{3a^3(4c+3d)\cos(e+fx)\sin(}{8f}$$
$$= \frac{5}{8}a^3(4c+3d)x-\frac{a^3(4c+3d)\cos(e+fx)}{f}+\frac{a^3(4c+3d)\cos^3(e+fx)}{12f}-\frac{3a^3}{}$$

Mathematica [A] time = 0.510178, size = 120, normalized size = 1.09

$$-\frac{a^3\cos(e+fx)\left(30(4c+3d)\sin^{-1}\left(\frac{\sqrt{1-\sin(e+fx)}}{\sqrt{2}}\right)+\sqrt{\cos^2(e+fx)}\left(8(c+3d)\sin^2(e+fx)+9(4c+5d)\sin(e+fx)+88c\right.\right.}{24f\sqrt{\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x]),x]
```

```
[Out] -(a^3*Cos[e + f*x]*(30*(4*c + 3*d)*ArcSin[Sqrt[1 - Sin[e + f*x]]/Sqrt[2]] +
 Sqrt[Cos[e + f*x]^2]*(88*c + 72*d + 9*(4*c + 5*d)*Sin[e + f*x] + 8*(c + 3*
d)*Sin[e + f*x]^2 + 6*d*Sin[e + f*x]^3)))/(24*f*Sqrt[Cos[e + f*x]^2])
```

Maple [A] time = 0.039, size = 178, normalized size = 1.6

$$\frac{1}{f}\left(-\frac{a^3c\left(2+\left(\sin\left(fx+e\right)\right)^2\right)\cos\left(fx+e\right)}{3}+a^3d\left(-\frac{\cos\left(fx+e\right)}{4}\left(\left(\sin\left(fx+e\right)\right)^3+\frac{3\sin\left(fx+e\right)}{2}\right)+\frac{3fx}{8}+\frac{3e}{8}\right)+3a^3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(c+d*sin(f*x+e)),x)
```

```
[Out] 1/f*(-1/3*a^3*c*(2+sin(f*x+e)^2)*cos(f*x+e)+a^3*d*(-1/4*(sin(f*x+e)^3+3/2*s
in(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)+3*a^3*c*(-1/2*sin(f*x+e)*cos(f*x+e)+1/
2*f*x+1/2*e)-a^3*d*(2+sin(f*x+e)^2)*cos(f*x+e)-3*a^3*c*cos(f*x+e)+3*a^3*d*(
-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)+a^3*c*(f*x+e)-a^3*d*cos(f*x+e))
```

Maxima [A] time = 1.1739, size = 231, normalized size = 2.1

$$\frac{32\left(\cos\left(fx+e\right)^3-3\cos\left(fx+e\right)\right)a^3c+72\left(2fx+2e-\sin\left(2fx+2e\right)\right)a^3c+96\left(fx+e\right)a^3c+96\left(\cos\left(fx+e\right)^3-3c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c+d*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/96*(32*(cos(f*x + e)^3 - 3*cos(f*x + e))*a^3*c + 72*(2*f*x + 2*e - sin(2*
f*x + 2*e))*a^3*c + 96*(f*x + e)*a^3*c + 96*(cos(f*x + e)^3 - 3*cos(f*x + e
))*a^3*d + 3*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*a^3*d
+ 72*(2*f*x + 2*e - sin(2*f*x + 2*e))*a^3*d - 288*a^3*c*cos(f*x + e) - 96*a
^3*d*cos(f*x + e))/f
```

Fricas [A] time = 1.99665, size = 252, normalized size = 2.29

$$\frac{8\left(a^3c+3a^3d\right)\cos\left(fx+e\right)^3+15\left(4a^3c+3a^3d\right)fx-96\left(a^3c+a^3d\right)\cos\left(fx+e\right)+3\left(2a^3d\cos\left(fx+e\right)^3-\left(12a^3c+\right.\right.}{24f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c+d*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/24*(8*(a^3*c + 3*a^3*d)*cos(f*x + e)^3 + 15*(4*a^3*c + 3*a^3*d)*f*x - 96*
(a^3*c + a^3*d)*cos(f*x + e) + 3*(2*a^3*d*cos(f*x + e)^3 - (12*a^3*c + 17*a
^3*d)*cos(f*x + e))*sin(f*x + e))/f
```

Sympy [A] time = 1.69122, size = 371, normalized size = 3.37

$$\begin{cases} \frac{3a^3cx\sin^2\left(e+fx\right)}{2}+\frac{3a^3cx\cos^2\left(e+fx\right)}{2}+a^3cx-\frac{a^3c\sin^2\left(e+fx\right)\cos\left(e+fx\right)}{f}-\frac{3a^3c\sin\left(e+fx\right)\cos\left(e+fx\right)}{2f}-\frac{2a^3c\cos^3\left(e+fx\right)}{3f}-\frac{3a^3c\cos\left(e+fx\right)}{f} \\ x\left(c+d\sin\left(e\right)\right)\left(a\sin\left(e\right)+a\right)^3 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(c+d*sin(f*x+e)),x)
```

```
[Out] Piecewise((3*a**3*c*x*sin(e + f*x)**2/2 + 3*a**3*c*x*cos(e + f*x)**2/2 + a*
*3*c*x - a**3*c*sin(e + f*x)**2*cos(e + f*x)/f - 3*a**3*c*sin(e + f*x)*cos(
e + f*x)/(2*f) - 2*a**3*c*cos(e + f*x)**3/(3*f) - 3*a**3*c*cos(e + f*x)/f +
 3*a**3*d*x*sin(e + f*x)**4/8 + 3*a**3*d*x*sin(e + f*x)**2*cos(e + f*x)**2/
4 + 3*a**3*d*x*sin(e + f*x)**2/2 + 3*a**3*d*x*cos(e + f*x)**4/8 + 3*a**3*d*
x*cos(e + f*x)**2/2 - 5*a**3*d*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 3*a**3*
d*sin(e + f*x)**2*cos(e + f*x)/f - 3*a**3*d*sin(e + f*x)*cos(e + f*x)**3/(8
*f) - 3*a**3*d*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*a**3*d*cos(e + f*x)**3/f
 - a**3*d*cos(e + f*x)/f, Ne(f, 0)), (x*(c + d*sin(e))*(a*sin(e) + a)**3, T
rue))
```

Giac [A] time = 1.37336, size = 186, normalized size = 1.69

$$a^3cx-\frac{a^3d\cos\left(fx+e\right)}{f}+\frac{a^3d\sin\left(4fx+4e\right)}{32f}+\frac{3}{8}\left(4a^3c+5a^3d\right)x+\frac{\left(a^3c+3a^3d\right)\cos\left(3fx+3e\right)}{12f}-\frac{3\left(5a^3c+3a^3d\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c+d*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] a^3*c*x - a^3*d*cos(f*x + e)/f + 1/32*a^3*d*sin(4*f*x + 4*e)/f + 3/8*(4*a^3
*c + 5*a^3*d)*x + 1/12*(a^3*c + 3*a^3*d)*cos(3*f*x + 3*e)/f - 3/4*(5*a^3*c
+ 3*a^3*d)*cos(f*x + e)/f - 1/4*(3*a^3*c + 4*a^3*d)*sin(2*f*x + 2*e)/f
```

3.447 $\int (a + a\sin(e + fx))^3\,dx$

Optimal. Leaf size=63

$$\frac{a^3\cos^3(e+fx)}{3f} - \frac{4a^3\cos(e+fx)}{f} - \frac{3a^3\sin(e+fx)\cos(e+fx)}{2f} + \frac{5a^3x}{2}$$

```
[Out] (5*a^3*x)/2 - (4*a^3*Cos[e + f*x])/f + (a^3*Cos[e + f*x]^3)/(3*f) - (3*a^3*
Cos[e + f*x]*Sin[e + f*x])/(2*f)
```

Rubi [A] time = 0.0503981, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {2645, 2638, 2635, 8, 2633}

$$\frac{a^3\cos^3(e+fx)}{3f} - \frac{4a^3\cos(e+fx)}{f} - \frac{3a^3\sin(e+fx)\cos(e+fx)}{2f} + \frac{5a^3x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3,x]
```


```
[Out] (5*a^3*x)/2 - (4*a^3*Cos[e + f*x])/f + (a^3*Cos[e + f*x]^3)/(3*f) - (3*a^3*
Cos[e + f*x]*Sin[e + f*x])/(2*f)
```


Rule 2645

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Int[ExpandTri
g[(a + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 -
b^2, 0] && IGtQ[n, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int (a + a\sin(e + fx))^3 \, dx = \int \left(a^3 + 3a^3\sin(e + fx) + 3a^3\sin^2(e + fx) + a^3\sin^3(e + fx)\right) dx$$
$$= a^3x + a^3 \int \sin^3(e + fx) \, dx + \left(3a^3\right) \int \sin(e + fx) \, dx + \left(3a^3\right) \int \sin^2(e + fx) \, dx$$
$$= a^3x - \frac{3a^3\cos(e + fx)}{f} - \frac{3a^3\cos(e + fx)\sin(e + fx)}{2f} + \frac{1}{2}\left(3a^3\right)\int 1 \, dx - \frac{a^3 \operatorname{Subst}\left(\int \left(1 - x^2\right.\right.}{}$$
$$= \frac{5a^3x}{2} - \frac{4a^3\cos(e + fx)}{f} + \frac{a^3\cos^3(e + fx)}{3f} - \frac{3a^3\cos(e + fx)\sin(e + fx)}{2f}$$

Mathematica [A] time = 0.343007, size = 44, normalized size = 0.7

$$\frac{a^3(-9\sin(2(e + fx)) - 45\cos(e + fx) + \cos(3(e + fx)) + 30e + 30fx)}{12f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3,x]
```

```
[Out] (a^3*(30*e + 30*f*x - 45*Cos[e + f*x] + Cos[3*(e + f*x)] - 9*Sin[2*(e + f*x
)]))/(12*f)
```

Maple [A] time = 0.029, size = 74, normalized size = 1.2

$$\frac{1}{f}\left(-\frac{a^3\left(2 + \left(\sin\left(fx + e\right)\right)^2\right)\cos\left(fx + e\right)}{3} + 3\,a^3\left(-1/2\,\sin\left(fx + e\right)\cos\left(fx + e\right) + 1/2\,fx + e/2\right) - 3\,a^3\cos\left(fx + e\right) + \left(f\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3,x)
```

```
[Out] 1/f*(-1/3*a^3*(2+sin(f*x+e)^2)*cos(f*x+e)+3*a^3*(-1/2*sin(f*x+e)*cos(f*x+e)
+1/2*f*x+1/2*e)-3*a^3*cos(f*x+e)+(f*x+e)*a^3)
```

Maxima [A] time = 1.1194, size = 97, normalized size = 1.54

$$a^3x + \frac{\left(\cos\left(fx + e\right)^3 - 3\,\cos\left(fx + e\right)\right)a^3}{3\,f} + \frac{3\left(2\,fx + 2\,e - \sin\left(2\,fx + 2\,e\right)\right)a^3}{4\,f} - \frac{3\,a^3\cos\left(fx + e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] a^3*x + 1/3*(cos(f*x + e)^3 - 3*cos(f*x + e))*a^3/f + 3/4*(2*f*x + 2*e - si
n(2*f*x + 2*e))*a^3/f - 3*a^3*cos(f*x + e)/f
```

Fricas [A] time = 1.91581, size = 134, normalized size = 2.13

$$\frac{2\,a^{3}\cos\left(fx+e\right)^{3}+15\,a^{3}fx-9\,a^{3}\cos\left(fx+e\right)\sin\left(fx+e\right)-24\,a^{3}\cos\left(fx+e\right)}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/6*(2*a^3*cos(f*x + e)^3 + 15*a^3*f*x - 9*a^3*cos(f*x + e)*sin(f*x + e) -
24*a^3*cos(f*x + e))/f
```

Sympy [A] time = 0.625299, size = 121, normalized size = 1.92

$$\begin{cases}\frac{3a^{3}x\sin^{2}\left(e+fx\right)}{2}+\frac{3a^{3}x\cos^{2}\left(e+fx\right)}{2}+a^{3}x-\frac{a^{3}\sin^{2}\left(e+fx\right)\cos\left(e+fx\right)}{f}-\frac{3a^{3}\sin\left(e+fx\right)\cos\left(e+fx\right)}{2f}-\frac{2a^{3}\cos^{3}\left(e+fx\right)}{3f}-\frac{3a^{3}\cos\left(e+fx\right)}{f} & \text{fo} \\ x\left(a\sin\left(e\right)+a\right)^{3} & \text{ot} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((3*a**3*x*sin(e + f*x)**2/2 + 3*a**3*x*cos(e + f*x)**2/2 + a**3*x
 - a**3*sin(e + f*x)**2*cos(e + f*x)/f - 3*a**3*sin(e + f*x)*cos(e + f*x)/(
2*f) - 2*a**3*cos(e + f*x)**3/(3*f) - 3*a**3*cos(e + f*x)/f, Ne(f, 0)), (x*
(a*sin(e) + a)**3, True))
```

Giac [A] time = 1.30665, size = 78, normalized size = 1.24

$$\frac{5}{2}\,a^{3}x+\frac{a^{3}\cos\left(3\,fx+3\,e\right)}{12\,f}-\frac{15\,a^{3}\cos\left(fx+e\right)}{4\,f}-\frac{3\,a^{3}\sin\left(2\,fx+2\,e\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 5/2*a^3*x + 1/12*a^3*cos(3*f*x + 3*e)/f - 15/4*a^3*cos(f*x + e)/f - 3/4*a^3
*sin(2*f*x + 2*e)/f
```

3.448 $\int \frac{(a+a\sin(e+fx))^3}{c+d\sin(e+fx)}\,dx$

Optimal. Leaf size=143

$$-\frac{2a^3(c-d)^3\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{d^3f\sqrt{c^2-d^2}}+\frac{a^3x\left(2c^2-6cd+7d^2\right)}{2d^3}+\frac{a^3(2c-5d)\cos(e+fx)}{2d^2f}-\frac{\cos(e+fx)\left(a^3\sin(e+fx)+a\right.}{2df}$$

```
[Out] (a^3*(2*c^2 - 6*c*d + 7*d^2)*x)/(2*d^3) - (2*a^3*(c - d)^3*ArcTan[(d + c*Ta
n[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(d^3*Sqrt[c^2 - d^2]*f) + (a^3*(2*c - 5*d
)*Cos[e + f*x])/(2*d^2*f) - (Cos[e + f*x]*(a^3 + a^3*Sin[e + f*x]))/(2*d*f)
```

Rubi [A] time = 0.38868, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {2763, 2968, 3023, 2735, 2660, 618, 204}

$$-\frac{2a^3(c-d)^3\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{d^3f\sqrt{c^2-d^2}}+\frac{a^3x\left(2c^2-6cd+7d^2\right)}{2d^3}+\frac{a^3(2c-5d)\cos(e+fx)}{2d^2f}-\frac{\cos(e+fx)\left(a^3\sin(e+fx)+a\right.}{2df}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3/(c + d*Sin[e + f*x]),x]
```


```
[Out] (a^3*(2*c^2 - 6*c*d + 7*d^2)*x)/(2*d^3) - (2*a^3*(c - d)^3*ArcTan[(d + c*Ta
n[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(d^3*Sqrt[c^2 - d^2]*f) + (a^3*(2*c - 5*d
)*Cos[e + f*x])/(2*d^2*f) - (Cos[e + f*x]*(a^3 + a^3*Sin[e + f*x]))/(2*d*f)
```


Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
```

```
!LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^3}{c+d\sin(e+fx)}\,dx = -\frac{\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{2df}+\frac{\int \frac{(a+a\sin(e+fx))\left(a^2(c+2d)-a^2(2c-5d)\sin(e+fx)\right)}{c+d\sin(e+fx)}\,dx}{2d}$$

$$= -\frac{\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{2df}+\frac{\int \frac{a^3(c+2d)+\left(-a^3(2c-5d)+a^3(c+2d)\right)\sin(e+fx)-a^3(2c-5d)\sin^2(e+\cdots}{c+d\sin(e+fx)}}{2d}\cdots$$

$$= \frac{a^3(2c-5d)\cos(e+fx)}{2d^2f}-\frac{\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{2df}+\frac{\int \frac{a^3d(c+2d)+a^3\left(2c^2-6cd+7d^2\right)\mathrm{si}\cdots}{c+d\sin(e+fx)}}{2d^2}\cdots$$

$$= \frac{a^3\left(2c^2-6cd+7d^2\right)x}{2d^3}+\frac{a^3(2c-5d)\cos(e+fx)}{2d^2f}-\frac{\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{2df}-\cdots$$

$$= \frac{a^3\left(2c^2-6cd+7d^2\right)x}{2d^3}+\frac{a^3(2c-5d)\cos(e+fx)}{2d^2f}-\frac{\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{2df}-\cdots$$

$$= \frac{a^3\left(2c^2-6cd+7d^2\right)x}{2d^3}+\frac{a^3(2c-5d)\cos(e+fx)}{2d^2f}-\frac{\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{2df}+\cdots$$

$$= \frac{a^3\left(2c^2-6cd+7d^2\right)x}{2d^3}-\frac{2a^3(c-d)^3\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{d^3\sqrt{c^2-d^2}\,f}+\frac{a^3(2c-5d)\cos(e+fx)}{2d^2f}-\cdots$$

Mathematica [A] time = 0.650172, size = 162, normalized size = 1.13

$$\frac{a^3(\sin(e+fx)+1)^3\left(\sqrt{c^2-d^2}\left(2\left(2c^2-6cd+7d^2\right)(e+fx)+4d(c-3d)\cos(e+fx)+d^2(-\sin(2(e+fx)))\right)-8(c-d)^3\right.}{4d^3f\sqrt{c^2-d^2}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3/(c + d*Sin[e + f*x]),x]
```

```
[Out] (a^3*(1 + Sin[e + f*x])^3*(-8*(c - d)^3*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqr
t[c^2 - d^2]] + Sqrt[c^2 - d^2]*(2*(2*c^2 - 6*c*d + 7*d^2)*(e + f*x) + 4*(c
 - 3*d)*d*Cos[e + f*x] - d^2*Sin[2*(e + f*x)])))/(4*d^3*Sqrt[c^2 - d^2]*f*(
Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6)
```

Maple [B] time = 0.101, size = 480, normalized size = 3.4

$$-2\,\frac{a^3c^3}{fd^3\sqrt{c^2-d^2}}\arctan\left(1/2\,\frac{2\,c\tan\left(1/2\,fx+e/2\right)+2\,d}{\sqrt{c^2-d^2}}\right)+6\,\frac{a^3c^2}{fd^2\sqrt{c^2-d^2}}\arctan\left(1/2\,\frac{2\,c\tan\left(1/2\,fx+e/2\right)+2\,d}{\sqrt{c^2-d^2}}\right)-6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e)),x)
```

```
[Out] -2/f*a^3/d^3/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d
^2)^(1/2))*c^3+6/f*a^3/d^2/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*
e)+2*d)/(c^2-d^2)^(1/2))*c^2-6/f*a^3/d/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(
1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*c+2/f*a^3/(c^2-d^2)^(1/2)*arctan(1/2*(
2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))+1/f*a^3/d/(1+tan(1/2*f*x+1/2*e
)^2)^2*tan(1/2*f*x+1/2*e)^3+2/f*a^3/d^2/(1+tan(1/2*f*x+1/2*e)^2)^2*tan(1/2*
f*x+1/2*e)^2*c-6/f*a^3/d/(1+tan(1/2*f*x+1/2*e)^2)^2*tan(1/2*f*x+1/2*e)^2-1/
f*a^3/d/(1+tan(1/2*f*x+1/2*e)^2)^2*tan(1/2*f*x+1/2*e)+2/f*a^3/d^2/(1+tan(1/
2*f*x+1/2*e)^2)^2*c-6/f*a^3/d/(1+tan(1/2*f*x+1/2*e)^2)^2+2/f*a^3/d^3*arctan
(tan(1/2*f*x+1/2*e))*c^2-6/f*a^3/d^2*arctan(tan(1/2*f*x+1/2*e))*c+7/f*a^3/d
*arctan(tan(1/2*f*x+1/2*e))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.21524, size = 909, normalized size = 6.36

$$\left[-\frac{a^3d^2\cos\left(fx+e\right)\sin\left(fx+e\right)-\left(2\,a^3c^2-6\,a^3cd+7\,a^3d^2\right)fx-\left(a^3c^2-2\,a^3cd+a^3d^2\right)\sqrt{-\frac{c-d}{c+d}}\log\left(\frac{\left(2\,c^2-d^2\right)\cos\left(fx+e\right)^2-}{}\right.}{2\,d^3f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] [-1/2*(a^3*d^2*cos(f*x + e)*sin(f*x + e) - (2*a^3*c^2 - 6*a^3*c*d + 7*a^3*d
^2)*f*x - (a^3*c^2 - 2*a^3*c*d + a^3*d^2)*sqrt(-(c - d)/(c + d))*log(((2*c^
2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 + 2*((c^2 + c*d)*c
os(f*x + e)*sin(f*x + e) + (c*d + d^2)*cos(f*x + e))*sqrt(-(c - d)/(c + d))
)/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) - 2*(a^3*c*d - 3*a
^3*d^2)*cos(f*x + e))/(d^3*f), -1/2*(a^3*d^2*cos(f*x + e)*sin(f*x + e) - (2
*a^3*c^2 - 6*a^3*c*d + 7*a^3*d^2)*f*x - 2*(a^3*c^2 - 2*a^3*c*d + a^3*d^2)*s
qrt((c - d)/(c + d))*arctan(-(c*sin(f*x + e) + d)*sqrt((c - d)/(c + d))/((c
 - d)*cos(f*x + e))) - 2*(a^3*c*d - 3*a^3*d^2)*cos(f*x + e))/(d^3*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3/(c+d*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.32861, size = 323, normalized size = 2.26

$$\frac{\frac{\left(2\,a^3c^2-6\,a^3cd+7\,a^3d^2\right)\left(fx+e\right)}{d^3}-\frac{4\left(a^3c^3-3\,a^3c^2d+3\,a^3cd^2-a^3d^3\right)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{\sqrt{c^2-d^2}\,d^3}+\frac{2\left(a^3d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+2\,a^3c\tan\left(\frac{1}{2}\right.\right.}{}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/2*((2*a^3*c^2 - 6*a^3*c*d + 7*a^3*d^2)*(f*x + e)/d^3 - 4*(a^3*c^3 - 3*a^3
*c^2*d + 3*a^3*c*d^2 - a^3*d^3)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) +
arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/(sqrt(c^2 - d^2)*d^3)
 + 2*(a^3*d*tan(1/2*f*x + 1/2*e)^3 + 2*a^3*c*tan(1/2*f*x + 1/2*e)^2 - 6*a^3
*d*tan(1/2*f*x + 1/2*e)^2 - a^3*d*tan(1/2*f*x + 1/2*e) + 2*a^3*c - 6*a^3*d)
/((tan(1/2*f*x + 1/2*e)^2 + 1)^2*d^2))/f
```

3.449 $\int \frac{(a+a\sin(e+fx))^3}{(c+d\sin(e+fx))^2}\,dx$

Optimal. Leaf size=161

$$\frac{2a^3(c-d)^2(2c+3d)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{d^3f(c+d)\sqrt{c^2-d^2}}-\frac{2a^3c\cos(e+fx)}{d^2f(c+d)}-\frac{a^3x(2c-3d)}{d^3}+\frac{(c-d)\cos(e+fx)\big(a^3\sin(e+fx)+a^3}{df(c+d)(c+d\sin(e+fx))}$$

```
[Out] -((a^3*(2*c - 3*d)*x)/d^3) + (2*a^3*(c - d)^2*(2*c + 3*d)*ArcTan[(d + c*Tan
[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(d^3*(c + d)*Sqrt[c^2 - d^2]*f) - (2*a^3*c
*Cos[e + f*x])/(d^2*(c + d)*f) + ((c - d)*Cos[e + f*x]*(a^3 + a^3*Sin[e + f
*x]))/(d*(c + d)*f*(c + d*Sin[e + f*x]))
```

Rubi [A] time = 0.38052, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {2762, 2968, 3023, 2735, 2660, 618, 204}

$$\frac{2a^3(c-d)^2(2c+3d)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{d^3f(c+d)\sqrt{c^2-d^2}}-\frac{2a^3c\cos(e+fx)}{d^2f(c+d)}-\frac{a^3x(2c-3d)}{d^3}+\frac{(c-d)\cos(e+fx)\big(a^3\sin(e+fx)+a^3}{df(c+d)(c+d\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3/(c + d*Sin[e + f*x])^2,x]
```

```
[Out] -((a^3*(2*c - 3*d)*x)/d^3) + (2*a^3*(c - d)^2*(2*c + 3*d)*ArcTan[(d + c*Tan
[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(d^3*(c + d)*Sqrt[c^2 - d^2]*f) - (2*a^3*c
*Cos[e + f*x])/(d^2*(c + d)*f) + ((c - d)*Cos[e + f*x]*(a^3 + a^3*Sin[e + f
*x]))/(d*(c + d)*f*(c + d*Sin[e + f*x]))
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
```

```
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^3}{(c+d\sin(e+fx))^2}\,dx = \frac{(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{d(c+d)f(c+d\sin(e+fx))} - \frac{a\int \frac{(a+a\sin(e+fx))(a(c-3d)-2ac\sin(e+fx))}{c+d\sin(e+fx)}\,dx}{d(c+d)}$$

$$= \frac{(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{d(c+d)f(c+d\sin(e+fx))} - \frac{a\int \frac{a^2(c-3d)+\left(-2a^2c+a^2(c-3d)\right)\sin(e+fx)-2a^2c\sin^2(e+fx}{c+d\sin(e+fx)}}{d(c+d)}$$

$$= -\frac{2a^3c\cos(e+fx)}{d^2(c+d)f} + \frac{(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{d(c+d)f(c+d\sin(e+fx))} - \frac{a\int \frac{a^2(c-3d)d+a^2(2c-3d)(c+d)}{c+d\sin(e+fx)}}{d^2(c+d)}$$

$$= -\frac{a^3(2c-3d)x}{d^3} - \frac{2a^3c\cos(e+fx)}{d^2(c+d)f} + \frac{(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{d(c+d)f(c+d\sin(e+fx))} + \frac{\left(a^3(c-\right.}{}$$

$$= -\frac{a^3(2c-3d)x}{d^3} - \frac{2a^3c\cos(e+fx)}{d^2(c+d)f} + \frac{(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{d(c+d)f(c+d\sin(e+fx))} + \frac{\left(2a^3(c-\right.}{}$$

$$= -\frac{a^3(2c-3d)x}{d^3} - \frac{2a^3c\cos(e+fx)}{d^2(c+d)f} + \frac{(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{d(c+d)f(c+d\sin(e+fx))} - \frac{\left(4a^3(c-\right.}{}$$

$$= -\frac{a^3(2c-3d)x}{d^3} + \frac{2a^3(c-d)^2(2c+3d)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{d^3(c+d)\sqrt{c^2-d^2}f} - \frac{2a^3c\cos(e+fx)}{d^2(c+d)f} + \frac{(c-}{}$$

Mathematica [A] time = 0.678351, size = 162, normalized size = 1.01

$$\frac{a^3(\sin(e+fx)+1)^3\left(\frac{2(2c+3d)(c-d)^2\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{(c+d)\sqrt{c^2-d^2}}+(3d-2c)(e+fx)-\frac{d(c-d)^2\cos(e+fx)}{(c+d)(c+d\sin(e+fx))}-d\cos(e+fx)\right)}{d^3f\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3/(c + d*Sin[e + f*x])^2,x]
```

```
[Out] (a^3*(1 + Sin[e + f*x])^3*((-2*c + 3*d)*(e + f*x) + (2*(c - d)^2*(2*c + 3*d
)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/((c + d)*Sqrt[c^2 - d^2
]) - d*Cos[e + f*x] - ((c - d)^2*d*Cos[e + f*x])/((c + d)*(c + d*Sin[e + f*
x]))))/(d^3*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6)
```

Maple [B] time = 0.131, size = 600, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^2,x)
```

```
[Out] -2/f*a^3/d/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)/(c+d)*c*tan(1/
2*f*x+1/2*e)+4/f*a^3/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)/(c+d
)*tan(1/2*f*x+1/2*e)-2/f*a^3*d/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)
*d+c)/(c+d)/c*tan(1/2*f*x+1/2*e)-2/f*a^3/d^2/(c*tan(1/2*f*x+1/2*e)^2+2*tan(
1/2*f*x+1/2*e)*d+c)/(c+d)*c^2+4/f*a^3/d/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f
*x+1/2*e)*d+c)/(c+d)*c-2/f*a^3/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)
*d+c)/(c+d)+4/f*a^3/d^3/(c+d)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1
/2*e)+2*d)/(c^2-d^2)^(1/2))*c^3-2/f*a^3/d^2/(c+d)/(c^2-d^2)^(1/2)*arctan(1/
2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*c^2-8/f*a^3/d/(c+d)/(c^2-d^
2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*c+6/f*a^3
/(c+d)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1
/2))-2/f*a^3/d^2/(1+tan(1/2*f*x+1/2*e)^2)-4/f*a^3/d^3*arctan(tan(1/2*f*x+1/
2*e))*c+6/f*a^3/d^2*arctan(tan(1/2*f*x+1/2*e))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.00415, size = 1389, normalized size = 8.63

$$\left[-\frac{2\left(2\,a^3c^3-a^3c^2d-3\,a^3cd^2\right)fx+\left(2\,a^3c^3+a^3c^2d-3\,a^3cd^2+\left(2\,a^3c^2d+a^3cd^2-3\,a^3d^3\right)\sin\left(fx+e\right)\right)\sqrt{-\frac{c-d}{c+d}}\log\left(\frac{(2c\cdots}{\cdots}\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(2*(2*a^3*c^3 - a^3*c^2*d - 3*a^3*c*d^2)*f*x + (2*a^3*c^3 + a^3*c^2*d
 - 3*a^3*c*d^2 + (2*a^3*c^2*d + a^3*c*d^2 - 3*a^3*d^3)*sin(f*x + e))*sqrt(-
(c - d)/(c + d))*log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c
^2 - d^2 + 2*((c^2 + c*d)*cos(f*x + e)*sin(f*x + e) + (c*d + d^2)*cos(f*x +
 e))*sqrt(-(c - d)/(c + d)))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2
 - d^2)) + 2*(2*a^3*c^2*d - a^3*c*d^2 + a^3*d^3)*cos(f*x + e) + 2*((2*a^3*c
^2*d - a^3*c*d^2 - 3*a^3*d^3)*f*x + (a^3*c*d^2 + a^3*d^3)*cos(f*x + e))*sin
(f*x + e))/((c*d^4 + d^5)*f*sin(f*x + e) + (c^2*d^3 + c*d^4)*f), -((2*a^3*c
^3 - a^3*c^2*d - 3*a^3*c*d^2)*f*x + (2*a^3*c^3 + a^3*c^2*d - 3*a^3*c*d^2 +
(2*a^3*c^2*d + a^3*c*d^2 - 3*a^3*d^3)*sin(f*x + e))*sqrt((c - d)/(c + d))*a
rctan(-(c*sin(f*x + e) + d)*sqrt((c - d)/(c + d))/((c - d)*cos(f*x + e))) +
 (2*a^3*c^2*d - a^3*c*d^2 + a^3*d^3)*cos(f*x + e) + ((2*a^3*c^2*d - a^3*c*d
^2 - 3*a^3*d^3)*f*x + (a^3*c*d^2 + a^3*d^3)*cos(f*x + e))*sin(f*x + e))/((c
*d^4 + d^5)*f*sin(f*x + e) + (c^2*d^3 + c*d^4)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3/(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.40815, size = 533, normalized size = 3.31

$$\frac{2\left(2\,a^3c^3-a^3c^2d-4\,a^3cd^2+3\,a^3d^3\right)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{\left(cd^3+d^4\right)\sqrt{c^2-d^2}}-\frac{2\left(a^3c^2d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-2\,a^3cd^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+a^3d^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\cdots\right.}{\left(c\tan\left(\frac{1}{2}\,f\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] (2*(2*a^3*c^3 - a^3*c^2*d - 4*a^3*c*d^2 + 3*a^3*d^3)*(pi*floor(1/2*(f*x + e
)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/
((c*d^3 + d^4)*sqrt(c^2 - d^2)) - 2*(a^3*c^2*d*tan(1/2*f*x + 1/2*e)^3 - 2*a
^3*c*d^2*tan(1/2*f*x + 1/2*e)^3 + a^3*d^3*tan(1/2*f*x + 1/2*e)^3 + 2*a^3*c^
3*tan(1/2*f*x + 1/2*e)^2 - a^3*c^2*d*tan(1/2*f*x + 1/2*e)^2 + a^3*c*d^2*tan
```

```
(1/2*f*x + 1/2*e)^2 + 3*a^3*c^2*d*tan(1/2*f*x + 1/2*e) + a^3*d^3*tan(1/2*f*
x + 1/2*e) + 2*a^3*c^3 - a^3*c^2*d + a^3*c*d^2)/((c*tan(1/2*f*x + 1/2*e)^4
+ 2*d*tan(1/2*f*x + 1/2*e)^3 + 2*c*tan(1/2*f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x
 + 1/2*e) + c)*(c^2*d^2 + c*d^3)) - (2*a^3*c - 3*a^3*d)*(f*x + e)/d^3)/f
```

3.450 $\int \frac{(a+a\sin(e+fx))^3}{(c+d\sin(e+fx))^3}\,dx$

Optimal. Leaf size=187

$$-\frac{a^3(c-d)\left(2c^2+6cd+7d^2\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{d^3f(c+d)^2\sqrt{c^2-d^2}}+\frac{a^3(c-d)(2c+5d)\cos(e+fx)}{2d^2f(c+d)^2(c+d\sin(e+fx))}+\frac{(c-d)\cos(e+fx)\left(a^3\sin(e\right.}{2df(c+d)(c+d\sin(e\ }$$

```
[Out] (a^3*x)/d^3 - (a^3*(c - d)*(2*c^2 + 6*c*d + 7*d^2)*ArcTan[(d + c*Tan[(e + f
*x)/2])/Sqrt[c^2 - d^2]])/(d^3*(c + d)^2*Sqrt[c^2 - d^2]*f) + ((c - d)*Cos[
e + f*x]*(a^3 + a^3*Sin[e + f*x]))/(2*d*(c + d)*f*(c + d*Sin[e + f*x])^2) +
 (a^3*(c - d)*(2*c + 5*d)*Cos[e + f*x])/(2*d^2*(c + d)^2*f*(c + d*Sin[e + f
*x]))
```

Rubi [A] time = 0.477478, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {2762, 2968, 3021, 2735, 2660, 618, 204}

$$-\frac{a^3(c-d)\left(2c^2+6cd+7d^2\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{d^3f(c+d)^2\sqrt{c^2-d^2}}+\frac{a^3(c-d)(2c+5d)\cos(e+fx)}{2d^2f(c+d)^2(c+d\sin(e+fx))}+\frac{(c-d)\cos(e+fx)\left(a^3\sin(e\right.}{2df(c+d)(c+d\sin(e\ }$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3/(c + d*Sin[e + f*x])^3,x]
```

```
[Out] (a^3*x)/d^3 - (a^3*(c - d)*(2*c^2 + 6*c*d + 7*d^2)*ArcTan[(d + c*Tan[(e + f
*x)/2])/Sqrt[c^2 - d^2]])/(d^3*(c + d)^2*Sqrt[c^2 - d^2]*f) + ((c - d)*Cos[
e + f*x]*(a^3 + a^3*Sin[e + f*x]))/(2*d*(c + d)*f*(c + d*Sin[e + f*x])^2) +
 (a^3*(c - d)*(2*c + 5*d)*Cos[e + f*x])/(2*d^2*(c + d)^2*f*(c + d*Sin[e + f
*x]))
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
```

```
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^3}{(c+d\sin(e+fx))^3}\,dx = \frac{(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{2d(c+d)f(c+d\sin(e+fx))^2} - \frac{a\int \frac{(a+a\sin(e+fx))(a(c-5d)-2a(c+d)\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx}{2d(c+d)}$$

$$= \frac{(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{2d(c+d)f(c+d\sin(e+fx))^2} - \frac{a\int \frac{a^2(c-5d)+\left(a^2(c-5d)-2a^2(c+d)\right)\sin(e+fx)-2a^2(c+d)\sin^2(e\cdots}{(c+d\sin(e+fx))^2}}{2d(c+d)}$$

$$= \frac{(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{2d(c+d)f(c+d\sin(e+fx))^2} + \frac{a^3(c-d)(2c+5d)\cos(e+fx)}{2d^2(c+d)^2f(c+d\sin(e+fx))} + \frac{a\int \frac{a^2(c-d)d(c+7\cdots}{\cdots}}{2(c\cdots}$$

$$= \frac{a^3x}{d^3} + \frac{(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{2d(c+d)f(c+d\sin(e+fx))^2} + \frac{a^3(c-d)(2c+5d)\cos(e+fx)}{2d^2(c+d)^2f(c+d\sin(e+fx))} - \frac{\left(a^3(c-\cdots\right.}{\cdots}$$

$$= \frac{a^3x}{d^3} + \frac{(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{2d(c+d)f(c+d\sin(e+fx))^2} + \frac{a^3(c-d)(2c+5d)\cos(e+fx)}{2d^2(c+d)^2f(c+d\sin(e+fx))} - \frac{\left(a^3(c-\cdots\right.}{\cdots}$$

$$= \frac{a^3x}{d^3} + \frac{(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{2d(c+d)f(c+d\sin(e+fx))^2} + \frac{a^3(c-d)(2c+5d)\cos(e+fx)}{2d^2(c+d)^2f(c+d\sin(e+fx))} + \frac{\left(2a^3(c-\cdots\right.}{\cdots}$$

$$= \frac{a^3x}{d^3} - \frac{a^3(c-d)\left(2c^2+6cd+7d^2\right)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{d^3(c+d)^2\sqrt{c^2-d^2}\,f} + \frac{(c-d)\cos(e+fx)\left(a^3+a^3\sin(\cdots\right.}{2d(c+d)f(c+d\sin(e+f\cdots}$$

Mathematica [A] time = 0.985872, size = 196, normalized size = 1.05

$$\frac{a^3(\sin(e+fx)+1)^3\left(-\frac{2(4c^2d+2c^3+cd^2-7d^3)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{(c+d)^2\sqrt{c^2-d^2}}+\frac{3d(c^2+cd-2d^2)\cos(e+fx)}{(c+d)^2(c+d\sin(e+fx))}-\frac{d(c-d)^2\cos(e+fx)}{(c+d)(c+d\sin(e+fx))^2}+2(e+fx)\right)}{2d^3f\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3/(c + d*Sin[e + f*x])^3,x]
```

```
[Out] (a^3*(1 + Sin[e + f*x])^3*(2*(e + f*x) - (2*(2*c^3 + 4*c^2*d + c*d^2 - 7*d^
3)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/((c + d)^2*Sqrt[c^2 -
d^2]) - ((c - d)^2*d*Cos[e + f*x])/((c + d)*(c + d*Sin[e + f*x])^2) + (3*d*
(c^2 + c*d - 2*d^2)*Cos[e + f*x])/((c + d)^2*(c + d*Sin[e + f*x]))))/(2*d^3
*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6)
```

Maple [B] time = 0.149, size = 1400, normalized size = 7.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^3,x)
```

```
[Out] 1/f*a^3/d/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^
2)*c^2*tan(1/2*f*x+1/2*e)^3+5/f*a^3/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1
/2*e)*d+c)^2/(c^2+2*c*d+d^2)*c*tan(1/2*f*x+1/2*e)^3-4/f*a^3*d/(c*tan(1/2*f*
x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3
-2/f*a^3*d^2/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d
+d^2)/c*tan(1/2*f*x+1/2*e)^3+2/f*a^3/d^2/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*
f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*c^3*tan(1/2*f*x+1/2*e)^2+4/f*a^3/d/(c*tan
(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*c^2*tan(1/2*f
*x+1/2*e)^2-1/f*a^3/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^
2+2*c*d+d^2)*c*tan(1/2*f*x+1/2*e)^2+7/f*a^3*d/(c*tan(1/2*f*x+1/2*e)^2+2*tan
(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^2-10/f*a^3*d^2/(c
*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)/c*tan(1/2
*f*x+1/2*e)^2-2/f*a^3*d^3/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)
^2/(c^2+2*c*d+d^2)/c^2*tan(1/2*f*x+1/2*e)^2+7/f*a^3/d/(c*tan(1/2*f*x+1/2*e)
^2+2*tan(1/2*f*x+1/2*e)*d+c)^2*c^2/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)+11/f*
a^3/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2*c/(c^2+2*c*d+d^2)*t
an(1/2*f*x+1/2*e)-16/f*a^3*d/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d
+c)^2/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)-2/f*a^3*d^2/(c*tan(1/2*f*x+1/2*e)^
2+2*tan(1/2*f*x+1/2*e)*d+c)^2/c/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)+2/f*a^3/
d^2/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*c^3
+4/f*a^3/d/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d
^2)*c^2-5/f*a^3/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*
c*d+d^2)*c-1/f*a^3*d/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c
^2+2*c*d+d^2)-2/f*a^3/d^3/(c^2+2*c*d+d^2)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*t
an(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*c^3-4/f*a^3/d^2/(c^2+2*c*d+d^2)/(c^
2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*c^2-1
/f*a^3/d/(c^2+2*c*d+d^2)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)
+2*d)/(c^2-d^2)^(1/2))*c+7/f*a^3/(c^2+2*c*d+d^2)/(c^2-d^2)^(1/2)*arctan(1/2
*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))+2/f*a^3/d^3*arctan(tan(1/2*f
```

```
*x+1/2*e))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^3,x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [B] time = 2.12509, size = 2256, normalized size = 12.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^3,x, algorithm="fricas")

[Out] [1/4*(4*(a^3*c^2*d^2 + 2*a^3*c*d^3 + a^3*d^4)*f*x*cos(f*x + e)^2 - 4*(a^3*c
^4 + 2*a^3*c^3*d + 2*a^3*c^2*d^2 + 2*a^3*c*d^3 + a^3*d^4)*f*x - (2*a^3*c^4
+ 6*a^3*c^3*d + 9*a^3*c^2*d^2 + 6*a^3*c*d^3 + 7*a^3*d^4 - (2*a^3*c^2*d^2 +
6*a^3*c*d^3 + 7*a^3*d^4)*cos(f*x + e)^2 + 2*(2*a^3*c^3*d + 6*a^3*c^2*d^2 +
7*a^3*c*d^3)*sin(f*x + e))*sqrt(-(c - d)/(c + d))*log(((2*c^2 - d^2)*cos(f*
x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 + 2*((c^2 + c*d)*cos(f*x + e)*sin
(f*x + e) + (c*d + d^2)*cos(f*x + e))*sqrt(-(c - d)/(c + d)))/(d^2*cos(f*x
+ e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) - 2*(2*a^3*c^3*d + 4*a^3*c^2*d^2
- 5*a^3*c*d^3 - a^3*d^4)*cos(f*x + e) - 2*(4*(a^3*c^3*d + 2*a^3*c^2*d^2 + a
^3*c*d^3)*f*x + 3*(a^3*c^2*d^2 + a^3*c*d^3 - 2*a^3*d^4)*cos(f*x + e))*sin(f
*x + e))/((c^2*d^5 + 2*c*d^6 + d^7)*f*cos(f*x + e)^2 - 2*(c^3*d^4 + 2*c^2*d
^5 + c*d^6)*f*sin(f*x + e) - (c^4*d^3 + 2*c^3*d^4 + 2*c^2*d^5 + 2*c*d^6 + d
^7)*f), 1/2*(2*(a^3*c^2*d^2 + 2*a^3*c*d^3 + a^3*d^4)*f*x*cos(f*x + e)^2 - 2
*(a^3*c^4 + 2*a^3*c^3*d + 2*a^3*c^2*d^2 + 2*a^3*c*d^3 + a^3*d^4)*f*x - (2*a
^3*c^4 + 6*a^3*c^3*d + 9*a^3*c^2*d^2 + 6*a^3*c*d^3 + 7*a^3*d^4 - (2*a^3*c^2
*d^2 + 6*a^3*c*d^3 + 7*a^3*d^4)*cos(f*x + e)^2 + 2*(2*a^3*c^3*d + 6*a^3*c^2
*d^2 + 7*a^3*c*d^3)*sin(f*x + e))*sqrt((c - d)/(c + d))*arctan(-(c*sin(f*x
+ e) + d)*sqrt((c - d)/(c + d))/((c - d)*cos(f*x + e))) - (2*a^3*c^3*d + 4*
a^3*c^2*d^2 - 5*a^3*c*d^3 - a^3*d^4)*cos(f*x + e) - (4*(a^3*c^3*d + 2*a^3*c
^2*d^2 + a^3*c*d^3)*f*x + 3*(a^3*c^2*d^2 + a^3*c*d^3 - 2*a^3*d^4)*cos(f*x +
 e))*sin(f*x + e))/((c^2*d^5 + 2*c*d^6 + d^7)*f*cos(f*x + e)^2 - 2*(c^3*d^4
 + 2*c^2*d^5 + c*d^6)*f*sin(f*x + e) - (c^4*d^3 + 2*c^3*d^4 + 2*c^2*d^5 + 2
*c*d^6 + d^7)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3/(c+d*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.48818, size = 705, normalized size = 3.77

$$\frac{(fx+e)a^3}{d^3} - \frac{(2a^3c^3+4a^3c^2d+a^3cd^2-7a^3d^3)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{(c^2d^3+2cd^4+d^5)\sqrt{c^2-d^2}} + \frac{a^3c^4d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+5a^3c^3d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-4a^3c^2d}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] ((f*x + e)*a^3/d^3 - (2*a^3*c^3 + 4*a^3*c^2*d + a^3*c*d^2 - 7*a^3*d^3)*(pi*
floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/
sqrt(c^2 - d^2)))/((c^2*d^3 + 2*c*d^4 + d^5)*sqrt(c^2 - d^2)) + (a^3*c^4*d*
tan(1/2*f*x + 1/2*e)^3 + 5*a^3*c^3*d^2*tan(1/2*f*x + 1/2*e)^3 - 4*a^3*c^2*d
^3*tan(1/2*f*x + 1/2*e)^3 - 2*a^3*c*d^4*tan(1/2*f*x + 1/2*e)^3 + 2*a^3*c^5*
tan(1/2*f*x + 1/2*e)^2 + 4*a^3*c^4*d*tan(1/2*f*x + 1/2*e)^2 - a^3*c^3*d^2*t
an(1/2*f*x + 1/2*e)^2 + 7*a^3*c^2*d^3*tan(1/2*f*x + 1/2*e)^2 - 10*a^3*c*d^4
*tan(1/2*f*x + 1/2*e)^2 - 2*a^3*d^5*tan(1/2*f*x + 1/2*e)^2 + 7*a^3*c^4*d*ta
n(1/2*f*x + 1/2*e) + 11*a^3*c^3*d^2*tan(1/2*f*x + 1/2*e) - 16*a^3*c^2*d^3*t
an(1/2*f*x + 1/2*e) - 2*a^3*c*d^4*tan(1/2*f*x + 1/2*e) + 2*a^3*c^5 + 4*a^3*
c^4*d - 5*a^3*c^3*d^2 - a^3*c^2*d^3)/((c^4*d^2 + 2*c^3*d^3 + c^2*d^4)*(c*ta
n(1/2*f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e) + c)^2))/f
```

3.451 $\int \frac{(a+a\sin(e+fx))^3}{(c+d\sin(e+fx))^4}\,dx$

Optimal. Leaf size=207

$$\frac{5a^3\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{f(c+d)^3\sqrt{c^2-d^2}}-\frac{a^3\left(2c^2+9cd+22d^2\right)\cos(e+fx)}{6d^2f(c+d)^3(c+d\sin(e+fx))}+\frac{a^3(c-d)(2c+7d)\cos(e+fx)}{6d^2f(c+d)^2(c+d\sin(e+fx))^2}+\frac{(c-d)\cos(e+fx\cdots}{3df(c+d)(c\cdots}$$

```
[Out] (5*a^3*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/((c + d)^3*Sqrt[c^
2 - d^2]*f) + ((c - d)*Cos[e + f*x]*(a^3 + a^3*Sin[e + f*x]))/(3*d*(c + d)*
f*(c + d*Sin[e + f*x])^3) + (a^3*(c - d)*(2*c + 7*d)*Cos[e + f*x])/(6*d^2*(
c + d)^2*f*(c + d*Sin[e + f*x])^2) - (a^3*(2*c^2 + 9*c*d + 22*d^2)*Cos[e +
f*x])/(6*d^2*(c + d)^3*f*(c + d*Sin[e + f*x]))
```

Rubi [A] time = 0.475085, antiderivative size = 207, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {2762, 2968, 3021, 2754, 12, 2660, 618, 204}

$$\frac{5a^3\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{f(c+d)^3\sqrt{c^2-d^2}}-\frac{a^3\left(2c^2+9cd+22d^2\right)\cos(e+fx)}{6d^2f(c+d)^3(c+d\sin(e+fx))}+\frac{a^3(c-d)(2c+7d)\cos(e+fx)}{6d^2f(c+d)^2(c+d\sin(e+fx))^2}+\frac{(c-d)\cos(e+fx\cdots}{3df(c+d)(c\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3/(c + d*Sin[e + f*x])^4,x]
```

```
[Out] (5*a^3*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/((c + d)^3*Sqrt[c^
2 - d^2]*f) + ((c - d)*Cos[e + f*x]*(a^3 + a^3*Sin[e + f*x]))/(3*d*(c + d)*
f*(c + d*Sin[e + f*x])^3) + (a^3*(c - d)*(2*c + 7*d)*Cos[e + f*x])/(6*d^2*(
c + d)^2*f*(c + d*Sin[e + f*x])^2) - (a^3*(2*c^2 + 9*c*d + 22*d^2)*Cos[e +
f*x])/(6*d^2*(c + d)^3*f*(c + d*Sin[e + f*x]))
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
```

```
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^3}{(c+d\sin(e+fx))^4}\,dx = \frac{(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{3d(c+d)f(c+d\sin(e+fx))^3} - \frac{a\int \frac{(a+a\sin(e+fx))(a(c-7d)-2a(c+2d)\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx}{3d(c+d)}$$

$$= \frac{(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{3d(c+d)f(c+d\sin(e+fx))^3} - \frac{a\int \frac{a^2(c-7d)+\left(a^2(c-7d)-2a^2(c+2d)\right)\sin(e+fx)-2a^2(c+2d)\sin^2 \cdots}{(c+d\sin(e+fx))^3}}{3d(c+d)}$$

$$= \frac{(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{3d(c+d)f(c+d\sin(e+fx))^3} + \frac{a^3(c-d)(2c+7d)\cos(e+fx)}{6d^2(c+d)^2f(c+d\sin(e+fx))^2} + \frac{a\int \frac{2a^2(c-d)d(c \cdots}{\cdots}}{\cdots}$$

$$= \frac{(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{3d(c+d)f(c+d\sin(e+fx))^3} + \frac{a^3(c-d)(2c+7d)\cos(e+fx)}{6d^2(c+d)^2f(c+d\sin(e+fx))^2} - \frac{a^3\left(2c^2+9cd \cdots\right.}{6d^2(c+d)^3 f \cdots}$$

$$= \frac{(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{3d(c+d)f(c+d\sin(e+fx))^3} + \frac{a^3(c-d)(2c+7d)\cos(e+fx)}{6d^2(c+d)^2f(c+d\sin(e+fx))^2} - \frac{a^3\left(2c^2+9cd \cdots\right.}{6d^2(c+d)^3 f \cdots}$$

$$= \frac{(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{3d(c+d)f(c+d\sin(e+fx))^3} + \frac{a^3(c-d)(2c+7d)\cos(e+fx)}{6d^2(c+d)^2f(c+d\sin(e+fx))^2} - \frac{a^3\left(2c^2+9cd \cdots\right.}{6d^2(c+d)^3 f \cdots}$$

$$= \frac{(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{3d(c+d)f(c+d\sin(e+fx))^3} + \frac{a^3(c-d)(2c+7d)\cos(e+fx)}{6d^2(c+d)^2f(c+d\sin(e+fx))^2} - \frac{a^3\left(2c^2+9cd \cdots\right.}{6d^2(c+d)^3 f \cdots}$$

$$= \frac{5a^3\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{(c+d)^3\sqrt{c^2-d^2}\,f} + \frac{(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{3d(c+d)f(c+d\sin(e+fx))^3} + \frac{a^3(c-d)(2c+7d \cdots}{6d^2(c+d)^2f(c+d \cdots}$$

Mathematica [A] time = 2.45778, size = 178, normalized size = 0.86

$$\frac{a^3\cos(e+fx)\left(-\frac{(\sin(e+fx)+1)^2}{(c+d\sin(e+fx))^3}-\frac{5(\sin(e+fx)+1)}{2(c+d)(c+d\sin(e+fx))^2}-\frac{15}{2(c+d)^2(c+d\sin(e+fx))}+\frac{15\tan^{-1}\left(\frac{\sqrt{d-c}\sqrt{1-\sin(e+fx)}}{\sqrt{-c-d}\sqrt{\sin(e+fx)+1}}\right)}{(-c-d)^{5/2}\sqrt{d-c}\sqrt{\cos^2(e+fx)}}\right)}{3f(c+d)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3/(c + d*Sin[e + f*x])^4,x]
```

```
[Out] (a^3*Cos[e + f*x]*((15*ArcTan[(Sqrt[-c + d]*Sqrt[1 - Sin[e + f*x]])/(Sqrt[-
c - d]*Sqrt[1 + Sin[e + f*x]])])/((-c - d)^(5/2)*Sqrt[-c + d]*Sqrt[Cos[e +
f*x]^2]) - (1 + Sin[e + f*x])^2/(c + d*Sin[e + f*x])^3 - (5*(1 + Sin[e + f*
x]))/(2*(c + d)*(c + d*Sin[e + f*x])^2) - 15/(2*(c + d)^2*(c + d*Sin[e + f*
x]))))/(3*(c + d)*f)
```

Maple [B] time = 0.17, size = 1924, normalized size = 9.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^4,x)
```

```
[Out] -18/f*a^3/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^3/c/(c^3+3*c^2*
d+3*c*d^2+d^3)*tan(1/2*f*x+1/2*e)^2*d^3-4/f*a^3/(c*tan(1/2*f*x+1/2*e)^2+2*t
an(1/2*f*x+1/2*e)*d+c)^3/c^2/(c^3+3*c^2*d+3*c*d^2+d^3)*tan(1/2*f*x+1/2*e)^2
```

```
*d^4-38/f*a^3/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^3*c/(c^3+3*
c^2*d+3*c*d^2+d^3)*tan(1/2*f*x+1/2*e)*d-2/f*a^3/(c*tan(1/2*f*x+1/2*e)^2+2*t
an(1/2*f*x+1/2*e)*d+c)^3/c/(c^3+3*c^2*d+3*c*d^2+d^3)*tan(1/2*f*x+1/2*e)*d^3
-6/f*a^3/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^3*c/(c^3+3*c^2*d
+3*c*d^2+d^3)*tan(1/2*f*x+1/2*e)^5*d-2/f*a^3/(c*tan(1/2*f*x+1/2*e)^2+2*tan(
1/2*f*x+1/2*e)*d+c)^3/c/(c^3+3*c^2*d+3*c*d^2+d^3)*tan(1/2*f*x+1/2*e)^5*d^3+
3/f*a^3/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^3*c/(c^3+3*c^2*d+
3*c*d^2+d^3)*tan(1/2*f*x+1/2*e)^4*d-18/f*a^3/(c*tan(1/2*f*x+1/2*e)^2+2*tan(
1/2*f*x+1/2*e)*d+c)^3/c/(c^3+3*c^2*d+3*c*d^2+d^3)*tan(1/2*f*x+1/2*e)^4*d^3-
4/f*a^3/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^3/c^2/(c^3+3*c^2*
d+3*c*d^2+d^3)*tan(1/2*f*x+1/2*e)^4*d^4-44/f*a^3/(c*tan(1/2*f*x+1/2*e)^2+2*
tan(1/2*f*x+1/2*e)*d+c)^3*c*d/(c^3+3*c^2*d+3*c*d^2+d^3)*tan(1/2*f*x+1/2*e)^
3-100/3/f*a^3/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^3/c*d^3/(c^
3+3*c^2*d+3*c*d^2+d^3)*tan(1/2*f*x+1/2*e)^3-12/f*a^3/(c*tan(1/2*f*x+1/2*e)^
2+2*tan(1/2*f*x+1/2*e)*d+c)^3/c^2*d^4/(c^3+3*c^2*d+3*c*d^2+d^3)*tan(1/2*f*x
+1/2*e)^3-6/f*a^3/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^3/(c^3+
3*c^2*d+3*c*d^2+d^3)*tan(1/2*f*x+1/2*e)^5*d^2-30/f*a^3/(c*tan(1/2*f*x+1/2*e
)^2+2*tan(1/2*f*x+1/2*e)*d+c)^3/(c^3+3*c^2*d+3*c*d^2+d^3)*tan(1/2*f*x+1/2*e
)^4*d^2-18/f*a^3/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^3*d^2/(c
^3+3*c^2*d+3*c*d^2+d^3)*tan(1/2*f*x+1/2*e)^3-12/f*a^3/(c*tan(1/2*f*x+1/2*e)
^2+2*tan(1/2*f*x+1/2*e)*d+c)^3/(c^3+3*c^2*d+3*c*d^2+d^3)*tan(1/2*f*x+1/2*e)
*d^2-60/f*a^3/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^3/(c^3+3*c^
2*d+3*c*d^2+d^3)*tan(1/2*f*x+1/2*e)^2*d^2+3/f*a^3/(c*tan(1/2*f*x+1/2*e)^2+2
*tan(1/2*f*x+1/2*e)*d+c)^3*c^2/(c^3+3*c^2*d+3*c*d^2+d^3)*tan(1/2*f*x+1/2*e)
^5-6/f*a^3/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^3*c^2/(c^3+3*c
^2*d+3*c*d^2+d^3)*tan(1/2*f*x+1/2*e)^4-16/f*a^3/(c*tan(1/2*f*x+1/2*e)^2+2*t
an(1/2*f*x+1/2*e)*d+c)^3*c^2/(c^3+3*c^2*d+3*c*d^2+d^3)*tan(1/2*f*x+1/2*e)^2
-3/f*a^3/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^3*c^2/(c^3+3*c^2
*d+3*c*d^2+d^3)*tan(1/2*f*x+1/2*e)-3/f*a^3/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/
2*f*x+1/2*e)*d+c)^3/(c^3+3*c^2*d+3*c*d^2+d^3)*c*d-22/3/f*a^3/(c*tan(1/2*f*x
+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^3/(c^3+3*c^2*d+3*c*d^2+d^3)*c^2-2/3/f*a
^3/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^3/(c^3+3*c^2*d+3*c*d^2
+d^3)*d^2-8/3/f*a^3/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^3/c^3
*d^5/(c^3+3*c^2*d+3*c*d^2+d^3)*tan(1/2*f*x+1/2*e)^3-12/f*a^3/(c*tan(1/2*f*x
+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^3*c/(c^3+3*c^2*d+3*c*d^2+d^3)*tan(1/2*f
*x+1/2*e)^2*d+5/f*a^3/(c^3+3*c^2*d+3*c*d^2+d^3)/(c^2-d^2)^(1/2)*arctan(1/2*
(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.14736, size = 2369, normalized size = 11.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^4,x, algorithm="fricas")
```

```
[Out] [-1/12*(2*(2*a^3*c^4 + 9*a^3*c^3*d + 20*a^3*c^2*d^2 - 9*a^3*c*d^3 - 22*a^3*
d^4)*cos(f*x + e)^3 - 6*(3*a^3*c^4 + 16*a^3*c^3*d - 16*a^3*c*d^3 - 3*a^3*d^
4)*cos(f*x + e)*sin(f*x + e) + 15*(3*a^3*c*d^2*cos(f*x + e)^2 - a^3*c^3 - 3
*a^3*c*d^2 + (a^3*d^3*cos(f*x + e)^2 - 3*a^3*c^2*d - a^3*d^3)*sin(f*x + e))
*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) -
c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x + e) + d*cos(f*x + e))*sqrt(-c^2 + d^
2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) - 12*(4*a^3*c^4
+ 3*a^3*c^3*d - 3*a^3*c*d^3 - 4*a^3*d^4)*cos(f*x + e))/(3*(c^6*d^2 + 3*c^5*
d^3 + 2*c^4*d^4 - 2*c^3*d^5 - 3*c^2*d^6 - c*d^7)*f*cos(f*x + e)^2 - (c^8 +
3*c^7*d + 5*c^6*d^2 + 7*c^5*d^3 + 3*c^4*d^4 - 7*c^3*d^5 - 9*c^2*d^6 - 3*c*d
^7)*f + ((c^5*d^3 + 3*c^4*d^4 + 2*c^3*d^5 - 2*c^2*d^6 - 3*c*d^7 - d^8)*f*co
s(f*x + e)^2 - (3*c^7*d + 9*c^6*d^2 + 7*c^5*d^3 - 3*c^4*d^4 - 7*c^3*d^5 - 5
*c^2*d^6 - 3*c*d^7 - d^8)*f)*sin(f*x + e)), -1/6*((2*a^3*c^4 + 9*a^3*c^3*d
+ 20*a^3*c^2*d^2 - 9*a^3*c*d^3 - 22*a^3*d^4)*cos(f*x + e)^3 - 3*(3*a^3*c^4
+ 16*a^3*c^3*d - 16*a^3*c*d^3 - 3*a^3*d^4)*cos(f*x + e)*sin(f*x + e) + 15*(
3*a^3*c*d^2*cos(f*x + e)^2 - a^3*c^3 - 3*a^3*c*d^2 + (a^3*d^3*cos(f*x + e)^
2 - 3*a^3*c^2*d - a^3*d^3)*sin(f*x + e))*sqrt(c^2 - d^2)*arctan(-(c*sin(f*x
 + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) - 6*(4*a^3*c^4 + 3*a^3*c^3*d - 3
*a^3*c*d^3 - 4*a^3*d^4)*cos(f*x + e))/(3*(c^6*d^2 + 3*c^5*d^3 + 2*c^4*d^4 -
 2*c^3*d^5 - 3*c^2*d^6 - c*d^7)*f*cos(f*x + e)^2 - (c^8 + 3*c^7*d + 5*c^6*d
^2 + 7*c^5*d^3 + 3*c^4*d^4 - 7*c^3*d^5 - 9*c^2*d^6 - 3*c*d^7)*f + ((c^5*d^3
 + 3*c^4*d^4 + 2*c^3*d^5 - 2*c^2*d^6 - 3*c*d^7 - d^8)*f*cos(f*x + e)^2 - (3
*c^7*d + 9*c^6*d^2 + 7*c^5*d^3 - 3*c^4*d^4 - 7*c^3*d^5 - 5*c^2*d^6 - 3*c*d^
7 - d^8)*f)*sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3/(c+d*sin(f*x+e))**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.48734, size = 900, normalized size = 4.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^4,x, algorithm="giac")
```

```
[Out] 1/3*(15*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x +
1/2*e) + d)/sqrt(c^2 - d^2)))*a^3/((c^3 + 3*c^2*d + 3*c*d^2 + d^3)*sqrt(c^2
 - d^2)) + (9*a^3*c^5*tan(1/2*f*x + 1/2*e)^5 - 18*a^3*c^4*d*tan(1/2*f*x + 1
/2*e)^5 - 18*a^3*c^3*d^2*tan(1/2*f*x + 1/2*e)^5 - 6*a^3*c^2*d^3*tan(1/2*f*x
 + 1/2*e)^5 - 18*a^3*c^5*tan(1/2*f*x + 1/2*e)^4 + 9*a^3*c^4*d*tan(1/2*f*x +
 1/2*e)^4 - 90*a^3*c^3*d^2*tan(1/2*f*x + 1/2*e)^4 - 54*a^3*c^2*d^3*tan(1/2*
f*x + 1/2*e)^4 - 12*a^3*c*d^4*tan(1/2*f*x + 1/2*e)^4 - 132*a^3*c^4*d*tan(1/
2*f*x + 1/2*e)^3 - 54*a^3*c^3*d^2*tan(1/2*f*x + 1/2*e)^3 - 100*a^3*c^2*d^3*
tan(1/2*f*x + 1/2*e)^3 - 36*a^3*c*d^4*tan(1/2*f*x + 1/2*e)^3 - 8*a^3*d^5*ta
n(1/2*f*x + 1/2*e)^3 - 48*a^3*c^5*tan(1/2*f*x + 1/2*e)^2 - 36*a^3*c^4*d*tan
(1/2*f*x + 1/2*e)^2 - 180*a^3*c^3*d^2*tan(1/2*f*x + 1/2*e)^2 - 54*a^3*c^2*d
```

```
^3*tan(1/2*f*x + 1/2*e)^2 - 12*a^3*c*d^4*tan(1/2*f*x + 1/2*e)^2 - 9*a^3*c^5
*tan(1/2*f*x + 1/2*e) - 114*a^3*c^4*d*tan(1/2*f*x + 1/2*e) - 36*a^3*c^3*d^2
*tan(1/2*f*x + 1/2*e) - 6*a^3*c^2*d^3*tan(1/2*f*x + 1/2*e) - 22*a^3*c^5 - 9
*a^3*c^4*d - 2*a^3*c^3*d^2)/((c^6 + 3*c^5*d + 3*c^4*d^2 + c^3*d^3)*(c*tan(1
/2*f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e) + c)^3))/f
```

3.452 $\int \frac{(a+a\sin(e+fx))^3}{(c+d\sin(e+fx))^5}\,dx$

Optimal. Leaf size=289

$$\frac{5a^3(4c-3d)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{4f(c-d)(c+d)^4\sqrt{c^2-d^2}}-\frac{a^3\left(12c^2d+2c^3+43cd^2-72d^3\right)\cos(e+fx)}{24d^2f(c-d)(c+d)^4(c+d\sin(e+fx))}-\frac{a^3\left(2c^2+12cd+45d^2\right)\cos(e+fx)}{24d^2f(c+d)^3(c+d\sin(e+fx))^2}$$

```
[Out] (5*a^3*(4*c - 3*d)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(4*(c
- d)*(c + d)^4*Sqrt[c^2 - d^2]*f) + ((c - d)*Cos[e + f*x]*(a^3 + a^3*Sin[e
+ f*x]))/(4*d*(c + d)*f*(c + d*Sin[e + f*x])^4) + (a^3*(c - d)*(2*c + 9*d)*
Cos[e + f*x])/(12*d^2*(c + d)^2*f*(c + d*Sin[e + f*x])^3) - (a^3*(2*c^2 + 1
2*c*d + 45*d^2)*Cos[e + f*x])/(24*d^2*(c + d)^3*f*(c + d*Sin[e + f*x])^2) -
 (a^3*(2*c^3 + 12*c^2*d + 43*c*d^2 - 72*d^3)*Cos[e + f*x])/(24*(c - d)*d^2*
(c + d)^4*f*(c + d*Sin[e + f*x]))
```

Rubi [A] time = 0.696139, antiderivative size = 289, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {2762, 2968, 3021, 2754, 12, 2660, 618, 204}

$$\frac{5a^3(4c-3d)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{4f(c-d)(c+d)^4\sqrt{c^2-d^2}}-\frac{a^3\left(12c^2d+2c^3+43cd^2-72d^3\right)\cos(e+fx)}{24d^2f(c-d)(c+d)^4(c+d\sin(e+fx))}-\frac{a^3\left(2c^2+12cd+45d^2\right)\cos(e+fx)}{24d^2f(c+d)^3(c+d\sin(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3/(c + d*Sin[e + f*x])^5,x]
```


```
[Out] (5*a^3*(4*c - 3*d)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(4*(c
- d)*(c + d)^4*Sqrt[c^2 - d^2]*f) + ((c - d)*Cos[e + f*x]*(a^3 + a^3*Sin[e
+ f*x]))/(4*d*(c + d)*f*(c + d*Sin[e + f*x])^4) + (a^3*(c - d)*(2*c + 9*d)*
Cos[e + f*x])/(12*d^2*(c + d)^2*f*(c + d*Sin[e + f*x])^3) - (a^3*(2*c^2 + 1
2*c*d + 45*d^2)*Cos[e + f*x])/(24*d^2*(c + d)^3*f*(c + d*Sin[e + f*x])^2) -
 (a^3*(2*c^3 + 12*c^2*d + 43*c*d^2 - 72*d^3)*Cos[e + f*x])/(24*(c - d)*d^2*
(c + d)^4*f*(c + d*Sin[e + f*x]))
```


Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^3}{(c+d\sin(e+fx))^5}\,dx = \frac{(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{4d(c+d)f(c+d\sin(e+fx))^4} - \frac{a\int \frac{(a+a\sin(e+fx))(a(c-9d)-2a(c+3d)\sin(e+fx))}{(c+d\sin(e+fx))^4}dx}{4d(c+d)}$$

$$= \frac{(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{4d(c+d)f(c+d\sin(e+fx))^4} - \frac{a\int \frac{a^2(c-9d)+\left(a^2(c-9d)-2a^2(c+3d)\right)\sin(e+fx)-2a^2(c+3d)\sin^2}{(c+d\sin(e+fx))^4}}{4d(c+d)}$$

$$= \frac{(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{4d(c+d)f(c+d\sin(e+fx))^4} + \frac{a^3(c-d)(2c+9d)\cos(e+fx)}{12d^2(c+d)^2f(c+d\sin(e+fx))^3} + \frac{a\int 3a^2(c-d)d(}{}$$

$$= \frac{(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{4d(c+d)f(c+d\sin(e+fx))^4} + \frac{a^3(c-d)(2c+9d)\cos(e+fx)}{12d^2(c+d)^2f(c+d\sin(e+fx))^3} - \frac{a^3\left(2c^2+12\right.}{24d^2(c+d)}$$

$$= \frac{(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{4d(c+d)f(c+d\sin(e+fx))^4} + \frac{a^3(c-d)(2c+9d)\cos(e+fx)}{12d^2(c+d)^2f(c+d\sin(e+fx))^3} - \frac{a^3\left(2c^2+12\right.}{24d^2(c+d)}$$

$$= \frac{(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{4d(c+d)f(c+d\sin(e+fx))^4} + \frac{a^3(c-d)(2c+9d)\cos(e+fx)}{12d^2(c+d)^2f(c+d\sin(e+fx))^3} - \frac{a^3\left(2c^2+12\right.}{24d^2(c+d)}$$

$$= \frac{(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{4d(c+d)f(c+d\sin(e+fx))^4} + \frac{a^3(c-d)(2c+9d)\cos(e+fx)}{12d^2(c+d)^2f(c+d\sin(e+fx))^3} - \frac{a^3\left(2c^2+12\right.}{24d^2(c+d)}$$

$$= \frac{(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{4d(c+d)f(c+d\sin(e+fx))^4} + \frac{a^3(c-d)(2c+9d)\cos(e+fx)}{12d^2(c+d)^2f(c+d\sin(e+fx))^3} - \frac{a^3\left(2c^2+12\right.}{24d^2(c+d)}$$

$$= \frac{5a^3(4c-3d)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{4(c-d)(c+d)^4\sqrt{c^2-d^2}f} + \frac{(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{4d(c+d)f(c+d\sin(e+fx))^4} + \frac{a^3(c-d}{12d^2(c+}$$

Mathematica [A] time = 3.1805, size = 240, normalized size = 0.83

$$\frac{a^3\cos(e+fx)\left(-\frac{(4c-3d)\left(-\frac{\sqrt{\cos^2(e+fx)}\left((2c^2+9cd+22d^2)\sin^2(e+fx)+(9c^2+48cd+9d^2)\sin(e+fx)+22c^2+9cd+2d^2\right)}{6(c+d)^3(c+d\sin(e+fx))^3}-\frac{5\tan^{-1}\left(\frac{\sqrt{d-c}\sqrt{1-\sin(e+fx)}}{\sqrt{-c-d}\sqrt{\sin(e+fx)+1}}\right)}{(-c-d)^{7/2}\sqrt{d-c}}\right)}{\sqrt{\cos^2(e+fx)}}-\frac{d(\sin(e+fx)+1)}{(c+d\sin(e+fx))}\right.}{4f(d-c)(c+d)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3/(c + d*Sin[e + f*x])^5,x]
```

```
[Out] (a^3*Cos[e + f*x]*(-((d*(1 + Sin[e + f*x])^3)/(c + d*Sin[e + f*x])^4) - ((4
*c - 3*d)*((-5*ArcTan[(Sqrt[-c + d]*Sqrt[1 - Sin[e + f*x]])/(Sqrt[-c - d]*S
qrt[1 + Sin[e + f*x]])])/((-c - d)^(7/2)*Sqrt[-c + d]) - (Sqrt[Cos[e + f*x]
^2]*(22*c^2 + 9*c*d + 2*d^2 + (9*c^2 + 48*c*d + 9*d^2)*Sin[e + f*x] + (2*c^
2 + 9*c*d + 22*d^2)*Sin[e + f*x]^2))/(6*(c + d)^3*(c + d*Sin[e + f*x])^3)))
/Sqrt[Cos[e + f*x]^2]))/(4*(-c + d)*(c + d)*f)
```

Maple [B] time = 0.233, size = 5149, normalized size = 17.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^5,x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.62538, size = 4343, normalized size = 15.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^5,x, algorithm="fricas")
```

```
[Out] [1/48*(2*(8*a^3*c^6 + 48*a^3*c^5*d + 164*a^3*c^4*d^2 - 276*a^3*c^3*d^3 - 21
7*a^3*c^2*d^4 + 228*a^3*c*d^5 + 45*a^3*d^6)*cos(f*x + e)^3 - 15*(4*a^3*c^5
- 3*a^3*c^4*d + 24*a^3*c^3*d^2 - 18*a^3*c^2*d^3 + 4*a^3*c*d^4 - 3*a^3*d^5 +
 (4*a^3*c*d^4 - 3*a^3*d^5)*cos(f*x + e)^4 - 2*(12*a^3*c^3*d^2 - 9*a^3*c^2*d
^3 + 4*a^3*c*d^4 - 3*a^3*d^5)*cos(f*x + e)^2 + 4*(4*a^3*c^4*d - 3*a^3*c^3*d
^2 + 4*a^3*c^2*d^3 - 3*a^3*c*d^4 - (4*a^3*c^2*d^3 - 3*a^3*c*d^4)*cos(f*x +
e)^2)*sin(f*x + e))*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*
c*d*sin(f*x + e) - c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x + e) + d*cos(f*x +
 e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2
)) - 6*(32*a^3*c^6 + 4*a^3*c^5*d + 13*a^3*c^4*d^2 - 88*a^3*c^3*d^3 - 62*a^3
*c^2*d^4 + 84*a^3*c*d^5 + 17*a^3*d^6)*cos(f*x + e) + 2*((2*a^3*c^5*d + 12*a
^3*c^4*d^2 + 41*a^3*c^3*d^3 - 84*a^3*c^2*d^4 - 43*a^3*c*d^5 + 72*a^3*d^6)*c
os(f*x + e)^3 - 3*(12*a^3*c^6 + 79*a^3*c^5*d - 72*a^3*c^4*d^2 - 98*a^3*c^3*
d^3 + 28*a^3*c^2*d^4 + 19*a^3*c*d^5 + 32*a^3*d^6)*cos(f*x + e))*sin(f*x + e
))/((c^7*d^4 + 3*c^6*d^5 + c^5*d^6 - 5*c^4*d^7 - 5*c^3*d^8 + c^2*d^9 + 3*c*
d^10 + d^11)*f*cos(f*x + e)^4 - 2*(3*c^9*d^2 + 9*c^8*d^3 + 4*c^7*d^4 - 12*c
^6*d^5 - 14*c^5*d^6 - 2*c^4*d^7 + 4*c^3*d^8 + 4*c^2*d^9 + 3*c*d^10 + d^11)*
f*cos(f*x + e)^2 + (c^11 + 3*c^10*d + 7*c^9*d^2 + 13*c^8*d^3 + 2*c^7*d^4 -
26*c^6*d^5 - 26*c^5*d^6 + 2*c^4*d^7 + 13*c^3*d^8 + 7*c^2*d^9 + 3*c*d^10 + d
^11)*f - 4*((c^8*d^3 + 3*c^7*d^4 + c^6*d^5 - 5*c^5*d^6 - 5*c^4*d^7 + c^3*d^
8 + 3*c^2*d^9 + c*d^10)*f*cos(f*x + e)^2 - (c^10*d + 3*c^9*d^2 + 2*c^8*d^3
- 2*c^7*d^4 - 4*c^6*d^5 - 4*c^5*d^6 - 2*c^4*d^7 + 2*c^3*d^8 + 3*c^2*d^9 + c
*d^10)*f)*sin(f*x + e)), 1/24*((8*a^3*c^6 + 48*a^3*c^5*d + 164*a^3*c^4*d^2
- 276*a^3*c^3*d^3 - 217*a^3*c^2*d^4 + 228*a^3*c*d^5 + 45*a^3*d^6)*cos(f*x +
 e)^3 - 15*(4*a^3*c^5 - 3*a^3*c^4*d + 24*a^3*c^3*d^2 - 18*a^3*c^2*d^3 + 4*a
^3*c*d^4 - 3*a^3*d^5 + (4*a^3*c*d^4 - 3*a^3*d^5)*cos(f*x + e)^4 - 2*(12*a^3
*c^3*d^2 - 9*a^3*c^2*d^3 + 4*a^3*c*d^4 - 3*a^3*d^5)*cos(f*x + e)^2 + 4*(4*a
^3*c^4*d - 3*a^3*c^3*d^2 + 4*a^3*c^2*d^3 - 3*a^3*c*d^4 - (4*a^3*c^2*d^3 - 3
*a^3*c*d^4)*cos(f*x + e)^2)*sin(f*x + e))*sqrt(c^2 - d^2)*arctan(-(c*sin(f*
x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) - 3*(32*a^3*c^6 + 4*a^3*c^5*d +
 13*a^3*c^4*d^2 - 88*a^3*c^3*d^3 - 62*a^3*c^2*d^4 + 84*a^3*c*d^5 + 17*a^3*d
^6)*cos(f*x + e) + ((2*a^3*c^5*d + 12*a^3*c^4*d^2 + 41*a^3*c^3*d^3 - 84*a^3
```

```
*c^2*d^4 - 43*a^3*c*d^5 + 72*a^3*d^6)*cos(f*x + e)^3 - 3*(12*a^3*c^6 + 79*a
^3*c^5*d - 72*a^3*c^4*d^2 - 98*a^3*c^3*d^3 + 28*a^3*c^2*d^4 + 19*a^3*c*d^5
+ 32*a^3*d^6)*cos(f*x + e))*sin(f*x + e))/((c^7*d^4 + 3*c^6*d^5 + c^5*d^6 -
 5*c^4*d^7 - 5*c^3*d^8 + c^2*d^9 + 3*c*d^10 + d^11)*f*cos(f*x + e)^4 - 2*(3
*c^9*d^2 + 9*c^8*d^3 + 4*c^7*d^4 - 12*c^6*d^5 - 14*c^5*d^6 - 2*c^4*d^7 + 4*
c^3*d^8 + 4*c^2*d^9 + 3*c*d^10 + d^11)*f*cos(f*x + e)^2 + (c^11 + 3*c^10*d
+ 7*c^9*d^2 + 13*c^8*d^3 + 2*c^7*d^4 - 26*c^6*d^5 - 26*c^5*d^6 + 2*c^4*d^7
+ 13*c^3*d^8 + 7*c^2*d^9 + 3*c*d^10 + d^11)*f - 4*((c^8*d^3 + 3*c^7*d^4 + c
^6*d^5 - 5*c^5*d^6 - 5*c^4*d^7 + c^3*d^8 + 3*c^2*d^9 + c*d^10)*f*cos(f*x +
e)^2 - (c^10*d + 3*c^9*d^2 + 2*c^8*d^3 - 2*c^7*d^4 - 4*c^6*d^5 - 4*c^5*d^6
- 2*c^4*d^7 + 2*c^3*d^8 + 3*c^2*d^9 + c*d^10)*f)*sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3/(c+d*sin(f*x+e))**5,x)

[Out] Timed out
```

Giac [B] time = 1.62097, size = 1806, normalized size = 6.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^5,x, algorithm="giac")

[Out] 1/12*(15*(4*a^3*c - 3*a^3*d)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arc
tan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/((c^5 + 3*c^4*d + 2*c^3*
d^2 - 2*c^2*d^3 - 3*c*d^4 - d^5)*sqrt(c^2 - d^2)) + (36*a^3*c^8*tan(1/2*f*x
 + 1/2*e)^7 - 117*a^3*c^7*d*tan(1/2*f*x + 1/2*e)^7 - 48*a^3*c^6*d^2*tan(1/2
*f*x + 1/2*e)^7 + 48*a^3*c^5*d^3*tan(1/2*f*x + 1/2*e)^7 + 72*a^3*c^4*d^4*ta
n(1/2*f*x + 1/2*e)^7 + 24*a^3*c^3*d^5*tan(1/2*f*x + 1/2*e)^7 - 72*a^3*c^8*t
an(1/2*f*x + 1/2*e)^6 + 132*a^3*c^7*d*tan(1/2*f*x + 1/2*e)^6 - 675*a^3*c^6*
d^2*tan(1/2*f*x + 1/2*e)^6 + 360*a^3*c^4*d^4*tan(1/2*f*x + 1/2*e)^6 + 288*a
^3*c^3*d^5*tan(1/2*f*x + 1/2*e)^6 + 72*a^3*c^2*d^6*tan(1/2*f*x + 1/2*e)^6 +
 36*a^3*c^8*tan(1/2*f*x + 1/2*e)^5 - 813*a^3*c^7*d*tan(1/2*f*x + 1/2*e)^5 +
 288*a^3*c^6*d^2*tan(1/2*f*x + 1/2*e)^5 - 892*a^3*c^5*d^3*tan(1/2*f*x + 1/2
*e)^5 + 552*a^3*c^4*d^4*tan(1/2*f*x + 1/2*e)^5 + 664*a^3*c^3*d^5*tan(1/2*f*
x + 1/2*e)^5 + 384*a^3*c^2*d^6*tan(1/2*f*x + 1/2*e)^5 + 96*a^3*c*d^7*tan(1/
2*f*x + 1/2*e)^5 - 264*a^3*c^8*tan(1/2*f*x + 1/2*e)^4 + 108*a^3*c^7*d*tan(1
/2*f*x + 1/2*e)^4 - 2001*a^3*c^6*d^2*tan(1/2*f*x + 1/2*e)^4 + 936*a^3*c^5*d
^3*tan(1/2*f*x + 1/2*e)^4 + 202*a^3*c^4*d^4*tan(1/2*f*x + 1/2*e)^4 + 864*a^
3*c^3*d^5*tan(1/2*f*x + 1/2*e)^4 + 440*a^3*c^2*d^6*tan(1/2*f*x + 1/2*e)^4 +
 192*a^3*c*d^7*tan(1/2*f*x + 1/2*e)^4 + 48*a^3*d^8*tan(1/2*f*x + 1/2*e)^4 -
 36*a^3*c^8*tan(1/2*f*x + 1/2*e)^3 - 1299*a^3*c^7*d*tan(1/2*f*x + 1/2*e)^3
+ 576*a^3*c^6*d^2*tan(1/2*f*x + 1/2*e)^3 - 1036*a^3*c^5*d^3*tan(1/2*f*x + 1
/2*e)^3 + 1176*a^3*c^4*d^4*tan(1/2*f*x + 1/2*e)^3 + 664*a^3*c^3*d^5*tan(1/2
*f*x + 1/2*e)^3 + 384*a^3*c^2*d^6*tan(1/2*f*x + 1/2*e)^3 + 96*a^3*c*d^7*tan
(1/2*f*x + 1/2*e)^3 - 280*a^3*c^8*tan(1/2*f*x + 1/2*e)^2 + 12*a^3*c^7*d*tan
(1/2*f*x + 1/2*e)^2 - 1289*a^3*c^6*d^2*tan(1/2*f*x + 1/2*e)^2 + 960*a^3*c^5
```

```
*d^3*tan(1/2*f*x + 1/2*e)^2 + 552*a^3*c^4*d^4*tan(1/2*f*x + 1/2*e)^2 + 288*
a^3*c^3*d^5*tan(1/2*f*x + 1/2*e)^2 + 72*a^3*c^2*d^6*tan(1/2*f*x + 1/2*e)^2
- 36*a^3*c^8*tan(1/2*f*x + 1/2*e) - 587*a^3*c^7*d*tan(1/2*f*x + 1/2*e) + 33
6*a^3*c^6*d^2*tan(1/2*f*x + 1/2*e) + 248*a^3*c^5*d^3*tan(1/2*f*x + 1/2*e) +
 120*a^3*c^4*d^4*tan(1/2*f*x + 1/2*e) + 24*a^3*c^3*d^5*tan(1/2*f*x + 1/2*e)
 - 88*a^3*c^8 + 36*a^3*c^7*d + 37*a^3*c^6*d^2 + 24*a^3*c^5*d^3 + 6*a^3*c^4*
d^4)/((c^9 + 3*c^8*d + 2*c^7*d^2 - 2*c^6*d^3 - 3*c^5*d^4 - c^4*d^5)*(c*tan(
1/2*f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e) + c)^4))/f
```

3.453 $\int \frac{(c+d\sin(e+fx))^4}{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=189

$$\frac{2d\left(-16c^2d+3c^3+12cd^2-4d^3\right)\cos(e+fx)}{3af}+\frac{d^2\left(6c^2-20cd+9d^2\right)\sin(e+fx)\cos(e+fx)}{6af}+\frac{dx\left(-12c^2d+8c^3+12cd\right.}{2a}$$

```
[Out] (d*(8*c^3 - 12*c^2*d + 12*c*d^2 - 3*d^3)*x)/(2*a) + (2*d*(3*c^3 - 16*c^2*d
+ 12*c*d^2 - 4*d^3)*Cos[e + f*x])/(3*a*f) + (d^2*(6*c^2 - 20*c*d + 9*d^2)*C
os[e + f*x]*Sin[e + f*x])/(6*a*f) + ((3*c - 4*d)*d*Cos[e + f*x]*(c + d*Sin[
e + f*x])^2)/(3*a*f) - ((c - d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(f*(a
+ a*Sin[e + f*x]))
```

Rubi [A] time = 0.22736, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2767, 2753, 2734}

$$\frac{2d\left(-16c^2d+3c^3+12cd^2-4d^3\right)\cos(e+fx)}{3af}+\frac{d^2\left(6c^2-20cd+9d^2\right)\sin(e+fx)\cos(e+fx)}{6af}+\frac{dx\left(-12c^2d+8c^3+12cd\right.}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^4/(a + a*Sin[e + f*x]),x]
```

```
[Out] (d*(8*c^3 - 12*c^2*d + 12*c*d^2 - 3*d^3)*x)/(2*a) + (2*d*(3*c^3 - 16*c^2*d
+ 12*c*d^2 - 4*d^3)*Cos[e + f*x])/(3*a*f) + (d^2*(6*c^2 - 20*c*d + 9*d^2)*C
os[e + f*x]*Sin[e + f*x])/(6*a*f) + ((3*c - 4*d)*d*Cos[e + f*x]*(c + d*Sin[
e + f*x])^2)/(3*a*f) - ((c - d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(f*(a
+ a*Sin[e + f*x]))
```

Rule 2767

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e + f
*x])^(n - 1))/(a*f*(a + b*Sin[e + f*x])), x] - Dist[d/(a*b), Int[(c + d*Sin
[e + f*x])^(n - 2)*Simp[b*d*(n - 1) - a*c*n + (b*c*(n - 1) - a*d*n)*Sin[e +
 f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && E
qQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && (IntegerQ[2*n] || EqQ
[c, 0])
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^4}{a+a\sin(e+fx)}dx = -\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^3}{f(a+a\sin(e+fx))} - \frac{d\int(-a(4c-3d)+a(3c-4d)\sin(e+fx))(c+}{a^2}$$

$$= \frac{(3c-4d)d\cos(e+fx)(c+d\sin(e+fx))^2}{3af} - \frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^3}{f(a+a\sin(e+fx))} - \frac{d\int}{}$$

$$= \frac{d\left(8c^3-12c^2d+12cd^2-3d^3\right)x}{2a} + \frac{2d\left(3c^3-16c^2d+12cd^2-4d^3\right)\cos(e+fx)}{3af} + \frac{d^2\left(6c^2\right.}{}$$

Mathematica [A] time = 0.41104, size = 234, normalized size = 1.24

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(-6d\left(12c^2d-8c^3-12cd^2+3d^3\right)(e+fx)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])^4/(a + a*Sin[e + f*x]),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(24*(c - d)^4*Sin[(e + f*x)/2] - 6*d
*(-8*c^3 + 12*c^2*d - 12*c*d^2 + 3*d^3)*(e + f*x)*(Cos[(e + f*x)/2] + Sin[(
e + f*x)/2]) - 3*d^2*(24*c^2 - 16*c*d + 7*d^2)*Cos[e + f*x]*(Cos[(e + f*x)/
2] + Sin[(e + f*x)/2]) + d^4*Cos[3*(e + f*x)]*(Cos[(e + f*x)/2] + Sin[(e +
f*x)/2]) - 3*(4*c - d)*d^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sin[2*(e +
 f*x)]))/(12*a*f*(1 + Sin[e + f*x]))
```

Maple [B] time = 0.069, size = 673, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^4/(a+a*sin(f*x+e)),x)
```

```
[Out] 4/a/f/(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)^5*c*d^3-1/a/f/(1+tan(1/
2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)^5*d^4-12/a/f/(1+tan(1/2*f*x+1/2*e)^2)^
3*tan(1/2*f*x+1/2*e)^4*c^2*d^2+8/a/f/(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x
+1/2*e)^4*c*d^3-2/a/f/(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)^4*d^4-2
4/a/f/(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)^2*c^2*d^2+16/a/f/(1+tan
(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)^2*c*d^3-8/a/f/(1+tan(1/2*f*x+1/2*e)
^2)^3*tan(1/2*f*x+1/2*e)^2*d^4-4/a/f/(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x
+1/2*e)*c*d^3+1/a/f/(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)*d^4-12/a/
f/(1+tan(1/2*f*x+1/2*e)^2)^3*c^2*d^2+8/a/f/(1+tan(1/2*f*x+1/2*e)^2)^3*c*d^3
-10/3/a/f/(1+tan(1/2*f*x+1/2*e)^2)^3*d^4+8/a/f*d*arctan(tan(1/2*f*x+1/2*e))
*c^3-12/a/f*arctan(tan(1/2*f*x+1/2*e))*c^2*d^2+12/a/f*arctan(tan(1/2*f*x+1/
2*e))*c*d^3-3/a/f*arctan(tan(1/2*f*x+1/2*e))*d^4-2/a/f/(tan(1/2*f*x+1/2*e)+
1)*c^4+8/a/f/(tan(1/2*f*x+1/2*e)+1)*c^3*d-12/a/f/(tan(1/2*f*x+1/2*e)+1)*c^2
*d^2+8/a/f/(tan(1/2*f*x+1/2*e)+1)*c*d^3-2/a/f/(tan(1/2*f*x+1/2*e)+1)*d^4
```

Maxima [B] time = 1.81284, size = 979, normalized size = 5.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^4/(a+a*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] -1/3*(d^4*((7*sin(f*x + e)/(cos(f*x + e) + 1) + 39*sin(f*x + e)^2/(cos(f*x
+ e) + 1)^2 + 24*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 24*sin(f*x + e)^4/(c
os(f*x + e) + 1)^4 + 9*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 9*sin(f*x + e)
^6/(cos(f*x + e) + 1)^6 + 16)/(a + a*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a*
sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 3*a*sin(f*x + e)^3/(cos(f*x + e) + 1)
^3 + 3*a*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 3*a*sin(f*x + e)^5/(cos(f*x
+ e) + 1)^5 + a*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + a*sin(f*x + e)^7/(cos
(f*x + e) + 1)^7) + 9*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a) - 12*c*d^3
*((sin(f*x + e)/(cos(f*x + e) + 1) + 5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2
+ 3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^4/(cos(f*x + e) +
1)^4 + 4)/(a + a*sin(f*x + e)/(cos(f*x + e) + 1) + 2*a*sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 + 2*a*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + a*sin(f*x + e)^
4/(cos(f*x + e) + 1)^4 + a*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 3*arctan(
sin(f*x + e)/(cos(f*x + e) + 1))/a) + 36*c^2*d^2*((sin(f*x + e)/(cos(f*x +
e) + 1) + sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2)/(a + a*sin(f*x + e)/(cos
(f*x + e) + 1) + a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a*sin(f*x + e)^3/(
cos(f*x + e) + 1)^3) + arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a) - 24*c^3*
d*(arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a + 1/(a + a*sin(f*x + e)/(cos(f
*x + e) + 1))) + 6*c^4/(a + a*sin(f*x + e)/(cos(f*x + e) + 1)))/f
```

Fricas [A] time = 1.68836, size = 799, normalized size = 4.23

$$\frac{2\,d^4\cos\left(fx+e\right)^4-6\,c^4+24\,c^3d-36\,c^2d^2+24\,cd^3-6\,d^4+\left(12\,cd^3-d^4\right)\cos\left(fx+e\right)^3+3\left(8\,c^3d-12\,c^2d^2+12\,cd^3-3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^4/(a+a*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*d^4*cos(f*x + e)^4 - 6*c^4 + 24*c^3*d - 36*c^2*d^2 + 24*c*d^3 - 6*d^
4 + (12*c*d^3 - d^4)*cos(f*x + e)^3 + 3*(8*c^3*d - 12*c^2*d^2 + 12*c*d^3 -
3*d^4)*f*x - 12*(3*c^2*d^2 - 2*c*d^3 + d^4)*cos(f*x + e)^2 - 3*(2*c^4 - 8*c
^3*d + 24*c^2*d^2 - 12*c*d^3 + 5*d^4 - (8*c^3*d - 12*c^2*d^2 + 12*c*d^3 - 3
*d^4)*f*x)*cos(f*x + e) + (2*d^4*cos(f*x + e)^3 + 6*c^4 - 24*c^3*d + 36*c^2
*d^2 - 24*c*d^3 + 6*d^4 + 3*(8*c^3*d - 12*c^2*d^2 + 12*c*d^3 - 3*d^4)*f*x -
 3*(4*c*d^3 - d^4)*cos(f*x + e)^2 - 3*(12*c^2*d^2 - 4*c*d^3 + 3*d^4)*cos(f*
x + e))*sin(f*x + e))/(a*f*cos(f*x + e) + a*f*sin(f*x + e) + a*f)
```

Sympy [A] time = 25.654, size = 8344, normalized size = 44.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**4/(a+a*sin(f*x+e)),x)
```

```
[Out] Piecewise((12*c**4*tan(e/2 + f*x/2)**7/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*t
an(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**
4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2
```

```
 + f*x/2) + 6*a*f) + 36*c**4*tan(e/2 + f*x/2)**5/(6*a*f*tan(e/2 + f*x/2)**7
 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2
+ f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a
*f*tan(e/2 + f*x/2) + 6*a*f) + 36*c**4*tan(e/2 + f*x/2)**3/(6*a*f*tan(e/2 +
 f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*
f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2
)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 12*c**4*tan(e/2 + f*x/2)/(6*a*f*ta
n(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5
+ 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2
+ f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 24*c**3*d*f*x*tan(e/2 + f*x
/2)**7/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(
e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 +
 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 24*c**3*d*f
*x*tan(e/2 + f*x/2)**6/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)*
*6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e
/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*
f) + 72*c**3*d*f*x*tan(e/2 + f*x/2)**5/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*t
an(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**
4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2
 + f*x/2) + 6*a*f) + 72*c**3*d*f*x*tan(e/2 + f*x/2)**4/(6*a*f*tan(e/2 + f*x
/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*ta
n(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2
 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 72*c**3*d*f*x*tan(e/2 + f*x/2)**3/(6*a
*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2
)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan
(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 72*c**3*d*f*x*tan(e/2
+ f*x/2)**2/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f
*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)
**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 24*c**
3*d*f*x*tan(e/2 + f*x/2)/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2
)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan
(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*
a*f) + 24*c**3*d*f*x/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6
 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2
 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f)
 - 48*c**3*d*tan(e/2 + f*x/2)**7/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2
 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18
*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x
/2) + 6*a*f) - 144*c**3*d*tan(e/2 + f*x/2)**5/(6*a*f*tan(e/2 + f*x/2)**7 +
6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f
*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*
tan(e/2 + f*x/2) + 6*a*f) - 144*c**3*d*tan(e/2 + f*x/2)**3/(6*a*f*tan(e/2 +
 f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*
f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2
)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 48*c**3*d*tan(e/2 + f*x/2)/(6*a*f*
tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**
5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/
2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 36*c**2*d**2*f*x*tan(e/2
+ f*x/2)**7/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f
*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)
**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 36*c**
2*d**2*f*x*tan(e/2 + f*x/2)**6/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 +
 f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a
*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2
) + 6*a*f) - 108*c**2*d**2*f*x*tan(e/2 + f*x/2)**5/(6*a*f*tan(e/2 + f*x/2)*
*7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/
2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6
*a*f*tan(e/2 + f*x/2) + 6*a*f) - 108*c**2*d**2*f*x*tan(e/2 + f*x/2)**4/(6*a
*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2
```

```
)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan
(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 108*c**2*d**2*f*x*tan(
e/2 + f*x/2)**3/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18
*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*
x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 10
8*c**2*d**2*f*x*tan(e/2 + f*x/2)**2/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(
e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 +
 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 +
f*x/2) + 6*a*f) - 36*c**2*d**2*f*x*tan(e/2 + f*x/2)/(6*a*f*tan(e/2 + f*x/2)
**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e
/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 +
6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 36*c**2*d**2*f*x/(6*a*f*tan(e/2 + f*x/2)*
*7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/
2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6
*a*f*tan(e/2 + f*x/2) + 6*a*f) + 72*c**2*d**2*tan(e/2 + f*x/2)**7/(6*a*f*ta
n(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5
+ 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2
+ f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 144*c**2*d**2*tan(e/2 + f*x
/2)**5/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(
e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 +
 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 72*c**2*d**
2*tan(e/2 + f*x/2)**4/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**
6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/
2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f
) + 72*c**2*d**2*tan(e/2 + f*x/2)**3/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan
(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4
+ 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 +
 f*x/2) + 6*a*f) - 144*c**2*d**2*tan(e/2 + f*x/2)**2/(6*a*f*tan(e/2 + f*x/2
)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(
e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 +
 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 72*c**2*d**2/(6*a*f*tan(e/2 + f*x/2)**7
+ 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 +
 f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*
f*tan(e/2 + f*x/2) + 6*a*f) + 36*c*d**3*f*x*tan(e/2 + f*x/2)**7/(6*a*f*tan(
e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 +
18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 +
f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 36*c*d**3*f*x*tan(e/2 + f*x/2
)**6/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/
2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 1
8*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 108*c*d**3*f*
x*tan(e/2 + f*x/2)**5/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**
6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/
2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f
) + 108*c*d**3*f*x*tan(e/2 + f*x/2)**4/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*t
an(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**
4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2
 + f*x/2) + 6*a*f) + 108*c*d**3*f*x*tan(e/2 + f*x/2)**3/(6*a*f*tan(e/2 + f*
x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*t
an(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**
2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 108*c*d**3*f*x*tan(e/2 + f*x/2)**2/(6
*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x
/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*t
an(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 36*c*d**3*f*x*tan(e/
2 + f*x/2)/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*
tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)*
*3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 36*c*d*
*3*f*x/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(
e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 +
 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 72*c*d**3*t
```

```
an(e/2 + f*x/2)**7/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 +
 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 +
 f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) -
 144*c*d**3*tan(e/2 + f*x/2)**5/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2
+ f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*
a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/
2) + 6*a*f) - 24*c*d**3*tan(e/2 + f*x/2)**4/(6*a*f*tan(e/2 + f*x/2)**7 + 6*
a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x
/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*ta
n(e/2 + f*x/2) + 6*a*f) - 120*c*d**3*tan(e/2 + f*x/2)**3/(6*a*f*tan(e/2 + f
*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*
tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)*
*2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 48*c*d**3*tan(e/2 + f*x/2)/(6*a*f*ta
n(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5
+ 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2
+ f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 24*c*d**3/(6*a*f*tan(e/2 +
f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f
*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)
**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 9*d**4*f*x*tan(e/2 + f*x/2)**7/(6*a
*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2
)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan
(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 9*d**4*f*x*tan(e/2 + f
*x/2)**6/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*ta
n(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3
 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 27*d**4*f
*x*tan(e/2 + f*x/2)**5/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)*
*6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e
/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*
f) - 27*d**4*f*x*tan(e/2 + f*x/2)**4/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan
(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4
+ 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 +
 f*x/2) + 6*a*f) - 27*d**4*f*x*tan(e/2 + f*x/2)**3/(6*a*f*tan(e/2 + f*x/2)*
*7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/
2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6
*a*f*tan(e/2 + f*x/2) + 6*a*f) - 27*d**4*f*x*tan(e/2 + f*x/2)**2/(6*a*f*tan
(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 +
 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 +
 f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 9*d**4*f*x*tan(e/2 + f*x/2)/
(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f
*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f
*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 9*d**4*f*x/(6*a*f*
tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**
5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/
2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 18*d**4*tan(e/2 + f*x/2)*
*7/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2
+ f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*
a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 36*d**4*tan(e/2
 + f*x/2)**5/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*
f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2
)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 6*d**
4*tan(e/2 + f*x/2)**4/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**
6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/
2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f
) + 6*d**4*tan(e/2 + f*x/2)**3/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 +
 f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a
*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2
) + 6*a*f) - 24*d**4*tan(e/2 + f*x/2)**2/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f
*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)
**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e
```

```
/2 + f*x/2) + 6*a*f) + 4*d**4*tan(e/2 + f*x/2)/(6*a*f*tan(e/2 + f*x/2)**7 +
 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 +
f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f
*tan(e/2 + f*x/2) + 6*a*f) - 14*d**4/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan
(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4
+ 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 +
 f*x/2) + 6*a*f), Ne(f, 0)), (x*(c + d*sin(e))**4/(a*sin(e) + a), True))
```

Giac [A] time = 1.36612, size = 413, normalized size = 2.19

$$\frac{\frac{3\left(8c^3d-12c^2d^2+12cd^3-3d^4\right)(fx+e)}{a}-\frac{12\left(c^4-4c^3d+6c^2d^2-4cd^3+d^4\right)}{a\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}+\frac{2\left(12cd^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5-3d^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5-36c^2d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+24cd^3\tan\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^4/(a+a*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/6*(3*(8*c^3*d - 12*c^2*d^2 + 12*c*d^3 - 3*d^4)*(f*x + e)/a - 12*(c^4 - 4*
c^3*d + 6*c^2*d^2 - 4*c*d^3 + d^4)/(a*(tan(1/2*f*x + 1/2*e) + 1)) + 2*(12*c
*d^3*tan(1/2*f*x + 1/2*e)^5 - 3*d^4*tan(1/2*f*x + 1/2*e)^5 - 36*c^2*d^2*tan
(1/2*f*x + 1/2*e)^4 + 24*c*d^3*tan(1/2*f*x + 1/2*e)^4 - 6*d^4*tan(1/2*f*x +
 1/2*e)^4 - 72*c^2*d^2*tan(1/2*f*x + 1/2*e)^2 + 48*c*d^3*tan(1/2*f*x + 1/2*
e)^2 - 24*d^4*tan(1/2*f*x + 1/2*e)^2 - 12*c*d^3*tan(1/2*f*x + 1/2*e) + 3*d^
4*tan(1/2*f*x + 1/2*e) - 36*c^2*d^2 + 24*c*d^3 - 10*d^4)/((tan(1/2*f*x + 1/
2*e)^2 + 1)^3*a))/f
```

3.454 $\int \frac{(c+d\sin(e+fx))^3}{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=121

$$\frac{2d\left(c^2-3cd+d^2\right)\cos(e+fx)}{af}+\frac{3dx\left(2c^2-2cd+d^2\right)}{2a}+\frac{d^2(2c-3d)\sin(e+fx)\cos(e+fx)}{2af}-\frac{(c-d)\cos(e+fx)(c+}{f(a\sin(e+f}$$

```
[Out] (3*d*(2*c^2 - 2*c*d + d^2)*x)/(2*a) + (2*d*(c^2 - 3*c*d + d^2)*Cos[e + f*x]
)/(a*f) + ((2*c - 3*d)*d^2*Cos[e + f*x]*Sin[e + f*x])/(2*a*f) - ((c - d)*Co
s[e + f*x]*(c + d*Sin[e + f*x])^2)/(f*(a + a*Sin[e + f*x]))
```

Rubi [A] time = 0.125398, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {2767, 2734}

$$\frac{2d\left(c^2-3cd+d^2\right)\cos(e+fx)}{af}+\frac{3dx\left(2c^2-2cd+d^2\right)}{2a}+\frac{d^2(2c-3d)\sin(e+fx)\cos(e+fx)}{2af}-\frac{(c-d)\cos(e+fx)(c+}{f(a\sin(e+f}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^3/(a + a*Sin[e + f*x]),x]
```

```
[Out] (3*d*(2*c^2 - 2*c*d + d^2)*x)/(2*a) + (2*d*(c^2 - 3*c*d + d^2)*Cos[e + f*x]
)/(a*f) + ((2*c - 3*d)*d^2*Cos[e + f*x]*Sin[e + f*x])/(2*a*f) - ((c - d)*Co
s[e + f*x]*(c + d*Sin[e + f*x])^2)/(f*(a + a*Sin[e + f*x]))
```

Rule 2767

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e + f
*x])^(n - 1))/(a*f*(a + b*Sin[e + f*x])), x] - Dist[d/(a*b), Int[(c + d*Sin
[e + f*x])^(n - 2)*Simp[b*d*(n - 1) - a*c*n + (b*c*(n - 1) - a*d*n)*Sin[e +
 f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && E
qQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && (IntegerQ[2*n] || EqQ
[c, 0])
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^3}{a+a\sin(e+fx)}\,dx=-\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^2}{f(a+a\sin(e+fx))}-\frac{d\int(-a(3c-2d)+a(2c-3d)\sin(e+fx))(c+}{a^2}$$
$$=\frac{3d\left(2c^2-2cd+d^2\right)x}{2a}+\frac{2d\left(c^2-3cd+d^2\right)\cos(e+fx)}{af}+\frac{(2c-3d)d^2\cos(e+fx)\sin(e+}{2af}$$

Mathematica [A] time = 0.561892, size = 192, normalized size = 1.59

$$\left(\sin\left(\tfrac{1}{2}(e+fx)\right)+\cos\left(\tfrac{1}{2}(e+fx)\right)\right)\left(d\cos\left(\tfrac{1}{2}(e+fx)\right)\left(6\left(2c^2-2cd+d^2\right)(e+fx)-4d(3c-d)\cos(e+fx)+d^2(-\sin\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])^3/(a + a*Sin[e + f*x]),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(d*Cos[(e + f*x)/2]*(6*(2*c^2 - 2*c*
d + d^2)*(e + f*x) - 4*(3*c - d)*d*Cos[e + f*x] - d^2*Sin[2*(e + f*x)]) + S
in[(e + f*x)/2]*(2*(4*c^3 + 6*c^2*d*(-2 + e + f*x) - 6*c*d^2*(-2 + e + f*x)
 + d^3*(-4 + 3*e + 3*f*x)) - 4*(3*c - d)*d^2*Cos[e + f*x] - d^3*Sin[2*(e +
f*x)])))/(4*a*f*(1 + Sin[e + f*x]))
```

Maple [B] time = 0.068, size = 364, normalized size = 3.

$$\frac{d^3}{af}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3\left(1+\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2\right)^{-2}-6\,\frac{c\left(\tan\left(1/2\,fx+e/2\right)\right)^2d^2}{af\left(1+\left(\tan\left(1/2\,fx+e/2\right)\right)^2\right)^2}+2\,\frac{\left(\tan\left(1/2\,fx+e/2\right)\right)^2d^3}{af\left(1+\left(\tan\left(1/2\,fx+e/2\right)\right)^2\right)^2}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^3/(a+a*sin(f*x+e)),x)
```

```
[Out] 1/a/f/(1+tan(1/2*f*x+1/2*e)^2)^2*tan(1/2*f*x+1/2*e)^3*d^3-6/a/f/(1+tan(1/2*
f*x+1/2*e)^2)^2*tan(1/2*f*x+1/2*e)^2*c*d^2+2/a/f/(1+tan(1/2*f*x+1/2*e)^2)^2
*tan(1/2*f*x+1/2*e)^2*d^3-1/a/f/(1+tan(1/2*f*x+1/2*e)^2)^2*tan(1/2*f*x+1/2*
e)*d^3-6/a/f/(1+tan(1/2*f*x+1/2*e)^2)^2*c*d^2+2/a/f/(1+tan(1/2*f*x+1/2*e)^2
)^2*d^3+6/a/f*d*arctan(tan(1/2*f*x+1/2*e))*c^2-6/a/f*arctan(tan(1/2*f*x+1/2
*e))*c*d^2+3/a/f*arctan(tan(1/2*f*x+1/2*e))*d^3-2/f*c^3/a/(tan(1/2*f*x+1/2*
e)+1)+6/a/f/(tan(1/2*f*x+1/2*e)+1)*c^2*d-6/a/f/(tan(1/2*f*x+1/2*e)+1)*c*d^2
+2/a/f/(tan(1/2*f*x+1/2*e)+1)*d^3
```

Maxima [B] time = 1.74515, size = 574, normalized size = 4.74

$$\frac{d^3\left(\frac{\frac{\sin(fx+e)}{\cos(fx+e)+1}+\frac{5\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+4}{a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}+\frac{2a\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{2a\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{a\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{a\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}+\frac{3\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a}\right)-6\,cd^2\left(\frac{\frac{\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^2}{(\cos(fx+e)+}}{a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}+\frac{a\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+}\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^3/(a+a*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] (d^3*((sin(f*x + e)/(cos(f*x + e) + 1) + 5*sin(f*x + e)^2/(cos(f*x + e) + 1
)^2 + 3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^4/(cos(f*x + e
) + 1)^4 + 4)/(a + a*sin(f*x + e)/(cos(f*x + e) + 1) + 2*a*sin(f*x + e)^2/(
cos(f*x + e) + 1)^2 + 2*a*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + a*sin(f*x +
 e)^4/(cos(f*x + e) + 1)^4 + a*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 3*arc
tan(sin(f*x + e)/(cos(f*x + e) + 1))/a) - 6*c*d^2*((sin(f*x + e)/(cos(f*x +
 e) + 1) + sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2)/(a + a*sin(f*x + e)/(co
s(f*x + e) + 1) + a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a*sin(f*x + e)^3/
(cos(f*x + e) + 1)^3) + arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a) + 6*c^2*
d*(arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a + 1/(a + a*sin(f*x + e)/(cos(f
*x + e) + 1))) - 2*c^3/(a + a*sin(f*x + e)/(cos(f*x + e) + 1)))/f
```

Fricas [B] time = 1.6493, size = 537, normalized size = 4.44

$$\frac{d^3 \cos\left(fx+e\right)^3 - 2c^3 + 6c^2d - 6cd^2 + 2d^3 + 3\left(2c^2d - 2cd^2 + d^3\right)fx - 2\left(3cd^2 - d^3\right)\cos\left(fx+e\right)^2 - \left(2c^3 - 6c^2d + \cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^3/(a+a*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/2*(d^3*cos(f*x + e)^3 - 2*c^3 + 6*c^2*d - 6*c*d^2 + 2*d^3 + 3*(2*c^2*d -
2*c*d^2 + d^3)*f*x - 2*(3*c*d^2 - d^3)*cos(f*x + e)^2 - (2*c^3 - 6*c^2*d +
12*c*d^2 - 3*d^3 - 3*(2*c^2*d - 2*c*d^2 + d^3)*f*x)*cos(f*x + e) - (d^3*cos
(f*x + e)^2 - 2*c^3 + 6*c^2*d - 6*c*d^2 + 2*d^3 - 3*(2*c^2*d - 2*c*d^2 + d^
3)*f*x + (6*c*d^2 - d^3)*cos(f*x + e))*sin(f*x + e))/(a*f*cos(f*x + e) + a*
f*sin(f*x + e) + a*f)
```

Sympy [A] time = 9.64225, size = 3499, normalized size = 28.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**3/(a+a*sin(f*x+e)),x)
```

```
[Out] Piecewise((-4*c**3*tan(e/2 + f*x/2)**4/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*t
an(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2
+ 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 8*c**3*tan(e/2 + f*x/2)**2/(2*a*f*tan(e
/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*
a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 4*c**3/(2*a*f*t
an(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3
+ 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 6*c**2*d*f*
x*tan(e/2 + f*x/2)**5/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**
4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 +
 f*x/2) + 2*a*f) + 6*c**2*d*f*x*tan(e/2 + f*x/2)**4/(2*a*f*tan(e/2 + f*x/2)
**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2
 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 12*c**2*d*f*x*tan(e/2 + f*
x/2)**3/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(
e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*
f) + 12*c**2*d*f*x*tan(e/2 + f*x/2)**2/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*t
an(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2
+ 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 6*c**2*d*f*x*tan(e/2 + f*x/2)/(2*a*f*ta
n(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 +
 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 6*c**2*d*f*x
/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f
*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 12
*c**2*d*tan(e/2 + f*x/2)**4/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*
x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan
(e/2 + f*x/2) + 2*a*f) + 24*c**2*d*tan(e/2 + f*x/2)**2/(2*a*f*tan(e/2 + f*x
/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(
e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 12*c**2*d/(2*a*f*tan(e/
2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a
*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 6*c*d**2*f*x*tan
(e/2 + f*x/2)**5/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4
*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/
```

```
2) + 2*a*f) - 6*c*d**2*f*x*tan(e/2 + f*x/2)**4/(2*a*f*tan(e/2 + f*x/2)**5 +
 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*
x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 12*c*d**2*f*x*tan(e/2 + f*x/2)*
*3/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 +
 f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) -
12*c*d**2*f*x*tan(e/2 + f*x/2)**2/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/
2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a
*f*tan(e/2 + f*x/2) + 2*a*f) - 6*c*d**2*f*x*tan(e/2 + f*x/2)/(2*a*f*tan(e/2
 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*
f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 6*c*d**2*f*x/(2*a
*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)
**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 12*c*d*
*2*tan(e/2 + f*x/2)**5/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)*
*4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2
+ f*x/2) + 2*a*f) + 12*c*d**2*tan(e/2 + f*x/2)**3/(2*a*f*tan(e/2 + f*x/2)**
5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 +
 f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 12*c*d**2*tan(e/2 + f*x/2)**
2/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 +
f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 1
2*c*d**2/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan
(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a
*f) + 3*d**3*f*x*tan(e/2 + f*x/2)**5/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan
(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 +
2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 3*d**3*f*x*tan(e/2 + f*x/2)**4/(2*a*f*tan
(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 +
4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 6*d**3*f*x*ta
n(e/2 + f*x/2)**3/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 +
4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x
/2) + 2*a*f) + 6*d**3*f*x*tan(e/2 + f*x/2)**2/(2*a*f*tan(e/2 + f*x/2)**5 +
2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x
/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 3*d**3*f*x*tan(e/2 + f*x/2)/(2*a
*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)
**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 3*d**3*
f*x/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2
+ f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) -
 2*d**3*tan(e/2 + f*x/2)**5/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*
x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan
(e/2 + f*x/2) + 2*a*f) + 4*d**3*tan(e/2 + f*x/2)**4/(2*a*f*tan(e/2 + f*x/2)
**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2
 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 2*d**3*tan(e/2 + f*x/2)**3
/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f
*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 6*
d**3*tan(e/2 + f*x/2)**2/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2
)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/
2 + f*x/2) + 2*a*f) + 6*d**3/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f
*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*ta
n(e/2 + f*x/2) + 2*a*f), Ne(f, 0)), (x*(c + d*sin(e))**3/(a*sin(e) + a), Tr
ue))
```

Giac [A] time = 1.36096, size = 232, normalized size = 1.92

$$\frac{\frac{3\left(2c^2d-2cd^2+d^3\right)(fx+e)}{a}-\frac{4\left(c^3-3c^2d+3cd^2-d^3\right)}{a\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}+\frac{2\left(d^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-6cd^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2d^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-d^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-6cd^2+2d^3\right)}{\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+1\right)^2a}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^3/(a+a*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/2*(3*(2*c^2*d - 2*c*d^2 + d^3)*(f*x + e)/a - 4*(c^3 - 3*c^2*d + 3*c*d^2 -
 d^3)/(a*(tan(1/2*f*x + 1/2*e) + 1)) + 2*(d^3*tan(1/2*f*x + 1/2*e)^3 - 6*c*
d^2*tan(1/2*f*x + 1/2*e)^2 + 2*d^3*tan(1/2*f*x + 1/2*e)^2 - d^3*tan(1/2*f*x
 + 1/2*e) - 6*c*d^2 + 2*d^3)/((tan(1/2*f*x + 1/2*e)^2 + 1)^2*a))/f
```

3.455 $\int \frac{(c+d\sin(e+fx))^2}{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=62

$$-\frac{(c-d)^2\cos(e+fx)}{af(\sin(e+fx)+1)}+\frac{dx(2c-d)}{a}-\frac{d^2\cos(e+fx)}{af}$$

```
[Out] ((2*c - d)*d*x)/a - (d^2*Cos[e + f*x])/(a*f) - ((c - d)^2*Cos[e + f*x])/(a*
f*(1 + Sin[e + f*x]))
```

Rubi [A] time = 0.136563, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2746, 2735, 2648}

$$-\frac{(c-d)^2\cos(e+fx)}{af(\sin(e+fx)+1)}+\frac{dx(2c-d)}{a}-\frac{d^2\cos(e+fx)}{af}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^2/(a + a*Sin[e + f*x]),x]
```

```
[Out] ((2*c - d)*d*x)/a - (d^2*Cos[e + f*x])/(a*f) - ((c - d)^2*Cos[e + f*x])/(a*
f*(1 + Sin[e + f*x]))
```

Rule 2746

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2/((c_.) + (d_.)*sin[(e_.) + (f
_.)*(x_)]), x_Symbol] :> -Simp[(b^2*Cos[e + f*x])/(d*f), x] + Dist[1/d, Int
[Simp[a^2*d - b*(b*c - 2*a*d)*Sin[e + f*x], x]/(c + d*Sin[e + f*x]), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(c+d\sin(e+fx))^2}{a+a\sin(e+fx)}\,dx &= -\frac{d^2\cos(e+fx)}{af}+\frac{\int \frac{ac^2+a(2c-d)d\sin(e+fx)}{a+a\sin(e+fx)}\,dx}{a}\\ &= \frac{(2c-d)dx}{a}-\frac{d^2\cos(e+fx)}{af}+(c-d)^2\int \frac{1}{a+a\sin(e+fx)}\,dx\\ &= \frac{(2c-d)dx}{a}-\frac{d^2\cos(e+fx)}{af}-\frac{(c-d)^2\cos(e+fx)}{f(a+a\sin(e+fx))}\end{aligned}$$

Mathematica [A] time = 0.448811, size = 122, normalized size = 1.97

$$-\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)\left(-2c^2-2cd(e+fx-2)+d^2(e+fx-2)+d^2\cos(e+fx)\right)+d\right.}{af(\sin(e+fx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])^2/(a + a*Sin[e + f*x]),x]
```

```
[Out] -(((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(d*Cos[(e + f*x)/2]*(-((2*c - d)*(
e + f*x)) + d*Cos[e + f*x]) + (-2*c^2 - 2*c*d*(-2 + e + f*x) + d^2*(-2 + e
+ f*x) + d^2*Cos[e + f*x])*Sin[(e + f*x)/2]))/(a*f*(1 + Sin[e + f*x])))
```

Maple [B] time = 0.059, size = 140, normalized size = 2.3

$$-2\frac{d^2}{af\left(1+\left(\tan\left(1/2\,fx+e/2\right)\right)^2\right)}+4\frac{d\arctan\left(\tan\left(1/2\,fx+e/2\right)\right)c}{af}-2\frac{\arctan\left(\tan\left(1/2\,fx+e/2\right)\right)d^2}{af}-2\frac{}{af\left(\tan\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^2/(a+a*sin(f*x+e)),x)
```

```
[Out] -2/a/f*d^2/(1+tan(1/2*f*x+1/2*e)^2)+4/a/f*d*arctan(tan(1/2*f*x+1/2*e))*c-2/
a/f*arctan(tan(1/2*f*x+1/2*e))*d^2-2/a/f/(tan(1/2*f*x+1/2*e)+1)*c^2+4/a/f/(
tan(1/2*f*x+1/2*e)+1)*c*d-2/a/f/(tan(1/2*f*x+1/2*e)+1)*d^2
```

Maxima [B] time = 1.71011, size = 282, normalized size = 4.55

$$-\frac{2\left(d^2\left(\frac{\frac{\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+2}{a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}+\frac{a\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}+\frac{\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a}\right)-2\,cd\left(\frac{\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a}+\frac{1}{a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}}\right)+\frac{c^2}{a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^2/(a+a*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] -2*(d^2*((sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^2/(cos(f*x + e) +
1)^2 + 2)/(a + a*sin(f*x + e)/(cos(f*x + e) + 1) + a*sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 + a*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + arctan(sin(f*x + e
)/(cos(f*x + e) + 1))/a) - 2*c*d*(arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a
 + 1/(a + a*sin(f*x + e)/(cos(f*x + e) + 1))) + c^2/(a + a*sin(f*x + e)/(co
s(f*x + e) + 1)))/f
```

Fricas [B] time = 1.58977, size = 321, normalized size = 5.18

$$-\frac{d^2\cos\left(fx+e\right)^2-\left(2\,cd-d^2\right)fx+c^2-2\,cd+d^2-\left(\left(2\,cd-d^2\right)fx-c^2+2\,cd-2\,d^2\right)\cos\left(fx+e\right)-\left(\left(2\,cd-d^2\right)fx-\right.}{af\cos\left(fx+e\right)+af\sin\left(fx+e\right)+af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^2/(a+a*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] -(d^2*cos(f*x + e)^2 - (2*c*d - d^2)*f*x + c^2 - 2*c*d + d^2 - ((2*c*d - d^
2)*f*x - c^2 + 2*c*d - 2*d^2)*cos(f*x + e) - ((2*c*d - d^2)*f*x - d^2*cos(f
*x + e) + c^2 - 2*c*d + d^2)*sin(f*x + e))/(a*f*cos(f*x + e) + a*f*sin(f*x
+ e) + a*f)
```

Sympy [A] time = 4.12448, size = 877, normalized size = 14.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**2/(a+a*sin(f*x+e)),x)
```

```
[Out] Piecewise((-2*c**2*tan(e/2 + f*x/2)**2/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e
/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) - 2*c**2/(a*f*tan(e/2 + f*x/2)
**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) + 2*c*d*f*x*tan
(e/2 + f*x/2)**3/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*t
an(e/2 + f*x/2) + a*f) + 2*c*d*f*x*tan(e/2 + f*x/2)**2/(a*f*tan(e/2 + f*x/2
)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) + 2*c*d*f*x*ta
n(e/2 + f*x/2)/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan
(e/2 + f*x/2) + a*f) + 2*c*d*f*x/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f
*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) + 4*c*d*tan(e/2 + f*x/2)**2/(a*f*tan
(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) +
4*c*d/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*
x/2) + a*f) - d**2*f*x*tan(e/2 + f*x/2)**3/(a*f*tan(e/2 + f*x/2)**3 + a*f*t
an(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) - d**2*f*x*tan(e/2 + f*x/2
)**2/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x
/2) + a*f) - d**2*f*x*tan(e/2 + f*x/2)/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e
/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) - d**2*f*x/(a*f*tan(e/2 + f*x/
2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) + 2*d**2*tan(
e/2 + f*x/2)**3/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*ta
n(e/2 + f*x/2) + a*f) - 2*d**2/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x
/2)**2 + a*f*tan(e/2 + f*x/2) + a*f), Ne(f, 0)), (x*(c + d*sin(e))**2/(a*si
n(e) + a), True))
```

Giac [B] time = 1.19535, size = 193, normalized size = 3.11

$$\frac{\frac{(2\,cd-d^2)(fx+e)}{a}-\frac{2\left(c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-2\,cd\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+d^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+d^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+c^2-2\,cd+2\,d^2\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)a}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^2/(a+a*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] ((2*c*d - d^2)*(f*x + e)/a - 2*(c^2*tan(1/2*f*x + 1/2*e)^2 - 2*c*d*tan(1/2*
f*x + 1/2*e)^2 + d^2*tan(1/2*f*x + 1/2*e)^2 + d^2*tan(1/2*f*x + 1/2*e) + c^
2 - 2*c*d + 2*d^2)/((tan(1/2*f*x + 1/2*e)^3 + tan(1/2*f*x + 1/2*e)^2 + tan(
1/2*f*x + 1/2*e) + 1)*a))/f
```

3.456 $\int \frac{c+d\sin(e+fx)}{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=35

$$\frac{dx}{a}-\frac{(c-d)\cos(e+fx)}{f(a\sin(e+fx)+a)}$$

```
[Out] (d*x)/a - ((c - d)*Cos[e + f*x])/(f*(a + a*Sin[e + f*x]))
```

Rubi [A] time = 0.047232, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2735, 2648}

$$\frac{dx}{a}-\frac{(c-d)\cos(e+fx)}{f(a\sin(e+fx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])/(a + a*Sin[e + f*x]),x]
```

```
[Out] (d*x)/a - ((c - d)*Cos[e + f*x])/(f*(a + a*Sin[e + f*x]))
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{c+d\sin(e+fx)}{a+a\sin(e+fx)}\,dx&=\frac{dx}{a}-(-c+d)\int \frac{1}{a+a\sin(e+fx)}\,dx\\&=\frac{dx}{a}-\frac{(c-d)\cos(e+fx)}{f(a+a\sin(e+fx))}\end{aligned}$$

Mathematica [B] time = 0.157543, size = 79, normalized size = 2.26

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)(2c+d(e+fx-2))+d(e+fx)\cos\left(\frac{1}{2}(e+fx)\right)\right)}{af(\sin(e+fx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])/(a + a*Sin[e + f*x]),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(d*(e + f*x)*Cos[(e + f*x)/2] + (2*c
 + d*(-2 + e + f*x))*Sin[(e + f*x)/2]))/(a*f*(1 + Sin[e + f*x]))
```

Maple [A] time = 0.036, size = 65, normalized size = 1.9

$$2\,\frac{d\arctan\left(\tan\left(1/2\,fx+e/2\right)\right)}{af}-2\,\frac{c}{af\left(\tan\left(1/2\,fx+e/2\right)+1\right)}+2\,\frac{d}{af\left(\tan\left(1/2\,fx+e/2\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))/(a+a*sin(f*x+e)),x)
```

```
[Out] 2/a/f*d*arctan(tan(1/2*f*x+1/2*e))-2/a/f/(tan(1/2*f*x+1/2*e)+1)*c+2/a/f/(ta
n(1/2*f*x+1/2*e)+1)*d
```

Maxima [B] time = 1.80311, size = 105, normalized size = 3.

$$\frac{2\left(d\left(\frac{\arctan\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}\right)}{a}+\frac{1}{a+\frac{a\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}}\right)-\frac{c}{a+\frac{a\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))/(a+a*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] 2*(d*(arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a + 1/(a + a*sin(f*x + e)/(co
s(f*x + e) + 1))) - c/(a + a*sin(f*x + e)/(cos(f*x + e) + 1)))/f
```

Fricas [A] time = 1.53784, size = 166, normalized size = 4.74

$$\frac{dfx+\left(dfx-c+d\right)\cos\left(fx+e\right)+\left(dfx+c-d\right)\sin\left(fx+e\right)-c+d}{af\cos\left(fx+e\right)+af\sin\left(fx+e\right)+af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))/(a+a*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] (d*f*x + (d*f*x - c + d)*cos(f*x + e) + (d*f*x + c - d)*sin(f*x + e) - c +
d)/(a*f*cos(f*x + e) + a*f*sin(f*x + e) + a*f)
```

Sympy [A] time = 1.81715, size = 109, normalized size = 3.11

$$\begin{cases}-\frac{2c}{af\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+af}+\frac{dfx\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{af\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+af}+\frac{dfx}{af\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+af}+\frac{2d}{af\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+af} & \text{for}\ f\neq 0\\ \frac{x\left(c+d\sin\left(e\right)\right)}{a\sin\left(e\right)+a} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))/(a+a*sin(f*x+e)),x)
```

```
[Out] Piecewise((-2*c/(a*f*tan(e/2 + f*x/2) + a*f) + d*f*x*tan(e/2 + f*x/2)/(a*f*
tan(e/2 + f*x/2) + a*f) + d*f*x/(a*f*tan(e/2 + f*x/2) + a*f) + 2*d/(a*f*tan
(e/2 + f*x/2) + a*f), Ne(f, 0)), (x*(c + d*sin(e))/(a*sin(e) + a), True))
```

Giac [A] time = 1.39979, size = 54, normalized size = 1.54

$$\frac{\frac{(fx+e)d}{a} - \frac{2(c-d)}{a\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))/(a+a*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] ((f*x + e)*d/a - 2*(c - d)/(a*(tan(1/2*f*x + 1/2*e) + 1)))/f
```

3.457 $\int \frac{1}{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=23

$$-\frac{\cos(e+fx)}{f(a\sin(e+fx)+a)}$$

```
[Out] -(Cos[e + f*x]/(f*(a + a*Sin[e + f*x])))
```

Rubi [A] time = 0.0124415, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2648}

$$-\frac{\cos(e+fx)}{f(a\sin(e+fx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(-1),x]
```

```
[Out] -(Cos[e + f*x]/(f*(a + a*Sin[e + f*x])))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{1}{a+a\sin(e+fx)}\,dx = -\frac{\cos(e+fx)}{f(a+a\sin(e+fx))}$$

Mathematica [B] time = 0.0423332, size = 48, normalized size = 2.09

$$\frac{2\sin\left(\frac{1}{2}(e+fx)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}{f(a\sin(e+fx)+a)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(-1),x]
```

```
[Out] (2*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))/(f*(a + a*Sin[e
+ f*x]))
```

Maple [A] time = 0.023, size = 22, normalized size = 1.

$$-2\,\frac{1}{af\left(\tan\left(1/2\,fx+e/2\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e)),x)
```

```
[Out] -2/f/a/(tan(1/2*f*x+1/2*e)+1)
```

Maxima [A] time = 1.06959, size = 36, normalized size = 1.57

$$-\frac{2}{\left(a+\frac{a\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] -2/((a + a*sin(f*x + e)/(cos(f*x + e) + 1))*f)
```

Fricas [A] time = 1.49856, size = 108, normalized size = 4.7

$$-\frac{\cos\left(fx+e\right)-\sin\left(fx+e\right)+1}{af\cos\left(fx+e\right)+af\sin\left(fx+e\right)+af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] -(cos(f*x + e) - sin(f*x + e) + 1)/(a*f*cos(f*x + e) + a*f*sin(f*x + e) + a
*f)
```

Sympy [A] time = 0.867783, size = 27, normalized size = 1.17

$$\begin{cases} -\frac{2}{af\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+af} & \text{for } f\neq 0 \\ \frac{x}{a\sin\left(e\right)+a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e)),x)
```

```
[Out] Piecewise((-2/(a*f*tan(e/2 + f*x/2) + a*f), Ne(f, 0)), (x/(a*sin(e) + a), T
rue))
```

Giac [A] time = 1.41652, size = 30, normalized size = 1.3

$$-\frac{2}{af\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] -2/(a*f*(tan(1/2*f*x + 1/2*e) + 1))
```

3.458 $\int \frac{1}{(a+a\sin(e+fx))(c+d\sin(e+fx))}\,dx$

Optimal. Leaf size=89

$$-\frac{2d\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{af(c-d)\sqrt{c^2-d^2}}-\frac{\cos(e+fx)}{f(c-d)(a\sin(e+fx)+a)}$$

```
[Out] (-2*d*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(a*(c - d)*Sqrt[c^2
 - d^2]*f) - Cos[e + f*x]/((c - d)*f*(a + a*Sin[e + f*x]))
```

Rubi [A] time = 0.12999, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2747, 2648, 2660, 618, 204}

$$-\frac{2d\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{af(c-d)\sqrt{c^2-d^2}}-\frac{\cos(e+fx)}{f(c-d)(a\sin(e+fx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])),x]
```

```
[Out] (-2*d*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(a*(c - d)*Sqrt[c^2
 - d^2]*f) - Cos[e + f*x]/((c - d)*f*(a + a*Sin[e + f*x]))
```

Rule 2747

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]),
x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))(c+d\sin(e+fx))}\,dx = \frac{\int \frac{1}{a+a\sin(e+fx)}\,dx}{c-d} - \frac{d\int \frac{1}{c+d\sin(e+fx)}\,dx}{a(c-d)}$$
$$= -\frac{\cos(e+fx)}{(c-d)f(a+a\sin(e+fx))} - \frac{(2d)\,\text{Subst}\left(\int \frac{1}{c+2dx+cx^2}\,dx, x, \tan\left(\frac{1}{2}(e+fx)\right)\right)}{a(c-d)f}$$
$$= -\frac{\cos(e+fx)}{(c-d)f(a+a\sin(e+fx))} + \frac{(4d)\,\text{Subst}\left(\int \frac{1}{-4(c^2-d^2)-x^2}\,dx, x, 2d+2c\tan\left(\frac{1}{2}(e+fx)\right)\right)}{a(c-d)f}$$
$$= -\frac{2d\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{a(c-d)\sqrt{c^2-d^2}f} - \frac{\cos(e+fx)}{(c-d)f(a+a\sin(e+fx))}$$

Mathematica [A] time = 0.471396, size = 115, normalized size = 1.29

$$\frac{\cos(e+fx)\left(\frac{1}{(d-c)(\sin(e+fx)+1)} + \frac{2d\tan^{-1}\left(\frac{\sqrt{d-c}\sqrt{1-\sin(e+fx)}}{\sqrt{-c-d}\sqrt{\sin(e+fx)+1}}\right)}{\sqrt{-c-d}(d-c)^{3/2}\sqrt{\cos^2(e+fx)}}\right)}{af}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])),x]
```

```
[Out] (Cos[e + f*x]*((2*d*ArcTan[(Sqrt[-c + d]*Sqrt[1 - Sin[e + f*x]])/(Sqrt[-c -
 d]*Sqrt[1 + Sin[e + f*x]])])/(Sqrt[-c - d]*(-c + d)^(3/2)*Sqrt[Cos[e + f*x
]^2]) + 1/((-c + d)*(1 + Sin[e + f*x]))))/(a*f)
```

Maple [A] time = 0.073, size = 87, normalized size = 1.

$$-2\,\frac{d}{af\,(c-d)\,\sqrt{c^2-d^2}}\arctan\left(1/2\,\frac{2\,c\tan\left(1/2\,fx+e/2\right)+2\,d}{\sqrt{c^2-d^2}}\right)-2\,\frac{1}{af\,(c-d)\left(\tan\left(1/2\,fx+e/2\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

```
[Out] -2/a/f*d/(c-d)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2
-d^2)^(1/2))-2/a/f/(c-d)/(tan(1/2*f*x+1/2*e)+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [B] time = 1.70122, size = 1089, normalized size = 12.24

$$\left[\frac{\sqrt{-c^2+d^2}\left(d\cos\left(fx+e\right)+d\sin\left(fx+e\right)+d\right)\log\left(\frac{\left(2c^2-d^2\right)\cos\left(fx+e\right)^2-2cd\sin\left(fx+e\right)-c^2-d^2+2\left(c\cos\left(fx+e\right)\sin\left(fx+e\right)+d\cos\left(fx+e\right.\right.}{d^2\cos\left(fx+e\right)^2-2cd\sin\left(fx+e\right)-c^2-d^2}\right.}{2\left(\left(ac^3-ac^2d-acd^2+ad^3\right)f\cos\left(fx+e\right)+\left(ac^3-ac^2d-acd^2+ad^3\right)f\sin\left(f\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm="fricas")

[Out] [1/2*(sqrt(-c^2 + d^2)*(d*cos(f*x + e) + d*sin(f*x + e) + d)*log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x + e) + d*cos(f*x + e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) - 2*c^2 + 2*d^2 - 2*(c^2 - d^2)*cos(f*x + e) + 2*(c^2 - d^2)*sin(f*x + e))/((a*c^3 - a*c^2*d - a*c*d^2 + a*d^3)*f*cos(f*x + e) + (a*c^3 - a*c^2*d - a*c*d^2 + a*d^3)*f*sin(f*x + e) + (a*c^3 - a*c^2*d - a*c*d^2 + a*d^3)*f), (sqrt(c^2 - d^2)*(d*cos(f*x + e) + d*sin(f*x + e) + d)*arctan(-(c*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) - c^2 + d^2 - (c^2 - d^2)*cos(f*x + e) + (c^2 - d^2)*sin(f*x + e))/((a*c^3 - a*c^2*d - a*c*d^2 + a*d^3)*f*cos(f*x + e) + (a*c^3 - a*c^2*d - a*c*d^2 + a*d^3)*f*sin(f*x + e) + (a*c^3 - a*c^2*d - a*c*d^2 + a*d^3)*f)]

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sin(f*x+e))/(c+d*sin(f*x+e)),x)

[Out] Timed out

Giac [A] time = 1.54384, size = 135, normalized size = 1.52

$$-\frac{2\left(\frac{\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+d}{\sqrt{c^2-d^2}}\right)\right)d}{(ac-ad)\sqrt{c^2-d^2}}+\frac{1}{(ac-ad)\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm="giac")

```
[Out] -2*((pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*
e) + d)/sqrt(c^2 - d^2)))*d/((a*c - a*d)*sqrt(c^2 - d^2)) + 1/((a*c - a*d)*
(tan(1/2*f*x + 1/2*e) + 1)))/f
```

3.459 $\int \frac{1}{(a+a\sin(e+fx))(c+d\sin(e+fx))^2}\,dx$

Optimal. Leaf size=150

$$-\frac{2d(2c+d)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{af(c-d)\left(c^2-d^2\right)^{3/2}}-\frac{d(c+2d)\cos(e+fx)}{af(c-d)^2(c+d)(c+d\sin(e+fx))}-\frac{\cos(e+fx)}{f(c-d)(a\sin(e+fx)+a)(c+d\sin(e+$$

```
[Out] (-2*d*(2*c + d)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(a*(c - d
)*(c^2 - d^2)^(3/2)*f) - (d*(c + 2*d)*Cos[e + f*x])/(a*(c - d)^2*(c + d)*f*
(c + d*Sin[e + f*x])) - Cos[e + f*x]/((c - d)*f*(a + a*Sin[e + f*x])*(c + d
*Sin[e + f*x]))
```

Rubi [A] time = 0.178282, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {2768, 2754, 12, 2660, 618, 204}

$$-\frac{2d(2c+d)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{af(c-d)\left(c^2-d^2\right)^{3/2}}-\frac{d(c+2d)\cos(e+fx)}{af(c-d)^2(c+d)(c+d\sin(e+fx))}-\frac{\cos(e+fx)}{f(c-d)(a\sin(e+fx)+a)(c+d\sin(e+$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^2),x]
```


```
[Out] (-2*d*(2*c + d)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(a*(c - d
)*(c^2 - d^2)^(3/2)*f) - (d*(c + 2*d)*Cos[e + f*x])/(a*(c - d)^2*(c + d)*f*
(c + d*Sin[e + f*x])) - Cos[e + f*x]/((c - d)*f*(a + a*Sin[e + f*x])*(c + d
*Sin[e + f*x]))
```


Rule 2768

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n
+ 1))/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x])), x] + Dist[d/(a*(b*c - a*d)),
Int[(c + d*Sin[e + f*x])^n*(a*n - b*(n + 1)*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))(c+d\sin(e+fx))^2}\,dx = -\frac{\cos(e+fx)}{(c-d)f(a+a\sin(e+fx))(c+d\sin(e+fx))}+\frac{d\int \frac{-2a+a\sin(e+fx)}{(c+d\sin(e+fx))^2}\,dx}{a^2(c-d)}$$

$$= -\frac{d(c+2d)\cos(e+fx)}{a(c-d)^2(c+d)f(c+d\sin(e+fx))}-\frac{\cos(e+fx)}{(c-d)f(a+a\sin(e+fx))(c+d\sin}$$

$$= -\frac{d(c+2d)\cos(e+fx)}{a(c-d)^2(c+d)f(c+d\sin(e+fx))}-\frac{\cos(e+fx)}{(c-d)f(a+a\sin(e+fx))(c+d\sin}$$

$$= -\frac{d(c+2d)\cos(e+fx)}{a(c-d)^2(c+d)f(c+d\sin(e+fx))}-\frac{\cos(e+fx)}{(c-d)f(a+a\sin(e+fx))(c+d\sin}$$

$$= -\frac{d(c+2d)\cos(e+fx)}{a(c-d)^2(c+d)f(c+d\sin(e+fx))}-\frac{\cos(e+fx)}{(c-d)f(a+a\sin(e+fx))(c+d\sin}$$

$$= -\frac{2d(2c+d)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{a(c-d)^2(c+d)\sqrt{c^2-d^2}f}-\frac{d(c+2d)\cos(e+fx)}{a(c-d)^2(c+d)f(c+d\sin(e+fx))}$$

Mathematica [A] time = 0.638198, size = 162, normalized size = 1.08

$$\frac{\cos(e+fx)\left(-\frac{d}{(\sin(e+fx)+1)(c+d\sin(e+fx))}+\frac{c+2d}{(c-d)(\sin(e+fx)+1)}-\frac{2d(2c+d)\tan^{-1}\left(\frac{\sqrt{d-c}\sqrt{1-\sin(e+fx)}}{\sqrt{-c-d}\sqrt{\sin(e+fx)+1}}\right)}{\sqrt{-c-d}(d-c)^{3/2}\sqrt{\cos^2(e+fx)}}\right)}{af(d-c)(c+d)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^2),x]
```

```
[Out] (Cos[e + f*x]*((-2*d*(2*c + d)*ArcTan[(Sqrt[-c + d]*Sqrt[1 - Sin[e + f*x]])
/(Sqrt[-c - d]*Sqrt[1 + Sin[e + f*x]])])/(Sqrt[-c - d]*(-c + d)^(3/2)*Sqrt[
Cos[e + f*x]^2]) + (c + 2*d)/((c - d)*(1 + Sin[e + f*x])) - d/((1 + Sin[e +
 f*x])*(c + d*Sin[e + f*x]))))/(a*(-c + d)*(c + d)*f)
```

Maple [A] time = 0.104, size = 273, normalized size = 1.8

$$-2\frac{d^3\tan\left(1/2\,fx+e/2\right)}{af\,(c-d)^2\left(c\left(\tan\left(1/2\,fx+e/2\right)\right)^2+2\,\tan\left(1/2\,fx+e/2\right)d+c\right)(c+d)\,c}-2\frac{d^2}{af\,(c-d)^2\left(c\left(\tan\left(1/2\,fx+e/2\right)\right)^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^2,x)
```

```
[Out] -2/a/f*d^3/(c-d)^2/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)/(c+d)/
c*tan(1/2*f*x+1/2*e)-2/a/f*d^2/(c-d)^2/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*
x+1/2*e)*d+c)/(c+d)-4/a/f*d/(c-d)^2/(c+d)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*t
an(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*c-2/a/f*d^2/(c-d)^2/(c+d)/(c^2-d^2)
^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))-2/a/f/(c-d)
^2/(tan(1/2*f*x+1/2*e)+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.92085, size = 2422, normalized size = 16.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*c^4 - 4*c^2*d^2 + 2*d^4 + 2*(c^3*d + 2*c^2*d^2 - c*d^3 - 2*d^4)*cos
(f*x + e)^2 + (2*c^2*d + 3*c*d^2 + d^3 - (2*c*d^2 + d^3)*cos(f*x + e)^2 + (
2*c^2*d + c*d^2)*cos(f*x + e) + (2*c^2*d + 3*c*d^2 + d^3 + (2*c*d^2 + d^3)*
cos(f*x + e))*sin(f*x + e))*sqrt(-c^2 + d^2)*log(-((2*c^2 - d^2)*cos(f*x +
e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 - 2*(c*cos(f*x + e)*sin(f*x + e) + d*
cos(f*x + e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) -
c^2 - d^2)) + 2*(c^4 + c^3*d - c*d^3 - d^4)*cos(f*x + e) - 2*(c^4 - 2*c^2*d
^2 + d^4 - (c^3*d + 2*c^2*d^2 - c*d^3 - 2*d^4)*cos(f*x + e))*sin(f*x + e))/
((a*c^5*d - a*c^4*d^2 - 2*a*c^3*d^3 + 2*a*c^2*d^4 + a*c*d^5 - a*d^6)*f*cos(
f*x + e)^2 - (a*c^6 - a*c^5*d - 2*a*c^4*d^2 + 2*a*c^3*d^3 + a*c^2*d^4 - a*c
*d^5)*f*cos(f*x + e) - (a*c^6 - 3*a*c^4*d^2 + 3*a*c^2*d^4 - a*d^6)*f - ((a*
c^5*d - a*c^4*d^2 - 2*a*c^3*d^3 + 2*a*c^2*d^4 + a*c*d^5 - a*d^6)*f*cos(f*x
+ e) + (a*c^6 - 3*a*c^4*d^2 + 3*a*c^2*d^4 - a*d^6)*f)*sin(f*x + e)), (c^4 -
 2*c^2*d^2 + d^4 + (c^3*d + 2*c^2*d^2 - c*d^3 - 2*d^4)*cos(f*x + e)^2 - (2*
c^2*d + 3*c*d^2 + d^3 - (2*c*d^2 + d^3)*cos(f*x + e)^2 + (2*c^2*d + c*d^2)*
cos(f*x + e) + (2*c^2*d + 3*c*d^2 + d^3 + (2*c*d^2 + d^3)*cos(f*x + e))*sin
(f*x + e))*sqrt(c^2 - d^2)*arctan(-(c*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*co
s(f*x + e))) + (c^4 + c^3*d - c*d^3 - d^4)*cos(f*x + e) - (c^4 - 2*c^2*d^2
+ d^4 - (c^3*d + 2*c^2*d^2 - c*d^3 - 2*d^4)*cos(f*x + e))*sin(f*x + e))/((a
```

```
*c^5*d - a*c^4*d^2 - 2*a*c^3*d^3 + 2*a*c^2*d^4 + a*c*d^5 - a*d^6)*f*cos(f*x
 + e)^2 - (a*c^6 - a*c^5*d - 2*a*c^4*d^2 + 2*a*c^3*d^3 + a*c^2*d^4 - a*c*d^
5)*f*cos(f*x + e) - (a*c^6 - 3*a*c^4*d^2 + 3*a*c^2*d^4 - a*d^6)*f - ((a*c^5
*d - a*c^4*d^2 - 2*a*c^3*d^3 + 2*a*c^2*d^4 + a*c*d^5 - a*d^6)*f*cos(f*x + e
) + (a*c^6 - 3*a*c^4*d^2 + 3*a*c^2*d^4 - a*d^6)*f)*sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))**2,x)

[Out] Timed out
```

Giac [B] time = 1.62021, size = 726, normalized size = 4.84

$$\frac{\frac{(2\,ac^4d-ac^3d^2-3\,ac^2d^3+acd^4+ad^5)\sqrt{-c^2+d^2}\log\left(\left|\left(d+\sqrt{-c^2+d^2}\right)\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+c\right|\right)}{a^2c^8-2\,a^2c^7d-2\,a^2c^6d^2+6\,a^2c^5d^3-6\,a^2c^3d^5+2\,a^2c^2d^6+2\,a^2cd^7-a^2d^8}-\frac{(2\,ac^4d-ac^3d^2-3\,ac^2d^3+acd^4+ad^5)\sqrt{-c^2+d^2}\log\left(\left|-\left(d-\sqrt{-c^2+d^2}\right)\tan\left(\frac{1}{2}\right.\right.\right.}{a^2c^8-2\,a^2c^7d-2\,a^2c^6d^2+6\,a^2c^5d^3-6\,a^2c^3d^5+2\,a^2c^2d^6+2\,a^2cd^7-a}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorithm="giac")

[Out] ((2*a*c^4*d - a*c^3*d^2 - 3*a*c^2*d^3 + a*c*d^4 + a*d^5)*sqrt(-c^2 + d^2)*l
og(abs((d + sqrt(-c^2 + d^2))*tan(1/2*f*x + 1/2*e) + c))/(a^2*c^8 - 2*a^2*c
^7*d - 2*a^2*c^6*d^2 + 6*a^2*c^5*d^3 - 6*a^2*c^3*d^5 + 2*a^2*c^2*d^6 + 2*a^
2*c*d^7 - a^2*d^8) - (2*a*c^4*d - a*c^3*d^2 - 3*a*c^2*d^3 + a*c*d^4 + a*d^5
)*sqrt(-c^2 + d^2)*log(abs(-(d - sqrt(-c^2 + d^2))*tan(1/2*f*x + 1/2*e) - c
))/(a^2*c^8 - 2*a^2*c^7*d - 2*a^2*c^6*d^2 + 6*a^2*c^5*d^3 - 6*a^2*c^3*d^5 +
 2*a^2*c^2*d^6 + 2*a^2*c*d^7 - a^2*d^8) - 2*(c^3*tan(1/2*f*x + 1/2*e)^2 + c
^2*d*tan(1/2*f*x + 1/2*e)^2 + d^3*tan(1/2*f*x + 1/2*e)^2 + 2*c^2*d*tan(1/2*
f*x + 1/2*e) + 3*c*d^2*tan(1/2*f*x + 1/2*e) + d^3*tan(1/2*f*x + 1/2*e) + c^
3 + c^2*d + c*d^2)/((a*c^4 - a*c^3*d - a*c^2*d^2 + a*c*d^3)*(c*tan(1/2*f*x
+ 1/2*e)^3 + c*tan(1/2*f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e)^2 + c*tan(
1/2*f*x + 1/2*e) + 2*d*tan(1/2*f*x + 1/2*e) + c)))/f
```

3.460 $\int \frac{1}{(a+a\sin(e+fx))(c+d\sin(e+fx))^3}\,dx$

Optimal. Leaf size=213

$$-\frac{3d\left(2c^2+2cd+d^2\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{af(c-d)\left(c^2-d^2\right)^{5/2}}-\frac{d(2c+d)(c+4d)\cos(e+fx)}{2af(c-d)^3(c+d)^2(c+d\sin(e+fx))}-\frac{d(2c+3d)\cos(e+fx)}{2af(c-d)^2(c+d)(c+d\sin(e}$$

```
[Out] (-3*d*(2*c^2 + 2*c*d + d^2)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]
])/(a*(c - d)*(c^2 - d^2)^(5/2)*f) - (d*(2*c + 3*d)*Cos[e + f*x])/(2*a*(c -
 d)^2*(c + d)*f*(c + d*Sin[e + f*x])^2) - Cos[e + f*x]/((c - d)*f*(a + a*Si
n[e + f*x])*(c + d*Sin[e + f*x])^2) - (d*(2*c + d)*(c + 4*d)*Cos[e + f*x])/
(2*a*(c - d)^3*(c + d)^2*f*(c + d*Sin[e + f*x]))
```

Rubi [A] time = 0.322788, antiderivative size = 213, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {2768, 2754, 12, 2660, 618, 204}

$$-\frac{3d\left(2c^2+2cd+d^2\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{af(c-d)\left(c^2-d^2\right)^{5/2}}-\frac{d(2c+d)(c+4d)\cos(e+fx)}{2af(c-d)^3(c+d)^2(c+d\sin(e+fx))}-\frac{d(2c+3d)\cos(e+fx)}{2af(c-d)^2(c+d)(c+d\sin(e}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^3),x]
```

```
[Out] (-3*d*(2*c^2 + 2*c*d + d^2)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]
])/(a*(c - d)*(c^2 - d^2)^(5/2)*f) - (d*(2*c + 3*d)*Cos[e + f*x])/(2*a*(c -
 d)^2*(c + d)*f*(c + d*Sin[e + f*x])^2) - Cos[e + f*x]/((c - d)*f*(a + a*Si
n[e + f*x])*(c + d*Sin[e + f*x])^2) - (d*(2*c + d)*(c + 4*d)*Cos[e + f*x])/
(2*a*(c - d)^3*(c + d)^2*f*(c + d*Sin[e + f*x]))
```

Rule 2768

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n
+ 1))/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x])), x] + Dist[d/(a*(b*c - a*d)),
Int[(c + d*Sin[e + f*x])^n*(a*n - b*(n + 1)*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))(c+d\sin(e+fx))^3}\,dx = -\frac{\cos(e+fx)}{(c-d)f(a+a\sin(e+fx))(c+d\sin(e+fx))^2} + \frac{d\int \frac{-3a+2a\sin(e+fx)}{(c+d\sin(e+fx))^3}\,dx}{a^2(c-d)}$$

$$= -\frac{d(2c+3d)\cos(e+fx)}{2a(c-d)^2(c+d)f(c+d\sin(e+fx))^2} - \frac{\cos(e+fx)}{(c-d)f(a+a\sin(e+fx))(c+}$$

$$= -\frac{d(2c+3d)\cos(e+fx)}{2a(c-d)^2(c+d)f(c+d\sin(e+fx))^2} - \frac{\cos(e+fx)}{(c-d)f(a+a\sin(e+fx))(c+}$$

$$= -\frac{d(2c+3d)\cos(e+fx)}{2a(c-d)^2(c+d)f(c+d\sin(e+fx))^2} - \frac{\cos(e+fx)}{(c-d)f(a+a\sin(e+fx))(c+}$$

$$= -\frac{d(2c+3d)\cos(e+fx)}{2a(c-d)^2(c+d)f(c+d\sin(e+fx))^2} - \frac{\cos(e+fx)}{(c-d)f(a+a\sin(e+fx))(c+}$$

$$= -\frac{d(2c+3d)\cos(e+fx)}{2a(c-d)^2(c+d)f(c+d\sin(e+fx))^2} - \frac{\cos(e+fx)}{(c-d)f(a+a\sin(e+fx))(c+}$$

$$= -\frac{3d\left(2c^2+2cd+d^2\right)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{a(c-d)^3(c+d)^2\sqrt{c^2-d^2}\,f} - \frac{d(2c+3d)\cos(e+}{2a(c-d)^2(c+d)f(c+d\,\mathrm{si}}$$

Mathematica [A] time = 1.34363, size = 232, normalized size = 1.09

$$\frac{\cos(e+fx)\left(\frac{\frac{2c^2+9cd+4d^2}{(c-d)^2(\sin(e+fx)+1)}+\frac{6d\left(2c^2+2cd+d^2\right)\tan^{-1}\left(\frac{\sqrt{d-c}\sqrt{1-\sin(e+fx)}}{\sqrt{-c-d}\sqrt{\sin(e+fx)+1}}\right)}{\sqrt{-c-d}(d-c)^{5/2}\sqrt{\cos^2(e+fx)}}}{c+d} - \frac{d(4c+d)}{(c-d)(c+d)(\sin(e+fx)+1)(c+d\sin(e+fx))} - \frac{d}{(\sin(e+fx)+1)(c+d\sin(e+fx))^2}\right.}{2af(d-c)(c+d)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^3),x]
```

```
[Out] (Cos[e + f*x]*(-(d/((1 + Sin[e + f*x])*(c + d*Sin[e + f*x])^2)) - (d*(4*c +
 d))/((c - d)*(c + d)*(1 + Sin[e + f*x])*(c + d*Sin[e + f*x])) + ((6*d*(2*c
^2 + 2*c*d + d^2)*ArcTan[(Sqrt[-c + d]*Sqrt[1 - Sin[e + f*x]])/(Sqrt[-c - d
]*Sqrt[1 + Sin[e + f*x]])])/(Sqrt[-c - d]*(-c + d)^(5/2)*Sqrt[Cos[e + f*x]^
2]) + (2*c^2 + 9*c*d + 4*d^2)/((c - d)^2*(1 + Sin[e + f*x])))/(c + d)))/(2*
a*(-c + d)*(c + d)*f)
```

Maple [B] time = 0.115, size = 1224, normalized size = 5.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^3,x)
```

```
[Out] -7/a/f*d^3/(c-d)^3/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2*c/(c
^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3-2/a/f*d^4/(c-d)^3/(c*tan(1/2*f*x+1/2*e)^
2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3+2/a/f*d^
5/(c-d)^3/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/c/(c^2+2*c*d+
d^2)*tan(1/2*f*x+1/2*e)^3-6/a/f*d^2/(c-d)^3/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1
/2*f*x+1/2*e)*d+c)^2*c^2/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^2-2/a/f*d^3/(c-
d)^3/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2*c/(c^2+2*c*d+d^2)*
tan(1/2*f*x+1/2*e)^2-11/a/f*d^4/(c-d)^3/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f
*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^2-4/a/f*d^5/(c-d)^3/(c*
tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/c/(c^2+2*c*d+d^2)*tan(1/2*
f*x+1/2*e)^2+2/a/f*d^6/(c-d)^3/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)
*d+c)^2/c^2/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^2-17/a/f*d^3/(c-d)^3/(c*tan(
1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*c*tan(1/2*f*x+
1/2*e)-6/a/f*d^4/(c-d)^3/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^
2/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)+2/a/f*d^5/(c-d)^3/(c*tan(1/2*f*x+1/2*e
)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)/c*tan(1/2*f*x+1/2*e)-6/a/f*
d^2/(c-d)^3/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+
d^2)*c^2-2/a/f*d^3/(c-d)^3/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c
)^2/(c^2+2*c*d+d^2)*c+1/a/f*d^4/(c-d)^3/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f
*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)-6/a/f*d/(c-d)^3/(c^2+2*c*d+d^2)/(c^2-d^2)^
(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*c^2-6/a/f*d^
2/(c-d)^3/(c^2+2*c*d+d^2)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e
)+2*d)/(c^2-d^2)^(1/2))*c-3/a/f*d^3/(c-d)^3/(c^2+2*c*d+d^2)/(c^2-d^2)^(1/2)
*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))-2/a/f/(c-d)^3/(ta
n(1/2*f*x+1/2*e)+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.38012, size = 5072, normalized size = 23.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] [1/4*(4*c^6 - 12*c^4*d^2 + 12*c^2*d^4 - 4*d^6 - 2*(2*c^4*d^2 + 9*c^3*d^3 +
2*c^2*d^4 - 9*c*d^5 - 4*d^6)*cos(f*x + e)^3 + 2*(4*c^5*d + 12*c^4*d^2 - 2*c
^3*d^3 - 15*c^2*d^4 - 2*c*d^5 + 3*d^6)*cos(f*x + e)^2 - 3*(2*c^4*d + 6*c^3*
d^2 + 7*c^2*d^3 + 4*c*d^4 + d^5 - (2*c^2*d^3 + 2*c*d^4 + d^5)*cos(f*x + e)^
3 - (4*c^3*d^2 + 6*c^2*d^3 + 4*c*d^4 + d^5)*cos(f*x + e)^2 + (2*c^4*d + 2*c
^3*d^2 + 3*c^2*d^3 + 2*c*d^4 + d^5)*cos(f*x + e) + (2*c^4*d + 6*c^3*d^2 + 7
*c^2*d^3 + 4*c*d^4 + d^5 - (2*c^2*d^3 + 2*c*d^4 + d^5)*cos(f*x + e)^2 + 2*(
2*c^3*d^2 + 2*c^2*d^3 + c*d^4)*cos(f*x + e))*sin(f*x + e))*sqrt(-c^2 + d^2)
*log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 + 2*(c*
cos(f*x + e)*sin(f*x + e) + d*cos(f*x + e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x
+ e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) + 2*(2*c^6 + 4*c^5*d + 8*c^4*d^2
+ 7*c^3*d^3 - 7*c^2*d^4 - 11*c*d^5 - 3*d^6)*cos(f*x + e) - 2*(2*c^6 - 6*c^4
*d^2 + 6*c^2*d^4 - 2*d^6 - (2*c^4*d^2 + 9*c^3*d^3 + 2*c^2*d^4 - 9*c*d^5 - 4
*d^6)*cos(f*x + e)^2 - (4*c^5*d + 14*c^4*d^2 + 7*c^3*d^3 - 13*c^2*d^4 - 11*
c*d^5 - d^6)*cos(f*x + e))*sin(f*x + e))/((a*c^7*d^2 - a*c^6*d^3 - 3*a*c^5*
d^4 + 3*a*c^4*d^5 + 3*a*c^3*d^6 - 3*a*c^2*d^7 - a*c*d^8 + a*d^9)*f*cos(f*x
+ e)^3 + (2*a*c^8*d - a*c^7*d^2 - 7*a*c^6*d^3 + 3*a*c^5*d^4 + 9*a*c^4*d^5 -
 3*a*c^3*d^6 - 5*a*c^2*d^7 + a*c*d^8 + a*d^9)*f*cos(f*x + e)^2 - (a*c^9 - a
*c^8*d - 2*a*c^7*d^2 + 2*a*c^6*d^3 + 2*a*c^3*d^6 - 2*a*c^2*d^7 - a*c*d^8 +
a*d^9)*f*cos(f*x + e) - (a*c^9 + a*c^8*d - 4*a*c^7*d^2 - 4*a*c^6*d^3 + 6*a*
c^5*d^4 + 6*a*c^4*d^5 - 4*a*c^3*d^6 - 4*a*c^2*d^7 + a*c*d^8 + a*d^9)*f + ((
a*c^7*d^2 - a*c^6*d^3 - 3*a*c^5*d^4 + 3*a*c^4*d^5 + 3*a*c^3*d^6 - 3*a*c^2*d
^7 - a*c*d^8 + a*d^9)*f*cos(f*x + e)^2 - 2*(a*c^8*d - a*c^7*d^2 - 3*a*c^6*d
^3 + 3*a*c^5*d^4 + 3*a*c^4*d^5 - 3*a*c^3*d^6 - a*c^2*d^7 + a*c*d^8)*f*cos(f
*x + e) - (a*c^9 + a*c^8*d - 4*a*c^7*d^2 - 4*a*c^6*d^3 + 6*a*c^5*d^4 + 6*a*
c^4*d^5 - 4*a*c^3*d^6 - 4*a*c^2*d^7 + a*c*d^8 + a*d^9)*f)*sin(f*x + e)), 1/
2*(2*c^6 - 6*c^4*d^2 + 6*c^2*d^4 - 2*d^6 - (2*c^4*d^2 + 9*c^3*d^3 + 2*c^2*d
^4 - 9*c*d^5 - 4*d^6)*cos(f*x + e)^3 + (4*c^5*d + 12*c^4*d^2 - 2*c^3*d^3 -
15*c^2*d^4 - 2*c*d^5 + 3*d^6)*cos(f*x + e)^2 - 3*(2*c^4*d + 6*c^3*d^2 + 7*c
^2*d^3 + 4*c*d^4 + d^5 - (2*c^2*d^3 + 2*c*d^4 + d^5)*cos(f*x + e)^3 - (4*c^
3*d^2 + 6*c^2*d^3 + 4*c*d^4 + d^5)*cos(f*x + e)^2 + (2*c^4*d + 2*c^3*d^2 +
3*c^2*d^3 + 2*c*d^4 + d^5)*cos(f*x + e) + (2*c^4*d + 6*c^3*d^2 + 7*c^2*d^3
+ 4*c*d^4 + d^5 - (2*c^2*d^3 + 2*c*d^4 + d^5)*cos(f*x + e)^2 + 2*(2*c^3*d^2
 + 2*c^2*d^3 + c*d^4)*cos(f*x + e))*sin(f*x + e))*sqrt(c^2 - d^2)*arctan(-(
c*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) + (2*c^6 + 4*c^5*d + 8*
c^4*d^2 + 7*c^3*d^3 - 7*c^2*d^4 - 11*c*d^5 - 3*d^6)*cos(f*x + e) - (2*c^6 -
 6*c^4*d^2 + 6*c^2*d^4 - 2*d^6 - (2*c^4*d^2 + 9*c^3*d^3 + 2*c^2*d^4 - 9*c*d
^5 - 4*d^6)*cos(f*x + e)^2 - (4*c^5*d + 14*c^4*d^2 + 7*c^3*d^3 - 13*c^2*d^4
 - 11*c*d^5 - d^6)*cos(f*x + e))*sin(f*x + e))/((a*c^7*d^2 - a*c^6*d^3 - 3*
a*c^5*d^4 + 3*a*c^4*d^5 + 3*a*c^3*d^6 - 3*a*c^2*d^7 - a*c*d^8 + a*d^9)*f*co
s(f*x + e)^3 + (2*a*c^8*d - a*c^7*d^2 - 7*a*c^6*d^3 + 3*a*c^5*d^4 + 9*a*c^4
*d^5 - 3*a*c^3*d^6 - 5*a*c^2*d^7 + a*c*d^8 + a*d^9)*f*cos(f*x + e)^2 - (a*c
^9 - a*c^8*d - 2*a*c^7*d^2 + 2*a*c^6*d^3 + 2*a*c^3*d^6 - 2*a*c^2*d^7 - a*c*
d^8 + a*d^9)*f*cos(f*x + e) - (a*c^9 + a*c^8*d - 4*a*c^7*d^2 - 4*a*c^6*d^3
+ 6*a*c^5*d^4 + 6*a*c^4*d^5 - 4*a*c^3*d^6 - 4*a*c^2*d^7 + a*c*d^8 + a*d^9)*
f + ((a*c^7*d^2 - a*c^6*d^3 - 3*a*c^5*d^4 + 3*a*c^4*d^5 + 3*a*c^3*d^6 - 3*a
*c^2*d^7 - a*c*d^8 + a*d^9)*f*cos(f*x + e)^2 - 2*(a*c^8*d - a*c^7*d^2 - 3*a
*c^6*d^3 + 3*a*c^5*d^4 + 3*a*c^4*d^5 - 3*a*c^3*d^6 - a*c^2*d^7 + a*c*d^8)*f
*cos(f*x + e) - (a*c^9 + a*c^8*d - 4*a*c^7*d^2 - 4*a*c^6*d^3 + 6*a*c^5*d^4
+ 6*a*c^4*d^5 - 4*a*c^3*d^6 - 4*a*c^2*d^7 + a*c*d^8 + a*d^9)*f)*sin(f*x + e
))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.47471, size = 651, normalized size = 3.06

$$-\frac{\frac{3\left(2c^2d+2cd^2+d^3\right)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{\left(ac^5-ac^4d-2ac^3d^2+2ac^2d^3+acd^4-ad^5\right)\sqrt{c^2-d^2}}+\frac{7c^3d^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+2c^2d^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-2cd^5\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+6c^4d^2\tan\cdots}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -(3*(2*c^2*d + 2*c*d^2 + d^3)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + ar
ctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/((a*c^5 - a*c^4*d - 2*a
*c^3*d^2 + 2*a*c^2*d^3 + a*c*d^4 - a*d^5)*sqrt(c^2 - d^2)) + (7*c^3*d^3*tan
(1/2*f*x + 1/2*e)^3 + 2*c^2*d^4*tan(1/2*f*x + 1/2*e)^3 - 2*c*d^5*tan(1/2*f*
x + 1/2*e)^3 + 6*c^4*d^2*tan(1/2*f*x + 1/2*e)^2 + 2*c^3*d^3*tan(1/2*f*x + 1
/2*e)^2 + 11*c^2*d^4*tan(1/2*f*x + 1/2*e)^2 + 4*c*d^5*tan(1/2*f*x + 1/2*e)^
2 - 2*d^6*tan(1/2*f*x + 1/2*e)^2 + 17*c^3*d^3*tan(1/2*f*x + 1/2*e) + 6*c^2*
d^4*tan(1/2*f*x + 1/2*e) - 2*c*d^5*tan(1/2*f*x + 1/2*e) + 6*c^4*d^2 + 2*c^3
*d^3 - c^2*d^4)/((a*c^7 - a*c^6*d - 2*a*c^5*d^2 + 2*a*c^4*d^3 + a*c^3*d^4 -
 a*c^2*d^5)*(c*tan(1/2*f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e) + c)^2) +
2/((a*c^3 - 3*a*c^2*d + 3*a*c*d^2 - a*d^3)*(tan(1/2*f*x + 1/2*e) + 1)))/f
```

3.461 $\int \frac{(c+d\sin(e+fx))^5}{(a+a\sin(e+fx))^2}\,dx$

Optimal. Leaf size=260

$$\frac{2d\left(-44c^2d^2+10c^3d+c^4+40cd^3-12d^4\right)\cos(e+fx)}{3a^2f}+\frac{d\left(c^2+10cd-12d^2\right)\cos(e+fx)(c+d\sin(e+fx))^2}{3a^2f}+\frac{d^2\left(20c^2\right.}{}$$

```
[Out] (5*(2*c - d)*d^2*(2*c^2 - 3*c*d + 2*d^2)*x)/(2*a^2) + (2*d*(c^4 + 10*c^3*d
- 44*c^2*d^2 + 40*c*d^3 - 12*d^4)*Cos[e + f*x])/(3*a^2*f) + (d^2*(2*c^3 + 2
0*c^2*d - 57*c*d^2 + 30*d^3)*Cos[e + f*x]*Sin[e + f*x])/(6*a^2*f) + (d*(c^2
 + 10*c*d - 12*d^2)*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(3*a^2*f) - ((c -
d)*(c + 10*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(3*a^2*f*(1 + Sin[e + f*
x])) - ((c - d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^4)/(3*f*(a + a*Sin[e + f*
x])^2)
```

Rubi [A] time = 0.495092, antiderivative size = 260, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2765, 2977, 2753, 2734}

$$\frac{2d\left(-44c^2d^2+10c^3d+c^4+40cd^3-12d^4\right)\cos(e+fx)}{3a^2f}+\frac{d\left(c^2+10cd-12d^2\right)\cos(e+fx)(c+d\sin(e+fx))^2}{3a^2f}+\frac{d^2\left(20c^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^5/(a + a*Sin[e + f*x])^2,x]
```


```
[Out] (5*(2*c - d)*d^2*(2*c^2 - 3*c*d + 2*d^2)*x)/(2*a^2) + (2*d*(c^4 + 10*c^3*d
- 44*c^2*d^2 + 40*c*d^3 - 12*d^4)*Cos[e + f*x])/(3*a^2*f) + (d^2*(2*c^3 + 2
0*c^2*d - 57*c*d^2 + 30*d^3)*Cos[e + f*x]*Sin[e + f*x])/(6*a^2*f) + (d*(c^2
 + 10*c*d - 12*d^2)*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(3*a^2*f) - ((c -
d)*(c + 10*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(3*a^2*f*(1 + Sin[e + f*
x])) - ((c - d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^4)/(3*f*(a + a*Sin[e + f*
x])^2)
```


Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
```

```
egerQ[2*n] || EqQ[c, 0])
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^5}{(a+a\sin(e+fx))^2}\,dx = -\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^4}{3f(a+a\sin(e+fx))^2} - \frac{\int \frac{(c+d\sin(e+fx))^3\left(-a\left(c^2+6cd-4d^2\right)+3a(c-2d)d\sin(e+fx)\right)}{a+a\sin(e+fx)}}{3a^2}$$

$$= -\frac{(c-d)(c+10d)\cos(e+fx)(c+d\sin(e+fx))^3}{3a^2f(1+\sin(e+fx))} - \frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))}{3f(a+a\sin(e+fx))^2}$$

$$= \frac{d\left(c^2+10cd-12d^2\right)\cos(e+fx)(c+d\sin(e+fx))^2}{3a^2f} - \frac{(c-d)(c+10d)\cos(e+fx)(c+d}{3a^2f(1+\sin(e+fx)}$$

$$= \frac{5(2c-d)d^2\left(2c^2-3cd+2d^2\right)x}{2a^2} + \frac{2d\left(c^4+10c^3d-44c^2d^2+40cd^3-12d^4\right)\cos(e+fx)}{3a^2f}$$

Mathematica [B] time = 1.79741, size = 837, normalized size = 3.22

$$\left(\cos\left(\tfrac{1}{2}(e+fx)\right)+\sin\left(\tfrac{1}{2}(e+fx)\right)\right)\left(48\sin\left(\tfrac{1}{2}(e+fx)\right)c^5+240d\sin\left(\tfrac{1}{2}(e+fx)\right)c^4-1440d^2\sin\left(\tfrac{1}{2}(e+fx)\right)c^3+72$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])^5/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(3*d*(80*c^4 + 80*c^3*d*(-4 + 3*e +
3*f*x) - 80*c^2*d^2*(-5 + 6*e + 6*f*x) + 35*c*d^3*(-7 + 12*e + 12*f*x) - 4*
d^4*(-13 + 30*e + 30*f*x))*Cos[(e + f*x)/2] - (16*c^5 + 160*c^4*d + 80*c^3*
d^2*(-10 + 3*e + 3*f*x) - 40*c^2*d^3*(-41 + 12*e + 12*f*x) - 6*d^5*(-57 + 2
0*e + 20*f*x) + 5*c*d^4*(-239 + 84*e + 84*f*x))*Cos[(3*(e + f*x))/2] + 120*
c^2*d^3*Cos[(5*(e + f*x))/2] - 75*c*d^4*Cos[(5*(e + f*x))/2] + 30*d^5*Cos[(
5*(e + f*x))/2] + 15*c*d^4*Cos[(7*(e + f*x))/2] - 3*d^5*Cos[(7*(e + f*x))/2
] - d^5*Cos[(9*(e + f*x))/2] + 48*c^5*Sin[(e + f*x)/2] + 240*c^4*d*Sin[(e +
 f*x)/2] - 1440*c^3*d^2*Sin[(e + f*x)/2] + 2640*c^2*d^3*Sin[(e + f*x)/2] -
1905*c*d^4*Sin[(e + f*x)/2] + 516*d^5*Sin[(e + f*x)/2] + 720*c^3*d^2*e*Sin[
(e + f*x)/2] - 1440*c^2*d^3*e*Sin[(e + f*x)/2] + 1260*c*d^4*e*Sin[(e + f*x)
/2] - 360*d^5*e*Sin[(e + f*x)/2] + 720*c^3*d^2*f*x*Sin[(e + f*x)/2] - 1440*
c^2*d^3*f*x*Sin[(e + f*x)/2] + 1260*c*d^4*f*x*Sin[(e + f*x)/2] - 360*d^5*f*
x*Sin[(e + f*x)/2] - 360*c^2*d^3*Sin[(3*(e + f*x))/2] + 315*c*d^4*Sin[(3*(e
 + f*x))/2] - 118*d^5*Sin[(3*(e + f*x))/2] + 240*c^3*d^2*e*Sin[(3*(e + f*x)
```

```
)/2] - 480*c^2*d^3*e*Sin[(3*(e + f*x))/2] + 420*c*d^4*e*Sin[(3*(e + f*x))/2
] - 120*d^5*e*Sin[(3*(e + f*x))/2] + 240*c^3*d^2*f*x*Sin[(3*(e + f*x))/2] -
 480*c^2*d^3*f*x*Sin[(3*(e + f*x))/2] + 420*c*d^4*f*x*Sin[(3*(e + f*x))/2]
- 120*d^5*f*x*Sin[(3*(e + f*x))/2] - 120*c^2*d^3*Sin[(5*(e + f*x))/2] + 75*
c*d^4*Sin[(5*(e + f*x))/2] - 30*d^5*Sin[(5*(e + f*x))/2] + 15*c*d^4*Sin[(7*
(e + f*x))/2] - 3*d^5*Sin[(7*(e + f*x))/2] + d^5*Sin[(9*(e + f*x))/2]))/(48
*a^2*f*(1 + Sin[e + f*x])^2)
```

Maple [B] time = 0.102, size = 982, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^5/(a+a*sin(f*x+e))^2,x)
```

```
[Out] 5/f/a^2*d^4/(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)^5*c-20/f/a^2*d^3/
(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)^4*c^2+20/f/a^2*d^4/(1+tan(1/2
*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)^4*c-40/f/a^2*d^3/(1+tan(1/2*f*x+1/2*e)^
2)^3*tan(1/2*f*x+1/2*e)^2*c^2+40/f/a^2*d^4/(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1
/2*f*x+1/2*e)^2*c-5/f/a^2*d^4/(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)
*c+10/f/a^2/(tan(1/2*f*x+1/2*e)+1)^2*c*d^4-2/f/a^2*d^5/(1+tan(1/2*f*x+1/2*e
)^2)^3*tan(1/2*f*x+1/2*e)^5-6/f/a^2*d^5/(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*
f*x+1/2*e)^4-16/f/a^2*d^5/(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)^2+2
/f/a^2*d^5/(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)-20/f/a^2*d^3/(1+ta
n(1/2*f*x+1/2*e)^2)^3*c^2+20/f/a^2*d^4/(1+tan(1/2*f*x+1/2*e)^2)^3*c+20/f/a^
2/(tan(1/2*f*x+1/2*e)+1)*c^3*d^2+20/f/a^2*d^2*arctan(tan(1/2*f*x+1/2*e))*c^
3-10/f/a^2/(tan(1/2*f*x+1/2*e)+1)^2*c^4*d+20/f/a^2/(tan(1/2*f*x+1/2*e)+1)^2
*c^3*d^2+2/f/a^2/(tan(1/2*f*x+1/2*e)+1)^2*c^5-2/f/a^2/(tan(1/2*f*x+1/2*e)+1
)^2*d^5-4/3/f/a^2/(tan(1/2*f*x+1/2*e)+1)^3*c^5+4/3/f/a^2/(tan(1/2*f*x+1/2*e
)+1)^3*d^5-22/3/f/a^2*d^5/(1+tan(1/2*f*x+1/2*e)^2)^3-10/f/a^2*d^5*arctan(ta
n(1/2*f*x+1/2*e))-2/f/a^2/(tan(1/2*f*x+1/2*e)+1)*c^5-8/f/a^2/(tan(1/2*f*x+1
/2*e)+1)*d^5-40/f/a^2*d^3*arctan(tan(1/2*f*x+1/2*e))*c^2+35/f/a^2*d^4*arcta
n(tan(1/2*f*x+1/2*e))*c-40/f/a^2/(tan(1/2*f*x+1/2*e)+1)*c^2*d^3+30/f/a^2/(t
an(1/2*f*x+1/2*e)+1)*c*d^4+20/3/f/a^2/(tan(1/2*f*x+1/2*e)+1)^3*c^4*d-40/3/f
/a^2/(tan(1/2*f*x+1/2*e)+1)^3*c^3*d^2+40/3/f/a^2/(tan(1/2*f*x+1/2*e)+1)^3*c
^2*d^3-20/3/f/a^2/(tan(1/2*f*x+1/2*e)+1)^3*c*d^4-20/f/a^2/(tan(1/2*f*x+1/2*
e)+1)^2*c^2*d^3
```

Maxima [B] time = 1.97013, size = 1771, normalized size = 6.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^5/(a+a*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/3*(5*c*d^4*((75*sin(f*x + e)/(cos(f*x + e) + 1) + 97*sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 + 126*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 98*sin(f*x + e)
^4/(cos(f*x + e) + 1)^4 + 63*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 21*sin(f
*x + e)^6/(cos(f*x + e) + 1)^6 + 32)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e
) + 1) + 5*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 7*a^2*sin(f*x + e)^3/(
cos(f*x + e) + 1)^3 + 7*a^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 5*a^2*sin
(f*x + e)^5/(cos(f*x + e) + 1)^5 + 3*a^2*sin(f*x + e)^6/(cos(f*x + e) + 1)^
```

```
6 + a^2*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) + 21*arctan(sin(f*x + e)/(cos(
f*x + e) + 1))/a^2) - 2*d^5*((57*sin(f*x + e)/(cos(f*x + e) + 1) + 99*sin(f
*x + e)^2/(cos(f*x + e) + 1)^2 + 155*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 +
153*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 135*sin(f*x + e)^5/(cos(f*x + e)
+ 1)^5 + 85*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 45*sin(f*x + e)^7/(cos(f*
x + e) + 1)^7 + 15*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 24)/(a^2 + 3*a^2*s
in(f*x + e)/(cos(f*x + e) + 1) + 6*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2
+ 10*a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 12*a^2*sin(f*x + e)^4/(cos(f
*x + e) + 1)^4 + 12*a^2*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 10*a^2*sin(f*
x + e)^6/(cos(f*x + e) + 1)^6 + 6*a^2*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 +
 3*a^2*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + a^2*sin(f*x + e)^9/(cos(f*x +
e) + 1)^9) + 15*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) - 40*c^2*d^3*(
(12*sin(f*x + e)/(cos(f*x + e) + 1) + 11*sin(f*x + e)^2/(cos(f*x + e) + 1)^
2 + 9*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^4/(cos(f*x + e)
+ 1)^4 + 5)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 4*a^2*sin(f*x +
e)^2/(cos(f*x + e) + 1)^2 + 4*a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*a
^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^2*sin(f*x + e)^5/(cos(f*x + e) +
 1)^5) + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) + 20*c^3*d^2*((9*si
n(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4)/
(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 3*arctan(sin(f*x
 + e)/(cos(f*x + e) + 1))/a^2) - 2*c^5*(3*sin(f*x + e)/(cos(f*x + e) + 1) +
 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2)/(a^2 + 3*a^2*sin(f*x + e)/(cos(
f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e
)^3/(cos(f*x + e) + 1)^3) - 10*c^4*d*(3*sin(f*x + e)/(cos(f*x + e) + 1) + 1
)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3))/f
```

Fricas [B] time = 1.81477, size = 1314, normalized size = 5.05

$$2\,d^5\cos\left(fx+e\right)^5+2\,c^5-10\,c^4d+20\,c^3d^2-20\,c^2d^3+10\,cd^4-2\,d^5-\left(15\,cd^4-4\,d^5\right)\cos\left(fx+e\right)^4-2\left(30\,c^2d^3-15\,c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^5/(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/6*(2*d^5*cos(f*x + e)^5 + 2*c^5 - 10*c^4*d + 20*c^3*d^2 - 20*c^2*d^3 + 10
*c*d^4 - 2*d^5 - (15*c*d^4 - 4*d^5)*cos(f*x + e)^4 - 2*(30*c^2*d^3 - 15*c*d
^4 + 8*d^5)*cos(f*x + e)^3 - 30*(4*c^3*d^2 - 8*c^2*d^3 + 7*c*d^4 - 2*d^5)*f
*x + (2*c^5 + 20*c^4*d - 100*c^3*d^2 + 220*c^2*d^3 - 155*c*d^4 + 46*d^5 + 1
5*(4*c^3*d^2 - 8*c^2*d^3 + 7*c*d^4 - 2*d^5)*f*x)*cos(f*x + e)^2 + (4*c^5 +
10*c^4*d - 80*c^3*d^2 + 260*c^2*d^3 - 190*c*d^4 + 62*d^5 - 15*(4*c^3*d^2 -
8*c^2*d^3 + 7*c*d^4 - 2*d^5)*f*x)*cos(f*x + e) - (2*d^5*cos(f*x + e)^4 + 2*
c^5 - 10*c^4*d + 20*c^3*d^2 - 20*c^2*d^3 + 10*c*d^4 - 2*d^5 + (15*c*d^4 - 2
*d^5)*cos(f*x + e)^3 + 30*(4*c^3*d^2 - 8*c^2*d^3 + 7*c*d^4 - 2*d^5)*f*x - 3
*(20*c^2*d^3 - 15*c*d^4 + 6*d^5)*cos(f*x + e)^2 - (2*c^5 + 20*c^4*d - 100*c
^3*d^2 + 280*c^2*d^3 - 200*c*d^4 + 64*d^5 - 15*(4*c^3*d^2 - 8*c^2*d^3 + 7*c
*d^4 - 2*d^5)*f*x)*cos(f*x + e))*sin(f*x + e))/(a^2*f*cos(f*x + e)^2 - a^2*
f*cos(f*x + e) - 2*a^2*f - (a^2*f*cos(f*x + e) + 2*a^2*f)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**5/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.53243, size = 1319, normalized size = 5.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^5/(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/6*(15*(4*c^3*d^2 - 8*c^2*d^3 + 7*c*d^4 - 2*d^5)*(f*x + e)/a^2 - 2*(6*c^5*
tan(1/2*f*x + 1/2*e)^8 - 60*c^3*d^2*tan(1/2*f*x + 1/2*e)^8 + 120*c^2*d^3*ta
n(1/2*f*x + 1/2*e)^8 - 105*c*d^4*tan(1/2*f*x + 1/2*e)^8 + 30*d^5*tan(1/2*f*
x + 1/2*e)^8 + 6*c^5*tan(1/2*f*x + 1/2*e)^7 + 30*c^4*d*tan(1/2*f*x + 1/2*e)
^7 - 180*c^3*d^2*tan(1/2*f*x + 1/2*e)^7 + 360*c^2*d^3*tan(1/2*f*x + 1/2*e)^
7 - 315*c*d^4*tan(1/2*f*x + 1/2*e)^7 + 90*d^5*tan(1/2*f*x + 1/2*e)^7 + 22*c
^5*tan(1/2*f*x + 1/2*e)^6 + 10*c^4*d*tan(1/2*f*x + 1/2*e)^6 - 260*c^3*d^2*t
an(1/2*f*x + 1/2*e)^6 + 680*c^2*d^3*tan(1/2*f*x + 1/2*e)^6 - 595*c*d^4*tan(
1/2*f*x + 1/2*e)^6 + 170*d^5*tan(1/2*f*x + 1/2*e)^6 + 18*c^5*tan(1/2*f*x +
1/2*e)^5 + 90*c^4*d*tan(1/2*f*x + 1/2*e)^5 - 540*c^3*d^2*tan(1/2*f*x + 1/2*
e)^5 + 1200*c^2*d^3*tan(1/2*f*x + 1/2*e)^5 - 945*c*d^4*tan(1/2*f*x + 1/2*e)
^5 + 270*d^5*tan(1/2*f*x + 1/2*e)^5 + 30*c^5*tan(1/2*f*x + 1/2*e)^4 + 30*c^
4*d*tan(1/2*f*x + 1/2*e)^4 - 420*c^3*d^2*tan(1/2*f*x + 1/2*e)^4 + 1200*c^2*
d^3*tan(1/2*f*x + 1/2*e)^4 - 975*c*d^4*tan(1/2*f*x + 1/2*e)^4 + 306*d^5*tan
(1/2*f*x + 1/2*e)^4 + 18*c^5*tan(1/2*f*x + 1/2*e)^3 + 90*c^4*d*tan(1/2*f*x
+ 1/2*e)^3 - 540*c^3*d^2*tan(1/2*f*x + 1/2*e)^3 + 1320*c^2*d^3*tan(1/2*f*x
+ 1/2*e)^3 - 1005*c*d^4*tan(1/2*f*x + 1/2*e)^3 + 310*d^5*tan(1/2*f*x + 1/2*
e)^3 + 18*c^5*tan(1/2*f*x + 1/2*e)^2 + 30*c^4*d*tan(1/2*f*x + 1/2*e)^2 - 30
0*c^3*d^2*tan(1/2*f*x + 1/2*e)^2 + 840*c^2*d^3*tan(1/2*f*x + 1/2*e)^2 - 645
*c*d^4*tan(1/2*f*x + 1/2*e)^2 + 198*d^5*tan(1/2*f*x + 1/2*e)^2 + 6*c^5*tan(
1/2*f*x + 1/2*e) + 30*c^4*d*tan(1/2*f*x + 1/2*e) - 180*c^3*d^2*tan(1/2*f*x
+ 1/2*e) + 480*c^2*d^3*tan(1/2*f*x + 1/2*e) - 375*c*d^4*tan(1/2*f*x + 1/2*e
) + 114*d^5*tan(1/2*f*x + 1/2*e) + 4*c^5 + 10*c^4*d - 80*c^3*d^2 + 200*c^2*
d^3 - 160*c*d^4 + 48*d^5)/((tan(1/2*f*x + 1/2*e)^3 + tan(1/2*f*x + 1/2*e)^2
 + tan(1/2*f*x + 1/2*e) + 1)^3*a^2))/f
```

3.462 $\int \frac{(c+d\sin(e+fx))^4}{(a+a\sin(e+fx))^2} dx$

Optimal. Leaf size=195

$$\frac{2d\left(8c^2d+c^3-20cd^2+8d^3\right)\cos(e+fx)}{3a^2f}+\frac{d^2\left(2c^2+16cd-21d^2\right)\sin(e+fx)\cos(e+fx)}{6a^2f}+\frac{d^2x\left(12c^2-16cd+7d^2\right)}{2a^2}$$

```
[Out] (d^2*(12*c^2 - 16*c*d + 7*d^2)*x)/(2*a^2) + (2*d*(c^3 + 8*c^2*d - 20*c*d^2
+ 8*d^3)*Cos[e + f*x])/(3*a^2*f) + (d^2*(2*c^2 + 16*c*d - 21*d^2)*Cos[e + f
*x]*Sin[e + f*x])/(6*a^2*f) - ((c - d)*(c + 8*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^2)/(3*a^2*f*(1 + Sin[e + f*x])) - ((c - d)*Cos[e + f*x]*(c + d*Sin[
e + f*x])^3)/(3*f*(a + a*Sin[e + f*x])^2)
```

Rubi [A] time = 0.362295, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2765, 2977, 2734}

$$\frac{2d\left(8c^2d+c^3-20cd^2+8d^3\right)\cos(e+fx)}{3a^2f}+\frac{d^2\left(2c^2+16cd-21d^2\right)\sin(e+fx)\cos(e+fx)}{6a^2f}+\frac{d^2x\left(12c^2-16cd+7d^2\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^4/(a + a*Sin[e + f*x])^2,x]
```


```
[Out] (d^2*(12*c^2 - 16*c*d + 7*d^2)*x)/(2*a^2) + (2*d*(c^3 + 8*c^2*d - 20*c*d^2
+ 8*d^3)*Cos[e + f*x])/(3*a^2*f) + (d^2*(2*c^2 + 16*c*d - 21*d^2)*Cos[e + f
*x]*Sin[e + f*x])/(6*a^2*f) - ((c - d)*(c + 8*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^2)/(3*a^2*f*(1 + Sin[e + f*x])) - ((c - d)*Cos[e + f*x]*(c + d*Sin[
e + f*x])^3)/(3*f*(a + a*Sin[e + f*x])^2)
```


Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^4}{(a+a\sin(e+fx))^2}\,dx = -\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^3}{3f(a+a\sin(e+fx))^2} - \frac{\int \frac{(c+d\sin(e+fx))^2\left(-a\left(c^2+5cd-3d^2\right)+a(2c-5d)d\sin(e+fx)\right)}{a+a\sin(e+fx)}\,dx}{3a^2}$$

$$= -\frac{(c-d)(c+8d)\cos(e+fx)(c+d\sin(e+fx))^2}{3a^2f(1+\sin(e+fx))} - \frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^3}{3f(a+a\sin(e+fx))^2} -$$

$$= \frac{d^2\left(12c^2-16cd+7d^2\right)x}{2a^2} + \frac{2d\left(c^3+8c^2d-20cd^2+8d^3\right)\cos(e+fx)}{3a^2f} + \frac{d^2\left(2c^2+16cd-21c\right.}{}$$

Mathematica [A] time = 1.93449, size = 378, normalized size = 1.94

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(3d\cos\left(\frac{1}{2}(e+fx)\right)\left(48c^2d(3e+3fx-4)+64c^3-32cd^2(6e+6fx-5)+7d^3(12e+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])^4/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(3*d*(64*c^3 + 48*c^2*d*(-4 + 3*e +
3*f*x) - 32*c*d^2*(-5 + 6*e + 6*f*x) + 7*d^3*(-7 + 12*e + 12*f*x))*Cos[(e +
 f*x)/2] - (16*c^4 + 128*c^3*d + 48*c^2*d^2*(-10 + 3*e + 3*f*x) - 16*c*d^3*
(-41 + 12*e + 12*f*x) + d^4*(-239 + 84*e + 84*f*x))*Cos[(3*(e + f*x))/2] +
3*((16*c - 5*d)*d^3*Cos[(5*(e + f*x))/2] + d^4*Cos[(7*(e + f*x))/2] + 2*(8*
c^4 + 32*c^3*d - 144*c^2*d^2 + 144*c*d^3 - 50*d^4 + 96*c^2*d^2*e - 128*c*d^
3*e + 56*d^4*e + 96*c^2*d^2*f*x - 128*c*d^3*f*x + 56*d^4*f*x + d^2*(48*c^2*
(e + f*x) - 64*c*d*(1 + e + f*x) + d^2*(27 + 28*e + 28*f*x))*Cos[e + f*x] -
 2*(8*c - 3*d)*d^3*Cos[2*(e + f*x)] + d^4*Cos[3*(e + f*x)])*Sin[(e + f*x)/2
])))/(48*a^2*f*(1 + Sin[e + f*x])^2)
```

Maple [B] time = 0.087, size = 618, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^4/(a+a*sin(f*x+e))^2,x)
```

```
[Out] 1/f/a^2*d^4/(1+tan(1/2*f*x+1/2*e)^2)^2*tan(1/2*f*x+1/2*e)^3-8/f/a^2*d^3/(1+
tan(1/2*f*x+1/2*e)^2)^2*tan(1/2*f*x+1/2*e)^2*c+4/f/a^2*d^4/(1+tan(1/2*f*x+1
/2*e)^2)^2*tan(1/2*f*x+1/2*e)^2-1/f/a^2*d^4/(1+tan(1/2*f*x+1/2*e)^2)^2*tan(
1/2*f*x+1/2*e)-8/f/a^2*d^3/(1+tan(1/2*f*x+1/2*e)^2)^2*c+4/f/a^2*d^4/(1+tan(
1/2*f*x+1/2*e)^2)^2+12/f/a^2*d^2*arctan(tan(1/2*f*x+1/2*e))*c^2-16/f/a^2*d^
3*arctan(tan(1/2*f*x+1/2*e))*c+7/f/a^2*d^4*arctan(tan(1/2*f*x+1/2*e))-2/f/a
^2/(tan(1/2*f*x+1/2*e)+1)*c^4+12/f/a^2/(tan(1/2*f*x+1/2*e)+1)*c^2*d^2-16/f/
a^2/(tan(1/2*f*x+1/2*e)+1)*c*d^3+6/f/a^2/(tan(1/2*f*x+1/2*e)+1)*d^4+2/f/a^2
/(tan(1/2*f*x+1/2*e)+1)^2*c^4-8/f/a^2/(tan(1/2*f*x+1/2*e)+1)^2*c^3*d+12/f/a
```

```
^2/(tan(1/2*f*x+1/2*e)+1)^2*c^2*d^2-8/f/a^2/(tan(1/2*f*x+1/2*e)+1)^2*c*d^3+
2/f/a^2/(tan(1/2*f*x+1/2*e)+1)^2*d^4-4/3/f/a^2/(tan(1/2*f*x+1/2*e)+1)^3*c^4
+16/3/f/a^2/(tan(1/2*f*x+1/2*e)+1)^3*c^3*d-8/f/a^2/(tan(1/2*f*x+1/2*e)+1)^3
*c^2*d^2+16/3/f/a^2/(tan(1/2*f*x+1/2*e)+1)^3*c*d^3-4/3/f/a^2/(tan(1/2*f*x+1
/2*e)+1)^3*d^4
```

Maxima [B] time = 1.7998, size = 1226, normalized size = 6.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^4/(a+a*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/3*(d^4*((75*sin(f*x + e)/(cos(f*x + e) + 1) + 97*sin(f*x + e)^2/(cos(f*x
+ e) + 1)^2 + 126*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 98*sin(f*x + e)^4/(
cos(f*x + e) + 1)^4 + 63*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 21*sin(f*x +
 e)^6/(cos(f*x + e) + 1)^6 + 32)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) +
1) + 5*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 7*a^2*sin(f*x + e)^3/(cos(
f*x + e) + 1)^3 + 7*a^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 5*a^2*sin(f*x
 + e)^5/(cos(f*x + e) + 1)^5 + 3*a^2*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 +
a^2*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) + 21*arctan(sin(f*x + e)/(cos(f*x
+ e) + 1))/a^2) - 16*c*d^3*((12*sin(f*x + e)/(cos(f*x + e) + 1) + 11*sin(f*
x + e)^2/(cos(f*x + e) + 1)^2 + 9*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*s
in(f*x + e)^4/(cos(f*x + e) + 1)^4 + 5)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x
+ e) + 1) + 4*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4*a^2*sin(f*x + e)^
3/(cos(f*x + e) + 1)^3 + 3*a^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^2*si
n(f*x + e)^5/(cos(f*x + e) + 1)^5) + 3*arctan(sin(f*x + e)/(cos(f*x + e) +
1))/a^2) + 12*c^2*d^2*((9*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^
2/(cos(f*x + e) + 1)^2 + 4)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) +
3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e
) + 1)^3) + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) - 2*c^4*(3*sin(f
*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2)/(a^
2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) - 8*c^3*d*(3*sin(f*x
+ e)/(cos(f*x + e) + 1) + 1)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) +
 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3))/f
```

Fricas [B] time = 1.67779, size = 1000, normalized size = 5.13

$$-\frac{3\,d^4\cos\left(fx+e\right)^4-2\,c^4+8\,c^3d-12\,c^2d^2+8\,cd^3-2\,d^4+6\left(4\,cd^3-d^4\right)\cos\left(fx+e\right)^3+6\left(12\,c^2d^2-16\,cd^3+7\,d^4\right)f}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^4/(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -1/6*(3*d^4*cos(f*x + e)^4 - 2*c^4 + 8*c^3*d - 12*c^2*d^2 + 8*c*d^3 - 2*d^4
 + 6*(4*c*d^3 - d^4)*cos(f*x + e)^3 + 6*(12*c^2*d^2 - 16*c*d^3 + 7*d^4)*f*x
 - (2*c^4 + 16*c^3*d - 60*c^2*d^2 + 88*c*d^3 - 31*d^4 + 3*(12*c^2*d^2 - 16*
c*d^3 + 7*d^4)*f*x)*cos(f*x + e)^2 - (4*c^4 + 8*c^3*d - 48*c^2*d^2 + 104*c*
d^3 - 38*d^4 - 3*(12*c^2*d^2 - 16*c*d^3 + 7*d^4)*f*x)*cos(f*x + e) + (3*d^4
```

```
*cos(f*x + e)^3 + 2*c^4 - 8*c^3*d + 12*c^2*d^2 - 8*c*d^3 + 2*d^4 + 6*(12*c^
2*d^2 - 16*c*d^3 + 7*d^4)*f*x - 3*(8*c*d^3 - 3*d^4)*cos(f*x + e)^2 - (2*c^4
 + 16*c^3*d - 60*c^2*d^2 + 112*c*d^3 - 40*d^4 - 3*(12*c^2*d^2 - 16*c*d^3 +
7*d^4)*f*x)*cos(f*x + e))*sin(f*x + e))/(a^2*f*cos(f*x + e)^2 - a^2*f*cos(f
*x + e) - 2*a^2*f - (a^2*f*cos(f*x + e) + 2*a^2*f)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**4/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.32684, size = 456, normalized size = 2.34

$$\frac{\frac{3\left(12c^2d^2-16cd^3+7d^4\right)(fx+e)}{a^2}+\frac{6\left(d^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-8cd^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+4d^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-d^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-8cd^3+4d^4\right)}{\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+1\right)^2a^2}-\frac{4\left(3c^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^4/(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/6*(3*(12*c^2*d^2 - 16*c*d^3 + 7*d^4)*(f*x + e)/a^2 + 6*(d^4*tan(1/2*f*x +
 1/2*e)^3 - 8*c*d^3*tan(1/2*f*x + 1/2*e)^2 + 4*d^4*tan(1/2*f*x + 1/2*e)^2 -
 d^4*tan(1/2*f*x + 1/2*e) - 8*c*d^3 + 4*d^4)/((tan(1/2*f*x + 1/2*e)^2 + 1)^
2*a^2) - 4*(3*c^4*tan(1/2*f*x + 1/2*e)^2 - 18*c^2*d^2*tan(1/2*f*x + 1/2*e)^
2 + 24*c*d^3*tan(1/2*f*x + 1/2*e)^2 - 9*d^4*tan(1/2*f*x + 1/2*e)^2 + 3*c^4*
tan(1/2*f*x + 1/2*e) + 12*c^3*d*tan(1/2*f*x + 1/2*e) - 54*c^2*d^2*tan(1/2*f
*x + 1/2*e) + 60*c*d^3*tan(1/2*f*x + 1/2*e) - 21*d^4*tan(1/2*f*x + 1/2*e) +
 2*c^4 + 4*c^3*d - 24*c^2*d^2 + 28*c*d^3 - 10*d^4)/(a^2*(tan(1/2*f*x + 1/2*
e) + 1)^3))/f
```

3.463 $\int \frac{(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^2}\,dx$

Optimal. Leaf size=120

$$\frac{d^2(c-4d)\cos(e+fx)}{3a^2f}+\frac{d^2x(3c-2d)}{a^2}-\frac{(c+6d)(c-d)^2\cos(e+fx)}{3a^2f(\sin(e+fx)+1)}-\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^2}{3f(a\sin(e+fx)+a)^2}$$

```
[Out] ((3*c - 2*d)*d^2*x)/a^2 + ((c - 4*d)*d^2*Cos[e + f*x])/(3*a^2*f) - ((c - d)
^2*(c + 6*d)*Cos[e + f*x])/(3*a^2*f*(1 + Sin[e + f*x])) - ((c - d)*Cos[e +
f*x]*(c + d*Sin[e + f*x])^2)/(3*f*(a + a*Sin[e + f*x])^2)
```

Rubi [A] time = 0.367196, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2765, 2968, 3023, 2735, 2648}

$$\frac{d^2(c-4d)\cos(e+fx)}{3a^2f}+\frac{d^2x(3c-2d)}{a^2}-\frac{(c+6d)(c-d)^2\cos(e+fx)}{3a^2f(\sin(e+fx)+1)}-\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^2}{3f(a\sin(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^3/(a + a*Sin[e + f*x])^2,x]
```


```
[Out] ((3*c - 2*d)*d^2*x)/a^2 + ((c - 4*d)*d^2*Cos[e + f*x])/(3*a^2*f) - ((c - d)
^2*(c + 6*d)*Cos[e + f*x])/(3*a^2*f*(1 + Sin[e + f*x])) - ((c - d)*Cos[e +
f*x]*(c + d*Sin[e + f*x])^2)/(3*f*(a + a*Sin[e + f*x])^2)
```


Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^2}\,dx = -\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^2}{3f(a+a\sin(e+fx))^2} - \frac{\int \frac{(c+d\sin(e+fx))\left(-a\left(c^2+4cd-2d^2\right)+a(c-4d)d\sin(e+fx)\right)}{a+a\sin(e+fx)}\,dx}{3a^2}$$

$$= -\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^2}{3f(a+a\sin(e+fx))^2} - \frac{\int \frac{-ac\left(c^2+4cd-2d^2\right)+\left(ac(c-4d)d-ad\left(c^2+4cd-2d^2\right)\right)\sin(e+fx)+a}{a+a\sin(e+fx)}}{3a^2}$$

$$= \frac{(c-4d)d^2\cos(e+fx)}{3a^2f} - \frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^2}{3f(a+a\sin(e+fx))^2} - \frac{\int \frac{-a^2c\left(c^2+4cd-2d^2\right)-3a^2(3c-2d}{a+a\sin(e+fx)}}{3a^3}$$

$$= \frac{(3c-2d)d^2x}{a^2} + \frac{(c-4d)d^2\cos(e+fx)}{3a^2f} - \frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^2}{3f(a+a\sin(e+fx))^2} + \frac{\left((c-d)^2(\right.}{}$$

$$= \frac{(3c-2d)d^2x}{a^2} + \frac{(c-4d)d^2\cos(e+fx)}{3a^2f} - \frac{(c-d)^2(c+6d)\cos(e+fx)}{3f\left(a^2+a^2\sin(e+fx)\right)} - \frac{(c-d)\cos(e+fx)(c}{3f(a+a\sin}$$

Mathematica [A] time = 0.368427, size = 212, normalized size = 1.77

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(3d^2(3c-2d)(e+fx)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3+2(c-d)^3\sin\left(\frac{1}{2}(e+fx\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])^3/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(2*(c - d)^3*Sin[(e + f*x)/2] - (c -
 d)^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + 2*(c - d)^2*(c + 8*d)*Sin[(e
+ f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + 3*(3*c - 2*d)*d^2*(e +
f*x)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 - 3*d^3*Cos[e + f*x]*(Cos[(e +
 f*x)/2] + Sin[(e + f*x)/2])^3))/(3*a^2*f*(1 + Sin[e + f*x])^2)
```

Maple [B] time = 0.076, size = 340, normalized size = 2.8

$$-2\,\frac{d^3}{a^2f\left(1+\left(\tan\left(1/2\,fx+e/2\right)\right)^2\right)}+6\,\frac{d^2\arctan\left(\tan\left(1/2\,fx+e/2\right)\right)c}{a^2f}-4\,\frac{d^3\arctan\left(\tan\left(1/2\,fx+e/2\right)\right)}{a^2f}-2\,\frac{}{a^2f\left(\tan\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^2,x)
```

```
[Out] -2/f/a^2*d^3/(1+tan(1/2*f*x+1/2*e)^2)+6/f/a^2*d^2*arctan(tan(1/2*f*x+1/2*e)
)*c-4/f/a^2*d^3*arctan(tan(1/2*f*x+1/2*e))-2/f/a^2/(tan(1/2*f*x+1/2*e)+1)*c
^3+6/f/a^2/(tan(1/2*f*x+1/2*e)+1)*c*d^2-4/f/a^2/(tan(1/2*f*x+1/2*e)+1)*d^3+
2/f/a^2/(tan(1/2*f*x+1/2*e)+1)^2*c^3-6/f/a^2/(tan(1/2*f*x+1/2*e)+1)^2*c^2*d
+6/f/a^2/(tan(1/2*f*x+1/2*e)+1)^2*c*d^2-2/f/a^2/(tan(1/2*f*x+1/2*e)+1)^2*d^
3-4/3/f/a^2/(tan(1/2*f*x+1/2*e)+1)^3*c^3+4/f/a^2/(tan(1/2*f*x+1/2*e)+1)^3*c
^2*d-4/f/a^2/(tan(1/2*f*x+1/2*e)+1)^3*c*d^2+4/3/f/a^2/(tan(1/2*f*x+1/2*e)+1
)^3*d^3
```

Maxima [B] time = 1.7316, size = 798, normalized size = 6.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] -2/3*(2*d^3*((12*sin(f*x + e)/(cos(f*x + e) + 1) + 11*sin(f*x + e)^2/(cos(f
*x + e) + 1)^2 + 9*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^4/(
cos(f*x + e) + 1)^4 + 5)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 4*a
^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4*a^2*sin(f*x + e)^3/(cos(f*x + e)
 + 1)^3 + 3*a^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^2*sin(f*x + e)^5/(c
os(f*x + e) + 1)^5) + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) - 3*c*
d^2*((9*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e) +
1)^2 + 4)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)
^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 3*arct
an(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) + c^3*(3*sin(f*x + e)/(cos(f*x + e
) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2)/(a^2 + 3*a^2*sin(f*x +
e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin
(f*x + e)^3/(cos(f*x + e) + 1)^3) + 3*c^2*d*(3*sin(f*x + e)/(cos(f*x + e) +
 1) + 1)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^
2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3))/f
```

Fricas [B] time = 1.62919, size = 699, normalized size = 5.82

$$-\frac{3\,d^3\cos\left(fx+e\right)^3 - c^3 + 3\,c^2d - 3\,cd^2 + d^3 + 6\left(3\,cd^2 - 2\,d^3\right)fx - \left(c^3 + 6\,c^2d - 15\,cd^2 + 11\,d^3 + 3\left(3\,cd^2 - 2\,d^3\right)fx\right)c\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -1/3*(3*d^3*cos(f*x + e)^3 - c^3 + 3*c^2*d - 3*c*d^2 + d^3 + 6*(3*c*d^2 - 2
*d^3)*f*x - (c^3 + 6*c^2*d - 15*c*d^2 + 11*d^3 + 3*(3*c*d^2 - 2*d^3)*f*x)*c
os(f*x + e)^2 - (2*c^3 + 3*c^2*d - 12*c*d^2 + 13*d^3 - 3*(3*c*d^2 - 2*d^3)*
f*x)*cos(f*x + e) - (3*d^3*cos(f*x + e)^2 - c^3 + 3*c^2*d - 3*c*d^2 + d^3 -
 6*(3*c*d^2 - 2*d^3)*f*x + (c^3 + 6*c^2*d - 15*c*d^2 + 14*d^3 - 3*(3*c*d^2
- 2*d^3)*f*x)*cos(f*x + e))*sin(f*x + e))/(a^2*f*cos(f*x + e)^2 - a^2*f*cos
(f*x + e) - 2*a^2*f - (a^2*f*cos(f*x + e) + 2*a^2*f)*sin(f*x + e))
```

Sympy [A] time = 29.065, size = 3471, normalized size = 28.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**3/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((2*c**3*tan(e/2 + f*x/2)**5/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**
2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2
 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 2*c**3*tan(e/2 + f*x
/2)**3/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a*
*2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2
 + f*x/2) + 3*a**2*f) - 2*c**3*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/
2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a
**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 2*c**3/
(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*ta
n(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/
2) + 3*a**2*f) - 18*c**2*d*tan(e/2 + f*x/2)**3/(3*a**2*f*tan(e/2 + f*x/2)**
5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*
f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 6*c**2*d*ta
n(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)
**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**
2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 18*c**2*d*tan(e/2 + f*x/2)/(3*a**2*f*tan
(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2
)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f
) - 6*c**2*d/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 +
 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*t
an(e/2 + f*x/2) + 3*a**2*f) + 9*c*d**2*f*x*tan(e/2 + f*x/2)**5/(3*a**2*f*ta
n(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/
2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*
f) + 27*c*d**2*f*x*tan(e/2 + f*x/2)**4/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a*
*2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/
2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 36*c*d**2*f*x*tan(e
/2 + f*x/2)**3/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4
 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f
*tan(e/2 + f*x/2) + 3*a**2*f) + 36*c*d**2*f*x*tan(e/2 + f*x/2)**2/(3*a**2*f
*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f
*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a*
*2*f) + 27*c*d**2*f*x*tan(e/2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a*
*2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/
2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 9*c*d**2*f*x/(3*a**
2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2
+ f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3
*a**2*f) - 6*c*d**2*tan(e/2 + f*x/2)**5/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a
**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e
/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 30*c*d**2*tan(e/2
+ f*x/2)**3/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 +
12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*ta
n(e/2 + f*x/2) + 3*a**2*f) + 18*c*d**2*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/
2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**
3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) +
 36*c*d**2*tan(e/2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/
2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)*
*2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 18*c*d**2/(3*a**2*f*tan(e/2 +
f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 +
12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 6*d
**3*f*x*tan(e/2 + f*x/2)**5/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/
2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)*
```

```
*2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 18*d**3*f*x*tan(e/2 + f*x/2)**
4/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*
tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*
x/2) + 3*a**2*f) - 24*d**3*f*x*tan(e/2 + f*x/2)**3/(3*a**2*f*tan(e/2 + f*x/
2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a
**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 24*d**3
*f*x*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 +
 f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2
+ 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 18*d**3*f*x*tan(e/2 + f*x/2)/(3*a
**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/
2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) +
 3*a**2*f) - 6*d**3*f*x/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 +
f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 +
 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 4*d**3*tan(e/2 + f*x/2)**5/(3*a**2
*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 +
 f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*
a**2*f) - 20*d**3*tan(e/2 + f*x/2)**3/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**
2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2
 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 28*d**3*tan(e/2 + f*
x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a
**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/
2 + f*x/2) + 3*a**2*f) - 36*d**3*tan(e/2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2
)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a*
*2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 16*d**3/
(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*ta
n(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/
2) + 3*a**2*f), Ne(f, 0)), (x*(c + d*sin(e))**3/(a*sin(e) + a)**2, True))
```

Giac [A] time = 1.28716, size = 282, normalized size = 2.35

$$\frac{\frac{3\left(3\,cd^2-2\,d^3\right)(fx+e)}{a^2}-\frac{6\,d^3}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)a^2}-\frac{2\left(3\,c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-9\,cd^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+6\,d^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+3\,c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+9\,c^2d\tan\left(\frac{1}{2}\right.\right.}{a^2\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/3*(3*(3*c*d^2 - 2*d^3)*(f*x + e)/a^2 - 6*d^3/((tan(1/2*f*x + 1/2*e)^2 + 1
)*a^2) - 2*(3*c^3*tan(1/2*f*x + 1/2*e)^2 - 9*c*d^2*tan(1/2*f*x + 1/2*e)^2 +
 6*d^3*tan(1/2*f*x + 1/2*e)^2 + 3*c^3*tan(1/2*f*x + 1/2*e) + 9*c^2*d*tan(1/
2*f*x + 1/2*e) - 27*c*d^2*tan(1/2*f*x + 1/2*e) + 15*d^3*tan(1/2*f*x + 1/2*e
) + 2*c^3 + 3*c^2*d - 12*c*d^2 + 7*d^3)/(a^2*(tan(1/2*f*x + 1/2*e) + 1)^3))
/f
```

3.464 $\int \frac{(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^2}\,dx$

Optimal. Leaf size=85

$$-\frac{(c-d)(c+4d)\cos(e+fx)}{3a^2f(\sin(e+fx)+1)}+\frac{d^2x}{a^2}-\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))}{3f(a\sin(e+fx)+a)^2}$$

```
[Out] (d^2*x)/a^2 - ((c - d)*(c + 4*d)*Cos[e + f*x])/(3*a^2*f*(1 + Sin[e + f*x]))
 - ((c - d)*Cos[e + f*x]*(c + d*Sin[e + f*x]))/(3*f*(a + a*Sin[e + f*x])^2)
```

Rubi [A] time = 0.141571, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2760, 2735, 2648}

$$-\frac{(c-d)(c+4d)\cos(e+fx)}{3a^2f(\sin(e+fx)+1)}+\frac{d^2x}{a^2}-\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))}{3f(a\sin(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^2/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] (d^2*x)/a^2 - ((c - d)*(c + 4*d)*Cos[e + f*x])/(3*a^2*f*(1 + Sin[e + f*x]))
 - ((c - d)*Cos[e + f*x]*(c + d*Sin[e + f*x]))/(3*f*(a + a*Sin[e + f*x])^2)
```

Rule 2760

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^2, x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^m*(c + d*Sin[e + f*x]))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*(2*m + 1)),
Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*c*d*(m - 1) + b*(d^2 + c^2*(m + 1))
 + d*(a*d*(m - 1) + b*c*(m + 2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^2}\,dx &= -\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))}{3f(a+a\sin(e+fx))^2}-\frac{\int \frac{-a\left(c^2+3cd-d^2\right)-3ad^2\sin(e+fx)}{a+a\sin(e+fx)}\,dx}{3a^2}\\ &= \frac{d^2x}{a^2}-\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))}{3f(a+a\sin(e+fx))^2}+\frac{((c-d)(c+4d))\int \frac{1}{a+a\sin(e+fx)}\,dx}{3a}\\ &= \frac{d^2x}{a^2}-\frac{(c-d)(c+4d)\cos(e+fx)}{3f\left(a^2+a^2\sin(e+fx)\right)}-\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))}{3f(a+a\sin(e+fx))^2}\end{aligned}$$

Mathematica [B] time = 0.283661, size = 172, normalized size = 2.02

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(2\left(c^2+4cd-5d^2\right)\sin\left(\frac{1}{2}(e+fx)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^2+2(c-\right.}{3a^2f(\sin(e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])^2/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(2*(c - d)^2*Sin[(e + f*x)/2] - (c -
 d)^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + 2*(c^2 + 4*c*d - 5*d^2)*Sin[(
e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + 3*d^2*(e + f*x)*(Cos[
(e + f*x)/2] + Sin[(e + f*x)/2])^3))/(3*a^2*f*(1 + Sin[e + f*x])^2)
```

Maple [B] time = 0.059, size = 213, normalized size = 2.5

$$2\,\frac{d^2\arctan\left(\tan\left(1/2\,fx+e/2\right)\right)}{a^2f}-2\,\frac{c^2}{a^2f\left(\tan\left(1/2\,fx+e/2\right)+1\right)}+2\,\frac{d^2}{a^2f\left(\tan\left(1/2\,fx+e/2\right)+1\right)}+2\,\frac{c}{a^2f\left(\tan\left(1/2\,f\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^2,x)
```

```
[Out] 2/f/a^2*d^2*arctan(tan(1/2*f*x+1/2*e))-2/f/a^2/(tan(1/2*f*x+1/2*e)+1)*c^2+2
/f/a^2/(tan(1/2*f*x+1/2*e)+1)*d^2+2/f/a^2/(tan(1/2*f*x+1/2*e)+1)^2*c^2-4/f/
a^2/(tan(1/2*f*x+1/2*e)+1)^2*c*d+2/f/a^2/(tan(1/2*f*x+1/2*e)+1)^2*d^2-4/3/f
/a^2/(tan(1/2*f*x+1/2*e)+1)^3*c^2+8/3/f/a^2/(tan(1/2*f*x+1/2*e)+1)^3*c*d-4/
3/f/a^2/(tan(1/2*f*x+1/2*e)+1)^3*d^2
```

Maxima [B] time = 1.70652, size = 486, normalized size = 5.72

$$\frac{2\left(d^2\left(\frac{\frac{9\sin(fx+e)}{\cos(fx+e)+1}+\frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+4}{a^2+\frac{3a^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{3a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}+\frac{3\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^2}\right)-\frac{c^2\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}+\frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+2\right)}{a^2+\frac{3a^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{3a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}-\frac{\cdots}{a^2+\frac{3a^2\sin}{\cos(f}}\right.}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 2/3*(d^2*((9*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x +
e) + 1)^2 + 4)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x
 + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 3
*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) - c^2*(3*sin(f*x + e)/(cos(f*
x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2)/(a^2 + 3*a^2*sin(f
*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^
2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) - 2*c*d*(3*sin(f*x + e)/(cos(f*x + e
) + 1) + 1)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x +
e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3))/f
```

Fricas [B] time = 1.62509, size = 454, normalized size = 5.34

$$-\frac{6\,d^2fx-\left(3\,d^2fx+c^2+4\,cd-5\,d^2\right)\cos\left(fx+e\right)^2-c^2+2\,cd-d^2+\left(3\,d^2fx-2\,c^2-2\,cd+4\,d^2\right)\cos\left(fx+e\right)+\left(6\,d^2f\right.}{3\left(a^2f\cos\left(fx+e\right)^2-a^2f\cos\left(fx+e\right)-2\,a^2f-\left(a^2f\cos\left(fx+e\right)\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -1/3*(6*d^2*f*x - (3*d^2*f*x + c^2 + 4*c*d - 5*d^2)*cos(f*x + e)^2 - c^2 +
2*c*d - d^2 + (3*d^2*f*x - 2*c^2 - 2*c*d + 4*d^2)*cos(f*x + e) + (6*d^2*f*x
 + c^2 - 2*c*d + d^2 + (3*d^2*f*x - c^2 - 4*c*d + 5*d^2)*cos(f*x + e))*sin(
f*x + e))/(a^2*f*cos(f*x + e)^2 - a^2*f*cos(f*x + e) - 2*a^2*f - (a^2*f*cos
(f*x + e) + 2*a^2*f)*sin(f*x + e))
```

Sympy [A] time = 9.74888, size = 853, normalized size = 10.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**2/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((2*c**2*tan(e/2 + f*x/2)**3/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**
2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 2*c**2/(3
*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e
/2 + f*x/2) + 3*a**2*f) + 4*c*d*tan(e/2 + f*x/2)**3/(3*a**2*f*tan(e/2 + f*x
/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*
f) + 12*c*d*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*ta
n(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 3*d**2*f*x*tan(
e/2 + f*x/2)**3/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**
2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 9*d**2*f*x*tan(e/2 + f*x/2)**2/
(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan
(e/2 + f*x/2) + 3*a**2*f) + 9*d**2*f*x*tan(e/2 + f*x/2)/(3*a**2*f*tan(e/2 +
 f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a
**2*f) + 3*d**2*f*x/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/
2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 6*d**2*tan(e/2 + f*x/2)**3/
(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan
(e/2 + f*x/2) + 3*a**2*f) - 12*d**2*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 +
 f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a
**2*f) + 2*d**2/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**
2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f), Ne(f, 0)), (x*(c + d*sin(e))**2/
(a*sin(e) + a)**2, True))
```

Giac [A] time = 1.2666, size = 178, normalized size = 2.09

$$\frac{\frac{3\left(fx+e\right)d^2}{a^2}-\frac{2\left(3\,c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-3\,d^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+3\,c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+6\,cd\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-9\,d^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+2\,c^2+2\,cd-4\,d^2\right)}{a^2\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^3}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/3*(3*(f*x + e)*d^2/a^2 - 2*(3*c^2*tan(1/2*f*x + 1/2*e)^2 - 3*d^2*tan(1/2*
f*x + 1/2*e)^2 + 3*c^2*tan(1/2*f*x + 1/2*e) + 6*c*d*tan(1/2*f*x + 1/2*e) -
9*d^2*tan(1/2*f*x + 1/2*e) + 2*c^2 + 2*c*d - 4*d^2)/(a^2*(tan(1/2*f*x + 1/2
*e) + 1)^3))/f
```

3.465 $\int \frac{c+d\sin(e+fx)}{(a+a\sin(e+fx))^2}\,dx$

Optimal. Leaf size=65

$$-\frac{(c+2d)\cos(e+fx)}{3f\left(a^2\sin(e+fx)+a^2\right)}-\frac{(c-d)\cos(e+fx)}{3f(a\sin(e+fx)+a)^2}$$

```
[Out] -((c - d)*Cos[e + f*x])/(3*f*(a + a*Sin[e + f*x])^2) - ((c + 2*d)*Cos[e + f
*x])/(3*f*(a^2 + a^2*Sin[e + f*x]))
```

Rubi [A] time = 0.0519345, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2750, 2648}

$$-\frac{(c+2d)\cos(e+fx)}{3f\left(a^2\sin(e+fx)+a^2\right)}-\frac{(c-d)\cos(e+fx)}{3f(a\sin(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] -((c - d)*Cos[e + f*x])/(3*f*(a + a*Sin[e + f*x])^2) - ((c + 2*d)*Cos[e + f
*x])/(3*f*(a^2 + a^2*Sin[e + f*x]))
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{c+d\sin(e+fx)}{(a+a\sin(e+fx))^2}\,dx &= -\frac{(c-d)\cos(e+fx)}{3f(a+a\sin(e+fx))^2}+\frac{(c+2d)\int \frac{1}{a+a\sin(e+fx)}\,dx}{3a}\\ &= -\frac{(c-d)\cos(e+fx)}{3f(a+a\sin(e+fx))^2}-\frac{(c+2d)\cos(e+fx)}{3f\left(a^2+a^2\sin(e+fx)\right)}\end{aligned}$$

Mathematica [A] time = 0.0569325, size = 43, normalized size = 0.66

$$-\frac{\cos(e+fx)((c+2d)\sin(e+fx)+2c+d)}{3a^2f(\sin(e+fx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] -(Cos[e + f*x]*(2*c + d + (c + 2*d)*Sin[e + f*x]))/(3*a^2*f*(1 + Sin[e + f*
x])^2)
```

Maple [A] time = 0.051, size = 70, normalized size = 1.1

$$2\,{\frac {1}{{a}^{2}f} \left(-{\frac {c}{\tan \left(1/2\,fx+e/2 \right) +1}}-1/2\,{\frac {-2\,c+2\,d}{ \left(\tan \left(1/2\,fx+e/2 \right) +1 \right) ^{2}}}-1/3\,{\frac {2\,c-2\,d}{ \left(\tan \left(1/2\,fx+e/2 \right) +1 \right) ^{3}}} \right) }$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))/(a+a*sin(f*x+e))^2,x)
```

```
[Out] 2/f/a^2*(-c/(tan(1/2*f*x+1/2*e)+1)-1/2*(-2*c+2*d)/(tan(1/2*f*x+1/2*e)+1)^2-
1/3*(2*c-2*d)/(tan(1/2*f*x+1/2*e)+1)^3)
```

Maxima [B] time = 1.17, size = 289, normalized size = 4.45

$$-\frac{2\left(\frac{c\left(\frac{3\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{3\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+2\right)}{a^{2}+\frac{3a^{2}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{3a^{2}\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{a^{2}\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}}+\frac{d\left(\frac{3\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+1\right)}{a^{2}+\frac{3a^{2}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{3a^{2}\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{a^{2}\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}}\right)}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))/(a+a*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] -2/3*(c*(3*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e)
 + 1)^2 + 2)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x +
 e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + d*(
3*sin(f*x + e)/(cos(f*x + e) + 1) + 1)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x +
 e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(
cos(f*x + e) + 1)^3))/f
```

Fricas [A] time = 1.52365, size = 288, normalized size = 4.43

$$\frac{\left(c+2\,d\right)\cos\left(fx+e\right)^{2}+\left(2\,c+d\right)\cos\left(fx+e\right)+\left(\left(c+2\,d\right)\cos\left(fx+e\right)-c+d\right)\sin\left(fx+e\right)+c-d}{3\left(a^{2}f\cos\left(fx+e\right)^{2}-a^{2}f\cos\left(fx+e\right)-2\,a^{2}f-\left(a^{2}f\cos\left(fx+e\right)+2\,a^{2}f\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))/(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/3*((c + 2*d)*cos(f*x + e)^2 + (2*c + d)*cos(f*x + e) + ((c + 2*d)*cos(f*x
 + e) - c + d)*sin(f*x + e) + c - d)/(a^2*f*cos(f*x + e)^2 - a^2*f*cos(f*x
+ e) - 2*a^2*f - (a^2*f*cos(f*x + e) + 2*a^2*f)*sin(f*x + e))
```

Sympy [A] time = 4.32901, size = 309, normalized size = 4.75

$$\begin{cases} \frac{2c\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+3a^2f} - \frac{2c}{3a^2f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+3a^2f} + \frac{}{3a^2f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+} \\ \frac{x(c+d\sin(e))}{(a\sin(e)+a)^2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((2*c*tan(e/2 + f*x/2)**3/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f
*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 2*c/(3*a**2*
f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f
*x/2) + 3*a**2*f) + 2*d*tan(e/2 + f*x/2)**3/(3*a**2*f*tan(e/2 + f*x/2)**3 +
 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 6*d
*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x
/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f), Ne(f, 0)), (x*(c + d*sin(e)
)/(a*sin(e) + a)**2, True))
```

Giac [A] time = 1.25629, size = 92, normalized size = 1.42

$$-\frac{2\left(3\,c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+3\,c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+3\,d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+2\,c+d\right)}{3\,a^2f\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))/(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] -2/3*(3*c*tan(1/2*f*x + 1/2*e)^2 + 3*c*tan(1/2*f*x + 1/2*e) + 3*d*tan(1/2*f
*x + 1/2*e) + 2*c + d)/(a^2*f*(tan(1/2*f*x + 1/2*e) + 1)^3)
```

3.466 $\int \frac{1}{(a+a\sin(e+fx))^2}\,dx$

Optimal. Leaf size=55

$$-\frac{\cos(e+fx)}{3f\left(a^2\sin(e+fx)+a^2\right)}-\frac{\cos(e+fx)}{3f(a\sin(e+fx)+a)^2}$$

```
[Out] -Cos[e + f*x]/(3*f*(a + a*Sin[e + f*x])^2) - Cos[e + f*x]/(3*f*(a^2 + a^2*S
in[e + f*x]))
```

Rubi [A] time = 0.0278865, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2650, 2648}

$$-\frac{\cos(e+fx)}{3f\left(a^2\sin(e+fx)+a^2\right)}-\frac{\cos(e+fx)}{3f(a\sin(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(-2),x]
```

```
[Out] -Cos[e + f*x]/(3*f*(a + a*Sin[e + f*x])^2) - Cos[e + f*x]/(3*f*(a^2 + a^2*S
in[e + f*x]))
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+a\sin(e+fx))^2}\,dx &= -\frac{\cos(e+fx)}{3f(a+a\sin(e+fx))^2}+\frac{\int \frac{1}{a+a\sin(e+fx)}\,dx}{3a}\\ &= -\frac{\cos(e+fx)}{3f(a+a\sin(e+fx))^2}-\frac{\cos(e+fx)}{3f\left(a^2+a^2\sin(e+fx)\right)}\end{aligned}$$

Mathematica [A] time = 0.0995097, size = 54, normalized size = 0.98

$$-\frac{-4\sin(e+fx)+\sin(2(e+fx))+4\cos(e+fx)+\cos(2(e+fx))-3}{6a^2f(\sin(e+fx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(-2),x]
```

```
[Out] -(-3 + 4*Cos[e + f*x] + Cos[2*(e + f*x)] - 4*Sin[e + f*x] + Sin[2*(e + f*x)
])/(6*a^2*f*(1 + Sin[e + f*x])^2)
```

Maple [A] time = 0.039, size = 53, normalized size = 1.

$$2\frac{1}{a^2f}\left(\left(\tan\left(1/2\,fx+e/2\right)+1\right)^{-2}-\left(\tan\left(1/2\,fx+e/2\right)+1\right)^{-1}-2/3\,\left(\tan\left(1/2\,fx+e/2\right)+1\right)^{-3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^2,x)
```

```
[Out] 2/f/a^2*(1/(tan(1/2*f*x+1/2*e)+1)^2-1/(tan(1/2*f*x+1/2*e)+1)-2/3/(tan(1/2*f
*x+1/2*e)+1)^3)
```

Maxima [B] time = 1.18346, size = 158, normalized size = 2.87

$$-\frac{2\left(\frac{3\,\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{3\,\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}+2\right)}{3\left(a^2+\frac{3\,a^2\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{3\,a^2\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}+\frac{a^2\sin\left(fx+e\right)^3}{\left(\cos\left(fx+e\right)+1\right)^3}\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] -2/3*(3*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e) +
1)^2 + 2)/((a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e
)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3)*f)
```

Fricas [A] time = 1.45571, size = 239, normalized size = 4.35

$$\frac{\cos\left(fx+e\right)^2+\left(\cos\left(fx+e\right)-1\right)\sin\left(fx+e\right)+2\,\cos\left(fx+e\right)+1}{3\left(a^2f\cos\left(fx+e\right)^2-a^2f\cos\left(fx+e\right)-2\,a^2f-\left(a^2f\cos\left(fx+e\right)+2\,a^2f\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(cos(f*x + e)^2 + (cos(f*x + e) - 1)*sin(f*x + e) + 2*cos(f*x + e) + 1)
/(a^2*f*cos(f*x + e)^2 - a^2*f*cos(f*x + e) - 2*a^2*f - (a^2*f*cos(f*x + e)
 + 2*a^2*f)*sin(f*x + e))
```

Sympy [A] time = 1.99466, size = 146, normalized size = 2.65

$$\begin{cases} \frac{2\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+3a^2f}-\frac{2}{3a^2f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+3a^2f} & \text{for } f\neq 0 \\ \frac{x}{\left(a\sin\left(e\right)+a\right)^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((2*tan(e/2 + f*x/2)**3/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*t
an(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 2/(3*a**2*f*ta
n(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2
) + 3*a**2*f), Ne(f, 0)), (x/(a*sin(e) + a)**2, True))
```

Giac [A] time = 1.28746, size = 68, normalized size = 1.24

$$-\frac{2\left(3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+2\right)}{3a^2f\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] -2/3*(3*tan(1/2*f*x + 1/2*e)^2 + 3*tan(1/2*f*x + 1/2*e) + 2)/(a^2*f*(tan(1/
2*f*x + 1/2*e) + 1)^3)
```

3.467 $\int \frac{1}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))} dx$

Optimal. Leaf size=131

$$\frac{2d^2\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{a^2f(c-d)^2\sqrt{c^2-d^2}}-\frac{(c-4d)\cos(e+fx)}{3a^2f(c-d)^2(\sin(e+fx)+1)}-\frac{\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2}$$

```
[Out] (2*d^2*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(a^2*(c - d)^2*Sqr
t[c^2 - d^2]*f) - ((c - 4*d)*Cos[e + f*x])/(3*a^2*(c - d)^2*f*(1 + Sin[e +
f*x])) - Cos[e + f*x]/(3*(c - d)*f*(a + a*Sin[e + f*x])^2)
```

Rubi [A] time = 0.282265, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {2766, 2978, 12, 2660, 618, 204}

$$\frac{2d^2\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{a^2f(c-d)^2\sqrt{c^2-d^2}}-\frac{(c-4d)\cos(e+fx)}{3a^2f(c-d)^2(\sin(e+fx)+1)}-\frac{\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])),x]
```

```
[Out] (2*d^2*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(a^2*(c - d)^2*Sqr
t[c^2 - d^2]*f) - ((c - 4*d)*Cos[e + f*x])/(3*a^2*(c - d)^2*f*(1 + Sin[e +
f*x])) - Cos[e + f*x]/(3*(c - d)*f*(a + a*Sin[e + f*x])^2)
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))}\,dx = -\frac{\cos(e+fx)}{3(c-d)f(a+a\sin(e+fx))^2} - \frac{\int \frac{-a(c-3d)-ad\sin(e+fx)}{(a+a\sin(e+fx))(c+d\sin(e+fx))}\,dx}{3a^2(c-d)}$$

$$= -\frac{(c-4d)\cos(e+fx)}{3a^2(c-d)^2f(1+\sin(e+fx))} - \frac{\cos(e+fx)}{3(c-d)f(a+a\sin(e+fx))^2} + \frac{\int \frac{\ }{c+}}{3}$$

$$= -\frac{(c-4d)\cos(e+fx)}{3a^2(c-d)^2f(1+\sin(e+fx))} - \frac{\cos(e+fx)}{3(c-d)f(a+a\sin(e+fx))^2} + \frac{d^2\int}{}$$

$$= -\frac{(c-4d)\cos(e+fx)}{3a^2(c-d)^2f(1+\sin(e+fx))} - \frac{\cos(e+fx)}{3(c-d)f(a+a\sin(e+fx))^2} + \frac{(2d^2}{}$$

$$= -\frac{(c-4d)\cos(e+fx)}{3a^2(c-d)^2f(1+\sin(e+fx))} - \frac{\cos(e+fx)}{3(c-d)f(a+a\sin(e+fx))^2} - \frac{(4d^2}{}$$

$$= \frac{2d^2\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{a^2(c-d)^2\sqrt{c^2-d^2}f} - \frac{(c-4d)\cos(e+fx)}{3a^2(c-d)^2f(1+\sin(e+fx))} - \frac{\ }{3(c-d)}$$

Mathematica [A] time = 0.363462, size = 204, normalized size = 1.56

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(\frac{6d^2\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{\sqrt{c^2-d^2}} + 2(c-d)\sin\left(\frac{1}{2}(e+fx)\right) + 2(c-\right.}{3a^2f(c-d)^2(\sin(e+}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(2*(c - d)*Sin[(e + f*x)/2] - (c - d
)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + 2*(c - 4*d)*Sin[(e + f*x)/2]*(Cos
[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + (6*d^2*ArcTan[(d + c*Tan[(e + f*x)/2]
)/Sqrt[c^2 - d^2]]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3)/Sqrt[c^2 - d^2]
```

```
))/(3*a^2*(c - d)^2*f*(1 + Sin[e + f*x])^2)
```

Maple [A] time = 0.093, size = 175, normalized size = 1.3

$$2\,{\frac {{d}^{2}}{{a}^{2}f \left(c-d \right) ^{2}\sqrt {{c}^{2}-{d}^{2}}}\arctan \left(1/2\,{\frac {2\,c\tan \left(1/2\,fx+e/2 \right) +2\,d}{\sqrt {{c}^{2}-{d}^{2}}}} \right) }-2\,{\frac {c}{{a}^{2}f \left(c-d \right) ^{2} \left(\tan \left(1/2\,fx+e/2 \right) +1 \right) }}+4\,{\frac {}{{a}^{2}f \left(c-d \right) ^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e)),x)
```

```
[Out] 2/f/a^2*d^2/(c-d)^2/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)
/(c^2-d^2)^(1/2))-2/f/a^2/(c-d)^2/(tan(1/2*f*x+1/2*e)+1)*c+4/f/a^2/(c-d)^2/
(tan(1/2*f*x+1/2*e)+1)*d-4/3/f/a^2/(c-d)/(tan(1/2*f*x+1/2*e)+1)^3+2/f/a^2/(
c-d)/(tan(1/2*f*x+1/2*e)+1)^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.82903, size = 2129, normalized size = 16.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] [1/6*(2*c^3 - 2*c^2*d - 2*c*d^2 + 2*d^3 + 2*(c^3 - 4*c^2*d - c*d^2 + 4*d^3)
*cos(f*x + e)^2 - 3*(d^2*cos(f*x + e)^2 - d^2*cos(f*x + e) - 2*d^2 - (d^2*c
os(f*x + e) + 2*d^2)*sin(f*x + e))*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(
f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x + e
) + d*cos(f*x + e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x +
 e) - c^2 - d^2)) + 2*(2*c^3 - 5*c^2*d - 2*c*d^2 + 5*d^3)*cos(f*x + e) - 2*
(c^3 - c^2*d - c*d^2 + d^3 - (c^3 - 4*c^2*d - c*d^2 + 4*d^3)*cos(f*x + e))*
sin(f*x + e))/((a^2*c^4 - 2*a^2*c^3*d + 2*a^2*c*d^3 - a^2*d^4)*f*cos(f*x +
e)^2 - (a^2*c^4 - 2*a^2*c^3*d + 2*a^2*c*d^3 - a^2*d^4)*f*cos(f*x + e) - 2*(
a^2*c^4 - 2*a^2*c^3*d + 2*a^2*c*d^3 - a^2*d^4)*f - ((a^2*c^4 - 2*a^2*c^3*d
+ 2*a^2*c*d^3 - a^2*d^4)*f*cos(f*x + e) + 2*(a^2*c^4 - 2*a^2*c^3*d + 2*a^2*
c*d^3 - a^2*d^4)*f)*sin(f*x + e)), 1/3*(c^3 - c^2*d - c*d^2 + d^3 + (c^3 -
4*c^2*d - c*d^2 + 4*d^3)*cos(f*x + e)^2 - 3*(d^2*cos(f*x + e)^2 - d^2*cos(f
*x + e) - 2*d^2 - (d^2*cos(f*x + e) + 2*d^2)*sin(f*x + e))*sqrt(c^2 - d^2)*
arctan(-(c*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) + (2*c^3 - 5*c
^2*d - 2*c*d^2 + 5*d^3)*cos(f*x + e) - (c^3 - c^2*d - c*d^2 + d^3 - (c^3 -
4*c^2*d - c*d^2 + 4*d^3)*cos(f*x + e))*sin(f*x + e))/((a^2*c^4 - 2*a^2*c^3*
```

```
d + 2*a^2*c*d^3 - a^2*d^4)*f*cos(f*x + e)^2 - (a^2*c^4 - 2*a^2*c^3*d + 2*a^
2*c*d^3 - a^2*d^4)*f*cos(f*x + e) - 2*(a^2*c^4 - 2*a^2*c^3*d + 2*a^2*c*d^3
- a^2*d^4)*f - ((a^2*c^4 - 2*a^2*c^3*d + 2*a^2*c*d^3 - a^2*d^4)*f*cos(f*x +
 e) + 2*(a^2*c^4 - 2*a^2*c^3*d + 2*a^2*c*d^3 - a^2*d^4)*f)*sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**2/(c+d*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.34709, size = 263, normalized size = 2.01

$$\frac{2\left(\frac{3\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+d}{\sqrt{c^2-d^2}}\right)\right)d^2}{\left(a^2c^2-2a^2cd+a^2d^2\right)\sqrt{c^2-d^2}}-\frac{3c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-6d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+3c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-9d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+2c-5d}{\left(a^2c^2-2a^2cd+a^2d^2\right)\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^3}\right)}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] 2/3*(3*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1
/2*e) + d)/sqrt(c^2 - d^2)))*d^2/((a^2*c^2 - 2*a^2*c*d + a^2*d^2)*sqrt(c^2
- d^2)) - (3*c*tan(1/2*f*x + 1/2*e)^2 - 6*d*tan(1/2*f*x + 1/2*e)^2 + 3*c*ta
n(1/2*f*x + 1/2*e) - 9*d*tan(1/2*f*x + 1/2*e) + 2*c - 5*d)/((a^2*c^2 - 2*a^
2*c*d + a^2*d^2)*(tan(1/2*f*x + 1/2*e) + 1)^3))/f
```

3.468 $\int \frac{1}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))^2}\,dx$

Optimal. Leaf size=221

$$\frac{2d^2(3c+2d)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{a^2f(c-d)^3(c+d)\sqrt{c^2-d^2}}-\frac{d\left(c^2-6cd-10d^2\right)\cos(e+fx)}{3a^2f(c-d)^3(c+d)(c+d\sin(e+fx))}-\frac{(c-6d)\cos(e+fx)}{3a^2f(c-d)^2(\sin(e+fx)+1)(c+d\sin(e}$$

```
[Out] (2*d^2*(3*c + 2*d)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(a^2*(
c - d)^3*(c + d)*Sqrt[c^2 - d^2]*f) - (d*(c^2 - 6*c*d - 10*d^2)*Cos[e + f*x
])/(3*a^2*(c - d)^3*(c + d)*f*(c + d*Sin[e + f*x])) - ((c - 6*d)*Cos[e + f*
x])/(3*a^2*(c - d)^2*f*(1 + Sin[e + f*x])*(c + d*Sin[e + f*x])) - Cos[e + f
*x]/(3*(c - d)*f*(a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x]))
```

Rubi [A] time = 0.417081, antiderivative size = 221, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {2766, 2978, 2754, 12, 2660, 618, 204}

$$\frac{2d^2(3c+2d)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{a^2f(c-d)^3(c+d)\sqrt{c^2-d^2}}-\frac{d\left(c^2-6cd-10d^2\right)\cos(e+fx)}{3a^2f(c-d)^3(c+d)(c+d\sin(e+fx))}-\frac{(c-6d)\cos(e+fx)}{3a^2f(c-d)^2(\sin(e+fx)+1)(c+d\sin(e}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^2),x]
```

```
[Out] (2*d^2*(3*c + 2*d)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(a^2*(
c - d)^3*(c + d)*Sqrt[c^2 - d^2]*f) - (d*(c^2 - 6*c*d - 10*d^2)*Cos[e + f*x
])/(3*a^2*(c - d)^3*(c + d)*f*(c + d*Sin[e + f*x])) - ((c - 6*d)*Cos[e + f*
x])/(3*a^2*(c - d)^2*f*(1 + Sin[e + f*x])*(c + d*Sin[e + f*x])) - Cos[e + f
*x]/(3*(c - d)*f*(a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x]))
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))^2}\,dx = -\frac{\cos(e+fx)}{3(c-d)f(a+a\sin(e+fx))^2(c+d\sin(e+fx))} - \frac{\int \frac{-a(c-4d)-2ad\,\mathrm{s}}{(a+a\sin(e+fx))(c+}}{3a^2(c-}$$

$$= -\frac{(c-6d)\cos(e+fx)}{3a^2(c-d)^2f(1+\sin(e+fx))(c+d\sin(e+fx))} - \frac{}{3(c-d)f(a+a\,\mathrm{s}}$$

$$= -\frac{d\left(c^2-6cd-10d^2\right)\cos(e+fx)}{3a^2(c-d)^3(c+d)f(c+d\sin(e+fx))} - \frac{(c-6d)\cos(}{3a^2(c-d)^2f(1+\sin(e+f}$$

$$= -\frac{d\left(c^2-6cd-10d^2\right)\cos(e+fx)}{3a^2(c-d)^3(c+d)f(c+d\sin(e+fx))} - \frac{(c-6d)\cos(}{3a^2(c-d)^2f(1+\sin(e+f}$$

$$= -\frac{d\left(c^2-6cd-10d^2\right)\cos(e+fx)}{3a^2(c-d)^3(c+d)f(c+d\sin(e+fx))} - \frac{(c-6d)\cos(}{3a^2(c-d)^2f(1+\sin(e+f}$$

$$= -\frac{d\left(c^2-6cd-10d^2\right)\cos(e+fx)}{3a^2(c-d)^3(c+d)f(c+d\sin(e+fx))} - \frac{(c-6d)\cos(}{3a^2(c-d)^2f(1+\sin(e+f}$$

$$= \frac{2d^2(3c+2d)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{a^2(c-d)^3(c+d)\sqrt{c^2-d^2}f} - \frac{d\left(c^2-6cd-10d^2\right)\cos(e+}{3a^2(c-d)^3(c+d)f(c+d\sin(}$$

Mathematica [A] time = 1.37593, size = 267, normalized size = 1.21

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(\frac{6d^2(3c+2d)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{(c+d)\sqrt{c^2-d^2}}+\frac{3d^3\cos(e+fx)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}\right.\right.}{(c+d)(c+d\sin(e+fx))}\right.}{3a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(2*(c - d)*Sin[(e + f*x)/2] - (c - d
)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + 2*(c - 7*d)*Sin[(e + f*x)/2]*(Cos
[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + (6*d^2*(3*c + 2*d)*ArcTan[(d + c*Tan[
(e + f*x)/2])/Sqrt[c^2 - d^2]]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3)/((c
 + d)*Sqrt[c^2 - d^2]) + (3*d^3*Cos[e + f*x]*(Cos[(e + f*x)/2] + Sin[(e + f
*x)/2])^3)/((c + d)*(c + d*Sin[e + f*x]))))/(3*a^2*(c - d)^3*f*(1 + Sin[e +
 f*x])^2)
```

Maple [A] time = 0.104, size = 361, normalized size = 1.6

$$2\,\frac{d^4\tan\left(1/2\,fx+e/2\right)}{a^2f\,(c-d)^3\left(c\left(\tan\left(1/2\,fx+e/2\right)\right)^2+2\,\tan\left(1/2\,fx+e/2\right)d+c\right)(c+d)\,c}+2\,\frac{d^3}{a^2f\,(c-d)^3\left(c\left(\tan\left(1/2\,fx+e/2\right)\right)^2+2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^2,x)
```

```
[Out] 2/f/a^2*d^4/(c-d)^3/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)/(c+d)
/c*tan(1/2*f*x+1/2*e)+2/f/a^2*d^3/(c-d)^3/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2
*f*x+1/2*e)*d+c)/(c+d)+6/f/a^2*d^2/(c-d)^3/(c+d)/(c^2-d^2)^(1/2)*arctan(1/2
*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*c+4/f/a^2*d^3/(c-d)^3/(c+d)/
(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))-2/
f/a^2/(c-d)^3/(tan(1/2*f*x+1/2*e)+1)*c+6/f/a^2/(c-d)^3/(tan(1/2*f*x+1/2*e)+
1)*d-4/3/f/a^2/(c-d)^2/(tan(1/2*f*x+1/2*e)+1)^3+2/f/a^2/(c-d)^2/(tan(1/2*f*
x+1/2*e)+1)^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.27953, size = 4787, normalized size = 21.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] [1/6*(2*c^5 - 2*c^4*d - 4*c^3*d^2 + 4*c^2*d^3 + 2*c*d^4 - 2*d^5 - 2*(c^4*d
- 6*c^3*d^2 - 11*c^2*d^3 + 6*c*d^4 + 10*d^5)*cos(f*x + e)^3 + 2*(c^5 - 5*c^
4*d - 8*c^3*d^2 + c^2*d^3 + 7*c*d^4 + 4*d^5)*cos(f*x + e)^2 - 3*(6*c^2*d^2
+ 10*c*d^3 + 4*d^4 - (3*c*d^3 + 2*d^4)*cos(f*x + e)^3 - (3*c^2*d^2 + 8*c*d^
3 + 4*d^4)*cos(f*x + e)^2 + (3*c^2*d^2 + 5*c*d^3 + 2*d^4)*cos(f*x + e) + (6
*c^2*d^2 + 10*c*d^3 + 4*d^4 - (3*c*d^3 + 2*d^4)*cos(f*x + e)^2 + (3*c^2*d^2
 + 5*c*d^3 + 2*d^4)*cos(f*x + e))*sin(f*x + e))*sqrt(-c^2 + d^2)*log(-((2*c
^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 - 2*(c*cos(f*x +
e)*sin(f*x + e) + d*cos(f*x + e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2
*c*d*sin(f*x + e) - c^2 - d^2)) + 2*(2*c^5 - 5*c^4*d - 16*c^3*d^2 - 8*c^2*d
^3 + 14*c*d^4 + 13*d^5)*cos(f*x + e) - 2*(c^5 - c^4*d - 2*c^3*d^2 + 2*c^2*d
^3 + c*d^4 - d^5 - (c^4*d - 6*c^3*d^2 - 11*c^2*d^3 + 6*c*d^4 + 10*d^5)*cos(
f*x + e)^2 - (c^5 - 4*c^4*d - 14*c^3*d^2 - 10*c^2*d^3 + 13*c*d^4 + 14*d^5)*
cos(f*x + e))*sin(f*x + e))/((a^2*c^6*d - 2*a^2*c^5*d^2 - a^2*c^4*d^3 + 4*a
^2*c^3*d^4 - a^2*c^2*d^5 - 2*a^2*c*d^6 + a^2*d^7)*f*cos(f*x + e)^3 + (a^2*c
^7 - 5*a^2*c^5*d^2 + 2*a^2*c^4*d^3 + 7*a^2*c^3*d^4 - 4*a^2*c^2*d^5 - 3*a^2*
c*d^6 + 2*a^2*d^7)*f*cos(f*x + e)^2 - (a^2*c^7 - a^2*c^6*d - 3*a^2*c^5*d^2
+ 3*a^2*c^4*d^3 + 3*a^2*c^3*d^4 - 3*a^2*c^2*d^5 - a^2*c*d^6 + a^2*d^7)*f*co
s(f*x + e) - 2*(a^2*c^7 - a^2*c^6*d - 3*a^2*c^5*d^2 + 3*a^2*c^4*d^3 + 3*a^2
*c^3*d^4 - 3*a^2*c^2*d^5 - a^2*c*d^6 + a^2*d^7)*f + ((a^2*c^6*d - 2*a^2*c^5
*d^2 - a^2*c^4*d^3 + 4*a^2*c^3*d^4 - a^2*c^2*d^5 - 2*a^2*c*d^6 + a^2*d^7)*f
*cos(f*x + e)^2 - (a^2*c^7 - a^2*c^6*d - 3*a^2*c^5*d^2 + 3*a^2*c^4*d^3 + 3*
a^2*c^3*d^4 - 3*a^2*c^2*d^5 - a^2*c*d^6 + a^2*d^7)*f*cos(f*x + e) - 2*(a^2*
c^7 - a^2*c^6*d - 3*a^2*c^5*d^2 + 3*a^2*c^4*d^3 + 3*a^2*c^3*d^4 - 3*a^2*c^2
*d^5 - a^2*c*d^6 + a^2*d^7)*f)*sin(f*x + e)), 1/3*(c^5 - c^4*d - 2*c^3*d^2
+ 2*c^2*d^3 + c*d^4 - d^5 - (c^4*d - 6*c^3*d^2 - 11*c^2*d^3 + 6*c*d^4 + 10*
d^5)*cos(f*x + e)^3 + (c^5 - 5*c^4*d - 8*c^3*d^2 + c^2*d^3 + 7*c*d^4 + 4*d^
5)*cos(f*x + e)^2 + 3*(6*c^2*d^2 + 10*c*d^3 + 4*d^4 - (3*c*d^3 + 2*d^4)*cos
(f*x + e)^3 - (3*c^2*d^2 + 8*c*d^3 + 4*d^4)*cos(f*x + e)^2 + (3*c^2*d^2 + 5
*c*d^3 + 2*d^4)*cos(f*x + e) + (6*c^2*d^2 + 10*c*d^3 + 4*d^4 - (3*c*d^3 + 2
*d^4)*cos(f*x + e)^2 + (3*c^2*d^2 + 5*c*d^3 + 2*d^4)*cos(f*x + e))*sin(f*x
+ e))*sqrt(c^2 - d^2)*arctan(-(c*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x
 + e))) + (2*c^5 - 5*c^4*d - 16*c^3*d^2 - 8*c^2*d^3 + 14*c*d^4 + 13*d^5)*co
s(f*x + e) - (c^5 - c^4*d - 2*c^3*d^2 + 2*c^2*d^3 + c*d^4 - d^5 - (c^4*d -
6*c^3*d^2 - 11*c^2*d^3 + 6*c*d^4 + 10*d^5)*cos(f*x + e)^2 - (c^5 - 4*c^4*d
- 14*c^3*d^2 - 10*c^2*d^3 + 13*c*d^4 + 14*d^5)*cos(f*x + e))*sin(f*x + e))/
((a^2*c^6*d - 2*a^2*c^5*d^2 - a^2*c^4*d^3 + 4*a^2*c^3*d^4 - a^2*c^2*d^5 - 2
*a^2*c*d^6 + a^2*d^7)*f*cos(f*x + e)^3 + (a^2*c^7 - 5*a^2*c^5*d^2 + 2*a^2*c
^4*d^3 + 7*a^2*c^3*d^4 - 4*a^2*c^2*d^5 - 3*a^2*c*d^6 + 2*a^2*d^7)*f*cos(f*x
 + e)^2 - (a^2*c^7 - a^2*c^6*d - 3*a^2*c^5*d^2 + 3*a^2*c^4*d^3 + 3*a^2*c^3*
d^4 - 3*a^2*c^2*d^5 - a^2*c*d^6 + a^2*d^7)*f*cos(f*x + e) - 2*(a^2*c^7 - a^
2*c^6*d - 3*a^2*c^5*d^2 + 3*a^2*c^4*d^3 + 3*a^2*c^3*d^4 - 3*a^2*c^2*d^5 - a
^2*c*d^6 + a^2*d^7)*f + ((a^2*c^6*d - 2*a^2*c^5*d^2 - a^2*c^4*d^3 + 4*a^2*c
^3*d^4 - a^2*c^2*d^5 - 2*a^2*c*d^6 + a^2*d^7)*f*cos(f*x + e)^2 - (a^2*c^7 -
 a^2*c^6*d - 3*a^2*c^5*d^2 + 3*a^2*c^4*d^3 + 3*a^2*c^3*d^4 - 3*a^2*c^2*d^5
- a^2*c*d^6 + a^2*d^7)*f*cos(f*x + e) - 2*(a^2*c^7 - a^2*c^6*d - 3*a^2*c^5*
d^2 + 3*a^2*c^4*d^3 + 3*a^2*c^3*d^4 - 3*a^2*c^2*d^5 - a^2*c*d^6 + a^2*d^7)*
f)*sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**2/(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.36131, size = 432, normalized size = 1.95

$$\frac{2\left(\frac{3\left(3\,cd^2+2\,d^3\right)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{\left(a^2c^4-2\,a^2c^3d+2\,a^2cd^3-a^2d^4\right)\sqrt{c^2-d^2}}+\frac{3\left(d^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+cd^3\right)}{\left(a^2c^5-2\,a^2c^4d+2\,a^2c^2d^3-a^2cd^4\right)\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+2\,d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+c\right)}-\frac{3\,c\tan\left(\frac{1}{2}\cdots\right.}{\cdots}\right.}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 2/3*(3*(3*c*d^2 + 2*d^3)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan(
(c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/((a^2*c^4 - 2*a^2*c^3*d + 2*
a^2*c*d^3 - a^2*d^4)*sqrt(c^2 - d^2)) + 3*(d^4*tan(1/2*f*x + 1/2*e) + c*d^3
)/((a^2*c^5 - 2*a^2*c^4*d + 2*a^2*c^2*d^3 - a^2*c*d^4)*(c*tan(1/2*f*x + 1/2
*e)^2 + 2*d*tan(1/2*f*x + 1/2*e) + c)) - (3*c*tan(1/2*f*x + 1/2*e)^2 - 9*d*
tan(1/2*f*x + 1/2*e)^2 + 3*c*tan(1/2*f*x + 1/2*e) - 15*d*tan(1/2*f*x + 1/2*
e) + 2*c - 8*d)/((a^2*c^3 - 3*a^2*c^2*d + 3*a^2*c*d^2 - a^2*d^3)*(tan(1/2*f
*x + 1/2*e) + 1)^3))/f
```

3.469 $\int \frac{1}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))^3}\,dx$

Optimal. Leaf size=294

$$\frac{d^2\left(12c^2+16cd+7d^2\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{a^2f(c-d)^4(c+d)^2\sqrt{c^2-d^2}}-\frac{d\left(-16c^2d+2c^3-59cd^2-32d^3\right)\cos(e+fx)}{6a^2f(c-d)^4(c+d)^2(c+d\sin(e+fx))}-\frac{d\left(2c^2-16cd-21\right.}{6a^2f(c-d)^3(c+d)(c}$$

```
[Out] (d^2*(12*c^2 + 16*c*d + 7*d^2)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d
^2]])/(a^2*(c - d)^4*(c + d)^2*Sqrt[c^2 - d^2]*f) - (d*(2*c^2 - 16*c*d - 21
*d^2)*Cos[e + f*x])/(6*a^2*(c - d)^3*(c + d)*f*(c + d*Sin[e + f*x])^2) - ((
c - 8*d)*Cos[e + f*x])/(3*a^2*(c - d)^2*f*(1 + Sin[e + f*x])*(c + d*Sin[e +
 f*x])^2) - Cos[e + f*x]/(3*(c - d)*f*(a + a*Sin[e + f*x])^2*(c + d*Sin[e +
 f*x])^2) - (d*(2*c^3 - 16*c^2*d - 59*c*d^2 - 32*d^3)*Cos[e + f*x])/(6*a^2*
(c - d)^4*(c + d)^2*f*(c + d*Sin[e + f*x]))
```

Rubi [A] time = 0.615488, antiderivative size = 294, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {2766, 2978, 2754, 12, 2660, 618, 204}

$$\frac{d^2\left(12c^2+16cd+7d^2\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{a^2f(c-d)^4(c+d)^2\sqrt{c^2-d^2}}-\frac{d\left(-16c^2d+2c^3-59cd^2-32d^3\right)\cos(e+fx)}{6a^2f(c-d)^4(c+d)^2(c+d\sin(e+fx))}-\frac{d\left(2c^2-16cd-21\right.}{6a^2f(c-d)^3(c+d)(c}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^3),x]
```

```
[Out] (d^2*(12*c^2 + 16*c*d + 7*d^2)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d
^2]])/(a^2*(c - d)^4*(c + d)^2*Sqrt[c^2 - d^2]*f) - (d*(2*c^2 - 16*c*d - 21
*d^2)*Cos[e + f*x])/(6*a^2*(c - d)^3*(c + d)*f*(c + d*Sin[e + f*x])^2) - ((
c - 8*d)*Cos[e + f*x])/(3*a^2*(c - d)^2*f*(1 + Sin[e + f*x])*(c + d*Sin[e +
 f*x])^2) - Cos[e + f*x]/(3*(c - d)*f*(a + a*Sin[e + f*x])^2*(c + d*Sin[e +
 f*x])^2) - (d*(2*c^3 - 16*c^2*d - 59*c*d^2 - 32*d^3)*Cos[e + f*x])/(6*a^2*
(c - d)^4*(c + d)^2*f*(c + d*Sin[e + f*x]))
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
```

```
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))^3}\,dx = -\frac{\cos(e+fx)}{3(c-d)f(a+a\sin(e+fx))^2(c+d\sin(e+fx))^2} - \frac{\int \frac{-a(c-5d)-3ad}{(a+a\sin(e+fx))(c}}{3a^2(c}$$

$$= -\frac{(c-8d)\cos(e+fx)}{3a^2(c-d)^2f(1+\sin(e+fx))(c+d\sin(e+fx))^2} - \frac{}{3(c-d)f(a+a}$$

$$= -\frac{d(2c^2-16cd-21d^2)\cos(e+fx)}{6a^2(c-d)^3(c+d)f(c+d\sin(e+fx))^2} - \frac{(c-8d)\cos}{3a^2(c-d)^2f(1+\sin(e+f}$$

$$= -\frac{d(2c^2-16cd-21d^2)\cos(e+fx)}{6a^2(c-d)^3(c+d)f(c+d\sin(e+fx))^2} - \frac{(c-8d)\cos}{3a^2(c-d)^2f(1+\sin(e+f}$$

$$= -\frac{d(2c^2-16cd-21d^2)\cos(e+fx)}{6a^2(c-d)^3(c+d)f(c+d\sin(e+fx))^2} - \frac{(c-8d)\cos}{3a^2(c-d)^2f(1+\sin(e+f}$$

$$= -\frac{d(2c^2-16cd-21d^2)\cos(e+fx)}{6a^2(c-d)^3(c+d)f(c+d\sin(e+fx))^2} - \frac{(c-8d)\cos}{3a^2(c-d)^2f(1+\sin(e+f}$$

$$= -\frac{d(2c^2-16cd-21d^2)\cos(e+fx)}{6a^2(c-d)^3(c+d)f(c+d\sin(e+fx))^2} - \frac{(c-8d)\cos}{3a^2(c-d)^2f(1+\sin(e+f}$$

$$= \frac{d^2(12c^2+16cd+7d^2)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{a^2(c-d)^4(c+d)^2\sqrt{c^2-d^2}\,f} - \frac{d(2c^2-16cd-2}{6a^2(c-d)^3(c+d)f(}$$

Mathematica [A] time = 1.18258, size = 338, normalized size = 1.15

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(\frac{6d^2(12c^2+16cd+7d^2)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{(c+d)^2\sqrt{c^2-d^2}} + \frac{3d^3(7c+4d)\cos(e+fx)(}{(c+d)^2}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^3),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(4*(c - d)*Sin[(e + f*x)/2] - 2*(c -
 d)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + 4*(c - 10*d)*Sin[(e + f*x)/2]*(
Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + (6*d^2*(12*c^2 + 16*c*d + 7*d^2)*A
rcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]]*(Cos[(e + f*x)/2] + Sin[(e
+ f*x)/2])^3)/((c + d)^2*Sqrt[c^2 - d^2]) + (3*(c - d)*d^3*Cos[e + f*x]*(Co
s[(e + f*x)/2] + Sin[(e + f*x)/2])^3)/((c + d)*(c + d*Sin[e + f*x])^2) + (3
*d^3*(7*c + 4*d)*Cos[e + f*x]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3)/((c
+ d)^2*(c + d*Sin[e + f*x]))))/(6*a^2*(c - d)^4*f*(1 + Sin[e + f*x])^2)
```

Maple [B] time = 0.127, size = 1313, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^3,x)
```

[Out] 9/f/a^2*d^4/(c-d)^4/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2*c/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3+4/f/a^2*d^5/(c-d)^4/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3-2/f/a^2*d^6/(c-d)^4/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/c/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3+8/f/a^2*d^3/(c-d)^4/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2*c^2/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^2+4/f/a^2*d^4/(c-d)^4/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2*c/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^2+15/f/a^2*d^5/(c-d)^4/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^2+8/f/a^2*d^6/(c-d)^4/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/c/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^2-2/f/a^2*d^7/(c-d)^4/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/c^2/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^2+23/f/a^2*d^4/(c-d)^4/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*c*tan(1/2*f*x+1/2*e)+12/f/a^2*d^5/(c-d)^4/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)-2/f/a^2*d^6/(c-d)^4/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)/c*tan(1/2*f*x+1/2*e)+8/f/a^2*d^3/(c-d)^4/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*c^2+4/f/a^2*d^4/(c-d)^4/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*c-1/f/a^2*d^5/(c-d)^4/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)+12/f/a^2*d^2/(c-d)^4/(c^2+2*c*d+d^2)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*c^2+16/f/a^2*d^3/(c-d)^4/(c^2+2*c*d+d^2)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*c+7/f/a^2*d^4/(c-d)^4/(c^2+2*c*d+d^2)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))-2/f/a^2/(c-d)^4/(tan(1/2*f*x+1/2*e)+1)*c+8/f/a^2/(c-d)^4/(tan(1/2*f*x+1/2*e)+1)*d-4/3/f/a^2/(c-d)^3/(tan(1/2*f*x+1/2*e)+1)^3+2/f/a^2/(c-d)^3/(tan(1/2*f*x+1/2*e)+1)^2

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^3,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [B] time = 2.81252, size = 7625, normalized size = 25.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^3,x, algorithm="fricas")

[Out] [-1/12*(4*c^7 - 4*c^6*d - 12*c^5*d^2 + 12*c^4*d^3 + 12*c^3*d^4 - 12*c^2*d^5 - 4*c*d^6 + 4*d^7 - 2*(2*c^5*d^2 - 16*c^4*d^3 - 61*c^3*d^4 - 16*c^2*d^5 + 59*c*d^6 + 32*d^7)*cos(f*x + e)^4 - 2*(4*c^6*d - 28*c^5*d^2 - 118*c^4*d^3 - 106*c^3*d^4 + 71*c^2*d^5 + 134*c*d^6 + 43*d^7)*cos(f*x + e)^3 + 2*(2*c^7 - 12*c^6*d - 36*c^5*d^2 - 54*c^4*d^3 - 39*c^3*d^4 + 39*c^2*d^5 + 73*c*d^6 + 27*d^7)*cos(f*x + e)^2 + 3*(24*c^4*d^2 + 80*c^3*d^3 + 102*c^2*d^4 + 60*c*d^5 + 14*d^6 + (12*c^2*d^4 + 16*c*d^5 + 7*d^6)*cos(f*x + e)^4 - (24*c^3*d^3 + 44*c^2*d^4 + 30*c*d^5 + 7*d^6)*cos(f*x + e)^3 - (12*c^4*d^2 + 64*c^3*d^3 + 107*c^2*d^4 + 76*c*d^5 + 21*d^6)*cos(f*x + e)^2 + (12*c^4*d^2 + 40*c^3*d^3

+ 51*c^2*d^4 + 30*c*d^5 + 7*d^6)*cos(f*x + e) + (24*c^4*d^2 + 80*c^3*d^3 + 102*c^2*d^4 + 60*c*d^5 + 14*d^6 - (12*c^2*d^4 + 16*c*d^5 + 7*d^6)*cos(f*x + e)^3 - 2*(12*c^3*d^3 + 28*c^2*d^4 + 23*c*d^5 + 7*d^6)*cos(f*x + e)^2 + (12*c^4*d^2 + 40*c^3*d^3 + 51*c^2*d^4 + 30*c*d^5 + 7*d^6)*cos(f*x + e))*sin(f*x + e))*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x + e) + d*cos(f*x + e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) + 4*(2*c^7 - 5*c^6*d - 36*c^5*d^2 - 75*c^4*d^3 - 39*c^3*d^4 + 60*c^2*d^5 + 73*c*d^6 + 20*d^7)*cos(f*x + e) - 2*(2*c^7 - 2*c^6*d - 6*c^5*d^2 + 6*c^4*d^3 + 6*c^3*d^4 - 6*c^2*d^5 - 2*c*d^6 + 2*d^7 + (2*c^5*d^2 - 16*c^4*d^3 - 61*c^3*d^4 - 16*c^2*d^5 + 59*c*d^6 + 32*d^7)*cos(f*x + e)^3 - (4*c^6*d - 30*c^5*d^2 - 102*c^4*d^3 - 45*c^3*d^4 + 87*c^2*d^5 + 75*c*d^6 + 11*d^7)*cos(f*x + e)^2 - 2*(c^7 - 4*c^6*d - 33*c^5*d^2 - 78*c^4*d^3 - 42*c^3*d^4 + 63*c^2*d^5 + 74*c*d^6 + 19*d^7)*cos(f*x + e))*sin(f*x + e))/((a^2*c^8*d^2 - 2*a^2*c^7*d^3 - 2*a^2*c^6*d^4 + 6*a^2*c^5*d^5 - 6*a^2*c^3*d^7 + 2*a^2*c^2*d^8 + 2*a^2*c*d^9 - a^2*d^10)*f*cos(f*x + e)^4 - (2*a^2*c^9*d - 3*a^2*c^8*d^2 - 6*a^2*c^7*d^3 + 10*a^2*c^6*d^4 + 6*a^2*c^5*d^5 - 12*a^2*c^4*d^6 - 2*a^2*c^3*d^7 + 6*a^2*c^2*d^8 - a^2*d^10)*f*cos(f*x + e)^3 - (a^2*c^10 + 2*a^2*c^9*d - 7*a^2*c^8*d^2 - 8*a^2*c^7*d^3 + 18*a^2*c^6*d^4 + 12*a^2*c^5*d^5 - 22*a^2*c^4*d^6 - 8*a^2*c^3*d^7 + 13*a^2*c^2*d^8 + 2*a^2*c*d^9 - 3*a^2*d^10)*f*cos(f*x + e)^2 + (a^2*c^10 - 5*a^2*c^8*d^2 + 10*a^2*c^6*d^4 - 10*a^2*c^4*d^6 + 5*a^2*c^2*d^8 - a^2*d^10)*f*cos(f*x + e) + 2*(a^2*c^10 - 5*a^2*c^8*d^2 + 10*a^2*c^6*d^4 - 10*a^2*c^4*d^6 + 5*a^2*c^2*d^8 - a^2*d^10)*f - ((a^2*c^8*d^2 - 2*a^2*c^7*d^3 - 2*a^2*c^6*d^4 + 6*a^2*c^5*d^5 - 6*a^2*c^3*d^7 + 2*a^2*c^2*d^8 + 2*a^2*c*d^9 - a^2*d^10)*f*cos(f*x + e)^3 + 2*(a^2*c^9*d - a^2*c^8*d^2 - 4*a^2*c^7*d^3 + 4*a^2*c^6*d^4 + 6*a^2*c^5*d^5 - 6*a^2*c^4*d^6 - 4*a^2*c^3*d^7 + 4*a^2*c^2*d^8 + a^2*c*d^9 - a^2*d^10)*f*cos(f*x + e)^2 - (a^2*c^10 - 5*a^2*c^8*d^2 + 10*a^2*c^6*d^4 - 10*a^2*c^4*d^6 + 5*a^2*c^2*d^8 - a^2*d^10)*f*cos(f*x + e) - 2*(a^2*c^10 - 5*a^2*c^8*d^2 + 10*a^2*c^6*d^4 - 10*a^2*c^4*d^6 + 5*a^2*c^2*d^8 - a^2*d^10)*f)*sin(f*x + e)), -1/6*(2*c^7 - 2*c^6*d - 6*c^5*d^2 + 6*c^4*d^3 + 6*c^3*d^4 - 6*c^2*d^5 - 2*c*d^6 + 2*d^7 - (2*c^5*d^2 - 16*c^4*d^3 - 61*c^3*d^4 - 16*c^2*d^5 + 59*c*d^6 + 32*d^7)*cos(f*x + e)^4 - (4*c^6*d - 28*c^5*d^2 - 118*c^4*d^3 - 106*c^3*d^4 + 71*c^2*d^5 + 134*c*d^6 + 43*d^7)*cos(f*x + e)^3 + (2*c^7 - 12*c^6*d - 36*c^5*d^2 - 54*c^4*d^3 - 39*c^3*d^4 + 39*c^2*d^5 + 73*c*d^6 + 27*d^7)*cos(f*x + e)^2 + 3*(24*c^4*d^2 + 80*c^3*d^3 + 102*c^2*d^4 + 60*c*d^5 + 14*d^6 + (12*c^2*d^4 + 16*c*d^5 + 7*d^6)*cos(f*x + e)^4 - (24*c^3*d^3 + 44*c^2*d^4 + 30*c*d^5 + 7*d^6)*cos(f*x + e)^3 - (12*c^4*d^2 + 64*c^3*d^3 + 107*c^2*d^4 + 76*c*d^5 + 21*d^6)*cos(f*x + e)^2 + (12*c^4*d^2 + 40*c^3*d^3 + 51*c^2*d^4 + 30*c*d^5 + 7*d^6)*cos(f*x + e) + (24*c^4*d^2 + 80*c^3*d^3 + 102*c^2*d^4 + 60*c*d^5 + 14*d^6 - (12*c^2*d^4 + 16*c*d^5 + 7*d^6)*cos(f*x + e)^3 - 2*(12*c^3*d^3 + 28*c^2*d^4 + 23*c*d^5 + 7*d^6)*cos(f*x + e)^2 + (12*c^4*d^2 + 40*c^3*d^3 + 51*c^2*d^4 + 30*c*d^5 + 7*d^6)*cos(f*x + e))*sin(f*x + e))*sqrt(c^2 - d^2)*arctan(-(c*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) + 2*(2*c^7 - 5*c^6*d - 36*c^5*d^2 - 75*c^4*d^3 - 39*c^3*d^4 + 60*c^2*d^5 + 73*c*d^6 + 20*d^7)*cos(f*x + e) - (2*c^7 - 2*c^6*d - 6*c^5*d^2 + 6*c^4*d^3 + 6*c^3*d^4 - 6*c^2*d^5 - 2*c*d^6 + 2*d^7 + (2*c^5*d^2 - 16*c^4*d^3 - 61*c^3*d^4 - 16*c^2*d^5 + 59*c*d^6 + 32*d^7)*cos(f*x + e)^3 - (4*c^6*d - 30*c^5*d^2 - 102*c^4*d^3 - 45*c^3*d^4 + 87*c^2*d^5 + 75*c*d^6 + 11*d^7)*cos(f*x + e)^2 - 2*(c^7 - 4*c^6*d - 33*c^5*d^2 - 78*c^4*d^3 - 42*c^3*d^4 + 63*c^2*d^5 + 74*c*d^6 + 19*d^7)*cos(f*x + e))*sin(f*x + e))/((a^2*c^8*d^2 - 2*a^2*c^7*d^3 - 2*a^2*c^6*d^4 + 6*a^2*c^5*d^5 - 6*a^2*c^3*d^7 + 2*a^2*c^2*d^8 + 2*a^2*c*d^9 - a^2*d^10)*f*cos(f*x + e)^4 - (2*a^2*c^9*d - 3*a^2*c^8*d^2 - 6*a^2*c^7*d^3 + 10*a^2*c^6*d^4 + 6*a^2*c^5*d^5 - 12*a^2*c^4*d^6 - 2*a^2*c^3*d^7 + 6*a^2*c^2*d^8 - a^2*d^10)*f*cos(f*x + e)^3 - (a^2*c^10 + 2*a^2*c^9*d - 7*a^2*c^8*d^2 - 8*a^2*c^7*d^3 + 18*a^2*c^6*d^4 + 12*a^2*c^5*d^5 - 22*a^2*c^4*d^6 - 8*a^2*c^3*d^7 + 13*a^2*c^2*d^8 + 2*a^2*c*d^9 - 3*a^2*d^10)*f*cos(f*x + e)^2 + (a^2*c^10 - 5*a^2*c^8*d^2 + 10*a^2*c^6*d^4 - 10*a^2*c^4*d^6 + 5*a^2*c^2*d^8 - a^2*d^10)*f*cos(f*x + e) + 2*(a^2*c^10 - 5*a^2*c^8*d^2 + 10*a^2*c^6*d^4 - 10*a^2*c^4*d^6 + 5*a

```
^2*c^2*d^8 - a^2*d^10)*f - ((a^2*c^8*d^2 - 2*a^2*c^7*d^3 - 2*a^2*c^6*d^4 +
6*a^2*c^5*d^5 - 6*a^2*c^3*d^7 + 2*a^2*c^2*d^8 + 2*a^2*c*d^9 - a^2*d^10)*f*c
os(f*x + e)^3 + 2*(a^2*c^9*d - a^2*c^8*d^2 - 4*a^2*c^7*d^3 + 4*a^2*c^6*d^4
+ 6*a^2*c^5*d^5 - 6*a^2*c^4*d^6 - 4*a^2*c^3*d^7 + 4*a^2*c^2*d^8 + a^2*c*d^9
 - a^2*d^10)*f*cos(f*x + e)^2 - (a^2*c^10 - 5*a^2*c^8*d^2 + 10*a^2*c^6*d^4
- 10*a^2*c^4*d^6 + 5*a^2*c^2*d^8 - a^2*d^10)*f*cos(f*x + e) - 2*(a^2*c^10 -
 5*a^2*c^8*d^2 + 10*a^2*c^6*d^4 - 10*a^2*c^4*d^6 + 5*a^2*c^2*d^8 - a^2*d^10
)*f)*sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**2/(c+d*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.41964, size = 829, normalized size = 2.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/3*(3*(12*c^2*d^2 + 16*c*d^3 + 7*d^4)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sg
n(c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/((a^2*c^6 - 2*
a^2*c^5*d - a^2*c^4*d^2 + 4*a^2*c^3*d^3 - a^2*c^2*d^4 - 2*a^2*c*d^5 + a^2*d
^6)*sqrt(c^2 - d^2)) + 3*(9*c^3*d^4*tan(1/2*f*x + 1/2*e)^3 + 4*c^2*d^5*tan(
1/2*f*x + 1/2*e)^3 - 2*c*d^6*tan(1/2*f*x + 1/2*e)^3 + 8*c^4*d^3*tan(1/2*f*x
 + 1/2*e)^2 + 4*c^3*d^4*tan(1/2*f*x + 1/2*e)^2 + 15*c^2*d^5*tan(1/2*f*x + 1
/2*e)^2 + 8*c*d^6*tan(1/2*f*x + 1/2*e)^2 - 2*d^7*tan(1/2*f*x + 1/2*e)^2 + 2
3*c^3*d^4*tan(1/2*f*x + 1/2*e) + 12*c^2*d^5*tan(1/2*f*x + 1/2*e) - 2*c*d^6*
tan(1/2*f*x + 1/2*e) + 8*c^4*d^3 + 4*c^3*d^4 - c^2*d^5)/((a^2*c^8 - 2*a^2*c
^7*d - a^2*c^6*d^2 + 4*a^2*c^5*d^3 - a^2*c^4*d^4 - 2*a^2*c^3*d^5 + a^2*c^2*
d^6)*(c*tan(1/2*f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e) + c)^2) - 2*(3*c*
tan(1/2*f*x + 1/2*e)^2 - 12*d*tan(1/2*f*x + 1/2*e)^2 + 3*c*tan(1/2*f*x + 1/
2*e) - 21*d*tan(1/2*f*x + 1/2*e) + 2*c - 11*d)/((a^2*c^4 - 4*a^2*c^3*d + 6*
a^2*c^2*d^2 - 4*a^2*c*d^3 + a^2*d^4)*(tan(1/2*f*x + 1/2*e) + 1)^3))/f
```

3.470 $\int \frac{(c+d\sin(e+fx))^6}{(a+a\sin(e+fx))^3}\,dx$

Optimal. Leaf size=354

$$\frac{2d\left(107c^3d^2-472c^2d^3+18c^4d+2c^5+456cd^4-136d^5\right)\cos(e+fx)}{15a^3f}-\frac{(c-d)\left(2c^2+18cd+115d^2\right)\cos(e+fx)(c+d}{15f\left(a^3\sin(e+fx)+a^3\right)}$$

```
[Out] (d^3*(40*c^3 - 90*c^2*d + 78*c*d^2 - 23*d^3)*x)/(2*a^3) + (2*d*(2*c^5 + 18*
c^4*d + 107*c^3*d^2 - 472*c^2*d^3 + 456*c*d^4 - 136*d^5)*Cos[e + f*x])/(15*
a^3*f) + (d^2*(4*c^4 + 36*c^3*d + 216*c^2*d^2 - 626*c*d^3 + 345*d^4)*Cos[e
+ f*x]*Sin[e + f*x])/(30*a^3*f) + (d*(2*c^3 + 18*c^2*d + 111*c*d^2 - 136*d^
3)*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(15*a^3*f) - ((c - d)*(2*c^2 + 18*c
*d + 115*d^2)*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(15*f*(a^3 + a^3*Sin[e +
 f*x])) - ((c - d)*(2*c + 13*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^4)/(15*a*
f*(a + a*Sin[e + f*x])^2) - ((c - d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^5)/(
5*f*(a + a*Sin[e + f*x])^3)
```

Rubi [A] time = 0.789744, antiderivative size = 354, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2765, 2977, 2753, 2734}

$$\frac{2d\left(107c^3d^2-472c^2d^3+18c^4d+2c^5+456cd^4-136d^5\right)\cos(e+fx)}{15a^3f}-\frac{(c-d)\left(2c^2+18cd+115d^2\right)\cos(e+fx)(c+d}{15f\left(a^3\sin(e+fx)+a^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^6/(a + a*Sin[e + f*x])^3,x]
```



```
[Out] (d^3*(40*c^3 - 90*c^2*d + 78*c*d^2 - 23*d^3)*x)/(2*a^3) + (2*d*(2*c^5 + 18*
c^4*d + 107*c^3*d^2 - 472*c^2*d^3 + 456*c*d^4 - 136*d^5)*Cos[e + f*x])/(15*
a^3*f) + (d^2*(4*c^4 + 36*c^3*d + 216*c^2*d^2 - 626*c*d^3 + 345*d^4)*Cos[e
+ f*x]*Sin[e + f*x])/(30*a^3*f) + (d*(2*c^3 + 18*c^2*d + 111*c*d^2 - 136*d^
3)*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(15*a^3*f) - ((c - d)*(2*c^2 + 18*c
*d + 115*d^2)*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(15*f*(a^3 + a^3*Sin[e +
 f*x])) - ((c - d)*(2*c + 13*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^4)/(15*a*
f*(a + a*Sin[e + f*x])^2) - ((c - d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^5)/(
5*f*(a + a*Sin[e + f*x])^3)
```


Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
```

```
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^6}{(a+a\sin(e+fx))^3}\,dx = -\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^5}{5f(a+a\sin(e+fx))^3} - \frac{\int \frac{(c+d\sin(e+fx))^4\left(-a\left(2c^2+8cd-5d^2\right)+a(3c-8d)d\sin(e+fx)\right)}{(a+a\sin(e+fx))^2}\,d}{5a^2}$$

$$= -\frac{(c-d)(2c+13d)\cos(e+fx)(c+d\sin(e+fx))^4}{15af(a+a\sin(e+fx))^2} - \frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^5}{5f(a+a\sin(e+fx))^3}$$

$$= -\frac{(c-d)\left(2c^2+18cd+115d^2\right)\cos(e+fx)(c+d\sin(e+fx))^3}{15f\left(a^3+a^3\sin(e+fx)\right)} - \frac{(c-d)(2c+13d)\cos(e+f}{15af(a+a\sin(}$$

$$= \frac{d\left(2c^3+18c^2d+111cd^2-136d^3\right)\cos(e+fx)(c+d\sin(e+fx))^2}{15a^3f} - \frac{(c-d)\left(2c^2+18cd+11\right.}{15f\left(a\right.}$$

$$= \frac{d^3\left(40c^3-90c^2d+78cd^2-23d^3\right)x}{2a^3} + \frac{2d\left(2c^5+18c^4d+107c^3d^2-472c^2d^3+456cd^4-136\right.}{15a^3f}$$

Mathematica [C] time = 2.96154, size = 560, normalized size = 1.58

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(16\left(2c^2+26cd+197d^2\right)(c-d)^4\sin\left(\frac{1}{2}(e+fx)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])^6/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(48*(c - d)^6*Sin[(e + f*x)/2] - 24*
(c - d)^6*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + 32*(c - d)^5*(c + 14*d)*S
in[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 - 16*(c - d)^5*(c +
 14*d)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 + 16*(c - d)^4*(2*c^2 + 26*c
*d + 197*d^2)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4 - 60
*d^3*(-40*c^3 + 90*c^2*d - 78*c*d^2 + 23*d^3)*(e + f*x)*(Cos[(e + f*x)/2] +
 Sin[(e + f*x)/2])^5 + 10*d^6*Cos[3*(e + f*x)]*(Cos[(e + f*x)/2] + Sin[(e +
```

```
f*x)/2])^5 - 45*d^4*(20*c^2 - 24*c*d + 9*d^2)*(Cos[(e + f*x)/2] + Sin[(e +
f*x)/2])^5*(Cos[e + f*x] - I*Sin[e + f*x]) - 45*d^4*(20*c^2 - 24*c*d + 9*d
^2)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5*(Cos[e + f*x] + I*Sin[e + f*x])
- (45*I)*(2*c - d)*d^5*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5*(Cos[2*(e +
f*x)] - I*Sin[2*(e + f*x)]) + (45*I)*(2*c - d)*d^5*(Cos[(e + f*x)/2] + Sin
[(e + f*x)/2])^5*(Cos[2*(e + f*x)] + I*Sin[2*(e + f*x)])))/(120*a^3*f*(1 +
Sin[e + f*x])^3)
```

Maple [B] time = 0.104, size = 1340, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^6/(a+a*sin(f*x+e))^3,x)
```

```
[Out] 3/f/a^3*d^6/(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)-30/f/a^3*d^4/(1+t
an(1/2*f*x+1/2*e)^2)^3*c^2+36/f/a^3*d^5/(1+tan(1/2*f*x+1/2*e)^2)^3*c+40/f/a
^3*d^3*arctan(tan(1/2*f*x+1/2*e))*c^3-90/f/a^3*d^4*arctan(tan(1/2*f*x+1/2*e
))*c^2+78/f/a^3*d^5*arctan(tan(1/2*f*x+1/2*e))*c+80/3/f/a^3/(tan(1/2*f*x+1/
2*e)+1)^3*c^3*d^3-8/f/a^3/(tan(1/2*f*x+1/2*e)+1)^3*c*d^5-24/f/a^3/(tan(1/2*
f*x+1/2*e)+1)^4*c^5*d+60/f/a^3/(tan(1/2*f*x+1/2*e)+1)^4*c^4*d^2-80/f/a^3/(t
an(1/2*f*x+1/2*e)+1)^4*c^3*d^3+60/f/a^3/(tan(1/2*f*x+1/2*e)+1)^4*c^2*d^4-28
/f/a^3*d^6/(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)^2-12/f/a^3/(tan(1/
2*f*x+1/2*e)+1)^2*c^5*d+40/f/a^3/(tan(1/2*f*x+1/2*e)+1)*c^3*d^3+40/f/a^3/(t
an(1/2*f*x+1/2*e)+1)^2*c^3*d^3-60/f/a^3/(tan(1/2*f*x+1/2*e)+1)^2*c^2*d^4-40
/3/f/a^3*d^6/(1+tan(1/2*f*x+1/2*e)^2)^3-23/f/a^3*d^6*arctan(tan(1/2*f*x+1/2
*e))-2/f/a^3/(tan(1/2*f*x+1/2*e)+1)*c^6-20/f/a^3/(tan(1/2*f*x+1/2*e)+1)*d^6
+4/f/a^3/(tan(1/2*f*x+1/2*e)+1)^2*c^6-8/f/a^3/(tan(1/2*f*x+1/2*e)+1)^2*d^6-
16/3/f/a^3/(tan(1/2*f*x+1/2*e)+1)^3*c^6+8/3/f/a^3/(tan(1/2*f*x+1/2*e)+1)^3*
d^6-24/f/a^3/(tan(1/2*f*x+1/2*e)+1)^5*c^4*d^2+32/f/a^3/(tan(1/2*f*x+1/2*e)+
1)^5*c^3*d^3+24/f/a^3/(tan(1/2*f*x+1/2*e)+1)^3*c^5*d-40/f/a^3/(tan(1/2*f*x+
1/2*e)+1)^3*c^4*d^2+36/f/a^3/(tan(1/2*f*x+1/2*e)+1)^2*c*d^5-24/f/a^3/(tan(1
/2*f*x+1/2*e)+1)^4*c*d^5+48/5/f/a^3/(tan(1/2*f*x+1/2*e)+1)^5*c^5*d-24/f/a^3
/(tan(1/2*f*x+1/2*e)+1)^5*c^2*d^4+48/5/f/a^3/(tan(1/2*f*x+1/2*e)+1)^5*c*d^5
-3/f/a^3*d^6/(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)^5-12/f/a^3*d^6/(
1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)^4-90/f/a^3/(tan(1/2*f*x+1/2*e)
+1)*c^2*d^4+72/f/a^3/(tan(1/2*f*x+1/2*e)+1)*c*d^5+36/f/a^3*d^5/(1+tan(1/2*f
*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)^4*c-60/f/a^3*d^4/(1+tan(1/2*f*x+1/2*e)^2)
^3*tan(1/2*f*x+1/2*e)^2*c^2+72/f/a^3*d^5/(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2
*f*x+1/2*e)^2*c-6/f/a^3*d^5/(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)*c
+4/f/a^3/(tan(1/2*f*x+1/2*e)+1)^4*c^6+4/f/a^3/(tan(1/2*f*x+1/2*e)+1)^4*d^6-
8/5/f/a^3/(tan(1/2*f*x+1/2*e)+1)^5*c^6-8/5/f/a^3/(tan(1/2*f*x+1/2*e)+1)^5*d
^6+6/f/a^3*d^5/(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)^5*c-30/f/a^3*d
^4/(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)^4*c^2
```

Maxima [B] time = 2.14224, size = 2691, normalized size = 7.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^6/(a+a*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] -1/15*(d^6*((2375*sin(f*x + e)/(cos(f*x + e) + 1) + 5347*sin(f*x + e)^2/(co
s(f*x + e) + 1)^2 + 9230*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 12622*sin(f*
x + e)^4/(cos(f*x + e) + 1)^4 + 13340*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 +
 11684*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 8050*sin(f*x + e)^7/(cos(f*x +
 e) + 1)^7 + 4370*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 1725*sin(f*x + e)^9
/(cos(f*x + e) + 1)^9 + 345*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 + 544)/(a
^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 13*a^3*sin(f*x + e)^2/(cos(f*x
 + e) + 1)^2 + 25*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 38*a^3*sin(f*x
+ e)^4/(cos(f*x + e) + 1)^4 + 46*a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 +
46*a^3*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 38*a^3*sin(f*x + e)^7/(cos(f*x
 + e) + 1)^7 + 25*a^3*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 13*a^3*sin(f*x
+ e)^9/(cos(f*x + e) + 1)^9 + 5*a^3*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 +
 a^3*sin(f*x + e)^11/(cos(f*x + e) + 1)^11) + 345*arctan(sin(f*x + e)/(cos(
f*x + e) + 1))/a^3) - 6*c*d^5*((1325*sin(f*x + e)/(cos(f*x + e) + 1) + 2673
*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 3805*sin(f*x + e)^3/(cos(f*x + e) +
1)^3 + 4329*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 3575*sin(f*x + e)^5/(cos(
f*x + e) + 1)^5 + 2275*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 975*sin(f*x +
e)^7/(cos(f*x + e) + 1)^7 + 195*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 304)/
(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 12*a^3*sin(f*x + e)^2/(cos(f
*x + e) + 1)^2 + 20*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 26*a^3*sin(f*
x + e)^4/(cos(f*x + e) + 1)^4 + 26*a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5
+ 20*a^3*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 12*a^3*sin(f*x + e)^7/(cos(f
*x + e) + 1)^7 + 5*a^3*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + a^3*sin(f*x +
e)^9/(cos(f*x + e) + 1)^9) + 195*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^
3) + 90*c^2*d^4*((105*sin(f*x + e)/(cos(f*x + e) + 1) + 189*sin(f*x + e)^2/
(cos(f*x + e) + 1)^2 + 200*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 160*sin(f*
x + e)^4/(cos(f*x + e) + 1)^4 + 75*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 15
*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 24)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f
*x + e) + 1) + 11*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 15*a^3*sin(f*x
+ e)^3/(cos(f*x + e) + 1)^3 + 15*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 +
11*a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 5*a^3*sin(f*x + e)^6/(cos(f*x
+ e) + 1)^6 + a^3*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) + 15*arctan(sin(f*x
+ e)/(cos(f*x + e) + 1))/a^3) - 40*c^3*d^3*((95*sin(f*x + e)/(cos(f*x + e)
+ 1) + 145*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 75*sin(f*x + e)^3/(cos(f*x
 + e) + 1)^3 + 15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 22)/(a^3 + 5*a^3*si
n(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2
+ 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*
x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 15*arctan(sin(f*
x + e)/(cos(f*x + e) + 1))/a^3) + 2*c^6*(20*sin(f*x + e)/(cos(f*x + e) + 1)
 + 40*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 30*sin(f*x + e)^3/(cos(f*x + e)
 + 1)^3 + 15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 7)/(a^3 + 5*a^3*sin(f*x
+ e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a
^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e)
 + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 60*c^4*d^2*(5*sin(f*x
+ e)/(cos(f*x + e) + 1) + 10*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/(a^3
+ 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x +
e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)
^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 36*c^5
*d*(5*sin(f*x + e)/(cos(f*x + e) + 1) + 5*sin(f*x + e)^2/(cos(f*x + e) + 1)
^2 + 5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(
cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin
(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^
4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5))/f
```

Fricas [B] time = 1.9088, size = 1949, normalized size = 5.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^6/(a+a*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/30*(10*d^6*cos(f*x + e)^6 + 6*c^6 - 36*c^5*d + 90*c^4*d^2 - 120*c^3*d^3 +
 90*c^2*d^4 - 36*c*d^5 + 6*d^6 + 15*(6*c*d^5 - d^6)*cos(f*x + e)^5 - 10*(45
*c^2*d^4 - 36*c*d^5 + 14*d^6)*cos(f*x + e)^4 - (4*c^6 + 36*c^5*d + 210*c^4*
d^2 - 1280*c^3*d^3 + 3510*c^2*d^4 - 2694*c*d^5 + 839*d^6 - 15*(40*c^3*d^3 -
 90*c^2*d^4 + 78*c*d^5 - 23*d^6)*f*x)*cos(f*x + e)^3 - 60*(40*c^3*d^3 - 90*
c^2*d^4 + 78*c*d^5 - 23*d^6)*f*x + (8*c^6 + 72*c^5*d - 30*c^4*d^2 - 760*c^3
*d^3 + 2520*c^2*d^4 - 2148*c*d^5 + 668*d^6 + 45*(40*c^3*d^3 - 90*c^2*d^4 +
78*c*d^5 - 23*d^6)*f*x)*cos(f*x + e)^2 + 6*(3*c^6 + 12*c^5*d + 45*c^4*d^2 -
 360*c^3*d^3 + 945*c^2*d^4 - 768*c*d^5 + 233*d^6 - 5*(40*c^3*d^3 - 90*c^2*d
^4 + 78*c*d^5 - 23*d^6)*f*x)*cos(f*x + e) + (10*d^6*cos(f*x + e)^5 - 6*c^6
+ 36*c^5*d - 90*c^4*d^2 + 120*c^3*d^3 - 90*c^2*d^4 + 36*c*d^5 - 6*d^6 - 5*(
18*c*d^5 - 5*d^6)*cos(f*x + e)^4 - 5*(90*c^2*d^4 - 54*c*d^5 + 23*d^6)*cos(f
*x + e)^3 - 60*(40*c^3*d^3 - 90*c^2*d^4 + 78*c*d^5 - 23*d^6)*f*x + (4*c^6 +
 36*c^5*d + 210*c^4*d^2 - 1280*c^3*d^3 + 3060*c^2*d^4 - 2424*c*d^5 + 724*d^
6 + 15*(40*c^3*d^3 - 90*c^2*d^4 + 78*c*d^5 - 23*d^6)*f*x)*cos(f*x + e)^2 +
6*(2*c^6 + 18*c^5*d + 30*c^4*d^2 - 340*c^3*d^3 + 930*c^2*d^4 - 762*c*d^5 +
232*d^6 - 5*(40*c^3*d^3 - 90*c^2*d^4 + 78*c*d^5 - 23*d^6)*f*x)*cos(f*x + e)
)*sin(f*x + e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 - 2*a^3*f*co
s(f*x + e) - 4*a^3*f + (a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3
*f)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**6/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.32225, size = 1048, normalized size = 2.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^6/(a+a*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/30*(15*(40*c^3*d^3 - 90*c^2*d^4 + 78*c*d^5 - 23*d^6)*(f*x + e)/a^3 + 10*(
18*c*d^5*tan(1/2*f*x + 1/2*e)^5 - 9*d^6*tan(1/2*f*x + 1/2*e)^5 - 90*c^2*d^4
*tan(1/2*f*x + 1/2*e)^4 + 108*c*d^5*tan(1/2*f*x + 1/2*e)^4 - 36*d^6*tan(1/2
*f*x + 1/2*e)^4 - 180*c^2*d^4*tan(1/2*f*x + 1/2*e)^2 + 216*c*d^5*tan(1/2*f*
x + 1/2*e)^2 - 84*d^6*tan(1/2*f*x + 1/2*e)^2 - 18*c*d^5*tan(1/2*f*x + 1/2*e
) + 9*d^6*tan(1/2*f*x + 1/2*e) - 90*c^2*d^4 + 108*c*d^5 - 40*d^6)/((tan(1/2
*f*x + 1/2*e)^2 + 1)^3*a^3) - 4*(15*c^6*tan(1/2*f*x + 1/2*e)^4 - 300*c^3*d^
3*tan(1/2*f*x + 1/2*e)^4 + 675*c^2*d^4*tan(1/2*f*x + 1/2*e)^4 - 540*c*d^5*t
an(1/2*f*x + 1/2*e)^4 + 150*d^6*tan(1/2*f*x + 1/2*e)^4 + 30*c^6*tan(1/2*f*x
 + 1/2*e)^3 + 90*c^5*d*tan(1/2*f*x + 1/2*e)^3 - 1500*c^3*d^3*tan(1/2*f*x +
1/2*e)^3 + 3150*c^2*d^4*tan(1/2*f*x + 1/2*e)^3 - 2430*c*d^5*tan(1/2*f*x + 1
```

```
/2*e)^3 + 660*d^6*tan(1/2*f*x + 1/2*e)^3 + 40*c^6*tan(1/2*f*x + 1/2*e)^2 +
90*c^5*d*tan(1/2*f*x + 1/2*e)^2 + 300*c^4*d^2*tan(1/2*f*x + 1/2*e)^2 - 2900
*c^3*d^3*tan(1/2*f*x + 1/2*e)^2 + 5400*c^2*d^4*tan(1/2*f*x + 1/2*e)^2 - 399
0*c*d^5*tan(1/2*f*x + 1/2*e)^2 + 1060*d^6*tan(1/2*f*x + 1/2*e)^2 + 20*c^6*t
an(1/2*f*x + 1/2*e) + 90*c^5*d*tan(1/2*f*x + 1/2*e) + 150*c^4*d^2*tan(1/2*f
*x + 1/2*e) - 1900*c^3*d^3*tan(1/2*f*x + 1/2*e) + 3600*c^2*d^4*tan(1/2*f*x
+ 1/2*e) - 2670*c*d^5*tan(1/2*f*x + 1/2*e) + 710*d^6*tan(1/2*f*x + 1/2*e) +
 7*c^6 + 18*c^5*d + 30*c^4*d^2 - 440*c^3*d^3 + 855*c^2*d^4 - 642*c*d^5 + 17
2*d^6)/(a^3*(tan(1/2*f*x + 1/2*e) + 1)^5))/f
```

3.471 $\int \frac{(c+d\sin(e+fx))^5}{(a+a\sin(e+fx))^3}\,dx$

Optimal. Leaf size=278

$$\frac{2d\left(72c^2d^2+15c^3d+2c^4-180cd^3+76d^4\right)\cos(e+fx)}{15a^3f}-\frac{(c-d)\left(2c^2+15cd+76d^2\right)\cos(e+fx)(c+d\sin(e+fx))^2}{15f\left(a^3\sin(e+fx)+a^3\right)}$$

```
[Out] (d^3*(20*c^2 - 30*c*d + 13*d^2)*x)/(2*a^3) + (2*d*(2*c^4 + 15*c^3*d + 72*c^
2*d^2 - 180*c*d^3 + 76*d^4)*Cos[e + f*x])/(15*a^3*f) + (d^2*(4*c^3 + 30*c^2
*d + 146*c*d^2 - 195*d^3)*Cos[e + f*x]*Sin[e + f*x])/(30*a^3*f) - ((c - d)*
(2*c^2 + 15*c*d + 76*d^2)*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(15*f*(a^3 +
 a^3*Sin[e + f*x])) - ((c - d)*(2*c + 11*d)*Cos[e + f*x]*(c + d*Sin[e + f*x
])^3)/(15*a*f*(a + a*Sin[e + f*x])^2) - ((c - d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^4)/(5*f*(a + a*Sin[e + f*x])^3)
```

Rubi [A] time = 0.614009, antiderivative size = 278, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2765, 2977, 2734}

$$\frac{2d\left(72c^2d^2+15c^3d+2c^4-180cd^3+76d^4\right)\cos(e+fx)}{15a^3f}-\frac{(c-d)\left(2c^2+15cd+76d^2\right)\cos(e+fx)(c+d\sin(e+fx))^2}{15f\left(a^3\sin(e+fx)+a^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^5/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] (d^3*(20*c^2 - 30*c*d + 13*d^2)*x)/(2*a^3) + (2*d*(2*c^4 + 15*c^3*d + 72*c^
2*d^2 - 180*c*d^3 + 76*d^4)*Cos[e + f*x])/(15*a^3*f) + (d^2*(4*c^3 + 30*c^2
*d + 146*c*d^2 - 195*d^3)*Cos[e + f*x]*Sin[e + f*x])/(30*a^3*f) - ((c - d)*
(2*c^2 + 15*c*d + 76*d^2)*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(15*f*(a^3 +
 a^3*Sin[e + f*x])) - ((c - d)*(2*c + 11*d)*Cos[e + f*x]*(c + d*Sin[e + f*x
])^3)/(15*a*f*(a + a*Sin[e + f*x])^2) - ((c - d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^4)/(5*f*(a + a*Sin[e + f*x])^3)
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
```

```
egerQ[2*n] || EqQ[c, 0])
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^5}{(a+a\sin(e+fx))^3}\,dx = -\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^4}{5f(a+a\sin(e+fx))^3} - \frac{\int \frac{(c+d\sin(e+fx))^3(-a(2c-d)(c+4d)+a(2c-7d)d\sin(e+fx))}{(a+a\sin(e+fx))^2}\,dx}{5a^2}$$

$$= -\frac{(c-d)(2c+11d)\cos(e+fx)(c+d\sin(e+fx))^3}{15af(a+a\sin(e+fx))^2} - \frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^4}{5f(a+a\sin(e+fx))^3}$$

$$= -\frac{(c-d)\left(2c^2+15cd+76d^2\right)\cos(e+fx)(c+d\sin(e+fx))^2}{15f\left(a^3+a^3\sin(e+fx)\right)} - \frac{(c-d)(2c+11d)\cos(e+fx)}{15af(a+a\sin(e}$$

$$= \frac{d^3\left(20c^2-30cd+13d^2\right)x}{2a^3} + \frac{2d\left(2c^4+15c^3d+72c^2d^2-180cd^3+76d^4\right)\cos(e+fx)}{15a^3f} + \frac{d^2(4}{}$$

Mathematica [B] time = 8.02663, size = 992, normalized size = 3.57

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(-160\cos\left(\frac{3}{2}(e+fx)\right)c^5+320\sin\left(\frac{1}{2}(e+fx)\right)c^5-32\sin\left(\frac{5}{2}(e+fx)\right)c^5+1200d\cos\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])^5/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(1200*c^4*d*Cos[(e + f*x)/2] + 4800*
c^3*d^2*Cos[(e + f*x)/2] - 21600*c^2*d^3*Cos[(e + f*x)/2] + 22500*c*d^4*Cos
[(e + f*x)/2] - 7560*d^5*Cos[(e + f*x)/2] + 12000*c^2*d^3*(e + f*x)*Cos[(e
+ f*x)/2] - 18000*c*d^4*(e + f*x)*Cos[(e + f*x)/2] + 7800*d^5*(e + f*x)*Cos
[(e + f*x)/2] - 160*c^5*Cos[(3*(e + f*x))/2] - 1200*c^4*d*Cos[(3*(e + f*x))
/2] - 3200*c^3*d^2*Cos[(3*(e + f*x))/2] + 18400*c^2*d^3*Cos[(3*(e + f*x))/2
] - 24300*c*d^4*Cos[(3*(e + f*x))/2] + 9230*d^5*Cos[(3*(e + f*x))/2] - 6000
*c^2*d^3*(e + f*x)*Cos[(3*(e + f*x))/2] + 9000*c*d^4*(e + f*x)*Cos[(3*(e +
f*x))/2] - 3900*d^5*(e + f*x)*Cos[(3*(e + f*x))/2] + 1500*c*d^4*Cos[(5*(e +
 f*x))/2] - 750*d^5*Cos[(5*(e + f*x))/2] - 1200*c^2*d^3*(e + f*x)*Cos[(5*(e
 + f*x))/2] + 1800*c*d^4*(e + f*x)*Cos[(5*(e + f*x))/2] - 780*d^5*(e + f*x)
*Cos[(5*(e + f*x))/2] + 300*c*d^4*Cos[(7*(e + f*x))/2] - 105*d^5*Cos[(7*(e
+ f*x))/2] - 15*d^5*Cos[(9*(e + f*x))/2] + 320*c^5*Sin[(e + f*x)/2] + 1200*
c^4*d*Sin[(e + f*x)/2] + 6400*c^3*d^2*Sin[(e + f*x)/2] - 29600*c^2*d^3*Sin[
(e + f*x)/2] + 35100*c*d^4*Sin[(e + f*x)/2] - 12760*d^5*Sin[(e + f*x)/2] +
12000*c^2*d^3*(e + f*x)*Sin[(e + f*x)/2] - 18000*c*d^4*(e + f*x)*Sin[(e + f
*x)/2] + 7800*d^5*(e + f*x)*Sin[(e + f*x)/2] + 2400*c^3*d^2*Sin[(3*(e + f*x
))/2] - 7200*c^2*d^3*Sin[(3*(e + f*x))/2] + 4500*c*d^4*Sin[(3*(e + f*x))/2]
 - 930*d^5*Sin[(3*(e + f*x))/2] + 6000*c^2*d^3*(e + f*x)*Sin[(3*(e + f*x))/
2] - 9000*c*d^4*(e + f*x)*Sin[(3*(e + f*x))/2] + 3900*d^5*(e + f*x)*Sin[(3*
(e + f*x))/2] - 32*c^5*Sin[(5*(e + f*x))/2] - 240*c^4*d*Sin[(5*(e + f*x))/2
] - 1120*c^3*d^2*Sin[(5*(e + f*x))/2] + 5120*c^2*d^3*Sin[(5*(e + f*x))/2] -
 7260*c*d^4*Sin[(5*(e + f*x))/2] + 2782*d^5*Sin[(5*(e + f*x))/2] - 1200*c^2
```

```
*d^3*(e + f*x)*Sin[(5*(e + f*x))/2] + 1800*c*d^4*(e + f*x)*Sin[(5*(e + f*x)
)/2] - 780*d^5*(e + f*x)*Sin[(5*(e + f*x))/2] + 300*c*d^4*Sin[(7*(e + f*x))
/2] - 105*d^5*Sin[(7*(e + f*x))/2] + 15*d^5*Sin[(9*(e + f*x))/2]))/(480*f*(
a + a*Sin[e + f*x])^3)
```

Maple [B] time = 0.101, size = 924, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^5/(a+a*sin(f*x+e))^3,x)
```

```
[Out] 6/f/a^3*d^5/(1+tan(1/2*f*x+1/2*e)^2)^2+13/f/a^3*d^5*arctan(tan(1/2*f*x+1/2*
e))-2/f/a^3/(tan(1/2*f*x+1/2*e)+1)*c^5+12/f/a^3/(tan(1/2*f*x+1/2*e)+1)*d^5+
4/f/a^3/(tan(1/2*f*x+1/2*e)+1)^2*c^5+6/f/a^3/(tan(1/2*f*x+1/2*e)+1)^2*d^5-1
6/3/f/a^3/(tan(1/2*f*x+1/2*e)+1)^3*c^5-4/3/f/a^3/(tan(1/2*f*x+1/2*e)+1)^3*d
^5+4/f/a^3/(tan(1/2*f*x+1/2*e)+1)^4*c^5-30/f/a^3/(tan(1/2*f*x+1/2*e)+1)*c*d
^4-8/f/a^3/(tan(1/2*f*x+1/2*e)+1)^5*c*d^4+1/f/a^3*d^5/(1+tan(1/2*f*x+1/2*e)
^2)^2*tan(1/2*f*x+1/2*e)^3+6/f/a^3*d^5/(1+tan(1/2*f*x+1/2*e)^2)^2*tan(1/2*f
*x+1/2*e)^2-1/f/a^3*d^5/(1+tan(1/2*f*x+1/2*e)^2)^2*tan(1/2*f*x+1/2*e)-10/f/
a^3*d^4/(1+tan(1/2*f*x+1/2*e)^2)^2*c+20/f/a^3*d^3*arctan(tan(1/2*f*x+1/2*e)
)*c^2-30/f/a^3*d^4*arctan(tan(1/2*f*x+1/2*e))*c-4/f/a^3/(tan(1/2*f*x+1/2*e)
+1)^4*d^5-8/5/f/a^3/(tan(1/2*f*x+1/2*e)+1)^5*c^5+8/5/f/a^3/(tan(1/2*f*x+1/2
*e)+1)^5*d^5-10/f/a^3/(tan(1/2*f*x+1/2*e)+1)^2*c^4*d+20/f/a^3/(tan(1/2*f*x+
1/2*e)+1)^2*c^2*d^3-20/f/a^3/(tan(1/2*f*x+1/2*e)+1)^2*c*d^4+20/f/a^3/(tan(1
/2*f*x+1/2*e)+1)^3*c^4*d-80/3/f/a^3/(tan(1/2*f*x+1/2*e)+1)^3*c^3*d^2+40/3/f
/a^3/(tan(1/2*f*x+1/2*e)+1)^3*c^2*d^3-20/f/a^3/(tan(1/2*f*x+1/2*e)+1)^4*c^4
*d+40/f/a^3/(tan(1/2*f*x+1/2*e)+1)^4*c^3*d^2-40/f/a^3/(tan(1/2*f*x+1/2*e)+1
)^4*c^2*d^3+20/f/a^3/(tan(1/2*f*x+1/2*e)+1)^4*c*d^4+8/f/a^3/(tan(1/2*f*x+1/
2*e)+1)^5*c^4*d-16/f/a^3/(tan(1/2*f*x+1/2*e)+1)^5*c^3*d^2+16/f/a^3/(tan(1/2
*f*x+1/2*e)+1)^5*c^2*d^3+20/f/a^3/(tan(1/2*f*x+1/2*e)+1)*c^2*d^3-10/f/a^3*d
^4/(1+tan(1/2*f*x+1/2*e)^2)^2*tan(1/2*f*x+1/2*e)^2*c
```

Maxima [B] time = 2.56399, size = 2030, normalized size = 7.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^5/(a+a*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 1/15*(d^5*((1325*sin(f*x + e)/(cos(f*x + e) + 1) + 2673*sin(f*x + e)^2/(cos
(f*x + e) + 1)^2 + 3805*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 4329*sin(f*x
+ e)^4/(cos(f*x + e) + 1)^4 + 3575*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 22
75*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 975*sin(f*x + e)^7/(cos(f*x + e) +
 1)^7 + 195*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 304)/(a^3 + 5*a^3*sin(f*x
 + e)/(cos(f*x + e) + 1) + 12*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 20*
a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 26*a^3*sin(f*x + e)^4/(cos(f*x +
e) + 1)^4 + 26*a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 20*a^3*sin(f*x + e
)^6/(cos(f*x + e) + 1)^6 + 12*a^3*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 5*a
^3*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + a^3*sin(f*x + e)^9/(cos(f*x + e) +
 1)^9) + 195*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^3) - 30*c*d^4*((105*
sin(f*x + e)/(cos(f*x + e) + 1) + 189*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 +
```

```
 200*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 160*sin(f*x + e)^4/(cos(f*x + e)
 + 1)^4 + 75*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 15*sin(f*x + e)^6/(cos(f
*x + e) + 1)^6 + 24)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 11*a^3*
sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 15*a^3*sin(f*x + e)^3/(cos(f*x + e) +
 1)^3 + 15*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 11*a^3*sin(f*x + e)^5/
(cos(f*x + e) + 1)^5 + 5*a^3*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + a^3*sin(
f*x + e)^7/(cos(f*x + e) + 1)^7) + 15*arctan(sin(f*x + e)/(cos(f*x + e) + 1
))/a^3) + 20*c^2*d^3*((95*sin(f*x + e)/(cos(f*x + e) + 1) + 145*sin(f*x + e
)^2/(cos(f*x + e) + 1)^2 + 75*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*sin(
f*x + e)^4/(cos(f*x + e) + 1)^4 + 22)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x +
e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^
3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*si
n(f*x + e)^5/(cos(f*x + e) + 1)^5) + 15*arctan(sin(f*x + e)/(cos(f*x + e) +
 1))/a^3) - 2*c^5*(20*sin(f*x + e)/(cos(f*x + e) + 1) + 40*sin(f*x + e)^2/(
cos(f*x + e) + 1)^2 + 30*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*sin(f*x +
 e)^4/(cos(f*x + e) + 1)^4 + 7)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1
) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos
(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x
+ e)^5/(cos(f*x + e) + 1)^5) - 40*c^3*d^2*(5*sin(f*x + e)/(cos(f*x + e) + 1
) + 10*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(
cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin
(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^
4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) - 30*c^4*d*(5*sin(f*x + e)/(co
s(f*x + e) + 1) + 5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 5*sin(f*x + e)^3/
(cos(f*x + e) + 1)^3 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10
*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x +
 e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5
/(cos(f*x + e) + 1)^5))/f
```

Fricas [B] time = 1.83992, size = 1548, normalized size = 5.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^5/(a+a*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/30*(15*d^5*cos(f*x + e)^5 + 6*c^5 - 30*c^4*d + 60*c^3*d^2 - 60*c^2*d^3 +
30*c*d^4 - 6*d^5 - 30*(5*c*d^4 - 2*d^5)*cos(f*x + e)^4 - (4*c^5 + 30*c^4*d
+ 140*c^3*d^2 - 640*c^2*d^3 + 1170*c*d^4 - 449*d^5 - 15*(20*c^2*d^3 - 30*c*
d^4 + 13*d^5)*f*x)*cos(f*x + e)^3 - 60*(20*c^2*d^3 - 30*c*d^4 + 13*d^5)*f*x
 + (8*c^5 + 60*c^4*d - 20*c^3*d^2 - 380*c^2*d^3 + 840*c*d^4 - 358*d^5 + 45*
(20*c^2*d^3 - 30*c*d^4 + 13*d^5)*f*x)*cos(f*x + e)^2 + 6*(3*c^5 + 10*c^4*d
+ 30*c^3*d^2 - 180*c^2*d^3 + 315*c*d^4 - 128*d^5 - 5*(20*c^2*d^3 - 30*c*d^4
 + 13*d^5)*f*x)*cos(f*x + e) - (15*d^5*cos(f*x + e)^4 + 6*c^5 - 30*c^4*d +
60*c^3*d^2 - 60*c^2*d^3 + 30*c*d^4 - 6*d^5 + 15*(10*c*d^4 - 3*d^5)*cos(f*x
+ e)^3 + 60*(20*c^2*d^3 - 30*c*d^4 + 13*d^5)*f*x - (4*c^5 + 30*c^4*d + 140*
c^3*d^2 - 640*c^2*d^3 + 1020*c*d^4 - 404*d^5 + 15*(20*c^2*d^3 - 30*c*d^4 +
13*d^5)*f*x)*cos(f*x + e)^2 - 6*(2*c^5 + 15*c^4*d + 20*c^3*d^2 - 170*c^2*d^
3 + 310*c*d^4 - 127*d^5 - 5*(20*c^2*d^3 - 30*c*d^4 + 13*d^5)*f*x)*cos(f*x +
 e))*sin(f*x + e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 - 2*a^3*f
*cos(f*x + e) - 4*a^3*f + (a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*
a^3*f)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**5/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.33615, size = 761, normalized size = 2.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^5/(a+a*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/30*(15*(20*c^2*d^3 - 30*c*d^4 + 13*d^5)*(f*x + e)/a^3 + 30*(d^5*tan(1/2*f
*x + 1/2*e)^3 - 10*c*d^4*tan(1/2*f*x + 1/2*e)^2 + 6*d^5*tan(1/2*f*x + 1/2*e
)^2 - d^5*tan(1/2*f*x + 1/2*e) - 10*c*d^4 + 6*d^5)/((tan(1/2*f*x + 1/2*e)^2
 + 1)^2*a^3) - 4*(15*c^5*tan(1/2*f*x + 1/2*e)^4 - 150*c^2*d^3*tan(1/2*f*x +
 1/2*e)^4 + 225*c*d^4*tan(1/2*f*x + 1/2*e)^4 - 90*d^5*tan(1/2*f*x + 1/2*e)^
4 + 30*c^5*tan(1/2*f*x + 1/2*e)^3 + 75*c^4*d*tan(1/2*f*x + 1/2*e)^3 - 750*c
^2*d^3*tan(1/2*f*x + 1/2*e)^3 + 1050*c*d^4*tan(1/2*f*x + 1/2*e)^3 - 405*d^5
*tan(1/2*f*x + 1/2*e)^3 + 40*c^5*tan(1/2*f*x + 1/2*e)^2 + 75*c^4*d*tan(1/2*
f*x + 1/2*e)^2 + 200*c^3*d^2*tan(1/2*f*x + 1/2*e)^2 - 1450*c^2*d^3*tan(1/2*
f*x + 1/2*e)^2 + 1800*c*d^4*tan(1/2*f*x + 1/2*e)^2 - 665*d^5*tan(1/2*f*x +
1/2*e)^2 + 20*c^5*tan(1/2*f*x + 1/2*e) + 75*c^4*d*tan(1/2*f*x + 1/2*e) + 10
0*c^3*d^2*tan(1/2*f*x + 1/2*e) - 950*c^2*d^3*tan(1/2*f*x + 1/2*e) + 1200*c*
d^4*tan(1/2*f*x + 1/2*e) - 445*d^5*tan(1/2*f*x + 1/2*e) + 7*c^5 + 15*c^4*d
+ 20*c^3*d^2 - 220*c^2*d^3 + 285*c*d^4 - 107*d^5)/(a^3*(tan(1/2*f*x + 1/2*e
) + 1)^5))/f
```

3.472 $\int \frac{(c+d\sin(e+fx))^4}{(a+a\sin(e+fx))^3}\,dx$

Optimal. Leaf size=195

$$\frac{d^2\left(2c^2+10cd-27d^2\right)\cos(e+fx)}{15a^3f}-\frac{(c-d)^2\left(2c^2+12cd+45d^2\right)\cos(e+fx)}{15f\left(a^3\sin(e+fx)+a^3\right)}+\frac{d^3x(4c-3d)}{a^3}-\frac{(c-d)\cos(e+fx)(c+d\ldots}{5f(a\sin(e+fx\ldots}$$

```
[Out] ((4*c - 3*d)*d^3*x)/a^3 + (d^2*(2*c^2 + 10*c*d - 27*d^2)*Cos[e + f*x])/(15*
a^3*f) - ((c - d)^2*(2*c^2 + 12*c*d + 45*d^2)*Cos[e + f*x])/(15*f*(a^3 + a^
3*Sin[e + f*x])) - ((c - d)*(2*c + 9*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^2
)/(15*a*f*(a + a*Sin[e + f*x])^2) - ((c - d)*Cos[e + f*x]*(c + d*Sin[e + f*
x])^3)/(5*f*(a + a*Sin[e + f*x])^3)
```

Rubi [A] time = 0.613012, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {2765, 2977, 2968, 3023, 2735, 2648}

$$\frac{d^2\left(2c^2+10cd-27d^2\right)\cos(e+fx)}{15a^3f}-\frac{(c-d)^2\left(2c^2+12cd+45d^2\right)\cos(e+fx)}{15f\left(a^3\sin(e+fx)+a^3\right)}+\frac{d^3x(4c-3d)}{a^3}-\frac{(c-d)\cos(e+fx)(c+d\ldots}{5f(a\sin(e+fx\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^4/(a + a*Sin[e + f*x])^3,x]
```


```
[Out] ((4*c - 3*d)*d^3*x)/a^3 + (d^2*(2*c^2 + 10*c*d - 27*d^2)*Cos[e + f*x])/(15*
a^3*f) - ((c - d)^2*(2*c^2 + 12*c*d + 45*d^2)*Cos[e + f*x])/(15*f*(a^3 + a^
3*Sin[e + f*x])) - ((c - d)*(2*c + 9*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^2
)/(15*a*f*(a + a*Sin[e + f*x])^2) - ((c - d)*Cos[e + f*x]*(c + d*Sin[e + f*
x])^3)/(5*f*(a + a*Sin[e + f*x])^3)
```


Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^4}{(a+a\sin(e+fx))^3}\,dx = -\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^3}{5f(a+a\sin(e+fx))^3} - \frac{\int \frac{(c+d\sin(e+fx))^2\left(-a\left(2c^2+6cd-3d^2\right)+a(c-6d)d\sin(e+fx)\right)}{(a+a\sin(e+fx))^2}}{5a^2}$$

$$= -\frac{(c-d)(2c+9d)\cos(e+fx)(c+d\sin(e+fx))^2}{15af(a+a\sin(e+fx))^2} - \frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))}{5f(a+a\sin(e+fx))^3}$$

$$= -\frac{(c-d)(2c+9d)\cos(e+fx)(c+d\sin(e+fx))^2}{15af(a+a\sin(e+fx))^2} - \frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))}{5f(a+a\sin(e+fx))^3}$$

$$= \frac{d^2\left(2c^2+10cd-27d^2\right)\cos(e+fx)}{15a^3f} - \frac{(c-d)(2c+9d)\cos(e+fx)(c+d\sin(e+fx))^2}{15af(a+a\sin(e+fx))^2} -$$

$$= \frac{(4c-3d)d^3x}{a^3} + \frac{d^2\left(2c^2+10cd-27d^2\right)\cos(e+fx)}{15a^3f} - \frac{(c-d)(2c+9d)\cos(e+fx)(c+d\sin}{15af(a+a\sin(e+fx))}$$

$$= \frac{(4c-3d)d^3x}{a^3} + \frac{d^2\left(2c^2+10cd-27d^2\right)\cos(e+fx)}{15a^3f} - \frac{(c-d)^2\left(2c^2+12cd+45d^2\right)\cos(e}{15f\left(a^3+a^3\sin(e+fx)\right)}$$

Mathematica [B] time = 1.42452, size = 683, normalized size = 3.5

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(960c^2d^2\sin\left(\frac{1}{2}(e+fx)\right)+360c^2d^2\sin\left(\frac{3}{2}(e+fx)\right)-168c^2d^2\sin\left(\frac{5}{2}(e+fx)\right)+1\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])^4/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(15*d*(16*c^3 + 48*c^2*d - 15*d^3*(-
5 + 4*e + 4*f*x) + 16*c*d^2*(-9 + 5*e + 5*f*x))*Cos[(e + f*x)/2] - 5*(8*c^4
 + 48*c^3*d + 96*c^2*d^2 - 9*d^4*(-27 + 10*e + 10*f*x) + 8*c*d^3*(-46 + 15*
e + 15*f*x))*Cos[(3*(e + f*x))/2] + 75*d^4*Cos[(5*(e + f*x))/2] - 120*c*d^3
*e*Cos[(5*(e + f*x))/2] + 90*d^4*e*Cos[(5*(e + f*x))/2] - 120*c*d^3*f*x*Cos
[(5*(e + f*x))/2] + 90*d^4*f*x*Cos[(5*(e + f*x))/2] + 15*d^4*Cos[(7*(e + f*
x))/2] + 80*c^4*Sin[(e + f*x)/2] + 240*c^3*d*Sin[(e + f*x)/2] + 960*c^2*d^2
*Sin[(e + f*x)/2] - 2960*c*d^3*Sin[(e + f*x)/2] + 1755*d^4*Sin[(e + f*x)/2]
 + 1200*c*d^3*e*Sin[(e + f*x)/2] - 900*d^4*e*Sin[(e + f*x)/2] + 1200*c*d^3*
f*x*Sin[(e + f*x)/2] - 900*d^4*f*x*Sin[(e + f*x)/2] + 360*c^2*d^2*Sin[(3*(e
 + f*x))/2] - 720*c*d^3*Sin[(3*(e + f*x))/2] + 225*d^4*Sin[(3*(e + f*x))/2]
 + 600*c*d^3*e*Sin[(3*(e + f*x))/2] - 450*d^4*e*Sin[(3*(e + f*x))/2] + 600*
c*d^3*f*x*Sin[(3*(e + f*x))/2] - 450*d^4*f*x*Sin[(3*(e + f*x))/2] - 8*c^4*S
in[(5*(e + f*x))/2] - 48*c^3*d*Sin[(5*(e + f*x))/2] - 168*c^2*d^2*Sin[(5*(e
 + f*x))/2] + 512*c*d^3*Sin[(5*(e + f*x))/2] - 363*d^4*Sin[(5*(e + f*x))/2]
 - 120*c*d^3*e*Sin[(5*(e + f*x))/2] + 90*d^4*e*Sin[(5*(e + f*x))/2] - 120*c
*d^3*f*x*Sin[(5*(e + f*x))/2] + 90*d^4*f*x*Sin[(5*(e + f*x))/2] + 15*d^4*Si
n[(7*(e + f*x))/2]))/(120*a^3*f*(1 + Sin[e + f*x])^3)
```

Maple [B] time = 0.092, size = 593, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^4/(a+a*sin(f*x+e))^3,x)
```

```
[Out] -2/f/a^3*d^4/(1+tan(1/2*f*x+1/2*e)^2)+8/f/a^3*d^3*arctan(tan(1/2*f*x+1/2*e)
)*c-6/f/a^3*d^4*arctan(tan(1/2*f*x+1/2*e))-2/f/a^3/(tan(1/2*f*x+1/2*e)+1)*c
^4+8/f/a^3/(tan(1/2*f*x+1/2*e)+1)*c*d^3-6/f/a^3/(tan(1/2*f*x+1/2*e)+1)*d^4+
4/f/a^3/(tan(1/2*f*x+1/2*e)+1)^2*c^4-8/f/a^3/(tan(1/2*f*x+1/2*e)+1)^2*c^3*d
+8/f/a^3/(tan(1/2*f*x+1/2*e)+1)^2*c*d^3-4/f/a^3/(tan(1/2*f*x+1/2*e)+1)^2*d^
4+4/f/a^3/(tan(1/2*f*x+1/2*e)+1)^4*c^4-16/f/a^3/(tan(1/2*f*x+1/2*e)+1)^4*c^
3*d+24/f/a^3/(tan(1/2*f*x+1/2*e)+1)^4*c^2*d^2-16/f/a^3/(tan(1/2*f*x+1/2*e)+
1)^4*c*d^3+4/f/a^3/(tan(1/2*f*x+1/2*e)+1)^4*d^4-8/5/f/a^3/(tan(1/2*f*x+1/2*
e)+1)^5*c^4+32/5/f/a^3/(tan(1/2*f*x+1/2*e)+1)^5*c^3*d-48/5/f/a^3/(tan(1/2*f
*x+1/2*e)+1)^5*c^2*d^2+32/5/f/a^3/(tan(1/2*f*x+1/2*e)+1)^5*c*d^3-8/5/f/a^3/
(tan(1/2*f*x+1/2*e)+1)^5*d^4-16/3/f/a^3*c^4/(tan(1/2*f*x+1/2*e)+1)^3+16/f/a
^3*c^3/(tan(1/2*f*x+1/2*e)+1)^3*d-16/f/a^3*c^2/(tan(1/2*f*x+1/2*e)+1)^3*d^2
+16/3/f/a^3*c/(tan(1/2*f*x+1/2*e)+1)^3*d^3
```

Maxima [B] time = 2.23886, size = 1486, normalized size = 7.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^4/(a+a*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] -2/15*(3*d^4*((105*sin(f*x + e)/(cos(f*x + e) + 1) + 189*sin(f*x + e)^2/(co
s(f*x + e) + 1)^2 + 200*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 160*sin(f*x +
 e)^4/(cos(f*x + e) + 1)^4 + 75*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 15*si
n(f*x + e)^6/(cos(f*x + e) + 1)^6 + 24)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x
+ e) + 1) + 11*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 15*a^3*sin(f*x + e
```

```
)^3/(cos(f*x + e) + 1)^3 + 15*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 11*
a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 5*a^3*sin(f*x + e)^6/(cos(f*x + e
) + 1)^6 + a^3*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) + 15*arctan(sin(f*x + e
)/(cos(f*x + e) + 1))/a^3) - 4*c*d^3*((95*sin(f*x + e)/(cos(f*x + e) + 1) +
 145*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 75*sin(f*x + e)^3/(cos(f*x + e)
+ 1)^3 + 15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 22)/(a^3 + 5*a^3*sin(f*x
+ e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a
^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e)
 + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 15*arctan(sin(f*x + e)
/(cos(f*x + e) + 1))/a^3) + c^4*(20*sin(f*x + e)/(cos(f*x + e) + 1) + 40*si
n(f*x + e)^2/(cos(f*x + e) + 1)^2 + 30*sin(f*x + e)^3/(cos(f*x + e) + 1)^3
+ 15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 7)/(a^3 + 5*a^3*sin(f*x + e)/(co
s(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f
*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4
+ a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 12*c^2*d^2*(5*sin(f*x + e)/(co
s(f*x + e) + 1) + 10*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/(a^3 + 5*a^3*
sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^
2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(
f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 12*c^3*d*(5*si
n(f*x + e)/(cos(f*x + e) + 1) + 5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 5*s
in(f*x + e)^3/(cos(f*x + e) + 1)^3 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x
+ e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e
)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*
sin(f*x + e)^5/(cos(f*x + e) + 1)^5))/f
```

Fricas [B] time = 1.7586, size = 1154, normalized size = 5.92

$$-\frac{15\,d^4\cos\left(fx+e\right)^4 - 3\,c^4 + 12\,c^3d - 18\,c^2d^2 + 12\,cd^3 - 3\,d^4 + \left(2\,c^4 + 12\,c^3d + 42\,c^2d^2 - 128\,cd^3 + 117\,d^4 - 15\left(4\,cd^3\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^4/(a+a*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -1/15*(15*d^4*cos(f*x + e)^4 - 3*c^4 + 12*c^3*d - 18*c^2*d^2 + 12*c*d^3 - 3
*d^4 + (2*c^4 + 12*c^3*d + 42*c^2*d^2 - 128*c*d^3 + 117*d^4 - 15*(4*c*d^3 -
 3*d^4)*f*x)*cos(f*x + e)^3 + 60*(4*c*d^3 - 3*d^4)*f*x - (4*c^4 + 24*c^3*d
- 6*c^2*d^2 - 76*c*d^3 + 84*d^4 + 45*(4*c*d^3 - 3*d^4)*f*x)*cos(f*x + e)^2
- 3*(3*c^4 + 8*c^3*d + 18*c^2*d^2 - 72*c*d^3 + 63*d^4 - 10*(4*c*d^3 - 3*d^4
)*f*x)*cos(f*x + e) + (15*d^4*cos(f*x + e)^3 + 3*c^4 - 12*c^3*d + 18*c^2*d^
2 - 12*c*d^3 + 3*d^4 + 60*(4*c*d^3 - 3*d^4)*f*x - (2*c^4 + 12*c^3*d + 42*c^
2*d^2 - 128*c*d^3 + 102*d^4 + 15*(4*c*d^3 - 3*d^4)*f*x)*cos(f*x + e)^2 - 6*
(c^4 + 6*c^3*d + 6*c^2*d^2 - 34*c*d^3 + 31*d^4 - 5*(4*c*d^3 - 3*d^4)*f*x)*c
os(f*x + e))*sin(f*x + e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 -
 2*a^3*f*cos(f*x + e) - 4*a^3*f + (a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x +
 e) - 4*a^3*f)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**4/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.2366, size = 533, normalized size = 2.73

$$-\frac{\frac{30\,d^4}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)a^3}-\frac{15\left(4\,cd^3-3\,d^4\right)\left(fx+e\right)}{a^3}+\frac{2\left(15\,c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-60\,cd^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+45\,d^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+30\,c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+60\,c^3d}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^4/(a+a*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -1/15*(30*d^4/((tan(1/2*f*x + 1/2*e)^2 + 1)*a^3) - 15*(4*c*d^3 - 3*d^4)*(f*
x + e)/a^3 + 2*(15*c^4*tan(1/2*f*x + 1/2*e)^4 - 60*c*d^3*tan(1/2*f*x + 1/2*
e)^4 + 45*d^4*tan(1/2*f*x + 1/2*e)^4 + 30*c^4*tan(1/2*f*x + 1/2*e)^3 + 60*c
^3*d*tan(1/2*f*x + 1/2*e)^3 - 300*c*d^3*tan(1/2*f*x + 1/2*e)^3 + 210*d^4*ta
n(1/2*f*x + 1/2*e)^3 + 40*c^4*tan(1/2*f*x + 1/2*e)^2 + 60*c^3*d*tan(1/2*f*x
 + 1/2*e)^2 + 120*c^2*d^2*tan(1/2*f*x + 1/2*e)^2 - 580*c*d^3*tan(1/2*f*x +
1/2*e)^2 + 360*d^4*tan(1/2*f*x + 1/2*e)^2 + 20*c^4*tan(1/2*f*x + 1/2*e) + 6
0*c^3*d*tan(1/2*f*x + 1/2*e) + 60*c^2*d^2*tan(1/2*f*x + 1/2*e) - 380*c*d^3*
tan(1/2*f*x + 1/2*e) + 240*d^4*tan(1/2*f*x + 1/2*e) + 7*c^4 + 12*c^3*d + 12
*c^2*d^2 - 88*c*d^3 + 57*d^4)/(a^3*(tan(1/2*f*x + 1/2*e) + 1)^5))/f
```

3.473 $\int \frac{(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^3}\,dx$

Optimal. Leaf size=142

$$-\frac{(c-d)\left(2c^2+11cd+29d^2\right)\cos(e+fx)}{15f\left(a^3\sin(e+fx)+a^3\right)}+\frac{d^3x}{a^3}-\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^2}{5f(a\sin(e+fx)+a)^3}-\frac{(c-d)^2(2c+7d)\cos(e+}{15af(a\sin(e+fx)+a)}$$

```
[Out] (d^3*x)/a^3 - ((c - d)^2*(2*c + 7*d)*Cos[e + f*x])/(15*a*f*(a + a*Sin[e + f
*x])^2) - ((c - d)*(2*c^2 + 11*c*d + 29*d^2)*Cos[e + f*x])/(15*f*(a^3 + a^3
*Sin[e + f*x])) - ((c - d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(5*f*(a + a
*Sin[e + f*x])^3)
```

Rubi [A] time = 0.332717, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2765, 2968, 3019, 2735, 2648}

$$-\frac{(c-d)\left(2c^2+11cd+29d^2\right)\cos(e+fx)}{15f\left(a^3\sin(e+fx)+a^3\right)}+\frac{d^3x}{a^3}-\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^2}{5f(a\sin(e+fx)+a)^3}-\frac{(c-d)^2(2c+7d)\cos(e+}{15af(a\sin(e+fx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^3/(a + a*Sin[e + f*x])^3,x]
```


```
[Out] (d^3*x)/a^3 - ((c - d)^2*(2*c + 7*d)*Cos[e + f*x])/(15*a*f*(a + a*Sin[e + f
*x])^2) - ((c - d)*(2*c^2 + 11*c*d + 29*d^2)*Cos[e + f*x])/(15*f*(a^3 + a^3
*Sin[e + f*x])) - ((c - d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(5*f*(a + a
*Sin[e + f*x])^3)
```


Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3019

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b - a
*B + b*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[1
/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*
B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^3}\,dx = -\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^2}{5f(a+a\sin(e+fx))^3} - \frac{\int \frac{(c+d\sin(e+fx))\left(-a\left(2c^2+5cd-2d^2\right)-5ad^2\sin(e+fx)\right)}{(a+a\sin(e+fx))^2}\,dx}{5a^2}$$

$$= -\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^2}{5f(a+a\sin(e+fx))^3} - \frac{\int \frac{-ac\left(2c^2+5cd-2d^2\right)+\left(-5acd^2-ad\left(2c^2+5cd-2d^2\right)\right)\sin(e+fx)-5a}{(a+a\sin(e+fx))^2}}{5a^2}$$

$$= -\frac{(c-d)^2(2c+7d)\cos(e+fx)}{15af(a+a\sin(e+fx))^2} - \frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^2}{5f(a+a\sin(e+fx))^3} + \frac{\int \frac{a^2\left(2c^3+9c^2d+18cd^2\right.}{a+a}}{}$$

$$= \frac{d^3x}{a^3} - \frac{(c-d)^2(2c+7d)\cos(e+fx)}{15af(a+a\sin(e+fx))^2} - \frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^2}{5f(a+a\sin(e+fx))^3} + \frac{\left((c-d)\left(2c^2\right.\right.}{}$$

$$= \frac{d^3x}{a^3} - \frac{(c-d)^2(2c+7d)\cos(e+fx)}{15af(a+a\sin(e+fx))^2} - \frac{(c-d)\left(2c^2+11cd+29d^2\right)\cos(e+fx)}{15f\left(a^3+a^3\sin(e+fx)\right)} - \frac{(c-d)\cos(}{5f(}$$

Mathematica [B] time = 5.59349, size = 408, normalized size = 2.87

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(30d\cos\left(\frac{1}{2}(e+fx)\right)\left(3c^2+6cd+d^2(5e+5fx-9)\right)-5\cos\left(\frac{3}{2}(e+fx)\right)\left(18c^2d+4c\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])^3/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(30*d*(3*c^2 + 6*c*d + d^2*(-9 + 5*e
 + 5*f*x))*Cos[(e + f*x)/2] - 5*(4*c^3 + 18*c^2*d + 24*c*d^2 + d^3*(-46 + 1
5*e + 15*f*x))*Cos[(3*(e + f*x))/2] - 15*d^3*e*Cos[(5*(e + f*x))/2] - 15*d^
3*f*x*Cos[(5*(e + f*x))/2] + 40*c^3*Sin[(e + f*x)/2] + 90*c^2*d*Sin[(e + f*
x)/2] + 240*c*d^2*Sin[(e + f*x)/2] - 370*d^3*Sin[(e + f*x)/2] + 150*d^3*e*S
in[(e + f*x)/2] + 150*d^3*f*x*Sin[(e + f*x)/2] + 90*c*d^2*Sin[(3*(e + f*x))
/2] - 90*d^3*Sin[(3*(e + f*x))/2] + 75*d^3*e*Sin[(3*(e + f*x))/2] + 75*d^3*
f*x*Sin[(3*(e + f*x))/2] - 4*c^3*Sin[(5*(e + f*x))/2] - 18*c^2*d*Sin[(5*(e
+ f*x))/2] - 42*c*d^2*Sin[(5*(e + f*x))/2] + 64*d^3*Sin[(5*(e + f*x))/2] -
15*d^3*e*Sin[(5*(e + f*x))/2] - 15*d^3*f*x*Sin[(5*(e + f*x))/2]))/(60*a^3*f
*(1 + Sin[e + f*x])^3)
```

Maple [B] time = 0.075, size = 438, normalized size = 3.1

$$2\,\frac{d^3\arctan\left(\tan\left(1/2\,fx+e/2\right)\right)}{fa^3} - 2\,\frac{c^3}{fa^3\left(\tan\left(1/2\,fx+e/2\right)+1\right)} + 2\,\frac{d^3}{fa^3\left(\tan\left(1/2\,fx+e/2\right)+1\right)} + 4\,\frac{c^3}{fa^3\left(\tan\left(1/2\,fx+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^3,x)
```

```
[Out] 2/f/a^3*d^3*arctan(tan(1/2*f*x+1/2*e))-2/f/a^3/(tan(1/2*f*x+1/2*e)+1)*c^3+2
/f/a^3/(tan(1/2*f*x+1/2*e)+1)*d^3+4/f/a^3/(tan(1/2*f*x+1/2*e)+1)^2*c^3-6/f/
a^3/(tan(1/2*f*x+1/2*e)+1)^2*c^2*d+2/f/a^3/(tan(1/2*f*x+1/2*e)+1)^2*d^3+4/f
/a^3/(tan(1/2*f*x+1/2*e)+1)^4*c^3-12/f/a^3/(tan(1/2*f*x+1/2*e)+1)^4*c^2*d+1
2/f/a^3/(tan(1/2*f*x+1/2*e)+1)^4*c*d^2-4/f/a^3/(tan(1/2*f*x+1/2*e)+1)^4*d^3
-8/5/f/a^3/(tan(1/2*f*x+1/2*e)+1)^5*c^3+24/5/f/a^3/(tan(1/2*f*x+1/2*e)+1)^5
*c^2*d-24/5/f/a^3/(tan(1/2*f*x+1/2*e)+1)^5*c*d^2+8/5/f/a^3/(tan(1/2*f*x+1/2
*e)+1)^5*d^3-16/3/f/a^3/(tan(1/2*f*x+1/2*e)+1)^3*c^3+12/f/a^3/(tan(1/2*f*x+
1/2*e)+1)^3*c^2*d-8/f/a^3/(tan(1/2*f*x+1/2*e)+1)^3*c*d^2+4/3/f/a^3/(tan(1/2
*f*x+1/2*e)+1)^3*d^3
```

Maxima [B] time = 2.6582, size = 1058, normalized size = 7.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 2/15*(d^3*((95*sin(f*x + e)/(cos(f*x + e) + 1) + 145*sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 + 75*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*sin(f*x + e)^4/
(cos(f*x + e) + 1)^4 + 22)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 1
0*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x
+ e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^
5/(cos(f*x + e) + 1)^5) + 15*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^3) -
 c^3*(20*sin(f*x + e)/(cos(f*x + e) + 1) + 40*sin(f*x + e)^2/(cos(f*x + e)
+ 1)^2 + 30*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*sin(f*x + e)^4/(cos(f*
x + e) + 1)^4 + 7)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*si
n(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1
)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f
*x + e) + 1)^5) - 6*c*d^2*(5*sin(f*x + e)/(cos(f*x + e) + 1) + 10*sin(f*x +
 e)^2/(cos(f*x + e) + 1)^2 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1
) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos
(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x
+ e)^5/(cos(f*x + e) + 1)^5) - 9*c^2*d*(5*sin(f*x + e)/(cos(f*x + e) + 1) +
 5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 5*sin(f*x + e)^3/(cos(f*x + e) + 1
)^3 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)
^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^
3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) +
1)^5))/f
```

Fricas [B] time = 1.64782, size = 824, normalized size = 5.8

$$-\frac{60\,d^3fx - \left(15\,d^3fx - 2\,c^3 - 9\,c^2d - 21\,cd^2 + 32\,d^3\right)\cos\left(fx+e\right)^3 - 3\,c^3 + 9\,c^2d - 9\,cd^2 + 3\,d^3 - \left(45\,d^3fx + 4\,c^3 + 18\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -1/15*(60*d^3*f*x - (15*d^3*f*x - 2*c^3 - 9*c^2*d - 21*c*d^2 + 32*d^3)*cos(
f*x + e)^3 - 3*c^3 + 9*c^2*d - 9*c*d^2 + 3*d^3 - (45*d^3*f*x + 4*c^3 + 18*c
^2*d - 3*c*d^2 - 19*d^3)*cos(f*x + e)^2 + 3*(10*d^3*f*x - 3*c^3 - 6*c^2*d -
 9*c*d^2 + 18*d^3)*cos(f*x + e) + (60*d^3*f*x + 3*c^3 - 9*c^2*d + 9*c*d^2 -
 3*d^3 - (15*d^3*f*x + 2*c^3 + 9*c^2*d + 21*c*d^2 - 32*d^3)*cos(f*x + e)^2
+ 3*(10*d^3*f*x - 2*c^3 - 9*c^2*d - 6*c*d^2 + 17*d^3)*cos(f*x + e))*sin(f*x
 + e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e
) - 4*a^3*f + (a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f)*sin(f
*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**3/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.32374, size = 378, normalized size = 2.66

$$\frac{\frac{15\left(fx+e\right)d^3}{a^3} - \frac{2\left(15c^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4 - 15d^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4 + 30c^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3 + 45c^2d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3 - 75d^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3 + 40c^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2 + \cdots}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/15*(15*(f*x + e)*d^3/a^3 - 2*(15*c^3*tan(1/2*f*x + 1/2*e)^4 - 15*d^3*tan(
1/2*f*x + 1/2*e)^4 + 30*c^3*tan(1/2*f*x + 1/2*e)^3 + 45*c^2*d*tan(1/2*f*x +
 1/2*e)^3 - 75*d^3*tan(1/2*f*x + 1/2*e)^3 + 40*c^3*tan(1/2*f*x + 1/2*e)^2 +
 45*c^2*d*tan(1/2*f*x + 1/2*e)^2 + 60*c*d^2*tan(1/2*f*x + 1/2*e)^2 - 145*d^
3*tan(1/2*f*x + 1/2*e)^2 + 20*c^3*tan(1/2*f*x + 1/2*e) + 45*c^2*d*tan(1/2*f
*x + 1/2*e) + 30*c*d^2*tan(1/2*f*x + 1/2*e) - 95*d^3*tan(1/2*f*x + 1/2*e) +
 7*c^3 + 9*c^2*d + 6*c*d^2 - 22*d^3)/(a^3*(tan(1/2*f*x + 1/2*e) + 1)^5))/f
```

3.474 $\int \frac{(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^3}\,dx$

Optimal. Leaf size=125

$$-\frac{(2c^2+6cd+7d^2)\cos(e+fx)}{15f(a^3\sin(e+fx)+a^3)}-\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))}{5f(a\sin(e+fx)+a)^3}-\frac{(c-d)(2c+5d)\cos(e+fx)}{15af(a\sin(e+fx)+a)^2}$$

```
[Out] -((c - d)*(2*c + 5*d)*Cos[e + f*x])/(15*a*f*(a + a*Sin[e + f*x])^2) - ((2*c
^2 + 6*c*d + 7*d^2)*Cos[e + f*x])/(15*f*(a^3 + a^3*Sin[e + f*x])) - ((c - d
)*Cos[e + f*x]*(c + d*Sin[e + f*x]))/(5*f*(a + a*Sin[e + f*x])^3)
```

Rubi [A] time = 0.181693, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2760, 2750, 2648}

$$-\frac{(2c^2+6cd+7d^2)\cos(e+fx)}{15f(a^3\sin(e+fx)+a^3)}-\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))}{5f(a\sin(e+fx)+a)^3}-\frac{(c-d)(2c+5d)\cos(e+fx)}{15af(a\sin(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^2/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] -((c - d)*(2*c + 5*d)*Cos[e + f*x])/(15*a*f*(a + a*Sin[e + f*x])^2) - ((2*c
^2 + 6*c*d + 7*d^2)*Cos[e + f*x])/(15*f*(a^3 + a^3*Sin[e + f*x])) - ((c - d
)*Cos[e + f*x]*(c + d*Sin[e + f*x]))/(5*f*(a + a*Sin[e + f*x])^3)
```

Rule 2760

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^2, x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^m*(c + d*Sin[e + f*x]))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*(2*m + 1)),
Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*c*d*(m - 1) + b*(d^2 + c^2*(m + 1))
 + d*(a*d*(m - 1) + b*c*(m + 2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^3}\,dx = -\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))}{5f(a+a\sin(e+fx))^3} - \frac{\int \frac{-a(2c^2+4cd-d^2)-ad(c+4d)\sin(e+fx)}{(a+a\sin(e+fx))^2}dx}{5a^2}$$

$$= -\frac{(c-d)(2c+5d)\cos(e+fx)}{15af(a+a\sin(e+fx))^2} - \frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))}{5f(a+a\sin(e+fx))^3} + \frac{(2c^2+6cd+7d^2)\int}{15a^2}$$

$$= -\frac{(c-d)(2c+5d)\cos(e+fx)}{15af(a+a\sin(e+fx))^2} - \frac{(2c^2+6cd+7d^2)\cos(e+fx)}{15f(a^3+a^3\sin(e+fx))} - \frac{(c-d)\cos(e+fx)(c+d\sin}{5f(a+a\sin(e+f}$$

Mathematica [A] time = 0.113856, size = 84, normalized size = 0.67

$$-\frac{\cos(e+fx)\left(\left(2c^2+6cd+7d^2\right)\sin^2(e+fx)+6\left(c^2+3cd+d^2\right)\sin(e+fx)+7c^2+6cd+2d^2\right)}{15a^3f(\sin(e+fx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])^2/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] -(Cos[e + f*x]*(7*c^2 + 6*c*d + 2*d^2 + 6*(c^2 + 3*c*d + d^2)*Sin[e + f*x]
+ (2*c^2 + 6*c*d + 7*d^2)*Sin[e + f*x]^2))/(15*a^3*f*(1 + Sin[e + f*x])^3)
```

Maple [A] time = 0.062, size = 139, normalized size = 1.1

$$2\,\frac{1}{fa^3}\left(-1/5\,\frac{4\,c^2-8\,cd+4\,d^2}{\left(\tan\left(1/2\,fx+e/2\right)+1\right)^5}+2\,\frac{c\,(c-d)}{\left(\tan\left(1/2\,fx+e/2\right)+1\right)^2}-1/4\,\frac{-8\,c^2+16\,cd-8\,d^2}{\left(\tan\left(1/2\,fx+e/2\right)+1\right)^4}-\frac{c^2}{\tan\left(1/2\,fx+e/2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^3,x)
```

```
[Out] 2/f/a^3*(-1/5*(4*c^2-8*c*d+4*d^2)/(tan(1/2*f*x+1/2*e)+1)^5+2*c*(c-d)/(tan(1
/2*f*x+1/2*e)+1)^2-1/4*(-8*c^2+16*c*d-8*d^2)/(tan(1/2*f*x+1/2*e)+1)^4-c^2/(
tan(1/2*f*x+1/2*e)+1)-1/3*(8*c^2-12*c*d+4*d^2)/(tan(1/2*f*x+1/2*e)+1)^3)
```

Maxima [B] time = 1.56218, size = 747, normalized size = 5.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] -2/15*(c^2*(20*sin(f*x + e)/(cos(f*x + e) + 1) + 40*sin(f*x + e)^2/(cos(f*x
 + e) + 1)^2 + 30*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*sin(f*x + e)^4/(
cos(f*x + e) + 1)^4 + 7)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*
a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/
(cos(f*x + e) + 1)^5) + 2*d^2*(5*sin(f*x + e)/(cos(f*x + e) + 1) + 10*sin(f
*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e)
```

```
 + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/
(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(
f*x + e)^5/(cos(f*x + e) + 1)^5) + 6*c*d*(5*sin(f*x + e)/(cos(f*x + e) + 1)
 + 5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 5*sin(f*x + e)^3/(cos(f*x + e) +
 1)^3 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x +
e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*
a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e)
+ 1)^5))/f
```

Fricas [B] time = 1.56926, size = 570, normalized size = 4.56

$$-\frac{\left(2\,c^2+6\,cd+7\,d^2\right)\cos\left(fx+e\right)^3-\left(4\,c^2+12\,cd-d^2\right)\cos\left(fx+e\right)^2-3\,c^2+6\,cd-3\,d^2-3\left(3\,c^2+4\,cd+3\,d^2\right)\cos\left(\right.}{15\left(a^3f\cos\left(fx+e\right)^3+3\,a^3f\cos\left(fx+e\right)^2-2\,a^3f\cos\left(fx+e\right)-4\,a^3f\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -1/15*((2*c^2 + 6*c*d + 7*d^2)*cos(f*x + e)^3 - (4*c^2 + 12*c*d - d^2)*cos(
f*x + e)^2 - 3*c^2 + 6*c*d - 3*d^2 - 3*(3*c^2 + 4*c*d + 3*d^2)*cos(f*x + e)
 - ((2*c^2 + 6*c*d + 7*d^2)*cos(f*x + e)^2 - 3*c^2 + 6*c*d - 3*d^2 + 6*(c^2
 + 3*c*d + d^2)*cos(f*x + e))*sin(f*x + e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f
*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f + (a^3*f*cos(f*x + e)^2 -
2*a^3*f*cos(f*x + e) - 4*a^3*f)*sin(f*x + e))
```

Sympy [A] time = 29.6415, size = 1248, normalized size = 9.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**2/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((6*c**2*tan(e/2 + f*x/2)**5/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a
**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan
(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 20*c**2*tan(e/
2 + f*x/2)**2/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**
4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a*
*3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 10*c**2*tan(e/2 + f*x/2)/(15*a**3*f*ta
n(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*
x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*
a**3*f) - 8*c**2/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2
)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75
*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 60*c*d*tan(e/2 + f*x/2)**3/(15*a**3
*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2
 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2)
+ 15*a**3*f) - 60*c*d*tan(e/2 + f*x/2)**2/(15*a**3*f*tan(e/2 + f*x/2)**5 +
75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f
*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 60*c*d*tan
(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**
4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a*
*3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 12*c*d/(15*a**3*f*tan(e/2 + f*x/2)**5
+ 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3
```

```
*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 40*d**2*
tan(e/2 + f*x/2)**2/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*
x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 +
 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 20*d**2*tan(e/2 + f*x/2)/(15*a**
3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/
2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2)
 + 15*a**3*f) - 4*d**2/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 +
 f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**
2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f), Ne(f, 0)), (x*(c + d*sin(e))**
2/(a*sin(e) + a)**3, True))
```

Giac [A] time = 1.26524, size = 244, normalized size = 1.95

$$-\frac{2\left(15\,c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+30\,c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+30\,cd\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+40\,c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+30\,cd\tan\left(\frac{1}{2}\,f\right.\right.}{15\,a^3f\left(\mathrm{ta}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -2/15*(15*c^2*tan(1/2*f*x + 1/2*e)^4 + 30*c^2*tan(1/2*f*x + 1/2*e)^3 + 30*c
*d*tan(1/2*f*x + 1/2*e)^3 + 40*c^2*tan(1/2*f*x + 1/2*e)^2 + 30*c*d*tan(1/2*
f*x + 1/2*e)^2 + 20*d^2*tan(1/2*f*x + 1/2*e)^2 + 20*c^2*tan(1/2*f*x + 1/2*e
) + 30*c*d*tan(1/2*f*x + 1/2*e) + 10*d^2*tan(1/2*f*x + 1/2*e) + 7*c^2 + 6*c
*d + 2*d^2)/(a^3*f*(tan(1/2*f*x + 1/2*e) + 1)^5)
```

3.475 $\int \frac{c+d\sin(e+fx)}{(a+a\sin(e+fx))^3}\,dx$

Optimal. Leaf size=102

$$-\frac{(2c+3d)\cos(e+fx)}{15f\left(a^3\sin(e+fx)+a^3\right)}-\frac{(2c+3d)\cos(e+fx)}{15af(a\sin(e+fx)+a)^2}-\frac{(c-d)\cos(e+fx)}{5f(a\sin(e+fx)+a)^3}$$

```
[Out] -((c - d)*Cos[e + f*x])/(5*f*(a + a*Sin[e + f*x])^3) - ((2*c + 3*d)*Cos[e +
 f*x])/(15*a*f*(a + a*Sin[e + f*x])^2) - ((2*c + 3*d)*Cos[e + f*x])/(15*f*(
a^3 + a^3*Sin[e + f*x]))
```

Rubi [A] time = 0.0753259, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {2750, 2650, 2648}

$$-\frac{(2c+3d)\cos(e+fx)}{15f\left(a^3\sin(e+fx)+a^3\right)}-\frac{(2c+3d)\cos(e+fx)}{15af(a\sin(e+fx)+a)^2}-\frac{(c-d)\cos(e+fx)}{5f(a\sin(e+fx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] -((c - d)*Cos[e + f*x])/(5*f*(a + a*Sin[e + f*x])^3) - ((2*c + 3*d)*Cos[e +
 f*x])/(15*a*f*(a + a*Sin[e + f*x])^2) - ((2*c + 3*d)*Cos[e + f*x])/(15*f*(
a^3 + a^3*Sin[e + f*x]))
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{c+d\sin(e+fx)}{(a+a\sin(e+fx))^3}\,dx = -\frac{(c-d)\cos(e+fx)}{5f(a+a\sin(e+fx))^3} + \frac{(2c+3d)\int \frac{1}{(a+a\sin(e+fx))^2}\,dx}{5a}$$
$$= -\frac{(c-d)\cos(e+fx)}{5f(a+a\sin(e+fx))^3} - \frac{(2c+3d)\cos(e+fx)}{15af(a+a\sin(e+fx))^2} + \frac{(2c+3d)\int \frac{1}{a+a\sin(e+fx)}\,dx}{15a^2}$$
$$= -\frac{(c-d)\cos(e+fx)}{5f(a+a\sin(e+fx))^3} - \frac{(2c+3d)\cos(e+fx)}{15af(a+a\sin(e+fx))^2} - \frac{(2c+3d)\cos(e+fx)}{15f\left(a^3+a^3\sin(e+fx)\right)}$$

Mathematica [A] time = 0.0698925, size = 63, normalized size = 0.62

$$-\frac{\cos(e+fx)\left((2c+3d)\sin^2(e+fx)+(6c+9d)\sin(e+fx)+7c+3d\right)}{15a^3f(\sin(e+fx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] -(Cos[e + f*x]*(7*c + 3*d + (6*c + 9*d)*Sin[e + f*x] + (2*c + 3*d)*Sin[e +
f*x]^2))/(15*a^3*f*(1 + Sin[e + f*x])^3)
```

Maple [A] time = 0.056, size = 114, normalized size = 1.1

$$2\,\frac{1}{fa^3}\left(-1/5\,\frac{4\,c-4\,d}{\left(\tan\left(1/2\,fx+e/2\right)+1\right)^5}-\frac{c}{\tan\left(1/2\,fx+e/2\right)+1}-1/3\,\frac{8\,c-6\,d}{\left(\tan\left(1/2\,fx+e/2\right)+1\right)^3}-1/2\,\frac{-4\,c+2\,d}{\left(\tan\left(1/2\,fx+e/2\right)\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))/(a+a*sin(f*x+e))^3,x)
```

```
[Out] 2/f/a^3*(-1/5*(4*c-4*d)/(tan(1/2*f*x+1/2*e)+1)^5-c/(tan(1/2*f*x+1/2*e)+1)-1
/3*(8*c-6*d)/(tan(1/2*f*x+1/2*e)+1)^3-1/2*(-4*c+2*d)/(tan(1/2*f*x+1/2*e)+1)
^2-1/4*(-8*c+8*d)/(tan(1/2*f*x+1/2*e)+1)^4)
```

Maxima [B] time = 1.68366, size = 522, normalized size = 5.12

$$-\frac{2\left(\frac{c\left(\frac{20\sin(fx+e)}{\cos(fx+e)+1}+\frac{40\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{30\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{15\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+7\right)}{a^3+\frac{5a^3\sin(fx+e)}{\cos(fx+e)+1}+\frac{10a^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{10a^3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{5a^3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{a^3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}+\frac{3d\left(\frac{5\sin(fx+e)}{\cos(fx+e)+1}+\frac{5\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{5\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\right.}{a^3+\frac{5a^3\sin(fx+e)}{\cos(fx+e)+1}+\frac{10a^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{10a^3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{5a^3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}}\right.}{15f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))/(a+a*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] -2/15*(c*(20*sin(f*x + e)/(cos(f*x + e) + 1) + 40*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 + 30*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*sin(f*x + e)^4/(co
s(f*x + e) + 1)^4 + 7)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^
3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e)
```

```
+ 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(c
os(f*x + e) + 1)^5) + 3*d*(5*sin(f*x + e)/(cos(f*x + e) + 1) + 5*sin(f*x +
e)^2/(cos(f*x + e) + 1)^2 + 5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 1)/(a^3
 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e
)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5))/f
```

Fricas [A] time = 1.52028, size = 466, normalized size = 4.57

$$-\frac{(2c+3d)\cos\left(fx+e\right)^3-2\,(2c+3d)\cos\left(fx+e\right)^2-3\,(3c+2d)\cos\left(fx+e\right)-\left((2c+3d)\cos\left(fx+e\right)^2+3\,(2c+\cdots}{15\left(a^3f\cos\left(fx+e\right)^3+3\,a^3f\cos\left(fx+e\right)^2-2\,a^3f\cos\left(fx+e\right)-4\,a^3f+\left(a^3f\cos\left(fx+e\right)^2-2\,a^3\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))/(a+a*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -1/15*((2*c + 3*d)*cos(f*x + e)^3 - 2*(2*c + 3*d)*cos(f*x + e)^2 - 3*(3*c +
 2*d)*cos(f*x + e) - ((2*c + 3*d)*cos(f*x + e)^2 + 3*(2*c + 3*d)*cos(f*x +
e) - 3*c + 3*d)*sin(f*x + e) - 3*c + 3*d)/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*c
os(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f + (a^3*f*cos(f*x + e)^2 - 2*
a^3*f*cos(f*x + e) - 4*a^3*f)*sin(f*x + e))
```

Sympy [A] time = 10.4863, size = 915, normalized size = 8.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((8*c*tan(e/2 + f*x/2)**5/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3
*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/
2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 10*c*tan(e/2 + f*
x/2)**4/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 15
0*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*t
an(e/2 + f*x/2) + 15*a**3*f) + 20*c*tan(e/2 + f*x/2)**3/(15*a**3*f*tan(e/2
+ f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**
3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f
) - 6*c/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 15
0*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*t
an(e/2 + f*x/2) + 15*a**3*f) + 6*d*tan(e/2 + f*x/2)**5/(15*a**3*f*tan(e/2 +
 f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3
 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f)
 + 30*d*tan(e/2 + f*x/2)**4/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(
e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x
/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 30*d*tan(e/2 + f*x/2)**3
/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*
f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2
+ f*x/2) + 15*a**3*f) + 30*d*tan(e/2 + f*x/2)**2/(15*a**3*f*tan(e/2 + f*x/2
)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150
*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f), Ne(f
, 0)), (x*(c + d*sin(e))/(a*sin(e) + a)**3, True))
```

Giac [A] time = 1.32448, size = 176, normalized size = 1.73

$$-\frac{2\left(15\,c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}+30\,c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+15\,d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+40\,c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+15\,d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\right.}{15\,a^{3}f\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))/(a+a*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -2/15*(15*c*tan(1/2*f*x + 1/2*e)^4 + 30*c*tan(1/2*f*x + 1/2*e)^3 + 15*d*tan
(1/2*f*x + 1/2*e)^3 + 40*c*tan(1/2*f*x + 1/2*e)^2 + 15*d*tan(1/2*f*x + 1/2*
e)^2 + 20*c*tan(1/2*f*x + 1/2*e) + 15*d*tan(1/2*f*x + 1/2*e) + 7*c + 3*d)/(
a^3*f*(tan(1/2*f*x + 1/2*e) + 1)^5)
```

3.476 $\int \frac{1}{(a+a\sin(e+fx))^3}\,dx$

Optimal. Leaf size=83

$$-\frac{2\cos(e+fx)}{15f\left(a^3\sin(e+fx)+a^3\right)}-\frac{2\cos(e+fx)}{15af(a\sin(e+fx)+a)^2}-\frac{\cos(e+fx)}{5f(a\sin(e+fx)+a)^3}$$

```
[Out] -Cos[e + f*x]/(5*f*(a + a*Sin[e + f*x])^3) - (2*Cos[e + f*x])/(15*a*f*(a +
a*Sin[e + f*x])^2) - (2*Cos[e + f*x])/(15*f*(a^3 + a^3*Sin[e + f*x]))
```

Rubi [A] time = 0.0469928, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2650, 2648}

$$-\frac{2\cos(e+fx)}{15f\left(a^3\sin(e+fx)+a^3\right)}-\frac{2\cos(e+fx)}{15af(a\sin(e+fx)+a)^2}-\frac{\cos(e+fx)}{5f(a\sin(e+fx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(-3),x]
```

```
[Out] -Cos[e + f*x]/(5*f*(a + a*Sin[e + f*x])^3) - (2*Cos[e + f*x])/(15*a*f*(a +
a*Sin[e + f*x])^2) - (2*Cos[e + f*x])/(15*f*(a^3 + a^3*Sin[e + f*x]))
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+a\sin(e+fx))^3}\,dx &= -\frac{\cos(e+fx)}{5f(a+a\sin(e+fx))^3}+\frac{2\int \frac{1}{(a+a\sin(e+fx))^2}\,dx}{5a}\\
&= -\frac{\cos(e+fx)}{5f(a+a\sin(e+fx))^3}-\frac{2\cos(e+fx)}{15af(a+a\sin(e+fx))^2}+\frac{2\int \frac{1}{a+a\sin(e+fx)}\,dx}{15a^2}\\
&= -\frac{\cos(e+fx)}{5f(a+a\sin(e+fx))^3}-\frac{2\cos(e+fx)}{15af(a+a\sin(e+fx))^2}-\frac{2\cos(e+fx)}{15f\left(a^3+a^3\sin(e+fx)\right)}
\end{aligned}$$

Mathematica [A] time = 0.122553, size = 76, normalized size = 0.92

$$\frac{15\sin(e+fx)-6\sin(2(e+fx))-\sin(3(e+fx))-15\cos(e+fx)-6\cos(2(e+fx))+\cos(3(e+fx))+10}{30a^3f(\sin(e+fx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(-3),x]
```

```
[Out] (10 - 15*Cos[e + f*x] - 6*Cos[2*(e + f*x)] + Cos[3*(e + f*x)] + 15*Sin[e +
f*x] - 6*Sin[2*(e + f*x)] - Sin[3*(e + f*x)])/(30*a^3*f*(1 + Sin[e + f*x])^
3)
```

Maple [A] time = 0.045, size = 85, normalized size = 1.

$$2\frac{1}{fa^3}\left(-4/5\left(\tan\left(1/2\,fx+e/2\right)+1\right)^{-5}+2\left(\tan\left(1/2\,fx+e/2\right)+1\right)^{-4}-\left(\tan\left(1/2\,fx+e/2\right)+1\right)^{-1}+2\left(\tan\left(1/2\,fx+e/2\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^3,x)
```

```
[Out] 2/f/a^3*(-4/5/(tan(1/2*f*x+1/2*e)+1)^5+2/(tan(1/2*f*x+1/2*e)+1)^4-1/(tan(1/
2*f*x+1/2*e)+1)+2/(tan(1/2*f*x+1/2*e)+1)^2-8/3/(tan(1/2*f*x+1/2*e)+1)^3)
```

Maxima [B] time = 1.7579, size = 274, normalized size = 3.3

$$-\frac{2\left(\frac{20\sin(fx+e)}{\cos(fx+e)+1}+\frac{40\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{30\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{15\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+7\right)}{15\left(a^3+\frac{5a^3\sin(fx+e)}{\cos(fx+e)+1}+\frac{10a^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{10a^3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{5a^3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{a^3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] -2/15*(20*sin(f*x + e)/(cos(f*x + e) + 1) + 40*sin(f*x + e)^2/(cos(f*x + e)
 + 1)^2 + 30*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*sin(f*x + e)^4/(cos(f
*x + e) + 1)^4 + 7)/((a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*
sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) +
 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos
(f*x + e) + 1)^5)*f)
```

Fricas [A] time = 1.4969, size = 369, normalized size = 4.45

$$-\frac{2\cos\left(fx+e\right)^3-4\cos\left(fx+e\right)^2-\left(2\cos\left(fx+e\right)^2+6\cos\left(fx+e\right)-3\right)\sin\left(fx+e\right)-9\cos\left(fx+e\right)\cdots}{15\left(a^3f\cos\left(fx+e\right)^3+3a^3f\cos\left(fx+e\right)^2-2a^3f\cos\left(fx+e\right)-4a^3f+\left(a^3f\cos\left(fx+e\right)^2-2a^3f\cos\left(fx+e\right)-4a^3\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -1/15*(2*cos(f*x + e)^3 - 4*cos(f*x + e)^2 - (2*cos(f*x + e)^2 + 6*cos(f*x
+ e) - 3)*sin(f*x + e) - 9*cos(f*x + e) - 3)/(a^3*f*cos(f*x + e)^3 + 3*a^3*
f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f + (a^3*f*cos(f*x + e)^2 -
 2*a^3*f*cos(f*x + e) - 4*a^3*f)*sin(f*x + e))
```

Sympy [A] time = 4.77631, size = 558, normalized size = 6.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((-30*tan(e/2 + f*x/2)**4/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3
*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/
2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 60*tan(e/2 + f*x/
2)**3/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*
a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan
(e/2 + f*x/2) + 15*a**3*f) - 80*tan(e/2 + f*x/2)**2/(15*a**3*f*tan(e/2 + f*
x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 +
150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) -
40*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*
x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 +
 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 14/(15*a**3*f*tan(e/2 + f*x/2)**
5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a*
*3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f), Ne(f, 0
)), (x/(a*sin(e) + a)**3, True))
```

Giac [A] time = 1.2323, size = 105, normalized size = 1.27

$$-\frac{2\left(15\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}+30\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{3}+40\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+20\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+7\right)}{15a^{3}f\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -2/15*(15*tan(1/2*f*x + 1/2*e)^4 + 30*tan(1/2*f*x + 1/2*e)^3 + 40*tan(1/2*f
*x + 1/2*e)^2 + 20*tan(1/2*f*x + 1/2*e) + 7)/(a^3*f*(tan(1/2*f*x + 1/2*e) +
 1)^5)
```

3.477 $\int \frac{1}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))}\,dx$

Optimal. Leaf size=186

$$-\frac{2d^3\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{a^3f(c-d)^3\sqrt{c^2-d^2}}-\frac{\left(2c^2-9cd+22d^2\right)\cos(e+fx)}{15f(c-d)^3\left(a^3\sin(e+fx)+a^3\right)}-\frac{(2c-7d)\cos(e+fx)}{15af(c-d)^2(a\sin(e+fx)+a)^2}-\frac{\cos(e+f}{5f(c-d)(a\sin(e+}$$

```
[Out] (-2*d^3*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(a^3*(c - d)^3*Sq
rt[c^2 - d^2]*f) - Cos[e + f*x]/(5*(c - d)*f*(a + a*Sin[e + f*x])^3) - ((2*
c - 7*d)*Cos[e + f*x])/(15*a*(c - d)^2*f*(a + a*Sin[e + f*x])^2) - ((2*c^2
- 9*c*d + 22*d^2)*Cos[e + f*x])/(15*(c - d)^3*f*(a^3 + a^3*Sin[e + f*x]))
```

Rubi [A] time = 0.521514, antiderivative size = 186, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {2766, 2978, 12, 2660, 618, 204}

$$-\frac{2d^3\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{a^3f(c-d)^3\sqrt{c^2-d^2}}-\frac{\left(2c^2-9cd+22d^2\right)\cos(e+fx)}{15f(c-d)^3\left(a^3\sin(e+fx)+a^3\right)}-\frac{(2c-7d)\cos(e+fx)}{15af(c-d)^2(a\sin(e+fx)+a)^2}-\frac{\cos(e+f}{5f(c-d)(a\sin(e+}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])),x]
```


```
[Out] (-2*d^3*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(a^3*(c - d)^3*Sq
rt[c^2 - d^2]*f) - Cos[e + f*x]/(5*(c - d)*f*(a + a*Sin[e + f*x])^3) - ((2*
c - 7*d)*Cos[e + f*x])/(15*a*(c - d)^2*f*(a + a*Sin[e + f*x])^2) - ((2*c^2
- 9*c*d + 22*d^2)*Cos[e + f*x])/(15*(c - d)^3*f*(a^3 + a^3*Sin[e + f*x]))
```


Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))}\,dx = -\frac{\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3} - \frac{\int \frac{-a(2c-5d)-2ad\sin(e+fx)}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))}\,dx}{5a^2(c-d)}$$

$$= -\frac{\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3} - \frac{(2c-7d)\cos(e+fx)}{15a(c-d)^2f(a+a\sin(e+fx))^2} + \int$$

$$= -\frac{\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3} - \frac{(2c-7d)\cos(e+fx)}{15a(c-d)^2f(a+a\sin(e+fx))^2} - \frac{(}{15}$$

$$= -\frac{\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3} - \frac{(2c-7d)\cos(e+fx)}{15a(c-d)^2f(a+a\sin(e+fx))^2} - \frac{(}{15}$$

$$= -\frac{\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3} - \frac{(2c-7d)\cos(e+fx)}{15a(c-d)^2f(a+a\sin(e+fx))^2} - \frac{(}{15}$$

$$= -\frac{\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3} - \frac{(2c-7d)\cos(e+fx)}{15a(c-d)^2f(a+a\sin(e+fx))^2} - \frac{(}{15}$$

$$= -\frac{2d^3\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{a^3(c-d)^3\sqrt{c^2-d^2}f} - \frac{\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3} - \frac{(2}{15a(c-}$$

Mathematica [A] time = 0.713297, size = 301, normalized size = 1.62

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(2\left(2c^2-9cd+22d^2\right)\sin\left(\frac{1}{2}(e+fx)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^4 - \frac{30d^3}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(6*(c - d)^2*Sin[(e + f*x)/2] - 3*(c
 - d)^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + 2*(2*c - 7*d)*(c - d)*Sin[(
e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + (c - d)*(-2*c + 7*d)*
(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 + 2*(2*c^2 - 9*c*d + 22*d^2)*Sin[(e
 + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4 - (30*d^3*ArcTan[(d + c*
Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)
/Sqrt[c^2 - d^2]))/(15*a^3*(c - d)^3*f*(1 + Sin[e + f*x])^3)
```

Maple [A] time = 0.095, size = 325, normalized size = 1.8

$$-2\,\frac{d^3}{fa^3\,(c-d)^3\,\sqrt{c^2-d^2}}\arctan\left(1/2\,\frac{2\,c\tan\left(1/2\,fx+e/2\right)+2\,d}{\sqrt{c^2-d^2}}\right)+4\,\frac{c}{fa^3\,(c-d)^2\left(\tan\left(1/2\,fx+e/2\right)+1\right)^2}-6\,\frac{}{fa^3\,(c-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e)),x)
```

```
[Out] -2/f/a^3*d^3/(c-d)^3/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d
)/(c^2-d^2)^(1/2))+4/f/a^3/(c-d)^2/(tan(1/2*f*x+1/2*e)+1)^2*c-6/f/a^3/(c-d)
^2/(tan(1/2*f*x+1/2*e)+1)^2*d-16/3/f/a^3/(c-d)^2/(tan(1/2*f*x+1/2*e)+1)^3*c
+20/3/f/a^3/(c-d)^2/(tan(1/2*f*x+1/2*e)+1)^3*d-2/f/a^3/(c-d)^3/(tan(1/2*f*x
+1/2*e)+1)*c^2+6/f/a^3/(c-d)^3/(tan(1/2*f*x+1/2*e)+1)*c*d-6/f/a^3/(c-d)^3/(
tan(1/2*f*x+1/2*e)+1)*d^2-8/5/f/a^3/(c-d)/(tan(1/2*f*x+1/2*e)+1)^5+4/f/a^3/
(c-d)/(tan(1/2*f*x+1/2*e)+1)^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.07473, size = 3800, normalized size = 20.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] [1/30*(6*c^4 - 12*c^3*d + 12*c*d^3 - 6*d^4 - 2*(2*c^4 - 9*c^3*d + 20*c^2*d^
2 + 9*c*d^3 - 22*d^4)*cos(f*x + e)^3 + 2*(4*c^4 - 18*c^3*d + 25*c^2*d^2 + 1
8*c*d^3 - 29*d^4)*cos(f*x + e)^2 + 15*(d^3*cos(f*x + e)^3 + 3*d^3*cos(f*x +
 e)^2 - 2*d^3*cos(f*x + e) - 4*d^3 + (d^3*cos(f*x + e)^2 - 2*d^3*cos(f*x +
e) - 4*d^3)*sin(f*x + e))*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(f*x + e)^
2 - 2*c*d*sin(f*x + e) - c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x + e) + d*cos
```

```
(f*x + e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2
 - d^2)) + 6*(3*c^4 - 11*c^3*d + 15*c^2*d^2 + 11*c*d^3 - 18*d^4)*cos(f*x +
e) - 2*(3*c^4 - 6*c^3*d + 6*c*d^3 - 3*d^4 - (2*c^4 - 9*c^3*d + 20*c^2*d^2 +
 9*c*d^3 - 22*d^4)*cos(f*x + e)^2 - 3*(2*c^4 - 9*c^3*d + 15*c^2*d^2 + 9*c*d
^3 - 17*d^4)*cos(f*x + e))*sin(f*x + e))/((a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^
3*d^2 + 2*a^3*c^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*f*cos(f*x + e)^3 + 3*(a^3*c^
5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2 + 2*a^3*c^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*f*
cos(f*x + e)^2 - 2*(a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2 + 2*a^3*c^2*d^3 -
 3*a^3*c*d^4 + a^3*d^5)*f*cos(f*x + e) - 4*(a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c
^3*d^2 + 2*a^3*c^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*f + ((a^3*c^5 - 3*a^3*c^4*d
 + 2*a^3*c^3*d^2 + 2*a^3*c^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*f*cos(f*x + e)^2
- 2*(a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2 + 2*a^3*c^2*d^3 - 3*a^3*c*d^4 +
a^3*d^5)*f*cos(f*x + e) - 4*(a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2 + 2*a^3*
c^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*f)*sin(f*x + e)), 1/15*(3*c^4 - 6*c^3*d +
6*c*d^3 - 3*d^4 - (2*c^4 - 9*c^3*d + 20*c^2*d^2 + 9*c*d^3 - 22*d^4)*cos(f*x
 + e)^3 + (4*c^4 - 18*c^3*d + 25*c^2*d^2 + 18*c*d^3 - 29*d^4)*cos(f*x + e)^
2 + 15*(d^3*cos(f*x + e)^3 + 3*d^3*cos(f*x + e)^2 - 2*d^3*cos(f*x + e) - 4*
d^3 + (d^3*cos(f*x + e)^2 - 2*d^3*cos(f*x + e) - 4*d^3)*sin(f*x + e))*sqrt(
c^2 - d^2)*arctan(-(c*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) + 3
*(3*c^4 - 11*c^3*d + 15*c^2*d^2 + 11*c*d^3 - 18*d^4)*cos(f*x + e) - (3*c^4
- 6*c^3*d + 6*c*d^3 - 3*d^4 - (2*c^4 - 9*c^3*d + 20*c^2*d^2 + 9*c*d^3 - 22*
d^4)*cos(f*x + e)^2 - 3*(2*c^4 - 9*c^3*d + 15*c^2*d^2 + 9*c*d^3 - 17*d^4)*c
os(f*x + e))*sin(f*x + e))/((a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2 + 2*a^3*
c^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*f*cos(f*x + e)^3 + 3*(a^3*c^5 - 3*a^3*c^4*
d + 2*a^3*c^3*d^2 + 2*a^3*c^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*f*cos(f*x + e)^2
 - 2*(a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2 + 2*a^3*c^2*d^3 - 3*a^3*c*d^4 +
 a^3*d^5)*f*cos(f*x + e) - 4*(a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2 + 2*a^3
*c^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*f + ((a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d
^2 + 2*a^3*c^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*f*cos(f*x + e)^2 - 2*(a^3*c^5 -
 3*a^3*c^4*d + 2*a^3*c^3*d^2 + 2*a^3*c^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*f*cos
(f*x + e) - 4*(a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2 + 2*a^3*c^2*d^3 - 3*a^
3*c*d^4 + a^3*d^5)*f)*sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**3/(c+d*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.40248, size = 491, normalized size = 2.64

$$-2\left(\frac{15\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+d}{\sqrt{c^2-d^2}}\right)\right)d^3}{\left(a^3c^3-3a^3c^2d+3a^3cd^2-a^3d^3\right)\sqrt{c^2-d^2}}+\frac{15c^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-45cd\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+45d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+30c^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] -2/15*(15*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x
+ 1/2*e) + d)/sqrt(c^2 - d^2)))*d^3/((a^3*c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2 -
 a^3*d^3)*sqrt(c^2 - d^2)) + (15*c^2*tan(1/2*f*x + 1/2*e)^4 - 45*c*d*tan(1/
2*f*x + 1/2*e)^4 + 45*d^2*tan(1/2*f*x + 1/2*e)^4 + 30*c^2*tan(1/2*f*x + 1/2
*e)^3 - 105*c*d*tan(1/2*f*x + 1/2*e)^3 + 135*d^2*tan(1/2*f*x + 1/2*e)^3 + 4
0*c^2*tan(1/2*f*x + 1/2*e)^2 - 135*c*d*tan(1/2*f*x + 1/2*e)^2 + 185*d^2*tan
(1/2*f*x + 1/2*e)^2 + 20*c^2*tan(1/2*f*x + 1/2*e) - 75*c*d*tan(1/2*f*x + 1/
2*e) + 115*d^2*tan(1/2*f*x + 1/2*e) + 7*c^2 - 24*c*d + 32*d^2)/((a^3*c^3 -
3*a^3*c^2*d + 3*a^3*c*d^2 - a^3*d^3)*(tan(1/2*f*x + 1/2*e) + 1)^5))/f
```

3.478 $\int \frac{1}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))^2}\,dx$

Optimal. Leaf size=298

$$-\frac{2d^3(4c+3d)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{a^3f(c-d)^4(c+d)\sqrt{c^2-d^2}}-\frac{d\left(-12c^2d+2c^3+43cd^2+72d^3\right)\cos(e+fx)}{15a^3f(c-d)^4(c+d)(c+d\sin(e+fx))}-\frac{\left(2c^2-12cd+45d^2\right.}{15f(c-d)^3\left(a^3\sin(e+fx)+\right.}$$

```
[Out] (-2*d^3*(4*c + 3*d)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(a^3*
(c - d)^4*(c + d)*Sqrt[c^2 - d^2]*f) - (d*(2*c^3 - 12*c^2*d + 43*c*d^2 + 72
*d^3)*Cos[e + f*x])/(15*a^3*(c - d)^4*(c + d)*f*(c + d*Sin[e + f*x])) - Cos
[e + f*x]/(5*(c - d)*f*(a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])) - ((2*c
 - 9*d)*Cos[e + f*x])/(15*a*(c - d)^2*f*(a + a*Sin[e + f*x])^2*(c + d*Sin[e
 + f*x])) - ((2*c^2 - 12*c*d + 45*d^2)*Cos[e + f*x])/(15*(c - d)^3*f*(a^3 +
 a^3*Sin[e + f*x])*(c + d*Sin[e + f*x]))
```

Rubi [A] time = 0.730008, antiderivative size = 298, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {2766, 2978, 2754, 12, 2660, 618, 204}

$$-\frac{2d^3(4c+3d)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{a^3f(c-d)^4(c+d)\sqrt{c^2-d^2}}-\frac{d\left(-12c^2d+2c^3+43cd^2+72d^3\right)\cos(e+fx)}{15a^3f(c-d)^4(c+d)(c+d\sin(e+fx))}-\frac{\left(2c^2-12cd+45d^2\right.}{15f(c-d)^3\left(a^3\sin(e+fx)+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^2),x]
```

```
[Out] (-2*d^3*(4*c + 3*d)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(a^3*
(c - d)^4*(c + d)*Sqrt[c^2 - d^2]*f) - (d*(2*c^3 - 12*c^2*d + 43*c*d^2 + 72
*d^3)*Cos[e + f*x])/(15*a^3*(c - d)^4*(c + d)*f*(c + d*Sin[e + f*x])) - Cos
[e + f*x]/(5*(c - d)*f*(a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])) - ((2*c
 - 9*d)*Cos[e + f*x])/(15*a*(c - d)^2*f*(a + a*Sin[e + f*x])^2*(c + d*Sin[e
 + f*x])) - ((2*c^2 - 12*c*d + 45*d^2)*Cos[e + f*x])/(15*(c - d)^3*f*(a^3 +
 a^3*Sin[e + f*x])*(c + d*Sin[e + f*x]))
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
```

```
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))^2}\,dx = -\frac{\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3(c+d\sin(e+fx))} - \frac{\int \frac{-2a(c-3d)-3ad}{(a+a\sin(e+fx))^2(c+}}{5a^2(c-}$$

$$= -\frac{\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3(c+d\sin(e+fx))} - \frac{(2c}{15a(c-d)^2f(a+}$$

$$= -\frac{\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3(c+d\sin(e+fx))} - \frac{(2c}{15a(c-d)^2f(a+}$$

$$= -\frac{d\left(2c^3-12c^2d+43cd^2+72d^3\right)\cos(e+fx)}{15a^3(c-d)^4(c+d)f(c+d\sin(e+fx))} - \frac{\cos}{5(c-d)f(a+a\sin(e}$$

$$= -\frac{d\left(2c^3-12c^2d+43cd^2+72d^3\right)\cos(e+fx)}{15a^3(c-d)^4(c+d)f(c+d\sin(e+fx))} - \frac{\cos}{5(c-d)f(a+a\sin(e}$$

$$= -\frac{d\left(2c^3-12c^2d+43cd^2+72d^3\right)\cos(e+fx)}{15a^3(c-d)^4(c+d)f(c+d\sin(e+fx))} - \frac{\cos}{5(c-d)f(a+a\sin(e}$$

$$= -\frac{d\left(2c^3-12c^2d+43cd^2+72d^3\right)\cos(e+fx)}{15a^3(c-d)^4(c+d)f(c+d\sin(e+fx))} - \frac{\cos}{5(c-d)f(a+a\sin(e}$$

$$= -\frac{2d^3(4c+3d)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{a^3(c-d)^4(c+d)\sqrt{c^2-d^2}f} - \frac{d\left(2c^3-12c^2d+43cd^2+72d\right.}{15a^3(c-d)^4(c+d)f(c+d}$$

Mathematica [A] time = 2.48324, size = 361, normalized size = 1.21

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(2\left(2c^2-14cd+57d^2\right)\sin\left(\frac{1}{2}(e+fx)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^4-\frac{30d}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(6*(c - d)^2*Sin[(e + f*x)/2] - 3*(c
 - d)^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + 4*(c - 6*d)*(c - d)*Sin[(e
+ f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 - 2*(c - 6*d)*(c - d)*(Co
s[(e + f*x)/2] + Sin[(e + f*x)/2])^3 + 2*(2*c^2 - 14*c*d + 57*d^2)*Sin[(e +
 f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4 - (30*d^3*(4*c + 3*d)*ArcT
an[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]]*(Cos[(e + f*x)/2] + Sin[(e + f
*x)/2])^5)/((c + d)*Sqrt[c^2 - d^2]) - (15*d^4*Cos[e + f*x]*(Cos[(e + f*x)/
2] + Sin[(e + f*x)/2])^5)/((c + d)*(c + d*Sin[e + f*x]))))/(15*a^3*(c - d)^
4*f*(1 + Sin[e + f*x])^3)
```

Maple [A] time = 0.118, size = 511, normalized size = 1.7

$$-2\,\frac{d^5\tan\left(1/2\,fx+e/2\right)}{fa^3\left(c-d\right)^4\left(c\left(\tan\left(1/2\,fx+e/2\right)\right)^2+2\,\tan\left(1/2\,fx+e/2\right)d+c\right)\left(c+d\right)c}-2\,\frac{d}{fa^3\left(c-d\right)^4\left(c\left(\tan\left(1/2\,fx+e/2\right)\right)^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^2,x)
```

```
[Out] -2/f/a^3*d^5/(c-d)^4/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)/(c+d
)/c*tan(1/2*f*x+1/2*e)-2/f/a^3*d^4/(c-d)^4/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/
2*f*x+1/2*e)*d+c)/(c+d)-8/f/a^3*d^3/(c-d)^4/(c+d)/(c^2-d^2)^(1/2)*arctan(1/
2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*c-6/f/a^3*d^4/(c-d)^4/(c+d)
/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))-1
6/3/f/a^3/(c-d)^3/(tan(1/2*f*x+1/2*e)+1)^3*c+8/f/a^3/(c-d)^3/(tan(1/2*f*x+1
/2*e)+1)^3*d-8/f/a^3/(c-d)^3/(tan(1/2*f*x+1/2*e)+1)^2*d+4/f/a^3/(c-d)^3/(ta
n(1/2*f*x+1/2*e)+1)^2*c-2/f/a^3/(c-d)^4/(tan(1/2*f*x+1/2*e)+1)*c^2+8/f/a^3/
(c-d)^4/(tan(1/2*f*x+1/2*e)+1)*c*d-12/f/a^3/(c-d)^4/(tan(1/2*f*x+1/2*e)+1)*
d^2-8/5/f/a^3/(c-d)^2/(tan(1/2*f*x+1/2*e)+1)^5+4/f/a^3/(c-d)^2/(tan(1/2*f*x
+1/2*e)+1)^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.63708, size = 6917, normalized size = 23.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] [-1/30*(6*c^6 - 12*c^5*d - 6*c^4*d^2 + 24*c^3*d^3 - 6*c^2*d^4 - 12*c*d^5 +
6*d^6 - 2*(2*c^5*d - 12*c^4*d^2 + 41*c^3*d^3 + 84*c^2*d^4 - 43*c*d^5 - 72*d
^6)*cos(f*x + e)^4 - 2*(2*c^6 - 6*c^5*d + 5*c^4*d^2 + 147*c^3*d^3 + 164*c^2
*d^4 - 141*c*d^5 - 171*d^6)*cos(f*x + e)^3 + 2*(4*c^6 - 19*c^5*d + 22*c^4*d
^2 + 128*c^3*d^3 + 64*c^2*d^4 - 109*c*d^5 - 90*d^6)*cos(f*x + e)^2 + 15*(16
*c^2*d^3 + 28*c*d^4 + 12*d^5 + (4*c*d^4 + 3*d^5)*cos(f*x + e)^4 - (4*c^2*d^
3 + 11*c*d^4 + 6*d^5)*cos(f*x + e)^3 - (12*c^2*d^3 + 29*c*d^4 + 15*d^5)*cos
(f*x + e)^2 + 2*(4*c^2*d^3 + 7*c*d^4 + 3*d^5)*cos(f*x + e) + (16*c^2*d^3 +
28*c*d^4 + 12*d^5 - (4*c*d^4 + 3*d^5)*cos(f*x + e)^3 - (4*c^2*d^3 + 15*c*d^
4 + 9*d^5)*cos(f*x + e)^2 + 2*(4*c^2*d^3 + 7*c*d^4 + 3*d^5)*cos(f*x + e))*s
in(f*x + e))*sqrt(-c^2 + d^2)*log(-((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*si
n(f*x + e) - c^2 - d^2 - 2*(c*cos(f*x + e)*sin(f*x + e) + d*cos(f*x + e))*s
qrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) + 6
*(3*c^6 - 11*c^5*d + 12*c^4*d^2 + 82*c^3*d^3 + 47*c^2*d^4 - 71*c*d^5 - 62*d
^6)*cos(f*x + e) - 2*(3*c^6 - 6*c^5*d - 3*c^4*d^2 + 12*c^3*d^3 - 3*c^2*d^4
- 6*c*d^5 + 3*d^6 + (2*c^5*d - 12*c^4*d^2 + 41*c^3*d^3 + 84*c^2*d^4 - 43*c*
d^5 - 72*d^6)*cos(f*x + e)^3 - (2*c^6 - 8*c^5*d + 17*c^4*d^2 + 106*c^3*d^3
+ 80*c^2*d^4 - 98*c*d^5 - 99*d^6)*cos(f*x + e)^2 - 3*(2*c^6 - 9*c^5*d + 13*
c^4*d^2 + 78*c^3*d^3 + 48*c^2*d^4 - 69*c*d^5 - 63*d^6)*cos(f*x + e))*sin(f*
x + e))/((a^3*c^7*d - 3*a^3*c^6*d^2 + a^3*c^5*d^3 + 5*a^3*c^4*d^4 - 5*a^3*c
```

```
^3*d^5 - a^3*c^2*d^6 + 3*a^3*c*d^7 - a^3*d^8)*f*cos(f*x + e)^4 - (a^3*c^8 -
 a^3*c^7*d - 5*a^3*c^6*d^2 + 7*a^3*c^5*d^3 + 5*a^3*c^4*d^4 - 11*a^3*c^3*d^5
 + a^3*c^2*d^6 + 5*a^3*c*d^7 - 2*a^3*d^8)*f*cos(f*x + e)^3 - (3*a^3*c^8 - 4
*a^3*c^7*d - 12*a^3*c^6*d^2 + 20*a^3*c^5*d^3 + 10*a^3*c^4*d^4 - 28*a^3*c^3*
d^5 + 4*a^3*c^2*d^6 + 12*a^3*c*d^7 - 5*a^3*d^8)*f*cos(f*x + e)^2 + 2*(a^3*c
^8 - 2*a^3*c^7*d - 2*a^3*c^6*d^2 + 6*a^3*c^5*d^3 - 6*a^3*c^3*d^5 + 2*a^3*c^
2*d^6 + 2*a^3*c*d^7 - a^3*d^8)*f*cos(f*x + e) + 4*(a^3*c^8 - 2*a^3*c^7*d -
2*a^3*c^6*d^2 + 6*a^3*c^5*d^3 - 6*a^3*c^3*d^5 + 2*a^3*c^2*d^6 + 2*a^3*c*d^7
 - a^3*d^8)*f - ((a^3*c^7*d - 3*a^3*c^6*d^2 + a^3*c^5*d^3 + 5*a^3*c^4*d^4 -
 5*a^3*c^3*d^5 - a^3*c^2*d^6 + 3*a^3*c*d^7 - a^3*d^8)*f*cos(f*x + e)^3 + (a
^3*c^8 - 8*a^3*c^6*d^2 + 8*a^3*c^5*d^3 + 10*a^3*c^4*d^4 - 16*a^3*c^3*d^5 +
8*a^3*c*d^7 - 3*a^3*d^8)*f*cos(f*x + e)^2 - 2*(a^3*c^8 - 2*a^3*c^7*d - 2*a^
3*c^6*d^2 + 6*a^3*c^5*d^3 - 6*a^3*c^3*d^5 + 2*a^3*c^2*d^6 + 2*a^3*c*d^7 - a
^3*d^8)*f*cos(f*x + e) - 4*(a^3*c^8 - 2*a^3*c^7*d - 2*a^3*c^6*d^2 + 6*a^3*c
^5*d^3 - 6*a^3*c^3*d^5 + 2*a^3*c^2*d^6 + 2*a^3*c*d^7 - a^3*d^8)*f)*sin(f*x
+ e)), -1/15*(3*c^6 - 6*c^5*d - 3*c^4*d^2 + 12*c^3*d^3 - 3*c^2*d^4 - 6*c*d^
5 + 3*d^6 - (2*c^5*d - 12*c^4*d^2 + 41*c^3*d^3 + 84*c^2*d^4 - 43*c*d^5 - 72
*d^6)*cos(f*x + e)^4 - (2*c^6 - 6*c^5*d + 5*c^4*d^2 + 147*c^3*d^3 + 164*c^2
*d^4 - 141*c*d^5 - 171*d^6)*cos(f*x + e)^3 + (4*c^6 - 19*c^5*d + 22*c^4*d^2
 + 128*c^3*d^3 + 64*c^2*d^4 - 109*c*d^5 - 90*d^6)*cos(f*x + e)^2 - 15*(16*c
^2*d^3 + 28*c*d^4 + 12*d^5 + (4*c*d^4 + 3*d^5)*cos(f*x + e)^4 - (4*c^2*d^3
+ 11*c*d^4 + 6*d^5)*cos(f*x + e)^3 - (12*c^2*d^3 + 29*c*d^4 + 15*d^5)*cos(f
*x + e)^2 + 2*(4*c^2*d^3 + 7*c*d^4 + 3*d^5)*cos(f*x + e) + (16*c^2*d^3 + 28
*c*d^4 + 12*d^5 - (4*c*d^4 + 3*d^5)*cos(f*x + e)^3 - (4*c^2*d^3 + 15*c*d^4
+ 9*d^5)*cos(f*x + e)^2 + 2*(4*c^2*d^3 + 7*c*d^4 + 3*d^5)*cos(f*x + e))*sin
(f*x + e))*sqrt(c^2 - d^2)*arctan(-(c*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*co
s(f*x + e))) + 3*(3*c^6 - 11*c^5*d + 12*c^4*d^2 + 82*c^3*d^3 + 47*c^2*d^4 -
 71*c*d^5 - 62*d^6)*cos(f*x + e) - (3*c^6 - 6*c^5*d - 3*c^4*d^2 + 12*c^3*d^
3 - 3*c^2*d^4 - 6*c*d^5 + 3*d^6 + (2*c^5*d - 12*c^4*d^2 + 41*c^3*d^3 + 84*c
^2*d^4 - 43*c*d^5 - 72*d^6)*cos(f*x + e)^3 - (2*c^6 - 8*c^5*d + 17*c^4*d^2
+ 106*c^3*d^3 + 80*c^2*d^4 - 98*c*d^5 - 99*d^6)*cos(f*x + e)^2 - 3*(2*c^6 -
 9*c^5*d + 13*c^4*d^2 + 78*c^3*d^3 + 48*c^2*d^4 - 69*c*d^5 - 63*d^6)*cos(f*
x + e))*sin(f*x + e))/((a^3*c^7*d - 3*a^3*c^6*d^2 + a^3*c^5*d^3 + 5*a^3*c^4
*d^4 - 5*a^3*c^3*d^5 - a^3*c^2*d^6 + 3*a^3*c*d^7 - a^3*d^8)*f*cos(f*x + e)^
4 - (a^3*c^8 - a^3*c^7*d - 5*a^3*c^6*d^2 + 7*a^3*c^5*d^3 + 5*a^3*c^4*d^4 -
11*a^3*c^3*d^5 + a^3*c^2*d^6 + 5*a^3*c*d^7 - 2*a^3*d^8)*f*cos(f*x + e)^3 -
(3*a^3*c^8 - 4*a^3*c^7*d - 12*a^3*c^6*d^2 + 20*a^3*c^5*d^3 + 10*a^3*c^4*d^4
 - 28*a^3*c^3*d^5 + 4*a^3*c^2*d^6 + 12*a^3*c*d^7 - 5*a^3*d^8)*f*cos(f*x + e
)^2 + 2*(a^3*c^8 - 2*a^3*c^7*d - 2*a^3*c^6*d^2 + 6*a^3*c^5*d^3 - 6*a^3*c^3*
d^5 + 2*a^3*c^2*d^6 + 2*a^3*c*d^7 - a^3*d^8)*f*cos(f*x + e) + 4*(a^3*c^8 -
2*a^3*c^7*d - 2*a^3*c^6*d^2 + 6*a^3*c^5*d^3 - 6*a^3*c^3*d^5 + 2*a^3*c^2*d^6
 + 2*a^3*c*d^7 - a^3*d^8)*f - ((a^3*c^7*d - 3*a^3*c^6*d^2 + a^3*c^5*d^3 + 5
*a^3*c^4*d^4 - 5*a^3*c^3*d^5 - a^3*c^2*d^6 + 3*a^3*c*d^7 - a^3*d^8)*f*cos(f
*x + e)^3 + (a^3*c^8 - 8*a^3*c^6*d^2 + 8*a^3*c^5*d^3 + 10*a^3*c^4*d^4 - 16*
a^3*c^3*d^5 + 8*a^3*c*d^7 - 3*a^3*d^8)*f*cos(f*x + e)^2 - 2*(a^3*c^8 - 2*a^
3*c^7*d - 2*a^3*c^6*d^2 + 6*a^3*c^5*d^3 - 6*a^3*c^3*d^5 + 2*a^3*c^2*d^6 + 2
*a^3*c*d^7 - a^3*d^8)*f*cos(f*x + e) - 4*(a^3*c^8 - 2*a^3*c^7*d - 2*a^3*c^6
*d^2 + 6*a^3*c^5*d^3 - 6*a^3*c^3*d^5 + 2*a^3*c^2*d^6 + 2*a^3*c*d^7 - a^3*d^
8)*f)*sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**3/(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.43095, size = 698, normalized size = 2.34

$$-\frac{2\left(\frac{15\left(4\,cd^3+3\,d^4\right)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{\left(a^3c^5-3\,a^3c^4d+2\,a^3c^3d^2+2\,a^3c^2d^3-3\,a^3cd^4+a^3d^5\right)\sqrt{c^2-d^2}}+\frac{15\left(d^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+cd^4\right)}{\left(a^3c^6-3\,a^3c^5d+2\,a^3c^4d^2+2\,a^3c^3d^3-3\,a^3c^2d^4+a^3cd^5\right)\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+2\,d\tan\left(\frac{1}{2}\,fx+\right.\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] -2/15*(15*(4*c*d^3 + 3*d^4)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arct
an((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/((a^3*c^5 - 3*a^3*c^4*d +
 2*a^3*c^3*d^2 + 2*a^3*c^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*sqrt(c^2 - d^2)) +
15*(d^5*tan(1/2*f*x + 1/2*e) + c*d^4)/((a^3*c^6 - 3*a^3*c^5*d + 2*a^3*c^4*d
^2 + 2*a^3*c^3*d^3 - 3*a^3*c^2*d^4 + a^3*c*d^5)*(c*tan(1/2*f*x + 1/2*e)^2 +
 2*d*tan(1/2*f*x + 1/2*e) + c)) + (15*c^2*tan(1/2*f*x + 1/2*e)^4 - 60*c*d*t
an(1/2*f*x + 1/2*e)^4 + 90*d^2*tan(1/2*f*x + 1/2*e)^4 + 30*c^2*tan(1/2*f*x
+ 1/2*e)^3 - 150*c*d*tan(1/2*f*x + 1/2*e)^3 + 300*d^2*tan(1/2*f*x + 1/2*e)^
3 + 40*c^2*tan(1/2*f*x + 1/2*e)^2 - 190*c*d*tan(1/2*f*x + 1/2*e)^2 + 420*d^
2*tan(1/2*f*x + 1/2*e)^2 + 20*c^2*tan(1/2*f*x + 1/2*e) - 110*c*d*tan(1/2*f*
x + 1/2*e) + 270*d^2*tan(1/2*f*x + 1/2*e) + 7*c^2 - 34*c*d + 72*d^2)/((a^3*
c^4 - 4*a^3*c^3*d + 6*a^3*c^2*d^2 - 4*a^3*c*d^3 + a^3*d^4)*(tan(1/2*f*x + 1
/2*e) + 1)^5))/f
```

3.479 $\int \frac{1}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))^3}\,dx$

Optimal. Leaf size=378

$$-\frac{d^3\left(20c^2+30cd+13d^2\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{a^3f(c-d)^5(c+d)^2\sqrt{c^2-d^2}}-\frac{d\left(142c^2d^2-30c^3d+4c^4+525cd^3+304d^4\right)\cos(e+fx)}{30a^3f(c-d)^5(c+d)^2(c+d\sin(e+fx))}-\frac{d\left(-3\cdots\right.}{30\cdots}$$

```
[Out] -((d^3*(20*c^2 + 30*c*d + 13*d^2)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2
- d^2]])/(a^3*(c - d)^5*(c + d)^2*Sqrt[c^2 - d^2]*f)) - (d*(4*c^3 - 30*c^2*
d + 146*c*d^2 + 195*d^3)*Cos[e + f*x])/(30*a^3*(c - d)^4*(c + d)*f*(c + d*S
in[e + f*x])^2) - Cos[e + f*x]/(5*(c - d)*f*(a + a*Sin[e + f*x])^3*(c + d*S
in[e + f*x])^2) - ((2*c - 11*d)*Cos[e + f*x])/(15*a*(c - d)^2*f*(a + a*Sin[
e + f*x])^2*(c + d*Sin[e + f*x])^2) - ((2*c^2 - 15*c*d + 76*d^2)*Cos[e + f*
x])/(15*(c - d)^3*f*(a^3 + a^3*Sin[e + f*x])*(c + d*Sin[e + f*x])^2) - (d*(
4*c^4 - 30*c^3*d + 142*c^2*d^2 + 525*c*d^3 + 304*d^4)*Cos[e + f*x])/(30*a^3
*(c - d)^5*(c + d)^2*f*(c + d*Sin[e + f*x]))
```

Rubi [A] time = 0.96211, antiderivative size = 378, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {2766, 2978, 2754, 12, 2660, 618, 204}

$$-\frac{d^3\left(20c^2+30cd+13d^2\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{a^3f(c-d)^5(c+d)^2\sqrt{c^2-d^2}}-\frac{d\left(142c^2d^2-30c^3d+4c^4+525cd^3+304d^4\right)\cos(e+fx)}{30a^3f(c-d)^5(c+d)^2(c+d\sin(e+fx))}-\frac{d\left(-3\cdots\right.}{30\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^3),x]
```

```
[Out] -((d^3*(20*c^2 + 30*c*d + 13*d^2)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2
- d^2]])/(a^3*(c - d)^5*(c + d)^2*Sqrt[c^2 - d^2]*f)) - (d*(4*c^3 - 30*c^2*
d + 146*c*d^2 + 195*d^3)*Cos[e + f*x])/(30*a^3*(c - d)^4*(c + d)*f*(c + d*S
in[e + f*x])^2) - Cos[e + f*x]/(5*(c - d)*f*(a + a*Sin[e + f*x])^3*(c + d*S
in[e + f*x])^2) - ((2*c - 11*d)*Cos[e + f*x])/(15*a*(c - d)^2*f*(a + a*Sin[
e + f*x])^2*(c + d*Sin[e + f*x])^2) - ((2*c^2 - 15*c*d + 76*d^2)*Cos[e + f*
x])/(15*(c - d)^3*f*(a^3 + a^3*Sin[e + f*x])*(c + d*Sin[e + f*x])^2) - (d*(
4*c^4 - 30*c^3*d + 142*c^2*d^2 + 525*c*d^3 + 304*d^4)*Cos[e + f*x])/(30*a^3
*(c - d)^5*(c + d)^2*f*(c + d*Sin[e + f*x]))
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
```

```
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))^3}\,dx = -\frac{\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3(c+d\sin(e+fx))^2} - \frac{\int \frac{-a(2c-7d)-4a \cdots}{(a+a\sin(e+fx))^2(\cdots}}{5a^2(c \cdots}$$

$$= -\frac{\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3(c+d\sin(e+fx))^2} - \frac{(2 \cdots}{15a(c-d)^2f(a+ \cdots}$$

$$= -\frac{\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3(c+d\sin(e+fx))^2} - \frac{(2 \cdots}{15a(c-d)^2f(a+ \cdots}$$

$$= -\frac{d\left(4c^3-30c^2d+146cd^2+195d^3\right)\cos(e+fx)}{30a^3(c-d)^4(c+d)f(c+d\sin(e+fx))^2} - \frac{\cdots}{5(c-d)f(a+a\sin \cdots}$$

$$= -\frac{d\left(4c^3-30c^2d+146cd^2+195d^3\right)\cos(e+fx)}{30a^3(c-d)^4(c+d)f(c+d\sin(e+fx))^2} - \frac{\cdots}{5(c-d)f(a+a\sin \cdots}$$

$$= -\frac{d\left(4c^3-30c^2d+146cd^2+195d^3\right)\cos(e+fx)}{30a^3(c-d)^4(c+d)f(c+d\sin(e+fx))^2} - \frac{\cdots}{5(c-d)f(a+a\sin \cdots}$$

$$= -\frac{d\left(4c^3-30c^2d+146cd^2+195d^3\right)\cos(e+fx)}{30a^3(c-d)^4(c+d)f(c+d\sin(e+fx))^2} - \frac{\cdots}{5(c-d)f(a+a\sin \cdots}$$

$$= -\frac{d\left(4c^3-30c^2d+146cd^2+195d^3\right)\cos(e+fx)}{30a^3(c-d)^4(c+d)f(c+d\sin(e+fx))^2} - \frac{\cdots}{5(c-d)f(a+a\sin \cdots}$$

$$= -\frac{d^3\left(20c^2+30cd+13d^2\right)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{a^3(c-d)^5(c+d)^2\sqrt{c^2-d^2}f} - \frac{d\left(4c^3-30c^2d+\cdots\right.}{30a^3(c-d)^4(c \cdots}$$

Mathematica [B] time = 6.26632, size = 914, normalized size = 2.42

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\frac{320\sin\left(\frac{1}{2}(e+fx)\right)c^6-32\sin\left(\frac{5}{2}(e+fx)\right)c^6+32d\cos\left(\frac{7}{2}(e+fx)\right)c^5-1520d\sin\left(\frac{1}{2}(e+fx)\right)c^5+80d\sin\left(\frac{5}{2}(e+fx)\right)c \cdots}{\cdots}\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^3),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*((-480*d^3*(20*c^2 + 30*c*d + 13*d^2
)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]]*(Cos[(e + f*x)/2] + Sin[
(e + f*x)/2])^5)/Sqrt[c^2 - d^2] + (10*d*(-40*c^5 + 340*c^4*d + 1934*c^3*d^
2 + 3040*c^2*d^3 + 1994*c*d^4 + 481*d^5)*Cos[(e + f*x)/2] - 2*(80*c^6 - 424
*c^5*d + 1200*c^4*d^2 + 9698*c^3*d^3 + 17640*c^2*d^4 + 12371*c*d^5 + 2905*d
^6)*Cos[(3*(e + f*x))/2] - 1260*c^3*d^3*Cos[(5*(e + f*x))/2] - 2640*c^2*d^4
*Cos[(5*(e + f*x))/2] - 2250*c*d^5*Cos[(5*(e + f*x))/2] - 870*d^6*Cos[(5*(e
 + f*x))/2] + 32*c^5*d*Cos[(7*(e + f*x))/2] - 200*c^4*d^2*Cos[(7*(e + f*x))
/2] + 836*c^3*d^3*Cos[(7*(e + f*x))/2] + 4480*c^2*d^4*Cos[(7*(e + f*x))/2]
+ 5747*c*d^5*Cos[(7*(e + f*x))/2] + 2200*d^6*Cos[(7*(e + f*x))/2] - 135*c*d
^5*Cos[(9*(e + f*x))/2] - 90*d^6*Cos[(9*(e + f*x))/2] + 320*c^6*Sin[(e + f*
x)/2] - 1520*c^5*d*Sin[(e + f*x)/2] + 4568*c^4*d^2*Sin[(e + f*x)/2] + 27340
*c^3*d^3*Sin[(e + f*x)/2] + 40904*c^2*d^4*Sin[(e + f*x)/2] + 26020*c*d^5*Si
n[(e + f*x)/2] + 6318*d^6*Sin[(e + f*x)/2] + 800*c^4*d^2*Sin[(3*(e + f*x))/
```

```
2] + 7500*c^3*d^3*Sin[(3*(e + f*x))/2] + 13280*c^2*d^4*Sin[(3*(e + f*x))/2]
 + 9690*c*d^5*Sin[(3*(e + f*x))/2] + 2750*d^6*Sin[(3*(e + f*x))/2] - 32*c^6
*Sin[(5*(e + f*x))/2] + 80*c^5*d*Sin[(5*(e + f*x))/2] - 32*c^4*d^2*Sin[(5*(
e + f*x))/2] - 6820*c^3*d^3*Sin[(5*(e + f*x))/2] - 18080*c^2*d^4*Sin[(5*(e
+ f*x))/2] - 15670*c*d^5*Sin[(5*(e + f*x))/2] - 4266*d^6*Sin[(5*(e + f*x))/
2] - 60*c^2*d^4*Sin[(7*(e + f*x))/2] + 135*c*d^5*Sin[(7*(e + f*x))/2] + 60*
d^6*Sin[(7*(e + f*x))/2] + 8*c^4*d^2*Sin[(9*(e + f*x))/2] - 60*c^3*d^3*Sin[
(9*(e + f*x))/2] + 284*c^2*d^4*Sin[(9*(e + f*x))/2] + 915*c*d^5*Sin[(9*(e +
 f*x))/2] + 518*d^6*Sin[(9*(e + f*x))/2])/(c + d*Sin[e + f*x])^2))/(480*a^3
*(c - d)^5*(c + d)^2*f*(1 + Sin[e + f*x])^3)
```

Maple [B] time = 0.141, size = 1462, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^3,x)
```

```
[Out] -20/f/a^3/(c-d)^5/(tan(1/2*f*x+1/2*e)+1)*d^2+4/f/a^3/(c-d)^4/(tan(1/2*f*x+1
/2*e)+1)^2*c-10/f/a^3/(c-d)^4/(tan(1/2*f*x+1/2*e)+1)^2*d-16/3/f/a^3/(c-d)^4
/(tan(1/2*f*x+1/2*e)+1)^3*c+28/3/f/a^3/(c-d)^4/(tan(1/2*f*x+1/2*e)+1)^3*d+2
/f/a^3*d^7/(c-d)^5/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2
+2*c*d+d^2)/c*tan(1/2*f*x+1/2*e)+2/f/a^3*d^7/(c-d)^5/(c*tan(1/2*f*x+1/2*e)^
2+2*tan(1/2*f*x+1/2*e)*d+c)^2/c/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3-10/f/a
^3*d^4/(c-d)^5/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2*c^2/(c^2
+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^2+2/f/a^3*d^8/(c-d)^5/(c*tan(1/2*f*x+1/2*e)^
2+2*tan(1/2*f*x+1/2*e)*d+c)^2/c^2/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^2-29/f
/a^3*d^5/(c-d)^5/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2
*c*d+d^2)*c*tan(1/2*f*x+1/2*e)-6/f/a^3*d^5/(c-d)^5/(c*tan(1/2*f*x+1/2*e)^2+
2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*c-13/f/a^3*d^5/(c-d)^5/(c^2+2*c
*d+d^2)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(
1/2))-6/f/a^3*d^6/(c-d)^5/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)
^2/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3-19/f/a^3*d^6/(c-d)^5/(c*tan(1/2*f*x
+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^2-
18/f/a^3*d^6/(c-d)^5/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c
^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)-10/f/a^3*d^4/(c-d)^5/(c*tan(1/2*f*x+1/2*e)
^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*c^2-12/f/a^3*d^7/(c-d)^5/(c*
tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/c/(c^2+2*c*d+d^2)*tan(1/2*
f*x+1/2*e)^2-6/f/a^3*d^5/(c-d)^5/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*
e)*d+c)^2*c/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^2-20/f/a^3*d^3/(c-d)^5/(c^2+
2*c*d+d^2)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2
)^(1/2))*c^2-30/f/a^3*d^4/(c-d)^5/(c^2+2*c*d+d^2)/(c^2-d^2)^(1/2)*arctan(1/
2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*c-11/f/a^3*d^5/(c-d)^5/(c*t
an(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2*c/(c^2+2*c*d+d^2)*tan(1/2*f
*x+1/2*e)^3-2/f/a^3/(c-d)^5/(tan(1/2*f*x+1/2*e)+1)*c^2+1/f/a^3*d^6/(c-d)^5/
(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)+10/f/a^
3/(c-d)^5/(tan(1/2*f*x+1/2*e)+1)*c*d+4/f/a^3/(c-d)^3/(tan(1/2*f*x+1/2*e)+1)
^4-8/5/f/a^3/(c-d)^3/(tan(1/2*f*x+1/2*e)+1)^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.51949, size = 11534, normalized size = 30.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] [-1/60*(12*c^8 - 24*c^7*d - 24*c^6*d^2 + 72*c^5*d^3 - 72*c^3*d^5 + 24*c^2*d
^6 + 24*c*d^7 - 12*d^8 + 2*(4*c^6*d^2 - 30*c^5*d^3 + 138*c^4*d^4 + 555*c^3*
d^5 + 162*c^2*d^6 - 525*c*d^7 - 304*d^8)*cos(f*x + e)^5 - 2*(8*c^7*d - 52*c
^6*d^2 + 216*c^5*d^3 + 1086*c^4*d^4 + 984*c^3*d^5 - 621*c^2*d^6 - 1208*c*d^
7 - 413*d^8)*cos(f*x + e)^4 - 2*(4*c^8 - 6*c^7*d - 20*c^6*d^2 + 768*c^5*d^3
 + 2676*c^4*d^4 + 2307*c^3*d^5 - 1573*c^2*d^6 - 3069*c*d^7 - 1087*d^8)*cos(
f*x + e)^3 + 4*(4*c^8 - 20*c^7*d + 19*c^6*d^2 + 330*c^5*d^3 + 699*c^4*d^4 +
 345*c^3*d^5 - 526*c^2*d^6 - 655*c*d^7 - 196*d^8)*cos(f*x + e)^2 - 15*(80*c
^4*d^3 + 280*c^3*d^4 + 372*c^2*d^5 + 224*c*d^6 + 52*d^7 + (20*c^2*d^5 + 30*
c*d^6 + 13*d^7)*cos(f*x + e)^5 + (40*c^3*d^4 + 120*c^2*d^5 + 116*c*d^6 + 39
*d^7)*cos(f*x + e)^4 - (20*c^4*d^3 + 110*c^3*d^4 + 193*c^2*d^5 + 142*c*d^6
+ 39*d^7)*cos(f*x + e)^3 - (60*c^4*d^3 + 290*c^3*d^4 + 479*c^2*d^5 + 340*c*
d^6 + 91*d^7)*cos(f*x + e)^2 + 2*(20*c^4*d^3 + 70*c^3*d^4 + 93*c^2*d^5 + 56
*c*d^6 + 13*d^7)*cos(f*x + e) + (80*c^4*d^3 + 280*c^3*d^4 + 372*c^2*d^5 + 2
24*c*d^6 + 52*d^7 + (20*c^2*d^5 + 30*c*d^6 + 13*d^7)*cos(f*x + e)^4 - 2*(20
*c^3*d^4 + 50*c^2*d^5 + 43*c*d^6 + 13*d^7)*cos(f*x + e)^3 - (20*c^4*d^3 + 1
50*c^3*d^4 + 293*c^2*d^5 + 228*c*d^6 + 65*d^7)*cos(f*x + e)^2 + 2*(20*c^4*d
^3 + 70*c^3*d^4 + 93*c^2*d^5 + 56*c*d^6 + 13*d^7)*cos(f*x + e))*sin(f*x + e
))*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e)
- c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x + e) + d*cos(f*x + e))*sqrt(-c^2 +
d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) + 12*(3*c^8 -
11*c^7*d + 9*c^6*d^2 + 213*c^5*d^3 + 475*c^4*d^4 + 237*c^3*d^5 - 359*c^2*d^
6 - 439*c*d^7 - 128*d^8)*cos(f*x + e) - 2*(6*c^8 - 12*c^7*d - 12*c^6*d^2 +
36*c^5*d^3 - 36*c^3*d^5 + 12*c^2*d^6 + 12*c*d^7 - 6*d^8 + (4*c^6*d^2 - 30*c
^5*d^3 + 138*c^4*d^4 + 555*c^3*d^5 + 162*c^2*d^6 - 525*c*d^7 - 304*d^8)*cos
(f*x + e)^4 + (8*c^7*d - 48*c^6*d^2 + 186*c^5*d^3 + 1224*c^4*d^4 + 1539*c^3
*d^5 - 459*c^2*d^6 - 1733*c*d^7 - 717*d^8)*cos(f*x + e)^3 - 2*(2*c^8 - 7*c^
7*d + 14*c^6*d^2 + 291*c^5*d^3 + 726*c^4*d^4 + 384*c^3*d^5 - 557*c^2*d^6 -
668*c*d^7 - 185*d^8)*cos(f*x + e)^2 - 6*(2*c^8 - 9*c^7*d + 11*c^6*d^2 + 207
*c^5*d^3 + 475*c^4*d^4 + 243*c^3*d^5 - 361*c^2*d^6 - 441*c*d^7 - 127*d^8)*c
os(f*x + e))*sin(f*x + e))/((a^3*c^9*d^2 - 3*a^3*c^8*d^3 + 8*a^3*c^6*d^5 -
6*a^3*c^5*d^6 - 6*a^3*c^4*d^7 + 8*a^3*c^3*d^8 - 3*a^3*c*d^10 + a^3*d^11)*f*
cos(f*x + e)^5 + (2*a^3*c^10*d - 3*a^3*c^9*d^2 - 9*a^3*c^8*d^3 + 16*a^3*c^7
*d^4 + 12*a^3*c^6*d^5 - 30*a^3*c^5*d^6 - 2*a^3*c^4*d^7 + 24*a^3*c^3*d^8 - 6
*a^3*c^2*d^9 - 7*a^3*c*d^10 + 3*a^3*d^11)*f*cos(f*x + e)^4 - (a^3*c^11 + a^
3*c^10*d - 9*a^3*c^9*d^2 - a^3*c^8*d^3 + 26*a^3*c^7*d^4 - 6*a^3*c^6*d^5 - 3
4*a^3*c^5*d^6 + 14*a^3*c^4*d^7 + 21*a^3*c^3*d^8 - 11*a^3*c^2*d^9 - 5*a^3*c*
d^10 + 3*a^3*d^11)*f*cos(f*x + e)^3 - (3*a^3*c^11 + a^3*c^10*d - 23*a^3*c^9
*d^2 + 3*a^3*c^8*d^3 + 62*a^3*c^7*d^4 - 22*a^3*c^6*d^5 - 78*a^3*c^5*d^6 + 3
8*a^3*c^4*d^7 + 47*a^3*c^3*d^8 - 27*a^3*c^2*d^9 - 11*a^3*c*d^10 + 7*a^3*d^1
1)*f*cos(f*x + e)^2 + 2*(a^3*c^11 - a^3*c^10*d - 5*a^3*c^9*d^2 + 5*a^3*c^8*
d^3 + 10*a^3*c^7*d^4 - 10*a^3*c^6*d^5 - 10*a^3*c^5*d^6 + 10*a^3*c^4*d^7 + 5
*a^3*c^3*d^8 - 5*a^3*c^2*d^9 - a^3*c*d^10 + a^3*d^11)*f*cos(f*x + e) + 4*(a
^3*c^11 - a^3*c^10*d - 5*a^3*c^9*d^2 + 5*a^3*c^8*d^3 + 10*a^3*c^7*d^4 - 10*
a^3*c^6*d^5 - 10*a^3*c^5*d^6 + 10*a^3*c^4*d^7 + 5*a^3*c^3*d^8 - 5*a^3*c^2*d
```

^9 - a^3*c*d^10 + a^3*d^11)*f + ((a^3*c^9*d^2 - 3*a^3*c^8*d^3 + 8*a^3*c^6*d
^5 - 6*a^3*c^5*d^6 - 6*a^3*c^4*d^7 + 8*a^3*c^3*d^8 - 3*a^3*c*d^10 + a^3*d^1
1)*f*cos(f*x + e)^4 - 2*(a^3*c^10*d - 2*a^3*c^9*d^2 - 3*a^3*c^8*d^3 + 8*a^3
*c^7*d^4 + 2*a^3*c^6*d^5 - 12*a^3*c^5*d^6 + 2*a^3*c^4*d^7 + 8*a^3*c^3*d^8 -
3*a^3*c^2*d^9 - 2*a^3*c*d^10 + a^3*d^11)*f*cos(f*x + e)^3 - (a^3*c^11 + 3*
a^3*c^10*d - 13*a^3*c^9*d^2 - 7*a^3*c^8*d^3 + 42*a^3*c^7*d^4 - 2*a^3*c^6*d^
5 - 58*a^3*c^5*d^6 + 18*a^3*c^4*d^7 + 37*a^3*c^3*d^8 - 17*a^3*c^2*d^9 - 9*a
^3*c*d^10 + 5*a^3*d^11)*f*cos(f*x + e)^2 + 2*(a^3*c^11 - a^3*c^10*d - 5*a^3
*c^9*d^2 + 5*a^3*c^8*d^3 + 10*a^3*c^7*d^4 - 10*a^3*c^6*d^5 - 10*a^3*c^5*d^6
+ 10*a^3*c^4*d^7 + 5*a^3*c^3*d^8 - 5*a^3*c^2*d^9 - a^3*c*d^10 + a^3*d^11)*
f*cos(f*x + e) + 4*(a^3*c^11 - a^3*c^10*d - 5*a^3*c^9*d^2 + 5*a^3*c^8*d^3 +
10*a^3*c^7*d^4 - 10*a^3*c^6*d^5 - 10*a^3*c^5*d^6 + 10*a^3*c^4*d^7 + 5*a^3*
c^3*d^8 - 5*a^3*c^2*d^9 - a^3*c*d^10 + a^3*d^11)*f)*sin(f*x + e)), -1/30*(6
*c^8 - 12*c^7*d - 12*c^6*d^2 + 36*c^5*d^3 - 36*c^3*d^5 + 12*c^2*d^6 + 12*c*
d^7 - 6*d^8 + (4*c^6*d^2 - 30*c^5*d^3 + 138*c^4*d^4 + 555*c^3*d^5 + 162*c^2
*d^6 - 525*c*d^7 - 304*d^8)*cos(f*x + e)^5 - (8*c^7*d - 52*c^6*d^2 + 216*c^
5*d^3 + 1086*c^4*d^4 + 984*c^3*d^5 - 621*c^2*d^6 - 1208*c*d^7 - 413*d^8)*co
s(f*x + e)^4 - (4*c^8 - 6*c^7*d - 20*c^6*d^2 + 768*c^5*d^3 + 2676*c^4*d^4 +
2307*c^3*d^5 - 1573*c^2*d^6 - 3069*c*d^7 - 1087*d^8)*cos(f*x + e)^3 + 2*(4
*c^8 - 20*c^7*d + 19*c^6*d^2 + 330*c^5*d^3 + 699*c^4*d^4 + 345*c^3*d^5 - 52
6*c^2*d^6 - 655*c*d^7 - 196*d^8)*cos(f*x + e)^2 - 15*(80*c^4*d^3 + 280*c^3*
d^4 + 372*c^2*d^5 + 224*c*d^6 + 52*d^7 + (20*c^2*d^5 + 30*c*d^6 + 13*d^7)*c
os(f*x + e)^5 + (40*c^3*d^4 + 120*c^2*d^5 + 116*c*d^6 + 39*d^7)*cos(f*x + e
)^4 - (20*c^4*d^3 + 110*c^3*d^4 + 193*c^2*d^5 + 142*c*d^6 + 39*d^7)*cos(f*x
+ e)^3 - (60*c^4*d^3 + 290*c^3*d^4 + 479*c^2*d^5 + 340*c*d^6 + 91*d^7)*cos
(f*x + e)^2 + 2*(20*c^4*d^3 + 70*c^3*d^4 + 93*c^2*d^5 + 56*c*d^6 + 13*d^7)*
cos(f*x + e) + (80*c^4*d^3 + 280*c^3*d^4 + 372*c^2*d^5 + 224*c*d^6 + 52*d^7
+ (20*c^2*d^5 + 30*c*d^6 + 13*d^7)*cos(f*x + e)^4 - 2*(20*c^3*d^4 + 50*c^2
*d^5 + 43*c*d^6 + 13*d^7)*cos(f*x + e)^3 - (20*c^4*d^3 + 150*c^3*d^4 + 293*
c^2*d^5 + 228*c*d^6 + 65*d^7)*cos(f*x + e)^2 + 2*(20*c^4*d^3 + 70*c^3*d^4 +
93*c^2*d^5 + 56*c*d^6 + 13*d^7)*cos(f*x + e))*sin(f*x + e))*sqrt(c^2 - d^2
)*arctan(-(c*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) + 6*(3*c^8 -
11*c^7*d + 9*c^6*d^2 + 213*c^5*d^3 + 475*c^4*d^4 + 237*c^3*d^5 - 359*c^2*d
^6 - 439*c*d^7 - 128*d^8)*cos(f*x + e) - (6*c^8 - 12*c^7*d - 12*c^6*d^2 + 3
6*c^5*d^3 - 36*c^3*d^5 + 12*c^2*d^6 + 12*c*d^7 - 6*d^8 + (4*c^6*d^2 - 30*c^
5*d^3 + 138*c^4*d^4 + 555*c^3*d^5 + 162*c^2*d^6 - 525*c*d^7 - 304*d^8)*cos(
f*x + e)^4 + (8*c^7*d - 48*c^6*d^2 + 186*c^5*d^3 + 1224*c^4*d^4 + 1539*c^3*
d^5 - 459*c^2*d^6 - 1733*c*d^7 - 717*d^8)*cos(f*x + e)^3 - 2*(2*c^8 - 7*c^7
*d + 14*c^6*d^2 + 291*c^5*d^3 + 726*c^4*d^4 + 384*c^3*d^5 - 557*c^2*d^6 - 6
68*c*d^7 - 185*d^8)*cos(f*x + e)^2 - 6*(2*c^8 - 9*c^7*d + 11*c^6*d^2 + 207*
c^5*d^3 + 475*c^4*d^4 + 243*c^3*d^5 - 361*c^2*d^6 - 441*c*d^7 - 127*d^8)*co
s(f*x + e))*sin(f*x + e))/((a^3*c^9*d^2 - 3*a^3*c^8*d^3 + 8*a^3*c^6*d^5 - 6
*a^3*c^5*d^6 - 6*a^3*c^4*d^7 + 8*a^3*c^3*d^8 - 3*a^3*c*d^10 + a^3*d^11)*f*c
os(f*x + e)^5 + (2*a^3*c^10*d - 3*a^3*c^9*d^2 - 9*a^3*c^8*d^3 + 16*a^3*c^7*
d^4 + 12*a^3*c^6*d^5 - 30*a^3*c^5*d^6 - 2*a^3*c^4*d^7 + 24*a^3*c^3*d^8 - 6*
a^3*c^2*d^9 - 7*a^3*c*d^10 + 3*a^3*d^11)*f*cos(f*x + e)^4 - (a^3*c^11 + a^3
*c^10*d - 9*a^3*c^9*d^2 - a^3*c^8*d^3 + 26*a^3*c^7*d^4 - 6*a^3*c^6*d^5 - 34
*a^3*c^5*d^6 + 14*a^3*c^4*d^7 + 21*a^3*c^3*d^8 - 11*a^3*c^2*d^9 - 5*a^3*c*d
^10 + 3*a^3*d^11)*f*cos(f*x + e)^3 - (3*a^3*c^11 + a^3*c^10*d - 23*a^3*c^9*
d^2 + 3*a^3*c^8*d^3 + 62*a^3*c^7*d^4 - 22*a^3*c^6*d^5 - 78*a^3*c^5*d^6 + 38
*a^3*c^4*d^7 + 47*a^3*c^3*d^8 - 27*a^3*c^2*d^9 - 11*a^3*c*d^10 + 7*a^3*d^11
)*f*cos(f*x + e)^2 + 2*(a^3*c^11 - a^3*c^10*d - 5*a^3*c^9*d^2 + 5*a^3*c^8*d
^3 + 10*a^3*c^7*d^4 - 10*a^3*c^6*d^5 - 10*a^3*c^5*d^6 + 10*a^3*c^4*d^7 + 5*
a^3*c^3*d^8 - 5*a^3*c^2*d^9 - a^3*c*d^10 + a^3*d^11)*f*cos(f*x + e) + 4*(a^
3*c^11 - a^3*c^10*d - 5*a^3*c^9*d^2 + 5*a^3*c^8*d^3 + 10*a^3*c^7*d^4 - 10*a
^3*c^6*d^5 - 10*a^3*c^5*d^6 + 10*a^3*c^4*d^7 + 5*a^3*c^3*d^8 - 5*a^3*c^2*d^
9 - a^3*c*d^10 + a^3*d^11)*f + ((a^3*c^9*d^2 - 3*a^3*c^8*d^3 + 8*a^3*c^6*d^
5 - 6*a^3*c^5*d^6 - 6*a^3*c^4*d^7 + 8*a^3*c^3*d^8 - 3*a^3*c*d^10 + a^3*d^11
)*f*cos(f*x + e)^4 - 2*(a^3*c^10*d - 2*a^3*c^9*d^2 - 3*a^3*c^8*d^3 + 8*a^3*

```
c^7*d^4 + 2*a^3*c^6*d^5 - 12*a^3*c^5*d^6 + 2*a^3*c^4*d^7 + 8*a^3*c^3*d^8 -
3*a^3*c^2*d^9 - 2*a^3*c*d^10 + a^3*d^11)*f*cos(f*x + e)^3 - (a^3*c^11 + 3*a
^3*c^10*d - 13*a^3*c^9*d^2 - 7*a^3*c^8*d^3 + 42*a^3*c^7*d^4 - 2*a^3*c^6*d^5
 - 58*a^3*c^5*d^6 + 18*a^3*c^4*d^7 + 37*a^3*c^3*d^8 - 17*a^3*c^2*d^9 - 9*a^
3*c*d^10 + 5*a^3*d^11)*f*cos(f*x + e)^2 + 2*(a^3*c^11 - a^3*c^10*d - 5*a^3*
c^9*d^2 + 5*a^3*c^8*d^3 + 10*a^3*c^7*d^4 - 10*a^3*c^6*d^5 - 10*a^3*c^5*d^6
+ 10*a^3*c^4*d^7 + 5*a^3*c^3*d^8 - 5*a^3*c^2*d^9 - a^3*c*d^10 + a^3*d^11)*f
*cos(f*x + e) + 4*(a^3*c^11 - a^3*c^10*d - 5*a^3*c^9*d^2 + 5*a^3*c^8*d^3 +
10*a^3*c^7*d^4 - 10*a^3*c^6*d^5 - 10*a^3*c^5*d^6 + 10*a^3*c^4*d^7 + 5*a^3*c
^3*d^8 - 5*a^3*c^2*d^9 - a^3*c*d^10 + a^3*d^11)*f)*sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**3/(c+d*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.56968, size = 1072, normalized size = 2.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -1/15*(15*(20*c^2*d^3 + 30*c*d^4 + 13*d^5)*(pi*floor(1/2*(f*x + e)/pi + 1/2
)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/((a^3*c^7
- 3*a^3*c^6*d + a^3*c^5*d^2 + 5*a^3*c^4*d^3 - 5*a^3*c^3*d^4 - a^3*c^2*d^5 +
 3*a^3*c*d^6 - a^3*d^7)*sqrt(c^2 - d^2)) + 15*(11*c^3*d^5*tan(1/2*f*x + 1/2
*e)^3 + 6*c^2*d^6*tan(1/2*f*x + 1/2*e)^3 - 2*c*d^7*tan(1/2*f*x + 1/2*e)^3 +
 10*c^4*d^4*tan(1/2*f*x + 1/2*e)^2 + 6*c^3*d^5*tan(1/2*f*x + 1/2*e)^2 + 19*
c^2*d^6*tan(1/2*f*x + 1/2*e)^2 + 12*c*d^7*tan(1/2*f*x + 1/2*e)^2 - 2*d^8*ta
n(1/2*f*x + 1/2*e)^2 + 29*c^3*d^5*tan(1/2*f*x + 1/2*e) + 18*c^2*d^6*tan(1/2
*f*x + 1/2*e) - 2*c*d^7*tan(1/2*f*x + 1/2*e) + 10*c^4*d^4 + 6*c^3*d^5 - c^2
*d^6)/((a^3*c^9 - 3*a^3*c^8*d + a^3*c^7*d^2 + 5*a^3*c^6*d^3 - 5*a^3*c^5*d^4
 - a^3*c^4*d^5 + 3*a^3*c^3*d^6 - a^3*c^2*d^7)*(c*tan(1/2*f*x + 1/2*e)^2 + 2
*d*tan(1/2*f*x + 1/2*e) + c)^2) + 2*(15*c^2*tan(1/2*f*x + 1/2*e)^4 - 75*c*d
*tan(1/2*f*x + 1/2*e)^4 + 150*d^2*tan(1/2*f*x + 1/2*e)^4 + 30*c^2*tan(1/2*f
*x + 1/2*e)^3 - 195*c*d*tan(1/2*f*x + 1/2*e)^3 + 525*d^2*tan(1/2*f*x + 1/2*
e)^3 + 40*c^2*tan(1/2*f*x + 1/2*e)^2 - 245*c*d*tan(1/2*f*x + 1/2*e)^2 + 745
*d^2*tan(1/2*f*x + 1/2*e)^2 + 20*c^2*tan(1/2*f*x + 1/2*e) - 145*c*d*tan(1/2
*f*x + 1/2*e) + 485*d^2*tan(1/2*f*x + 1/2*e) + 7*c^2 - 44*c*d + 127*d^2)/((
a^3*c^5 - 5*a^3*c^4*d + 10*a^3*c^3*d^2 - 10*a^3*c^2*d^3 + 5*a^3*c*d^4 - a^3
*d^5)*(tan(1/2*f*x + 1/2*e) + 1)^5))/f
```

3.480 $\int \frac{A+B\sin(x)}{(1+\sin(x))^4}\,dx$

Optimal. Leaf size=75

$$-\frac{2(3A+4B)\cos(x)}{105(\sin(x)+1)}-\frac{2(3A+4B)\cos(x)}{105(\sin(x)+1)^2}-\frac{(3A+4B)\cos(x)}{35(\sin(x)+1)^3}-\frac{(A-B)\cos(x)}{7(\sin(x)+1)^4}$$

```
[Out] -((A - B)*Cos[x])/(7*(1 + Sin[x])^4) - ((3*A + 4*B)*Cos[x])/(35*(1 + Sin[x]
)^3) - (2*(3*A + 4*B)*Cos[x])/(105*(1 + Sin[x])^2) - (2*(3*A + 4*B)*Cos[x])
/(105*(1 + Sin[x]))
```

Rubi [A] time = 0.0567041, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2750, 2650, 2648}

$$-\frac{2(3A+4B)\cos(x)}{105(\sin(x)+1)}-\frac{2(3A+4B)\cos(x)}{105(\sin(x)+1)^2}-\frac{(3A+4B)\cos(x)}{35(\sin(x)+1)^3}-\frac{(A-B)\cos(x)}{7(\sin(x)+1)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[x])/(1 + Sin[x])^4,x]
```


```
[Out] -((A - B)*Cos[x])/(7*(1 + Sin[x])^4) - ((3*A + 4*B)*Cos[x])/(35*(1 + Sin[x]
)^3) - (2*(3*A + 4*B)*Cos[x])/(105*(1 + Sin[x])^2) - (2*(3*A + 4*B)*Cos[x])
/(105*(1 + Sin[x]))
```


Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\sin(x)}{(1+\sin(x))^4}\,dx &= -\frac{(A-B)\cos(x)}{7(1+\sin(x))^4}+\frac{1}{7}(3A+4B)\int \frac{1}{(1+\sin(x))^3}\,dx\\ &= -\frac{(A-B)\cos(x)}{7(1+\sin(x))^4}-\frac{(3A+4B)\cos(x)}{35(1+\sin(x))^3}+\frac{1}{35}(2(3A+4B))\int \frac{1}{(1+\sin(x))^2}\,dx\\ &= -\frac{(A-B)\cos(x)}{7(1+\sin(x))^4}-\frac{(3A+4B)\cos(x)}{35(1+\sin(x))^3}-\frac{2(3A+4B)\cos(x)}{105(1+\sin(x))^2}+\frac{1}{105}(2(3A+4B))\int \frac{1}{1+\sin(x)}\,dx\\ &= -\frac{(A-B)\cos(x)}{7(1+\sin(x))^4}-\frac{(3A+4B)\cos(x)}{35(1+\sin(x))^3}-\frac{2(3A+4B)\cos(x)}{105(1+\sin(x))^2}-\frac{2(3A+4B)\cos(x)}{105(1+\sin(x))}\end{aligned}$$

Mathematica [A] time = 0.0510955, size = 55, normalized size = 0.73

$$-\frac{\cos(x)\left((6A+8B)\sin^3(x)+8(3A+4B)\sin^2(x)+13(3A+4B)\sin(x)+36A+13B\right)}{105(\sin(x)+1)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[x])/(1 + Sin[x])^4,x]
```

```
[Out] -(Cos[x]*(36*A + 13*B + 13*(3*A + 4*B)*Sin[x] + 8*(3*A + 4*B)*Sin[x]^2 + (6
*A + 8*B)*Sin[x]^3))/(105*(1 + Sin[x])^4)
```

Maple [A] time = 0.035, size = 115, normalized size = 1.5

$$-\frac{-24\,A+24\,B}{3}\left(\tan\left(\frac{x}{2}\right)+1\right)^{-6}-\frac{72\,A-64\,B}{5}\left(\tan\left(\frac{x}{2}\right)+1\right)^{-5}-\frac{-32\,A+24\,B}{2}\left(\tan\left(\frac{x}{2}\right)+1\right)^{-4}-2\,\frac{A}{\tan(x/2)+1}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(x))/(1+sin(x))^4,x)
```

```
[Out] -1/3*(-24*A+24*B)/(tan(1/2*x)+1)^6-2/5*(36*A-32*B)/(tan(1/2*x)+1)^5-1/2*(-3
2*A+24*B)/(tan(1/2*x)+1)^4-2*A/(tan(1/2*x)+1)-2/7*(8*A-8*B)/(tan(1/2*x)+1)^
7-2/3*(18*A-10*B)/(tan(1/2*x)+1)^3-(-6*A+2*B)/(tan(1/2*x)+1)^2
```

Maxima [B] time = 1.84997, size = 417, normalized size = 5.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(x))/(1+sin(x))^4,x, algorithm="maxima")
```

```
[Out] -2/105*B*(91*sin(x)/(cos(x) + 1) + 168*sin(x)^2/(cos(x) + 1)^2 + 280*sin(x)
^3/(cos(x) + 1)^3 + 175*sin(x)^4/(cos(x) + 1)^4 + 105*sin(x)^5/(cos(x) + 1)
^5 + 13)/(7*sin(x)/(cos(x) + 1) + 21*sin(x)^2/(cos(x) + 1)^2 + 35*sin(x)^3/
(cos(x) + 1)^3 + 35*sin(x)^4/(cos(x) + 1)^4 + 21*sin(x)^5/(cos(x) + 1)^5 +
7*sin(x)^6/(cos(x) + 1)^6 + sin(x)^7/(cos(x) + 1)^7 + 1) - 2/35*A*(49*sin(x
)/(cos(x) + 1) + 147*sin(x)^2/(cos(x) + 1)^2 + 210*sin(x)^3/(cos(x) + 1)^3
+ 210*sin(x)^4/(cos(x) + 1)^4 + 105*sin(x)^5/(cos(x) + 1)^5 + 35*sin(x)^6/(
cos(x) + 1)^6 + 12)/(7*sin(x)/(cos(x) + 1) + 21*sin(x)^2/(cos(x) + 1)^2 + 3
5*sin(x)^3/(cos(x) + 1)^3 + 35*sin(x)^4/(cos(x) + 1)^4 + 21*sin(x)^5/(cos(x
```

```
) + 1)^5 + 7*sin(x)^6/(cos(x) + 1)^6 + sin(x)^7/(cos(x) + 1)^7 + 1)
```

Fricas [B] time = 1.55325, size = 428, normalized size = 5.71

$$\frac{2\,(3\,A+4\,B)\cos{(x)}^{4}+8\,(3\,A+4\,B)\cos{(x)}^{3}-9\,(3\,A+4\,B)\cos{(x)}^{2}-15\,(4\,A+3\,B)\cos{(x)}+\left(2\,(3\,A+4\,B)\cos{(x)}^{3}-6\right.}{105\left(\cos{(x)}^{4}-3\,\cos{(x)}^{3}-8\,\cos{(x)}^{2}-\left(\cos{(x)}^{3}+4\,\cos{(x)}^{2}-4\,\cos\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(x))/(1+sin(x))^4,x, algorithm="fricas")
```

```
[Out] 1/105*(2*(3*A + 4*B)*cos(x)^4 + 8*(3*A + 4*B)*cos(x)^3 - 9*(3*A + 4*B)*cos(
x)^2 - 15*(4*A + 3*B)*cos(x) + (2*(3*A + 4*B)*cos(x)^3 - 6*(3*A + 4*B)*cos(
x)^2 - 15*(3*A + 4*B)*cos(x) + 15*A - 15*B)*sin(x) - 15*A + 15*B)/(cos(x)^4
 - 3*cos(x)^3 - 8*cos(x)^2 - (cos(x)^3 + 4*cos(x)^2 - 4*cos(x) - 8)*sin(x)
+ 4*cos(x) + 8)
```

Sympy [B] time = 18.9729, size = 818, normalized size = 10.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(x))/(1+sin(x))**4,x)
```

```
[Out] 30*A*tan(x/2)**7/(105*tan(x/2)**7 + 735*tan(x/2)**6 + 2205*tan(x/2)**5 + 36
75*tan(x/2)**4 + 3675*tan(x/2)**3 + 2205*tan(x/2)**2 + 735*tan(x/2) + 105)
- 210*A*tan(x/2)**4/(105*tan(x/2)**7 + 735*tan(x/2)**6 + 2205*tan(x/2)**5 +
 3675*tan(x/2)**4 + 3675*tan(x/2)**3 + 2205*tan(x/2)**2 + 735*tan(x/2) + 10
5) - 210*A*tan(x/2)**3/(105*tan(x/2)**7 + 735*tan(x/2)**6 + 2205*tan(x/2)**
5 + 3675*tan(x/2)**4 + 3675*tan(x/2)**3 + 2205*tan(x/2)**2 + 735*tan(x/2) +
 105) - 252*A*tan(x/2)**2/(105*tan(x/2)**7 + 735*tan(x/2)**6 + 2205*tan(x/2
)**5 + 3675*tan(x/2)**4 + 3675*tan(x/2)**3 + 2205*tan(x/2)**2 + 735*tan(x/2
) + 105) - 84*A*tan(x/2)/(105*tan(x/2)**7 + 735*tan(x/2)**6 + 2205*tan(x/2)
**5 + 3675*tan(x/2)**4 + 3675*tan(x/2)**3 + 2205*tan(x/2)**2 + 735*tan(x/2)
 + 105) - 42*A/(105*tan(x/2)**7 + 735*tan(x/2)**6 + 2205*tan(x/2)**5 + 3675
*tan(x/2)**4 + 3675*tan(x/2)**3 + 2205*tan(x/2)**2 + 735*tan(x/2) + 105) -
210*B*tan(x/2)**5/(105*tan(x/2)**7 + 735*tan(x/2)**6 + 2205*tan(x/2)**5 + 3
675*tan(x/2)**4 + 3675*tan(x/2)**3 + 2205*tan(x/2)**2 + 735*tan(x/2) + 105)
 - 350*B*tan(x/2)**4/(105*tan(x/2)**7 + 735*tan(x/2)**6 + 2205*tan(x/2)**5
+ 3675*tan(x/2)**4 + 3675*tan(x/2)**3 + 2205*tan(x/2)**2 + 735*tan(x/2) + 1
05) - 560*B*tan(x/2)**3/(105*tan(x/2)**7 + 735*tan(x/2)**6 + 2205*tan(x/2)*
*5 + 3675*tan(x/2)**4 + 3675*tan(x/2)**3 + 2205*tan(x/2)**2 + 735*tan(x/2)
+ 105) - 336*B*tan(x/2)**2/(105*tan(x/2)**7 + 735*tan(x/2)**6 + 2205*tan(x/
2)**5 + 3675*tan(x/2)**4 + 3675*tan(x/2)**3 + 2205*tan(x/2)**2 + 735*tan(x/
2) + 105) - 182*B*tan(x/2)/(105*tan(x/2)**7 + 735*tan(x/2)**6 + 2205*tan(x/
2)**5 + 3675*tan(x/2)**4 + 3675*tan(x/2)**3 + 2205*tan(x/2)**2 + 735*tan(x/
2) + 105) - 26*B/(105*tan(x/2)**7 + 735*tan(x/2)**6 + 2205*tan(x/2)**5 + 36
75*tan(x/2)**4 + 3675*tan(x/2)**3 + 2205*tan(x/2)**2 + 735*tan(x/2) + 105)
```

Giac [A] time = 1.34411, size = 151, normalized size = 2.01

$$-\frac{2\left(105\,A\tan\left(\frac{1}{2}\,x\right)^{6}+315\,A\tan\left(\frac{1}{2}\,x\right)^{5}+105\,B\tan\left(\frac{1}{2}\,x\right)^{5}+630\,A\tan\left(\frac{1}{2}\,x\right)^{4}+175\,B\tan\left(\frac{1}{2}\,x\right)^{4}+630\,A\tan\left(\frac{1}{2}\,x\right)\right.}{105\left(\tan\left(\frac{1}{2}\,x\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(x))/(1+sin(x))^4,x, algorithm="giac")
```

```
[Out] -2/105*(105*A*tan(1/2*x)^6 + 315*A*tan(1/2*x)^5 + 105*B*tan(1/2*x)^5 + 630*
A*tan(1/2*x)^4 + 175*B*tan(1/2*x)^4 + 630*A*tan(1/2*x)^3 + 280*B*tan(1/2*x)
^3 + 441*A*tan(1/2*x)^2 + 168*B*tan(1/2*x)^2 + 147*A*tan(1/2*x) + 91*B*tan(
1/2*x) + 36*A + 13*B)/(tan(1/2*x) + 1)^7
```

3.481 $\int \frac{A+B\sin(x)}{(1-\sin(x))^4}\,dx$

Optimal. Leaf size=81

$$\frac{2(3A-4B)\cos(x)}{105(1-\sin(x))}+\frac{2(3A-4B)\cos(x)}{105(1-\sin(x))^2}+\frac{(3A-4B)\cos(x)}{35(1-\sin(x))^3}+\frac{(A+B)\cos(x)}{7(1-\sin(x))^4}$$

```
[Out] ((A + B)*Cos[x])/(7*(1 - Sin[x])^4) + ((3*A - 4*B)*Cos[x])/(35*(1 - Sin[x])
^3) + (2*(3*A - 4*B)*Cos[x])/(105*(1 - Sin[x])^2) + (2*(3*A - 4*B)*Cos[x])/
(105*(1 - Sin[x]))
```

Rubi [A] time = 0.0663277, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2750, 2650, 2648}

$$\frac{2(3A-4B)\cos(x)}{105(1-\sin(x))}+\frac{2(3A-4B)\cos(x)}{105(1-\sin(x))^2}+\frac{(3A-4B)\cos(x)}{35(1-\sin(x))^3}+\frac{(A+B)\cos(x)}{7(1-\sin(x))^4}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[x])/(1 - Sin[x])^4,x]
```

```
[Out] ((A + B)*Cos[x])/(7*(1 - Sin[x])^4) + ((3*A - 4*B)*Cos[x])/(35*(1 - Sin[x])
^3) + (2*(3*A - 4*B)*Cos[x])/(105*(1 - Sin[x])^2) + (2*(3*A - 4*B)*Cos[x])/
(105*(1 - Sin[x]))
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+B\sin(x)}{(1-\sin(x))^4}\,dx &= \frac{(A+B)\cos(x)}{7(1-\sin(x))^4} + \frac{1}{7}(3A-4B)\int \frac{1}{(1-\sin(x))^3}\,dx \\
&= \frac{(A+B)\cos(x)}{7(1-\sin(x))^4} + \frac{(3A-4B)\cos(x)}{35(1-\sin(x))^3} + \frac{1}{35}(2(3A-4B))\int \frac{1}{(1-\sin(x))^2}\,dx \\
&= \frac{(A+B)\cos(x)}{7(1-\sin(x))^4} + \frac{(3A-4B)\cos(x)}{35(1-\sin(x))^3} + \frac{2(3A-4B)\cos(x)}{105(1-\sin(x))^2} + \frac{1}{105}(2(3A-4B))\int \frac{1}{1-\sin(x)}\,dx \\
&= \frac{(A+B)\cos(x)}{7(1-\sin(x))^4} + \frac{(3A-4B)\cos(x)}{35(1-\sin(x))^3} + \frac{2(3A-4B)\cos(x)}{105(1-\sin(x))^2} + \frac{2(3A-4B)\cos(x)}{105(1-\sin(x))}
\end{aligned}
$$

Mathematica [A] time = 0.0650276, size = 54, normalized size = 0.67

$$
\frac{\cos(x)\left((8B-6A)\sin^3(x)+8(3A-4B)\sin^2(x)+(52B-39A)\sin(x)+36A-13B\right)}{105(\sin(x)-1)^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[x])/(1 - Sin[x])^4,x]
```

```
[Out] (Cos[x]*(36*A - 13*B + (-39*A + 52*B)*Sin[x] + 8*(3*A - 4*B)*Sin[x]^2 + (-6
*A + 8*B)*Sin[x]^3))/(105*(-1 + Sin[x])^4)
```

Maple [A] time = 0.036, size = 115, normalized size = 1.4

$$
-(6A+2B)\left(\tan\left(\frac{x}{2}\right)-1\right)^{-2} - \frac{24A+24B}{3}\left(\tan\left(\frac{x}{2}\right)-1\right)^{-6} - \frac{72A+64B}{5}\left(\tan\left(\frac{x}{2}\right)-1\right)^{-5} - \frac{32A+24B}{2}\left(\tan\left(\frac{x}{2}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(x))/(1-sin(x))^4,x)
```

```
[Out] -(6*A+2*B)/(tan(1/2*x)-1)^2-1/3*(24*A+24*B)/(tan(1/2*x)-1)^6-2/5*(36*A+32*B
)/(tan(1/2*x)-1)^5-1/2*(32*A+24*B)/(tan(1/2*x)-1)^4-2/7*(8*A+8*B)/(tan(1/2*
x)-1)^7-2/3*(18*A+10*B)/(tan(1/2*x)-1)^3-2*A/(tan(1/2*x)-1)
```

Maxima [B] time = 1.53041, size = 417, normalized size = 5.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(x))/(1-sin(x))^4,x, algorithm="maxima")
```

```
[Out] -2/105*B*(91*sin(x)/(cos(x) + 1) - 168*sin(x)^2/(cos(x) + 1)^2 + 280*sin(x)
^3/(cos(x) + 1)^3 - 175*sin(x)^4/(cos(x) + 1)^4 + 105*sin(x)^5/(cos(x) + 1)
^5 - 13)/(7*sin(x)/(cos(x) + 1) - 21*sin(x)^2/(cos(x) + 1)^2 + 35*sin(x)^3/
(cos(x) + 1)^3 - 35*sin(x)^4/(cos(x) + 1)^4 + 21*sin(x)^5/(cos(x) + 1)^5 -
7*sin(x)^6/(cos(x) + 1)^6 + sin(x)^7/(cos(x) + 1)^7 - 1) + 2/35*A*(49*sin(x
)/(cos(x) + 1) - 147*sin(x)^2/(cos(x) + 1)^2 + 210*sin(x)^3/(cos(x) + 1)^3
- 210*sin(x)^4/(cos(x) + 1)^4 + 105*sin(x)^5/(cos(x) + 1)^5 - 35*sin(x)^6/(
cos(x) + 1)^6 - 12)/(7*sin(x)/(cos(x) + 1) - 21*sin(x)^2/(cos(x) + 1)^2 + 3
5*sin(x)^3/(cos(x) + 1)^3 - 35*sin(x)^4/(cos(x) + 1)^4 + 21*sin(x)^5/(cos(x
```

```
) + 1)^5 - 7*sin(x)^6/(cos(x) + 1)^6 + sin(x)^7/(cos(x) + 1)^7 - 1)
```

Fricas [B] time = 1.53891, size = 429, normalized size = 5.3

$$-\frac{2\,(3\,A-4\,B)\cos{(x)}^{4}+8\,(3\,A-4\,B)\cos{(x)}^{3}-9\,(3\,A-4\,B)\cos{(x)}^{2}-15\,(4\,A-3\,B)\cos{(x)}-\left(2\,(3\,A-4\,B)\cos{(x)}^{3}-\right.}{105\left(\cos{(x)}^{4}-3\,\cos{(x)}^{3}-8\,\cos{(x)}^{2}+\left(\cos{(x)}^{3}+4\,\cos{(x)}^{2}-4\,\cos\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(x))/(1-sin(x))^4,x, algorithm="fricas")
```

```
[Out] -1/105*(2*(3*A - 4*B)*cos(x)^4 + 8*(3*A - 4*B)*cos(x)^3 - 9*(3*A - 4*B)*cos
(x)^2 - 15*(4*A - 3*B)*cos(x) - (2*(3*A - 4*B)*cos(x)^3 - 6*(3*A - 4*B)*cos
(x)^2 - 15*(3*A - 4*B)*cos(x) + 15*A + 15*B)*sin(x) - 15*A - 15*B)/(cos(x)^
4 - 3*cos(x)^3 - 8*cos(x)^2 + (cos(x)^3 + 4*cos(x)^2 - 4*cos(x) - 8)*sin(x)
 + 4*cos(x) + 8)
```

Sympy [B] time = 19.0602, size = 818, normalized size = 10.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(x))/(1-sin(x))**4,x)
```

```
[Out] -30*A*tan(x/2)**7/(105*tan(x/2)**7 - 735*tan(x/2)**6 + 2205*tan(x/2)**5 - 3
675*tan(x/2)**4 + 3675*tan(x/2)**3 - 2205*tan(x/2)**2 + 735*tan(x/2) - 105)
 - 210*A*tan(x/2)**4/(105*tan(x/2)**7 - 735*tan(x/2)**6 + 2205*tan(x/2)**5
- 3675*tan(x/2)**4 + 3675*tan(x/2)**3 - 2205*tan(x/2)**2 + 735*tan(x/2) - 1
05) + 210*A*tan(x/2)**3/(105*tan(x/2)**7 - 735*tan(x/2)**6 + 2205*tan(x/2)*
*5 - 3675*tan(x/2)**4 + 3675*tan(x/2)**3 - 2205*tan(x/2)**2 + 735*tan(x/2)
- 105) - 252*A*tan(x/2)**2/(105*tan(x/2)**7 - 735*tan(x/2)**6 + 2205*tan(x/
2)**5 - 3675*tan(x/2)**4 + 3675*tan(x/2)**3 - 2205*tan(x/2)**2 + 735*tan(x/
2) - 105) + 84*A*tan(x/2)/(105*tan(x/2)**7 - 735*tan(x/2)**6 + 2205*tan(x/2
)**5 - 3675*tan(x/2)**4 + 3675*tan(x/2)**3 - 2205*tan(x/2)**2 + 735*tan(x/2
) - 105) - 42*A/(105*tan(x/2)**7 - 735*tan(x/2)**6 + 2205*tan(x/2)**5 - 367
5*tan(x/2)**4 + 3675*tan(x/2)**3 - 2205*tan(x/2)**2 + 735*tan(x/2) - 105) -
 210*B*tan(x/2)**5/(105*tan(x/2)**7 - 735*tan(x/2)**6 + 2205*tan(x/2)**5 -
3675*tan(x/2)**4 + 3675*tan(x/2)**3 - 2205*tan(x/2)**2 + 735*tan(x/2) - 105
) + 350*B*tan(x/2)**4/(105*tan(x/2)**7 - 735*tan(x/2)**6 + 2205*tan(x/2)**5
 - 3675*tan(x/2)**4 + 3675*tan(x/2)**3 - 2205*tan(x/2)**2 + 735*tan(x/2) -
105) - 560*B*tan(x/2)**3/(105*tan(x/2)**7 - 735*tan(x/2)**6 + 2205*tan(x/2)
**5 - 3675*tan(x/2)**4 + 3675*tan(x/2)**3 - 2205*tan(x/2)**2 + 735*tan(x/2)
 - 105) + 336*B*tan(x/2)**2/(105*tan(x/2)**7 - 735*tan(x/2)**6 + 2205*tan(x
/2)**5 - 3675*tan(x/2)**4 + 3675*tan(x/2)**3 - 2205*tan(x/2)**2 + 735*tan(x
/2) - 105) - 182*B*tan(x/2)/(105*tan(x/2)**7 - 735*tan(x/2)**6 + 2205*tan(x
/2)**5 - 3675*tan(x/2)**4 + 3675*tan(x/2)**3 - 2205*tan(x/2)**2 + 735*tan(x
/2) - 105) + 26*B/(105*tan(x/2)**7 - 735*tan(x/2)**6 + 2205*tan(x/2)**5 - 3
675*tan(x/2)**4 + 3675*tan(x/2)**3 - 2205*tan(x/2)**2 + 735*tan(x/2) - 105)
```

Giac [A] time = 1.32475, size = 151, normalized size = 1.86

$$-\frac{2\left(105\,A\tan\left(\frac{1}{2}\,x\right)^{6}-315\,A\tan\left(\frac{1}{2}\,x\right)^{5}+105\,B\tan\left(\frac{1}{2}\,x\right)^{5}+630\,A\tan\left(\frac{1}{2}\,x\right)^{4}-175\,B\tan\left(\frac{1}{2}\,x\right)^{4}-630\,A\tan\left(\frac{1}{2}\,x\right)^{3}}{105\left(\tan\left(\frac{1}{2}\,x\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(x))/(1-sin(x))^4,x, algorithm="giac")
```

```
[Out] -2/105*(105*A*tan(1/2*x)^6 - 315*A*tan(1/2*x)^5 + 105*B*tan(1/2*x)^5 + 630*
A*tan(1/2*x)^4 - 175*B*tan(1/2*x)^4 - 630*A*tan(1/2*x)^3 + 280*B*tan(1/2*x)
^3 + 441*A*tan(1/2*x)^2 - 168*B*tan(1/2*x)^2 - 147*A*tan(1/2*x) + 91*B*tan(
1/2*x) + 36*A - 13*B)/(tan(1/2*x) - 1)^7
```

3.482 $\int (a + a\sin(e + fx))(c + d\sin(e + fx))^{5/2}\,dx$

Optimal. Leaf size=290

$$-\frac{2a\left(15c^2+56cd+25d^2\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{105f}-\frac{2a\left(c^2-d^2\right)\left(15c^2+56cd+25d^2\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\right.\right.}{105df\sqrt{c+d\sin(e+fx)}}$$

```
[Out] (-2*a*(15*c^2 + 56*c*d + 25*d^2)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(10
5*f) - (2*a*(5*c + 7*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(3/2))/(35*f) - (
2*a*Cos[e + f*x]*(c + d*Sin[e + f*x])^(5/2))/(7*f) + (2*a*(15*c^3 + 161*c^2
*d + 145*c*d^2 + 63*d^3)*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[
c + d*Sin[e + f*x]])/(105*d*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) - (2*a*(c
^2 - d^2)*(15*c^2 + 56*c*d + 25*d^2)*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c
 + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(105*d*f*Sqrt[c + d*Sin[e + f*x]
])
```

Rubi [A] time = 0.442811, antiderivative size = 290, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {2753, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2a\left(15c^2+56cd+25d^2\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{105f}-\frac{2a\left(c^2-d^2\right)\left(15c^2+56cd+25d^2\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\right.\right.}{105df\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (-2*a*(15*c^2 + 56*c*d + 25*d^2)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(10
5*f) - (2*a*(5*c + 7*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(3/2))/(35*f) - (
2*a*Cos[e + f*x]*(c + d*Sin[e + f*x])^(5/2))/(7*f) + (2*a*(15*c^3 + 161*c^2
*d + 145*c*d^2 + 63*d^3)*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[
c + d*Sin[e + f*x]])/(105*d*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) - (2*a*(c
^2 - d^2)*(15*c^2 + 56*c*d + 25*d^2)*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c
 + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(105*d*f*Sqrt[c + d*Sin[e + f*x]
])
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))(c+d\sin(e+fx))^{5/2}\,dx = -\frac{2a\cos(e+fx)(c+d\sin(e+fx))^{5/2}}{7f}+\frac{2}{7}\int (c+d\sin(e+fx))^{3/2}\left(\frac{1}{2}a\right.$$

$$= -\frac{2a(5c+7d)\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{35f}-\frac{2a\cos(e+fx)(c+d}{7f}$$

$$= -\frac{2a\left(15c^2+56cd+25d^2\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{105f}-\frac{2a(5c+7}{}$$

$$= -\frac{2a\left(15c^2+56cd+25d^2\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{105f}-\frac{2a(5c+7}{}$$

$$= -\frac{2a\left(15c^2+56cd+25d^2\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{105f}-\frac{2a(5c+7}{}$$

$$= -\frac{2a\left(15c^2+56cd+25d^2\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{105f}-\frac{2a(5c+7}{}$$

Mathematica [C] time = 6.72639, size = 3531, normalized size = 12.18

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] a*((c^3*Sec[e]*(1 + Sin[e + f*x])*(-((AppellF1[-1/2, -1/2, -1/2, 1/2, -((Cs
c[e]*(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d*Sqrt[1
 + Cot[e]^2]*(1 - (c*Csc[e])/(d*Sqrt[1 + Cot[e]^2])))), -((Csc[e]*(c + d*Co
```

```
s[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d*Sqrt[1 + Cot[e]^2]*(
-1 - (c*Csc[e])/(d*Sqrt[1 + Cot[e]^2]))))]*Cot[e]*Sin[f*x - ArcTan[Cot[e]]]
)/(Sqrt[1 + Cot[e]^2]*Sqrt[(d*Sqrt[1 + Cot[e]^2] + d*Cos[f*x - ArcTan[Cot[e
]]]*Sqrt[1 + Cot[e]^2])/(d*Sqrt[1 + Cot[e]^2] - c*Csc[e])]*Sqrt[(d*Sqrt[1 +
 Cot[e]^2] - d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2])/(d*Sqrt[1 + Co
t[e]^2] + c*Csc[e])]*Sqrt[c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2
]*Sin[e]])) - ((2*d*Sin[e]*(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]
^2]*Sin[e]))/(d^2*Cos[e]^2 + d^2*Sin[e]^2) - (Cot[e]*Sin[f*x - ArcTan[Cot[e
]]])/Sqrt[1 + Cot[e]^2])/Sqrt[c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[
e]^2]*Sin[e]]))/(7*f*(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^2) + (23*c^2
*d*Sec[e]*(1 + Sin[e + f*x])*(-((AppellF1[-1/2, -1/2, -1/2, 1/2, -((Csc[e]*
(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d*Sqrt[1 + Co
t[e]^2]*(1 - (c*Csc[e])/(d*Sqrt[1 + Cot[e]^2])))), -((Csc[e]*(c + d*Cos[f*x
 - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d*Sqrt[1 + Cot[e]^2]*(-1 -
(c*Csc[e])/(d*Sqrt[1 + Cot[e]^2]))))]*Cot[e]*Sin[f*x - ArcTan[Cot[e]]])/(Sq
rt[1 + Cot[e]^2]*Sqrt[(d*Sqrt[1 + Cot[e]^2] + d*Cos[f*x - ArcTan[Cot[e]]]*S
qrt[1 + Cot[e]^2])/(d*Sqrt[1 + Cot[e]^2] - c*Csc[e])]*Sqrt[(d*Sqrt[1 + Cot[
e]^2] - d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2])/(d*Sqrt[1 + Cot[e]^
2] + c*Csc[e])]*Sqrt[c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin
[e]])) - ((2*d*Sin[e]*(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*S
in[e]))/(d^2*Cos[e]^2 + d^2*Sin[e]^2) - (Cot[e]*Sin[f*x - ArcTan[Cot[e]]])/
Sqrt[1 + Cot[e]^2])/Sqrt[c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]
*Sin[e]]))/(15*f*(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^2) + (29*c*d^2*S
ec[e]*(1 + Sin[e + f*x])*(-((AppellF1[-1/2, -1/2, -1/2, 1/2, -((Csc[e]*(c +
 d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d*Sqrt[1 + Cot[e]
^2]*(1 - (c*Csc[e])/(d*Sqrt[1 + Cot[e]^2])))), -((Csc[e]*(c + d*Cos[f*x - A
rcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d*Sqrt[1 + Cot[e]^2]*(-1 - (c*C
sc[e])/(d*Sqrt[1 + Cot[e]^2]))))]*Cot[e]*Sin[f*x - ArcTan[Cot[e]]])/(Sqrt[1
 + Cot[e]^2]*Sqrt[(d*Sqrt[1 + Cot[e]^2] + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[
1 + Cot[e]^2])/(d*Sqrt[1 + Cot[e]^2] - c*Csc[e])]*Sqrt[(d*Sqrt[1 + Cot[e]^2
] - d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2])/(d*Sqrt[1 + Cot[e]^2] +
 c*Csc[e])]*Sqrt[c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]]
)) - ((2*d*Sin[e]*(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e
]))/(d^2*Cos[e]^2 + d^2*Sin[e]^2) - (Cot[e]*Sin[f*x - ArcTan[Cot[e]]])/Sqrt
[1 + Cot[e]^2])/Sqrt[c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin
[e]]))/(21*f*(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^2) + (3*d^3*Sec[e]*(
1 + Sin[e + f*x])*(-((AppellF1[-1/2, -1/2, -1/2, 1/2, -((Csc[e]*(c + d*Cos[
f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d*Sqrt[1 + Cot[e]^2]*(1
- (c*Csc[e])/(d*Sqrt[1 + Cot[e]^2])))), -((Csc[e]*(c + d*Cos[f*x - ArcTan[C
ot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d*Sqrt[1 + Cot[e]^2]*(-1 - (c*Csc[e])/
(d*Sqrt[1 + Cot[e]^2]))))]*Cot[e]*Sin[f*x - ArcTan[Cot[e]]])/(Sqrt[1 + Cot[
e]^2]*Sqrt[(d*Sqrt[1 + Cot[e]^2] + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot
[e]^2])/(d*Sqrt[1 + Cot[e]^2] - c*Csc[e])]*Sqrt[(d*Sqrt[1 + Cot[e]^2] - d*C
os[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2])/(d*Sqrt[1 + Cot[e]^2] + c*Csc[
e])]*Sqrt[c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]])) - ((
2*d*Sin[e]*(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d^
2*Cos[e]^2 + d^2*Sin[e]^2) - (Cot[e]*Sin[f*x - ArcTan[Cot[e]]])/Sqrt[1 + Co
t[e]^2])/Sqrt[c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]]))/
(5*f*(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^2) + ((1 + Sin[e + f*x])*Sqr
t[c + d*Sin[e + f*x]]*(-((180*c^2 + 308*c*d + 115*d^2)*Cos[e]*Cos[f*x])/(21
0*f) + (d^2*Cos[3*e]*Cos[3*f*x])/(14*f) - (d*(15*c + 7*d)*Cos[2*f*x]*Sin[2*
e])/(35*f) + ((180*c^2 + 308*c*d + 115*d^2)*Sin[e]*Sin[f*x])/(210*f) - (d*(
15*c + 7*d)*Cos[2*e]*Sin[2*f*x])/(35*f) - (d^2*Sin[3*e]*Sin[3*f*x])/(14*f)
+ (2*(15*c^3 + 161*c^2*d + 145*c*d^2 + 63*d^3)*Tan[e])/(105*d*f)))/(Cos[e/2
 + (f*x)/2] + Sin[e/2 + (f*x)/2])^2 + (18*c^2*AppellF1[1/2, 1/2, 1/2, 3/2,
-((Sec[e]*(c + d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]))/(d*S
qrt[1 + Tan[e]^2]*(1 - (c*Sec[e])/(d*Sqrt[1 + Tan[e]^2])))), -((Sec[e]*(c +
 d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]))/(d*Sqrt[1 + Tan[e]
^2]*(-1 - (c*Sec[e])/(d*Sqrt[1 + Tan[e]^2]))))]*Sec[e]*Sec[f*x + ArcTan[Tan
```

```
[e]]]*(1 + Sin[e + f*x])*Sqrt[(d*Sqrt[1 + Tan[e]^2] - d*Sin[f*x + ArcTan[Ta
n[e]]]*Sqrt[1 + Tan[e]^2])/(c*Sec[e] + d*Sqrt[1 + Tan[e]^2])]*Sqrt[(d*Sqrt[
1 + Tan[e]^2] + d*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2])/(-(c*Sec[e]
) + d*Sqrt[1 + Tan[e]^2])]*Sqrt[c + d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*Sqrt
[1 + Tan[e]^2]])/(7*f*(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^2*Sqrt[1 +
Tan[e]^2]) + (2*c^3*AppellF1[1/2, 1/2, 1/2, 3/2, -((Sec[e]*(c + d*Cos[e]*Si
n[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]))/(d*Sqrt[1 + Tan[e]^2]*(1 - (c*
Sec[e])/(d*Sqrt[1 + Tan[e]^2])))), -((Sec[e]*(c + d*Cos[e]*Sin[f*x + ArcTan
[Tan[e]]]*Sqrt[1 + Tan[e]^2]))/(d*Sqrt[1 + Tan[e]^2]*(-1 - (c*Sec[e])/(d*Sq
rt[1 + Tan[e]^2]))))]*Sec[e]*Sec[f*x + ArcTan[Tan[e]]]*(1 + Sin[e + f*x])*S
qrt[(d*Sqrt[1 + Tan[e]^2] - d*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2])
/(c*Sec[e] + d*Sqrt[1 + Tan[e]^2])]*Sqrt[(d*Sqrt[1 + Tan[e]^2] + d*Sin[f*x
+ ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2])/(-(c*Sec[e]) + d*Sqrt[1 + Tan[e]^2])]
*Sqrt[c + d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]])/(d*f*(Cos
[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^2*Sqrt[1 + Tan[e]^2]) + (34*c*d*Appel
lF1[1/2, 1/2, 1/2, 3/2, -((Sec[e]*(c + d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*S
qrt[1 + Tan[e]^2]))/(d*Sqrt[1 + Tan[e]^2]*(1 - (c*Sec[e])/(d*Sqrt[1 + Tan[e
]^2])))), -((Sec[e]*(c + d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]
^2]))/(d*Sqrt[1 + Tan[e]^2]*(-1 - (c*Sec[e])/(d*Sqrt[1 + Tan[e]^2]))))]*Sec
[e]*Sec[f*x + ArcTan[Tan[e]]]*(1 + Sin[e + f*x])*Sqrt[(d*Sqrt[1 + Tan[e]^2]
 - d*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2])/(c*Sec[e] + d*Sqrt[1 + T
an[e]^2])]*Sqrt[(d*Sqrt[1 + Tan[e]^2] + d*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1
+ Tan[e]^2])/(-(c*Sec[e]) + d*Sqrt[1 + Tan[e]^2])]*Sqrt[c + d*Cos[e]*Sin[f*
x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]])/(15*f*(Cos[e/2 + (f*x)/2] + Sin[e/
2 + (f*x)/2])^2*Sqrt[1 + Tan[e]^2]) + (10*d^2*AppellF1[1/2, 1/2, 1/2, 3/2,
-((Sec[e]*(c + d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]))/(d*S
qrt[1 + Tan[e]^2]*(1 - (c*Sec[e])/(d*Sqrt[1 + Tan[e]^2])))), -((Sec[e]*(c +
 d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]))/(d*Sqrt[1 + Tan[e]
^2]*(-1 - (c*Sec[e])/(d*Sqrt[1 + Tan[e]^2]))))]*Sec[e]*Sec[f*x + ArcTan[Tan
[e]]]*(1 + Sin[e + f*x])*Sqrt[(d*Sqrt[1 + Tan[e]^2] - d*Sin[f*x + ArcTan[Ta
n[e]]]*Sqrt[1 + Tan[e]^2])/(c*Sec[e] + d*Sqrt[1 + Tan[e]^2])]*Sqrt[(d*Sqrt[
1 + Tan[e]^2] + d*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2])/(-(c*Sec[e]
) + d*Sqrt[1 + Tan[e]^2])]*Sqrt[c + d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*Sqrt
[1 + Tan[e]^2]])/(21*f*(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^2*Sqrt[1 +
 Tan[e]^2]))
```

Maple [B] time = 1.273, size = 1315, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(c+d*sin(f*x+e))^(5/2),x)
```

```
[Out] 2/105*a*(60*c*d^4*sin(f*x+e)^4-77*c^2*d^3-25*c*d^4+176*((c+d*sin(f*x+e))/(c
-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)
*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c^3*d^2+90*c
^2*d^3*sin(f*x+e)^3+98*c*d^4*sin(f*x+e)^3+45*c^3*d^2*sin(f*x+e)^2+77*c^2*d^
3*sin(f*x+e)^2-35*c*d^4*sin(f*x+e)^2-90*c^2*d^3*sin(f*x+e)-98*c*d^4*sin(f*x
+e)+15*d^5*sin(f*x+e)^5+21*d^5*sin(f*x+e)^4+10*d^5*sin(f*x+e)^3-21*d^5*sin(
f*x+e)^2-25*d^5*sin(f*x+e)-45*c^3*d^2-88*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(
-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c
+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*d^5-15*((c+d*sin(f*x+e))/(
c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2
)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c^5+63*((c+
d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+
e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/
```

```
2))*d^5-32*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*
(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((
c-d)/(c+d))^(1/2))*c^2*d^3-176*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x
+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x
+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c*d^4-161*((c+d*sin(f*x+e))/(c-d))^(
1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*Ellip
ticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c^4*d-130*((c+d*si
n(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/
(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*
c^3*d^2+98*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*
(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((
c-d)/(c+d))^(1/2))*c^2*d^3+145*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x
+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x
+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c*d^4+120*((c+d*sin(f*x+e))/(c-d))^(
1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*Ellip
ticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c^4*d)/d^2/cos(f*x
+e)/(c+d*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(\left(ac^2+2\,acd+ad^2-\left(2\,acd+ad^2\right)\cos\left(fx+e\right)^2-\left(ad^2\cos\left(fx+e\right)^2-ac^2-2\,acd-ad^2\right)\sin\left(fx+e\right)\right)\sqrt{d\sin\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((a*c^2 + 2*a*c*d + a*d^2 - (2*a*c*d + a*d^2)*cos(f*x + e)^2 - (a*d
^2*cos(f*x + e)^2 - a*c^2 - 2*a*c*d - a*d^2)*sin(f*x + e))*sqrt(d*sin(f*x +
 e) + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.483 $\int (a+a\sin(e+fx))(c+d\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=231

$$-\frac{2a(3c+5d)\left(c^2-d^2\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{15df\sqrt{c+d\sin(e+fx)}}+\frac{2a\left(3c^2+20cd+9d^2\right)\sqrt{c+d\sin(e+fx)}E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right.\right.}{15df\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

```
[Out] (-2*a*(3*c + 5*d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(15*f) - (2*a*Cos[
e + f*x]*(c + d*Sin[e + f*x])^(3/2))/(5*f) + (2*a*(3*c^2 + 20*c*d + 9*d^2)*
EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(15*
d*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) - (2*a*(3*c + 5*d)*(c^2 - d^2)*Elli
pticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)]
)/(15*d*f*Sqrt[c + d*Sin[e + f*x]])
```

Rubi [A] time = 0.317024, antiderivative size = 231, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {2753, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2a(3c+5d)\left(c^2-d^2\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{15df\sqrt{c+d\sin(e+fx)}}+\frac{2a\left(3c^2+20cd+9d^2\right)\sqrt{c+d\sin(e+fx)}E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right.\right.}{15df\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-2*a*(3*c + 5*d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(15*f) - (2*a*Cos[
e + f*x]*(c + d*Sin[e + f*x])^(3/2))/(5*f) + (2*a*(3*c^2 + 20*c*d + 9*d^2)*
EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(15*
d*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) - (2*a*(3*c + 5*d)*(c^2 - d^2)*Elli
pticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)]
)/(15*d*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
```

```
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))(c+d\sin(e+fx))^{3/2}\,dx = -\frac{2a\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{5f}+\frac{2}{5}\int \sqrt{c+d\sin(e+fx)}\left(\frac{1}{2}a(5$$

$$= -\frac{2a(3c+5d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15f}-\frac{2a\cos(e+fx)(c+d\,\mathrm{si}}{5f}$$

$$= -\frac{2a(3c+5d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15f}-\frac{2a\cos(e+fx)(c+d\,\mathrm{si}}{5f}$$

$$= -\frac{2a(3c+5d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15f}-\frac{2a\cos(e+fx)(c+d\,\mathrm{si}}{5f}$$

$$= -\frac{2a(3c+5d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15f}-\frac{2a\cos(e+fx)(c+d\,\mathrm{si}}{5f}$$

Mathematica [C] time = 6.36819, size = 2625, normalized size = 11.36

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] a*((c^2*Sec[e]*(1 + Sin[e + f*x])*(-((AppellF1[-1/2, -1/2, -1/2, 1/2, -((Cs
c[e]*(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d*Sqrt[1
 + Cot[e]^2]*(1 - (c*Csc[e])/(d*Sqrt[1 + Cot[e]^2])))), -((Csc[e]*(c + d*Co
s[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d*Sqrt[1 + Cot[e]^2]*(
-1 - (c*Csc[e])/(d*Sqrt[1 + Cot[e]^2]))))]*Cot[e]*Sin[f*x - ArcTan[Cot[e]]]
)/(Sqrt[1 + Cot[e]^2]*Sqrt[(d*Sqrt[1 + Cot[e]^2] + d*Cos[f*x - ArcTan[Cot[e
]]]*Sqrt[1 + Cot[e]^2])/(d*Sqrt[1 + Cot[e]^2] - c*Csc[e])]*Sqrt[(d*Sqrt[1 +
 Cot[e]^2] - d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2])/(d*Sqrt[1 + Co
```

t[e]^2] + c*Csc[e])]*Sqrt[c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2
]*Sin[e]])) - ((2*d*Sin[e]*(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]
^2]*Sin[e]))/(d^2*Cos[e]^2 + d^2*Sin[e]^2) - (Cot[e]*Sin[f*x - ArcTan[Cot[e
]]])/Sqrt[1 + Cot[e]^2])/Sqrt[c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[
e]^2]*Sin[e]]))/(5*f*(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^2) + (4*c*d*
Sec[e]*(1 + Sin[e + f*x])*(-((AppellF1[-1/2, -1/2, -1/2, 1/2, -((Csc[e]*(c
+ d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d*Sqrt[1 + Cot[e
]^2]*(1 - (c*Csc[e])/(d*Sqrt[1 + Cot[e]^2])))), -((Csc[e]*(c + d*Cos[f*x -
ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d*Sqrt[1 + Cot[e]^2]*(-1 - (c*
Csc[e])/(d*Sqrt[1 + Cot[e]^2]))))]*Cot[e]*Sin[f*x - ArcTan[Cot[e]]])/(Sqrt[
1 + Cot[e]^2]*Sqrt[(d*Sqrt[1 + Cot[e]^2] + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt
[1 + Cot[e]^2])/(d*Sqrt[1 + Cot[e]^2] - c*Csc[e])]*Sqrt[(d*Sqrt[1 + Cot[e]^
2] - d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2])/(d*Sqrt[1 + Cot[e]^2]
+ c*Csc[e])]*Sqrt[c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]
])) - ((2*d*Sin[e]*(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[
e]))/(d^2*Cos[e]^2 + d^2*Sin[e]^2) - (Cot[e]*Sin[f*x - ArcTan[Cot[e]]])/Sqr
t[1 + Cot[e]^2])/Sqrt[c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Si
n[e]]))/(3*f*(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^2) + (3*d^2*Sec[e]*(
1 + Sin[e + f*x])*(-((AppellF1[-1/2, -1/2, -1/2, 1/2, -((Csc[e]*(c + d*Cos[
f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d*Sqrt[1 + Cot[e]^2]*(1
- (c*Csc[e])/(d*Sqrt[1 + Cot[e]^2])))), -((Csc[e]*(c + d*Cos[f*x - ArcTan[C
ot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d*Sqrt[1 + Cot[e]^2]*(-1 - (c*Csc[e])/
(d*Sqrt[1 + Cot[e]^2]))))]*Cot[e]*Sin[f*x - ArcTan[Cot[e]]])/(Sqrt[1 + Cot[
e]^2]*Sqrt[(d*Sqrt[1 + Cot[e]^2] + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot
[e]^2])/(d*Sqrt[1 + Cot[e]^2] - c*Csc[e])]*Sqrt[(d*Sqrt[1 + Cot[e]^2] - d*C
os[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2])/(d*Sqrt[1 + Cot[e]^2] + c*Csc[
e])]*Sqrt[c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]])) - ((
2*d*Sin[e]*(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d^
2*Cos[e]^2 + d^2*Sin[e]^2) - (Cot[e]*Sin[f*x - ArcTan[Cot[e]]])/Sqrt[1 + Co
t[e]^2])/Sqrt[c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]]))/
(5*f*(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^2) + ((1 + Sin[e + f*x])*Sqr
t[c + d*Sin[e + f*x]]*((-2*(6*c + 5*d)*Cos[e]*Cos[f*x])/(15*f) - (d*Cos[2*f
*x]*Sin[2*e])/(5*f) + (2*(6*c + 5*d)*Sin[e]*Sin[f*x])/(15*f) - (d*Cos[2*e]*
Sin[2*f*x])/(5*f) + (2*(3*c^2 + 20*c*d + 9*d^2)*Tan[e])/(15*d*f)))/(Cos[e/2
 + (f*x)/2] + Sin[e/2 + (f*x)/2])^2 + (8*c*AppellF1[1/2, 1/2, 1/2, 3/2, -((
Sec[e]*(c + d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]))/(d*Sqrt
[1 + Tan[e]^2]*(1 - (c*Sec[e])/(d*Sqrt[1 + Tan[e]^2])))), -((Sec[e]*(c + d*
Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]))/(d*Sqrt[1 + Tan[e]^2]
*(-1 - (c*Sec[e])/(d*Sqrt[1 + Tan[e]^2]))))]*Sec[e]*Sec[f*x + ArcTan[Tan[e]
]]*(1 + Sin[e + f*x])*Sqrt[(d*Sqrt[1 + Tan[e]^2] - d*Sin[f*x + ArcTan[Tan[e
]]]*Sqrt[1 + Tan[e]^2])/(c*Sec[e] + d*Sqrt[1 + Tan[e]^2])]*Sqrt[(d*Sqrt[1 +
 Tan[e]^2] + d*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2])/(-(c*Sec[e]) +
 d*Sqrt[1 + Tan[e]^2])]*Sqrt[c + d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1
+ Tan[e]^2]])/(5*f*(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^2*Sqrt[1 + Tan
[e]^2]) + (2*c^2*AppellF1[1/2, 1/2, 1/2, 3/2, -((Sec[e]*(c + d*Cos[e]*Sin[f
*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]))/(d*Sqrt[1 + Tan[e]^2]*(1 - (c*Sec
[e])/(d*Sqrt[1 + Tan[e]^2])))), -((Sec[e]*(c + d*Cos[e]*Sin[f*x + ArcTan[Ta
n[e]]]*Sqrt[1 + Tan[e]^2]))/(d*Sqrt[1 + Tan[e]^2]*(-1 - (c*Sec[e])/(d*Sqrt[
1 + Tan[e]^2]))))]*Sec[e]*Sec[f*x + ArcTan[Tan[e]]]*(1 + Sin[e + f*x])*Sqrt
[(d*Sqrt[1 + Tan[e]^2] - d*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2])/(c
*Sec[e] + d*Sqrt[1 + Tan[e]^2])]*Sqrt[(d*Sqrt[1 + Tan[e]^2] + d*Sin[f*x + A
rcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2])/(-(c*Sec[e]) + d*Sqrt[1 + Tan[e]^2])]*Sq
rt[c + d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]])/(d*f*(Cos[e/
2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^2*Sqrt[1 + Tan[e]^2]) + (2*d*AppellF1[1/
2, 1/2, 1/2, 3/2, -((Sec[e]*(c + d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1
+ Tan[e]^2]))/(d*Sqrt[1 + Tan[e]^2]*(1 - (c*Sec[e])/(d*Sqrt[1 + Tan[e]^2]))
)), -((Sec[e]*(c + d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]))/
(d*Sqrt[1 + Tan[e]^2]*(-1 - (c*Sec[e])/(d*Sqrt[1 + Tan[e]^2]))))]*Sec[e]*Se
c[f*x + ArcTan[Tan[e]]]*(1 + Sin[e + f*x])*Sqrt[(d*Sqrt[1 + Tan[e]^2] - d*S

```
in[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2])/(c*Sec[e] + d*Sqrt[1 + Tan[e]^
2])]*Sqrt[(d*Sqrt[1 + Tan[e]^2] + d*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[
e]^2])/(-(c*Sec[e]) + d*Sqrt[1 + Tan[e]^2])]*Sqrt[c + d*Cos[e]*Sin[f*x + Ar
cTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]])/(3*f*(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*
x)/2])^2*Sqrt[1 + Tan[e]^2]))
```

Maple [B] time = 0.927, size = 1034, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(c+d*sin(f*x+e))^(3/2),x)
```

```
[Out] 2/15*a*(18*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*
(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((
c-d)/(c+d))^(1/2))*c^3*d+14*c^2*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*
x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*
x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*d^2-18*c*((c+d*sin(f*x+e))/(c-d))^(
1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*Ellip
ticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*d^3-14*((c+d*sin(f
*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-
d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*d^4
-3*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+s
in(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+
d))^(1/2))*c^4-20*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))
^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(
1/2),((c-d)/(c+d))^(1/2))*c^3*d-6*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(
f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(
f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c^2*d^2+20*((c+d*sin(f*x+e))/(c-d
))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*E
llipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c*d^3+9*((c+d*
sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e)
)/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)
)*d^4+3*d^4*sin(f*x+e)^4+9*c*d^3*sin(f*x+e)^3+5*d^4*sin(f*x+e)^3+6*c^2*d^2*
sin(f*x+e)^2+5*c*d^3*sin(f*x+e)^2-3*d^4*sin(f*x+e)^2-9*c*d^3*sin(f*x+e)-5*d
^4*sin(f*x+e)-6*c^2*d^2-5*c*d^3)/d^2/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a \sin\left(fx+e\right)+a\right)\left(d \sin\left(fx+e\right)+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\left(ad\cos\left(fx+e\right)^{2}-ac-ad-(ac+ad)\sin\left(fx+e\right)\right)\sqrt{d\sin\left(fx+e\right)+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-(a*d*cos(f*x + e)^2 - a*c - a*d - (a*c + a*d)*sin(f*x + e))*sqrt(
d*sin(f*x + e) + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int c\sqrt{c+d\sin\left(e+fx\right)}\,dx+\int c\sqrt{c+d\sin\left(e+fx\right)}\sin\left(e+fx\right)dx+\int d\sqrt{c+d\sin\left(e+fx\right)}\sin\left(e+fx\right)dx+\int d\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e))**(3/2),x)
```

```
[Out] a*(Integral(c*sqrt(c + d*sin(e + f*x)), x) + Integral(c*sqrt(c + d*sin(e +
f*x))*sin(e + f*x), x) + Integral(d*sqrt(c + d*sin(e + f*x))*sin(e + f*x),
x) + Integral(d*sqrt(c + d*sin(e + f*x))*sin(e + f*x)**2, x))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.484 $\int (a+a\sin(e+fx))\sqrt{c+d\sin(e+fx)}\,dx$

Optimal. Leaf size=179

$$-\frac{2a\left(c^2-d^2\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3df\sqrt{c+d\sin(e+fx)}}-\frac{2a\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3f}+\frac{2a(c+3d)\sqrt{c+d\sin(e+fx}}{3df\sqrt{\frac{c+d\,\mathrm{s}}{}}}$$

```
[Out] (-2*a*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(3*f) + (2*a*(c + 3*d)*Ellipti
cE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(3*d*f*Sqrt
[(c + d*Sin[e + f*x])/(c + d)]) - (2*a*(c^2 - d^2)*EllipticF[(e - Pi/2 + f*
x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(3*d*f*Sqrt[c + d*
Sin[e + f*x]])
```

Rubi [A] time = 0.197774, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {2753, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2a\left(c^2-d^2\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3df\sqrt{c+d\sin(e+fx)}}-\frac{2a\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3f}+\frac{2a(c+3d)\sqrt{c+d\sin(e+fx}}{3df\sqrt{\frac{c+d\,\mathrm{s}}{}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]],x]
```


```
[Out] (-2*a*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(3*f) + (2*a*(c + 3*d)*Ellipti
cE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(3*d*f*Sqrt
[(c + d*Sin[e + f*x])/(c + d)]) - (2*a*(c^2 - d^2)*EllipticF[(e - Pi/2 + f*
x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(3*d*f*Sqrt[c + d*
Sin[e + f*x]])
```


Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))\sqrt{c+d\sin(e+fx)}\,dx = -\frac{2a\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3f}+\frac{2}{3}\int\frac{\frac{1}{2}a(3c+d)+\frac{1}{2}a(c+3d)\sin(e+}{\sqrt{c+d\sin(e+fx)}}$$

$$= -\frac{2a\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3f}+\frac{(a(c+3d))\int\sqrt{c+d\sin(e+fx)}\,dx}{3d} -$$

$$= -\frac{2a\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3f}+\frac{\left(a(c+3d)\sqrt{c+d\sin(e+fx)}\right)\int\sqrt{\frac{c}{c+}}}{3d\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

$$= -\frac{2a\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3f}+\frac{2a(c+3d)E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2d}{c+d}\right)\sqrt{c}}{3df\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

Mathematica [C] time = 6.28371, size = 1736, normalized size = 9.7

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] a*((c*Sec[e]*(1 + Sin[e + f*x])*(-((AppellF1[-1/2, -1/2, -1/2, 1/2, -((Csc[
e]*(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d*Sqrt[1 +
 Cot[e]^2]*(1 - (c*Csc[e])/(d*Sqrt[1 + Cot[e]^2])))), -((Csc[e]*(c + d*Cos[
f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d*Sqrt[1 + Cot[e]^2]*(-1
 - (c*Csc[e])/(d*Sqrt[1 + Cot[e]^2]))))]*Cot[e]*Sin[f*x - ArcTan[Cot[e]]])/
(Sqrt[1 + Cot[e]^2]*Sqrt[(d*Sqrt[1 + Cot[e]^2] + d*Cos[f*x - ArcTan[Cot[e]]
]*Sqrt[1 + Cot[e]^2])/(d*Sqrt[1 + Cot[e]^2] - c*Csc[e])]*Sqrt[(d*Sqrt[1 + C
ot[e]^2] - d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2])/(d*Sqrt[1 + Cot[
e]^2] + c*Csc[e])]*Sqrt[c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*
Sin[e]])) - ((2*d*Sin[e]*(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2
]*Sin[e]))/(d^2*Cos[e]^2 + d^2*Sin[e]^2) - (Cot[e]*Sin[f*x - ArcTan[Cot[e]]
])/Sqrt[1 + Cot[e]^2])/Sqrt[c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]
^2]*Sin[e]]))/(3*f*(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^2) + (d*Sec[e]
```

```
*(1 + Sin[e + f*x])*(-((AppellF1[-1/2, -1/2, -1/2, 1/2, -((Csc[e]*(c + d*Co
s[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d*Sqrt[1 + Cot[e]^2]*(
1 - (c*Csc[e])/(d*Sqrt[1 + Cot[e]^2])))), -((Csc[e]*(c + d*Cos[f*x - ArcTan
[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d*Sqrt[1 + Cot[e]^2]*(-1 - (c*Csc[e]
)/(d*Sqrt[1 + Cot[e]^2]))))]*Cot[e]*Sin[f*x - ArcTan[Cot[e]]])/(Sqrt[1 + Co
t[e]^2]*Sqrt[(d*Sqrt[1 + Cot[e]^2] + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + C
ot[e]^2])/(d*Sqrt[1 + Cot[e]^2] - c*Csc[e])]*Sqrt[(d*Sqrt[1 + Cot[e]^2] - d
*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2])/(d*Sqrt[1 + Cot[e]^2] + c*Cs
c[e])]*Sqrt[c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]])) -
((2*d*Sin[e]*(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(
d^2*Cos[e]^2 + d^2*Sin[e]^2) - (Cot[e]*Sin[f*x - ArcTan[Cot[e]]])/Sqrt[1 +
Cot[e]^2])/Sqrt[c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]])
)/(f*(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^2) + ((1 + Sin[e + f*x])*Sqr
t[c + d*Sin[e + f*x]]*((-2*Cos[e]*Cos[f*x])/(3*f) + (2*Sin[e]*Sin[f*x])/(3*
f) + (2*(c + 3*d)*Tan[e])/(3*d*f)))/(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2
])^2 + (2*AppellF1[1/2, 1/2, 1/2, 3/2, -((Sec[e]*(c + d*Cos[e]*Sin[f*x + Ar
cTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]))/(d*Sqrt[1 + Tan[e]^2]*(1 - (c*Sec[e])/(d
*Sqrt[1 + Tan[e]^2])))), -((Sec[e]*(c + d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*
Sqrt[1 + Tan[e]^2]))/(d*Sqrt[1 + Tan[e]^2]*(-1 - (c*Sec[e])/(d*Sqrt[1 + Tan
[e]^2]))))]*Sec[e]*Sec[f*x + ArcTan[Tan[e]]]*(1 + Sin[e + f*x])*Sqrt[(d*Sqr
t[1 + Tan[e]^2] - d*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2])/(c*Sec[e]
 + d*Sqrt[1 + Tan[e]^2])]*Sqrt[(d*Sqrt[1 + Tan[e]^2] + d*Sin[f*x + ArcTan[T
an[e]]]*Sqrt[1 + Tan[e]^2])/(-(c*Sec[e]) + d*Sqrt[1 + Tan[e]^2])]*Sqrt[c +
d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]])/(3*f*(Cos[e/2 + (f*
x)/2] + Sin[e/2 + (f*x)/2])^2*Sqrt[1 + Tan[e]^2]) + (2*c*AppellF1[1/2, 1/2,
 1/2, 3/2, -((Sec[e]*(c + d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e
]^2]))/(d*Sqrt[1 + Tan[e]^2]*(1 - (c*Sec[e])/(d*Sqrt[1 + Tan[e]^2])))), -((
Sec[e]*(c + d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]))/(d*Sqrt
[1 + Tan[e]^2]*(-1 - (c*Sec[e])/(d*Sqrt[1 + Tan[e]^2]))))]*Sec[e]*Sec[f*x +
 ArcTan[Tan[e]]]*(1 + Sin[e + f*x])*Sqrt[(d*Sqrt[1 + Tan[e]^2] - d*Sin[f*x
+ ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2])/(c*Sec[e] + d*Sqrt[1 + Tan[e]^2])]*Sq
rt[(d*Sqrt[1 + Tan[e]^2] + d*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2])/
(-(c*Sec[e]) + d*Sqrt[1 + Tan[e]^2])]*Sqrt[c + d*Cos[e]*Sin[f*x + ArcTan[Ta
n[e]]]*Sqrt[1 + Tan[e]^2]])/(d*f*(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^
2*Sqrt[1 + Tan[e]^2]))
```

Maple [B] time = 0.967, size = 657, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(c+d*sin(f*x+e))^(1/2),x)
```

```
[Out] 2/3*a*(4*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-
d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-
d)/(c+d))^(1/2))*c^2*d-4*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d
/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(
c-d))^(1/2),((c-d)/(c+d))^(1/2))*d^3-((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+s
in(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*s
in(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c^3-3*((c+d*sin(f*x+e))/(c-d))
^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*Ell
ipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c^2*d+((c+d*sin(
f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c
-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c*
d^2+3*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(
1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/
```

```
(c+d))^(1/2))*d^3+d^3*sin(f*x+e)^3+c*d^2*sin(f*x+e)^2-d^3*sin(f*x+e)-c*d^2)
/d^2/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)\sqrt{d\sin\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a\sin\left(fx+e\right)+a\right)\sqrt{d\sin\left(fx+e\right)+c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int\sqrt{c+d\sin\left(e+fx\right)}\sin\left(e+fx\right)dx+\int\sqrt{c+d\sin\left(e+fx\right)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] a*(Integral(sqrt(c + d*sin(e + f*x))*sin(e + f*x), x) + Integral(sqrt(c + d
*sin(e + f*x)), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)\sqrt{d\sin\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c), x)
```

3.485 $\int \frac{a+a\sin(e+fx)}{\sqrt{c+d\sin(e+fx)}}\,dx$

Optimal. Leaf size=138

$$\frac{2a\sqrt{c+d\sin(e+fx)}E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{df\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}-\frac{2a(c-d)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{df\sqrt{c+d\sin(e+fx)}}$$

```
[Out] (2*a*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])
/(d*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) - (2*a*(c - d)*EllipticF[(e - Pi/
2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(d*f*Sqrt[c
+ d*Sin[e + f*x]])
```

Rubi [A] time = 0.122268, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2752, 2663, 2661, 2655, 2653}

$$\frac{2a\sqrt{c+d\sin(e+fx)}E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{df\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}-\frac{2a(c-d)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{df\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])/Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] (2*a*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])
/(d*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) - (2*a*(c - d)*EllipticF[(e - Pi/
2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(d*f*Sqrt[c
+ d*Sin[e + f*x]])
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
```

```
0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{a+a\sin(e+fx)}{\sqrt{c+d\sin(e+fx)}}\,dx = \frac{a\int \sqrt{c+d\sin(e+fx)}\,dx}{d} + \frac{(-ac+ad)\int \frac{1}{\sqrt{c+d\sin(e+fx)}}\,dx}{d}$$

$$= \frac{\left(a\sqrt{c+d\sin(e+fx)}\right)\int \sqrt{\frac{c}{c+d}+\frac{d\sin(e+fx)}{c+d}}\,dx}{d\sqrt{\frac{c+d\sin(e+fx)}{c+d}}} + \frac{\left((-ac+ad)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\right)\int \frac{1}{\sqrt{\frac{c}{c+d}+\frac{d\sin(e+fx)}{c+d}}}\,d}{d\sqrt{c+d\sin(e+fx)}}$$

$$= \frac{2aE\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2d}{c+d}\right)\sqrt{c+d\sin(e+fx)}}{df\sqrt{\frac{c+d\sin(e+fx)}{c+d}}} - \frac{2a(c-d)F\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2d}{c+d}\right)\sqrt{\frac{c+d\sin(e+fx}{c+d}}}{df\sqrt{c+d\sin(e+fx)}}$$

Mathematica [C] time = 6.24661, size = 880, normalized size = 6.38

$$a\left(\frac{\sec(e)\left(-\frac{F_1\left(-\frac{1}{2};-\frac{1}{2},-\frac{1}{2};\frac{1}{2};-\frac{\csc(e)\left(c+d\cos\left(fx-\tan^{-1}(\cot(e))\right)\sqrt{\cot^2(e)+1}\sin(e)\right)}{d\sqrt{\cot^2(e)+1}\left(1-\frac{c\csc(e)}{d\sqrt{\cot^2(e)+1}}\right)},-\frac{\csc(e)\left(c+d\cos\left(fx-\tan^{-1}(\cot(e))\right)\sqrt{\cot^2(e)+1}\sin(e)\right)}{d\sqrt{\cot^2(e)+1}\left(-\frac{c\csc(e)}{d\sqrt{\cot^2(e)+1}}-1\right)}\right)\cot(e)\sin\left(fx-\tan^{-1}(\ldots\right.}{\sqrt{\cot^2(e)+1}\sqrt{\frac{\cos\left(fx-\tan^{-1}(\cot(e))\right)\sqrt{\cot^2(e)+1}d+\sqrt{\cot^2(e)+1}d}{d\sqrt{\cot^2(e)+1}-c\csc(e)}}\sqrt{\frac{d\sqrt{\cot^2(e)+1}-d\cos\left(fx-\tan^{-1}(\cot(e))\right)\sqrt{\cot^2(e)+1}}{\sqrt{\cot^2(e)+1}d+c\csc(e)}}\sqrt{c+d\cos\left(fx-\tan^{-1}(\cot(e))\right)\sqrt{\cot\ldots}}}\right.}{f\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)+\sin\left(\frac{e}{2}+\frac{fx}{2}\right.\ldots}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])/Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] a*((Sec[e]*(1 + Sin[e + f*x])*(-((AppellF1[-1/2, -1/2, -1/2, 1/2, -((Csc[e]
*(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d*Sqrt[1 + C
ot[e]^2]*(1 - (c*Csc[e])/(d*Sqrt[1 + Cot[e]^2])))), -((Csc[e]*(c + d*Cos[f*
x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d*Sqrt[1 + Cot[e]^2]*(-1 -
 (c*Csc[e])/(d*Sqrt[1 + Cot[e]^2]))))]*Cot[e]*Sin[f*x - ArcTan[Cot[e]]])/(S
qrt[1 + Cot[e]^2]*Sqrt[(d*Sqrt[1 + Cot[e]^2] + d*Cos[f*x - ArcTan[Cot[e]]]*
Sqrt[1 + Cot[e]^2])/(d*Sqrt[1 + Cot[e]^2] - c*Csc[e])]*Sqrt[(d*Sqrt[1 + Cot
[e]^2] - d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2])/(d*Sqrt[1 + Cot[e]
^2] + c*Csc[e])]*Sqrt[c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Si
n[e]])) - ((2*d*Sin[e]*(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*
Sin[e]))/(d^2*Cos[e]^2 + d^2*Sin[e]^2) - (Cot[e]*Sin[f*x - ArcTan[Cot[e]]])
/Sqrt[1 + Cot[e]^2])/Sqrt[c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2
```

```
]*Sin[e]]))/(f*(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^2) + (2*(1 + Sin[e
 + f*x])*Sqrt[c + d*Sin[e + f*x]]*Tan[e])/(d*f*(Cos[e/2 + (f*x)/2] + Sin[e/
2 + (f*x)/2])^2) + (2*AppellF1[1/2, 1/2, 1/2, 3/2, -((Sec[e]*(c + d*Cos[e]*
Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]))/(d*Sqrt[1 + Tan[e]^2]*(1 - (
c*Sec[e])/(d*Sqrt[1 + Tan[e]^2])))), -((Sec[e]*(c + d*Cos[e]*Sin[f*x + ArcT
an[Tan[e]]]*Sqrt[1 + Tan[e]^2]))/(d*Sqrt[1 + Tan[e]^2]*(-1 - (c*Sec[e])/(d*
Sqrt[1 + Tan[e]^2]))))]*Sec[e]*Sec[f*x + ArcTan[Tan[e]]]*(1 + Sin[e + f*x])
*Sqrt[(d*Sqrt[1 + Tan[e]^2] - d*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2
])/(c*Sec[e] + d*Sqrt[1 + Tan[e]^2])]*Sqrt[(d*Sqrt[1 + Tan[e]^2] + d*Sin[f*
x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2])/(-(c*Sec[e]) + d*Sqrt[1 + Tan[e]^2]
)]*Sqrt[c + d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]])/(d*f*(C
os[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^2*Sqrt[1 + Tan[e]^2]))
```

Maple [A] time = 0.8, size = 203, normalized size = 1.5

$$-2\frac{a(c-d)}{d^2\cos\left(fx+e\right)\sqrt{c+d\sin\left(fx+e\right)}f}\sqrt{\frac{c+d\sin\left(fx+e\right)}{c-d}}\sqrt{-\frac{\left(-1+\sin\left(fx+e\right)\right)d}{c+d}}\sqrt{-\frac{d\left(1+\sin\left(fx+e\right)\right)}{c-d}}\left(\text{Ellipt}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(1/2),x)
```

```
[Out] -2*a*(c-d)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*
(-d*(1+sin(f*x+e))/(c-d))^(1/2)*(EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),(
(c-d)/(c+d))^(1/2))*c+EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d)
)^(1/2))*d-2*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*
d)/d^2/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a\sin\left(fx+e\right)+a}{\sqrt{d\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)/sqrt(d*sin(f*x + e) + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{a\sin\left(fx+e\right)+a}{\sqrt{d\sin\left(fx+e\right)+c}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)/sqrt(d*sin(f*x + e) + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int \frac{\sin\left(e+fx\right)}{\sqrt{c+d\sin\left(e+fx\right)}}\,dx+\int \frac{1}{\sqrt{c+d\sin\left(e+fx\right)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] a*(Integral(sin(e + f*x)/sqrt(c + d*sin(e + f*x)), x) + Integral(1/sqrt(c +
 d*sin(e + f*x)), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a\sin\left(fx+e\right)+a}{\sqrt{d\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)/sqrt(d*sin(f*x + e) + c), x)
```

3.486 $\int \frac{a+a\sin(e+fx)}{(c+d\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=169

$$-\frac{2a\cos(e+fx)}{f(c+d)\sqrt{c+d\sin(e+fx)}}+\frac{2a\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{df\sqrt{c+d\sin(e+fx)}}-\frac{2a\sqrt{c+d\sin(e+fx)}E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{df(c+d)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

```
[Out] (-2*a*Cos[e + f*x])/((c + d)*f*Sqrt[c + d*Sin[e + f*x]]) - (2*a*EllipticE[(
e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(d*(c + d)*f*Sq
rt[(c + d*Sin[e + f*x])/(c + d)]) + (2*a*EllipticF[(e - Pi/2 + f*x)/2, (2*d
)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(d*f*Sqrt[c + d*Sin[e + f*x]
])
```

Rubi [A] time = 0.205837, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {2754, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2a\cos(e+fx)}{f(c+d)\sqrt{c+d\sin(e+fx)}}+\frac{2a\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{df\sqrt{c+d\sin(e+fx)}}-\frac{2a\sqrt{c+d\sin(e+fx)}E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{df(c+d)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])/(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-2*a*Cos[e + f*x])/((c + d)*f*Sqrt[c + d*Sin[e + f*x]]) - (2*a*EllipticE[(
e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(d*(c + d)*f*Sq
rt[(c + d*Sin[e + f*x])/(c + d)]) + (2*a*EllipticF[(e - Pi/2 + f*x)/2, (2*d
)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(d*f*Sqrt[c + d*Sin[e + f*x]
])
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{a+a\sin(e+fx)}{(c+d\sin(e+fx))^{3/2}}\,dx = -\frac{2a\cos(e+fx)}{(c+d)f\sqrt{c+d\sin(e+fx)}} - \frac{2\int \frac{-\frac{1}{2}a(c-d)+\frac{1}{2}a(c-d)\sin(e+fx)}{\sqrt{c+d\sin(e+fx)}}\,dx}{c^2-d^2}$$

$$= -\frac{2a\cos(e+fx)}{(c+d)f\sqrt{c+d\sin(e+fx)}} + \frac{a\int \frac{1}{\sqrt{c+d\sin(e+fx)}}\,dx}{d} - \frac{a\int \sqrt{c+d\sin(e+fx)}\,dx}{d(c+d)}$$

$$= -\frac{2a\cos(e+fx)}{(c+d)f\sqrt{c+d\sin(e+fx)}} - \frac{\left(a\sqrt{c+d\sin(e+fx)}\right)\int \sqrt{\frac{c}{c+d}+\frac{d\sin(e+fx)}{c+d}}\,dx}{d(c+d)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}} + \frac{\left(a\sqrt{\frac{c+d\sin(\ldots}{c+\ldots}}\right.}{d\sqrt{\ldots}}$$

$$= -\frac{2a\cos(e+fx)}{(c+d)f\sqrt{c+d\sin(e+fx)}} - \frac{2aE\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2d}{c+d}\right)\sqrt{c+d\sin(e+fx)}}{d(c+d)f\sqrt{\frac{c+d\sin(e+fx)}{c+d}}} + \frac{2aF\left(\frac{1}{2}\left(e-\ldots\right.\right.}{df\ldots}$$

Mathematica [C] time = 6.39223, size = 938, normalized size = 5.55

$$a\left(\frac{\sqrt{c+d\sin(e+fx)}\left(\frac{2\csc(e)(c\cos(e)+d\sin(fx))}{d(c+d)f(c+d\sin(e+fx))}-\frac{2\csc(e)\sec(e)}{d(c+d)f}\right)(\sin(e+fx)+1)}{\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)+\sin\left(\frac{e}{2}+\frac{fx}{2}\right)\right)^2} - \frac{\sec(e)\left(-\frac{F_1\left(-\frac{1}{2};-\frac{1}{2},-\frac{1}{2};\frac{1}{2};-\frac{\csc(e)\left(c+d\cos\left(fx-\tan\ldots\right.\right.}{d\sqrt{\cot^2(e)\ldots}}\right)}{\sqrt{\cot^2(e)+1}\sqrt{\frac{\cos\left(fx-\tan^{-1}(\cot(e))\right)\sqrt{\cot^2(e\ldots}}{d\sqrt{\cot^2(e)+1}-c\,c\ldots}}}\right.}{\ldots}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])/(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] a*(((1 + Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]]*((-2*Csc[e]*Sec[e])/(d*(c +
 d)*f) + (2*Csc[e]*(c*Cos[e] + d*Sin[f*x]))/(d*(c + d)*f*(c + d*Sin[e + f*x
]))))/(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^2 - (Sec[e]*(1 + Sin[e + f*
x])*(-((AppellF1[-1/2, -1/2, -1/2, 1/2, -((Csc[e]*(c + d*Cos[f*x - ArcTan[C
ot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d*Sqrt[1 + Cot[e]^2]*(1 - (c*Csc[e])/(
d*Sqrt[1 + Cot[e]^2])))), -((Csc[e]*(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1
 + Cot[e]^2]*Sin[e]))/(d*Sqrt[1 + Cot[e]^2]*(-1 - (c*Csc[e])/(d*Sqrt[1 + Co
t[e]^2]))))]*Cot[e]*Sin[f*x - ArcTan[Cot[e]]])/(Sqrt[1 + Cot[e]^2]*Sqrt[(d*
Sqrt[1 + Cot[e]^2] + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2])/(d*Sqr
t[1 + Cot[e]^2] - c*Csc[e])]*Sqrt[(d*Sqrt[1 + Cot[e]^2] - d*Cos[f*x - ArcTa
n[Cot[e]]]*Sqrt[1 + Cot[e]^2])/(d*Sqrt[1 + Cot[e]^2] + c*Csc[e])]*Sqrt[c +
d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]])) - ((2*d*Sin[e]*(c
+ d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d^2*Cos[e]^2 + d
^2*Sin[e]^2) - (Cot[e]*Sin[f*x - ArcTan[Cot[e]]])/Sqrt[1 + Cot[e]^2])/Sqrt[
c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]]))/((c + d)*f*(Co
s[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^2) + (2*AppellF1[1/2, 1/2, 1/2, 3/2,
 -((Sec[e]*(c + d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]))/(d*
Sqrt[1 + Tan[e]^2]*(1 - (c*Sec[e])/(d*Sqrt[1 + Tan[e]^2])))), -((Sec[e]*(c
+ d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]))/(d*Sqrt[1 + Tan[e
]^2]*(-1 - (c*Sec[e])/(d*Sqrt[1 + Tan[e]^2]))))]*Sec[e]*Sec[f*x + ArcTan[Ta
n[e]]]*(1 + Sin[e + f*x])*Sqrt[(d*Sqrt[1 + Tan[e]^2] - d*Sin[f*x + ArcTan[T
an[e]]]*Sqrt[1 + Tan[e]^2])/(c*Sec[e] + d*Sqrt[1 + Tan[e]^2])]*Sqrt[(d*Sqrt
[1 + Tan[e]^2] + d*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2])/(-(c*Sec[e
]) + d*Sqrt[1 + Tan[e]^2])]*Sqrt[c + d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*Sqr
t[1 + Tan[e]^2]])/(d*(c + d)*f*(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^2*
Sqrt[1 + Tan[e]^2]))
```

Maple [A] time = 0.955, size = 246, normalized size = 1.5

$$2\,\frac{a}{d^2(c+d)\cos(fx+e)\sqrt{c+d\sin(fx+e)}f}\left(\sqrt{\frac{c+d\sin(fx+e)}{c-d}}\sqrt{-\frac{(-1+\sin(fx+e))d}{c+d}}\sqrt{-\frac{d(1+\sin(fx+e))}{c-d}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(3/2),x)
```

```
[Out] 2*(((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+s
in(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+
d))^(1/2))*c^2-((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1
/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2
),((c-d)/(c+d))^(1/2))*d^2+d^2*sin(f*x+e)^2-d^2)/d^2*a/(c+d)/cos(f*x+e)/(c+
d*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a\sin(fx+e)+a}{(d\sin(fx+e)+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)/(d*sin(f*x + e) + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(a\sin\left(fx+e\right)+a\right)\sqrt{d\sin\left(fx+e\right)+c}}{d^2\cos\left(fx+e\right)^2-2\,cd\sin\left(fx+e\right)-c^2-d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)/(d^2*cos(f*x + e)^2
 - 2*c*d*sin(f*x + e) - c^2 - d^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{a\sin\left(fx+e\right)+a}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)/(d*sin(f*x + e) + c)^(3/2), x)
```

3.487 $\int \frac{a+a\sin(e+fx)}{(c+d\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=237

$$-\frac{2a(c-3d)\cos(e+fx)}{3f(c-d)(c+d)^2\sqrt{c+d\sin(e+fx)}}-\frac{2a\cos(e+fx)}{3f(c+d)(c+d\sin(e+fx))^{3/2}}+\frac{2a\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3df(c+d)\sqrt{c+d\sin(e+fx)}}-\frac{2a}{}$$

```
[Out] (-2*a*Cos[e + f*x])/(3*(c + d)*f*(c + d*Sin[e + f*x])^(3/2)) - (2*a*(c - 3*
d)*Cos[e + f*x])/(3*(c - d)*(c + d)^2*f*Sqrt[c + d*Sin[e + f*x]]) - (2*a*(c
 - 3*d)*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x
]])/(3*(c - d)*d*(c + d)^2*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + (2*a*Ell
ipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)
])/(3*d*(c + d)*f*Sqrt[c + d*Sin[e + f*x]])
```

Rubi [A] time = 0.336074, antiderivative size = 237, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {2754, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2a(c-3d)\cos(e+fx)}{3f(c-d)(c+d)^2\sqrt{c+d\sin(e+fx)}}-\frac{2a\cos(e+fx)}{3f(c+d)(c+d\sin(e+fx))^{3/2}}+\frac{2a\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3df(c+d)\sqrt{c+d\sin(e+fx)}}-\frac{2a}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])/(c + d*Sin[e + f*x])^(5/2),x]
```


```
[Out] (-2*a*Cos[e + f*x])/(3*(c + d)*f*(c + d*Sin[e + f*x])^(3/2)) - (2*a*(c - 3*
d)*Cos[e + f*x])/(3*(c - d)*(c + d)^2*f*Sqrt[c + d*Sin[e + f*x]]) - (2*a*(c
 - 3*d)*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x
]])/(3*(c - d)*d*(c + d)^2*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + (2*a*Ell
ipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)
])/(3*d*(c + d)*f*Sqrt[c + d*Sin[e + f*x]])
```


Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
```

```
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{a+a\sin(e+fx)}{(c+d\sin(e+fx))^{5/2}}\,dx = -\frac{2a\cos(e+fx)}{3(c+d)f(c+d\sin(e+fx))^{3/2}} - \frac{2\int \frac{-\frac{3}{2}a(c-d)-\frac{1}{2}a(c-d)\sin(e+fx)}{(c+d\sin(e+fx))^{3/2}}\,dx}{3\left(c^2-d^2\right)}$$

$$= -\frac{2a\cos(e+fx)}{3(c+d)f(c+d\sin(e+fx))^{3/2}} - \frac{2a(c-3d)\cos(e+fx)}{3(c-d)(c+d)^2f\sqrt{c+d\sin(e+fx)}} + \frac{4\int \frac{\frac{1}{4}a(c-d)(3c-d)-}{\sqrt{c+}}}{3(c}$$

$$= -\frac{2a\cos(e+fx)}{3(c+d)f(c+d\sin(e+fx))^{3/2}} - \frac{2a(c-3d)\cos(e+fx)}{3(c-d)(c+d)^2f\sqrt{c+d\sin(e+fx)}} - \frac{(a(c-3d))\int\sqrt{c}}{3(c-d)}$$

$$= -\frac{2a\cos(e+fx)}{3(c+d)f(c+d\sin(e+fx))^{3/2}} - \frac{2a(c-3d)\cos(e+fx)}{3(c-d)(c+d)^2f\sqrt{c+d\sin(e+fx)}} - \frac{\left(a(c-3d)\sqrt{c+d}\right.}{3(c-}$$

$$= -\frac{2a\cos(e+fx)}{3(c+d)f(c+d\sin(e+fx))^{3/2}} - \frac{2a(c-3d)\cos(e+fx)}{3(c-d)(c+d)^2f\sqrt{c+d\sin(e+fx)}} - \frac{2a(c-3d)E\left(\frac{1}{2}\left(e\right.\right.}{3(c-c}$$

Mathematica [C] time = 6.69321, size = 1870, normalized size = 7.89

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])/(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] a*(((1 + Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]]*((-2*(c - 3*d)*Csc[e]*Sec[e
])/(3*(c - d)*d*(c + d)^2*f) + (2*Csc[e]*(c*Cos[e] + d*Sin[f*x]))/(3*d*(c +
 d)*f*(c + d*Sin[e + f*x])^2) - (2*Csc[e]*(3*c*Cos[e] - d*Cos[e] - c*Sin[f*
x] + 3*d*Sin[f*x]))/(3*(c - d)*(c + d)^2*f*(c + d*Sin[e + f*x]))))/(Cos[e/2
 + (f*x)/2] + Sin[e/2 + (f*x)/2])^2 - (c*Sec[e]*(1 + Sin[e + f*x])*(-((Appe
llF1[-1/2, -1/2, -1/2, 1/2, -((Csc[e]*(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt
```

```
[1 + Cot[e]^2]*Sin[e]))/(d*Sqrt[1 + Cot[e]^2]*(1 - (c*Csc[e])/(d*Sqrt[1 + C
ot[e]^2])))), -((Csc[e]*(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]
*Sin[e]))/(d*Sqrt[1 + Cot[e]^2]*(-1 - (c*Csc[e])/(d*Sqrt[1 + Cot[e]^2]))))]
*Cot[e]*Sin[f*x - ArcTan[Cot[e]]])/(Sqrt[1 + Cot[e]^2]*Sqrt[(d*Sqrt[1 + Cot
[e]^2] + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2])/(d*Sqrt[1 + Cot[e]
^2] - c*Csc[e])]*Sqrt[(d*Sqrt[1 + Cot[e]^2] - d*Cos[f*x - ArcTan[Cot[e]]]*S
qrt[1 + Cot[e]^2])/(d*Sqrt[1 + Cot[e]^2] + c*Csc[e])]*Sqrt[c + d*Cos[f*x -
ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]])) - ((2*d*Sin[e]*(c + d*Cos[f*x
- ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d^2*Cos[e]^2 + d^2*Sin[e]^2)
 - (Cot[e]*Sin[f*x - ArcTan[Cot[e]]])/Sqrt[1 + Cot[e]^2])/Sqrt[c + d*Cos[f*
x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]]))/(3*(c - d)*(c + d)^2*f*(Co
s[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^2) + (d*Sec[e]*(1 + Sin[e + f*x])*(-
((AppellF1[-1/2, -1/2, -1/2, 1/2, -((Csc[e]*(c + d*Cos[f*x - ArcTan[Cot[e]]
]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d*Sqrt[1 + Cot[e]^2]*(1 - (c*Csc[e])/(d*Sqrt
[1 + Cot[e]^2])))), -((Csc[e]*(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot
[e]^2]*Sin[e]))/(d*Sqrt[1 + Cot[e]^2]*(-1 - (c*Csc[e])/(d*Sqrt[1 + Cot[e]^2
]))))]*Cot[e]*Sin[f*x - ArcTan[Cot[e]]])/(Sqrt[1 + Cot[e]^2]*Sqrt[(d*Sqrt[1
 + Cot[e]^2] + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2])/(d*Sqrt[1 +
Cot[e]^2] - c*Csc[e])]*Sqrt[(d*Sqrt[1 + Cot[e]^2] - d*Cos[f*x - ArcTan[Cot[
e]]]*Sqrt[1 + Cot[e]^2])/(d*Sqrt[1 + Cot[e]^2] + c*Csc[e])]*Sqrt[c + d*Cos[
f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]])) - ((2*d*Sin[e]*(c + d*Co
s[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d^2*Cos[e]^2 + d^2*Sin
[e]^2) - (Cot[e]*Sin[f*x - ArcTan[Cot[e]]])/Sqrt[1 + Cot[e]^2])/Sqrt[c + d*
Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]]))/((c - d)*(c + d)^2*f
*(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^2) - (2*AppellF1[1/2, 1/2, 1/2,
3/2, -((Sec[e]*(c + d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]))
/(d*Sqrt[1 + Tan[e]^2]*(1 - (c*Sec[e])/(d*Sqrt[1 + Tan[e]^2])))), -((Sec[e]
*(c + d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]))/(d*Sqrt[1 + T
an[e]^2]*(-1 - (c*Sec[e])/(d*Sqrt[1 + Tan[e]^2]))))]*Sec[e]*Sec[f*x + ArcTa
n[Tan[e]]]*(1 + Sin[e + f*x])*Sqrt[(d*Sqrt[1 + Tan[e]^2] - d*Sin[f*x + ArcT
an[Tan[e]]]*Sqrt[1 + Tan[e]^2])/(c*Sec[e] + d*Sqrt[1 + Tan[e]^2])]*Sqrt[(d*
Sqrt[1 + Tan[e]^2] + d*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2])/(-(c*S
ec[e]) + d*Sqrt[1 + Tan[e]^2])]*Sqrt[c + d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]
*Sqrt[1 + Tan[e]^2]])/(3*(c - d)*(c + d)^2*f*(Cos[e/2 + (f*x)/2] + Sin[e/2
+ (f*x)/2])^2*Sqrt[1 + Tan[e]^2]) + (2*c*AppellF1[1/2, 1/2, 1/2, 3/2, -((Se
c[e]*(c + d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]))/(d*Sqrt[1
 + Tan[e]^2]*(1 - (c*Sec[e])/(d*Sqrt[1 + Tan[e]^2])))), -((Sec[e]*(c + d*Co
s[e]*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]))/(d*Sqrt[1 + Tan[e]^2]*(
-1 - (c*Sec[e])/(d*Sqrt[1 + Tan[e]^2]))))]*Sec[e]*Sec[f*x + ArcTan[Tan[e]]]
*(1 + Sin[e + f*x])*Sqrt[(d*Sqrt[1 + Tan[e]^2] - d*Sin[f*x + ArcTan[Tan[e]]
]*Sqrt[1 + Tan[e]^2])/(c*Sec[e] + d*Sqrt[1 + Tan[e]^2])]*Sqrt[(d*Sqrt[1 + T
an[e]^2] + d*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2])/(-(c*Sec[e]) + d
*Sqrt[1 + Tan[e]^2])]*Sqrt[c + d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 +
Tan[e]^2]])/((c - d)*d*(c + d)^2*f*(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2]
)^2*Sqrt[1 + Tan[e]^2]))
```

Maple [B] time = 3.656, size = 884, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(5/2),x)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*a*(1/d*(2*d*cos(f*x+e)^2/(c^2-d^2)/
(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2*c/(c^2-d^2)*(c/d-1)*((c+d*sin(f*x
+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^
```

```
(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(
c-d))^(1/2),((c-d)/(c+d))^(1/2))+2/(c^2-d^2)*d*(c/d-1)*((c+d*sin(f*x+e))/(c
-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(
-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e)
)/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2
),((c-d)/(c+d))^(1/2))))+(-c+d)/d*(2/3/(c^2-d^2)/d*(-(-d*sin(f*x+e)-c)*cos(
f*x+e)^2)^(1/2)/(sin(f*x+e)+c/d)^2+8/3*d*cos(f*x+e)^2/(c^2-d^2)^2*c/(-(-d*s
in(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2*(3*c^2+d^2)/(3*c^4-6*c^2*d^2+3*d^4)*(c/d
-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*
x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((
c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+8/3*c*d/(c^2-d^2)^2*(c/d-
1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x
+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*Ell
ipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*
sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))))/cos(f*x+e)/(c+d*sin(f*x+e)
)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a \sin\left(fx+e\right)+a}{\left(d \sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)/(d*sin(f*x + e) + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-\frac{\left(a \sin\left(fx+e\right)+a\right)\sqrt{d \sin\left(fx+e\right)+c}}{3\,cd^2 \cos\left(fx+e\right)^2-c^3-3\,cd^2+\left(d^3 \cos\left(fx+e\right)^2-3\,c^2d-d^3\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)/(3*c*d^2*cos(f*x +
e)^2 - c^3 - 3*c*d^2 + (d^3*cos(f*x + e)^2 - 3*c^2*d - d^3)*sin(f*x + e)),
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a \sin \left(fx+e\right)+a}{\left(d \sin \left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)/(d*sin(f*x + e) + c)^(5/2), x)
```

3.488 $\int \frac{a+a\sin(e+fx)}{(c+d\sin(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=318

$$-\frac{2a\left(3c^2-20cd+9d^2\right)\cos(e+fx)}{15f(c-d)^2(c+d)^3\sqrt{c+d\sin(e+fx)}}-\frac{2a\left(3c^2-20cd+9d^2\right)\sqrt{c+d\sin(e+fx)}E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{15df(c-d)^2(c+d)^3\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}-\frac{2a(}{15f(c-d)(}$$

```
[Out] (-2*a*Cos[e + f*x])/(5*(c + d)*f*(c + d*Sin[e + f*x])^(5/2)) - (2*a*(3*c -
5*d)*Cos[e + f*x])/(15*(c - d)*(c + d)^2*f*(c + d*Sin[e + f*x])^(3/2)) - (2
*a*(3*c^2 - 20*c*d + 9*d^2)*Cos[e + f*x])/(15*(c - d)^2*(c + d)^3*f*Sqrt[c
+ d*Sin[e + f*x]]) - (2*a*(3*c^2 - 20*c*d + 9*d^2)*EllipticE[(e - Pi/2 + f*
x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(15*(c - d)^2*d*(c + d)^3*f*
Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + (2*a*(3*c - 5*d)*EllipticF[(e - Pi/2
+ f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(15*(c - d)*d*
(c + d)^2*f*Sqrt[c + d*Sin[e + f*x]])
```

Rubi [A] time = 0.506882, antiderivative size = 318, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {2754, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2a\left(3c^2-20cd+9d^2\right)\cos(e+fx)}{15f(c-d)^2(c+d)^3\sqrt{c+d\sin(e+fx)}}-\frac{2a\left(3c^2-20cd+9d^2\right)\sqrt{c+d\sin(e+fx)}E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{15df(c-d)^2(c+d)^3\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}-\frac{2a(}{15f(c-d)(}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])/(c + d*Sin[e + f*x])^(7/2),x]
```

```
[Out] (-2*a*Cos[e + f*x])/(5*(c + d)*f*(c + d*Sin[e + f*x])^(5/2)) - (2*a*(3*c -
5*d)*Cos[e + f*x])/(15*(c - d)*(c + d)^2*f*(c + d*Sin[e + f*x])^(3/2)) - (2
*a*(3*c^2 - 20*c*d + 9*d^2)*Cos[e + f*x])/(15*(c - d)^2*(c + d)^3*f*Sqrt[c
+ d*Sin[e + f*x]]) - (2*a*(3*c^2 - 20*c*d + 9*d^2)*EllipticE[(e - Pi/2 + f*
x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(15*(c - d)^2*d*(c + d)^3*f*
Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + (2*a*(3*c - 5*d)*EllipticF[(e - Pi/2
+ f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(15*(c - d)*d*
(c + d)^2*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{a+a\sin(e+fx)}{(c+d\sin(e+fx))^{7/2}}\,dx = -\frac{2a\cos(e+fx)}{5(c+d)f(c+d\sin(e+fx))^{5/2}} - \frac{2\int \frac{-\frac{5}{2}a(c-d)-\frac{3}{2}a(c-d)\sin(e+fx)}{(c+d\sin(e+fx))^{5/2}}\,dx}{5\left(c^2-d^2\right)}$$

$$= -\frac{2a\cos(e+fx)}{5(c+d)f(c+d\sin(e+fx))^{5/2}} - \frac{2a(3c-5d)\cos(e+fx)}{15(c-d)(c+d)^2f(c+d\sin(e+fx))^{3/2}} + 4\int \frac{\frac{3}{4}a(5c-3 \ldots}{\ldots}$$

$$= -\frac{2a\cos(e+fx)}{5(c+d)f(c+d\sin(e+fx))^{5/2}} - \frac{2a(3c-5d)\cos(e+fx)}{15(c-d)(c+d)^2f(c+d\sin(e+fx))^{3/2}} - \frac{2a\left(3c^2 \ldots\right.}{15(c-d)^2 \ldots}$$

$$= -\frac{2a\cos(e+fx)}{5(c+d)f(c+d\sin(e+fx))^{5/2}} - \frac{2a(3c-5d)\cos(e+fx)}{15(c-d)(c+d)^2f(c+d\sin(e+fx))^{3/2}} - \frac{2a\left(3c^2 \ldots\right.}{15(c-d)^2 \ldots}$$

$$= -\frac{2a\cos(e+fx)}{5(c+d)f(c+d\sin(e+fx))^{5/2}} - \frac{2a(3c-5d)\cos(e+fx)}{15(c-d)(c+d)^2f(c+d\sin(e+fx))^{3/2}} - \frac{2a\left(3c^2 \ldots\right.}{15(c-d)^2 \ldots}$$

$$= -\frac{2a\cos(e+fx)}{5(c+d)f(c+d\sin(e+fx))^{5/2}} - \frac{2a(3c-5d)\cos(e+fx)}{15(c-d)(c+d)^2f(c+d\sin(e+fx))^{3/2}} - \frac{2a\left(3c^2 \ldots\right.}{15(c-d)^2 \ldots}$$

Mathematica [C] time = 7.06736, size = 2815, normalized size = 8.85

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])/(c + d*Sin[e + f*x])^(7/2),x]
```

```
[Out] a*(((1 + Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]]*((-2*(3*c^2 - 20*c*d + 9*d^
2)*Csc[e]*Sec[e])/(15*(c - d)^2*d*(c + d)^3*f) + (2*Csc[e]*(c*Cos[e] + d*Si
n[f*x]))/(5*d*(c + d)*f*(c + d*Sin[e + f*x])^3) - (2*Csc[e]*(5*c*Cos[e] - 3
*d*Cos[e] - 3*c*Sin[f*x] + 5*d*Sin[f*x]))/(15*(c - d)*(c + d)^2*f*(c + d*Si
n[e + f*x])^2) - (2*Csc[e]*(15*c^2*Cos[e] - 12*c*d*Cos[e] + 5*d^2*Cos[e] -
3*c^2*Sin[f*x] + 20*c*d*Sin[f*x] - 9*d^2*Sin[f*x]))/(15*(c - d)^2*(c + d)^3
*f*(c + d*Sin[e + f*x]))))/(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^2 - (c
^2*Sec[e]*(1 + Sin[e + f*x])*(-((AppellF1[-1/2, -1/2, -1/2, 1/2, -((Csc[e]*
(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d*Sqrt[1 + Co
t[e]^2]*(1 - (c*Csc[e])/(d*Sqrt[1 + Cot[e]^2])))), -((Csc[e]*(c + d*Cos[f*x
 - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d*Sqrt[1 + Cot[e]^2]*(-1 -
(c*Csc[e])/(d*Sqrt[1 + Cot[e]^2]))))]*Cot[e]*Sin[f*x - ArcTan[Cot[e]]])/(Sq
rt[1 + Cot[e]^2]*Sqrt[(d*Sqrt[1 + Cot[e]^2] + d*Cos[f*x - ArcTan[Cot[e]]]*S
qrt[1 + Cot[e]^2])/(d*Sqrt[1 + Cot[e]^2] - c*Csc[e])]*Sqrt[(d*Sqrt[1 + Cot[
e]^2] - d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2])/(d*Sqrt[1 + Cot[e]^
2] + c*Csc[e])]*Sqrt[c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin
[e]])) - ((2*d*Sin[e]*(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*S
in[e]))/(d^2*Cos[e]^2 + d^2*Sin[e]^2) - (Cot[e]*Sin[f*x - ArcTan[Cot[e]]])/
Sqrt[1 + Cot[e]^2])/Sqrt[c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]
*Sin[e]]))/(5*(c - d)^2*(c + d)^3*f*(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2
])^2) + (4*c*d*Sec[e]*(1 + Sin[e + f*x])*(-((AppellF1[-1/2, -1/2, -1/2, 1/2
, -((Csc[e]*(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d
*Sqrt[1 + Cot[e]^2]*(1 - (c*Csc[e])/(d*Sqrt[1 + Cot[e]^2])))), -((Csc[e]*(c
 + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d*Sqrt[1 + Cot[
e]^2]*(-1 - (c*Csc[e])/(d*Sqrt[1 + Cot[e]^2]))))]*Cot[e]*Sin[f*x - ArcTan[C
ot[e]]])/(Sqrt[1 + Cot[e]^2]*Sqrt[(d*Sqrt[1 + Cot[e]^2] + d*Cos[f*x - ArcTa
n[Cot[e]]]*Sqrt[1 + Cot[e]^2])/(d*Sqrt[1 + Cot[e]^2] - c*Csc[e])]*Sqrt[(d*S
qrt[1 + Cot[e]^2] - d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2])/(d*Sqrt
[1 + Cot[e]^2] + c*Csc[e])]*Sqrt[c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + C
ot[e]^2]*Sin[e]])) - ((2*d*Sin[e]*(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 +
 Cot[e]^2]*Sin[e]))/(d^2*Cos[e]^2 + d^2*Sin[e]^2) - (Cot[e]*Sin[f*x - ArcTa
n[Cot[e]]])/Sqrt[1 + Cot[e]^2])/Sqrt[c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1
 + Cot[e]^2]*Sin[e]]))/(3*(c - d)^2*(c + d)^3*f*(Cos[e/2 + (f*x)/2] + Sin[e
/2 + (f*x)/2])^2) - (3*d^2*Sec[e]*(1 + Sin[e + f*x])*(-((AppellF1[-1/2, -1/
2, -1/2, 1/2, -((Csc[e]*(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]
*Sin[e]))/(d*Sqrt[1 + Cot[e]^2]*(1 - (c*Csc[e])/(d*Sqrt[1 + Cot[e]^2])))),
-((Csc[e]*(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d*S
qrt[1 + Cot[e]^2]*(-1 - (c*Csc[e])/(d*Sqrt[1 + Cot[e]^2]))))]*Cot[e]*Sin[f*
x - ArcTan[Cot[e]]])/(Sqrt[1 + Cot[e]^2]*Sqrt[(d*Sqrt[1 + Cot[e]^2] + d*Cos
[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2])/(d*Sqrt[1 + Cot[e]^2] - c*Csc[e]
)]*Sqrt[(d*Sqrt[1 + Cot[e]^2] - d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]
^2])/(d*Sqrt[1 + Cot[e]^2] + c*Csc[e])]*Sqrt[c + d*Cos[f*x - ArcTan[Cot[e]]
]*Sqrt[1 + Cot[e]^2]*Sin[e]])) - ((2*d*Sin[e]*(c + d*Cos[f*x - ArcTan[Cot[e
]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d^2*Cos[e]^2 + d^2*Sin[e]^2) - (Cot[e]*Sin
[f*x - ArcTan[Cot[e]]])/Sqrt[1 + Cot[e]^2])/Sqrt[c + d*Cos[f*x - ArcTan[Cot
[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]]))/(5*(c - d)^2*(c + d)^3*f*(Cos[e/2 + (f*x
)/2] + Sin[e/2 + (f*x)/2])^2) - (8*c*AppellF1[1/2, 1/2, 1/2, 3/2, -((Sec[e]
*(c + d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]))/(d*Sqrt[1 + T
an[e]^2]*(1 - (c*Sec[e])/(d*Sqrt[1 + Tan[e]^2])))), -((Sec[e]*(c + d*Cos[e]
*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]))/(d*Sqrt[1 + Tan[e]^2]*(-1 -
 (c*Sec[e])/(d*Sqrt[1 + Tan[e]^2]))))]*Sec[e]*Sec[f*x + ArcTan[Tan[e]]]*(1
+ Sin[e + f*x])*Sqrt[(d*Sqrt[1 + Tan[e]^2] - d*Sin[f*x + ArcTan[Tan[e]]]*Sq
rt[1 + Tan[e]^2])/(c*Sec[e] + d*Sqrt[1 + Tan[e]^2])]*Sqrt[(d*Sqrt[1 + Tan[e
]^2] + d*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2])/(-(c*Sec[e]) + d*Sqr
t[1 + Tan[e]^2])]*Sqrt[c + d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[
e]^2]])/(5*(c - d)^2*(c + d)^3*f*(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^
2*Sqrt[1 + Tan[e]^2]) + (2*c^2*AppellF1[1/2, 1/2, 1/2, 3/2, -((Sec[e]*(c +
d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]))/(d*Sqrt[1 + Tan[e]^
2]*(1 - (c*Sec[e])/(d*Sqrt[1 + Tan[e]^2])))), -((Sec[e]*(c + d*Cos[e]*Sin[f
```

```
*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]))/(d*Sqrt[1 + Tan[e]^2]*(-1 - (c*Se
c[e])/(d*Sqrt[1 + Tan[e]^2]))))]*Sec[e]*Sec[f*x + ArcTan[Tan[e]]]*(1 + Sin[
e + f*x])*Sqrt[(d*Sqrt[1 + Tan[e]^2] - d*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 +
 Tan[e]^2])/(c*Sec[e] + d*Sqrt[1 + Tan[e]^2])]*Sqrt[(d*Sqrt[1 + Tan[e]^2] +
 d*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2])/(-(c*Sec[e]) + d*Sqrt[1 +
Tan[e]^2])]*Sqrt[c + d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]]
)/((c - d)^2*d*(c + d)^3*f*(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^2*Sqrt
[1 + Tan[e]^2]) + (2*d*AppellF1[1/2, 1/2, 1/2, 3/2, -((Sec[e]*(c + d*Cos[e]
*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]))/(d*Sqrt[1 + Tan[e]^2]*(1 -
(c*Sec[e])/(d*Sqrt[1 + Tan[e]^2])))), -((Sec[e]*(c + d*Cos[e]*Sin[f*x + Arc
Tan[Tan[e]]]*Sqrt[1 + Tan[e]^2]))/(d*Sqrt[1 + Tan[e]^2]*(-1 - (c*Sec[e])/(d
*Sqrt[1 + Tan[e]^2]))))]*Sec[e]*Sec[f*x + ArcTan[Tan[e]]]*(1 + Sin[e + f*x]
)*Sqrt[(d*Sqrt[1 + Tan[e]^2] - d*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^
2])/(c*Sec[e] + d*Sqrt[1 + Tan[e]^2])]*Sqrt[(d*Sqrt[1 + Tan[e]^2] + d*Sin[f
*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2])/(-(c*Sec[e]) + d*Sqrt[1 + Tan[e]^2
])]*Sqrt[c + d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]])/(3*(c
- d)^2*(c + d)^3*f*(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^2*Sqrt[1 + Tan
[e]^2]))
```

Maple [B] time = 5.059, size = 1046, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(7/2),x)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*a*((-c+d)/d*(2/5/(c^2-d^2)/d^2*(-(-
d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(sin(f*x+e)+c/d)^3+16/15*c/(c^2-d^2)^2/
d*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(sin(f*x+e)+c/d)^2+2/15*d*cos(f*x
+e)^2/(c^2-d^2)^3*(23*c^2+9*d^2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2*
(15*c^3+17*c*d^2)/(15*c^6-45*c^4*d^2+45*c^2*d^4-15*d^6)*(c/d-1)*((c+d*sin(f
*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d)
)^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))
/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+2/15*d*(23*c^2+9*d^2)/(c^2-d^2)^3*(c/d-1
)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+
e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*Elli
pticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*s
in(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+1/d*(2/3/(c^2-d^2)/d*(-(-d*s
in(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(sin(f*x+e)+c/d)^2+8/3*d*cos(f*x+e)^2/(c^2
-d^2)^2*c/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2*(3*c^2+d^2)/(3*c^4-6*c^
2*d^2+3*d^4)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d)
)^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(
1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+8/3*c*d/
(c^2-d^2)^2*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))
^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1
/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))
+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))))/cos(f*x+e
)/(c+d*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a\sin\left(fx+e\right)+a}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)/(d*sin(f*x + e) + c)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a\sin\left(fx+e\right)+a\right)\sqrt{d\sin\left(fx+e\right)+c}}{d^4\cos\left(fx+e\right)^4+c^4+6\,c^2d^2+d^4-2\left(3\,c^2d^2+d^4\right)\cos\left(fx+e\right)^2-4\left(cd^3\cos\left(fx+e\right)^2-c^3d-cd^3\right)\sin\left(fx+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(7/2),x, algorithm="fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)/(d^4*cos(f*x + e)^4
+ c^4 + 6*c^2*d^2 + d^4 - 2*(3*c^2*d^2 + d^4)*cos(f*x + e)^2 - 4*(c*d^3*cos
(f*x + e)^2 - c^3*d - c*d^3)*sin(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a\sin\left(fx+e\right)+a}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)/(d*sin(f*x + e) + c)^(7/2), x)
```

3.489 $\int (a+a\sin(e+fx))^2(c+d\sin(e+fx))^{5/2}\,dx$

Optimal. Leaf size=378

$$\frac{4a^2\left(-45c^2d+5c^3-141cd^2-75d^3\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{315df}+\frac{4a^2\left(c^2-d^2\right)\left(-45c^2d+5c^3-141cd^2-75d^3\right)\sqrt{}}{315d^2f\sqrt{c+d\sin(e}}$$

```
[Out] (4*a^2*(5*c^3 - 45*c^2*d - 141*c*d^2 - 75*d^3)*Cos[e + f*x]*Sqrt[c + d*Sin[
e + f*x]])/(315*d*f) + (4*a^2*(5*c*(c - 9*d) - 56*d^2)*Cos[e + f*x]*(c + d*
Sin[e + f*x])^(3/2))/(315*d*f) + (4*a^2*(c - 9*d)*Cos[e + f*x]*(c + d*Sin[e
 + f*x])^(5/2))/(63*d*f) - (2*a^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(7/2))/
(9*d*f) - (4*a^2*(5*c^4 - 45*c^3*d - 381*c^2*d^2 - 435*c*d^3 - 168*d^4)*Ell
ipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(315*d^
2*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + (4*a^2*(c^2 - d^2)*(5*c^3 - 45*c^
2*d - 141*c*d^2 - 75*d^3)*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt
[(c + d*Sin[e + f*x])/(c + d)])/(315*d^2*f*Sqrt[c + d*Sin[e + f*x]])
```

Rubi [A] time = 0.669623, antiderivative size = 378, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2763, 2753, 2752, 2663, 2661, 2655, 2653}

$$\frac{4a^2\left(-45c^2d+5c^3-141cd^2-75d^3\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{315df}+\frac{4a^2\left(c^2-d^2\right)\left(-45c^2d+5c^3-141cd^2-75d^3\right)\sqrt{}}{315d^2f\sqrt{c+d\sin(e}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^(5/2),x]
```


```
[Out] (4*a^2*(5*c^3 - 45*c^2*d - 141*c*d^2 - 75*d^3)*Cos[e + f*x]*Sqrt[c + d*Sin[
e + f*x]])/(315*d*f) + (4*a^2*(5*c*(c - 9*d) - 56*d^2)*Cos[e + f*x]*(c + d*
Sin[e + f*x])^(3/2))/(315*d*f) + (4*a^2*(c - 9*d)*Cos[e + f*x]*(c + d*Sin[e
 + f*x])^(5/2))/(63*d*f) - (2*a^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(7/2))/
(9*d*f) - (4*a^2*(5*c^4 - 45*c^3*d - 381*c^2*d^2 - 435*c*d^3 - 168*d^4)*Ell
ipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(315*d^
2*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + (4*a^2*(c^2 - d^2)*(5*c^3 - 45*c^
2*d - 141*c*d^2 - 75*d^3)*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt
[(c + d*Sin[e + f*x])/(c + d)])/(315*d^2*f*Sqrt[c + d*Sin[e + f*x]])
```


Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
```

```
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int (a + a\sin(e + fx))^2(c + d\sin(e + fx))^{5/2}\,dx = -\frac{2a^2\cos(e + fx)(c + d\sin(e + fx))^{7/2}}{9df} + \frac{2\int \left(8a^2d - a^2(c - 9d)\sin(e\right.}{}$$

$$= \frac{4a^2(c - 9d)\cos(e + fx)(c + d\sin(e + fx))^{5/2}}{63df} - \frac{2a^2\cos(e + fx)(c + d}{9df}$$

$$= \frac{4a^2\left(5c(c - 9d) - 56d^2\right)\cos(e + fx)(c + d\sin(e + fx))^{3/2}}{315df} + \frac{4a^2(c - 9}{}$$

$$= \frac{4a^2\left(5c^3 - 45c^2d - 141cd^2 - 75d^3\right)\cos(e + fx)\sqrt{c + d\sin(e + fx)}}{315df} +$$

$$= \frac{4a^2\left(5c^3 - 45c^2d - 141cd^2 - 75d^3\right)\cos(e + fx)\sqrt{c + d\sin(e + fx)}}{315df} +$$

$$= \frac{4a^2\left(5c^3 - 45c^2d - 141cd^2 - 75d^3\right)\cos(e + fx)\sqrt{c + d\sin(e + fx)}}{315df} +$$

$$= \frac{4a^2\left(5c^3 - 45c^2d - 141cd^2 - 75d^3\right)\cos(e + fx)\sqrt{c + d\sin(e + fx)}}{315df} +$$

Mathematica [A] time = 1.97777, size = 322, normalized size = 0.85

$$\frac{a^2(\sin(e + fx) + 1)^2\left(16\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\left(\left(-381c^2d^2 - 45c^3d + 5c^4 - 435cd^3 - 168d^4\right)\left((c + d)E\left(\frac{1}{4}(-2e - 2fx + \pi)|\frac{2d}{c+d}\right)\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (a^2*(1 + Sin[e + f*x])^2*(16*(-(d^2*(235*c^3 + 405*c^2*d + 309*c*d^2 + 75*
d^3)*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]) + (5*c^4 - 45*c^3*d -
 381*c^2*d^2 - 435*c*d^3 - 168*d^4)*((c + d)*EllipticE[(-2*e + Pi - 2*f*x)/
4, (2*d)/(c + d)] - c*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]))*Sqr
t[(c + d*Sin[e + f*x])/(c + d)] - d*(c + d*Sin[e + f*x])*(2*(20*c^3 + 1080*
c^2*d + 1671*c*d^2 + 690*d^3)*Cos[e + f*x] + 2*d*(-5*d*(19*c + 18*d)*Cos[3*
(e + f*x)] + (150*c^2 + 540*c*d + 259*d^2 - 35*d^2*Cos[2*(e + f*x)])*Sin[2*
(e + f*x)]))))/(1260*d^2*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*Sqrt[c +
 d*Sin[e + f*x]])
```

Maple [B] time = 1.148, size = 1614, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^(5/2),x)
```

```
[Out] -2/315*a^2*(486*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(
1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/
```

```
2),((c-d)/(c+d))^(1/2))*d^6-10*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x
+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x
+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c^6-336*((c+d*sin(f*x+e))/(c-d))^(1/
2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*Ellipti
cE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*d^6+5*c^4*d^2+10*((c
+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x
+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1
/2))*c^5*d+80*c^3*d^3*sin(f*x+e)+540*c^2*d^4*sin(f*x+e)+506*c*d^5*sin(f*x+e
)-130*c*d^5*sin(f*x+e)^5-170*c^2*d^4*sin(f*x+e)^4-360*c*d^5*sin(f*x+e)^4-80
*c^3*d^3*sin(f*x+e)^3-540*c^2*d^4*sin(f*x+e)^3-376*c*d^5*sin(f*x+e)^3-5*c^4
*d^2*sin(f*x+e)^2-270*c^3*d^3*sin(f*x+e)^2-224*c^2*d^4*sin(f*x+e)^2+210*c*d
^5*sin(f*x+e)^2-570*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d
))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))
^(1/2),((c-d)/(c+d))^(1/2))*c^4*d^2-1012*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(
-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c
+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c^3*d^3+84*((c+d*sin(f*x+e
))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^
(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c^2*d^4
+1002*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(
1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/
(c+d))^(1/2))*c*d^5+90*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(
c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-
d))^(1/2),((c-d)/(c+d))^(1/2))*c^5*d+772*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(
-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c
+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c^4*d^2+780*((c+d*sin(f*x+
e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))
^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c^3*d^
3-426*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(
1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/
(c+d))^(1/2))*c^2*d^4-870*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*
d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/
(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c*d^5+394*c^2*d^4-90*d^6*sin(f*x+e)^5-77*
d^6*sin(f*x+e)^4-60*d^6*sin(f*x+e)^3+112*d^6*sin(f*x+e)^2+150*d^6*sin(f*x+e
)-35*d^6*sin(f*x+e)^6+270*c^3*d^3+150*c*d^5)/d^3/cos(f*x+e)/(c+d*sin(f*x+e)
)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^{2}\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^2*(d*sin(f*x + e) + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

integral $\left(\left(a^2d^2\cos\left(fx+e\right)^4+2\,a^2c^2+4\,a^2cd+2\,a^2d^2-\left(a^2c^2+4\,a^2cd+3\,a^2d^2\right)\cos\left(fx+e\right)^2+2\left(a^2c^2+2\,a^2cd+a^2d\right.\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*d^2*cos(f*x + e)^4 + 2*a^2*c^2 + 4*a^2*c*d + 2*a^2*d^2 - (a^2
*c^2 + 4*a^2*c*d + 3*a^2*d^2)*cos(f*x + e)^2 + 2*(a^2*c^2 + 2*a^2*c*d + a^2
*d^2 - (a^2*c*d + a^2*d^2)*cos(f*x + e)^2)*sin(f*x + e))*sqrt(d*sin(f*x + e
) + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(c+d*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^{2}\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^2*(d*sin(f*x + e) + c)^(5/2), x)
```

3.490 $\int (a+a\sin(e+fx))^2(c+d\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=298

$$\frac{4a^2\left(c^2-7cd-10d^2\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{35df}+\frac{4a^2\left(c^2-7cd-10d^2\right)\left(c^2-d^2\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{35d^2f\sqrt{c+d\sin(e+fx)}}$$

```
[Out] (4*a^2*(c^2 - 7*c*d - 10*d^2)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(35*d*
f) + (4*a^2*(c - 7*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(3/2))/(35*d*f) - (
2*a^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(5/2))/(7*d*f) - (4*a^2*(c + 3*d)*(
c^2 - 10*c*d - 7*d^2)*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c +
 d*Sin[e + f*x]])/(35*d^2*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + (4*a^2*(c
^2 - 7*c*d - 10*d^2)*(c^2 - d^2)*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d
)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(35*d^2*f*Sqrt[c + d*Sin[e + f*x]])
```

Rubi [A] time = 0.478873, antiderivative size = 298, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2763, 2753, 2752, 2663, 2661, 2655, 2653}

$$\frac{4a^2\left(c^2-7cd-10d^2\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{35df}+\frac{4a^2\left(c^2-7cd-10d^2\right)\left(c^2-d^2\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{35d^2f\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] (4*a^2*(c^2 - 7*c*d - 10*d^2)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(35*d*
f) + (4*a^2*(c - 7*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(3/2))/(35*d*f) - (
2*a^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(5/2))/(7*d*f) - (4*a^2*(c + 3*d)*(
c^2 - 10*c*d - 7*d^2)*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c +
 d*Sin[e + f*x]])/(35*d^2*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + (4*a^2*(c
^2 - 7*c*d - 10*d^2)*(c^2 - d^2)*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d
)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(35*d^2*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

```
 && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))^2(c+d\sin(e+fx))^{3/2}\,dx = -\frac{2a^2\cos(e+fx)(c+d\sin(e+fx))^{5/2}}{7df} + \frac{2\int \left(6a^2d-a^2(c-7d)\sin(e\right.}{}$$

$$= \frac{4a^2(c-7d)\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{35df} - \frac{2a^2\cos(e+fx)(c+d}{7df}$$

$$= \frac{4a^2\left(c^2-7cd-10d^2\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{35df} + \frac{4a^2(c-7d)\,\mathrm{co}}{}$$

$$= \frac{4a^2\left(c^2-7cd-10d^2\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{35df} + \frac{4a^2(c-7d)\,\mathrm{co}}{}$$

$$= \frac{4a^2\left(c^2-7cd-10d^2\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{35df} + \frac{4a^2(c-7d)\,\mathrm{co}}{}$$

$$= \frac{4a^2\left(c^2-7cd-10d^2\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{35df} + \frac{4a^2(c-7d)\,\mathrm{co}}{}$$

Mathematica [A] time = 2.10408, size = 262, normalized size = 0.88

$$a^2\left(d\cos(e+fx)\left(-d\left(36c^2+168cd+95d^2\right)\sin(e+fx)-112c^2d-4c^3+2d^2(13c+14d)\cos(2(e+fx))-106cd^2+5d^3\,\mathrm{s}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] (a^2*(8*(c^4 - 6*c^3*d - 44*c^2*d^2 - 58*c*d^3 - 21*d^4)*EllipticE[(-2*e +
Pi - 2*f*x)/4, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)] - 8*(c^4 -
 7*c^3*d - 11*c^2*d^2 + 7*c*d^3 + 10*d^4)*EllipticF[(-2*e + Pi - 2*f*x)/4,
(2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)] + d*Cos[e + f*x]*(-4*c^3
- 112*c^2*d - 106*c*d^2 - 28*d^3 + 2*d^2*(13*c + 14*d)*Cos[2*(e + f*x)] - d
*(36*c^2 + 168*c*d + 95*d^2)*Sin[e + f*x] + 5*d^3*Sin[3*(e + f*x)])))/(70*d
^2*f*Sqrt[c + d*Sin[e + f*x]])
```

Maple [B] time = 1.149, size = 1316, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^(3/2),x)
```

```
[Out] -2/35*a^2*(-13*c*d^4*sin(f*x+e)^4+28*c^2*d^3+20*c*d^4-68*((c+d*sin(f*x+e))/
(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/
2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c^3*d^2-9*
c^2*d^3*sin(f*x+e)^3-42*c*d^4*sin(f*x+e)^3-c^3*d^2*sin(f*x+e)^2-28*c^2*d^3*
sin(f*x+e)^2-7*c*d^4*sin(f*x+e)^2+9*c^2*d^3*sin(f*x+e)+42*c*d^4*sin(f*x+e)-
5*d^5*sin(f*x+e)^5-14*d^5*sin(f*x+e)^4-15*d^5*sin(f*x+e)^3+14*d^5*sin(f*x+e
)^2+20*d^5*sin(f*x+e)+c^3*d^2+62*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f
*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f
*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*d^5-2*((c+d*sin(f*x+e))/(c-d))^(1/
2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*Ellipti
cE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c^5-42*((c+d*sin(f*x
+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d)
)^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*d^5-6
4*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+si
n(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d
))^(1/2))*c^2*d^3+68*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+
d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d)
)^(1/2),((c-d)/(c+d))^(1/2))*c*d^4+14*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+
sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*
sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c^4*d+76*((c+d*sin(f*x+e))/(c
-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)
*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c^3*d^2+28*(
(c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f
*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^
(1/2))*c^2*d^3-74*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))
^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(
1/2),((c-d)/(c+d))^(1/2))*c*d^4+2*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(
f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(
f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c^4*d)/d^3/cos(f*x+e)/(c+d*sin(f*
x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^{2}\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^2*(d*sin(f*x + e) + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(2\,a^{2}c+2\,a^{2}d-\left(a^{2}c+2\,a^{2}d\right)\cos\left(fx+e\right)^{2}-\left(a^{2}d\cos\left(fx+e\right)^{2}-2\,a^{2}c-2\,a^{2}d\right)\sin\left(fx+e\right)\right)\sqrt{d\sin\left(fx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((2*a^2*c + 2*a^2*d - (a^2*c + 2*a^2*d)*cos(f*x + e)^2 - (a^2*d*cos
(f*x + e)^2 - 2*a^2*c - 2*a^2*d)*sin(f*x + e))*sqrt(d*sin(f*x + e) + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^{2}\left(\int c\sqrt{c+d\sin\left(e+fx\right)}\,dx+\int 2c\sqrt{c+d\sin\left(e+fx\right)}\sin\left(e+fx\right)dx+\int c\sqrt{c+d\sin\left(e+fx\right)}\sin^{2}\left(e+fx\right)dx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(c+d*sin(f*x+e))**(3/2),x)
```

```
[Out] a**2*(Integral(c*sqrt(c + d*sin(e + f*x)), x) + Integral(2*c*sqrt(c + d*sin
(e + f*x))*sin(e + f*x), x) + Integral(c*sqrt(c + d*sin(e + f*x))*sin(e + f
*x)**2, x) + Integral(d*sqrt(c + d*sin(e + f*x))*sin(e + f*x), x) + Integra
l(2*d*sqrt(c + d*sin(e + f*x))*sin(e + f*x)**2, x) + Integral(d*sqrt(c + d*
sin(e + f*x))*sin(e + f*x)**3, x))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.491 $\int (a + a\sin(e + fx))^2 \sqrt{c + d\sin(e + fx)}\, dx$

Optimal. Leaf size=239

$$\frac{4a^2(c-5d)\left(c^2-d^2\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{15d^2f\sqrt{c+d\sin(e+fx)}}-\frac{4a^2\left(c^2-5cd-12d^2\right)\sqrt{c+d\sin(e+fx)}E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{15d^2f\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

```
[Out] (4*a^2*(c - 5*d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(15*d*f) - (2*a^2*C
os[e + f*x]*(c + d*Sin[e + f*x])^(3/2))/(5*d*f) - (4*a^2*(c^2 - 5*c*d - 12*
d^2)*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])
/(15*d^2*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + (4*a^2*(c - 5*d)*(c^2 - d^
2)*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(
c + d)])/(15*d^2*f*Sqrt[c + d*Sin[e + f*x]])
```

Rubi [A] time = 0.336093, antiderivative size = 239, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2763, 2753, 2752, 2663, 2661, 2655, 2653}

$$\frac{4a^2(c-5d)\left(c^2-d^2\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{15d^2f\sqrt{c+d\sin(e+fx)}}-\frac{4a^2\left(c^2-5cd-12d^2\right)\sqrt{c+d\sin(e+fx)}E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{15d^2f\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2*Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] (4*a^2*(c - 5*d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(15*d*f) - (2*a^2*C
os[e + f*x]*(c + d*Sin[e + f*x])^(3/2))/(5*d*f) - (4*a^2*(c^2 - 5*c*d - 12*
d^2)*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])
/(15*d^2*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + (4*a^2*(c - 5*d)*(c^2 - d^
2)*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(
c + d)])/(15*d^2*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))^2\sqrt{c+d\sin(e+fx)}\,dx = -\frac{2a^2\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{5df} + \frac{2\int \left(4a^2d-a^2(c-5d)\sin(e+\right.}{5d}$$

$$= \frac{4a^2(c-5d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15df} - \frac{2a^2\cos(e+fx)(c+d\sin(}{5df}$$

$$= \frac{4a^2(c-5d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15df} - \frac{2a^2\cos(e+fx)(c+d\sin(}{5df}$$

$$= \frac{4a^2(c-5d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15df} - \frac{2a^2\cos(e+fx)(c+d\sin(}{5df}$$

$$= \frac{4a^2(c-5d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15df} - \frac{2a^2\cos(e+fx)(c+d\sin(}{5df}$$

Mathematica [A] time = 1.42886, size = 244, normalized size = 1.02

$$-\frac{a^2(\sin(e+fx)+1)^2\left(-d\cos(e+fx)\left(-2c^2-4d(2c+5d)\sin(e+fx)-20cd+3d^2\cos(2(e+fx))-3d^2\right)+4\left(-5c^2d\right.\right.}{15d^2f\left(\sin\left(\frac{1}{2}(e+fx)\right)\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2*Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] -(a^2*(1 + Sin[e + f*x])^2*(-4*(c^3 - 4*c^2*d - 17*c*d^2 - 12*d^3)*Elliptic
E[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)]
+ 4*(c^3 - 5*c^2*d - c*d^2 + 5*d^3)*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/
(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)] - d*Cos[e + f*x]*(-2*c^2 - 20*c
*d - 3*d^2 + 3*d^2*Cos[2*(e + f*x)] - 4*d*(2*c + 5*d)*Sin[e + f*x])))/(15*d
^2*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*Sqrt[c + d*Sin[e + f*x]])
```

Maple [B] time = 1.029, size = 1035, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^(1/2),x)
```

```
[Out] -2/15*a^2*(2*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2
)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),
((c-d)/(c+d))^(1/2))*c^3*d-34*c^2*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(
f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(
f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*d^2-2*c*((c+d*sin(f*x+e))/(c-d))^
(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*Elli
pticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*d^3+34*((c+d*sin(
f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c
-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*d^
4-2*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+
sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c
+d))^(1/2))*c^4+10*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d)
)^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^
(1/2),((c-d)/(c+d))^(1/2))*c^3*d+26*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+si
n(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*si
n(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c^2*d^2-10*((c+d*sin(f*x+e))/(c
-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)
*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c*d^3-24*((c
+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x
+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1
/2))*d^4-3*d^4*sin(f*x+e)^4-4*c*d^3*sin(f*x+e)^3-10*d^4*sin(f*x+e)^3-c^2*d^
2*sin(f*x+e)^2-10*c*d^3*sin(f*x+e)^2+3*d^4*sin(f*x+e)^2+4*c*d^3*sin(f*x+e)+
10*d^4*sin(f*x+e)+c^2*d^2+10*c*d^3)/d^3/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^2\sqrt{d\sin\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^2*sqrt(d*sin(f*x + e) + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(a^2\cos\left(fx+e\right)^2-2\,a^2\sin\left(fx+e\right)-2\,a^2\right)\sqrt{d\sin\left(fx+e\right)+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-(a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x + e) - 2*a^2)*sqrt(d*sin(f*x
+ e) + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2\left(\int 2\sqrt{c+d\sin\left(e+fx\right)}\sin\left(e+fx\right)dx+\int\sqrt{c+d\sin\left(e+fx\right)}\sin^2\left(e+fx\right)dx+\int\sqrt{c+d\sin\left(e+fx\right)}dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] a**2*(Integral(2*sqrt(c + d*sin(e + f*x))*sin(e + f*x), x) + Integral(sqrt(
c + d*sin(e + f*x))*sin(e + f*x)**2, x) + Integral(sqrt(c + d*sin(e + f*x))
, x))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.492 $\int \frac{(a+a\sin(e+fx))^2}{\sqrt{c+d\sin(e+fx)}}\,dx$

Optimal. Leaf size=189

$$\frac{4a^2(c-2d)(c-d)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3d^2f\sqrt{c+d\sin(e+fx)}}-\frac{4a^2(c-3d)\sqrt{c+d\sin(e+fx)}E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3d^2f\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}-\frac{2a^2\cos}{}$$

```
[Out] (-2*a^2*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(3*d*f) - (4*a^2*(c - 3*d)*E
llipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(3*d^
2*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + (4*a^2*(c - 2*d)*(c - d)*Elliptic
F[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(3
*d^2*f*Sqrt[c + d*Sin[e + f*x]])
```

Rubi [A] time = 0.246494, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2763, 2752, 2663, 2661, 2655, 2653}

$$\frac{4a^2(c-2d)(c-d)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3d^2f\sqrt{c+d\sin(e+fx)}}-\frac{4a^2(c-3d)\sqrt{c+d\sin(e+fx)}E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3d^2f\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}-\frac{2a^2\cos}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2/Sqrt[c + d*Sin[e + f*x]],x]
```


```
[Out] (-2*a^2*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(3*d*f) - (4*a^2*(c - 3*d)*E
llipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(3*d^
2*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + (4*a^2*(c - 2*d)*(c - d)*Elliptic
F[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(3
*d^2*f*Sqrt[c + d*Sin[e + f*x]])
```


Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
```

```
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^2}{\sqrt{c+d\sin(e+fx)}}\,dx = -\frac{2a^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3df} + \frac{2\int \frac{2a^2d-a^2(c-3d)\sin(e+fx)}{\sqrt{c+d\sin(e+fx)}}\,dx}{3d}$$

$$= -\frac{2a^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3df} - \frac{\left(2a^2(c-3d)\right)\int \sqrt{c+d\sin(e+fx)}\,dx}{3d^2} + \frac{\left(2a^2(c-\right.}{}$$

$$= -\frac{2a^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3df} - \frac{\left(2a^2(c-3d)\sqrt{c+d\sin(e+fx)}\right)\int \sqrt{\frac{c}{c+d}+\frac{d\sin(e+}{c+d}}}{3d^2\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

$$= -\frac{2a^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3df} - \frac{4a^2(c-3d)E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2d}{c+d}\right)\sqrt{c+d\sin(e+}}{3d^2f\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

Mathematica [A] time = 1.08974, size = 193, normalized size = 1.02

$$-\frac{2a^2(\sin(e+fx)+1)^2\left(2\left(c^2-3cd+2d^2\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{4}(-2e-2fx+\pi)|\frac{2d}{c+d}\right)-2\left(c^2-2cd-3d^2\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}E\right.}{3d^2f\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^4\sqrt{c+d\sin(e+f}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2/Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] (-2*a^2*(1 + Sin[e + f*x])^2*(d*Cos[e + f*x]*(c + d*Sin[e + f*x]) - 2*(c^2
- 2*c*d - 3*d^2)*EllipticE[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]*Sqrt[(c +
d*Sin[e + f*x])/(c + d)] + 2*(c^2 - 3*c*d + 2*d^2)*EllipticF[(-2*e + Pi - 2
*f*x)/4, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)]))/(3*d^2*f*(Cos[
(e + f*x)/2] + Sin[(e + f*x)/2])^4*Sqrt[c + d*Sin[e + f*x]])
```

Maple [B] time = 0.998, size = 758, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^(1/2),x)
```

```
[Out] -2/3*a^2*(2*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)
*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),(
(c-d)/(c+d))^(1/2))*c^2*d-12*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e
))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e
))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*d^2*c+10*((c+d*sin(f*x+e))/(c-d))^(1/2
)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*Elliptic
F(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*d^3-2*((c+d*sin(f*x+e
))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^
(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c^3+6*(
(c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f
*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^
(1/2))*c^2*d+2*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1
/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2
),((c-d)/(c+d))^(1/2))*c*d^2-6*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x
+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x
+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*d^3-d^3*sin(f*x+e)^3-c*d^2*sin(f*x+e
)^2+d^3*sin(f*x+e)+c*d^2)/d^3/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a \sin\left(fx+e\right)+a\right)^2}{\sqrt{d \sin\left(fx+e\right)+c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^2/sqrt(d*sin(f*x + e) + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{a^2 \cos\left(fx+e\right)^2 - 2\,a^2 \sin\left(fx+e\right) - 2\,a^2}{\sqrt{d \sin\left(fx+e\right)+c}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-(a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x + e) - 2*a^2)/sqrt(d*sin(f*x
+ e) + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2\left(\int \frac{2\sin\left(e+fx\right)}{\sqrt{c+d\sin\left(e+fx\right)}}\,dx+\int \frac{\sin^2\left(e+fx\right)}{\sqrt{c+d\sin\left(e+fx\right)}}\,dx+\int \frac{1}{\sqrt{c+d\sin\left(e+fx\right)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2/(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] a**2*(Integral(2*sin(e + f*x)/sqrt(c + d*sin(e + f*x)), x) + Integral(sin(e
 + f*x)**2/sqrt(c + d*sin(e + f*x)), x) + Integral(1/sqrt(c + d*sin(e + f*x
)), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^2}{\sqrt{d\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^2/sqrt(d*sin(f*x + e) + c), x)
```

3.493 $\int \frac{(a+a\sin(e+fx))^2}{(c+d\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=189

$$-\frac{4a^2(c-d)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{d^2f\sqrt{c+d\sin(e+fx)}}+\frac{4a^2c\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{d^2f(c+d)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}+\frac{2a^2(c-d)\cos(e+fx)}{df(c+d)\sqrt{c+d\sin(e+fx)}}$$

```
[Out] (2*a^2*(c - d)*Cos[e + f*x])/(d*(c + d)*f*Sqrt[c + d*Sin[e + f*x]]) + (4*a^
2*c*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/
(d^2*(c + d)*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) - (4*a^2*(c - d)*Ellipti
cF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(
d^2*f*Sqrt[c + d*Sin[e + f*x]])
```

Rubi [A] time = 0.239003, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2762, 2752, 2663, 2661, 2655, 2653}

$$-\frac{4a^2(c-d)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{d^2f\sqrt{c+d\sin(e+fx)}}+\frac{4a^2c\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{d^2f(c+d)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}+\frac{2a^2(c-d)\cos(e+fx)}{df(c+d)\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2/(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] (2*a^2*(c - d)*Cos[e + f*x])/(d*(c + d)*f*Sqrt[c + d*Sin[e + f*x]]) + (4*a^
2*c*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/
(d^2*(c + d)*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) - (4*a^2*(c - d)*Ellipti
cF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(
d^2*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
```

```
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^2}{(c+d\sin(e+fx))^{3/2}}\,dx = \frac{2a^2(c-d)\cos(e+fx)}{d(c+d)f\sqrt{c+d\sin(e+fx)}} - \frac{(2a)\int \frac{-ad-ac\sin(e+fx)}{\sqrt{c+d\sin(e+fx)}}\,dx}{d(c+d)}$$

$$= \frac{2a^2(c-d)\cos(e+fx)}{d(c+d)f\sqrt{c+d\sin(e+fx)}} - \frac{\left(2a^2(c-d)\right)\int \frac{1}{\sqrt{c+d\sin(e+fx)}}\,dx}{d^2} + \frac{\left(2a^2c\right)\int \sqrt{c+d\sin(e}{d^2(c+d)}$$

$$= \frac{2a^2(c-d)\cos(e+fx)}{d(c+d)f\sqrt{c+d\sin(e+fx)}} + \frac{\left(2a^2c\sqrt{c+d\sin(e+fx)}\right)\int \sqrt{\frac{c}{c+d}+\frac{d\sin(e+fx)}{c+d}}\,dx}{d^2(c+d)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}} - \frac{\left(2a^2\right.}{}$$

$$= \frac{2a^2(c-d)\cos(e+fx)}{d(c+d)f\sqrt{c+d\sin(e+fx)}} + \frac{4a^2cE\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2d}{c+d}\right)\sqrt{c+d\sin(e+fx)}}{d^2(c+d)f\sqrt{\frac{c+d\sin(e+fx)}{c+d}}} - \frac{4a^2(c}{}$$

Mathematica [A] time = 0.92012, size = 175, normalized size = 0.93

$$-\frac{2a^2(\sin(e+fx)+1)^2\left(2c(c+d)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}E\left(\frac{1}{4}(-2e-2fx+\pi)|\frac{2d}{c+d}\right)-(c-d)\left(2(c+d)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{4}(-2e-2\right.\right.\right.}{d^2f(c+d)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^4\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2/(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-2*a^2*(1 + Sin[e + f*x])^2*(2*c*(c + d)*EllipticE[(-2*e + Pi - 2*f*x)/4,
(2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)] - (c - d)*(d*Cos[e + f*x]
 + 2*(c + d)*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]*Sqrt[(c + d*Si
n[e + f*x])/(c + d)])))/(d^2*(c + d)*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)^4*Sqrt[c + d*Sin[e + f*x]])
```

Maple [A] time = 0.826, size = 463, normalized size = 2.5

$$-2\frac{a^2}{d^3(c+d)\cos(fx+e)\sqrt{c+d\sin(fx+e)}f}\left(2\sqrt{\frac{c+d\sin(fx+e)}{c-d}}\sqrt{-\frac{(-1+\sin(fx+e))d}{c+d}}\sqrt{-\frac{d(1+\sin(fx+e))}{c-d}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^(3/2),x)
```

```
[Out] -2*(2*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(
1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/
(c+d))^(1/2))*c^3-2*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d
))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))
^(1/2),((c-d)/(c+d))^(1/2))*c*d^2-2*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+si
n(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*si
n(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c^2*d+2*((c+d*sin(f*x+e))/(c-d)
)^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*El
lipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*d^3+c*d^2*sin(f
*x+e)^2-d^3*sin(f*x+e)^2-c*d^2+d^3)/d^3*a^2/(c+d)/cos(f*x+e)/(c+d*sin(f*x+e
))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\sin(fx+e)+a)^2}{(d\sin(fx+e)+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^2/(d*sin(f*x + e) + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^2\cos(fx+e)^2-2\,a^2\sin(fx+e)-2\,a^2\right)\sqrt{d\sin(fx+e)+c}}{d^2\cos(fx+e)^2-2\,cd\sin(fx+e)-c^2-d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x + e) - 2*a^2)*sqrt(d*sin(f*x +
 e) + c)/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2/(c+d*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^2}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^2/(d*sin(f*x + e) + c)^(3/2), x)
```

3.494 $\int \frac{(a+a\sin(e+fx))^2}{(c+d\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=247

$$\frac{4a^2(c+2d)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3d^2f(c+d)\sqrt{c+d\sin(e+fx)}}-\frac{4a^2(c+3d)\sqrt{c+d\sin(e+fx)}E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3d^2f(c+d)^2\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}-\frac{4a^2(c+3a}{3df(c+d)^2\sqrt{}}$$

```
[Out] (2*a^2*(c - d)*Cos[e + f*x])/(3*d*(c + d)*f*(c + d*Sin[e + f*x])^(3/2)) - (
4*a^2*(c + 3*d)*Cos[e + f*x])/(3*d*(c + d)^2*f*Sqrt[c + d*Sin[e + f*x]]) -
(4*a^2*(c + 3*d)*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Si
n[e + f*x]])/(3*d^2*(c + d)^2*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + (4*a^
2*(c + 2*d)*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e
+ f*x])/(c + d)])/(3*d^2*(c + d)*f*Sqrt[c + d*Sin[e + f*x]])
```

Rubi [A] time = 0.369142, antiderivative size = 247, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2762, 2754, 2752, 2663, 2661, 2655, 2653}

$$\frac{4a^2(c+2d)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3d^2f(c+d)\sqrt{c+d\sin(e+fx)}}-\frac{4a^2(c+3d)\sqrt{c+d\sin(e+fx)}E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3d^2f(c+d)^2\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}-\frac{4a^2(c+3a}{3df(c+d)^2\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2/(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (2*a^2*(c - d)*Cos[e + f*x])/(3*d*(c + d)*f*(c + d*Sin[e + f*x])^(3/2)) - (
4*a^2*(c + 3*d)*Cos[e + f*x])/(3*d*(c + d)^2*f*Sqrt[c + d*Sin[e + f*x]]) -
(4*a^2*(c + 3*d)*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Si
n[e + f*x]])/(3*d^2*(c + d)^2*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + (4*a^
2*(c + 2*d)*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e
+ f*x])/(c + d)])/(3*d^2*(c + d)*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^2}{(c+d\sin(e+fx))^{5/2}}\,dx = \frac{2a^2(c-d)\cos(e+fx)}{3d(c+d)f(c+d\sin(e+fx))^{3/2}} - \frac{(2a)\int \frac{-3ad-a(c+2d)\sin(e+fx)}{(c+d\sin(e+fx))^{3/2}}\,dx}{3d(c+d)}$$

$$= \frac{2a^2(c-d)\cos(e+fx)}{3d(c+d)f(c+d\sin(e+fx))^{3/2}} - \frac{4a^2(c+3d)\cos(e+fx)}{3d(c+d)^2f\sqrt{c+d\sin(e+fx)}} + \frac{(4a)\int \frac{a(c-d)d-\frac{1}{2}a(c-d \cdots}{\sqrt{c+d\,\mathrm{s}\cdots}}}{3(c-d)d(\cdots}$$

$$= \frac{2a^2(c-d)\cos(e+fx)}{3d(c+d)f(c+d\sin(e+fx))^{3/2}} - \frac{4a^2(c+3d)\cos(e+fx)}{3d(c+d)^2f\sqrt{c+d\sin(e+fx)}} + \frac{\left(2a^2(c+2d)\right)\int \frac{\cdots}{\sqrt{c+\cdots}}}{3d^2(c+\cdots}$$

$$= \frac{2a^2(c-d)\cos(e+fx)}{3d(c+d)f(c+d\sin(e+fx))^{3/2}} - \frac{4a^2(c+3d)\cos(e+fx)}{3d(c+d)^2f\sqrt{c+d\sin(e+fx)}} - \frac{\left(2a^2(c+3d)\sqrt{c+d\cdots}\right.}{3d^2(\cdots}$$

$$= \frac{2a^2(c-d)\cos(e+fx)}{3d(c+d)f(c+d\sin(e+fx))^{3/2}} - \frac{4a^2(c+3d)\cos(e+fx)}{3d(c+d)^2f\sqrt{c+d\sin(e+fx)}} - \frac{4a^2(c+3d)E\left(\frac{1}{2}\left(e\cdots\right.\right.}{3d^2(c\cdots}$$

Mathematica [A] time = 1.76248, size = 207, normalized size = 0.84

$$-\frac{2a^2(\sin(e+fx)+1)^2\left(d\cos(e+fx)\left(c^2+2d(c+3d)\sin(e+fx)+6cd+d^2\right)+2(c+2d)(c+d)^2\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{3/2}F\left(\frac{1}{4}(-\right.\right.}{3d^2f(c+d)^2\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^4(c+}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2/(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (-2*a^2*(1 + Sin[e + f*x])^2*(-2*(c + d)^2*(c + 3*d)*EllipticE[(-2*e + Pi -
 2*f*x)/4, (2*d)/(c + d)]*((c + d*Sin[e + f*x])/(c + d))^(3/2) + 2*(c + d)^
2*(c + 2*d)*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]*((c + d*Sin[e +
 f*x])/(c + d))^(3/2) + d*Cos[e + f*x]*(c^2 + 6*c*d + d^2 + 2*d*(c + 3*d)*S
in[e + f*x])))/(3*d^2*(c + d)^2*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*(
c + d*Sin[e + f*x])^(3/2))
```

Maple [B] time = 1.007, size = 1221, normalized size = 4.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^(5/2),x)
```

```
[Out] -2/3*a^2*((2*c*d^3+6*d^4)*sin(f*x+e)*cos(f*x+e)^2-2*(-d/(c+d)*sin(f*x+e)+d/
(c+d))^(1/2)*(-d/(c-d)*sin(f*x+e)-d/(c-d))^(1/2)*(d/(c-d)*sin(f*x+e)+1/(c-d
)*c)^(1/2)*d*(EllipticE((d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2),((c-d)/(c+d))^
(1/2))*c^3+3*EllipticE((d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2),((c-d)/(c+d))^(
1/2))*c^2*d-EllipticE((d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2),((c-d)/(c+d))^(1
/2))*c*d^2-3*EllipticE((d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2),((c-d)/(c+d))^(
1/2))*d^3-EllipticF((d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2),((c-d)/(c+d))^(1/2
))*c^2*d+EllipticF((d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2),((c-d)/(c+d))^(1/2)
)*d^3)*sin(f*x+e)+(c^2*d^2+6*c*d^3+d^4)*cos(f*x+e)^2+2*(d/(c-d)*sin(f*x+e)+
1/(c-d)*c)^(1/2)*(-d/(c+d)*sin(f*x+e)+d/(c+d))^(1/2)*(-d/(c-d)*sin(f*x+e)-d
/(c-d))^(1/2)*EllipticF((d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2),((c-d)/(c+d))^
(1/2))*c^3*d-2*(d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2)*(-d/(c+d)*sin(f*x+e)+d/
(c+d))^(1/2)*(-d/(c-d)*sin(f*x+e)-d/(c-d))^(1/2)*EllipticF((d/(c-d)*sin(f*x
+e)+1/(c-d)*c)^(1/2),((c-d)/(c+d))^(1/2))*c*d^3-2*(d/(c-d)*sin(f*x+e)+1/(c-
d)*c)^(1/2)*(-d/(c+d)*sin(f*x+e)+d/(c+d))^(1/2)*(-d/(c-d)*sin(f*x+e)-d/(c-d
))^(1/2)*EllipticE((d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2),((c-d)/(c+d))^(1/2)
)*c^4-6*(d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2)*(-d/(c+d)*sin(f*x+e)+d/(c+d))^
(1/2)*(-d/(c-d)*sin(f*x+e)-d/(c-d))^(1/2)*EllipticE((d/(c-d)*sin(f*x+e)+1/(
c-d)*c)^(1/2),((c-d)/(c+d))^(1/2))*c^3*d+2*(d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(
1/2)*(-d/(c+d)*sin(f*x+e)+d/(c+d))^(1/2)*(-d/(c-d)*sin(f*x+e)-d/(c-d))^(1/2
)*EllipticE((d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2),((c-d)/(c+d))^(1/2))*c^2*d
^2+6*(d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2)*(-d/(c+d)*sin(f*x+e)+d/(c+d))^(1/
2)*(-d/(c-d)*sin(f*x+e)-d/(c-d))^(1/2)*EllipticE((d/(c-d)*sin(f*x+e)+1/(c-d
)*c)^(1/2),((c-d)/(c+d))^(1/2))*c*d^3)/(c+d)^2/(c+d*sin(f*x+e))^(3/2)/d^3/c
os(f*x+e)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{2}}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^2/(d*sin(f*x + e) + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^{2}\cos\left(fx+e\right)^{2}-2\,a^{2}\sin\left(fx+e\right)-2\,a^{2}\right)\sqrt{d\sin\left(fx+e\right)+c}}{3\,cd^{2}\cos\left(fx+e\right)^{2}-c^{3}-3\,cd^{2}+\left(d^{3}\cos\left(fx+e\right)^{2}-3\,c^{2}d-d^{3}\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x + e) - 2*a^2)*sqrt(d*sin(f*x +
 e) + c)/(3*c*d^2*cos(f*x + e)^2 - c^3 - 3*c*d^2 + (d^3*cos(f*x + e)^2 - 3*
c^2*d - d^3)*sin(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2/(c+d*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{2}}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^2/(d*sin(f*x + e) + c)^(5/2), x)
```

3.495 $\int \frac{(a+a\sin(e+fx))^2}{(c+d\sin(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=320

$$-\frac{4a^2\left(c^2+5cd-12d^2\right)\cos(e+fx)}{15df(c-d)(c+d)^3\sqrt{c+d\sin(e+fx)}}-\frac{4a^2\left(c^2+5cd-12d^2\right)\sqrt{c+d\sin(e+fx)}E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{15d^2f(c-d)(c+d)^3\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}+\frac{4a^2(c+5d)}{15d}$$

```
[Out] (2*a^2*(c - d)*Cos[e + f*x])/(5*d*(c + d)*f*(c + d*Sin[e + f*x])^(5/2)) - (
4*a^2*(c + 5*d)*Cos[e + f*x])/(15*d*(c + d)^2*f*(c + d*Sin[e + f*x])^(3/2))
 - (4*a^2*(c^2 + 5*c*d - 12*d^2)*Cos[e + f*x])/(15*(c - d)*d*(c + d)^3*f*Sq
rt[c + d*Sin[e + f*x]]) - (4*a^2*(c^2 + 5*c*d - 12*d^2)*EllipticE[(e - Pi/2
 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(15*(c - d)*d^2*(c + d)
^3*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + (4*a^2*(c + 5*d)*EllipticF[(e -
Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(15*d^2*(
c + d)^2*f*Sqrt[c + d*Sin[e + f*x]])
```

Rubi [A] time = 0.577853, antiderivative size = 320, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2762, 2754, 2752, 2663, 2661, 2655, 2653}

$$-\frac{4a^2\left(c^2+5cd-12d^2\right)\cos(e+fx)}{15df(c-d)(c+d)^3\sqrt{c+d\sin(e+fx)}}-\frac{4a^2\left(c^2+5cd-12d^2\right)\sqrt{c+d\sin(e+fx)}E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{15d^2f(c-d)(c+d)^3\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}+\frac{4a^2(c+5d)}{15d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2/(c + d*Sin[e + f*x])^(7/2),x]
```

```
[Out] (2*a^2*(c - d)*Cos[e + f*x])/(5*d*(c + d)*f*(c + d*Sin[e + f*x])^(5/2)) - (
4*a^2*(c + 5*d)*Cos[e + f*x])/(15*d*(c + d)^2*f*(c + d*Sin[e + f*x])^(3/2))
 - (4*a^2*(c^2 + 5*c*d - 12*d^2)*Cos[e + f*x])/(15*(c - d)*d*(c + d)^3*f*Sq
rt[c + d*Sin[e + f*x]]) - (4*a^2*(c^2 + 5*c*d - 12*d^2)*EllipticE[(e - Pi/2
 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(15*(c - d)*d^2*(c + d)
^3*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + (4*a^2*(c + 5*d)*EllipticF[(e -
Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(15*d^2*(
c + d)^2*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
```

```
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^2}{(c+d\sin(e+fx))^{7/2}}\,dx = \frac{2a^2(c-d)\cos(e+fx)}{5d(c+d)f(c+d\sin(e+fx))^{5/2}} - \frac{(2a)\int \frac{-5ad-a(c+4d)\sin(e+fx)}{(c+d\sin(e+fx))^{5/2}}\,dx}{5d(c+d)}$$

$$= \frac{2a^2(c-d)\cos(e+fx)}{5d(c+d)f(c+d\sin(e+fx))^{5/2}} - \frac{4a^2(c+5d)\cos(e+fx)}{15d(c+d)^2f(c+d\sin(e+fx))^{3/2}} + \frac{(4a)\int \frac{6a(c-d)d+\frac{1}{2}a(c-}{(c+d\sin}}{15(c-d)}$$

$$= \frac{2a^2(c-d)\cos(e+fx)}{5d(c+d)f(c+d\sin(e+fx))^{5/2}} - \frac{4a^2(c+5d)\cos(e+fx)}{15d(c+d)^2f(c+d\sin(e+fx))^{3/2}} - \frac{4a^2\left(c^2+5cd-1\right.}{15(c-d)d(c+d)^3f}$$

$$= \frac{2a^2(c-d)\cos(e+fx)}{5d(c+d)f(c+d\sin(e+fx))^{5/2}} - \frac{4a^2(c+5d)\cos(e+fx)}{15d(c+d)^2f(c+d\sin(e+fx))^{3/2}} - \frac{4a^2\left(c^2+5cd-1\right.}{15(c-d)d(c+d)^3f}$$

$$= \frac{2a^2(c-d)\cos(e+fx)}{5d(c+d)f(c+d\sin(e+fx))^{5/2}} - \frac{4a^2(c+5d)\cos(e+fx)}{15d(c+d)^2f(c+d\sin(e+fx))^{3/2}} - \frac{4a^2\left(c^2+5cd-1\right.}{15(c-d)d(c+d)^3f}$$

$$= \frac{2a^2(c-d)\cos(e+fx)}{5d(c+d)f(c+d\sin(e+fx))^{5/2}} - \frac{4a^2(c+5d)\cos(e+fx)}{15d(c+d)^2f(c+d\sin(e+fx))^{3/2}} - \frac{4a^2\left(c^2+5cd-1\right.}{15(c-d)d(c+d)^3f}$$

Mathematica [A] time = 2.02668, size = 283, normalized size = 0.88

$$\frac{2a^2(\sin(e+fx)+1)^2\left(d\cos(e+fx)\left(-2\left(c^2+5cd-12d^2\right)(c+d\sin(e+fx))^2-2(c-d)(c+5d)(c+d)(c+d\sin(e+fx)\right.\right.}{15d^2f(c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2/(c + d*Sin[e + f*x])^(7/2),x]
```

```
[Out] (2*a^2*(1 + Sin[e + f*x])^2*(-2*((11*c - 5*d)*d^2*EllipticF[(-2*e + Pi - 2*
f*x)/4, (2*d)/(c + d)] - (c^2 + 5*c*d - 12*d^2)*((c + d)*EllipticE[(-2*e +
Pi - 2*f*x)/4, (2*d)/(c + d)] - c*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c
 + d)]))*(c + d*Sin[e + f*x])^2*Sqrt[(c + d*Sin[e + f*x])/(c + d)] + d*Cos[
e + f*x]*(3*(c - d)^2*(c + d)^2 - 2*(c - d)*(c + d)*(c + 5*d)*(c + d*Sin[e
+ f*x]) - 2*(c^2 + 5*c*d - 12*d^2)*(c + d*Sin[e + f*x])^2)))/(15*(c - d)*d^
2*(c + d)^3*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*(c + d*Sin[e + f*x])^
(5/2))
```

Maple [B] time = 5.667, size = 1436, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^(7/2),x)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*a^2*((c^2-2*c*d+d^2)/d^2*(2/5/(c^2-
d^2)/d^2*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(sin(f*x+e)+c/d)^3+16/15*c
/(c^2-d^2)^2/d*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(sin(f*x+e)+c/d)^2+2
/15*d*cos(f*x+e)^2/(c^2-d^2)^3*(23*c^2+9*d^2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e
```

```
)^2)^(1/2)+2*(15*c^3+17*c*d^2)/(15*c^6-45*c^4*d^2+45*c^2*d^4-15*d^6)*(c/d-1
)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+
e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+
d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+2/15*d*(23*c^2+9*d^2)/(c^2-
d^2)^3*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2
)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*(
(-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+Elli
pticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+1/d^2*(2*d*cos(
f*x+e)^2/(c^2-d^2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2*c/(c^2-d^2)*(c
/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(
f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(
((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+2/(c^2-d^2)*d*(c/d-1)*(
(c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-
1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*Ellipti
cE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(
f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+2*(-c+d)/d^2*(2/3/(c^2-d^2)/d*(
-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(sin(f*x+e)+c/d)^2+8/3*d*cos(f*x+e)^
2/(c^2-d^2)^2*c/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2*(3*c^2+d^2)/(3*c^
4-6*c^2*d^2+3*d^4)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))
/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e
)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+8/
3*c*d/(c^2-d^2)^2*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/
(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)
^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^
(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))))/cos
(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^2}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^2/(d*sin(f*x + e) + c)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(a^2\cos\left(fx+e\right)^2-2\,a^2\sin\left(fx+e\right)-2\,a^2\right)\sqrt{d\sin\left(fx+e\right)+c}}{d^4\cos\left(fx+e\right)^4+c^4+6\,c^2d^2+d^4-2\left(3\,c^2d^2+d^4\right)\cos\left(fx+e\right)^2-4\left(cd^3\cos\left(fx+e\right)^2-c^3d-cd^3\right)\sin\left(\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^(7/2),x, algorithm="fricas")
```

```
[Out] integral(-(a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x + e) - 2*a^2)*sqrt(d*sin(f*x
+ e) + c)/(d^4*cos(f*x + e)^4 + c^4 + 6*c^2*d^2 + d^4 - 2*(3*c^2*d^2 + d^4)
*cos(f*x + e)^2 - 4*(c*d^3*cos(f*x + e)^2 - c^3*d - c*d^3)*sin(f*x + e)), x
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2/(c+d*sin(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{2}}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^2/(d*sin(f*x + e) + c)^(7/2), x)
```

3.496 $\int (a+a\sin(e+fx))^3(c+d\sin(e+fx))^{5/2}\,dx$

Optimal. Leaf size=467

$$-\frac{4a^3\left(4c^2-33cd+189d^2\right)\cos(e+fx)(c+d\sin(e+fx))^{5/2}}{693d^2f}-\frac{4a^3\left(-33c^2d+4c^3+182cd^2+231d^3\right)\cos(e+fx)(c+d}{693d^2f}$$

```
[Out] (-4*a^3*(4*c^4 - 33*c^3*d + 177*c^2*d^2 + 561*c*d^3 + 315*d^4)*Cos[e + f*x]
*Sqrt[c + d*Sin[e + f*x]])/(693*d^2*f) - (4*a^3*(4*c^3 - 33*c^2*d + 182*c*d
^2 + 231*d^3)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(3/2))/(693*d^2*f) - (4*a^3
*(4*c^2 - 33*c*d + 189*d^2)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(5/2))/(693*d
^2*f) + (8*a^3*(c - 6*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(7/2))/(99*d^2*f
) - (2*Cos[e + f*x]*(a^3 + a^3*Sin[e + f*x])*(c + d*Sin[e + f*x])^(7/2))/(1
1*d*f) + (4*a^3*(c + 3*d)*(4*c^4 - 45*c^3*d + 309*c^2*d^2 + 525*c*d^3 + 231
*d^4)*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]]
)/(693*d^3*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) - (4*a^3*(c^2 - d^2)*(4*c^
4 - 33*c^3*d + 177*c^2*d^2 + 561*c*d^3 + 315*d^4)*EllipticF[(e - Pi/2 + f*x
)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(693*d^3*f*Sqrt[c +
 d*Sin[e + f*x]])
```

Rubi [A] time = 1.03167, antiderivative size = 467, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2763, 2968, 3023, 2753, 2752, 2663, 2661, 2655, 2653}

$$-\frac{4a^3\left(4c^2-33cd+189d^2\right)\cos(e+fx)(c+d\sin(e+fx))^{5/2}}{693d^2f}-\frac{4a^3\left(-33c^2d+4c^3+182cd^2+231d^3\right)\cos(e+fx)(c+d}{693d^2f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (-4*a^3*(4*c^4 - 33*c^3*d + 177*c^2*d^2 + 561*c*d^3 + 315*d^4)*Cos[e + f*x]
*Sqrt[c + d*Sin[e + f*x]])/(693*d^2*f) - (4*a^3*(4*c^3 - 33*c^2*d + 182*c*d
^2 + 231*d^3)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(3/2))/(693*d^2*f) - (4*a^3
*(4*c^2 - 33*c*d + 189*d^2)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(5/2))/(693*d
^2*f) + (8*a^3*(c - 6*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(7/2))/(99*d^2*f
) - (2*Cos[e + f*x]*(a^3 + a^3*Sin[e + f*x])*(c + d*Sin[e + f*x])^(7/2))/(1
1*d*f) + (4*a^3*(c + 3*d)*(4*c^4 - 45*c^3*d + 309*c^2*d^2 + 525*c*d^3 + 231
*d^4)*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]]
)/(693*d^3*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) - (4*a^3*(c^2 - d^2)*(4*c^
4 - 33*c^3*d + 177*c^2*d^2 + 561*c*d^3 + 315*d^4)*EllipticF[(e - Pi/2 + f*x
)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(693*d^3*f*Sqrt[c +
 d*Sin[e + f*x]])
```

Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
```

```
0]))
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))^3(c+d\sin(e+fx))^{5/2}\,dx = -\frac{2\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)(c+d\sin(e+fx))^{7/2}}{11df}+\frac{2\int(a+$$

$$= -\frac{2\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)(c+d\sin(e+fx))^{7/2}}{11df}+\frac{2\int(c+$$

$$= \frac{8a^3(c-6d)\cos(e+fx)(c+d\sin(e+fx))^{7/2}}{99d^2f}-\frac{2\cos(e+fx)\left(a^3+a^3\right.}{}$$

$$= -\frac{4a^3\left(4c^2-33cd+189d^2\right)\cos(e+fx)(c+d\sin(e+fx))^{5/2}}{693d^2f}+\frac{8a^3(c}{}$$

$$= -\frac{4a^3\left(4c^3-33c^2d+182cd^2+231d^3\right)\cos(e+fx)(c+d\sin(e+fx))^3}{693d^2f}$$

$$= -\frac{4a^3\left(4c^4-33c^3d+177c^2d^2+561cd^3+315d^4\right)\cos(e+fx)\sqrt{c+d\,\mathrm{s}}}{693d^2f}$$

$$= -\frac{4a^3\left(4c^4-33c^3d+177c^2d^2+561cd^3+315d^4\right)\cos(e+fx)\sqrt{c+d\,\mathrm{s}}}{693d^2f}$$

$$= -\frac{4a^3\left(4c^4-33c^3d+177c^2d^2+561cd^3+315d^4\right)\cos(e+fx)\sqrt{c+d\,\mathrm{s}}}{693d^2f}$$

$$= -\frac{4a^3\left(4c^4-33c^3d+177c^2d^2+561cd^3+315d^4\right)\cos(e+fx)\sqrt{c+d\,\mathrm{s}}}{693d^2f}$$

Mathematica [A] time = 1.89943, size = 377, normalized size = 0.81

$$\frac{a^3(\sin(e+fx)+1)^3\left(d(c+d\sin(e+fx))\left(-4d\left(990c^2d+6c^3+2401cd^2+1155d^3\right)\sin(2(e+fx))+d^2\left(452c^2+2508\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (a^3*(1 + Sin[e + f*x])^3*(-32*(d^2*(c^4 + 858*c^3*d + 1668*c^2*d^2 + 1254*
c*d^3 + 315*d^4)*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)] + (4*c^5 -
 33*c^4*d + 174*c^3*d^2 + 1452*c^2*d^3 + 1806*c*d^4 + 693*d^5)*((c + d)*Ell
ipticE[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)] - c*EllipticF[(-2*e + Pi - 2*f
*x)/4, (2*d)/(c + d)]))*Sqrt[(c + d*Sin[e + f*x])/(c + d)] + d*(c + d*Sin[e
 + f*x])*(2*(32*c^4 - 264*c^3*d - 8994*c^2*d^2 - 13926*c*d^3 - 5859*d^4)*Co
s[e + f*x] + d^2*(452*c^2 + 2508*c*d + 1701*d^2)*Cos[3*(e + f*x)] - 63*d^4*
Cos[5*(e + f*x)] - 4*d*(6*c^3 + 990*c^2*d + 2401*c*d^2 + 1155*d^3)*Sin[2*(e
 + f*x)] + 14*d^3*(23*c + 33*d)*Sin[4*(e + f*x)])))/(5544*d^3*f*(Cos[(e + f
*x)/2] + Sin[(e + f*x)/2])^6*Sqrt[c + d*Sin[e + f*x]])
```

Maple [B] time = 1.256, size = 1926, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(c+d*sin(f*x+e))^(5/2),x)
```

```
[Out] 2/693*a^3*(224*c*d^6*sin(f*x+e)^6-72*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+s
in(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*s
in(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c^5*d^2+274*c^2*d^5*sin(f*x+e)
^5+858*c*d^6*sin(f*x+e)^5+116*c^3*d^4*sin(f*x+e)^4+1122*c^2*d^5*sin(f*x+e)^
4+1274*c*d^6*sin(f*x+e)^4-c^4*d^3*sin(f*x+e)^3+528*c^3*d^4*sin(f*x+e)^3+194
2*c^2*d^5*sin(f*x+e)^3+1188*c*d^6*sin(f*x+e)^3-4*c^5*d^2*sin(f*x+e)^2+33*c^
4*d^3*sin(f*x+e)^2+980*c^3*d^4*sin(f*x+e)^2+462*c^2*d^5*sin(f*x+e)^2-868*c*
d^6*sin(f*x+e)^2+c^4*d^3*sin(f*x+e)-528*c^3*d^4*sin(f*x+e)-2216*c^2*d^5*sin
(f*x+e)-2046*c*d^6*sin(f*x+e)-2016*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin
(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin
(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*d^7-8*((c+d*sin(f*x+e))/(c-d))^(
1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*Ellip
ticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c^7+1386*((c+d*sin
(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(
c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*d
^7+63*d^7*sin(f*x+e)^7+231*d^7*sin(f*x+e)^6+315*d^7*sin(f*x+e)^5+231*d^7*si
n(f*x+e)^4+252*d^7*sin(f*x+e)^3-462*d^7*sin(f*x+e)^2-630*d^7*sin(f*x+e)-158
4*c^2*d^5-630*c*d^6-33*c^4*d^3+4*c^5*d^2-1096*c^3*d^4+2128*((c+d*sin(f*x+e)
)/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(
1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c^4*d^3+
4176*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1
+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(
c+d))^(1/2))*c^3*d^4-120*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d
/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(
c-d))^(1/2),((c-d)/(c+d))^(1/2))*c^2*d^5-4104*((c+d*sin(f*x+e))/(c-d))^(1/2
)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*Elliptic
F(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c*d^6+66*((c+d*sin(f*
x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d
))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c^6*
d-340*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(
1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/
(c+d))^(1/2))*c^5*d^2-2970*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))
*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))
/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c^4*d^3-3264*((c+d*sin(f*x+e))/(c-d))^(1
/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*Ellipt
icE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c^3*d^4+1518*((c+d*
sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e)
)/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)
)*c^2*d^5+3612*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1
/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2
),((c-d)/(c+d))^(1/2))*c*d^6+8*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x
+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x
+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c^6*d)/d^4/cos(f*x+e)/(c+d*sin(f*x+e
))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^3 \left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c+d*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^3*(d*sin(f*x + e) + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(4\,a^3c^2 + 8\,a^3cd + 4\,a^3d^2 + \left(2\,a^3cd + 3\,a^3d^2\right)\cos\left(fx+e\right)^4 - \left(3\,a^3c^2 + 10\,a^3cd + 7\,a^3d^2\right)\cos\left(fx+e\right)^2 + \left(a^3\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c+d*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((4*a^3*c^2 + 8*a^3*c*d + 4*a^3*d^2 + (2*a^3*c*d + 3*a^3*d^2)*cos(f
*x + e)^4 - (3*a^3*c^2 + 10*a^3*c*d + 7*a^3*d^2)*cos(f*x + e)^2 + (a^3*d^2*
cos(f*x + e)^4 + 4*a^3*c^2 + 8*a^3*c*d + 4*a^3*d^2 - (a^3*c^2 + 6*a^3*c*d +
 5*a^3*d^2)*cos(f*x + e)^2)*sin(f*x + e))*sqrt(d*sin(f*x + e) + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(c+d*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^3\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c+d*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^3*(d*sin(f*x + e) + c)^(5/2), x)
```

3.497 $\int (a + a\sin(e + fx))^3(c + d\sin(e + fx))^{3/2}\,dx$

Optimal. Leaf size=390

$$-\frac{4a^3\left(4c^2-27cd+119d^2\right)\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{315d^2f}-\frac{4a^3\left(-27c^2d+4c^3+114cd^2+165d^3\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{315d^2f}$$

```
[Out] (-4*a^3*(4*c^3 - 27*c^2*d + 114*c*d^2 + 165*d^3)*Cos[e + f*x]*Sqrt[c + d*Si
n[e + f*x]])/(315*d^2*f) - (4*a^3*(4*c^2 - 27*c*d + 119*d^2)*Cos[e + f*x]*(
c + d*Sin[e + f*x])^(3/2))/(315*d^2*f) + (8*a^3*(c - 5*d)*Cos[e + f*x]*(c +
 d*Sin[e + f*x])^(5/2))/(63*d^2*f) - (2*Cos[e + f*x]*(a^3 + a^3*Sin[e + f*x
])*(c + d*Sin[e + f*x])^(5/2))/(9*d*f) + (4*a^3*(4*c^4 - 27*c^3*d + 111*c^2
*d^2 + 579*c*d^3 + 357*d^4)*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sq
rt[c + d*Sin[e + f*x]])/(315*d^3*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) - (4
*a^3*(c^2 - d^2)*(4*c^3 - 27*c^2*d + 114*c*d^2 + 165*d^3)*EllipticF[(e - Pi
/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(315*d^3*f*
Sqrt[c + d*Sin[e + f*x]])
```

Rubi [A] time = 0.775134, antiderivative size = 390, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2763, 2968, 3023, 2753, 2752, 2663, 2661, 2655, 2653}

$$-\frac{4a^3\left(4c^2-27cd+119d^2\right)\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{315d^2f}-\frac{4a^3\left(-27c^2d+4c^3+114cd^2+165d^3\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{315d^2f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^(3/2),x]
```


```
[Out] (-4*a^3*(4*c^3 - 27*c^2*d + 114*c*d^2 + 165*d^3)*Cos[e + f*x]*Sqrt[c + d*Si
n[e + f*x]])/(315*d^2*f) - (4*a^3*(4*c^2 - 27*c*d + 119*d^2)*Cos[e + f*x]*(
c + d*Sin[e + f*x])^(3/2))/(315*d^2*f) + (8*a^3*(c - 5*d)*Cos[e + f*x]*(c +
 d*Sin[e + f*x])^(5/2))/(63*d^2*f) - (2*Cos[e + f*x]*(a^3 + a^3*Sin[e + f*x
])*(c + d*Sin[e + f*x])^(5/2))/(9*d*f) + (4*a^3*(4*c^4 - 27*c^3*d + 111*c^2
*d^2 + 579*c*d^3 + 357*d^4)*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sq
rt[c + d*Sin[e + f*x]])/(315*d^3*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) - (4
*a^3*(c^2 - d^2)*(4*c^3 - 27*c^2*d + 114*c*d^2 + 165*d^3)*EllipticF[(e - Pi
/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(315*d^3*f*
Sqrt[c + d*Sin[e + f*x]])
```


Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))^3(c+d\sin(e+fx))^{3/2}\,dx = -\frac{2\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)(c+d\sin(e+fx))^{5/2}}{9df}+\frac{2\int(a+a\sin\cdots}{\cdots}$$

$$= -\frac{2\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)(c+d\sin(e+fx))^{5/2}}{9df}+\frac{2\int(c+d\sin\cdots}{\cdots}$$

$$= \frac{8a^3(c-5d)\cos(e+fx)(c+d\sin(e+fx))^{5/2}}{63d^2f}-\frac{2\cos(e+fx)\left(a^3+a^3\sin\cdots\right.}{\cdots}$$

$$= -\frac{4a^3\left(4c^2-27cd+119d^2\right)\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{315d^2f}+\frac{8a^3(c-5\cdots}{\cdots}$$

$$= -\frac{4a^3\left(4c^3-27c^2d+114cd^2+165d^3\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{315d^2f}-\cdots$$

$$= -\frac{4a^3\left(4c^3-27c^2d+114cd^2+165d^3\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{315d^2f}-\cdots$$

$$= -\frac{4a^3\left(4c^3-27c^2d+114cd^2+165d^3\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{315d^2f}-\cdots$$

$$= -\frac{4a^3\left(4c^3-27c^2d+114cd^2+165d^3\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{315d^2f}-\cdots$$

Mathematica [A] time = 2.30345, size = 318, normalized size = 0.82

$$\frac{a^3(\sin(e+fx)+1)^3\left(d(c+d\sin(e+fx))\left(\left(-216c^2d+32c^3-3828cd^2-2910d^3\right)\cos(e+fx)+2d\left(5d(10c+27d)\cos(3(\cdots\right.\right.\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] (a^3*(1 + Sin[e + f*x])^3*(-16*(d^2*(c^3 + 387*c^2*d + 471*c*d^2 + 165*d^3)
*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)] + (4*c^4 - 27*c^3*d + 111*
c^2*d^2 + 579*c*d^3 + 357*d^4)*((c + d)*EllipticE[(-2*e + Pi - 2*f*x)/4, (2
*d)/(c + d)] - c*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]))*Sqrt[(c
+ d*Sin[e + f*x])/(c + d)] + d*(c + d*Sin[e + f*x])*((32*c^3 - 216*c^2*d -
3828*c*d^2 - 2910*d^3)*Cos[e + f*x] + 2*d*(5*d*(10*c + 27*d)*Cos[3*(e + f*x
)] - (6*c^2 + 432*c*d + 511*d^2 - 35*d^2*Cos[2*(e + f*x)])*Sin[2*(e + f*x)]
))))/(1260*d^3*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6*Sqrt[c + d*Sin[e +
 f*x]])
```

Maple [B] time = 1.2, size = 1613, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(c+d*sin(f*x+e))^(3/2),x)
```

```
[Out] 2/315*a^3*(-1044*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^
(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1
/2),((c-d)/(c+d))^(1/2))*d^6-8*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x
+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x
+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c^6+714*((c+d*sin(f*x+e))/(c-d))^(1/
2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*Ellipti
cE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*d^6+4*c^4*d^2+8*((c+
d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+
e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/
2))*c^5*d+c^3*d^3*sin(f*x+e)-243*c^2*d^4*sin(f*x+e)-704*c*d^5*sin(f*x+e)+85
*c*d^5*sin(f*x+e)^5+53*c^2*d^4*sin(f*x+e)^4+351*c*d^5*sin(f*x+e)^4-c^3*d^3*
sin(f*x+e)^3+243*c^2*d^4*sin(f*x+e)^3+619*c*d^5*sin(f*x+e)^3-4*c^4*d^2*sin(
f*x+e)^2+27*c^3*d^3*sin(f*x+e)^2+413*c^2*d^4*sin(f*x+e)^2-21*c*d^5*sin(f*x+
e)^2-60*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d
*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d
)/(c+d))^(1/2))*c^4*d^2+1048*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e
))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e
))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c^3*d^3+1104*((c+d*sin(f*x+e))/(c-d))^
(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*Elli
pticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c^2*d^4-1056*((c+
d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+
e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/
2))*c*d^5+54*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2
)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),
((c-d)/(c+d))^(1/2))*c^5*d-214*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x
+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x
+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c^4*d^2-1212*((c+d*sin(f*x+e))/(c-d)
)^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*El
lipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c^3*d^3-492*((c
+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x
+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1
/2))*c^2*d^4+1158*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))
^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(
1/2),((c-d)/(c+d))^(1/2))*c*d^5-466*c^2*d^4+135*d^6*sin(f*x+e)^5+203*d^6*si
n(f*x+e)^4+195*d^6*sin(f*x+e)^3-238*d^6*sin(f*x+e)^2-330*d^6*sin(f*x+e)+35*
d^6*sin(f*x+e)^6-27*c^3*d^3-330*c*d^5)/d^4/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2
)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a \sin\left(fx+e\right)+a\right)^{3} \left(d \sin\left(fx+e\right)+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c+d*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^3*(d*sin(f*x + e) + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

integral $\left(\left(a^{3} d \cos\left(fx+e\right)^{4} + 4\,a^{3} c + 4\,a^{3} d - \left(3\,a^{3} c + 5\,a^{3} d\right) \cos\left(fx+e\right)^{2} + \left(4\,a^{3} c + 4\,a^{3} d - \left(a^{3} c + 3\,a^{3} d\right) \cos\left(fx+\right.\right.\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c+d*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a^3*d*cos(f*x + e)^4 + 4*a^3*c + 4*a^3*d - (3*a^3*c + 5*a^3*d)*co
s(f*x + e)^2 + (4*a^3*c + 4*a^3*d - (a^3*c + 3*a^3*d)*cos(f*x + e)^2)*sin(f
*x + e))*sqrt(d*sin(f*x + e) + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(c+d*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^{3}\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c+d*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^3*(d*sin(f*x + e) + c)^(3/2), x)
```

3.498 $\int (a+a\sin(e+fx))^3\sqrt{c+d\sin(e+fx)}\,dx$

Optimal. Leaf size=318

$$-\frac{4a^3\left(4c^2-21cd+65d^2\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{105d^2f}-\frac{4a^3\left(c^2-d^2\right)\left(4c^2-21cd+65d^2\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+f\right.\right.}{105d^3f\sqrt{c+d\sin(e+fx)}}$$

```
[Out] (-4*a^3*(4*c^2 - 21*c*d + 65*d^2)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(1
05*d^2*f) + (8*a^3*(c - 4*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(3/2))/(35*d
^2*f) - (2*Cos[e + f*x]*(a^3 + a^3*Sin[e + f*x])*(c + d*Sin[e + f*x])^(3/2)
)/(7*d*f) + (4*a^3*(4*c^3 - 21*c^2*d + 62*c*d^2 + 147*d^3)*EllipticE[(e - P
i/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(105*d^3*f*Sqrt[(c +
 d*Sin[e + f*x])/(c + d)]) - (4*a^3*(c^2 - d^2)*(4*c^2 - 21*c*d + 65*d^2)*E
llipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c +
d)])/(105*d^3*f*Sqrt[c + d*Sin[e + f*x]])
```

Rubi [A] time = 0.579793, antiderivative size = 318, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2763, 2968, 3023, 2753, 2752, 2663, 2661, 2655, 2653}

$$-\frac{4a^3\left(4c^2-21cd+65d^2\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{105d^2f}-\frac{4a^3\left(c^2-d^2\right)\left(4c^2-21cd+65d^2\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+f\right.\right.}{105d^3f\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3*Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] (-4*a^3*(4*c^2 - 21*c*d + 65*d^2)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(1
05*d^2*f) + (8*a^3*(c - 4*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(3/2))/(35*d
^2*f) - (2*Cos[e + f*x]*(a^3 + a^3*Sin[e + f*x])*(c + d*Sin[e + f*x])^(3/2)
)/(7*d*f) + (4*a^3*(4*c^3 - 21*c^2*d + 62*c*d^2 + 147*d^3)*EllipticE[(e - P
i/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(105*d^3*f*Sqrt[(c +
 d*Sin[e + f*x])/(c + d)]) - (4*a^3*(c^2 - d^2)*(4*c^2 - 21*c*d + 65*d^2)*E
llipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c +
d)])/(105*d^3*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
```

```
x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))^3\sqrt{c+d\sin(e+fx)}\,dx = -\frac{2\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)(c+d\sin(e+fx))^{3/2}}{7df}+\frac{2\int(a+a\text{s}}{}$$

$$= -\frac{2\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)(c+d\sin(e+fx))^{3/2}}{7df}+\frac{2\int\sqrt{c+d}}{}$$

$$= \frac{8a^3(c-4d)\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{35d^2f}-\frac{2\cos(e+fx)\left(a^3+a^3\text{s}\right.}{}$$

$$= -\frac{4a^3\left(4c^2-21cd+65d^2\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{105d^2f}+\frac{8a^3(c-4d)}{}$$

$$= -\frac{4a^3\left(4c^2-21cd+65d^2\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{105d^2f}+\frac{8a^3(c-4d)}{}$$

$$= -\frac{4a^3\left(4c^2-21cd+65d^2\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{105d^2f}+\frac{8a^3(c-4d)}{}$$

$$= -\frac{4a^3\left(4c^2-21cd+65d^2\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{105d^2f}+\frac{8a^3(c-4d)}{}$$

Mathematica [A] time = 2.74104, size = 266, normalized size = 0.84

$$-\frac{a^3\left(-2d\cos(e+fx)\left(d\left(4c^2-336cd-565d^2\right)\sin(e+fx)-84c^2d+16c^3+18d^2(2c+7d)\cos(2(e+fx))-556cd^2+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3*Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] -(a^3*(16*(4*c^4 - 17*c^3*d + 41*c^2*d^2 + 209*c*d^3 + 147*d^4)*EllipticE[(
-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)] - 1
6*(4*c^4 - 21*c^3*d + 61*c^2*d^2 + 21*c*d^3 - 65*d^4)*EllipticF[(-2*e + Pi
- 2*f*x)/4, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)] - 2*d*Cos[e +
 f*x]*(16*c^3 - 84*c^2*d - 556*c*d^2 - 126*d^3 + 18*d^2*(2*c + 7*d)*Cos[2*(
e + f*x)] + d*(4*c^2 - 336*c*d - 565*d^2)*Sin[e + f*x] + 15*d^3*Sin[3*(e +
f*x)])))/(420*d^3*f*Sqrt[c + d*Sin[e + f*x]])
```

Maple [B] time = 1.246, size = 1316, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(c+d*sin(f*x+e))^(1/2),x)
```

```
[Out] 2/105*a^3*(18*c*d^4*sin(f*x+e)^4-21*c^2*d^3-130*c*d^4-48*((c+d*sin(f*x+e))/
(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/
2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c^3*d^2-c^
2*d^3*sin(f*x+e)^3+84*c*d^4*sin(f*x+e)^3-4*c^3*d^2*sin(f*x+e)^2+21*c^2*d^3*
```

```
sin(f*x+e)^2+112*c*d^4*sin(f*x+e)^2+c^2*d^3*sin(f*x+e)-84*c*d^4*sin(f*x+e)+
15*d^5*sin(f*x+e)^5+63*d^5*sin(f*x+e)^4+115*d^5*sin(f*x+e)^3-63*d^5*sin(f*x
+e)^2-130*d^5*sin(f*x+e)+4*c^3*d^2-424*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1
+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d
*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*d^5-8*((c+d*sin(f*x+e))/(c-d
))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*E
llipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c^5+294*((c+d*
sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e)
)/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)
)*d^5+416*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(
-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c
-d)/(c+d))^(1/2))*c^2*d^3+48*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e
))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e
))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c*d^4+42*((c+d*sin(f*x+e))/(c-d))^(1/2
)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*Elliptic
E(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c^4*d-116*((c+d*sin(f
*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-
d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c^3
*d^2-336*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-
d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-
d)/(c+d))^(1/2))*c^2*d^3+124*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e
))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e
))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c*d^4+8*((c+d*sin(f*x+e))/(c-d))^(1/2)
*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF
(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c^4*d)/d^4/cos(f*x+e)/
(c+d*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^{3}\sqrt{d\sin\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c+d*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^3*sqrt(d*sin(f*x + e) + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\left(3\,a^{3}\cos\left(fx+e\right)^{2}-4\,a^{3}+\left(a^{3}\cos\left(fx+e\right)^{2}-4\,a^{3}\right)\sin\left(fx+e\right)\right)\sqrt{d\sin\left(fx+e\right)+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c+d*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-(3*a^3*cos(f*x + e)^2 - 4*a^3 + (a^3*cos(f*x + e)^2 - 4*a^3)*sin(
f*x + e))*sqrt(d*sin(f*x + e) + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^{3}\left(\int 3\sqrt{c+d\sin\left(e+fx\right)}\sin\left(e+fx\right)dx+\int 3\sqrt{c+d\sin\left(e+fx\right)}\sin^{2}\left(e+fx\right)dx+\int \sqrt{c+d\sin\left(e+fx\right)}\sin^{3}\left(e+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] a**3*(Integral(3*sqrt(c + d*sin(e + f*x))*sin(e + f*x), x) + Integral(3*sqr
t(c + d*sin(e + f*x))*sin(e + f*x)**2, x) + Integral(sqrt(c + d*sin(e + f*x
))*sin(e + f*x)**3, x) + Integral(sqrt(c + d*sin(e + f*x)), x))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c+d*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.499 $\int \frac{(a+a\sin(e+fx))^3}{\sqrt{c+d\sin(e+fx)}}\,dx$

Optimal. Leaf size=258

$$-\frac{4a^3(c-d)\left(4c^2-11cd+15d^2\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{15d^3f\sqrt{c+d\sin(e+fx)}}+\frac{4a^3\left(4c^2-15cd+27d^2\right)\sqrt{c+d\sin(e+fx)}E\left(\frac{1}{2}\left(\right.\right.}{15d^3f\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

```
[Out] (8*a^3*(c - 3*d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(15*d^2*f) - (2*Cos
[e + f*x]*(a^3 + a^3*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]])/(5*d*f) + (4*a
^3*(4*c^2 - 15*c*d + 27*d^2)*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*S
qrt[c + d*Sin[e + f*x]])/(15*d^3*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) - (4
*a^3*(c - d)*(4*c^2 - 11*c*d + 15*d^2)*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/
(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(15*d^3*f*Sqrt[c + d*Sin[e + f
*x]])
```

Rubi [A] time = 0.476269, antiderivative size = 258, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2763, 2968, 3023, 2752, 2663, 2661, 2655, 2653}

$$-\frac{4a^3(c-d)\left(4c^2-11cd+15d^2\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{15d^3f\sqrt{c+d\sin(e+fx)}}+\frac{4a^3\left(4c^2-15cd+27d^2\right)\sqrt{c+d\sin(e+fx)}E\left(\frac{1}{2}\left(\right.\right.}{15d^3f\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3/Sqrt[c + d*Sin[e + f*x]],x]
```


```
[Out] (8*a^3*(c - 3*d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(15*d^2*f) - (2*Cos
[e + f*x]*(a^3 + a^3*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]])/(5*d*f) + (4*a
^3*(4*c^2 - 15*c*d + 27*d^2)*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*S
qrt[c + d*Sin[e + f*x]])/(15*d^3*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) - (4
*a^3*(c - d)*(4*c^2 - 11*c*d + 15*d^2)*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/
(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(15*d^3*f*Sqrt[c + d*Sin[e + f
*x]])
```


Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^3}{\sqrt{c+d\sin(e+fx)}}\,dx = -\frac{2\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)\sqrt{c+d\sin(e+fx)}}{5df} + \frac{2\int \frac{(a+a\sin(e+fx))\left(a^2(c+3d)-2a^2(c-3d)\,\mathrm{s}\right.}{\sqrt{c+d\sin(e+fx)}}}{5d}$$

$$= -\frac{2\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)\sqrt{c+d\sin(e+fx)}}{5df} + \frac{2\int \frac{a^3(c+3d)+\left(-2a^3(c-3d)+a^3(c+3d)\right)\sin(}{\sqrt{c+d\sin(e+}}}{5d}$$

$$= \frac{8a^3(c-3d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15d^2f} - \frac{2\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)\sqrt{c+d\sin(}}{5df}$$

$$= \frac{8a^3(c-3d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15d^2f} - \frac{2\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)\sqrt{c+d\sin(}}{5df}$$

$$= \frac{8a^3(c-3d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15d^2f} - \frac{2\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)\sqrt{c+d\sin(}}{5df}$$

$$= \frac{8a^3(c-3d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15d^2f} - \frac{2\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)\sqrt{c+d\sin(}}{5df}$$

Mathematica [A] time = 1.63038, size = 246, normalized size = 0.95

$$-\frac{a^3(\sin(e+fx)+1)^3\left(-d\cos(e+fx)\left(8c^2+2d(c-15d)\sin(e+fx)-30cd+3d^2\cos(2(e+fx))-3d^2\right)-4\left(-15c^2d+4\right.\right.}{15d^3f\left(\sin\left(\frac{1}{2}(e+fx)\right)\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3/Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] -(a^3*(1 + Sin[e + f*x])^3*(4*(4*c^3 - 11*c^2*d + 12*c*d^2 + 27*d^3)*Ellipt
icE[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)
] - 4*(4*c^3 - 15*c^2*d + 26*c*d^2 - 15*d^3)*EllipticF[(-2*e + Pi - 2*f*x)/
4, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)] - d*Cos[e + f*x]*(8*c^
2 - 30*c*d - 3*d^2 + 3*d^2*Cos[2*(e + f*x)] + 2*(c - 15*d)*d*Sin[e + f*x]))
)/(15*d^3*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6*Sqrt[c + d*Sin[e + f*x]
])
```

Maple [B] time = 1.255, size = 1035, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^(1/2),x)
```

```
[Out] 2/15*a^3*(8*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)
*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),(
(c-d)/(c+d))^(1/2))*c^3*d-36*c^2*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f
*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f
*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*d^2+112*c*((c+d*sin(f*x+e))/(c-d))
```

```
^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*Ell
ipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*d^3-84*((c+d*sin
(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(
c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*d
^4-8*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1
+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(
c+d))^(1/2))*c^4+30*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d
))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))
^(1/2),((c-d)/(c+d))^(1/2))*c^3*d-46*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+s
in(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*s
in(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c^2*d^2-30*((c+d*sin(f*x+e))/(
c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2
)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c*d^3+54*((
c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*
x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(
1/2))*d^4+3*d^4*sin(f*x+e)^4-c*d^3*sin(f*x+e)^3+15*d^4*sin(f*x+e)^3-4*c^2*d
^2*sin(f*x+e)^2+15*c*d^3*sin(f*x+e)^2-3*d^4*sin(f*x+e)^2+c*d^3*sin(f*x+e)-1
5*d^4*sin(f*x+e)+4*c^2*d^2-15*c*d^3)/d^4/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/
f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{3}}{\sqrt{d\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^3/sqrt(d*sin(f*x + e) + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{3\,a^{3}\cos\left(fx+e\right)^{2}-4\,a^{3}+\left(a^{3}\cos\left(fx+e\right)^{2}-4\,a^{3}\right)\sin\left(fx+e\right)}{\sqrt{d\sin\left(fx+e\right)+c}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-(3*a^3*cos(f*x + e)^2 - 4*a^3 + (a^3*cos(f*x + e)^2 - 4*a^3)*sin(
f*x + e))/sqrt(d*sin(f*x + e) + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^{3}\left(\int \frac{3\sin\left(e+fx\right)}{\sqrt{c+d\sin\left(e+fx\right)}}\,dx+\int \frac{3\sin^{2}\left(e+fx\right)}{\sqrt{c+d\sin\left(e+fx\right)}}\,dx+\int \frac{\sin^{3}\left(e+fx\right)}{\sqrt{c+d\sin\left(e+fx\right)}}\,dx+\int \frac{1}{\sqrt{c+d\sin\left(e+fx\right)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3/(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] a**3*(Integral(3*sin(e + f*x)/sqrt(c + d*sin(e + f*x)), x) + Integral(3*sin
(e + f*x)**2/sqrt(c + d*sin(e + f*x)), x) + Integral(sin(e + f*x)**3/sqrt(c
 + d*sin(e + f*x)), x) + Integral(1/sqrt(c + d*sin(e + f*x)), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^3}{\sqrt{d\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^3/sqrt(d*sin(f*x + e) + c), x)
```

3.500 $\int \frac{(a+a\sin(e+fx))^3}{(c+d\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=270

$$-\frac{4a^3\left(4c^2-5cd-3d^2\right)\sqrt{c+d\sin(e+fx)}E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3d^3f(c+d)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}-\frac{4a^3(2c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3d^2f(c+d)}+\frac{4a^3(}{}$$

```
[Out] (2*(c - d)*Cos[e + f*x]*(a^3 + a^3*Sin[e + f*x]))/(d*(c + d)*f*Sqrt[c + d*S
in[e + f*x]]) - (4*a^3*(2*c - d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(3*
d^2*(c + d)*f) - (4*a^3*(4*c^2 - 5*c*d - 3*d^2)*EllipticE[(e - Pi/2 + f*x)/
2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(3*d^3*(c + d)*f*Sqrt[(c + d*Si
n[e + f*x])/(c + d)]) + (4*a^3*(4*c - 5*d)*(c - d)*EllipticF[(e - Pi/2 + f*
x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(3*d^3*f*Sqrt[c +
d*Sin[e + f*x]])
```

Rubi [A] time = 0.480551, antiderivative size = 270, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2762, 2968, 3023, 2752, 2663, 2661, 2655, 2653}

$$-\frac{4a^3\left(4c^2-5cd-3d^2\right)\sqrt{c+d\sin(e+fx)}E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3d^3f(c+d)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}-\frac{4a^3(2c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3d^2f(c+d)}+\frac{4a^3(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3/(c + d*Sin[e + f*x])^(3/2),x]
```


```
[Out] (2*(c - d)*Cos[e + f*x]*(a^3 + a^3*Sin[e + f*x]))/(d*(c + d)*f*Sqrt[c + d*S
in[e + f*x]]) - (4*a^3*(2*c - d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(3*
d^2*(c + d)*f) - (4*a^3*(4*c^2 - 5*c*d - 3*d^2)*EllipticE[(e - Pi/2 + f*x)/
2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(3*d^3*(c + d)*f*Sqrt[(c + d*Si
n[e + f*x])/(c + d)]) + (4*a^3*(4*c - 5*d)*(c - d)*EllipticF[(e - Pi/2 + f*
x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(3*d^3*f*Sqrt[c +
d*Sin[e + f*x]])
```


Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^3}{(c+d\sin(e+fx))^{3/2}}\,dx = \frac{2(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{d(c+d)f\sqrt{c+d\sin(e+fx)}} - \frac{(2a)\int \frac{(a+a\sin(e+fx))(a(c-2d)-a(2c-d)\sin(e+fx))}{\sqrt{c+d\sin(e+fx)}}\,d}{d(c+d)}$$

$$= \frac{2(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{d(c+d)f\sqrt{c+d\sin(e+fx)}} - \frac{(2a)\int \frac{a^2(c-2d)+\left(a^2(c-2d)-a^2(2c-d)\right)\sin(e+fx)-a^2(2c}{\sqrt{c+d\sin(e+fx)}}}{d(c+d)}$$

$$= \frac{2(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{d(c+d)f\sqrt{c+d\sin(e+fx)}} - \frac{4a^3(2c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3d^2(c+d)f}$$

$$= \frac{2(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{d(c+d)f\sqrt{c+d\sin(e+fx)}} - \frac{4a^3(2c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3d^2(c+d)f}$$

$$= \frac{2(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{d(c+d)f\sqrt{c+d\sin(e+fx)}} - \frac{4a^3(2c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3d^2(c+d)f}$$

$$= \frac{2(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{d(c+d)f\sqrt{c+d\sin(e+fx)}} - \frac{4a^3(2c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3d^2(c+d)f}$$

Mathematica [A] time = 1.43319, size = 234, normalized size = 0.87

$$-\frac{2a^3(\sin(e+fx)+1)^3\left(d\cos(e+fx)\left(4c^2+d(c+d)\sin(e+fx)-5cd+3d^2\right)+2\left(-5c^2d+4c^3-4cd^2+5d^3\right)\sqrt{\frac{c+d\sin}{c}}\right.}{3d^3f(c+d)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3/(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-2*a^3*(1 + Sin[e + f*x])^3*(-2*(4*c^3 - c^2*d - 8*c*d^2 - 3*d^3)*Elliptic
E[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)]
+ 2*(4*c^3 - 5*c^2*d - 4*c*d^2 + 5*d^3)*EllipticF[(-2*e + Pi - 2*f*x)/4, (2
*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)] + d*Cos[e + f*x]*(4*c^2 - 5
*c*d + 3*d^2 + d*(c + d)*Sin[e + f*x])))/(3*d^3*(c + d)*f*(Cos[(e + f*x)/2]
 + Sin[(e + f*x)/2])^6*Sqrt[c + d*Sin[e + f*x]])
```

Maple [B] time = 1.208, size = 1031, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^(3/2),x)
```

```
[Out] -2/3*(8*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d
*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d
)/(c+d))^(1/2))*c^3*d-16*c^2*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e
))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e
))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*d^2-8*c*((c+d*sin(f*x+e))/(c-d))^(1/2)
*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF
```

```
(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*d^3+16*((c+d*sin(f*x+e
))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^
(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*d^4-8*(
(c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f
*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^
(1/2))*c^4+10*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/
2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2)
,((c-d)/(c+d))^(1/2))*c^3*d+14*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x
+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x
+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c^2*d^2-10*((c+d*sin(f*x+e))/(c-d))^
(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*Elli
pticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c*d^3-6*((c+d*sin
(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(
c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*d
^4-c*d^3*sin(f*x+e)^3-d^4*sin(f*x+e)^3-4*c^2*d^2*sin(f*x+e)^2+5*c*d^3*sin(f
*x+e)^2-3*d^4*sin(f*x+e)^2+c*d^3*sin(f*x+e)+d^4*sin(f*x+e)+4*c^2*d^2-5*c*d^
3+3*d^4)*a^3/d^4/(c+d)/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^3}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^3/(d*sin(f*x + e) + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(3\,a^3\cos\left(fx+e\right)^2-4\,a^3+\left(a^3\cos\left(fx+e\right)^2-4\,a^3\right)\sin\left(fx+e\right)\right)\sqrt{d\sin\left(fx+e\right)+c}}{d^2\cos\left(fx+e\right)^2-2\,cd\sin\left(fx+e\right)-c^2-d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((3*a^3*cos(f*x + e)^2 - 4*a^3 + (a^3*cos(f*x + e)^2 - 4*a^3)*sin(f
*x + e))*sqrt(d*sin(f*x + e) + c)/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e)
- c^2 - d^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3/(c+d*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{3}}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^3/(d*sin(f*x + e) + c)^(3/2), x)
```

3.501 $\int \frac{(a+a\sin(e+fx))^3}{(c+d\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=280

$$\frac{4a^3\left(4c^2+5cd-3d^2\right)\sqrt{c+d\sin(e+fx)}E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3d^3f(c+d)^2\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}+\frac{8a^3(c-d)(c+2d)\cos(e+fx)}{3d^2f(c+d)^2\sqrt{c+d\sin(e+fx)}}-\frac{4a^3(c-d)(4c+5d)}{3d^3f(c}$$

```
[Out] (2*(c - d)*Cos[e + f*x]*(a^3 + a^3*Sin[e + f*x]))/(3*d*(c + d)*f*(c + d*Sin
[e + f*x])^(3/2)) + (8*a^3*(c - d)*(c + 2*d)*Cos[e + f*x])/(3*d^2*(c + d)^2
*f*Sqrt[c + d*Sin[e + f*x]]) + (4*a^3*(4*c^2 + 5*c*d - 3*d^2)*EllipticE[(e
- Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(3*d^3*(c + d)^2*
f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) - (4*a^3*(c - d)*(4*c + 5*d)*Elliptic
F[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(3
*d^3*(c + d)*f*Sqrt[c + d*Sin[e + f*x]])
```

Rubi [A] time = 0.577285, antiderivative size = 280, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2762, 2968, 3021, 2752, 2663, 2661, 2655, 2653}

$$\frac{4a^3\left(4c^2+5cd-3d^2\right)\sqrt{c+d\sin(e+fx)}E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3d^3f(c+d)^2\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}+\frac{8a^3(c-d)(c+2d)\cos(e+fx)}{3d^2f(c+d)^2\sqrt{c+d\sin(e+fx)}}-\frac{4a^3(c-d)(4c+5d)}{3d^3f(c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3/(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (2*(c - d)*Cos[e + f*x]*(a^3 + a^3*Sin[e + f*x]))/(3*d*(c + d)*f*(c + d*Sin
[e + f*x])^(3/2)) + (8*a^3*(c - d)*(c + 2*d)*Cos[e + f*x])/(3*d^2*(c + d)^2
*f*Sqrt[c + d*Sin[e + f*x]]) + (4*a^3*(4*c^2 + 5*c*d - 3*d^2)*EllipticE[(e
- Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(3*d^3*(c + d)^2*
f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) - (4*a^3*(c - d)*(4*c + 5*d)*Elliptic
F[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(3
*d^3*(c + d)*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^3}{(c+d\sin(e+fx))^{5/2}}\,dx = \frac{2(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{3d(c+d)f(c+d\sin(e+fx))^{3/2}} - \frac{(2a)\int \frac{(a+a\sin(e+fx))(a(c-4d)-a(2c+d)\sin(e+fx))}{(c+d\sin(e+fx))^{3/2}}dx}{3d(c+d)}$$

$$= \frac{2(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{3d(c+d)f(c+d\sin(e+fx))^{3/2}} - \frac{(2a)\int \frac{a^2(c-4d)+\left(a^2(c-4d)-a^2(2c+d)\right)\sin(e+fx)-a^2(2c+d)}{(c+d\sin(e+fx))^{3/2}}}{3d(c+d)}$$

$$= \frac{2(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{3d(c+d)f(c+d\sin(e+fx))^{3/2}} + \frac{8a^3(c-d)(c+2d)\cos(e+fx)}{3d^2(c+d)^2f\sqrt{c+d\sin(e+fx)}} + \frac{(4a)\int \frac{\frac{1}{2}a^2(c}{}}{}$$

$$= \frac{2(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{3d(c+d)f(c+d\sin(e+fx))^{3/2}} + \frac{8a^3(c-d)(c+2d)\cos(e+fx)}{3d^2(c+d)^2f\sqrt{c+d\sin(e+fx)}} - \frac{\left(2a^3(c-d)(\right.}{}$$

$$= \frac{2(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{3d(c+d)f(c+d\sin(e+fx))^{3/2}} + \frac{8a^3(c-d)(c+2d)\cos(e+fx)}{3d^2(c+d)^2f\sqrt{c+d\sin(e+fx)}} + \frac{\left(2a^3\left(4c^2+\right.\right.}{}$$

$$= \frac{2(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{3d(c+d)f(c+d\sin(e+fx))^{3/2}} + \frac{8a^3(c-d)(c+2d)\cos(e+fx)}{3d^2(c+d)^2f\sqrt{c+d\sin(e+fx)}} + \frac{4a^3\left(4c^2+5\right.}{}$$

Mathematica [A] time = 1.52587, size = 232, normalized size = 0.83

$$-\frac{2a^3(\sin(e+fx)+1)^3\left(d(d-c)\cos(e+fx)\left(4c^2+d(5c+9d)\sin(e+fx)+9cd+d^2\right)+2(c+d)\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{3/2}\left(\left(4c^2+\right.\right.\right.}{3d^3f(c+d)^2\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3/(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (-2*a^3*(1 + Sin[e + f*x])^3*(2*(c + d)*(d^2*(c + 5*d)*EllipticF[(-2*e + Pi
 - 2*f*x)/4, (2*d)/(c + d)] + (4*c^2 + 5*c*d - 3*d^2)*((c + d)*EllipticE[(-
2*e + Pi - 2*f*x)/4, (2*d)/(c + d)] - c*EllipticF[(-2*e + Pi - 2*f*x)/4, (2
*d)/(c + d)]))*((c + d*Sin[e + f*x])/(c + d))^(3/2) + d*(-c + d)*Cos[e + f*
x]*(4*c^2 + 9*c*d + d^2 + d*(5*c + 9*d)*Sin[e + f*x])))/(3*d^3*(c + d)^2*f*
(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6*(c + d*Sin[e + f*x])^(3/2))
```

Maple [B] time = 4.573, size = 1257, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^(5/2),x)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*a^3*(-2/d^4/(cos(f*x+e)^2*sin(f*x+e
)*d+c*cos(f*x+e)^2)^(1/2)*(-d/(c-d)*sin(f*x+e)-d/(c-d))^(1/2)*(-d/(c+d)*sin
(f*x+e)+d/(c+d))^(1/2)*(d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2)*(EllipticE((d/(
c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2),((c-d)/(c+d))^(1/2))*c^2-EllipticE((d/(c-d
)*sin(f*x+e)+1/(c-d)*c)^(1/2),((c-d)/(c+d))^(1/2))*d^2+2*EllipticF((d/(c-d)
*sin(f*x+e)+1/(c-d)*c)^(1/2),((c-d)/(c+d))^(1/2))*c^2-6*EllipticF((d/(c-d)*
```

```
sin(f*x+e)+1/(c-d)*c)^(1/2),((c-d)/(c+d))^(1/2))*c*d+4*EllipticF((d/(c-d)*s
in(f*x+e)+1/(c-d)*c)^(1/2),((c-d)/(c+d))^(1/2))*d^2)+3/d^3*(c^2-2*c*d+d^2)*
(2*d*cos(f*x+e)^2/(c^2-d^2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2*c/(c^
2-d^2)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2
)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*E
llipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+2/(c^2-d^2)*d*
(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-si
n(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1
)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF((
(c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+1/d^3*(-c^3+3*c^2*d-3*
c*d^2+d^3)*(2/3/(c^2-d^2)/d*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(sin(f*
x+e)+c/d)^2+8/3*d*cos(f*x+e)^2/(c^2-d^2)^2*c/(-(-d*sin(f*x+e)-c)*cos(f*x+e)
^2)^(1/2)+2*(3*c^2+d^2)/(3*c^4-6*c^2*d^2+3*d^4)*(c/d-1)*((c+d*sin(f*x+e))/(
c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/
(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^
(1/2),((c-d)/(c+d))^(1/2))+8/3*c*d/(c^2-d^2)^2*(c/d-1)*((c+d*sin(f*x+e))/(c
-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(
-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e)
)/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2
),((c-d)/(c+d))^(1/2)))))/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{3}}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^3/(d*sin(f*x + e) + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(3\,a^{3}\cos\left(fx+e\right)^{2}-4\,a^{3}+\left(a^{3}\cos\left(fx+e\right)^{2}-4\,a^{3}\right)\sin\left(fx+e\right)\right)\sqrt{d\sin\left(fx+e\right)+c}}{3\,cd^{2}\cos\left(fx+e\right)^{2}-c^{3}-3\,cd^{2}+\left(d^{3}\cos\left(fx+e\right)^{2}-3\,c^{2}d-d^{3}\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((3*a^3*cos(f*x + e)^2 - 4*a^3 + (a^3*cos(f*x + e)^2 - 4*a^3)*sin(f
*x + e))*sqrt(d*sin(f*x + e) + c)/(3*c*d^2*cos(f*x + e)^2 - c^3 - 3*c*d^2 +
 (d^3*cos(f*x + e)^2 - 3*c^2*d - d^3)*sin(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3/(c+d*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^3}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^3/(d*sin(f*x + e) + c)^(5/2), x)
```

3.502 $\int \frac{(a+a\sin(e+fx))^3}{(c+d\sin(e+fx))^{7/2}} dx$

Optimal. Leaf size=336

$$-\frac{4a^3(4c^2+15cd+27d^2)\cos(e+fx)}{15d^2f(c+d)^3\sqrt{c+d\sin(e+fx)}}+\frac{4a^3(4c^2+11cd+15d^2)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{15d^3f(c+d)^2\sqrt{c+d\sin(e+fx)}}-\frac{4a^3(4c^2+15\cdots}{}$$

```
[Out] (2*(c - d)*Cos[e + f*x]*(a^3 + a^3*Sin[e + f*x]))/(5*d*(c + d)*f*(c + d*Sin
[e + f*x])^(5/2)) + (8*a^3*(c - d)*(c + 3*d)*Cos[e + f*x])/(15*d^2*(c + d)^
2*f*(c + d*Sin[e + f*x])^(3/2)) - (4*a^3*(4*c^2 + 15*c*d + 27*d^2)*Cos[e +
f*x])/(15*d^2*(c + d)^3*f*Sqrt[c + d*Sin[e + f*x]]) - (4*a^3*(4*c^2 + 15*c*
d + 27*d^2)*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e +
 f*x]])/(15*d^3*(c + d)^3*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + (4*a^3*(4
*c^2 + 11*c*d + 15*d^2)*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(
c + d*Sin[e + f*x])/(c + d)])/(15*d^3*(c + d)^2*f*Sqrt[c + d*Sin[e + f*x]])
```

Rubi [A] time = 0.720415, antiderivative size = 336, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2762, 2968, 3021, 2754, 2752, 2663, 2661, 2655, 2653}

$$-\frac{4a^3(4c^2+15cd+27d^2)\cos(e+fx)}{15d^2f(c+d)^3\sqrt{c+d\sin(e+fx)}}+\frac{4a^3(4c^2+11cd+15d^2)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{15d^3f(c+d)^2\sqrt{c+d\sin(e+fx)}}-\frac{4a^3(4c^2+15\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3/(c + d*Sin[e + f*x])^(7/2),x]
```


```
[Out] (2*(c - d)*Cos[e + f*x]*(a^3 + a^3*Sin[e + f*x]))/(5*d*(c + d)*f*(c + d*Sin
[e + f*x])^(5/2)) + (8*a^3*(c - d)*(c + 3*d)*Cos[e + f*x])/(15*d^2*(c + d)^
2*f*(c + d*Sin[e + f*x])^(3/2)) - (4*a^3*(4*c^2 + 15*c*d + 27*d^2)*Cos[e +
f*x])/(15*d^2*(c + d)^3*f*Sqrt[c + d*Sin[e + f*x]]) - (4*a^3*(4*c^2 + 15*c*
d + 27*d^2)*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e +
 f*x]])/(15*d^3*(c + d)^3*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + (4*a^3*(4
*c^2 + 11*c*d + 15*d^2)*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(
c + d*Sin[e + f*x])/(c + d)])/(15*d^3*(c + d)^2*f*Sqrt[c + d*Sin[e + f*x]])
```


Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
```

```
x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^3}{(c+d\sin(e+fx))^{7/2}}dx = \frac{2(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{5d(c+d)f(c+d\sin(e+fx))^{5/2}} - \frac{(2a)\int \frac{(a+a\sin(e+fx))(a(c-6d)-a(2c+3d)\sin(e+fx))}{(c+d\sin(e+fx))^{5/2}}}{5d(c+d)}$$

$$= \frac{2(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{5d(c+d)f(c+d\sin(e+fx))^{5/2}} - \frac{(2a)\int \frac{a^2(c-6d)+\left(a^2(c-6d)-a^2(2c+3d)\right)\sin(e+fx)-a^2(}{(c+d\sin(e+fx))^{5/2}}}{5d(c+d)}$$

$$= \frac{2(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{5d(c+d)f(c+d\sin(e+fx))^{5/2}} + \frac{8a^3(c-d)(c+3d)\cos(e+fx)}{15d^2(c+d)^2f(c+d\sin(e+fx))^{3/2}} + \frac{(4a)}{}$$

$$= \frac{2(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{5d(c+d)f(c+d\sin(e+fx))^{5/2}} + \frac{8a^3(c-d)(c+3d)\cos(e+fx)}{15d^2(c+d)^2f(c+d\sin(e+fx))^{3/2}} - \frac{4a^3(}{15a}$$

$$= \frac{2(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{5d(c+d)f(c+d\sin(e+fx))^{5/2}} + \frac{8a^3(c-d)(c+3d)\cos(e+fx)}{15d^2(c+d)^2f(c+d\sin(e+fx))^{3/2}} - \frac{4a^3(}{15a}$$

$$= \frac{2(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{5d(c+d)f(c+d\sin(e+fx))^{5/2}} + \frac{8a^3(c-d)(c+3d)\cos(e+fx)}{15d^2(c+d)^2f(c+d\sin(e+fx))^{3/2}} - \frac{4a^3(}{15a}$$

$$= \frac{2(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{5d(c+d)f(c+d\sin(e+fx))^{5/2}} + \frac{8a^3(c-d)(c+3d)\cos(e+fx)}{15d^2(c+d)^2f(c+d\sin(e+fx))^{3/2}} - \frac{4a^3(}{15a}$$

Mathematica [A] time = 2.10717, size = 298, normalized size = 0.89

$$-\frac{2a^3(\sin(e+fx)+1)^3\left(d\cos(e+fx)\left(2d^2\left(4c^2+15cd+27d^2\right)\sin^2(e+fx)+d\left(45c^2d+9c^3+115cd^2+15d^3\right)\sin(e\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3/(c + d*Sin[e + f*x])^(7/2),x]
```

```
[Out] (-2*a^3*(1 + Sin[e + f*x])^3*(-2*((c - 15*d)*d^2*EllipticF[(-2*e + Pi - 2*f
*x)/4, (2*d)/(c + d)] + (4*c^2 + 15*c*d + 27*d^2)*((c + d)*EllipticE[(-2*e
+ Pi - 2*f*x)/4, (2*d)/(c + d)] - c*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/
(c + d)]))*(c + d*Sin[e + f*x])^2*Sqrt[(c + d*Sin[e + f*x])/(c + d)] + d*Co
s[e + f*x]*(4*c^4 + 15*c^3*d + 55*c^2*d^2 + 15*c*d^3 + 3*d^4 + d*(9*c^3 + 4
5*c^2*d + 115*c*d^2 + 15*d^3)*Sin[e + f*x] + 2*d^2*(4*c^2 + 15*c*d + 27*d^2
)*Sin[e + f*x]^2)))/(15*d^3*(c + d)^3*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2
])^6*(c + d*Sin[e + f*x])^(5/2))
```

Maple [B] time = 6.027, size = 1589, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^(7/2),x)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*a^3*(2/d^3*(c/d-1)*((c+d*sin(f*x+e)
)/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/
```

```
2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d
))^(1/2),((c-d)/(c+d))^(1/2))+1/d^3*(-c^3+3*c^2*d-3*c*d^2+d^3)*(2/5/(c^2-d^
2)/d^2*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(sin(f*x+e)+c/d)^3+16/15*c/(
c^2-d^2)^2/d*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(sin(f*x+e)+c/d)^2+2/1
5*d*cos(f*x+e)^2/(c^2-d^2)^3*(23*c^2+9*d^2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^
2)^(1/2)+2*(15*c^3+17*c*d^2)/(15*c^6-45*c^4*d^2+45*c^2*d^4-15*d^6)*(c/d-1)*
((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)
-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*
sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+2/15*d*(23*c^2+9*d^2)/(c^2-d^
2)^3*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*
((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-
c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+Ellipt
icF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+3*(-c+d)/d^3*(2*d
*cos(f*x+e)^2/(c^2-d^2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2*c/(c^2-d^
2)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((
-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*Ellip
ticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+2/(c^2-d^2)*d*(c/d
-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*
x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*El
lipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d
*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+3/d^3*(c^2-2*c*d+d^2)*(2/3
/(c^2-d^2)/d*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(sin(f*x+e)+c/d)^2+8/3
*d*cos(f*x+e)^2/(c^2-d^2)^2*c/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2*(3*
c^2+d^2)/(3*c^4-6*c^2*d^2+3*d^4)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*
(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e
)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c
+d))^(1/2))+8/3*c*d/(c^2-d^2)^2*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(
1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)
-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),
((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))
^(1/2)))))/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{3}}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^3/(d*sin(f*x + e) + c)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(3\,a^{3}\cos\left(fx+e\right)^{2}-4\,a^{3}+\left(a^{3}\cos\left(fx+e\right)^{2}-4\,a^{3}\right)\sin\left(fx+e\right)\right)\sqrt{d\sin\left(fx+e\right)+c}}{d^{4}\cos\left(fx+e\right)^{4}+c^{4}+6\,c^{2}d^{2}+d^{4}-2\left(3\,c^{2}d^{2}+d^{4}\right)\cos\left(fx+e\right)^{2}-4\left(cd^{3}\cos\left(fx+e\right)^{2}-c^{3}d-cd^{3}\right)\sin\left(fx\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^(7/2),x, algorithm="fricas")
```

```
[Out] integral(-(3*a^3*cos(f*x + e)^2 - 4*a^3 + (a^3*cos(f*x + e)^2 - 4*a^3)*sin(
f*x + e))*sqrt(d*sin(f*x + e) + c)/(d^4*cos(f*x + e)^4 + c^4 + 6*c^2*d^2 +
d^4 - 2*(3*c^2*d^2 + d^4)*cos(f*x + e)^2 - 4*(c*d^3*cos(f*x + e)^2 - c^3*d
- c*d^3)*sin(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3/(c+d*sin(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^3}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^3/(d*sin(f*x + e) + c)^(7/2), x)
```

3.503 $\int \frac{(a+a\sin(e+fx))^3}{(c+d\sin(e+fx))^{9/2}}\,dx$

Optimal. Leaf size=419

$$-\frac{4a^3\left(21c^2d+4c^3+62cd^2-147d^3\right)\cos(e+fx)}{105d^2f(c-d)(c+d)^4\sqrt{c+d\sin(e+fx)}}-\frac{4a^3\left(4c^2+21cd+65d^2\right)\cos(e+fx)}{105d^2f(c+d)^3(c+d\sin(e+fx))^{3/2}}+\frac{4a^3\left(4c^2+21cd+65d^2\right)\sqrt{\frac{c+d}{}}}{105d^3f(c+d)^3\sqrt{}}$$

```
[Out] (2*(c - d)*Cos[e + f*x]*(a^3 + a^3*Sin[e + f*x]))/(7*d*(c + d)*f*(c + d*Sin
[e + f*x])^(7/2)) + (8*a^3*(c - d)*(c + 4*d)*Cos[e + f*x])/(35*d^2*(c + d)^
2*f*(c + d*Sin[e + f*x])^(5/2)) - (4*a^3*(4*c^2 + 21*c*d + 65*d^2)*Cos[e +
f*x])/(105*d^2*(c + d)^3*f*(c + d*Sin[e + f*x])^(3/2)) - (4*a^3*(4*c^3 + 21
*c^2*d + 62*c*d^2 - 147*d^3)*Cos[e + f*x])/(105*(c - d)*d^2*(c + d)^4*f*Sqr
t[c + d*Sin[e + f*x]]) - (4*a^3*(4*c^3 + 21*c^2*d + 62*c*d^2 - 147*d^3)*Ell
ipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(105*(c
 - d)*d^3*(c + d)^4*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + (4*a^3*(4*c^2 +
 21*c*d + 65*d^2)*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*
Sin[e + f*x])/(c + d)])/(105*d^3*(c + d)^3*f*Sqrt[c + d*Sin[e + f*x]])
```

Rubi [A] time = 0.928286, antiderivative size = 419, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2762, 2968, 3021, 2754, 2752, 2663, 2661, 2655, 2653}

$$-\frac{4a^3\left(21c^2d+4c^3+62cd^2-147d^3\right)\cos(e+fx)}{105d^2f(c-d)(c+d)^4\sqrt{c+d\sin(e+fx)}}-\frac{4a^3\left(4c^2+21cd+65d^2\right)\cos(e+fx)}{105d^2f(c+d)^3(c+d\sin(e+fx))^{3/2}}+\frac{4a^3\left(4c^2+21cd+65d^2\right)\sqrt{\frac{c+d}{}}}{105d^3f(c+d)^3\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3/(c + d*Sin[e + f*x])^(9/2),x]
```


```
[Out] (2*(c - d)*Cos[e + f*x]*(a^3 + a^3*Sin[e + f*x]))/(7*d*(c + d)*f*(c + d*Sin
[e + f*x])^(7/2)) + (8*a^3*(c - d)*(c + 4*d)*Cos[e + f*x])/(35*d^2*(c + d)^
2*f*(c + d*Sin[e + f*x])^(5/2)) - (4*a^3*(4*c^2 + 21*c*d + 65*d^2)*Cos[e +
f*x])/(105*d^2*(c + d)^3*f*(c + d*Sin[e + f*x])^(3/2)) - (4*a^3*(4*c^3 + 21
*c^2*d + 62*c*d^2 - 147*d^3)*Cos[e + f*x])/(105*(c - d)*d^2*(c + d)^4*f*Sqr
t[c + d*Sin[e + f*x]]) - (4*a^3*(4*c^3 + 21*c^2*d + 62*c*d^2 - 147*d^3)*Ell
ipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(105*(c
 - d)*d^3*(c + d)^4*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + (4*a^3*(4*c^2 +
 21*c*d + 65*d^2)*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*
Sin[e + f*x])/(c + d)])/(105*d^3*(c + d)^3*f*Sqrt[c + d*Sin[e + f*x]])
```


Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^3}{(c+d\sin(e+fx))^{9/2}}dx = \frac{2(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{7d(c+d)f(c+d\sin(e+fx))^{7/2}} - \frac{(2a)\int \frac{(a+a\sin(e+fx))(a(c-8d)-a(2c+5d)\sin(e+fx))}{(c+d\sin(e+fx))^{7/2}}dx}{7d(c+d)}$$

$$= \frac{2(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{7d(c+d)f(c+d\sin(e+fx))^{7/2}} - \frac{(2a)\int \frac{a^2(c-8d)+\left(a^2(c-8d)-a^2(2c+5d)\right)\sin(e+fx)-a^2(2c+5}{(c+d\sin(e+fx))^{7/2}}}{7d(c+d)}$$

$$= \frac{2(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{7d(c+d)f(c+d\sin(e+fx))^{7/2}} + \frac{8a^3(c-d)(c+4d)\cos(e+fx)}{35d^2(c+d)^2f(c+d\sin(e+fx))^{5/2}} + \frac{(4a)\int \frac{5}{2}}{}$$

$$= \frac{2(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{7d(c+d)f(c+d\sin(e+fx))^{7/2}} + \frac{8a^3(c-d)(c+4d)\cos(e+fx)}{35d^2(c+d)^2f(c+d\sin(e+fx))^{5/2}} - \frac{4a^3\left(4c^2\right.}{105d^2(c}$$

$$= \frac{2(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{7d(c+d)f(c+d\sin(e+fx))^{7/2}} + \frac{8a^3(c-d)(c+4d)\cos(e+fx)}{35d^2(c+d)^2f(c+d\sin(e+fx))^{5/2}} - \frac{4a^3\left(4c^2\right.}{105d^2(c}$$

$$= \frac{2(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{7d(c+d)f(c+d\sin(e+fx))^{7/2}} + \frac{8a^3(c-d)(c+4d)\cos(e+fx)}{35d^2(c+d)^2f(c+d\sin(e+fx))^{5/2}} - \frac{4a^3\left(4c^2\right.}{105d^2(c}$$

$$= \frac{2(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{7d(c+d)f(c+d\sin(e+fx))^{7/2}} + \frac{8a^3(c-d)(c+4d)\cos(e+fx)}{35d^2(c+d)^2f(c+d\sin(e+fx))^{5/2}} - \frac{4a^3\left(4c^2\right.}{105d^2(c}$$

$$= \frac{2(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{7d(c+d)f(c+d\sin(e+fx))^{7/2}} + \frac{8a^3(c-d)(c+4d)\cos(e+fx)}{35d^2(c+d)^2f(c+d\sin(e+fx))^{5/2}} - \frac{4a^3\left(4c^2\right.}{105d^2(c}$$

Mathematica [A] time = 3.56264, size = 351, normalized size = 0.84

$$-\frac{2a^3(\sin(e+fx)+1)^3\left(d\cos(e+fx)\left(2(c-d)\left(4c^2+21cd+65d^2\right)(c+d)(c+d\sin(e+fx))^2+2\left(21c^2d+4c^3+62cd^2\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3/(c + d*Sin[e + f*x])^(9/2),x]
```

```
[Out] (-2*a^3*(1 + Sin[e + f*x])^3*(-2*(d^2*(c^2 - 126*c*d + 65*d^2)*EllipticF[(-
2*e + Pi - 2*f*x)/4, (2*d)/(c + d)] + (4*c^3 + 21*c^2*d + 62*c*d^2 - 147*d^
3)*((c + d)*EllipticE[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)] - c*EllipticF[(
-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]))*(c + d*Sin[e + f*x])^3*Sqrt[(c + d*S
in[e + f*x])/(c + d)] + d*Cos[e + f*x]*(15*(c - d)^3*(c + d)^3 - 9*(c - d)^
2*(c + d)^2*(3*c + 7*d)*(c + d*Sin[e + f*x]) + 2*(c - d)*(c + d)*(4*c^2 + 2
1*c*d + 65*d^2)*(c + d*Sin[e + f*x])^2 + 2*(4*c^3 + 21*c^2*d + 62*c*d^2 - 1
47*d^3)*(c + d*Sin[e + f*x])^3)))/(105*(c - d)*d^3*(c + d)^4*f*(Cos[(e + f*
x)/2] + Sin[(e + f*x)/2])^6*(c + d*Sin[e + f*x])^(7/2))
```

Maple [B] time = 8.848, size = 2079, normalized size = 5.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^(9/2),x)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*a^3*((-c^3+3*c^2*d-3*c*d^2+d^3)/d^3
*(2/7/(c^2-d^2)/d^3*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(sin(f*x+e)+c/d
)^4+24/35/(c^2-d^2)^2/d^2*c*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(sin(f*
x+e)+c/d)^3+2/105*(71*c^2+25*d^2)/d/(c^2-d^2)^3*(-(-d*sin(f*x+e)-c)*cos(f*x
+e)^2)^(1/2)/(sin(f*x+e)+c/d)^2+32/105*d*cos(f*x+e)^2/(c^2-d^2)^4*c*(11*c^2
+13*d^2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2*(105*c^4+254*c^2*d^2+25*
d^4)/(105*c^8-420*c^6*d^2+630*c^4*d^4-420*c^2*d^6+105*d^8)*(c/d-1)*((c+d*si
n(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c
-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+
e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+32/105*c*d*(11*c^2+13*d^2)/(c^2-d^2)^
4*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-
sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d
-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF
(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+3*(c^2-2*c*d+d^2)/d^
3*(2/5/(c^2-d^2)/d^2*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(sin(f*x+e)+c/
d)^3+16/15*c/(c^2-d^2)^2/d*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(sin(f*x
+e)+c/d)^2+2/15*d*cos(f*x+e)^2/(c^2-d^2)^3*(23*c^2+9*d^2)/(-(-d*sin(f*x+e)-
c)*cos(f*x+e)^2)^(1/2)+2*(15*c^3+17*c*d^2)/(15*c^6-45*c^4*d^2+45*c^2*d^4-15
*d^6)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)
*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*El
lipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+2/15*d*(23*c^2+
9*d^2)/(c^2-d^2)^3*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))
/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e
)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))
^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+1/d
^3*(2*d*cos(f*x+e)^2/(c^2-d^2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2*c/
(c^2-d^2)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(
1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2
)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+2/(c^2-d^2)
*d*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((
-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/
d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+Elliptic
F(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+3*(-c+d)/d^3*(2/3/(
c^2-d^2)/d*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(sin(f*x+e)+c/d)^2+8/3*d
*cos(f*x+e)^2/(c^2-d^2)^2*c/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2*(3*c^
2+d^2)/(3*c^4-6*c^2*d^2+3*d^4)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1
-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-
c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d
))^(1/2))+8/3*c*d/(c^2-d^2)^2*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-
sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c
)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((
c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(
1/2)))))/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{3}}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^3/(d*sin(f*x + e) + c)^(9/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(3\,a^3\cos\left(fx+e\right)^2-4\,a^3+\left(a^3\cos\left(fx+e\right)^2-4\,a^3\right)\sin\left(fx+e\right)\right)\sqrt{d\sin}}{5\,cd^4\cos\left(fx+e\right)^4+c^5+10\,c^3d^2+5\,cd^4-10\left(c^3d^2+cd^4\right)\cos\left(fx+e\right)^2+\left(d^5\cos\left(fx+e\right)^4+5\,c^4d+10\,c^2d\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^(9/2),x, algorithm="fricas")
```

```
[Out] integral(-(3*a^3*cos(f*x + e)^2 - 4*a^3 + (a^3*cos(f*x + e)^2 - 4*a^3)*sin(
f*x + e))*sqrt(d*sin(f*x + e) + c)/(5*c*d^4*cos(f*x + e)^4 + c^5 + 10*c^3*d
^2 + 5*c*d^4 - 10*(c^3*d^2 + c*d^4)*cos(f*x + e)^2 + (d^5*cos(f*x + e)^4 +
5*c^4*d + 10*c^2*d^3 + d^5 - 2*(5*c^2*d^3 + d^5)*cos(f*x + e)^2)*sin(f*x +
e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3/(c+d*sin(f*x+e))**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^3}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^(9/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^3/(d*sin(f*x + e) + c)^(9/2), x)
```

3.504 $\int \frac{(c+d\sin(e+fx))^{5/2}}{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=246

$$\frac{(3c-5d)\left(c^2-d^2\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3af\sqrt{c+d\sin(e+fx)}}-\frac{\left(3c^2-20cd+9d^2\right)\sqrt{c+d\sin(e+fx)}E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3af\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

```
[Out] ((3*c - 5*d)*d*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(3*a*f) - ((c - d)*Co
s[e + f*x]*(c + d*Sin[e + f*x])^(3/2))/(f*(a + a*Sin[e + f*x])) - ((3*c^2 -
 20*c*d + 9*d^2)*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Si
n[e + f*x]])/(3*a*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + ((3*c - 5*d)*(c^2
 - d^2)*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*
x])/(c + d)])/(3*a*f*Sqrt[c + d*Sin[e + f*x]])
```

Rubi [A] time = 0.381304, antiderivative size = 246, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2767, 2753, 2752, 2663, 2661, 2655, 2653}

$$\frac{(3c-5d)\left(c^2-d^2\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3af\sqrt{c+d\sin(e+fx)}}-\frac{\left(3c^2-20cd+9d^2\right)\sqrt{c+d\sin(e+fx)}E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3af\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^(5/2)/(a + a*Sin[e + f*x]),x]
```

```
[Out] ((3*c - 5*d)*d*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(3*a*f) - ((c - d)*Co
s[e + f*x]*(c + d*Sin[e + f*x])^(3/2))/(f*(a + a*Sin[e + f*x])) - ((3*c^2 -
 20*c*d + 9*d^2)*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Si
n[e + f*x]])/(3*a*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + ((3*c - 5*d)*(c^2
 - d^2)*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*
x])/(c + d)])/(3*a*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2767

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e + f
*x])^(n - 1))/(a*f*(a + b*Sin[e + f*x])), x] - Dist[d/(a*b), Int[(c + d*Sin
[e + f*x])^(n - 2)*Simp[b*d*(n - 1) - a*c*n + (b*c*(n - 1) - a*d*n)*Sin[e +
 f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && E
qQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && (IntegerQ[2*n] || EqQ
[c, 0])
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^{5/2}}{a+a\sin(e+fx)}\,dx = -\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{f(a+a\sin(e+fx))} - \frac{d\int\left(-\frac{1}{2}a(5c-3d)+\frac{1}{2}a(3c-5d)\sin(e+fx)\right)\cdots}{a^2}$$

$$= \frac{(3c-5d)d\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3af} - \frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{f(a+a\sin(e+fx))} - \frac{(2d\cdots}{\cdots}$$

$$= \frac{(3c-5d)d\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3af} - \frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{f(a+a\sin(e+fx))} + \frac{((3c\cdots}{\cdots}$$

$$= \frac{(3c-5d)d\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3af} - \frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{f(a+a\sin(e+fx))} - \frac{((3c\cdots}{\cdots}$$

$$= \frac{(3c-5d)d\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3af} - \frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{f(a+a\sin(e+fx))} - \frac{(3c^2\cdots}{\cdots}$$

Mathematica [A] time = 1.36459, size = 298, normalized size = 1.21

$$\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^2\left(-d\left(15c^2-12cd+5d^2\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{4}(-2e-2fx+\pi)|\frac{2d}{c+d}\right)+\left(3c^2-20cd+9d\cdots\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])^(5/2)/(a + a*Sin[e + f*x]),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2*(-3*(c - d)^2*(c + d*Sin[e + f*x])
 - 2*d^2*Cos[e + f*x]*(c + d*Sin[e + f*x]) + (6*(c - d)^2*Sin[(e + f*x)/2]*
(c + d*Sin[e + f*x]))/(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) - d*(15*c^2 - 1
2*c*d + 5*d^2)*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]*Sqrt[(c + d*
Sin[e + f*x])/(c + d)] + (3*c^2 - 20*c*d + 9*d^2)*((c + d)*EllipticE[(-2*e
+ Pi - 2*f*x)/4, (2*d)/(c + d)] - c*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/
(c + d)])*Sqrt[(c + d*Sin[e + f*x])/(c + d)]))/(3*a*f*(1 + Sin[e + f*x])*Sq
rt[c + d*Sin[e + f*x]])
```

Maple [B] time = 1.304, size = 1372, normalized size = 5.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e)),x)
```

```
[Out] 1/3*(cos(f*x+e)^2*sin(f*x+e)*d+c*cos(f*x+e)^2)^(1/2)*(12*(d/(c-d)*sin(f*x+e
)+1/(c-d)*c)^(1/2)*(-d/(c+d)*sin(f*x+e)+d/(c+d))^(1/2)*(-d/(c-d)*sin(f*x+e)
-d/(c-d))^(1/2)*EllipticF((d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2),((c-d)/(c+d)
)^(1/2))*c^3*d-4*(d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2)*(-d/(c+d)*sin(f*x+e)+
d/(c+d))^(1/2)*(-d/(c-d)*sin(f*x+e)-d/(c-d))^(1/2)*EllipticF((d/(c-d)*sin(f
*x+e)+1/(c-d)*c)^(1/2),((c-d)/(c+d))^(1/2))*c^2*d^2-12*(d/(c-d)*sin(f*x+e)+
1/(c-d)*c)^(1/2)*(-d/(c+d)*sin(f*x+e)+d/(c+d))^(1/2)*(-d/(c-d)*sin(f*x+e)-d
/(c-d))^(1/2)*EllipticF((d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2),((c-d)/(c+d))^
(1/2))*c*d^3+4*(d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2)*(-d/(c+d)*sin(f*x+e)+d/
(c+d))^(1/2)*(-d/(c-d)*sin(f*x+e)-d/(c-d))^(1/2)*EllipticF((d/(c-d)*sin(f*x
+e)+1/(c-d)*c)^(1/2),((c-d)/(c+d))^(1/2))*d^4+3*(d/(c-d)*sin(f*x+e)+1/(c-d)
*c)^(1/2)*(-d/(c+d)*sin(f*x+e)+d/(c+d))^(1/2)*(-d/(c-d)*sin(f*x+e)-d/(c-d))
^(1/2)*EllipticE((d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2),((c-d)/(c+d))^(1/2))*
c^4-20*(d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2)*(-d/(c+d)*sin(f*x+e)+d/(c+d))^(
1/2)*(-d/(c-d)*sin(f*x+e)-d/(c-d))^(1/2)*EllipticE((d/(c-d)*sin(f*x+e)+1/(c
-d)*c)^(1/2),((c-d)/(c+d))^(1/2))*c^3*d+6*(d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1
/2)*(-d/(c+d)*sin(f*x+e)+d/(c+d))^(1/2)*(-d/(c-d)*sin(f*x+e)-d/(c-d))^(1/2)
*EllipticE((d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2),((c-d)/(c+d))^(1/2))*c^2*d^
2+20*(d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2)*(-d/(c+d)*sin(f*x+e)+d/(c+d))^(1/
2)*(-d/(c-d)*sin(f*x+e)-d/(c-d))^(1/2)*EllipticE((d/(c-d)*sin(f*x+e)+1/(c-d
)*c)^(1/2),((c-d)/(c+d))^(1/2))*c*d^3-9*(d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2
)*(-d/(c+d)*sin(f*x+e)+d/(c+d))^(1/2)*(-d/(c-d)*sin(f*x+e)-d/(c-d))^(1/2)*E
llipticE((d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2),((c-d)/(c+d))^(1/2))*d^4-2*d^
4*sin(f*x+e)*cos(f*x+e)^2-3*c^2*cos(f*x+e)^2*d^2+4*cos(f*x+e)^2*c*d^3-3*cos
(f*x+e)^2*d^4+3*c^3*d*sin(f*x+e)-9*c^2*d^2*sin(f*x+e)+9*c*d^3*sin(f*x+e)-3*
d^4*sin(f*x+e)-3*c^3*d+9*c^2*d^2-9*c*d^3+3*d^4)/d/(-(c+d*sin(f*x+e))*(-1+si
n(f*x+e))*(1+sin(f*x+e)))^(1/2)/a/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}{a\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(5/2)/(a*sin(f*x + e) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(d^2\cos\left(fx+e\right)^2-2\,cd\sin\left(fx+e\right)-c^2-d^2\right)\sqrt{d\sin\left(fx+e\right)+c}}{a\sin\left(fx+e\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral(-(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)*sqrt(d*sin(
f*x + e) + c)/(a*sin(f*x + e) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(5/2)/(a+a*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}{a\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(5/2)/(a*sin(f*x + e) + a), x)
```

3.505 $\int \frac{(c+d\sin(e+fx))^{3/2}}{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=186

$$\frac{\left(c^2-d^2\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{af\sqrt{c+d\sin(e+fx)}}-\frac{(c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{f(a\sin(e+fx)+a)}-\frac{(c-3d)\sqrt{c+d\sin(e+fx)}E}{af\sqrt{\frac{c+d\sin(}{c+}}}$$

```
[Out] -(((c - d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(f*(a + a*Sin[e + f*x])))
 - ((c - 3*d)*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e
 + f*x]])/(a*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + ((c^2 - d^2)*EllipticF
[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(a*
f*Sqrt[c + d*Sin[e + f*x]])
```

Rubi [A] time = 0.257038, antiderivative size = 186, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2767, 2752, 2663, 2661, 2655, 2653}

$$\frac{\left(c^2-d^2\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{af\sqrt{c+d\sin(e+fx)}}-\frac{(c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{f(a\sin(e+fx)+a)}-\frac{(c-3d)\sqrt{c+d\sin(e+fx)}E}{af\sqrt{\frac{c+d\sin(}{c+}}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^(3/2)/(a + a*Sin[e + f*x]),x]
```


```
[Out] -(((c - d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(f*(a + a*Sin[e + f*x])))
 - ((c - 3*d)*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e
 + f*x]])/(a*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + ((c^2 - d^2)*EllipticF
[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(a*
f*Sqrt[c + d*Sin[e + f*x]])
```


Rule 2767

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e + f
*x])^(n - 1))/(a*f*(a + b*Sin[e + f*x])), x] - Dist[d/(a*b), Int[(c + d*Sin
[e + f*x])^(n - 2)*Simp[b*d*(n - 1) - a*c*n + (b*c*(n - 1) - a*d*n)*Sin[e +
 f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && E
qQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && (IntegerQ[2*n] || EqQ
[c, 0])
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
```

```
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^{3/2}}{a+a\sin(e+fx)}\,dx = -\frac{(c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{f(a+a\sin(e+fx))} - \frac{d\int \frac{-\frac{1}{2}a(3c-d)+\frac{1}{2}a(c-3d)\sin(e+fx)}{\sqrt{c+d\sin(e+fx)}}\,dx}{a^2}$$

$$= -\frac{(c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{f(a+a\sin(e+fx))} - \frac{(c-3d)\int \sqrt{c+d\sin(e+fx)}\,dx}{2a} + \frac{\left(c^2-d^2\right)\int \frac{}{\sqrt{}}}{2}$$

$$= -\frac{(c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{f(a+a\sin(e+fx))} - \frac{\left((c-3d)\sqrt{c+d\sin(e+fx)}\right)\int \sqrt{\frac{c}{c+d}+\frac{d\sin(e+fx)}{c+d}}}{2a\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

$$= -\frac{(c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{f(a+a\sin(e+fx))} - \frac{(c-3d)E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2d}{c+d}\right)\sqrt{c+d\sin(e+fx)}}{af\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

Mathematica [A] time = 1.56371, size = 223, normalized size = 1.2

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(2(c-d)\sin\left(\frac{1}{2}(e+fx)\right)(c+d\sin(e+fx))-\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(\right.\right.}{af(\sin(e+}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])^(3/2)/(a + a*Sin[e + f*x]),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(2*(c - d)*Sin[(e + f*x)/2]*(c + d*S
in[e + f*x]) - (Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-((c^2 - 2*c*d - 3*d^
2)*EllipticE[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x]
)/(c + d)]) + (c - d)*(c + d*Sin[e + f*x] + (c + d)*EllipticF[(-2*e + Pi -
2*f*x)/4, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)]))))/(a*f*(1 + S
in[e + f*x])*Sqrt[c + d*Sin[e + f*x]])
```

Maple [B] time = 1.28, size = 925, normalized size = 5.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e)),x)
```

```
[Out] (cos(f*x+e)^2*sin(f*x+e)*d+c*cos(f*x+e)^2)^(1/2)*((d/(c-d)*sin(f*x+e)+1/(c-
d)*c)^(1/2)*(-d/(c+d)*sin(f*x+e)+d/(c+d))^(1/2)*(-d/(c-d)*sin(f*x+e)-d/(c-d
))^(1/2)*EllipticE((d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2),((c-d)/(c+d))^(1/2)
)*c^3-3*(d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2)*(-d/(c+d)*sin(f*x+e)+d/(c+d))^
(1/2)*(-d/(c-d)*sin(f*x+e)-d/(c-d))^(1/2)*EllipticE((d/(c-d)*sin(f*x+e)+1/(
c-d)*c)^(1/2),((c-d)/(c+d))^(1/2))*c^2*d-(d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/
2)*(-d/(c+d)*sin(f*x+e)+d/(c+d))^(1/2)*(-d/(c-d)*sin(f*x+e)-d/(c-d))^(1/2)*
EllipticE((d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2),((c-d)/(c+d))^(1/2))*c*d^2+3
*(d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2)*(-d/(c+d)*sin(f*x+e)+d/(c+d))^(1/2)*(
-d/(c-d)*sin(f*x+e)-d/(c-d))^(1/2)*EllipticE((d/(c-d)*sin(f*x+e)+1/(c-d)*c)
^(1/2),((c-d)/(c+d))^(1/2))*d^3+2*(d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2)*(-d/
(c+d)*sin(f*x+e)+d/(c+d))^(1/2)*(-d/(c-d)*sin(f*x+e)-d/(c-d))^(1/2)*Ellipti
cF((d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2),((c-d)/(c+d))^(1/2))*c^2*d-2*(d/(c-
d)*sin(f*x+e)+1/(c-d)*c)^(1/2)*(-d/(c+d)*sin(f*x+e)+d/(c+d))^(1/2)*(-d/(c-d
)*sin(f*x+e)-d/(c-d))^(1/2)*EllipticF((d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2),
((c-d)/(c+d))^(1/2))*d^3-c*cos(f*x+e)^2*d^2+cos(f*x+e)^2*d^3+c^2*d*sin(f*x+
e)-2*sin(f*x+e)*d^2*c+d^3*sin(f*x+e)-c^2*d+2*c*d^2-d^3)/d/(-(c+d*sin(f*x+e)
)*(-1+sin(f*x+e))*(1+sin(f*x+e)))^(1/2)/a/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)
/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}{a\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(3/2)/(a*sin(f*x + e) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}{a\sin\left(fx+e\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral((d*sin(f*x + e) + c)^(3/2)/(a*sin(f*x + e) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{c\sqrt{c+d\sin(e+fx)}}{\sin(e+fx)+1}\,dx + \int \frac{d\sqrt{c+d\sin(e+fx)}\sin(e+fx)}{\sin(e+fx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(3/2)/(a+a*sin(f*x+e)),x)
```

```
[Out] (Integral(c*sqrt(c + d*sin(e + f*x))/(sin(e + f*x) + 1), x) + Integral(d*sq
rt(c + d*sin(e + f*x))*sin(e + f*x)/(sin(e + f*x) + 1), x))/a
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d\sin(fx+e)+c)^{\frac{3}{2}}}{a\sin(fx+e)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(3/2)/(a*sin(f*x + e) + a), x)
```

3.506 $\int \frac{\sqrt{c+d\sin(e+fx)}}{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=170

$$-\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{f(a\sin(e+fx)+a)}+\frac{(c+d)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{af\sqrt{c+d\sin(e+fx)}}-\frac{\sqrt{c+d\sin(e+fx)}E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\right.}{af\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

```
[Out] -((Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(f*(a + a*Sin[e + f*x]))) - (Elli
pticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(a*f*Sqr
t[(c + d*Sin[e + f*x])/(c + d)]) + ((c + d)*EllipticF[(e - Pi/2 + f*x)/2, (
2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(a*f*Sqrt[c + d*Sin[e + f
*x]])
```

Rubi [A] time = 0.207021, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2769, 2752, 2663, 2661, 2655, 2653}

$$-\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{f(a\sin(e+fx)+a)}+\frac{(c+d)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{af\sqrt{c+d\sin(e+fx)}}-\frac{\sqrt{c+d\sin(e+fx)}E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\right.}{af\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*Sin[e + f*x]]/(a + a*Sin[e + f*x]),x]
```


```
[Out] -((Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(f*(a + a*Sin[e + f*x]))) - (Elli
pticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(a*f*Sqr
t[(c + d*Sin[e + f*x])/(c + d)]) + ((c + d)*EllipticF[(e - Pi/2 + f*x)/2, (
2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(a*f*Sqrt[c + d*Sin[e + f
*x]])
```


Rule 2769

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(a
*f*(a + b*Sin[e + f*x])), x] + Dist[(d*n)/(a*b), Int[(c + d*Sin[e + f*x])^(
n - 1)*(a - b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (IntegerQ[2*n
] || EqQ[c, 0])
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{c+d\sin(e+fx)}}{a+a\sin(e+fx)}\,dx &= -\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{f(a+a\sin(e+fx))}+\frac{d\int \frac{a-a\sin(e+fx)}{\sqrt{c+d\sin(e+fx)}}\,dx}{2a^2}\\
&= -\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{f(a+a\sin(e+fx))}-\frac{\int \sqrt{c+d\sin(e+fx)}\,dx}{2a}+\frac{(c+d)\int \frac{1}{\sqrt{c+d\sin(e+fx)}}\,dx}{2a}\\
&= -\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{f(a+a\sin(e+fx))}-\frac{\sqrt{c+d\sin(e+fx)}\int \sqrt{\frac{c}{c+d}+\frac{d\sin(e+fx)}{c+d}}\,dx}{2a\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}+\frac{\left((c+d)\sqrt{\frac{c+d}{}}\right.}{2a}\\
&= -\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{f(a+a\sin(e+fx))}-\frac{E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2d}{c+d}\right)\sqrt{c+d\sin(e+fx)}}{af\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}+\frac{(c+d)F\left(\frac{1}{2}\right.}{a}
\end{aligned}
$$

Mathematica [A] time = 1.09891, size = 201, normalized size = 1.18

$$
\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(2\sin\left(\frac{1}{2}(e+fx)\right)(c+d\sin(e+fx))-\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left((c+d)\right.\right.}{af(\sin(e+fx)+1)\sqrt{c+}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*Sin[e + f*x]]/(a + a*Sin[e + f*x]),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(2*Sin[(e + f*x)/2]*(c + d*Sin[e + f
*x]) - (Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(c + d*Sin[e + f*x] - (c + d)*
EllipticE[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(
c + d)] + (c + d)*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]*Sqrt[(c +
 d*Sin[e + f*x])/(c + d)])))/(a*f*(1 + Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x
]])
```

Maple [A] time = 1.27, size = 382, normalized size = 2.3

$$\frac{1}{da\cos\left(fx+e\right)f}\sqrt{\left(\cos\left(fx+e\right)\right)^{2}\sin\left(fx+e\right)d+c\left(\cos\left(fx+e\right)\right)^{2}}\left(\sqrt{\frac{d\sin\left(fx+e\right)}{c-d}+\frac{c}{c-d}}\sqrt{-\frac{d\sin\left(fx+e\right)}{c+d}+\frac{}{c}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e)),x)
```

```
[Out] (cos(f*x+e)^2*sin(f*x+e)*d+c*cos(f*x+e)^2)^(1/2)*((d/(c-d)*sin(f*x+e)+1/(c-
d)*c)^(1/2)*(-d/(c+d)*sin(f*x+e)+d/(c+d))^(1/2)*(-d/(c-d)*sin(f*x+e)-d/(c-d
))^(1/2)*EllipticE((d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2),((c-d)/(c+d))^(1/2)
)*c^2-(d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2)*(-d/(c+d)*sin(f*x+e)+d/(c+d))^(1
/2)*(-d/(c-d)*sin(f*x+e)-d/(c-d))^(1/2)*EllipticE((d/(c-d)*sin(f*x+e)+1/(c-
d)*c)^(1/2),((c-d)/(c+d))^(1/2))*d^2-cos(f*x+e)^2*d^2+sin(f*x+e)*c*d-sin(f*
x+e)*d^2-c*d+d^2)/d/(-(c+d*sin(f*x+e))*(-1+sin(f*x+e))*(1+sin(f*x+e)))^(1/2
)/a/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{d\sin\left(fx+e\right)+c}}{a\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*sin(f*x + e) + c)/(a*sin(f*x + e) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{d\sin\left(fx+e\right)+c}}{a\sin\left(fx+e\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*sin(f*x + e) + c)/(a*sin(f*x + e) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{\sqrt{c+d\sin\left(e+fx\right)}}{\sin\left(e+fx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(1/2)/(a+a*sin(f*x+e)),x)
```

```
[Out] Integral(sqrt(c + d*sin(e + f*x))/(sin(e + f*x) + 1), x)/a
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d \sin\left(fx+e\right)+c}}{a \sin\left(fx+e\right)+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*sin(f*x + e) + c)/(a*sin(f*x + e) + a), x)
```

3.507 $\int \frac{1}{(a+a\sin(e+fx))\sqrt{c+d\sin(e+fx)}}\,dx$

Optimal. Leaf size=181

$$-\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{f(c-d)(a\sin(e+fx)+a)}+\frac{\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{af\sqrt{c+d\sin(e+fx)}}-\frac{\sqrt{c+d\sin(e+fx)}E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{af(c-d)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

```
[Out] -((Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/((c - d)*f*(a + a*Sin[e + f*x])))
 - (EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/
(a*(c - d)*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + (EllipticF[(e - Pi/2 + f
*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(a*f*Sqrt[c + d*S
in[e + f*x]])
```

Rubi [A] time = 0.214433, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2768, 2752, 2663, 2661, 2655, 2653}

$$-\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{f(c-d)(a\sin(e+fx)+a)}+\frac{\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{af\sqrt{c+d\sin(e+fx)}}-\frac{\sqrt{c+d\sin(e+fx)}E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{af(c-d)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]]),x]
```

```
[Out] -((Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/((c - d)*f*(a + a*Sin[e + f*x])))
 - (EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/
(a*(c - d)*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + (EllipticF[(e - Pi/2 + f
*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(a*f*Sqrt[c + d*S
in[e + f*x]])
```

Rule 2768

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n
+ 1))/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x])), x] + Dist[d/(a*(b*c - a*d)),
Int[(c + d*Sin[e + f*x])^n*(a*n - b*(n + 1)*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))\sqrt{c+d\sin(e+fx)}}\,dx = -\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{(c-d)f(a+a\sin(e+fx))}+\frac{d\int \frac{-\frac{a}{2}-\frac{1}{2}a\sin(e+fx)}{\sqrt{c+d\sin(e+fx)}}\,dx}{a^2(c-d)}$$

$$= -\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{(c-d)f(a+a\sin(e+fx))}+\frac{\int \frac{1}{\sqrt{c+d\sin(e+fx)}}\,dx}{2a}-\frac{\int \sqrt{c+d\sin(e+}}{2a(c-d)}$$

$$= -\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{(c-d)f(a+a\sin(e+fx))}-\frac{\sqrt{c+d\sin(e+fx)}\int \sqrt{\frac{c}{c+d}+\frac{d\sin(e+fx)}{c+d}}}{2a(c-d)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

$$= -\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{(c-d)f(a+a\sin(e+fx))}-\frac{E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2d}{c+d}\right)\sqrt{c+d\sin(e+}}{a(c-d)f\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

Mathematica [A] time = 1.09524, size = 210, normalized size = 1.16

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(2\sin\left(\frac{1}{2}(e+fx)\right)(c+d\sin(e+fx))-\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left((c-d)\right.\right.}{af(c-d)(\sin(e+fx)+1)\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]]),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(2*Sin[(e + f*x)/2]*(c + d*Sin[e + f
*x]) - (Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(c + d*Sin[e + f*x] - (c + d)*
EllipticE[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(
c + d)] + (c - d)*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]*Sqrt[(c +
 d*Sin[e + f*x])/(c + d)])))/(a*(c - d)*f*(1 + Sin[e + f*x])*Sqrt[c + d*Sin
[e + f*x]])
```

Maple [A] time = 3.408, size = 443, normalized size = 2.5

$$\frac{1}{af\cos\left(fx+e\right)}\sqrt{-\left(-d\sin\left(fx+e\right)-c\right)\left(\cos\left(fx+e\right)\right)^{2}}\left(-\frac{-\left(\sin\left(fx+e\right)\right)^{2}d-c\sin\left(fx+e\right)+d\sin\left(fx+e\right)+c}{c-d}\ -\ \sqrt{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(1/2),x)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/a*(-(-sin(f*x+e)^2*d-c*sin(f*x+e)+d
*sin(f*x+e)+c)/(c-d)/((-d*sin(f*x+e)-c)*(-1+sin(f*x+e))*(1+sin(f*x+e)))^(1/
2)-2*d/(2*c-2*d)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(
c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^
2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))-d/(c
-d)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*(
(-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c
/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+Ellipti
cF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))/cos(f*x+e)/(c+d*si
n(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\sin\left(fx+e\right)+a\right)\sqrt{d\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{d\sin\left(fx+e\right)+c}}{ad\cos\left(fx+e\right)^{2}-ac-ad-(ac+ad)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(d*sin(f*x + e) + c)/(a*d*cos(f*x + e)^2 - a*c - a*d - (a*c +
 a*d)*sin(f*x + e)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{\sqrt{c+d\sin\left(e+fx\right)}\sin\left(e+fx\right)+\sqrt{c+d\sin\left(e+fx\right)}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(c + d*sin(e + f*x))*sin(e + f*x) + sqrt(c + d*sin(e + f*x)
)), x)/a
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a \sin\left(fx+e\right)+a\right)\sqrt{d \sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)), x)
```

3.508 $\int \frac{1}{(a+a\sin(e+fx))(c+d\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=244

$$-\frac{d(c+3d)\cos(e+fx)}{af(c-d)^2(c+d)\sqrt{c+d\sin(e+fx)}}-\frac{\cos(e+fx)}{f(c-d)(a\sin(e+fx)+a)\sqrt{c+d\sin(e+fx)}}+\frac{\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\right.\right.}{af(c-d)\sqrt{c+d\sin(e+}}$$

```
[Out] -((d*(c + 3*d)*Cos[e + f*x])/(a*(c - d)^2*(c + d)*f*Sqrt[c + d*Sin[e + f*x]
])) - Cos[e + f*x]/((c - d)*f*(a + a*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]]
) - ((c + 3*d)*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[
e + f*x]])/(a*(c - d)^2*(c + d)*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + (El
lipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d
)])/(a*(c - d)*f*Sqrt[c + d*Sin[e + f*x]])
```

Rubi [A] time = 0.326915, antiderivative size = 244, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2768, 2754, 2752, 2663, 2661, 2655, 2653}

$$-\frac{d(c+3d)\cos(e+fx)}{af(c-d)^2(c+d)\sqrt{c+d\sin(e+fx)}}-\frac{\cos(e+fx)}{f(c-d)(a\sin(e+fx)+a)\sqrt{c+d\sin(e+fx)}}+\frac{\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\right.\right.}{af(c-d)\sqrt{c+d\sin(e+}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^(3/2)),x]
```

```
[Out] -((d*(c + 3*d)*Cos[e + f*x])/(a*(c - d)^2*(c + d)*f*Sqrt[c + d*Sin[e + f*x]
])) - Cos[e + f*x]/((c - d)*f*(a + a*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]]
) - ((c + 3*d)*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[
e + f*x]])/(a*(c - d)^2*(c + d)*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + (El
lipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d
)])/(a*(c - d)*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2768

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n
+ 1))/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x])), x] + Dist[d/(a*(b*c - a*d)),
Int[(c + d*Sin[e + f*x])^n*(a*n - b*(n + 1)*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
```

```
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))(c+d\sin(e+fx))^{3/2}}\,dx = -\frac{\cos(e+fx)}{(c-d)f(a+a\sin(e+fx))\sqrt{c+d\sin(e+fx)}} + \frac{d\int \frac{-\frac{3a}{2}+\frac{1}{2}a\sin(e+fx)}{(c+d\sin(e+fx))^{3/2}}\,dx}{a^2(c-d)}$$

$$= -\frac{d(c+3d)\cos(e+fx)}{a(c-d)^2(c+d)f\sqrt{c+d\sin(e+fx)}} - \frac{\cos(e+fx)}{(c-d)f(a+a\sin(e+fx))\sqrt{c+}}$$

$$= -\frac{d(c+3d)\cos(e+fx)}{a(c-d)^2(c+d)f\sqrt{c+d\sin(e+fx)}} - \frac{\cos(e+fx)}{(c-d)f(a+a\sin(e+fx))\sqrt{c+}}$$

$$= -\frac{d(c+3d)\cos(e+fx)}{a(c-d)^2(c+d)f\sqrt{c+d\sin(e+fx)}} - \frac{\cos(e+fx)}{(c-d)f(a+a\sin(e+fx))\sqrt{c+}}$$

$$= -\frac{d(c+3d)\cos(e+fx)}{a(c-d)^2(c+d)f\sqrt{c+d\sin(e+fx)}} - \frac{\cos(e+fx)}{(c-d)f(a+a\sin(e+fx))\sqrt{c+}}$$

Mathematica [A] time = 1.99742, size = 264, normalized size = 1.08

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^2\left(-\left(c^2-d^2\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{4}(-2e-2fx+\pi)|\frac{2d}{c+d}\right)+\left(c^2+4cd+3d^2\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\right.}{af(c-d)^2(c+d)(\sin(e+fx)+}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^(3/2)),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2*((-2*((c + d)^2*Cos[(e + f*x)/2] +
 d*(2*(c + d) + (c + 3*d)*Cos[e + f*x])*Sin[(e + f*x)/2]))/(Cos[(e + f*x)/2
] + Sin[(e + f*x)/2]) + (c + 3*d)*(c + d*Sin[e + f*x]) + (c^2 + 4*c*d + 3*d
^2)*EllipticE[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x
])/(c + d)] - (c^2 - d^2)*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]*S
qrt[(c + d*Sin[e + f*x])/(c + d)]))/(a*(c - d)^2*(c + d)*f*(1 + Sin[e + f*x
])*Sqrt[c + d*Sin[e + f*x]])
```

Maple [B] time = 1.509, size = 925, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(3/2),x)
```

```
[Out] (cos(f*x+e)^2*sin(f*x+e)*d+c*cos(f*x+e)^2)^(1/2)*((d/(c-d)*sin(f*x+e)+1/(c-
d)*c)^(1/2)*(-d/(c+d)*sin(f*x+e)+d/(c+d))^(1/2)*(-d/(c-d)*sin(f*x+e)-d/(c-d
))^(1/2)*EllipticE((d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2),((c-d)/(c+d))^(1/2)
)*c^3+3*(d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2)*(-d/(c+d)*sin(f*x+e)+d/(c+d))^
(1/2)*(-d/(c-d)*sin(f*x+e)-d/(c-d))^(1/2)*EllipticE((d/(c-d)*sin(f*x+e)+1/(
c-d)*c)^(1/2),((c-d)/(c+d))^(1/2))*c^2*d-(d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/
2)*(-d/(c+d)*sin(f*x+e)+d/(c+d))^(1/2)*(-d/(c-d)*sin(f*x+e)-d/(c-d))^(1/2)*
EllipticE((d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2),((c-d)/(c+d))^(1/2))*c*d^2-3
*(d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2)*(-d/(c+d)*sin(f*x+e)+d/(c+d))^(1/2)*(
-d/(c-d)*sin(f*x+e)-d/(c-d))^(1/2)*EllipticE((d/(c-d)*sin(f*x+e)+1/(c-d)*c)
^(1/2),((c-d)/(c+d))^(1/2))*d^3-4*(d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2)*(-d/
(c+d)*sin(f*x+e)+d/(c+d))^(1/2)*(-d/(c-d)*sin(f*x+e)-d/(c-d))^(1/2)*Ellipti
cF((d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2),((c-d)/(c+d))^(1/2))*c^2*d+4*(d/(c-
d)*sin(f*x+e)+1/(c-d)*c)^(1/2)*(-d/(c+d)*sin(f*x+e)+d/(c+d))^(1/2)*(-d/(c-d
)*sin(f*x+e)-d/(c-d))^(1/2)*EllipticF((d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2),
((c-d)/(c+d))^(1/2))*d^3-c*cos(f*x+e)^2*d^2-3*cos(f*x+e)^2*d^3+c^2*d*sin(f*
x+e)-d^3*sin(f*x+e)-c^2*d+d^3)/d/(-(c+d*sin(f*x+e))*(-1+sin(f*x+e))*(1+sin(
f*x+e)))^(1/2)/(c^2-d^2)/(c-d)/a/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a \sin\left(fx + e\right) + a\right)\left(d \sin\left(fx + e\right) + c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{d \sin\left(fx + e\right) + c}}{ac^2 + 2\,acd + ad^2 - \left(2\,acd + ad^2\right)\cos\left(fx + e\right)^2 - \left(ad^2 \cos\left(fx + e\right)^2 - ac^2 - 2\,acd - ad^2\right)\sin\left(fx + e\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*sin(f*x + e) + c)/(a*c^2 + 2*a*c*d + a*d^2 - (2*a*c*d + a*d
^2)*cos(f*x + e)^2 - (a*d^2*cos(f*x + e)^2 - a*c^2 - 2*a*c*d - a*d^2)*sin(f
*x + e)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{c\sqrt{c+d\sin\left(e+fx\right)}\sin\left(e+fx\right)+c\sqrt{c+d\sin\left(e+fx\right)}+d\sqrt{c+d\sin\left(e+fx\right)}\sin^{2}\left(e+fx\right)+d\sqrt{c+d\sin\left(e+fx\right)}\sin\left(e+fx\right)}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral(1/(c*sqrt(c + d*sin(e + f*x))*sin(e + f*x) + c*sqrt(c + d*sin(e +
f*x)) + d*sqrt(c + d*sin(e + f*x))*sin(e + f*x)**2 + d*sqrt(c + d*sin(e + f
*x))*sin(e + f*x)), x)/a
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^(3/2)), x)
```

3.509 $\int \frac{1}{(a+a\sin(e+fx))(c+d\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=333

$$-\frac{d\left(3c^2+20cd+9d^2\right)\cos(e+fx)}{3af(c-d)^3(c+d)^2\sqrt{c+d\sin(e+fx)}}-\frac{\left(3c^2+20cd+9d^2\right)\sqrt{c+d\sin(e+fx)}E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3af(c-d)^3(c+d)^2\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}-\frac{d(\cdot}{3af(c-d)^{\cdot}}$$

```
[Out] -(d*(3*c + 5*d)*Cos[e + f*x])/(3*a*(c - d)^2*(c + d)*f*(c + d*Sin[e + f*x])
^(3/2)) - Cos[e + f*x]/((c - d)*f*(a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])
^(3/2)) - (d*(3*c^2 + 20*c*d + 9*d^2)*Cos[e + f*x])/(3*a*(c - d)^3*(c + d)^
2*f*Sqrt[c + d*Sin[e + f*x]]) - ((3*c^2 + 20*c*d + 9*d^2)*EllipticE[(e - Pi
/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(3*a*(c - d)^3*(c + d
)^2*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + ((3*c + 5*d)*EllipticF[(e - Pi/
2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(3*a*(c - d)
^2*(c + d)*f*Sqrt[c + d*Sin[e + f*x]])
```

Rubi [A] time = 0.488061, antiderivative size = 333, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2768, 2754, 2752, 2663, 2661, 2655, 2653}

$$-\frac{d\left(3c^2+20cd+9d^2\right)\cos(e+fx)}{3af(c-d)^3(c+d)^2\sqrt{c+d\sin(e+fx)}}-\frac{\left(3c^2+20cd+9d^2\right)\sqrt{c+d\sin(e+fx)}E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3af(c-d)^3(c+d)^2\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}-\frac{d(\cdot}{3af(c-d)^{\cdot}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^(5/2)),x]
```



```
[Out] -(d*(3*c + 5*d)*Cos[e + f*x])/(3*a*(c - d)^2*(c + d)*f*(c + d*Sin[e + f*x])
^(3/2)) - Cos[e + f*x]/((c - d)*f*(a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])
^(3/2)) - (d*(3*c^2 + 20*c*d + 9*d^2)*Cos[e + f*x])/(3*a*(c - d)^3*(c + d)^
2*f*Sqrt[c + d*Sin[e + f*x]]) - ((3*c^2 + 20*c*d + 9*d^2)*EllipticE[(e - Pi
/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(3*a*(c - d)^3*(c + d
)^2*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + ((3*c + 5*d)*EllipticF[(e - Pi/
2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(3*a*(c - d)
^2*(c + d)*f*Sqrt[c + d*Sin[e + f*x]])
```


Rule 2768

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n
+ 1))/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x])), x] + Dist[d/(a*(b*c - a*d)),
Int[(c + d*Sin[e + f*x])^n*(a*n - b*(n + 1)*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))(c+d\sin(e+fx))^{5/2}}\,dx = -\frac{\cos(e+fx)}{(c-d)f(a+a\sin(e+fx))(c+d\sin(e+fx))^{3/2}} + \frac{d\int \frac{-\frac{5a}{2}+\frac{3}{2}a\sin(e+fx)}{(c+d\sin(e+fx))^{5/2}}\,c}{a^2(c-d)}$$

$$= -\frac{d(3c+5d)\cos(e+fx)}{3a(c-d)^2(c+d)f(c+d\sin(e+fx))^{3/2}} - \frac{\cos(e+fx)}{(c-d)f(a+a\sin(e+fx))(c}$$

$$= -\frac{d(3c+5d)\cos(e+fx)}{3a(c-d)^2(c+d)f(c+d\sin(e+fx))^{3/2}} - \frac{\cos(e+fx)}{(c-d)f(a+a\sin(e+fx))(c}$$

$$= -\frac{d(3c+5d)\cos(e+fx)}{3a(c-d)^2(c+d)f(c+d\sin(e+fx))^{3/2}} - \frac{\cos(e+fx)}{(c-d)f(a+a\sin(e+fx))(c}$$

$$= -\frac{d(3c+5d)\cos(e+fx)}{3a(c-d)^2(c+d)f(c+d\sin(e+fx))^{3/2}} - \frac{\cos(e+fx)}{(c-d)f(a+a\sin(e+fx))(c}$$

$$= -\frac{d(3c+5d)\cos(e+fx)}{3a(c-d)^2(c+d)f(c+d\sin(e+fx))^{3/2}} - \frac{\cos(e+fx)}{(c-d)f(a+a\sin(e+fx))(c}$$

Mathematica [A] time = 4.22967, size = 367, normalized size = 1.1

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^2\left(2(c+d\sin(e+fx))\left(\frac{3\sin\left(\frac{1}{2}(e+fx)\right)}{\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)}-\frac{\frac{d^2\cos(e+fx)\left(8c^2+d(7c+3d)\sin(e+fx)+3cd-d^2\right)}{(c+d\sin(e+fx))^2}+}{(c+d)^2}\right.\right.}{3af(c}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^(5/2)),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2*(((3*c^2 + 20*c*d + 9*d^2)*(c + d*
Sin[e + f*x]) + d*(15*c^2 + 12*c*d + 5*d^2)*EllipticF[(-2*e + Pi - 2*f*x)/4
, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)] + (3*c^2 + 20*c*d + 9*d
^2)*((c + d)*EllipticE[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)] - c*EllipticF[
(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)])*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/
(c + d)^2 + 2*(c + d*Sin[e + f*x])*((3*Sin[(e + f*x)/2])/(Cos[(e + f*x)/2]
+ Sin[(e + f*x)/2]) - (3*c^2 + 13*c*d + 6*d^2 + (d^2*Cos[e + f*x]*(8*c^2 +
3*c*d - d^2 + d*(7*c + 3*d)*Sin[e + f*x]))/(c + d*Sin[e + f*x])^2)/(c + d)^
2)))/(3*a*(c - d)^3*f*(1 + Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]])
```

Maple [B] time = 5.365, size = 1291, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(5/2),x)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/a*(-d/(c-d)^2*(2*d*cos(f*x+e)^2/(c^
2-d^2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2*c/(c^2-d^2)*(c/d-1)*((c+d*
sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/
(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*
x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+2/(c^2-d^2)*d*(c/d-1)*((c+d*sin(f*x
+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^
(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin
(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d
))^(1/2),((c-d)/(c+d))^(1/2))))-d/(c-d)*(2/3/(c^2-d^2)/d*(-(-d*sin(f*x+e)-c
)*cos(f*x+e)^2)^(1/2)/(sin(f*x+e)+c/d)^2+8/3*d*cos(f*x+e)^2/(c^2-d^2)^2*c/(
-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2*(3*c^2+d^2)/(3*c^4-6*c^2*d^2+3*d^4
)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-
sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*Ellipt
icF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+8/3*c*d/(c^2-d^2)^2
*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-s
in(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-
1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(
((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+1/(c-d)^2*(-(-sin(f*x
+e)^2*d-c*sin(f*x+e)+d*sin(f*x+e)+c)/(c-d)/((-d*sin(f*x+e)-c)*(-1+sin(f*x+e
))*(1+sin(f*x+e)))^(1/2)-2*d/(2*c-2*d)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/
2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin
(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c
-d)/(c+d))^(1/2))-d/(c-d)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(
f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*co
s(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)
/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)
))))/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{d\sin\left(fx+e\right)+c}}{ad^3\cos\left(fx+e\right)^4+ac^3+3\,ac^2d+3\,acd^2+ad^3-\left(3\,ac^2d+3\,acd^2+2\,ad^3\right)\cos\left(fx+e\right)^2+\left(ac^3+3\,ac^2d+3\,a\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*sin(f*x + e) + c)/(a*d^3*cos(f*x + e)^4 + a*c^3 + 3*a*c^2*d
 + 3*a*c*d^2 + a*d^3 - (3*a*c^2*d + 3*a*c*d^2 + 2*a*d^3)*cos(f*x + e)^2 + (
a*c^3 + 3*a*c^2*d + 3*a*c*d^2 + a*d^3 - (3*a*c*d^2 + a*d^3)*cos(f*x + e)^2)
*sin(f*x + e)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{c^2\sqrt{c+d\sin\left(e+fx\right)}\sin\left(e+fx\right)+c^2\sqrt{c+d\sin\left(e+fx\right)}+2cd\sqrt{c+d\sin\left(e+fx\right)}\sin^2\left(e+fx\right)+2cd\sqrt{c+d\sin\left(e+fx\right)}\sin\left(e+fx\right)+d^2\sqrt{c+d\sin\left(e+fx\right)}\sin^3\left(e+fx\right)+d^2\sqrt{}}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))**(5/2),x)
```

```
[Out] Integral(1/(c**2*sqrt(c + d*sin(e + f*x))*sin(e + f*x) + c**2*sqrt(c + d*si
n(e + f*x)) + 2*c*d*sqrt(c + d*sin(e + f*x))*sin(e + f*x)**2 + 2*c*d*sqrt(c
 + d*sin(e + f*x))*sin(e + f*x) + d**2*sqrt(c + d*sin(e + f*x))*sin(e + f*x
)**3 + d**2*sqrt(c + d*sin(e + f*x))*sin(e + f*x)**2), x)/a
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^(5/2)), x)
```

3.510 $\int \frac{(c+d\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^2}\,dx$

Optimal. Leaf size=256

$$\frac{(c+5d)\left(c^2-d^2\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3a^2f\sqrt{c+d\sin(e+fx)}}-\frac{\left(c^2+5cd-12d^2\right)\sqrt{c+d\sin(e+fx)}E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3a^2f\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

```
[Out] -((c - d)*(c + 5*d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(3*a^2*f*(1 + Si
n[e + f*x])) - ((c - d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(3/2))/(3*f*(a +
a*Sin[e + f*x])^2) - ((c^2 + 5*c*d - 12*d^2)*EllipticE[(e - Pi/2 + f*x)/2,
(2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(3*a^2*f*Sqrt[(c + d*Sin[e + f*x])
/(c + d)]) + ((c + 5*d)*(c^2 - d^2)*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c
+ d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(3*a^2*f*Sqrt[c + d*Sin[e + f*x]]
)
```

Rubi [A] time = 0.553451, antiderivative size = 256, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2765, 2977, 2752, 2663, 2661, 2655, 2653}

$$\frac{(c+5d)\left(c^2-d^2\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3a^2f\sqrt{c+d\sin(e+fx)}}-\frac{\left(c^2+5cd-12d^2\right)\sqrt{c+d\sin(e+fx)}E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3a^2f\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^(5/2)/(a + a*Sin[e + f*x])^2,x]
```


```
[Out] -((c - d)*(c + 5*d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(3*a^2*f*(1 + Si
n[e + f*x])) - ((c - d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(3/2))/(3*f*(a +
a*Sin[e + f*x])^2) - ((c^2 + 5*c*d - 12*d^2)*EllipticE[(e - Pi/2 + f*x)/2,
(2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(3*a^2*f*Sqrt[(c + d*Sin[e + f*x])
/(c + d)]) + ((c + 5*d)*(c^2 - d^2)*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c
+ d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(3*a^2*f*Sqrt[c + d*Sin[e + f*x]]
)
```


Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
```

```
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^2}\,dx = -\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{3f(a+a\sin(e+fx))^2} - \frac{\int \frac{\sqrt{c+d\sin(e+fx)}\left(-\frac{1}{2}a\left(2c^2+7cd-3d^2\right)+\frac{1}{2}a(c-7d)d\sin(e+fx\right.}{a+a\sin(e+fx)}}{3a^2}$$

$$= -\frac{(c-d)(c+5d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3a^2f(1+\sin(e+fx))} - \frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{3f(a+a\sin(e+fx))^2}$$

$$= -\frac{(c-d)(c+5d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3a^2f(1+\sin(e+fx))} - \frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{3f(a+a\sin(e+fx))^2}$$

$$= -\frac{(c-d)(c+5d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3a^2f(1+\sin(e+fx))} - \frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{3f(a+a\sin(e+fx))^2}$$

$$= -\frac{(c-d)(c+5d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3a^2f(1+\sin(e+fx))} - \frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{3f(a+a\sin(e+fx))^2}$$

Mathematica [A] time = 2.62961, size = 310, normalized size = 1.21

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^4\left(-\left(c^2+5cd-6d^2\right)(c+d\sin(e+fx))+\left(c^2+5cd-12d^2\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\left((c+d\right.\right.}{}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*Sin[e + f*x])^(5/2)/(a + a*Sin[e + f*x])^2,x]

[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*(-((c^2 + 5*c*d - 6*d^2)*(c + d*Si
n[e + f*x])) + ((c - d)*(7*d*Cos[(e + f*x)/2] - (c + 6*d)*Cos[(3*(e + f*x))
/2] + (3*c + 11*d)*Sin[(e + f*x)/2])*(c + d*Sin[e + f*x]))/(Cos[(e + f*x)/2
] + Sin[(e + f*x)/2])^3 + d^2*(-11*c + 5*d)*EllipticF[(-2*e + Pi - 2*f*x)/4
, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)] + (c^2 + 5*c*d - 12*d^2
)*((c + d)*EllipticE[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)] - c*EllipticF[(-
2*e + Pi - 2*f*x)/4, (2*d)/(c + d)])*Sqrt[(c + d*Sin[e + f*x])/(c + d)]))/(
3*a^2*f*(1 + Sin[e + f*x])^2*Sqrt[c + d*Sin[e + f*x]])

Maple [B] time = 4.717, size = 1372, normalized size = 5.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c+d*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^2,x)

[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/a^2*(2*d^3*(c/d-1)*((c+d*sin(f*x+e)
)/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/
2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*
x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^
(1/2),((c-d)/(c+d))^(1/2)))+6*c*d^2*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*
(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*
x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)
/(c+d))^(1/2))-4*d^3*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e
))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x
+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+
(c^3-3*c^2*d+3*c*d^2-d^3)*(-1/3/(c-d)*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/
2)/(1+sin(f*x+e))^2-1/3*(-sin(f*x+e)^2*d-c*sin(f*x+e)+d*sin(f*x+e)+c)/(c-d)
^2*(c-3*d)/((-d*sin(f*x+e)-c)*(-1+sin(f*x+e))*(1+sin(f*x+e)))^(1/2)+2*d^2/(
3*c^2-6*c*d+3*d^2)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))
/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e
)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))-1/
3*d*(c-3*d)/(c-d)^2*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e)
)/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+
e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d)
)^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+3*
d*(c^2-2*c*d+d^2)*(-(-sin(f*x+e)^2*d-c*sin(f*x+e)+d*sin(f*x+e)+c)/(c-d)/((-
d*sin(f*x+e)-c)*(-1+sin(f*x+e))*(1+sin(f*x+e)))^(1/2)-2*d/(2*c-2*d)*(c/d-1)
*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e
)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d
*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))-d/(c-d)*(c/d-1)*((c+d*sin(f*
x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))
^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*si
n(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-

```
d))^(1/2),((c-d)/(c+d))^(1/2)))))/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}{\left(a\sin\left(fx+e\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(5/2)/(a*sin(f*x + e) + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(d^{2}\cos\left(fx+e\right)^{2}-2\,cd\sin\left(fx+e\right)-c^{2}-d^{2}\right)\sqrt{d\sin\left(fx+e\right)+c}}{a^{2}\cos\left(fx+e\right)^{2}-2\,a^{2}\sin\left(fx+e\right)-2\,a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral((d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)*sqrt(d*sin(f
*x + e) + c)/(a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x + e) - 2*a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(5/2)/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}{\left(a\sin\left(fx+e\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(5/2)/(a*sin(f*x + e) + a)^2, x)
```

3.511 $\int \frac{(c+d\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^2}\,dx$

Optimal. Leaf size=237

$$-\frac{(c+3d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3a^2f(\sin(e+fx)+1)}+\frac{(c+d)(c+2d)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3a^2f\sqrt{c+d\sin(e+fx)}}-\frac{(c+3d)\sqrt{c+d\sin(e}{3a^2j}$$

```
[Out] -((c + 3*d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(3*a^2*f*(1 + Sin[e + f*
x])) - ((c - d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(3*f*(a + a*Sin[e +
f*x])^2) - ((c + 3*d)*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c +
 d*Sin[e + f*x]])/(3*a^2*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + ((c + d)*(
c + 2*d)*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f
*x])/(c + d)])/(3*a^2*f*Sqrt[c + d*Sin[e + f*x]])
```

Rubi [A] time = 0.537778, antiderivative size = 237, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2765, 2978, 2752, 2663, 2661, 2655, 2653}

$$-\frac{(c+3d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3a^2f(\sin(e+fx)+1)}+\frac{(c+d)(c+2d)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3a^2f\sqrt{c+d\sin(e+fx)}}-\frac{(c+3d)\sqrt{c+d\sin(e}{3a^2j}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^(3/2)/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] -((c + 3*d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(3*a^2*f*(1 + Sin[e + f*
x])) - ((c - d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(3*f*(a + a*Sin[e +
f*x])^2) - ((c + 3*d)*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c +
 d*Sin[e + f*x]])/(3*a^2*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + ((c + d)*(
c + 2*d)*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f
*x])/(c + d)])/(3*a^2*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
```

```
&& !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] && !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^2}\,dx = -\frac{(c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3f(a+a\sin(e+fx))^2} - \frac{\int \frac{-\frac{1}{2}a\left(2c^2+5cd-d^2\right)-\frac{1}{2}ad(c+5d)\sin(e+fx)}{(a+a\sin(e+fx))\sqrt{c+d\sin(e+fx)}}\,dx}{3a^2}$$

$$= -\frac{(c+3d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3a^2f(1+\sin(e+fx))} - \frac{(c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3f(a+a\sin(e+fx))^2} + \frac{\int \frac{a^2(c-}{}}{}$$

$$= -\frac{(c+3d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3a^2f(1+\sin(e+fx))} - \frac{(c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3f(a+a\sin(e+fx))^2} + \frac{((c+d}{}$$

$$= -\frac{(c+3d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3a^2f(1+\sin(e+fx))} - \frac{(c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3f(a+a\sin(e+fx))^2} - \frac{((c+3}{}$$

$$= -\frac{(c+3d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3a^2f(1+\sin(e+fx))} - \frac{(c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3f(a+a\sin(e+fx))^2} - \frac{(c+3d}{}$$

Mathematica [A] time = 2.7908, size = 283, normalized size = 1.19

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^4\left(-2d^2\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{4}(-2e-2fx+\pi)|\frac{2d}{c+d}\right)-(c+3d)(c+d\sin(e+fx))+\frac{(c+}{}\right.}{3a^2f(}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])^(3/2)/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*(-((c + 3*d)*(c + d*Sin[e + f*x]))
 + ((4*d*Cos[(e + f*x)/2] - (c + 3*d)*Cos[(3*(e + f*x))/2] + (3*c + 5*d)*Si
n[(e + f*x)/2])*(c + d*Sin[e + f*x]))/(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])
^3 - 2*d^2*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]*Sqrt[(c + d*Sin[
e + f*x])/(c + d)] + (c + 3*d)*((c + d)*EllipticE[(-2*e + Pi - 2*f*x)/4, (2
*d)/(c + d)] - c*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)])*Sqrt[(c +
 d*Sin[e + f*x])/(c + d)]))/(3*a^2*f*(1 + Sin[e + f*x])^2*Sqrt[c + d*Sin[e
+ f*x]])
```

Maple [B] time = 4.859, size = 1049, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^2,x)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/a^2*(2*d^2*(c/d-1)*((c+d*sin(f*x+e)
)/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/
2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d
))^(1/2),((c-d)/(c+d))^(1/2))+(c^2-2*c*d+d^2)*(-1/3/(c-d)*(-(-d*sin(f*x+e)-
c)*cos(f*x+e)^2)^(1/2)/(1+sin(f*x+e))^2-1/3*(-sin(f*x+e)^2*d-c*sin(f*x+e)+d
*sin(f*x+e)+c)/(c-d)^2*(c-3*d)/((-d*sin(f*x+e)-c)*(-1+sin(f*x+e))*(1+sin(f*
x+e)))^(1/2)+2*d^2/(3*c^2-6*c*d+3*d^2)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/
2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin
(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c
-d)/(c+d))^(1/2))-1/3*d*(c-3*d)/(c-d)^2*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1
/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*si
n(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d)
)^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d
)/(c+d))^(1/2))))+2*d*(c-d)*(-(-sin(f*x+e)^2*d-c*sin(f*x+e)+d*sin(f*x+e)+c)
/(c-d)/((-d*sin(f*x+e)-c)*(-1+sin(f*x+e))*(1+sin(f*x+e)))^(1/2)-2*d/(2*c-2*
d)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((
-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*Ellip
ticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))-d/(c-d)*(c/d-1)*((
c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1
)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*Elliptic
E(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f
*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))))/cos(f*x+e)/(c+d*sin(f*x+e))^(1/
2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}{\left(a\sin\left(fx+e\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(3/2)/(a*sin(f*x + e) + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}{a^{2}\cos\left(fx+e\right)^{2}-2\,a^{2}\sin\left(fx+e\right)-2\,a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral(-(d*sin(f*x + e) + c)^(3/2)/(a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x +
e) - 2*a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(3/2)/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}{\left(a\sin\left(fx+e\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(3/2)/(a*sin(f*x + e) + a)^2, x)
```

3.512 $\int \frac{\sqrt{c+d\sin(e+fx)}}{(a+a\sin(e+fx))^2}\,dx$

Optimal. Leaf size=233

$$-\frac{c\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3a^2f(c-d)(\sin(e+fx)+1)}+\frac{(c+d)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3a^2f\sqrt{c+d\sin(e+fx)}}-\frac{c\sqrt{c+d\sin(e+fx)}E\left(\frac{1}{2}\left(e+fx-\right.\right.}{3a^2f(c-d)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

```
[Out] -(c*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(3*a^2*(c - d)*f*(1 + Sin[e + f*
x])) - (Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(3*f*(a + a*Sin[e + f*x])^2)
 - (c*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]]
)/(3*a^2*(c - d)*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + ((c + d)*EllipticF
[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(3*
a^2*f*Sqrt[c + d*Sin[e + f*x]])
```

Rubi [A] time = 0.410489, antiderivative size = 233, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2764, 2978, 2752, 2663, 2661, 2655, 2653}

$$-\frac{c\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3a^2f(c-d)(\sin(e+fx)+1)}+\frac{(c+d)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3a^2f\sqrt{c+d\sin(e+fx)}}-\frac{c\sqrt{c+d\sin(e+fx)}E\left(\frac{1}{2}\left(e+fx-\right.\right.}{3a^2f(c-d)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*Sin[e + f*x]]/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] -(c*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(3*a^2*(c - d)*f*(1 + Sin[e + f*
x])) - (Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(3*f*(a + a*Sin[e + f*x])^2)
 - (c*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]]
)/(3*a^2*(c - d)*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + ((c + d)*EllipticF
[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(3*
a^2*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2764

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m
*(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a*d*n - b*c*
(m + 1) - b*d*(m + n + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && L
tQ[m, -1] && LtQ[0, n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c,
 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
```

```
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\sqrt{c+d\sin(e+fx)}}{(a+a\sin(e+fx))^2}\,dx = -\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3f(a+a\sin(e+fx))^2}+\frac{\int \frac{\frac{1}{2}a(2c+d)+\frac{1}{2}ad\sin(e+fx)}{(a+a\sin(e+fx))\sqrt{c+d\sin(e+fx)}}\,dx}{3a^2}$$

$$= -\frac{c\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3a^2(c-d)f(1+\sin(e+fx))}-\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3f(a+a\sin(e+fx))^2}-\frac{\int \frac{\frac{a^2d^2}{2}+\frac{1}{2}a^2cd\sin(e+fx)}{\sqrt{c+d\sin(e+fx)}}\,d}{3a^4(c-d)}$$

$$= -\frac{c\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3a^2(c-d)f(1+\sin(e+fx))}-\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3f(a+a\sin(e+fx))^2}-\frac{c\int\sqrt{c+d\sin(e+fx}}{6a^2(c-d)}$$

$$= -\frac{c\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3a^2(c-d)f(1+\sin(e+fx))}-\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3f(a+a\sin(e+fx))^2}-\frac{\left(c\sqrt{c+d\sin(e+fx)}\right)}{6a^2(c-d)\sqrt{}}$$

$$= -\frac{c\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3a^2(c-d)f(1+\sin(e+fx))}-\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3f(a+a\sin(e+fx))^2}-\frac{cE\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2d}{c+d}\right.}{3a^2(c-d)f\sqrt{}}$$

Mathematica [A] time = 2.56064, size = 256, normalized size = 1.1

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^4\left(-\left(c^2-d^2\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{4}(-2e-2fx+\pi)|\frac{2d}{c+d}\right)-c(c+d\sin(e+fx))+\frac{(c+d}{}\right.}{3a^2f(c-d)(\sin(e+fx)+1)^2\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*Sin[e + f*x]]/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*(-(c*(c + d*Sin[e + f*x])) + ((d*C
os[(e + f*x)/2] - c*Cos[(3*(e + f*x))/2] + (3*c - d)*Sin[(e + f*x)/2])*(c +
 d*Sin[e + f*x]))/(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 + c*(c + d)*Ellip
ticE[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d
)] - (c^2 - d^2)*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]*Sqrt[(c +
d*Sin[e + f*x])/(c + d)]))/(3*a^2*(c - d)*f*(1 + Sin[e + f*x])^2*Sqrt[c + d
*Sin[e + f*x]])
```

Maple [B] time = 4.282, size = 906, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^2,x)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/a^2*((c-d)*(-1/3/(c-d)*(-(-d*sin(f*
x+e)-c)*cos(f*x+e)^2)^(1/2)/(1+sin(f*x+e))^2-1/3*(-sin(f*x+e)^2*d-c*sin(f*x
+e)+d*sin(f*x+e)+c)/(c-d)^2*(c-3*d)/((-d*sin(f*x+e)-c)*(-1+sin(f*x+e))*(1+s
in(f*x+e)))^(1/2)+2*d^2/(3*c^2-6*c*d+3*d^2)*(c/d-1)*((c+d*sin(f*x+e))/(c-d)
)^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-
d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2
),((c-d)/(c+d))^(1/2))-1/3*d*(c-3*d)/(c-d)^2*(c/d-1)*((c+d*sin(f*x+e))/(c-d
))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(
-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/
(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),
((c-d)/(c+d))^(1/2))))+d*(-(-sin(f*x+e)^2*d-c*sin(f*x+e)+d*sin(f*x+e)+c)/(c
-d)/((-d*sin(f*x+e)-c)*(-1+sin(f*x+e))*(1+sin(f*x+e)))^(1/2)-2*d/(2*c-2*d)*
(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-si
n(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*Elliptic
F(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))-d/(c-d)*(c/d-1)*((c+d
*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d
/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE((
(c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+
e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))))/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/
f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d\sin\left(fx+e\right)+c}}{\left(a\sin\left(fx+e\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*sin(f*x + e) + c)/(a*sin(f*x + e) + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{d\sin\left(fx+e\right)+c}}{a^2\cos\left(fx+e\right)^2-2\,a^2\sin\left(fx+e\right)-2\,a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral(-sqrt(d*sin(f*x + e) + c)/(a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x + e)
 - 2*a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sqrt{c+d\sin\left(e+fx\right)}}{\sin^2\left(e+fx\right)+2\sin\left(e+fx\right)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(1/2)/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Integral(sqrt(c + d*sin(e + f*x))/(sin(e + f*x)**2 + 2*sin(e + f*x) + 1), x
)/a**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d\sin\left(fx+e\right)+c}}{\left(a\sin\left(fx+e\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*sin(f*x + e) + c)/(a*sin(f*x + e) + a)^2, x)
```

3.513 $\int \frac{1}{(a+a\sin(e+fx))^2\sqrt{c+d\sin(e+fx)}}\,dx$

Optimal. Leaf size=257

$$-\frac{(c-3d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3a^2f(c-d)^2(\sin(e+fx)+1)}+\frac{(c-2d)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3a^2f(c-d)\sqrt{c+d\sin(e+fx)}}-\frac{(c-3d)\sqrt{c+d\sin(e+fx)}}{3a^2f(c-d)^2\sqrt{}}$$

```
[Out] -((c - 3*d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(3*a^2*(c - d)^2*f*(1 +
Sin[e + f*x])) - (Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(3*(c - d)*f*(a +
a*Sin[e + f*x])^2) - ((c - 3*d)*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)
]*Sqrt[c + d*Sin[e + f*x]])/(3*a^2*(c - d)^2*f*Sqrt[(c + d*Sin[e + f*x])/(c
 + d)]) + ((c - 2*d)*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c +
 d*Sin[e + f*x])/(c + d)])/(3*a^2*(c - d)*f*Sqrt[c + d*Sin[e + f*x]])
```

Rubi [A] time = 0.441346, antiderivative size = 257, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2766, 2978, 2752, 2663, 2661, 2655, 2653}

$$-\frac{(c-3d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3a^2f(c-d)^2(\sin(e+fx)+1)}+\frac{(c-2d)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3a^2f(c-d)\sqrt{c+d\sin(e+fx)}}-\frac{(c-3d)\sqrt{c+d\sin(e+fx)}}{3a^2f(c-d)^2\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^2*Sqrt[c + d*Sin[e + f*x]]),x]
```


```
[Out] -((c - 3*d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(3*a^2*(c - d)^2*f*(1 +
Sin[e + f*x])) - (Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(3*(c - d)*f*(a +
a*Sin[e + f*x])^2) - ((c - 3*d)*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)
]*Sqrt[c + d*Sin[e + f*x]])/(3*a^2*(c - d)^2*f*Sqrt[(c + d*Sin[e + f*x])/(c
 + d)]) + ((c - 2*d)*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c +
 d*Sin[e + f*x])/(c + d)])/(3*a^2*(c - d)*f*Sqrt[c + d*Sin[e + f*x]])
```


Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
```

```
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^2\sqrt{c+d\sin(e+fx)}}\,dx = -\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3(c-d)f(a+a\sin(e+fx))^2} - \frac{\int \frac{-\frac{1}{2}a(2c-5d)-\frac{1}{2}ad\sin(e+fx)}{(a+a\sin(e+fx))\sqrt{c+d\sin(e+fx)}}\,dx}{3a^2(c-d)}$$

$$= -\frac{(c-3d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3a^2(c-d)^2f(1+\sin(e+fx))} - \frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3(c-d)f(a+a\sin(e+fx))^2}$$

$$= -\frac{(c-3d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3a^2(c-d)^2f(1+\sin(e+fx))} - \frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3(c-d)f(a+a\sin(e+fx))^2}$$

$$= -\frac{(c-3d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3a^2(c-d)^2f(1+\sin(e+fx))} - \frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3(c-d)f(a+a\sin(e+fx))^2}$$

$$= -\frac{(c-3d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3a^2(c-d)^2f(1+\sin(e+fx))} - \frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3(c-d)f(a+a\sin(e+fx))^2}$$

Mathematica [A] time = 2.46164, size = 290, normalized size = 1.13

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^4\left(-2d^2\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{4}(-2e-2fx+\pi)|\frac{2d}{c+d}\right)-(c-3d)(c+d\sin(e+fx))-\frac{(c+d}{}\right.}{3a^2f(c-}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^2*Sqrt[c + d*Sin[e + f*x]]),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*(-((c - 3*d)*(c + d*Sin[e + f*x]))
 - ((2*d*Cos[(e + f*x)/2] + (c - 3*d)*Cos[(3*(e + f*x))/2] + (-3*c + 7*d)*S
in[(e + f*x)/2])*(c + d*Sin[e + f*x]))/(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)^3 - 2*d^2*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]*Sqrt[(c + d*Sin
[e + f*x])/(c + d)] + (c - 3*d)*((c + d)*EllipticE[(-2*e + Pi - 2*f*x)/4, (
2*d)/(c + d)] - c*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)])*Sqrt[(c
+ d*Sin[e + f*x])/(c + d)]))/(3*a^2*(c - d)^2*f*(1 + Sin[e + f*x])^2*Sqrt[c
 + d*Sin[e + f*x]])
```

Maple [A] time = 3.622, size = 507, normalized size = 2.

$$\frac{1}{a^2\cos\left(fx+e\right)f}\sqrt{-\left(-d\sin\left(fx+e\right)-c\right)\left(\cos\left(fx+e\right)\right)^2}\left(-\frac{1}{\left(3c-3d\right)\left(1+\sin\left(fx+e\right)\right)^2}\sqrt{-\left(-d\sin\left(fx+e\right)-c\right)\left(\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^(1/2),x)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/a^2*(-1/3/(c-d)*(-(-d*sin(f*x+e)-c)
*cos(f*x+e)^2)^(1/2)/(1+sin(f*x+e))^2-1/3*(-sin(f*x+e)^2*d-c*sin(f*x+e)+d*s
in(f*x+e)+c)/(c-d)^2*(c-3*d)/((-d*sin(f*x+e)-c)*(-1+sin(f*x+e))*(1+sin(f*x+
e)))^(1/2)+2*d^2/(3*c^2-6*c*d+3*d^2)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)
*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f
*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d
)/(c+d))^(1/2))-1/3*d*(c-3*d)/(c-d)^2*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2
)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(
f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^
(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/
(c+d))^(1/2))))/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\sin\left(fx+e\right)+a\right)^2\sqrt{d\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)^2*sqrt(d*sin(f*x + e) + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{d\sin\left(fx+e\right)+c}}{2\,a^2c+2\,a^2d-\left(a^2c+2\,a^2d\right)\cos\left(fx+e\right)^2-\left(a^2d\cos\left(fx+e\right)^2-2\,a^2c-2\,a^2d\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(d*sin(f*x + e) + c)/(2*a^2*c + 2*a^2*d - (a^2*c + 2*a^2*d)*co
s(f*x + e)^2 - (a^2*d*cos(f*x + e)^2 - 2*a^2*c - 2*a^2*d)*sin(f*x + e)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{\sqrt{c+d\sin\left(e+fx\right)}\sin^2\left(e+fx\right)+2\sqrt{c+d\sin\left(e+fx\right)}\sin\left(e+fx\right)+\sqrt{c+d\sin\left(e+fx\right)}}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**2/(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(c + d*sin(e + f*x))*sin(e + f*x)**2 + 2*sqrt(c + d*sin(e +
 f*x))*sin(e + f*x) + sqrt(c + d*sin(e + f*x))), x)/a**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\sin\left(fx+e\right)+a\right)^2\sqrt{d\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)^2*sqrt(d*sin(f*x + e) + c)), x)
```

3.514 $\int \frac{1}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=326

$$-\frac{d\left(c^2-5cd-12d^2\right)\cos(e+fx)}{3a^2f(c-d)^3(c+d)\sqrt{c+d\sin(e+fx)}}-\frac{\left(c^2-5cd-12d^2\right)\sqrt{c+d\sin(e+fx)}E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3a^2f(c-d)^3(c+d)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}-\frac{}{3a^2f(c-d)^2}$$

```
[Out] -(d*(c^2 - 5*c*d - 12*d^2)*Cos[e + f*x])/(3*a^2*(c - d)^3*(c + d)*f*Sqrt[c
+ d*Sin[e + f*x]]) - ((c - 5*d)*Cos[e + f*x])/(3*a^2*(c - d)^2*f*(1 + Sin[e
 + f*x])*Sqrt[c + d*Sin[e + f*x]]) - Cos[e + f*x]/(3*(c - d)*f*(a + a*Sin[e
 + f*x])^2*Sqrt[c + d*Sin[e + f*x]]) - ((c^2 - 5*c*d - 12*d^2)*EllipticE[(e
 - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(3*a^2*(c - d)^3
*(c + d)*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + ((c - 5*d)*EllipticF[(e -
Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(3*a^2*(c
 - d)^2*f*Sqrt[c + d*Sin[e + f*x]])
```

Rubi [A] time = 0.641699, antiderivative size = 326, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2766, 2978, 2754, 2752, 2663, 2661, 2655, 2653}

$$-\frac{d\left(c^2-5cd-12d^2\right)\cos(e+fx)}{3a^2f(c-d)^3(c+d)\sqrt{c+d\sin(e+fx)}}-\frac{\left(c^2-5cd-12d^2\right)\sqrt{c+d\sin(e+fx)}E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3a^2f(c-d)^3(c+d)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}-\frac{}{3a^2f(c-d)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^(3/2)),x]
```


```
[Out] -(d*(c^2 - 5*c*d - 12*d^2)*Cos[e + f*x])/(3*a^2*(c - d)^3*(c + d)*f*Sqrt[c
+ d*Sin[e + f*x]]) - ((c - 5*d)*Cos[e + f*x])/(3*a^2*(c - d)^2*f*(1 + Sin[e
 + f*x])*Sqrt[c + d*Sin[e + f*x]]) - Cos[e + f*x]/(3*(c - d)*f*(a + a*Sin[e
 + f*x])^2*Sqrt[c + d*Sin[e + f*x]]) - ((c^2 - 5*c*d - 12*d^2)*EllipticE[(e
 - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(3*a^2*(c - d)^3
*(c + d)*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + ((c - 5*d)*EllipticF[(e -
Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(3*a^2*(c
 - d)^2*f*Sqrt[c + d*Sin[e + f*x]])
```


Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
```

```
Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))^{3/2}}\,dx = -\frac{\cos(e+fx)}{3(c-d)f(a+a\sin(e+fx))^2\sqrt{c+d\sin(e+fx)}} - \frac{\int \frac{-\frac{1}{2}a(2c-7d)-\cdots}{(a+a\sin(e+fx))(\cdots}}{3a^2(\cdots}$$

$$= -\frac{(c-5d)\cos(e+fx)}{3a^2(c-d)^2f(1+\sin(e+fx))\sqrt{c+d\sin(e+fx)}} - \frac{\cdots}{3(c-d)f(a+a\cdots}$$

$$= -\frac{d\left(c^2-5cd-12d^2\right)\cos(e+fx)}{3a^2(c-d)^3(c+d)f\sqrt{c+d\sin(e+fx)}} - \frac{(c-5d)\cos\cdots}{3a^2(c-d)^2f(1+\sin(e+\cdots}$$

$$= -\frac{d\left(c^2-5cd-12d^2\right)\cos(e+fx)}{3a^2(c-d)^3(c+d)f\sqrt{c+d\sin(e+fx)}} - \frac{(c-5d)\cos\cdots}{3a^2(c-d)^2f(1+\sin(e+\cdots}$$

$$= -\frac{d\left(c^2-5cd-12d^2\right)\cos(e+fx)}{3a^2(c-d)^3(c+d)f\sqrt{c+d\sin(e+fx)}} - \frac{(c-5d)\cos\cdots}{3a^2(c-d)^2f(1+\sin(e+\cdots}$$

$$= -\frac{d\left(c^2-5cd-12d^2\right)\cos(e+fx)}{3a^2(c-d)^3(c+d)f\sqrt{c+d\sin(e+fx)}} - \frac{(c-5d)\cos\cdots}{3a^2(c-d)^2f(1+\sin(e+\cdots}$$

Mathematica [A] time = 4.77666, size = 405, normalized size = 1.24

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^4\left((c+d\sin(e+fx))\left(-\frac{2\left(c^2-5cd-9d^2\right)}{c+d}+\frac{6d^3\cos(e+fx)}{(c+d)(c+d\sin(e+fx))}+\frac{2(c-6d)\sin\left(\frac{1}{2}(e+fx)\right)}{\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)}\cdots\right.\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^(3/2)),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*((c + d*Sin[e + f*x])*((-2*(c^2 -
5*c*d - 9*d^2))/(c + d) + (2*(c - d)*Sin[(e + f*x)/2])/(Cos[(e + f*x)/2] +
Sin[(e + f*x)/2])^3 + (-c + d)/(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + (2
*(c - 6*d)*Sin[(e + f*x)/2])/(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + (6*d^3
*Cos[e + f*x])/((c + d)*(c + d*Sin[e + f*x]))) + ((c^2 - 5*c*d - 12*d^2)*(c
 + d*Sin[e + f*x]) - d^2*(11*c + 5*d)*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d
)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)] + (c^2 - 5*c*d - 12*d^2)*((c
+ d)*EllipticE[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)] - c*EllipticF[(-2*e +
Pi - 2*f*x)/4, (2*d)/(c + d)])*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(c + d))
)/(3*a^2*(c - d)^3*f*(1 + Sin[e + f*x])^2*Sqrt[c + d*Sin[e + f*x]])
```

Maple [B] time = 5.844, size = 1299, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^(3/2),x)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/a^2*(1/(c-d)*(-1/3/(c-d)*(-(-d*sin(
f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(1+sin(f*x+e))^2-1/3*(-sin(f*x+e)^2*d-c*sin(f
```

```
*x+e)+d*sin(f*x+e)+c)/(c-d)^2*(c-3*d)/((-d*sin(f*x+e)-c)*(-1+sin(f*x+e))*(1
+sin(f*x+e)))^(1/2)+2*d^2/(3*c^2-6*c*d+3*d^2)*(c/d-1)*((c+d*sin(f*x+e))/(c-
d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-
(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1
/2),((c-d)/(c+d))^(1/2))-1/3*d*(c-3*d)/(c-d)^2*(c/d-1)*((c+d*sin(f*x+e))/(c
-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(
-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e)
)/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2
),((c-d)/(c+d))^(1/2))))+d^2/(c-d)^2*(2*d*cos(f*x+e)^2/(c^2-d^2)/(-(-d*sin(
f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2*c/(c^2-d^2)*(c/d-1)*((c+d*sin(f*x+e))/(c-d)
)^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-
d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2
),((c-d)/(c+d))^(1/2))+2/(c^2-d^2)*d*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)
*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f
*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(
1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(
c+d))^(1/2))))-d/(c-d)^2*(-(-sin(f*x+e)^2*d-c*sin(f*x+e)+d*sin(f*x+e)+c)/(c
-d)/((-d*sin(f*x+e)-c)*(-1+sin(f*x+e))*(1+sin(f*x+e)))^(1/2)-2*d/(2*c-2*d)*
(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-si
n(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*Elliptic
F(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))-d/(c-d)*(c/d-1)*((c+d
*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d
/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE((
(c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+
e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))))/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/
f
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^(3/2),x, algorithm="maxima"
)
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{d\sin\left(fx+e\right)+c}}{a^2d^2\cos\left(fx+e\right)^4+2\,a^2c^2+4\,a^2cd+2\,a^2d^2-\left(a^2c^2+4\,a^2cd+3\,a^2d^2\right)\cos\left(fx+e\right)^2+2\left(a^2c^2+2\,a^2cd+a^2d^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^(3/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(d*sin(f*x + e) + c)/(a^2*d^2*cos(f*x + e)^4 + 2*a^2*c^2 + 4*a
^2*c*d + 2*a^2*d^2 - (a^2*c^2 + 4*a^2*c*d + 3*a^2*d^2)*cos(f*x + e)^2 + 2*(
a^2*c^2 + 2*a^2*c*d + a^2*d^2 - (a^2*c*d + a^2*d^2)*cos(f*x + e)^2)*sin(f*x
 + e)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{c\sqrt{c+d\sin(e+fx)}\sin^2(e+fx)+2c\sqrt{c+d\sin(e+fx)}\sin(e+fx)+c\sqrt{c+d\sin(e+fx)}+d\sqrt{c+d\sin(e+fx)}\sin^3(e+fx)+2d\sqrt{c+d\sin(e+fx)}\sin^2(e+fx)+d\sqrt{c+d\sin(e+fx)}\sin(e+fx)}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**2/(c+d*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral(1/(c*sqrt(c + d*sin(e + f*x))*sin(e + f*x)**2 + 2*c*sqrt(c + d*sin
(e + f*x))*sin(e + f*x) + c*sqrt(c + d*sin(e + f*x)) + d*sqrt(c + d*sin(e +
 f*x))*sin(e + f*x)**3 + 2*d*sqrt(c + d*sin(e + f*x))*sin(e + f*x)**2 + d*s
qrt(c + d*sin(e + f*x))*sin(e + f*x)), x)/a**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\sin\left(fx+e\right)+a\right)^2\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)^2*(d*sin(f*x + e) + c)^(3/2)), x)
```

3.515 $\int \frac{1}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=405

$$-\frac{d(c+3d)\left(c^2-10cd-7d^2\right)\cos(e+fx)}{3a^2f(c-d)^4(c+d)^2\sqrt{c+d\sin(e+fx)}}-\frac{d\left(c^2-7cd-10d^2\right)\cos(e+fx)}{3a^2f(c-d)^3(c+d)(c+d\sin(e+fx))^{3/2}}+\frac{\left(c^2-7cd-10d^2\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\right.}{3a^2f(c-d)^3(c+d)\sqrt{c+c}}$$

```
[Out] -(d*(c^2 - 7*c*d - 10*d^2)*Cos[e + f*x])/(3*a^2*(c - d)^3*(c + d)*f*(c + d*
Sin[e + f*x])^(3/2)) - ((c - 7*d)*Cos[e + f*x])/(3*a^2*(c - d)^2*f*(1 + Sin
[e + f*x])*(c + d*Sin[e + f*x])^(3/2)) - Cos[e + f*x]/(3*(c - d)*f*(a + a*S
in[e + f*x])^2*(c + d*Sin[e + f*x])^(3/2)) - (d*(c + 3*d)*(c^2 - 10*c*d - 7
*d^2)*Cos[e + f*x])/(3*a^2*(c - d)^4*(c + d)^2*f*Sqrt[c + d*Sin[e + f*x]])
 - ((c + 3*d)*(c^2 - 10*c*d - 7*d^2)*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c
 + d)]*Sqrt[c + d*Sin[e + f*x]])/(3*a^2*(c - d)^4*(c + d)^2*f*Sqrt[(c + d*Si
n[e + f*x])/(c + d)]) + ((c^2 - 7*c*d - 10*d^2)*EllipticF[(e - Pi/2 + f*x)/
2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(3*a^2*(c - d)^3*(c +
 d)*f*Sqrt[c + d*Sin[e + f*x]])
```

Rubi [A] time = 0.833648, antiderivative size = 405, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2766, 2978, 2754, 2752, 2663, 2661, 2655, 2653}

$$-\frac{d(c+3d)\left(c^2-10cd-7d^2\right)\cos(e+fx)}{3a^2f(c-d)^4(c+d)^2\sqrt{c+d\sin(e+fx)}}-\frac{d\left(c^2-7cd-10d^2\right)\cos(e+fx)}{3a^2f(c-d)^3(c+d)(c+d\sin(e+fx))^{3/2}}+\frac{\left(c^2-7cd-10d^2\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\right.}{3a^2f(c-d)^3(c+d)\sqrt{c+c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^(5/2)),x]
```

```
[Out] -(d*(c^2 - 7*c*d - 10*d^2)*Cos[e + f*x])/(3*a^2*(c - d)^3*(c + d)*f*(c + d*
Sin[e + f*x])^(3/2)) - ((c - 7*d)*Cos[e + f*x])/(3*a^2*(c - d)^2*f*(1 + Sin
[e + f*x])*(c + d*Sin[e + f*x])^(3/2)) - Cos[e + f*x]/(3*(c - d)*f*(a + a*S
in[e + f*x])^2*(c + d*Sin[e + f*x])^(3/2)) - (d*(c + 3*d)*(c^2 - 10*c*d - 7
*d^2)*Cos[e + f*x])/(3*a^2*(c - d)^4*(c + d)^2*f*Sqrt[c + d*Sin[e + f*x]])
 - ((c + 3*d)*(c^2 - 10*c*d - 7*d^2)*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c
 + d)]*Sqrt[c + d*Sin[e + f*x]])/(3*a^2*(c - d)^4*(c + d)^2*f*Sqrt[(c + d*Si
n[e + f*x])/(c + d)]) + ((c^2 - 7*c*d - 10*d^2)*EllipticF[(e - Pi/2 + f*x)/
2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(3*a^2*(c - d)^3*(c +
 d)*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))^{5/2}}\,dx = -\frac{\cos(e+fx)}{3(c-d)f(a+a\sin(e+fx))^2(c+d\sin(e+fx))^{3/2}} - \frac{\int \frac{-\frac{1}{2}a(2c-9d)-\frac{5}{2}\cdots}{(a+a\sin(e+fx))(c+\cdots}}{3a^2(c\cdots}$$

$$= -\frac{(c-7d)\cos(e+fx)}{3a^2(c-d)^2f(1+\sin(e+fx))(c+d\sin(e+fx))^{3/2}} - \frac{\cdots}{3(c-d)f(a+a\,\mathrm{s}\cdots}$$

$$= -\frac{d\left(c^2-7cd-10d^2\right)\cos(e+fx)}{3a^2(c-d)^3(c+d)f(c+d\sin(e+fx))^{3/2}} - \frac{(c-7d)\cos\cdots}{3a^2(c-d)^2f(1+\sin(e+f\cdots}$$

$$= -\frac{d\left(c^2-7cd-10d^2\right)\cos(e+fx)}{3a^2(c-d)^3(c+d)f(c+d\sin(e+fx))^{3/2}} - \frac{(c-7d)\cos\cdots}{3a^2(c-d)^2f(1+\sin(e+f\cdots}$$

$$= -\frac{d\left(c^2-7cd-10d^2\right)\cos(e+fx)}{3a^2(c-d)^3(c+d)f(c+d\sin(e+fx))^{3/2}} - \frac{(c-7d)\cos\cdots}{3a^2(c-d)^2f(1+\sin(e+f\cdots}$$

$$= -\frac{d\left(c^2-7cd-10d^2\right)\cos(e+fx)}{3a^2(c-d)^3(c+d)f(c+d\sin(e+fx))^{3/2}} - \frac{(c-7d)\cos\cdots}{3a^2(c-d)^2f(1+\sin(e+f\cdots}$$

$$= -\frac{d\left(c^2-7cd-10d^2\right)\cos(e+fx)}{3a^2(c-d)^3(c+d)f(c+d\sin(e+fx))^{3/2}} - \frac{(c-7d)\cos\cdots}{3a^2(c-d)^2f(1+\sin(e+f\cdots}$$

Mathematica [A] time = 6.62043, size = 674, normalized size = 1.66

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^4\sqrt{c+d\sin(e+fx)}\left(-\frac{2\left(-7c^2d+c^3-27cd^2-15d^3\right)}{3(c-d)^4(c+d)^2}+\frac{2d^3\cos(e+fx)}{3(c-d)^3(c+d)(c+d\sin(e+fx))^2}+\frac{4\left(5cd^3\cos(e+fx\cdots\right.}{3(c-d)^4(c+d)^2(\cdots}\right.}{f(a\sin(e+f\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^(5/2)),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*Sqrt[c + d*Sin[e + f*x]]*((-2*(c^3
 - 7*c^2*d - 27*c*d^2 - 15*d^3))/(3*(c - d)^4*(c + d)^2) + (2*Sin[(e + f*x)
/2])/(3*(c - d)^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3) - 1/(3*(c - d)^3
*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2) + (2*(c*Sin[(e + f*x)/2] - 9*d*Si
n[(e + f*x)/2]))/(3*(c - d)^4*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])) + (2*d
^3*Cos[e + f*x])/(3*(c - d)^3*(c + d)*(c + d*Sin[e + f*x])^2) + (4*(5*c*d^3
*Cos[e + f*x] + 3*d^4*Cos[e + f*x]))/(3*(c - d)^4*(c + d)^2*(c + d*Sin[e +
f*x]))))/(f*(a + a*Sin[e + f*x])^2) + (d*(Cos[(e + f*x)/2] + Sin[(e + f*x)/
2])^4*((-2*(26*c^2*d + 28*c*d^2 + 10*d^3)*EllipticF[(-e + Pi/2 - f*x)/2, (2
*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/Sqrt[c + d*Sin[e + f*x]] +
 (2*(c^3 - 7*c^2*d - 37*c*d^2 - 21*d^3)*Cos[e + f*x]^2*Sqrt[c + d*Sin[e + f
*x]])/(d*(1 - Sin[e + f*x]^2)) - ((-c^3 + 7*c^2*d + 37*c*d^2 + 21*d^3)*((2*
(c + d)*EllipticE[(-e + Pi/2 - f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f
*x])/(c + d)])/Sqrt[c + d*Sin[e + f*x]] - (2*c*EllipticF[(-e + Pi/2 - f*x)/
2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/Sqrt[c + d*Sin[e + f*
x]]))/d))/(6*(c - d)^4*(c + d)^2*f*(a + a*Sin[e + f*x])^2)
```

Maple [B] time = 7.269, size = 1758, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^(5/2),x)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/a^2*(d^2/(c-d)^2*(2/3/(c^2-d^2)/d*(
-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(sin(f*x+e)+c/d)^2+8/3*d*cos(f*x+e)^
2/(c^2-d^2)^2*c/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2*(3*c^2+d^2)/(3*c^
4-6*c^2*d^2+3*d^4)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))
/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e
)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+8/
3*c*d/(c^2-d^2)^2*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/
(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)
^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^
(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+1/(c
-d)^2*(-1/3/(c-d)*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(1+sin(f*x+e))^2-
1/3*(-sin(f*x+e)^2*d-c*sin(f*x+e)+d*sin(f*x+e)+c)/(c-d)^2*(c-3*d)/((-d*sin(
f*x+e)-c)*(-1+sin(f*x+e))*(1+sin(f*x+e)))^(1/2)+2*d^2/(3*c^2-6*c*d+3*d^2)*(
c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin
(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF
(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))-1/3*d*(c-3*d)/(c-d)^2*
(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-si
n(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1
)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF((
(c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+2*d^2/(c-d)^3*(2*d*cos
(f*x+e)^2/(c^2-d^2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2*c/(c^2-d^2)*(
c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin
(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF
(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+2/(c^2-d^2)*d*(c/d-1)*
((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)
-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*Ellipt
icE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin
(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))-2/(c-d)^3*d*(-(-sin(f*x+e)^2*d
-c*sin(f*x+e)+d*sin(f*x+e)+c)/(c-d)/((-d*sin(f*x+e)-c)*(-1+sin(f*x+e))*(1+s
in(f*x+e)))^(1/2)-2*d/(2*c-2*d)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(
1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)
-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+
d))^(1/2))-d/(c-d)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))
/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e
)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))
^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))))/co
s(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^(5/2),x, algorithm="maxima"
)
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{}}{2\,a^2c^3+6\,a^2c^2d+6\,a^2cd^2+2\,a^2d^3+\left(3\,a^2cd^2+2\,a^2d^3\right)\cos\left(fx+e\right)^4-\left(a^2c^3+6\,a^2c^2d+9\,a^2cd^2+4\,a^2d^3\right)\cos}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^(5/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(d*sin(f*x + e) + c)/(2*a^2*c^3 + 6*a^2*c^2*d + 6*a^2*c*d^2 +
2*a^2*d^3 + (3*a^2*c*d^2 + 2*a^2*d^3)*cos(f*x + e)^4 - (a^2*c^3 + 6*a^2*c^2
*d + 9*a^2*c*d^2 + 4*a^2*d^3)*cos(f*x + e)^2 + (a^2*d^3*cos(f*x + e)^4 + 2*
a^2*c^3 + 6*a^2*c^2*d + 6*a^2*c*d^2 + 2*a^2*d^3 - 3*(a^2*c^2*d + 2*a^2*c*d^
2 + a^2*d^3)*cos(f*x + e)^2)*sin(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**2/(c+d*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\sin\left(fx+e\right)+a\right)^2\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)^2*(d*sin(f*x + e) + c)^(5/2)), x)
```

3.516 $\int \frac{(c+d\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^3}\,dx$

Optimal. Leaf size=322

$$-\frac{\left(4c^2+15cd+27d^2\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{30f\left(a^3\sin(e+fx)+a^3\right)}+\frac{(c+d)\left(4c^2+11cd+15d^2\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{30a^3f\sqrt{c+d\sin(e+fx)}}$$

```
[Out] (-2*(c - d)*(c + 3*d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(15*a*f*(a + a
*Sin[e + f*x])^2) - ((4*c^2 + 15*c*d + 27*d^2)*Cos[e + f*x]*Sqrt[c + d*Sin[
e + f*x]])/(30*f*(a^3 + a^3*Sin[e + f*x])) - ((c - d)*Cos[e + f*x]*(c + d*S
in[e + f*x])^(3/2))/(5*f*(a + a*Sin[e + f*x])^3) - ((4*c^2 + 15*c*d + 27*d^
2)*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(
30*a^3*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + ((c + d)*(4*c^2 + 11*c*d + 1
5*d^2)*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x
])/(c + d)])/(30*a^3*f*Sqrt[c + d*Sin[e + f*x]])
```

Rubi [A] time = 0.833098, antiderivative size = 322, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2765, 2977, 2978, 2752, 2663, 2661, 2655, 2653}

$$-\frac{\left(4c^2+15cd+27d^2\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{30f\left(a^3\sin(e+fx)+a^3\right)}+\frac{(c+d)\left(4c^2+11cd+15d^2\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{30a^3f\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^(5/2)/(a + a*Sin[e + f*x])^3,x]
```


```
[Out] (-2*(c - d)*(c + 3*d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(15*a*f*(a + a
*Sin[e + f*x])^2) - ((4*c^2 + 15*c*d + 27*d^2)*Cos[e + f*x]*Sqrt[c + d*Sin[
e + f*x]])/(30*f*(a^3 + a^3*Sin[e + f*x])) - ((c - d)*Cos[e + f*x]*(c + d*S
in[e + f*x])^(3/2))/(5*f*(a + a*Sin[e + f*x])^3) - ((4*c^2 + 15*c*d + 27*d^
2)*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(
30*a^3*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + ((c + d)*(4*c^2 + 11*c*d + 1
5*d^2)*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x
])/(c + d)])/(30*a^3*f*Sqrt[c + d*Sin[e + f*x]])
```


Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
```

```
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^3}\,dx = -\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{5f(a+a\sin(e+fx))^3} - \frac{\int \frac{\sqrt{c+d\sin(e+fx)}\left(-\frac{1}{2}a\left(4c^2+9cd-3d^2\right)-\frac{1}{2}ad(c+9d)\sin(e\right.}{(a+a\sin(e+fx))^2}}{5a^2}$$

$$= -\frac{2(c-d)(c+3d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15af(a+a\sin(e+fx))^2} - \frac{(c-d)\cos(e+fx)(c+d\sin(e+fx)}{5f(a+a\sin(e+fx))^3}$$

$$= -\frac{2(c-d)(c+3d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15af(a+a\sin(e+fx))^2} - \frac{\left(4c^2+15cd+27d^2\right)\cos(e+fx)\sqrt{c}}{30f\left(a^3+a^3\sin(e+fx\right.}$$

$$= -\frac{2(c-d)(c+3d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15af(a+a\sin(e+fx))^2} - \frac{\left(4c^2+15cd+27d^2\right)\cos(e+fx)\sqrt{c}}{30f\left(a^3+a^3\sin(e+fx\right.}$$

$$= -\frac{2(c-d)(c+3d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15af(a+a\sin(e+fx))^2} - \frac{\left(4c^2+15cd+27d^2\right)\cos(e+fx)\sqrt{c}}{30f\left(a^3+a^3\sin(e+fx\right.}$$

$$= -\frac{2(c-d)(c+3d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15af(a+a\sin(e+fx))^2} - \frac{\left(4c^2+15cd+27d^2\right)\cos(e+fx)\sqrt{c}}{30f\left(a^3+a^3\sin(e+fx\right.}$$

Mathematica [A] time = 5.92663, size = 385, normalized size = 1.2

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^6\left(-\left(4c^2+15cd+27d^2\right)(c+d\sin(e+fx)) - \frac{(c+d\sin(e+fx))\left(\left(20c^2+74cd+90d^2\right)\cos\left(\frac{3}{2}(e+f\right.\right.}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])^(5/2)/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6*(-((4*c^2 + 15*c*d + 27*d^2)*(c +
d*Sin[e + f*x])) - ((-2*d*(35*c + 57*d)*Cos[(e + f*x)/2] + (20*c^2 + 74*c*d
 + 90*d^2)*Cos[(3*(e + f*x))/2] + 2*(-3*(6*c^2 + 11*c*d + 29*d^2) + 2*(2*c^
2 + 7*c*d - 9*d^2)*Cos[e + f*x] + (4*c^2 + 15*c*d + 27*d^2)*Cos[2*(e + f*x)
])*Sin[(e + f*x)/2])*(c + d*Sin[e + f*x]))/(2*(Cos[(e + f*x)/2] + Sin[(e +
f*x)/2])^5) + (c - 15*d)*d^2*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)
]*Sqrt[(c + d*Sin[e + f*x])/(c + d)] + (4*c^2 + 15*c*d + 27*d^2)*((c + d)*E
llipticE[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)] - c*EllipticF[(-2*e + Pi - 2
*f*x)/4, (2*d)/(c + d)])*Sqrt[(c + d*Sin[e + f*x])/(c + d)]))/(30*a^3*f*(1
+ Sin[e + f*x])^3*Sqrt[c + d*Sin[e + f*x]])
```

Maple [B] time = 6.812, size = 1615, normalized size = 5.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^3,x)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/a^3*(2*d^3*(c/d-1)*((c+d*sin(f*x+e)
)/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/
```

```
2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d
))^(1/2),((c-d)/(c+d))^(1/2))+(c^3-3*c^2*d+3*c*d^2-d^3)*(-1/5/(c-d)*(-(-d*s
in(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(1+sin(f*x+e))^3-2/15*(c-3*d)/(c-d)^2*(-(-
d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(1+sin(f*x+e))^2-1/30*(-sin(f*x+e)^2*d-
c*sin(f*x+e)+d*sin(f*x+e)+c)/(c-d)^3*(4*c^2-15*c*d+27*d^2)/((-d*sin(f*x+e)-
c)*(-1+sin(f*x+e))*(1+sin(f*x+e)))^(1/2)+2*(-c*d^2-15*d^3)/(60*c^3-180*c^2*
d+180*c*d^2-60*d^3)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e)
)/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+
e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))-1
/30*d*(4*c^2-15*c*d+27*d^2)/(c-d)^3*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*
(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*
x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1
/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c
+d))^(1/2))))+3*d*(c^2-2*c*d+d^2)*(-1/3/(c-d)*(-(-d*sin(f*x+e)-c)*cos(f*x+e
)^2)^(1/2)/(1+sin(f*x+e))^2-1/3*(-sin(f*x+e)^2*d-c*sin(f*x+e)+d*sin(f*x+e)+
c)/(c-d)^2*(c-3*d)/((-d*sin(f*x+e)-c)*(-1+sin(f*x+e))*(1+sin(f*x+e)))^(1/2)
+2*d^2/(3*c^2-6*c*d+3*d^2)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin
(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*c
os(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(
1/2))-1/3*d*(c-3*d)/(c-d)^2*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-si
n(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*
cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-
d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/
2))))+3*d^2*(c-d)*(-(-sin(f*x+e)^2*d-c*sin(f*x+e)+d*sin(f*x+e)+c)/(c-d)/((-
d*sin(f*x+e)-c)*(-1+sin(f*x+e))*(1+sin(f*x+e)))^(1/2)-2*d/(2*c-2*d)*(c/d-1)
*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e
)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d
*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))-d/(c-d)*(c/d-1)*((c+d*sin(f*
x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))
^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*si
n(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-
d))^(1/2),((c-d)/(c+d))^(1/2)))))/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(d^2\cos\left(fx+e\right)^2-2\,cd\sin\left(fx+e\right)-c^2-d^2\right)\sqrt{d\sin\left(fx+e\right)+c}}{3\,a^3\cos\left(fx+e\right)^2-4\,a^3+\left(a^3\cos\left(fx+e\right)^2-4\,a^3\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] integral((d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)*sqrt(d*sin(f
*x + e) + c)/(3*a^3*cos(f*x + e)^2 - 4*a^3 + (a^3*cos(f*x + e)^2 - 4*a^3)*s
```

```
in(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(5/2)/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}{\left(a\sin\left(fx+e\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(5/2)/(a*sin(f*x + e) + a)^3, x)
```

3.517 $\int \frac{(c+d\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^3}\,dx$

Optimal. Leaf size=323

$$-\frac{\left(4c^2+5cd-3d^2\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{30f(c-d)\left(a^3\sin(e+fx)+a^3\right)}-\frac{\left(4c^2+5cd-3d^2\right)\sqrt{c+d\sin(e+fx)}E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{30a^3f(c-d)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}+\frac{(c+}{}$$

```
[Out] -((c - d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(5*f*(a + a*Sin[e + f*x])^
3) - (2*(c + 2*d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(15*a*f*(a + a*Sin
[e + f*x])^2) - ((4*c^2 + 5*c*d - 3*d^2)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*
x]])/(30*(c - d)*f*(a^3 + a^3*Sin[e + f*x])) - ((4*c^2 + 5*c*d - 3*d^2)*Ell
ipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(30*a^3
*(c - d)*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + ((c + d)*(4*c + 5*d)*Ellip
ticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])
/(30*a^3*f*Sqrt[c + d*Sin[e + f*x]])
```

Rubi [A] time = 0.876879, antiderivative size = 323, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2765, 2978, 2752, 2663, 2661, 2655, 2653}

$$-\frac{\left(4c^2+5cd-3d^2\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{30f(c-d)\left(a^3\sin(e+fx)+a^3\right)}-\frac{\left(4c^2+5cd-3d^2\right)\sqrt{c+d\sin(e+fx)}E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{30a^3f(c-d)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}+\frac{(c+}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^(3/2)/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] -((c - d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(5*f*(a + a*Sin[e + f*x])^
3) - (2*(c + 2*d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(15*a*f*(a + a*Sin
[e + f*x])^2) - ((4*c^2 + 5*c*d - 3*d^2)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*
x]])/(30*(c - d)*f*(a^3 + a^3*Sin[e + f*x])) - ((4*c^2 + 5*c*d - 3*d^2)*Ell
ipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(30*a^3
*(c - d)*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + ((c + d)*(4*c + 5*d)*Ellip
ticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])
/(30*a^3*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
```

```
Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^3}\,dx = -\frac{(c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{5f(a+a\sin(e+fx))^3} - \frac{\int \frac{-\frac{1}{2}a(4c^2+7cd-d^2)-\frac{1}{2}ad(3c+7d)\sin(e+fx)}{(a+a\sin(e+fx))^2\sqrt{c+d\sin(e+fx)}}\,dx}{5a^2}$$

$$= -\frac{(c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{5f(a+a\sin(e+fx))^3} - \frac{2(c+2d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15af(a+a\sin(e+fx))^2} + \frac{\int \frac{1}{2}a^2 \cdots}{\cdots}$$

$$= -\frac{(c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{5f(a+a\sin(e+fx))^3} - \frac{2(c+2d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15af(a+a\sin(e+fx))^2} - \frac{(4c^2 \cdots}{\cdots}$$

$$= -\frac{(c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{5f(a+a\sin(e+fx))^3} - \frac{2(c+2d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15af(a+a\sin(e+fx))^2} - \frac{(4c^2 \cdots}{\cdots}$$

$$= -\frac{(c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{5f(a+a\sin(e+fx))^3} - \frac{2(c+2d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15af(a+a\sin(e+fx))^2} - \frac{(4c^2 \cdots}{\cdots}$$

$$= -\frac{(c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{5f(a+a\sin(e+fx))^3} - \frac{2(c+2d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15af(a+a\sin(e+fx))^2} - \frac{(4c^2 \cdots}{\cdots}$$

Mathematica [A] time = 6.24505, size = 631, normalized size = 1.95

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^6\sqrt{c+d\sin(e+fx)}\left(\frac{4c^2\sin\left(\frac{1}{2}(e+fx)\right)+5cd\sin\left(\frac{1}{2}(e+fx)\right)-3d^2\sin\left(\frac{1}{2}(e+fx)\right)}{15(c-d)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}+\frac{d-c}{5\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2} \cdots\right.\right.}\cdots\right.}{f(a\sin(e+fx)+a)^3 \cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])^(3/2)/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6*((-c + d)/(5*(Cos[(e + f*x)/2] + S
in[(e + f*x)/2])^4) - (2*(c + 2*d))/(15*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2
])^2) + (2*(c*Sin[(e + f*x)/2] - d*Sin[(e + f*x)/2]))/(5*(Cos[(e + f*x)/2]
+ Sin[(e + f*x)/2])^5) + (4*(c*Sin[(e + f*x)/2] + 2*d*Sin[(e + f*x)/2]))/(1
5*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3) + (4*c^2*Sin[(e + f*x)/2] + 5*c*
d*Sin[(e + f*x)/2] - 3*d^2*Sin[(e + f*x)/2])/(15*(c - d)*(Cos[(e + f*x)/2]
+ Sin[(e + f*x)/2])))*Sqrt[c + d*Sin[e + f*x]])/(f*(a + a*Sin[e + f*x])^3)
- (d*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6*((-2*(c*d + 5*d^2)*EllipticF[(
-e + Pi/2 - f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/Sqrt
[c + d*Sin[e + f*x]] + (2*(4*c^2 + 5*c*d - 3*d^2)*Cos[e + f*x]^2*Sqrt[c + d
*Sin[e + f*x]])/(d*(1 - Sin[e + f*x]^2)) - ((4*c^2 + 5*c*d - 3*d^2)*((2*(c
+ d)*EllipticE[(-e + Pi/2 - f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x]
)/(c + d)])/Sqrt[c + d*Sin[e + f*x]] - (2*c*EllipticF[(-e + Pi/2 - f*x)/2,
(2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/Sqrt[c + d*Sin[e + f*x]]
))/d))/(60*(c - d)*f*(a + a*Sin[e + f*x])^3)
```

Maple [B] time = 6.334, size = 1462, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^3,x)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/a^3*((c^2-2*c*d+d^2)*(-1/5/(c-d)*(-
(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(1+sin(f*x+e))^3-2/15*(c-3*d)/(c-d)^2
*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(1+sin(f*x+e))^2-1/30*(-sin(f*x+e)
^2*d-c*sin(f*x+e)+d*sin(f*x+e)+c)/(c-d)^3*(4*c^2-15*c*d+27*d^2)/((-d*sin(f*
x+e)-c)*(-1+sin(f*x+e))*(1+sin(f*x+e)))^(1/2)+2*(-c*d^2-15*d^3)/(60*c^3-180
*c^2*d+180*c*d^2-60*d^3)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f
*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos
(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/
2))-1/30*d*(4*c^2-15*c*d+27*d^2)/(c-d)^3*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(
1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*s
in(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d
))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-
d)/(c+d))^(1/2))))+2*d*(c-d)*(-1/3/(c-d)*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^
(1/2)/(1+sin(f*x+e))^2-1/3*(-sin(f*x+e)^2*d-c*sin(f*x+e)+d*sin(f*x+e)+c)/(c
-d)^2*(c-3*d)/((-d*sin(f*x+e)-c)*(-1+sin(f*x+e))*(1+sin(f*x+e)))^(1/2)+2*d^
2/(3*c^2-6*c*d+3*d^2)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+
e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*
x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))
-1/3*d*(c-3*d)/(c-d)^2*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x
+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f
*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c
+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))
+d^2*(-(-sin(f*x+e)^2*d-c*sin(f*x+e)+d*sin(f*x+e)+c)/(c-d)/((-d*sin(f*x+e)-
c)*(-1+sin(f*x+e))*(1+sin(f*x+e)))^(1/2)-2*d/(2*c-2*d)*(c/d-1)*((c+d*sin(f*
x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))
^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/
(c-d))^(1/2),((c-d)/(c+d))^(1/2))-d/(c-d)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^
(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*
sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-
d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c
-d)/(c+d))^(1/2)))))/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}{\left(a\sin\left(fx+e\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(3/2)/(a*sin(f*x + e) + a)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}{3\,a^{3}\cos\left(fx+e\right)^{2}-4\,a^{3}+\left(a^{3}\cos\left(fx+e\right)^{2}-4\,a^{3}\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] integral(-(d*sin(f*x + e) + c)^(3/2)/(3*a^3*cos(f*x + e)^2 - 4*a^3 + (a^3*c
os(f*x + e)^2 - 4*a^3)*sin(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(3/2)/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d \sin\left(fx + e\right) + c\right)^{\frac{3}{2}}}{\left(a \sin\left(fx + e\right) + a\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(3/2)/(a*sin(f*x + e) + a)^3, x)
```

3.518 $\int \frac{\sqrt{c+d\sin(e+fx)}}{(a+a\sin(e+fx))^3}\,dx$

Optimal. Leaf size=334

$$-\frac{(4c^2-5cd-3d^2)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{30f(c-d)^2(a^3\sin(e+fx)+a^3)}-\frac{(4c^2-5cd-3d^2)\sqrt{c+d\sin(e+fx)}E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{30a^3f(c-d)^2\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}+$$

```
[Out] -(Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(5*f*(a + a*Sin[e + f*x])^3) - ((2
*c - d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(15*a*(c - d)*f*(a + a*Sin[e
 + f*x])^2) - ((4*c^2 - 5*c*d - 3*d^2)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]
])/(30*(c - d)^2*f*(a^3 + a^3*Sin[e + f*x])) - ((4*c^2 - 5*c*d - 3*d^2)*Ell
ipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(30*a^3
*(c - d)^2*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + ((4*c - 5*d)*(c + d)*Ell
ipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)
])/(30*a^3*(c - d)*f*Sqrt[c + d*Sin[e + f*x]])
```

Rubi [A] time = 0.767721, antiderivative size = 334, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2764, 2978, 2752, 2663, 2661, 2655, 2653}

$$-\frac{(4c^2-5cd-3d^2)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{30f(c-d)^2(a^3\sin(e+fx)+a^3)}-\frac{(4c^2-5cd-3d^2)\sqrt{c+d\sin(e+fx)}E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{30a^3f(c-d)^2\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}+$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*Sin[e + f*x]]/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] -(Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(5*f*(a + a*Sin[e + f*x])^3) - ((2
*c - d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(15*a*(c - d)*f*(a + a*Sin[e
 + f*x])^2) - ((4*c^2 - 5*c*d - 3*d^2)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]
])/(30*(c - d)^2*f*(a^3 + a^3*Sin[e + f*x])) - ((4*c^2 - 5*c*d - 3*d^2)*Ell
ipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(30*a^3
*(c - d)^2*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + ((4*c - 5*d)*(c + d)*Ell
ipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)
])/(30*a^3*(c - d)*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2764

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m
*(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a*d*n - b*c*
(m + 1) - b*d*(m + n + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && L
tQ[m, -1] && LtQ[0, n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c,
 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
```

```
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\sqrt{c+d\sin(e+fx)}}{(a+a\sin(e+fx))^3}\,dx = -\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{5f(a+a\sin(e+fx))^3} + \frac{\int \frac{\frac{1}{2}a(4c+d)+\frac{3}{2}ad\sin(e+fx)}{(a+a\sin(e+fx))^2\sqrt{c+d\sin(e+fx)}}\,dx}{5a^2}$$

$$= -\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{5f(a+a\sin(e+fx))^3} - \frac{(2c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15a(c-d)f(a+a\sin(e+fx))^2} - \frac{\int \frac{-\frac{1}{2}a^2(4c^2-3\ldots}{(a+a\sin\ldots}}{}$$

$$= -\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{5f(a+a\sin(e+fx))^3} - \frac{(2c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15a(c-d)f(a+a\sin(e+fx))^2} - \frac{(4c^2-5cd-\ldots}{30(c\ldots}$$

$$= -\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{5f(a+a\sin(e+fx))^3} - \frac{(2c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15a(c-d)f(a+a\sin(e+fx))^2} - \frac{(4c^2-5cd-\ldots}{30(c\ldots}$$

$$= -\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{5f(a+a\sin(e+fx))^3} - \frac{(2c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15a(c-d)f(a+a\sin(e+fx))^2} - \frac{(4c^2-5cd-\ldots}{30(c\ldots}$$

$$= -\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{5f(a+a\sin(e+fx))^3} - \frac{(2c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15a(c-d)f(a+a\sin(e+fx))^2} - \frac{(4c^2-5cd-\ldots}{30(c\ldots}$$

Mathematica [A] time = 5.95959, size = 449, normalized size = 1.34

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^6\left(-\left(4c^2-5cd-3d^2\right)(c+d\sin(e+fx))+\frac{2(c+d\sin(e+fx))\left(\left(4c^2-5cd-3d^2\right)\sin\left(\frac{1}{2}(e+fx)\right)\left(\sin\ldots\right.\right.}{\ldots}\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*Sin[e + f*x]]/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6*(-((4*c^2 - 5*c*d - 3*d^2)*(c + d*
Sin[e + f*x])) + (2*(6*(c - d)^2*Sin[(e + f*x)/2] - 3*(c - d)^2*(Cos[(e + f
*x)/2] + Sin[(e + f*x)/2]) + 2*(c - d)*(2*c - d)*Sin[(e + f*x)/2]*(Cos[(e +
 f*x)/2] + Sin[(e + f*x)/2])^2 - (c - d)*(2*c - d)*(Cos[(e + f*x)/2] + Sin[
(e + f*x)/2])^3 + (4*c^2 - 5*c*d - 3*d^2)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2
] + Sin[(e + f*x)/2])^4)*(c + d*Sin[e + f*x]))/(Cos[(e + f*x)/2] + Sin[(e +
 f*x)/2])^5 + (c - 5*d)*d^2*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]
*Sqrt[(c + d*Sin[e + f*x])/(c + d)] + (4*c^2 - 5*c*d - 3*d^2)*((c + d)*Elli
pticE[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)] - c*EllipticF[(-2*e + Pi - 2*f*
x)/4, (2*d)/(c + d)])*Sqrt[(c + d*Sin[e + f*x])/(c + d)]))/(30*a^3*(c - d)^
2*f*(1 + Sin[e + f*x])^3*Sqrt[c + d*Sin[e + f*x]])
```

Maple [B] time = 5.708, size = 1056, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^3,x)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/a^3*((c-d)*(-1/5/(c-d)*(-(-d*sin(f*
x+e)-c)*cos(f*x+e)^2)^(1/2)/(1+sin(f*x+e))^3-2/15*(c-3*d)/(c-d)^2*(-(-d*sin
(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(1+sin(f*x+e))^2-1/30*(-sin(f*x+e)^2*d-c*sin
(f*x+e)+d*sin(f*x+e)+c)/(c-d)^3*(4*c^2-15*c*d+27*d^2)/((-d*sin(f*x+e)-c)*(-
1+sin(f*x+e))*(1+sin(f*x+e)))^(1/2)+2*(-c*d^2-15*d^3)/(60*c^3-180*c^2*d+180
*c*d^2-60*d^3)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+
d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)
^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))-1/30*d
*(4*c^2-15*c*d+27*d^2)/(c-d)^3*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1
-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-
c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),(
(c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^
(1/2))))+d*(-1/3/(c-d)*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(1+sin(f*x+e
))^2-1/3*(-sin(f*x+e)^2*d-c*sin(f*x+e)+d*sin(f*x+e)+c)/(c-d)^2*(c-3*d)/((-d
*sin(f*x+e)-c)*(-1+sin(f*x+e))*(1+sin(f*x+e)))^(1/2)+2*d^2/(3*c^2-6*c*d+3*d
^2)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*(
(-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*Elli
pticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))-1/3*d*(c-3*d)/(c-
d)^2*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*
((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-
c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+Ellipt
icF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))))/cos(f*x+e)/(c+d*
sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d \sin\left(fx+e\right)+c}}{\left(a \sin\left(fx+e\right)+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*sin(f*x + e) + c)/(a*sin(f*x + e) + a)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{d \sin\left(fx+e\right)+c}}{3\,a^3 \cos\left(fx+e\right)^2-4\,a^3+\left(a^3 \cos\left(fx+e\right)^2-4\,a^3\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] integral(-sqrt(d*sin(f*x + e) + c)/(3*a^3*cos(f*x + e)^2 - 4*a^3 + (a^3*cos
(f*x + e)^2 - 4*a^3)*sin(f*x + e)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sqrt{c+d\sin\left(e+fx\right)}}{\sin^3\left(e+fx\right)+3\sin^2\left(e+fx\right)+3\sin\left(e+fx\right)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(1/2)/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Integral(sqrt(c + d*sin(e + f*x))/(sin(e + f*x)**3 + 3*sin(e + f*x)**2 + 3*
sin(e + f*x) + 1), x)/a**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d\sin\left(fx+e\right)+c}}{\left(a\sin\left(fx+e\right)+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*sin(f*x + e) + c)/(a*sin(f*x + e) + a)^3, x)
```

3.519 $\int \frac{1}{(a+a\sin(e+fx))^3\sqrt{c+d\sin(e+fx)}}\,dx$

Optimal. Leaf size=344

$$-\frac{\left(4c^2-15cd+27d^2\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{30f(c-d)^3\left(a^3\sin(e+fx)+a^3\right)}+\frac{\left(4c^2-11cd+15d^2\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{30a^3f(c-d)^2\sqrt{c+d\sin(e+fx)}}-\frac{\left(4c^2\right.}{}$$

```
[Out] -(Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(5*(c - d)*f*(a + a*Sin[e + f*x])^
3) - (2*(c - 3*d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(15*a*(c - d)^2*f*
(a + a*Sin[e + f*x])^2) - ((4*c^2 - 15*c*d + 27*d^2)*Cos[e + f*x]*Sqrt[c +
d*Sin[e + f*x]])/(30*(c - d)^3*f*(a^3 + a^3*Sin[e + f*x])) - ((4*c^2 - 15*c
*d + 27*d^2)*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e
+ f*x]])/(30*a^3*(c - d)^3*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + ((4*c^2
- 11*c*d + 15*d^2)*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d
*Sin[e + f*x])/(c + d)])/(30*a^3*(c - d)^2*f*Sqrt[c + d*Sin[e + f*x]])
```

Rubi [A] time = 0.769078, antiderivative size = 344, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2766, 2978, 2752, 2663, 2661, 2655, 2653}

$$-\frac{\left(4c^2-15cd+27d^2\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{30f(c-d)^3\left(a^3\sin(e+fx)+a^3\right)}+\frac{\left(4c^2-11cd+15d^2\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{30a^3f(c-d)^2\sqrt{c+d\sin(e+fx)}}-\frac{\left(4c^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^3*Sqrt[c + d*Sin[e + f*x]]),x]
```

```
[Out] -(Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(5*(c - d)*f*(a + a*Sin[e + f*x])^
3) - (2*(c - 3*d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(15*a*(c - d)^2*f*
(a + a*Sin[e + f*x])^2) - ((4*c^2 - 15*c*d + 27*d^2)*Cos[e + f*x]*Sqrt[c +
d*Sin[e + f*x]])/(30*(c - d)^3*f*(a^3 + a^3*Sin[e + f*x])) - ((4*c^2 - 15*c
*d + 27*d^2)*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e
+ f*x]])/(30*a^3*(c - d)^3*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + ((4*c^2
- 11*c*d + 15*d^2)*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d
*Sin[e + f*x])/(c + d)])/(30*a^3*(c - d)^2*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
```

```
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^3\sqrt{c+d\sin(e+fx)}}\,dx = -\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{5(c-d)f(a+a\sin(e+fx))^3} - \frac{\int \frac{-\frac{1}{2}a(4c-9d)-\frac{3}{2}ad\sin(e+fx)}{(a+a\sin(e+fx))^2\sqrt{c+d\sin(e+fx)}}\,dx}{5a^2(c-d)}$$

$$= -\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{5(c-d)f(a+a\sin(e+fx))^3} - \frac{2(c-3d)\cos(e+fx)\sqrt{c+d\sin(e+f}}{15a(c-d)^2f(a+a\sin(e+fx))^2}$$

$$= -\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{5(c-d)f(a+a\sin(e+fx))^3} - \frac{2(c-3d)\cos(e+fx)\sqrt{c+d\sin(e+f}}{15a(c-d)^2f(a+a\sin(e+fx))^2}$$

$$= -\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{5(c-d)f(a+a\sin(e+fx))^3} - \frac{2(c-3d)\cos(e+fx)\sqrt{c+d\sin(e+f}}{15a(c-d)^2f(a+a\sin(e+fx))^2}$$

$$= -\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{5(c-d)f(a+a\sin(e+fx))^3} - \frac{2(c-3d)\cos(e+fx)\sqrt{c+d\sin(e+f}}{15a(c-d)^2f(a+a\sin(e+fx))^2}$$

$$= -\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{5(c-d)f(a+a\sin(e+fx))^3} - \frac{2(c-3d)\cos(e+fx)\sqrt{c+d\sin(e+f}}{15a(c-d)^2f(a+a\sin(e+fx))^2}$$

Mathematica [A] time = 6.33633, size = 638, normalized size = 1.85

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^6\sqrt{c+d\sin(e+fx)}\left(\frac{4c^2\sin\left(\frac{1}{2}(e+fx)\right)-15cd\sin\left(\frac{1}{2}(e+fx)\right)+27d^2\sin\left(\frac{1}{2}(e+fx)\right)}{15(c-d)^3\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)} - \frac{2(c-}{15(c-d)^2\left(\sin\left(\frac{1}{2}(e+f\right.\right.}\right.}{f(a\sin(e+f}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^3*Sqrt[c + d*Sin[e + f*x]]),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6*((2*Sin[(e + f*x)/2])/(5*(c - d)*(
Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5) - 1/(5*(c - d)*(Cos[(e + f*x)/2] +
Sin[(e + f*x)/2])^4) - (2*(c - 3*d))/(15*(c - d)^2*(Cos[(e + f*x)/2] + Sin[
(e + f*x)/2])^2) + (4*(c*Sin[(e + f*x)/2] - 3*d*Sin[(e + f*x)/2]))/(15*(c -
 d)^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3) + (4*c^2*Sin[(e + f*x)/2] -
15*c*d*Sin[(e + f*x)/2] + 27*d^2*Sin[(e + f*x)/2])/(15*(c - d)^3*(Cos[(e +
f*x)/2] + Sin[(e + f*x)/2])))*Sqrt[c + d*Sin[e + f*x]])/(f*(a + a*Sin[e + f
*x])^3) - (d*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6*((-2*(c*d + 15*d^2)*El
lipticF[(-e + Pi/2 - f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c +
d)])/Sqrt[c + d*Sin[e + f*x]] + (2*(4*c^2 - 15*c*d + 27*d^2)*Cos[e + f*x]^2
*Sqrt[c + d*Sin[e + f*x]])/(d*(1 - Sin[e + f*x]^2)) - ((4*c^2 - 15*c*d + 27
*d^2)*((2*(c + d)*EllipticE[(-e + Pi/2 - f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d
*Sin[e + f*x])/(c + d)])/Sqrt[c + d*Sin[e + f*x]] - (2*c*EllipticF[(-e + Pi
/2 - f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/Sqrt[c + d*
Sin[e + f*x]]))/d))/(60*(c - d)^3*f*(a + a*Sin[e + f*x])^3)
```

Maple [A] time = 4.102, size = 593, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^(1/2),x)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/a^3*(-1/5/(c-d)*(-(-d*sin(f*x+e)-c)
*cos(f*x+e)^2)^(1/2)/(1+sin(f*x+e))^3-2/15*(c-3*d)/(c-d)^2*(-(-d*sin(f*x+e)
-c)*cos(f*x+e)^2)^(1/2)/(1+sin(f*x+e))^2-1/30*(-sin(f*x+e)^2*d-c*sin(f*x+e)
+d*sin(f*x+e)+c)/(c-d)^3*(4*c^2-15*c*d+27*d^2)/((-d*sin(f*x+e)-c)*(-1+sin(f
*x+e))*(1+sin(f*x+e)))^(1/2)+2*(-c*d^2-15*d^3)/(60*c^3-180*c^2*d+180*c*d^2-
60*d^3)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/
2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*
EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))-1/30*d*(4*c^2
-15*c*d+27*d^2)/(c-d)^3*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*
x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(
f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(
c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))
)/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^(1/2),x, algorithm="maxima"
)
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{d\sin\left(fx+e\right)+c}}{a^3d\cos\left(fx+e\right)^4+4\,a^3c+4\,a^3d-\left(3\,a^3c+5\,a^3d\right)\cos\left(fx+e\right)^2+\left(4\,a^3c+4\,a^3d-\left(a^3c+3\,a^3d\right)\cos\left(fx+e\right)^2\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(d*sin(f*x + e) + c)/(a^3*d*cos(f*x + e)^4 + 4*a^3*c + 4*a^3*d
 - (3*a^3*c + 5*a^3*d)*cos(f*x + e)^2 + (4*a^3*c + 4*a^3*d - (a^3*c + 3*a^3
*d)*cos(f*x + e)^2)*sin(f*x + e)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{\sqrt{c+d\sin\left(e+fx\right)}\sin^3\left(e+fx\right)+3\sqrt{c+d\sin\left(e+fx\right)}\sin^2\left(e+fx\right)+3\sqrt{c+d\sin\left(e+fx\right)}\sin\left(e+fx\right)+\sqrt{c+d\sin\left(e+fx\right)}}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**3/(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(c + d*sin(e + f*x))*sin(e + f*x)**3 + 3*sqrt(c + d*sin(e +
 f*x))*sin(e + f*x)**2 + 3*sqrt(c + d*sin(e + f*x))*sin(e + f*x) + sqrt(c +
 d*sin(e + f*x))), x)/a**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\sin\left(fx+e\right)+a\right)^{3}\sqrt{d\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)^3*sqrt(d*sin(f*x + e) + c)), x)
```

3.520 $\int \frac{1}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=423

$$-\frac{d\left(-21c^2d+4c^3+62cd^2+147d^3\right)\cos(e+fx)}{30a^3f(c-d)^4(c+d)\sqrt{c+d\sin(e+fx)}}-\frac{\left(4c^2-21cd+65d^2\right)\cos(e+fx)}{30f(c-d)^3\left(a^3\sin(e+fx)+a^3\right)\sqrt{c+d\sin(e+fx)}}+\frac{\left(4c^2-21cd\right.}{}$$

```
[Out] -(d*(4*c^3 - 21*c^2*d + 62*c*d^2 + 147*d^3)*Cos[e + f*x])/(30*a^3*(c - d)^4
*(c + d)*f*Sqrt[c + d*Sin[e + f*x]]) - Cos[e + f*x]/(5*(c - d)*f*(a + a*Sin
[e + f*x])^3*Sqrt[c + d*Sin[e + f*x]]) - (2*(c - 4*d)*Cos[e + f*x])/(15*a*(
c - d)^2*f*(a + a*Sin[e + f*x])^2*Sqrt[c + d*Sin[e + f*x]]) - ((4*c^2 - 21*
c*d + 65*d^2)*Cos[e + f*x])/(30*(c - d)^3*f*(a^3 + a^3*Sin[e + f*x])*Sqrt[c
 + d*Sin[e + f*x]]) - ((4*c^3 - 21*c^2*d + 62*c*d^2 + 147*d^3)*EllipticE[(e
 - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(30*a^3*(c - d)^
4*(c + d)*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + ((4*c^2 - 21*c*d + 65*d^2
)*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c
 + d)])/(30*a^3*(c - d)^3*f*Sqrt[c + d*Sin[e + f*x]])
```

Rubi [A] time = 1.02531, antiderivative size = 423, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2766, 2978, 2754, 2752, 2663, 2661, 2655, 2653}

$$-\frac{d\left(-21c^2d+4c^3+62cd^2+147d^3\right)\cos(e+fx)}{30a^3f(c-d)^4(c+d)\sqrt{c+d\sin(e+fx)}}-\frac{\left(4c^2-21cd+65d^2\right)\cos(e+fx)}{30f(c-d)^3\left(a^3\sin(e+fx)+a^3\right)\sqrt{c+d\sin(e+fx)}}+\frac{\left(4c^2-21cd\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^(3/2)),x]
```


```
[Out] -(d*(4*c^3 - 21*c^2*d + 62*c*d^2 + 147*d^3)*Cos[e + f*x])/(30*a^3*(c - d)^4
*(c + d)*f*Sqrt[c + d*Sin[e + f*x]]) - Cos[e + f*x]/(5*(c - d)*f*(a + a*Sin
[e + f*x])^3*Sqrt[c + d*Sin[e + f*x]]) - (2*(c - 4*d)*Cos[e + f*x])/(15*a*(
c - d)^2*f*(a + a*Sin[e + f*x])^2*Sqrt[c + d*Sin[e + f*x]]) - ((4*c^2 - 21*
c*d + 65*d^2)*Cos[e + f*x])/(30*(c - d)^3*f*(a^3 + a^3*Sin[e + f*x])*Sqrt[c
 + d*Sin[e + f*x]]) - ((4*c^3 - 21*c^2*d + 62*c*d^2 + 147*d^3)*EllipticE[(e
 - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(30*a^3*(c - d)^
4*(c + d)*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + ((4*c^2 - 21*c*d + 65*d^2
)*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c
 + d)])/(30*a^3*(c - d)^3*f*Sqrt[c + d*Sin[e + f*x]])
```


Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))^{3/2}}\,dx = -\frac{\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3\sqrt{c+d\sin(e+fx)}} - \frac{\int \frac{-\frac{1}{2}a(4c-11d)-}{(a+a\sin(e+fx))^2(}}{5a^2(}$$

$$= -\frac{\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3\sqrt{c+d\sin(e+fx)}} - \frac{2}{15a(c-d)^2f(a}$$

$$= -\frac{\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3\sqrt{c+d\sin(e+fx)}} - \frac{2}{15a(c-d)^2f(a}$$

$$= -\frac{d\left(4c^3-21c^2d+62cd^2+147d^3\right)\cos(e+fx)}{30a^3(c-d)^4(c+d)f\sqrt{c+d\sin(e+fx)}} - \frac{c}{5(c-d)f(a+a\sin}$$

$$= -\frac{d\left(4c^3-21c^2d+62cd^2+147d^3\right)\cos(e+fx)}{30a^3(c-d)^4(c+d)f\sqrt{c+d\sin(e+fx)}} - \frac{c}{5(c-d)f(a+a\sin}$$

$$= -\frac{d\left(4c^3-21c^2d+62cd^2+147d^3\right)\cos(e+fx)}{30a^3(c-d)^4(c+d)f\sqrt{c+d\sin(e+fx)}} - \frac{c}{5(c-d)f(a+a\sin}$$

$$= -\frac{d\left(4c^3-21c^2d+62cd^2+147d^3\right)\cos(e+fx)}{30a^3(c-d)^4(c+d)f\sqrt{c+d\sin(e+fx)}} - \frac{c}{5(c-d)f(a+a\sin}$$

Mathematica [A] time = 6.54493, size = 745, normalized size = 1.76

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^6\sqrt{c+d\sin(e+fx)}\left(\frac{4c^2\sin\left(\frac{1}{2}(e+fx)\right)-25cd\sin\left(\frac{1}{2}(e+fx)\right)+87d^2\sin\left(\frac{1}{2}(e+fx)\right)}{15(c-d)^4\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)} - \frac{-21c^2d+4c^3+62c}{15(c-d)^4(c}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^(3/2)),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6*Sqrt[c + d*Sin[e + f*x]]*(-(4*c^3
- 21*c^2*d + 62*c*d^2 + 117*d^3)/(15*(c - d)^4*(c + d)) + (2*Sin[(e + f*x)/
2])/(5*(c - d)^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5) - 1/(5*(c - d)^2*
(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4) + (-2*c + 11*d)/(15*(c - d)^3*(Cos
[(e + f*x)/2] + Sin[(e + f*x)/2])^2) + (2*(2*c*Sin[(e + f*x)/2] - 11*d*Sin[
(e + f*x)/2]))/(15*(c - d)^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3) + (4*
c^2*Sin[(e + f*x)/2] - 25*c*d*Sin[(e + f*x)/2] + 87*d^2*Sin[(e + f*x)/2])/(
15*(c - d)^4*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])) - (2*d^4*Cos[e + f*x])/
((c - d)^4*(c + d)*(c + d*Sin[e + f*x]))))/(f*(a + a*Sin[e + f*x])^3) + (d*
(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6*((-2*(-(c^2*d) - 126*c*d^2 - 65*d^3
)*EllipticF[(-e + Pi/2 - f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(
c + d)])/Sqrt[c + d*Sin[e + f*x]] + (2*(4*c^3 - 21*c^2*d + 62*c*d^2 + 147*d
^3)*Cos[e + f*x]^2*Sqrt[c + d*Sin[e + f*x]])/(d*(1 - Sin[e + f*x]^2)) - ((-
4*c^3 + 21*c^2*d - 62*c*d^2 - 147*d^3)*((2*(c + d)*EllipticE[(-e + Pi/2 - f
*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/Sqrt[c + d*Sin[e
+ f*x]] - (2*c*EllipticF[(-e + Pi/2 - f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Si
n[e + f*x])/(c + d)])/Sqrt[c + d*Sin[e + f*x]]))/d))/(60*(c - d)^4*(c + d)*
```

```
f*(a + a*Sin[e + f*x])^3)
```

Maple [B] time = 7.014, size = 1851, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^(3/2),x)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/a^3*(1/(c-d)*(-1/5/(c-d)*(-(-d*sin(
f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(1+sin(f*x+e))^3-2/15*(c-3*d)/(c-d)^2*(-(-d*s
in(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(1+sin(f*x+e))^2-1/30*(-sin(f*x+e)^2*d-c*s
in(f*x+e)+d*sin(f*x+e)+c)/(c-d)^3*(4*c^2-15*c*d+27*d^2)/((-d*sin(f*x+e)-c)*
(-1+sin(f*x+e))*(1+sin(f*x+e)))^(1/2)+2*(-c*d^2-15*d^3)/(60*c^3-180*c^2*d+1
80*c*d^2-60*d^3)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(
c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^
2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))-1/30
*d*(4*c^2-15*c*d+27*d^2)/(c-d)^3*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*
(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e
)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2)
,((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d)
)^(1/2))))-d/(c-d)^2*(-1/3/(c-d)*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(1
+sin(f*x+e))^2-1/3*(-sin(f*x+e)^2*d-c*sin(f*x+e)+d*sin(f*x+e)+c)/(c-d)^2*(c
-3*d)/((-d*sin(f*x+e)-c)*(-1+sin(f*x+e))*(1+sin(f*x+e)))^(1/2)+2*d^2/(3*c^2
-6*c*d+3*d^2)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d
))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^
(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))-1/3*d*(
c-3*d)/(c-d)^2*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+
d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)
^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/
2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))-d^3/(c-
d)^3*(2*d*cos(f*x+e)^2/(c^2-d^2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2*
c/(c^2-d^2)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))
^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1
/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+2/(c^2-d^
2)*d*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*
((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-
c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+Ellipt
icF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+d^2/(c-d)^3*(-(-s
in(f*x+e)^2*d-c*sin(f*x+e)+d*sin(f*x+e)+c)/(c-d)/((-d*sin(f*x+e)-c)*(-1+sin
(f*x+e))*(1+sin(f*x+e)))^(1/2)-2*d/(2*c-2*d)*(c/d-1)*((c+d*sin(f*x+e))/(c-d
))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(
-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/
2),((c-d)/(c+d))^(1/2))-d/(c-d)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(
1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)
-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),
((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))
^(1/2)))))/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^(3/2),x, algorithm="maxima"
)
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{d\sin\left(fx+e\right)}}{4\,a^3c^2+8\,a^3cd+4\,a^3d^2+\left(2\,a^3cd+3\,a^3d^2\right)\cos\left(fx+e\right)^4-\left(3\,a^3c^2+10\,a^3cd+7\,a^3d^2\right)\cos\left(fx+e\right)^2+\left(a^3c\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^(3/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(d*sin(f*x + e) + c)/(4*a^3*c^2 + 8*a^3*c*d + 4*a^3*d^2 + (2*a
^3*c*d + 3*a^3*d^2)*cos(f*x + e)^4 - (3*a^3*c^2 + 10*a^3*c*d + 7*a^3*d^2)*c
os(f*x + e)^2 + (a^3*d^2*cos(f*x + e)^4 + 4*a^3*c^2 + 8*a^3*c*d + 4*a^3*d^2
 - (a^3*c^2 + 6*a^3*c*d + 5*a^3*d^2)*cos(f*x + e)^2)*sin(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**3/(c+d*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\sin\left(fx+e\right)+a\right)^3\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)^3*(d*sin(f*x + e) + c)^(3/2)), x)
```

3.521 $\int \frac{1}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=518

$$-\frac{d\left(111c^2d^2-27c^3d+4c^4+579cd^3+357d^4\right)\cos(e+fx)}{30a^3f(c-d)^5(c+d)^2\sqrt{c+d\sin(e+fx)}}-\frac{d\left(-27c^2d+4c^3+114cd^2+165d^3\right)\cos(e+fx)}{30a^3f(c-d)^4(c+d)(c+d\sin(e+fx))^{3/2}}-\frac{}{30f(c-d}$$

```
[Out] -(d*(4*c^3 - 27*c^2*d + 114*c*d^2 + 165*d^3)*Cos[e + f*x])/(30*a^3*(c - d)^
4*(c + d)*f*(c + d*Sin[e + f*x])^(3/2)) - Cos[e + f*x]/(5*(c - d)*f*(a + a*
Sin[e + f*x])^3*(c + d*Sin[e + f*x])^(3/2)) - (2*(c - 5*d)*Cos[e + f*x])/(1
5*a*(c - d)^2*f*(a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^(3/2)) - ((4*c^
2 - 27*c*d + 119*d^2)*Cos[e + f*x])/(30*(c - d)^3*f*(a^3 + a^3*Sin[e + f*x]
)*(c + d*Sin[e + f*x])^(3/2)) - (d*(4*c^4 - 27*c^3*d + 111*c^2*d^2 + 579*c*
d^3 + 357*d^4)*Cos[e + f*x])/(30*a^3*(c - d)^5*(c + d)^2*f*Sqrt[c + d*Sin[e
 + f*x]]) - ((4*c^4 - 27*c^3*d + 111*c^2*d^2 + 579*c*d^3 + 357*d^4)*Ellipti
cE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(30*a^3*(c
- d)^5*(c + d)^2*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + ((4*c^3 - 27*c^2*d
 + 114*c*d^2 + 165*d^3)*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(
c + d*Sin[e + f*x])/(c + d)])/(30*a^3*(c - d)^4*(c + d)*f*Sqrt[c + d*Sin[e
+ f*x]])
```

Rubi [A] time = 1.21486, antiderivative size = 518, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2766, 2978, 2754, 2752, 2663, 2661, 2655, 2653}

$$-\frac{d\left(111c^2d^2-27c^3d+4c^4+579cd^3+357d^4\right)\cos(e+fx)}{30a^3f(c-d)^5(c+d)^2\sqrt{c+d\sin(e+fx)}}-\frac{d\left(-27c^2d+4c^3+114cd^2+165d^3\right)\cos(e+fx)}{30a^3f(c-d)^4(c+d)(c+d\sin(e+fx))^{3/2}}-\frac{}{30f(c-d}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^(5/2)),x]
```


```
[Out] -(d*(4*c^3 - 27*c^2*d + 114*c*d^2 + 165*d^3)*Cos[e + f*x])/(30*a^3*(c - d)^
4*(c + d)*f*(c + d*Sin[e + f*x])^(3/2)) - Cos[e + f*x]/(5*(c - d)*f*(a + a*
Sin[e + f*x])^3*(c + d*Sin[e + f*x])^(3/2)) - (2*(c - 5*d)*Cos[e + f*x])/(1
5*a*(c - d)^2*f*(a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^(3/2)) - ((4*c^
2 - 27*c*d + 119*d^2)*Cos[e + f*x])/(30*(c - d)^3*f*(a^3 + a^3*Sin[e + f*x]
)*(c + d*Sin[e + f*x])^(3/2)) - (d*(4*c^4 - 27*c^3*d + 111*c^2*d^2 + 579*c*
d^3 + 357*d^4)*Cos[e + f*x])/(30*a^3*(c - d)^5*(c + d)^2*f*Sqrt[c + d*Sin[e
 + f*x]]) - ((4*c^4 - 27*c^3*d + 111*c^2*d^2 + 579*c*d^3 + 357*d^4)*Ellipti
cE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(30*a^3*(c
- d)^5*(c + d)^2*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + ((4*c^3 - 27*c^2*d
 + 114*c*d^2 + 165*d^3)*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(
c + d*Sin[e + f*x])/(c + d)])/(30*a^3*(c - d)^4*(c + d)*f*Sqrt[c + d*Sin[e
+ f*x]])
```


Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
```

```
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))^{5/2}}\,dx = -\frac{\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3(c+d\sin(e+fx))^{3/2}} - \frac{\int \frac{-\frac{1}{2}a(4c-13d)-\ldots}{(a+a\sin(e+fx))^2(c\ldots}}{5a^2(c\ldots}$$

$$= -\frac{\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3(c+d\sin(e+fx))^{3/2}} - \frac{2\ldots}{15a(c-d)^2f(a+\ldots}$$

$$= -\frac{\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3(c+d\sin(e+fx))^{3/2}} - \frac{2\ldots}{15a(c-d)^2f(a+\ldots}$$

$$= -\frac{d\left(4c^3-27c^2d+114cd^2+165d^3\right)\cos(e+fx)}{30a^3(c-d)^4(c+d)f(c+d\sin(e+fx))^{3/2}} - \frac{\mathrm{c}\ldots}{5(c-d)f(a+a\sin(e\ldots}$$

$$= -\frac{d\left(4c^3-27c^2d+114cd^2+165d^3\right)\cos(e+fx)}{30a^3(c-d)^4(c+d)f(c+d\sin(e+fx))^{3/2}} - \frac{\mathrm{c}\ldots}{5(c-d)f(a+a\sin(e\ldots}$$

$$= -\frac{d\left(4c^3-27c^2d+114cd^2+165d^3\right)\cos(e+fx)}{30a^3(c-d)^4(c+d)f(c+d\sin(e+fx))^{3/2}} - \frac{\mathrm{c}\ldots}{5(c-d)f(a+a\sin(e\ldots}$$

$$= -\frac{d\left(4c^3-27c^2d+114cd^2+165d^3\right)\cos(e+fx)}{30a^3(c-d)^4(c+d)f(c+d\sin(e+fx))^{3/2}} - \frac{\mathrm{c}\ldots}{5(c-d)f(a+a\sin(e\ldots}$$

$$= -\frac{d\left(4c^3-27c^2d+114cd^2+165d^3\right)\cos(e+fx)}{30a^3(c-d)^4(c+d)f(c+d\sin(e+fx))^{3/2}} - \frac{\mathrm{c}\ldots}{5(c-d)f(a+a\sin(e\ldots}$$

Mathematica [A] time = 6.83506, size = 828, normalized size = 1.6

$$\frac{\sqrt{c+d\sin(e+fx)}\left(-\frac{2\cos(e+fx)d^4}{3(c-d)^4(c+d)(c+d\sin(e+fx))^2} - \frac{4c^4-27dc^3+111d^2c^2+449d^3c+267d^4}{15(c-d)^5(c+d)^2} + \frac{4\left(c\sin\left(\frac{1}{2}(e+fx)\right)-8d\sin\left(\frac{1}{2}(e+fx)\right)\right)}{15(c-d)^4\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3} + \frac{4\sin\left(\frac{1}{2}(e+\ldots\right.}{1\ldots}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^(5/2)),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6*Sqrt[c + d*Sin[e + f*x]]*(-(4*c^4
- 27*c^3*d + 111*c^2*d^2 + 449*c*d^3 + 267*d^4)/(15*(c - d)^5*(c + d)^2) +
(2*Sin[(e + f*x)/2])/(5*(c - d)^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)
- 1/(5*(c - d)^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4) - (2*(c - 8*d))/(
15*(c - d)^4*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2) + (4*(c*Sin[(e + f*x)
/2] - 8*d*Sin[(e + f*x)/2]))/(15*(c - d)^4*(Cos[(e + f*x)/2] + Sin[(e + f*x
)/2])^3) + (4*c^2*Sin[(e + f*x)/2] - 35*c*d*Sin[(e + f*x)/2] + 177*d^2*Sin[
(e + f*x)/2])/(15*(c - d)^5*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])) - (2*d^4
*Cos[e + f*x])/(3*(c - d)^4*(c + d)*(c + d*Sin[e + f*x])^2) - (2*(13*c*d^4*
Cos[e + f*x] + 9*d^5*Cos[e + f*x]))/(3*(c - d)^5*(c + d)^2*(c + d*Sin[e + f
*x]))))/(f*(a + a*Sin[e + f*x])^3) + (d*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2
])^6*((-2*(-(c^3*d) - 387*c^2*d^2 - 471*c*d^3 - 165*d^4)*EllipticF[(-e + Pi
/2 - f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/Sqrt[c + d*
Sin[e + f*x]] + (2*(4*c^4 - 27*c^3*d + 111*c^2*d^2 + 579*c*d^3 + 357*d^4)*C
```

```
os[e + f*x]^2*Sqrt[c + d*Sin[e + f*x]])/(d*(1 - Sin[e + f*x]^2)) - ((-4*c^4
 + 27*c^3*d - 111*c^2*d^2 - 579*c*d^3 - 357*d^4)*((2*(c + d)*EllipticE[(-e
+ Pi/2 - f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/Sqrt[c
+ d*Sin[e + f*x]] - (2*c*EllipticF[(-e + Pi/2 - f*x)/2, (2*d)/(c + d)]*Sqrt
[(c + d*Sin[e + f*x])/(c + d)])/Sqrt[c + d*Sin[e + f*x]]))/d))/(60*(c - d)^
5*(c + d)^2*f*(a + a*Sin[e + f*x])^3)
```

Maple [B] time = 9.667, size = 2311, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^(5/2),x)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/a^3*(-d^3/(c-d)^3*(2/3/(c^2-d^2)/d*
(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(sin(f*x+e)+c/d)^2+8/3*d*cos(f*x+e)
^2/(c^2-d^2)^2*c/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2*(3*c^2+d^2)/(3*c
^4-6*c^2*d^2+3*d^4)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e)
)/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+
e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+8
/3*c*d/(c^2-d^2)^2*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))
/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e
)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))
^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+1/(
c-d)^2*(-1/5/(c-d)*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(1+sin(f*x+e))^3
-2/15*(c-3*d)/(c-d)^2*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(1+sin(f*x+e)
)^2-1/30*(-sin(f*x+e)^2*d-c*sin(f*x+e)+d*sin(f*x+e)+c)/(c-d)^3*(4*c^2-15*c*
d+27*d^2)/((-d*sin(f*x+e)-c)*(-1+sin(f*x+e))*(1+sin(f*x+e)))^(1/2)+2*(-c*d^
2-15*d^3)/(60*c^3-180*c^2*d+180*c*d^2-60*d^3)*(c/d-1)*((c+d*sin(f*x+e))/(c-
d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-
(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1
/2),((c-d)/(c+d))^(1/2))-1/30*d*(4*c^2-15*c*d+27*d^2)/(c-d)^3*(c/d-1)*((c+d
*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d
/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE((
(c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+
e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))-2/(c-d)^3*d*(-1/3/(c-d)*(-(-d*sin(f
*x+e)-c)*cos(f*x+e)^2)^(1/2)/(1+sin(f*x+e))^2-1/3*(-sin(f*x+e)^2*d-c*sin(f*
x+e)+d*sin(f*x+e)+c)/(c-d)^2*(c-3*d)/((-d*sin(f*x+e)-c)*(-1+sin(f*x+e))*(1+
sin(f*x+e)))^(1/2)+2*d^2/(3*c^2-6*c*d+3*d^2)*(c/d-1)*((c+d*sin(f*x+e))/(c-d
))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(
-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/
2),((c-d)/(c+d))^(1/2))-1/3*d*(c-3*d)/(c-d)^2*(c/d-1)*((c+d*sin(f*x+e))/(c-
d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-
(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))
/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2)
,((c-d)/(c+d))^(1/2))))-3*d^3/(c-d)^4*(2*d*cos(f*x+e)^2/(c^2-d^2)/(-(-d*sin
(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2*c/(c^2-d^2)*(c/d-1)*((c+d*sin(f*x+e))/(c-d
))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(
-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/
2),((c-d)/(c+d))^(1/2))+2/(c^2-d^2)*d*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2
)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(
f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^
(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/
(c+d))^(1/2))))+3/(c-d)^4*d^2*(-(-sin(f*x+e)^2*d-c*sin(f*x+e)+d*sin(f*x+e)+
c)/(c-d)/((-d*sin(f*x+e)-c)*(-1+sin(f*x+e))*(1+sin(f*x+e)))^(1/2)-2*d/(2*c-
2*d)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*
```

```
((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*Ell
ipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))-d/(c-d)*(c/d-1)*
((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)
-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*Ellipt
icE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin
(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))))/cos(f*x+e)/(c+d*sin(f*x+e))^(
1/2)/f
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^(5/2),x, algorithm="maxima"
)
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{}{a^3d^3\cos\left(fx+e\right)^6-4\,a^3c^3-12\,a^3c^2d-12\,a^3cd^2-4\,a^3d^3-3\left(a^3c^2d+3\,a^3cd^2+2\,a^3d^3\right)\cos\left(fx+e\right)^4+3\left(a^3\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^(5/2),x, algorithm="fricas"
)
```

```
[Out] integral(-sqrt(d*sin(f*x + e) + c)/(a^3*d^3*cos(f*x + e)^6 - 4*a^3*c^3 - 12
*a^3*c^2*d - 12*a^3*c*d^2 - 4*a^3*d^3 - 3*(a^3*c^2*d + 3*a^3*c*d^2 + 2*a^3*
d^3)*cos(f*x + e)^4 + 3*(a^3*c^3 + 5*a^3*c^2*d + 7*a^3*c*d^2 + 3*a^3*d^3)*c
os(f*x + e)^2 - (4*a^3*c^3 + 12*a^3*c^2*d + 12*a^3*c*d^2 + 4*a^3*d^3 + 3*(a
^3*c*d^2 + a^3*d^3)*cos(f*x + e)^4 - (a^3*c^3 + 9*a^3*c^2*d + 15*a^3*c*d^2
+ 7*a^3*d^3)*cos(f*x + e)^2)*sin(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**3/(c+d*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\sin\left(fx+e\right)+a\right)^3\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)^3*(d*sin(f*x + e) + c)^(5/2)), x)
```

3.522 $\int \sqrt{a + a\sin(e + fx)}(c + d\sin(e + fx))^3 \, dx$

Optimal. Leaf size=161

$$-\frac{4a(c+d)\left(15c^2+10cd+7d^2\right)\cos(e+fx)}{35f\sqrt{a\sin(e+fx)+a}}-\frac{12d^2(c+d)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{35af}-\frac{2a\cos(e+fx)(c+d\sin(e-}{7f\sqrt{a\sin(e+fx)+a}}$$

```
[Out] (-4*a*(c + d)*(15*c^2 + 10*c*d + 7*d^2)*Cos[e + f*x])/(35*f*Sqrt[a + a*Sin[
e + f*x]]) - (8*(5*c - d)*d*(c + d)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/
(35*f) - (12*d^2*(c + d)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(35*a*f)
- (2*a*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(7*f*Sqrt[a + a*Sin[e + f*x]])
```

Rubi [A] time = 0.278762, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2770, 2761, 2751, 2646}

$$-\frac{4a(c+d)\left(15c^2+10cd+7d^2\right)\cos(e+fx)}{35f\sqrt{a\sin(e+fx)+a}}-\frac{12d^2(c+d)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{35af}-\frac{2a\cos(e+fx)(c+d\sin(e-}{7f\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^3,x]
```


```
[Out] (-4*a*(c + d)*(15*c^2 + 10*c*d + 7*d^2)*Cos[e + f*x])/(35*f*Sqrt[a + a*Sin[
e + f*x]]) - (8*(5*c - d)*d*(c + d)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/
(35*f) - (12*d^2*(c + d)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(35*a*f)
- (2*a*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(7*f*Sqrt[a + a*Sin[e + f*x]])
```


Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2761

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^2, x_Symbol] :> -Simp[(d^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m
 + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*S
imp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int \sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^3\,dx = -\frac{2a\cos(e+fx)(c+d\sin(e+fx))^3}{7f\sqrt{a+a\sin(e+fx)}} + \frac{1}{7}(6(c+d))\int \sqrt{a+a\sin(e+f$$

$$= -\frac{12d^2(c+d)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{35af} - \frac{2a\cos(e+fx)(c+d\,\text{s}}{7f\sqrt{a+a\sin(e}}$$

$$= -\frac{8(5c-d)d(c+d)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{35f} - \frac{12d^2(c+d)\cos(e}{}$$

$$= -\frac{4a(c+d)\left(15c^2+10cd+7d^2\right)\cos(e+fx)}{35f\sqrt{a+a\sin(e+fx)}} - \frac{8(5c-d)d(c+d)\cos(e+}{35}$$

Mathematica [A] time = 0.520631, size = 146, normalized size = 0.91

$$-\frac{\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(d\left(140c^2+112cd+47d^2\right)\sin(e+fx)+280c^2d+140c^3-6}{70f\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^3,x]
```

```
[Out] -((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(140*c^3
 + 280*c^2*d + 266*c*d^2 + 76*d^3 - 6*d^2*(7*c + 2*d)*Cos[2*(e + f*x)] + d*
(140*c^2 + 112*c*d + 47*d^2)*Sin[e + f*x] - 5*d^3*Sin[3*(e + f*x)]))/(70*f*
(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))
```

Maple [A] time = 0.647, size = 141, normalized size = 0.9

$$\frac{(2+2\sin(fx+e))\,a\,(-1+\sin(fx+e))\left(5d^3(\sin(fx+e))^3+21cd^2(\sin(fx+e))^2+6d^3(\sin(fx+e))^2+35c^2a}{35f\cos(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^3,x)
```

```
[Out] 2/35*(1+sin(f*x+e))*a*(-1+sin(f*x+e))*(5*d^3*sin(f*x+e)^3+21*c*d^2*sin(f*x+
e)^2+6*d^3*sin(f*x+e)^2+35*c^2*d*sin(f*x+e)+28*sin(f*x+e)*d^2*c+8*d^3*sin(f
*x+e)+35*c^3+70*c^2*d+56*c*d^2+16*d^3)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\sin(fx+e)+a}\,(d\sin(fx+e)+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^3, x)
```

Fricas [A] time = 1.96251, size = 576, normalized size = 3.58

$$\frac{2\left(5\,d^3\cos\left(fx+e\right)^4+3\left(7\,cd^2+2\,d^3\right)\cos\left(fx+e\right)^3-35\,c^3-35\,c^2d-49\,cd^2-9\,d^3-\left(35\,c^2d+7\,cd^2+12\,d^3\right)\cos\left(fx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 2/35*(5*d^3*cos(f*x + e)^4 + 3*(7*c*d^2 + 2*d^3)*cos(f*x + e)^3 - 35*c^3 -
35*c^2*d - 49*c*d^2 - 9*d^3 - (35*c^2*d + 7*c*d^2 + 12*d^3)*cos(f*x + e)^2
- (35*c^3 + 70*c^2*d + 77*c*d^2 + 22*d^3)*cos(f*x + e) + (5*d^3*cos(f*x + e
)^3 + 35*c^3 + 35*c^2*d + 49*c*d^2 + 9*d^3 - (21*c*d^2 + d^3)*cos(f*x + e)^
2 - (35*c^2*d + 28*c*d^2 + 13*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f
*x + e) + a)/(f*cos(f*x + e) + f*sin(f*x + e) + f)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\left(\sin\left(e+fx\right)+1\right)}\left(c+d\sin\left(e+fx\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(1/2)*(c+d*sin(f*x+e))**3,x)
```

```
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))*(c + d*sin(e + f*x))**3, x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] Timed out
```

3.523 $\int \sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^2\,dx$

Optimal. Leaf size=112

$$-\frac{2a\left(15c^2+10cd+7d^2\right)\cos(e+fx)}{15f\sqrt{a\sin(e+fx)+a}}-\frac{4d(5c-d)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{15f}-\frac{2d^2\cos(e+fx)(a\sin(e+fx)+a)}{5af}$$

```
[Out] (-2*a*(15*c^2 + 10*c*d + 7*d^2)*Cos[e + f*x])/(15*f*Sqrt[a + a*Sin[e + f*x]
]) - (4*(5*c - d)*d*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(15*f) - (2*d^2*
Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(5*a*f)
```

Rubi [A] time = 0.168584, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2761, 2751, 2646}

$$-\frac{2a\left(15c^2+10cd+7d^2\right)\cos(e+fx)}{15f\sqrt{a\sin(e+fx)+a}}-\frac{4d(5c-d)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{15f}-\frac{2d^2\cos(e+fx)(a\sin(e+fx)+a)}{5af}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^2,x]
```

```
[Out] (-2*a*(15*c^2 + 10*c*d + 7*d^2)*Cos[e + f*x])/(15*f*Sqrt[a + a*Sin[e + f*x]
]) - (4*(5*c - d)*d*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(15*f) - (2*d^2*
Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(5*a*f)
```

Rule 2761

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^2, x_Symbol] :> -Simp[(d^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m
 + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*S
imp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int \sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^2\,dx = -\frac{2d^2\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{5af} + \frac{2\int \sqrt{a+a\sin(e+fx)}\left(\frac{1}{2}a\left(5c^2\right.\right.}{}$$

$$= -\frac{4(5c-d)d\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{15f} - \frac{2d^2\cos(e+fx)(a+a\sin(e}{5af}$$

$$= -\frac{2a\left(15c^2+10cd+7d^2\right)\cos(e+fx)}{15f\sqrt{a+a\sin(e+fx)}} - \frac{4(5c-d)d\cos(e+fx)\sqrt{a+a\sin(}{15f}$$

Mathematica [A] time = 0.286979, size = 111, normalized size = 0.99

$$-\frac{\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(30c^2+4d(5c+2d)\sin(e+fx)+40cd-3d^2\cos(2(e+fx))+\right.}{15f\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^2,x]
```

```
[Out] -((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(30*c^2
+ 40*c*d + 19*d^2 - 3*d^2*Cos[2*(e + f*x)] + 4*d*(5*c + 2*d)*Sin[e + f*x]))
/(15*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))
```

Maple [A] time = 0.648, size = 92, normalized size = 0.8

$$\frac{\left(2+2\sin\left(fx+e\right)\right)a\left(-1+\sin\left(fx+e\right)\right)\left(3d^2\left(\sin\left(fx+e\right)\right)^2+10\sin\left(fx+e\right)cd+4\sin\left(fx+e\right)d^2+15c^2+20cd+\right.}{15f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^2,x)
```

```
[Out] 2/15*(1+sin(f*x+e))*a*(-1+sin(f*x+e))*(3*d^2*sin(f*x+e)^2+10*sin(f*x+e)*c*d
+4*sin(f*x+e)*d^2+15*c^2+20*c*d+8*d^2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\sin\left(fx+e\right)+a}\left(d\sin\left(fx+e\right)+c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^2, x)
```

Fricas [A] time = 1.87626, size = 390, normalized size = 3.48

$$\frac{2\left(3\,d^{2}\cos\left(fx+e\right)^{3}-\left(10\,cd+d^{2}\right)\cos\left(fx+e\right)^{2}-15\,c^{2}-10\,cd-7\,d^{2}-\left(15\,c^{2}+20\,cd+11\,d^{2}\right)\cos\left(fx+e\right)-\left(3\,d^{2}\,\mathrm{c}\right.\right.}{15\left(f\cos\left(fx+e\right)+f\sin\left(fx\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 2/15*(3*d^2*cos(f*x + e)^3 - (10*c*d + d^2)*cos(f*x + e)^2 - 15*c^2 - 10*c*
d - 7*d^2 - (15*c^2 + 20*c*d + 11*d^2)*cos(f*x + e) - (3*d^2*cos(f*x + e)^2
 - 15*c^2 - 10*c*d - 7*d^2 + 2*(5*c*d + 2*d^2)*cos(f*x + e))*sin(f*x + e))*
sqrt(a*sin(f*x + e) + a)/(f*cos(f*x + e) + f*sin(f*x + e) + f)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\left(\sin\left(e+fx\right)+1\right)}\left(c+d\sin\left(e+fx\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(1/2)*(c+d*sin(f*x+e))**2,x)
```

```
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))*(c + d*sin(e + f*x))**2, x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.524 $\int \sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))\,dx$

Optimal. Leaf size=62

$$-\frac{2a(3c+d)\cos(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}-\frac{2d\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}$$

```
[Out] (-2*a*(3*c + d)*Cos[e + f*x])/(3*f*Sqrt[a + a*Sin[e + f*x]]) - (2*d*Cos[e +
 f*x]*Sqrt[a + a*Sin[e + f*x]])/(3*f)
```

Rubi [A] time = 0.0554464, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {2751, 2646}

$$-\frac{2a(3c+d)\cos(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}-\frac{2d\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x]),x]
```

```
[Out] (-2*a*(3*c + d)*Cos[e + f*x])/(3*f*Sqrt[a + a*Sin[e + f*x]]) - (2*d*Cos[e +
 f*x]*Sqrt[a + a*Sin[e + f*x]])/(3*f)
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))\,dx &= -\frac{2d\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3f}+\frac{1}{3}(3c+d)\int \sqrt{a+a\sin(e+fx)}\,dx\\ &= -\frac{2a(3c+d)\cos(e+fx)}{3f\sqrt{a+a\sin(e+fx)}}-\frac{2d\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3f}\end{aligned}$$

Mathematica [A] time = 0.123096, size = 82, normalized size = 1.32

$$-\frac{2\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(3c+d\sin(e+fx)+2d)}{3f\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x]),x]
```

```
[Out] (-2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(3*c +
 2*d + d*Sin[e + f*x]))/(3*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))
```

Maple [A] time = 0.614, size = 58, normalized size = 0.9

$$\frac{\left(2+2\sin\left(fx+e\right)\right)a\left(-1+\sin\left(fx+e\right)\right)\left(d\sin\left(fx+e\right)+3c+2d\right)}{3f\cos\left(fx+e\right)}\frac{1}{\sqrt{a+a\sin\left(fx+e\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e)),x)
```

```
[Out] 2/3*(1+sin(f*x+e))*a*(-1+sin(f*x+e))*(d*sin(f*x+e)+3*c+2*d)/cos(f*x+e)/(a+a
*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a\sin\left(fx+e\right)+a}\left(d\sin\left(fx+e\right)+c\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) + c), x)
```

Fricas [A] time = 1.83614, size = 225, normalized size = 3.63

$$-\frac{2\left(d\cos\left(fx+e\right)^{2}+(3c+2d)\cos\left(fx+e\right)+\left(d\cos\left(fx+e\right)-3c-d\right)\sin\left(fx+e\right)+3c+d\right)\sqrt{a\sin\left(fx+e\right)+a}}{3\left(f\cos\left(fx+e\right)+f\sin\left(fx+e\right)+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] -2/3*(d*cos(f*x + e)^2 + (3*c + 2*d)*cos(f*x + e) + (d*cos(f*x + e) - 3*c -
 d)*sin(f*x + e) + 3*c + d)*sqrt(a*sin(f*x + e) + a)/(f*cos(f*x + e) + f*si
n(f*x + e) + f)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a\left(\sin\left(e+fx\right)+1\right)}\left(c+d\sin\left(e+fx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(1/2)*(c+d*sin(f*x+e)),x)
```

```
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))*(c + d*sin(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\sin\left(fx+e\right)+a}\left(d\sin\left(fx+e\right)+c\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) + c), x)
```

3.525 $\int \sqrt{a + a\sin(e + fx)}\,dx$

Optimal. Leaf size=26

$$-\frac{2a\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}$$

```
[Out] (-2*a*Cos[e + f*x])/(f*Sqrt[a + a*Sin[e + f*x]])
```

Rubi [A] time = 0.0138016, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2646}

$$-\frac{2a\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] (-2*a*Cos[e + f*x])/(f*Sqrt[a + a*Sin[e + f*x]])
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int \sqrt{a+a\sin(e+fx)}\,dx = -\frac{2a\cos(e+fx)}{f\sqrt{a+a\sin(e+fx)}}$$

Mathematica [B] time = 0.0334138, size = 65, normalized size = 2.5

$$\frac{2\sqrt{a(\sin(e+fx)+1)}\left(\sin\left(\frac{1}{2}(e+fx)\right)-\cos\left(\frac{1}{2}(e+fx)\right)\right)}{f\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] (2*(-Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])])/(f*(C
os[(e + f*x)/2] + Sin[(e + f*x)/2]))
```

Maple [A] time = 0.467, size = 43, normalized size = 1.7

$$2\,\frac{a\left(1+\sin\left(fx+e\right)\right)\left(-1+\sin\left(fx+e\right)\right)}{\cos\left(fx+e\right)\sqrt{a+a\sin\left(fx+e\right)}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] 2*(1+sin(f*x+e))*a*(-1+sin(f*x+e))/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a), x)
```

Fricas [B] time = 1.77519, size = 136, normalized size = 5.23

$$-\frac{2\sqrt{a\sin\left(fx+e\right)+a}\left(\cos\left(fx+e\right)-\sin\left(fx+e\right)+1\right)}{f\cos\left(fx+e\right)+f\sin\left(fx+e\right)+f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(a*sin(f*x + e) + a)*(cos(f*x + e) - sin(f*x + e) + 1)/(f*cos(f*x +
e) + f*sin(f*x + e) + f)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\sin\left(e+fx\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a*sin(e + f*x) + a), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a), x)
```

3.526 $\int \frac{\sqrt{a+a\sin(e+fx)}}{c+d\sin(e+fx)}\,dx$

Optimal. Leaf size=61

$$-\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{d}f\sqrt{c+d}}$$

```
[Out] (-2*Sqrt[a]*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]*Sqrt[a + a*
Sin[e + f*x]])])/(Sqrt[d]*Sqrt[c + d]*f)
```

Rubi [A] time = 0.115007, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2773, 208}

$$-\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{d}f\sqrt{c+d}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]/(c + d*Sin[e + f*x]),x]
```

```
[Out] (-2*Sqrt[a]*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]*Sqrt[a + a*
Sin[e + f*x]])])/(Sqrt[d]*Sqrt[c + d]*f)
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+a\sin(e+fx)}}{c+d\sin(e+fx)}\,dx &= -\frac{(2a)\,\text{Subst}\left(\int \frac{1}{ac+ad-dx^2}\,dx, x, \frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{f}\\ &= -\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{d}\sqrt{c+d}f}\end{aligned}$$

Mathematica [C] time = 5.1734, size = 657, normalized size = 10.77

$$\left(\frac{1}{8}+\frac{i}{8}\right)\left(\cos\left(\frac{e}{2}\right)+i\sin\left(\frac{e}{2}\right)\right)\sqrt{a(\sin(e+fx)+1)}\left(\text{RootSum}\left[2i\#1^2ce^{ie}+\#1^4de^{2ie}-d\&,\frac{\#1^3\left(-\sqrt{d}\right)e^{ie}fx\sqrt{c+d}-2i\#1^3\sqrt{d}e^{ie}\sqrt{c+d}\log\cdots}{\cdots}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]/(c + d*Sin[e + f*x]),x]
```

```
[Out] ((1/8 + I/8)*(RootSum[-d + (2*I)*c*E^(I*e)*#1^2 + d*E^((2*I)*e)*#1^4 & , ((
1 + I)*d*Sqrt[E^((-I)*e)]*f*x - (2 - 2*I)*d*Sqrt[E^((-I)*e)]*Log[E^((I/2)*f
*x) - #1] - I*Sqrt[d]*Sqrt[c + d]*f*x*#1 + 2*Sqrt[d]*Sqrt[c + d]*Log[E^((I/
2)*f*x) - #1]*#1 + ((1 - I)*c*f*x*#1^2)/Sqrt[E^((-I)*e)] + ((2 + 2*I)*c*Log
[E^((I/2)*f*x) - #1]*#1^2)/Sqrt[E^((-I)*e)] - Sqrt[d]*Sqrt[c + d]*E^(I*e)*f
*x*#1^3 - (2*I)*Sqrt[d]*Sqrt[c + d]*E^(I*e)*Log[E^((I/2)*f*x) - #1]*#1^3)/(
d - I*c*E^(I*e)*#1^2) & ] - I*RootSum[-d + (2*I)*c*E^(I*e)*#1^2 + d*E^((2*I
)*e)*#1^4 & , ((1 - I)*d*Sqrt[E^((-I)*e)]*f*x + (2 + 2*I)*d*Sqrt[E^((-I)*e)
]*Log[E^((I/2)*f*x) - #1] + Sqrt[d]*Sqrt[c + d]*f*x*#1 + (2*I)*Sqrt[d]*Sqrt
[c + d]*Log[E^((I/2)*f*x) - #1]*#1 - ((1 + I)*c*f*x*#1^2)/Sqrt[E^((-I)*e)]
+ ((2 - 2*I)*c*Log[E^((I/2)*f*x) - #1]*#1^2)/Sqrt[E^((-I)*e)] - I*Sqrt[d]*S
qrt[c + d]*E^(I*e)*f*x*#1^3 + 2*Sqrt[d]*Sqrt[c + d]*E^(I*e)*Log[E^((I/2)*f*
x) - #1]*#1^3)/(d - I*c*E^(I*e)*#1^2) & ])*(Cos[e/2] + I*Sin[e/2])*Sqrt[a*(
1 + Sin[e + f*x])])/(Sqrt[d]*Sqrt[c + d]*f*(Cos[(e + f*x)/2] + Sin[(e + f*x
)/2]))
```

Maple [A] time = 0.654, size = 80, normalized size = 1.3

$$-2\,\frac{a\left(1+\sin\left(fx+e\right)\right)\sqrt{-a\left(-1+\sin\left(fx+e\right)\right)}}{\sqrt{a\left(c+d\right)d}\cos\left(fx+e\right)\sqrt{a+a\sin\left(fx+e\right)}f}\text{Artanh}\left(\frac{\sqrt{-a\left(-1+\sin\left(fx+e\right)\right)}d}{\sqrt{a\left(c+d\right)d}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e)),x)
```

```
[Out] -2*a*(1+sin(f*x+e))*(-a*(-1+sin(f*x+e)))^(1/2)/(a*(c+d)*d)^(1/2)*arctanh((-
a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))/cos(f*x+e)/(a+a*sin(f*x+e))^(
1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{a\sin\left(fx+e\right)+a}}{d\sin\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)/(d*sin(f*x + e) + c), x)
```

Fricas [A] time = 2.50714, size = 1061, normalized size = 17.39

$$\left[\sqrt{\frac{a}{cd+d^2}}\log\left(\frac{ad^2\cos\left(fx+e\right)^3-ac^2-2\,acd-ad^2-\left(6\,acd+7\,ad^2\right)\cos\left(fx+e\right)^2+4\left(c^2d+4\,cd^2+3\,d^3-\left(cd^2+d^3\right)\cos\left(fx+e\right)^2+\left(c^2d+3\,cd^2+2\,d^3\right)\cos\left(fx+e\right)-\left(c^2d+\cdots\right.\right.}{d^2\cos\left(fx+e\right)^3+\left(2\,cd+d^2\right)\cos\left(fx+e\right)^2-c^2-2\,c\cdots}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(a/(c*d + d^2))*log((a*d^2*cos(f*x + e)^3 - a*c^2 - 2*a*c*d - a*d^
2 - (6*a*c*d + 7*a*d^2)*cos(f*x + e)^2 + 4*(c^2*d + 4*c*d^2 + 3*d^3 - (c*d^
2 + d^3)*cos(f*x + e)^2 + (c^2*d + 3*c*d^2 + 2*d^3)*cos(f*x + e) - (c^2*d +
 4*c*d^2 + 3*d^3 + (c*d^2 + d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x
 + e) + a)*sqrt(a/(c*d + d^2)) - (a*c^2 + 8*a*c*d + 9*a*d^2)*cos(f*x + e) +
 (a*d^2*cos(f*x + e)^2 - a*c^2 - 2*a*c*d - a*d^2 + 2*(3*a*c*d + 4*a*d^2)*co
s(f*x + e))*sin(f*x + e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^
2 - c^2 - 2*c*d - d^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - 2*
c*d*cos(f*x + e) - c^2 - 2*c*d - d^2)*sin(f*x + e)))/f, -sqrt(-a/(c*d + d^2
))*arctan(1/2*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) - c - 2*d)*sqrt(-a/(
c*d + d^2))/(a*cos(f*x + e)))/f]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(1/2)/(c+d*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.527 $\int \frac{\sqrt{a+a\sin(e+fx)}}{(c+d\sin(e+fx))^2}\,dx$

Optimal. Leaf size=105

$$-\frac{a\cos(e+fx)}{f(c+d)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{d}f(c+d)^{3/2}}$$

```
[Out] -((Sqrt[a]*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]*Sqrt[a + a*S
in[e + f*x]])])/(Sqrt[d]*(c + d)^(3/2)*f)) - (a*Cos[e + f*x])/((c + d)*f*Sq
rt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x]))
```

Rubi [A] time = 0.185632, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2772, 2773, 208}

$$-\frac{a\cos(e+fx)}{f(c+d)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{d}f(c+d)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]/(c + d*Sin[e + f*x])^2,x]
```

```
[Out] -((Sqrt[a]*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]*Sqrt[a + a*S
in[e + f*x]])])/(Sqrt[d]*(c + d)^(3/2)*f)) - (a*Cos[e + f*x])/((c + d)*f*Sq
rt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x]))
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{a+a\sin(e+fx)}}{(c+d\sin(e+fx))^2}\,dx &= -\frac{a\cos(e+fx)}{(c+d)f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))}+\frac{\int \frac{\sqrt{a+a\sin(e+fx)}}{c+d\sin(e+fx)}\,dx}{2(c+d)} \\
&= -\frac{a\cos(e+fx)}{(c+d)f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))}-\frac{a\,\mathrm{Subst}\left(\int \frac{1}{ac+ad-dx^2}\,dx,x,\frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx}}\right.}{(c+d)f} \\
&= -\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{d}(c+d)^{3/2}f}-\frac{a\cos(e+fx)}{(c+d)f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))}
\end{aligned}
$$

Mathematica [C] time = 5.83129, size = 871, normalized size = 8.3

$$
\frac{\left(\frac{1}{4}+\frac{i}{4}\right)\sqrt{a(\sin(e+fx)+1)}\left(\frac{\left(\cos\left(\frac{e}{2}\right)+i\sin\left(\frac{e}{2}\right)\right)\left((-1+i)x\cos(e)+(1+i)x\sin(e)+\frac{\mathrm{RootSum}\left[de^{2ie}\#1^4+2ice^{ie}\#1^2-d\&,\frac{-\sqrt{d}\sqrt{c+d}e^{ie}fx\#1^3-2i\sqrt{d}\sqrt{c+d}e^{ie}\log\left(e^{\frac{ifx}{2}}\right.}{}\right.}{}\right.}{\sqrt{d}(c}\right.}{}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]/(c + d*Sin[e + f*x])^2,x]
```

```
[Out] ((1/4 + I/4)*Sqrt[a*(1 + Sin[e + f*x])]*(((Cos[e/2] + I*Sin[e/2])*((-1 + I)
*x*Cos[e] + (RootSum[-d + (2*I)*c*E^(I*e)*#1^2 + d*E^((2*I)*e)*#1^4 & , ((1
 + I)*d*Sqrt[E^((-I)*e)]*f*x - (2 - 2*I)*d*Sqrt[E^((-I)*e)]*Log[E^((I/2)*f*
x) - #1] - I*Sqrt[d]*Sqrt[c + d]*f*x*#1 + 2*Sqrt[d]*Sqrt[c + d]*Log[E^((I/2
)*f*x) - #1]*#1 + ((1 - I)*c*f*x*#1^2)/Sqrt[E^((-I)*e)] + ((2 + 2*I)*c*Log[
E^((I/2)*f*x) - #1]*#1^2)/Sqrt[E^((-I)*e)] - Sqrt[d]*Sqrt[c + d]*E^(I*e)*f*
x*#1^3 - (2*I)*Sqrt[d]*Sqrt[c + d]*E^(I*e)*Log[E^((I/2)*f*x) - #1]*#1^3)/(d
 - I*c*E^(I*e)*#1^2) & ]*(Cos[e] + I*(-1 + Sin[e]))*Sqrt[Cos[e] - I*Sin[e]]
)/(4*f) + (1 + I)*x*Sin[e]))/(Sqrt[d]*(c + d)^(3/2)*(Cos[e] + I*(-1 + Sin[e
]))*Sqrt[Cos[e] - I*Sin[e]]) + ((Cos[e/2] + I*Sin[e/2])*((1 - I)*x*Cos[e] -
 (1 + I)*x*Sin[e] + (RootSum[-d + (2*I)*c*E^(I*e)*#1^2 + d*E^((2*I)*e)*#1^4
 & , ((1 - I)*d*Sqrt[E^((-I)*e)]*f*x + (2 + 2*I)*d*Sqrt[E^((-I)*e)]*Log[E^(
(I/2)*f*x) - #1] + Sqrt[d]*Sqrt[c + d]*f*x*#1 + (2*I)*Sqrt[d]*Sqrt[c + d]*L
og[E^((I/2)*f*x) - #1]*#1 - ((1 + I)*c*f*x*#1^2)/Sqrt[E^((-I)*e)] + ((2 - 2
*I)*c*Log[E^((I/2)*f*x) - #1]*#1^2)/Sqrt[E^((-I)*e)] - I*Sqrt[d]*Sqrt[c + d
]*E^(I*e)*f*x*#1^3 + 2*Sqrt[d]*Sqrt[c + d]*E^(I*e)*Log[E^((I/2)*f*x) - #1]*
```

```
#1^3)/(d - I*c*E^(I*e)*#1^2) & ]*Sqrt[Cos[e] - I*Sin[e]]*(-1 - I*Cos[e] + S
in[e]))/(4*f)))/(Sqrt[d]*(c + d)^(3/2)*(Cos[e] + I*(-1 + Sin[e]))*Sqrt[Cos[
e] - I*Sin[e]]) - ((2 - 2*I)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))/((c + d
)*f*(c + d*Sin[e + f*x]))))/(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])
```

Maple [A] time = 1.078, size = 155, normalized size = 1.5

$$-\frac{1+\sin\left(fx+e\right)}{\left(c+d\right)\left(c+d\sin\left(fx+e\right)\right)\cos\left(fx+e\right)f}\sqrt{-a\left(-1+\sin\left(fx+e\right)\right)}\left(\mathrm{Artanh}\left(d\sqrt{-a\left(-1+\sin\left(fx+e\right)\right)}\frac{1}{\sqrt{a\left(c+d\right)d}}\right)\mathrm{si}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^2,x)
```

```
[Out] -(1+sin(f*x+e))*(-a*(-1+sin(f*x+e)))^(1/2)*(arctanh((-a*(-1+sin(f*x+e)))^(1
/2)*d/(a*(c+d)*d)^(1/2))*sin(f*x+e)*a*d+arctanh((-a*(-1+sin(f*x+e)))^(1/2)*
d/(a*(c+d)*d)^(1/2))*a*c+(-a*(-1+sin(f*x+e)))^(1/2)*(a*(c+d)*d)^(1/2))/(c+d
)/(c+d*sin(f*x+e))/(a*(c+d)*d)^(1/2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a\sin\left(fx+e\right)+a}}{\left(d\sin\left(fx+e\right)+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)/(d*sin(f*x + e) + c)^2, x)
```

Fricas [B] time = 2.61356, size = 1874, normalized size = 17.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] [1/4*((d*cos(f*x + e)^2 - c*cos(f*x + e) - (d*cos(f*x + e) + c + d)*sin(f*x
 + e) - c - d)*sqrt(a/(c*d + d^2))*log((a*d^2*cos(f*x + e)^3 - a*c^2 - 2*a*
c*d - a*d^2 - (6*a*c*d + 7*a*d^2)*cos(f*x + e)^2 + 4*(c^2*d + 4*c*d^2 + 3*d
^3 - (c*d^2 + d^3)*cos(f*x + e)^2 + (c^2*d + 3*c*d^2 + 2*d^3)*cos(f*x + e)
- (c^2*d + 4*c*d^2 + 3*d^3 + (c*d^2 + d^3)*cos(f*x + e))*sin(f*x + e))*sqrt
(a*sin(f*x + e) + a)*sqrt(a/(c*d + d^2)) - (a*c^2 + 8*a*c*d + 9*a*d^2)*cos(
f*x + e) + (a*d^2*cos(f*x + e)^2 - a*c^2 - 2*a*c*d - a*d^2 + 2*(3*a*c*d + 4
*a*d^2)*cos(f*x + e))*sin(f*x + e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos
(f*x + e)^2 - c^2 - 2*c*d - d^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x +
 e)^2 - 2*c*d*cos(f*x + e) - c^2 - 2*c*d - d^2)*sin(f*x + e))) + 4*sqrt(a*s
in(f*x + e) + a)*(cos(f*x + e) - sin(f*x + e) + 1))/((c*d + d^2)*f*cos(f*x
+ e)^2 - (c^2 + c*d)*f*cos(f*x + e) - (c^2 + 2*c*d + d^2)*f - ((c*d + d^2)*
```

```
f*cos(f*x + e) + (c^2 + 2*c*d + d^2)*f)*sin(f*x + e)), -1/2*((d*cos(f*x + e
)^2 - c*cos(f*x + e) - (d*cos(f*x + e) + c + d)*sin(f*x + e) - c - d)*sqrt(
-a/(c*d + d^2))*arctan(1/2*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) - c - 2
*d)*sqrt(-a/(c*d + d^2))/(a*cos(f*x + e))) - 2*sqrt(a*sin(f*x + e) + a)*(co
s(f*x + e) - sin(f*x + e) + 1))/((c*d + d^2)*f*cos(f*x + e)^2 - (c^2 + c*d)
*f*cos(f*x + e) - (c^2 + 2*c*d + d^2)*f - ((c*d + d^2)*f*cos(f*x + e) + (c^
2 + 2*c*d + d^2)*f)*sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(1/2)/(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.528 $\int \frac{\sqrt{a+a\sin(e+fx)}}{(c+d\sin(e+fx))^3}\,dx$

Optimal. Leaf size=154

$$-\frac{3a\cos(e+fx)}{4f(c+d)^2\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}-\frac{a\cos(e+fx)}{2f(c+d)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^2}-\frac{3\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}}{\sqrt{c+d}}\right.}{4\sqrt{d}f(c+}$$

```
[Out] (-3*Sqrt[a]*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]*Sqrt[a + a*
Sin[e + f*x]])])/(4*Sqrt[d]*(c + d)^(5/2)*f) - (a*Cos[e + f*x])/(2*(c + d)*
f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^2) - (3*a*Cos[e + f*x])/(4*
(c + d)^2*f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x]))
```

Rubi [A] time = 0.269156, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2772, 2773, 208}

$$-\frac{3a\cos(e+fx)}{4f(c+d)^2\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}-\frac{a\cos(e+fx)}{2f(c+d)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^2}-\frac{3\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}}{\sqrt{c+d}}\right.}{4\sqrt{d}f(c+}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]/(c + d*Sin[e + f*x])^3,x]
```


```
[Out] (-3*Sqrt[a]*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]*Sqrt[a + a*
Sin[e + f*x]])])/(4*Sqrt[d]*(c + d)^(5/2)*f) - (a*Cos[e + f*x])/(2*(c + d)*
f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^2) - (3*a*Cos[e + f*x])/(4*
(c + d)^2*f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x]))
```


Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sin(e+fx)}}{(c+d\sin(e+fx))^3}\,dx = -\frac{a\cos(e+fx)}{2(c+d)f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^2} + \frac{3\int \frac{\sqrt{a+a\sin(e+fx)}}{(c+d\sin(e+fx))^2}\,dx}{4(c+d)}$$

$$= -\frac{a\cos(e+fx)}{2(c+d)f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^2} - \frac{3a\cos(e+fx)}{4(c+d)^2 f\sqrt{a+a\sin(e+fx)}(c+d\,\mathrm{s}}$$

$$= -\frac{a\cos(e+fx)}{2(c+d)f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^2} - \frac{3a\cos(e+fx)}{4(c+d)^2 f\sqrt{a+a\sin(e+fx)}(c+d\,\mathrm{s}}$$

$$= -\frac{3\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{4\sqrt{d}(c+d)^{5/2}f} - \frac{a\cos(e+fx)}{2(c+d)f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^2} - \frac{}{4}$$

Mathematica [C] time = 7.24634, size = 920, normalized size = 5.97

$$\left(\frac{1}{16}+\frac{i}{16}\right)\sqrt{a(\sin(e+fx)+1)}\left(\frac{3\left(\cos\left(\frac{e}{2}\right)+i\sin\left(\frac{e}{2}\right)\right)\left((-1+i)x\cos(e)+(1+i)x\sin(e)+\frac{\text{RootSum}\left[de^{2ie}\#1^4+2ice^{ie}\#1^2-d\&,\frac{-\sqrt{d}\sqrt{c+d}e^{ie}fx\#1^3-2i\sqrt{d}\sqrt{c+d}e^{ie}\log\left(\right.}{}\right]}{}\right.}{\sqrt{}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]/(c + d*Sin[e + f*x])^3,x]
```

```
[Out] ((1/16 + I/16)*Sqrt[a*(1 + Sin[e + f*x])]*((3*(Cos[e/2] + I*Sin[e/2])*((-1
+ I)*x*Cos[e] + (RootSum[-d + (2*I)*c*E^(I*e)*#1^2 + d*E^((2*I)*e)*#1^4 & ,
 ((1 + I)*d*Sqrt[E^((-I)*e)]*f*x - (2 - 2*I)*d*Sqrt[E^((-I)*e)]*Log[E^((I/2
)*f*x) - #1] - I*Sqrt[d]*Sqrt[c + d]*f*x*#1 + 2*Sqrt[d]*Sqrt[c + d]*Log[E^(
(I/2)*f*x) - #1]*#1 + ((1 - I)*c*f*x*#1^2)/Sqrt[E^((-I)*e)] + ((2 + 2*I)*c*
Log[E^((I/2)*f*x) - #1]*#1^2)/Sqrt[E^((-I)*e)] - Sqrt[d]*Sqrt[c + d]*E^(I*e
)*f*x*#1^3 - (2*I)*Sqrt[d]*Sqrt[c + d]*E^(I*e)*Log[E^((I/2)*f*x) - #1]*#1^3
)/(d - I*c*E^(I*e)*#1^2) & ]*(Cos[e] + I*(-1 + Sin[e]))*Sqrt[Cos[e] - I*Sin
[e]])/(4*f) + (1 + I)*x*Sin[e]))/(Sqrt[d]*(c + d)^(5/2)*(Cos[e] + I*(-1 + S
in[e]))*Sqrt[Cos[e] - I*Sin[e]]) + (3*(Cos[e/2] + I*Sin[e/2])*((1 - I)*x*Co
s[e] - (1 + I)*x*Sin[e] + (RootSum[-d + (2*I)*c*E^(I*e)*#1^2 + d*E^((2*I)*e
)*#1^4 & , ((1 - I)*d*Sqrt[E^((-I)*e)]*f*x + (2 + 2*I)*d*Sqrt[E^((-I)*e)]*L
```

```
og[E^((I/2)*f*x) - #1] + Sqrt[d]*Sqrt[c + d]*f*x*#1 + (2*I)*Sqrt[d]*Sqrt[c
+ d]*Log[E^((I/2)*f*x) - #1]*#1 - ((1 + I)*c*f*x*#1^2)/Sqrt[E^((-I)*e)] + (
(2 - 2*I)*c*Log[E^((I/2)*f*x) - #1]*#1^2)/Sqrt[E^((-I)*e)] - I*Sqrt[d]*Sqrt
[c + d]*E^(I*e)*f*x*#1^3 + 2*Sqrt[d]*Sqrt[c + d]*E^(I*e)*Log[E^((I/2)*f*x)
- #1]*#1^3)/(d - I*c*E^(I*e)*#1^2) & ]*Sqrt[Cos[e] - I*Sin[e]]*(-1 - I*Cos[
e] + Sin[e]))/(4*f)))/(Sqrt[d]*(c + d)^(5/2)*(Cos[e] + I*(-1 + Sin[e]))*Sqr
t[Cos[e] - I*Sin[e]]) - ((4 - 4*I)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))/(
(c + d)*f*(c + d*Sin[e + f*x])^2) - ((6 - 6*I)*(Cos[(e + f*x)/2] - Sin[(e +
 f*x)/2]))/((c + d)^2*f*(c + d*Sin[e + f*x]))))/(Cos[(e + f*x)/2] + Sin[(e
+ f*x)/2])
```

Maple [A] time = 1., size = 254, normalized size = 1.7

$$-\frac{1+\sin\left(fx+e\right)}{4\left(c+d\right)^{2}\left(c+d\sin\left(fx+e\right)\right)^{2}\cos\left(fx+e\right)f}\sqrt{-a\left(-1+\sin\left(fx+e\right)\right)}\left(3\,\mathrm{Artanh}\left(\frac{\sqrt{-a\left(-1+\sin\left(fx+e\right)\right)}d}{\sqrt{a\left(c+d\right)d}}\right)\left(\sin\left(fx\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^3,x)
```

```
[Out] -1/4*(1+sin(f*x+e))*(-a*(-1+sin(f*x+e)))^(1/2)*(3*arctanh((-a*(-1+sin(f*x+e
)))^(1/2)*d/(a*(c+d)*d)^(1/2))*sin(f*x+e)^2*a*d^2+6*arctanh((-a*(-1+sin(f*x
+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*sin(f*x+e)*a*c*d+3*(-a*(-1+sin(f*x+e)))^(1
/2)*(a*(c+d)*d)^(1/2)*sin(f*x+e)*d+3*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(
a*(c+d)*d)^(1/2))*a*c^2+5*(-a*(-1+sin(f*x+e)))^(1/2)*(a*(c+d)*d)^(1/2)*c+2*
(-a*(-1+sin(f*x+e)))^(1/2)*(a*(c+d)*d)^(1/2)*d)/(c+d)^2/(c+d*sin(f*x+e))^2/
(a*(c+d)*d)^(1/2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a\sin\left(fx+e\right)+a}}{\left(d\sin\left(fx+e\right)+c\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)/(d*sin(f*x + e) + c)^3, x)
```

Fricas [B] time = 3.13945, size = 2890, normalized size = 18.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] [1/16*(3*(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^2 - 2*c*d -
 d^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - 2*c*d*cos(f*x + e)
```

```
- c^2 - 2*c*d - d^2)*sin(f*x + e))*sqrt(a/(c*d + d^2))*log((a*d^2*cos(f*x +
 e)^3 - a*c^2 - 2*a*c*d - a*d^2 - (6*a*c*d + 7*a*d^2)*cos(f*x + e)^2 + 4*(c
^2*d + 4*c*d^2 + 3*d^3 - (c*d^2 + d^3)*cos(f*x + e)^2 + (c^2*d + 3*c*d^2 +
2*d^3)*cos(f*x + e) - (c^2*d + 4*c*d^2 + 3*d^3 + (c*d^2 + d^3)*cos(f*x + e)
)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(a/(c*d + d^2)) - (a*c^2 + 8*a
*c*d + 9*a*d^2)*cos(f*x + e) + (a*d^2*cos(f*x + e)^2 - a*c^2 - 2*a*c*d - a*
d^2 + 2*(3*a*c*d + 4*a*d^2)*cos(f*x + e))*sin(f*x + e))/(d^2*cos(f*x + e)^3
 + (2*c*d + d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 - (c^2 + d^2)*cos(f*x +
 e) + (d^2*cos(f*x + e)^2 - 2*c*d*cos(f*x + e) - c^2 - 2*c*d - d^2)*sin(f*x
 + e))) + 4*(3*d*cos(f*x + e)^2 + (5*c + 2*d)*cos(f*x + e) + (3*d*cos(f*x +
 e) - 5*c + d)*sin(f*x + e) + 5*c - d)*sqrt(a*sin(f*x + e) + a))/((c^2*d^2
+ 2*c*d^3 + d^4)*f*cos(f*x + e)^3 + (2*c^3*d + 5*c^2*d^2 + 4*c*d^3 + d^4)*f
*cos(f*x + e)^2 - (c^4 + 2*c^3*d + 2*c^2*d^2 + 2*c*d^3 + d^4)*f*cos(f*x + e
) - (c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^3 + d^4)*f + ((c^2*d^2 + 2*c*d^3 + d
^4)*f*cos(f*x + e)^2 - 2*(c^3*d + 2*c^2*d^2 + c*d^3)*f*cos(f*x + e) - (c^4
+ 4*c^3*d + 6*c^2*d^2 + 4*c*d^3 + d^4)*f)*sin(f*x + e)), -1/8*(3*(d^2*cos(f
*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 - (c^2 + d^2)*
cos(f*x + e) + (d^2*cos(f*x + e)^2 - 2*c*d*cos(f*x + e) - c^2 - 2*c*d - d^2
)*sin(f*x + e))*sqrt(-a/(c*d + d^2))*arctan(1/2*sqrt(a*sin(f*x + e) + a)*(d
*sin(f*x + e) - c - 2*d)*sqrt(-a/(c*d + d^2))/(a*cos(f*x + e))) - 2*(3*d*co
s(f*x + e)^2 + (5*c + 2*d)*cos(f*x + e) + (3*d*cos(f*x + e) - 5*c + d)*sin(
f*x + e) + 5*c - d)*sqrt(a*sin(f*x + e) + a))/((c^2*d^2 + 2*c*d^3 + d^4)*f*
cos(f*x + e)^3 + (2*c^3*d + 5*c^2*d^2 + 4*c*d^3 + d^4)*f*cos(f*x + e)^2 - (
c^4 + 2*c^3*d + 2*c^2*d^2 + 2*c*d^3 + d^4)*f*cos(f*x + e) - (c^4 + 4*c^3*d
+ 6*c^2*d^2 + 4*c*d^3 + d^4)*f + ((c^2*d^2 + 2*c*d^3 + d^4)*f*cos(f*x + e)^
2 - 2*(c^3*d + 2*c^2*d^2 + c*d^3)*f*cos(f*x + e) - (c^4 + 4*c^3*d + 6*c^2*d
^2 + 4*c*d^3 + d^4)*f)*sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(1/2)/(c+d*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.529 $\int (a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^3\,dx$

Optimal. Leaf size=231

$$\frac{4a^2(c-17d)(c+d)\left(15c^2+10cd+7d^2\right)\cos(e+fx)}{315df\sqrt{a\sin(e+fx)+a}}-\frac{2a^2\cos(e+fx)(c+d\sin(e+fx))^4}{9df\sqrt{a\sin(e+fx)+a}}+\frac{2a^2(c-17d)\cos(e+fx)(c+}{63df\sqrt{a\sin(e+f}}$$

```
[Out] (4*a^2*(c - 17*d)*(c + d)*(15*c^2 + 10*c*d + 7*d^2)*Cos[e + f*x])/(315*d*f*
Sqrt[a + a*Sin[e + f*x]]) + (8*a*(c - 17*d)*(5*c - d)*(c + d)*Cos[e + f*x]*
Sqrt[a + a*Sin[e + f*x]])/(315*f) + (4*(c - 17*d)*d*(c + d)*Cos[e + f*x]*(a
 + a*Sin[e + f*x])^(3/2))/(105*f) + (2*a^2*(c - 17*d)*Cos[e + f*x]*(c + d*S
in[e + f*x])^3)/(63*d*f*Sqrt[a + a*Sin[e + f*x]]) - (2*a^2*Cos[e + f*x]*(c
+ d*Sin[e + f*x])^4)/(9*d*f*Sqrt[a + a*Sin[e + f*x]])
```

Rubi [A] time = 0.384428, antiderivative size = 231, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2763, 21, 2770, 2761, 2751, 2646}

$$\frac{4a^2(c-17d)(c+d)\left(15c^2+10cd+7d^2\right)\cos(e+fx)}{315df\sqrt{a\sin(e+fx)+a}}-\frac{2a^2\cos(e+fx)(c+d\sin(e+fx))^4}{9df\sqrt{a\sin(e+fx)+a}}+\frac{2a^2(c-17d)\cos(e+fx)(c+}{63df\sqrt{a\sin(e+f}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^3,x]
```


```
[Out] (4*a^2*(c - 17*d)*(c + d)*(15*c^2 + 10*c*d + 7*d^2)*Cos[e + f*x])/(315*d*f*
Sqrt[a + a*Sin[e + f*x]]) + (8*a*(c - 17*d)*(5*c - d)*(c + d)*Cos[e + f*x]*
Sqrt[a + a*Sin[e + f*x]])/(315*f) + (4*(c - 17*d)*d*(c + d)*Cos[e + f*x]*(a
 + a*Sin[e + f*x])^(3/2))/(105*f) + (2*a^2*(c - 17*d)*Cos[e + f*x]*(c + d*S
in[e + f*x])^3)/(63*d*f*Sqrt[a + a*Sin[e + f*x]]) - (2*a^2*Cos[e + f*x]*(c
+ d*Sin[e + f*x])^4)/(9*d*f*Sqrt[a + a*Sin[e + f*x]])
```


Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
```

```
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2761

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^2, x_Symbol] :> -Simp[(d^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m
 + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*S
imp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^3\,dx = -\frac{2a^2\cos(e+fx)(c+d\sin(e+fx))^4}{9df\sqrt{a+a\sin(e+fx)}} + \frac{2\int \frac{\left(-\frac{1}{2}a^2(c-17d)-\frac{1}{2}a^2(c-17d)\sin(e\cdots\right.}{\sqrt{a+a\sin(e+\cdots}}}{9d}$$

$$= -\frac{2a^2\cos(e+fx)(c+d\sin(e+fx))^4}{9df\sqrt{a+a\sin(e+fx)}} - \frac{(a(c-17d))\int\sqrt{a+a\sin(e+\cdots}}{9d}$$

$$= \frac{2a^2(c-17d)\cos(e+fx)(c+d\sin(e+fx))^3}{63df\sqrt{a+a\sin(e+fx)}} - \frac{2a^2\cos(e+fx)(c+d\cdots}{9df\sqrt{a+a\sin(\cdots}}$$

$$= \frac{4(c-17d)d(c+d)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{105f} + \frac{2a^2(c-17d)\,\mathrm{c}\cdots}{63d\cdots}$$

$$= \frac{8a(c-17d)(5c-d)(c+d)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{315f} + \frac{4(c-17\cdots}{\cdots}$$

$$= \frac{4a^2(c-17d)(c+d)\left(15c^2+10cd+7d^2\right)\cos(e+fx)}{315df\sqrt{a+a\sin(e+fx)}} + \frac{8a(c-17d)(5\cdots}{\cdots}$$

Mathematica [A] time = 1.67798, size = 203, normalized size = 0.88

$$-\frac{a\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(-4d\left(189c^2+351cd+137d^2\right)\cos(2(e+fx))+4536c^2d\,\mathrm{si}\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^3,x]
```

```
[Out] -(a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(4200*
c^3 + 9828*c^2*d + 8892*c*d^2 + 2689*d^3 - 4*d*(189*c^2 + 351*c*d + 137*d^2
)*Cos[2*(e + f*x)] + 35*d^3*Cos[4*(e + f*x)] + 840*c^3*Sin[e + f*x] + 4536*
c^2*d*Sin[e + f*x] + 4554*c*d^2*Sin[e + f*x] + 1598*d^3*Sin[e + f*x] - 270*
c*d^2*Sin[3*(e + f*x)] - 170*d^3*Sin[3*(e + f*x)]))/(1260*f*(Cos[(e + f*x)/
2] + Sin[(e + f*x)/2]))
```

Maple [A] time = 0.596, size = 195, normalized size = 0.8

$$\frac{\left(2+2\sin\left(fx+e\right)\right)a^2\left(-1+\sin\left(fx+e\right)\right)\left(35\,d^3\left(\sin\left(fx+e\right)\right)^4+135\,cd^2\left(\sin\left(fx+e\right)\right)^3+85\,d^3\left(\sin\left(fx+e\right)\right)^3+189\,\cdots\right)}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^3,x)
```

```
[Out] 2/315*(1+sin(f*x+e))*a^2*(-1+sin(f*x+e))*(35*d^3*sin(f*x+e)^4+135*c*d^2*sin
(f*x+e)^3+85*d^3*sin(f*x+e)^3+189*c^2*d*sin(f*x+e)^2+351*c*d^2*sin(f*x+e)^2
+102*d^3*sin(f*x+e)^2+105*c^3*sin(f*x+e)+567*c^2*d*sin(f*x+e)+468*sin(f*x+e
)*d^2*c+136*d^3*sin(f*x+e)+525*c^3+1134*c^2*d+936*c*d^2+272*d^3)/cos(f*x+e)
/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(d\sin\left(fx+e\right)+c\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e) + c)^3, x)
```

Fricas [A] time = 1.73393, size = 872, normalized size = 3.77

$$-\frac{2\left(35\,ad^3\cos\left(fx+e\right)^5-5\left(27\,acd^2+10\,ad^3\right)\cos\left(fx+e\right)^4+420\,ac^3+756\,ac^2d+684\,acd^2+188\,ad^3-\left(189\,ac^2d+35\cdots\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -2/315*(35*a*d^3*cos(f*x + e)^5 - 5*(27*a*c*d^2 + 10*a*d^3)*cos(f*x + e)^4
+ 420*a*c^3 + 756*a*c^2*d + 684*a*c*d^2 + 188*a*d^3 - (189*a*c^2*d + 351*a*
c*d^2 + 172*a*d^3)*cos(f*x + e)^3 + (105*a*c^3 + 378*a*c^2*d + 387*a*c*d^2
+ 134*a*d^3)*cos(f*x + e)^2 + (525*a*c^3 + 1323*a*c^2*d + 1287*a*c*d^2 + 40
9*a*d^3)*cos(f*x + e) - (35*a*d^3*cos(f*x + e)^4 + 420*a*c^3 + 756*a*c^2*d
+ 684*a*c*d^2 + 188*a*d^3 + 5*(27*a*c*d^2 + 17*a*d^3)*cos(f*x + e)^3 - 3*(6
3*a*c^2*d + 72*a*c*d^2 + 29*a*d^3)*cos(f*x + e)^2 - (105*a*c^3 + 567*a*c^2*
d + 603*a*c*d^2 + 221*a*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e
```

```
) + a)/(f*cos(f*x + e) + f*sin(f*x + e) + f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*(c+d*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] Timed out
```

3.530 $\int (a + a\sin(e + fx))^{3/2}(c + d\sin(e + fx))^2\,dx$

Optimal. Leaf size=157

$$-\frac{8a^2\left(35c^2+42cd+19d^2\right)\cos(e+fx)}{105f\sqrt{a\sin(e+fx)+a}}-\frac{2a\left(35c^2+42cd+19d^2\right)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{105f}-\frac{4d(7c-d)\cos(e+f\ldots}{3\ldots}$$

```
[Out] (-8*a^2*(35*c^2 + 42*c*d + 19*d^2)*Cos[e + f*x])/(105*f*Sqrt[a + a*Sin[e +
f*x]]) - (2*a*(35*c^2 + 42*c*d + 19*d^2)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*
x]])/(105*f) - (4*(7*c - d)*d*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(35*
f) - (2*d^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(7*a*f)
```

Rubi [A] time = 0.229463, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2761, 2751, 2647, 2646}

$$-\frac{8a^2\left(35c^2+42cd+19d^2\right)\cos(e+fx)}{105f\sqrt{a\sin(e+fx)+a}}-\frac{2a\left(35c^2+42cd+19d^2\right)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{105f}-\frac{4d(7c-d)\cos(e+f\ldots}{3\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^2,x]
```


```
[Out] (-8*a^2*(35*c^2 + 42*c*d + 19*d^2)*Cos[e + f*x])/(105*f*Sqrt[a + a*Sin[e +
f*x]]) - (2*a*(35*c^2 + 42*c*d + 19*d^2)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*
x]])/(105*f) - (4*(7*c - d)*d*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(35*
f) - (2*d^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(7*a*f)
```


Rule 2761

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^2, x_Symbol] :> -Simp[(d^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m
 + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*S
imp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
```

```
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^2\,dx = -\frac{2d^2\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{7af} + \frac{2\int (a+a\sin(e+fx))^{3/2}\left(\frac{1}{2}\right.}{}$$

$$= -\frac{4(7c-d)d\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{35f} - \frac{2d^2\cos(e+fx)(a+}{7af}$$

$$= -\frac{2a\left(35c^2+42cd+19d^2\right)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{105f} - \frac{4(7c-d)}{}$$

$$= -\frac{8a^2\left(35c^2+42cd+19d^2\right)\cos(e+fx)}{105f\sqrt{a+a\sin(e+fx)}} - \frac{2a\left(35c^2+42cd+19d^2\right)\cos}{10}$$

Mathematica [A] time = 0.875155, size = 136, normalized size = 0.87

$$-\frac{a\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\left(140c^2+504cd+253d^2\right)\sin(e+fx)+700c^2-6d(14c+\right.}{210f\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^2,x]
```

```
[Out] -(a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(700*c
^2 + 1092*c*d + 494*d^2 - 6*d*(14*c + 13*d)*Cos[2*(e + f*x)] + (140*c^2 + 5
04*c*d + 253*d^2)*Sin[e + f*x] - 15*d^2*Sin[3*(e + f*x)]))/(210*f*(Cos[(e +
 f*x)/2] + Sin[(e + f*x)/2]))
```

Maple [A] time = 0.668, size = 130, normalized size = 0.8

$$\frac{\left(2+2\sin\left(fx+e\right)\right)a^2\left(-1+\sin\left(fx+e\right)\right)\left(15\,d^2\left(\sin\left(fx+e\right)\right)^3+42\,cd\left(\sin\left(fx+e\right)\right)^2+39\,d^2\left(\sin\left(fx+e\right)\right)^2+35\,\right.}{105\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^2,x)
```

```
[Out] 2/105*(1+sin(f*x+e))*a^2*(-1+sin(f*x+e))*(15*d^2*sin(f*x+e)^3+42*c*d*sin(f*
x+e)^2+39*d^2*sin(f*x+e)^2+35*c^2*sin(f*x+e)+126*sin(f*x+e)*c*d+52*sin(f*x+
e)*d^2+175*c^2+252*c*d+104*d^2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(d\sin\left(fx+e\right)+c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e) + c)^2, x)
```

Fricas [A] time = 1.62153, size = 590, normalized size = 3.76

$$\frac{2\left(15\,ad^2\cos\left(fx+e\right)^4+3\left(14\,acd+13\,ad^2\right)\cos\left(fx+e\right)^3-140\,ac^2-168\,acd-76\,ad^2-\left(35\,ac^2+84\,acd+43\,ad^2\right)\cos\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 2/105*(15*a*d^2*cos(f*x + e)^4 + 3*(14*a*c*d + 13*a*d^2)*cos(f*x + e)^3 - 1
40*a*c^2 - 168*a*c*d - 76*a*d^2 - (35*a*c^2 + 84*a*c*d + 43*a*d^2)*cos(f*x
+ e)^2 - (175*a*c^2 + 294*a*c*d + 143*a*d^2)*cos(f*x + e) + (15*a*d^2*cos(f
*x + e)^3 + 140*a*c^2 + 168*a*c*d + 76*a*d^2 - 6*(7*a*c*d + 4*a*d^2)*cos(f*
x + e)^2 - (35*a*c^2 + 126*a*c*d + 67*a*d^2)*cos(f*x + e))*sin(f*x + e))*sq
rt(a*sin(f*x + e) + a)/(f*cos(f*x + e) + f*sin(f*x + e) + f)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}\left(c+d\sin\left(e+fx\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*(c+d*sin(f*x+e))**2,x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**(3/2)*(c + d*sin(e + f*x))**2, x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.531 $\int (a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))\,dx$

Optimal. Leaf size=101

$$-\frac{8a^2(5c+3d)\cos(e+fx)}{15f\sqrt{a\sin(e+fx)+a}}-\frac{2a(5c+3d)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{15f}-\frac{2d\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5f}$$

```
[Out] (-8*a^2*(5*c + 3*d)*Cos[e + f*x])/(15*f*Sqrt[a + a*Sin[e + f*x]]) - (2*a*(5
*c + 3*d)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(15*f) - (2*d*Cos[e + f*x]
*(a + a*Sin[e + f*x])^(3/2))/(5*f)
```

Rubi [A] time = 0.0842337, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2751, 2647, 2646}

$$-\frac{8a^2(5c+3d)\cos(e+fx)}{15f\sqrt{a\sin(e+fx)+a}}-\frac{2a(5c+3d)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{15f}-\frac{2d\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x]),x]
```



```
[Out] (-8*a^2*(5*c + 3*d)*Cos[e + f*x])/(15*f*Sqrt[a + a*Sin[e + f*x]]) - (2*a*(5
*c + 3*d)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(15*f) - (2*d*Cos[e + f*x]
*(a + a*Sin[e + f*x])^(3/2))/(5*f)
```


Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))\,dx = -\frac{2d\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{5f}+\frac{1}{5}(5c+3d)\int (a+a\sin(e+fx))^{3/2}$$
$$= -\frac{2a(5c+3d)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{15f}-\frac{2d\cos(e+fx)(a+a\sin(e$$
$$= -\frac{8a^2(5c+3d)\cos(e+fx)}{15f\sqrt{a+a\sin(e+fx)}}-\frac{2a(5c+3d)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{15f}$$

Mathematica [A] time = 0.411252, size = 101, normalized size = 1.

$$-\frac{a\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(2(5c+9d)\sin(e+fx)+50c-3d\cos(2(e+fx))+39d)}{15f\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x]),x]
```

```
[Out] -(a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(50*c
+ 39*d - 3*d*Cos[2*(e + f*x)] + 2*(5*c + 9*d)*Sin[e + f*x]))/(15*f*(Cos[(e
+ f*x)/2] + Sin[(e + f*x)/2]))
```

Maple [A] time = 0.665, size = 77, normalized size = 0.8

$$\frac{(2+2\sin(fx+e))a^2(-1+\sin(fx+e))\left(\sin(fx+e)(5c+9d)-3(\cos(fx+e))^2d+25c+21d\right)}{15f\cos(fx+e)}\frac{1}{\sqrt{a+a\sin(fx+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e)),x)
```

```
[Out] 2/15*(1+sin(f*x+e))*a^2*(-1+sin(f*x+e))*(sin(f*x+e)*(5*c+9*d)-3*cos(f*x+e)^
2*d+25*c+21*d)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(fx+e)+a)^{\frac{3}{2}}(d\sin(fx+e)+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e) + c), x)
```

Fricas [A] time = 1.56504, size = 355, normalized size = 3.51

$$\frac{2\left(3ad\cos(fx+e)^3-(5ac+6ad)\cos(fx+e)^2-20ac-12ad-(25ac+21ad)\cos(fx+e)-\left(3ad\cos(fx+e)^2-2\right.\right.}{15\left(f\cos(fx+e)+f\sin(fx+e)+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] 2/15*(3*a*d*cos(f*x + e)^3 - (5*a*c + 6*a*d)*cos(f*x + e)^2 - 20*a*c - 12*a
*d - (25*a*c + 21*a*d)*cos(f*x + e) - (3*a*d*cos(f*x + e)^2 - 20*a*c - 12*a
*d + (5*a*c + 9*a*d)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)/(
f*cos(f*x + e) + f*sin(f*x + e) + f)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a \left(\sin \left(e + fx\right) + 1\right)\right)^{\frac{3}{2}} \left(c + d \sin \left(e + fx\right)\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*(c+d*sin(f*x+e)),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**(3/2)*(c + d*sin(e + f*x)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.532 $\int (a+a\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=59

$$-\frac{8a^2\cos(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}-\frac{2a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}$$

```
[Out] (-8*a^2*Cos[e + f*x])/(3*f*Sqrt[a + a*Sin[e + f*x]]) - (2*a*Cos[e + f*x]*Sq
rt[a + a*Sin[e + f*x]])/(3*f)
```

Rubi [A] time = 0.0294619, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2647, 2646}

$$-\frac{8a^2\cos(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}-\frac{2a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-8*a^2*Cos[e + f*x])/(3*f*Sqrt[a + a*Sin[e + f*x]]) - (2*a*Cos[e + f*x]*Sq
rt[a + a*Sin[e + f*x]])/(3*f)
```

Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int (a+a\sin(e+fx))^{3/2}\,dx &= -\frac{2a\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3f}+\frac{1}{3}(4a)\int \sqrt{a+a\sin(e+fx)}\,dx\\ &= -\frac{8a^2\cos(e+fx)}{3f\sqrt{a+a\sin(e+fx)}}-\frac{2a\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3f}\end{aligned}$$

Mathematica [A] time = 0.142243, size = 89, normalized size = 1.51

$$-\frac{(a(\sin(e+fx)+1))^{3/2}\left(-9\sin\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{3}{2}(e+fx)\right)+9\cos\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{3}{2}(e+fx)\right)\right)}{3f\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] -((a*(1 + Sin[e + f*x]))^(3/2)*(9*Cos[(e + f*x)/2] + Cos[(3*(e + f*x))/2] -
 9*Sin[(e + f*x)/2] + Sin[(3*(e + f*x))/2]))/(3*f*(Cos[(e + f*x)/2] + Sin[(
e + f*x)/2])^3)
```

Maple [A] time = 0.46, size = 53, normalized size = 0.9

$$\frac{\left(2+2\sin\left(fx+e\right)\right)a^{2}\left(-1+\sin\left(fx+e\right)\right)\left(\sin\left(fx+e\right)+5\right)}{3f\cos\left(fx+e\right)}\frac{1}{\sqrt{a+a\sin\left(fx+e\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2),x)
```

```
[Out] 2/3*(1+sin(f*x+e))*a^2*(-1+sin(f*x+e))*(sin(f*x+e)+5)/cos(f*x+e)/(a+a*sin(f
*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2), x)
```

Fricas [A] time = 1.51968, size = 204, normalized size = 3.46

$$-\frac{2\left(a\cos\left(fx+e\right)^{2}+5\,a\cos\left(fx+e\right)+\left(a\cos\left(fx+e\right)-4\,a\right)\sin\left(fx+e\right)+4\,a\right)\sqrt{a\sin\left(fx+e\right)+a}}{3\left(f\cos\left(fx+e\right)+f\sin\left(fx+e\right)+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] -2/3*(a*cos(f*x + e)^2 + 5*a*cos(f*x + e) + (a*cos(f*x + e) - 4*a)*sin(f*x
+ e) + 4*a)*sqrt(a*sin(f*x + e) + a)/(f*cos(f*x + e) + f*sin(f*x + e) + f)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(e+fx\right)+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral((a*sin(e + f*x) + a)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2), x)
```

3.533 $\int \frac{(a+a\sin(e+fx))^{3/2}}{c+d\sin(e+fx)}\,dx$

Optimal. Leaf size=98

$$\frac{2a^{3/2}(c-d)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{d^{3/2}f\sqrt{c+d}}-\frac{2a^2\cos(e+fx)}{df\sqrt{a\sin(e+fx)+a}}$$

```
[Out] (2*a^(3/2)*(c - d)*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]*Sqrt
[a + a*Sin[e + f*x]])])/(d^(3/2)*Sqrt[c + d]*f) - (2*a^2*Cos[e + f*x])/(d*f
*Sqrt[a + a*Sin[e + f*x]])
```

Rubi [A] time = 0.1998, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2763, 21, 2773, 208}

$$\frac{2a^{3/2}(c-d)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{d^{3/2}f\sqrt{c+d}}-\frac{2a^2\cos(e+fx)}{df\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)/(c + d*Sin[e + f*x]),x]
```

```
[Out] (2*a^(3/2)*(c - d)*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]*Sqrt
[a + a*Sin[e + f*x]])])/(d^(3/2)*Sqrt[c + d]*f) - (2*a^2*Cos[e + f*x])/(d*f
*Sqrt[a + a*Sin[e + f*x]])
```

Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{3/2}}{c+d\sin(e+fx)}\,dx = -\frac{2a^2\cos(e+fx)}{df\sqrt{a+a\sin(e+fx)}}+\frac{2\int \frac{-\frac{1}{2}a^2(c-d)-\frac{1}{2}a^2(c-d)\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))}\,dx}{d}$$
$$= -\frac{2a^2\cos(e+fx)}{df\sqrt{a+a\sin(e+fx)}}-\frac{(a(c-d))\int \frac{\sqrt{a+a\sin(e+fx)}}{c+d\sin(e+fx)}\,dx}{d}$$
$$= -\frac{2a^2\cos(e+fx)}{df\sqrt{a+a\sin(e+fx)}}+\frac{\left(2a^2(c-d)\right)\operatorname{Subst}\left(\int \frac{1}{ac+ad-dx^2}\,dx,x,\frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{df}$$
$$= \frac{2a^{3/2}(c-d)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{d^{3/2}\sqrt{c+d}f}-\frac{2a^2\cos(e+fx)}{df\sqrt{a+a\sin(e+fx)}}$$

Mathematica [B] time = 2.20081, size = 233, normalized size = 2.38

$$\frac{(a(\sin(e+fx)+1))^{3/2}\left(2\sqrt{d}\sqrt{c+d}\sin\left(\frac{1}{2}(e+fx)\right)-2\sqrt{d}\sqrt{c+d}\cos\left(\frac{1}{2}(e+fx)\right)+(c-d)\left(\log\left(-\sec^2\left(\frac{1}{4}(e+fx)\right)\left(-\sqrt{d}\right.\right.\right.\right.}{d^{3/2}f\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)/(c + d*Sin[e + f*x]),x]
```

```
[Out] ((-2*Sqrt[d]*Sqrt[c + d]*Cos[(e + f*x)/2] + (c - d)*(Log[-(Sec[(e + f*x)/4]
^2*(c + d + Sqrt[d]*Sqrt[c + d]*Cos[(e + f*x)/2] - Sqrt[d]*Sqrt[c + d]*Sin[
(e + f*x)/2]))] - Log[(c + d)*Sec[(e + f*x)/4]^2 + Sqrt[d]*Sqrt[c + d]*(-1
+ 2*Tan[(e + f*x)/4] + Tan[(e + f*x)/4]^2)]) + 2*Sqrt[d]*Sqrt[c + d]*Sin[(e
 + f*x)/2])*(a*(1 + Sin[e + f*x]))^(3/2))/(d^(3/2)*Sqrt[c + d]*f*(Cos[(e +
f*x)/2] + Sin[(e + f*x)/2])^3)
```

Maple [A] time = 0.934, size = 137, normalized size = 1.4

$$-2\,\frac{a\left(1+\sin\left(fx+e\right)\right)\sqrt{-a\left(-1+\sin\left(fx+e\right)\right)}}{d\sqrt{a\left(c+d\right)d}\cos\left(fx+e\right)\sqrt{a+a\sin\left(fx+e\right)}f}\left(-\mathrm{Artanh}\left(\frac{\sqrt{-a\left(-1+\sin\left(fx+e\right)\right)}d}{\sqrt{a\left(c+d\right)d}}\right)ac+a\mathrm{Artanh}\left(\frac{\sqrt{-a\left(-1+\mathrm{si}\right.}}{\sqrt{a\left(c\right.}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e)),x)
```

```
[Out] -2*a*(1+sin(f*x+e))*(-a*(-1+sin(f*x+e)))^(1/2)*(-arctanh((-a*(-1+sin(f*x+e)
))^(1/2)*d/(a*(c+d)*d)^(1/2))*a*c+a*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a
*(c+d)*d)^(1/2))*d+(-a*(-1+sin(f*x+e)))^(1/2)*(a*(c+d)*d)^(1/2))/d/(a*(c+d)
*d)^(1/2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}{d\sin\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)/(d*sin(f*x + e) + c), x)
```

Fricas [B] time = 2.34015, size = 1539, normalized size = 15.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] [-1/2*((a*c - a*d + (a*c - a*d)*cos(f*x + e) + (a*c - a*d)*sin(f*x + e))*sq
rt(a/(c*d + d^2))*log((a*d^2*cos(f*x + e)^3 - a*c^2 - 2*a*c*d - a*d^2 - (6*
a*c*d + 7*a*d^2)*cos(f*x + e)^2 + 4*(c^2*d + 4*c*d^2 + 3*d^3 - (c*d^2 + d^3
)*cos(f*x + e)^2 + (c^2*d + 3*c*d^2 + 2*d^3)*cos(f*x + e) - (c^2*d + 4*c*d^
2 + 3*d^3 + (c*d^2 + d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) +
 a)*sqrt(a/(c*d + d^2)) - (a*c^2 + 8*a*c*d + 9*a*d^2)*cos(f*x + e) + (a*d^2
*cos(f*x + e)^2 - a*c^2 - 2*a*c*d - a*d^2 + 2*(3*a*c*d + 4*a*d^2)*cos(f*x +
 e))*sin(f*x + e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^2
 - 2*c*d - d^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - 2*c*d*cos
(f*x + e) - c^2 - 2*c*d - d^2)*sin(f*x + e))) + 4*(a*cos(f*x + e) - a*sin(f
*x + e) + a)*sqrt(a*sin(f*x + e) + a))/(d*f*cos(f*x + e) + d*f*sin(f*x + e)
 + d*f), ((a*c - a*d + (a*c - a*d)*cos(f*x + e) + (a*c - a*d)*sin(f*x + e))
*sqrt(-a/(c*d + d^2))*arctan(1/2*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) -
 c - 2*d)*sqrt(-a/(c*d + d^2))/(a*cos(f*x + e))) - 2*(a*cos(f*x + e) - a*si
n(f*x + e) + a)*sqrt(a*sin(f*x + e) + a))/(d*f*cos(f*x + e) + d*f*sin(f*x +
 e) + d*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)/(c+d*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.534 $\int \frac{(a+a\sin(e+fx))^{3/2}}{(c+d\sin(e+fx))^2}\,dx$

Optimal. Leaf size=119

$$\frac{a^2(c-d)\cos(e+fx)}{df(c+d)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}-\frac{a^{3/2}(c+3d)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{d^{3/2}f(c+d)^{3/2}}$$

```
[Out] -((a^(3/2)*(c + 3*d)*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]*Sq
rt[a + a*Sin[e + f*x]])])/(d^(3/2)*(c + d)^(3/2)*f)) + (a^2*(c - d)*Cos[e +
 f*x])/(d*(c + d)*f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x]))
```

Rubi [A] time = 0.192916, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2762, 21, 2773, 208}

$$\frac{a^2(c-d)\cos(e+fx)}{df(c+d)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}-\frac{a^{3/2}(c+3d)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{d^{3/2}f(c+d)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)/(c + d*Sin[e + f*x])^2,x]
```

```
[Out] -((a^(3/2)*(c + 3*d)*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]*Sq
rt[a + a*Sin[e + f*x]])])/(d^(3/2)*(c + d)^(3/2)*f)) + (a^2*(c - d)*Cos[e +
 f*x])/(d*(c + d)*f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x]))
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{3/2}}{(c+d\sin(e+fx))^2}\,dx = \frac{a^2(c-d)\cos(e+fx)}{d(c+d)f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))} - \frac{a\int \frac{-\frac{1}{2}a(c+3d)-\frac{1}{2}a(c+3d)\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))}\,dx}{d(c+d)}$$

$$= \frac{a^2(c-d)\cos(e+fx)}{d(c+d)f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))} + \frac{(a(c+3d))\int \frac{\sqrt{a+a\sin(e+fx)}}{c+d\sin(e+fx)}\,dx}{2d(c+d)}$$

$$= \frac{a^2(c-d)\cos(e+fx)}{d(c+d)f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))} - \frac{\left(a^2(c+3d)\right)\mathrm{Subst}\left(\int \frac{1}{ac+ad-dx^2}\,dx, x, \frac{a}{\sqrt{a\cdot}}\right.}{d(c+d)f}$$

$$= -\frac{a^{3/2}(c+3d)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{d^{3/2}(c+d)^{3/2}f} + \frac{a^2(c-d)\cos(e+fx)}{d(c+d)f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))}$$

Mathematica [B] time = 2.35363, size = 268, normalized size = 2.25

$$-\frac{(a(\sin(e+fx)+1))^{3/2}\left(2\sqrt{d}(c-d)\sqrt{c+d}\sin\left(\frac{1}{2}(e+fx)\right)-2\sqrt{d}(c-d)\sqrt{c+d}\cos\left(\frac{1}{2}(e+fx)\right)+(c+3d)(c+d\sin(e+\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)/(c + d*Sin[e + f*x])^2,x]
```

```
[Out] -((a*(1 + Sin[e + f*x]))^(3/2)*(-2*(c - d)*Sqrt[d]*Sqrt[c + d]*Cos[(e + f*x
)/2] + 2*(c - d)*Sqrt[d]*Sqrt[c + d]*Sin[(e + f*x)/2] + (c + 3*d)*(Log[-(Se
c[(e + f*x)/4]^2*(c + d + Sqrt[d]*Sqrt[c + d]*Cos[(e + f*x)/2] - Sqrt[d]*Sq
rt[c + d]*Sin[(e + f*x)/2]))] - Log[(c + d)*Sec[(e + f*x)/4]^2 + Sqrt[d]*Sq
rt[c + d]*(-1 + 2*Tan[(e + f*x)/4] + Tan[(e + f*x)/4]^2)])*(c + d*Sin[e + f
*x])))/(2*d^(3/2)*(c + d)^(3/2)*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*(
c + d*Sin[e + f*x]))
```

Maple [B] time = 1.197, size = 233, normalized size = 2.

$$\frac{a\left(1+\sin\left(fx+e\right)\right)}{d\left(c+d\right)\left(c+d\sin\left(fx+e\right)\right)\cos\left(fx+e\right)f}\sqrt{-a\left(-1+\sin\left(fx+e\right)\right)}\left(-\sin\left(fx+e\right)\mathrm{Artanh}\left(d\sqrt{a-a\sin\left(fx+e\right)}\frac{1}{\sqrt{acd}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^2,x)
```

```
[Out] a*(1+sin(f*x+e))*(-a*(-1+sin(f*x+e)))^(1/2)*(-sin(f*x+e)*arctanh((a-a*sin(f
*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a*d*(c+3*d)-arctanh((a-a*sin(f*x+e))^(1
/2)*d/(a*c*d+a*d^2)^(1/2))*a*c^2-3*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+
a*d^2)^(1/2))*a*c*d+(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*c-(a-a*sin(f*x
+e))^(1/2)*(a*(c+d)*d)^(1/2)*d)/d/(c+d)/(c+d*sin(f*x+e))/(a*(c+d)*d)^(1/2)/
```

```
cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\sin(fx+e)+a)^{\frac{3}{2}}}{(d\sin(fx+e)+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)/(d*sin(f*x + e) + c)^2, x)
```

Fricas [B] time = 2.60626, size = 2241, normalized size = 18.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] [-1/4*((a*c^2 + 4*a*c*d + 3*a*d^2 - (a*c*d + 3*a*d^2)*cos(f*x + e)^2 + (a*c
^2 + 3*a*c*d)*cos(f*x + e) + (a*c^2 + 4*a*c*d + 3*a*d^2 + (a*c*d + 3*a*d^2)
*cos(f*x + e))*sin(f*x + e))*sqrt(a/(c*d + d^2))*log((a*d^2*cos(f*x + e)^3
- a*c^2 - 2*a*c*d - a*d^2 - (6*a*c*d + 7*a*d^2)*cos(f*x + e)^2 + 4*(c^2*d +
 4*c*d^2 + 3*d^3 - (c*d^2 + d^3)*cos(f*x + e)^2 + (c^2*d + 3*c*d^2 + 2*d^3)
*cos(f*x + e) - (c^2*d + 4*c*d^2 + 3*d^3 + (c*d^2 + d^3)*cos(f*x + e))*sin(
f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(a/(c*d + d^2)) - (a*c^2 + 8*a*c*d +
 9*a*d^2)*cos(f*x + e) + (a*d^2*cos(f*x + e)^2 - a*c^2 - 2*a*c*d - a*d^2 +
2*(3*a*c*d + 4*a*d^2)*cos(f*x + e))*sin(f*x + e))/(d^2*cos(f*x + e)^3 + (2*
c*d + d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 - (c^2 + d^2)*cos(f*x + e) +
(d^2*cos(f*x + e)^2 - 2*c*d*cos(f*x + e) - c^2 - 2*c*d - d^2)*sin(f*x + e))
) + 4*(a*c - a*d + (a*c - a*d)*cos(f*x + e) - (a*c - a*d)*sin(f*x + e))*sqr
t(a*sin(f*x + e) + a))/((c*d^2 + d^3)*f*cos(f*x + e)^2 - (c^2*d + c*d^2)*f*
cos(f*x + e) - (c^2*d + 2*c*d^2 + d^3)*f - ((c*d^2 + d^3)*f*cos(f*x + e) +
(c^2*d + 2*c*d^2 + d^3)*f)*sin(f*x + e)), 1/2*((a*c^2 + 4*a*c*d + 3*a*d^2 -
 (a*c*d + 3*a*d^2)*cos(f*x + e)^2 + (a*c^2 + 3*a*c*d)*cos(f*x + e) + (a*c^2
 + 4*a*c*d + 3*a*d^2 + (a*c*d + 3*a*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(-
a/(c*d + d^2))*arctan(1/2*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) - c - 2*
d)*sqrt(-a/(c*d + d^2))/(a*cos(f*x + e))) - 2*(a*c - a*d + (a*c - a*d)*cos(
f*x + e) - (a*c - a*d)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/((c*d^2 + d^
3)*f*cos(f*x + e)^2 - (c^2*d + c*d^2)*f*cos(f*x + e) - (c^2*d + 2*c*d^2 + d
^3)*f - ((c*d^2 + d^3)*f*cos(f*x + e) + (c^2*d + 2*c*d^2 + d^3)*f)*sin(f*x
+ e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)/(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.535 $\int \frac{(a+a\sin(e+fx))^{3/2}}{(c+d\sin(e+fx))^3}\,dx$

Optimal. Leaf size=179

$$-\frac{a^{3/2}(c+7d)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{4d^{3/2}f(c+d)^{5/2}}-\frac{a^2(c+7d)\cos(e+fx)}{4df(c+d)^2\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}+\frac{a^2(c-d}{2df(c+d)\sqrt{a\sin(e+}}$$

```
[Out] -(a^(3/2)*(c + 7*d)*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]*Sqr
t[a + a*Sin[e + f*x]])])/(4*d^(3/2)*(c + d)^(5/2)*f) + (a^2*(c - d)*Cos[e +
 f*x])/(2*d*(c + d)*f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^2) - (a
^2*(c + 7*d)*Cos[e + f*x])/(4*d*(c + d)^2*f*Sqrt[a + a*Sin[e + f*x]]*(c + d
*Sin[e + f*x]))
```

Rubi [A] time = 0.290403, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2762, 21, 2772, 2773, 208}

$$-\frac{a^{3/2}(c+7d)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{4d^{3/2}f(c+d)^{5/2}}-\frac{a^2(c+7d)\cos(e+fx)}{4df(c+d)^2\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}+\frac{a^2(c-d}{2df(c+d)\sqrt{a\sin(e+}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)/(c + d*Sin[e + f*x])^3,x]
```


```
[Out] -(a^(3/2)*(c + 7*d)*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]*Sqr
t[a + a*Sin[e + f*x]])])/(4*d^(3/2)*(c + d)^(5/2)*f) + (a^2*(c - d)*Cos[e +
 f*x])/(2*d*(c + d)*f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^2) - (a
^2*(c + 7*d)*Cos[e + f*x])/(4*d*(c + d)^2*f*Sqrt[a + a*Sin[e + f*x]]*(c + d
*Sin[e + f*x]))
```


Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
```

```
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{3/2}}{(c+d\sin(e+fx))^3}\,dx = \frac{a^2(c-d)\cos(e+fx)}{2d(c+d)f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^2} - \frac{a\int \frac{-\frac{1}{2}a(c+7d)-\frac{1}{2}a(c+7d)\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^2}\,dx}{2d(c+d)}$$

$$= \frac{a^2(c-d)\cos(e+fx)}{2d(c+d)f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^2} + \frac{(a(c+7d))\int \frac{\sqrt{a+a\sin(e+fx)}}{(c+d\sin(e+fx))^2}\,dx}{4d(c+d)}$$

$$= \frac{a^2(c-d)\cos(e+fx)}{2d(c+d)f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^2} - \frac{a^2(c+7d)\cos(e+fx)}{4d(c+d)^2f\sqrt{a+a\sin(e+fx)}(c+d\,\mathrm{si}}$$

$$= \frac{a^2(c-d)\cos(e+fx)}{2d(c+d)f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^2} - \frac{a^2(c+7d)\cos(e+fx)}{4d(c+d)^2f\sqrt{a+a\sin(e+fx)}(c+d\,\mathrm{si}}$$

$$= -\frac{a^{3/2}(c+7d)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{4d^{3/2}(c+d)^{5/2}f} + \frac{a^2(c-d)\cos(e+fx)}{2d(c+d)f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx}$$

Mathematica [A] time = 3.95965, size = 313, normalized size = 1.75

$$\frac{(a(\sin(e+fx)+1))^{3/2}\left(\frac{4\sqrt{d}\sin\left(\frac{1}{2}(e+fx)\right)\left(-c^2+d(c+7d)\sin(e+fx)+7cd+2d^2\right)}{(c+d)^2(c+d\sin(e+fx))^2} - \frac{4\sqrt{d}\cos\left(\frac{1}{2}(e+fx)\right)\left(-c^2+d(c+7d)\sin(e+fx)+7cd+2d^2\right)}{(c+d)^2(c+d\sin(e+fx))^2} - \frac{2(c+7d)(\log}{}\right.}{16d^{3/2}f\left(\sin\left(\frac{1}{2}(e+fx)\right)\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)/(c + d*Sin[e + f*x])^3,x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^(3/2)*((-2*(c + 7*d)*(Log[-(Sec[(e + f*x)/4]^2*(c +
 d + Sqrt[d]*Sqrt[c + d]*Cos[(e + f*x)/2] - Sqrt[d]*Sqrt[c + d]*Sin[(e + f*
x)/2]))] - Log[(c + d)*Sec[(e + f*x)/4]^2 + Sqrt[d]*Sqrt[c + d]*(-1 + 2*Tan
[(e + f*x)/4] + Tan[(e + f*x)/4]^2)]))/(c + d)^(5/2) - (4*Sqrt[d]*Cos[(e +
f*x)/2]*(-c^2 + 7*c*d + 2*d^2 + d*(c + 7*d)*Sin[e + f*x]))/((c + d)^2*(c +
d*Sin[e + f*x])^2) + (4*Sqrt[d]*Sin[(e + f*x)/2]*(-c^2 + 7*c*d + 2*d^2 + d*
(c + 7*d)*Sin[e + f*x]))/((c + d)^2*(c + d*Sin[e + f*x])^2)))/(16*d^(3/2)*f
```

```
*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3)
```

Maple [B] time = 1.422, size = 429, normalized size = 2.4

$$\frac{1+\sin\left(fx+e\right)}{4\left(c+d\sin\left(fx+e\right)\right)^{2}(c+d)^{2}d\cos\left(fx+e\right)f}\left(-\mathrm{Artanh}\left(d\sqrt{-a\left(-1+\sin\left(fx+e\right)\right)}\frac{1}{\sqrt{a(c+d)d}}\right)\left(\sin\left(fx+e\right)\right)^{2}a^{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^3,x)
```

```
[Out] 1/4*(-arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*sin(f*x+e)^2*
a^2*c*d^2-7*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*sin(f*x
+e)^2*a^2*d^3-2*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*sin
(f*x+e)*a^2*c^2*d-14*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2)
)*sin(f*x+e)*a^2*c*d^2+(-a*(-1+sin(f*x+e)))^(3/2)*(a*(c+d)*d)^(1/2)*c*d+7*(
-a*(-1+sin(f*x+e)))^(3/2)*(a*(c+d)*d)^(1/2)*d^2-arctanh((-a*(-1+sin(f*x+e))
)^(1/2)*d/(a*(c+d)*d)^(1/2))*a^2*c^3-7*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d
/(a*(c+d)*d)^(1/2))*a^2*c^2*d+(-a*(-1+sin(f*x+e)))^(1/2)*(a*(c+d)*d)^(1/2)*
a*c^2-8*(-a*(-1+sin(f*x+e)))^(1/2)*(a*(c+d)*d)^(1/2)*a*c*d-9*(-a*(-1+sin(f*
x+e)))^(1/2)*(a*(c+d)*d)^(1/2)*a*d^2)*(-a*(-1+sin(f*x+e)))^(1/2)*(1+sin(f*x
+e))/(a*(c+d)*d)^(1/2)/(c+d*sin(f*x+e))^2/(c+d)^2/d/cos(f*x+e)/(a+a*sin(f*x
+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}{\left(d\sin\left(fx+e\right)+c\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)/(d*sin(f*x + e) + c)^3, x)
```

Fricas [B] time = 3.1653, size = 3536, normalized size = 19.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] [-1/16*((a*c^3 + 9*a*c^2*d + 15*a*c*d^2 + 7*a*d^3 - (a*c*d^2 + 7*a*d^3)*cos
(f*x + e)^3 - (2*a*c^2*d + 15*a*c*d^2 + 7*a*d^3)*cos(f*x + e)^2 + (a*c^3 +
7*a*c^2*d + a*c*d^2 + 7*a*d^3)*cos(f*x + e) + (a*c^3 + 9*a*c^2*d + 15*a*c*d
^2 + 7*a*d^3 - (a*c*d^2 + 7*a*d^3)*cos(f*x + e)^2 + 2*(a*c^2*d + 7*a*c*d^2)
*cos(f*x + e))*sin(f*x + e))*sqrt(a/(c*d + d^2))*log((a*d^2*cos(f*x + e)^3
- a*c^2 - 2*a*c*d - a*d^2 - (6*a*c*d + 7*a*d^2)*cos(f*x + e)^2 + 4*(c^2*d +
```

```
 4*c*d^2 + 3*d^3 - (c*d^2 + d^3)*cos(f*x + e)^2 + (c^2*d + 3*c*d^2 + 2*d^3)
*cos(f*x + e) - (c^2*d + 4*c*d^2 + 3*d^3 + (c*d^2 + d^3)*cos(f*x + e))*sin(
f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(a/(c*d + d^2)) - (a*c^2 + 8*a*c*d +
 9*a*d^2)*cos(f*x + e) + (a*d^2*cos(f*x + e)^2 - a*c^2 - 2*a*c*d - a*d^2 +
2*(3*a*c*d + 4*a*d^2)*cos(f*x + e))*sin(f*x + e))/(d^2*cos(f*x + e)^3 + (2*
c*d + d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 - (c^2 + d^2)*cos(f*x + e) +
(d^2*cos(f*x + e)^2 - 2*c*d*cos(f*x + e) - c^2 - 2*c*d - d^2)*sin(f*x + e))
) + 4*(a*c^2 - 6*a*c*d + 5*a*d^2 - (a*c*d + 7*a*d^2)*cos(f*x + e)^2 + (a*c^
2 - 7*a*c*d - 2*a*d^2)*cos(f*x + e) - (a*c^2 - 6*a*c*d + 5*a*d^2 + (a*c*d +
 7*a*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/((c^2*d^3 +
 2*c*d^4 + d^5)*f*cos(f*x + e)^3 + (2*c^3*d^2 + 5*c^2*d^3 + 4*c*d^4 + d^5)*
f*cos(f*x + e)^2 - (c^4*d + 2*c^3*d^2 + 2*c^2*d^3 + 2*c*d^4 + d^5)*f*cos(f*
x + e) - (c^4*d + 4*c^3*d^2 + 6*c^2*d^3 + 4*c*d^4 + d^5)*f + ((c^2*d^3 + 2*
c*d^4 + d^5)*f*cos(f*x + e)^2 - 2*(c^3*d^2 + 2*c^2*d^3 + c*d^4)*f*cos(f*x +
 e) - (c^4*d + 4*c^3*d^2 + 6*c^2*d^3 + 4*c*d^4 + d^5)*f)*sin(f*x + e)), 1/8
*((a*c^3 + 9*a*c^2*d + 15*a*c*d^2 + 7*a*d^3 - (a*c*d^2 + 7*a*d^3)*cos(f*x +
 e)^3 - (2*a*c^2*d + 15*a*c*d^2 + 7*a*d^3)*cos(f*x + e)^2 + (a*c^3 + 7*a*c^
2*d + a*c*d^2 + 7*a*d^3)*cos(f*x + e) + (a*c^3 + 9*a*c^2*d + 15*a*c*d^2 + 7
*a*d^3 - (a*c*d^2 + 7*a*d^3)*cos(f*x + e)^2 + 2*(a*c^2*d + 7*a*c*d^2)*cos(f
*x + e))*sin(f*x + e))*sqrt(-a/(c*d + d^2))*arctan(1/2*sqrt(a*sin(f*x + e)
+ a)*(d*sin(f*x + e) - c - 2*d)*sqrt(-a/(c*d + d^2))/(a*cos(f*x + e))) - 2*
(a*c^2 - 6*a*c*d + 5*a*d^2 - (a*c*d + 7*a*d^2)*cos(f*x + e)^2 + (a*c^2 - 7*
a*c*d - 2*a*d^2)*cos(f*x + e) - (a*c^2 - 6*a*c*d + 5*a*d^2 + (a*c*d + 7*a*d
^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/((c^2*d^3 + 2*c*d
^4 + d^5)*f*cos(f*x + e)^3 + (2*c^3*d^2 + 5*c^2*d^3 + 4*c*d^4 + d^5)*f*cos(
f*x + e)^2 - (c^4*d + 2*c^3*d^2 + 2*c^2*d^3 + 2*c*d^4 + d^5)*f*cos(f*x + e)
 - (c^4*d + 4*c^3*d^2 + 6*c^2*d^3 + 4*c*d^4 + d^5)*f + ((c^2*d^3 + 2*c*d^4
+ d^5)*f*cos(f*x + e)^2 - 2*(c^3*d^2 + 2*c^2*d^3 + c*d^4)*f*cos(f*x + e) -
(c^4*d + 4*c^3*d^2 + 6*c^2*d^3 + 4*c*d^4 + d^5)*f)*sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)/(c+d*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.536 $\int (a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^3\,dx$

Optimal. Leaf size=328

$$-\frac{2a^3\left(3c^2-38cd+355d^2\right)\cos(e+fx)(c+d\sin(e+fx))^3}{693d^2f\sqrt{a\sin(e+fx)+a}}-\frac{8a^2(5c-d)(c+d)\left(3c^2-38cd+355d^2\right)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3465df}$$

```
[Out] (-4*a^3*(c + d)*(15*c^2 + 10*c*d + 7*d^2)*(3*c^2 - 38*c*d + 355*d^2)*Cos[e
+ f*x])/(3465*d^2*f*Sqrt[a + a*Sin[e + f*x]]) - (8*a^2*(5*c - d)*(c + d)*(3
*c^2 - 38*c*d + 355*d^2)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(3465*d*f)
- (4*a*(c + d)*(3*c^2 - 38*c*d + 355*d^2)*Cos[e + f*x]*(a + a*Sin[e + f*x])
^(3/2))/(1155*f) - (2*a^3*(3*c^2 - 38*c*d + 355*d^2)*Cos[e + f*x]*(c + d*Si
n[e + f*x])^3)/(693*d^2*f*Sqrt[a + a*Sin[e + f*x]]) + (2*a^3*(3*c - 23*d)*C
os[e + f*x]*(c + d*Sin[e + f*x])^4)/(99*d^2*f*Sqrt[a + a*Sin[e + f*x]]) - (
2*a^2*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^4)/(11*d*f
)
```

Rubi [A] time = 0.656315, antiderivative size = 328, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2763, 2981, 2770, 2761, 2751, 2646}

$$-\frac{2a^3\left(3c^2-38cd+355d^2\right)\cos(e+fx)(c+d\sin(e+fx))^3}{693d^2f\sqrt{a\sin(e+fx)+a}}-\frac{8a^2(5c-d)(c+d)\left(3c^2-38cd+355d^2\right)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3465df}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])^3,x]
```


```
[Out] (-4*a^3*(c + d)*(15*c^2 + 10*c*d + 7*d^2)*(3*c^2 - 38*c*d + 355*d^2)*Cos[e
+ f*x])/(3465*d^2*f*Sqrt[a + a*Sin[e + f*x]]) - (8*a^2*(5*c - d)*(c + d)*(3
*c^2 - 38*c*d + 355*d^2)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(3465*d*f)
- (4*a*(c + d)*(3*c^2 - 38*c*d + 355*d^2)*Cos[e + f*x]*(a + a*Sin[e + f*x])
^(3/2))/(1155*f) - (2*a^3*(3*c^2 - 38*c*d + 355*d^2)*Cos[e + f*x]*(c + d*Si
n[e + f*x])^3)/(693*d^2*f*Sqrt[a + a*Sin[e + f*x]]) + (2*a^3*(3*c - 23*d)*C
os[e + f*x]*(c + d*Sin[e + f*x])^4)/(99*d^2*f*Sqrt[a + a*Sin[e + f*x]]) - (
2*a^2*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^4)/(11*d*f
)
```


Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
```

```
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2761

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^2, x_Symbol] :> -Simp[(d^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m
 + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*S
imp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^3\,dx = -\frac{2a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^4}{11df}+\frac{2\int\sqrt{a+a\,\mathrm{si}}}{}$$

$$=\frac{2a^3(3c-23d)\cos(e+fx)(c+d\sin(e+fx))^4}{99d^2f\sqrt{a+a\sin(e+fx)}}-\frac{2a^2\cos(e+fx)\sqrt{a+a\,\mathrm{s}}}{}$$

$$=-\frac{2a^3\left(3c^2-38cd+355d^2\right)\cos(e+fx)(c+d\sin(e+fx))^3}{693d^2f\sqrt{a+a\sin(e+fx)}}+\frac{2a^3(3c-2}{}$$

$$=-\frac{4a(c+d)\left(3c^2-38cd+355d^2\right)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{1155f}-\frac{2a^3}{}$$

$$=-\frac{8a^2(5c-d)(c+d)\left(3c^2-38cd+355d^2\right)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3465df}$$

$$=-\frac{4a^3(c+d)\left(15c^2+10cd+7d^2\right)\left(3c^2-38cd+355d^2\right)\cos(e+fx)}{3465d^2f\sqrt{a+a\sin(e+fx)}}-\frac{8a}{}$$

Mathematica [A] time = 6.35429, size = 246, normalized size = 0.75

$$-\frac{a^2\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(-8\left(5940c^2d+693c^3+8382cd^2+3250d^3\right)\cos(2(e+fx\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])^3,x]
```

```
[Out] -(a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(164
472*c^3 + 411840*c^2*d + 373098*c*d^2 + 114640*d^3 - 8*(693*c^3 + 5940*c^2*
d + 8382*c*d^2 + 3250*d^3)*Cos[2*(e + f*x)] + 70*d^2*(33*c + 32*d)*Cos[4*(e
 + f*x)] + 51744*c^3*Sin[e + f*x] + 199980*c^2*d*Sin[e + f*x] + 205656*c*d^
2*Sin[e + f*x] + 69890*d^3*Sin[e + f*x] - 5940*c^2*d*Sin[3*(e + f*x)] - 171
60*c*d^2*Sin[3*(e + f*x)] - 8675*d^3*Sin[3*(e + f*x)] + 315*d^3*Sin[5*(e +
f*x)]))/(27720*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))
```

Maple [A] time = 0.668, size = 249, normalized size = 0.8

$$\frac{\left(2+2\sin\left(fx+e\right)\right)a^3\left(-1+\sin\left(fx+e\right)\right)\left(315\,d^3\left(\sin\left(fx+e\right)\right)^5+1155\,cd^2\left(\sin\left(fx+e\right)\right)^4+1120\,d^3\left(\sin\left(fx+e\right)\right)^4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^3,x)
```

```
[Out] 2/3465*(1+sin(f*x+e))*a^3*(-1+sin(f*x+e))*(315*d^3*sin(f*x+e)^5+1155*c*d^2*
sin(f*x+e)^4+1120*d^3*sin(f*x+e)^4+1485*c^2*d*sin(f*x+e)^3+4290*c*d^2*sin(f
*x+e)^3+1775*d^3*sin(f*x+e)^3+693*c^3*sin(f*x+e)^2+5940*c^2*d*sin(f*x+e)^2+
7227*c*d^2*sin(f*x+e)^2+2130*d^3*sin(f*x+e)^2+3234*c^3*sin(f*x+e)+11385*c^2
*d*sin(f*x+e)+9636*sin(f*x+e)*d^2*c+2840*d^3*sin(f*x+e)+9933*c^3+22770*c^2*
d+19272*c*d^2+5680*d^3)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\left(d\sin\left(fx+e\right)+c\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)*(d*sin(f*x + e) + c)^3, x)
```

Fricas [A] time = 1.81451, size = 1227, normalized size = 3.74

$$-\frac{2\left(315\,a^2d^3\cos\left(fx+e\right)^6+35\left(33\,a^2cd^2+32\,a^2d^3\right)\cos\left(fx+e\right)^5+7392\,a^2c^3+15840\,a^2c^2d+13728\,a^2cd^2+4000\,a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -2/3465*(315*a^2*d^3*cos(f*x + e)^6 + 35*(33*a^2*c*d^2 + 32*a^2*d^3)*cos(f*
x + e)^5 + 7392*a^2*c^3 + 15840*a^2*c^2*d + 13728*a^2*c*d^2 + 4000*a^2*d^3
- 5*(297*a^2*c^2*d + 627*a^2*c*d^2 + 320*a^2*d^3)*cos(f*x + e)^4 - (693*a^2
*c^3 + 5940*a^2*c^2*d + 9537*a^2*c*d^2 + 4370*a^2*d^3)*cos(f*x + e)^3 + (25
41*a^2*c^3 + 8415*a^2*c^2*d + 8679*a^2*c*d^2 + 2965*a^2*d^3)*cos(f*x + e)^2
 + 2*(5313*a^2*c^3 + 14355*a^2*c^2*d + 13827*a^2*c*d^2 + 4465*a^2*d^3)*cos(
f*x + e) + (315*a^2*d^3*cos(f*x + e)^5 - 7392*a^2*c^3 - 15840*a^2*c^2*d - 1
3728*a^2*c*d^2 - 4000*a^2*d^3 - 35*(33*a^2*c*d^2 + 23*a^2*d^3)*cos(f*x + e)
^4 - 5*(297*a^2*c^2*d + 858*a^2*c*d^2 + 481*a^2*d^3)*cos(f*x + e)^3 + 3*(23
1*a^2*c^3 + 1485*a^2*c^2*d + 1749*a^2*c*d^2 + 655*a^2*d^3)*cos(f*x + e)^2 +
 2*(1617*a^2*c^3 + 6435*a^2*c^2*d + 6963*a^2*c*d^2 + 2465*a^2*d^3)*cos(f*x
+ e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)/(f*cos(f*x + e) + f*sin(f*x +
e) + f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*(c+d*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] Timed out
```

3.537 $\int (a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^2\,dx$

Optimal. Leaf size=202

$$-\frac{16a^2\left(21c^2+30cd+13d^2\right)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{315f}-\frac{64a^3\left(21c^2+30cd+13d^2\right)\cos(e+fx)}{315f\sqrt{a\sin(e+fx)+a}}-\frac{2a\left(21c^2+30c\right.}{}$$

```
[Out] (-64*a^3*(21*c^2 + 30*c*d + 13*d^2)*Cos[e + f*x])/(315*f*Sqrt[a + a*Sin[e +
 f*x]]) - (16*a^2*(21*c^2 + 30*c*d + 13*d^2)*Cos[e + f*x]*Sqrt[a + a*Sin[e
+ f*x]])/(315*f) - (2*a*(21*c^2 + 30*c*d + 13*d^2)*Cos[e + f*x]*(a + a*Sin[
e + f*x])^(3/2))/(105*f) - (4*(9*c - d)*d*Cos[e + f*x]*(a + a*Sin[e + f*x])
^(5/2))/(63*f) - (2*d^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(9*a*f)
```

Rubi [A] time = 0.273899, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2761, 2751, 2647, 2646}

$$-\frac{16a^2\left(21c^2+30cd+13d^2\right)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{315f}-\frac{64a^3\left(21c^2+30cd+13d^2\right)\cos(e+fx)}{315f\sqrt{a\sin(e+fx)+a}}-\frac{2a\left(21c^2+30c\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])^2,x]
```


```
[Out] (-64*a^3*(21*c^2 + 30*c*d + 13*d^2)*Cos[e + f*x])/(315*f*Sqrt[a + a*Sin[e +
 f*x]]) - (16*a^2*(21*c^2 + 30*c*d + 13*d^2)*Cos[e + f*x]*Sqrt[a + a*Sin[e
+ f*x]])/(315*f) - (2*a*(21*c^2 + 30*c*d + 13*d^2)*Cos[e + f*x]*(a + a*Sin[
e + f*x])^(3/2))/(105*f) - (4*(9*c - d)*d*Cos[e + f*x]*(a + a*Sin[e + f*x])
^(5/2))/(63*f) - (2*d^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(9*a*f)
```


Rule 2761

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^2, x_Symbol] :> -Simp[(d^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m
 + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*S
imp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int (a + a\sin(e + fx))^{5/2}(c + d\sin(e + fx))^2\,dx = -\frac{2d^2\cos(e + fx)(a + a\sin(e + fx))^{7/2}}{9af} + \frac{2\int (a + a\sin(e + fx))^{5/2}\left(\frac{1}{2}a\left(9\right.\right.}{}$$
$$= -\frac{4(9c - d)d\cos(e + fx)(a + a\sin(e + fx))^{5/2}}{63f} - \frac{2d^2\cos(e + fx)(a + a\sin}{9af}$$
$$= -\frac{2a\left(21c^2 + 30cd + 13d^2\right)\cos(e + fx)(a + a\sin(e + fx))^{3/2}}{105f} - \frac{4(9c - d)d}{}$$
$$= -\frac{16a^2\left(21c^2 + 30cd + 13d^2\right)\cos(e + fx)\sqrt{a + a\sin(e + fx)}}{315f} - \frac{2a\left(21c^2 +\right.}{}$$
$$= -\frac{64a^3\left(21c^2 + 30cd + 13d^2\right)\cos(e + fx)}{315f\sqrt{a + a\sin(e + fx)}} - \frac{16a^2\left(21c^2 + 30cd + 13d^2\right)\mathrm{co}}{31}$$

Mathematica [A] time = 3.35411, size = 180, normalized size = 0.89

$$-\frac{a^2\sqrt{a(\sin(e + fx) + 1)}\left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right)\right)\left(-4\left(63c^2 + 360cd + 254d^2\right)\cos(2(e + fx)) + 2352c^2\sin(e +\right.}{1260f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])^2,x]
```

```
[Out] -(a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(747
6*c^2 + 12480*c*d + 5653*d^2 - 4*(63*c^2 + 360*c*d + 254*d^2)*Cos[2*(e + f*
x)] + 35*d^2*Cos[4*(e + f*x)] + 2352*c^2*Sin[e + f*x] + 6060*c*d*Sin[e + f*
x] + 3116*d^2*Sin[e + f*x] - 180*c*d*Sin[3*(e + f*x)] - 260*d^2*Sin[3*(e +
f*x)]))/(1260*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))
```

Maple [A] time = 0.638, size = 168, normalized size = 0.8

$$\frac{(2 + 2\sin(fx + e))a^3(-1 + \sin(fx + e))\left(35d^2(\sin(fx + e))^4 + 90cd(\sin(fx + e))^3 + 130d^2(\sin(fx + e))^3 + 63c^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^2,x)
```

```
[Out] 2/315*(1+sin(f*x+e))*a^3*(-1+sin(f*x+e))*(35*d^2*sin(f*x+e)^4+90*c*d*sin(f*
x+e)^3+130*d^2*sin(f*x+e)^3+63*c^2*sin(f*x+e)^2+360*c*d*sin(f*x+e)^2+219*d^
2*sin(f*x+e)^2+294*c^2*sin(f*x+e)+690*sin(f*x+e)*c*d+292*sin(f*x+e)*d^2+903
*c^2+1380*c*d+584*d^2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\left(d\sin\left(fx+e\right)+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)*(d*sin(f*x + e) + c)^2, x)
```

Fricas [A] time = 1.67319, size = 822, normalized size = 4.07

$$-\frac{2\left(35\,a^{2}d^{2}\cos\left(fx+e\right)^{5}-5\left(18\,a^{2}cd+19\,a^{2}d^{2}\right)\cos\left(fx+e\right)^{4}+672\,a^{2}c^{2}+960\,a^{2}cd+416\,a^{2}d^{2}-\left(63\,a^{2}c^{2}+360\,a^{2}c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -2/315*(35*a^2*d^2*cos(f*x + e)^5 - 5*(18*a^2*c*d + 19*a^2*d^2)*cos(f*x + e
)^4 + 672*a^2*c^2 + 960*a^2*c*d + 416*a^2*d^2 - (63*a^2*c^2 + 360*a^2*c*d +
 289*a^2*d^2)*cos(f*x + e)^3 + (231*a^2*c^2 + 510*a^2*c*d + 263*a^2*d^2)*co
s(f*x + e)^2 + 2*(483*a^2*c^2 + 870*a^2*c*d + 419*a^2*d^2)*cos(f*x + e) - (
35*a^2*d^2*cos(f*x + e)^4 + 672*a^2*c^2 + 960*a^2*c*d + 416*a^2*d^2 + 10*(9
*a^2*c*d + 13*a^2*d^2)*cos(f*x + e)^3 - 3*(21*a^2*c^2 + 90*a^2*c*d + 53*a^2
*d^2)*cos(f*x + e)^2 - 2*(147*a^2*c^2 + 390*a^2*c*d + 211*a^2*d^2)*cos(f*x
+ e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)/(f*cos(f*x + e) + f*sin(f*x +
e) + f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.538 $\int (a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))\,dx$

Optimal. Leaf size=138

$$-\frac{64a^3(7c+5d)\cos(e+fx)}{105f\sqrt{a\sin(e+fx)+a}}-\frac{16a^2(7c+5d)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{105f}-\frac{2a(7c+5d)\cos(e+fx)(a\sin(e+fx)+a)}{35f}$$

```
[Out] (-64*a^3*(7*c + 5*d)*Cos[e + f*x])/(105*f*Sqrt[a + a*Sin[e + f*x]]) - (16*a
^2*(7*c + 5*d)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(105*f) - (2*a*(7*c +
 5*d)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(35*f) - (2*d*Cos[e + f*x]*(
a + a*Sin[e + f*x])^(5/2))/(7*f)
```

Rubi [A] time = 0.107962, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2751, 2647, 2646}

$$-\frac{64a^3(7c+5d)\cos(e+fx)}{105f\sqrt{a\sin(e+fx)+a}}-\frac{16a^2(7c+5d)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{105f}-\frac{2a(7c+5d)\cos(e+fx)(a\sin(e+fx)+a)}{35f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x]),x]
```


```
[Out] (-64*a^3*(7*c + 5*d)*Cos[e + f*x])/(105*f*Sqrt[a + a*Sin[e + f*x]]) - (16*a
^2*(7*c + 5*d)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(105*f) - (2*a*(7*c +
 5*d)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(35*f) - (2*d*Cos[e + f*x]*(
a + a*Sin[e + f*x])^(5/2))/(7*f)
```


Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int (a + a\sin(e + fx))^{5/2}(c + d\sin(e + fx))\,dx = -\frac{2d\cos(e + fx)(a + a\sin(e + fx))^{5/2}}{7f} + \frac{1}{7}(7c + 5d)\int (a + a\sin(e + f$$
$$= -\frac{2a(7c + 5d)\cos(e + fx)(a + a\sin(e + fx))^{3/2}}{35f} - \frac{2d\cos(e + fx)(a + a}{7f}$$
$$= -\frac{16a^2(7c + 5d)\cos(e + fx)\sqrt{a + a\sin(e + fx)}}{105f} - \frac{2a(7c + 5d)\cos(e +}{}$$
$$= -\frac{64a^3(7c + 5d)\cos(e + fx)}{105f\sqrt{a + a\sin(e + fx)}} - \frac{16a^2(7c + 5d)\cos(e + fx)\sqrt{a + a\sin(e}}{105f}$$

Mathematica [A] time = 1.50671, size = 119, normalized size = 0.86

$$-\frac{a^2\sqrt{a(\sin(e + fx) + 1)}\left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right)\right)((392c + 505d)\sin(e + fx) - 6(7c + 20d)\cos(2(e + fx)) +}{210f\left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x]),x]
```

```
[Out] -(a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(124
6*c + 1040*d - 6*(7*c + 20*d)*Cos[2*(e + f*x)] + (392*c + 505*d)*Sin[e + f*
x] - 15*d*Sin[3*(e + f*x)]))/(210*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))
```

Maple [A] time = 0.743, size = 99, normalized size = 0.7

$$\frac{(2 + 2\sin(fx + e))\,a^3\,(-1 + \sin(fx + e))\left(-15\,(\cos(fx + e))^2\sin(fx + e)\,d + (98\,c + 130\,d)\sin(fx + e) + (-21\,c -}{105\,f\cos(fx + e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e)),x)
```

```
[Out] 2/105*(1+sin(f*x+e))*a^3*(-1+sin(f*x+e))*(-15*cos(f*x+e)^2*sin(f*x+e)*d+(98
*c+130*d)*sin(f*x+e)+(-21*c-60*d)*cos(f*x+e)^2+322*c+290*d)/cos(f*x+e)/(a+a
*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(fx + e) + a)^{\frac{5}{2}}(d\sin(fx + e) + c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)*(d*sin(f*x + e) + c), x)
```

Fricas [A] time = 1.64812, size = 518, normalized size = 3.75

$$2\left(15\,a^2d\cos\left(fx+e\right)^4+3\left(7\,a^2c+20\,a^2d\right)\cos\left(fx+e\right)^3-224\,a^2c-160\,a^2d-\left(77\,a^2c+85\,a^2d\right)\cos\left(fx+e\right)^2-2\left(161\,a\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] 2/105*(15*a^2*d*cos(f*x + e)^4 + 3*(7*a^2*c + 20*a^2*d)*cos(f*x + e)^3 - 22
4*a^2*c - 160*a^2*d - (77*a^2*c + 85*a^2*d)*cos(f*x + e)^2 - 2*(161*a^2*c +
 145*a^2*d)*cos(f*x + e) + (15*a^2*d*cos(f*x + e)^3 + 224*a^2*c + 160*a^2*d
 - 3*(7*a^2*c + 15*a^2*d)*cos(f*x + e)^2 - 2*(49*a^2*c + 65*a^2*d)*cos(f*x
+ e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)/(f*cos(f*x + e) + f*sin(f*x +
e) + f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*(c+d*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.539 $\int (a + a\sin(e + fx))^{5/2}\, dx$

Optimal. Leaf size=89

$$-\frac{64a^3\cos(e+fx)}{15f\sqrt{a\sin(e+fx)+a}}-\frac{16a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{15f}-\frac{2a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5f}$$

```
[Out] (-64*a^3*Cos[e + f*x])/(15*f*Sqrt[a + a*Sin[e + f*x]]) - (16*a^2*Cos[e + f*
x]*Sqrt[a + a*Sin[e + f*x]])/(15*f) - (2*a*Cos[e + f*x]*(a + a*Sin[e + f*x]
)^(3/2))/(5*f)
```

Rubi [A] time = 0.0487699, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2647, 2646}

$$-\frac{64a^3\cos(e+fx)}{15f\sqrt{a\sin(e+fx)+a}}-\frac{16a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{15f}-\frac{2a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] (-64*a^3*Cos[e + f*x])/(15*f*Sqrt[a + a*Sin[e + f*x]]) - (16*a^2*Cos[e + f*
x]*Sqrt[a + a*Sin[e + f*x]])/(15*f) - (2*a*Cos[e + f*x]*(a + a*Sin[e + f*x]
)^(3/2))/(5*f)
```

Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int (a+a\sin(e+fx))^{5/2}\,dx &= -\frac{2a\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{5f}+\frac{1}{5}(8a)\int (a+a\sin(e+fx))^{3/2}\,dx\\ &= -\frac{16a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{15f}-\frac{2a\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{5f}+\frac{1}{15}\left(32a^2\right.\\ &= -\frac{64a^3\cos(e+fx)}{15f\sqrt{a+a\sin(e+fx)}}-\frac{16a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{15f}-\frac{2a\cos(e+fx)(a+a\sin}{5f}\end{aligned}$$

Mathematica [A] time = 0.319158, size = 117, normalized size = 1.31

$$-\frac{(a(\sin(e+fx)+1))^{5/2}\left(-150\sin\left(\frac{1}{2}(e+fx)\right)+25\sin\left(\frac{3}{2}(e+fx)\right)+3\sin\left(\frac{5}{2}(e+fx)\right)+150\cos\left(\frac{1}{2}(e+fx)\right)+25\cos\right.}{30f\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] -((a*(1 + Sin[e + f*x]))^(5/2)*(150*Cos[(e + f*x)/2] + 25*Cos[(3*(e + f*x))
/2] - 3*Cos[(5*(e + f*x))/2] - 150*Sin[(e + f*x)/2] + 25*Sin[(3*(e + f*x))/
2] + 3*Sin[(5*(e + f*x))/2]))/(30*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5
)
```

Maple [A] time = 0.592, size = 65, normalized size = 0.7

$$\frac{(2+2\sin(fx+e))\,a^3\,(-1+\sin(fx+e))\left(3(\sin(fx+e))^2+14\sin(fx+e)+43\right)}{15f\cos(fx+e)}\frac{1}{\sqrt{a+a\sin(fx+e)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2),x)
```

```
[Out] 2/15*(1+sin(f*x+e))*a^3*(-1+sin(f*x+e))*(3*sin(f*x+e)^2+14*sin(f*x+e)+43)/c
os(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(fx+e)+a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2), x)
```

Fricas [A] time = 1.52946, size = 292, normalized size = 3.28

$$\frac{2\left(3a^2\cos(fx+e)^3-11a^2\cos(fx+e)^2-46a^2\cos(fx+e)-32a^2-\left(3a^2\cos(fx+e)^2+14a^2\cos(fx+e)-32a^2\right)\right.}{15\left(f\cos(fx+e)+f\sin(fx+e)+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*a^2*cos(f*x + e)^3 - 11*a^2*cos(f*x + e)^2 - 46*a^2*cos(f*x + e) -
32*a^2 - (3*a^2*cos(f*x + e)^2 + 14*a^2*cos(f*x + e) - 32*a^2)*sin(f*x + e)
)*sqrt(a*sin(f*x + e) + a)/(f*cos(f*x + e) + f*sin(f*x + e) + f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.540 $\int \frac{(a+a\sin(e+fx))^{5/2}}{c+d\sin(e+fx)}\,dx$

Optimal. Leaf size=142

$$\frac{2a^3(3c-7d)\cos(e+fx)}{3d^2f\sqrt{a\sin(e+fx)+a}}-\frac{2a^{5/2}(c-d)^2\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{d^{5/2}f\sqrt{c+d}}-\frac{2a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3df}$$

```
[Out] (-2*a^(5/2)*(c - d)^2*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]*S
qrt[a + a*Sin[e + f*x]])])/(d^(5/2)*Sqrt[c + d]*f) + (2*a^3*(3*c - 7*d)*Cos
[e + f*x])/(3*d^2*f*Sqrt[a + a*Sin[e + f*x]]) - (2*a^2*Cos[e + f*x]*Sqrt[a
+ a*Sin[e + f*x]])/(3*d*f)
```

Rubi [A] time = 0.411399, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2763, 2981, 2773, 208}

$$\frac{2a^3(3c-7d)\cos(e+fx)}{3d^2f\sqrt{a\sin(e+fx)+a}}-\frac{2a^{5/2}(c-d)^2\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{d^{5/2}f\sqrt{c+d}}-\frac{2a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3df}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)/(c + d*Sin[e + f*x]),x]
```


```
[Out] (-2*a^(5/2)*(c - d)^2*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]*S
qrt[a + a*Sin[e + f*x]])])/(d^(5/2)*Sqrt[c + d]*f) + (2*a^3*(3*c - 7*d)*Cos
[e + f*x])/(3*d^2*f*Sqrt[a + a*Sin[e + f*x]]) - (2*a^2*Cos[e + f*x]*Sqrt[a
+ a*Sin[e + f*x]])/(3*d*f)
```


Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
```

```
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{5/2}}{c+d\sin(e+fx)}\,dx = -\frac{2a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3df}+\frac{2\int \frac{\sqrt{a+a\sin(e+fx)}\left(\frac{1}{2}a^2(c+3d)-\frac{1}{2}a^2(3c-7d)\sin(e+fx)\right)}{c+d\sin(e+fx)}\,dx}{3d}$$

$$= \frac{2a^3(3c-7d)\cos(e+fx)}{3d^2f\sqrt{a+a\sin(e+fx)}}-\frac{2a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3df}+\frac{\left(a^2(c-d)^2\right)\int \frac{\sqrt{a+a\sin}}{c+d\sin(}}{d^2}$$

$$= \frac{2a^3(3c-7d)\cos(e+fx)}{3d^2f\sqrt{a+a\sin(e+fx)}}-\frac{2a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3df}-\frac{\left(2a^3(c-d)^2\right)\mathrm{Subst}\left(\right.}{}$$

$$= -\frac{2a^{5/2}(c-d)^2\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{d^{5/2}\sqrt{c+d}f}+\frac{2a^3(3c-7d)\cos(e+fx)}{3d^2f\sqrt{a+a\sin(e+fx)}}-\frac{2a^2\cos(e+fx}{}$$

Mathematica [B] time = 3.59531, size = 330, normalized size = 2.32

$$\frac{(a(\sin(e+fx)+1))^{5/2}\left(6\sqrt{d}(5d-2c)\sin\left(\frac{1}{2}(e+fx)\right)+6\sqrt{d}(2c-5d)\cos\left(\frac{1}{2}(e+fx)\right)+\frac{3(c-d)^2\left(2\log\left(\sqrt{d}\sqrt{c+d}\left(\tan^2\left(\frac{1}{4}(e+fx)\right)\right.\right.\right.}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)/(c + d*Sin[e + f*x]),x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^(5/2)*(6*(2*c - 5*d)*Sqrt[d]*Cos[(e + f*x)/2] - 2*d
^(3/2)*Cos[(3*(e + f*x))/2] - (3*(c - d)^2*(e + f*x - 2*Log[Sec[(e + f*x)/4
]^2] + 2*Log[-(Sec[(e + f*x)/4]^2*(c + d + Sqrt[d]*Sqrt[c + d]*Cos[(e + f*x
)/2] - Sqrt[d]*Sqrt[c + d]*Sin[(e + f*x)/2]))]))/Sqrt[c + d] + (3*(c - d)^2
*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + 2*Log[(c + d)*Sec[(e + f*x)/4]^2 +
Sqrt[d]*Sqrt[c + d]*(-1 + 2*Tan[(e + f*x)/4] + Tan[(e + f*x)/4]^2)]))/Sqrt[
c + d] + 6*Sqrt[d]*(-2*c + 5*d)*Sin[(e + f*x)/2] - 2*d^(3/2)*Sin[(3*(e + f*
x))/2]))/(6*d^(5/2)*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)
```

Maple [A] time = 0.804, size = 228, normalized size = 1.6

$$\frac{2a\left(1+\sin\left(fx+e\right)\right)}{3d^2\cos\left(fx+e\right)f}\sqrt{-a\left(-1+\sin\left(fx+e\right)\right)}\left(\left(-a\left(-1+\sin\left(fx+e\right)\right)\right)^{\frac{3}{2}}\sqrt{a\left(c+d\right)d}\,d-3\,\mathrm{Artanh}\left(\frac{\sqrt{-a\left(-1+\sin\left(\right.\right.}}{\sqrt{a\left(c+d\right.}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e)),x)
```

```
[Out] 2/3*a*(1+sin(f*x+e))*(-a*(-1+sin(f*x+e)))^(1/2)*((-a*(-1+sin(f*x+e)))^(3/2)
*(a*(c+d)*d)^(1/2)*d-3*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/
2))*a^2*c^2+6*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^2*c
*d-3*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^2*d^2+3*(-a*
(-1+sin(f*x+e)))^(1/2)*(a*(c+d)*d)^(1/2)*a*c-9*(-a*(-1+sin(f*x+e)))^(1/2)*(
a*(c+d)*d)^(1/2)*a*d)/d^2/(a*(c+d)*d)^(1/2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/
2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}{d\sin\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)/(d*sin(f*x + e) + c), x)
```

Fricas [B] time = 2.52201, size = 1987, normalized size = 13.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(a^2*c^2 - 2*a^2*c*d + a^2*d^2 + (a^2*c^2 - 2*a^2*c*d + a^2*d^2)*co
s(f*x + e) + (a^2*c^2 - 2*a^2*c*d + a^2*d^2)*sin(f*x + e))*sqrt(a/(c*d + d^
2))*log((a*d^2*cos(f*x + e)^3 - a*c^2 - 2*a*c*d - a*d^2 - (6*a*c*d + 7*a*d^
2)*cos(f*x + e)^2 + 4*(c^2*d + 4*c*d^2 + 3*d^3 - (c*d^2 + d^3)*cos(f*x + e)
^2 + (c^2*d + 3*c*d^2 + 2*d^3)*cos(f*x + e) - (c^2*d + 4*c*d^2 + 3*d^3 + (c
*d^2 + d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(a/(c*
d + d^2)) - (a*c^2 + 8*a*c*d + 9*a*d^2)*cos(f*x + e) + (a*d^2*cos(f*x + e)^
2 - a*c^2 - 2*a*c*d - a*d^2 + 2*(3*a*c*d + 4*a*d^2)*cos(f*x + e))*sin(f*x +
 e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2
 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - 2*c*d*cos(f*x + e) - c^
2 - 2*c*d - d^2)*sin(f*x + e))) - 4*(a^2*d*cos(f*x + e)^2 - 3*a^2*c + 7*a^2
*d - (3*a^2*c - 8*a^2*d)*cos(f*x + e) + (a^2*d*cos(f*x + e) + 3*a^2*c - 7*a
^2*d)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/(d^2*f*cos(f*x + e) + d^2*f*s
in(f*x + e) + d^2*f), -1/3*(3*(a^2*c^2 - 2*a^2*c*d + a^2*d^2 + (a^2*c^2 - 2
*a^2*c*d + a^2*d^2)*cos(f*x + e) + (a^2*c^2 - 2*a^2*c*d + a^2*d^2)*sin(f*x
+ e))*sqrt(-a/(c*d + d^2))*arctan(1/2*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x +
 e) - c - 2*d)*sqrt(-a/(c*d + d^2))/(a*cos(f*x + e))) + 2*(a^2*d*cos(f*x +
e)^2 - 3*a^2*c + 7*a^2*d - (3*a^2*c - 8*a^2*d)*cos(f*x + e) + (a^2*d*cos(f*
x + e) + 3*a^2*c - 7*a^2*d)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/(d^2*f*
cos(f*x + e) + d^2*f*sin(f*x + e) + d^2*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)/(c+d*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.541 $\int \frac{(a+a\sin(e+fx))^{5/2}}{(c+d\sin(e+fx))^2}\,dx$

Optimal. Leaf size=166

$$-\frac{a^3(3c+d)\cos(e+fx)}{d^2f(c+d)\sqrt{a\sin(e+fx)+a}}+\frac{a^{5/2}(c-d)(3c+5d)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{d^{5/2}f(c+d)^{3/2}}+\frac{a^2(c-d)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{df(c+d)(c+d\sin(e+fx))}$$

```
[Out] (a^(5/2)*(c - d)*(3*c + 5*d)*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c
 + d]*Sqrt[a + a*Sin[e + f*x]])])/(d^(5/2)*(c + d)^(3/2)*f) - (a^3*(3*c + d
)*Cos[e + f*x])/(d^2*(c + d)*f*Sqrt[a + a*Sin[e + f*x]]) + (a^2*(c - d)*Cos
[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(d*(c + d)*f*(c + d*Sin[e + f*x]))
```

Rubi [A] time = 0.388863, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2762, 2981, 2773, 208}

$$-\frac{a^3(3c+d)\cos(e+fx)}{d^2f(c+d)\sqrt{a\sin(e+fx)+a}}+\frac{a^{5/2}(c-d)(3c+5d)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{d^{5/2}f(c+d)^{3/2}}+\frac{a^2(c-d)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{df(c+d)(c+d\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)/(c + d*Sin[e + f*x])^2,x]
```

```
[Out] (a^(5/2)*(c - d)*(3*c + 5*d)*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c
 + d]*Sqrt[a + a*Sin[e + f*x]])])/(d^(5/2)*(c + d)^(3/2)*f) - (a^3*(3*c + d
)*Cos[e + f*x])/(d^2*(c + d)*f*Sqrt[a + a*Sin[e + f*x]]) + (a^2*(c - d)*Cos
[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(d*(c + d)*f*(c + d*Sin[e + f*x]))
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
```

```
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{5/2}}{(c+d\sin(e+fx))^2}\,dx = \frac{a^2(c-d)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{d(c+d)f(c+d\sin(e+fx))} - \frac{a\int \frac{\sqrt{a+a\sin(e+fx)}\left(\frac{1}{2}a(c-5d)-\frac{1}{2}a(3c+d)\sin(e+fx)\right)}{c+d\sin(e+fx)}\,dx}{d(c+d)}$$

$$= -\frac{a^3(3c+d)\cos(e+fx)}{d^2(c+d)f\sqrt{a+a\sin(e+fx)}} + \frac{a^2(c-d)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{d(c+d)f(c+d\sin(e+fx))} - \frac{\left(a^2(c-d)(\right.}{}$$

$$= -\frac{a^3(3c+d)\cos(e+fx)}{d^2(c+d)f\sqrt{a+a\sin(e+fx)}} + \frac{a^2(c-d)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{d(c+d)f(c+d\sin(e+fx))} + \frac{\left(a^3(c-d)(\right.}{}$$

$$= \frac{a^{5/2}(c-d)(3c+5d)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{d^{5/2}(c+d)^{3/2}f} - \frac{a^3(3c+d)\cos(e+fx)}{d^2(c+d)f\sqrt{a+a\sin(e+fx)}} + \frac{a^2(c}{}$$

Mathematica [B] time = 4.11335, size = 350, normalized size = 2.11

$$(a(\sin(e+fx)+1))^{5/2}\left(\frac{\left(-3c^2-2cd+5d^2\right)\left(2\log\left(\sqrt{d}\sqrt{c+d}\left(\tan^2\left(\frac{1}{4}(e+fx)\right)+2\tan\left(\frac{1}{4}(e+fx)\right)-1\right)+(c+d)\sec^2\left(\frac{1}{4}(e+fx)\right)\right)-2\log\left(\sec^2\left(\frac{1}{4}(e+fx)\right)\right)+e+fx\right.}{(c+d)^{3/2}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)/(c + d*Sin[e + f*x])^2,x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^(5/2)*(-8*Sqrt[d]*Cos[(e + f*x)/2] + ((3*c^2 + 2*c*
d - 5*d^2)*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + 2*Log[-(Sec[(e + f*x)/4]^
2*(c + d + Sqrt[d]*Sqrt[c + d]*Cos[(e + f*x)/2] - Sqrt[d]*Sqrt[c + d]*Sin[(
e + f*x)/2]))]))/(c + d)^(3/2) + ((-3*c^2 - 2*c*d + 5*d^2)*(e + f*x - 2*Log
[Sec[(e + f*x)/4]^2] + 2*Log[(c + d)*Sec[(e + f*x)/4]^2 + Sqrt[d]*Sqrt[c +
d]*(-1 + 2*Tan[(e + f*x)/4] + Tan[(e + f*x)/4]^2)]))/(c + d)^(3/2) + 8*Sqrt
[d]*Sin[(e + f*x)/2] - (4*(c - d)^2*Sqrt[d]*(Cos[(e + f*x)/2] - Sin[(e + f*
x)/2]))/((c + d)*(c + d*Sin[e + f*x]))))/(4*d^(5/2)*f*(Cos[(e + f*x)/2] + S
in[(e + f*x)/2])^5)
```

Maple [B] time = 1.224, size = 392, normalized size = 2.4

$$-\frac{a^2\left(1+\sin\left(fx+e\right)\right)}{(c+d)\,d^2\left(c+d\sin\left(fx+e\right)\right)\cos\left(fx+e\right)f}\sqrt{-a\left(-1+\sin\left(fx+e\right)\right)}\left(\sin\left(fx+e\right)d\left(-3\,\mathrm{Artanh}\left(\frac{\sqrt{a-a\sin\left(fx+\right.}}{\sqrt{acd+ad^2}}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^2,x)
```

```
[Out] -a^2*(1+sin(f*x+e))*(-a*(-1+sin(f*x+e)))^(1/2)*(sin(f*x+e)*d*(-3*arctanh((a
-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a*c^2-2*arctanh((a-a*sin(f*x+e)
)^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a*c*d+5*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*
c*d+a*d^2)^(1/2))*a*d^2+2*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*c+2*(a-a
*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*d)-3*arctanh((a-a*sin(f*x+e))^(1/2)*d/
(a*c*d+a*d^2)^(1/2))*a*c^3-2*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)
^(1/2))*a*c^2*d+5*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a*c
*d^2+3*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*c^2+(a-a*sin(f*x+e))^(1/2)*
(a*(c+d)*d)^(1/2)*d^2)/d^2/(c+d)/(c+d*sin(f*x+e))/(a*(c+d)*d)^(1/2)/cos(f*x
+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}{\left(d\sin\left(fx+e\right)+c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)/(d*sin(f*x + e) + c)^2, x)
```

Fricas [B] time = 2.70633, size = 2916, normalized size = 17.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] [1/4*((3*a^2*c^3 + 5*a^2*c^2*d - 3*a^2*c*d^2 - 5*a^2*d^3 - (3*a^2*c^2*d + 2
*a^2*c*d^2 - 5*a^2*d^3)*cos(f*x + e)^2 + (3*a^2*c^3 + 2*a^2*c^2*d - 5*a^2*c
*d^2)*cos(f*x + e) + (3*a^2*c^3 + 5*a^2*c^2*d - 3*a^2*c*d^2 - 5*a^2*d^3 + (
3*a^2*c^2*d + 2*a^2*c*d^2 - 5*a^2*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a/(
c*d + d^2))*log((a*d^2*cos(f*x + e)^3 - a*c^2 - 2*a*c*d - a*d^2 - (6*a*c*d
+ 7*a*d^2)*cos(f*x + e)^2 + 4*(c^2*d + 4*c*d^2 + 3*d^3 - (c*d^2 + d^3)*cos(
f*x + e)^2 + (c^2*d + 3*c*d^2 + 2*d^3)*cos(f*x + e) - (c^2*d + 4*c*d^2 + 3*
d^3 + (c*d^2 + d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sq
rt(a/(c*d + d^2)) - (a*c^2 + 8*a*c*d + 9*a*d^2)*cos(f*x + e) + (a*d^2*cos(f
*x + e)^2 - a*c^2 - 2*a*c*d - a*d^2 + 2*(3*a*c*d + 4*a*d^2)*cos(f*x + e))*s
in(f*x + e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^2 - 2*c
*d - d^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - 2*c*d*cos(f*x +
 e) - c^2 - 2*c*d - d^2)*sin(f*x + e))) + 4*(3*a^2*c^2 - 2*a^2*c*d - a^2*d^
2 + 2*(a^2*c*d + a^2*d^2)*cos(f*x + e)^2 + (3*a^2*c^2 + a^2*d^2)*cos(f*x +
e) - (3*a^2*c^2 - 2*a^2*c*d - a^2*d^2 - 2*(a^2*c*d + a^2*d^2)*cos(f*x + e))
*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/((c*d^3 + d^4)*f*cos(f*x + e)^2 -
(c^2*d^2 + c*d^3)*f*cos(f*x + e) - (c^2*d^2 + 2*c*d^3 + d^4)*f - ((c*d^3 +
d^4)*f*cos(f*x + e) + (c^2*d^2 + 2*c*d^3 + d^4)*f)*sin(f*x + e)), -1/2*((3*
a^2*c^3 + 5*a^2*c^2*d - 3*a^2*c*d^2 - 5*a^2*d^3 - (3*a^2*c^2*d + 2*a^2*c*d^
2 - 5*a^2*d^3)*cos(f*x + e)^2 + (3*a^2*c^3 + 2*a^2*c^2*d - 5*a^2*c*d^2)*cos
(f*x + e) + (3*a^2*c^3 + 5*a^2*c^2*d - 3*a^2*c*d^2 - 5*a^2*d^3 + (3*a^2*c^2
```

```
*d + 2*a^2*c*d^2 - 5*a^2*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(-a/(c*d + d^
2))*arctan(1/2*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) - c - 2*d)*sqrt(-a/
(c*d + d^2))/(a*cos(f*x + e))) - 2*(3*a^2*c^2 - 2*a^2*c*d - a^2*d^2 + 2*(a^
2*c*d + a^2*d^2)*cos(f*x + e)^2 + (3*a^2*c^2 + a^2*d^2)*cos(f*x + e) - (3*a
^2*c^2 - 2*a^2*c*d - a^2*d^2 - 2*(a^2*c*d + a^2*d^2)*cos(f*x + e))*sin(f*x
+ e))*sqrt(a*sin(f*x + e) + a))/((c*d^3 + d^4)*f*cos(f*x + e)^2 - (c^2*d^2
+ c*d^3)*f*cos(f*x + e) - (c^2*d^2 + 2*c*d^3 + d^4)*f - ((c*d^3 + d^4)*f*co
s(f*x + e) + (c^2*d^2 + 2*c*d^3 + d^4)*f)*sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)/(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.542 $\int \frac{(a+a\sin(e+fx))^{5/2}}{(c+d\sin(e+fx))^3}\,dx$

Optimal. Leaf size=194

$$-\frac{a^{5/2}\left(3c^2+10cd+19d^2\right)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{4d^{5/2}f(c+d)^{5/2}}+\frac{3a^3(c-d)(c+3d)\cos(e+fx)}{4d^2f(c+d)^2\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}+\frac{a^2(c-d)\,\mathrm{cc}}{2df(c}$$

```
[Out] -(a^(5/2)*(3*c^2 + 10*c*d + 19*d^2)*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/
(Sqrt[c + d]*Sqrt[a + a*Sin[e + f*x]])])/(4*d^(5/2)*(c + d)^(5/2)*f) + (a^2
*(c - d)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(2*d*(c + d)*f*(c + d*Sin[e
 + f*x])^2) + (3*a^3*(c - d)*(c + 3*d)*Cos[e + f*x])/(4*d^2*(c + d)^2*f*Sqr
t[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x]))
```

Rubi [A] time = 0.442532, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2762, 2980, 2773, 208}

$$-\frac{a^{5/2}\left(3c^2+10cd+19d^2\right)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{4d^{5/2}f(c+d)^{5/2}}+\frac{3a^3(c-d)(c+3d)\cos(e+fx)}{4d^2f(c+d)^2\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}+\frac{a^2(c-d)\,\mathrm{cc}}{2df(c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)/(c + d*Sin[e + f*x])^3,x]
```

```
[Out] -(a^(5/2)*(3*c^2 + 10*c*d + 19*d^2)*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/
(Sqrt[c + d]*Sqrt[a + a*Sin[e + f*x]])])/(4*d^(5/2)*(c + d)^(5/2)*f) + (a^2
*(c - d)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(2*d*(c + d)*f*(c + d*Sin[e
 + f*x])^2) + (3*a^3*(c - d)*(c + 3*d)*Cos[e + f*x])/(4*d^2*(c + d)^2*f*Sqr
t[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x]))
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{5/2}}{(c+d\sin(e+fx))^3}\,dx = \frac{a^2(c-d)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{2d(c+d)f(c+d\sin(e+fx))^2} - \frac{a\int \frac{\sqrt{a+a\sin(e+fx)}\left(\frac{1}{2}a(c-9d)-\frac{1}{2}a(3c+5d)\sin(e+fx)\right)}{(c+d\sin(e+fx))^2}\,dx}{2d(c+d)}$$

$$= \frac{a^2(c-d)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{2d(c+d)f(c+d\sin(e+fx))^2} + \frac{3a^3(c-d)(c+3d)\cos(e+fx)}{4d^2(c+d)^2f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+}$$

$$= \frac{a^2(c-d)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{2d(c+d)f(c+d\sin(e+fx))^2} + \frac{3a^3(c-d)(c+3d)\cos(e+fx)}{4d^2(c+d)^2f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+}$$

$$= -\frac{a^{5/2}\left(3c^2+10cd+19d^2\right)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{4d^{5/2}(c+d)^{5/2}f} + \frac{a^2(c-d)\cos(e+fx)\sqrt{a+a\sin}}{2d(c+d)f(c+d\sin(e+f}$$

Mathematica [A] time = 4.723, size = 379, normalized size = 1.95

$$(a(\sin(e+fx)+1))^{5/2}\left(-\frac{4\sqrt{d}\left(-5c^2-6cd+11d^2\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}{(c+d)^2(c+d\sin(e+fx))} + \frac{\left(3c^2+10cd+19d^2\right)\left(2\log\left(\sqrt{d}\sqrt{c+d}\left(\tan^2\left(\frac{1}{4}(e+fx)\right)+2\tan\left(\frac{1}{4}(e+f\right.\right.\right.\right.}{(c+d)}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)/(c + d*Sin[e + f*x])^3,x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^(5/2)*(-(((3*c^2 + 10*c*d + 19*d^2)*(e + f*x - 2*Lo
g[Sec[(e + f*x)/4]^2] + 2*Log[-(Sec[(e + f*x)/4]^2*(c + d + Sqrt[d]*Sqrt[c
+ d]*Cos[(e + f*x)/2] - Sqrt[d]*Sqrt[c + d]*Sin[(e + f*x)/2]))]))/(c + d)^(
5/2)) + ((3*c^2 + 10*c*d + 19*d^2)*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + 2
*Log[(c + d)*Sec[(e + f*x)/4]^2 + Sqrt[d]*Sqrt[c + d]*(-1 + 2*Tan[(e + f*x)
/4] + Tan[(e + f*x)/4]^2)]))/(c + d)^(5/2) - (8*(c - d)^2*Sqrt[d]*(Cos[(e +
 f*x)/2] - Sin[(e + f*x)/2]))/((c + d)*(c + d*Sin[e + f*x])^2) - (4*Sqrt[d]
*(-5*c^2 - 6*c*d + 11*d^2)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))/((c + d)^
2*(c + d*Sin[e + f*x]))))/(16*d^(5/2)*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2
])^5)
```

Maple [B] time = 1.523, size = 567, normalized size = 2.9

$$-\frac{a\left(1+\sin\left(fx+e\right)\right)}{4\left(c+d\sin\left(fx+e\right)\right)^2(c+d)^2d^2\cos\left(fx+e\right)f}\left(2\sin\left(fx+e\right)\mathrm{Artanh}\left(\frac{\sqrt{a-a\sin\left(fx+e\right)}d}{\sqrt{acd+ad^2}}\right)a^2cd\left(3c^2+10cd+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^3,x)
```

```
[Out] -1/4*a*(2*sin(f*x+e)*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*
a^2*c*d*(3*c^2+10*c*d+19*d^2)-arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2
)^(1/2))*a^2*d^2*(3*c^2+10*c*d+19*d^2)*cos(f*x+e)^2+5*(a-a*sin(f*x+e))^(3/2
)*(a*(c+d)*d)^(1/2)*c^2*d+6*(a-a*sin(f*x+e))^(3/2)*(a*(c+d)*d)^(1/2)*c*d^2-
11*(a-a*sin(f*x+e))^(3/2)*(a*(c+d)*d)^(1/2)*d^3+3*arctanh((a-a*sin(f*x+e))^
(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*c^4+10*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a
*c*d+a*d^2)^(1/2))*a^2*c^3*d+22*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d
^2)^(1/2))*a^2*c^2*d^2+10*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1
/2))*a^2*c*d^3+19*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2
*d^4-3*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a*c^3-13*(a-a*sin(f*x+e))^(
1/2)*(a*(c+d)*d)^(1/2)*a*c^2*d+3*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a
*c*d^2+13*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a*d^3)*(-a*(-1+sin(f*x+e
)))^(1/2)*(1+sin(f*x+e))/(a*(c+d)*d)^(1/2)/(c+d*sin(f*x+e))^2/(c+d)^2/d^2/c
os(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 3.4697, size = 4378, normalized size = 22.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] [-1/16*((3*a^2*c^4 + 16*a^2*c^3*d + 42*a^2*c^2*d^2 + 48*a^2*c*d^3 + 19*a^2*
d^4 - (3*a^2*c^2*d^2 + 10*a^2*c*d^3 + 19*a^2*d^4)*cos(f*x + e)^3 - (6*a^2*c
^3*d + 23*a^2*c^2*d^2 + 48*a^2*c*d^3 + 19*a^2*d^4)*cos(f*x + e)^2 + (3*a^2*
c^4 + 10*a^2*c^3*d + 22*a^2*c^2*d^2 + 10*a^2*c*d^3 + 19*a^2*d^4)*cos(f*x +
e) + (3*a^2*c^4 + 16*a^2*c^3*d + 42*a^2*c^2*d^2 + 48*a^2*c*d^3 + 19*a^2*d^4
 - (3*a^2*c^2*d^2 + 10*a^2*c*d^3 + 19*a^2*d^4)*cos(f*x + e)^2 + 2*(3*a^2*c^
3*d + 10*a^2*c^2*d^2 + 19*a^2*c*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a/(c*
d + d^2))*log((a*d^2*cos(f*x + e)^3 - a*c^2 - 2*a*c*d - a*d^2 - (6*a*c*d +
7*a*d^2)*cos(f*x + e)^2 + 4*(c^2*d + 4*c*d^2 + 3*d^3 - (c*d^2 + d^3)*cos(f*
x + e)^2 + (c^2*d + 3*c*d^2 + 2*d^3)*cos(f*x + e) - (c^2*d + 4*c*d^2 + 3*d^
3 + (c*d^2 + d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt
(a/(c*d + d^2)) - (a*c^2 + 8*a*c*d + 9*a*d^2)*cos(f*x + e) + (a*d^2*cos(f*x
 + e)^2 - a*c^2 - 2*a*c*d - a*d^2 + 2*(3*a*c*d + 4*a*d^2)*cos(f*x + e))*sin
(f*x + e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^2 - 2*c*d
 - d^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - 2*c*d*cos(f*x + e
) - c^2 - 2*c*d - d^2)*sin(f*x + e))) + 4*(3*a^2*c^3 + 3*a^2*c^2*d - 15*a^2
*c*d^2 + 9*a^2*d^3 + (5*a^2*c^2*d + 6*a^2*c*d^2 - 11*a^2*d^3)*cos(f*x + e)^
```

```
2 + (3*a^2*c^3 + 8*a^2*c^2*d - 9*a^2*c*d^2 - 2*a^2*d^3)*cos(f*x + e) - (3*a
^2*c^3 + 3*a^2*c^2*d - 15*a^2*c*d^2 + 9*a^2*d^3 - (5*a^2*c^2*d + 6*a^2*c*d^
2 - 11*a^2*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/((c^2
*d^4 + 2*c*d^5 + d^6)*f*cos(f*x + e)^3 + (2*c^3*d^3 + 5*c^2*d^4 + 4*c*d^5 +
 d^6)*f*cos(f*x + e)^2 - (c^4*d^2 + 2*c^3*d^3 + 2*c^2*d^4 + 2*c*d^5 + d^6)*
f*cos(f*x + e) - (c^4*d^2 + 4*c^3*d^3 + 6*c^2*d^4 + 4*c*d^5 + d^6)*f + ((c^
2*d^4 + 2*c*d^5 + d^6)*f*cos(f*x + e)^2 - 2*(c^3*d^3 + 2*c^2*d^4 + c*d^5)*f
*cos(f*x + e) - (c^4*d^2 + 4*c^3*d^3 + 6*c^2*d^4 + 4*c*d^5 + d^6)*f)*sin(f*
x + e)), 1/8*((3*a^2*c^4 + 16*a^2*c^3*d + 42*a^2*c^2*d^2 + 48*a^2*c*d^3 + 1
9*a^2*d^4 - (3*a^2*c^2*d^2 + 10*a^2*c*d^3 + 19*a^2*d^4)*cos(f*x + e)^3 - (6
*a^2*c^3*d + 23*a^2*c^2*d^2 + 48*a^2*c*d^3 + 19*a^2*d^4)*cos(f*x + e)^2 + (
3*a^2*c^4 + 10*a^2*c^3*d + 22*a^2*c^2*d^2 + 10*a^2*c*d^3 + 19*a^2*d^4)*cos(
f*x + e) + (3*a^2*c^4 + 16*a^2*c^3*d + 42*a^2*c^2*d^2 + 48*a^2*c*d^3 + 19*a
^2*d^4 - (3*a^2*c^2*d^2 + 10*a^2*c*d^3 + 19*a^2*d^4)*cos(f*x + e)^2 + 2*(3*
a^2*c^3*d + 10*a^2*c^2*d^2 + 19*a^2*c*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt
(-a/(c*d + d^2))*arctan(1/2*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) - c -
2*d)*sqrt(-a/(c*d + d^2))/(a*cos(f*x + e))) - 2*(3*a^2*c^3 + 3*a^2*c^2*d -
15*a^2*c*d^2 + 9*a^2*d^3 + (5*a^2*c^2*d + 6*a^2*c*d^2 - 11*a^2*d^3)*cos(f*x
 + e)^2 + (3*a^2*c^3 + 8*a^2*c^2*d - 9*a^2*c*d^2 - 2*a^2*d^3)*cos(f*x + e)
- (3*a^2*c^3 + 3*a^2*c^2*d - 15*a^2*c*d^2 + 9*a^2*d^3 - (5*a^2*c^2*d + 6*a^
2*c*d^2 - 11*a^2*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))
/((c^2*d^4 + 2*c*d^5 + d^6)*f*cos(f*x + e)^3 + (2*c^3*d^3 + 5*c^2*d^4 + 4*c
*d^5 + d^6)*f*cos(f*x + e)^2 - (c^4*d^2 + 2*c^3*d^3 + 2*c^2*d^4 + 2*c*d^5 +
 d^6)*f*cos(f*x + e) - (c^4*d^2 + 4*c^3*d^3 + 6*c^2*d^4 + 4*c*d^5 + d^6)*f
+ ((c^2*d^4 + 2*c*d^5 + d^6)*f*cos(f*x + e)^2 - 2*(c^3*d^3 + 2*c^2*d^4 + c*
d^5)*f*cos(f*x + e) - (c^4*d^2 + 4*c^3*d^3 + 6*c^2*d^4 + 4*c*d^5 + d^6)*f)*
sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)/(c+d*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.543 $\int \frac{(c+d\sin(e+fx))^3}{\sqrt{a+a\sin(e+fx)}}\,dx$

Optimal. Leaf size=178

$$-\frac{4d\left(21c^2-12cd+7d^2\right)\cos(e+fx)}{15f\sqrt{a\sin(e+fx)+a}}-\frac{2d^2(9c-d)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{15af}-\frac{2d\cos(e+fx)(c+d\sin(e+fx))^2}{5f\sqrt{a\sin(e+fx)+a}}-$$

```
[Out] -((Sqrt[2]*(c - d)^3*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin
[e + f*x]])])/(Sqrt[a]*f)) - (4*d*(21*c^2 - 12*c*d + 7*d^2)*Cos[e + f*x])/(
15*f*Sqrt[a + a*Sin[e + f*x]]) - (2*(9*c - d)*d^2*Cos[e + f*x]*Sqrt[a + a*S
in[e + f*x]])/(15*a*f) - (2*d*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(5*f*Sqr
t[a + a*Sin[e + f*x]])
```

Rubi [A] time = 0.439525, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2778, 2968, 3023, 2751, 2649, 206}

$$-\frac{4d\left(21c^2-12cd+7d^2\right)\cos(e+fx)}{15f\sqrt{a\sin(e+fx)+a}}-\frac{2d^2(9c-d)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{15af}-\frac{2d\cos(e+fx)(c+d\sin(e+fx))^2}{5f\sqrt{a\sin(e+fx)+a}}-$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^3/Sqrt[a + a*Sin[e + f*x]],x]
```



```
[Out] -((Sqrt[2]*(c - d)^3*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin
[e + f*x]])])/(Sqrt[a]*f)) - (4*d*(21*c^2 - 12*c*d + 7*d^2)*Cos[e + f*x])/(
15*f*Sqrt[a + a*Sin[e + f*x]]) - (2*(9*c - d)*d^2*Cos[e + f*x]*Sqrt[a + a*S
in[e + f*x]])/(15*a*f) - (2*d*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(5*f*Sqr
t[a + a*Sin[e + f*x]])
```


Rule 2778

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> Simp[(-2*d*Cos[e + f*x]*(c + d*Sin[e + f*x])
^(n - 1))/(f*(2*n - 1)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(b*(2*n - 1))
, Int[((c + d*Sin[e + f*x])^(n - 2)*Simp[a*c*d - b*(2*d^2*(n - 1) + c^2*(2*
n - 1)) + d*(a*d - b*c*(4*n - 3))*Sin[e + f*x], x])/Sqrt[a + b*Sin[e + f*x]
], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
```

```
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^3}{\sqrt{a+a\sin(e+fx)}}\,dx = -\frac{2d\cos(e+fx)(c+d\sin(e+fx))^2}{5f\sqrt{a+a\sin(e+fx)}} - \frac{\int \frac{(c+d\sin(e+fx))\left(-a\left(5c^2-cd+4d^2\right)-a(9c-d)d\sin(e+fx)\right)}{\sqrt{a+a\sin(e+fx)}}\,dx}{5a}$$

$$= -\frac{2d\cos(e+fx)(c+d\sin(e+fx))^2}{5f\sqrt{a+a\sin(e+fx)}} - \frac{\int \frac{-ac\left(5c^2-cd+4d^2\right)+\left(-ac(9c-d)d-ad\left(5c^2-cd+4d^2\right)\right)\sin(e+fx)-a \cdots}{\sqrt{a+a\sin(e+fx)}}}{5a}$$

$$= -\frac{2(9c-d)d^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{15af} - \frac{2d\cos(e+fx)(c+d\sin(e+fx))^2}{5f\sqrt{a+a\sin(e+fx)}} - \frac{2\int \cdots}{\cdots}$$

$$= -\frac{4d\left(21c^2-12cd+7d^2\right)\cos(e+fx)}{15f\sqrt{a+a\sin(e+fx)}} - \frac{2(9c-d)d^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{15af} - \frac{2d \cdots}{\cdots}$$

$$= -\frac{4d\left(21c^2-12cd+7d^2\right)\cos(e+fx)}{15f\sqrt{a+a\sin(e+fx)}} - \frac{2(9c-d)d^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{15af} - \frac{2d \cdots}{\cdots}$$

$$= -\frac{\sqrt{2}(c-d)^3\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}f} - \frac{4d\left(21c^2-12cd+7d^2\right)\cos(e+fx)}{15f\sqrt{a+a\sin(e+fx)}} - \frac{2(9c-d)d \cdots}{\cdots}$$

Mathematica [C] time = 0.59945, size = 155, normalized size = 0.87

$$-\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(-2d\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(-90c^2-2d(15c-d)\sin(e+fx)+30 \cdots\right.\right.}{30f\sqrt{a(\sin(e+ \cdots}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])^3/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] -((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*((-60 - 60*I)*(-1)^(3/4)*(c - d)^3*
ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])] - 2*d*(Cos[(e + f*x
```

```
)/2] - Sin[(e + f*x)/2])*(-90*c^2 + 30*c*d - 29*d^2 + 3*d^2*Cos[2*(e + f*x)
] - 2*(15*c - d)*d*Sin[e + f*x])))/(30*f*Sqrt[a*(1 + Sin[e + f*x])])
```

Maple [A] time = 0.859, size = 285, normalized size = 1.6

$$-\frac{1+\sin\left(fx+e\right)}{15\,a^{3}\cos\left(fx+e\right)f}\sqrt{-a\left(-1+\sin\left(fx+e\right)\right)}\left(15\,a^{5/2}\sqrt{2}\mathrm{Artanh}\left(1/2\,\frac{\sqrt{a-a\sin\left(fx+e\right)}\sqrt{2}}{\sqrt{a}}\right)c^{3}-45\,a^{5/2}\sqrt{2}\mathrm{Artanh}\left(1/\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] -1/15*(1+sin(f*x+e))*(-a*(-1+sin(f*x+e)))^(1/2)*(15*a^(5/2)*2^(1/2)*arctanh
(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*c^3-45*a^(5/2)*2^(1/2)*arctanh
(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*c^2*d+45*a^(5/2)*2^(1/2)*arcta
nh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*c*d^2-15*a^(5/2)*2^(1/2)*arc
tanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*d^3+6*d^3*(a-a*sin(f*x+e))
^(5/2)-30*(a-a*sin(f*x+e))^(3/2)*a*c*d^2-10*(a-a*sin(f*x+e))^(3/2)*a*d^3+90
*c^2*d*a^2*(a-a*sin(f*x+e))^(1/2)+30*a^2*d^3*(a-a*sin(f*x+e))^(1/2))/a^3/co
s(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{3}}{\sqrt{a\sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*sin(f*x + e) + c)^3/sqrt(a*sin(f*x + e) + a), x)
```

Fricas [B] time = 1.72624, size = 965, normalized size = 5.42

$$-\frac{\frac{15\sqrt{2}\left(ac^{3}-3\,ac^{2}d+3\,acd^{2}-ad^{3}+\left(ac^{3}-3\,ac^{2}d+3\,acd^{2}-ad^{3}\right)\cos\left(fx+e\right)+\left(ac^{3}-3\,ac^{2}d+3\,acd^{2}-ad^{3}\right)\sin\left(fx+e\right)\right)\log\left(-\frac{\cos\left(fx+e\right)^{2}-\left(\cos\left(fx+e\right)-2\right)\sin\left(fx+e\right)+\frac{2\sqrt{2}\sqrt{a\sin}}{}}{\cos\left(fx+e\right)^{2}-\left(\cos\left(fx+e\right)+\right.}\right.}{\sqrt{a}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/30*(15*sqrt(2)*(a*c^3 - 3*a*c^2*d + 3*a*c*d^2 - a*d^3 + (a*c^3 - 3*a*c^2
*d + 3*a*c*d^2 - a*d^3)*cos(f*x + e) + (a*c^3 - 3*a*c^2*d + 3*a*c*d^2 - a*d
^3)*sin(f*x + e))*log(-(cos(f*x + e)^2 - (cos(f*x + e) - 2)*sin(f*x + e) +
2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*(cos(f*x + e) - sin(f*x + e) + 1)/sqrt(a
) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) -
```

```
cos(f*x + e) - 2))/sqrt(a) - 4*(3*d^3*cos(f*x + e)^3 - 45*c^2*d + 30*c*d^2
 - 17*d^3 - (15*c*d^2 - 4*d^3)*cos(f*x + e)^2 - (45*c^2*d - 15*c*d^2 + 16*d
^3)*cos(f*x + e) - (3*d^3*cos(f*x + e)^2 - 45*c^2*d + 30*c*d^2 - 17*d^3 + (
15*c*d^2 - d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/(a*f*
cos(f*x + e) + a*f*sin(f*x + e) + a*f)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c + d\sin\left(e + fx\right)\right)^3}{\sqrt{a\left(\sin\left(e + fx\right) + 1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**3/(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((c + d*sin(e + f*x))**3/sqrt(a*(sin(e + f*x) + 1)), x)
```

Giac [B] time = 2.96252, size = 1029, normalized size = 5.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] 1/60*(120*sqrt(2)*(c^3 - 3*c^2*d + 3*c*d^2 - d^3)*arctan(-1/2*sqrt(2)*(sqrt
(a)*tan(1/2*f*x + 1/2*e) - sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a) + sqrt(a))/sq
rt(-a))/(sqrt(-a)*sgn(tan(1/2*f*x + 1/2*e) + 1)) + ((((((45*a^2*c^2*d*sgn(t
an(1/2*f*x + 1/2*e) + 1) - 15*a^2*c*d^2*sgn(tan(1/2*f*x + 1/2*e) + 1) + 13*
a^2*d^3*sgn(tan(1/2*f*x + 1/2*e) + 1))*tan(1/2*f*x + 1/2*e)/a^9 - 15*(3*a^2
*c^2*d*sgn(tan(1/2*f*x + 1/2*e) + 1) - 3*a^2*c*d^2*sgn(tan(1/2*f*x + 1/2*e)
 + 1) + a^2*d^3*sgn(tan(1/2*f*x + 1/2*e) + 1))/a^9)*tan(1/2*f*x + 1/2*e) +
10*(9*a^2*c^2*d*sgn(tan(1/2*f*x + 1/2*e) + 1) - 6*a^2*c*d^2*sgn(tan(1/2*f*x
 + 1/2*e) + 1) + 4*a^2*d^3*sgn(tan(1/2*f*x + 1/2*e) + 1))/a^9)*tan(1/2*f*x
+ 1/2*e) - 10*(9*a^2*c^2*d*sgn(tan(1/2*f*x + 1/2*e) + 1) - 6*a^2*c*d^2*sgn(
tan(1/2*f*x + 1/2*e) + 1) + 4*a^2*d^3*sgn(tan(1/2*f*x + 1/2*e) + 1))/a^9)*t
an(1/2*f*x + 1/2*e) + 15*(3*a^2*c^2*d*sgn(tan(1/2*f*x + 1/2*e) + 1) - 3*a^2
*c*d^2*sgn(tan(1/2*f*x + 1/2*e) + 1) + a^2*d^3*sgn(tan(1/2*f*x + 1/2*e) + 1
))/a^9)*tan(1/2*f*x + 1/2*e) - (45*a^2*c^2*d*sgn(tan(1/2*f*x + 1/2*e) + 1)
- 15*a^2*c*d^2*sgn(tan(1/2*f*x + 1/2*e) + 1) + 13*a^2*d^3*sgn(tan(1/2*f*x +
 1/2*e) + 1))/a^9)/(a*tan(1/2*f*x + 1/2*e)^2 + a)^(5/2) - (120*sqrt(2)*a^10
*c^3*arctan(sqrt(a)/sqrt(-a)) - 360*sqrt(2)*a^10*c^2*d*arctan(sqrt(a)/sqrt(
-a)) + 360*sqrt(2)*a^10*c*d^2*arctan(sqrt(a)/sqrt(-a)) - 120*sqrt(2)*a^10*d
^3*arctan(sqrt(a)/sqrt(-a)) - 45*sqrt(2)*sqrt(-a)*sqrt(a)*c^2*d + 30*sqrt(2
)*sqrt(-a)*sqrt(a)*c*d^2 - 17*sqrt(2)*sqrt(-a)*sqrt(a)*d^3)*sgn(tan(1/2*f*x
 + 1/2*e) + 1)/(sqrt(-a)*a^10))/f
```

3.544 $\int \frac{(c+d\sin(e+fx))^2}{\sqrt{a+a\sin(e+fx)}}\,dx$

Optimal. Leaf size=123

$$-\frac{4d(3c-d)\cos(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}-\frac{\sqrt{2}(c-d)^2\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a}f}-\frac{2d^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3af}$$

```
[Out] -((Sqrt[2]*(c - d)^2*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin
[e + f*x]])])/(Sqrt[a]*f)) - (4*(3*c - d)*d*Cos[e + f*x])/(3*f*Sqrt[a + a*S
in[e + f*x]]) - (2*d^2*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(3*a*f)
```

Rubi [A] time = 0.200624, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2761, 2751, 2649, 206}

$$-\frac{4d(3c-d)\cos(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}-\frac{\sqrt{2}(c-d)^2\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a}f}-\frac{2d^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3af}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^2/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] -((Sqrt[2]*(c - d)^2*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin
[e + f*x]])])/(Sqrt[a]*f)) - (4*(3*c - d)*d*Cos[e + f*x])/(3*f*Sqrt[a + a*S
in[e + f*x]]) - (2*d^2*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(3*a*f)
```

Rule 2761

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^2, x_Symbol] :> -Simp[(d^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m
 + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*S
imp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^2}{\sqrt{a+a\sin(e+fx)}}\,dx = -\frac{2d^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3af} + \frac{2\int \frac{\frac{1}{2}a(3c^2+d^2)+a(3c-d)d\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx}{3a}$$

$$= -\frac{4(3c-d)d\cos(e+fx)}{3f\sqrt{a+a\sin(e+fx)}} - \frac{2d^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3af} + (c-d)^2\int \frac{1}{\sqrt{a+a\sin(e}$$

$$= -\frac{4(3c-d)d\cos(e+fx)}{3f\sqrt{a+a\sin(e+fx)}} - \frac{2d^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3af} - \frac{(2(c-d)^2)\,\mathrm{Subst}\left(\int \frac{1}{2a-}\right.}{f}$$

$$= -\frac{\sqrt{2}(c-d)^2\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}f} - \frac{4(3c-d)d\cos(e+fx)}{3f\sqrt{a+a\sin(e+fx)}} - \frac{2d^2\cos(e+fx)\sqrt{a+a}}{3af}$$

Mathematica [C] time = 0.384894, size = 125, normalized size = 1.02

$$-\frac{2\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(d\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(6c+d\sin(e+fx)-d)-(3+3i)(-1)^{3/}\right.}{3f\sqrt{a(\sin(e+fx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])^2/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] (-2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*((-3 - 3*I)*(-1)^(3/4)*(c - d)^2*
ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])] + d*(Cos[(e + f*x)/
2] - Sin[(e + f*x)/2])*(6*c - d + d*Sin[e + f*x])))/(3*f*Sqrt[a*(1 + Sin[e
+ f*x])])
```

Maple [A] time = 0.839, size = 185, normalized size = 1.5

$$-\frac{1+\sin(fx+e)}{3a^2\cos(fx+e)f}\sqrt{-a(-1+\sin(fx+e))}\left(3a^{3/2}\sqrt{2}\mathrm{Artanh}\left(1/2\frac{\sqrt{a-a\sin(fx+e)}\sqrt{2}}{\sqrt{a}}\right)c^2-6a^{3/2}\sqrt{2}\mathrm{Artanh}\left(1/\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] -1/3*(1+sin(f*x+e))*(-a*(-1+sin(f*x+e)))^(1/2)*(3*a^(3/2)*2^(1/2)*arctanh(1
/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*c^2-6*a^(3/2)*2^(1/2)*arctanh(1/
2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*c*d+3*a^(3/2)*2^(1/2)*arctanh(1/2
*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*d^2-2*(a-a*sin(f*x+e))^(3/2)*d^2+1
2*a*c*d*(a-a*sin(f*x+e))^(1/2))/a^2/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d\sin(fx+e)+c)^2}{\sqrt{a\sin(fx+e)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*sin(f*x + e) + c)^2/sqrt(a*sin(f*x + e) + a), x)
```

Fricas [B] time = 1.69025, size = 741, normalized size = 6.02

$$\frac{\frac{3\sqrt{2}\left(ac^2-2\,acd+ad^2+\left(ac^2-2\,acd+ad^2\right)\cos\left(fx+e\right)+\left(ac^2-2\,acd+ad^2\right)\sin\left(fx+e\right)\right)\log\left(-\frac{\cos\left(fx+e\right)^2-\left(\cos\left(fx+e\right)-2\right)\sin\left(fx+e\right)-\frac{2\sqrt{2}\sqrt{a\sin\left(fx+e\right)+a}\left(\cos\left(fx+e\right)-\sin\left(fx+e\right)+\cdots\right.}{\sqrt{a}}}{\cos\left(fx+e\right)^2-\left(\cos\left(fx+e\right)+2\right)\sin\left(fx+e\right)-\cos\left(fx+e\right)-2}\right.}{\sqrt{a}}}{6\left(af\cos\left(\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*(3*sqrt(2)*(a*c^2 - 2*a*c*d + a*d^2 + (a*c^2 - 2*a*c*d + a*d^2)*cos(f*x
 + e) + (a*c^2 - 2*a*c*d + a*d^2)*sin(f*x + e))*log(-(cos(f*x + e)^2 - (cos
(f*x + e) - 2)*sin(f*x + e) - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*(cos(f*x +
 e) - sin(f*x + e) + 1)/sqrt(a) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2 - (co
s(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2))/sqrt(a) - 4*(d^2*cos(f*x
+ e)^2 + 6*c*d - 2*d^2 + (6*c*d - d^2)*cos(f*x + e) + (d^2*cos(f*x + e) - 6
*c*d + 2*d^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/(a*f*cos(f*x + e) + a
*f*sin(f*x + e) + a*f)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c + d\sin\left(e + fx\right)\right)^2}{\sqrt{a\left(\sin\left(e + fx\right) + 1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**2/(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((c + d*sin(e + f*x))**2/sqrt(a*(sin(e + f*x) + 1)), x)
```

Giac [B] time = 2.33619, size = 583, normalized size = 4.74

$$\frac{\frac{6\sqrt{2}\left(c^2-2\,cd+d^2\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{a}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-\sqrt{a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+a}+\sqrt{a}\right)}{2\sqrt{-a}}\right)}{\sqrt{-a}\operatorname{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)}+\frac{\left(\left(\left(\frac{\left(6\,acd\operatorname{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)-ad^2\operatorname{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)\right)\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{a^6}-\frac{3\left(2\,acd\operatorname{sgn}\cdots\right.}{\cdots}\right.\right.\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*(6*sqrt(2)*(c^2 - 2*c*d + d^2)*arctan(-1/2*sqrt(2)*(sqrt(a)*tan(1/2*f*x
 + 1/2*e) - sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a) + sqrt(a))/sqrt(-a))/(sqrt(-
a)*sgn(tan(1/2*f*x + 1/2*e) + 1)) + ((((6*a*c*d*sgn(tan(1/2*f*x + 1/2*e) +
1) - a*d^2*sgn(tan(1/2*f*x + 1/2*e) + 1))*tan(1/2*f*x + 1/2*e)/a^6 - 3*(2*a
*c*d*sgn(tan(1/2*f*x + 1/2*e) + 1) - a*d^2*sgn(tan(1/2*f*x + 1/2*e) + 1))/a
^6)*tan(1/2*f*x + 1/2*e) + 3*(2*a*c*d*sgn(tan(1/2*f*x + 1/2*e) + 1) - a*d^2
*sgn(tan(1/2*f*x + 1/2*e) + 1))/a^6)*tan(1/2*f*x + 1/2*e) - (6*a*c*d*sgn(ta
n(1/2*f*x + 1/2*e) + 1) - a*d^2*sgn(tan(1/2*f*x + 1/2*e) + 1))/a^6)/(a*tan(
1/2*f*x + 1/2*e)^2 + a)^(3/2) - 2*(3*sqrt(2)*a^7*c^2*arctan(sqrt(a)/sqrt(-a
)) - 6*sqrt(2)*a^7*c*d*arctan(sqrt(a)/sqrt(-a)) + 3*sqrt(2)*a^7*d^2*arctan(
sqrt(a)/sqrt(-a)) - 3*sqrt(2)*sqrt(-a)*sqrt(a)*c*d + sqrt(2)*sqrt(-a)*sqrt(
a)*d^2)*sgn(tan(1/2*f*x + 1/2*e) + 1)/(sqrt(-a)*a^7))/f
```

3.545 $\int \frac{c+d\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx$

Optimal. Leaf size=79

$$-\frac{\sqrt{2}(c-d)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a}f}-\frac{2d\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}$$

```
[Out] -((Sqrt[2]*(c - d)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e
 + f*x]])])/(Sqrt[a]*f)) - (2*d*Cos[e + f*x])/(f*Sqrt[a + a*Sin[e + f*x]])
```

Rubi [A] time = 0.0699759, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2751, 2649, 206}

$$-\frac{\sqrt{2}(c-d)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a}f}-\frac{2d\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] -((Sqrt[2]*(c - d)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e
 + f*x]])])/(Sqrt[a]*f)) - (2*d*Cos[e + f*x])/(f*Sqrt[a + a*Sin[e + f*x]])
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{c+d\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx = -\frac{2d\cos(e+fx)}{f\sqrt{a+a\sin(e+fx)}} + (c-d)\int \frac{1}{\sqrt{a+a\sin(e+fx)}}\,dx$$
$$= -\frac{2d\cos(e+fx)}{f\sqrt{a+a\sin(e+fx)}} - \frac{(2(c-d))\,\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{f}$$
$$= -\frac{\sqrt{2}(c-d)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}f} - \frac{2d\cos(e+fx)}{f\sqrt{a+a\sin(e+fx)}}$$

Mathematica [C] time = 0.213114, size = 106, normalized size = 1.34

$$\frac{2\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(d\left(\sin\left(\frac{1}{2}(e+fx)\right)-\cos\left(\frac{1}{2}(e+fx)\right)\right)+(1+i)(-1)^{3/4}(c-d)\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(\cdots\right.\right.}{f\sqrt{a(\sin(e+fx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] (2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*((1 + I)*(-1)^(3/4)*(c - d)*ArcTan
h[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])] + d*(-Cos[(e + f*x)/2] +
Sin[(e + f*x)/2])))/(f*Sqrt[a*(1 + Sin[e + f*x])])
```

Maple [A] time = 0.741, size = 128, normalized size = 1.6

$$-\frac{1+\sin\left(fx+e\right)}{af\cos\left(fx+e\right)}\sqrt{-a\left(-1+\sin\left(fx+e\right)\right)}\left(\sqrt{a}\sqrt{2}\text{Artanh}\left(\frac{\sqrt{2}}{2}\sqrt{a-a\sin\left(fx+e\right)}\frac{1}{\sqrt{a}}\right)c-\sqrt{a}\sqrt{2}\text{Artanh}\left(\frac{\sqrt{2}}{2}\sqrt{a-\cdots}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] -(1+sin(f*x+e))*(-a*(-1+sin(f*x+e)))^(1/2)*(a^(1/2)*2^(1/2)*arctanh(1/2*(a-
a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*c-a^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin
(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*d+2*(a-a*sin(f*x+e))^(1/2)*d)/a/cos(f*x+e)/
(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{d\sin\left(fx+e\right)+c}{\sqrt{a\sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*sin(f*x + e) + c)/sqrt(a*sin(f*x + e) + a), x)
```

Fricas [B] time = 1.64717, size = 578, normalized size = 7.32

$$-\frac{\frac{\sqrt{2}\left(ac-ad+(ac-ad)\cos\left(fx+e\right)+(ac-ad)\sin\left(fx+e\right)\right)\log\left(-\frac{\cos\left(fx+e\right)^{2}-\left(\cos\left(fx+e\right)-2\right)\sin\left(fx+e\right)+\frac{2\sqrt{2}\sqrt{a\sin\left(fx+e\right)+a}\left(\cos\left(fx+e\right)-\sin\left(fx+e\right)+1\right)}{\sqrt{a}}+3\cos\left(fx+e\right)+2}{\cos\left(fx+e\right)^{2}-\left(\cos\left(fx+e\right)+2\right)\sin\left(fx+e\right)-\cos\left(fx+e\right)-2}\right)}{\sqrt{a}}+4\left(\right.}{2\left(af\cos\left(fx+e\right)+af\sin\left(fx+e\right)+af\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(2)*(a*c - a*d + (a*c - a*d)*cos(f*x + e) + (a*c - a*d)*sin(f*x +
 e))*log(-(cos(f*x + e)^2 - (cos(f*x + e) - 2)*sin(f*x + e) + 2*sqrt(2)*sqr
t(a*sin(f*x + e) + a)*(cos(f*x + e) - sin(f*x + e) + 1)/sqrt(a) + 3*cos(f*x
 + e) + 2)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e)
 - 2))/sqrt(a) + 4*(d*cos(f*x + e) - d*sin(f*x + e) + d)*sqrt(a*sin(f*x + e
) + a))/(a*f*cos(f*x + e) + a*f*sin(f*x + e) + a*f)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{c+d\sin\left(e+fx\right)}{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))/(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((c + d*sin(e + f*x))/sqrt(a*(sin(e + f*x) + 1)), x)
```

Giac [B] time = 2.18192, size = 297, normalized size = 3.76

$$\frac{2\left(\frac{\sqrt{2}(c-d)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+a}+\sqrt{a}\right)}{2\sqrt{-a}}\right)}{\sqrt{-a}\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}+\frac{\frac{d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}-\frac{d}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}}{\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+a}}-\frac{\left(\sqrt{2}ac\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)-\sqrt{2}ad\arctan\left(\right.\right.}{}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] 2*(sqrt(2)*(c - d)*arctan(-1/2*sqrt(2)*(sqrt(a)*tan(1/2*f*x + 1/2*e) - sqrt
(a*tan(1/2*f*x + 1/2*e)^2 + a) + sqrt(a))/sqrt(-a))/(sqrt(-a)*sgn(tan(1/2*f
*x + 1/2*e) + 1)) + (d*tan(1/2*f*x + 1/2*e)/sgn(tan(1/2*f*x + 1/2*e) + 1) -
 d/sgn(tan(1/2*f*x + 1/2*e) + 1))/sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a) - (sqr
t(2)*a*c*arctan(sqrt(a)/sqrt(-a)) - sqrt(2)*a*d*arctan(sqrt(a)/sqrt(-a)) -
sqrt(2)*sqrt(-a)*sqrt(a)*d)*sgn(tan(1/2*f*x + 1/2*e) + 1)/(sqrt(-a)*a))/f
```

3.546 $\int \frac{1}{\sqrt{a+a\sin(e+fx)}}\,dx$

Optimal. Leaf size=47

$$-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a}f}$$

```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]]
)])/(Sqrt[a]*f))
```

Rubi [A] time = 0.0227762, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2649, 206}

$$-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a}f}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]]
)])/(Sqrt[a]*f))
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{a+a\sin(e+fx)}}\,dx &= -\frac{2\,\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{f}\\
&= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}f}
\end{aligned}$$

Mathematica [C] time = 0.0476718, size = 73, normalized size = 1.55

$$\frac{(2+2i)(-1)^{3/4}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(\tan\left(\frac{1}{4}(e+fx)\right)-1\right)\right)}{f\sqrt{a(\sin(e+fx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] ((2 + 2*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4]
)]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))/(f*Sqrt[a*(1 + Sin[e + f*x])])
```

Maple [A] time = 0.421, size = 75, normalized size = 1.6

$$-\frac{\left(1+\sin\left(fx+e\right)\right)\sqrt{2}}{f\cos\left(fx+e\right)}\sqrt{-a\left(-1+\sin\left(fx+e\right)\right)}\mathrm{Artanh}\left(\frac{\sqrt{2}}{2}\sqrt{-a\left(-1+\sin\left(fx+e\right)\right)}\frac{1}{\sqrt{a}}\right)\frac{1}{\sqrt{a}}\frac{1}{\sqrt{a+a\sin\left(fx+e\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] -(1+sin(f*x+e))*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2)*arctanh(1/2*(-a*
(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(a*sin(f*x + e) + a), x)
```

Fricas [A] time = 1.60793, size = 463, normalized size = 9.85

$$\left[\frac{\sqrt{2}\log\left(-\frac{\cos\left(fx+e\right)^{2}-\left(\cos\left(fx+e\right)-2\right)\sin\left(fx+e\right)-\frac{2\sqrt{2}\sqrt{a\sin\left(fx+e\right)+a}\left(\cos\left(fx+e\right)-\sin\left(fx+e\right)+1\right)}{\sqrt{a}}+3\cos\left(fx+e\right)+2}{\cos\left(fx+e\right)^{2}-\left(\cos\left(fx+e\right)+2\right)\sin\left(fx+e\right)-\cos\left(fx+e\right)-2}\right)}{2\sqrt{a}f}, \frac{\sqrt{2}\sqrt{-\frac{1}{a}}\arctan\left(\frac{\sqrt{2}\sqrt{a\sin\left(fx+e\right)+}}{\cos\left(fx+e\right)}\right.}{f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(2)*log(-(cos(f*x + e)^2 - (cos(f*x + e) - 2)*sin(f*x + e) - 2*sqr
t(2)*sqrt(a*sin(f*x + e) + a)*(cos(f*x + e) - sin(f*x + e) + 1)/sqrt(a) + 3
*cos(f*x + e) + 2)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(
f*x + e) - 2))/(sqrt(a)*f), sqrt(2)*sqrt(-1/a)*arctan(sqrt(2)*sqrt(a*sin(f*
x + e) + a)*sqrt(-1/a)/cos(f*x + e))/f]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\sin\left(e+fx\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*sin(e + f*x) + a), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(a*sin(f*x + e) + a), x)
```

3.547 $\int \frac{1}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))}\,dx$

Optimal. Leaf size=123

$$\frac{2\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a}f(c-d)\sqrt{c+d}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a}f(c-d)}$$

```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]]
)])/(Sqrt[a]*(c - d)*f)) + (2*Sqrt[d]*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x]
)/(Sqrt[c + d]*Sqrt[a + a*Sin[e + f*x]])])/(Sqrt[a]*(c - d)*Sqrt[c + d]*f)
```

Rubi [A] time = 0.214057, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2780, 2649, 206, 2773, 208}

$$\frac{2\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a}f(c-d)\sqrt{c+d}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a}f(c-d)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])),x]
```


```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]]
)])/(Sqrt[a]*(c - d)*f)) + (2*Sqrt[d]*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x]
)/(Sqrt[c + d]*Sqrt[a + a*Sin[e + f*x]])])/(Sqrt[a]*(c - d)*Sqrt[c + d]*f)
```


Rule 2780

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e +
f*x]], x], x] - Dist[d/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))}\,dx = \frac{\int \frac{1}{\sqrt{a+a\sin(e+fx)}}\,dx}{c-d} - \frac{d\int \frac{\sqrt{a+a\sin(e+fx)}}{c+d\sin(e+fx)}\,dx}{a(c-d)}$$

$$= -\frac{2\,\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{(c-d)f} + \frac{(2d)\,\text{Subst}\left(\int \frac{1}{ac+ad-dx^2}\,dx, x,\right.}{(c-d)f}$$

$$= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}(c-d)f} + \frac{2\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}(c-d)\sqrt{c+d}f}$$

Mathematica [C] time = 1.72989, size = 215, normalized size = 1.75

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(\sqrt{d}\left(\log\left(\sec^2\left(\frac{1}{4}(e+fx)\right)\left(\sqrt{c+d}-\sqrt{d}\sin\left(\frac{1}{2}(e+fx)\right)+\sqrt{d}\cos\left(\frac{1}{2}(e+fx)\right)\right)\right)\right.\right.}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])),x]
```

```
[Out] (((2 + 2*I)*(-1)^(3/4)*Sqrt[c + d]*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan
[(e + f*x)/4])] + Sqrt[d]*(Log[Sec[(e + f*x)/4]^2*(Sqrt[c + d] + Sqrt[d]*Co
s[(e + f*x)/2] - Sqrt[d]*Sin[(e + f*x)/2])] - Log[Sec[(e + f*x)/4]^2*(Sqrt[
c + d] - Sqrt[d]*Cos[(e + f*x)/2] + Sqrt[d]*Sin[(e + f*x)/2])]))*(Cos[(e +
f*x)/2] + Sin[(e + f*x)/2]))/((c - d)*Sqrt[c + d]*f*Sqrt[a*(1 + Sin[e + f*x
])])
```

Maple [A] time = 0.947, size = 131, normalized size = 1.1

$$-\frac{1+\sin(fx+e)}{(c-d)\cos(fx+e)f}\sqrt{-a(-1+\sin(fx+e))}\left(-2\,d\,\text{Artanh}\left(\frac{\sqrt{-a(-1+\sin(fx+e))}\,d}{\sqrt{a(c+d)d}}\right)a^{3/2}+\sqrt{2}\,\text{Artanh}\left(\frac{\sqrt{2}}{2}\sqrt{-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e)),x)
```

```
[Out] -(1+sin(f*x+e))*(-a*(-1+sin(f*x+e)))^(1/2)*(-2*d*arctanh((-a*(-1+sin(f*x+e)
))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(3/2)+2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e
)))^(1/2)*2^(1/2)/a^(1/2))*a*(a*(c+d)*d)^(1/2))/(c-d)/(a*(c+d)*d)^(1/2)/a^(
3/2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\sin(fx+e)+a}\left(d\sin(fx+e)+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)), x)
```

Fricas [B] time = 2.5517, size = 1701, normalized size = 13.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(d/(a*c + a*d))*log((d^2*cos(f*x + e)^3 - (6*c*d + 7*d^2)*cos(f*
x + e)^2 - c^2 - 2*c*d - d^2 - 4*((c*d + d^2)*cos(f*x + e)^2 - c^2 - 4*c*d
- 3*d^2 - (c^2 + 3*c*d + 2*d^2)*cos(f*x + e) + (c^2 + 4*c*d + 3*d^2 + (c*d
+ d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d/(a*c + a
*d)) - (c^2 + 8*c*d + 9*d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - c^2 - 2*c
*d - d^2 + 2*(3*c*d + 4*d^2)*cos(f*x + e))*sin(f*x + e))/(d^2*cos(f*x + e)^
3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 - (c^2 + d^2)*cos(f*x
+ e) + (d^2*cos(f*x + e)^2 - 2*c*d*cos(f*x + e) - c^2 - 2*c*d - d^2)*sin(f*
x + e))) + sqrt(2)*log(-(cos(f*x + e)^2 - (cos(f*x + e) - 2)*sin(f*x + e) +
 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*(cos(f*x + e) - sin(f*x + e) + 1)/sqrt(
a) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e)
- cos(f*x + e) - 2))/sqrt(a))/((c - d)*f), 1/2*(2*sqrt(-d/(a*c + a*d))*arct
an(1/2*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) - c - 2*d)*sqrt(-d/(a*c + a
*d))/(d*cos(f*x + e))) - sqrt(2)*log(-(cos(f*x + e)^2 - (cos(f*x + e) - 2)*
sin(f*x + e) + 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*(cos(f*x + e) - sin(f*x +
 e) + 1)/sqrt(a) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)
*sin(f*x + e) - cos(f*x + e) - 2))/sqrt(a))/((c - d)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**(1/2)/(c+d*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.548 $\int \frac{1}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^2}\,dx$

Optimal. Leaf size=175

$$\frac{d\cos(e+fx)}{f(c^2-d^2)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a}f(c-d)^2}+\frac{\sqrt{d}(3c+d)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)}}\right)}{\sqrt{a}f(c-d)^2(c+d)^{3/2}}$$

```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]]
)])/(Sqrt[a]*(c - d)^2*f)) + (Sqrt[d]*(3*c + d)*ArcTanh[(Sqrt[a]*Sqrt[d]*Co
s[e + f*x])/(Sqrt[c + d]*Sqrt[a + a*Sin[e + f*x]])])/(Sqrt[a]*(c - d)^2*(c
+ d)^(3/2)*f) + (d*Cos[e + f*x])/((c^2 - d^2)*f*Sqrt[a + a*Sin[e + f*x]]*(c
 + d*Sin[e + f*x]))
```

Rubi [A] time = 0.424737, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2779, 2985, 2649, 206, 2773, 208}

$$\frac{d\cos(e+fx)}{f(c^2-d^2)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a}f(c-d)^2}+\frac{\sqrt{d}(3c+d)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)}}\right)}{\sqrt{a}f(c-d)^2(c+d)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^2),x]
```


```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]]
)])/(Sqrt[a]*(c - d)^2*f)) + (Sqrt[d]*(3*c + d)*ArcTanh[(Sqrt[a]*Sqrt[d]*Co
s[e + f*x])/(Sqrt[c + d]*Sqrt[a + a*Sin[e + f*x]])])/(Sqrt[a]*(c - d)^2*(c
+ d)^(3/2)*f) + (d*Cos[e + f*x])/((c^2 - d^2)*f*Sqrt[a + a*Sin[e + f*x]]*(c
 + d*Sin[e + f*x]))
```


Rule 2779

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> -Simp[(d*Cos[e + f*x]*(c + d*Sin[e + f*x])^(
n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(2*b*
(n + 1)*(c^2 - d^2)), Int[((c + d*Sin[e + f*x])^(n + 1)*Simp[a*d - 2*b*c*(n
 + 1) + b*d*(2*n + 3)*Sin[e + f*x], x])/Sqrt[a + b*Sin[e + f*x]], x], x] /;
 FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
```

```
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^2}\,dx = \frac{d\cos(e+fx)}{\left(c^2-d^2\right)f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))} + \frac{\int \frac{a(2c+d)-ad\sin(e+fx}{\sqrt{a+a\sin(e+fx)}(c+d\sin(}}{2a\left(c^2-d^2\right)}$$

$$= \frac{d\cos(e+fx)}{\left(c^2-d^2\right)f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))} + \frac{\int \frac{1}{\sqrt{a+a\sin(e+fx)}}\,dx}{(c-d)^2} - \cdots$$

$$= \frac{d\cos(e+fx)}{\left(c^2-d^2\right)f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))} - \frac{2\,\mathrm{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x\right.}{(c-d)^2}$$

$$= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}(c-d)^2f} + \frac{\sqrt{d}(3c+d)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}(c-d)^2(c+d)^{3/2}f}$$

Mathematica [C] time = 3.3845, size = 324, normalized size = 1.85

$$\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(\frac{4d(c-d)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}{(c+d)(c+d\sin(e+fx))} + \frac{\sqrt{d}(3c+d)\left(2\log\left(\sec^2\left(\frac{1}{4}(e+fx)\right)\left(\sqrt{c+d}-\sqrt{d}\sin\left(\frac{1}{2}(e+fx)\right)+\sqrt{d}\cos\right.\right.\right.}{(c+d)^{3/2}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*((8 + 8*I)*(-1)^(3/4)*ArcTanh[(1/2 +
 I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])] + (Sqrt[d]*(3*c + d)*(e + f*x - 2
*Log[Sec[(e + f*x)/4]^2] + 2*Log[Sec[(e + f*x)/4]^2*(Sqrt[c + d] + Sqrt[d]*
Cos[(e + f*x)/2] - Sqrt[d]*Sin[(e + f*x)/2])]))/(c + d)^(3/2) - (Sqrt[d]*(3
*c + d)*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + 2*Log[Sec[(e + f*x)/4]^2*(Sq
rt[c + d] - Sqrt[d]*Cos[(e + f*x)/2] + Sqrt[d]*Sin[(e + f*x)/2])]))/(c + d)
^(3/2) + (4*(c - d)*d*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))/((c + d)*(c +
d*Sin[e + f*x]))))/(4*(c - d)^2*f*Sqrt[a*(1 + Sin[e + f*x])])
```

Maple [B] time = 1.22, size = 453, normalized size = 2.6

$$\frac{1+\sin\left(fx+e\right)}{\left(c-d\right)^{2}\left(c+d\right)\left(c+d\sin\left(fx+e\right)\right)\cos\left(fx+e\right)f}\sqrt{-a\left(-1+\sin\left(fx+e\right)\right)}\left(\sin\left(fx+e\right)\left(3\,\mathrm{Artanh}\left(\frac{\sqrt{a-a\sin\left(fx+\right.}}{\sqrt{acd+ad^{2}}}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^2,x)
```

```
[Out] (1+sin(f*x+e))*(-a*(-1+sin(f*x+e)))^(1/2)/a^(7/2)*(sin(f*x+e)*(3*arctanh((a
-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^(7/2)*c*d^2+a^(7/2)*arctanh((
a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*d^3-(a*(c+d)*d)^(1/2)*2^(1/2)*
arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^3*c*d-(a*(c+d)*d)^(1/
2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^3*d^2)+3*a
rctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^(7/2)*c^2*d+arctanh(
(a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^(7/2)*c*d^2+(a-a*sin(f*x+e)
)^(1/2)*(a*(c+d)*d)^(1/2)*a^(5/2)*c*d-(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1
/2)*a^(5/2)*d^2-(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2
)*2^(1/2)/a^(1/2))*a^3*c^2-(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f
*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^3*c*d)/(c-d)^2/(c+d)/(c+d*sin(f*x+e))/(a*(c
+d)*d)^(1/2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^2,x, algorithm="maxima"
)
```

```
[Out] Timed out
```

Fricas [B] time = 4.34319, size = 3519, normalized size = 20.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^2,x, algorithm="fricas"
)
```

```
[Out] [-1/4*((3*a*c^2 + 4*a*c*d + a*d^2 - (3*a*c*d + a*d^2)*cos(f*x + e)^2 + (3*a
*c^2 + a*c*d)*cos(f*x + e) + (3*a*c^2 + 4*a*c*d + a*d^2 + (3*a*c*d + a*d^2)
*cos(f*x + e))*sin(f*x + e))*sqrt(d/(a*c + a*d))*log((d^2*cos(f*x + e)^3 -
(6*c*d + 7*d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 + 4*((c*d + d^2)*cos(f*x
 + e)^2 - c^2 - 4*c*d - 3*d^2 - (c^2 + 3*c*d + 2*d^2)*cos(f*x + e) + (c^2 +
 4*c*d + 3*d^2 + (c*d + d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e
) + a)*sqrt(d/(a*c + a*d)) - (c^2 + 8*c*d + 9*d^2)*cos(f*x + e) + (d^2*cos(
f*x + e)^2 - c^2 - 2*c*d - d^2 + 2*(3*c*d + 4*d^2)*cos(f*x + e))*sin(f*x +
e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2
```

```
- (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - 2*c*d*cos(f*x + e) - c^2
 - 2*c*d - d^2)*sin(f*x + e))) + 2*sqrt(2)*(a*c^2 + 2*a*c*d + a*d^2 - (a*c*
d + a*d^2)*cos(f*x + e)^2 + (a*c^2 + a*c*d)*cos(f*x + e) + (a*c^2 + 2*a*c*d
 + a*d^2 + (a*c*d + a*d^2)*cos(f*x + e))*sin(f*x + e))*log(-(cos(f*x + e)^2
 - (cos(f*x + e) - 2)*sin(f*x + e) - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*(co
s(f*x + e) - sin(f*x + e) + 1)/sqrt(a) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^
2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2))/sqrt(a) + 4*(c*d -
 d^2 + (c*d - d^2)*cos(f*x + e) - (c*d - d^2)*sin(f*x + e))*sqrt(a*sin(f*x
+ e) + a))/((a*c^3*d - a*c^2*d^2 - a*c*d^3 + a*d^4)*f*cos(f*x + e)^2 - (a*c
^4 - a*c^3*d - a*c^2*d^2 + a*c*d^3)*f*cos(f*x + e) - (a*c^4 - 2*a*c^2*d^2 +
 a*d^4)*f - ((a*c^3*d - a*c^2*d^2 - a*c*d^3 + a*d^4)*f*cos(f*x + e) + (a*c^
4 - 2*a*c^2*d^2 + a*d^4)*f)*sin(f*x + e)), -1/2*((3*a*c^2 + 4*a*c*d + a*d^2
 - (3*a*c*d + a*d^2)*cos(f*x + e)^2 + (3*a*c^2 + a*c*d)*cos(f*x + e) + (3*a
*c^2 + 4*a*c*d + a*d^2 + (3*a*c*d + a*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt
(-d/(a*c + a*d))*arctan(1/2*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) - c -
2*d)*sqrt(-d/(a*c + a*d))/(d*cos(f*x + e))) + sqrt(2)*(a*c^2 + 2*a*c*d + a*
d^2 - (a*c*d + a*d^2)*cos(f*x + e)^2 + (a*c^2 + a*c*d)*cos(f*x + e) + (a*c^
2 + 2*a*c*d + a*d^2 + (a*c*d + a*d^2)*cos(f*x + e))*sin(f*x + e))*log(-(cos
(f*x + e)^2 - (cos(f*x + e) - 2)*sin(f*x + e) - 2*sqrt(2)*sqrt(a*sin(f*x +
e) + a)*(cos(f*x + e) - sin(f*x + e) + 1)/sqrt(a) + 3*cos(f*x + e) + 2)/(co
s(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2))/sqrt(a)
 + 2*(c*d - d^2 + (c*d - d^2)*cos(f*x + e) - (c*d - d^2)*sin(f*x + e))*sqrt
(a*sin(f*x + e) + a))/((a*c^3*d - a*c^2*d^2 - a*c*d^3 + a*d^4)*f*cos(f*x +
e)^2 - (a*c^4 - a*c^3*d - a*c^2*d^2 + a*c*d^3)*f*cos(f*x + e) - (a*c^4 - 2*
a*c^2*d^2 + a*d^4)*f - ((a*c^3*d - a*c^2*d^2 - a*c*d^3 + a*d^4)*f*cos(f*x +
 e) + (a*c^4 - 2*a*c^2*d^2 + a*d^4)*f)*sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**(1/2)/(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.549 $\int \frac{1}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^3}\,dx$

Optimal. Leaf size=247

$$\frac{d(7c+d)\cos(e+fx)}{4f\left(c^2-d^2\right)^2\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}+\frac{d\cos(e+fx)}{2f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^2}+\frac{\sqrt{d}\left(15c^2\right.}{}$$

```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]]
)])/(Sqrt[a]*(c - d)^3*f)) + (Sqrt[d]*(15*c^2 + 10*c*d + 7*d^2)*ArcTanh[(Sq
rt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]*Sqrt[a + a*Sin[e + f*x]])])/(4*Sqr
t[a]*(c - d)^3*(c + d)^(5/2)*f) + (d*Cos[e + f*x])/(2*(c^2 - d^2)*f*Sqrt[a
+ a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^2) + (d*(7*c + d)*Cos[e + f*x])/(4*(
c^2 - d^2)^2*f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x]))
```

Rubi [A] time = 0.732143, antiderivative size = 247, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2779, 2984, 2985, 2649, 206, 2773, 208}

$$\frac{d(7c+d)\cos(e+fx)}{4f\left(c^2-d^2\right)^2\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}+\frac{d\cos(e+fx)}{2f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^2}+\frac{\sqrt{d}\left(15c^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^3),x]
```


```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]]
)])/(Sqrt[a]*(c - d)^3*f)) + (Sqrt[d]*(15*c^2 + 10*c*d + 7*d^2)*ArcTanh[(Sq
rt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]*Sqrt[a + a*Sin[e + f*x]])])/(4*Sqr
t[a]*(c - d)^3*(c + d)^(5/2)*f) + (d*Cos[e + f*x])/(2*(c^2 - d^2)*f*Sqrt[a
+ a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^2) + (d*(7*c + d)*Cos[e + f*x])/(4*(
c^2 - d^2)^2*f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x]))
```


Rule 2779

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> -Simp[(d*Cos[e + f*x]*(c + d*Sin[e + f*x])^(
n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(2*b*
(n + 1)*(c^2 - d^2)), Int[((c + d*Sin[e + f*x])^(n + 1)*Simp[a*d - 2*b*c*(n
 + 1) + b*d*(2*n + 3)*Sin[e + f*x], x])/Sqrt[a + b*Sin[e + f*x]], x], x] /;
 FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
```

```
+ 1/2, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^3}\,dx = \frac{d\cos(e+fx)}{2\left(c^2-d^2\right)f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^2} + \frac{\int \frac{a(4c+d)-3ad\sin(}{\sqrt{a+a\sin(e+fx)}(c+d\,s}}{4a\left(c^2-d\right.}$$

$$= \frac{d\cos(e+fx)}{2\left(c^2-d^2\right)f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^2} + \frac{d(7c}{4\left(c^2-d^2\right)^2 f\sqrt{a+}}$$

$$= \frac{d\cos(e+fx)}{2\left(c^2-d^2\right)f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^2} + \frac{d(7c}{4\left(c^2-d^2\right)^2 f\sqrt{a+}}$$

$$= \frac{d\cos(e+fx)}{2\left(c^2-d^2\right)f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^2} + \frac{d(7c}{4\left(c^2-d^2\right)^2 f\sqrt{a+}}$$

$$= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}(c-d)^3 f} + \frac{\sqrt{d}\left(15c^2+10cd+7d^2\right)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{c+d}\sqrt{a}}\right.}{4\sqrt{a}(c-d)^3(c+d)^{5/2}f}$$

Mathematica [C] time = 4.9607, size = 414, normalized size = 1.68

$$\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(\frac{\sqrt{d}(15c^2+10cd+7d^2)\left(2\log\left(\sec^2\left(\frac{1}{4}(e+fx)\right)\left(\sqrt{c+d}-\sqrt{d}\sin\left(\frac{1}{2}(e+fx)\right)+\sqrt{d}\cos\left(\frac{1}{2}(e+fx)\right)\right)\right)-2\log\left(\sec^2\left(\frac{1}{4}(e\right.\right.\right.}{(c-d)^3(c+d)^{5/2}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^3),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(((32 + 32*I)*(-1)^(3/4)*ArcTanh[(1/
2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])])/(c - d)^3 + (Sqrt[d]*(15*c^2
+ 10*c*d + 7*d^2)*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + 2*Log[Sec[(e + f*x
)/4]^2*(Sqrt[c + d] + Sqrt[d]*Cos[(e + f*x)/2] - Sqrt[d]*Sin[(e + f*x)/2])]
))/((c - d)^3*(c + d)^(5/2)) + (Sqrt[d]*(15*c^2 + 10*c*d + 7*d^2)*(e + f*x
- 2*Log[Sec[(e + f*x)/4]^2] + 2*Log[Sec[(e + f*x)/4]^2*(Sqrt[c + d] - Sqrt[
d]*Cos[(e + f*x)/2] + Sqrt[d]*Sin[(e + f*x)/2])]))/((-c + d)^3*(c + d)^(5/2
)) + (8*d*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))/((c - d)*(c + d)*(c + d*Si
n[e + f*x])^2) + (4*d*(7*c + d)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))/((c
- d)^2*(c + d)^2*(c + d*Sin[e + f*x]))))/(16*f*Sqrt[a*(1 + Sin[e + f*x])])
```

Maple [B] time = 1.689, size = 1066, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^3,x)
```

```
[Out] -1/4*(8*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^
(1/2)/a^(1/2))*a^5*c^3*d-7*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)
^(1/2))*a^(11/2)*sin(f*x+e)^2*d^5-15*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(
a*(c+d)*d)^(1/2))*a^(11/2)*c^4*d-10*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a
*(c+d)*d)^(1/2))*a^(11/2)*c^3*d^2-7*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a
*(c+d)*d)^(1/2))*a^(11/2)*c^2*d^3-(-a*(-1+sin(f*x+e)))^(1/2)*(a*(c+d)*d)^(1
/2)*a^(9/2)*d^4-(-a*(-1+sin(f*x+e)))^(3/2)*(a*(c+d)*d)^(1/2)*a^(7/2)*d^4-15
*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(11/2)*sin(f*x+e
)^2*c^2*d^3-10*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(1
1/2)*sin(f*x+e)^2*c*d^4-30*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)
^(1/2))*a^(11/2)*sin(f*x+e)*c^3*d^2-20*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d
/(a*(c+d)*d)^(1/2))*a^(11/2)*sin(f*x+e)*c^2*d^3-14*arctanh((-a*(-1+sin(f*x+
e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(11/2)*sin(f*x+e)*c*d^4-9*(-a*(-1+sin(f*x
+e)))^(1/2)*(a*(c+d)*d)^(1/2)*a^(9/2)*c^3*d+(-a*(-1+sin(f*x+e)))^(1/2)*(a*(
c+d)*d)^(1/2)*a^(9/2)*c^2*d^2+9*(-a*(-1+sin(f*x+e)))^(1/2)*(a*(c+d)*d)^(1/2
)*a^(9/2)*c*d^3+7*(-a*(-1+sin(f*x+e)))^(3/2)*(a*(c+d)*d)^(1/2)*a^(7/2)*c^2*
d^2-6*(-a*(-1+sin(f*x+e)))^(3/2)*(a*(c+d)*d)^(1/2)*a^(7/2)*c*d^3+4*(a*(c+d)
*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*a
^5*c^4+4*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2
^(1/2)/a^(1/2))*sin(f*x+e)^2*a^5*c^2*d^2+8*(a*(c+d)*d)^(1/2)*2^(1/2)*arctan
h(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*a^5*c*d^3+8*
(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^
(1/2))*sin(f*x+e)*a^5*c^3*d+16*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(-
1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a^5*c^2*d^2+8*(a*(c+d)*d)^
(1/2)*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*sin(f
*x+e)*a^5*c*d^3+4*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e))
)^(1/2)*2^(1/2)/a^(1/2))*a^5*c^2*d^2+4*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/
```

```
2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*a^5*d^4)/a^(11/2
)*(-a*(-1+sin(f*x+e)))^(1/2)*(1+sin(f*x+e))/(a*(c+d)*d)^(1/2)/(c+d*sin(f*x+
e))^2/(c+d)^2/(c-d)^3/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^3,x, algorithm="maxima"
)
```

```
[Out] Timed out
```

Fricas [B] time = 7.32945, size = 6543, normalized size = 26.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^3,x, algorithm="fricas"
)
```

```
[Out] [1/16*((15*a*c^4 + 40*a*c^3*d + 42*a*c^2*d^2 + 24*a*c*d^3 + 7*a*d^4 - (15*a
*c^2*d^2 + 10*a*c*d^3 + 7*a*d^4)*cos(f*x + e)^3 - (30*a*c^3*d + 35*a*c^2*d^
2 + 24*a*c*d^3 + 7*a*d^4)*cos(f*x + e)^2 + (15*a*c^4 + 10*a*c^3*d + 22*a*c^
2*d^2 + 10*a*c*d^3 + 7*a*d^4)*cos(f*x + e) + (15*a*c^4 + 40*a*c^3*d + 42*a*
c^2*d^2 + 24*a*c*d^3 + 7*a*d^4 - (15*a*c^2*d^2 + 10*a*c*d^3 + 7*a*d^4)*cos(
f*x + e)^2 + 2*(15*a*c^3*d + 10*a*c^2*d^2 + 7*a*c*d^3)*cos(f*x + e))*sin(f*
x + e))*sqrt(d/(a*c + a*d))*log((d^2*cos(f*x + e)^3 - (6*c*d + 7*d^2)*cos(f
*x + e)^2 - c^2 - 2*c*d - d^2 - 4*((c*d + d^2)*cos(f*x + e)^2 - c^2 - 4*c*d
 - 3*d^2 - (c^2 + 3*c*d + 2*d^2)*cos(f*x + e) + (c^2 + 4*c*d + 3*d^2 + (c*d
 + d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d/(a*c +
a*d)) - (c^2 + 8*c*d + 9*d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - c^2 - 2*
c*d - d^2 + 2*(3*c*d + 4*d^2)*cos(f*x + e))*sin(f*x + e))/(d^2*cos(f*x + e)
^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 - (c^2 + d^2)*cos(f*x
 + e) + (d^2*cos(f*x + e)^2 - 2*c*d*cos(f*x + e) - c^2 - 2*c*d - d^2)*sin(f
*x + e))) + 8*sqrt(2)*(a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 + a*d^4
- (a*c^2*d^2 + 2*a*c*d^3 + a*d^4)*cos(f*x + e)^3 - (2*a*c^3*d + 5*a*c^2*d^2
 + 4*a*c*d^3 + a*d^4)*cos(f*x + e)^2 + (a*c^4 + 2*a*c^3*d + 2*a*c^2*d^2 + 2
*a*c*d^3 + a*d^4)*cos(f*x + e) + (a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d
^3 + a*d^4 - (a*c^2*d^2 + 2*a*c*d^3 + a*d^4)*cos(f*x + e)^2 + 2*(a*c^3*d +
2*a*c^2*d^2 + a*c*d^3)*cos(f*x + e))*sin(f*x + e))*log(-(cos(f*x + e)^2 - (
cos(f*x + e) - 2)*sin(f*x + e) + 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*(cos(f*
x + e) - sin(f*x + e) + 1)/sqrt(a) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2 -
(cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2))/sqrt(a) - 4*(9*c^3*d -
 15*c^2*d^2 + 3*c*d^3 + 3*d^4 + (7*c^2*d^2 - 6*c*d^3 - d^4)*cos(f*x + e)^2
+ (9*c^3*d - 8*c^2*d^2 - 3*c*d^3 + 2*d^4)*cos(f*x + e) - (9*c^3*d - 15*c^2*
d^2 + 3*c*d^3 + 3*d^4 - (7*c^2*d^2 - 6*c*d^3 - d^4)*cos(f*x + e))*sin(f*x +
 e))*sqrt(a*sin(f*x + e) + a))/((a*c^5*d^2 - a*c^4*d^3 - 2*a*c^3*d^4 + 2*a*
c^2*d^5 + a*c*d^6 - a*d^7)*f*cos(f*x + e)^3 + (2*a*c^6*d - a*c^5*d^2 - 5*a*
c^4*d^3 + 2*a*c^3*d^4 + 4*a*c^2*d^5 - a*c*d^6 - a*d^7)*f*cos(f*x + e)^2 - (
a*c^7 - a*c^6*d - a*c^5*d^2 + a*c^4*d^3 - a*c^3*d^4 + a*c^2*d^5 + a*c*d^6 -
 a*d^7)*f*cos(f*x + e) - (a*c^7 + a*c^6*d - 3*a*c^5*d^2 - 3*a*c^4*d^3 + 3*a
```

```
*c^3*d^4 + 3*a*c^2*d^5 - a*c*d^6 - a*d^7)*f + ((a*c^5*d^2 - a*c^4*d^3 - 2*a
*c^3*d^4 + 2*a*c^2*d^5 + a*c*d^6 - a*d^7)*f*cos(f*x + e)^2 - 2*(a*c^6*d - a
*c^5*d^2 - 2*a*c^4*d^3 + 2*a*c^3*d^4 + a*c^2*d^5 - a*c*d^6)*f*cos(f*x + e)
- (a*c^7 + a*c^6*d - 3*a*c^5*d^2 - 3*a*c^4*d^3 + 3*a*c^3*d^4 + 3*a*c^2*d^5
- a*c*d^6 - a*d^7)*f)*sin(f*x + e)), -1/8*((15*a*c^4 + 40*a*c^3*d + 42*a*c^
2*d^2 + 24*a*c*d^3 + 7*a*d^4 - (15*a*c^2*d^2 + 10*a*c*d^3 + 7*a*d^4)*cos(f*
x + e)^3 - (30*a*c^3*d + 35*a*c^2*d^2 + 24*a*c*d^3 + 7*a*d^4)*cos(f*x + e)^
2 + (15*a*c^4 + 10*a*c^3*d + 22*a*c^2*d^2 + 10*a*c*d^3 + 7*a*d^4)*cos(f*x +
 e) + (15*a*c^4 + 40*a*c^3*d + 42*a*c^2*d^2 + 24*a*c*d^3 + 7*a*d^4 - (15*a*
c^2*d^2 + 10*a*c*d^3 + 7*a*d^4)*cos(f*x + e)^2 + 2*(15*a*c^3*d + 10*a*c^2*d
^2 + 7*a*c*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(-d/(a*c + a*d))*arctan(1/2
*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) - c - 2*d)*sqrt(-d/(a*c + a*d))/(
d*cos(f*x + e))) - 4*sqrt(2)*(a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 +
 a*d^4 - (a*c^2*d^2 + 2*a*c*d^3 + a*d^4)*cos(f*x + e)^3 - (2*a*c^3*d + 5*a*
c^2*d^2 + 4*a*c*d^3 + a*d^4)*cos(f*x + e)^2 + (a*c^4 + 2*a*c^3*d + 2*a*c^2*
d^2 + 2*a*c*d^3 + a*d^4)*cos(f*x + e) + (a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 +
4*a*c*d^3 + a*d^4 - (a*c^2*d^2 + 2*a*c*d^3 + a*d^4)*cos(f*x + e)^2 + 2*(a*c
^3*d + 2*a*c^2*d^2 + a*c*d^3)*cos(f*x + e))*sin(f*x + e))*log(-(cos(f*x + e
)^2 - (cos(f*x + e) - 2)*sin(f*x + e) + 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*
(cos(f*x + e) - sin(f*x + e) + 1)/sqrt(a) + 3*cos(f*x + e) + 2)/(cos(f*x +
e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2))/sqrt(a) + 2*(9*
c^3*d - 15*c^2*d^2 + 3*c*d^3 + 3*d^4 + (7*c^2*d^2 - 6*c*d^3 - d^4)*cos(f*x
+ e)^2 + (9*c^3*d - 8*c^2*d^2 - 3*c*d^3 + 2*d^4)*cos(f*x + e) - (9*c^3*d -
15*c^2*d^2 + 3*c*d^3 + 3*d^4 - (7*c^2*d^2 - 6*c*d^3 - d^4)*cos(f*x + e))*si
n(f*x + e))*sqrt(a*sin(f*x + e) + a))/((a*c^5*d^2 - a*c^4*d^3 - 2*a*c^3*d^4
 + 2*a*c^2*d^5 + a*c*d^6 - a*d^7)*f*cos(f*x + e)^3 + (2*a*c^6*d - a*c^5*d^2
 - 5*a*c^4*d^3 + 2*a*c^3*d^4 + 4*a*c^2*d^5 - a*c*d^6 - a*d^7)*f*cos(f*x + e
)^2 - (a*c^7 - a*c^6*d - a*c^5*d^2 + a*c^4*d^3 - a*c^3*d^4 + a*c^2*d^5 + a*
c*d^6 - a*d^7)*f*cos(f*x + e) - (a*c^7 + a*c^6*d - 3*a*c^5*d^2 - 3*a*c^4*d^
3 + 3*a*c^3*d^4 + 3*a*c^2*d^5 - a*c*d^6 - a*d^7)*f + ((a*c^5*d^2 - a*c^4*d^
3 - 2*a*c^3*d^4 + 2*a*c^2*d^5 + a*c*d^6 - a*d^7)*f*cos(f*x + e)^2 - 2*(a*c^
6*d - a*c^5*d^2 - 2*a*c^4*d^3 + 2*a*c^3*d^4 + a*c^2*d^5 - a*c*d^6)*f*cos(f*
x + e) - (a*c^7 + a*c^6*d - 3*a*c^5*d^2 - 3*a*c^4*d^3 + 3*a*c^3*d^4 + 3*a*c
^2*d^5 - a*c*d^6 - a*d^7)*f)*sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**(1/2)/(c+d*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.550 $\int \frac{(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=192

$$\frac{d^2(3c-7d)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{6a^2f}-\frac{(c+11d)(c-d)^2\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{2\sqrt{2}a^{3/2}f}+\frac{d\left(3c^2-24cd+13d^2\right)\cos(e+}{3af\sqrt{a\sin(e+fx)+a}}$$

```
[Out] -((c - d)^2*(c + 11*d)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*S
in[e + f*x]])])/(2*Sqrt[2]*a^(3/2)*f) + (d*(3*c^2 - 24*c*d + 13*d^2)*Cos[e
+ f*x])/(3*a*f*Sqrt[a + a*Sin[e + f*x]]) + ((3*c - 7*d)*d^2*Cos[e + f*x]*Sq
rt[a + a*Sin[e + f*x]])/(6*a^2*f) - ((c - d)*Cos[e + f*x]*(c + d*Sin[e + f*
x])^2)/(2*f*(a + a*Sin[e + f*x])^(3/2))
```

Rubi [A] time = 0.462758, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2765, 2968, 3023, 2751, 2649, 206}

$$\frac{d^2(3c-7d)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{6a^2f}-\frac{(c+11d)(c-d)^2\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{2\sqrt{2}a^{3/2}f}+\frac{d\left(3c^2-24cd+13d^2\right)\cos(e+}{3af\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^3/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] -((c - d)^2*(c + 11*d)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*S
in[e + f*x]])])/(2*Sqrt[2]*a^(3/2)*f) + (d*(3*c^2 - 24*c*d + 13*d^2)*Cos[e
+ f*x])/(3*a*f*Sqrt[a + a*Sin[e + f*x]]) + ((3*c - 7*d)*d^2*Cos[e + f*x]*Sq
rt[a + a*Sin[e + f*x]])/(6*a^2*f) - ((c - d)*Cos[e + f*x]*(c + d*Sin[e + f*
x])^2)/(2*f*(a + a*Sin[e + f*x])^(3/2))
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
```

```
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^{3/2}}\,dx = -\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^2}{2f(a+a\sin(e+fx))^{3/2}} - \frac{\int \frac{(c+d\sin(e+fx))\left(-\frac{1}{2}a\left(c^2+7cd-4d^2\right)+\frac{1}{2}a(3c-7d)d\sin(e+\ldots}{\sqrt{a+a\sin(e+fx)}}}{2a^2}$$

$$= -\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^2}{2f(a+a\sin(e+fx))^{3/2}} - \frac{\int \frac{-\frac{1}{2}ac\left(c^2+7cd-4d^2\right)+\left(\frac{1}{2}ac(3c-7d)d-\frac{1}{2}ad\left(c^2+7cd-4d^2\right)\right)\ldots}{\sqrt{a+a\sin(e+fx)}}}{2a^2}$$

$$= \frac{(3c-7d)d^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{6a^2f} - \frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^2}{2f(a+a\sin(e+fx))^{3/2}} - \ldots$$

$$= \frac{d\left(3c^2-24cd+13d^2\right)\cos(e+fx)}{3af\sqrt{a+a\sin(e+fx)}} + \frac{(3c-7d)d^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{6a^2f} - \frac{(c-\ldots}{\ldots}$$

$$= \frac{d\left(3c^2-24cd+13d^2\right)\cos(e+fx)}{3af\sqrt{a+a\sin(e+fx)}} + \frac{(3c-7d)d^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{6a^2f} - \frac{(c-\ldots}{\ldots}$$

$$= -\frac{(c-d)^2(c+11d)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{2\sqrt{2}a^{3/2}f} + \frac{d\left(3c^2-24cd+13d^2\right)\cos(e+fx)}{3af\sqrt{a+a\sin(e+fx)}} + \frac{(3c\ldots}{\ldots}$$

Mathematica [C] time = 0.54195, size = 328, normalized size = 1.71

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(-18d^2(2c-d)\cos\left(\frac{1}{2}(e+fx)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^2+18d^2(2c-\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])^3/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(6*(c - d)^3*Sin[(e + f*x)/2] - 3*(c
 - d)^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + (3 + 3*I)*(-1)^(3/4)*(c - d
)^2*(c + 11*d)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])]*(Cos
[(e + f*x)/2] + Sin[(e + f*x)/2])^2 - 18*(2*c - d)*d^2*Cos[(e + f*x)/2]*(Co
s[(e + f*x)/2] + Sin[(e + f*x)/2])^2 - 2*d^3*Cos[(3*(e + f*x))/2]*(Cos[(e +
 f*x)/2] + Sin[(e + f*x)/2])^2 + 18*(2*c - d)*d^2*Sin[(e + f*x)/2]*(Cos[(e
+ f*x)/2] + Sin[(e + f*x)/2])^2 - 2*d^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2
])^2*Sin[(3*(e + f*x))/2]))/(6*f*(a*(1 + Sin[e + f*x]))^(3/2))
```

Maple [B] time = 0.769, size = 490, normalized size = 2.6

$$-\frac{1}{12 f \cos\left(fx+e\right)}\left(\sin\left(fx+e\right)\left(3\sqrt{2}\mathrm{Artanh}\left(1/2\frac{\sqrt{a-a\sin\left(fx+e\right)}\sqrt{2}}{\sqrt{a}}\right)a^2c^3+27\sqrt{2}\mathrm{Artanh}\left(1/2\frac{\sqrt{a-a\sin\left(fx+\right.}}{\sqrt{a}}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(3/2),x)
```

```
[Out] -1/12*(sin(f*x+e)*(3*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(
1/2))*a^2*c^3+27*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2)
)*a^2*c^2*d-63*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*
a^2*c*d^2+33*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^
2*d^3-8*(a-a*sin(f*x+e))^(3/2)*d^3*a^(1/2)+72*a^(3/2)*c*d^2*(a-a*sin(f*x+e)
)^(1/2)-24*d^3*a^(3/2)*(a-a*sin(f*x+e))^(1/2))+3*2^(1/2)*arctanh(1/2*(a-a*s
in(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^3+27*2^(1/2)*arctanh(1/2*(a-a*sin(f
*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^2*d-63*2^(1/2)*arctanh(1/2*(a-a*sin(f*x
+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c*d^2+33*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e
))^(1/2)*2^(1/2)/a^(1/2))*a^2*d^3-8*(a-a*sin(f*x+e))^(3/2)*d^3*a^(1/2)+6*(a
-a*sin(f*x+e))^(1/2)*a^(3/2)*c^3-18*(a-a*sin(f*x+e))^(1/2)*a^(3/2)*c^2*d+90
*a^(3/2)*c*d^2*(a-a*sin(f*x+e))^(1/2)-30*d^3*a^(3/2)*(a-a*sin(f*x+e))^(1/2)
)*(-a*(-1+sin(f*x+e)))^(1/2)/a^(7/2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^3}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*sin(f*x + e) + c)^3/(a*sin(f*x + e) + a)^(3/2), x)
```

Fricas [B] time = 1.76277, size = 1211, normalized size = 6.31

$$-\frac{3\sqrt{2}\left(2c^3+18c^2d-42cd^2+22d^3-\left(c^3+9c^2d-21cd^2+11d^3\right)\cos\left(fx+e\right)^2+\left(c^3+9c^2d-21cd^2+11d^3\right)\cos\left(fx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/24*(3*sqrt(2)*(2*c^3 + 18*c^2*d - 42*c*d^2 + 22*d^3 - (c^3 + 9*c^2*d - 2
1*c*d^2 + 11*d^3)*cos(f*x + e)^2 + (c^3 + 9*c^2*d - 21*c*d^2 + 11*d^3)*cos(
f*x + e) + (2*c^3 + 18*c^2*d - 42*c*d^2 + 22*d^3 + (c^3 + 9*c^2*d - 21*c*d^
2 + 11*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a)*log(-(a*cos(f*x + e)^2 - 2*
sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e) - sin(f*x + e) + 1)
+ 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2*a)/(cos(f*x +
e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) + 4*(4*d^3*cos(
f*x + e)^3 - 3*c^3 + 9*c^2*d - 9*c*d^2 + 3*d^3 - 4*(9*c*d^2 - 4*d^3)*cos(f*
x + e)^2 - 3*(c^3 - 3*c^2*d + 15*c*d^2 - 5*d^3)*cos(f*x + e) - (4*d^3*cos(f
*x + e)^2 - 3*c^3 + 9*c^2*d - 9*c*d^2 + 3*d^3 + 12*(3*c*d^2 - d^3)*cos(f*x
+ e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/(a^2*f*cos(f*x + e)^2 - a^2*f
*cos(f*x + e) - 2*a^2*f - (a^2*f*cos(f*x + e) + 2*a^2*f)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**3/(a+a*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] sage2
```

3.551 $\int \frac{(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=138

$$-\frac{(c-d)(c+7d)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{2\sqrt{2}a^{3/2}f}-\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))}{2f(a\sin(e+fx)+a)^{3/2}}+\frac{d(c-5d)\cos(e+fx)}{2af\sqrt{a\sin(e+fx)+a}}$$

```
[Out] -((c - d)*(c + 7*d)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[
e + f*x]])])/(2*Sqrt[2]*a^(3/2)*f) + ((c - 5*d)*d*Cos[e + f*x])/(2*a*f*Sqrt
[a + a*Sin[e + f*x]]) - ((c - d)*Cos[e + f*x]*(c + d*Sin[e + f*x]))/(2*f*(a
 + a*Sin[e + f*x])^(3/2))
```

Rubi [A] time = 0.215946, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2760, 2751, 2649, 206}

$$-\frac{(c-d)(c+7d)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{2\sqrt{2}a^{3/2}f}-\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))}{2f(a\sin(e+fx)+a)^{3/2}}+\frac{d(c-5d)\cos(e+fx)}{2af\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^2/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] -((c - d)*(c + 7*d)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[
e + f*x]])])/(2*Sqrt[2]*a^(3/2)*f) + ((c - 5*d)*d*Cos[e + f*x])/(2*a*f*Sqrt
[a + a*Sin[e + f*x]]) - ((c - d)*Cos[e + f*x]*(c + d*Sin[e + f*x]))/(2*f*(a
 + a*Sin[e + f*x])^(3/2))
```

Rule 2760

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^2, x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^m*(c + d*Sin[e + f*x]))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*(2*m + 1)),
Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*c*d*(m - 1) + b*(d^2 + c^2*(m + 1))
 + d*(a*d*(m - 1) + b*c*(m + 2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^{3/2}}\,dx = -\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))}{2f(a+a\sin(e+fx))^{3/2}} - \frac{\int \frac{-\frac{1}{2}a\left(c^2+5cd-2d^2\right)+\frac{1}{2}a(c-5d)d\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx}{2a^2}$$

$$= \frac{(c-5d)d\cos(e+fx)}{2af\sqrt{a+a\sin(e+fx)}} - \frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))}{2f(a+a\sin(e+fx))^{3/2}} + \frac{((c-d)(c+7d))\int \frac{}{\sqrt{a}}}{4a}$$

$$= \frac{(c-5d)d\cos(e+fx)}{2af\sqrt{a+a\sin(e+fx)}} - \frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))}{2f(a+a\sin(e+fx))^{3/2}} - \frac{((c-d)(c+7d))\,\mathrm{Subs}}{}$$

$$= -\frac{(c-d)(c+7d)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{2\sqrt{2}a^{3/2}f} + \frac{(c-5d)d\cos(e+fx)}{2af\sqrt{a+a\sin(e+fx)}} - \frac{(c-d)\cos(e+f}{2f(a+a\,\mathrm{s}}$$

Mathematica [C] time = 0.332217, size = 239, normalized size = 1.73

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left((1+i)(-1)^{3/4}\left(c^2+6cd-7d^2\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^2\tanh^{-1}\left(\left(\frac{1}{2}\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])^2/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(2*(c - d)^2*Sin[(e + f*x)/2] - (c -
 d)^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + (1 + I)*(-1)^(3/4)*(c^2 + 6*c
*d - 7*d^2)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])]*(Cos[(e
 + f*x)/2] + Sin[(e + f*x)/2])^2 - 4*d^2*Cos[(e + f*x)/2]*(Cos[(e + f*x)/2]
 + Sin[(e + f*x)/2])^2 + 4*d^2*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e
+ f*x)/2])^2))/(2*f*(a*(1 + Sin[e + f*x]))^(3/2))
```

Maple [B] time = 0.78, size = 316, normalized size = 2.3

$$-\frac{1}{4f\cos(fx+e)}\left(\sin(fx+e)\left(\sqrt{2}\mathrm{Artanh}\left(\frac{\sqrt{2}}{2}\sqrt{a-a\sin(fx+e)}\frac{1}{\sqrt{a}}\right)ac^2+6\sqrt{2}\mathrm{Artanh}\left(1/2\frac{\sqrt{a-a\sin(fx+e)}}{\sqrt{a}}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(3/2),x)
```

```
[Out] -1/4/a^(5/2)*(sin(f*x+e)*(2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2
)/a^(1/2))*a*c^2+6*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/
2))*a*c*d-7*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a*d
^2+8*(a-a*sin(f*x+e))^(1/2)*a^(1/2)*d^2)+2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e
))^(1/2)*2^(1/2)/a^(1/2))*a*c^2+6*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2
)*2^(1/2)/a^(1/2))*a*c*d-7*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/
```

```
2)/a^(1/2))*a*d^2+2*(a-a*sin(f*x+e))^(1/2)*a^(1/2)*c^2-4*(a-a*sin(f*x+e))^(
1/2)*a^(1/2)*c*d+10*(a-a*sin(f*x+e))^(1/2)*a^(1/2)*d^2)*(-a*(-1+sin(f*x+e))
)^(1/2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d \sin\left(fx+e\right)+c\right)^{2}}{\left(a \sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*sin(f*x + e) + c)^2/(a*sin(f*x + e) + a)^(3/2), x)
```

Fricas [B] time = 1.69076, size = 953, normalized size = 6.91

$$-\frac{\sqrt{2}\left(\left(c^{2}+6\,cd-7\,d^{2}\right)\cos\left(fx+e\right)^{2}-2\,c^{2}-12\,cd+14\,d^{2}-\left(c^{2}+6\,cd-7\,d^{2}\right)\cos\left(fx+e\right)-\left(2\,c^{2}+12\,cd-14\,d^{2}+\left(c^{2}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/8*(sqrt(2)*((c^2 + 6*c*d - 7*d^2)*cos(f*x + e)^2 - 2*c^2 - 12*c*d + 14*d
^2 - (c^2 + 6*c*d - 7*d^2)*cos(f*x + e) - (2*c^2 + 12*c*d - 14*d^2 + (c^2 +
 6*c*d - 7*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a)*log(-(a*cos(f*x + e)^2
+ 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e) - sin(f*x + e) +
 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2*a)/(cos(f*
x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) - 4*(4*d^2*
cos(f*x + e)^2 + c^2 - 2*c*d + d^2 + (c^2 - 2*c*d + 5*d^2)*cos(f*x + e) + (
4*d^2*cos(f*x + e) - c^2 + 2*c*d - d^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) +
 a))/(a^2*f*cos(f*x + e)^2 - a^2*f*cos(f*x + e) - 2*a^2*f - (a^2*f*cos(f*x
+ e) + 2*a^2*f)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**2/(a+a*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 3.34741, size = 911, normalized size = 6.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(4*(d^2*tan(1/2*f*x + 1/2*e)/(a*sgn(tan(1/2*f*x + 1/2*e) + 1)) - d^2/(a
*sgn(tan(1/2*f*x + 1/2*e) + 1)))/sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a) + sqrt(
2)*(c^2 + 6*c*d - 7*d^2)*arctan(-1/2*sqrt(2)*(sqrt(a)*tan(1/2*f*x + 1/2*e)
- sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a) + sqrt(a))/sqrt(-a))/(sqrt(-a)*a*sgn(t
an(1/2*f*x + 1/2*e) + 1)) + 2*(3*(sqrt(a)*tan(1/2*f*x + 1/2*e) - sqrt(a*tan
(1/2*f*x + 1/2*e)^2 + a))^3*c^2 - 6*(sqrt(a)*tan(1/2*f*x + 1/2*e) - sqrt(a*
tan(1/2*f*x + 1/2*e)^2 + a))^3*c*d + 3*(sqrt(a)*tan(1/2*f*x + 1/2*e) - sqrt
(a*tan(1/2*f*x + 1/2*e)^2 + a))^3*d^2 + (sqrt(a)*tan(1/2*f*x + 1/2*e) - sqr
t(a*tan(1/2*f*x + 1/2*e)^2 + a))^2*sqrt(a)*c^2 - 2*(sqrt(a)*tan(1/2*f*x + 1
/2*e) - sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a))^2*sqrt(a)*c*d + (sqrt(a)*tan(1/
2*f*x + 1/2*e) - sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a))^2*sqrt(a)*d^2 - (sqrt(
a)*tan(1/2*f*x + 1/2*e) - sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a))*a*c^2 + 2*(sq
rt(a)*tan(1/2*f*x + 1/2*e) - sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a))*a*c*d - (s
qrt(a)*tan(1/2*f*x + 1/2*e) - sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a))*a*d^2 + a
^(3/2)*c^2 - 2*a^(3/2)*c*d + a^(3/2)*d^2)/(((sqrt(a)*tan(1/2*f*x + 1/2*e) -
 sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a))^2 + 2*(sqrt(a)*tan(1/2*f*x + 1/2*e) -
sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a))*sqrt(a) - a)^2*a*sgn(tan(1/2*f*x + 1/2*
e) + 1)))/f
```

3.552 $\int \frac{c+d\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=87

$$-\frac{(c+3d)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{2\sqrt{2}a^{3/2}f}-\frac{(c-d)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}}$$

```
[Out] -((c + 3*d)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]
])])/(2*Sqrt[2]*a^(3/2)*f) - ((c - d)*Cos[e + f*x])/(2*f*(a + a*Sin[e + f*x
])^(3/2))
```

Rubi [A] time = 0.0727975, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2750, 2649, 206}

$$-\frac{(c+3d)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{2\sqrt{2}a^{3/2}f}-\frac{(c-d)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] -((c + 3*d)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]
])])/(2*Sqrt[2]*a^(3/2)*f) - ((c - d)*Cos[e + f*x])/(2*f*(a + a*Sin[e + f*x
])^(3/2))
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{c+d\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx = -\frac{(c-d)\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}} + \frac{(c+3d)\int \frac{1}{\sqrt{a+a\sin(e+fx)}}\,dx}{4a}$$

$$= -\frac{(c-d)\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}} - \frac{(c+3d)\,\mathrm{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{2af}$$

$$= -\frac{(c+3d)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{2\sqrt{2}a^{3/2}f} - \frac{(c-d)\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}}$$

Mathematica [C] time = 0.197537, size = 150, normalized size = 1.72

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(2(c-d)\sin\left(\frac{1}{2}(e+fx)\right)+(d-c)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)+(1+i)(-\right.}{2f(a(\sin(e+fx)+1))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(2*(c - d)*Sin[(e + f*x)/2] + (-c +
d)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + (1 + I)*(-1)^(3/4)*(c + 3*d)*Arc
Tanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] + Si
n[(e + f*x)/2])^2))/(2*f*(a*(1 + Sin[e + f*x]))^(3/2))
```

Maple [B] time = 0.54, size = 176, normalized size = 2.

$$-\frac{1}{4f\cos(fx+e)}\left(\sin(fx+e)\sqrt{2}\mathrm{Artanh}\left(\frac{\sqrt{2}}{2}\sqrt{a-a\sin(fx+e)}\frac{1}{\sqrt{a}}\right)a(c+3d)+\sqrt{2}\mathrm{Artanh}\left(\frac{\sqrt{2}}{2}\sqrt{a-a\sin(fx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2),x)
```

```
[Out] -1/4/a^(5/2)*(sin(f*x+e)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)
/a^(1/2))*a*(c+3*d)+2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1
/2))*a*c+3*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a*d+
2*(a-a*sin(f*x+e))^(1/2)*a^(1/2)*c-2*(a-a*sin(f*x+e))^(1/2)*a^(1/2)*d)*(-a*
(-1+sin(f*x+e)))^(1/2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{d\sin(fx+e)+c}{(a\sin(fx+e)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*sin(f*x + e) + c)/(a*sin(f*x + e) + a)^(3/2), x)
```

Fricas [B] time = 1.69843, size = 755, normalized size = 8.68

$$\frac{\sqrt{2}\left((c+3\,d)\cos\left(fx+e\right)^{2}-(c+3\,d)\cos\left(fx+e\right)-\left((c+3\,d)\cos\left(fx+e\right)+2\,c+6\,d\right)\sin\left(fx+e\right)-2\,c-6\,d\right)\sqrt{a}\log\left(-\right.}{8\left(a^{2}f\cos\left(fx+e\right)^{2}-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/8*(sqrt(2)*((c + 3*d)*cos(f*x + e)^2 - (c + 3*d)*cos(f*x + e) - ((c + 3*d
)*cos(f*x + e) + 2*c + 6*d)*sin(f*x + e) - 2*c - 6*d)*sqrt(a)*log(-(a*cos(f
*x + e)^2 - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e) - sin(
f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2*
a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) +
 4*((c - d)*cos(f*x + e) - (c - d)*sin(f*x + e) + c - d)*sqrt(a*sin(f*x + e
) + a))/(a^2*f*cos(f*x + e)^2 - a^2*f*cos(f*x + e) - 2*a^2*f - (a^2*f*cos(f
*x + e) + 2*a^2*f)*sin(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{c+d\sin\left(e+fx\right)}{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))/(a+a*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral((c + d*sin(e + f*x))/(a*(sin(e + f*x) + 1))**(3/2), x)
```

Giac [B] time = 2.93497, size = 599, normalized size = 6.89

$$\frac{\sqrt{2}(c+3\,d)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{a}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-\sqrt{a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+a}+\sqrt{a}\right)}{2\,\sqrt{-a}}\right)}{\sqrt{-a}a\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)}+\frac{2\left(3\left(\sqrt{a}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-\sqrt{a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+a}\right)^{3}c-3\left(\sqrt{a}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-\sqrt{a\tan\left(\frac{1}{2}\,f\right.}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(sqrt(2)*(c + 3*d)*arctan(-1/2*sqrt(2)*(sqrt(a)*tan(1/2*f*x + 1/2*e) -
sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a) + sqrt(a))/sqrt(-a))/(sqrt(-a)*a*sgn(tan
(1/2*f*x + 1/2*e) + 1)) + 2*(3*(sqrt(a)*tan(1/2*f*x + 1/2*e) - sqrt(a*tan(1
/2*f*x + 1/2*e)^2 + a))^3*c - 3*(sqrt(a)*tan(1/2*f*x + 1/2*e) - sqrt(a*tan(
```

```
1/2*f*x + 1/2*e)^2 + a))^3*d + (sqrt(a)*tan(1/2*f*x + 1/2*e) - sqrt(a*tan(1
/2*f*x + 1/2*e)^2 + a))^2*sqrt(a)*c - (sqrt(a)*tan(1/2*f*x + 1/2*e) - sqrt(
a*tan(1/2*f*x + 1/2*e)^2 + a))^2*sqrt(a)*d - (sqrt(a)*tan(1/2*f*x + 1/2*e)
- sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a))*a*c + (sqrt(a)*tan(1/2*f*x + 1/2*e) -
 sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a))*a*d + a^(3/2)*c - a^(3/2)*d)/(((sqrt(a
)*tan(1/2*f*x + 1/2*e) - sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a))^2 + 2*(sqrt(a)
*tan(1/2*f*x + 1/2*e) - sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a))*sqrt(a) - a)^2*
a*sgn(tan(1/2*f*x + 1/2*e) + 1)))/f
```

3.553 $\int \frac{1}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=77

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{2\sqrt{2}a^{3/2}f}-\frac{\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}}$$

```
[Out] -ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])]/(2*Sqrt
[2]*a^(3/2)*f) - Cos[e + f*x]/(2*f*(a + a*Sin[e + f*x])^(3/2))
```

Rubi [A] time = 0.0405825, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2650, 2649, 206}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{2\sqrt{2}a^{3/2}f}-\frac{\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(-3/2),x]
```

```
[Out] -ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])]/(2*Sqrt
[2]*a^(3/2)*f) - Cos[e + f*x]/(2*f*(a + a*Sin[e + f*x])^(3/2))
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^{3/2}}\,dx = -\frac{\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}} + \frac{\int \frac{1}{\sqrt{a+a\sin(e+fx)}}\,dx}{4a}$$
$$= -\frac{\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}} - \frac{\operatorname{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{2af}$$
$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{2\sqrt{2}a^{3/2}f} - \frac{\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}}$$

Mathematica [C] time = 0.178384, size = 108, normalized size = 1.4

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)-\cos\left(\frac{1}{2}(e+fx)\right)+(1+i)(-1)^{3/4}(\sin(e+fx)+1)\tanh^{-1}\left(\left(\frac{1}{2}+\right.\right.\right.}{2f(a(\sin(e+fx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(-3/2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-Cos[(e + f*x)/2] + Sin[(e + f*x)/2
] + (1 + I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4
])]*(1 + Sin[e + f*x])))/(2*f*(a*(1 + Sin[e + f*x]))^(3/2))
```

Maple [A] time = 0.609, size = 125, normalized size = 1.6

$$-\frac{1}{4f\cos(fx+e)}\left(\sqrt{2}\mathrm{Artanh}\left(\frac{\sqrt{2}}{2}\sqrt{a-a\sin(fx+e)}\frac{1}{\sqrt{a}}\right)a^2\sin(fx+e)+2\sqrt{a-a\sin(fx+e)}a^{3/2}+\sqrt{2}\mathrm{Artanh}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^(3/2),x)
```

```
[Out] -1/4/a^(7/2)*(2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a
^2*sin(f*x+e)+2*(a-a*sin(f*x+e))^(1/2)*a^(3/2)+2^(1/2)*arctanh(1/2*(a-a*sin
(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2)*(-a*(-1+sin(f*x+e)))^(1/2)/cos(f*x+e)/
(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\sin(fx+e)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(-3/2), x)
```

Fricas [B] time = 1.62785, size = 666, normalized size = 8.65

$$\frac{\sqrt{2}\left(\cos\left(fx+e\right)^2-\left(\cos\left(fx+e\right)+2\right)\sin\left(fx+e\right)-\cos\left(fx+e\right)-2\right)\sqrt{a}\log\left(-\frac{a\cos\left(fx+e\right)^2-2\sqrt{2}\sqrt{a\sin\left(fx+e\right)+a}\sqrt{a}\left(\cos\left(fx+e\right)-\cdots\right.}{\cos\left(fx+e\right)^2-\left(\cos\left(fx\cdots\right.\right.}\right.}{8\left(a^2f\cos\left(fx+e\right)^2-a^2f\cos\left(fx+e\right)-2a^2f-\left(a\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/8*(sqrt(2)*(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x +
e) - 2)*sqrt(a)*log(-(a*cos(f*x + e)^2 - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)
*sqrt(a)*(cos(f*x + e) - sin(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x
+ e) - 2*a)*sin(f*x + e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*
x + e) - cos(f*x + e) - 2)) + 4*sqrt(a*sin(f*x + e) + a)*(cos(f*x + e) - si
n(f*x + e) + 1))/(a^2*f*cos(f*x + e)^2 - a^2*f*cos(f*x + e) - 2*a^2*f - (a^
2*f*cos(f*x + e) + 2*a^2*f)*sin(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\sin\left(e+fx\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral((a*sin(e + f*x) + a)**(-3/2), x)
```

Giac [B] time = 2.6763, size = 414, normalized size = 5.38

$$\frac{\frac{\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}+\sqrt{a}\right)}{2\sqrt{-a}}\right)}{\sqrt{-a}a\operatorname{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}+\frac{2\left(3\left(\sqrt{a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^3+\left(\sqrt{a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+\cdots}\right.\right.}{\left(\left(\sqrt{a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2+2\left(\sqrt{a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{a\tan\left(\cdots\right.}\right.\right.}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(a)*tan(1/2*f*x + 1/2*e) - sqrt(a*tan
(1/2*f*x + 1/2*e)^2 + a) + sqrt(a))/sqrt(-a))/(sqrt(-a)*a*sgn(tan(1/2*f*x +
 1/2*e) + 1)) + 2*(3*(sqrt(a)*tan(1/2*f*x + 1/2*e) - sqrt(a*tan(1/2*f*x + 1
/2*e)^2 + a))^3 + (sqrt(a)*tan(1/2*f*x + 1/2*e) - sqrt(a*tan(1/2*f*x + 1/2*
e)^2 + a))^2*sqrt(a) - (sqrt(a)*tan(1/2*f*x + 1/2*e) - sqrt(a*tan(1/2*f*x +
 1/2*e)^2 + a))*a + a^(3/2))/(((sqrt(a)*tan(1/2*f*x + 1/2*e) - sqrt(a*tan(1
/2*f*x + 1/2*e)^2 + a))^2 + 2*(sqrt(a)*tan(1/2*f*x + 1/2*e) - sqrt(a*tan(1/
2*f*x + 1/2*e)^2 + a))*sqrt(a) - a)^2*a*sgn(tan(1/2*f*x + 1/2*e) + 1)))/f
```

3.554 $\int \frac{1}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))}\,dx$

Optimal. Leaf size=164

$$-\frac{2d^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{a^{3/2}f(c-d)^2\sqrt{c+d}}-\frac{(c-5d)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{2\sqrt{2}a^{3/2}f(c-d)^2}-\frac{\cos(e+fx)}{2f(c-d)(a\sin(e+fx)+a)^{3/2}}$$

```
[Out] -((c - 5*d)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]
])])/(2*Sqrt[2]*a^(3/2)*(c - d)^2*f) - (2*d^(3/2)*ArcTanh[(Sqrt[a]*Sqrt[d]*
Cos[e + f*x])/(Sqrt[c + d]*Sqrt[a + a*Sin[e + f*x]])])/(a^(3/2)*(c - d)^2*S
qrt[c + d]*f) - Cos[e + f*x]/(2*(c - d)*f*(a + a*Sin[e + f*x])^(3/2))
```

Rubi [A] time = 0.416588, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2766, 2985, 2649, 206, 2773, 208}

$$-\frac{2d^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{a^{3/2}f(c-d)^2\sqrt{c+d}}-\frac{(c-5d)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{2\sqrt{2}a^{3/2}f(c-d)^2}-\frac{\cos(e+fx)}{2f(c-d)(a\sin(e+fx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])),x]
```

```
[Out] -((c - 5*d)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]
])])/(2*Sqrt[2]*a^(3/2)*(c - d)^2*f) - (2*d^(3/2)*ArcTanh[(Sqrt[a]*Sqrt[d]*
Cos[e + f*x])/(Sqrt[c + d]*Sqrt[a + a*Sin[e + f*x]])])/(a^(3/2)*(c - d)^2*S
qrt[c + d]*f) - Cos[e + f*x]/(2*(c - d)*f*(a + a*Sin[e + f*x])^(3/2))
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))}\,dx = -\frac{\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}} - \frac{\int \frac{-\frac{1}{2}a(c-4d)-\frac{1}{2}ad\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))}\,dx}{2a^2(c-d)}$$

$$= -\frac{\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}} + \frac{(c-5d)\int \frac{1}{\sqrt{a+a\sin(e+fx)}}\,dx}{4a(c-d)^2} + \frac{d^2\int \frac{\sqrt{a}}{c+}}{a^2}$$

$$= -\frac{\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}} - \frac{(c-5d)\,\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \frac{a\cos(e}{\sqrt{a+a\sin}}\right.}{2a(c-d)^2f}$$

$$= -\frac{(c-5d)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{2\sqrt{2}a^{3/2}(c-d)^2f} - \frac{2d^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{a^{3/2}(c-d)^2\sqrt{c+d}f} -$$

Mathematica [C] time = 1.78281, size = 385, normalized size = 2.35

$$\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(-\frac{d^{3/2}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^2\left(2\log\left(\sec^2\left(\frac{1}{4}(e+fx)\right)\left(\sqrt{c+d}-\sqrt{d}\sin\left(\frac{1}{2}(e+fx)\right)+\sqrt{d}\cos\left(\frac{1}{2}(e+fx)\right)\right)\right)-2\right.}{\sqrt{c+d}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(2*(c - d)*Sin[(e + f*x)/2] - (c - d
)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + (1 + I)*(-1)^(3/4)*(c - 5*d)*ArcT
anh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] + Sin
[(e + f*x)/2])^2 - (d^(3/2)*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + 2*Log[Se
c[(e + f*x)/4]^2*(Sqrt[c + d] + Sqrt[d]*Cos[(e + f*x)/2] - Sqrt[d]*Sin[(e +
 f*x)/2])])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2)/Sqrt[c + d] + (d^(3/2)
*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + 2*Log[Sec[(e + f*x)/4]^2*(Sqrt[c +
d] - Sqrt[d]*Cos[(e + f*x)/2] + Sqrt[d]*Sin[(e + f*x)/2])])*(Cos[(e + f*x)/
2] + Sin[(e + f*x)/2])^2)/Sqrt[c + d]))/(2*(c - d)^2*f*(a*(1 + Sin[e + f*x]
))^(3/2))
```

Maple [B] time = 0.836, size = 338, normalized size = 2.1

$$-\frac{1}{4 (c-d)^2 \cos \left(f x+e\right) f}\left(\sin \left(f x+e\right)\left(8 d^2 \mathrm{Artanh}\left(\frac{\sqrt{a-a \sin \left(f x+e\right)}\, d}{\sqrt{a c d+a d^2}}\right) a^{3/2}+\sqrt{a (c+d) d}\,\sqrt{2}\,\mathrm{Artanh}\left(\frac{\sqrt{2}}{2}\sqrt{a-\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e)),x)
```

```
[Out] -1/4/a^(5/2)*(sin(f*x+e)*(8*d^2*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d
^2)^(1/2))*a^(3/2)+(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(
1/2)*2^(1/2)/a^(1/2))*a*c-5*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(
f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a*d)+8*d^2*arctanh((a-a*sin(f*x+e))^(1/2)*d/
(a*c*d+a*d^2)^(1/2))*a^(3/2)+(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin
(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a*c-5*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2
*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a*d+2*(a*(c+d)*d)^(1/2)*(a-a*sin(f
*x+e))^(1/2)*a^(1/2)*c-2*(a*(c+d)*d)^(1/2)*(a-a*sin(f*x+e))^(1/2)*a^(1/2)*d
)*(-a*(-1+sin(f*x+e)))^(1/2)/(a*(c+d)*d)^(1/2)/(c-d)^2/cos(f*x+e)/(a+a*sin(
f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a \sin \left(f x+e\right)+a\right)^{\frac{3}{2}}\left(d \sin \left(f x+e\right)+c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e) + c)), x)
```

Fricas [B] time = 3.96197, size = 3133, normalized size = 19.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] [-1/8*(sqrt(2)*((c - 5*d)*cos(f*x + e)^2 - (c - 5*d)*cos(f*x + e) - ((c - 5
*d)*cos(f*x + e) + 2*c - 10*d)*sin(f*x + e) - 2*c + 10*d)*sqrt(a)*log(-(a*c
os(f*x + e)^2 + 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e) -
sin(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e)
+ 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2
)) - 4*(a*d*cos(f*x + e)^2 - a*d*cos(f*x + e) - 2*a*d - (a*d*cos(f*x + e) +
 2*a*d)*sin(f*x + e))*sqrt(d/(a*c + a*d))*log((d^2*cos(f*x + e)^3 - (6*c*d
+ 7*d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 - 4*((c*d + d^2)*cos(f*x + e)^2
 - c^2 - 4*c*d - 3*d^2 - (c^2 + 3*c*d + 2*d^2)*cos(f*x + e) + (c^2 + 4*c*d
+ 3*d^2 + (c*d + d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*
sqrt(d/(a*c + a*d)) - (c^2 + 8*c*d + 9*d^2)*cos(f*x + e) + (d^2*cos(f*x + e
)^2 - c^2 - 2*c*d - d^2 + 2*(3*c*d + 4*d^2)*cos(f*x + e))*sin(f*x + e))/(d^
2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 - (c^2
```

```
+ d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - 2*c*d*cos(f*x + e) - c^2 - 2*c*
d - d^2)*sin(f*x + e))) - 4*((c - d)*cos(f*x + e) - (c - d)*sin(f*x + e) +
c - d)*sqrt(a*sin(f*x + e) + a))/((a^2*c^2 - 2*a^2*c*d + a^2*d^2)*f*cos(f*x
 + e)^2 - (a^2*c^2 - 2*a^2*c*d + a^2*d^2)*f*cos(f*x + e) - 2*(a^2*c^2 - 2*a
^2*c*d + a^2*d^2)*f - ((a^2*c^2 - 2*a^2*c*d + a^2*d^2)*f*cos(f*x + e) + 2*(
a^2*c^2 - 2*a^2*c*d + a^2*d^2)*f)*sin(f*x + e)), -1/8*(sqrt(2)*((c - 5*d)*c
os(f*x + e)^2 - (c - 5*d)*cos(f*x + e) - ((c - 5*d)*cos(f*x + e) + 2*c - 10
*d)*sin(f*x + e) - 2*c + 10*d)*sqrt(a)*log(-(a*cos(f*x + e)^2 + 2*sqrt(2)*s
qrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e) - sin(f*x + e) + 1) + 3*a*cos
(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2*a)/(cos(f*x + e)^2 - (c
os(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) + 8*(a*d*cos(f*x + e)^2
- a*d*cos(f*x + e) - 2*a*d - (a*d*cos(f*x + e) + 2*a*d)*sin(f*x + e))*sqrt(
-d/(a*c + a*d))*arctan(1/2*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) - c - 2
*d)*sqrt(-d/(a*c + a*d))/(d*cos(f*x + e))) - 4*((c - d)*cos(f*x + e) - (c -
 d)*sin(f*x + e) + c - d)*sqrt(a*sin(f*x + e) + a))/((a^2*c^2 - 2*a^2*c*d +
 a^2*d^2)*f*cos(f*x + e)^2 - (a^2*c^2 - 2*a^2*c*d + a^2*d^2)*f*cos(f*x + e)
 - 2*(a^2*c^2 - 2*a^2*c*d + a^2*d^2)*f - ((a^2*c^2 - 2*a^2*c*d + a^2*d^2)*f
*cos(f*x + e) + 2*(a^2*c^2 - 2*a^2*c*d + a^2*d^2)*f)*sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**(3/2)/(c+d*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.555 $\int \frac{1}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^2}\,dx$

Optimal. Leaf size=243

$$-\frac{d^{3/2}(5c+3d)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{a^{3/2}f(c-d)^3(c+d)^{3/2}}-\frac{(c-9d)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{2\sqrt{2}a^{3/2}f(c-d)^3}-\frac{d(c+3d)\cos(e+f}{2af(c-d)^2(c+d)\sqrt{a\sin(e+fx)+}}$$

```
[Out] -((c - 9*d)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]
])])/(2*Sqrt[2]*a^(3/2)*(c - d)^3*f) - (d^(3/2)*(5*c + 3*d)*ArcTanh[(Sqrt[a
]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]*Sqrt[a + a*Sin[e + f*x]])])/(a^(3/2)*(
c - d)^3*(c + d)^(3/2)*f) - Cos[e + f*x]/(2*(c - d)*f*(a + a*Sin[e + f*x])^
(3/2)*(c + d*Sin[e + f*x])) - (d*(c + 3*d)*Cos[e + f*x])/(2*a*(c - d)^2*(c
+ d)*f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x]))
```

Rubi [A] time = 0.742985, antiderivative size = 243, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2766, 2984, 2985, 2649, 206, 2773, 208}

$$-\frac{d^{3/2}(5c+3d)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{a^{3/2}f(c-d)^3(c+d)^{3/2}}-\frac{(c-9d)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{2\sqrt{2}a^{3/2}f(c-d)^3}-\frac{d(c+3d)\cos(e+f}{2af(c-d)^2(c+d)\sqrt{a\sin(e+fx)+}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^2),x]
```

```
[Out] -((c - 9*d)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]
])])/(2*Sqrt[2]*a^(3/2)*(c - d)^3*f) - (d^(3/2)*(5*c + 3*d)*ArcTanh[(Sqrt[a
]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]*Sqrt[a + a*Sin[e + f*x]])])/(a^(3/2)*(
c - d)^3*(c + d)^(3/2)*f) - Cos[e + f*x]/(2*(c - d)*f*(a + a*Sin[e + f*x])^
(3/2)*(c + d*Sin[e + f*x])) - (d*(c + 3*d)*Cos[e + f*x])/(2*a*(c - d)^2*(c
+ d)*f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x]))
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
```

```
+ 1/2, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^2}\,dx = -\frac{\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))} - \frac{\int \frac{-\frac{1}{2}a(c-6d)-\frac{3}{2}ad}{\sqrt{a+a\sin(e+fx)}(c+}}{2a^2(c-}$$

$$= -\frac{\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))} - \frac{}{2a(c-d)^2(c+d)f}$$

$$= -\frac{\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))} - \frac{}{2a(c-d)^2(c+d)f}$$

$$= -\frac{\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))} - \frac{}{2a(c-d)^2(c+d)f}$$

$$= -\frac{(c-9d)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{2\sqrt{2}a^{3/2}(c-d)^3f} - \frac{d^{3/2}(5c+3d)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(}{\sqrt{c+d}\sqrt{a+a\sin}}\right)}{a^{3/2}(c-d)^3(c+d)^{3/2}f}$$

Mathematica [C] time = 4.54375, size = 491, normalized size = 2.02

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(-\frac{4d^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^2}{(c-d)^2(c+d)(c+d\sin(e+fx))}+\frac{d^{3/2}(5c+3d)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\right.}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*((4*Sin[(e + f*x)/2])/(c - d)^2 - (2
*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))/(c - d)^2 + ((2 + 2*I)*(-1)^(3/4)*(
c - 9*d)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])]*(Cos[(e +
f*x)/2] + Sin[(e + f*x)/2])^2)/(c - d)^3 + (d^(3/2)*(5*c + 3*d)*(e + f*x -
2*Log[Sec[(e + f*x)/4]^2] + 2*Log[Sec[(e + f*x)/4]^2*(Sqrt[c + d] + Sqrt[d]
*Cos[(e + f*x)/2] - Sqrt[d]*Sin[(e + f*x)/2])])*(Cos[(e + f*x)/2] + Sin[(e
+ f*x)/2])^2)/((-c + d)^3*(c + d)^(3/2)) + (d^(3/2)*(5*c + 3*d)*(e + f*x -
2*Log[Sec[(e + f*x)/4]^2] + 2*Log[Sec[(e + f*x)/4]^2*(Sqrt[c + d] - Sqrt[d]
*Cos[(e + f*x)/2] + Sqrt[d]*Sin[(e + f*x)/2])])*(Cos[(e + f*x)/2] + Sin[(e
+ f*x)/2])^2)/((c - d)^3*(c + d)^(3/2)) - (4*d^2*(Cos[(e + f*x)/2] - Sin[(e
 + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2)/((c - d)^2*(c + d)*(c
+ d*Sin[e + f*x]))))/(4*f*(a*(1 + Sin[e + f*x]))^(3/2))
```

Maple [B] time = 1.296, size = 978, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^2,x)
```

```
[Out] -1/4/a^(5/2)*((a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1
/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*a*c^2*d-8*(a*(c+d)*d)^(1/2)*2^(1/2)*arcta
nh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*a*c*d^2-9*(
a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(
1/2))*sin(f*x+e)^2*a*d^3+20*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d
)^(1/2))*a^(3/2)*sin(f*x+e)^2*c*d^3+12*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d
/(a*(c+d)*d)^(1/2))*a^(3/2)*sin(f*x+e)^2*d^4+(a*(c+d)*d)^(1/2)*2^(1/2)*arct
anh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a*c^3-7*(a*(
c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2
))*sin(f*x+e)*a*c^2*d-17*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(-1+sin(
f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a*c*d^2-9*(a*(c+d)*d)^(1/2)*2^(1
/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a*d^
3+20*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(3/2)*sin(f*
x+e)*c^2*d^2+32*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(
3/2)*sin(f*x+e)*c*d^3+12*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(
1/2))*a^(3/2)*sin(f*x+e)*d^4+2*(-a*(-1+sin(f*x+e)))^(1/2)*(a*(c+d)*d)^(1/2)
*a^(1/2)*sin(f*x+e)*c^2*d+4*(-a*(-1+sin(f*x+e)))^(1/2)*(a*(c+d)*d)^(1/2)*a^
(1/2)*sin(f*x+e)*c*d^2-6*(-a*(-1+sin(f*x+e)))^(1/2)*(a*(c+d)*d)^(1/2)*a^(1/
2)*sin(f*x+e)*d^3+(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e))
)^(1/2)*2^(1/2)/a^(1/2))*a*c^3-8*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*
(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*a*c^2*d-9*(a*(c+d)*d)^(1/2)*2^(1/2)
*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*a*c*d^2+20*arctanh
((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(3/2)*c^2*d^2+12*arctanh
((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(3/2)*c*d^3+2*(-a*(-1+si
n(f*x+e)))^(1/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*c^3+2*(-a*(-1+sin(f*x+e)))^(1/2)
```

```
*(a*(c+d)*d)^(1/2)*a^(1/2)*c*d^2-4*(-a*(-1+sin(f*x+e)))^(1/2)*(a*(c+d)*d)^(
1/2)*a^(1/2)*d^3)*(-a*(-1+sin(f*x+e)))^(1/2)/(a*(c+d)*d)^(1/2)/(c+d*sin(f*x
+e))/(c+d)/(c-d)^3/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^2,x, algorithm="maxima"
)

[Out] Timed out
```

Fricas [B] time = 7.39202, size = 5644, normalized size = 23.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^2,x, algorithm="fricas"
)

[Out] [1/8*(sqrt(2)*((c^2*d - 8*c*d^2 - 9*d^3)*cos(f*x + e)^3 - 2*c^3 + 14*c^2*d
+ 34*c*d^2 + 18*d^3 + (c^3 - 6*c^2*d - 25*c*d^2 - 18*d^3)*cos(f*x + e)^2 -
(c^3 - 7*c^2*d - 17*c*d^2 - 9*d^3)*cos(f*x + e) - (2*c^3 - 14*c^2*d - 34*c*
d^2 - 18*d^3 - (c^2*d - 8*c*d^2 - 9*d^3)*cos(f*x + e)^2 + (c^3 - 7*c^2*d -
17*c*d^2 - 9*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a)*log(-(a*cos(f*x + e)^
2 - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e) - sin(f*x + e)
 + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2*a)/(cos(
f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) + 2*(10*a
*c^2*d + 16*a*c*d^2 + 6*a*d^3 - (5*a*c*d^2 + 3*a*d^3)*cos(f*x + e)^3 - (5*a
*c^2*d + 13*a*c*d^2 + 6*a*d^3)*cos(f*x + e)^2 + (5*a*c^2*d + 8*a*c*d^2 + 3*
a*d^3)*cos(f*x + e) + (10*a*c^2*d + 16*a*c*d^2 + 6*a*d^3 - (5*a*c*d^2 + 3*a
*d^3)*cos(f*x + e)^2 + (5*a*c^2*d + 8*a*c*d^2 + 3*a*d^3)*cos(f*x + e))*sin(
f*x + e))*sqrt(d/(a*c + a*d))*log((d^2*cos(f*x + e)^3 - (6*c*d + 7*d^2)*cos
(f*x + e)^2 - c^2 - 2*c*d - d^2 + 4*((c*d + d^2)*cos(f*x + e)^2 - c^2 - 4*c
*d - 3*d^2 - (c^2 + 3*c*d + 2*d^2)*cos(f*x + e) + (c^2 + 4*c*d + 3*d^2 + (c
*d + d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d/(a*c
+ a*d)) - (c^2 + 8*c*d + 9*d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - c^2 -
2*c*d - d^2 + 2*(3*c*d + 4*d^2)*cos(f*x + e))*sin(f*x + e))/(d^2*cos(f*x +
e)^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 - (c^2 + d^2)*cos(f
*x + e) + (d^2*cos(f*x + e)^2 - 2*c*d*cos(f*x + e) - c^2 - 2*c*d - d^2)*sin
(f*x + e))) + 4*(c^3 - c^2*d - c*d^2 + d^3 + (c^2*d + 2*c*d^2 - 3*d^3)*cos(
f*x + e)^2 + (c^3 + c*d^2 - 2*d^3)*cos(f*x + e) - (c^3 - c^2*d - c*d^2 + d^
3 - (c^2*d + 2*c*d^2 - 3*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x +
e) + a))/((a^2*c^4*d - 2*a^2*c^3*d^2 + 2*a^2*c*d^4 - a^2*d^5)*f*cos(f*x + e
)^3 + (a^2*c^5 - 4*a^2*c^3*d^2 + 2*a^2*c^2*d^3 + 3*a^2*c*d^4 - 2*a^2*d^5)*f
*cos(f*x + e)^2 - (a^2*c^5 - a^2*c^4*d - 2*a^2*c^3*d^2 + 2*a^2*c^2*d^3 + a^
2*c*d^4 - a^2*d^5)*f*cos(f*x + e) - 2*(a^2*c^5 - a^2*c^4*d - 2*a^2*c^3*d^2
+ 2*a^2*c^2*d^3 + a^2*c*d^4 - a^2*d^5)*f + ((a^2*c^4*d - 2*a^2*c^3*d^2 + 2*
a^2*c*d^4 - a^2*d^5)*f*cos(f*x + e)^2 - (a^2*c^5 - a^2*c^4*d - 2*a^2*c^3*d^
2 + 2*a^2*c^2*d^3 + a^2*c*d^4 - a^2*d^5)*f*cos(f*x + e) - 2*(a^2*c^5 - a^2*
c^4*d - 2*a^2*c^3*d^2 + 2*a^2*c^2*d^3 + a^2*c*d^4 - a^2*d^5)*f)*sin(f*x + e
```

```
)), 1/8*(sqrt(2)*((c^2*d - 8*c*d^2 - 9*d^3)*cos(f*x + e)^3 - 2*c^3 + 14*c^2
*d + 34*c*d^2 + 18*d^3 + (c^3 - 6*c^2*d - 25*c*d^2 - 18*d^3)*cos(f*x + e)^2
 - (c^3 - 7*c^2*d - 17*c*d^2 - 9*d^3)*cos(f*x + e) - (2*c^3 - 14*c^2*d - 34
*c*d^2 - 18*d^3 - (c^2*d - 8*c*d^2 - 9*d^3)*cos(f*x + e)^2 + (c^3 - 7*c^2*d
 - 17*c*d^2 - 9*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a)*log(-(a*cos(f*x +
e)^2 - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e) - sin(f*x +
 e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2*a)/(c
os(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) + 4*(1
0*a*c^2*d + 16*a*c*d^2 + 6*a*d^3 - (5*a*c*d^2 + 3*a*d^3)*cos(f*x + e)^3 - (
5*a*c^2*d + 13*a*c*d^2 + 6*a*d^3)*cos(f*x + e)^2 + (5*a*c^2*d + 8*a*c*d^2 +
 3*a*d^3)*cos(f*x + e) + (10*a*c^2*d + 16*a*c*d^2 + 6*a*d^3 - (5*a*c*d^2 +
3*a*d^3)*cos(f*x + e)^2 + (5*a*c^2*d + 8*a*c*d^2 + 3*a*d^3)*cos(f*x + e))*s
in(f*x + e))*sqrt(-d/(a*c + a*d))*arctan(1/2*sqrt(a*sin(f*x + e) + a)*(d*si
n(f*x + e) - c - 2*d)*sqrt(-d/(a*c + a*d))/(d*cos(f*x + e))) + 4*(c^3 - c^2
*d - c*d^2 + d^3 + (c^2*d + 2*c*d^2 - 3*d^3)*cos(f*x + e)^2 + (c^3 + c*d^2
- 2*d^3)*cos(f*x + e) - (c^3 - c^2*d - c*d^2 + d^3 - (c^2*d + 2*c*d^2 - 3*d
^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/((a^2*c^4*d - 2*a
^2*c^3*d^2 + 2*a^2*c*d^4 - a^2*d^5)*f*cos(f*x + e)^3 + (a^2*c^5 - 4*a^2*c^3
*d^2 + 2*a^2*c^2*d^3 + 3*a^2*c*d^4 - 2*a^2*d^5)*f*cos(f*x + e)^2 - (a^2*c^5
 - a^2*c^4*d - 2*a^2*c^3*d^2 + 2*a^2*c^2*d^3 + a^2*c*d^4 - a^2*d^5)*f*cos(f
*x + e) - 2*(a^2*c^5 - a^2*c^4*d - 2*a^2*c^3*d^2 + 2*a^2*c^2*d^3 + a^2*c*d^
4 - a^2*d^5)*f + ((a^2*c^4*d - 2*a^2*c^3*d^2 + 2*a^2*c*d^4 - a^2*d^5)*f*cos
(f*x + e)^2 - (a^2*c^5 - a^2*c^4*d - 2*a^2*c^3*d^2 + 2*a^2*c^2*d^3 + a^2*c*
d^4 - a^2*d^5)*f*cos(f*x + e) - 2*(a^2*c^5 - a^2*c^4*d - 2*a^2*c^3*d^2 + 2*
a^2*c^2*d^3 + a^2*c*d^4 - a^2*d^5)*f)*sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**(3/2)/(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.556 $\int \frac{1}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^3}\,dx$

Optimal. Leaf size=318

$$-\frac{d^{3/2}\left(35c^2+42cd+19d^2\right)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{4a^{3/2}f(c-d)^4(c+d)^{5/2}}-\frac{(c-13d)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{2\sqrt{2}a^{3/2}f(c-d)^4}-\frac{d(2c+d)(c}{4af(c-d)^3(c+d)^2\sqrt{a\sin}}$$

```
[Out] -((c - 13*d)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x
]])])/(2*Sqrt[2]*a^(3/2)*(c - d)^4*f) - (d^(3/2)*(35*c^2 + 42*c*d + 19*d^2)
*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]*Sqrt[a + a*Sin[e + f*x
]])])/(4*a^(3/2)*(c - d)^4*(c + d)^(5/2)*f) - Cos[e + f*x]/(2*(c - d)*f*(a
+ a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^2) - (d*(c + 2*d)*Cos[e + f*x]
)/(2*a*(c - d)^2*(c + d)*f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^2)
 - (d*(2*c + d)*(c + 7*d)*Cos[e + f*x])/(4*a*(c - d)^3*(c + d)^2*f*Sqrt[a +
 a*Sin[e + f*x]]*(c + d*Sin[e + f*x]))
```

Rubi [A] time = 1.10809, antiderivative size = 318, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2766, 2984, 2985, 2649, 206, 2773, 208}

$$-\frac{d^{3/2}\left(35c^2+42cd+19d^2\right)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{4a^{3/2}f(c-d)^4(c+d)^{5/2}}-\frac{(c-13d)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{2\sqrt{2}a^{3/2}f(c-d)^4}-\frac{d(2c+d)(c}{4af(c-d)^3(c+d)^2\sqrt{a\sin}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^3),x]
```

```
[Out] -((c - 13*d)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x
]])])/(2*Sqrt[2]*a^(3/2)*(c - d)^4*f) - (d^(3/2)*(35*c^2 + 42*c*d + 19*d^2)
*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]*Sqrt[a + a*Sin[e + f*x
]])])/(4*a^(3/2)*(c - d)^4*(c + d)^(5/2)*f) - Cos[e + f*x]/(2*(c - d)*f*(a
+ a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^2) - (d*(c + 2*d)*Cos[e + f*x]
)/(2*a*(c - d)^2*(c + d)*f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^2)
 - (d*(2*c + d)*(c + 7*d)*Cos[e + f*x])/(4*a*(c - d)^3*(c + d)^2*f*Sqrt[a +
 a*Sin[e + f*x]]*(c + d*Sin[e + f*x]))
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
```

```
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^3}\,dx = -\frac{\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^2} - \frac{\int \frac{-\frac{1}{2}a(c-8d)-\frac{5}{2}a\cdots}{\sqrt{a+a\sin(e+fx)}(c\cdots}}{2a^2(c\cdots}$$

$$= -\frac{\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^2} - \frac{\cdots}{2a(c-d)^2(c+d)\cdots}$$

$$= -\frac{\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^2} - \frac{\cdots}{2a(c-d)^2(c+d)\cdots}$$

$$= -\frac{\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^2} - \frac{\cdots}{2a(c-d)^2(c+d)\cdots}$$

$$= -\frac{\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^2} - \frac{\cdots}{2a(c-d)^2(c+d)\cdots}$$

$$= -\frac{(c-13d)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{2\sqrt{2}a^{3/2}(c-d)^4 f} - \frac{d^{3/2}\left(35c^2+42cd+19d^2\right)\tanh^{\cdots}}{4a^{3/2}(c-d)^4(c+\cdots}$$

Mathematica [C] time = 6.26712, size = 570, normalized size = 1.79

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(-\frac{d^{3/2}\left(35c^2+42cd+19d^2\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^2\left(2\log\left(\sec^2\left(\frac{1}{4}(e+fx)\right)\left(\sqrt{c+d}-\sqrt{d}\sin\left(\frac{1}{2}(e+fx)\right)+\sqrt{d}\cdots\right.\right.\right.}{(c+d)^{5/2}}\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^3),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(16*(c - d)*Sin[(e + f*x)/2] - 8*(c
- d)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + (8 + 8*I)*(-1)^(3/4)*(c - 13*d
)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2]
 + Sin[(e + f*x)/2])^2 - (d^(3/2)*(35*c^2 + 42*c*d + 19*d^2)*(e + f*x - 2*L
og[Sec[(e + f*x)/4]^2] + 2*Log[Sec[(e + f*x)/4]^2*(Sqrt[c + d] + Sqrt[d]*Co
s[(e + f*x)/2] - Sqrt[d]*Sin[(e + f*x)/2])])*(Cos[(e + f*x)/2] + Sin[(e + f
*x)/2])^2)/(c + d)^(5/2) + (d^(3/2)*(35*c^2 + 42*c*d + 19*d^2)*(e + f*x - 2
*Log[Sec[(e + f*x)/4]^2] + 2*Log[Sec[(e + f*x)/4]^2*(Sqrt[c + d] - Sqrt[d]*
Cos[(e + f*x)/2] + Sqrt[d]*Sin[(e + f*x)/2])])*(Cos[(e + f*x)/2] + Sin[(e +
 f*x)/2])^2)/(c + d)^(5/2) - (8*(c - d)^2*d^2*(Cos[(e + f*x)/2] - Sin[(e +
f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2)/((c + d)*(c + d*Sin[e + f
*x])^2) - (4*(c - d)*d^2*(11*c + 5*d)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])
*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2)/((c + d)^2*(c + d*Sin[e + f*x]))
)/(16*(c - d)^4*f*(a*(1 + Sin[e + f*x]))^(3/2))
```

Maple [B] time = 1.677, size = 2222, normalized size = 7.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^3,x)
```

```
[Out] 1/4/a^(7/2)*(-a*(-1+sin(f*x+e)))^(1/2)*(61*(a*(c+d)*d)^(1/2)*2^(1/2)*arctan
h(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*a^2*c^2*d^3+
26*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)
/a^(1/2))*sin(f*x+e)*a^2*c*d^4-(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(-
1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^3*a^2*c^3*d^2-19*arctanh((
-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)^3*d^6-19*
arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)^
2*d^6-(-a*(-1+sin(f*x+e)))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*c^2*d^3+13*(-a*(
-1+sin(f*x+e)))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*c*d^4-5*(-a*(-1+sin(f*x+e))
)^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*sin(f*x+e)*d^5+11*(-a*(-1+sin(f*x+e)))^(3
/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*c^2*d^3-6*(-a*(-1+sin(f*x+e)))^(3/2)*(a*(c+d)
*d)^(1/2)*a^(1/2)*c*d^4-(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f
*x+e)))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^5-35*arctanh((-a*(-1+sin(f*x+e)))^(1/2
)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)^3*c^2*d^4-42*arctanh((-a*(-1+sin(
f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)^3*c*d^5-70*arctanh((
-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)^2*c^3*d^3
-119*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*
x+e)^2*c^2*d^4-80*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*a
^(5/2)*sin(f*x+e)^2*c*d^5+2*(-a*(-1+sin(f*x+e)))^(1/2)*(a*(c+d)*d)^(1/2)*a^
(3/2)*sin(f*x+e)^2*d^5-35*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^
(1/2))*a^(5/2)*sin(f*x+e)*c^4*d^2-112*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/
(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)*c^3*d^3-103*arctanh((-a*(-1+sin(f*x+e
)))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)*c^2*d^4-38*arctanh((-a*(-
1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)*c*d^5+3*(-a*(-
1+sin(f*x+e)))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*sin(f*x+e)*d^5-2*(-a*(-1+sin
(f*x+e)))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*c^4*d-11*(-a*(-1+sin(f*x+e)))^(1/
2)*(a*(c+d)*d)^(1/2)*a^(3/2)*c^3*d^2-(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*
(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a^2*c^5-35*arctanh((
-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*c^4*d^2-42*arctanh((
-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*c^3*d^3-19*arctanh((
-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*c^2*d^4-2*(-a*(-1+si
n(f*x+e)))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*c^5+3*(-a*(-1+sin(f*x+e)))^(1/2)
*(a*(c+d)*d)^(1/2)*a^(3/2)*d^5-5*(-a*(-1+sin(f*x+e)))^(3/2)*(a*(c+d)*d)^(1/
2)*a^(1/2)*d^5+47*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e))
)^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a^2*c^3*d^2+11*(a*(c+d)*d)^(1/2)*2^(1/2
)*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^3*a^2*
c^2*d^3+25*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)
*2^(1/2)/a^(1/2))*sin(f*x+e)^3*a^2*c*d^4-2*(a*(c+d)*d)^(1/2)*2^(1/2)*arctan
h(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*a^2*c^4*d+21
*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a
^(1/2))*sin(f*x+e)^2*a^2*c^3*d^2+63*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(
-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a^2*c^2*d^3+11*(a*(c+
d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))
*a^2*c^4*d+25*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1
/2)*2^(1/2)/a^(1/2))*a^2*c^3*d^2+13*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(
-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^2*d^3-2*(-a*(-1+sin(f*x+e)
))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*sin(f*x+e)^2*c^3*d^2+2*(-a*(-1+sin(f*x+e
)))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*sin(f*x+e)^2*c*d^4-4*(-a*(-1+sin(f*x+e)
))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*sin(f*x+e)*c^4*d-17*(-a*(-1+sin(f*x+e)))
^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*sin(f*x+e)*c^3*d^2+(-a*(-1+sin(f*x+e)))^(1
/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*sin(f*x+e)*c^2*d^3+17*(-a*(-1+sin(f*x+e)))^(1
/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*sin(f*x+e)*c*d^4+13*(a*(c+d)*d)^(1/2)*2^(1/2)
*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^3*a^2*d
^5+13*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1
/2)/a^(1/2))*sin(f*x+e)^2*a^2*d^5+11*(-a*(-1+sin(f*x+e)))^(3/2)*(a*(c+d)*d)
^(1/2)*a^(1/2)*sin(f*x+e)*c^2*d^3-2*(-a*(-1+sin(f*x+e)))^(1/2)*(a*(c+d)*d)^
(1/2)*a^(3/2)*sin(f*x+e)^2*c^2*d^3-6*(-a*(-1+sin(f*x+e)))^(3/2)*(a*(c+d)*d)
^(1/2)*a^(1/2)*sin(f*x+e)*c*d^4+51*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-
a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*a^2*c*d^4+9*(a*(c+d)
```

```
*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*s
in(f*x+e)*a^2*c^4*d)/(a*(c+d)*d)^(1/2)/(c+d*sin(f*x+e))^2/(c+d)^2/(c-d)^4/c
os(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^3,x, algorithm="maxima"
)
```

```
[Out] Timed out
```

Fricas [B] time = 13.286, size = 9231, normalized size = 29.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^3,x, algorithm="fricas"
)
```

```
[Out] [-1/16*(2*sqrt(2)*(2*c^5 - 18*c^4*d - 92*c^3*d^2 - 148*c^2*d^3 - 102*c*d^4
- 26*d^5 + (c^3*d^2 - 11*c^2*d^3 - 25*c*d^4 - 13*d^5)*cos(f*x + e)^4 - (2*c
^4*d - 21*c^3*d^2 - 61*c^2*d^3 - 51*c*d^4 - 13*d^5)*cos(f*x + e)^3 - (c^5 -
 7*c^4*d - 66*c^3*d^2 - 146*c^2*d^3 - 127*c*d^4 - 39*d^5)*cos(f*x + e)^2 +
(c^5 - 9*c^4*d - 46*c^3*d^2 - 74*c^2*d^3 - 51*c*d^4 - 13*d^5)*cos(f*x + e)
+ (2*c^5 - 18*c^4*d - 92*c^3*d^2 - 148*c^2*d^3 - 102*c*d^4 - 26*d^5 - (c^3*
d^2 - 11*c^2*d^3 - 25*c*d^4 - 13*d^5)*cos(f*x + e)^3 - 2*(c^4*d - 10*c^3*d^
2 - 36*c^2*d^3 - 38*c*d^4 - 13*d^5)*cos(f*x + e)^2 + (c^5 - 9*c^4*d - 46*c^
3*d^2 - 74*c^2*d^3 - 51*c*d^4 - 13*d^5)*cos(f*x + e))*sin(f*x + e))*sqrt(a)
*log(-(a*cos(f*x + e)^2 + 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f
*x + e) - sin(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin
(f*x + e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*
x + e) - 2)) - (70*a*c^4*d + 224*a*c^3*d^2 + 276*a*c^2*d^3 + 160*a*c*d^4 +
38*a*d^5 + (35*a*c^2*d^3 + 42*a*c*d^4 + 19*a*d^5)*cos(f*x + e)^4 - (70*a*c^
3*d^2 + 119*a*c^2*d^3 + 80*a*c*d^4 + 19*a*d^5)*cos(f*x + e)^3 - (35*a*c^4*d
 + 182*a*c^3*d^2 + 292*a*c^2*d^3 + 202*a*c*d^4 + 57*a*d^5)*cos(f*x + e)^2 +
 (35*a*c^4*d + 112*a*c^3*d^2 + 138*a*c^2*d^3 + 80*a*c*d^4 + 19*a*d^5)*cos(f
*x + e) + (70*a*c^4*d + 224*a*c^3*d^2 + 276*a*c^2*d^3 + 160*a*c*d^4 + 38*a*
d^5 - (35*a*c^2*d^3 + 42*a*c*d^4 + 19*a*d^5)*cos(f*x + e)^3 - 2*(35*a*c^3*d
^2 + 77*a*c^2*d^3 + 61*a*c*d^4 + 19*a*d^5)*cos(f*x + e)^2 + (35*a*c^4*d + 1
12*a*c^3*d^2 + 138*a*c^2*d^3 + 80*a*c*d^4 + 19*a*d^5)*cos(f*x + e))*sin(f*x
 + e))*sqrt(d/(a*c + a*d))*log((d^2*cos(f*x + e)^3 - (6*c*d + 7*d^2)*cos(f*
x + e)^2 - c^2 - 2*c*d - d^2 - 4*((c*d + d^2)*cos(f*x + e)^2 - c^2 - 4*c*d
- 3*d^2 - (c^2 + 3*c*d + 2*d^2)*cos(f*x + e) + (c^2 + 4*c*d + 3*d^2 + (c*d
+ d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d/(a*c + a
*d)) - (c^2 + 8*c*d + 9*d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - c^2 - 2*c
*d - d^2 + 2*(3*c*d + 4*d^2)*cos(f*x + e))*sin(f*x + e))/(d^2*cos(f*x + e)^
3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 - (c^2 + d^2)*cos(f*x
+ e) + (d^2*cos(f*x + e)^2 - 2*c*d*cos(f*x + e) - c^2 - 2*c*d - d^2)*sin(f*
x + e))) + 4*(2*c^5 - 2*c^4*d - 4*c^3*d^2 + 4*c^2*d^3 + 2*c*d^4 - 2*d^5 - (
2*c^3*d^2 + 13*c^2*d^3 - 8*c*d^4 - 7*d^5)*cos(f*x + e)^3 + (4*c^4*d + 15*c^
```

3*d^2 - 14*c^2*d^3 - 9*c*d^4 + 4*d^5)*cos(f*x + e)^2 + (2*c^5 + 2*c^4*d + 1
3*c^3*d^2 + 3*c^2*d^3 - 15*c*d^4 - 5*d^5)*cos(f*x + e) - (2*c^5 - 2*c^4*d -
 4*c^3*d^2 + 4*c^2*d^3 + 2*c*d^4 - 2*d^5 - (2*c^3*d^2 + 13*c^2*d^3 - 8*c*d^
4 - 7*d^5)*cos(f*x + e)^2 - (4*c^4*d + 17*c^3*d^2 - c^2*d^3 - 17*c*d^4 - 3*
d^5)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/((a^2*c^6*d^2 -
2*a^2*c^5*d^3 - a^2*c^4*d^4 + 4*a^2*c^3*d^5 - a^2*c^2*d^6 - 2*a^2*c*d^7 + a
^2*d^8)*f*cos(f*x + e)^4 - (2*a^2*c^7*d - 3*a^2*c^6*d^2 - 4*a^2*c^5*d^3 + 7
*a^2*c^4*d^4 + 2*a^2*c^3*d^5 - 5*a^2*c^2*d^6 + a^2*d^8)*f*cos(f*x + e)^3 -
(a^2*c^8 + 2*a^2*c^7*d - 6*a^2*c^6*d^2 - 6*a^2*c^5*d^3 + 12*a^2*c^4*d^4 + 6
*a^2*c^3*d^5 - 10*a^2*c^2*d^6 - 2*a^2*c*d^7 + 3*a^2*d^8)*f*cos(f*x + e)^2 +
 (a^2*c^8 - 4*a^2*c^6*d^2 + 6*a^2*c^4*d^4 - 4*a^2*c^2*d^6 + a^2*d^8)*f*cos(
f*x + e) + 2*(a^2*c^8 - 4*a^2*c^6*d^2 + 6*a^2*c^4*d^4 - 4*a^2*c^2*d^6 + a^2
*d^8)*f - ((a^2*c^6*d^2 - 2*a^2*c^5*d^3 - a^2*c^4*d^4 + 4*a^2*c^3*d^5 - a^2
*c^2*d^6 - 2*a^2*c*d^7 + a^2*d^8)*f*cos(f*x + e)^3 + 2*(a^2*c^7*d - a^2*c^6
*d^2 - 3*a^2*c^5*d^3 + 3*a^2*c^4*d^4 + 3*a^2*c^3*d^5 - 3*a^2*c^2*d^6 - a^2*
c*d^7 + a^2*d^8)*f*cos(f*x + e)^2 - (a^2*c^8 - 4*a^2*c^6*d^2 + 6*a^2*c^4*d^
4 - 4*a^2*c^2*d^6 + a^2*d^8)*f*cos(f*x + e) - 2*(a^2*c^8 - 4*a^2*c^6*d^2 +
6*a^2*c^4*d^4 - 4*a^2*c^2*d^6 + a^2*d^8)*f)*sin(f*x + e)), -1/8*(sqrt(2)*(2
*c^5 - 18*c^4*d - 92*c^3*d^2 - 148*c^2*d^3 - 102*c*d^4 - 26*d^5 + (c^3*d^2
- 11*c^2*d^3 - 25*c*d^4 - 13*d^5)*cos(f*x + e)^4 - (2*c^4*d - 21*c^3*d^2 -
61*c^2*d^3 - 51*c*d^4 - 13*d^5)*cos(f*x + e)^3 - (c^5 - 7*c^4*d - 66*c^3*d^
2 - 146*c^2*d^3 - 127*c*d^4 - 39*d^5)*cos(f*x + e)^2 + (c^5 - 9*c^4*d - 46*
c^3*d^2 - 74*c^2*d^3 - 51*c*d^4 - 13*d^5)*cos(f*x + e) + (2*c^5 - 18*c^4*d
- 92*c^3*d^2 - 148*c^2*d^3 - 102*c*d^4 - 26*d^5 - (c^3*d^2 - 11*c^2*d^3 - 2
5*c*d^4 - 13*d^5)*cos(f*x + e)^3 - 2*(c^4*d - 10*c^3*d^2 - 36*c^2*d^3 - 38*
c*d^4 - 13*d^5)*cos(f*x + e)^2 + (c^5 - 9*c^4*d - 46*c^3*d^2 - 74*c^2*d^3 -
 51*c*d^4 - 13*d^5)*cos(f*x + e))*sin(f*x + e))*sqrt(a)*log(-(a*cos(f*x + e
)^2 + 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e) - sin(f*x +
e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2*a)/(co
s(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) + (70*a
*c^4*d + 224*a*c^3*d^2 + 276*a*c^2*d^3 + 160*a*c*d^4 + 38*a*d^5 + (35*a*c^2
*d^3 + 42*a*c*d^4 + 19*a*d^5)*cos(f*x + e)^4 - (70*a*c^3*d^2 + 119*a*c^2*d^
3 + 80*a*c*d^4 + 19*a*d^5)*cos(f*x + e)^3 - (35*a*c^4*d + 182*a*c^3*d^2 + 2
92*a*c^2*d^3 + 202*a*c*d^4 + 57*a*d^5)*cos(f*x + e)^2 + (35*a*c^4*d + 112*a
*c^3*d^2 + 138*a*c^2*d^3 + 80*a*c*d^4 + 19*a*d^5)*cos(f*x + e) + (70*a*c^4*
d + 224*a*c^3*d^2 + 276*a*c^2*d^3 + 160*a*c*d^4 + 38*a*d^5 - (35*a*c^2*d^3
+ 42*a*c*d^4 + 19*a*d^5)*cos(f*x + e)^3 - 2*(35*a*c^3*d^2 + 77*a*c^2*d^3 +
61*a*c*d^4 + 19*a*d^5)*cos(f*x + e)^2 + (35*a*c^4*d + 112*a*c^3*d^2 + 138*a
*c^2*d^3 + 80*a*c*d^4 + 19*a*d^5)*cos(f*x + e))*sin(f*x + e))*sqrt(-d/(a*c
+ a*d))*arctan(1/2*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) - c - 2*d)*sqrt
(-d/(a*c + a*d))/(d*cos(f*x + e))) + 2*(2*c^5 - 2*c^4*d - 4*c^3*d^2 + 4*c^2
*d^3 + 2*c*d^4 - 2*d^5 - (2*c^3*d^2 + 13*c^2*d^3 - 8*c*d^4 - 7*d^5)*cos(f*x
 + e)^3 + (4*c^4*d + 15*c^3*d^2 - 14*c^2*d^3 - 9*c*d^4 + 4*d^5)*cos(f*x + e
)^2 + (2*c^5 + 2*c^4*d + 13*c^3*d^2 + 3*c^2*d^3 - 15*c*d^4 - 5*d^5)*cos(f*x
 + e) - (2*c^5 - 2*c^4*d - 4*c^3*d^2 + 4*c^2*d^3 + 2*c*d^4 - 2*d^5 - (2*c^3
*d^2 + 13*c^2*d^3 - 8*c*d^4 - 7*d^5)*cos(f*x + e)^2 - (4*c^4*d + 17*c^3*d^2
 - c^2*d^3 - 17*c*d^4 - 3*d^5)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x +
 e) + a))/((a^2*c^6*d^2 - 2*a^2*c^5*d^3 - a^2*c^4*d^4 + 4*a^2*c^3*d^5 - a^2
*c^2*d^6 - 2*a^2*c*d^7 + a^2*d^8)*f*cos(f*x + e)^4 - (2*a^2*c^7*d - 3*a^2*c
^6*d^2 - 4*a^2*c^5*d^3 + 7*a^2*c^4*d^4 + 2*a^2*c^3*d^5 - 5*a^2*c^2*d^6 + a^
2*d^8)*f*cos(f*x + e)^3 - (a^2*c^8 + 2*a^2*c^7*d - 6*a^2*c^6*d^2 - 6*a^2*c^
5*d^3 + 12*a^2*c^4*d^4 + 6*a^2*c^3*d^5 - 10*a^2*c^2*d^6 - 2*a^2*c*d^7 + 3*a
^2*d^8)*f*cos(f*x + e)^2 + (a^2*c^8 - 4*a^2*c^6*d^2 + 6*a^2*c^4*d^4 - 4*a^2
*c^2*d^6 + a^2*d^8)*f*cos(f*x + e) + 2*(a^2*c^8 - 4*a^2*c^6*d^2 + 6*a^2*c^4
*d^4 - 4*a^2*c^2*d^6 + a^2*d^8)*f - ((a^2*c^6*d^2 - 2*a^2*c^5*d^3 - a^2*c^4
*d^4 + 4*a^2*c^3*d^5 - a^2*c^2*d^6 - 2*a^2*c*d^7 + a^2*d^8)*f*cos(f*x + e)^
3 + 2*(a^2*c^7*d - a^2*c^6*d^2 - 3*a^2*c^5*d^3 + 3*a^2*c^4*d^4 + 3*a^2*c^3*
d^5 - 3*a^2*c^2*d^6 - a^2*c*d^7 + a^2*d^8)*f*cos(f*x + e)^2 - (a^2*c^8 - 4*
a^2*c^6*d^2 + 6*a^2*c^4*d^4 - 4*a^2*c^2*d^6 + a^2*d^8)*f*cos(f*x + e) - 2*(

```
a^2*c^8 - 4*a^2*c^6*d^2 + 6*a^2*c^4*d^4 - 4*a^2*c^2*d^6 + a^2*d^8)*f)*sin(f
*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**(3/2)/(c+d*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.557 $\int \frac{(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=194

$$-\frac{3\left(c^2+6cd+25d^2\right)(c-d)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{16\sqrt{2}a^{5/2}f}+\frac{d^2(c-9d)\cos(e+fx)}{4a^2f\sqrt{a\sin(e+fx)+a}}-\frac{(3c+13d)(c-d)^2\cos(e+fx)}{16af(a\sin(e+fx)+a)^{3/2}}-($$

```
[Out] (-3*(c - d)*(c^2 + 6*c*d + 25*d^2)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*
Sqrt[a + a*Sin[e + f*x]])])/(16*Sqrt[2]*a^(5/2)*f) - ((c - d)^2*(3*c + 13*d
)*Cos[e + f*x])/(16*a*f*(a + a*Sin[e + f*x])^(3/2)) + ((c - 9*d)*d^2*Cos[e
+ f*x])/(4*a^2*f*Sqrt[a + a*Sin[e + f*x]]) - ((c - d)*Cos[e + f*x]*(c + d*S
in[e + f*x])^2)/(4*f*(a + a*Sin[e + f*x])^(5/2))
```

Rubi [A] time = 0.469194, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2765, 2968, 3019, 2751, 2649, 206}

$$-\frac{3\left(c^2+6cd+25d^2\right)(c-d)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{16\sqrt{2}a^{5/2}f}+\frac{d^2(c-9d)\cos(e+fx)}{4a^2f\sqrt{a\sin(e+fx)+a}}-\frac{(3c+13d)(c-d)^2\cos(e+fx)}{16af(a\sin(e+fx)+a)^{3/2}}-($$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^3/(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] (-3*(c - d)*(c^2 + 6*c*d + 25*d^2)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*
Sqrt[a + a*Sin[e + f*x]])])/(16*Sqrt[2]*a^(5/2)*f) - ((c - d)^2*(3*c + 13*d
)*Cos[e + f*x])/(16*a*f*(a + a*Sin[e + f*x])^(3/2)) + ((c - 9*d)*d^2*Cos[e
+ f*x])/(4*a^2*f*Sqrt[a + a*Sin[e + f*x]]) - ((c - d)*Cos[e + f*x]*(c + d*S
in[e + f*x])^2)/(4*f*(a + a*Sin[e + f*x])^(5/2))
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3019

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b - a
*B + b*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[1
```

```
/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*
B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^{5/2}}\,dx = -\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^2}{4f(a+a\sin(e+fx))^{5/2}} - \frac{\int \frac{(c+d\sin(e+fx))\left(-\frac{1}{2}a\left(3c^2+9cd-4d^2\right)+\frac{1}{2}a(c-9d)d\sin(e+fx)\right)}{(a+a\sin(e+fx))^{3/2}}}{4a^2}$$

$$= -\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^2}{4f(a+a\sin(e+fx))^{5/2}} - \frac{\int \frac{-\frac{1}{2}ac\left(3c^2+9cd-4d^2\right)+\left(\frac{1}{2}ac(c-9d)d-\frac{1}{2}ad\left(3c^2+9cd-4d^2\right)\right)\sin}{(a+a\sin(e+fx))^{3/2}}}{4a^2}$$

$$= -\frac{(c-d)^2(3c+13d)\cos(e+fx)}{16af(a+a\sin(e+fx))^{3/2}} - \frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^2}{4f(a+a\sin(e+fx))^{5/2}} + \int \frac{\frac{1}{4}a^2\left(3c^3+15c^2d\right.}{}$$

$$= -\frac{(c-d)^2(3c+13d)\cos(e+fx)}{16af(a+a\sin(e+fx))^{3/2}} + \frac{(c-9d)d^2\cos(e+fx)}{4a^2f\sqrt{a+a\sin(e+fx)}} - \frac{(c-d)\cos(e+fx)(c+d\sin(}{4f(a+a\sin(e+fx))}$$

$$= -\frac{(c-d)^2(3c+13d)\cos(e+fx)}{16af(a+a\sin(e+fx))^{3/2}} + \frac{(c-9d)d^2\cos(e+fx)}{4a^2f\sqrt{a+a\sin(e+fx)}} - \frac{(c-d)\cos(e+fx)(c+d\sin(}{4f(a+a\sin(e+fx))}$$

$$= -\frac{3(c-d)\left(c^2+6cd+25d^2\right)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{16\sqrt{2}a^{5/2}f} - \frac{(c-d)^2(3c+13d)\cos(e+fx)}{16af(a+a\sin(e+fx))^{3/2}} + \frac{}{4}$$

Mathematica [C] time = 0.768338, size = 400, normalized size = 2.06

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left((6+6i)(-1)^{3/4}\left(5c^2d+c^3+19cd^2-25d^3\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^4\tan\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])^3/(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-11*c^3*Cos[(e + f*x)/2] + 9*c^2*d*
Cos[(e + f*x)/2] + 15*c*d^2*Cos[(e + f*x)/2] - 45*d^3*Cos[(e + f*x)/2] - 3*
c^3*Cos[(3*(e + f*x))/2] - 15*c^2*d*Cos[(3*(e + f*x))/2] + 39*c*d^2*Cos[(3*
(e + f*x))/2] - 69*d^3*Cos[(3*(e + f*x))/2] + 16*d^3*Cos[(5*(e + f*x))/2] +
 11*c^3*Sin[(e + f*x)/2] - 9*c^2*d*Sin[(e + f*x)/2] - 15*c*d^2*Sin[(e + f*x
)/2] + 45*d^3*Sin[(e + f*x)/2] + (6 + 6*I)*(-1)^(3/4)*(c^3 + 5*c^2*d + 19*c
*d^2 - 25*d^3)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])]*(Cos
[(e + f*x)/2] + Sin[(e + f*x)/2])^4 - 3*c^3*Sin[(3*(e + f*x))/2] - 15*c^2*d
*Sin[(3*(e + f*x))/2] + 39*c*d^2*Sin[(3*(e + f*x))/2] - 69*d^3*Sin[(3*(e +
f*x))/2] - 16*d^3*Sin[(5*(e + f*x))/2]))/(32*f*(a*(1 + Sin[e + f*x]))^(5/2)
)
```

Maple [B] time = 1.017, size = 688, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(5/2),x)
```

```
[Out] 1/32/a^(9/2)*(-2*sin(f*x+e)*(3*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2
^(1/2)/a^(1/2))*a^2*c^3+15*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/
2)/a^(1/2))*a^2*c^2*d+57*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)
/a^(1/2))*a^2*c*d^2-75*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a
^(1/2))*a^2*d^3+64*d^3*a^(3/2)*(a-a*sin(f*x+e))^(1/2))+(3*2^(1/2)*arctanh(1
/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^3+15*2^(1/2)*arctanh(1/2*(
a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^2*d+57*2^(1/2)*arctanh(1/2*(a-
a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c*d^2-75*2^(1/2)*arctanh(1/2*(a-a*
sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*d^3+64*d^3*a^(3/2)*(a-a*sin(f*x+e))^
(1/2))*cos(f*x+e)^2-6*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^
(1/2))*a^2*c^3-30*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2
))*a^2*c^2*d-114*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2)
)*a^2*c*d^2+150*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))
*a^2*d^3+6*(a-a*sin(f*x+e))^(3/2)*a^(1/2)*c^3+30*(a-a*sin(f*x+e))^(3/2)*a^(
1/2)*c^2*d-78*(a-a*sin(f*x+e))^(3/2)*a^(1/2)*c*d^2+42*(a-a*sin(f*x+e))^(3/2
)*d^3*a^(1/2)-20*(a-a*sin(f*x+e))^(1/2)*a^(3/2)*c^3-36*(a-a*sin(f*x+e))^(1/
2)*a^(3/2)*c^2*d+132*a^(3/2)*c*d^2*(a-a*sin(f*x+e))^(1/2)-204*d^3*a^(3/2)*(
a-a*sin(f*x+e))^(1/2))*(-a*(-1+sin(f*x+e)))^(1/2)/(1+sin(f*x+e))/cos(f*x+e)
/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d \sin\left(fx + e\right) + c\right)^{3}}{\left(a \sin\left(fx + e\right) + a\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*sin(f*x + e) + c)^3/(a*sin(f*x + e) + a)^(5/2), x)
```

Fricas [B] time = 1.80719, size = 1497, normalized size = 7.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/64*(3*sqrt(2)*((c^3 + 5*c^2*d + 19*c*d^2 - 25*d^3)*cos(f*x + e)^3 - 4*c^
3 - 20*c^2*d - 76*c*d^2 + 100*d^3 + 3*(c^3 + 5*c^2*d + 19*c*d^2 - 25*d^3)*c
os(f*x + e)^2 - 2*(c^3 + 5*c^2*d + 19*c*d^2 - 25*d^3)*cos(f*x + e) - (4*c^3
 + 20*c^2*d + 76*c*d^2 - 100*d^3 - (c^3 + 5*c^2*d + 19*c*d^2 - 25*d^3)*cos(
f*x + e)^2 + 2*(c^3 + 5*c^2*d + 19*c*d^2 - 25*d^3)*cos(f*x + e))*sin(f*x +
e))*sqrt(a)*log(-(a*cos(f*x + e)^2 + 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqr
t(a)*(cos(f*x + e) - sin(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e)
 - 2*a)*sin(f*x + e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x +
e) - cos(f*x + e) - 2)) + 4*(32*d^3*cos(f*x + e)^3 - 4*c^3 + 12*c^2*d - 12*
c*d^2 + 4*d^3 - (3*c^3 + 15*c^2*d - 39*c*d^2 + 53*d^3)*cos(f*x + e)^2 - (7*
c^3 + 3*c^2*d - 27*c*d^2 + 81*d^3)*cos(f*x + e) - (32*d^3*cos(f*x + e)^2 -
4*c^3 + 12*c^2*d - 12*c*d^2 + 4*d^3 + (3*c^3 + 15*c^2*d - 39*c*d^2 + 85*d^3
)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/(a^3*f*cos(f*x + e)
^3 + 3*a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f + (a^3*f*cos(f
*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**3/(a+a*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

sage_2

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] sage2
```

3.558 $\int \frac{(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=147

$$-\frac{\left(3c^2+10cd+19d^2\right)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{16\sqrt{2}a^{5/2}f}-\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))}{4f(a\sin(e+fx)+a)^{5/2}}-\frac{3(c-d)(c+3d)\cos(e+fx)}{16af(a\sin(e+fx)+a)^{3/2}}$$

```
[Out] -((3*c^2 + 10*c*d + 19*d^2)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a
+ a*Sin[e + f*x]])])/(16*Sqrt[2]*a^(5/2)*f) - (3*(c - d)*(c + 3*d)*Cos[e +
f*x])/(16*a*f*(a + a*Sin[e + f*x])^(3/2)) - ((c - d)*Cos[e + f*x]*(c + d*Si
n[e + f*x]))/(4*f*(a + a*Sin[e + f*x])^(5/2))
```

Rubi [A] time = 0.229836, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2760, 2750, 2649, 206}

$$-\frac{\left(3c^2+10cd+19d^2\right)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{16\sqrt{2}a^{5/2}f}-\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))}{4f(a\sin(e+fx)+a)^{5/2}}-\frac{3(c-d)(c+3d)\cos(e+fx)}{16af(a\sin(e+fx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^2/(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] -((3*c^2 + 10*c*d + 19*d^2)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a
+ a*Sin[e + f*x]])])/(16*Sqrt[2]*a^(5/2)*f) - (3*(c - d)*(c + 3*d)*Cos[e +
f*x])/(16*a*f*(a + a*Sin[e + f*x])^(3/2)) - ((c - d)*Cos[e + f*x]*(c + d*Si
n[e + f*x]))/(4*f*(a + a*Sin[e + f*x])^(5/2))
```

Rule 2760

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^2, x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^m*(c + d*Sin[e + f*x]))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*(2*m + 1)),
Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*c*d*(m - 1) + b*(d^2 + c^2*(m + 1))
 + d*(a*d*(m - 1) + b*c*(m + 2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^{5/2}}\,dx = -\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))}{4f(a+a\sin(e+fx))^{5/2}} - \frac{\int \frac{-\frac{1}{2}a(3c^2+7cd-2d^2)-\frac{1}{2}ad(c+7d)\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx}{4a^2}$$

$$= -\frac{3(c-d)(c+3d)\cos(e+fx)}{16af(a+a\sin(e+fx))^{3/2}} - \frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))}{4f(a+a\sin(e+fx))^{5/2}} + \frac{(3c^2+10cd+19a\ldots}{3\ldots}$$

$$= -\frac{3(c-d)(c+3d)\cos(e+fx)}{16af(a+a\sin(e+fx))^{3/2}} - \frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))}{4f(a+a\sin(e+fx))^{5/2}} - \frac{(3c^2+10cd+19d\ldots}{\ldots}$$

$$= -\frac{(3c^2+10cd+19d^2)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{16\sqrt{2}a^{5/2}f} - \frac{3(c-d)(c+3d)\cos(e+fx)}{16af(a+a\sin(e+fx))^{3/2}} - \frac{(c-d)\cos\ldots}{4f\ldots}$$

Mathematica [C] time = 0.553697, size = 252, normalized size = 1.71

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(2\left(3c^2+10cd-13d^2\right)\sin\left(\frac{1}{2}(e+fx)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^2+(1+i)\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])^2/(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(8*(c - d)^2*Sin[(e + f*x)/2] - 4*(c
 - d)^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + 2*(3*c^2 + 10*c*d - 13*d^2)
*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 - (c - d)*(3*c +
13*d)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 + (1 + I)*(-1)^(3/4)*(3*c^2 +
 10*c*d + 19*d^2)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])]*(
Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4))/(16*f*(a*(1 + Sin[e + f*x]))^(5/2)
)
```

Maple [B] time = 0.929, size = 379, normalized size = 2.6

$$-\frac{1}{\left(32+32\sin\left(fx+e\right)\right)\cos\left(fx+e\right)f}\left(2\sin\left(fx+e\right)\sqrt{2}\mathrm{Artanh}\left(1/2\frac{\sqrt{a-a\sin\left(fx+e\right)}\sqrt{2}}{\sqrt{a}}\right)a^2\left(3c^2+10cd+19d^2\right)\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(5/2),x)
```

```
[Out] -1/32*(2*sin(f*x+e)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1
/2))*a^2*(3*c^2+10*c*d+19*d^2)-2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2
^(1/2)/a^(1/2))*a^2*(3*c^2+10*c*d+19*d^2)*cos(f*x+e)^2+6*2^(1/2)*arctanh(1/
2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^2+20*2^(1/2)*arctanh(1/2*(a
-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c*d+38*2^(1/2)*arctanh(1/2*(a-a*s
in(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*d^2+20*(a-a*sin(f*x+e))^(1/2)*a^(3/2)
```

```
*c^2+24*(a-a*sin(f*x+e))^(1/2)*a^(3/2)*c*d-44*(a-a*sin(f*x+e))^(1/2)*a^(3/2
)*d^2-6*(a-a*sin(f*x+e))^(3/2)*a^(1/2)*c^2-20*(a-a*sin(f*x+e))^(3/2)*a^(1/2
)*c*d+26*(a-a*sin(f*x+e))^(3/2)*a^(1/2)*d^2)*(-a*(-1+sin(f*x+e)))^(1/2)/a^(
9/2)/(1+sin(f*x+e))/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{2}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*sin(f*x + e) + c)^2/(a*sin(f*x + e) + a)^(5/2), x)
```

Fricas [B] time = 1.87514, size = 1234, normalized size = 8.39

$$\sqrt{2}\left(\left(3\,c^{2}+10\,cd+19\,d^{2}\right)\cos\left(fx+e\right)^{3}+3\left(3\,c^{2}+10\,cd+19\,d^{2}\right)\cos\left(fx+e\right)^{2}-12\,c^{2}-40\,cd-76\,d^{2}-2\left(3\,c^{2}+10\,c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/64*(sqrt(2)*((3*c^2 + 10*c*d + 19*d^2)*cos(f*x + e)^3 + 3*(3*c^2 + 10*c*d
 + 19*d^2)*cos(f*x + e)^2 - 12*c^2 - 40*c*d - 76*d^2 - 2*(3*c^2 + 10*c*d +
19*d^2)*cos(f*x + e) + ((3*c^2 + 10*c*d + 19*d^2)*cos(f*x + e)^2 - 12*c^2 -
 40*c*d - 76*d^2 - 2*(3*c^2 + 10*c*d + 19*d^2)*cos(f*x + e))*sin(f*x + e))*
sqrt(a)*log(-(a*cos(f*x + e)^2 - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)
*(cos(f*x + e) - sin(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2
*a)*sin(f*x + e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) -
 cos(f*x + e) - 2)) + 4*((3*c^2 + 10*c*d - 13*d^2)*cos(f*x + e)^2 + 4*c^2 -
 8*c*d + 4*d^2 + (7*c^2 + 2*c*d - 9*d^2)*cos(f*x + e) - (4*c^2 - 8*c*d + 4*
d^2 - (3*c^2 + 10*c*d - 13*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x
+ e) + a))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x
 + e) - 4*a^3*f + (a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f)*s
in(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**2/(a+a*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 4.75015, size = 1540, normalized size = 10.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] 1/16*(sqrt(2)*(3*c^2 + 10*c*d + 19*d^2)*arctan(-1/2*sqrt(2)*(sqrt(a)*tan(1/
2*f*x + 1/2*e) - sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a) + sqrt(a))/sqrt(-a))/(s
qrt(-a)*a^2*sgn(tan(1/2*f*x + 1/2*e) + 1)) + 2*(29*(sqrt(a)*tan(1/2*f*x + 1
/2*e) - sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a))^7*c^2 - 10*(sqrt(a)*tan(1/2*f*x
 + 1/2*e) - sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a))^7*c*d - 19*(sqrt(a)*tan(1/2
*f*x + 1/2*e) - sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a))^7*d^2 + 75*(sqrt(a)*tan
(1/2*f*x + 1/2*e) - sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a))^6*sqrt(a)*c^2 + 58*
(sqrt(a)*tan(1/2*f*x + 1/2*e) - sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a))^6*sqrt(
a)*c*d - 133*(sqrt(a)*tan(1/2*f*x + 1/2*e) - sqrt(a*tan(1/2*f*x + 1/2*e)^2
+ a))^6*sqrt(a)*d^2 + 55*(sqrt(a)*tan(1/2*f*x + 1/2*e) - sqrt(a*tan(1/2*f*x
 + 1/2*e)^2 + a))^5*a*c^2 + 34*(sqrt(a)*tan(1/2*f*x + 1/2*e) - sqrt(a*tan(1
/2*f*x + 1/2*e)^2 + a))^5*a*c*d - 89*(sqrt(a)*tan(1/2*f*x + 1/2*e) - sqrt(a
*tan(1/2*f*x + 1/2*e)^2 + a))^5*a*d^2 - 91*(sqrt(a)*tan(1/2*f*x + 1/2*e) -
sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a))^4*a^(3/2)*c^2 - 26*(sqrt(a)*tan(1/2*f*x
 + 1/2*e) - sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a))^4*a^(3/2)*c*d + 117*(sqrt(a
)*tan(1/2*f*x + 1/2*e) - sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a))^4*a^(3/2)*d^2
- (sqrt(a)*tan(1/2*f*x + 1/2*e) - sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a))^3*a^2
*c^2 + 18*(sqrt(a)*tan(1/2*f*x + 1/2*e) - sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a
))^3*a^2*c*d - 17*(sqrt(a)*tan(1/2*f*x + 1/2*e) - sqrt(a*tan(1/2*f*x + 1/2*
e)^2 + a))^3*a^2*d^2 + 65*(sqrt(a)*tan(1/2*f*x + 1/2*e) - sqrt(a*tan(1/2*f*
x + 1/2*e)^2 + a))^2*a^(5/2)*c^2 - 18*(sqrt(a)*tan(1/2*f*x + 1/2*e) - sqrt(
a*tan(1/2*f*x + 1/2*e)^2 + a))^2*a^(5/2)*c*d - 47*(sqrt(a)*tan(1/2*f*x + 1/
2*e) - sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a))^2*a^(5/2)*d^2 - 27*(sqrt(a)*tan(
1/2*f*x + 1/2*e) - sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a))*a^3*c^2 - 26*(sqrt(a
)*tan(1/2*f*x + 1/2*e) - sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a))*a^3*c*d + 53*(
sqrt(a)*tan(1/2*f*x + 1/2*e) - sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a))*a^3*d^2
+ 7*a^(7/2)*c^2 + 2*a^(7/2)*c*d - 9*a^(7/2)*d^2)/(((sqrt(a)*tan(1/2*f*x + 1
/2*e) - sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a))^2 + 2*(sqrt(a)*tan(1/2*f*x + 1/
2*e) - sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a))*sqrt(a) - a)^4*a^2*sgn(tan(1/2*f
*x + 1/2*e) + 1)))/f
```

3.559 $\int \frac{c+d\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=126

$$-\frac{(3c+5d)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{16\sqrt{2}a^{5/2}f}-\frac{(3c+5d)\cos(e+fx)}{16af(a\sin(e+fx)+a)^{3/2}}-\frac{(c-d)\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}}$$

```
[Out] -((3*c + 5*d)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*
x]])])/(16*Sqrt[2]*a^(5/2)*f) - ((c - d)*Cos[e + f*x])/(4*f*(a + a*Sin[e +
f*x])^(5/2)) - ((3*c + 5*d)*Cos[e + f*x])/(16*a*f*(a + a*Sin[e + f*x])^(3/2
))
```

Rubi [A] time = 0.0979192, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2750, 2650, 2649, 206}

$$-\frac{(3c+5d)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{16\sqrt{2}a^{5/2}f}-\frac{(3c+5d)\cos(e+fx)}{16af(a\sin(e+fx)+a)^{3/2}}-\frac{(c-d)\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])/(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] -((3*c + 5*d)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*
x]])])/(16*Sqrt[2]*a^(5/2)*f) - ((c - d)*Cos[e + f*x])/(4*f*(a + a*Sin[e +
f*x])^(5/2)) - ((3*c + 5*d)*Cos[e + f*x])/(16*a*f*(a + a*Sin[e + f*x])^(3/2
))
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{c+d\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}}\,dx = -\frac{(c-d)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}} + \frac{(3c+5d)\int \frac{1}{(a+a\sin(e+fx))^{3/2}}\,dx}{8a}$$

$$= -\frac{(c-d)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}} - \frac{(3c+5d)\cos(e+fx)}{16af(a+a\sin(e+fx))^{3/2}} + \frac{(3c+5d)\int \frac{1}{\sqrt{a+a\sin(e+fx)}}\,dx}{32a^2}$$

$$= -\frac{(c-d)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}} - \frac{(3c+5d)\cos(e+fx)}{16af(a+a\sin(e+fx))^{3/2}} - \frac{(3c+5d)\,\mathrm{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, -\sqrt{\ldots}\right)}{16a^2 f}$$

$$= -\frac{(3c+5d)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{16\sqrt{2}a^{5/2}f} - \frac{(c-d)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}} - \frac{(3c+5d)\cos(e+fx)}{16af(a+a\sin(e+fx))^{3\ldots}}$$

Mathematica [C] time = 0.352523, size = 227, normalized size = 1.8

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(8(c-d)\sin\left(\frac{1}{2}(e+fx)\right)-(3c+5d)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3+2(3c+5\ldots\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])/(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(8*(c - d)*Sin[(e + f*x)/2] + 4*(-c
+ d)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + 2*(3*c + 5*d)*Sin[(e + f*x)/2]
*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 - (3*c + 5*d)*(Cos[(e + f*x)/2] +
Sin[(e + f*x)/2])^3 + (1 + I)*(-1)^(3/4)*(3*c + 5*d)*ArcTanh[(1/2 + I/2)*(-
1)^(3/4)*(-1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4))
/(16*f*(a*(1 + Sin[e + f*x]))^(5/2))
```

Maple [B] time = 1.065, size = 279, normalized size = 2.2

$$-\frac{1}{\left(32+32\sin\left(fx+e\right)\right)\cos\left(fx+e\right)f}\left(2\sin\left(fx+e\right)\mathrm{Artanh}\left(1/2\,\frac{\sqrt{a-a\sin\left(fx+e\right)}\sqrt{2}}{\sqrt{a}}\right)\sqrt{2}a^3\left(3c+5d\right)-\mathrm{Artanh}\left(\ldots\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2),x)
```

```
[Out] -1/32*(2*sin(f*x+e)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*2^(
1/2)*a^3*(3*c+5*d)-arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*2^(1
/2)*a^3*(3*c+5*d)*cos(f*x+e)^2+20*(a-a*sin(f*x+e))^(1/2)*a^(5/2)*c+12*(a-a*
sin(f*x+e))^(1/2)*a^(5/2)*d-6*(a-a*sin(f*x+e))^(3/2)*a^(3/2)*c-10*(a-a*sin(
f*x+e))^(3/2)*a^(3/2)*d+6*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2
)/a^(1/2))*a^3*c+10*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1
/2))*a^3*d)*(-a*(-1+sin(f*x+e)))^(1/2)/a^(11/2)/(1+sin(f*x+e))/cos(f*x+e)/(
a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{d\sin\left(fx+e\right)+c}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*sin(f*x + e) + c)/(a*sin(f*x + e) + a)^(5/2), x)
```

Fricas [B] time = 2.00946, size = 1015, normalized size = 8.06

$$\frac{\sqrt{2}\left((3\,c+5\,d)\cos\left(fx+e\right)^{3}+3\,(3\,c+5\,d)\cos\left(fx+e\right)^{2}-2\,(3\,c+5\,d)\cos\left(fx+e\right)+\left((3\,c+5\,d)\cos\left(fx+e\right)^{2}-2\,(3\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/64*(sqrt(2)*((3*c + 5*d)*cos(f*x + e)^3 + 3*(3*c + 5*d)*cos(f*x + e)^2 -
2*(3*c + 5*d)*cos(f*x + e) + ((3*c + 5*d)*cos(f*x + e)^2 - 2*(3*c + 5*d)*co
s(f*x + e) - 12*c - 20*d)*sin(f*x + e) - 12*c - 20*d)*sqrt(a)*log(-(a*cos(f
*x + e)^2 - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e) - sin(
f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2*
a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) +
 4*((3*c + 5*d)*cos(f*x + e)^2 + (7*c + d)*cos(f*x + e) + ((3*c + 5*d)*cos(
f*x + e) - 4*c + 4*d)*sin(f*x + e) + 4*c - 4*d)*sqrt(a*sin(f*x + e) + a))/(
a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^
3*f + (a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))/(a+a*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

sage_2

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] sage2
```

3.560 $\int \frac{1}{(a+a\sin(e+fx))^{5/2}} dx$

Optimal. Leaf size=107

$$-\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{16\sqrt{2}a^{5/2}f}-\frac{3\cos(e+fx)}{16af(a\sin(e+fx)+a)^{3/2}}-\frac{\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}}$$

```
[Out] (-3*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])])/(16
*Sqrt[2]*a^(5/2)*f) - Cos[e + f*x]/(4*f*(a + a*Sin[e + f*x])^(5/2)) - (3*Co
s[e + f*x])/(16*a*f*(a + a*Sin[e + f*x])^(3/2))
```

Rubi [A] time = 0.0594574, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2650, 2649, 206}

$$-\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{16\sqrt{2}a^{5/2}f}-\frac{3\cos(e+fx)}{16af(a\sin(e+fx)+a)^{3/2}}-\frac{\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(-5/2),x]
```


```
[Out] (-3*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])])/(16
*Sqrt[2]*a^(5/2)*f) - Cos[e + f*x]/(4*f*(a + a*Sin[e + f*x])^(5/2)) - (3*Co
s[e + f*x])/(16*a*f*(a + a*Sin[e + f*x])^(3/2))
```


Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^{5/2}}\,dx = -\frac{\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}} + \frac{3\int \frac{1}{(a+a\sin(e+fx))^{3/2}}\,dx}{8a}$$

$$= -\frac{\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}} - \frac{3\cos(e+fx)}{16af(a+a\sin(e+fx))^{3/2}} + \frac{3\int \frac{1}{\sqrt{a+a\sin(e+fx)}}\,dx}{32a^2}$$

$$= -\frac{\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}} - \frac{3\cos(e+fx)}{16af(a+a\sin(e+fx))^{3/2}} - \frac{3\,\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \frac{a\cos(e+f}{\sqrt{a+a\sin(e+}}\right.}{16a^2f}$$

$$= -\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{16\sqrt{2}a^{5/2}f} - \frac{\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}} - \frac{3\cos(e+fx)}{16af(a+a\sin(e+fx))^{3/2}}$$

Mathematica [C] time = 0.154643, size = 196, normalized size = 1.83

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(8\sin\left(\frac{1}{2}(e+fx)\right)-3\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3+6\sin\left(\frac{1}{2}(e+fx)\right)\left(\sin\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(-5/2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(8*Sin[(e + f*x)/2] - 4*(Cos[(e + f*
x)/2] + Sin[(e + f*x)/2]) + 6*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e +
 f*x)/2])^2 - 3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 + (3 + 3*I)*(-1)^(3
/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/
2] + Sin[(e + f*x)/2])^4))/(16*f*(a*(1 + Sin[e + f*x]))^(5/2))
```

Maple [B] time = 0.7, size = 195, normalized size = 1.8

$$-\frac{1}{\left(32+32\sin\left(fx+e\right)\right)\cos\left(fx+e\right)f}\left(\sin\left(fx+e\right)\left(6\sqrt{a-a\sin\left(fx+e\right)}a^{3/2}+6\sqrt{2}\text{Artanh}\left(1/2\frac{\sqrt{a-a\sin\left(fx+e\right)}\sqrt{}}{\sqrt{a}}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^(5/2),x)
```

```
[Out] -1/32/a^(9/2)*(sin(f*x+e)*(6*(a-a*sin(f*x+e))^(1/2)*a^(3/2)+6*2^(1/2)*arcta
nh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2)-3*2^(1/2)*arctanh(1/2*(
a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*cos(f*x+e)^2+14*(a-a*sin(f*x+e))
^(1/2)*a^(3/2)+6*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2)
)*a^2)*(-a*(-1+sin(f*x+e)))^(1/2)/(1+sin(f*x+e))/cos(f*x+e)/(a+a*sin(f*x+e)
)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(-5/2), x)
```

Fricas [B] time = 1.88877, size = 851, normalized size = 7.95

$$\frac{3\sqrt{2}\left(\cos\left(fx+e\right)^{3}+3\cos\left(fx+e\right)^{2}+\left(\cos\left(fx+e\right)^{2}-2\cos\left(fx+e\right)-4\right)\sin\left(fx+e\right)-2\cos\left(fx+e\right)-4\right)\sqrt{a}\log}{64\left(a^{3}f\cos\left(fx+e\right)^{3}+3a^{3}f\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/64*(3*sqrt(2)*(cos(f*x + e)^3 + 3*cos(f*x + e)^2 + (cos(f*x + e)^2 - 2*co
s(f*x + e) - 4)*sin(f*x + e) - 2*cos(f*x + e) - 4)*sqrt(a)*log(-(a*cos(f*x
+ e)^2 - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e) - sin(f*x
 + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2*a)/
(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) + 4*
(3*cos(f*x + e)^2 + (3*cos(f*x + e) - 4)*sin(f*x + e) + 7*cos(f*x + e) + 4)
*sqrt(a*sin(f*x + e) + a))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 -
 2*a^3*f*cos(f*x + e) - 4*a^3*f + (a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x +
 e) - 4*a^3*f)*sin(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\sin\left(e+fx\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**(5/2),x)
```

```
[Out] Integral((a*sin(e + f*x) + a)**(-5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] sage2
```

3.561 $\int \frac{1}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))}\,dx$

Optimal. Leaf size=218

$$-\frac{\left(3c^2-14cd+43d^2\right)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{16\sqrt{2}a^{5/2}f(c-d)^3}+\frac{2d^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{a^{5/2}f(c-d)^3\sqrt{c+d}}-\frac{(3c-11d)\cos(e+fx)}{16af(c-d)^2(a\sin(e+fx)+a)^{3/2}}-$$

```
[Out] -((3*c^2 - 14*c*d + 43*d^2)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a
+ a*Sin[e + f*x]])])/(16*Sqrt[2]*a^(5/2)*(c - d)^3*f) + (2*d^(5/2)*ArcTanh[
(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]*Sqrt[a + a*Sin[e + f*x]])])/(a^
(5/2)*(c - d)^3*Sqrt[c + d]*f) - Cos[e + f*x]/(4*(c - d)*f*(a + a*Sin[e + f
*x])^(5/2)) - ((3*c - 11*d)*Cos[e + f*x])/(16*a*(c - d)^2*f*(a + a*Sin[e +
f*x])^(3/2))
```

Rubi [A] time = 0.741456, antiderivative size = 218, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2766, 2978, 2985, 2649, 206, 2773, 208}

$$-\frac{\left(3c^2-14cd+43d^2\right)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{16\sqrt{2}a^{5/2}f(c-d)^3}+\frac{2d^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{a^{5/2}f(c-d)^3\sqrt{c+d}}-\frac{(3c-11d)\cos(e+fx)}{16af(c-d)^2(a\sin(e+fx)+a)^{3/2}}-$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])),x]
```


```
[Out] -((3*c^2 - 14*c*d + 43*d^2)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a
+ a*Sin[e + f*x]])])/(16*Sqrt[2]*a^(5/2)*(c - d)^3*f) + (2*d^(5/2)*ArcTanh[
(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]*Sqrt[a + a*Sin[e + f*x]])])/(a^
(5/2)*(c - d)^3*Sqrt[c + d]*f) - Cos[e + f*x]/(4*(c - d)*f*(a + a*Sin[e + f
*x])^(5/2)) - ((3*c - 11*d)*Cos[e + f*x])/(16*a*(c - d)^2*f*(a + a*Sin[e +
f*x])^(3/2))
```


Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
```

```
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))}\,dx = -\frac{\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}} - \frac{\int \frac{-\frac{1}{2}a(3c-8d)-\frac{3}{2}ad\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))}\,dx}{4a^2(c-d)}$$

$$= -\frac{\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}} - \frac{(3c-11d)\cos(e+fx)}{16a(c-d)^2f(a+a\sin(e+fx))^{3/2}}$$

$$= -\frac{\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}} - \frac{(3c-11d)\cos(e+fx)}{16a(c-d)^2f(a+a\sin(e+fx))^{3/2}}$$

$$= -\frac{\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}} - \frac{(3c-11d)\cos(e+fx)}{16a(c-d)^2f(a+a\sin(e+fx))^{3/2}}$$

$$= -\frac{\left(3c^2-14cd+43d^2\right)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{16\sqrt{2}a^{5/2}(c-d)^3f} + \frac{2d^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}}{\sqrt{c+d}}\right.}{a^{5/2}(c-d)^3}$$

Mathematica [C] time = 3.23491, size = 501, normalized size = 2.3

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(\frac{(1+i)(-1)^{3/4}\left(3c^2-14cd+43d^2\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^4\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(\tan\left(\frac{1}{4}(e+fx)\right)-1\right)\right)}{(c-d)^3}+\cdots\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*((8*Sin[(e + f*x)/2])/(c - d) - (4*(
Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))/(c - d) + (2*(3*c - 11*d)*Sin[(e + f*
x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2)/(c - d)^2 + ((-3*c + 11*d)*(
Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3)/(c - d)^2 + ((1 + I)*(-1)^(3/4)*(3*
c^2 - 14*c*d + 43*d^2)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4
])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4)/(c - d)^3 + (8*d^(5/2)*(e + f*
x - 2*Log[Sec[(e + f*x)/4]^2] + 2*Log[Sec[(e + f*x)/4]^2*(Sqrt[c + d] + Sqr
t[d]*Cos[(e + f*x)/2] - Sqrt[d]*Sin[(e + f*x)/2])])*(Cos[(e + f*x)/2] + Sin
[(e + f*x)/2])^4)/((c - d)^3*Sqrt[c + d]) + (8*d^(5/2)*(e + f*x - 2*Log[Sec
[(e + f*x)/4]^2] + 2*Log[Sec[(e + f*x)/4]^2*(Sqrt[c + d] - Sqrt[d]*Cos[(e +
 f*x)/2] + Sqrt[d]*Sin[(e + f*x)/2])])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)^4)/((-c + d)^3*Sqrt[c + d])))/(16*f*(a*(1 + Sin[e + f*x]))^(5/2))
```

Maple [B] time = 1.442, size = 732, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e)),x)
```

```
[Out] -1/32*(sin(f*x+e)*(-128*d^3*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^
(1/2))*a^(5/2)+6*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/
2)*2^(1/2)/a^(1/2))*a^2*c^2-28*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*s
in(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c*d+86*(a*(c+d)*d)^(1/2)*2^(1/2)*arct
anh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*d^2)+(64*d^3*arctanh((a
-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^(5/2)-3*(a*(c+d)*d)^(1/2)*2^(
1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^2+14*(a*(c+d
)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*
c*d-43*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)
/a^(1/2))*a^2*d^2)*cos(f*x+e)^2-128*d^3*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a
*c*d+a*d^2)^(1/2))*a^(5/2)+6*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin
(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^2-28*(a*(c+d)*d)^(1/2)*2^(1/2)*arctan
h(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c*d+86*(a*(c+d)*d)^(1/2)*
2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*d^2+20*(a*(
c+d)*d)^(1/2)*(a-a*sin(f*x+e))^(1/2)*a^(3/2)*c^2-72*(a*(c+d)*d)^(1/2)*(a-a*
sin(f*x+e))^(1/2)*a^(3/2)*c*d+52*(a*(c+d)*d)^(1/2)*(a-a*sin(f*x+e))^(1/2)*a
^(3/2)*d^2-6*(a*(c+d)*d)^(1/2)*(a-a*sin(f*x+e))^(3/2)*a^(1/2)*c^2+28*(a*(c+
d)*d)^(1/2)*(a-a*sin(f*x+e))^(3/2)*a^(1/2)*c*d-22*(a*(c+d)*d)^(1/2)*(a-a*si
n(f*x+e))^(3/2)*a^(1/2)*d^2)*(-a*(-1+sin(f*x+e)))^(1/2)/a^(9/2)/(1+sin(f*x+
e))/(a*(c+d)*d)^(1/2)/(c-d)^3/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 6.60575, size = 4721, normalized size = 21.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] [-1/64*(sqrt(2)*((3*c^2 - 14*c*d + 43*d^2)*cos(f*x + e)^3 + 3*(3*c^2 - 14*c
*d + 43*d^2)*cos(f*x + e)^2 - 12*c^2 + 56*c*d - 172*d^2 - 2*(3*c^2 - 14*c*d
 + 43*d^2)*cos(f*x + e) + ((3*c^2 - 14*c*d + 43*d^2)*cos(f*x + e)^2 - 12*c^
2 + 56*c*d - 172*d^2 - 2*(3*c^2 - 14*c*d + 43*d^2)*cos(f*x + e))*sin(f*x +
e))*sqrt(a)*log(-(a*cos(f*x + e)^2 + 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqr
t(a)*(cos(f*x + e) - sin(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e)
 - 2*a)*sin(f*x + e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x +
e) - cos(f*x + e) - 2)) + 32*(a*d^2*cos(f*x + e)^3 + 3*a*d^2*cos(f*x + e)^2
 - 2*a*d^2*cos(f*x + e) - 4*a*d^2 + (a*d^2*cos(f*x + e)^2 - 2*a*d^2*cos(f*x
 + e) - 4*a*d^2)*sin(f*x + e))*sqrt(d/(a*c + a*d))*log((d^2*cos(f*x + e)^3
- (6*c*d + 7*d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 - 4*((c*d + d^2)*cos(f
*x + e)^2 - c^2 - 4*c*d - 3*d^2 - (c^2 + 3*c*d + 2*d^2)*cos(f*x + e) + (c^2
 + 4*c*d + 3*d^2 + (c*d + d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x +
 e) + a)*sqrt(d/(a*c + a*d)) - (c^2 + 8*c*d + 9*d^2)*cos(f*x + e) + (d^2*co
s(f*x + e)^2 - c^2 - 2*c*d - d^2 + 2*(3*c*d + 4*d^2)*cos(f*x + e))*sin(f*x
+ e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^
2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - 2*c*d*cos(f*x + e) - c
^2 - 2*c*d - d^2)*sin(f*x + e))) - 4*((3*c^2 - 14*c*d + 11*d^2)*cos(f*x + e
)^2 + 4*c^2 - 8*c*d + 4*d^2 + (7*c^2 - 22*c*d + 15*d^2)*cos(f*x + e) - (4*c
^2 - 8*c*d + 4*d^2 - (3*c^2 - 14*c*d + 11*d^2)*cos(f*x + e))*sin(f*x + e))*
sqrt(a*sin(f*x + e) + a))/((a^3*c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2 - a^3*d^3)*
f*cos(f*x + e)^3 + 3*(a^3*c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2 - a^3*d^3)*f*cos(
f*x + e)^2 - 2*(a^3*c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2 - a^3*d^3)*f*cos(f*x +
e) - 4*(a^3*c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2 - a^3*d^3)*f + ((a^3*c^3 - 3*a^
3*c^2*d + 3*a^3*c*d^2 - a^3*d^3)*f*cos(f*x + e)^2 - 2*(a^3*c^3 - 3*a^3*c^2*
d + 3*a^3*c*d^2 - a^3*d^3)*f*cos(f*x + e) - 4*(a^3*c^3 - 3*a^3*c^2*d + 3*a^
3*c*d^2 - a^3*d^3)*f)*sin(f*x + e)), -1/64*(sqrt(2)*((3*c^2 - 14*c*d + 43*d
^2)*cos(f*x + e)^3 + 3*(3*c^2 - 14*c*d + 43*d^2)*cos(f*x + e)^2 - 12*c^2 +
56*c*d - 172*d^2 - 2*(3*c^2 - 14*c*d + 43*d^2)*cos(f*x + e) + ((3*c^2 - 14*
c*d + 43*d^2)*cos(f*x + e)^2 - 12*c^2 + 56*c*d - 172*d^2 - 2*(3*c^2 - 14*c*
d + 43*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a)*log(-(a*cos(f*x + e)^2 + 2*
sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e) - sin(f*x + e) + 1)
+ 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2*a)/(cos(f*x +
e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) - 64*(a*d^2*cos
(f*x + e)^3 + 3*a*d^2*cos(f*x + e)^2 - 2*a*d^2*cos(f*x + e) - 4*a*d^2 + (a*
d^2*cos(f*x + e)^2 - 2*a*d^2*cos(f*x + e) - 4*a*d^2)*sin(f*x + e))*sqrt(-d/
(a*c + a*d))*arctan(1/2*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) - c - 2*d)
*sqrt(-d/(a*c + a*d))/(d*cos(f*x + e))) - 4*((3*c^2 - 14*c*d + 11*d^2)*cos(
f*x + e)^2 + 4*c^2 - 8*c*d + 4*d^2 + (7*c^2 - 22*c*d + 15*d^2)*cos(f*x + e)
 - (4*c^2 - 8*c*d + 4*d^2 - (3*c^2 - 14*c*d + 11*d^2)*cos(f*x + e))*sin(f*x
 + e))*sqrt(a*sin(f*x + e) + a))/((a^3*c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2 - a^
3*d^3)*f*cos(f*x + e)^3 + 3*(a^3*c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2 - a^3*d^3)
*f*cos(f*x + e)^2 - 2*(a^3*c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2 - a^3*d^3)*f*cos
```

```
(f*x + e) - 4*(a^3*c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2 - a^3*d^3)*f + ((a^3*c^3
 - 3*a^3*c^2*d + 3*a^3*c*d^2 - a^3*d^3)*f*cos(f*x + e)^2 - 2*(a^3*c^3 - 3*a
^3*c^2*d + 3*a^3*c*d^2 - a^3*d^3)*f*cos(f*x + e) - 4*(a^3*c^3 - 3*a^3*c^2*d
 + 3*a^3*c*d^2 - a^3*d^3)*f)*sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**(5/2)/(c+d*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.562 $\int \frac{1}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^2}\,dx$

Optimal. Leaf size=313

$$-\frac{\left(3c^2-22cd+115d^2\right)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{16\sqrt{2}a^{5/2}f(c-d)^4}+\frac{d^{5/2}(7c+5d)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{a^{5/2}f(c-d)^4(c+d)^{3/2}}-\frac{d(c-7d)}{16a^2f(c-d)^3(c+d)\sqrt{}}$$

```
[Out] -((3*c^2 - 22*c*d + 115*d^2)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a
 + a*Sin[e + f*x]])])/(16*Sqrt[2]*a^(5/2)*(c - d)^4*f) + (d^(5/2)*(7*c + 5*
d)*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]*Sqrt[a + a*Sin[e + f
*x]])])/(a^(5/2)*(c - d)^4*(c + d)^(3/2)*f) - Cos[e + f*x]/(4*(c - d)*f*(a
+ a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])) - (3*(c - 5*d)*Cos[e + f*x])/
(16*a*(c - d)^2*f*(a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])) - ((c -
7*d)*d*(3*c + 5*d)*Cos[e + f*x])/(16*a^2*(c - d)^3*(c + d)*f*Sqrt[a + a*Sin
[e + f*x]]*(c + d*Sin[e + f*x]))
```

Rubi [A] time = 1.09969, antiderivative size = 313, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2766, 2978, 2984, 2985, 2649, 206, 2773, 208}

$$-\frac{\left(3c^2-22cd+115d^2\right)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{16\sqrt{2}a^{5/2}f(c-d)^4}+\frac{d^{5/2}(7c+5d)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{a^{5/2}f(c-d)^4(c+d)^{3/2}}-\frac{d(c-7d)}{16a^2f(c-d)^3(c+d)\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])^2),x]
```


```
[Out] -((3*c^2 - 22*c*d + 115*d^2)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a
 + a*Sin[e + f*x]])])/(16*Sqrt[2]*a^(5/2)*(c - d)^4*f) + (d^(5/2)*(7*c + 5*
d)*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]*Sqrt[a + a*Sin[e + f
*x]])])/(a^(5/2)*(c - d)^4*(c + d)^(3/2)*f) - Cos[e + f*x]/(4*(c - d)*f*(a
+ a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])) - (3*(c - 5*d)*Cos[e + f*x])/
(16*a*(c - d)^2*f*(a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])) - ((c -
7*d)*d*(3*c + 5*d)*Cos[e + f*x])/(16*a^2*(c - d)^3*(c + d)*f*Sqrt[a + a*Sin
[e + f*x]]*(c + d*Sin[e + f*x]))
```


Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
```

```
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^2}\,dx = -\frac{\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))} - \frac{\int \frac{-\frac{1}{2}a(3c-10d}{(a+a\sin(e+fx))}}{4a^2}$$

$$= -\frac{\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))} - \frac{}{16a(c-d)^2f(a}$$

$$= -\frac{\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))} - \frac{}{16a(c-d)^2f(a}$$

$$= -\frac{\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))} - \frac{}{16a(c-d)^2f(a}$$

$$= -\frac{\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))} - \frac{}{16a(c-d)^2f(a}$$

$$= -\frac{(3c^2-22cd+115d^2)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{16\sqrt{2}a^{5/2}(c-d)^4f} + \frac{d^{5/2}(7c+5d)\tan}{a^{5/2}(c-}$$

Mathematica [C] time = 5.76415, size = 570, normalized size = 1.82

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left((1+i)(-1)^{3/4}\left(3c^2-22cd+115d^2\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^4\tanh^{-1}(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])^2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(8*(c - d)^2*Sin[(e + f*x)/2] - 4*(c
 - d)^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + 2*(3*c - 19*d)*(c - d)*Sin[
(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + (c - d)*(-3*c + 19*d
)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 + (1 + I)*(-1)^(3/4)*(3*c^2 - 22*
c*d + 115*d^2)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])]*(Cos
[(e + f*x)/2] + Sin[(e + f*x)/2])^4 + (4*d^(5/2)*(7*c + 5*d)*(e + f*x - 2*L
og[Sec[(e + f*x)/4]^2] + 2*Log[Sec[(e + f*x)/4]^2*(Sqrt[c + d] + Sqrt[d]*Co
s[(e + f*x)/2] - Sqrt[d]*Sin[(e + f*x)/2])]])*(Cos[(e + f*x)/2] + Sin[(e + f
*x)/2])^4)/(c + d)^(3/2) - (4*d^(5/2)*(7*c + 5*d)*(e + f*x - 2*Log[Sec[(e +
 f*x)/4]^2] + 2*Log[Sec[(e + f*x)/4]^2*(Sqrt[c + d] - Sqrt[d]*Cos[(e + f*x)
/2] + Sqrt[d]*Sin[(e + f*x)/2])]])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4)/
(c + d)^(3/2) + (16*(c - d)*d^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[
(e + f*x)/2] + Sin[(e + f*x)/2])^4)/((c + d)*(c + d*Sin[e + f*x]))))/(16*(c
 - d)^4*f*(a*(1 + Sin[e + f*x]))^(5/2))
```

Maple [B] time = 1.894, size = 1972, normalized size = 6.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^2,x)
```

```
[Out] -1/32*(115*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)
*2^(1/2)/a^(1/2))*sin(f*x+e)^3*a^2*d^4+55*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh
(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*a^2*c^2*d^2+3
*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a
^(1/2))*sin(f*x+e)^3*a^2*c^3*d-19*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a
*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^3*a^2*c^2*d^2+301*(a*(c
+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2)
)*sin(f*x+e)^2*a^2*c*d^3-35*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(-1+s
in(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a^2*c^3*d+167*(a*(c+d)*d)^(1/
2)*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+
e)*a^2*c^2*d^2+323*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)
))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a^2*c*d^3+93*(a*(c+d)*d)^(1/2)*2^(1/2)
*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^3*a^2*c
*d^3-13*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^
(1/2)/a^(1/2))*sin(f*x+e)^2*a^2*c^3*d-160*arctanh((-a*(-1+sin(f*x+e)))^(1/2
)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)^3*d^5-320*arctanh((-a*(-1+sin(f*x
+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)^2*d^5-160*arctanh((-a*(
-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)*d^5-224*arcta
nh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*c^2*d^3-160*arct
anh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*c*d^4+20*(-a*(-
1+sin(f*x+e)))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*c^4+32*(-a*(-1+sin(f*x+e)))^
(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*d^4-6*(-a*(-1+sin(f*x+e)))^(3/2)*(a*(c+d)*d
)^(1/2)*a^(1/2)*c^4-32*(-a*(-1+sin(f*x+e)))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)
*sin(f*x+e)^2*c*d^3+20*(-a*(-1+sin(f*x+e)))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)
*sin(f*x+e)*c^3*d-84*(-a*(-1+sin(f*x+e)))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*s
in(f*x+e)*c^2*d^2-84*(-a*(-1+sin(f*x+e)))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*s
in(f*x+e)*c*d^3+3*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e))
)^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*a^2*c^4-6*(-a*(-1+sin(f*x+e)))^(3/2)*
(a*(c+d)*d)^(1/2)*a^(1/2)*sin(f*x+e)*c^3*d+38*(-a*(-1+sin(f*x+e)))^(3/2)*(a
*(c+d)*d)^(1/2)*a^(1/2)*sin(f*x+e)*c^2*d^2+6*(-a*(-1+sin(f*x+e)))^(3/2)*(a*
(c+d)*d)^(1/2)*a^(1/2)*sin(f*x+e)*c*d^3+6*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh
(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a^2*c^4+115*(a*
(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/
2))*sin(f*x+e)*a^2*d^4-19*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(-1+sin
(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^3*d+93*(a*(c+d)*d)^(1/2)*2^(1/2)*arc
tanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^2*d^2+115*(a*(c+
d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))
*a^2*c*d^3+230*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(
1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*a^2*d^4+3*(a*(c+d)*d)^(1/2)*2^(1/2)*arct
anh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^4-224*arctanh((-a
*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)^3*c*d^4-38*
(-a*(-1+sin(f*x+e)))^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*sin(f*x+e)*d^4+52*(-a*
(-1+sin(f*x+e)))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*c*d^3-608*arctanh((-a*(-1+
sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)^2*c*d^4+32*(-a*(
-1+sin(f*x+e)))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*sin(f*x+e)^2*d^4-448*arctan
h((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)*c^2*d^
3-544*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f
*x+e)*c*d^4+148*(-a*(-1+sin(f*x+e)))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*sin(f*
x+e)*d^4-84*(-a*(-1+sin(f*x+e)))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*c^3*d-20*(
-a*(-1+sin(f*x+e)))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*c^2*d^2-224*arctanh((-a
*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)^2*c^2*d^3+3
8*(-a*(-1+sin(f*x+e)))^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*c^3*d+6*(-a*(-1+sin(
f*x+e)))^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*c^2*d^2-38*(-a*(-1+sin(f*x+e)))^(3
/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*c*d^3)*(-a*(-1+sin(f*x+e)))^(1/2)/a^(9/2)/(1+
sin(f*x+e))/(a*(c+d)*d)^(1/2)/(c+d*sin(f*x+e))/(c+d)/(c-d)^4/cos(f*x+e)/(a+
a*sin(f*x+e))^(1/2)/f
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^2,x, algorithm="maxima"
)
```

```
[Out] Timed out
```

Fricas [B] time = 12.7231, size = 8342, normalized size = 26.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^2,x, algorithm="fricas"
)
```

```
[Out] [1/64*(sqrt(2)*((3*c^3*d - 19*c^2*d^2 + 93*c*d^3 + 115*d^4)*cos(f*x + e)^4
+ 12*c^4 - 64*c^3*d + 296*c^2*d^2 + 832*c*d^3 + 460*d^4 - (3*c^4 - 13*c^3*d
 + 55*c^2*d^2 + 301*c*d^3 + 230*d^4)*cos(f*x + e)^3 - (9*c^4 - 42*c^3*d + 1
84*c^2*d^2 + 810*c*d^3 + 575*d^4)*cos(f*x + e)^2 + 2*(3*c^4 - 16*c^3*d + 74
*c^2*d^2 + 208*c*d^3 + 115*d^4)*cos(f*x + e) + (12*c^4 - 64*c^3*d + 296*c^2
*d^2 + 832*c*d^3 + 460*d^4 - (3*c^3*d - 19*c^2*d^2 + 93*c*d^3 + 115*d^4)*co
s(f*x + e)^3 - (3*c^4 - 10*c^3*d + 36*c^2*d^2 + 394*c*d^3 + 345*d^4)*cos(f*
x + e)^2 + 2*(3*c^4 - 16*c^3*d + 74*c^2*d^2 + 208*c*d^3 + 115*d^4)*cos(f*x
+ e))*sin(f*x + e))*sqrt(a)*log(-(a*cos(f*x + e)^2 - 2*sqrt(2)*sqrt(a*sin(f
*x + e) + a)*sqrt(a)*(cos(f*x + e) - sin(f*x + e) + 1) + 3*a*cos(f*x + e) -
 (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e)
 + 2)*sin(f*x + e) - cos(f*x + e) - 2)) + 16*(28*a*c^2*d^2 + 48*a*c*d^3 + 2
0*a*d^4 + (7*a*c*d^3 + 5*a*d^4)*cos(f*x + e)^4 - (7*a*c^2*d^2 + 19*a*c*d^3
+ 10*a*d^4)*cos(f*x + e)^3 - (21*a*c^2*d^2 + 50*a*c*d^3 + 25*a*d^4)*cos(f*x
 + e)^2 + 2*(7*a*c^2*d^2 + 12*a*c*d^3 + 5*a*d^4)*cos(f*x + e) + (28*a*c^2*d
^2 + 48*a*c*d^3 + 20*a*d^4 - (7*a*c*d^3 + 5*a*d^4)*cos(f*x + e)^3 - (7*a*c^
2*d^2 + 26*a*c*d^3 + 15*a*d^4)*cos(f*x + e)^2 + 2*(7*a*c^2*d^2 + 12*a*c*d^3
 + 5*a*d^4)*cos(f*x + e))*sin(f*x + e))*sqrt(d/(a*c + a*d))*log((d^2*cos(f*
x + e)^3 - (6*c*d + 7*d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 + 4*((c*d + d
^2)*cos(f*x + e)^2 - c^2 - 4*c*d - 3*d^2 - (c^2 + 3*c*d + 2*d^2)*cos(f*x +
e) + (c^2 + 4*c*d + 3*d^2 + (c*d + d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*
sin(f*x + e) + a)*sqrt(d/(a*c + a*d)) - (c^2 + 8*c*d + 9*d^2)*cos(f*x + e)
+ (d^2*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 + 2*(3*c*d + 4*d^2)*cos(f*x + e))
*sin(f*x + e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^2 - 2
*c*d - d^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - 2*c*d*cos(f*x
 + e) - c^2 - 2*c*d - d^2)*sin(f*x + e))) - 4*(4*c^4 - 8*c^3*d + 8*c*d^3 -
4*d^4 - (3*c^3*d - 19*c^2*d^2 - 19*c*d^3 + 35*d^4)*cos(f*x + e)^3 + (3*c^4
- 15*c^3*d - 7*c^2*d^2 - c*d^3 + 20*d^4)*cos(f*x + e)^2 + (7*c^4 - 20*c^3*d
 - 26*c^2*d^2 - 12*c*d^3 + 51*d^4)*cos(f*x + e) - (4*c^4 - 8*c^3*d + 8*c*d^
3 - 4*d^4 - (3*c^3*d - 19*c^2*d^2 - 19*c*d^3 + 35*d^4)*cos(f*x + e)^2 - (3*
c^4 - 12*c^3*d - 26*c^2*d^2 - 20*c*d^3 + 55*d^4)*cos(f*x + e))*sin(f*x + e)
)*sqrt(a*sin(f*x + e) + a))/((a^3*c^5*d - 3*a^3*c^4*d^2 + 2*a^3*c^3*d^3 + 2
*a^3*c^2*d^4 - 3*a^3*c*d^5 + a^3*d^6)*f*cos(f*x + e)^4 - (a^3*c^6 - a^3*c^5
*d - 4*a^3*c^4*d^2 + 6*a^3*c^3*d^3 + a^3*c^2*d^4 - 5*a^3*c*d^5 + 2*a^3*d^6)
*f*cos(f*x + e)^3 - (3*a^3*c^6 - 4*a^3*c^5*d - 9*a^3*c^4*d^2 + 16*a^3*c^3*d
^3 + a^3*c^2*d^4 - 12*a^3*c*d^5 + 5*a^3*d^6)*f*cos(f*x + e)^2 + 2*(a^3*c^6
- 2*a^3*c^5*d - a^3*c^4*d^2 + 4*a^3*c^3*d^3 - a^3*c^2*d^4 - 2*a^3*c*d^5 + a
```

```
^3*d^6)*f*cos(f*x + e) + 4*(a^3*c^6 - 2*a^3*c^5*d - a^3*c^4*d^2 + 4*a^3*c^3
*d^3 - a^3*c^2*d^4 - 2*a^3*c*d^5 + a^3*d^6)*f - ((a^3*c^5*d - 3*a^3*c^4*d^2
 + 2*a^3*c^3*d^3 + 2*a^3*c^2*d^4 - 3*a^3*c*d^5 + a^3*d^6)*f*cos(f*x + e)^3
+ (a^3*c^6 - 7*a^3*c^4*d^2 + 8*a^3*c^3*d^3 + 3*a^3*c^2*d^4 - 8*a^3*c*d^5 +
3*a^3*d^6)*f*cos(f*x + e)^2 - 2*(a^3*c^6 - 2*a^3*c^5*d - a^3*c^4*d^2 + 4*a^
3*c^3*d^3 - a^3*c^2*d^4 - 2*a^3*c*d^5 + a^3*d^6)*f*cos(f*x + e) - 4*(a^3*c^
6 - 2*a^3*c^5*d - a^3*c^4*d^2 + 4*a^3*c^3*d^3 - a^3*c^2*d^4 - 2*a^3*c*d^5 +
 a^3*d^6)*f)*sin(f*x + e)), 1/64*(sqrt(2)*((3*c^3*d - 19*c^2*d^2 + 93*c*d^3
 + 115*d^4)*cos(f*x + e)^4 + 12*c^4 - 64*c^3*d + 296*c^2*d^2 + 832*c*d^3 +
460*d^4 - (3*c^4 - 13*c^3*d + 55*c^2*d^2 + 301*c*d^3 + 230*d^4)*cos(f*x + e
)^3 - (9*c^4 - 42*c^3*d + 184*c^2*d^2 + 810*c*d^3 + 575*d^4)*cos(f*x + e)^2
 + 2*(3*c^4 - 16*c^3*d + 74*c^2*d^2 + 208*c*d^3 + 115*d^4)*cos(f*x + e) + (
12*c^4 - 64*c^3*d + 296*c^2*d^2 + 832*c*d^3 + 460*d^4 - (3*c^3*d - 19*c^2*d
^2 + 93*c*d^3 + 115*d^4)*cos(f*x + e)^3 - (3*c^4 - 10*c^3*d + 36*c^2*d^2 +
394*c*d^3 + 345*d^4)*cos(f*x + e)^2 + 2*(3*c^4 - 16*c^3*d + 74*c^2*d^2 + 20
8*c*d^3 + 115*d^4)*cos(f*x + e))*sin(f*x + e))*sqrt(a)*log(-(a*cos(f*x + e)
^2 - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e) - sin(f*x + e
) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2*a)/(cos
(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) + 32*(28
*a*c^2*d^2 + 48*a*c*d^3 + 20*a*d^4 + (7*a*c*d^3 + 5*a*d^4)*cos(f*x + e)^4 -
 (7*a*c^2*d^2 + 19*a*c*d^3 + 10*a*d^4)*cos(f*x + e)^3 - (21*a*c^2*d^2 + 50*
a*c*d^3 + 25*a*d^4)*cos(f*x + e)^2 + 2*(7*a*c^2*d^2 + 12*a*c*d^3 + 5*a*d^4)
*cos(f*x + e) + (28*a*c^2*d^2 + 48*a*c*d^3 + 20*a*d^4 - (7*a*c*d^3 + 5*a*d^
4)*cos(f*x + e)^3 - (7*a*c^2*d^2 + 26*a*c*d^3 + 15*a*d^4)*cos(f*x + e)^2 +
2*(7*a*c^2*d^2 + 12*a*c*d^3 + 5*a*d^4)*cos(f*x + e))*sin(f*x + e))*sqrt(-d/
(a*c + a*d))*arctan(1/2*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) - c - 2*d)
*sqrt(-d/(a*c + a*d))/(d*cos(f*x + e))) - 4*(4*c^4 - 8*c^3*d + 8*c*d^3 - 4*
d^4 - (3*c^3*d - 19*c^2*d^2 - 19*c*d^3 + 35*d^4)*cos(f*x + e)^3 + (3*c^4 -
15*c^3*d - 7*c^2*d^2 - c*d^3 + 20*d^4)*cos(f*x + e)^2 + (7*c^4 - 20*c^3*d -
 26*c^2*d^2 - 12*c*d^3 + 51*d^4)*cos(f*x + e) - (4*c^4 - 8*c^3*d + 8*c*d^3
- 4*d^4 - (3*c^3*d - 19*c^2*d^2 - 19*c*d^3 + 35*d^4)*cos(f*x + e)^2 - (3*c^
4 - 12*c^3*d - 26*c^2*d^2 - 20*c*d^3 + 55*d^4)*cos(f*x + e))*sin(f*x + e))*
sqrt(a*sin(f*x + e) + a))/((a^3*c^5*d - 3*a^3*c^4*d^2 + 2*a^3*c^3*d^3 + 2*a
^3*c^2*d^4 - 3*a^3*c*d^5 + a^3*d^6)*f*cos(f*x + e)^4 - (a^3*c^6 - a^3*c^5*d
 - 4*a^3*c^4*d^2 + 6*a^3*c^3*d^3 + a^3*c^2*d^4 - 5*a^3*c*d^5 + 2*a^3*d^6)*f
*cos(f*x + e)^3 - (3*a^3*c^6 - 4*a^3*c^5*d - 9*a^3*c^4*d^2 + 16*a^3*c^3*d^3
 + a^3*c^2*d^4 - 12*a^3*c*d^5 + 5*a^3*d^6)*f*cos(f*x + e)^2 + 2*(a^3*c^6 -
2*a^3*c^5*d - a^3*c^4*d^2 + 4*a^3*c^3*d^3 - a^3*c^2*d^4 - 2*a^3*c*d^5 + a^3
*d^6)*f*cos(f*x + e) + 4*(a^3*c^6 - 2*a^3*c^5*d - a^3*c^4*d^2 + 4*a^3*c^3*d
^3 - a^3*c^2*d^4 - 2*a^3*c*d^5 + a^3*d^6)*f - ((a^3*c^5*d - 3*a^3*c^4*d^2 +
 2*a^3*c^3*d^3 + 2*a^3*c^2*d^4 - 3*a^3*c*d^5 + a^3*d^6)*f*cos(f*x + e)^3 +
(a^3*c^6 - 7*a^3*c^4*d^2 + 8*a^3*c^3*d^3 + 3*a^3*c^2*d^4 - 8*a^3*c*d^5 + 3*
a^3*d^6)*f*cos(f*x + e)^2 - 2*(a^3*c^6 - 2*a^3*c^5*d - a^3*c^4*d^2 + 4*a^3*
c^3*d^3 - a^3*c^2*d^4 - 2*a^3*c*d^5 + a^3*d^6)*f*cos(f*x + e) - 4*(a^3*c^6
- 2*a^3*c^5*d - a^3*c^4*d^2 + 4*a^3*c^3*d^3 - a^3*c^2*d^4 - 2*a^3*c*d^5 + a
^3*d^6)*f)*sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**(5/2)/(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.563 $\int \frac{1}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^3}\,dx$

Optimal. Leaf size=400

$$-\frac{3d(c+3d)\left(c^2-10cd-7d^2\right)\cos(e+fx)}{16a^2f(c-d)^4(c+d)^2\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}-\frac{d\left(3c^2-20cd-31d^2\right)\cos(e+fx)}{16a^2f(c-d)^3(c+d)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}$$

```
[Out] (-3*(c^2 - 10*c*d + 73*d^2)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a
+ a*Sin[e + f*x]])]])/(16*Sqrt[2]*a^(5/2)*(c - d)^5*f) + (3*d^(5/2)*(21*c^2
+ 30*c*d + 13*d^2)*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]*Sqrt
[a + a*Sin[e + f*x]])])/(4*a^(5/2)*(c - d)^5*(c + d)^(5/2)*f) - Cos[e + f*x
]/(4*(c - d)*f*(a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])^2) - ((3*c -
 19*d)*Cos[e + f*x])/(16*a*(c - d)^2*f*(a + a*Sin[e + f*x])^(3/2)*(c + d*Si
n[e + f*x])^2) - (d*(3*c^2 - 20*c*d - 31*d^2)*Cos[e + f*x])/(16*a^2*(c - d)
^3*(c + d)*f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^2) - (3*d*(c + 3
*d)*(c^2 - 10*c*d - 7*d^2)*Cos[e + f*x])/(16*a^2*(c - d)^4*(c + d)^2*f*Sqrt
[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x]))
```

Rubi [A] time = 1.51505, antiderivative size = 400, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2766, 2978, 2984, 2985, 2649, 206, 2773, 208}

$$-\frac{3d(c+3d)\left(c^2-10cd-7d^2\right)\cos(e+fx)}{16a^2f(c-d)^4(c+d)^2\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}-\frac{d\left(3c^2-20cd-31d^2\right)\cos(e+fx)}{16a^2f(c-d)^3(c+d)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])^3),x]
```


```
[Out] (-3*(c^2 - 10*c*d + 73*d^2)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a
+ a*Sin[e + f*x]])]])/(16*Sqrt[2]*a^(5/2)*(c - d)^5*f) + (3*d^(5/2)*(21*c^2
+ 30*c*d + 13*d^2)*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]*Sqrt
[a + a*Sin[e + f*x]])])/(4*a^(5/2)*(c - d)^5*(c + d)^(5/2)*f) - Cos[e + f*x
]/(4*(c - d)*f*(a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])^2) - ((3*c -
 19*d)*Cos[e + f*x])/(16*a*(c - d)^2*f*(a + a*Sin[e + f*x])^(3/2)*(c + d*Si
n[e + f*x])^2) - (d*(3*c^2 - 20*c*d - 31*d^2)*Cos[e + f*x])/(16*a^2*(c - d)
^3*(c + d)*f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^2) - (3*d*(c + 3
*d)*(c^2 - 10*c*d - 7*d^2)*Cos[e + f*x])/(16*a^2*(c - d)^4*(c + d)^2*f*Sqrt
[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x]))
```


Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^3}\,dx = -\frac{\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^2} - \frac{\int \frac{-\frac{3}{2}a(c-4d)-\frac{7}{2}\cdots}{(a+a\sin(e+fx))^{3/2}\cdots}}{4a^2(c\cdots}$$

$$= -\frac{\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^2} - \frac{(3\cdots}{16a(c-d)^2f(a+\cdots}$$

$$= -\frac{\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^2} - \frac{(3\cdots}{16a(c-d)^2f(a+\cdots}$$

$$= -\frac{\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^2} - \frac{(3\cdots}{16a(c-d)^2f(a+\cdots}$$

$$= -\frac{\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^2} - \frac{(3\cdots}{16a(c-d)^2f(a+\cdots}$$

$$= -\frac{\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^2} - \frac{(3\cdots}{16a(c-d)^2f(a+\cdots}$$

$$= -\frac{3\left(c^2-10cd+73d^2\right)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{16\sqrt{2}a^{5/2}(c-d)^5f} + \frac{3d^{5/2}\left(21c^2+30cd+\cdots\right.}{4a^{5/2}\cdots}$$

Mathematica [C] time = 9.30076, size = 958, normalized size = 2.4

$$\frac{3\left(3\cos\left(\frac{1}{2}(e+fx)\right)d^4-3\sin\left(\frac{1}{2}(e+fx)\right)d^4+5c\cos\left(\frac{1}{2}(e+fx)\right)d^3-5c\sin\left(\frac{1}{2}(e+fx)\right)d^3\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e\cdots\right.\right.}{4(c-d)^4(c+d)^2f(a(\sin(e+fx)+1))^{5/2}(c+d\sin(e+fx))}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])^3),x]
```

```
[Out] (Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))/(2*(c - d)^3*f*(a*
(1 + Sin[e + f*x]))^(5/2)) - (Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2/(4*(c
- d)^3*f*(a*(1 + Sin[e + f*x]))^(5/2)) - (3*(c - 9*d)*(Cos[(e + f*x)/2] + S
in[(e + f*x)/2])^4)/(16*(c - d)^4*f*(a*(1 + Sin[e + f*x]))^(5/2)) + ((3 + 3
*I)*(c^2 - 10*c*d + 73*d^2)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*Sec[(e + f*x)/4]
*(Cos[(e + f*x)/4] - Sin[(e + f*x)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2
])^5)/((16*(-1)^(1/4)*c^5 - 80*(-1)^(1/4)*c^4*d + 160*(-1)^(1/4)*c^3*d^2 -
160*(-1)^(1/4)*c^2*d^3 + 80*(-1)^(1/4)*c*d^4 - 16*(-1)^(1/4)*d^5)*f*(a*(1 +
 Sin[e + f*x]))^(5/2)) + (3*d^(5/2)*(21*c^2 + 30*c*d + 13*d^2)*(e + f*x - 2
*Log[Sec[(e + f*x)/4]^2] + 2*Log[Sec[(e + f*x)/4]^2*(Sqrt[c + d] + Sqrt[d]*
Cos[(e + f*x)/2] - Sqrt[d]*Sin[(e + f*x)/2])])*(Cos[(e + f*x)/2] + Sin[(e +
 f*x)/2])^5)/(16*(c - d)^5*(c + d)^(5/2)*f*(a*(1 + Sin[e + f*x]))^(5/2)) +
(3*d^(5/2)*(21*c^2 + 30*c*d + 13*d^2)*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2]
+ 2*Log[Sec[(e + f*x)/4]^2*(Sqrt[c + d] - Sqrt[d]*Cos[(e + f*x)/2] + Sqrt[d
]*Sin[(e + f*x)/2])])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)/(16*(-c + d)
^5*(c + d)^(5/2)*f*(a*(1 + Sin[e + f*x]))^(5/2)) + (3*(Cos[(e + f*x)/2] + S
in[(e + f*x)/2])^3*(c*Sin[(e + f*x)/2] - 9*d*Sin[(e + f*x)/2]))/(8*(c - d)^
4*f*(a*(1 + Sin[e + f*x]))^(5/2)) + ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^
5*(d^3*Cos[(e + f*x)/2] - d^3*Sin[(e + f*x)/2]))/(2*(c - d)^3*(c + d)*f*(a*
(1 + Sin[e + f*x]))^(5/2)*(c + d*Sin[e + f*x])^2) + (3*(Cos[(e + f*x)/2] +
Sin[(e + f*x)/2])^5*(5*c*d^3*Cos[(e + f*x)/2] + 3*d^4*Cos[(e + f*x)/2] - 5*
c*d^3*Sin[(e + f*x)/2] - 3*d^4*Sin[(e + f*x)/2]))/(4*(c - d)^4*(c + d)^2*f*
```

```
(a*(1 + Sin[e + f*x]))^(5/2)*(c + d*Sin[e + f*x]))
```

Maple [B] time = 2.497, size = 3535, normalized size = 8.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^3,x)
```

```
[Out] -1/32*(-a*(-1+sin(f*x+e)))^(1/2)*(408*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2
*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^3*d^3+20*(-a*(-1+sin(f*x
+e)))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*c^6+56*(-a*(-1+sin(f*x+e)))^(1/2)*(a*
(c+d)*d)^(1/2)*a^(3/2)*d^6-312*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d
)*d)^(1/2))*a^(5/2)*sin(f*x+e)^4*d^7-2448*arctanh((-a*(-1+sin(f*x+e)))^(1/2
)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)^3*c^2*d^5-2064*arctanh((-a*(-1+si
n(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)^3*c*d^6-504*arctan
h((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)^2*c^4*
d^3-2736*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*si
n(f*x+e)^2*c^3*d^4-3696*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1
/2))*a^(5/2)*sin(f*x+e)^2*c^2*d^5-1968*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d
/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)^2*c*d^6+172*(-a*(-1+sin(f*x+e)))^(1/
2)*(a*(c+d)*d)^(1/2)*a^(3/2)*sin(f*x+e)^2*d^6-1008*arctanh((-a*(-1+sin(f*x+
e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)*c^4*d^3-2448*arctanh((-a
*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)*c^3*d^4+69*
(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^
(1/2))*sin(f*x+e)^2*a^2*c^4*d^2+1032*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*
(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*a^2*c^3*d^3+2013*(
a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(
1/2))*sin(f*x+e)^2*a^2*c^2*d^4+1284*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(
-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*a^2*c*d^5+3*(a*(c+d
)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*
sin(f*x+e)^4*a^2*c^4*d^2-24*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(-1+s
in(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^4*a^2*c^3*d^3+162*(a*(c+d)*d)
^(1/2)*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*sin(
f*x+e)^4*a^2*c^2*d^4-624*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(
1/2))*a^(5/2)*sin(f*x+e)^3*d^7-312*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*
(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)^2*d^7-6*(-a*(-1+sin(f*x+e)))^(3/2)*(a*(c
+d)*d)^(1/2)*a^(1/2)*c^6-72*(-a*(-1+sin(f*x+e)))^(3/2)*(a*(c+d)*d)^(1/2)*a^
(1/2)*d^6-504*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/
2)*c^4*d^3-720*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5
/2)*c^3*d^4-312*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(
5/2)*c^2*d^5+60*(-a*(-1+sin(f*x+e)))^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*c^4*d^
2-48*(-a*(-1+sin(f*x+e)))^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*c^3*d^3+66*(-a*(-
1+sin(f*x+e)))^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*c^2*d^4-48*(-a*(-1+sin(f*x+e
)))^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*c*d^5+3*(a*(c+d)*d)^(1/2)*2^(1/2)*arcta
nh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^6-2064*arctanh((-a
*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)*c^2*d^5-624
*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)
*c*d^6+112*(-a*(-1+sin(f*x+e)))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*sin(f*x+e)*
d^6-96*(-a*(-1+sin(f*x+e)))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*c^5*d-136*(-a*(
-1+sin(f*x+e)))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*c^4*d^2-40*(-a*(-1+sin(f*x+
e)))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*c^3*d^3+60*(-a*(-1+sin(f*x+e)))^(1/2)*
(a*(c+d)*d)^(1/2)*a^(3/2)*c^2*d^4+136*(-a*(-1+sin(f*x+e)))^(1/2)*(a*(c+d)*d
)^(1/2)*a^(3/2)*c*d^5-504*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^
(1/2))*a^(5/2)*sin(f*x+e)^4*c^2*d^5-720*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*
d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)^4*c*d^6-1008*arctanh((-a*(-1+sin(f*
```

```
x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)^3*c^3*d^4-126*(-a*(-1+
sin(f*x+e)))^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*sin(f*x+e)^2*d^6-144*(-a*(-1+s
in(f*x+e)))^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*sin(f*x+e)*d^6+48*(-a*(-1+sin(f
*x+e)))^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*c^5*d+408*(a*(c+d)*d)^(1/2)*2^(1/2)
*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^4*a^2*c
*d^5+6*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(
1/2)/a^(1/2))*sin(f*x+e)^3*a^2*c^5*d-42*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1
/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^3*a^2*c^4*d^2-42*
(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^
(1/2))*sin(f*x+e)*a^2*c^5*d+276*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(
-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a^2*c^4*d^2+1140*(a*(c+d)
*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*s
in(f*x+e)*a^2*c^3*d^3+1254*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(-1+si
n(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a^2*c^2*d^4+438*(a*(c+d)*d)^(1
/2)*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x
+e)*a^2*c*d^5+276*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e))
)^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^3*a^2*c^3*d^3+1140*(a*(c+d)*d)^(1/2)*2^
(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^3*
a^2*c^2*d^4+1254*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)))
^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^3*a^2*c*d^5-12*(a*(c+d)*d)^(1/2)*2^(1/2)
*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*a^2*c
^5*d+219*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2
^(1/2)/a^(1/2))*a^2*c^2*d^4+219*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(
-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^4*a^2*d^6-6*(-a*(-1+sin(f
*x+e)))^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*sin(f*x+e)^2*c^4*d^2+48*(-a*(-1+sin
(f*x+e)))^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*sin(f*x+e)^2*c^3*d^3+180*(-a*(-1+
sin(f*x+e)))^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*sin(f*x+e)^2*c^2*d^4-96*(-a*(-
1+sin(f*x+e)))^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*sin(f*x+e)^2*c*d^5+3*(a*(c+d
)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*
sin(f*x+e)^2*a^2*c^6+219*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(-1+sin(
f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*a^2*d^6+20*(-a*(-1+sin(f*x+e))
)^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*sin(f*x+e)^2*c^4*d^2-12*(-a*(-1+sin(f*x+e
)))^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*sin(f*x+e)*c^5*d+96*(-a*(-1+sin(f*x+e))
)^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*sin(f*x+e)*c^4*d^2+120*(-a*(-1+sin(f*x+e)
))^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*sin(f*x+e)*c^3*d^3+144*(-a*(-1+sin(f*x+e
)))^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*sin(f*x+e)*c^2*d^4-204*(-a*(-1+sin(f*x+
e)))^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*sin(f*x+e)*c*d^5+6*(a*(c+d)*d)^(1/2)*2
^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a
^2*c^6-232*(-a*(-1+sin(f*x+e)))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*sin(f*x+e)^
2*c^3*d^3-192*(-a*(-1+sin(f*x+e)))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*sin(f*x+
e)^2*c^2*d^4+232*(-a*(-1+sin(f*x+e)))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*sin(f
*x+e)^2*c*d^5+40*(-a*(-1+sin(f*x+e)))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*sin(f
*x+e)*c^5*d-192*(-a*(-1+sin(f*x+e)))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*sin(f*
x+e)*c^4*d^2-544*(-a*(-1+sin(f*x+e)))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*sin(f
*x+e)*c^3*d^3+80*(-a*(-1+sin(f*x+e)))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*sin(f
*x+e)*c^2*d^4+504*(-a*(-1+sin(f*x+e)))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*sin(
f*x+e)*c*d^5-24*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)))^
(1/2)*2^(1/2)/a^(1/2))*a^2*c^5*d+162*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*
(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^4*d^2+438*(a*(c+d)*d)^(1/
2)*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+
e)^3*a^2*d^6)/a^(9/2)/(1+sin(f*x+e))/(a*(c+d)*d)^(1/2)/(c+d*sin(f*x+e))^2/(
c+d)^2/(c-d)^5/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^3,x, algorithm="maxima"
)
```

```
[Out] Timed out
```

Fricas [B] time = 24.1367, size = 13377, normalized size = 33.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^3,x, algorithm="fricas"
)
```

```
[Out] [-1/64*(3*sqrt(2)*(4*c^6 - 24*c^5*d + 156*c^4*d^2 + 944*c^3*d^3 + 1596*c^2*
d^4 + 1128*c*d^5 + 292*d^6 + (c^4*d^2 - 8*c^3*d^3 + 54*c^2*d^4 + 136*c*d^5
+ 73*d^6)*cos(f*x + e)^5 + (2*c^5*d - 13*c^4*d^2 + 84*c^3*d^3 + 434*c^2*d^4
 + 554*c*d^5 + 219*d^6)*cos(f*x + e)^4 - (c^6 - 4*c^5*d + 25*c^4*d^2 + 328*
c^3*d^3 + 779*c^2*d^4 + 700*c*d^5 + 219*d^6)*cos(f*x + e)^3 - (3*c^6 - 14*c
^5*d + 89*c^4*d^2 + 892*c^3*d^3 + 1957*c^2*d^4 + 1682*c*d^5 + 511*d^6)*cos(
f*x + e)^2 + 2*(c^6 - 6*c^5*d + 39*c^4*d^2 + 236*c^3*d^3 + 399*c^2*d^4 + 28
2*c*d^5 + 73*d^6)*cos(f*x + e) + (4*c^6 - 24*c^5*d + 156*c^4*d^2 + 944*c^3*
d^3 + 1596*c^2*d^4 + 1128*c*d^5 + 292*d^6 + (c^4*d^2 - 8*c^3*d^3 + 54*c^2*d
^4 + 136*c*d^5 + 73*d^6)*cos(f*x + e)^4 - 2*(c^5*d - 7*c^4*d^2 + 46*c^3*d^3
 + 190*c^2*d^4 + 209*c*d^5 + 73*d^6)*cos(f*x + e)^3 - (c^6 - 2*c^5*d + 11*c
^4*d^2 + 420*c^3*d^3 + 1159*c^2*d^4 + 1118*c*d^5 + 365*d^6)*cos(f*x + e)^2
+ 2*(c^6 - 6*c^5*d + 39*c^4*d^2 + 236*c^3*d^3 + 399*c^2*d^4 + 282*c*d^5 + 7
3*d^6)*cos(f*x + e))*sin(f*x + e))*sqrt(a)*log(-(a*cos(f*x + e)^2 + 2*sqrt(
2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e) - sin(f*x + e) + 1) + 3*a
*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2*a)/(cos(f*x + e)^2
- (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) + 12*(84*a*c^4*d^2 +
 288*a*c^3*d^3 + 376*a*c^2*d^4 + 224*a*c*d^5 + 52*a*d^6 + (21*a*c^2*d^4 + 3
0*a*c*d^5 + 13*a*d^6)*cos(f*x + e)^5 + (42*a*c^3*d^3 + 123*a*c^2*d^4 + 116*
a*c*d^5 + 39*a*d^6)*cos(f*x + e)^4 - (21*a*c^4*d^2 + 114*a*c^3*d^3 + 196*a*
c^2*d^4 + 142*a*c*d^5 + 39*a*d^6)*cos(f*x + e)^3 - (63*a*c^4*d^2 + 300*a*c^
3*d^3 + 486*a*c^2*d^4 + 340*a*c*d^5 + 91*a*d^6)*cos(f*x + e)^2 + 2*(21*a*c^
4*d^2 + 72*a*c^3*d^3 + 94*a*c^2*d^4 + 56*a*c*d^5 + 13*a*d^6)*cos(f*x + e) +
 (84*a*c^4*d^2 + 288*a*c^3*d^3 + 376*a*c^2*d^4 + 224*a*c*d^5 + 52*a*d^6 + (
21*a*c^2*d^4 + 30*a*c*d^5 + 13*a*d^6)*cos(f*x + e)^4 - 2*(21*a*c^3*d^3 + 51
*a*c^2*d^4 + 43*a*c*d^5 + 13*a*d^6)*cos(f*x + e)^3 - (21*a*c^4*d^2 + 156*a*
c^3*d^3 + 298*a*c^2*d^4 + 228*a*c*d^5 + 65*a*d^6)*cos(f*x + e)^2 + 2*(21*a*
c^4*d^2 + 72*a*c^3*d^3 + 94*a*c^2*d^4 + 56*a*c*d^5 + 13*a*d^6)*cos(f*x + e)
)*sin(f*x + e))*sqrt(d/(a*c + a*d))*log((d^2*cos(f*x + e)^3 - (6*c*d + 7*d^
2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 - 4*((c*d + d^2)*cos(f*x + e)^2 - c^2
 - 4*c*d - 3*d^2 - (c^2 + 3*c*d + 2*d^2)*cos(f*x + e) + (c^2 + 4*c*d + 3*d^
2 + (c*d + d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d
/(a*c + a*d)) - (c^2 + 8*c*d + 9*d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 -
c^2 - 2*c*d - d^2 + 2*(3*c*d + 4*d^2)*cos(f*x + e))*sin(f*x + e))/(d^2*cos(
f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 - (c^2 + d^2)
*cos(f*x + e) + (d^2*cos(f*x + e)^2 - 2*c*d*cos(f*x + e) - c^2 - 2*c*d - d^
2)*sin(f*x + e))) + 4*(4*c^6 - 8*c^5*d - 4*c^4*d^2 + 16*c^3*d^3 - 4*c^2*d^4
 - 8*c*d^5 + 4*d^6 - 3*(c^4*d^2 - 8*c^3*d^3 - 30*c^2*d^4 + 16*c*d^5 + 21*d^
6)*cos(f*x + e)^4 - (6*c^5*d - 41*c^4*d^2 - 152*c^3*d^3 - 78*c^2*d^4 + 170*
c*d^5 + 95*d^6)*cos(f*x + e)^3 + (3*c^6 - 16*c^5*d - 31*c^4*d^2 - 84*c^3*d^
3 - 23*c^2*d^4 + 100*c*d^5 + 51*d^6)*cos(f*x + e)^2 + (7*c^6 - 18*c^5*d - 7
9*c^4*d^2 - 196*c^3*d^3 - 15*c^2*d^4 + 214*c*d^5 + 87*d^6)*cos(f*x + e) - (
4*c^6 - 8*c^5*d - 4*c^4*d^2 + 16*c^3*d^3 - 4*c^2*d^4 - 8*c*d^5 + 4*d^6 + 3*
```

(c^4*d^2 - 8*c^3*d^3 - 30*c^2*d^4 + 16*c*d^5 + 21*d^6)*cos(f*x + e)^3 - 2*(
3*c^5*d - 22*c^4*d^2 - 64*c^3*d^3 + 6*c^2*d^4 + 61*c*d^5 + 16*d^6)*cos(f*x
+ e)^2 - (3*c^6 - 10*c^5*d - 75*c^4*d^2 - 212*c^3*d^3 - 11*c^2*d^4 + 222*c*
d^5 + 83*d^6)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/((a^3*c
^7*d^2 - 3*a^3*c^6*d^3 + a^3*c^5*d^4 + 5*a^3*c^4*d^5 - 5*a^3*c^3*d^6 - a^3*
c^2*d^7 + 3*a^3*c*d^8 - a^3*d^9)*f*cos(f*x + e)^5 + (2*a^3*c^8*d - 3*a^3*c^
7*d^2 - 7*a^3*c^6*d^3 + 13*a^3*c^5*d^4 + 5*a^3*c^4*d^5 - 17*a^3*c^3*d^6 + 3
*a^3*c^2*d^7 + 7*a^3*c*d^8 - 3*a^3*d^9)*f*cos(f*x + e)^4 - (a^3*c^9 + a^3*c
^8*d - 8*a^3*c^7*d^2 + 18*a^3*c^5*d^4 - 6*a^3*c^4*d^5 - 16*a^3*c^3*d^6 + 8*
a^3*c^2*d^7 + 5*a^3*c*d^8 - 3*a^3*d^9)*f*cos(f*x + e)^3 - (3*a^3*c^9 + a^3*
c^8*d - 20*a^3*c^7*d^2 + 4*a^3*c^6*d^3 + 42*a^3*c^5*d^4 - 18*a^3*c^4*d^5 -
36*a^3*c^3*d^6 + 20*a^3*c^2*d^7 + 11*a^3*c*d^8 - 7*a^3*d^9)*f*cos(f*x + e)^
2 + 2*(a^3*c^9 - a^3*c^8*d - 4*a^3*c^7*d^2 + 4*a^3*c^6*d^3 + 6*a^3*c^5*d^4
- 6*a^3*c^4*d^5 - 4*a^3*c^3*d^6 + 4*a^3*c^2*d^7 + a^3*c*d^8 - a^3*d^9)*f*co
s(f*x + e) + 4*(a^3*c^9 - a^3*c^8*d - 4*a^3*c^7*d^2 + 4*a^3*c^6*d^3 + 6*a^3
*c^5*d^4 - 6*a^3*c^4*d^5 - 4*a^3*c^3*d^6 + 4*a^3*c^2*d^7 + a^3*c*d^8 - a^3*
d^9)*f + ((a^3*c^7*d^2 - 3*a^3*c^6*d^3 + a^3*c^5*d^4 + 5*a^3*c^4*d^5 - 5*a^
3*c^3*d^6 - a^3*c^2*d^7 + 3*a^3*c*d^8 - a^3*d^9)*f*cos(f*x + e)^4 - 2*(a^3*
c^8*d - 2*a^3*c^7*d^2 - 2*a^3*c^6*d^3 + 6*a^3*c^5*d^4 - 6*a^3*c^3*d^6 + 2*a
^3*c^2*d^7 + 2*a^3*c*d^8 - a^3*d^9)*f*cos(f*x + e)^3 - (a^3*c^9 + 3*a^3*c^8
*d - 12*a^3*c^7*d^2 - 4*a^3*c^6*d^3 + 30*a^3*c^5*d^4 - 6*a^3*c^4*d^5 - 28*a
^3*c^3*d^6 + 12*a^3*c^2*d^7 + 9*a^3*c*d^8 - 5*a^3*d^9)*f*cos(f*x + e)^2 + 2
*(a^3*c^9 - a^3*c^8*d - 4*a^3*c^7*d^2 + 4*a^3*c^6*d^3 + 6*a^3*c^5*d^4 - 6*a
^3*c^4*d^5 - 4*a^3*c^3*d^6 + 4*a^3*c^2*d^7 + a^3*c*d^8 - a^3*d^9)*f*cos(f*x
 + e) + 4*(a^3*c^9 - a^3*c^8*d - 4*a^3*c^7*d^2 + 4*a^3*c^6*d^3 + 6*a^3*c^5*
d^4 - 6*a^3*c^4*d^5 - 4*a^3*c^3*d^6 + 4*a^3*c^2*d^7 + a^3*c*d^8 - a^3*d^9)*
f)*sin(f*x + e)), -1/64*(3*sqrt(2)*(4*c^6 - 24*c^5*d + 156*c^4*d^2 + 944*c^
3*d^3 + 1596*c^2*d^4 + 1128*c*d^5 + 292*d^6 + (c^4*d^2 - 8*c^3*d^3 + 54*c^2
*d^4 + 136*c*d^5 + 73*d^6)*cos(f*x + e)^5 + (2*c^5*d - 13*c^4*d^2 + 84*c^3*
d^3 + 434*c^2*d^4 + 554*c*d^5 + 219*d^6)*cos(f*x + e)^4 - (c^6 - 4*c^5*d +
25*c^4*d^2 + 328*c^3*d^3 + 779*c^2*d^4 + 700*c*d^5 + 219*d^6)*cos(f*x + e)^
3 - (3*c^6 - 14*c^5*d + 89*c^4*d^2 + 892*c^3*d^3 + 1957*c^2*d^4 + 1682*c*d^
5 + 511*d^6)*cos(f*x + e)^2 + 2*(c^6 - 6*c^5*d + 39*c^4*d^2 + 236*c^3*d^3 +
 399*c^2*d^4 + 282*c*d^5 + 73*d^6)*cos(f*x + e) + (4*c^6 - 24*c^5*d + 156*c
^4*d^2 + 944*c^3*d^3 + 1596*c^2*d^4 + 1128*c*d^5 + 292*d^6 + (c^4*d^2 - 8*c
^3*d^3 + 54*c^2*d^4 + 136*c*d^5 + 73*d^6)*cos(f*x + e)^4 - 2*(c^5*d - 7*c^4
*d^2 + 46*c^3*d^3 + 190*c^2*d^4 + 209*c*d^5 + 73*d^6)*cos(f*x + e)^3 - (c^6
 - 2*c^5*d + 11*c^4*d^2 + 420*c^3*d^3 + 1159*c^2*d^4 + 1118*c*d^5 + 365*d^6
)*cos(f*x + e)^2 + 2*(c^6 - 6*c^5*d + 39*c^4*d^2 + 236*c^3*d^3 + 399*c^2*d^
4 + 282*c*d^5 + 73*d^6)*cos(f*x + e))*sin(f*x + e))*sqrt(a)*log(-(a*cos(f*x
 + e)^2 + 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e) - sin(f*
x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2*a)
/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) - 2
4*(84*a*c^4*d^2 + 288*a*c^3*d^3 + 376*a*c^2*d^4 + 224*a*c*d^5 + 52*a*d^6 +
(21*a*c^2*d^4 + 30*a*c*d^5 + 13*a*d^6)*cos(f*x + e)^5 + (42*a*c^3*d^3 + 123
*a*c^2*d^4 + 116*a*c*d^5 + 39*a*d^6)*cos(f*x + e)^4 - (21*a*c^4*d^2 + 114*a
*c^3*d^3 + 196*a*c^2*d^4 + 142*a*c*d^5 + 39*a*d^6)*cos(f*x + e)^3 - (63*a*c
^4*d^2 + 300*a*c^3*d^3 + 486*a*c^2*d^4 + 340*a*c*d^5 + 91*a*d^6)*cos(f*x +
e)^2 + 2*(21*a*c^4*d^2 + 72*a*c^3*d^3 + 94*a*c^2*d^4 + 56*a*c*d^5 + 13*a*d^
6)*cos(f*x + e) + (84*a*c^4*d^2 + 288*a*c^3*d^3 + 376*a*c^2*d^4 + 224*a*c*d
^5 + 52*a*d^6 + (21*a*c^2*d^4 + 30*a*c*d^5 + 13*a*d^6)*cos(f*x + e)^4 - 2*(
21*a*c^3*d^3 + 51*a*c^2*d^4 + 43*a*c*d^5 + 13*a*d^6)*cos(f*x + e)^3 - (21*a
*c^4*d^2 + 156*a*c^3*d^3 + 298*a*c^2*d^4 + 228*a*c*d^5 + 65*a*d^6)*cos(f*x
+ e)^2 + 2*(21*a*c^4*d^2 + 72*a*c^3*d^3 + 94*a*c^2*d^4 + 56*a*c*d^5 + 13*a*
d^6)*cos(f*x + e))*sin(f*x + e))*sqrt(-d/(a*c + a*d))*arctan(1/2*sqrt(a*sin
(f*x + e) + a)*(d*sin(f*x + e) - c - 2*d)*sqrt(-d/(a*c + a*d))/(d*cos(f*x +
 e))) + 4*(4*c^6 - 8*c^5*d - 4*c^4*d^2 + 16*c^3*d^3 - 4*c^2*d^4 - 8*c*d^5 +
 4*d^6 - 3*(c^4*d^2 - 8*c^3*d^3 - 30*c^2*d^4 + 16*c*d^5 + 21*d^6)*cos(f*x +
 e)^4 - (6*c^5*d - 41*c^4*d^2 - 152*c^3*d^3 - 78*c^2*d^4 + 170*c*d^5 + 95*d

```
^6)*cos(f*x + e)^3 + (3*c^6 - 16*c^5*d - 31*c^4*d^2 - 84*c^3*d^3 - 23*c^2*d
^4 + 100*c*d^5 + 51*d^6)*cos(f*x + e)^2 + (7*c^6 - 18*c^5*d - 79*c^4*d^2 -
196*c^3*d^3 - 15*c^2*d^4 + 214*c*d^5 + 87*d^6)*cos(f*x + e) - (4*c^6 - 8*c^
5*d - 4*c^4*d^2 + 16*c^3*d^3 - 4*c^2*d^4 - 8*c*d^5 + 4*d^6 + 3*(c^4*d^2 - 8
*c^3*d^3 - 30*c^2*d^4 + 16*c*d^5 + 21*d^6)*cos(f*x + e)^3 - 2*(3*c^5*d - 22
*c^4*d^2 - 64*c^3*d^3 + 6*c^2*d^4 + 61*c*d^5 + 16*d^6)*cos(f*x + e)^2 - (3*
c^6 - 10*c^5*d - 75*c^4*d^2 - 212*c^3*d^3 - 11*c^2*d^4 + 222*c*d^5 + 83*d^6
)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/((a^3*c^7*d^2 - 3*a
^3*c^6*d^3 + a^3*c^5*d^4 + 5*a^3*c^4*d^5 - 5*a^3*c^3*d^6 - a^3*c^2*d^7 + 3*
a^3*c*d^8 - a^3*d^9)*f*cos(f*x + e)^5 + (2*a^3*c^8*d - 3*a^3*c^7*d^2 - 7*a^
3*c^6*d^3 + 13*a^3*c^5*d^4 + 5*a^3*c^4*d^5 - 17*a^3*c^3*d^6 + 3*a^3*c^2*d^7
 + 7*a^3*c*d^8 - 3*a^3*d^9)*f*cos(f*x + e)^4 - (a^3*c^9 + a^3*c^8*d - 8*a^3
*c^7*d^2 + 18*a^3*c^5*d^4 - 6*a^3*c^4*d^5 - 16*a^3*c^3*d^6 + 8*a^3*c^2*d^7
+ 5*a^3*c*d^8 - 3*a^3*d^9)*f*cos(f*x + e)^3 - (3*a^3*c^9 + a^3*c^8*d - 20*a
^3*c^7*d^2 + 4*a^3*c^6*d^3 + 42*a^3*c^5*d^4 - 18*a^3*c^4*d^5 - 36*a^3*c^3*d
^6 + 20*a^3*c^2*d^7 + 11*a^3*c*d^8 - 7*a^3*d^9)*f*cos(f*x + e)^2 + 2*(a^3*c
^9 - a^3*c^8*d - 4*a^3*c^7*d^2 + 4*a^3*c^6*d^3 + 6*a^3*c^5*d^4 - 6*a^3*c^4*
d^5 - 4*a^3*c^3*d^6 + 4*a^3*c^2*d^7 + a^3*c*d^8 - a^3*d^9)*f*cos(f*x + e) +
 4*(a^3*c^9 - a^3*c^8*d - 4*a^3*c^7*d^2 + 4*a^3*c^6*d^3 + 6*a^3*c^5*d^4 - 6
*a^3*c^4*d^5 - 4*a^3*c^3*d^6 + 4*a^3*c^2*d^7 + a^3*c*d^8 - a^3*d^9)*f + ((a
^3*c^7*d^2 - 3*a^3*c^6*d^3 + a^3*c^5*d^4 + 5*a^3*c^4*d^5 - 5*a^3*c^3*d^6 -
a^3*c^2*d^7 + 3*a^3*c*d^8 - a^3*d^9)*f*cos(f*x + e)^4 - 2*(a^3*c^8*d - 2*a^
3*c^7*d^2 - 2*a^3*c^6*d^3 + 6*a^3*c^5*d^4 - 6*a^3*c^3*d^6 + 2*a^3*c^2*d^7 +
 2*a^3*c*d^8 - a^3*d^9)*f*cos(f*x + e)^3 - (a^3*c^9 + 3*a^3*c^8*d - 12*a^3*
c^7*d^2 - 4*a^3*c^6*d^3 + 30*a^3*c^5*d^4 - 6*a^3*c^4*d^5 - 28*a^3*c^3*d^6 +
 12*a^3*c^2*d^7 + 9*a^3*c*d^8 - 5*a^3*d^9)*f*cos(f*x + e)^2 + 2*(a^3*c^9 -
a^3*c^8*d - 4*a^3*c^7*d^2 + 4*a^3*c^6*d^3 + 6*a^3*c^5*d^4 - 6*a^3*c^4*d^5 -
 4*a^3*c^3*d^6 + 4*a^3*c^2*d^7 + a^3*c*d^8 - a^3*d^9)*f*cos(f*x + e) + 4*(a
^3*c^9 - a^3*c^8*d - 4*a^3*c^7*d^2 + 4*a^3*c^6*d^3 + 6*a^3*c^5*d^4 - 6*a^3*
c^4*d^5 - 4*a^3*c^3*d^6 + 4*a^3*c^2*d^7 + a^3*c*d^8 - a^3*d^9)*f)*sin(f*x +
 e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**(5/2)/(c+d*sin(f*x+e))**3,x)

[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^3,x, algorithm="giac")

[Out] Exception raised: TypeError
```

3.564 $\int \sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^{5/2}\,dx$

Optimal. Leaf size=203

$$-\frac{5a(c+d)^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{8f\sqrt{a\sin(e+fx)+a}}-\frac{5a(c+d)\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{12f\sqrt{a\sin(e+fx)+a}}-\frac{a\cos(e+fx)(c+d\sin(e+fx}{3f\sqrt{a\sin(e+fx)+a}}$$

```
[Out] (-5*Sqrt[a]*(c + d)^3*ArcTan[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[a + a*Sin
[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])])/(8*Sqrt[d]*f) - (5*a*(c + d)^2*Cos[e
 + f*x]*Sqrt[c + d*Sin[e + f*x]])/(8*f*Sqrt[a + a*Sin[e + f*x]]) - (5*a*(c
+ d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(3/2))/(12*f*Sqrt[a + a*Sin[e + f*x]
]) - (a*Cos[e + f*x]*(c + d*Sin[e + f*x])^(5/2))/(3*f*Sqrt[a + a*Sin[e + f*
x]])
```

Rubi [A] time = 0.414528, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2770, 2775, 205}

$$-\frac{5a(c+d)^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{8f\sqrt{a\sin(e+fx)+a}}-\frac{5a(c+d)\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{12f\sqrt{a\sin(e+fx)+a}}-\frac{a\cos(e+fx)(c+d\sin(e+fx}{3f\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (-5*Sqrt[a]*(c + d)^3*ArcTan[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[a + a*Sin
[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])])/(8*Sqrt[d]*f) - (5*a*(c + d)^2*Cos[e
 + f*x]*Sqrt[c + d*Sin[e + f*x]])/(8*f*Sqrt[a + a*Sin[e + f*x]]) - (5*a*(c
+ d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(3/2))/(12*f*Sqrt[a + a*Sin[e + f*x]
]) - (a*Cos[e + f*x]*(c + d*Sin[e + f*x])^(5/2))/(3*f*Sqrt[a + a*Sin[e + f*
x]])
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^{5/2}\,dx = -\frac{a\cos(e+fx)(c+d\sin(e+fx))^{5/2}}{3f\sqrt{a+a\sin(e+fx)}}+\frac{1}{6}(5(c+d))\int \sqrt{a+a\sin(e+$$

$$= -\frac{5a(c+d)\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{12f\sqrt{a+a\sin(e+fx)}}-\frac{a\cos(e+fx)(c+d\sin}{3f\sqrt{a+a\sin(e}}$$

$$= -\frac{5a(c+d)^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{8f\sqrt{a+a\sin(e+fx)}}-\frac{5a(c+d)\cos(e+fx)(c}{12f\sqrt{a+a\sin}}$$

$$= -\frac{5a(c+d)^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{8f\sqrt{a+a\sin(e+fx)}}-\frac{5a(c+d)\cos(e+fx)(c}{12f\sqrt{a+a\sin}}$$

$$= -\frac{5\sqrt{a}(c+d)^3\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{8\sqrt{d}f}-\frac{5a(c+d)^2\cos(e+}{8f\sqrt{a+}}$$

Mathematica [C] time = 3.77205, size = 391, normalized size = 1.93

$$-\frac{\sqrt{a(\sin(e+fx)+1)}\left(2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(c+d\sin(e+fx))\left(33c^2+2d(13c+5d)\sin(e+fx)+40\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] -(Sqrt[a*(1 + Sin[e + f*x])]*((15*(c + d)^3*(Log[(2*(-1)^(1/4)*c - 2*(-1)^(
3/4)*d*E^(I*(e + f*x)) + 2*Sqrt[d]*Sqrt[2*c*E^(I*(e + f*x)) - I*d*(-1 + E^(
(2*I)*(e + f*x)))])/(Sqrt[d]*E^(I*e))] - Log[(2*E^((I/2)*(e - 2*f*x))*((-1)
^(3/4)*d + (-1)^(1/4)*c*E^(I*(e + f*x)) + I*Sqrt[d]*Sqrt[2*c*E^(I*(e + f*x)
) - I*d*(-1 + E^((2*I)*(e + f*x)))])*f)/Sqrt[d]])*(I*Cos[(e + f*x)/2] + Sin
[(e + f*x)/2])*Sqrt[(Cos[e + f*x] + I*Sin[e + f*x])*(c + d*Sin[e + f*x])])/
Sqrt[d] + 2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(c + d*Sin[e + f*x])*(33*
c^2 + 40*c*d + 19*d^2 - 4*d^2*Cos[2*(e + f*x)] + 2*d*(13*c + 5*d)*Sin[e + f
*x])))/(48*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c + d*Sin[e + f*x]]
)
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int \sqrt{a+a\sin\left(fx+e\right)}\left(c+d\sin\left(fx+e\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^(5/2),x)
```

```
[Out] int((a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\sin(fx+e)+a}\,(d\sin(fx+e)+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^(5/2), x)
```

Fricas [B] time = 5.82899, size = 3033, normalized size = 14.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] [1/192*(15*(c^3 + 3*c^2*d + 3*c*d^2 + d^3 + (c^3 + 3*c^2*d + 3*c*d^2 + d^3)
*cos(f*x + e) + (c^3 + 3*c^2*d + 3*c*d^2 + d^3)*sin(f*x + e))*sqrt(-a/d)*lo
g((128*a*d^4*cos(f*x + e)^5 + a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 +
 a*d^4 + 128*(2*a*c*d^3 - a*d^4)*cos(f*x + e)^4 - 32*(5*a*c^2*d^2 - 14*a*c*
d^3 + 13*a*d^4)*cos(f*x + e)^3 - 32*(a*c^3*d - 2*a*c^2*d^2 + 9*a*c*d^3 - 4*
a*d^4)*cos(f*x + e)^2 - 8*(16*d^4*cos(f*x + e)^4 - c^3*d + 17*c^2*d^2 - 59*
c*d^3 + 51*d^4 + 24*(c*d^3 - d^4)*cos(f*x + e)^3 - 2*(5*c^2*d^2 - 26*c*d^3
+ 33*d^4)*cos(f*x + e)^2 - (c^3*d - 7*c^2*d^2 + 31*c*d^3 - 25*d^4)*cos(f*x
+ e) + (16*d^4*cos(f*x + e)^3 + c^3*d - 17*c^2*d^2 + 59*c*d^3 - 51*d^4 - 8*
(3*c*d^3 - 5*d^4)*cos(f*x + e)^2 - 2*(5*c^2*d^2 - 14*c*d^3 + 13*d^4)*cos(f*
x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sqr
t(-a/d) + (a*c^4 - 28*a*c^3*d + 230*a*c^2*d^2 - 476*a*c*d^3 + 289*a*d^4)*co
s(f*x + e) + (128*a*d^4*cos(f*x + e)^4 + a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 +
4*a*c*d^3 + a*d^4 - 256*(a*c*d^3 - a*d^4)*cos(f*x + e)^3 - 32*(5*a*c^2*d^2
- 6*a*c*d^3 + 5*a*d^4)*cos(f*x + e)^2 + 32*(a*c^3*d - 7*a*c^2*d^2 + 15*a*c*
d^3 - 9*a*d^4)*cos(f*x + e))*sin(f*x + e))/(cos(f*x + e) + sin(f*x + e) + 1
)) + 8*(8*d^2*cos(f*x + e)^3 - 2*(13*c*d + d^2)*cos(f*x + e)^2 - 33*c^2 - 1
4*c*d - 13*d^2 - (33*c^2 + 40*c*d + 23*d^2)*cos(f*x + e) - (8*d^2*cos(f*x +
 e)^2 - 33*c^2 - 14*c*d - 13*d^2 + 2*(13*c*d + 5*d^2)*cos(f*x + e))*sin(f*x
 + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/(f*cos(f*x + e) +
 f*sin(f*x + e) + f), 1/96*(15*(c^3 + 3*c^2*d + 3*c*d^2 + d^3 + (c^3 + 3*c^
2*d + 3*c*d^2 + d^3)*cos(f*x + e) + (c^3 + 3*c^2*d + 3*c*d^2 + d^3)*sin(f*x
 + e))*sqrt(a/d)*arctan(1/4*(8*d^2*cos(f*x + e)^2 - c^2 + 6*c*d - 9*d^2 - 8
*(c*d - d^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c
)*sqrt(a/d)/(2*a*d^2*cos(f*x + e)^3 - (3*a*c*d - a*d^2)*cos(f*x + e)*sin(f*
x + e) - (a*c^2 - a*c*d + 2*a*d^2)*cos(f*x + e))) + 4*(8*d^2*cos(f*x + e)^3
 - 2*(13*c*d + d^2)*cos(f*x + e)^2 - 33*c^2 - 14*c*d - 13*d^2 - (33*c^2 + 4
0*c*d + 23*d^2)*cos(f*x + e) - (8*d^2*cos(f*x + e)^2 - 33*c^2 - 14*c*d - 13
*d^2 + 2*(13*c*d + 5*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) +
 a)*sqrt(d*sin(f*x + e) + c))/(f*cos(f*x + e) + f*sin(f*x + e) + f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(1/2)*(c+d*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError
```

3.565 $\int \sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=156

$$-\frac{3a(c+d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4f\sqrt{a\sin(e+fx)+a}}-\frac{a\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}-\frac{3\sqrt{a}(c+d)^2\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{4\sqrt{d}f}$$

```
[Out] (-3*Sqrt[a]*(c + d)^2*ArcTan[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[a + a*Sin
[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])])/(4*Sqrt[d]*f) - (3*a*(c + d)*Cos[e +
 f*x]*Sqrt[c + d*Sin[e + f*x]])/(4*f*Sqrt[a + a*Sin[e + f*x]]) - (a*Cos[e +
 f*x]*(c + d*Sin[e + f*x])^(3/2))/(2*f*Sqrt[a + a*Sin[e + f*x]])
```

Rubi [A] time = 0.290306, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2770, 2775, 205}

$$-\frac{3a(c+d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4f\sqrt{a\sin(e+fx)+a}}-\frac{a\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}-\frac{3\sqrt{a}(c+d)^2\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{4\sqrt{d}f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-3*Sqrt[a]*(c + d)^2*ArcTan[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[a + a*Sin
[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])])/(4*Sqrt[d]*f) - (3*a*(c + d)*Cos[e +
 f*x]*Sqrt[c + d*Sin[e + f*x]])/(4*f*Sqrt[a + a*Sin[e + f*x]]) - (a*Cos[e +
 f*x]*(c + d*Sin[e + f*x])^(3/2))/(2*f*Sqrt[a + a*Sin[e + f*x]])
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^{3/2}\,dx = -\frac{a\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{2f\sqrt{a+a\sin(e+fx)}} + \frac{1}{4}(3(c+d))\int \sqrt{a+a\sin(e+$$

$$= -\frac{3a(c+d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4f\sqrt{a+a\sin(e+fx)}} - \frac{a\cos(e+fx)(c+d\sin(e}{2f\sqrt{a+a\sin(e+}$$

$$= -\frac{3a(c+d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4f\sqrt{a+a\sin(e+fx)}} - \frac{a\cos(e+fx)(c+d\sin(e}{2f\sqrt{a+a\sin(e+}$$

$$= -\frac{3\sqrt{a}(c+d)^2\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{4\sqrt{d}f} - \frac{3a(c+d)\cos(e+f}{4f\sqrt{a+a}$$

Mathematica [C] time = 2.07325, size = 365, normalized size = 2.34

$$\frac{\sqrt{a(\sin(e+fx)+1)}\left(-2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(c+d\sin(e+fx))(5c+2d\sin(e+fx)+3d)-\frac{3i(c+d)^2(\cos}{}\right.}{8f(\sin}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] (Sqrt[a*(1 + Sin[e + f*x])]*(((-3*I)*(c + d)^2*(Log[(2*(-1)^(1/4)*c - 2*(-1
)^(3/4)*d*E^(I*(e + f*x)) + 2*Sqrt[d]*Sqrt[2*c*E^(I*(e + f*x)) - I*d*(-1 +
E^((2*I)*(e + f*x)))]])/(Sqrt[d]*E^(I*e))] - Log[(2*E^((I/2)*(e - 2*f*x))*((
-1)^(3/4)*d + (-1)^(1/4)*c*E^(I*(e + f*x)) + I*Sqrt[d]*Sqrt[2*c*E^(I*(e + f
*x)) - I*d*(-1 + E^((2*I)*(e + f*x)))]])*f)/Sqrt[d]])*(Cos[(e + f*x)/2] - I*
Sin[(e + f*x)/2])*Sqrt[(Cos[e + f*x] + I*Sin[e + f*x])*(c + d*Sin[e + f*x])
])/Sqrt[d] - 2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(c + d*Sin[e + f*x])*(
5*c + 3*d + 2*d*Sin[e + f*x])))/(8*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*
Sqrt[c + d*Sin[e + f*x]])
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int \sqrt{a+a\sin(fx+e)}\,(c+d\sin(fx+e))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^(3/2),x)
```

```
[Out] int((a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\sin(fx+e)+a}\,(d\sin(fx+e)+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^(3/2), x)
```

Fricas [B] time = 5.21785, size = 2600, normalized size = 16.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] [1/32*(3*(c^2 + 2*c*d + d^2 + (c^2 + 2*c*d + d^2)*cos(f*x + e) + (c^2 + 2*c
*d + d^2)*sin(f*x + e))*sqrt(-a/d)*log((128*a*d^4*cos(f*x + e)^5 + a*c^4 +
4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 + a*d^4 + 128*(2*a*c*d^3 - a*d^4)*cos(f
*x + e)^4 - 32*(5*a*c^2*d^2 - 14*a*c*d^3 + 13*a*d^4)*cos(f*x + e)^3 - 32*(a
*c^3*d - 2*a*c^2*d^2 + 9*a*c*d^3 - 4*a*d^4)*cos(f*x + e)^2 - 8*(16*d^4*cos(
f*x + e)^4 - c^3*d + 17*c^2*d^2 - 59*c*d^3 + 51*d^4 + 24*(c*d^3 - d^4)*cos(
f*x + e)^3 - 2*(5*c^2*d^2 - 26*c*d^3 + 33*d^4)*cos(f*x + e)^2 - (c^3*d - 7*
c^2*d^2 + 31*c*d^3 - 25*d^4)*cos(f*x + e) + (16*d^4*cos(f*x + e)^3 + c^3*d
- 17*c^2*d^2 + 59*c*d^3 - 51*d^4 - 8*(3*c*d^3 - 5*d^4)*cos(f*x + e)^2 - 2*(
5*c^2*d^2 - 14*c*d^3 + 13*d^4)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x +
 e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt(-a/d) + (a*c^4 - 28*a*c^3*d + 230*a*
c^2*d^2 - 476*a*c*d^3 + 289*a*d^4)*cos(f*x + e) + (128*a*d^4*cos(f*x + e)^4
 + a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 + a*d^4 - 256*(a*c*d^3 - a*d
^4)*cos(f*x + e)^3 - 32*(5*a*c^2*d^2 - 6*a*c*d^3 + 5*a*d^4)*cos(f*x + e)^2
+ 32*(a*c^3*d - 7*a*c^2*d^2 + 15*a*c*d^3 - 9*a*d^4)*cos(f*x + e))*sin(f*x +
 e))/(cos(f*x + e) + sin(f*x + e) + 1)) - 8*(2*d*cos(f*x + e)^2 + (5*c + 3*
d)*cos(f*x + e) + (2*d*cos(f*x + e) - 5*c - d)*sin(f*x + e) + 5*c + d)*sqrt
(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/(f*cos(f*x + e) + f*sin(f*x
+ e) + f), 1/16*(3*(c^2 + 2*c*d + d^2 + (c^2 + 2*c*d + d^2)*cos(f*x + e) +
(c^2 + 2*c*d + d^2)*sin(f*x + e))*sqrt(a/d)*arctan(1/4*(8*d^2*cos(f*x + e)^
2 - c^2 + 6*c*d - 9*d^2 - 8*(c*d - d^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) +
 a)*sqrt(d*sin(f*x + e) + c)*sqrt(a/d)/(2*a*d^2*cos(f*x + e)^3 - (3*a*c*d -
 a*d^2)*cos(f*x + e)*sin(f*x + e) - (a*c^2 - a*c*d + 2*a*d^2)*cos(f*x + e))
) - 4*(2*d*cos(f*x + e)^2 + (5*c + 3*d)*cos(f*x + e) + (2*d*cos(f*x + e) -
5*c - d)*sin(f*x + e) + 5*c + d)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x +
e) + c))/(f*cos(f*x + e) + f*sin(f*x + e) + f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(1/2)*(c+d*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: AttributeError
```

3.566 $\int \sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}\,dx$

Optimal. Leaf size=105

$$-\frac{a\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}-\frac{\sqrt{a}(c+d)\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{\sqrt{d}f}$$

```
[Out] -((Sqrt[a]*(c + d)*ArcTan[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[a + a*Sin[e
+ f*x]]*Sqrt[c + d*Sin[e + f*x]])]))/(Sqrt[d]*f)) - (a*Cos[e + f*x]*Sqrt[c +
 d*Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f*x]])
```

Rubi [A] time = 0.182373, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2770, 2775, 205}

$$-\frac{a\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}-\frac{\sqrt{a}(c+d)\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{\sqrt{d}f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] -((Sqrt[a]*(c + d)*ArcTan[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[a + a*Sin[e
+ f*x]]*Sqrt[c + d*Sin[e + f*x]])]))/(Sqrt[d]*f)) - (a*Cos[e + f*x]*Sqrt[c +
 d*Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f*x]])
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}\,dx = -\frac{a\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{f\sqrt{a+a\sin(e+fx)}} + \frac{1}{2}(c+d)\int \frac{\sqrt{a+a\sin(e+fx)}}{\sqrt{c+d\sin(e+fx)}}\,dx$$

$$= -\frac{a\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{f\sqrt{a+a\sin(e+fx)}} - \frac{(a(c+d))\,\text{Subst}\left(\int \frac{1}{a+dx^2}\,dx, x, \frac{}{\sqrt{a\cdots}}\right)}{f}$$

$$= -\frac{\sqrt{a}(c+d)\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{\sqrt{d}f} - \frac{a\cos(e+fx)\sqrt{c+d\sin\cdots}}{f\sqrt{a+a\sin(e+\cdots}}$$

Mathematica [C] time = 1.29864, size = 350, normalized size = 3.33

$$\frac{\sqrt{a(\sin(e+fx)+1)}\left(-\frac{2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(c+d\sin(e+fx))}{f} - \frac{i(c+d)\left(\cos\left(\frac{1}{2}(e+fx)\right)-i\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\log\left(\frac{e^{-ie}\left(2\sqrt{d}\sqrt{2ce^{i(e+fx)}-id\left(-1+e^{2i\cdots}\right)}\cdots\right)}{\cdots}\right)\cdots\right)}{\cdots}\right)}{2\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+\cdots\right)\right)\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] (Sqrt[a*(1 + Sin[e + f*x])]*((-2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(c +
 d*Sin[e + f*x]))/f - (I*(c + d)*(Log[(2*(-1)^(1/4)*c - 2*(-1)^(3/4)*d*E^(I
*(e + f*x)) + 2*Sqrt[d]*Sqrt[2*c*E^(I*(e + f*x)) - I*d*(-1 + E^((2*I)*(e +
f*x)))])/(Sqrt[d]*E^(I*e))] - Log[(2*E^((I/2)*(e - 2*f*x))*((-1)^(3/4)*d +
(-1)^(1/4)*c*E^(I*(e + f*x)) + I*Sqrt[d]*Sqrt[2*c*E^(I*(e + f*x)) - I*d*(-1
 + E^((2*I)*(e + f*x)))])*f)/Sqrt[d]])*(Cos[(e + f*x)/2] - I*Sin[(e + f*x)/
2])*Sqrt[(Cos[e + f*x] + I*Sin[e + f*x])*(c + d*Sin[e + f*x])])/(Sqrt[d]*f)
))/(2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c + d*Sin[e + f*x]])
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int \sqrt{a+a\sin\left(fx+e\right)}\sqrt{c+d\sin\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^(1/2),x)
```

```
[Out] int((a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\sin\left(fx+e\right)+a}\sqrt{d\sin\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c), x)
```

Fricas [B] time = 5.20041, size = 2311, normalized size = 22.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] [1/8*(((c + d)*cos(f*x + e) + (c + d)*sin(f*x + e) + c + d)*sqrt(-a/d)*log(
(128*a*d^4*cos(f*x + e)^5 + a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 + a
*d^4 + 128*(2*a*c*d^3 - a*d^4)*cos(f*x + e)^4 - 32*(5*a*c^2*d^2 - 14*a*c*d^
3 + 13*a*d^4)*cos(f*x + e)^3 - 32*(a*c^3*d - 2*a*c^2*d^2 + 9*a*c*d^3 - 4*a*
d^4)*cos(f*x + e)^2 - 8*(16*d^4*cos(f*x + e)^4 - c^3*d + 17*c^2*d^2 - 59*c*
d^3 + 51*d^4 + 24*(c*d^3 - d^4)*cos(f*x + e)^3 - 2*(5*c^2*d^2 - 26*c*d^3 +
33*d^4)*cos(f*x + e)^2 - (c^3*d - 7*c^2*d^2 + 31*c*d^3 - 25*d^4)*cos(f*x +
e) + (16*d^4*cos(f*x + e)^3 + c^3*d - 17*c^2*d^2 + 59*c*d^3 - 51*d^4 - 8*(3
*c*d^3 - 5*d^4)*cos(f*x + e)^2 - 2*(5*c^2*d^2 - 14*c*d^3 + 13*d^4)*cos(f*x
+ e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt(
-a/d) + (a*c^4 - 28*a*c^3*d + 230*a*c^2*d^2 - 476*a*c*d^3 + 289*a*d^4)*cos(
f*x + e) + (128*a*d^4*cos(f*x + e)^4 + a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*
a*c*d^3 + a*d^4 - 256*(a*c*d^3 - a*d^4)*cos(f*x + e)^3 - 32*(5*a*c^2*d^2 -
6*a*c*d^3 + 5*a*d^4)*cos(f*x + e)^2 + 32*(a*c^3*d - 7*a*c^2*d^2 + 15*a*c*d^
3 - 9*a*d^4)*cos(f*x + e))*sin(f*x + e))/(cos(f*x + e) + sin(f*x + e) + 1))
 - 8*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*(cos(f*x + e) - sin(
f*x + e) + 1))/(f*cos(f*x + e) + f*sin(f*x + e) + f), 1/4*(((c + d)*cos(f*x
 + e) + (c + d)*sin(f*x + e) + c + d)*sqrt(a/d)*arctan(1/4*(8*d^2*cos(f*x +
 e)^2 - c^2 + 6*c*d - 9*d^2 - 8*(c*d - d^2)*sin(f*x + e))*sqrt(a*sin(f*x +
e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt(a/d)/(2*a*d^2*cos(f*x + e)^3 - (3*a*c
*d - a*d^2)*cos(f*x + e)*sin(f*x + e) - (a*c^2 - a*c*d + 2*a*d^2)*cos(f*x +
 e))) - 4*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*(cos(f*x + e) -
 sin(f*x + e) + 1))/(f*cos(f*x + e) + f*sin(f*x + e) + f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\left(\sin\left(e+fx\right)+1\right)}\sqrt{c+d\sin\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(1/2)*(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))*sqrt(c + d*sin(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\sin\left(fx+e\right)+a}\sqrt{d\sin\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c), x)
```

3.567 $\int \frac{\sqrt{a+a\sin(e+fx)}}{\sqrt{c+d\sin(e+fx)}}\,dx$

Optimal. Leaf size=61

$$-\frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{\sqrt{d}f}$$

```
[Out] (-2*Sqrt[a]*ArcTan[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]
*Sqrt[c + d*Sin[e + f*x]])])/(Sqrt[d]*f)
```

Rubi [A] time = 0.0925582, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2775, 205}

$$-\frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{\sqrt{d}f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] (-2*Sqrt[a]*ArcTan[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]
*Sqrt[c + d*Sin[e + f*x]])])/(Sqrt[d]*f)
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+a\sin(e+fx)}}{\sqrt{c+d\sin(e+fx)}}\,dx &= -\frac{(2a)\operatorname{Subst}\left(\int \frac{1}{a+dx^2}\,dx, x, \frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{f}\\
&= -\frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{\sqrt{d}f}
\end{aligned}$$

Mathematica [C] time = 1.2394, size = 305, normalized size = 5.

$$-\frac{i\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-i\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\log\left(\frac{e^{-ie}\left(2\sqrt{d}\sqrt{2ce^{i(e+fx)}-id\left(-1+e^{2i(e+fx)}\right)}+2\sqrt[4]{-1}c-2(-1)^{3/4}de^{i(e+fx)}\right)}{\sqrt{d}}\right)-\text{lo}}{\sqrt{d}f\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] ((-I)*(Log[(2*(-1)^(1/4)*c - 2*(-1)^(3/4)*d*E^(I*(e + f*x)) + 2*Sqrt[d]*Sqr
t[2*c*E^(I*(e + f*x)) - I*d*(-1 + E^((2*I)*(e + f*x)))])/(Sqrt[d]*E^(I*e))]
 - Log[((-1 - I)*E^((I/2)*(e - 2*f*x))*(-((-1)^(1/4)*d) + (-1)^(3/4)*c*E^(I
*(e + f*x)) - Sqrt[d]*Sqrt[2*c*E^(I*(e + f*x)) - I*d*(-1 + E^((2*I)*(e + f*
x)))])*f)/Sqrt[d]])*(Cos[(e + f*x)/2] - I*Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin
[e + f*x])]*Sqrt[(Cos[e + f*x] + I*Sin[e + f*x])*(c + d*Sin[e + f*x])])/(Sq
rt[d]*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c + d*Sin[e + f*x]])
```

Maple [B] time = 0.378, size = 2707, normalized size = 44.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2),x)
```

```
[Out] -1/f/d^2/(-(d^2/c^2)^(1/2)*c)^(1/2)/(c^2-2*c*d+d^2)*(c+d*sin(f*x+e))^(1/2)*
(a*(1+sin(f*x+e)))^(1/2)*(d*(c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+
d))^(1/2)*(sin(f*x+e)*cos(f*x+e)*(d^2/c^2)^(1/2)*(-(d^2/c^2)^(1/2)*c)^(1/2)
*(((d^2/c^2)^(1/2)*c^4+6*(d^2/c^2)^(1/2)*d^2*c^2+d^4*(d^2/c^2)^(1/2)-4*c^2*
d^2-4*d^4)*c)^(1/2)*arctan(((d^2/c^2)^(1/2)*c^2-d^2)*c*((d^2/c^2)^(1/2)-1)/
(((d^2/c^2)^(1/2)*c^4+6*(d^2/c^2)^(1/2)*d^2*c^2+d^4*(d^2/c^2)^(1/2)-4*c^2*d
^2-4*d^4)*c)^(1/2)*((d^2/c^2)^(1/2)*c*sin(f*x+e)+d*cos(f*x+e)-d)/(d*(c+d*si
n(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d))^(1/2)/((d^2/c^2)^(1/2)*c*sin(f*
x+e)-d*cos(f*x+e)+d))*c*d-cos(f*x+e)*(d^2/c^2)^(1/2)*(-(d^2/c^2)^(1/2)*c)^(
1/2)*(((d^2/c^2)^(1/2)*c^4+6*(d^2/c^2)^(1/2)*d^2*c^2+d^4*(d^2/c^2)^(1/2)-4*
c^2*d^2-4*d^4)*c)^(1/2)*arctan(((d^2/c^2)^(1/2)*c^2-d^2)*c*((d^2/c^2)^(1/2)
-1)/(((d^2/c^2)^(1/2)*c^4+6*(d^2/c^2)^(1/2)*d^2*c^2+d^4*(d^2/c^2)^(1/2)-4*c
^2*d^2-4*d^4)*c)^(1/2)*((d^2/c^2)^(1/2)*c*sin(f*x+e)+d*cos(f*x+e)-d)/(d*(c+
d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d))^(1/2)/((d^2/c^2)^(1/2)*c*si
n(f*x+e)-d*cos(f*x+e)+d))*c^2-arctan(1/(-(d^2/c^2)^(1/2)*c)^(1/2)*(d*(c+d*s
in(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d))^(1/2))*d^5+sin(f*x+e)*cos(f*x+
e)*(-(d^2/c^2)^(1/2)*c)^(1/2)*(((d^2/c^2)^(1/2)*c^4+6*(d^2/c^2)^(1/2)*d^2*c
^2+d^4*(d^2/c^2)^(1/2)-4*c^2*d^2-4*d^4)*c)^(1/2)*arctan(((d^2/c^2)^(1/2)*c^
2-d^2)*c*((d^2/c^2)^(1/2)-1)/(((d^2/c^2)^(1/2)*c^4+6*(d^2/c^2)^(1/2)*d^2*c^
2+d^4*(d^2/c^2)^(1/2)-4*c^2*d^2-4*d^4)*c)^(1/2)*((d^2/c^2)^(1/2)*c*sin(f*x+
e)+d*cos(f*x+e)-d)/(d*(c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d))^(1
/2)/((d^2/c^2)^(1/2)*c*sin(f*x+e)-d*cos(f*x+e)+d))*d^2+arctan(1/(-(d^2/c^2)
^(1/2)*c)^(1/2)*(d*(c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d))^(1/2)
)*c*d^4+cos(f*x+e)^2*arctan(1/(-(d^2/c^2)^(1/2)*c)^(1/2)*(d*(c+d*sin(f*x+e)
)/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d))^(1/2))*d^5+sin(f*x+e)*arctan(1/(-(d^2/c
^2)^(1/2)*c)^(1/2)*(d*(c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d))^(1
/2))*d^5-arctan(1/(-(d^2/c^2)^(1/2)*c)^(1/2)*(d*(c+d*sin(f*x+e))/((d^2/c^2)
^(1/2)*c*sin(f*x+e)+d))^(1/2))*c^3*d^2+arctan(1/(-(d^2/c^2)^(1/2)*c)^(1/2)*
(d*(c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d))^(1/2))*c^2*d^3+cos(f*
x+e)^2*arctan(1/(-(d^2/c^2)^(1/2)*c)^(1/2)*(d*(c+d*sin(f*x+e))/((d^2/c^2)^(
1/2)*c*sin(f*x+e)+d))^(1/2))*c^2*d^3-2*cos(f*x+e)^2*arctan(1/(-(d^2/c^2)^(1
/2)*c)^(1/2)*(d*(c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d))^(1/2))*c
*d^4+sin(f*x+e)*arctan(1/(-(d^2/c^2)^(1/2)*c)^(1/2)*(d*(c+d*sin(f*x+e))/((d
^2/c^2)^(1/2)*c*sin(f*x+e)+d))^(1/2))*c^3*d^2-sin(f*x+e)*arctan(1/(-(d^2/c^
2)^(1/2)*c)^(1/2)*(d*(c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d))^(1/
2))*c^2*d^3-sin(f*x+e)*arctan(1/(-(d^2/c^2)^(1/2)*c)^(1/2)*(d*(c+d*sin(f*x+
e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d))^(1/2))*c*d^4+(d^2/c^2)^(1/2)*arctan(1
```

```
/(-(d^2/c^2)^(1/2)*c)^(1/2)*(d*(c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+
e)+d))^(1/2))*c^4*d-(d^2/c^2)^(1/2)*arctan(1/(-(d^2/c^2)^(1/2)*c)^(1/2)*(d*
(c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d))^(1/2))*c^3*d^2-(d^2/c^2)
^(1/2)*arctan(1/(-(d^2/c^2)^(1/2)*c)^(1/2)*(d*(c+d*sin(f*x+e))/((d^2/c^2)^(
1/2)*c*sin(f*x+e)+d))^(1/2))*c^2*d^3+(d^2/c^2)^(1/2)*arctan(1/(-(d^2/c^2)^(
1/2)*c)^(1/2)*(d*(c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d))^(1/2))*
c*d^4-cos(f*x+e)*(-(d^2/c^2)^(1/2)*c)^(1/2)*(((d^2/c^2)^(1/2)*c^4+6*(d^2/c^
2)^(1/2)*d^2*c^2+d^4*(d^2/c^2)^(1/2)-4*c^2*d^2-4*d^4)*c)^(1/2)*arctan(((d^2
/c^2)^(1/2)*c^2-d^2)*c*((d^2/c^2)^(1/2)-1)/(((d^2/c^2)^(1/2)*c^4+6*(d^2/c^2
)^(1/2)*d^2*c^2+d^4*(d^2/c^2)^(1/2)-4*c^2*d^2-4*d^4)*c)^(1/2)*((d^2/c^2)^(1
/2)*c*sin(f*x+e)+d*cos(f*x+e)-d)/(d*(c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin
(f*x+e)+d))^(1/2)/((d^2/c^2)^(1/2)*c*sin(f*x+e)-d*cos(f*x+e)+d))*c*d-cos(f*
x+e)^2*(d^2/c^2)^(1/2)*arctan(1/(-(d^2/c^2)^(1/2)*c)^(1/2)*(d*(c+d*sin(f*x+
e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d))^(1/2))*c^3*d^2+2*cos(f*x+e)^2*(d^2/c^
2)^(1/2)*arctan(1/(-(d^2/c^2)^(1/2)*c)^(1/2)*(d*(c+d*sin(f*x+e))/((d^2/c^2)
^(1/2)*c*sin(f*x+e)+d))^(1/2))*c^2*d^3-cos(f*x+e)^2*(d^2/c^2)^(1/2)*arctan(
1/(-(d^2/c^2)^(1/2)*c)^(1/2)*(d*(c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x
+e)+d))^(1/2))*c*d^4-sin(f*x+e)*(d^2/c^2)^(1/2)*arctan(1/(-(d^2/c^2)^(1/2)*
c)^(1/2)*(d*(c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d))^(1/2))*c^4*d
+sin(f*x+e)*(d^2/c^2)^(1/2)*arctan(1/(-(d^2/c^2)^(1/2)*c)^(1/2)*(d*(c+d*sin
(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d))^(1/2))*c^3*d^2+sin(f*x+e)*(d^2/c
^2)^(1/2)*arctan(1/(-(d^2/c^2)^(1/2)*c)^(1/2)*(d*(c+d*sin(f*x+e))/((d^2/c^2
)^(1/2)*c*sin(f*x+e)+d))^(1/2))*c^2*d^3-sin(f*x+e)*(d^2/c^2)^(1/2)*arctan(1
/(-(d^2/c^2)^(1/2)*c)^(1/2)*(d*(c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+
e)+d))^(1/2))*c*d^4)/cos(f*x+e)/(cos(f*x+e)^2*d^2+c^2-d^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a\sin\left(fx+e\right)+a}}{\sqrt{d\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)/sqrt(d*sin(f*x + e) + c), x)
```

Fricas [B] time = 5.65712, size = 1825, normalized size = 29.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] [1/4*sqrt(-a/d)*log((128*a*d^4*cos(f*x + e)^5 + a*c^4 + 4*a*c^3*d + 6*a*c^2
*d^2 + 4*a*c*d^3 + a*d^4 + 128*(2*a*c*d^3 - a*d^4)*cos(f*x + e)^4 - 32*(5*a
*c^2*d^2 - 14*a*c*d^3 + 13*a*d^4)*cos(f*x + e)^3 - 32*(a*c^3*d - 2*a*c^2*d^
2 + 9*a*c*d^3 - 4*a*d^4)*cos(f*x + e)^2 - 8*(16*d^4*cos(f*x + e)^4 - c^3*d
+ 17*c^2*d^2 - 59*c*d^3 + 51*d^4 + 24*(c*d^3 - d^4)*cos(f*x + e)^3 - 2*(5*c
^2*d^2 - 26*c*d^3 + 33*d^4)*cos(f*x + e)^2 - (c^3*d - 7*c^2*d^2 + 31*c*d^3
```

```
- 25*d^4)*cos(f*x + e) + (16*d^4*cos(f*x + e)^3 + c^3*d - 17*c^2*d^2 + 59*c
*d^3 - 51*d^4 - 8*(3*c*d^3 - 5*d^4)*cos(f*x + e)^2 - 2*(5*c^2*d^2 - 14*c*d^
3 + 13*d^4)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin
(f*x + e) + c)*sqrt(-a/d) + (a*c^4 - 28*a*c^3*d + 230*a*c^2*d^2 - 476*a*c*d
^3 + 289*a*d^4)*cos(f*x + e) + (128*a*d^4*cos(f*x + e)^4 + a*c^4 + 4*a*c^3*
d + 6*a*c^2*d^2 + 4*a*c*d^3 + a*d^4 - 256*(a*c*d^3 - a*d^4)*cos(f*x + e)^3
- 32*(5*a*c^2*d^2 - 6*a*c*d^3 + 5*a*d^4)*cos(f*x + e)^2 + 32*(a*c^3*d - 7*a
*c^2*d^2 + 15*a*c*d^3 - 9*a*d^4)*cos(f*x + e))*sin(f*x + e))/(cos(f*x + e)
+ sin(f*x + e) + 1))/f, 1/2*sqrt(a/d)*arctan(1/4*(8*d^2*cos(f*x + e)^2 - c^
2 + 6*c*d - 9*d^2 - 8*(c*d - d^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sq
rt(d*sin(f*x + e) + c)*sqrt(a/d)/(2*a*d^2*cos(f*x + e)^3 - (3*a*c*d - a*d^2
)*cos(f*x + e)*sin(f*x + e) - (a*c^2 - a*c*d + 2*a*d^2)*cos(f*x + e)))/f]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}}{\sqrt{c+d\sin\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(1/2)/(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))/sqrt(c + d*sin(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a\sin\left(fx+e\right)+a}}{\sqrt{d\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)/sqrt(d*sin(f*x + e) + c), x)
```

3.568 $\int \frac{\sqrt{a+a\sin(e+fx)}}{(c+d\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=45

$$-\frac{2a\cos(e+fx)}{f(c+d)\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}$$

```
[Out] (-2*a*Cos[e + f*x])/((c + d)*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c + d*Sin[e +
f*x]])
```

Rubi [A] time = 0.092265, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {2771}

$$-\frac{2a\cos(e+fx)}{f(c+d)\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]/(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-2*a*Cos[e + f*x])/((c + d)*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c + d*Sin[e +
f*x]])
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sin(e+fx)}}{(c+d\sin(e+fx))^{3/2}}\,dx = -\frac{2a\cos(e+fx)}{(c+d)f\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}$$

Mathematica [A] time = 0.195997, size = 84, normalized size = 1.87

$$-\frac{2\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}{f(c+d)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]/(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])])/((c +
 d)*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c + d*Sin[e + f*x]])
```

Maple [B] time = 0.205, size = 99, normalized size = 2.2

$$2\,\frac{\sqrt{a\left(1+\sin\left(fx+e\right)\right)}\sqrt{c+d\sin\left(fx+e\right)}\left(\left(\cos\left(fx+e\right)\right)^{2}d+c\sin\left(fx+e\right)+d\sin\left(fx+e\right)-c-d\right)}{f\left(c+d\right)\cos\left(fx+e\right)\left(\left(\cos\left(fx+e\right)\right)^{2}d^{2}+c^{2}-d^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(3/2),x)
```

```
[Out] 2/f/(c+d)*(a*(1+sin(f*x+e)))^(1/2)*(c+d*sin(f*x+e))^(1/2)*(cos(f*x+e)^2*d+c
*sin(f*x+e)+d*sin(f*x+e)-c-d)/cos(f*x+e)/(cos(f*x+e)^2*d^2+c^2-d^2)
```

Maxima [B] time = 1.83052, size = 242, normalized size = 5.38

$$-\frac{2\left(\sqrt{a}c-\frac{\sqrt{a}(c-2d)\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{\sqrt{a}(c-2d)\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}-\frac{\sqrt{a}c\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}\right)\left(\frac{\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+1\right)}{\left(c+d+\frac{(c+d)\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}\right)\left(c+\frac{2d\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{c\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}\right)^{\frac{3}{2}}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] -2*(sqrt(a)*c - sqrt(a)*(c - 2*d)*sin(f*x + e)/(cos(f*x + e) + 1) + sqrt(a)
*(c - 2*d)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - sqrt(a)*c*sin(f*x + e)^3/(
cos(f*x + e) + 1)^3)*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/((c + d + (c
 + d)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2)*(c + 2*d*sin(f*x + e)/(cos(f*x +
 e) + 1) + c*sin(f*x + e)^2/(cos(f*x + e) + 1)^2)^(3/2)*f)
```

Fricas [B] time = 2.28305, size = 323, normalized size = 7.18

$$\frac{2\sqrt{a\sin\left(fx+e\right)+a}\sqrt{d\sin\left(fx+e\right)+c}\left(\cos\left(fx+e\right)-\sin\left(fx+e\right)+1\right)}{\left(cd+d^{2}\right)f\cos\left(fx+e\right)^{2}-\left(c^{2}+cd\right)f\cos\left(fx+e\right)-\left(c^{2}+2\,cd+d^{2}\right)f-\left(\left(cd+d^{2}\right)f\cos\left(fx+e\right)+\left(c^{2}+2\,cd+d^{2}\right)f\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] 2*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*(cos(f*x + e) - sin(f*x
 + e) + 1)/((c*d + d^2)*f*cos(f*x + e)^2 - (c^2 + c*d)*f*cos(f*x + e) - (c^
2 + 2*c*d + d^2)*f - ((c*d + d^2)*f*cos(f*x + e) + (c^2 + 2*c*d + d^2)*f)*s
in(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}}{\left(c+d\sin\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(1/2)/(c+d*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))/(c + d*sin(e + f*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a\sin\left(fx+e\right)+a}}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)/(d*sin(f*x + e) + c)^(3/2), x)
```

3.569 $\int \frac{\sqrt{a+a\sin(e+fx)}}{(c+d\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=95

$$-\frac{4a\cos(e+fx)}{3f(c+d)^2\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}-\frac{2a\cos(e+fx)}{3f(c+d)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^{3/2}}$$

```
[Out] (-2*a*Cos[e + f*x])/(3*(c + d)*f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*
x])^(3/2)) - (4*a*Cos[e + f*x])/(3*(c + d)^2*f*Sqrt[a + a*Sin[e + f*x]]*Sqr
t[c + d*Sin[e + f*x]])
```

Rubi [A] time = 0.192069, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2772, 2771}

$$-\frac{4a\cos(e+fx)}{3f(c+d)^2\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}-\frac{2a\cos(e+fx)}{3f(c+d)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]/(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (-2*a*Cos[e + f*x])/(3*(c + d)*f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*
x])^(3/2)) - (4*a*Cos[e + f*x])/(3*(c + d)^2*f*Sqrt[a + a*Sin[e + f*x]]*Sqr
t[c + d*Sin[e + f*x]])
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+a\sin(e+fx)}}{(c+d\sin(e+fx))^{5/2}}\,dx &= -\frac{2a\cos(e+fx)}{3(c+d)f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^{3/2}}+\frac{2\int \frac{\sqrt{a+a\sin(e+fx)}}{(c+d\sin(e+fx))^{3/2}}\,dx}{3(c+d)}\\ &= -\frac{2a\cos(e+fx)}{3(c+d)f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^{3/2}}-\frac{4a\cos(e+fx)}{3(c+d)^2f\sqrt{a+a\sin(e+fx)}\sqrt{c+}}\end{aligned}$$

Mathematica [A] time = 0.273571, size = 100, normalized size = 1.05

$$-\frac{2\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(3c+2d\sin(e+fx)+d)}{3f(c+d)^2\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)(c+d\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]/(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (-2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(3*c +
 d + 2*d*Sin[e + f*x]))/(3*(c + d)^2*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)*(c + d*Sin[e + f*x])^(3/2))
```

Maple [B] time = 0.215, size = 222, normalized size = 2.3

$$\frac{4\left(\cos\left(fx+e\right)\right)^4 d^3+2\sin\left(fx+e\right)\left(\cos\left(fx+e\right)\right)^2 cd^2+2\sin\left(fx+e\right)\left(\cos\left(fx+e\right)\right)^2 d^3+8c^2\left(\cos\left(fx+e\right)\right)^2 d+2c}{3f(c+d)^2 c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(5/2),x)
```

```
[Out] 2/3/f/(c+d)^2*(a*(1+sin(f*x+e)))^(1/2)*(c+d*sin(f*x+e))^(1/2)*(2*cos(f*x+e)
^4*d^3+sin(f*x+e)*cos(f*x+e)^2*c*d^2+sin(f*x+e)*cos(f*x+e)^2*d^3+4*c^2*cos(
f*x+e)^2*d+c*cos(f*x+e)^2*d^2-3*cos(f*x+e)^2*d^3+3*c^3*sin(f*x+e)+5*c^2*d*s
in(f*x+e)+sin(f*x+e)*d^2*c-d^3*sin(f*x+e)-3*c^3-5*c^2*d-c*d^2+d^3)/cos(f*x+
e)/(cos(f*x+e)^2*d^2+c^2-d^2)^2
```

Maxima [B] time = 1.95327, size = 459, normalized size = 4.83

$$-\frac{2\left(\left(3c^2+cd\right)\sqrt{a}-\frac{\left(3c^2-9cd-2d^2\right)\sqrt{a}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{2\left(3c^2-4cd+3d^2\right)\sqrt{a}\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}-\frac{2\left(3c^2-4cd+3d^2\right)\sqrt{a}\sin\left(fx+e\right)^3}{\left(\cos\left(fx+e\right)+1\right)^3}+\frac{\left(3c^2-9cd-2d^2\right)\sqrt{a}\sin(}{\left(\cos\left(fx+e\right)+1\right)^4}\right.}{3\left(c^2+2cd+d^2+\frac{2\left(c^2+2cd+d^2\right)\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}+\frac{\left(c^2+2cd+d^2\right)\sin\left(fx+e\right)^4}{\left(\cos\left(fx+e\right)+1\right)^4}\right)\left(c+\frac{2d\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{c}{(co}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] -2/3*((3*c^2 + c*d)*sqrt(a) - (3*c^2 - 9*c*d - 2*d^2)*sqrt(a)*sin(f*x + e)/
(cos(f*x + e) + 1) + 2*(3*c^2 - 4*c*d + 3*d^2)*sqrt(a)*sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 - 2*(3*c^2 - 4*c*d + 3*d^2)*sqrt(a)*sin(f*x + e)^3/(cos(f*x
 + e) + 1)^3 + (3*c^2 - 9*c*d - 2*d^2)*sqrt(a)*sin(f*x + e)^4/(cos(f*x + e)
 + 1)^4 - (3*c^2 + c*d)*sqrt(a)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)*(sin(f
*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^2/((c^2 + 2*c*d + d^2 + 2*(c^2 + 2*c*d
+ d^2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + (c^2 + 2*c*d + d^2)*sin(f*x +
e)^4/(cos(f*x + e) + 1)^4)*(c + 2*d*sin(f*x + e)/(cos(f*x + e) + 1) + c*sin
(f*x + e)^2/(cos(f*x + e) + 1)^2)^(5/2)*f)
```

Fricas [B] time = 2.36704, size = 703, normalized size = 7.4

$$\frac{2\left(2\,d\cos\left(fx+e\right)^{2}+(3\,c+d)\cos\left(\right.\right.}{3\left(\left(c^{2}d^{2}+2\,cd^{3}+d^{4}\right)f\cos\left(fx+e\right)^{3}+\left(2\,c^{3}d+5\,c^{2}d^{2}+4\,cd^{3}+d^{4}\right)f\cos\left(fx+e\right)^{2}-\left(c^{4}+2\,c^{3}d+2\,c^{2}d^{2}+2\,cd^{3}+d^{4}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] 2/3*(2*d*cos(f*x + e)^2 + (3*c + d)*cos(f*x + e) + (2*d*cos(f*x + e) - 3*c
+ d)*sin(f*x + e) + 3*c - d)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) +
 c)/((c^2*d^2 + 2*c*d^3 + d^4)*f*cos(f*x + e)^3 + (2*c^3*d + 5*c^2*d^2 + 4*
c*d^3 + d^4)*f*cos(f*x + e)^2 - (c^4 + 2*c^3*d + 2*c^2*d^2 + 2*c*d^3 + d^4)
*f*cos(f*x + e) - (c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^3 + d^4)*f + ((c^2*d^2
 + 2*c*d^3 + d^4)*f*cos(f*x + e)^2 - 2*(c^3*d + 2*c^2*d^2 + c*d^3)*f*cos(f*
x + e) - (c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^3 + d^4)*f)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(1/2)/(c+d*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a\sin\left(fx+e\right)+a}}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)/(d*sin(f*x + e) + c)^(5/2), x)
```

3.570 $\int \frac{\sqrt{a+a\sin(e+fx)}}{(c+d\sin(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=142

$$-\frac{16a\cos(e+fx)}{15f(c+d)^3\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}-\frac{8a\cos(e+fx)}{15f(c+d)^2\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^{3/2}}-\frac{}{5f(c+d)\sqrt{a}}$$

```
[Out] (-2*a*Cos[e + f*x])/(5*(c + d)*f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*
x])^(5/2)) - (8*a*Cos[e + f*x])/(15*(c + d)^2*f*Sqrt[a + a*Sin[e + f*x]]*(c
 + d*Sin[e + f*x])^(3/2)) - (16*a*Cos[e + f*x])/(15*(c + d)^3*f*Sqrt[a + a*
Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])
```

Rubi [A] time = 0.294739, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2772, 2771}

$$-\frac{16a\cos(e+fx)}{15f(c+d)^3\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}-\frac{8a\cos(e+fx)}{15f(c+d)^2\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^{3/2}}-\frac{}{5f(c+d)\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]/(c + d*Sin[e + f*x])^(7/2),x]
```

```
[Out] (-2*a*Cos[e + f*x])/(5*(c + d)*f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*
x])^(5/2)) - (8*a*Cos[e + f*x])/(15*(c + d)^2*f*Sqrt[a + a*Sin[e + f*x]]*(c
 + d*Sin[e + f*x])^(3/2)) - (16*a*Cos[e + f*x])/(15*(c + d)^3*f*Sqrt[a + a*
Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sin(e+fx)}}{(c+d\sin(e+fx))^{7/2}}\,dx = -\frac{2a\cos(e+fx)}{5(c+d)f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^{5/2}} + \frac{4\int \frac{\sqrt{a+a\sin(e+fx)}}{(c+d\sin(e+fx))^{5/2}}dx}{5(c+d)}$$

$$= -\frac{2a\cos(e+fx)}{5(c+d)f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^{5/2}} - \frac{8a\cos(e+fx)}{15(c+d)^2f\sqrt{a+a\sin(e+fx)}(c+}$$

$$= -\frac{2a\cos(e+fx)}{5(c+d)f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^{5/2}} - \frac{8a\cos(e+fx)}{15(c+d)^2f\sqrt{a+a\sin(e+fx)}(c+}$$

Mathematica [A] time = 0.393021, size = 128, normalized size = 0.9

$$-\frac{2\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(15c^2+4d(5c+d)\sin(e+fx)+10cd+8d^2\sin^2(e+fx)+\right.}{15f(c+d)^3\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)(c+d\sin(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]/(c + d*Sin[e + f*x])^(7/2),x]
```

```
[Out] (-2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(15*c^
2 + 10*c*d + 3*d^2 + 4*d*(5*c + d)*Sin[e + f*x] + 8*d^2*Sin[e + f*x]^2))/(1
5*(c + d)^3*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(c + d*Sin[e + f*x])^(5
/2))
```

Maple [B] time = 0.244, size = 430, normalized size = 3.

$$\frac{-4c\left(\cos\left(fx+e\right)\right)^4d^4-14d^5-30\left(\cos\left(fx+e\right)\right)^2c^2d^3-70c^4d-12c^2d^3-22cd^4-46\left(\cos\left(fx+e\right)\right)^4d^5+44\left(\cos\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(7/2),x)
```

```
[Out] 2/15/f/(c+d)^3*(a*(1+sin(f*x+e)))^(1/2)*(c+d*sin(f*x+e))^(1/2)*(-2*c*cos(f*
x+e)^4*d^4-7*d^5-15*cos(f*x+e)^2*c^2*d^3-35*c^4*d-6*c^2*d^3-11*c*d^4-23*cos
(f*x+e)^4*d^5+22*cos(f*x+e)^2*d^5+8*cos(f*x+e)^6*d^5+15*c^5*sin(f*x+e)+6*c^
2*d^3*sin(f*x+e)+11*c*d^4*sin(f*x+e)+7*d^5*sin(f*x+e)-22*c^3*d^2+4*sin(f*x+
e)*cos(f*x+e)^4*c*d^4+7*sin(f*x+e)*cos(f*x+e)^2*c^3*d^2+3*sin(f*x+e)*cos(f*
x+e)^2*c^2*d^3-15*sin(f*x+e)*cos(f*x+e)^2*c*d^4+19*c^3*cos(f*x+e)^2*d^2+13*
c*d^4*cos(f*x+e)^2+4*sin(f*x+e)*cos(f*x+e)^4*d^5-11*sin(f*x+e)*cos(f*x+e)^2
*d^5+25*cos(f*x+e)^2*c^4*d+35*sin(f*x+e)*c^4*d+22*sin(f*x+e)*c^3*d^2+21*cos
(f*x+e)^4*c^2*d^3-15*c^5)/cos(f*x+e)/(cos(f*x+e)^2*d^2+c^2-d^2)^3
```

Maxima [B] time = 2.27793, size = 734, normalized size = 5.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(7/2),x, algorithm="maxim
a")
```

```
[Out] -2/15*((15*c^3 + 10*c^2*d + 3*c*d^2)*sqrt(a) - (15*c^3 - 60*c^2*d - 25*c*d^
2 - 6*d^3)*sqrt(a)*sin(f*x + e)/(cos(f*x + e) + 1) + (45*c^3 - 40*c^2*d + 9
3*c*d^2 + 10*d^3)*sqrt(a)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 5*(9*c^3 -
22*c^2*d + 13*c*d^2 - 12*d^3)*sqrt(a)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 +
 5*(9*c^3 - 22*c^2*d + 13*c*d^2 - 12*d^3)*sqrt(a)*sin(f*x + e)^4/(cos(f*x +
 e) + 1)^4 - (45*c^3 - 40*c^2*d + 93*c*d^2 + 10*d^3)*sqrt(a)*sin(f*x + e)^5
/(cos(f*x + e) + 1)^5 + (15*c^3 - 60*c^2*d - 25*c*d^2 - 6*d^3)*sqrt(a)*sin(
f*x + e)^6/(cos(f*x + e) + 1)^6 - (15*c^3 + 10*c^2*d + 3*c*d^2)*sqrt(a)*sin
(f*x + e)^7/(cos(f*x + e) + 1)^7)*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)
^3/((c^3 + 3*c^2*d + 3*c*d^2 + d^3 + 3*(c^3 + 3*c^2*d + 3*c*d^2 + d^3)*sin(
f*x + e)^2/(cos(f*x + e) + 1)^2 + 3*(c^3 + 3*c^2*d + 3*c*d^2 + d^3)*sin(f*x
 + e)^4/(cos(f*x + e) + 1)^4 + (c^3 + 3*c^2*d + 3*c*d^2 + d^3)*sin(f*x + e)
^6/(cos(f*x + e) + 1)^6)*(c + 2*d*sin(f*x + e)/(cos(f*x + e) + 1) + c*sin(f
*x + e)^2/(cos(f*x + e) + 1)^2)^(7/2)*f)
```

Fricas [B] time = 2.60363, size = 1264, normalized size = 8.9

$$\overline{15\left(\left(c^3d^3+3\,c^2d^4+3\,cd^5+d^6\right)f\cos\left(fx+e\right)^4-3\left(c^4d^2+3\,c^3d^3+3\,c^2d^4+cd^5\right)f\cos\left(fx+e\right)^3-\left(3\,c^5d+12\,c^4d^2+20\,c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(7/2),x, algorithm="frica
s")
```

```
[Out] 2/15*(8*d^2*cos(f*x + e)^3 - 4*(5*c*d - d^2)*cos(f*x + e)^2 - 15*c^2 + 10*c
*d - 7*d^2 - (15*c^2 + 10*c*d + 11*d^2)*cos(f*x + e) - (8*d^2*cos(f*x + e)^
2 - 15*c^2 + 10*c*d - 7*d^2 + 4*(5*c*d + d^2)*cos(f*x + e))*sin(f*x + e))*s
qrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)/(((c^3*d^3 + 3*c^2*d^4 + 3*
c*d^5 + d^6)*f*cos(f*x + e)^4 - 3*(c^4*d^2 + 3*c^3*d^3 + 3*c^2*d^4 + c*d^5)
*f*cos(f*x + e)^3 - (3*c^5*d + 12*c^4*d^2 + 20*c^3*d^3 + 18*c^2*d^4 + 9*c*d
^5 + 2*d^6)*f*cos(f*x + e)^2 + (c^6 + 3*c^5*d + 6*c^4*d^2 + 10*c^3*d^3 + 9*
c^2*d^4 + 3*c*d^5)*f*cos(f*x + e) + (c^6 + 6*c^5*d + 15*c^4*d^2 + 20*c^3*d^
3 + 15*c^2*d^4 + 6*c*d^5 + d^6)*f - ((c^3*d^3 + 3*c^2*d^4 + 3*c*d^5 + d^6)*
f*cos(f*x + e)^3 + (3*c^4*d^2 + 10*c^3*d^3 + 12*c^2*d^4 + 6*c*d^5 + d^6)*f*
cos(f*x + e)^2 - (3*c^5*d + 9*c^4*d^2 + 10*c^3*d^3 + 6*c^2*d^4 + 3*c*d^5 +
d^6)*f*cos(f*x + e) - (c^6 + 6*c^5*d + 15*c^4*d^2 + 20*c^3*d^3 + 15*c^2*d^4
 + 6*c*d^5 + d^6)*f)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(1/2)/(c+d*sin(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a \sin \left(fx+e\right)+a}}{\left(d \sin \left(fx+e\right)+c\right)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(7/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)/(d*sin(f*x + e) + c)^(7/2), x)
```

3.571 $\int (a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{5/2}\,dx$

Optimal. Leaf size=285

$$\frac{5a^{3/2}(c-15d)(c+d)^3\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{64d^{3/2}f}-\frac{a^2\cos(e+fx)(c+d\sin(e+fx))^{7/2}}{4df\sqrt{a\sin(e+fx)+a}}+\frac{a^2(c-15d)\cos(e+fx)(}{24df\sqrt{a\sin(e}}$$

```
[Out] (5*a^(3/2)*(c - 15*d)*(c + d)^3*ArcTan[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt
[a + a*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])])/(64*d^(3/2)*f) + (5*a^2*(c
 - 15*d)*(c + d)^2*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(64*d*f*Sqrt[a +
a*Sin[e + f*x]]) + (5*a^2*(c - 15*d)*(c + d)*Cos[e + f*x]*(c + d*Sin[e + f*
x])^(3/2))/(96*d*f*Sqrt[a + a*Sin[e + f*x]]) + (a^2*(c - 15*d)*Cos[e + f*x]
*(c + d*Sin[e + f*x])^(5/2))/(24*d*f*Sqrt[a + a*Sin[e + f*x]]) - (a^2*Cos[e
 + f*x]*(c + d*Sin[e + f*x])^(7/2))/(4*d*f*Sqrt[a + a*Sin[e + f*x]])
```

Rubi [A] time = 0.56667, antiderivative size = 285, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2763, 21, 2770, 2775, 205}

$$\frac{5a^{3/2}(c-15d)(c+d)^3\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{64d^{3/2}f}-\frac{a^2\cos(e+fx)(c+d\sin(e+fx))^{7/2}}{4df\sqrt{a\sin(e+fx)+a}}+\frac{a^2(c-15d)\cos(e+fx)(}{24df\sqrt{a\sin(e}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (5*a^(3/2)*(c - 15*d)*(c + d)^3*ArcTan[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt
[a + a*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])])/(64*d^(3/2)*f) + (5*a^2*(c
 - 15*d)*(c + d)^2*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(64*d*f*Sqrt[a +
a*Sin[e + f*x]]) + (5*a^2*(c - 15*d)*(c + d)*Cos[e + f*x]*(c + d*Sin[e + f*
x])^(3/2))/(96*d*f*Sqrt[a + a*Sin[e + f*x]]) + (a^2*(c - 15*d)*Cos[e + f*x]
*(c + d*Sin[e + f*x])^(5/2))/(24*d*f*Sqrt[a + a*Sin[e + f*x]]) - (a^2*Cos[e
 + f*x]*(c + d*Sin[e + f*x])^(7/2))/(4*d*f*Sqrt[a + a*Sin[e + f*x]])
```

Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{5/2}\,dx = -\frac{a^2\cos(e+fx)(c+d\sin(e+fx))^{7/2}}{4df\sqrt{a+a\sin(e+fx)}} + \frac{\int \frac{\left(-\frac{1}{2}a^2(c-15d)-\frac{1}{2}a^2(c-15d)\sin(e\cdots\right)}{\sqrt{a+a\sin(e+\cdots}}}{4d}$$

$$= -\frac{a^2\cos(e+fx)(c+d\sin(e+fx))^{7/2}}{4df\sqrt{a+a\sin(e+fx)}} - \frac{(a(c-15d))\int\sqrt{a+a\sin(e\cdots}}{\cdots}$$

$$= \frac{a^2(c-15d)\cos(e+fx)(c+d\sin(e+fx))^{5/2}}{24df\sqrt{a+a\sin(e+fx)}} - \frac{a^2\cos(e+fx)(c+d\cdots}{4df\sqrt{a+a\sin\cdots}}$$

$$= \frac{5a^2(c-15d)(c+d)\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{96df\sqrt{a+a\sin(e+fx)}} + \frac{a^2(c-15d)\cdots}{24\cdots}$$

$$= \frac{5a^2(c-15d)(c+d)^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{64df\sqrt{a+a\sin(e+fx)}} + \frac{5a^2(c-15d)(\cdots}{\cdots}$$

$$= \frac{5a^2(c-15d)(c+d)^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{64df\sqrt{a+a\sin(e+fx)}} + \frac{5a^2(c-15d)(\cdots}{\cdots}$$

$$= \frac{5a^{3/2}(c-15d)(c+d)^3\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{64d^{3/2}f} + \frac{5a^2(c-1\cdots}{\cdots}$$

Mathematica [A] time = 1.3732, size = 318, normalized size = 1.12

$$\frac{(a(\sin(e+fx)+1))^{3/2}\left(-\frac{2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{c+d\sin(e+fx)}\left(2d\left(59c^2+190cd+93d^2\right)\sin(e+fx)+455c^2d+15c^3-4d^2(17c+15d)\cos(2(e+\cdots\right.}{3d}\cdots\right.}{128f\left(\sin\left(\cdots\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^(5/2),x]
```

[Out] ((a*(1 + Sin[e + f*x]))^(3/2)*((-5*(c - 15*d)*(c + d)^3*(2*ArcTan[(Sqrt[2]*Sqrt[d]*Sin[(2*e - Pi + 2*f*x)/4])/Sqrt[c + d*Sin[e + f*x]]] + ArcTanh[(Sqrt[2]*Sqrt[d]*Cos[(2*e - Pi + 2*f*x)/4])/Sqrt[c + d*Sin[e + f*x]]] - Log[Sqrt[2]*Sqrt[d]*Cos[(2*e - Pi + 2*f*x)/4] + Sqrt[c + d*Sin[e + f*x]]]))/d^(3/2) - (2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[c + d*Sin[e + f*x]]*(15*c^3 + 455*c^2*d + 653*c*d^2 + 285*d^3 - 4*d^2*(17*c + 15*d)*Cos[2*(e + f*x)] + 2*d*(59*c^2 + 190*c*d + 93*d^2)*Sin[e + f*x] - 12*d^3*Sin[3*(e + f*x)]))/(3*d)))/(128*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3)

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int \left(a+a\sin\left(fx+e\right)\right)^{\frac{3}{2}}\left(c+d\sin\left(fx+e\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^(5/2),x)

[Out] int((a+a*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^(5/2),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^(5/2),x, algorithm="maxima")

[Out] integrate((a*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e) + c)^(5/2), x)

Fricas [B] time = 14.0633, size = 3841, normalized size = 13.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^(5/2),x, algorithm="fricas")

[Out] [-1/1536*(15*(a*c^4 - 12*a*c^3*d - 42*a*c^2*d^2 - 44*a*c*d^3 - 15*a*d^4 + (a*c^4 - 12*a*c^3*d - 42*a*c^2*d^2 - 44*a*c*d^3 - 15*a*d^4)*cos(f*x + e) + (a*c^4 - 12*a*c^3*d - 42*a*c^2*d^2 - 44*a*c*d^3 - 15*a*d^4)*sin(f*x + e))*sqrt(-a/d)*log((128*a*d^4*cos(f*x + e)^5 + a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 + a*d^4 + 128*(2*a*c*d^3 - a*d^4)*cos(f*x + e)^4 - 32*(5*a*c^2*d^2 - 14*a*c*d^3 + 13*a*d^4)*cos(f*x + e)^3 - 32*(a*c^3*d - 2*a*c^2*d^2 + 9*a*c*d^3 - 4*a*d^4)*cos(f*x + e)^2 - 8*(16*d^4*cos(f*x + e)^4 - c^3*d + 17*c^2*d^2 - 59*c*d^3 + 51*d^4 + 24*(c*d^3 - d^4)*cos(f*x + e)^3 - 2*(5*c^2*d^2 - 26*c*d^3 + 33*d^4)*cos(f*x + e)^2 - (c^3*d - 7*c^2*d^2 + 31*c*d^3 - 25*d^4)*cos(f*x + e) + (16*d^4*cos(f*x + e)^3 + c^3*d - 17*c^2*d^2 + 59*c*d^3 - 51*d^4 - 8*(3*c*d^3 - 5*d^4)*cos(f*x + e)^2 - 2*(5*c^2*d^2 - 14*c*d^3 + 13*

```
d^4)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x +
e) + c)*sqrt(-a/d) + (a*c^4 - 28*a*c^3*d + 230*a*c^2*d^2 - 476*a*c*d^3 + 28
9*a*d^4)*cos(f*x + e) + (128*a*d^4*cos(f*x + e)^4 + a*c^4 + 4*a*c^3*d + 6*a
*c^2*d^2 + 4*a*c*d^3 + a*d^4 - 256*(a*c*d^3 - a*d^4)*cos(f*x + e)^3 - 32*(5
*a*c^2*d^2 - 6*a*c*d^3 + 5*a*d^4)*cos(f*x + e)^2 + 32*(a*c^3*d - 7*a*c^2*d^
2 + 15*a*c*d^3 - 9*a*d^4)*cos(f*x + e))*sin(f*x + e))/(cos(f*x + e) + sin(f
*x + e) + 1)) - 8*(48*a*d^3*cos(f*x + e)^4 - 15*a*c^3 - 337*a*c^2*d - 341*a
*c*d^2 - 147*a*d^3 + 8*(17*a*c*d^2 + 15*a*d^3)*cos(f*x + e)^3 - 2*(59*a*c^2
*d + 122*a*c*d^2 + 63*a*d^3)*cos(f*x + e)^2 - (15*a*c^3 + 455*a*c^2*d + 721
*a*c*d^2 + 345*a*d^3)*cos(f*x + e) + (48*a*d^3*cos(f*x + e)^3 + 15*a*c^3 +
337*a*c^2*d + 341*a*c*d^2 + 147*a*d^3 - 8*(17*a*c*d^2 + 9*a*d^3)*cos(f*x +
e)^2 - 2*(59*a*c^2*d + 190*a*c*d^2 + 99*a*d^3)*cos(f*x + e))*sin(f*x + e))*
sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/(d*f*cos(f*x + e) + d*f*
sin(f*x + e) + d*f), -1/768*(15*(a*c^4 - 12*a*c^3*d - 42*a*c^2*d^2 - 44*a*c
*d^3 - 15*a*d^4 + (a*c^4 - 12*a*c^3*d - 42*a*c^2*d^2 - 44*a*c*d^3 - 15*a*d^
4)*cos(f*x + e) + (a*c^4 - 12*a*c^3*d - 42*a*c^2*d^2 - 44*a*c*d^3 - 15*a*d^
4)*sin(f*x + e))*sqrt(a/d)*arctan(1/4*(8*d^2*cos(f*x + e)^2 - c^2 + 6*c*d -
 9*d^2 - 8*(c*d - d^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*
x + e) + c)*sqrt(a/d)/(2*a*d^2*cos(f*x + e)^3 - (3*a*c*d - a*d^2)*cos(f*x +
 e)*sin(f*x + e) - (a*c^2 - a*c*d + 2*a*d^2)*cos(f*x + e))) - 4*(48*a*d^3*c
os(f*x + e)^4 - 15*a*c^3 - 337*a*c^2*d - 341*a*c*d^2 - 147*a*d^3 + 8*(17*a*
c*d^2 + 15*a*d^3)*cos(f*x + e)^3 - 2*(59*a*c^2*d + 122*a*c*d^2 + 63*a*d^3)*
cos(f*x + e)^2 - (15*a*c^3 + 455*a*c^2*d + 721*a*c*d^2 + 345*a*d^3)*cos(f*x
 + e) + (48*a*d^3*cos(f*x + e)^3 + 15*a*c^3 + 337*a*c^2*d + 341*a*c*d^2 + 1
47*a*d^3 - 8*(17*a*c*d^2 + 9*a*d^3)*cos(f*x + e)^2 - 2*(59*a*c^2*d + 190*a*
c*d^2 + 99*a*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt
(d*sin(f*x + e) + c))/(d*f*cos(f*x + e) + d*f*sin(f*x + e) + d*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*(c+d*sin(f*x+e))**(5/2),x)

[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^(5/2),x, algorithm="giac"
)

[Out] Exception raised: TypeError
```

3.572 $\int (a + a\sin(e + fx))^{3/2}(c + d\sin(e + fx))^{3/2}\,dx$

Optimal. Leaf size=228

$$\frac{a^{3/2}(c-11d)(c+d)^2\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{8d^{3/2}f} - \frac{a^2\cos(e+fx)(c+d\sin(e+fx))^{5/2}}{3df\sqrt{a\sin(e+fx)+a}} + \frac{a^2(c-11d)\cos(e+fx)(c}{12df\sqrt{a\sin(e+}}$$

```
[Out] (a^(3/2)*(c - 11*d)*(c + d)^2*ArcTan[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[a
 + a*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])])/(8*d^(3/2)*f) + (a^2*(c - 11
*d)*(c + d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(8*d*f*Sqrt[a + a*Sin[e
+ f*x]]) + (a^2*(c - 11*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(3/2))/(12*d*f
*Sqrt[a + a*Sin[e + f*x]]) - (a^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(5/2))/
(3*d*f*Sqrt[a + a*Sin[e + f*x]])
```

Rubi [A] time = 0.43496, antiderivative size = 228, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2763, 21, 2770, 2775, 205}

$$\frac{a^{3/2}(c-11d)(c+d)^2\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{8d^{3/2}f} - \frac{a^2\cos(e+fx)(c+d\sin(e+fx))^{5/2}}{3df\sqrt{a\sin(e+fx)+a}} + \frac{a^2(c-11d)\cos(e+fx)(c}{12df\sqrt{a\sin(e+}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] (a^(3/2)*(c - 11*d)*(c + d)^2*ArcTan[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[a
 + a*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])])/(8*d^(3/2)*f) + (a^2*(c - 11
*d)*(c + d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(8*d*f*Sqrt[a + a*Sin[e
+ f*x]]) + (a^2*(c - 11*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(3/2))/(12*d*f
*Sqrt[a + a*Sin[e + f*x]]) - (a^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(5/2))/
(3*d*f*Sqrt[a + a*Sin[e + f*x]])
```

Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
```

```
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{3/2}\,dx = -\frac{a^2\cos(e+fx)(c+d\sin(e+fx))^{5/2}}{3df\sqrt{a+a\sin(e+fx)}} + \frac{\int \frac{\left(-\frac{1}{2}a^2(c-11d)-\frac{1}{2}a^2(c-11d)\sin(e\cdots\right.}{\sqrt{a+a\sin(e+\cdots}}}{3d}$$

$$= -\frac{a^2\cos(e+fx)(c+d\sin(e+fx))^{5/2}}{3df\sqrt{a+a\sin(e+fx)}} - \frac{(a(c-11d))\int\sqrt{a+a\sin(e\cdots}}{\cdots}$$

$$= \frac{a^2(c-11d)\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{12df\sqrt{a+a\sin(e+fx)}} - \frac{a^2\cos(e+fx)(c+d\cdots}{3df\sqrt{a+a\sin\cdots}}$$

$$= \frac{a^2(c-11d)(c+d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{8df\sqrt{a+a\sin(e+fx)}} + \frac{a^2(c-11d)\cos(\cdots}{12df\cdots}$$

$$= \frac{a^2(c-11d)(c+d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{8df\sqrt{a+a\sin(e+fx)}} + \frac{a^2(c-11d)\cos(\cdots}{12df\cdots}$$

$$= \frac{a^{3/2}(c-11d)(c+d)^2\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{8d^{3/2}f} + \frac{a^2(c-11d\cdots}{\cdots}$$

Mathematica [A] time = 0.834184, size = 281, normalized size = 1.23

$$\frac{(a(\sin(e+fx)+1))^{3/2}\left(\frac{(c-11d)(c+d)^2\left(-2\tan^{-1}\left(\frac{\sqrt{2}\sqrt{d}\sin\left(\frac{1}{4}(2e+2fx-\pi)\right)}{\sqrt{c+d\sin(e+fx)}}\right)+\log\left(\sqrt{c+d\sin(e+fx)}+\sqrt{2}\sqrt{d}\cos\left(\frac{1}{4}(2e+2fx-\pi)\right)\right)-\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{d}\cos\left(\frac{1}{4}(2e\cdots\right)}{\sqrt{c+d\sin(e\cdots}}\right)\cdots\right)}{d^{3/2}}\cdots\right)}{16f\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e\cdots\right)\right)\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^(3/2)*(((c - 11*d)*(c + d)^2*(-2*ArcTan[(Sqrt[2]*Sq
rt[d]*Sin[(2*e - Pi + 2*f*x)/4])/Sqrt[c + d*Sin[e + f*x]]] - ArcTanh[(Sqrt[
2]*Sqrt[d]*Cos[(2*e - Pi + 2*f*x)/4])/Sqrt[c + d*Sin[e + f*x]]] + Log[Sqrt[
2]*Sqrt[d]*Cos[(2*e - Pi + 2*f*x)/4] + Sqrt[c + d*Sin[e + f*x]]]))/d^(3/2)
```

```
- (2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[c + d*Sin[e + f*x]]*(3*c^2
+ 52*c*d + 37*d^2 - 4*d^2*Cos[2*(e + f*x)] + 2*d*(7*c + 11*d)*Sin[e + f*x])
)/(3*d)))/(16*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3)
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int \left(a + a\sin\left(fx + e\right)\right)^{\frac{3}{2}} \left(c + d\sin\left(fx + e\right)\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^(3/2),x)
```

```
[Out] int((a+a*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx + e\right) + a\right)^{\frac{3}{2}} \left(d\sin\left(fx + e\right) + c\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e) + c)^(3/2), x)
```

Fricas [B] time = 9.25408, size = 3214, normalized size = 14.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] [-1/192*(3*(a*c^3 - 9*a*c^2*d - 21*a*c*d^2 - 11*a*d^3 + (a*c^3 - 9*a*c^2*d
 - 21*a*c*d^2 - 11*a*d^3)*cos(f*x + e) + (a*c^3 - 9*a*c^2*d - 21*a*c*d^2 - 1
1*a*d^3)*sin(f*x + e))*sqrt(-a/d)*log((128*a*d^4*cos(f*x + e)^5 + a*c^4 + 4
*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 + a*d^4 + 128*(2*a*c*d^3 - a*d^4)*cos(f*
x + e)^4 - 32*(5*a*c^2*d^2 - 14*a*c*d^3 + 13*a*d^4)*cos(f*x + e)^3 - 32*(a*
c^3*d - 2*a*c^2*d^2 + 9*a*c*d^3 - 4*a*d^4)*cos(f*x + e)^2 - 8*(16*d^4*cos(f
*x + e)^4 - c^3*d + 17*c^2*d^2 - 59*c*d^3 + 51*d^4 + 24*(c*d^3 - d^4)*cos(f
*x + e)^3 - 2*(5*c^2*d^2 - 26*c*d^3 + 33*d^4)*cos(f*x + e)^2 - (c^3*d - 7*c
^2*d^2 + 31*c*d^3 - 25*d^4)*cos(f*x + e) + (16*d^4*cos(f*x + e)^3 + c^3*d -
 17*c^2*d^2 + 59*c*d^3 - 51*d^4 - 8*(3*c*d^3 - 5*d^4)*cos(f*x + e)^2 - 2*(5
*c^2*d^2 - 14*c*d^3 + 13*d^4)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x +
e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt(-a/d) + (a*c^4 - 28*a*c^3*d + 230*a*c
^2*d^2 - 476*a*c*d^3 + 289*a*d^4)*cos(f*x + e) + (128*a*d^4*cos(f*x + e)^4
+ a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 + a*d^4 - 256*(a*c*d^3 - a*d^
4)*cos(f*x + e)^3 - 32*(5*a*c^2*d^2 - 6*a*c*d^3 + 5*a*d^4)*cos(f*x + e)^2 +
 32*(a*c^3*d - 7*a*c^2*d^2 + 15*a*c*d^3 - 9*a*d^4)*cos(f*x + e))*sin(f*x +
```

```
e))/(cos(f*x + e) + sin(f*x + e) + 1)) - 8*(8*a*d^2*cos(f*x + e)^3 - 3*a*c^
2 - 38*a*c*d - 19*a*d^2 - 14*(a*c*d + a*d^2)*cos(f*x + e)^2 - (3*a*c^2 + 52
*a*c*d + 41*a*d^2)*cos(f*x + e) - (8*a*d^2*cos(f*x + e)^2 - 3*a*c^2 - 38*a*
c*d - 19*a*d^2 + 2*(7*a*c*d + 11*a*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*
sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/(d*f*cos(f*x + e) + d*f*sin(f*x
 + e) + d*f), -1/96*(3*(a*c^3 - 9*a*c^2*d - 21*a*c*d^2 - 11*a*d^3 + (a*c^3
- 9*a*c^2*d - 21*a*c*d^2 - 11*a*d^3)*cos(f*x + e) + (a*c^3 - 9*a*c^2*d - 21
*a*c*d^2 - 11*a*d^3)*sin(f*x + e))*sqrt(a/d)*arctan(1/4*(8*d^2*cos(f*x + e)
^2 - c^2 + 6*c*d - 9*d^2 - 8*(c*d - d^2)*sin(f*x + e))*sqrt(a*sin(f*x + e)
+ a)*sqrt(d*sin(f*x + e) + c)*sqrt(a/d)/(2*a*d^2*cos(f*x + e)^3 - (3*a*c*d
- a*d^2)*cos(f*x + e)*sin(f*x + e) - (a*c^2 - a*c*d + 2*a*d^2)*cos(f*x + e)
)) - 4*(8*a*d^2*cos(f*x + e)^3 - 3*a*c^2 - 38*a*c*d - 19*a*d^2 - 14*(a*c*d
+ a*d^2)*cos(f*x + e)^2 - (3*a*c^2 + 52*a*c*d + 41*a*d^2)*cos(f*x + e) - (8
*a*d^2*cos(f*x + e)^2 - 3*a*c^2 - 38*a*c*d - 19*a*d^2 + 2*(7*a*c*d + 11*a*d
^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e
) + c))/(d*f*cos(f*x + e) + d*f*sin(f*x + e) + d*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*(c+d*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError
```

3.573 $\int (a+a\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}\,dx$

Optimal. Leaf size=171

$$\frac{a^{3/2}(c-7d)(c+d)\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{4d^{3/2}f}-\frac{a^2\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{2df\sqrt{a\sin(e+fx)+a}}+\frac{a^2(c-7d)\cos(e+fx)\sqrt{c+}}{4df\sqrt{a\sin(e+fx)}}$$

```
[Out] (a^(3/2)*(c - 7*d)*(c + d)*ArcTan[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[a +
a*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])])/(4*d^(3/2)*f) + (a^2*(c - 7*d)*
Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(4*d*f*Sqrt[a + a*Sin[e + f*x]]) - (
a^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(3/2))/(2*d*f*Sqrt[a + a*Sin[e + f*x]
])
```

Rubi [A] time = 0.310861, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2763, 21, 2770, 2775, 205}

$$\frac{a^{3/2}(c-7d)(c+d)\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{4d^{3/2}f}-\frac{a^2\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{2df\sqrt{a\sin(e+fx)+a}}+\frac{a^2(c-7d)\cos(e+fx)\sqrt{c+}}{4df\sqrt{a\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]],x]
```


```
[Out] (a^(3/2)*(c - 7*d)*(c + d)*ArcTan[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[a +
a*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])])/(4*d^(3/2)*f) + (a^2*(c - 7*d)*
Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(4*d*f*Sqrt[a + a*Sin[e + f*x]]) - (
a^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(3/2))/(2*d*f*Sqrt[a + a*Sin[e + f*x]
])
```


Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
```

```
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}\,dx = -\frac{a^2\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{2df\sqrt{a+a\sin(e+fx)}} + \frac{\int \frac{\left(-\frac{1}{2}a^2(c-7d)-\frac{1}{2}a^2(c-7d)\sin(e+fx)\right)}{\sqrt{a+a\sin(e+fx)}}}{2d}$$

$$= -\frac{a^2\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{2df\sqrt{a+a\sin(e+fx)}} - \frac{(a(c-7d))\int \sqrt{a+a\sin(e+f}}{4d}$$

$$= \frac{a^2(c-7d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4df\sqrt{a+a\sin(e+fx)}} - \frac{a^2\cos(e+fx)(c+d\sin(e}{2df\sqrt{a+a\sin(e+}}$$

$$= \frac{a^2(c-7d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4df\sqrt{a+a\sin(e+fx)}} - \frac{a^2\cos(e+fx)(c+d\sin(e}{2df\sqrt{a+a\sin(e+}}$$

$$= \frac{a^{3/2}(c-7d)(c+d)\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{4d^{3/2}f} + \frac{a^2(c-7d)\cos}{4df\sqrt{}}$$

Mathematica [A] time = 0.619842, size = 247, normalized size = 1.44

$$\frac{(a(\sin(e+fx)+1))^{3/2}\left(\frac{(c-7d)(c+d)\left(-2\tan^{-1}\left(\frac{\sqrt{2}\sqrt{d}\sin\left(\frac{1}{4}(2e+2fx-\pi)\right)}{\sqrt{c+d\sin(e+fx)}}\right)+\log\left(\sqrt{c+d\sin(e+fx)}+\sqrt{2}\sqrt{d}\cos\left(\frac{1}{4}(2e+2fx-\pi)\right)\right)-\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{d}\cos\left(\frac{1}{4}(2e+2\right.}{\sqrt{c+d\sin(e+f}}\right.}{d^{3/2}}\right.}{8f\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^(3/2)*(((c - 7*d)*(c + d)*(-2*ArcTan[(Sqrt[2]*Sqrt[
d]*Sin[(2*e - Pi + 2*f*x)/4])/Sqrt[c + d*Sin[e + f*x]]] - ArcTanh[(Sqrt[2]*
Sqrt[d]*Cos[(2*e - Pi + 2*f*x)/4])/Sqrt[c + d*Sin[e + f*x]]] + Log[Sqrt[2]*
Sqrt[d]*Cos[(2*e - Pi + 2*f*x)/4] + Sqrt[c + d*Sin[e + f*x]]]))/d^(3/2) - (
2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[c + d*Sin[e + f*x]]*(c + 7*d +
 2*d*Sin[e + f*x]))/d))/(8*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3)
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int \left(a + a\sin\left(fx + e\right)\right)^{\frac{3}{2}} \sqrt{c + d\sin\left(fx + e\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^(1/2),x)
```

```
[Out] int((a+a*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx + e\right) + a\right)^{\frac{3}{2}} \sqrt{d\sin\left(fx + e\right) + c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)*sqrt(d*sin(f*x + e) + c), x)
```

Fricas [B] time = 7.05944, size = 2716, normalized size = 15.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] [-1/32*((a*c^2 - 6*a*c*d - 7*a*d^2 + (a*c^2 - 6*a*c*d - 7*a*d^2)*cos(f*x +
e) + (a*c^2 - 6*a*c*d - 7*a*d^2)*sin(f*x + e))*sqrt(-a/d)*log((128*a*d^4*co
s(f*x + e)^5 + a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 + a*d^4 + 128*(2
*a*c*d^3 - a*d^4)*cos(f*x + e)^4 - 32*(5*a*c^2*d^2 - 14*a*c*d^3 + 13*a*d^4)
*cos(f*x + e)^3 - 32*(a*c^3*d - 2*a*c^2*d^2 + 9*a*c*d^3 - 4*a*d^4)*cos(f*x
+ e)^2 - 8*(16*d^4*cos(f*x + e)^4 - c^3*d + 17*c^2*d^2 - 59*c*d^3 + 51*d^4
+ 24*(c*d^3 - d^4)*cos(f*x + e)^3 - 2*(5*c^2*d^2 - 26*c*d^3 + 33*d^4)*cos(f
*x + e)^2 - (c^3*d - 7*c^2*d^2 + 31*c*d^3 - 25*d^4)*cos(f*x + e) + (16*d^4*
cos(f*x + e)^3 + c^3*d - 17*c^2*d^2 + 59*c*d^3 - 51*d^4 - 8*(3*c*d^3 - 5*d^
4)*cos(f*x + e)^2 - 2*(5*c^2*d^2 - 14*c*d^3 + 13*d^4)*cos(f*x + e))*sin(f*x
 + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt(-a/d) + (a*c^
4 - 28*a*c^3*d + 230*a*c^2*d^2 - 476*a*c*d^3 + 289*a*d^4)*cos(f*x + e) + (1
28*a*d^4*cos(f*x + e)^4 + a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 + a*d
^4 - 256*(a*c*d^3 - a*d^4)*cos(f*x + e)^3 - 32*(5*a*c^2*d^2 - 6*a*c*d^3 + 5
*a*d^4)*cos(f*x + e)^2 + 32*(a*c^3*d - 7*a*c^2*d^2 + 15*a*c*d^3 - 9*a*d^4)*
cos(f*x + e))*sin(f*x + e))/(cos(f*x + e) + sin(f*x + e) + 1)) + 8*(2*a*d*c
os(f*x + e)^2 + a*c + 5*a*d + (a*c + 7*a*d)*cos(f*x + e) + (2*a*d*cos(f*x +
 e) - a*c - 5*a*d)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x +
e) + c))/(d*f*cos(f*x + e) + d*f*sin(f*x + e) + d*f), -1/16*((a*c^2 - 6*a*c
*d - 7*a*d^2 + (a*c^2 - 6*a*c*d - 7*a*d^2)*cos(f*x + e) + (a*c^2 - 6*a*c*d
- 7*a*d^2)*sin(f*x + e))*sqrt(a/d)*arctan(1/4*(8*d^2*cos(f*x + e)^2 - c^2 +
 6*c*d - 9*d^2 - 8*(c*d - d^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(
```

```
d*sin(f*x + e) + c)*sqrt(a/d)/(2*a*d^2*cos(f*x + e)^3 - (3*a*c*d - a*d^2)*c
os(f*x + e)*sin(f*x + e) - (a*c^2 - a*c*d + 2*a*d^2)*cos(f*x + e))) + 4*(2*
a*d*cos(f*x + e)^2 + a*c + 5*a*d + (a*c + 7*a*d)*cos(f*x + e) + (2*a*d*cos(
f*x + e) - a*c - 5*a*d)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f
*x + e) + c))/(d*f*cos(f*x + e) + d*f*sin(f*x + e) + d*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] sage2
```

3.574 $\int \frac{(a+a\sin(e+fx))^{3/2}}{\sqrt{c+d\sin(e+fx)}}\,dx$

Optimal. Leaf size=111

$$\frac{a^{3/2}(c-3d)\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{d^{3/2}f}-\frac{a^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{df\sqrt{a\sin(e+fx)+a}}$$

```
[Out] (a^(3/2)*(c - 3*d)*ArcTan[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[a + a*Sin[e
+ f*x]]*Sqrt[c + d*Sin[e + f*x]])])/(d^(3/2)*f) - (a^2*Cos[e + f*x]*Sqrt[c
+ d*Sin[e + f*x]])/(d*f*Sqrt[a + a*Sin[e + f*x]])
```

Rubi [A] time = 0.20945, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2763, 21, 2775, 205}

$$\frac{a^{3/2}(c-3d)\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{d^{3/2}f}-\frac{a^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{df\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)/Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] (a^(3/2)*(c - 3*d)*ArcTan[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[a + a*Sin[e
+ f*x]]*Sqrt[c + d*Sin[e + f*x]])])/(d^(3/2)*f) - (a^2*Cos[e + f*x]*Sqrt[c
+ d*Sin[e + f*x]])/(d*f*Sqrt[a + a*Sin[e + f*x]])
```

Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{3/2}}{\sqrt{c+d\sin(e+fx)}}\,dx = -\frac{a^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{df\sqrt{a+a\sin(e+fx)}} + \frac{\int \frac{-\frac{1}{2}a^2(c-3d)-\frac{1}{2}a^2(c-3d)\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\,dx}{d}$$

$$= -\frac{a^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{df\sqrt{a+a\sin(e+fx)}} - \frac{(a(c-3d))\int \frac{\sqrt{a+a\sin(e+fx)}}{\sqrt{c+d\sin(e+fx)}}\,dx}{2d}$$

$$= -\frac{a^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{df\sqrt{a+a\sin(e+fx)}} + \frac{\left(a^2(c-3d)\right)\text{Subst}\left(\int \frac{1}{a+dx^2}\,dx, x, \frac{a\cos(e+f}{\sqrt{a+a\sin(e+fx)}\sqrt{c+}}\right.}{df}$$

$$= \frac{a^{3/2}(c-3d)\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{d^{3/2}f} - \frac{a^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{df\sqrt{a+a\sin(e+fx)}}$$

Mathematica [B] time = 0.600104, size = 301, normalized size = 2.71

$$\frac{(a(\sin(e+fx)+1))^{3/2}\left(2\sqrt{d}\sin\left(\frac{1}{2}(e+fx)\right)\sqrt{c+d\sin(e+fx)}-2(c-3d)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{d}\sin\left(\frac{1}{4}(2e+2fx-\pi)\right)}{\sqrt{c+d\sin(e+fx)}}\right)-2\sqrt{d}\cos\left(\frac{1}{2}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)/Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^(3/2)*(-2*(c - 3*d)*ArcTan[(Sqrt[2]*Sqrt[d]*Sin[(2*
e - Pi + 2*f*x)/4])/Sqrt[c + d*Sin[e + f*x]]] - (c - 3*d)*ArcTanh[(Sqrt[2]*
Sqrt[d]*Cos[(2*e - Pi + 2*f*x)/4])/Sqrt[c + d*Sin[e + f*x]]] + c*Log[Sqrt[2
]*Sqrt[d]*Cos[(2*e - Pi + 2*f*x)/4] + Sqrt[c + d*Sin[e + f*x]]] - 3*d*Log[S
qrt[2]*Sqrt[d]*Cos[(2*e - Pi + 2*f*x)/4] + Sqrt[c + d*Sin[e + f*x]]] - 2*Sq
rt[d]*Cos[(e + f*x)/2]*Sqrt[c + d*Sin[e + f*x]] + 2*Sqrt[d]*Sin[(e + f*x)/2
]*Sqrt[c + d*Sin[e + f*x]]))/(2*d^(3/2)*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)
/2])^3)
```

Maple [F] time = 0.29, size = 0, normalized size = 0.

$$\int \left(a+a\sin\left(fx+e\right)\right)^{\frac{3}{2}}\frac{1}{\sqrt{c+d\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(1/2),x)
```

```
[Out] int((a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}{\sqrt{d\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)/sqrt(d*sin(f*x + e) + c), x)
```

Fricas [B] time = 6.79816, size = 2390, normalized size = 21.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] [-1/8*((a*c - 3*a*d + (a*c - 3*a*d)*cos(f*x + e) + (a*c - 3*a*d)*sin(f*x +
e))*sqrt(-a/d)*log((128*a*d^4*cos(f*x + e)^5 + a*c^4 + 4*a*c^3*d + 6*a*c^2*
d^2 + 4*a*c*d^3 + a*d^4 + 128*(2*a*c*d^3 - a*d^4)*cos(f*x + e)^4 - 32*(5*a*
c^2*d^2 - 14*a*c*d^3 + 13*a*d^4)*cos(f*x + e)^3 - 32*(a*c^3*d - 2*a*c^2*d^2
 + 9*a*c*d^3 - 4*a*d^4)*cos(f*x + e)^2 - 8*(16*d^4*cos(f*x + e)^4 - c^3*d +
 17*c^2*d^2 - 59*c*d^3 + 51*d^4 + 24*(c*d^3 - d^4)*cos(f*x + e)^3 - 2*(5*c^
2*d^2 - 26*c*d^3 + 33*d^4)*cos(f*x + e)^2 - (c^3*d - 7*c^2*d^2 + 31*c*d^3 -
 25*d^4)*cos(f*x + e) + (16*d^4*cos(f*x + e)^3 + c^3*d - 17*c^2*d^2 + 59*c*
d^3 - 51*d^4 - 8*(3*c*d^3 - 5*d^4)*cos(f*x + e)^2 - 2*(5*c^2*d^2 - 14*c*d^3
 + 13*d^4)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(
f*x + e) + c)*sqrt(-a/d) + (a*c^4 - 28*a*c^3*d + 230*a*c^2*d^2 - 476*a*c*d^
3 + 289*a*d^4)*cos(f*x + e) + (128*a*d^4*cos(f*x + e)^4 + a*c^4 + 4*a*c^3*d
 + 6*a*c^2*d^2 + 4*a*c*d^3 + a*d^4 - 256*(a*c*d^3 - a*d^4)*cos(f*x + e)^3 -
 32*(5*a*c^2*d^2 - 6*a*c*d^3 + 5*a*d^4)*cos(f*x + e)^2 + 32*(a*c^3*d - 7*a*
c^2*d^2 + 15*a*c*d^3 - 9*a*d^4)*cos(f*x + e))*sin(f*x + e))/(cos(f*x + e) +
 sin(f*x + e) + 1)) + 8*(a*cos(f*x + e) - a*sin(f*x + e) + a)*sqrt(a*sin(f*
x + e) + a)*sqrt(d*sin(f*x + e) + c))/(d*f*cos(f*x + e) + d*f*sin(f*x + e)
+ d*f), -1/4*((a*c - 3*a*d + (a*c - 3*a*d)*cos(f*x + e) + (a*c - 3*a*d)*sin
(f*x + e))*sqrt(a/d)*arctan(1/4*(8*d^2*cos(f*x + e)^2 - c^2 + 6*c*d - 9*d^2
 - 8*(c*d - d^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e)
 + c)*sqrt(a/d)/(2*a*d^2*cos(f*x + e)^3 - (3*a*c*d - a*d^2)*cos(f*x + e)*si
n(f*x + e) - (a*c^2 - a*c*d + 2*a*d^2)*cos(f*x + e))) + 4*(a*cos(f*x + e) -
 a*sin(f*x + e) + a)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/(d*
f*cos(f*x + e) + d*f*sin(f*x + e) + d*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}{\sqrt{c+d\sin\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)/(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**(3/2)/sqrt(c + d*sin(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a \sin \left(fx+e\right)+a\right)^{\frac{3}{2}}}{\sqrt{d \sin \left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)/sqrt(d*sin(f*x + e) + c), x)
```

3.575 $\int \frac{(a+a\sin(e+fx))^{3/2}}{(c+d\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=117

$$\frac{2a^2(c-d)\cos(e+fx)}{df(c+d)\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}-\frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{d^{3/2}f}$$

```
[Out] (-2*a^(3/2)*ArcTan[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]
*Sqrt[c + d*Sin[e + f*x]])])/(d^(3/2)*f) + (2*a^2*(c - d)*Cos[e + f*x])/(d*
(c + d)*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])
```

Rubi [A] time = 0.213653, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2762, 21, 2775, 205}

$$\frac{2a^2(c-d)\cos(e+fx)}{df(c+d)\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}-\frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{d^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)/(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-2*a^(3/2)*ArcTan[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]
*Sqrt[c + d*Sin[e + f*x]])])/(d^(3/2)*f) + (2*a^2*(c - d)*Cos[e + f*x])/(d*
(c + d)*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{3/2}}{(c+d\sin(e+fx))^{3/2}}\,dx = \frac{2a^2(c-d)\cos(e+fx)}{d(c+d)f\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}} - \frac{(2a)\int \frac{-\frac{1}{2}a(c+d)-\frac{1}{2}a(c+d)\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\,dx}{d(c+d)}$$

$$= \frac{2a^2(c-d)\cos(e+fx)}{d(c+d)f\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}} + \frac{a\int \frac{\sqrt{a+a\sin(e+fx)}}{\sqrt{c+d\sin(e+fx)}}\,dx}{d}$$

$$= \frac{2a^2(c-d)\cos(e+fx)}{d(c+d)f\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}} - \frac{(2a^2)\,\text{Subst}\left(\int \frac{1}{a+dx^2}\,dx, x, \frac{a\,c}{\sqrt{a+a\sin(e+}}\right.}{df}$$

$$= -\frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{d^{3/2}f} + \frac{2a^2(c-d)\cos(e+fx)}{d(c+d)f\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+}}$$

Mathematica [B] time = 7.27412, size = 377, normalized size = 3.22

$$\frac{(a(\sin(e+fx)+1))^{3/2}\left(-2c\sqrt{d}\sin\left(\frac{1}{2}(e+fx)\right)+2c\sqrt{d}\cos\left(\frac{1}{2}(e+fx)\right)+2(c+d)\sqrt{c+d\sin(e+fx)}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{d}\sin\left(\frac{1}{4}\right.}{\sqrt{c+d\,\text{si}}}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)/(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^(3/2)*(2*c*Sqrt[d]*Cos[(e + f*x)/2] - 2*d^(3/2)*Cos
[(e + f*x)/2] - 2*c*Sqrt[d]*Sin[(e + f*x)/2] + 2*d^(3/2)*Sin[(e + f*x)/2] +
 2*(c + d)*ArcTan[(Sqrt[2]*Sqrt[d]*Sin[(2*e - Pi + 2*f*x)/4])/Sqrt[c + d*Si
n[e + f*x]]]*Sqrt[c + d*Sin[e + f*x]] + (c + d)*ArcTanh[(Sqrt[2]*Sqrt[d]*Co
s[(2*e - Pi + 2*f*x)/4])/Sqrt[c + d*Sin[e + f*x]]]*Sqrt[c + d*Sin[e + f*x]]
 - c*Log[Sqrt[2]*Sqrt[d]*Cos[(2*e - Pi + 2*f*x)/4] + Sqrt[c + d*Sin[e + f*x
]]]*Sqrt[c + d*Sin[e + f*x]] - d*Log[Sqrt[2]*Sqrt[d]*Cos[(2*e - Pi + 2*f*x)
/4] + Sqrt[c + d*Sin[e + f*x]]]*Sqrt[c + d*Sin[e + f*x]]))/(d^(3/2)*(c + d)
*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*Sqrt[c + d*Sin[e + f*x]])
```

Maple [B] time = 0.428, size = 6627, normalized size = 56.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)/(d*sin(f*x + e) + c)^(3/2), x)
```

Fricas [B] time = 7.01481, size = 3043, normalized size = 26.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] [-1/4*((a*c^2 + 2*a*c*d + a*d^2 - (a*c*d + a*d^2)*cos(f*x + e)^2 + (a*c^2 +
 a*c*d)*cos(f*x + e) + (a*c^2 + 2*a*c*d + a*d^2 + (a*c*d + a*d^2)*cos(f*x +
 e))*sin(f*x + e))*sqrt(-a/d)*log((128*a*d^4*cos(f*x + e)^5 + a*c^4 + 4*a*c
^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 + a*d^4 + 128*(2*a*c*d^3 - a*d^4)*cos(f*x +
e)^4 - 32*(5*a*c^2*d^2 - 14*a*c*d^3 + 13*a*d^4)*cos(f*x + e)^3 - 32*(a*c^3*
d - 2*a*c^2*d^2 + 9*a*c*d^3 - 4*a*d^4)*cos(f*x + e)^2 - 8*(16*d^4*cos(f*x +
 e)^4 - c^3*d + 17*c^2*d^2 - 59*c*d^3 + 51*d^4 + 24*(c*d^3 - d^4)*cos(f*x +
 e)^3 - 2*(5*c^2*d^2 - 26*c*d^3 + 33*d^4)*cos(f*x + e)^2 - (c^3*d - 7*c^2*d
^2 + 31*c*d^3 - 25*d^4)*cos(f*x + e) + (16*d^4*cos(f*x + e)^3 + c^3*d - 17*
c^2*d^2 + 59*c*d^3 - 51*d^4 - 8*(3*c*d^3 - 5*d^4)*cos(f*x + e)^2 - 2*(5*c^2
*d^2 - 14*c*d^3 + 13*d^4)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) +
 a)*sqrt(d*sin(f*x + e) + c)*sqrt(-a/d) + (a*c^4 - 28*a*c^3*d + 230*a*c^2*d
^2 - 476*a*c*d^3 + 289*a*d^4)*cos(f*x + e) + (128*a*d^4*cos(f*x + e)^4 + a*
c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 + a*d^4 - 256*(a*c*d^3 - a*d^4)*c
os(f*x + e)^3 - 32*(5*a*c^2*d^2 - 6*a*c*d^3 + 5*a*d^4)*cos(f*x + e)^2 + 32*
(a*c^3*d - 7*a*c^2*d^2 + 15*a*c*d^3 - 9*a*d^4)*cos(f*x + e))*sin(f*x + e))/
(cos(f*x + e) + sin(f*x + e) + 1)) + 8*(a*c - a*d + (a*c - a*d)*cos(f*x + e
) - (a*c - a*d)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e)
+ c))/((c*d^2 + d^3)*f*cos(f*x + e)^2 - (c^2*d + c*d^2)*f*cos(f*x + e) - (c
^2*d + 2*c*d^2 + d^3)*f - ((c*d^2 + d^3)*f*cos(f*x + e) + (c^2*d + 2*c*d^2
+ d^3)*f)*sin(f*x + e)), -1/2*((a*c^2 + 2*a*c*d + a*d^2 - (a*c*d + a*d^2)*c
os(f*x + e)^2 + (a*c^2 + a*c*d)*cos(f*x + e) + (a*c^2 + 2*a*c*d + a*d^2 + (
a*c*d + a*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a/d)*arctan(1/4*(8*d^2*cos(
f*x + e)^2 - c^2 + 6*c*d - 9*d^2 - 8*(c*d - d^2)*sin(f*x + e))*sqrt(a*sin(f
*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt(a/d)/(2*a*d^2*cos(f*x + e)^3 - (
3*a*c*d - a*d^2)*cos(f*x + e)*sin(f*x + e) - (a*c^2 - a*c*d + 2*a*d^2)*cos(
f*x + e))) + 4*(a*c - a*d + (a*c - a*d)*cos(f*x + e) - (a*c - a*d)*sin(f*x
+ e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/((c*d^2 + d^3)*f*c
os(f*x + e)^2 - (c^2*d + c*d^2)*f*cos(f*x + e) - (c^2*d + 2*c*d^2 + d^3)*f
- ((c*d^2 + d^3)*f*cos(f*x + e) + (c^2*d + 2*c*d^2 + d^3)*f)*sin(f*x + e))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}{\left(c+d\sin\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)/(c+d*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**(3/2)/(c + d*sin(e + f*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)/(d*sin(f*x + e) + c)^(3/2), x)
```

3.576 $\int \frac{(a+a\sin(e+fx))^{3/2}}{(c+d\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=115

$$\frac{2a^2(c-d)\cos(e+fx)}{3df(c+d)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^{3/2}}-\frac{2a^2(c+5d)\cos(e+fx)}{3df(c+d)^2\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}$$

```
[Out] (2*a^2*(c - d)*Cos[e + f*x])/(3*d*(c + d)*f*Sqrt[a + a*Sin[e + f*x]]*(c + d
*Sin[e + f*x])^(3/2)) - (2*a^2*(c + 5*d)*Cos[e + f*x])/(3*d*(c + d)^2*f*Sqr
t[a + a*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])
```

Rubi [A] time = 0.214542, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2762, 21, 2771}

$$\frac{2a^2(c-d)\cos(e+fx)}{3df(c+d)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^{3/2}}-\frac{2a^2(c+5d)\cos(e+fx)}{3df(c+d)^2\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)/(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (2*a^2*(c - d)*Cos[e + f*x])/(3*d*(c + d)*f*Sqrt[a + a*Sin[e + f*x]]*(c + d
*Sin[e + f*x])^(3/2)) - (2*a^2*(c + 5*d)*Cos[e + f*x])/(3*d*(c + d)^2*f*Sqr
t[a + a*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{3/2}}{(c+d\sin(e+fx))^{5/2}}\,dx = \frac{2a^2(c-d)\cos(e+fx)}{3d(c+d)f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^{3/2}} - \frac{(2a)\int \frac{-\frac{1}{2}a(c+5d)-\frac{1}{2}a(c+5d)\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^{3/2}}}{3d(c+d)}$$

$$= \frac{2a^2(c-d)\cos(e+fx)}{3d(c+d)f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^{3/2}} + \frac{(a(c+5d))\int \frac{\sqrt{a+a\sin(e+fx)}}{(c+d\sin(e+fx))^{3/2}}dx}{3d(c+d)}$$

$$= \frac{2a^2(c-d)\cos(e+fx)}{3d(c+d)f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^{3/2}} - \frac{2a^2(c+5d)\cos(e+f}{3d(c+d)^2f\sqrt{a+a\sin(e+fx)}\sqrt{c}}$$

Mathematica [A] time = 0.586816, size = 104, normalized size = 0.9

$$-\frac{2a\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)((c+5d)\sin(e+fx)+5c+d)}{3f(c+d)^2\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)(c+d\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)/(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (-2*a*a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(5*c
 + d + (c + 5*d)*Sin[e + f*x]))/(3*(c + d)^2*f*(Cos[(e + f*x)/2] + Sin[(e +
 f*x)/2])*(c + d*Sin[e + f*x])^(3/2))
```

Maple [B] time = 0.208, size = 345, normalized size = 3.

$$-\frac{2\sin(fx+e)(\cos(fx+e))^4cd^2+10\sin(fx+e)(\cos(fx+e))^4d^3-4c^2(\cos(fx+e))^4d-14c(\cos(fx+e))^4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(5/2),x)
```

```
[Out] -2/3/f/(c+d)^2*(a*(1+sin(f*x+e)))^(3/2)*(c+d*sin(f*x+e))^(1/2)*(sin(f*x+e)*
cos(f*x+e)^4*c*d^2+5*sin(f*x+e)*cos(f*x+e)^4*d^3-2*c^2*cos(f*x+e)^4*d-7*c*c
os(f*x+e)^4*d^2-9*cos(f*x+e)^4*d^3-sin(f*x+e)*cos(f*x+e)^2*c^3+sin(f*x+e)*c
os(f*x+e)^2*c^2*d-11*sin(f*x+e)*cos(f*x+e)^2*c*d^2-13*sin(f*x+e)*cos(f*x+e)
^2*d^3-3*c^3*cos(f*x+e)^2-5*c^2*cos(f*x+e)^2*d+15*c*cos(f*x+e)^2*d^2+17*cos
(f*x+e)^2*d^3-8*c^3*sin(f*x+e)-8*c^2*d*sin(f*x+e)+8*sin(f*x+e)*d^2*c+8*d^3*
sin(f*x+e)+8*c^3+8*c^2*d-8*c*d^2-8*d^3)/cos(f*x+e)^3/(cos(f*x+e)^2*d^2+c^2-
d^2)^2
```

Maxima [B] time = 1.7358, size = 414, normalized size = 3.6

$$-\frac{2\left((5c^2+cd)a^{\frac{3}{2}} - \frac{(3c^2-19cd-2d^2)a^{\frac{3}{2}}\sin(fx+e)}{\cos(fx+e)+1} + \frac{2(4c^2-7cd+9d^2)a^{\frac{3}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2} - \frac{2(4c^2-7cd+9d^2)a^{\frac{3}{2}}\sin(fx+e)^3}{(\cos(fx+e)+1)^3} + \frac{(3c^2-19cd-2d^2)a}{(\cos(fx+e)}\right.}{3\left(c^2+2cd+d^2+\frac{(c^2+2cd+d^2)\sin(fx+e)^2}{(\cos(fx+e)+1)^2}\right)\left(c+\frac{2d\sin(fx+e)}{\cos(fx+e)+1}+\frac{c\sin(fx+e)^2}{(\cos(fx+e)+1}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] -2/3*((5*c^2 + c*d)*a^(3/2) - (3*c^2 - 19*c*d - 2*d^2)*a^(3/2)*sin(f*x + e)
/(cos(f*x + e) + 1) + 2*(4*c^2 - 7*c*d + 9*d^2)*a^(3/2)*sin(f*x + e)^2/(cos
(f*x + e) + 1)^2 - 2*(4*c^2 - 7*c*d + 9*d^2)*a^(3/2)*sin(f*x + e)^3/(cos(f*
x + e) + 1)^3 + (3*c^2 - 19*c*d - 2*d^2)*a^(3/2)*sin(f*x + e)^4/(cos(f*x +
e) + 1)^4 - (5*c^2 + c*d)*a^(3/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)*(sin
(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/((c^2 + 2*c*d + d^2 + (c^2 + 2*c*d +
d^2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2)*(c + 2*d*sin(f*x + e)/(cos(f*x +
e) + 1) + c*sin(f*x + e)^2/(cos(f*x + e) + 1)^2)^(5/2)*f)
```

Fricas [B] time = 2.69004, size = 752, normalized size = 6.54

$$\frac{2\left(\left(ac+5\,ad\right)\cos\left(fx+e\right)^{2}+4\,ac-4\,ad+\left(5\,ac+a\right.\right.}{3\left(\left(c^{2}d^{2}+2\,cd^{3}+d^{4}\right)f\cos\left(fx+e\right)^{3}+\left(2\,c^{3}d+5\,c^{2}d^{2}+4\,cd^{3}+d^{4}\right)f\cos\left(fx+e\right)^{2}-\left(c^{4}+2\,c^{3}d+2\,c^{2}d^{2}+2\,cd^{3}+d^{4}\right)f\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] 2/3*((a*c + 5*a*d)*cos(f*x + e)^2 + 4*a*c - 4*a*d + (5*a*c + a*d)*cos(f*x +
 e) - (4*a*c - 4*a*d - (a*c + 5*a*d)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin
(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)/((c^2*d^2 + 2*c*d^3 + d^4)*f*cos(f*
x + e)^3 + (2*c^3*d + 5*c^2*d^2 + 4*c*d^3 + d^4)*f*cos(f*x + e)^2 - (c^4 +
2*c^3*d + 2*c^2*d^2 + 2*c*d^3 + d^4)*f*cos(f*x + e) - (c^4 + 4*c^3*d + 6*c^
2*d^2 + 4*c*d^3 + d^4)*f + ((c^2*d^2 + 2*c*d^3 + d^4)*f*cos(f*x + e)^2 - 2*
(c^3*d + 2*c^2*d^2 + c*d^3)*f*cos(f*x + e) - (c^4 + 4*c^3*d + 6*c^2*d^2 + 4
*c*d^3 + d^4)*f)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)/(c+d*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)/(d*sin(f*x + e) + c)^(5/2), x)
```

3.577 $\int \frac{(a+a\sin(e+fx))^{3/2}}{(c+d\sin(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=172

$$-\frac{4a^2(c+9d)\cos(e+fx)}{15df(c+d)^3\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}-\frac{2a^2(c+9d)\cos(e+fx)}{15df(c+d)^2\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^{3/2}}+\frac{}{5df(c+d)}$$

```
[Out] (2*a^2*(c - d)*Cos[e + f*x])/(5*d*(c + d)*f*Sqrt[a + a*Sin[e + f*x]]*(c + d
*Sin[e + f*x])^(5/2)) - (2*a^2*(c + 9*d)*Cos[e + f*x])/(15*d*(c + d)^2*f*Sq
rt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(3/2)) - (4*a^2*(c + 9*d)*Cos[e
 + f*x])/(15*d*(c + d)^3*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]
])
```

Rubi [A] time = 0.324019, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2762, 21, 2772, 2771}

$$-\frac{4a^2(c+9d)\cos(e+fx)}{15df(c+d)^3\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}-\frac{2a^2(c+9d)\cos(e+fx)}{15df(c+d)^2\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^{3/2}}+\frac{}{5df(c+d)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)/(c + d*Sin[e + f*x])^(7/2),x]
```


```
[Out] (2*a^2*(c - d)*Cos[e + f*x])/(5*d*(c + d)*f*Sqrt[a + a*Sin[e + f*x]]*(c + d
*Sin[e + f*x])^(5/2)) - (2*a^2*(c + 9*d)*Cos[e + f*x])/(15*d*(c + d)^2*f*Sq
rt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(3/2)) - (4*a^2*(c + 9*d)*Cos[e
 + f*x])/(15*d*(c + d)^3*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]
])
```


Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

```
&& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{3/2}}{(c+d\sin(e+fx))^{7/2}}\,dx = \frac{2a^2(c-d)\cos(e+fx)}{5d(c+d)f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^{5/2}} - \frac{(2a)\int \frac{-\frac{1}{2}a(c+9d)-\frac{1}{2}a(c+9d)\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^{5/2}}}{5d(c+d)}$$

$$= \frac{2a^2(c-d)\cos(e+fx)}{5d(c+d)f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^{5/2}} + \frac{(a(c+9d))\int \frac{\sqrt{a+a\sin(e+fx)}}{(c+d\sin(e+fx))^{5/2}}\,dx}{5d(c+d)}$$

$$= \frac{2a^2(c-d)\cos(e+fx)}{5d(c+d)f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^{5/2}} - \frac{2a^2(c+9d)\cos(e+}{15d(c+d)^2f\sqrt{a+a\sin(e+fx)}(c}$$

$$= \frac{2a^2(c-d)\cos(e+fx)}{5d(c+d)f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^{5/2}} - \frac{2a^2(c+9d)\cos(e+}{15d(c+d)^2f\sqrt{a+a\sin(e+fx)}(c}$$

Mathematica [A] time = 0.888506, size = 140, normalized size = 0.81

$$-\frac{2a\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\left(5c^2+46cd+9d^2\right)\sin(e+fx)+25c^2-d(c+9d)\cos(2\right.}{15f(c+d)^3\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)(c+d\sin(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)/(c + d*Sin[e + f*x])^(7/2),x]
```

```
[Out] (-2*a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(25*
c^2 + 13*c*d + 12*d^2 - d*(c + 9*d)*Cos[2*(e + f*x)] + (5*c^2 + 46*c*d + 9*
d^2)*Sin[e + f*x]))/(15*(c + d)^3*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(
c + d*Sin[e + f*x])^(5/2))
```

Maple [B] time = 0.234, size = 625, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(7/2),x)
```

```
[Out] -2/15/f/(c+d)^3*(a*(1+sin(f*x+e)))^(3/2)*(c+d*sin(f*x+e))^(1/2)*(23*c*cos(f
*x+e)^4*d^4+24*d^5+186*cos(f*x+e)^2*c^2*d^3+56*c^4*d-80*c^2*d^3+8*c*d^4+78*
cos(f*x+e)^4*d^5-75*cos(f*x+e)^2*d^5-27*cos(f*x+e)^6*d^5-40*c^5*sin(f*x+e)+
80*c^2*d^3*sin(f*x+e)-8*c*d^4*sin(f*x+e)-cos(f*x+e)^6*c^2*d^3-12*cos(f*x+e)
^6*c*d^4-13*cos(f*x+e)^4*c^4*d-63*cos(f*x+e)^4*c^3*d^2-5*sin(f*x+e)*cos(f*x
```

```
+e)^2*c^5-24*d^5*sin(f*x+e)+18*sin(f*x+e)*cos(f*x+e)^6*d^5-48*c^3*d^2+3*sin
(f*x+e)*cos(f*x+e)^2*c^4*d+2*sin(f*x+e)*cos(f*x+e)^6*c*d^4+9*sin(f*x+e)*cos
(f*x+e)^4*c^3*d^2+57*sin(f*x+e)*cos(f*x+e)^4*c^2*d^3-9*sin(f*x+e)*cos(f*x+e
)^4*c*d^4-90*sin(f*x+e)*cos(f*x+e)^2*c^3*d^2-146*sin(f*x+e)*cos(f*x+e)^2*c^
2*d^3+15*sin(f*x+e)*cos(f*x+e)^2*c*d^4+114*c^3*cos(f*x+e)^2*d^2-19*c*d^4*co
s(f*x+e)^2-57*sin(f*x+e)*cos(f*x+e)^4*d^5+63*sin(f*x+e)*cos(f*x+e)^2*d^5-31
*cos(f*x+e)^2*c^4*d-56*sin(f*x+e)*c^4*d+48*sin(f*x+e)*c^3*d^2-105*cos(f*x+e
)^4*c^2*d^3+40*c^5-15*cos(f*x+e)^2*c^5)/cos(f*x+e)^3/(cos(f*x+e)^2*d^2+c^2-
d^2)^3
```

Maxima [B] time = 2.08734, size = 682, normalized size = 3.97

$$-\frac{2\left(\left(25\,c^3+12\,c^2d+3\,cd^2\right)a^{\frac{3}{2}}-\frac{\left(15\,c^3-130\,c^2d-39\,cd^2-6\,d^3\right)a^{\frac{3}{2}}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{\left(65\,c^3-78\,c^2d+223\,cd^2+30\,d^3\right)a^{\frac{3}{2}}\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}-\frac{5\left(11\,c^3-44\,c^2d+33\,c\cdots\right.}{\left(\cos\left(\cdots\right.\right.}\cdots\right.}{15\left(c^3+3\,c^2d+3\,cd^2+d^3+\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(7/2),x, algorithm="maxim
a")
```

```
[Out] -2/15*((25*c^3 + 12*c^2*d + 3*c*d^2)*a^(3/2) - (15*c^3 - 130*c^2*d - 39*c*d
^2 - 6*d^3)*a^(3/2)*sin(f*x + e)/(cos(f*x + e) + 1) + (65*c^3 - 78*c^2*d +
223*c*d^2 + 30*d^3)*a^(3/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 5*(11*c^3
 - 44*c^2*d + 33*c*d^2 - 24*d^3)*a^(3/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^
3 + 5*(11*c^3 - 44*c^2*d + 33*c*d^2 - 24*d^3)*a^(3/2)*sin(f*x + e)^4/(cos(f
*x + e) + 1)^4 - (65*c^3 - 78*c^2*d + 223*c*d^2 + 30*d^3)*a^(3/2)*sin(f*x +
 e)^5/(cos(f*x + e) + 1)^5 + (15*c^3 - 130*c^2*d - 39*c*d^2 - 6*d^3)*a^(3/2
)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - (25*c^3 + 12*c^2*d + 3*c*d^2)*a^(3/
2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7)*(sin(f*x + e)^2/(cos(f*x + e) + 1)^
2 + 1)^2/((c^3 + 3*c^2*d + 3*c*d^2 + d^3 + 2*(c^3 + 3*c^2*d + 3*c*d^2 + d^3
)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + (c^3 + 3*c^2*d + 3*c*d^2 + d^3)*sin
(f*x + e)^4/(cos(f*x + e) + 1)^4)*(c + 2*d*sin(f*x + e)/(cos(f*x + e) + 1)
+ c*sin(f*x + e)^2/(cos(f*x + e) + 1)^2)^(7/2)*f)
```

Fricas [B] time = 3.32449, size = 1369, normalized size = 7.96

$$\frac{2\left(\cdots\right.}{15\left(\left(c^3d^3+3\,c^2d^4+3\,cd^5+d^6\right)f\cos\left(fx+e\right)^4-3\left(c^4d^2+3\,c^3d^3+3\,c^2d^4+cd^5\right)f\cos\left(fx+e\right)^3-\left(3\,c^5d+12\,c^4d^2+20\,c\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(7/2),x, algorithm="frica
s")
```

```
[Out] 2/15*(2*(a*c*d + 9*a*d^2)*cos(f*x + e)^3 - 20*a*c^2 + 32*a*c*d - 12*a*d^2 -
 (5*a*c^2 + 44*a*c*d - 9*a*d^2)*cos(f*x + e)^2 - (25*a*c^2 + 14*a*c*d + 21*
a*d^2)*cos(f*x + e) + (20*a*c^2 - 32*a*c*d + 12*a*d^2 - 2*(a*c*d + 9*a*d^2)
*cos(f*x + e)^2 - (5*a*c^2 + 46*a*c*d + 9*a*d^2)*cos(f*x + e))*sin(f*x + e)
)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)/((c^3*d^3 + 3*c^2*d^4 +
 3*c*d^5 + d^6)*f*cos(f*x + e)^4 - 3*(c^4*d^2 + 3*c^3*d^3 + 3*c^2*d^4 + c*d
```

```
^5)*f*cos(f*x + e)^3 - (3*c^5*d + 12*c^4*d^2 + 20*c^3*d^3 + 18*c^2*d^4 + 9*
c*d^5 + 2*d^6)*f*cos(f*x + e)^2 + (c^6 + 3*c^5*d + 6*c^4*d^2 + 10*c^3*d^3 +
 9*c^2*d^4 + 3*c*d^5)*f*cos(f*x + e) + (c^6 + 6*c^5*d + 15*c^4*d^2 + 20*c^3
*d^3 + 15*c^2*d^4 + 6*c*d^5 + d^6)*f - ((c^3*d^3 + 3*c^2*d^4 + 3*c*d^5 + d^
6)*f*cos(f*x + e)^3 + (3*c^4*d^2 + 10*c^3*d^3 + 12*c^2*d^4 + 6*c*d^5 + d^6)
*f*cos(f*x + e)^2 - (3*c^5*d + 9*c^4*d^2 + 10*c^3*d^3 + 6*c^2*d^4 + 3*c*d^5
 + d^6)*f*cos(f*x + e) - (c^6 + 6*c^5*d + 15*c^4*d^2 + 20*c^3*d^3 + 15*c^2*
d^4 + 6*c*d^5 + d^6)*f)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)/(c+d*sin(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(7/2),x, algorithm="giac"
)
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)/(d*sin(f*x + e) + c)^(7/2), x)
```

3.578 $\int \frac{(a+a\sin(e+fx))^{3/2}}{(c+d\sin(e+fx))^{9/2}}\,dx$

Optimal. Leaf size=229

$$-\frac{16a^2(c+13d)\cos(e+fx)}{105df(c+d)^4\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}-\frac{8a^2(c+13d)\cos(e+fx)}{105df(c+d)^3\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^{3/2}}-\frac{}{35df(c}$$

```
[Out] (2*a^2*(c - d)*Cos[e + f*x])/(7*d*(c + d)*f*Sqrt[a + a*Sin[e + f*x]]*(c + d
*Sin[e + f*x])^(7/2)) - (2*a^2*(c + 13*d)*Cos[e + f*x])/(35*d*(c + d)^2*f*S
qrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(5/2)) - (8*a^2*(c + 13*d)*Cos
[e + f*x])/(105*d*(c + d)^3*f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])
^(3/2)) - (16*a^2*(c + 13*d)*Cos[e + f*x])/(105*d*(c + d)^4*f*Sqrt[a + a*Si
n[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])
```

Rubi [A] time = 0.447942, antiderivative size = 229, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2762, 21, 2772, 2771}

$$-\frac{16a^2(c+13d)\cos(e+fx)}{105df(c+d)^4\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}-\frac{8a^2(c+13d)\cos(e+fx)}{105df(c+d)^3\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^{3/2}}-\frac{}{35df(c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)/(c + d*Sin[e + f*x])^(9/2),x]
```

```
[Out] (2*a^2*(c - d)*Cos[e + f*x])/(7*d*(c + d)*f*Sqrt[a + a*Sin[e + f*x]]*(c + d
*Sin[e + f*x])^(7/2)) - (2*a^2*(c + 13*d)*Cos[e + f*x])/(35*d*(c + d)^2*f*S
qrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(5/2)) - (8*a^2*(c + 13*d)*Cos
[e + f*x])/(105*d*(c + d)^3*f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])
^(3/2)) - (16*a^2*(c + 13*d)*Cos[e + f*x])/(105*d*(c + d)^4*f*Sqrt[a + a*Si
n[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
```

```
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{3/2}}{(c+d\sin(e+fx))^{9/2}}\,dx = \frac{2a^2(c-d)\cos(e+fx)}{7d(c+d)f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^{7/2}} - \frac{(2a)\int \frac{-\frac{1}{2}a(c+13d)-\frac{1}{2}a(c+13d)\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^{7/2}}}{7d(c+d)}$$

$$= \frac{2a^2(c-d)\cos(e+fx)}{7d(c+d)f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^{7/2}} + \frac{(a(c+13d))\int \frac{\sqrt{a+a\sin(e+fx)}}{(c+d\sin(e+fx))^{7/2}}\,dx}{7d(c+d)}$$

$$= \frac{2a^2(c-d)\cos(e+fx)}{7d(c+d)f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^{7/2}} - \frac{2a^2(c+13d)\cos(e+}{35d(c+d)^2f\sqrt{a+a\sin(e+fx)}(c}$$

$$= \frac{2a^2(c-d)\cos(e+fx)}{7d(c+d)f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^{7/2}} - \frac{2a^2(c+13d)\cos(e+}{35d(c+d)^2f\sqrt{a+a\sin(e+fx)}(c}$$

$$= \frac{2a^2(c-d)\cos(e+fx)}{7d(c+d)f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^{7/2}} - \frac{2a^2(c+13d)\cos(e+}{35d(c+d)^2f\sqrt{a+a\sin(e+fx)}(c}$$

Mathematica [A] time = 1.53433, size = 193, normalized size = 0.84

$$-\frac{2a\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\left(469c^2d+35c^3+191cd^2+117d^3\right)\sin(e+fx)-2d\left(7c^2\right.\right.}{105f(c+d)^4\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)/(c + d*Sin[e + f*x])^(9/2),x]
```

```
[Out] (-2*a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(175
*c^3 + 147*c^2*d + 253*c*d^2 + 41*d^3 - 2*d*(7*c^2 + 92*c*d + 13*d^2)*Cos[2
*(e + f*x)] + (35*c^3 + 469*c^2*d + 191*c*d^2 + 117*d^3)*Sin[e + f*x] - 2*c
*d^2*Sin[3*(e + f*x)] - 26*d^3*Sin[3*(e + f*x)]))/(105*(c + d)^4*f*(Cos[(e
+ f*x)/2] + Sin[(e + f*x)/2])*(c + d*Sin[e + f*x])^(7/2))
```

Maple [B] time = 0.319, size = 979, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(9/2),x)
```

```
[Out] -2/105/f/(c+d)^4*(c+d*sin(f*x+e))^(1/2)*(a*(1+sin(f*x+e)))^(3/2)*(-64*cos(f
*x+e)^8*c*d^6+104*sin(f*x+e)*cos(f*x+e)^8*d^7+8*sin(f*x+e)*cos(f*x+e)^8*c*d
^6+29*sin(f*x+e)*cos(f*x+e)^6*c^3*d^4+371*sin(f*x+e)*cos(f*x+e)^6*c^2*d^5-1
13*sin(f*x+e)*cos(f*x+e)^6*c*d^6+106*sin(f*x+e)*cos(f*x+e)^4*c^5*d^2+754*si
n(f*x+e)*cos(f*x+e)^4*c^4*d^3+72*sin(f*x+e)*cos(f*x+e)^4*c^3*d^4-944*sin(f*
x+e)*cos(f*x+e)^4*c^2*d^5+382*sin(f*x+e)*cos(f*x+e)^4*c*d^6+7*sin(f*x+e)*co
s(f*x+e)^2*c^6*d-887*sin(f*x+e)*cos(f*x+e)^2*c^5*d^2-1797*sin(f*x+e)*cos(f*
x+e)^2*c^4*d^3-25*sin(f*x+e)*cos(f*x+e)^2*c^3*d^4+997*sin(f*x+e)*cos(f*x+e)
^2*c^2*d^5-397*sin(f*x+e)*cos(f*x+e)^2*c*d^6+280*c^7+504*c^6*d+424*c^2*d^5-
120*c*d^6-776*c^4*d^3-152*d^7-296*c^5*d^2+136*c^3*d^4-455*sin(f*x+e)*cos(f*
x+e)^6*d^7+4*cos(f*x+e)^6*c^4*d^3-149*cos(f*x+e)^6*c^3*d^4-443*cos(f*x+e)^6
*c^2*d^5+345*cos(f*x+e)^6*c*d^6+750*sin(f*x+e)*cos(f*x+e)^4*d^7-112*cos(f*x
+e)^4*c^6*d-670*cos(f*x+e)^4*c^5*d^2-1398*cos(f*x+e)^4*c^4*d^3-4*cos(f*x+e)
^8*c^2*d^5+56*cos(f*x+e)^4*c^3*d^4+1232*cos(f*x+e)^4*c^2*d^5-618*cos(f*x+e)
^4*c*d^6-35*sin(f*x+e)*cos(f*x+e)^2*c^7-551*sin(f*x+e)*cos(f*x+e)^2*d^7-259
*cos(f*x+e)^2*c^6*d+1035*cos(f*x+e)^2*c^5*d^2+2185*cos(f*x+e)^2*c^4*d^3-43*
cos(f*x+e)^2*c^3*d^4-1209*cos(f*x+e)^2*c^2*d^5+457*cos(f*x+e)^2*c*d^6-504*s
in(f*x+e)*c^6*d+296*sin(f*x+e)*c^5*d^2+776*sin(f*x+e)*c^4*d^3-136*sin(f*x+e
)*c^3*d^4-424*sin(f*x+e)*c^2*d^5+120*sin(f*x+e)*c*d^6-156*cos(f*x+e)^8*d^7+
635*cos(f*x+e)^6*d^7-954*cos(f*x+e)^4*d^7-105*cos(f*x+e)^2*c^7+627*cos(f*x+
e)^2*d^7-280*sin(f*x+e)*c^7+152*sin(f*x+e)*d^7)/cos(f*x+e)^3/(cos(f*x+e)^2*
d^2+c^2-d^2)^4
```

Maxima [B] time = 2.41802, size = 1013, normalized size = 4.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(9/2),x, algorithm="maxim
a")
```

```
[Out] -2/105*((175*c^4 + 133*c^3*d + 69*c^2*d^2 + 15*c*d^3)*a^(3/2) - 3*(35*c^4 -
 385*c^3*d - 189*c^2*d^2 - 67*c*d^3 - 10*d^4)*a^(3/2)*sin(f*x + e)/(cos(f*x
 + e) + 1) + 18*(35*c^4 - 28*c^3*d + 166*c^2*d^2 + 44*c*d^3 + 7*d^4)*a^(3/2
)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 14*(35*c^4 - 220*c^3*d + 102*c^2*d^
2 - 244*c*d^3 - 25*d^4)*a^(3/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 42*(2
0*c^4 - 61*c^3*d + 117*c^2*d^2 - 55*c*d^3 + 35*d^4)*a^(3/2)*sin(f*x + e)^4/
(cos(f*x + e) + 1)^4 - 42*(20*c^4 - 61*c^3*d + 117*c^2*d^2 - 55*c*d^3 + 35*
d^4)*a^(3/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 14*(35*c^4 - 220*c^3*d +
 102*c^2*d^2 - 244*c*d^3 - 25*d^4)*a^(3/2)*sin(f*x + e)^6/(cos(f*x + e) + 1
)^6 - 18*(35*c^4 - 28*c^3*d + 166*c^2*d^2 + 44*c*d^3 + 7*d^4)*a^(3/2)*sin(f
*x + e)^7/(cos(f*x + e) + 1)^7 + 3*(35*c^4 - 385*c^3*d - 189*c^2*d^2 - 67*c
*d^3 - 10*d^4)*a^(3/2)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - (175*c^4 + 133
*c^3*d + 69*c^2*d^2 + 15*c*d^3)*a^(3/2)*sin(f*x + e)^9/(cos(f*x + e) + 1)^9
)*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^3/((c^4 + 4*c^3*d + 6*c^2*d^2 +
 4*c*d^3 + d^4 + 3*(c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^3 + d^4)*sin(f*x + e)
^2/(cos(f*x + e) + 1)^2 + 3*(c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^3 + d^4)*sin
(f*x + e)^4/(cos(f*x + e) + 1)^4 + (c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^3 + d
^4)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6)*(c + 2*d*sin(f*x + e)/(cos(f*x + e
) + 1) + c*sin(f*x + e)^2/(cos(f*x + e) + 1)^2)^(9/2)*f)
```

Fricas [B] time = 3.65345, size = 2163, normalized size = 9.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(9/2),x, algorithm="frica
s")
```

```
[Out] 2/105*(8*(a*c*d^2 + 13*a*d^3)*cos(f*x + e)^4 - 140*a*c^3 + 308*a*c^2*d - 24
4*a*c*d^2 + 76*a*d^3 + 4*(7*a*c^2*d + 92*a*c*d^2 + 13*a*d^3)*cos(f*x + e)^3
 - (35*a*c^3 + 441*a*c^2*d - 167*a*c*d^2 + 195*a*d^3)*cos(f*x + e)^2 - (175
*a*c^3 + 161*a*c^2*d + 437*a*c*d^2 + 67*a*d^3)*cos(f*x + e) + (140*a*c^3 -
308*a*c^2*d + 244*a*c*d^2 - 76*a*d^3 + 8*(a*c*d^2 + 13*a*d^3)*cos(f*x + e)^
3 - 4*(7*a*c^2*d + 90*a*c*d^2 - 13*a*d^3)*cos(f*x + e)^2 - (35*a*c^3 + 469*
a*c^2*d + 193*a*c*d^2 + 143*a*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f
*x + e) + a)*sqrt(d*sin(f*x + e) + c)/((c^4*d^4 + 4*c^3*d^5 + 6*c^2*d^6 + 4
*c*d^7 + d^8)*f*cos(f*x + e)^5 + (4*c^5*d^3 + 17*c^4*d^4 + 28*c^3*d^5 + 22*
c^2*d^6 + 8*c*d^7 + d^8)*f*cos(f*x + e)^4 - 2*(3*c^6*d^2 + 12*c^5*d^3 + 19*
c^4*d^4 + 16*c^3*d^5 + 9*c^2*d^6 + 4*c*d^7 + d^8)*f*cos(f*x + e)^3 - 2*(2*c
^7*d + 11*c^6*d^2 + 28*c^5*d^3 + 43*c^4*d^4 + 42*c^3*d^5 + 25*c^2*d^6 + 8*c
*d^7 + d^8)*f*cos(f*x + e)^2 + (c^8 + 4*c^7*d + 12*c^6*d^2 + 28*c^5*d^3 + 3
8*c^4*d^4 + 28*c^3*d^5 + 12*c^2*d^6 + 4*c*d^7 + d^8)*f*cos(f*x + e) + (c^8
+ 8*c^7*d + 28*c^6*d^2 + 56*c^5*d^3 + 70*c^4*d^4 + 56*c^3*d^5 + 28*c^2*d^6
+ 8*c*d^7 + d^8)*f + ((c^4*d^4 + 4*c^3*d^5 + 6*c^2*d^6 + 4*c*d^7 + d^8)*f*c
os(f*x + e)^4 - 4*(c^5*d^3 + 4*c^4*d^4 + 6*c^3*d^5 + 4*c^2*d^6 + c*d^7)*f*c
os(f*x + e)^3 - 2*(3*c^6*d^2 + 14*c^5*d^3 + 27*c^4*d^4 + 28*c^3*d^5 + 17*c^
2*d^6 + 6*c*d^7 + d^8)*f*cos(f*x + e)^2 + 4*(c^7*d + 4*c^6*d^2 + 7*c^5*d^3
+ 8*c^4*d^4 + 7*c^3*d^5 + 4*c^2*d^6 + c*d^7)*f*cos(f*x + e) + (c^8 + 8*c^7*
d + 28*c^6*d^2 + 56*c^5*d^3 + 70*c^4*d^4 + 56*c^3*d^5 + 28*c^2*d^6 + 8*c*d^
7 + d^8)*f)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)/(c+d*sin(f*x+e))**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(9/2),x, algorithm="giac"
)
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)/(d*sin(f*x + e) + c)^(9/2), x)
```

3.579 $\int (a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^{5/2}\,dx$

Optimal. Leaf size=377

$$-\frac{a^3\left(3c^2-34cd+283d^2\right)\cos(e+fx)(c+d\sin(e+fx))^{5/2}}{240d^2f\sqrt{a\sin(e+fx)+a}}-\frac{a^3(c+d)\left(3c^2-34cd+283d^2\right)\cos(e+fx)(c+d\sin(e+fx$$

```
[Out] -(a^(5/2)*(c + d)^3*(3*c^2 - 34*c*d + 283*d^2)*ArcTan[(Sqrt[a]*Sqrt[d]*Cos[
e + f*x])/(Sqrt[a + a*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])])/(128*d^(5/2
)*f) - (a^3*(c + d)^2*(3*c^2 - 34*c*d + 283*d^2)*Cos[e + f*x]*Sqrt[c + d*Si
n[e + f*x]])/(128*d^2*f*Sqrt[a + a*Sin[e + f*x]]) - (a^3*(c + d)*(3*c^2 - 3
4*c*d + 283*d^2)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(3/2))/(192*d^2*f*Sqrt[a
 + a*Sin[e + f*x]]) - (a^3*(3*c^2 - 34*c*d + 283*d^2)*Cos[e + f*x]*(c + d*S
in[e + f*x])^(5/2))/(240*d^2*f*Sqrt[a + a*Sin[e + f*x]]) + (3*a^3*(c - 7*d)
*Cos[e + f*x]*(c + d*Sin[e + f*x])^(7/2))/(40*d^2*f*Sqrt[a + a*Sin[e + f*x]
]) - (a^2*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(7/2))
/(5*d*f)
```

Rubi [A] time = 0.854936, antiderivative size = 377, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2763, 2981, 2770, 2775, 205}

$$-\frac{a^3\left(3c^2-34cd+283d^2\right)\cos(e+fx)(c+d\sin(e+fx))^{5/2}}{240d^2f\sqrt{a\sin(e+fx)+a}}-\frac{a^3(c+d)\left(3c^2-34cd+283d^2\right)\cos(e+fx)(c+d\sin(e+fx$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] -(a^(5/2)*(c + d)^3*(3*c^2 - 34*c*d + 283*d^2)*ArcTan[(Sqrt[a]*Sqrt[d]*Cos[
e + f*x])/(Sqrt[a + a*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])])/(128*d^(5/2
)*f) - (a^3*(c + d)^2*(3*c^2 - 34*c*d + 283*d^2)*Cos[e + f*x]*Sqrt[c + d*Si
n[e + f*x]])/(128*d^2*f*Sqrt[a + a*Sin[e + f*x]]) - (a^3*(c + d)*(3*c^2 - 3
4*c*d + 283*d^2)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(3/2))/(192*d^2*f*Sqrt[a
 + a*Sin[e + f*x]]) - (a^3*(3*c^2 - 34*c*d + 283*d^2)*Cos[e + f*x]*(c + d*S
in[e + f*x])^(5/2))/(240*d^2*f*Sqrt[a + a*Sin[e + f*x]]) + (3*a^3*(c - 7*d)
*Cos[e + f*x]*(c + d*Sin[e + f*x])^(7/2))/(40*d^2*f*Sqrt[a + a*Sin[e + f*x]
]) - (a^2*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(7/2))
/(5*d*f)
```

Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^{5/2}\,dx = -\frac{a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^{7/2}}{5df} + \frac{\int \sqrt{a+a \cdots}}{\cdots}$$

$$= \frac{3a^3(c-7d)\cos(e+fx)(c+d\sin(e+fx))^{7/2}}{40d^2f\sqrt{a+a\sin(e+fx)}} - \frac{a^2\cos(e+fx)\sqrt{a+\cdots}}{\cdots}$$

$$= -\frac{a^3\left(3c^2-34cd+283d^2\right)\cos(e+fx)(c+d\sin(e+fx))^{5/2}}{240d^2f\sqrt{a+a\sin(e+fx)}} + \frac{3a^3(c\cdots}{\cdots}$$

$$= -\frac{a^3(c+d)\left(3c^2-34cd+283d^2\right)\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{192d^2f\sqrt{a+a\sin(e+fx)}} - \cdots$$

$$= -\frac{a^3(c+d)^2\left(3c^2-34cd+283d^2\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{128d^2f\sqrt{a+a\sin(e+fx)}} - \cdots$$

$$= -\frac{a^3(c+d)^2\left(3c^2-34cd+283d^2\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{128d^2f\sqrt{a+a\sin(e+fx)}} - \cdots$$

$$= -\frac{a^{5/2}(c+d)^3\left(3c^2-34cd+283d^2\right)\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\cdots\right.}{128d^{5/2}f}$$

Mathematica [A] time = 2.96373, size = 395, normalized size = 1.05

$$(a(\sin(e+fx)+1))^{5/2}\left(\frac{2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{c+d\sin(e+fx)}\left(-3322c^2d^2\sin(e+fx)+4d^2\left(93c^2+488cd+331d^2\right)\cos(2(e+fx))-8396c^2d^2-30c^3d\right.}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^(5/2)*(((c + d)^3*(3*c^2 - 34*c*d + 283*d^2)*(2*Arc
Tan[(Sqrt[2]*Sqrt[d]*Sin[(2*e - Pi + 2*f*x)/4])/Sqrt[c + d*Sin[e + f*x]]] +
 ArcTanh[(Sqrt[2]*Sqrt[d]*Cos[(2*e - Pi + 2*f*x)/4])/Sqrt[c + d*Sin[e + f*x
]]] - Log[Sqrt[2]*Sqrt[d]*Cos[(2*e - Pi + 2*f*x)/4] + Sqrt[c + d*Sin[e + f*
x]]]))/d^(5/2) + (2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[c + d*Sin[e
+ f*x]]*(45*c^4 - 390*c^3*d - 8396*c^2*d^2 - 12762*c*d^3 - 5521*d^4 + 4*d^2
*(93*c^2 + 488*c*d + 331*d^2)*Cos[2*(e + f*x)] - 48*d^4*Cos[4*(e + f*x)] -
30*c^3*d*Sin[e + f*x] - 3322*c^2*d^2*Sin[e + f*x] - 7774*c*d^3*Sin[e + f*x]
 - 3874*d^4*Sin[e + f*x] + 252*c*d^3*Sin[3*(e + f*x)] + 348*d^4*Sin[3*(e +
f*x)]))/(15*d^2)))/(256*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int \left(a+a\sin\left(fx+e\right)\right)^{\frac{5}{2}}\left(c+d\sin\left(fx+e\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^(5/2),x)
```

```
[Out] int((a+a*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)*(d*sin(f*x + e) + c)^(5/2), x)
```

Fricas [B] time = 21.0004, size = 4905, normalized size = 13.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] [1/15360*(15*(3*a^2*c^5 - 25*a^2*c^4*d + 190*a^2*c^3*d^2 + 750*a^2*c^2*d^3
+ 815*a^2*c*d^4 + 283*a^2*d^5 + (3*a^2*c^5 - 25*a^2*c^4*d + 190*a^2*c^3*d^2
 + 750*a^2*c^2*d^3 + 815*a^2*c*d^4 + 283*a^2*d^5)*cos(f*x + e) + (3*a^2*c^5
 - 25*a^2*c^4*d + 190*a^2*c^3*d^2 + 750*a^2*c^2*d^3 + 815*a^2*c*d^4 + 283*a
^2*d^5)*sin(f*x + e))*sqrt(-a/d)*log((128*a*d^4*cos(f*x + e)^5 + a*c^4 + 4*
a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 + a*d^4 + 128*(2*a*c*d^3 - a*d^4)*cos(f*x
 + e)^4 - 32*(5*a*c^2*d^2 - 14*a*c*d^3 + 13*a*d^4)*cos(f*x + e)^3 - 32*(a*c
^3*d - 2*a*c^2*d^2 + 9*a*c*d^3 - 4*a*d^4)*cos(f*x + e)^2 - 8*(16*d^4*cos(f*
x + e)^4 - c^3*d + 17*c^2*d^2 - 59*c*d^3 + 51*d^4 + 24*(c*d^3 - d^4)*cos(f*
x + e)^3 - 2*(5*c^2*d^2 - 26*c*d^3 + 33*d^4)*cos(f*x + e)^2 - (c^3*d - 7*c^
2*d^2 + 31*c*d^3 - 25*d^4)*cos(f*x + e) + (16*d^4*cos(f*x + e)^3 + c^3*d -
17*c^2*d^2 + 59*c*d^3 - 51*d^4 - 8*(3*c*d^3 - 5*d^4)*cos(f*x + e)^2 - 2*(5*
c^2*d^2 - 14*c*d^3 + 13*d^4)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e
) + a)*sqrt(d*sin(f*x + e) + c)*sqrt(-a/d) + (a*c^4 - 28*a*c^3*d + 230*a*c^
2*d^2 - 476*a*c*d^3 + 289*a*d^4)*cos(f*x + e) + (128*a*d^4*cos(f*x + e)^4 +
 a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 + a*d^4 - 256*(a*c*d^3 - a*d^4
)*cos(f*x + e)^3 - 32*(5*a*c^2*d^2 - 6*a*c*d^3 + 5*a*d^4)*cos(f*x + e)^2 +
32*(a*c^3*d - 7*a*c^2*d^2 + 15*a*c*d^3 - 9*a*d^4)*cos(f*x + e))*sin(f*x + e
))/(cos(f*x + e) + sin(f*x + e) + 1)) - 8*(384*a^2*d^4*cos(f*x + e)^5 - 45*
a^2*c^4 + 360*a^2*c^3*d + 5446*a^2*c^2*d^2 + 6688*a^2*c*d^3 + 2671*a^2*d^4
- 1008*(a^2*c*d^3 + a^2*d^4)*cos(f*x + e)^4 - 8*(93*a^2*c^2*d^2 + 488*a^2*c
*d^3 + 379*a^2*d^4)*cos(f*x + e)^3 + 2*(15*a^2*c^3*d + 1289*a^2*c^2*d^2 + 2
565*a^2*c*d^3 + 1291*a^2*d^4)*cos(f*x + e)^2 - (45*a^2*c^4 - 390*a^2*c^3*d
- 8768*a^2*c^2*d^2 - 14714*a^2*c*d^3 - 6893*a^2*d^4)*cos(f*x + e) - (384*a^
2*d^4*cos(f*x + e)^4 - 45*a^2*c^4 + 360*a^2*c^3*d + 5446*a^2*c^2*d^2 + 6688
*a^2*c*d^3 + 2671*a^2*d^4 + 48*(21*a^2*c*d^3 + 29*a^2*d^4)*cos(f*x + e)^3 -
 8*(93*a^2*c^2*d^2 + 362*a^2*c*d^3 + 205*a^2*d^4)*cos(f*x + e)^2 - 2*(15*a^
2*c^3*d + 1661*a^2*c^2*d^2 + 4013*a^2*c*d^3 + 2111*a^2*d^4)*cos(f*x + e))*s
in(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/(d^2*f*cos(
f*x + e) + d^2*f*sin(f*x + e) + d^2*f), 1/7680*(15*(3*a^2*c^5 - 25*a^2*c^4*
d + 190*a^2*c^3*d^2 + 750*a^2*c^2*d^3 + 815*a^2*c*d^4 + 283*a^2*d^5 + (3*a^
2*c^5 - 25*a^2*c^4*d + 190*a^2*c^3*d^2 + 750*a^2*c^2*d^3 + 815*a^2*c*d^4 +
283*a^2*d^5)*cos(f*x + e) + (3*a^2*c^5 - 25*a^2*c^4*d + 190*a^2*c^3*d^2 + 7
50*a^2*c^2*d^3 + 815*a^2*c*d^4 + 283*a^2*d^5)*sin(f*x + e))*sqrt(a/d)*arcta
n(1/4*(8*d^2*cos(f*x + e)^2 - c^2 + 6*c*d - 9*d^2 - 8*(c*d - d^2)*sin(f*x +
 e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt(a/d)/(2*a*d^2*c
os(f*x + e)^3 - (3*a*c*d - a*d^2)*cos(f*x + e)*sin(f*x + e) - (a*c^2 - a*c*
d + 2*a*d^2)*cos(f*x + e))) - 4*(384*a^2*d^4*cos(f*x + e)^5 - 45*a^2*c^4 +
360*a^2*c^3*d + 5446*a^2*c^2*d^2 + 6688*a^2*c*d^3 + 2671*a^2*d^4 - 1008*(a^
2*c*d^3 + a^2*d^4)*cos(f*x + e)^4 - 8*(93*a^2*c^2*d^2 + 488*a^2*c*d^3 + 379
*a^2*d^4)*cos(f*x + e)^3 + 2*(15*a^2*c^3*d + 1289*a^2*c^2*d^2 + 2565*a^2*c*
d^3 + 1291*a^2*d^4)*cos(f*x + e)^2 - (45*a^2*c^4 - 390*a^2*c^3*d - 8768*a^2
*c^2*d^2 - 14714*a^2*c*d^3 - 6893*a^2*d^4)*cos(f*x + e) - (384*a^2*d^4*cos(
f*x + e)^4 - 45*a^2*c^4 + 360*a^2*c^3*d + 5446*a^2*c^2*d^2 + 6688*a^2*c*d^3
 + 2671*a^2*d^4 + 48*(21*a^2*c*d^3 + 29*a^2*d^4)*cos(f*x + e)^3 - 8*(93*a^2
*c^2*d^2 + 362*a^2*c*d^3 + 205*a^2*d^4)*cos(f*x + e)^2 - 2*(15*a^2*c^3*d +
1661*a^2*c^2*d^2 + 4013*a^2*c*d^3 + 2111*a^2*d^4)*cos(f*x + e))*sin(f*x + e
))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/(d^2*f*cos(f*x + e) +
 d^2*f*sin(f*x + e) + d^2*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*(c+d*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: AttributeError
```

3.580 $\int (a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=312

$$-\frac{a^3\left(3c^2-26cd+163d^2\right)\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{96d^2f\sqrt{a\sin(e+fx)+a}}-\frac{a^3(c+d)\left(3c^2-26cd+163d^2\right)\cos(e+fx)\sqrt{c+d\sin(e\cdots}}{64d^2f\sqrt{a\sin(e+fx)+a}}$$

```
[Out] -(a^(5/2)*(c + d)^2*(3*c^2 - 26*c*d + 163*d^2)*ArcTan[(Sqrt[a]*Sqrt[d]*Cos[
e + f*x])/(Sqrt[a + a*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])])/(64*d^(5/2)
*f) - (a^3*(c + d)*(3*c^2 - 26*c*d + 163*d^2)*Cos[e + f*x]*Sqrt[c + d*Sin[e
 + f*x]])/(64*d^2*f*Sqrt[a + a*Sin[e + f*x]]) - (a^3*(3*c^2 - 26*c*d + 163*
d^2)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(3/2))/(96*d^2*f*Sqrt[a + a*Sin[e +
f*x]]) + (a^3*(3*c - 17*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(5/2))/(24*d^2
*f*Sqrt[a + a*Sin[e + f*x]]) - (a^2*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(5/2))/(4*d*f)
```

Rubi [A] time = 0.726103, antiderivative size = 312, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2763, 2981, 2770, 2775, 205}

$$-\frac{a^3\left(3c^2-26cd+163d^2\right)\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{96d^2f\sqrt{a\sin(e+fx)+a}}-\frac{a^3(c+d)\left(3c^2-26cd+163d^2\right)\cos(e+fx)\sqrt{c+d\sin(e\cdots}}{64d^2f\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] -(a^(5/2)*(c + d)^2*(3*c^2 - 26*c*d + 163*d^2)*ArcTan[(Sqrt[a]*Sqrt[d]*Cos[
e + f*x])/(Sqrt[a + a*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])])/(64*d^(5/2)
*f) - (a^3*(c + d)*(3*c^2 - 26*c*d + 163*d^2)*Cos[e + f*x]*Sqrt[c + d*Sin[e
 + f*x]])/(64*d^2*f*Sqrt[a + a*Sin[e + f*x]]) - (a^3*(3*c^2 - 26*c*d + 163*
d^2)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(3/2))/(96*d^2*f*Sqrt[a + a*Sin[e +
f*x]]) + (a^3*(3*c - 17*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(5/2))/(24*d^2
*f*Sqrt[a + a*Sin[e + f*x]]) - (a^2*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(5/2))/(4*d*f)
```

Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
```

```
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^{3/2}\,dx = -\frac{a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^{5/2}}{4df}+\frac{\int \sqrt{a+a\,\mathrm{si}}}{}$$

$$= \frac{a^3(3c-17d)\cos(e+fx)(c+d\sin(e+fx))^{5/2}}{24d^2f\sqrt{a+a\sin(e+fx)}}-\frac{a^2\cos(e+fx)\sqrt{a+a\,\mathrm{s}}}{}$$

$$= -\frac{a^3\left(3c^2-26cd+163d^2\right)\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{96d^2f\sqrt{a+a\sin(e+fx)}}+\frac{a^3(3c-1}{}$$

$$= -\frac{a^3(c+d)\left(3c^2-26cd+163d^2\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{64d^2f\sqrt{a+a\sin(e+fx)}}-\frac{a^3\,(3}{}$$

$$= -\frac{a^3(c+d)\left(3c^2-26cd+163d^2\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{64d^2f\sqrt{a+a\sin(e+fx)}}-\frac{a^3\,(3}{}$$

$$= -\frac{a^{5/2}(c+d)^2\left(3c^2-26cd+163d^2\right)\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{64d^{5/2}f}-$$

Mathematica [A] time = 1.76093, size = 327, normalized size = 1.05

$$\frac{(a(\sin(e+fx)+1))^{5/2}\left(\frac{2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{c+d\sin(e+fx)}\left(-2d\left(3c^2+158cd+181d^2\right)\sin(e+fx)-63c^2d+9c^3+4d^2(9c+23d)\cos(2(e+fx))-773}{3d^2}\right.}{128f\left(\sin\left(\frac{1}{2}\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^(5/2)*(((c + d)^2*(3*c^2 - 26*c*d + 163*d^2)*(2*Arc
Tan[(Sqrt[2]*Sqrt[d]*Sin[(2*e - Pi + 2*f*x)/4])/Sqrt[c + d*Sin[e + f*x]]] +
 ArcTanh[(Sqrt[2]*Sqrt[d]*Cos[(2*e - Pi + 2*f*x)/4])/Sqrt[c + d*Sin[e + f*x
]]] - Log[Sqrt[2]*Sqrt[d]*Cos[(2*e - Pi + 2*f*x)/4] + Sqrt[c + d*Sin[e + f*
x]]]))/d^(5/2) + (2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[c + d*Sin[e
+ f*x]]*(9*c^3 - 63*c^2*d - 773*c*d^2 - 581*d^3 + 4*d^2*(9*c + 23*d)*Cos[2*
(e + f*x)] - 2*d*(3*c^2 + 158*c*d + 181*d^2)*Sin[e + f*x] + 12*d^3*Sin[3*(e
 + f*x)]))/(3*d^2)))/(128*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int \left(a + a\sin\left(fx + e\right)\right)^{\frac{5}{2}} \left(c + d\sin\left(fx + e\right)\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^(3/2),x)
```

```
[Out] int((a+a*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx + e\right) + a\right)^{\frac{5}{2}} \left(d\sin\left(fx + e\right) + c\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)*(d*sin(f*x + e) + c)^(3/2), x)
```

Fricas [B] time = 14.7684, size = 4084, normalized size = 13.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] [1/1536*(3*(3*a^2*c^4 - 20*a^2*c^3*d + 114*a^2*c^2*d^2 + 300*a^2*c*d^3 + 16
3*a^2*d^4 + (3*a^2*c^4 - 20*a^2*c^3*d + 114*a^2*c^2*d^2 + 300*a^2*c*d^3 + 1
63*a^2*d^4)*cos(f*x + e) + (3*a^2*c^4 - 20*a^2*c^3*d + 114*a^2*c^2*d^2 + 30
0*a^2*c*d^3 + 163*a^2*d^4)*sin(f*x + e))*sqrt(-a/d)*log((128*a*d^4*cos(f*x
+ e)^5 + a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 + a*d^4 + 128*(2*a*c*d
^3 - a*d^4)*cos(f*x + e)^4 - 32*(5*a*c^2*d^2 - 14*a*c*d^3 + 13*a*d^4)*cos(f
*x + e)^3 - 32*(a*c^3*d - 2*a*c^2*d^2 + 9*a*c*d^3 - 4*a*d^4)*cos(f*x + e)^2
 - 8*(16*d^4*cos(f*x + e)^4 - c^3*d + 17*c^2*d^2 - 59*c*d^3 + 51*d^4 + 24*(
c*d^3 - d^4)*cos(f*x + e)^3 - 2*(5*c^2*d^2 - 26*c*d^3 + 33*d^4)*cos(f*x + e
```

```
)^2 - (c^3*d - 7*c^2*d^2 + 31*c*d^3 - 25*d^4)*cos(f*x + e) + (16*d^4*cos(f*
x + e)^3 + c^3*d - 17*c^2*d^2 + 59*c*d^3 - 51*d^4 - 8*(3*c*d^3 - 5*d^4)*cos
(f*x + e)^2 - 2*(5*c^2*d^2 - 14*c*d^3 + 13*d^4)*cos(f*x + e))*sin(f*x + e))
*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt(-a/d) + (a*c^4 - 28
*a*c^3*d + 230*a*c^2*d^2 - 476*a*c*d^3 + 289*a*d^4)*cos(f*x + e) + (128*a*d
^4*cos(f*x + e)^4 + a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 + a*d^4 - 2
56*(a*c*d^3 - a*d^4)*cos(f*x + e)^3 - 32*(5*a*c^2*d^2 - 6*a*c*d^3 + 5*a*d^4
)*cos(f*x + e)^2 + 32*(a*c^3*d - 7*a*c^2*d^2 + 15*a*c*d^3 - 9*a*d^4)*cos(f*
x + e))*sin(f*x + e))/(cos(f*x + e) + sin(f*x + e) + 1)) + 8*(48*a^2*d^3*co
s(f*x + e)^4 + 9*a^2*c^3 - 57*a^2*c^2*d - 493*a^2*c*d^2 - 299*a^2*d^3 + 8*(
9*a^2*c*d^2 + 23*a^2*d^3)*cos(f*x + e)^3 - 2*(3*a^2*c^2*d + 122*a^2*c*d^2 +
 119*a^2*d^3)*cos(f*x + e)^2 + (9*a^2*c^3 - 63*a^2*c^2*d - 809*a^2*c*d^2 -
673*a^2*d^3)*cos(f*x + e) + (48*a^2*d^3*cos(f*x + e)^3 - 9*a^2*c^3 + 57*a^2
*c^2*d + 493*a^2*c*d^2 + 299*a^2*d^3 - 8*(9*a^2*c*d^2 + 17*a^2*d^3)*cos(f*x
 + e)^2 - 2*(3*a^2*c^2*d + 158*a^2*c*d^2 + 187*a^2*d^3)*cos(f*x + e))*sin(f
*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/(d^2*f*cos(f*x
+ e) + d^2*f*sin(f*x + e) + d^2*f), 1/768*(3*(3*a^2*c^4 - 20*a^2*c^3*d + 11
4*a^2*c^2*d^2 + 300*a^2*c*d^3 + 163*a^2*d^4 + (3*a^2*c^4 - 20*a^2*c^3*d + 1
14*a^2*c^2*d^2 + 300*a^2*c*d^3 + 163*a^2*d^4)*cos(f*x + e) + (3*a^2*c^4 - 2
0*a^2*c^3*d + 114*a^2*c^2*d^2 + 300*a^2*c*d^3 + 163*a^2*d^4)*sin(f*x + e))*
sqrt(a/d)*arctan(1/4*(8*d^2*cos(f*x + e)^2 - c^2 + 6*c*d - 9*d^2 - 8*(c*d -
 d^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt(
a/d)/(2*a*d^2*cos(f*x + e)^3 - (3*a*c*d - a*d^2)*cos(f*x + e)*sin(f*x + e)
- (a*c^2 - a*c*d + 2*a*d^2)*cos(f*x + e))) + 4*(48*a^2*d^3*cos(f*x + e)^4 +
 9*a^2*c^3 - 57*a^2*c^2*d - 493*a^2*c*d^2 - 299*a^2*d^3 + 8*(9*a^2*c*d^2 +
23*a^2*d^3)*cos(f*x + e)^3 - 2*(3*a^2*c^2*d + 122*a^2*c*d^2 + 119*a^2*d^3)*
cos(f*x + e)^2 + (9*a^2*c^3 - 63*a^2*c^2*d - 809*a^2*c*d^2 - 673*a^2*d^3)*c
os(f*x + e) + (48*a^2*d^3*cos(f*x + e)^3 - 9*a^2*c^3 + 57*a^2*c^2*d + 493*a
^2*c*d^2 + 299*a^2*d^3 - 8*(9*a^2*c*d^2 + 17*a^2*d^3)*cos(f*x + e)^2 - 2*(3
*a^2*c^2*d + 158*a^2*c*d^2 + 187*a^2*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(
a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/(d^2*f*cos(f*x + e) + d^2*f*s
in(f*x + e) + d^2*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*(c+d*sin(f*x+e))**(3/2),x)

[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^(3/2),x, algorithm="giac"
)

[Out] Exception raised: TypeError
```

3.581 $\int (a+a\sin(e+fx))^{5/2}\sqrt{c+d\sin(e+fx)}\,dx$

Optimal. Leaf size=241

$$-\frac{a^3(c^2-6cd+25d^2)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{8d^2f\sqrt{a\sin(e+fx)+a}}-\frac{a^{5/2}(c+d)(c^2-6cd+25d^2)\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{8d^{5/2}f}$$

```
[Out] -(a^(5/2)*(c + d)*(c^2 - 6*c*d + 25*d^2)*ArcTan[(Sqrt[a]*Sqrt[d]*Cos[e + f*
x])/(Sqrt[a + a*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])])/(8*d^(5/2)*f) - (
a^3*(c^2 - 6*c*d + 25*d^2)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(8*d^2*f*
Sqrt[a + a*Sin[e + f*x]]) + (a^3*(3*c - 13*d)*Cos[e + f*x]*(c + d*Sin[e + f
*x])^(3/2))/(12*d^2*f*Sqrt[a + a*Sin[e + f*x]]) - (a^2*Cos[e + f*x]*Sqrt[a
+ a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(3/2))/(3*d*f)
```

Rubi [A] time = 0.57596, antiderivative size = 241, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2763, 2981, 2770, 2775, 205}

$$-\frac{a^3(c^2-6cd+25d^2)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{8d^2f\sqrt{a\sin(e+fx)+a}}-\frac{a^{5/2}(c+d)(c^2-6cd+25d^2)\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{8d^{5/2}f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)*Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] -(a^(5/2)*(c + d)*(c^2 - 6*c*d + 25*d^2)*ArcTan[(Sqrt[a]*Sqrt[d]*Cos[e + f*
x])/(Sqrt[a + a*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])])/(8*d^(5/2)*f) - (
a^3*(c^2 - 6*c*d + 25*d^2)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(8*d^2*f*
Sqrt[a + a*Sin[e + f*x]]) + (a^3*(3*c - 13*d)*Cos[e + f*x]*(c + d*Sin[e + f
*x])^(3/2))/(12*d^2*f*Sqrt[a + a*Sin[e + f*x]]) - (a^2*Cos[e + f*x]*Sqrt[a
+ a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(3/2))/(3*d*f)
```

Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{5/2}\sqrt{c+d\sin(e+fx)}\,dx = -\frac{a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^{3/2}}{3df}+\frac{\int\sqrt{a+a\sin(\ldots}{\ldots}$$

$$=\frac{a^3(3c-13d)\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{12d^2f\sqrt{a+a\sin(e+fx)}}-\frac{a^2\cos(e+fx)\sqrt{a+a\sin\ldots}}{\ldots}$$

$$=-\frac{a^3\left(c^2-6cd+25d^2\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{8d^2f\sqrt{a+a\sin(e+fx)}}+\frac{a^3(3c-13d)\cos\ldots}{12d^2f\ldots}$$

$$=-\frac{a^3\left(c^2-6cd+25d^2\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{8d^2f\sqrt{a+a\sin(e+fx)}}+\frac{a^3(3c-13d)\cos\ldots}{12d^2f\ldots}$$

$$=-\frac{a^{5/2}(c+d)\left(c^2-6cd+25d^2\right)\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{8d^{5/2}f}-\frac{a^3\left(c^2\ldots\right.}{\ldots}$$

Mathematica [A] time = 0.920727, size = 285, normalized size = 1.18

$$\frac{(a(\sin(e+fx)+1))^{5/2}\left(\frac{2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{c+d\sin(e+fx)}\left(3c^2-2d(c+17d)\sin(e+fx)-16cd+4d^2\cos(2(e+fx))-79d^2\right)}{3d^2}+\frac{(c+d)\left(c^2-6cd+25\ldots\right.}{\ldots}\right.}{16f\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+\ldots\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)*Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^(5/2)*(((c + d)*(c^2 - 6*c*d + 25*d^2)*(2*ArcTan[(S
qrt[2]*Sqrt[d]*Sin[(2*e - Pi + 2*f*x)/4])/Sqrt[c + d*Sin[e + f*x]]] + ArcTa
nh[(Sqrt[2]*Sqrt[d]*Cos[(2*e - Pi + 2*f*x)/4])/Sqrt[c + d*Sin[e + f*x]]] -
```

```
Log[Sqrt[2]*Sqrt[d]*Cos[(2*e - Pi + 2*f*x)/4] + Sqrt[c + d*Sin[e + f*x]]]))
/d^(5/2) + (2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[c + d*Sin[e + f*x]
]*(3*c^2 - 16*c*d - 79*d^2 + 4*d^2*Cos[2*(e + f*x)] - 2*d*(c + 17*d)*Sin[e
+ f*x]))/(3*d^2)))/(16*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int \left(a + a\sin\left(fx + e\right)\right)^{\frac{5}{2}} \sqrt{c + d\sin\left(fx + e\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^(1/2),x)
```

```
[Out] int((a+a*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx + e\right) + a\right)^{\frac{5}{2}} \sqrt{d\sin\left(fx + e\right) + c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)*sqrt(d*sin(f*x + e) + c), x)
```

Fricas [B] time = 9.82851, size = 3374, normalized size = 14.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] [1/192*(3*(a^2*c^3 - 5*a^2*c^2*d + 19*a^2*c*d^2 + 25*a^2*d^3 + (a^2*c^3 - 5
*a^2*c^2*d + 19*a^2*c*d^2 + 25*a^2*d^3)*cos(f*x + e) + (a^2*c^3 - 5*a^2*c^2
*d + 19*a^2*c*d^2 + 25*a^2*d^3)*sin(f*x + e))*sqrt(-a/d)*log((128*a*d^4*cos
(f*x + e)^5 + a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 + a*d^4 + 128*(2*
a*c*d^3 - a*d^4)*cos(f*x + e)^4 - 32*(5*a*c^2*d^2 - 14*a*c*d^3 + 13*a*d^4)*
cos(f*x + e)^3 - 32*(a*c^3*d - 2*a*c^2*d^2 + 9*a*c*d^3 - 4*a*d^4)*cos(f*x +
 e)^2 - 8*(16*d^4*cos(f*x + e)^4 - c^3*d + 17*c^2*d^2 - 59*c*d^3 + 51*d^4 +
 24*(c*d^3 - d^4)*cos(f*x + e)^3 - 2*(5*c^2*d^2 - 26*c*d^3 + 33*d^4)*cos(f*
x + e)^2 - (c^3*d - 7*c^2*d^2 + 31*c*d^3 - 25*d^4)*cos(f*x + e) + (16*d^4*c
os(f*x + e)^3 + c^3*d - 17*c^2*d^2 + 59*c*d^3 - 51*d^4 - 8*(3*c*d^3 - 5*d^4
)*cos(f*x + e)^2 - 2*(5*c^2*d^2 - 14*c*d^3 + 13*d^4)*cos(f*x + e))*sin(f*x
+ e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt(-a/d) + (a*c^4
 - 28*a*c^3*d + 230*a*c^2*d^2 - 476*a*c*d^3 + 289*a*d^4)*cos(f*x + e) + (12
8*a*d^4*cos(f*x + e)^4 + a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 + a*d^
4 - 256*(a*c*d^3 - a*d^4)*cos(f*x + e)^3 - 32*(5*a*c^2*d^2 - 6*a*c*d^3 + 5*
```

```
a*d^4)*cos(f*x + e)^2 + 32*(a*c^3*d - 7*a*c^2*d^2 + 15*a*c*d^3 - 9*a*d^4)*c
os(f*x + e))*sin(f*x + e))/(cos(f*x + e) + sin(f*x + e) + 1)) + 8*(8*a^2*d^
2*cos(f*x + e)^3 + 3*a^2*c^2 - 14*a^2*c*d - 49*a^2*d^2 - 2*(a^2*c*d + 13*a^
2*d^2)*cos(f*x + e)^2 + (3*a^2*c^2 - 16*a^2*c*d - 83*a^2*d^2)*cos(f*x + e)
 - (8*a^2*d^2*cos(f*x + e)^2 + 3*a^2*c^2 - 14*a^2*c*d - 49*a^2*d^2 + 2*(a^2*
c*d + 17*a^2*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt
(d*sin(f*x + e) + c))/(d^2*f*cos(f*x + e) + d^2*f*sin(f*x + e) + d^2*f), 1/
96*(3*(a^2*c^3 - 5*a^2*c^2*d + 19*a^2*c*d^2 + 25*a^2*d^3 + (a^2*c^3 - 5*a^2
*c^2*d + 19*a^2*c*d^2 + 25*a^2*d^3)*cos(f*x + e) + (a^2*c^3 - 5*a^2*c^2*d +
 19*a^2*c*d^2 + 25*a^2*d^3)*sin(f*x + e))*sqrt(a/d)*arctan(1/4*(8*d^2*cos(f
*x + e)^2 - c^2 + 6*c*d - 9*d^2 - 8*(c*d - d^2)*sin(f*x + e))*sqrt(a*sin(f*
x + e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt(a/d)/(2*a*d^2*cos(f*x + e)^3 - (3
*a*c*d - a*d^2)*cos(f*x + e)*sin(f*x + e) - (a*c^2 - a*c*d + 2*a*d^2)*cos(f
*x + e))) + 4*(8*a^2*d^2*cos(f*x + e)^3 + 3*a^2*c^2 - 14*a^2*c*d - 49*a^2*d
^2 - 2*(a^2*c*d + 13*a^2*d^2)*cos(f*x + e)^2 + (3*a^2*c^2 - 16*a^2*c*d - 83
*a^2*d^2)*cos(f*x + e) - (8*a^2*d^2*cos(f*x + e)^2 + 3*a^2*c^2 - 14*a^2*c*d
 - 49*a^2*d^2 + 2*(a^2*c*d + 17*a^2*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a
*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/(d^2*f*cos(f*x + e) + d^2*f*si
n(f*x + e) + d^2*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError
```

3.582 $\int \frac{(a+a\sin(e+fx))^{5/2}}{\sqrt{c+d\sin(e+fx)}}\,dx$

Optimal. Leaf size=178

$$-\frac{a^{5/2}\left(3c^2-10cd+19d^2\right)\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{4d^{5/2}f}+\frac{3a^3(c-3d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4d^2f\sqrt{a\sin(e+fx)+a}}-\frac{a^2\cos(e+$$

```
[Out] -(a^(5/2)*(3*c^2 - 10*c*d + 19*d^2)*ArcTan[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(
Sqrt[a + a*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])])/(4*d^(5/2)*f) + (3*a^3
*(c - 3*d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(4*d^2*f*Sqrt[a + a*Sin[e
 + f*x]]) - (a^2*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f
*x]])/(2*d*f)
```

Rubi [A] time = 0.433055, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2763, 2981, 2775, 205}

$$-\frac{a^{5/2}\left(3c^2-10cd+19d^2\right)\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{4d^{5/2}f}+\frac{3a^3(c-3d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4d^2f\sqrt{a\sin(e+fx)+a}}-\frac{a^2\cos(e+$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)/Sqrt[c + d*Sin[e + f*x]],x]
```


```
[Out] -(a^(5/2)*(3*c^2 - 10*c*d + 19*d^2)*ArcTan[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(
Sqrt[a + a*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])])/(4*d^(5/2)*f) + (3*a^3
*(c - 3*d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(4*d^2*f*Sqrt[a + a*Sin[e
 + f*x]]) - (a^2*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f
*x]])/(2*d*f)
```


Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{5/2}}{\sqrt{c+d\sin(e+fx)}}\,dx = -\frac{a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}{2df} + \frac{\int \frac{\sqrt{a+a\sin(e+fx)}\left(\frac{1}{2}a^2(c+5d)-\frac{3}{2}a^2(c-3d)\sin\right.}{\sqrt{c+d\sin(e+fx)}}}{2d}$$

$$= \frac{3a^3(c-3d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4d^2f\sqrt{a+a\sin(e+fx)}} - \frac{a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e}}{2df}$$

$$= \frac{3a^3(c-3d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4d^2f\sqrt{a+a\sin(e+fx)}} - \frac{a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e}}{2df}$$

$$= -\frac{a^{5/2}\left(3c^2-10cd+19d^2\right)\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{4d^{5/2}f} + \frac{3a^3(c-3d)\cos(e+fx)\sqrt{c+}}{4d^2f\sqrt{a+a\sin(e-}}$$

Mathematica [A] time = 0.87528, size = 256, normalized size = 1.44

$$\frac{(a(\sin(e+fx)+1))^{5/2}\left(\frac{\left(3c^2-10cd+19d^2\right)\left(2\tan^{-1}\left(\frac{\sqrt{2}\sqrt{d}\sin\left(\frac{1}{4}(2e+2fx-\pi)\right)}{\sqrt{c+d\sin(e+fx)}}\right)-\log\left(\sqrt{c+d\sin(e+fx)}+\sqrt{2}\sqrt{d}\cos\left(\frac{1}{4}(2e+2fx-\pi)\right)\right)+\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{d}\cos\left(\frac{1}{4}(2e+2\right.}{\sqrt{c+d\sin(e+f}}\right.\right.}{d^{5/2}}\right.}{8f\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)/Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^(5/2)*(((3*c^2 - 10*c*d + 19*d^2)*(2*ArcTan[(Sqrt[2
]*Sqrt[d]*Sin[(2*e - Pi + 2*f*x)/4])/Sqrt[c + d*Sin[e + f*x]]] + ArcTanh[(S
qrt[2]*Sqrt[d]*Cos[(2*e - Pi + 2*f*x)/4])/Sqrt[c + d*Sin[e + f*x]]] - Log[S
qrt[2]*Sqrt[d]*Cos[(2*e - Pi + 2*f*x)/4] + Sqrt[c + d*Sin[e + f*x]]]))/d^(5
/2) + (2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(3*c - 11*d - 2*d*Sin[e + f*
x])*Sqrt[c + d*Sin[e + f*x]])/d^2))/(8*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/
2])^5)
```

Maple [F] time = 0.237, size = 0, normalized size = 0.

$$\int \left(a+a\sin\left(fx+e\right)\right)^{\frac{5}{2}}\frac{1}{\sqrt{c+d\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(1/2),x)
```

```
[Out] int((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}{\sqrt{d\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)/sqrt(d*sin(f*x + e) + c), x)
```

Fricas [B] time = 6.69849, size = 2873, normalized size = 16.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] [1/32*((3*a^2*c^2 - 10*a^2*c*d + 19*a^2*d^2 + (3*a^2*c^2 - 10*a^2*c*d + 19*
a^2*d^2)*cos(f*x + e) + (3*a^2*c^2 - 10*a^2*c*d + 19*a^2*d^2)*sin(f*x + e))
*sqrt(-a/d)*log((128*a*d^4*cos(f*x + e)^5 + a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2
 + 4*a*c*d^3 + a*d^4 + 128*(2*a*c*d^3 - a*d^4)*cos(f*x + e)^4 - 32*(5*a*c^2
*d^2 - 14*a*c*d^3 + 13*a*d^4)*cos(f*x + e)^3 - 32*(a*c^3*d - 2*a*c^2*d^2 +
9*a*c*d^3 - 4*a*d^4)*cos(f*x + e)^2 - 8*(16*d^4*cos(f*x + e)^4 - c^3*d + 17
*c^2*d^2 - 59*c*d^3 + 51*d^4 + 24*(c*d^3 - d^4)*cos(f*x + e)^3 - 2*(5*c^2*d
^2 - 26*c*d^3 + 33*d^4)*cos(f*x + e)^2 - (c^3*d - 7*c^2*d^2 + 31*c*d^3 - 25
*d^4)*cos(f*x + e) + (16*d^4*cos(f*x + e)^3 + c^3*d - 17*c^2*d^2 + 59*c*d^3
 - 51*d^4 - 8*(3*c*d^3 - 5*d^4)*cos(f*x + e)^2 - 2*(5*c^2*d^2 - 14*c*d^3 +
13*d^4)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x
 + e) + c)*sqrt(-a/d) + (a*c^4 - 28*a*c^3*d + 230*a*c^2*d^2 - 476*a*c*d^3 +
 289*a*d^4)*cos(f*x + e) + (128*a*d^4*cos(f*x + e)^4 + a*c^4 + 4*a*c^3*d +
6*a*c^2*d^2 + 4*a*c*d^3 + a*d^4 - 256*(a*c*d^3 - a*d^4)*cos(f*x + e)^3 - 32
*(5*a*c^2*d^2 - 6*a*c*d^3 + 5*a*d^4)*cos(f*x + e)^2 + 32*(a*c^3*d - 7*a*c^2
*d^2 + 15*a*c*d^3 - 9*a*d^4)*cos(f*x + e))*sin(f*x + e))/(cos(f*x + e) + si
n(f*x + e) + 1)) - 8*(2*a^2*d*cos(f*x + e)^2 - 3*a^2*c + 9*a^2*d - (3*a^2*c
 - 11*a^2*d)*cos(f*x + e) + (2*a^2*d*cos(f*x + e) + 3*a^2*c - 9*a^2*d)*sin(
f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/(d^2*f*cos(f*x
 + e) + d^2*f*sin(f*x + e) + d^2*f), 1/16*((3*a^2*c^2 - 10*a^2*c*d + 19*a^2
*d^2 + (3*a^2*c^2 - 10*a^2*c*d + 19*a^2*d^2)*cos(f*x + e) + (3*a^2*c^2 - 10
*a^2*c*d + 19*a^2*d^2)*sin(f*x + e))*sqrt(a/d)*arctan(1/4*(8*d^2*cos(f*x +
e)^2 - c^2 + 6*c*d - 9*d^2 - 8*(c*d - d^2)*sin(f*x + e))*sqrt(a*sin(f*x + e
) + a)*sqrt(d*sin(f*x + e) + c)*sqrt(a/d)/(2*a*d^2*cos(f*x + e)^3 - (3*a*c*
d - a*d^2)*cos(f*x + e)*sin(f*x + e) - (a*c^2 - a*c*d + 2*a*d^2)*cos(f*x +
e))) - 4*(2*a^2*d*cos(f*x + e)^2 - 3*a^2*c + 9*a^2*d - (3*a^2*c - 11*a^2*d)
```

```
*cos(f*x + e) + (2*a^2*d*cos(f*x + e) + 3*a^2*c - 9*a^2*d)*sin(f*x + e))*sq
rt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/(d^2*f*cos(f*x + e) + d^2*
f*sin(f*x + e) + d^2*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)/(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}{\sqrt{d\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)/sqrt(d*sin(f*x + e) + c), x)
```

3.583 $\int \frac{(a+a\sin(e+fx))^{5/2}}{(c+d\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=180

$$-\frac{a^3(3c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{d^2f(c+d)\sqrt{a\sin(e+fx)+a}}+\frac{a^{5/2}(3c-5d)\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{d^{5/2}f}+\frac{2a^2(c-d)\cos(e+fx)}{df(c+d)\sqrt{c+d}}$$

```
[Out] (a^(5/2)*(3*c - 5*d)*ArcTan[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[a + a*Sin[
e + f*x]]*Sqrt[c + d*Sin[e + f*x]])])/(d^(5/2)*f) + (2*a^2*(c - d)*Cos[e +
f*x]*Sqrt[a + a*Sin[e + f*x]])/(d*(c + d)*f*Sqrt[c + d*Sin[e + f*x]]) - (a^
3*(3*c - d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(d^2*(c + d)*f*Sqrt[a +
a*Sin[e + f*x]])
```

Rubi [A] time = 0.438198, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2762, 2981, 2775, 205}

$$-\frac{a^3(3c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{d^2f(c+d)\sqrt{a\sin(e+fx)+a}}+\frac{a^{5/2}(3c-5d)\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{d^{5/2}f}+\frac{2a^2(c-d)\cos(e+fx)}{df(c+d)\sqrt{c+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)/(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] (a^(5/2)*(3*c - 5*d)*ArcTan[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[a + a*Sin[
e + f*x]]*Sqrt[c + d*Sin[e + f*x]])])/(d^(5/2)*f) + (2*a^2*(c - d)*Cos[e +
f*x]*Sqrt[a + a*Sin[e + f*x]])/(d*(c + d)*f*Sqrt[c + d*Sin[e + f*x]]) - (a^
3*(3*c - d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(d^2*(c + d)*f*Sqrt[a +
a*Sin[e + f*x]])
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{5/2}}{(c+d\sin(e+fx))^{3/2}}\,dx = \frac{2a^2(c-d)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{d(c+d)f\sqrt{c+d\sin(e+fx)}} - \frac{(2a)\int \frac{\sqrt{a+a\sin(e+fx)}\left(\frac{1}{2}a(c-3d)-\frac{1}{2}a(3c-d)\sin(e+fx)\right)}{\sqrt{c+d\sin(e+fx)}}\,dx}{d(c+d)}$$

$$= \frac{2a^2(c-d)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{d(c+d)f\sqrt{c+d\sin(e+fx)}} - \frac{a^3(3c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{d^2(c+d)f\sqrt{a+a\sin(e+fx)}} - \frac{(a^{\cdots}}{\cdots}$$

$$= \frac{2a^2(c-d)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{d(c+d)f\sqrt{c+d\sin(e+fx)}} - \frac{a^3(3c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{d^2(c+d)f\sqrt{a+a\sin(e+fx)}} + \frac{(a^{\cdots}}{\cdots}$$

$$= \frac{a^{5/2}(3c-5d)\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{d^{5/2}f} + \frac{2a^2(c-d)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{d(c+d)f\sqrt{c+d\sin(e+fx)}}$$

Mathematica [A] time = 1.00875, size = 263, normalized size = 1.46

$$\frac{(a(\sin(e+fx)+1))^{5/2}\left(\frac{(5d-3c)\left(2\tan^{-1}\left(\frac{\sqrt{2}\sqrt{d}\sin\left(\frac{1}{4}(2e+2fx-\pi)\right)}{\sqrt{c+d\sin(e+fx)}}\right)-\log\left(\sqrt{c+d\sin(e+fx)}+\sqrt{2}\sqrt{d}\cos\left(\frac{1}{4}(2e+2fx-\pi)\right)\right)+\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{d}\cos\left(\frac{1}{4}(2e+2fx-\pi)\right)}{\sqrt{c+d\sin(e+fx)}}\right)\right)}{d^{5/2}} - \cdots\right)}{2f\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)/(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^(5/2)*(((-3*c + 5*d)*(2*ArcTan[(Sqrt[2]*Sqrt[d]*Sin
[(2*e - Pi + 2*f*x)/4])/Sqrt[c + d*Sin[e + f*x]]] + ArcTanh[(Sqrt[2]*Sqrt[d
]*Cos[(2*e - Pi + 2*f*x)/4])/Sqrt[c + d*Sin[e + f*x]]] - Log[Sqrt[2]*Sqrt[d
]*Cos[(2*e - Pi + 2*f*x)/4] + Sqrt[c + d*Sin[e + f*x]]]))/d^(5/2) - (2*(Cos
[(e + f*x)/2] - Sin[(e + f*x)/2])*(3*c^2 - 3*c*d + 2*d^2 + d*(c + d)*Sin[e
+ f*x]))/(d^2*(c + d)*Sqrt[c + d*Sin[e + f*x]])))/(2*f*(Cos[(e + f*x)/2] +
Sin[(e + f*x)/2])^5)
```

Maple [F] time = 0.252, size = 0, normalized size = 0.

$$\int \left(a+a\sin\left(fx+e\right)\right)^{\frac{5}{2}}\left(c+d\sin\left(fx+e\right)\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(3/2),x)
```

```
[Out] int((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)/(d*sin(f*x + e) + c)^(3/2), x)
```

Fricas [B] time = 5.57539, size = 3758, normalized size = 20.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] [1/8*((3*a^2*c^3 + a^2*c^2*d - 7*a^2*c*d^2 - 5*a^2*d^3 - (3*a^2*c^2*d - 2*a
^2*c*d^2 - 5*a^2*d^3)*cos(f*x + e)^2 + (3*a^2*c^3 - 2*a^2*c^2*d - 5*a^2*c*d
^2)*cos(f*x + e) + (3*a^2*c^3 + a^2*c^2*d - 7*a^2*c*d^2 - 5*a^2*d^3 + (3*a^
2*c^2*d - 2*a^2*c*d^2 - 5*a^2*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(-a/d)*l
og((128*a*d^4*cos(f*x + e)^5 + a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3
 + a*d^4 + 128*(2*a*c*d^3 - a*d^4)*cos(f*x + e)^4 - 32*(5*a*c^2*d^2 - 14*a*c
*d^3 + 13*a*d^4)*cos(f*x + e)^3 - 32*(a*c^3*d - 2*a*c^2*d^2 + 9*a*c*d^3 - 4
*a*d^4)*cos(f*x + e)^2 - 8*(16*d^4*cos(f*x + e)^4 - c^3*d + 17*c^2*d^2 - 59
*c*d^3 + 51*d^4 + 24*(c*d^3 - d^4)*cos(f*x + e)^3 - 2*(5*c^2*d^2 - 26*c*d^3
 + 33*d^4)*cos(f*x + e)^2 - (c^3*d - 7*c^2*d^2 + 31*c*d^3 - 25*d^4)*cos(f*x
 + e) + (16*d^4*cos(f*x + e)^3 + c^3*d - 17*c^2*d^2 + 59*c*d^3 - 51*d^4 - 8
*(3*c*d^3 - 5*d^4)*cos(f*x + e)^2 - 2*(5*c^2*d^2 - 14*c*d^3 + 13*d^4)*cos(f
*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sq
rt(-a/d) + (a*c^4 - 28*a*c^3*d + 230*a*c^2*d^2 - 476*a*c*d^3 + 289*a*d^4)*c
os(f*x + e) + (128*a*d^4*cos(f*x + e)^4 + a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 +
 4*a*c*d^3 + a*d^4 - 256*(a*c*d^3 - a*d^4)*cos(f*x + e)^3 - 32*(5*a*c^2*d^2
 - 6*a*c*d^3 + 5*a*d^4)*cos(f*x + e)^2 + 32*(a*c^3*d - 7*a*c^2*d^2 + 15*a*c
*d^3 - 9*a*d^4)*cos(f*x + e))*sin(f*x + e))/(cos(f*x + e) + sin(f*x + e) +
1)) + 8*(3*a^2*c^2 - 4*a^2*c*d + a^2*d^2 + (a^2*c*d + a^2*d^2)*cos(f*x + e)
^2 + (3*a^2*c^2 - 3*a^2*c*d + 2*a^2*d^2)*cos(f*x + e) - (3*a^2*c^2 - 4*a^2*
c*d + a^2*d^2 - (a^2*c*d + a^2*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(
f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/((c*d^3 + d^4)*f*cos(f*x + e)^2 - (
c^2*d^2 + c*d^3)*f*cos(f*x + e) - (c^2*d^2 + 2*c*d^3 + d^4)*f - ((c*d^3 + d
^4)*f*cos(f*x + e) + (c^2*d^2 + 2*c*d^3 + d^4)*f)*sin(f*x + e)), 1/4*((3*a^
2*c^3 + a^2*c^2*d - 7*a^2*c*d^2 - 5*a^2*d^3 - (3*a^2*c^2*d - 2*a^2*c*d^2 -
5*a^2*d^3)*cos(f*x + e)^2 + (3*a^2*c^3 - 2*a^2*c^2*d - 5*a^2*c*d^2)*cos(f*x
 + e) + (3*a^2*c^3 + a^2*c^2*d - 7*a^2*c*d^2 - 5*a^2*d^3 + (3*a^2*c^2*d - 2
*a^2*c*d^2 - 5*a^2*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a/d)*arctan(1/4*(8
```

```
*d^2*cos(f*x + e)^2 - c^2 + 6*c*d - 9*d^2 - 8*(c*d - d^2)*sin(f*x + e))*sqr
t(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt(a/d)/(2*a*d^2*cos(f*x +
 e)^3 - (3*a*c*d - a*d^2)*cos(f*x + e)*sin(f*x + e) - (a*c^2 - a*c*d + 2*a*
d^2)*cos(f*x + e))) + 4*(3*a^2*c^2 - 4*a^2*c*d + a^2*d^2 + (a^2*c*d + a^2*d
^2)*cos(f*x + e)^2 + (3*a^2*c^2 - 3*a^2*c*d + 2*a^2*d^2)*cos(f*x + e) - (3*
a^2*c^2 - 4*a^2*c*d + a^2*d^2 - (a^2*c*d + a^2*d^2)*cos(f*x + e))*sin(f*x +
 e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/((c*d^3 + d^4)*f*co
s(f*x + e)^2 - (c^2*d^2 + c*d^3)*f*cos(f*x + e) - (c^2*d^2 + 2*c*d^3 + d^4)
*f - ((c*d^3 + d^4)*f*cos(f*x + e) + (c^2*d^2 + 2*c*d^3 + d^4)*f)*sin(f*x +
 e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)/(c+d*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)/(d*sin(f*x + e) + c)^(3/2), x)
```

3.584 $\int \frac{(a+a\sin(e+fx))^{5/2}}{(c+d\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=183

$$\frac{2a^3(c-d)(3c+7d)\cos(e+fx)}{3d^2f(c+d)^2\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}-\frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{d^{5/2}f}+\frac{2a^2(c-d)\cos(e+fx)}{3df(c+d)(c+d\,\mathrm{s}}$$

```
[Out] (-2*a^(5/2)*ArcTan[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]
*Sqrt[c + d*Sin[e + f*x]])])/(d^(5/2)*f) + (2*a^2*(c - d)*Cos[e + f*x]*Sqrt
[a + a*Sin[e + f*x]])/(3*d*(c + d)*f*(c + d*Sin[e + f*x])^(3/2)) + (2*a^3*(
c - d)*(3*c + 7*d)*Cos[e + f*x])/(3*d^2*(c + d)^2*f*Sqrt[a + a*Sin[e + f*x]
]*Sqrt[c + d*Sin[e + f*x]])
```

Rubi [A] time = 0.443984, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2762, 2980, 2775, 205}

$$\frac{2a^3(c-d)(3c+7d)\cos(e+fx)}{3d^2f(c+d)^2\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}-\frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{d^{5/2}f}+\frac{2a^2(c-d)\cos(e+fx)}{3df(c+d)(c+d\,\mathrm{s}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)/(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (-2*a^(5/2)*ArcTan[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]
*Sqrt[c + d*Sin[e + f*x]])])/(d^(5/2)*f) + (2*a^2*(c - d)*Cos[e + f*x]*Sqrt
[a + a*Sin[e + f*x]])/(3*d*(c + d)*f*(c + d*Sin[e + f*x])^(3/2)) + (2*a^3*(
c - d)*(3*c + 7*d)*Cos[e + f*x])/(3*d^2*(c + d)^2*f*Sqrt[a + a*Sin[e + f*x]
]*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{5/2}}{(c+d\sin(e+fx))^{5/2}}\,dx = \frac{2a^2(c-d)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3d(c+d)f(c+d\sin(e+fx))^{3/2}} - \frac{(2a)\int \frac{\sqrt{a+a\sin(e+fx)}\left(\frac{1}{2}a(c-7d)-\frac{3}{2}a(c+d)\sin(e+fx)\right)}{(c+d\sin(e+fx))^{3/2}}\,dx}{3d(c+d)}$$

$$= \frac{2a^2(c-d)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3d(c+d)f(c+d\sin(e+fx))^{3/2}} + \frac{2a^3(c-d)(3c+7d)\cos(e+fx)}{3d^2(c+d)^2f\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+}$$

$$= \frac{2a^2(c-d)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3d(c+d)f(c+d\sin(e+fx))^{3/2}} + \frac{2a^3(c-d)(3c+7d)\cos(e+fx)}{3d^2(c+d)^2f\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+}$$

$$= -\frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{d^{5/2}f} + \frac{2a^2(c-d)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3d(c+d)f(c+d\sin(e+fx))^{3/2}} + \frac{}{3d}$$

Mathematica [A] time = 7.88622, size = 261, normalized size = 1.43

$$\frac{(a(\sin(e+fx)+1))^{5/2}\left(\frac{2(c-d)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(3c^2+4d(c+2d)\sin(e+fx)+8cd+d^2\right)}{3d^2(c+d)^2(c+d\sin(e+fx))^{3/2}} + \frac{2\tan^{-1}\left(\frac{\sqrt{2}\sqrt{d}\sin\left(\frac{1}{4}(2e+2fx-\pi)\right)}{\sqrt{c+d\sin(e+fx)}}\right)-\log\left(\sqrt{c+d\sin(e+}\right.}{}\right.}{f\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)/(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^(5/2)*((2*ArcTan[(Sqrt[2]*Sqrt[d]*Sin[(2*e - Pi + 2
*f*x)/4])/Sqrt[c + d*Sin[e + f*x]]] + ArcTanh[(Sqrt[2]*Sqrt[d]*Cos[(2*e - P
i + 2*f*x)/4])/Sqrt[c + d*Sin[e + f*x]]] - Log[Sqrt[2]*Sqrt[d]*Cos[(2*e - P
i + 2*f*x)/4] + Sqrt[c + d*Sin[e + f*x]]])/d^(5/2) + (2*(c - d)*(Cos[(e + f
*x)/2] - Sin[(e + f*x)/2])*(3*c^2 + 8*c*d + d^2 + 4*d*(c + 2*d)*Sin[e + f*x
]))/(3*d^2*(c + d)^2*(c + d*Sin[e + f*x])^(3/2))))/(f*(Cos[(e + f*x)/2] + S
in[(e + f*x)/2])^5)
```

Maple [B] time = 0.435, size = 16223, normalized size = 88.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)/(d*sin(f*x + e) + c)^(5/2), x)
```

Fricas [B] time = 5.4359, size = 5053, normalized size = 27.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] [-1/12*(3*(a^2*c^4 + 4*a^2*c^3*d + 6*a^2*c^2*d^2 + 4*a^2*c*d^3 + a^2*d^4 -
(a^2*c^2*d^2 + 2*a^2*c*d^3 + a^2*d^4)*cos(f*x + e)^3 - (2*a^2*c^3*d + 5*a^2
*c^2*d^2 + 4*a^2*c*d^3 + a^2*d^4)*cos(f*x + e)^2 + (a^2*c^4 + 2*a^2*c^3*d +
 2*a^2*c^2*d^2 + 2*a^2*c*d^3 + a^2*d^4)*cos(f*x + e) + (a^2*c^4 + 4*a^2*c^3
*d + 6*a^2*c^2*d^2 + 4*a^2*c*d^3 + a^2*d^4 - (a^2*c^2*d^2 + 2*a^2*c*d^3 + a
^2*d^4)*cos(f*x + e)^2 + 2*(a^2*c^3*d + 2*a^2*c^2*d^2 + a^2*c*d^3)*cos(f*x
+ e))*sin(f*x + e))*sqrt(-a/d)*log((128*a*d^4*cos(f*x + e)^5 + a*c^4 + 4*a*
c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 + a*d^4 + 128*(2*a*c*d^3 - a*d^4)*cos(f*x +
 e)^4 - 32*(5*a*c^2*d^2 - 14*a*c*d^3 + 13*a*d^4)*cos(f*x + e)^3 - 32*(a*c^3
*d - 2*a*c^2*d^2 + 9*a*c*d^3 - 4*a*d^4)*cos(f*x + e)^2 - 8*(16*d^4*cos(f*x
+ e)^4 - c^3*d + 17*c^2*d^2 - 59*c*d^3 + 51*d^4 + 24*(c*d^3 - d^4)*cos(f*x
+ e)^3 - 2*(5*c^2*d^2 - 26*c*d^3 + 33*d^4)*cos(f*x + e)^2 - (c^3*d - 7*c^2*
d^2 + 31*c*d^3 - 25*d^4)*cos(f*x + e) + (16*d^4*cos(f*x + e)^3 + c^3*d - 17
*c^2*d^2 + 59*c*d^3 - 51*d^4 - 8*(3*c*d^3 - 5*d^4)*cos(f*x + e)^2 - 2*(5*c^
2*d^2 - 14*c*d^3 + 13*d^4)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e)
+ a)*sqrt(d*sin(f*x + e) + c)*sqrt(-a/d) + (a*c^4 - 28*a*c^3*d + 230*a*c^2*
d^2 - 476*a*c*d^3 + 289*a*d^4)*cos(f*x + e) + (128*a*d^4*cos(f*x + e)^4 + a
*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 + a*d^4 - 256*(a*c*d^3 - a*d^4)*
cos(f*x + e)^3 - 32*(5*a*c^2*d^2 - 6*a*c*d^3 + 5*a*d^4)*cos(f*x + e)^2 + 32
*(a*c^3*d - 7*a*c^2*d^2 + 15*a*c*d^3 - 9*a*d^4)*cos(f*x + e))*sin(f*x + e))
/(cos(f*x + e) + sin(f*x + e) + 1)) + 8*(3*a^2*c^3 + a^2*c^2*d - 11*a^2*c*d
^2 + 7*a^2*d^3 + 4*(a^2*c^2*d + a^2*c*d^2 - 2*a^2*d^3)*cos(f*x + e)^2 + (3*
a^2*c^3 + 5*a^2*c^2*d - 7*a^2*c*d^2 - a^2*d^3)*cos(f*x + e) - (3*a^2*c^3 +
a^2*c^2*d - 11*a^2*c*d^2 + 7*a^2*d^3 - 4*(a^2*c^2*d + a^2*c*d^2 - 2*a^2*d^3
)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e)
+ c))/((c^2*d^4 + 2*c*d^5 + d^6)*f*cos(f*x + e)^3 + (2*c^3*d^3 + 5*c^2*d^4
+ 4*c*d^5 + d^6)*f*cos(f*x + e)^2 - (c^4*d^2 + 2*c^3*d^3 + 2*c^2*d^4 + 2*c*
d^5 + d^6)*f*cos(f*x + e) - (c^4*d^2 + 4*c^3*d^3 + 6*c^2*d^4 + 4*c*d^5 + d^
```

```
6)*f + ((c^2*d^4 + 2*c*d^5 + d^6)*f*cos(f*x + e)^2 - 2*(c^3*d^3 + 2*c^2*d^4
 + c*d^5)*f*cos(f*x + e) - (c^4*d^2 + 4*c^3*d^3 + 6*c^2*d^4 + 4*c*d^5 + d^6
)*f)*sin(f*x + e)), -1/6*(3*(a^2*c^4 + 4*a^2*c^3*d + 6*a^2*c^2*d^2 + 4*a^2*
c*d^3 + a^2*d^4 - (a^2*c^2*d^2 + 2*a^2*c*d^3 + a^2*d^4)*cos(f*x + e)^3 - (2
*a^2*c^3*d + 5*a^2*c^2*d^2 + 4*a^2*c*d^3 + a^2*d^4)*cos(f*x + e)^2 + (a^2*c
^4 + 2*a^2*c^3*d + 2*a^2*c^2*d^2 + 2*a^2*c*d^3 + a^2*d^4)*cos(f*x + e) + (a
^2*c^4 + 4*a^2*c^3*d + 6*a^2*c^2*d^2 + 4*a^2*c*d^3 + a^2*d^4 - (a^2*c^2*d^2
 + 2*a^2*c*d^3 + a^2*d^4)*cos(f*x + e)^2 + 2*(a^2*c^3*d + 2*a^2*c^2*d^2 + a
^2*c*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a/d)*arctan(1/4*(8*d^2*cos(f*x +
 e)^2 - c^2 + 6*c*d - 9*d^2 - 8*(c*d - d^2)*sin(f*x + e))*sqrt(a*sin(f*x +
e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt(a/d)/(2*a*d^2*cos(f*x + e)^3 - (3*a*c
*d - a*d^2)*cos(f*x + e)*sin(f*x + e) - (a*c^2 - a*c*d + 2*a*d^2)*cos(f*x +
 e))) + 4*(3*a^2*c^3 + a^2*c^2*d - 11*a^2*c*d^2 + 7*a^2*d^3 + 4*(a^2*c^2*d
+ a^2*c*d^2 - 2*a^2*d^3)*cos(f*x + e)^2 + (3*a^2*c^3 + 5*a^2*c^2*d - 7*a^2*
c*d^2 - a^2*d^3)*cos(f*x + e) - (3*a^2*c^3 + a^2*c^2*d - 11*a^2*c*d^2 + 7*a
^2*d^3 - 4*(a^2*c^2*d + a^2*c*d^2 - 2*a^2*d^3)*cos(f*x + e))*sin(f*x + e))*
sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/((c^2*d^4 + 2*c*d^5 + d^
6)*f*cos(f*x + e)^3 + (2*c^3*d^3 + 5*c^2*d^4 + 4*c*d^5 + d^6)*f*cos(f*x + e
)^2 - (c^4*d^2 + 2*c^3*d^3 + 2*c^2*d^4 + 2*c*d^5 + d^6)*f*cos(f*x + e) - (c
^4*d^2 + 4*c^3*d^3 + 6*c^2*d^4 + 4*c*d^5 + d^6)*f + ((c^2*d^4 + 2*c*d^5 + d
^6)*f*cos(f*x + e)^2 - 2*(c^3*d^3 + 2*c^2*d^4 + c*d^5)*f*cos(f*x + e) - (c^
4*d^2 + 4*c^3*d^3 + 6*c^2*d^4 + 4*c*d^5 + d^6)*f)*sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)/(c+d*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)/(d*sin(f*x + e) + c)^(5/2), x)
```

3.585 $\int \frac{(a+a\sin(e+fx))^{5/2}}{(c+d\sin(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=189

$$-\frac{2a^3\left(3c^2+14cd+43d^2\right)\cos(e+fx)}{15d^2f(c+d)^3\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}+\frac{2a^3(c-d)(3c+11d)\cos(e+fx)}{15d^2f(c+d)^2\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^{3/2}}+\frac{2a^2(c$$

```
[Out] (2*a^2*(c - d)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(5*d*(c + d)*f*(c + d
*Sin[e + f*x])^(5/2)) + (2*a^3*(c - d)*(3*c + 11*d)*Cos[e + f*x])/(15*d^2*(
c + d)^2*f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(3/2)) - (2*a^3*(3
*c^2 + 14*c*d + 43*d^2)*Cos[e + f*x])/(15*d^2*(c + d)^3*f*Sqrt[a + a*Sin[e
+ f*x]]*Sqrt[c + d*Sin[e + f*x]])
```

Rubi [A] time = 0.484717, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2762, 2980, 2771}

$$-\frac{2a^3\left(3c^2+14cd+43d^2\right)\cos(e+fx)}{15d^2f(c+d)^3\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}+\frac{2a^3(c-d)(3c+11d)\cos(e+fx)}{15d^2f(c+d)^2\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^{3/2}}+\frac{2a^2(c$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)/(c + d*Sin[e + f*x])^(7/2),x]
```


```
[Out] (2*a^2*(c - d)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(5*d*(c + d)*f*(c + d
*Sin[e + f*x])^(5/2)) + (2*a^3*(c - d)*(3*c + 11*d)*Cos[e + f*x])/(15*d^2*(
c + d)^2*f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(3/2)) - (2*a^3*(3
*c^2 + 14*c*d + 43*d^2)*Cos[e + f*x])/(15*d^2*(c + d)^3*f*Sqrt[a + a*Sin[e
+ f*x]]*Sqrt[c + d*Sin[e + f*x]])
```


Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
```

```
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{5/2}}{(c+d\sin(e+fx))^{7/2}}\,dx = \frac{2a^2(c-d)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{5d(c+d)f(c+d\sin(e+fx))^{5/2}} - \frac{(2a)\int \frac{\sqrt{a+a\sin(e+fx)}\left(\frac{1}{2}a(c-11d)-\frac{1}{2}a(3c+7d)\sin(e+fx)\right)}{(c+d\sin(e+fx))^{5/2}}}{5d(c+d)}$$

$$= \frac{2a^2(c-d)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{5d(c+d)f(c+d\sin(e+fx))^{5/2}} + \frac{2a^3(c-d)(3c+11d)\cos(e+fx)}{15d^2(c+d)^2f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+}$$

$$= \frac{2a^2(c-d)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{5d(c+d)f(c+d\sin(e+fx))^{5/2}} + \frac{2a^3(c-d)(3c+11d)\cos(e+fx)}{15d^2(c+d)^2f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+}$$

Mathematica [A] time = 2.05218, size = 152, normalized size = 0.8

$$-\frac{a^2\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(4\left(7c^2+46cd+7d^2\right)\sin(e+fx)-\left(3c^2+14cd+43d^2\right)\cos\right.}{15f(c+d)^3\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)(c+d\sin(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)/(c + d*Sin[e + f*x])^(7/2),x]
```

```
[Out] -(a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(89*
c^2 + 42*c*d + 49*d^2 - (3*c^2 + 14*c*d + 43*d^2)*Cos[2*(e + f*x)] + 4*(7*c
^2 + 46*c*d + 7*d^2)*Sin[e + f*x]))/(15*(c + d)^3*f*(Cos[(e + f*x)/2] + Sin
[(e + f*x)/2])*(c + d*Sin[e + f*x])^(5/2))
```

Maple [B] time = 0.262, size = 793, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(7/2),x)
```

```
[Out] -2/15/f/(c+d)^3*(a*(1+sin(f*x+e)))^(5/2)*(c+d*sin(f*x+e))^(1/2)*(439*c*cos(
f*x+e)^4*d^4+128*d^5+544*cos(f*x+e)^2*c^2*d^3+128*c^4*d-256*c^2*d^3+128*c*d
^4+523*cos(f*x+e)^4*d^5-432*cos(f*x+e)^2*d^5-262*cos(f*x+e)^6*d^5-128*c^5*s
in(f*x+e)+256*c^2*d^3*sin(f*x+e)-128*c*d^4*sin(f*x+e)-cos(f*x+e)^6*c^2*d^3-
181*cos(f*x+e)^6*c*d^4+7*cos(f*x+e)^4*c^4*d-194*cos(f*x+e)^4*c^3*d^2+16*sin
(f*x+e)*cos(f*x+e)^2*c^5-128*d^5*sin(f*x+e)+115*sin(f*x+e)*cos(f*x+e)^6*d^5
+3*cos(f*x+e)^8*c^2*d^3+14*cos(f*x+e)^8*c*d^4-9*cos(f*x+e)^6*c^4*d-27*cos(f
*x+e)^6*c^3*d^2-256*c^3*d^2+48*sin(f*x+e)*cos(f*x+e)^2*c^4*d+79*sin(f*x+e)*
cos(f*x+e)^6*c*d^4+50*sin(f*x+e)*cos(f*x+e)^4*c^3*d^2+114*sin(f*x+e)*cos(f*
x+e)^4*c^2*d^3-287*sin(f*x+e)*cos(f*x+e)^4*c*d^4-352*sin(f*x+e)*cos(f*x+e)^
2*c^3*d^2-416*sin(f*x+e)*cos(f*x+e)^2*c^2*d^3+336*sin(f*x+e)*cos(f*x+e)^2*c
*d^4+480*c^3*cos(f*x+e)^2*d^2-400*c*d^4*cos(f*x+e)^2-355*sin(f*x+e)*cos(f*x
+e)^4*d^5+368*sin(f*x+e)*cos(f*x+e)^2*d^5-112*cos(f*x+e)^2*c^4*d-128*sin(f*
x+e)*c^4*d+256*sin(f*x+e)*c^3*d^2-290*cos(f*x+e)^4*c^2*d^3+9*sin(f*x+e)*cos
(f*x+e)^6*c^3*d^2+37*sin(f*x+e)*cos(f*x+e)^6*c^2*d^3+sin(f*x+e)*cos(f*x+e)^
```

```
4*c^4*d+128*c^5+43*cos(f*x+e)^8*d^5-5*cos(f*x+e)^4*c^5-80*cos(f*x+e)^2*c^5-
3*sin(f*x+e)*cos(f*x+e)^4*c^5)/cos(f*x+e)^5/(cos(f*x+e)^2*d^2+c^2-d^2)^3
```

Maxima [B] time = 2.04848, size = 626, normalized size = 3.31

$$-\frac{2\left(\left(43\,c^3+14\,c^2d+3\,cd^2\right)a^{\frac{5}{2}}-\frac{\left(15\,c^3-256\,c^2d-53\,cd^2-6\,d^3\right)a^{\frac{5}{2}}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{\left(113\,c^3-116\,c^2d+493\,cd^2+50\,d^3\right)a^{\frac{5}{2}}\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}-\frac{5\left(17\,c^3-82\,c^2 \cdots\right.}{\cdots}\right.}{15\left(c^3+3 \cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(7/2),x, algorithm="maxim
a")
```

```
[Out] -2/15*((43*c^3 + 14*c^2*d + 3*c*d^2)*a^(5/2) - (15*c^3 - 256*c^2*d - 53*c*d
^2 - 6*d^3)*a^(5/2)*sin(f*x + e)/(cos(f*x + e) + 1) + (113*c^3 - 116*c^2*d
+ 493*c*d^2 + 50*d^3)*a^(5/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 5*(17*c
^3 - 82*c^2*d + 65*c*d^2 - 60*d^3)*a^(5/2)*sin(f*x + e)^3/(cos(f*x + e) + 1
)^3 + 5*(17*c^3 - 82*c^2*d + 65*c*d^2 - 60*d^3)*a^(5/2)*sin(f*x + e)^4/(cos
(f*x + e) + 1)^4 - (113*c^3 - 116*c^2*d + 493*c*d^2 + 50*d^3)*a^(5/2)*sin(f
*x + e)^5/(cos(f*x + e) + 1)^5 + (15*c^3 - 256*c^2*d - 53*c*d^2 - 6*d^3)*a^
(5/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - (43*c^3 + 14*c^2*d + 3*c*d^2)*a
^(5/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7)*(sin(f*x + e)^2/(cos(f*x + e) +
 1)^2 + 1)/((c^3 + 3*c^2*d + 3*c*d^2 + d^3 + (c^3 + 3*c^2*d + 3*c*d^2 + d^3
)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2)*(c + 2*d*sin(f*x + e)/(cos(f*x + e)
+ 1) + c*sin(f*x + e)^2/(cos(f*x + e) + 1)^2)^(7/2)*f)
```

Fricas [B] time = 2.08252, size = 1467, normalized size = 7.76

$$-\frac{2\left(32\,a^2c^2-64\,a^2cd+\cdots\right.}{15\left(\left(c^3d^3+3\,c^2d^4+3\,cd^5+d^6\right)f\cos\left(fx+e\right)^4-3\left(c^4d^2+3\,c^3d^3+3\,c^2d^4+cd^5\right)f\cos\left(fx+e\right)^3-\left(3\,c^5d+12\,c^4d^2+\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(7/2),x, algorithm="frica
s")
```

```
[Out] -2/15*(32*a^2*c^2 - 64*a^2*c*d + 32*a^2*d^2 - (3*a^2*c^2 + 14*a^2*c*d + 43*
a^2*d^2)*cos(f*x + e)^3 + (11*a^2*c^2 + 78*a^2*c*d - 29*a^2*d^2)*cos(f*x +
e)^2 + 2*(23*a^2*c^2 + 14*a^2*c*d + 23*a^2*d^2)*cos(f*x + e) - (32*a^2*c^2
- 64*a^2*c*d + 32*a^2*d^2 - (3*a^2*c^2 + 14*a^2*c*d + 43*a^2*d^2)*cos(f*x +
 e)^2 - 2*(7*a^2*c^2 + 46*a^2*c*d + 7*a^2*d^2)*cos(f*x + e))*sin(f*x + e))*
sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)/((c^3*d^3 + 3*c^2*d^4 + 3
*c*d^5 + d^6)*f*cos(f*x + e)^4 - 3*(c^4*d^2 + 3*c^3*d^3 + 3*c^2*d^4 + c*d^5
)*f*cos(f*x + e)^3 - (3*c^5*d + 12*c^4*d^2 + 20*c^3*d^3 + 18*c^2*d^4 + 9*c*
d^5 + 2*d^6)*f*cos(f*x + e)^2 + (c^6 + 3*c^5*d + 6*c^4*d^2 + 10*c^3*d^3 + 9
*c^2*d^4 + 3*c*d^5)*f*cos(f*x + e) + (c^6 + 6*c^5*d + 15*c^4*d^2 + 20*c^3*d
^3 + 15*c^2*d^4 + 6*c*d^5 + d^6)*f - ((c^3*d^3 + 3*c^2*d^4 + 3*c*d^5 + d^6)
*f*cos(f*x + e)^3 + (3*c^4*d^2 + 10*c^3*d^3 + 12*c^2*d^4 + 6*c*d^5 + d^6)*f
*cos(f*x + e)^2 - (3*c^5*d + 9*c^4*d^2 + 10*c^3*d^3 + 6*c^2*d^4 + 3*c*d^5 +
 d^6)*f*cos(f*x + e) - (c^6 + 6*c^5*d + 15*c^4*d^2 + 20*c^3*d^3 + 15*c^2*d^
```

```
4 + 6*c*d^5 + d^6)*f)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)/(c+d*sin(f*x+e))**(7/2),x)

[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\sin(fx+e)+a)^{\frac{5}{2}}}{(d\sin(fx+e)+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(7/2),x, algorithm="giac"
)

[Out] integrate((a*sin(f*x + e) + a)^(5/2)/(d*sin(f*x + e) + c)^(7/2), x)
```

3.586 $\int \frac{(a+a\sin(e+fx))^{5/2}}{(c+d\sin(e+fx))^{9/2}}\,dx$

Optimal. Leaf size=254

$$-\frac{4a^3\left(3c^2+22cd+115d^2\right)\cos(e+fx)}{105d^2f(c+d)^4\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}-\frac{2a^3\left(3c^2+22cd+115d^2\right)\cos(e+fx)}{105d^2f(c+d)^3\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^{3/2}}+\frac{}{35}$$

```
[Out] (2*a^2*(c - d)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(7*d*(c + d)*f*(c + d
*Sin[e + f*x])^(7/2)) + (6*a^3*(c - d)*(c + 5*d)*Cos[e + f*x])/(35*d^2*(c +
 d)^2*f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(5/2)) - (2*a^3*(3*c^
2 + 22*c*d + 115*d^2)*Cos[e + f*x])/(105*d^2*(c + d)^3*f*Sqrt[a + a*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(3/2)) - (4*a^3*(3*c^2 + 22*c*d + 115*d^2)*Cos[
e + f*x])/(105*d^2*(c + d)^4*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c + d*Sin[e +
f*x]])
```

Rubi [A] time = 0.625349, antiderivative size = 254, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2762, 2980, 2772, 2771}

$$-\frac{4a^3\left(3c^2+22cd+115d^2\right)\cos(e+fx)}{105d^2f(c+d)^4\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}-\frac{2a^3\left(3c^2+22cd+115d^2\right)\cos(e+fx)}{105d^2f(c+d)^3\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^{3/2}}+\frac{}{35}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)/(c + d*Sin[e + f*x])^(9/2),x]
```


```
[Out] (2*a^2*(c - d)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(7*d*(c + d)*f*(c + d
*Sin[e + f*x])^(7/2)) + (6*a^3*(c - d)*(c + 5*d)*Cos[e + f*x])/(35*d^2*(c +
 d)^2*f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(5/2)) - (2*a^3*(3*c^
2 + 22*c*d + 115*d^2)*Cos[e + f*x])/(105*d^2*(c + d)^3*f*Sqrt[a + a*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(3/2)) - (4*a^3*(3*c^2 + 22*c*d + 115*d^2)*Cos[
e + f*x])/(105*d^2*(c + d)^4*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c + d*Sin[e +
f*x]])
```


Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{5/2}}{(c+d\sin(e+fx))^{9/2}}\,dx = \frac{2a^2(c-d)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{7d(c+d)f(c+d\sin(e+fx))^{7/2}} - \frac{(2a)\int \frac{\sqrt{a+a\sin(e+fx)}\left(\frac{1}{2}a(c-15d)-\frac{1}{2}a(3c+11d)\sin(e+fx)\right)}{(c+d\sin(e+fx))^{7/2}}}{7d(c+d)}$$

$$= \frac{2a^2(c-d)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{7d(c+d)f(c+d\sin(e+fx))^{7/2}} + \frac{6a^3(c-d)(c+5d)\cos(e+fx)}{35d^2(c+d)^2f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+}$$

$$= \frac{2a^2(c-d)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{7d(c+d)f(c+d\sin(e+fx))^{7/2}} + \frac{6a^3(c-d)(c+5d)\cos(e+fx)}{35d^2(c+d)^2f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+}$$

$$= \frac{2a^2(c-d)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{7d(c+d)f(c+d\sin(e+fx))^{7/2}} + \frac{6a^3(c-d)(c+5d)\cos(e+fx)}{35d^2(c+d)^2f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+}$$

Mathematica [A] time = 4.20083, size = 216, normalized size = 0.85

$$\frac{a^2\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(-\left(1865c^2d+196c^3+694cd^2+465d^3\right)\sin(e+fx)+\left(157c^2d\right.\right.}{105f(c+d)^4\left(\sin\left(\frac{1}{2}(e\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)/(c + d*Sin[e + f*x])^(9/2),x]
```

```
[Out] (a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(-623
*c^3 - 495*c^2*d - 977*c*d^2 - 145*d^3 + (21*c^3 + 157*c^2*d + 827*c*d^2 +
115*d^3)*Cos[2*(e + f*x)] - (196*c^3 + 1865*c^2*d + 694*c*d^2 + 465*d^3)*Si
n[e + f*x] + 3*c^2*d*Sin[3*(e + f*x)] + 22*c*d^2*Sin[3*(e + f*x)] + 115*d^3
*Sin[3*(e + f*x)]))/(105*(c + d)^4*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*
(c + d*Sin[e + f*x])^(7/2))
```

Maple [B] time = 0.374, size = 1223, normalized size = 4.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(9/2),x)
```

```
[Out] -2/105/f/(c+d)^4*(c+d*sin(f*x+e))^(1/2)*(a*(1+sin(f*x+e)))^(5/2)*(-485*cos(
f*x+e)^8*c*d^6-78*cos(f*x+e)^6*c^6*d+575*sin(f*x+e)*cos(f*x+e)^8*d^7+225*si
n(f*x+e)*cos(f*x+e)^8*c*d^6+1408*sin(f*x+e)*cos(f*x+e)^6*c^3*d^4+2392*sin(f
*x+e)*cos(f*x+e)^6*c^2*d^5-950*sin(f*x+e)*cos(f*x+e)^6*c*d^6+479*sin(f*x+e)
*cos(f*x+e)^4*c^5*d^2+1261*sin(f*x+e)*cos(f*x+e)^4*c^4*d^3-4367*sin(f*x+e)*
cos(f*x+e)^4*c^3*d^4-7117*sin(f*x+e)*cos(f*x+e)^4*c^2*d^5+1669*sin(f*x+e)*c
os(f*x+e)^4*c*d^6+368*sin(f*x+e)*cos(f*x+e)^2*c^6*d-3344*sin(f*x+e)*cos(f*x
+e)^2*c^5*d^2-5008*sin(f*x+e)*cos(f*x+e)^2*c^4*d^3+4560*sin(f*x+e)*cos(f*x+
e)^2*c^3*d^4+7120*sin(f*x+e)*cos(f*x+e)^2*c^2*d^5-1328*sin(f*x+e)*cos(f*x+e
)^2*c*d^6+896*c^7+1152*c^6*d+2432*c^2*d^5-384*c*d^6-2944*c^4*d^3-640*d^7-21
76*c^5*d^2+1664*c^3*d^4-2350*sin(f*x+e)*cos(f*x+e)^6*d^7-172*cos(f*x+e)^6*c
^4*d^3-2968*cos(f*x+e)^6*c^3*d^4-5370*cos(f*x+e)^6*c^2*d^5+1590*cos(f*x+e)^
6*c*d^6+3615*sin(f*x+e)*cos(f*x+e)^4*d^7+39*cos(f*x+e)^4*c^6*d-1879*cos(f*x
+e)^4*c^5*d^2-3397*cos(f*x+e)^4*c^4*d^3+895*cos(f*x+e)^8*c^2*d^5+6439*cos(f
*x+e)^4*c^3*d^4+10373*cos(f*x+e)^4*c^2*d^5-2285*cos(f*x+e)^4*c*d^6+112*sin(
f*x+e)*cos(f*x+e)^2*c^7-2480*sin(f*x+e)*cos(f*x+e)^2*d^7-944*cos(f*x+e)^2*c
^6*d+4432*cos(f*x+e)^2*c^5*d^2+6480*cos(f*x+e)^2*c^4*d^3-5392*cos(f*x+e)^2*
c^3*d^4-8336*cos(f*x+e)^2*c^2*d^5+1520*cos(f*x+e)^2*c*d^6-1152*sin(f*x+e)*c
^6*d+2176*sin(f*x+e)*c^5*d^2+2944*sin(f*x+e)*c^4*d^3-1664*sin(f*x+e)*c^3*d^
4-2432*sin(f*x+e)*c^2*d^5+384*sin(f*x+e)*c*d^6-1555*cos(f*x+e)^8*d^7+3940*c
os(f*x+e)^6*d^7-4775*cos(f*x+e)^4*d^7-560*cos(f*x+e)^2*c^7+2800*cos(f*x+e)^
2*d^7-896*sin(f*x+e)*c^7+640*sin(f*x+e)*d^7+230*cos(f*x+e)^10*d^7-35*cos(f*
x+e)^4*c^7-302*cos(f*x+e)^6*c^5*d^2+6*cos(f*x+e)^10*c^2*d^5+44*cos(f*x+e)^1
0*c*d^6+48*cos(f*x+e)^8*c^4*d^3+257*cos(f*x+e)^8*c^3*d^4-21*sin(f*x+e)*cos(
f*x+e)^4*c^7+3*sin(f*x+e)*cos(f*x+e)^8*c^3*d^4+37*sin(f*x+e)*cos(f*x+e)^8*c
^2*d^5+102*sin(f*x+e)*cos(f*x+e)^6*c^5*d^2+518*sin(f*x+e)*cos(f*x+e)^6*c^4*
d^3+sin(f*x+e)*cos(f*x+e)^4*c^6*d)/cos(f*x+e)^5/(cos(f*x+e)^2*d^2+c^2-d^2)^
4
```

Maxima [B] time = 2.25458, size = 949, normalized size = 3.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(9/2),x, algorithm="maxim
a")
```

```
[Out] -2/105*((301*c^4 + 169*c^3*d + 75*c^2*d^2 + 15*c*d^3)*a^(5/2) - 3*(35*c^4 -
 763*c^3*d - 297*c^2*d^2 - 85*c*d^3 - 10*d^4)*a^(5/2)*sin(f*x + e)/(cos(f*x
 + e) + 1) + 6*(182*c^4 - 127*c^3*d + 1059*c^2*d^2 + 251*c*d^3 + 35*d^4)*a^
(5/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 14*(50*c^4 - 421*c^3*d + 201*c^
2*d^2 - 535*c*d^3 - 55*d^4)*a^(5/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 1
26*(11*c^4 - 36*c^3*d + 80*c^2*d^2 - 40*c*d^3 + 25*d^4)*a^(5/2)*sin(f*x + e
)^4/(cos(f*x + e) + 1)^4 - 126*(11*c^4 - 36*c^3*d + 80*c^2*d^2 - 40*c*d^3 +
 25*d^4)*a^(5/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 14*(50*c^4 - 421*c^3
*d + 201*c^2*d^2 - 535*c*d^3 - 55*d^4)*a^(5/2)*sin(f*x + e)^6/(cos(f*x + e)
 + 1)^6 - 6*(182*c^4 - 127*c^3*d + 1059*c^2*d^2 + 251*c*d^3 + 35*d^4)*a^(5/
2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 3*(35*c^4 - 763*c^3*d - 297*c^2*d^
2 - 85*c*d^3 - 10*d^4)*a^(5/2)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - (301*c
^4 + 169*c^3*d + 75*c^2*d^2 + 15*c*d^3)*a^(5/2)*sin(f*x + e)^9/(cos(f*x + e
) + 1)^9)*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^2/((c^4 + 4*c^3*d + 6*c
^2*d^2 + 4*c*d^3 + d^4 + 2*(c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^3 + d^4)*sin(
f*x + e)^2/(cos(f*x + e) + 1)^2 + (c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^3 + d^
4)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4)*(c + 2*d*sin(f*x + e)/(cos(f*x + e)
```

```
+ 1) + c*sin(f*x + e)^2/(cos(f*x + e) + 1)^2)^(9/2)*f)
```

Fricas [B] time = 2.37057, size = 2342, normalized size = 9.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(9/2),x, algorithm="frica
s")
```

```
[Out] -2/105*(224*a^2*c^3 - 608*a^2*c^2*d + 544*a^2*c*d^2 - 160*a^2*d^3 - 2*(3*a^
2*c^2*d + 22*a^2*c*d^2 + 115*a^2*d^3)*cos(f*x + e)^4 - (21*a^2*c^3 + 157*a^
2*c^2*d + 827*a^2*c*d^2 + 115*a^2*d^3)*cos(f*x + e)^3 + (77*a^2*c^3 + 783*a
^2*c^2*d - 425*a^2*c*d^2 + 405*a^2*d^3)*cos(f*x + e)^2 + 2*(161*a^2*c^3 + 1
63*a^2*c^2*d + 451*a^2*c*d^2 + 65*a^2*d^3)*cos(f*x + e) - (224*a^2*c^3 - 60
8*a^2*c^2*d + 544*a^2*c*d^2 - 160*a^2*d^3 + 2*(3*a^2*c^2*d + 22*a^2*c*d^2 +
 115*a^2*d^3)*cos(f*x + e)^3 - (21*a^2*c^3 + 151*a^2*c^2*d + 783*a^2*c*d^2
- 115*a^2*d^3)*cos(f*x + e)^2 - 2*(49*a^2*c^3 + 467*a^2*c^2*d + 179*a^2*c*d
^2 + 145*a^2*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt
(d*sin(f*x + e) + c)/((c^4*d^4 + 4*c^3*d^5 + 6*c^2*d^6 + 4*c*d^7 + d^8)*f*c
os(f*x + e)^5 + (4*c^5*d^3 + 17*c^4*d^4 + 28*c^3*d^5 + 22*c^2*d^6 + 8*c*d^7
 + d^8)*f*cos(f*x + e)^4 - 2*(3*c^6*d^2 + 12*c^5*d^3 + 19*c^4*d^4 + 16*c^3*
d^5 + 9*c^2*d^6 + 4*c*d^7 + d^8)*f*cos(f*x + e)^3 - 2*(2*c^7*d + 11*c^6*d^2
 + 28*c^5*d^3 + 43*c^4*d^4 + 42*c^3*d^5 + 25*c^2*d^6 + 8*c*d^7 + d^8)*f*cos
(f*x + e)^2 + (c^8 + 4*c^7*d + 12*c^6*d^2 + 28*c^5*d^3 + 38*c^4*d^4 + 28*c^
3*d^5 + 12*c^2*d^6 + 4*c*d^7 + d^8)*f*cos(f*x + e) + (c^8 + 8*c^7*d + 28*c^
6*d^2 + 56*c^5*d^3 + 70*c^4*d^4 + 56*c^3*d^5 + 28*c^2*d^6 + 8*c*d^7 + d^8)*
f + ((c^4*d^4 + 4*c^3*d^5 + 6*c^2*d^6 + 4*c*d^7 + d^8)*f*cos(f*x + e)^4 - 4
*(c^5*d^3 + 4*c^4*d^4 + 6*c^3*d^5 + 4*c^2*d^6 + c*d^7)*f*cos(f*x + e)^3 - 2
*(3*c^6*d^2 + 14*c^5*d^3 + 27*c^4*d^4 + 28*c^3*d^5 + 17*c^2*d^6 + 6*c*d^7 +
 d^8)*f*cos(f*x + e)^2 + 4*(c^7*d + 4*c^6*d^2 + 7*c^5*d^3 + 8*c^4*d^4 + 7*c
^3*d^5 + 4*c^2*d^6 + c*d^7)*f*cos(f*x + e) + (c^8 + 8*c^7*d + 28*c^6*d^2 +
56*c^5*d^3 + 70*c^4*d^4 + 56*c^3*d^5 + 28*c^2*d^6 + 8*c*d^7 + d^8)*f)*sin(f
*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)/(c+d*sin(f*x+e))**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a \sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}{\left(d \sin\left(fx+e\right)+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(9/2),x, algorithm="giac"
)
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)/(d*sin(f*x + e) + c)^(9/2), x)
```

3.587 $\int \frac{(a+a\sin(e+fx))^{5/2}}{(c+d\sin(e+fx))^{11/2}}\,dx$

Optimal. Leaf size=317

$$-\frac{16a^3\left(c^2+10cd+73d^2\right)\cos(e+fx)}{315d^2f(c+d)^5\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}-\frac{8a^3\left(c^2+10cd+73d^2\right)\cos(e+fx)}{315d^2f(c+d)^4\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^{3/2}}-\frac{}{105d^2}$$

```
[Out] (2*a^2*(c - d)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(9*d*(c + d)*f*(c + d
*Sin[e + f*x])^(9/2)) + (2*a^3*(c - d)*(3*c + 19*d)*Cos[e + f*x])/(63*d^2*(
c + d)^2*f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(7/2)) - (2*a^3*(c
^2 + 10*c*d + 73*d^2)*Cos[e + f*x])/(105*d^2*(c + d)^3*f*Sqrt[a + a*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(5/2)) - (8*a^3*(c^2 + 10*c*d + 73*d^2)*Cos[e +
 f*x])/(315*d^2*(c + d)^4*f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(
3/2)) - (16*a^3*(c^2 + 10*c*d + 73*d^2)*Cos[e + f*x])/(315*d^2*(c + d)^5*f*
Sqrt[a + a*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])
```

Rubi [A] time = 0.775709, antiderivative size = 317, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2762, 2980, 2772, 2771}

$$-\frac{16a^3\left(c^2+10cd+73d^2\right)\cos(e+fx)}{315d^2f(c+d)^5\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}-\frac{8a^3\left(c^2+10cd+73d^2\right)\cos(e+fx)}{315d^2f(c+d)^4\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^{3/2}}-\frac{}{105d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)/(c + d*Sin[e + f*x])^(11/2),x]
```


```
[Out] (2*a^2*(c - d)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(9*d*(c + d)*f*(c + d
*Sin[e + f*x])^(9/2)) + (2*a^3*(c - d)*(3*c + 19*d)*Cos[e + f*x])/(63*d^2*(
c + d)^2*f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(7/2)) - (2*a^3*(c
^2 + 10*c*d + 73*d^2)*Cos[e + f*x])/(105*d^2*(c + d)^3*f*Sqrt[a + a*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(5/2)) - (8*a^3*(c^2 + 10*c*d + 73*d^2)*Cos[e +
 f*x])/(315*d^2*(c + d)^4*f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(
3/2)) - (16*a^3*(c^2 + 10*c*d + 73*d^2)*Cos[e + f*x])/(315*d^2*(c + d)^5*f*
Sqrt[a + a*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])
```


Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
```

```
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{5/2}}{(c+d\sin(e+fx))^{11/2}}\,dx = \frac{2a^2(c-d)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{9d(c+d)f(c+d\sin(e+fx))^{9/2}} - \frac{(2a)\int \frac{\sqrt{a+a\sin(e+fx)}\left(\frac{1}{2}a(c-19d)-\frac{3}{2}a(c+5d)\sin(e+f}{(c+d\sin(e+fx))^{9/2}}}{9d(c+d)}$$

$$= \frac{2a^2(c-d)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{9d(c+d)f(c+d\sin(e+fx))^{9/2}} + \frac{2a^3(c-d)(3c+19d)\cos(e+fx}{63d^2(c+d)^2f\sqrt{a+a\sin(e+fx)}(c+d\sin}$$

$$= \frac{2a^2(c-d)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{9d(c+d)f(c+d\sin(e+fx))^{9/2}} + \frac{2a^3(c-d)(3c+19d)\cos(e+fx}{63d^2(c+d)^2f\sqrt{a+a\sin(e+fx)}(c+d\sin}$$

$$= \frac{2a^2(c-d)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{9d(c+d)f(c+d\sin(e+fx))^{9/2}} + \frac{2a^3(c-d)(3c+19d)\cos(e+fx}{63d^2(c+d)^2f\sqrt{a+a\sin(e+fx)}(c+d\sin}$$

$$= \frac{2a^2(c-d)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{9d(c+d)f(c+d\sin(e+fx))^{9/2}} + \frac{2a^3(c-d)(3c+19d)\cos(e+fx}{63d^2(c+d)^2f\sqrt{a+a\sin(e+fx)}(c+d\sin}$$

Mathematica [A] time = 6.53461, size = 616, normalized size = 1.94

$$\frac{(a(\sin(e+fx)+1))^{5/2}\sqrt{c+d\sin(e+fx)}\left(-\frac{16\left(-c^2\sin\left(\frac{1}{2}(e+fx)\right)+c^2\cos\left(\frac{1}{2}(e+fx)\right)-10cd\sin\left(\frac{1}{2}(e+fx)\right)+10cd\cos\left(\frac{1}{2}(e+fx)\right)-73d^2\sin\left(\frac{1}{2}(e+f}{315d^2(c+d)^5(c+d\sin(e+fx))}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)/(c + d*Sin[e + f*x])^(11/2),x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^(5/2)*Sqrt[c + d*Sin[e + f*x]]*((-2*(c^2*Cos[(e + f
*x)/2] - 2*c*d*Cos[(e + f*x)/2] + d^2*Cos[(e + f*x)/2] - c^2*Sin[(e + f*x)/
2] + 2*c*d*Sin[(e + f*x)/2] - d^2*Sin[(e + f*x)/2]))/(9*d^2*(c + d)*(c + d*
Sin[e + f*x])^5) - (4*(-5*c^2*Cos[(e + f*x)/2] - 8*c*d*Cos[(e + f*x)/2] + 1
3*d^2*Cos[(e + f*x)/2] + 5*c^2*Sin[(e + f*x)/2] + 8*c*d*Sin[(e + f*x)/2] -
13*d^2*Sin[(e + f*x)/2]))/(63*d^2*(c + d)^2*(c + d*Sin[e + f*x])^4) - (2*(c
```

```
^2*Cos[(e + f*x)/2] + 10*c*d*Cos[(e + f*x)/2] + 73*d^2*Cos[(e + f*x)/2] - c
^2*Sin[(e + f*x)/2] - 10*c*d*Sin[(e + f*x)/2] - 73*d^2*Sin[(e + f*x)/2]))/(
105*d^2*(c + d)^3*(c + d*Sin[e + f*x])^3) - (8*(c^2*Cos[(e + f*x)/2] + 10*c
*d*Cos[(e + f*x)/2] + 73*d^2*Cos[(e + f*x)/2] - c^2*Sin[(e + f*x)/2] - 10*c
*d*Sin[(e + f*x)/2] - 73*d^2*Sin[(e + f*x)/2]))/(315*d^2*(c + d)^4*(c + d*S
in[e + f*x])^2) - (16*(c^2*Cos[(e + f*x)/2] + 10*c*d*Cos[(e + f*x)/2] + 73*
d^2*Cos[(e + f*x)/2] - c^2*Sin[(e + f*x)/2] - 10*c*d*Sin[(e + f*x)/2] - 73*
d^2*Sin[(e + f*x)/2]))/(315*d^2*(c + d)^5*(c + d*Sin[e + f*x]))))/(f*(Cos[(
e + f*x)/2] + Sin[(e + f*x)/2])^5)
```

Maple [B] time = 0.395, size = 1730, normalized size = 5.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(11/2),x)
```

```
[Out] -2/315/f/(c+d)^5*(a*(1+sin(f*x+e)))^(5/2)*(c+d*sin(f*x+e))^(1/2)*(11711*cos
(f*x+e)^4*c*d^8-51540*cos(f*x+e)^4*c^2*d^7+336*sin(f*x+e)*cos(f*x+e)^2*c^9+
8112*sin(f*x+e)*cos(f*x+e)^2*d^9-3312*cos(f*x+e)^2*c^8*d+16192*cos(f*x+e)^2
*c^7*d^2+8*cos(f*x+e)^12*c^2*d^7+80*cos(f*x+e)^12*c*d^8+1460*sin(f*x+e)*cos
(f*x+e)^10*d^9+37*cos(f*x+e)^10*c^4*d^5+360*cos(f*x+e)^10*c^3*d^6+2538*cos(
f*x+e)^10*c^2*d^7-1360*cos(f*x+e)^10*c*d^8-7535*sin(f*x+e)*cos(f*x+e)^8*d^9
+310*cos(f*x+e)^8*c^6*d^3+1875*cos(f*x+e)^8*c^5*d^4+6805*cos(f*x+e)^8*c^4*d
^5-2930*cos(f*x+e)^8*c^3*d^6-16320*cos(f*x+e)^8*c^2*d^7+6095*cos(f*x+e)^8*c
*d^8+15474*sin(f*x+e)*cos(f*x+e)^6*d^9+28864*cos(f*x+e)^2*c^6*d^3-23904*cos
(f*x+e)^2*c^5*d^4-47520*cos(f*x+e)^2*c^4*d^5+15168*cos(f*x+e)^2*c^3*d^6+309
12*cos(f*x+e)^2*c^2*d^7-5776*cos(f*x+e)^2*c*d^8-4224*sin(f*x+e)*c^8*d+7168*
sin(f*x+e)*c^7*d^2+12288*sin(f*x+e)*c^6*d^3-7424*sin(f*x+e)*c^5*d^4+4224*c^
8*d-12288*c^6*d^3-13568*sin(f*x+e)*c^4*d^5+4096*sin(f*x+e)*c^3*d^6+7168*sin
(f*x+e)*c^2*d^7-1152*sin(f*x+e)*c*d^8-1482*cos(f*x+e)^6*c^6*d^3-17010*cos(f
*x+e)^6*c^5*d^4-35980*cos(f*x+e)^6*c^4*d^5+11406*cos(f*x+e)^6*c^3*d^6+41570
*cos(f*x+e)^6*c^2*d^7-11902*cos(f*x+e)^6*c*d^8-63*sin(f*x+e)*cos(f*x+e)^4*c
^9-15847*sin(f*x+e)*cos(f*x+e)^4*d^9+87*cos(f*x+e)^4*c^8*d-7220*cos(f*x+e)^
4*c^7*d^2-15204*cos(f*x+e)^4*c^6*d^3+31650*cos(f*x+e)^4*c^5*d^4+63090*cos(f
*x+e)^4*c^4*d^5-19908*cos(f*x+e)^4*c^3*d^6-279*cos(f*x+e)^6*c^8*d-1310*cos(
f*x+e)^6*c^7*d^2+13568*c^4*d^5+2688*c^9-4096*d^6*c^3-7168*d^7*c^2+1152*d^8*
c-7168*c^7*d^2+7424*c^5*d^4-15*sin(f*x+e)*cos(f*x+e)^4*c^8*d+1812*sin(f*x+e
)*cos(f*x+e)^4*c^7*d^2+5380*sin(f*x+e)*cos(f*x+e)^4*c^6*d^3-22482*sin(f*x+e
)*cos(f*x+e)^4*c^5*d^4-44418*sin(f*x+e)*cos(f*x+e)^4*c^4*d^5+13860*sin(f*x+
e)*cos(f*x+e)^4*c^3*d^6+38772*sin(f*x+e)*cos(f*x+e)^4*c^2*d^7-9255*sin(f*x+
e)*cos(f*x+e)^4*c*d^8+4*sin(f*x+e)*cos(f*x+e)^10*c^3*d^6+60*sin(f*x+e)*cos(
f*x+e)^10*c^2*d^7+492*sin(f*x+e)*cos(f*x+e)^10*c*d^8-35*sin(f*x+e)*cos(f*x+
e)^8*c^5*d^4+5*sin(f*x+e)*cos(f*x+e)^8*c^4*d^5+1650*sin(f*x+e)*cos(f*x+e)^8
*c^3*d^6+5850*sin(f*x+e)*cos(f*x+e)^8*c^2*d^7-3295*sin(f*x+e)*cos(f*x+e)^8*
c*d^8+1200*sin(f*x+e)*cos(f*x+e)^2*c^8*d-12608*sin(f*x+e)*cos(f*x+e)^2*c^7*
d^2-22720*sin(f*x+e)*cos(f*x+e)^2*c^6*d^3+20192*sin(f*x+e)*cos(f*x+e)^2*c^5
*d^4+40736*sin(f*x+e)*cos(f*x+e)^2*c^4*d^5-13120*sin(f*x+e)*cos(f*x+e)^2*c^
3*d^6-27328*sin(f*x+e)*cos(f*x+e)^2*c^2*d^7+5200*sin(f*x+e)*cos(f*x+e)^2*c*
d^8+458*sin(f*x+e)*cos(f*x+e)^6*c^7*d^2+2730*sin(f*x+e)*cos(f*x+e)^6*c^6*d^
3+8774*sin(f*x+e)*cos(f*x+e)^6*c^5*d^4+17070*sin(f*x+e)*cos(f*x+e)^6*c^4*d^
5-6490*sin(f*x+e)*cos(f*x+e)^6*c^3*d^6-24522*sin(f*x+e)*cos(f*x+e)^6*c^2*d^
7+8010*sin(f*x+e)*cos(f*x+e)^6*c*d^8+1664*d^9+584*cos(f*x+e)^12*d^9-4599*co
s(f*x+e)^10*d^9+14245*cos(f*x+e)^8*d^9-22645*cos(f*x+e)^6*d^9-105*cos(f*x+e
)^4*c^9+19695*cos(f*x+e)^4*d^9-1680*cos(f*x+e)^2*c^9-8944*cos(f*x+e)^2*d^9-
2688*sin(f*x+e)*c^9-1664*sin(f*x+e)*d^9)/cos(f*x+e)^5/(cos(f*x+e)^2*d^2+c^2
```

```
-d^2)^5
```

Maxima [B] time = 2.36932, size = 1328, normalized size = 4.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(11/2),x, algorithm="maxi
ma")
```

```
[Out] -2/315*((903*c^5 + 720*c^4*d + 494*c^3*d^2 + 200*c^2*d^3 + 35*c*d^4)*a^(5/2
) - (315*c^5 - 8358*c^4*d - 4770*c^3*d^2 - 2284*c^2*d^3 - 625*c*d^4 - 70*d^
5)*a^(5/2)*sin(f*x + e)/(cos(f*x + e) + 1) + (4179*c^5 - 1710*c^4*d + 30878
*c^3*d^2 + 11540*c^2*d^3 + 3383*c*d^4 + 450*d^5)*a^(5/2)*sin(f*x + e)^2/(co
s(f*x + e) + 1)^2 - 3*(805*c^5 - 9912*c^4*d + 2330*c^3*d^2 - 18504*c^2*d^3
- 3895*c*d^4 - 504*d^5)*a^(5/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 6*(12
39*c^5 - 3100*c^4*d + 12918*c^3*d^2 - 3560*c^2*d^3 + 8043*c*d^4 + 700*d^5)*
a^(5/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 42*(149*c^5 - 894*c^4*d + 140
2*c^3*d^2 - 2052*c^2*d^3 + 745*c*d^4 - 390*d^5)*a^(5/2)*sin(f*x + e)^5/(cos
(f*x + e) + 1)^5 + 42*(149*c^5 - 894*c^4*d + 1402*c^3*d^2 - 2052*c^2*d^3 +
745*c*d^4 - 390*d^5)*a^(5/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 6*(1239*
c^5 - 3100*c^4*d + 12918*c^3*d^2 - 3560*c^2*d^3 + 8043*c*d^4 + 700*d^5)*a^(
5/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 3*(805*c^5 - 9912*c^4*d + 2330*c
^3*d^2 - 18504*c^2*d^3 - 3895*c*d^4 - 504*d^5)*a^(5/2)*sin(f*x + e)^8/(cos(
f*x + e) + 1)^8 - (4179*c^5 - 1710*c^4*d + 30878*c^3*d^2 + 11540*c^2*d^3 +
3383*c*d^4 + 450*d^5)*a^(5/2)*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + (315*c^
5 - 8358*c^4*d - 4770*c^3*d^2 - 2284*c^2*d^3 - 625*c*d^4 - 70*d^5)*a^(5/2)*
sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - (903*c^5 + 720*c^4*d + 494*c^3*d^2
+ 200*c^2*d^3 + 35*c*d^4)*a^(5/2)*sin(f*x + e)^11/(cos(f*x + e) + 1)^11)*(s
in(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^3/((c^5 + 5*c^4*d + 10*c^3*d^2 + 10
*c^2*d^3 + 5*c*d^4 + d^5 + 3*(c^5 + 5*c^4*d + 10*c^3*d^2 + 10*c^2*d^3 + 5*c
*d^4 + d^5)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 3*(c^5 + 5*c^4*d + 10*c^3
*d^2 + 10*c^2*d^3 + 5*c*d^4 + d^5)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + (c
^5 + 5*c^4*d + 10*c^3*d^2 + 10*c^2*d^3 + 5*c*d^4 + d^5)*sin(f*x + e)^6/(cos
(f*x + e) + 1)^6)*(c + 2*d*sin(f*x + e)/(cos(f*x + e) + 1) + c*sin(f*x + e)
^2/(cos(f*x + e) + 1)^2)^(11/2)*f)
```

Fricas [B] time = 2.94489, size = 3441, normalized size = 10.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(11/2),x, algorithm="fric
as")
```

```
[Out] 2/315*(672*a^2*c^4 - 2304*a^2*c^3*d + 3008*a^2*c^2*d^2 - 1792*a^2*c*d^3 + 4
16*a^2*d^4 + 8*(a^2*c^2*d^2 + 10*a^2*c*d^3 + 73*a^2*d^4)*cos(f*x + e)^5 - 4
*(9*a^2*c^3*d + 89*a^2*c^2*d^2 + 647*a^2*c*d^3 - 73*a^2*d^4)*cos(f*x + e)^4
 - (63*a^2*c^4 + 648*a^2*c^3*d + 4798*a^2*c^2*d^2 + 1504*a^2*c*d^3 + 1387*a
^2*d^4)*cos(f*x + e)^3 + (231*a^2*c^4 + 3060*a^2*c^3*d - 2158*a^2*c^2*d^2 +
 4580*a^2*c*d^3 - 673*a^2*d^4)*cos(f*x + e)^2 + 2*(483*a^2*c^4 + 684*a^2*c^
3*d + 2642*a^2*c^2*d^2 + 812*a^2*c*d^3 + 419*a^2*d^4)*cos(f*x + e) - (672*a
^2*c^4 - 2304*a^2*c^3*d + 3008*a^2*c^2*d^2 - 1792*a^2*c*d^3 + 416*a^2*d^4 +
```

```
 8*(a^2*c^2*d^2 + 10*a^2*c*d^3 + 73*a^2*d^4)*cos(f*x + e)^4 + 4*(9*a^2*c^3*
d + 91*a^2*c^2*d^2 + 667*a^2*c*d^3 + 73*a^2*d^4)*cos(f*x + e)^3 - 3*(21*a^2
*c^4 + 204*a^2*c^3*d + 1478*a^2*c^2*d^2 - 388*a^2*c*d^3 + 365*a^2*d^4)*cos(
f*x + e)^2 - 2*(147*a^2*c^4 + 1836*a^2*c^3*d + 1138*a^2*c^2*d^2 + 1708*a^2*
c*d^3 + 211*a^2*d^4)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*s
qrt(d*sin(f*x + e) + c)/((c^5*d^5 + 5*c^4*d^6 + 10*c^3*d^7 + 10*c^2*d^8 + 5
*c*d^9 + d^10)*f*cos(f*x + e)^6 - 5*(c^6*d^4 + 5*c^5*d^5 + 10*c^4*d^6 + 10*
c^3*d^7 + 5*c^2*d^8 + c*d^9)*f*cos(f*x + e)^5 - (10*c^7*d^3 + 55*c^6*d^4 +
128*c^5*d^5 + 165*c^4*d^6 + 130*c^3*d^7 + 65*c^2*d^8 + 20*c*d^9 + 3*d^10)*f
*cos(f*x + e)^4 + 10*(c^8*d^2 + 5*c^7*d^3 + 11*c^6*d^4 + 15*c^5*d^5 + 15*c^
4*d^6 + 11*c^3*d^7 + 5*c^2*d^8 + c*d^9)*f*cos(f*x + e)^3 + (5*c^9*d + 35*c^
8*d^2 + 120*c^7*d^3 + 260*c^6*d^4 + 378*c^5*d^5 + 370*c^4*d^6 + 240*c^3*d^7
 + 100*c^2*d^8 + 25*c*d^9 + 3*d^10)*f*cos(f*x + e)^2 - (c^10 + 5*c^9*d + 20
*c^8*d^2 + 60*c^7*d^3 + 110*c^6*d^4 + 126*c^5*d^5 + 100*c^4*d^6 + 60*c^3*d^
7 + 25*c^2*d^8 + 5*c*d^9)*f*cos(f*x + e) - (c^10 + 10*c^9*d + 45*c^8*d^2 +
120*c^7*d^3 + 210*c^6*d^4 + 252*c^5*d^5 + 210*c^4*d^6 + 120*c^3*d^7 + 45*c^
2*d^8 + 10*c*d^9 + d^10)*f - ((c^5*d^5 + 5*c^4*d^6 + 10*c^3*d^7 + 10*c^2*d^
8 + 5*c*d^9 + d^10)*f*cos(f*x + e)^5 + (5*c^6*d^4 + 26*c^5*d^5 + 55*c^4*d^6
 + 60*c^3*d^7 + 35*c^2*d^8 + 10*c*d^9 + d^10)*f*cos(f*x + e)^4 - 2*(5*c^7*d
^3 + 25*c^6*d^4 + 51*c^5*d^5 + 55*c^4*d^6 + 35*c^3*d^7 + 15*c^2*d^8 + 5*c*d
^9 + d^10)*f*cos(f*x + e)^3 - 2*(5*c^8*d^2 + 30*c^7*d^3 + 80*c^6*d^4 + 126*
c^5*d^5 + 130*c^4*d^6 + 90*c^3*d^7 + 40*c^2*d^8 + 10*c*d^9 + d^10)*f*cos(f*
x + e)^2 + (5*c^9*d + 25*c^8*d^2 + 60*c^7*d^3 + 100*c^6*d^4 + 126*c^5*d^5 +
 110*c^4*d^6 + 60*c^3*d^7 + 20*c^2*d^8 + 5*c*d^9 + d^10)*f*cos(f*x + e) + (
c^10 + 10*c^9*d + 45*c^8*d^2 + 120*c^7*d^3 + 210*c^6*d^4 + 252*c^5*d^5 + 21
0*c^4*d^6 + 120*c^3*d^7 + 45*c^2*d^8 + 10*c*d^9 + d^10)*f)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)/(c+d*sin(f*x+e))**(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(11/2),x, algorithm="giac
")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)/(d*sin(f*x + e) + c)^(11/2), x)
```

3.588 $\int \frac{(c+d\sin(e+fx))^{5/2}}{\sqrt{a+a\sin(e+fx)}}\,dx$

Optimal. Leaf size=249

$$-\frac{\sqrt{d}\left(15c^2-10cd+7d^2\right)\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{4\sqrt{a}f}-\frac{d\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}-\frac{d(7c-d)\cos(e+f}{4f\sqrt{a\sin}}$$

```
[Out] -(Sqrt[d]*(15*c^2 - 10*c*d + 7*d^2)*ArcTan[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(
Sqrt[a + a*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])])/(4*Sqrt[a]*f) - (Sqrt[
2]*(c - d)^(5/2)*ArcTanh[(Sqrt[a]*Sqrt[c - d]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a
 + a*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])])/(Sqrt[a]*f) - ((7*c - d)*d*C
os[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(4*f*Sqrt[a + a*Sin[e + f*x]]) - (d*C
os[e + f*x]*(c + d*Sin[e + f*x])^(3/2))/(2*f*Sqrt[a + a*Sin[e + f*x]])
```

Rubi [A] time = 0.928763, antiderivative size = 249, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2778, 2983, 2982, 2782, 208, 2775, 205}

$$-\frac{\sqrt{d}\left(15c^2-10cd+7d^2\right)\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{4\sqrt{a}f}-\frac{d\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}-\frac{d(7c-d)\cos(e+f}{4f\sqrt{a\sin}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^(5/2)/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] -(Sqrt[d]*(15*c^2 - 10*c*d + 7*d^2)*ArcTan[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(
Sqrt[a + a*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])])/(4*Sqrt[a]*f) - (Sqrt[
2]*(c - d)^(5/2)*ArcTanh[(Sqrt[a]*Sqrt[c - d]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a
 + a*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])])/(Sqrt[a]*f) - ((7*c - d)*d*C
os[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(4*f*Sqrt[a + a*Sin[e + f*x]]) - (d*C
os[e + f*x]*(c + d*Sin[e + f*x])^(3/2))/(2*f*Sqrt[a + a*Sin[e + f*x]])
```

Rule 2778

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> Simp[(-2*d*Cos[e + f*x]*(c + d*Sin[e + f*x])
^(n - 1))/(f*(2*n - 1)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(b*(2*n - 1))
, Int[((c + d*Sin[e + f*x])^(n - 2)*Simp[a*c*d - b*(2*d^2*(n - 1) + c^2*(2*
n - 1)) + d*(a*d - b*c*(4*n - 3))*Sin[e + f*x], x])/Sqrt[a + b*Sin[e + f*x]
], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^{5/2}}{\sqrt{a+a\sin(e+fx)}}\,dx = -\frac{d\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{2f\sqrt{a+a\sin(e+fx)}} - \frac{\int \frac{\sqrt{c+d\sin(e+fx)}\left(-a\left(4c^2-cd+3d^2\right)-a(7c-d)d\sin(e+fx)\right)}{\sqrt{a+a\sin(e+fx)}}\,dx}{4a}$$

$$= -\frac{(7c-d)d\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4f\sqrt{a+a\sin(e+fx)}} - \frac{d\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{2f\sqrt{a+a\sin(e+fx)}} - \frac{\int \frac{-\frac{1}{2}a^2(8c \ldots}{\ldots}}{\ldots}$$

$$= -\frac{(7c-d)d\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4f\sqrt{a+a\sin(e+fx)}} - \frac{d\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{2f\sqrt{a+a\sin(e+fx)}} + (c-d)^3 \ldots$$

$$= -\frac{(7c-d)d\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4f\sqrt{a+a\sin(e+fx)}} - \frac{d\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{2f\sqrt{a+a\sin(e+fx)}} - \frac{\left(2a(c-d \ldots\right.}{\ldots}$$

$$= -\frac{\sqrt{d}\left(15c^2-10cd+7d^2\right)\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{4\sqrt{a}f} - \frac{\sqrt{2}(c-d)^{5/2}\tanh^{-1}\left(\frac{\ldots}{\sqrt{2}\sqrt{a+a \ldots}}\right.}{\sqrt{a}f}$$

Mathematica [C] time = 17.3567, size = 1893, normalized size = 7.6

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*Sin[e + f*x])^(5/2)/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c + d*Sin[e + f*x]]*((d*(-9*c +
 2*d)*Cos[(e + f*x)/2])/4 - (d^2*Cos[(3*(e + f*x))/2])/4 - (d*(-9*c + 2*d)*
Sin[(e + f*x)/2])/4 - (d^2*Sin[(3*(e + f*x))/2])/4))/(f*Sqrt[a*(1 + Sin[e +
 f*x])]) + ((Sqrt[2]*(c - d)^(5/2)*Log[1 + Tan[(e + f*x)/2]] - Sqrt[2]*(c -
 d)^(5/2)*Log[c - d + 2*Sqrt[c - d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c +
d*Sin[e + f*x]] + (-c + d)*Tan[(e + f*x)/2]] - (I/8)*Sqrt[d]*(15*c^2 - 10*c
*d + 7*d^2)*Log[(2*(c - I*(d + (1 + I)*Sqrt[2]*Sqrt[d]*Sqrt[(1 + Cos[e + f*
x])^(-1)]*Sqrt[c + d*Sin[e + f*x]]) + ((-I)*c + d)*Tan[(e + f*x)/2]))/(d^(3
/2)*(15*c^2 - 10*c*d + 7*d^2)*(I + Tan[(e + f*x)/2]))] + (I/8)*Sqrt[d]*(15*
c^2 - 10*c*d + 7*d^2)*Log[(2*(c + I*d + (1 + I)*Sqrt[2]*Sqrt[d]*Sqrt[(1 + C
os[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + (I*c + d)*Tan[(e + f*x)/2]))/
(d^(3/2)*(15*c^2 - 10*c*d + 7*d^2)*(-I + Tan[(e + f*x)/2]))])*(Cos[(e + f*x
)/2] + Sin[(e + f*x)/2])*(c^3/((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c
 + d*Sin[e + f*x]]) - (9*c^2*d)/(8*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sq
rt[c + d*Sin[e + f*x]]) + (7*c*d^2)/(4*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)*Sqrt[c + d*Sin[e + f*x]]) - d^3/(8*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*
Sqrt[c + d*Sin[e + f*x]]) + (15*c^2*d*Sin[e + f*x])/(8*(Cos[(e + f*x)/2] +
Sin[(e + f*x)/2])*Sqrt[c + d*Sin[e + f*x]]) - (5*c*d^2*Sin[e + f*x])/(4*(Co
s[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c + d*Sin[e + f*x]]) + (7*d^3*Sin[e
 + f*x])/(8*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c + d*Sin[e + f*x]])
))/(f*Sqrt[a*(1 + Sin[e + f*x])]*(((c - d)^(5/2)*Sec[(e + f*x)/2]^2)/(Sqrt[
2]*(1 + Tan[(e + f*x)/2])) - (Sqrt[2]*(c - d)^(5/2)*(((-c + d)*Sec[(e + f*x
)/2]^2)/2 + (Sqrt[c - d]*d*Cos[e + f*x]*Sqrt[(1 + Cos[e + f*x])^(-1)])/Sqrt
[c + d*Sin[e + f*x]] + Sqrt[c - d]*((1 + Cos[e + f*x])^(-1))^(3/2)*Sin[e +
f*x]*Sqrt[c + d*Sin[e + f*x]]))/(c - d + 2*Sqrt[c - d]*Sqrt[(1 + Cos[e + f*
x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + (-c + d)*Tan[(e + f*x)/2]) - ((I/16)*d
^2*(15*c^2 - 10*c*d + 7*d^2)^2*(I + Tan[(e + f*x)/2])*((2*((((-I)*c + d)*Se
c[(e + f*x)/2]^2)/2 - I*(((1 + I)*d^(3/2)*Cos[e + f*x]*Sqrt[(1 + Cos[e + f*
x])^(-1)])/(Sqrt[2]*Sqrt[c + d*Sin[e + f*x]]) + ((1 + I)*Sqrt[d]*((1 + Cos[
e + f*x])^(-1))^(3/2)*Sin[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/Sqrt[2])))/(d^
(3/2)*(15*c^2 - 10*c*d + 7*d^2)*(I + Tan[(e + f*x)/2])) - (Sec[(e + f*x)/2]
^2*(c - I*(d + (1 + I)*Sqrt[2]*Sqrt[d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c
 + d*Sin[e + f*x]]) + ((-I)*c + d)*Tan[(e + f*x)/2]))/(d^(3/2)*(15*c^2 - 10
*c*d + 7*d^2)*(I + Tan[(e + f*x)/2])^2)))/(c - I*(d + (1 + I)*Sqrt[2]*Sqrt[
d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]]) + ((-I)*c + d)*T
an[(e + f*x)/2]) + ((I/16)*d^2*(15*c^2 - 10*c*d + 7*d^2)^2*(-I + Tan[(e + f
*x)/2])*((2*(((I*c + d)*Sec[(e + f*x)/2]^2)/2 + ((1 + I)*d^(3/2)*Cos[e + f*
x]*Sqrt[(1 + Cos[e + f*x])^(-1)])/(Sqrt[2]*Sqrt[c + d*Sin[e + f*x]]) + ((1
+ I)*Sqrt[d]*((1 + Cos[e + f*x])^(-1))^(3/2)*Sin[e + f*x]*Sqrt[c + d*Sin[e
+ f*x]])/Sqrt[2]))/(d^(3/2)*(15*c^2 - 10*c*d + 7*d^2)*(-I + Tan[(e + f*x)/2
])) - (Sec[(e + f*x)/2]^2*(c + I*d + (1 + I)*Sqrt[2]*Sqrt[d]*Sqrt[(1 + Cos[
e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + (I*c + d)*Tan[(e + f*x)/2]))/(d^
(3/2)*(15*c^2 - 10*c*d + 7*d^2)*(-I + Tan[(e + f*x)/2])^2)))/(c + I*d + (1
+ I)*Sqrt[2]*Sqrt[d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]]
 + (I*c + d)*Tan[(e + f*x)/2])))
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int \left(c + d \sin \left(f x + e\right)\right)^{\frac{5}{2}} \frac{1}{\sqrt{a + a \sin \left(f x + e\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x)

[Out] int((c+d*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}{\sqrt{a\sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x, algorithm="maxim
a")

[Out] integrate((d*sin(f*x + e) + c)^(5/2)/sqrt(a*sin(f*x + e) + a), x)
```

Fricas [B] time = 6.91909, size = 7116, normalized size = 28.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x, algorithm="frica
s")

[Out] [1/32*(16*sqrt(2)*(a*c^2 - 2*a*c*d + a*d^2 + (a*c^2 - 2*a*c*d + a*d^2)*cos(
f*x + e) + (a*c^2 - 2*a*c*d + a*d^2)*sin(f*x + e))*sqrt((c - d)/a)*log((2*s
qrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt((c - d)/a)*(c
os(f*x + e) - sin(f*x + e) + 1) - (c - 3*d)*cos(f*x + e)^2 - (3*c - d)*cos(
f*x + e) + ((c - 3*d)*cos(f*x + e) - 2*c - 2*d)*sin(f*x + e) - 2*c - 2*d)/(
cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) + (15
*a*c^2 - 10*a*c*d + 7*a*d^2 + (15*a*c^2 - 10*a*c*d + 7*a*d^2)*cos(f*x + e)
+ (15*a*c^2 - 10*a*c*d + 7*a*d^2)*sin(f*x + e))*sqrt(-d/a)*log((128*d^4*cos
(f*x + e)^5 + 128*(2*c*d^3 - d^4)*cos(f*x + e)^4 + c^4 + 4*c^3*d + 6*c^2*d^
2 + 4*c*d^3 + d^4 - 32*(5*c^2*d^2 - 14*c*d^3 + 13*d^4)*cos(f*x + e)^3 - 32*
(c^3*d - 2*c^2*d^2 + 9*c*d^3 - 4*d^4)*cos(f*x + e)^2 - 8*(16*d^3*cos(f*x +
e)^4 + 24*(c*d^2 - d^3)*cos(f*x + e)^3 - c^3 + 17*c^2*d - 59*c*d^2 + 51*d^3
 - 2*(5*c^2*d - 26*c*d^2 + 33*d^3)*cos(f*x + e)^2 - (c^3 - 7*c^2*d + 31*c*d
^2 - 25*d^3)*cos(f*x + e) + (16*d^3*cos(f*x + e)^3 + c^3 - 17*c^2*d + 59*c*
d^2 - 51*d^3 - 8*(3*c*d^2 - 5*d^3)*cos(f*x + e)^2 - 2*(5*c^2*d - 14*c*d^2 +
 13*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*
x + e) + c)*sqrt(-d/a) + (c^4 - 28*c^3*d + 230*c^2*d^2 - 476*c*d^3 + 289*d^
4)*cos(f*x + e) + (128*d^4*cos(f*x + e)^4 + c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c
*d^3 + d^4 - 256*(c*d^3 - d^4)*cos(f*x + e)^3 - 32*(5*c^2*d^2 - 6*c*d^3 + 5
*d^4)*cos(f*x + e)^2 + 32*(c^3*d - 7*c^2*d^2 + 15*c*d^3 - 9*d^4)*cos(f*x +
e))*sin(f*x + e))/(cos(f*x + e) + sin(f*x + e) + 1)) - 8*(2*d^2*cos(f*x + e
)^2 + 9*c*d - 3*d^2 + (9*c*d - d^2)*cos(f*x + e) + (2*d^2*cos(f*x + e) - 9*
c*d + 3*d^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c
))/(a*f*cos(f*x + e) + a*f*sin(f*x + e) + a*f), 1/16*(8*sqrt(2)*(a*c^2 - 2*
a*c*d + a*d^2 + (a*c^2 - 2*a*c*d + a*d^2)*cos(f*x + e) + (a*c^2 - 2*a*c*d +
 a*d^2)*sin(f*x + e))*sqrt((c - d)/a)*log((2*sqrt(2)*sqrt(a*sin(f*x + e) +
a)*sqrt(d*sin(f*x + e) + c)*sqrt((c - d)/a)*(cos(f*x + e) - sin(f*x + e) +
1) - (c - 3*d)*cos(f*x + e)^2 - (3*c - d)*cos(f*x + e) + ((c - 3*d)*cos(f*x
```

```
+ e) - 2*c - 2*d)*sin(f*x + e) - 2*c - 2*d)/(cos(f*x + e)^2 - (cos(f*x + e
) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) + (15*a*c^2 - 10*a*c*d + 7*a*d^2 +
 (15*a*c^2 - 10*a*c*d + 7*a*d^2)*cos(f*x + e) + (15*a*c^2 - 10*a*c*d + 7*a*
d^2)*sin(f*x + e))*sqrt(d/a)*arctan(1/4*(8*d^2*cos(f*x + e)^2 - c^2 + 6*c*d
 - 9*d^2 - 8*(c*d - d^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(
f*x + e) + c)*sqrt(d/a)/(2*d^3*cos(f*x + e)^3 - (3*c*d^2 - d^3)*cos(f*x + e
)*sin(f*x + e) - (c^2*d - c*d^2 + 2*d^3)*cos(f*x + e))) - 4*(2*d^2*cos(f*x
+ e)^2 + 9*c*d - 3*d^2 + (9*c*d - d^2)*cos(f*x + e) + (2*d^2*cos(f*x + e) -
 9*c*d + 3*d^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e)
+ c))/(a*f*cos(f*x + e) + a*f*sin(f*x + e) + a*f), -1/32*(32*sqrt(2)*(a*c^2
 - 2*a*c*d + a*d^2 + (a*c^2 - 2*a*c*d + a*d^2)*cos(f*x + e) + (a*c^2 - 2*a*
c*d + a*d^2)*sin(f*x + e))*sqrt(-(c - d)/a)*arctan(-sqrt(2)*sqrt(a*sin(f*x
+ e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt(-(c - d)/a)/((c - d)*cos(f*x + e)))
 - (15*a*c^2 - 10*a*c*d + 7*a*d^2 + (15*a*c^2 - 10*a*c*d + 7*a*d^2)*cos(f*x
 + e) + (15*a*c^2 - 10*a*c*d + 7*a*d^2)*sin(f*x + e))*sqrt(-d/a)*log((128*d
^4*cos(f*x + e)^5 + 128*(2*c*d^3 - d^4)*cos(f*x + e)^4 + c^4 + 4*c^3*d + 6*
c^2*d^2 + 4*c*d^3 + d^4 - 32*(5*c^2*d^2 - 14*c*d^3 + 13*d^4)*cos(f*x + e)^3
 - 32*(c^3*d - 2*c^2*d^2 + 9*c*d^3 - 4*d^4)*cos(f*x + e)^2 - 8*(16*d^3*cos(
f*x + e)^4 + 24*(c*d^2 - d^3)*cos(f*x + e)^3 - c^3 + 17*c^2*d - 59*c*d^2 +
51*d^3 - 2*(5*c^2*d - 26*c*d^2 + 33*d^3)*cos(f*x + e)^2 - (c^3 - 7*c^2*d +
31*c*d^2 - 25*d^3)*cos(f*x + e) + (16*d^3*cos(f*x + e)^3 + c^3 - 17*c^2*d +
 59*c*d^2 - 51*d^3 - 8*(3*c*d^2 - 5*d^3)*cos(f*x + e)^2 - 2*(5*c^2*d - 14*c
*d^2 + 13*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*
sin(f*x + e) + c)*sqrt(-d/a) + (c^4 - 28*c^3*d + 230*c^2*d^2 - 476*c*d^3 +
289*d^4)*cos(f*x + e) + (128*d^4*cos(f*x + e)^4 + c^4 + 4*c^3*d + 6*c^2*d^2
 + 4*c*d^3 + d^4 - 256*(c*d^3 - d^4)*cos(f*x + e)^3 - 32*(5*c^2*d^2 - 6*c*d
^3 + 5*d^4)*cos(f*x + e)^2 + 32*(c^3*d - 7*c^2*d^2 + 15*c*d^3 - 9*d^4)*cos(
f*x + e))*sin(f*x + e))/(cos(f*x + e) + sin(f*x + e) + 1)) + 8*(2*d^2*cos(f
*x + e)^2 + 9*c*d - 3*d^2 + (9*c*d - d^2)*cos(f*x + e) + (2*d^2*cos(f*x + e
) - 9*c*d + 3*d^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x +
e) + c))/(a*f*cos(f*x + e) + a*f*sin(f*x + e) + a*f), -1/16*(16*sqrt(2)*(a*
c^2 - 2*a*c*d + a*d^2 + (a*c^2 - 2*a*c*d + a*d^2)*cos(f*x + e) + (a*c^2 - 2
*a*c*d + a*d^2)*sin(f*x + e))*sqrt(-(c - d)/a)*arctan(-sqrt(2)*sqrt(a*sin(f
*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt(-(c - d)/a)/((c - d)*cos(f*x + e
))) - (15*a*c^2 - 10*a*c*d + 7*a*d^2 + (15*a*c^2 - 10*a*c*d + 7*a*d^2)*cos(
f*x + e) + (15*a*c^2 - 10*a*c*d + 7*a*d^2)*sin(f*x + e))*sqrt(d/a)*arctan(1
/4*(8*d^2*cos(f*x + e)^2 - c^2 + 6*c*d - 9*d^2 - 8*(c*d - d^2)*sin(f*x + e)
)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt(d/a)/(2*d^3*cos(f*
x + e)^3 - (3*c*d^2 - d^3)*cos(f*x + e)*sin(f*x + e) - (c^2*d - c*d^2 + 2*d
^3)*cos(f*x + e))) + 4*(2*d^2*cos(f*x + e)^2 + 9*c*d - 3*d^2 + (9*c*d - d^2
)*cos(f*x + e) + (2*d^2*cos(f*x + e) - 9*c*d + 3*d^2)*sin(f*x + e))*sqrt(a*
sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/(a*f*cos(f*x + e) + a*f*sin(f*x
 + e) + a*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(5/2)/(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}{\sqrt{a\sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate((d*sin(f*x + e) + c)^(5/2)/sqrt(a*sin(f*x + e) + a), x)
```

3.589 $\int \frac{(c+d\sin(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}}\,dx$

Optimal. Leaf size=188

$$-\frac{d\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}-\frac{\sqrt{d}(3c-d)\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{\sqrt{a}f}-\frac{\sqrt{2}(c-d)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-}}{\sqrt{2}\sqrt{a\sin(e+f}}\right.}{\sqrt{a}f}$$

```
[Out] -(((3*c - d)*Sqrt[d]*ArcTan[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[a + a*Sin[
e + f*x]]*Sqrt[c + d*Sin[e + f*x]])])/(Sqrt[a]*f)) - (Sqrt[2]*(c - d)^(3/2)
*ArcTanh[(Sqrt[a]*Sqrt[c - d]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]])])/(Sqrt[a]*f) - (d*Cos[e + f*x]*Sqrt[c + d*Sin
[e + f*x]])/(f*Sqrt[a + a*Sin[e + f*x]])
```

Rubi [A] time = 0.597811, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2778, 2982, 2782, 208, 2775, 205}

$$-\frac{d\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}-\frac{\sqrt{d}(3c-d)\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{\sqrt{a}f}-\frac{\sqrt{2}(c-d)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-}}{\sqrt{2}\sqrt{a\sin(e+f}}\right.}{\sqrt{a}f}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^(3/2)/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] -(((3*c - d)*Sqrt[d]*ArcTan[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[a + a*Sin[
e + f*x]]*Sqrt[c + d*Sin[e + f*x]])])/(Sqrt[a]*f)) - (Sqrt[2]*(c - d)^(3/2)
*ArcTanh[(Sqrt[a]*Sqrt[c - d]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]])])/(Sqrt[a]*f) - (d*Cos[e + f*x]*Sqrt[c + d*Sin
[e + f*x]])/(f*Sqrt[a + a*Sin[e + f*x]])
```

Rule 2778

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> Simp[(-2*d*Cos[e + f*x]*(c + d*Sin[e + f*x])
^(n - 1))/(f*(2*n - 1)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(b*(2*n - 1))
, Int[((c + d*Sin[e + f*x])^(n - 2)*Simp[a*c*d - b*(2*d^2*(n - 1) + c^2*(2*
n - 1)) + d*(a*d - b*c*(4*n - 3))*Sin[e + f*x], x])/Sqrt[a + b*Sin[e + f*x]
], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
```

```
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}}\,dx = -\frac{d\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{f\sqrt{a+a\sin(e+fx)}} - \frac{\int \frac{-a\left(2c^2-cd+d^2\right)-a(3c-d)d\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\,dx}{2a}$$

$$= -\frac{d\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{f\sqrt{a+a\sin(e+fx)}} + (c-d)^2\int \frac{1}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\,dx +$$

$$= -\frac{d\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{f\sqrt{a+a\sin(e+fx)}} - \frac{\left(2a(c-d)^2\right)\text{Subst}\left(\int \frac{1}{2a^2-(ac-ad)x^2}\,dx, x, \frac{a\cos(e}{\sqrt{a+a\sin(e+fx)}\sqrt{}}\right.}{f}$$

$$= -\frac{(3c-d)\sqrt{d}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{\sqrt{a}f} - \frac{\sqrt{2}(c-d)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+f}{\sqrt{2}\sqrt{a+a\sin(e+fx)}\sqrt{c+d}}\right.}{\sqrt{a}f}$$

Mathematica [C] time = 16.9243, size = 1639, normalized size = 8.72

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*Sin[e + f*x])^(3/2)/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-(d*Cos[(e + f*x)/2]) + d*Sin[(e +
f*x)/2])*Sqrt[c + d*Sin[e + f*x]])/(f*Sqrt[a*(1 + Sin[e + f*x])]) + ((Sqrt[
2]*(c - d)^(3/2)*Log[1 + Tan[(e + f*x)/2]] - Sqrt[2]*(c - d)^(3/2)*Log[c -
d + 2*Sqrt[c - d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] +
(-c + d)*Tan[(e + f*x)/2]] + (I/2)*Sqrt[d]*(-3*c + d)*(Log[((2*I)*(I*c + d
+ (1 + I)*Sqrt[2]*Sqrt[d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e +
f*x]] + (c + I*d)*Tan[(e + f*x)/2]))/(d^(3/2)*(-3*c + d)*(I + Tan[(e + f*x)
/2]))] - Log[(-2*(c + I*d + (1 + I)*Sqrt[2]*Sqrt[d]*Sqrt[(1 + Cos[e + f*x])
^(-1)]*Sqrt[c + d*Sin[e + f*x]] + (I*c + d)*Tan[(e + f*x)/2]))/(d^(3/2)*(-3
*c + d)*(-I + Tan[(e + f*x)/2]))]))*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(
```

```
c^2/((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c + d*Sin[e + f*x]]) - (c*d
)/(2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c + d*Sin[e + f*x]]) + d^2/
(2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c + d*Sin[e + f*x]]) + (3*c*d
*Sin[e + f*x])/(2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c + d*Sin[e +
f*x]]) - (d^2*Sin[e + f*x])/(2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c
 + d*Sin[e + f*x]])))/(f*Sqrt[a*(1 + Sin[e + f*x])]*(((c - d)^(3/2)*Sec[(e
+ f*x)/2]^2)/(Sqrt[2]*(1 + Tan[(e + f*x)/2])) - (Sqrt[2]*(c - d)^(3/2)*(((-
c + d)*Sec[(e + f*x)/2]^2)/2 + (Sqrt[c - d]*d*Cos[e + f*x]*Sqrt[(1 + Cos[e
+ f*x])^(-1)])/Sqrt[c + d*Sin[e + f*x]] + Sqrt[c - d]*((1 + Cos[e + f*x])^(
-1))^(3/2)*Sin[e + f*x]*Sqrt[c + d*Sin[e + f*x]]))/(c - d + 2*Sqrt[c - d]*S
qrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + (-c + d)*Tan[(e + f
*x)/2]) + (I/2)*Sqrt[d]*(-3*c + d)*(((-I/2)*d^(3/2)*(-3*c + d)*(I + Tan[(e
+ f*x)/2])*(((2*I)*(((c + I*d)*Sec[(e + f*x)/2]^2)/2 + ((1 + I)*d^(3/2)*Cos
[e + f*x]*Sqrt[(1 + Cos[e + f*x])^(-1)])/(Sqrt[2]*Sqrt[c + d*Sin[e + f*x]])
 + ((1 + I)*Sqrt[d]*((1 + Cos[e + f*x])^(-1))^(3/2)*Sin[e + f*x]*Sqrt[c + d
*Sin[e + f*x]])/Sqrt[2]))/(d^(3/2)*(-3*c + d)*(I + Tan[(e + f*x)/2])) - (I*
Sec[(e + f*x)/2]^2*(I*c + d + (1 + I)*Sqrt[2]*Sqrt[d]*Sqrt[(1 + Cos[e + f*x
])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + (c + I*d)*Tan[(e + f*x)/2]))/(d^(3/2)*(
-3*c + d)*(I + Tan[(e + f*x)/2])^2)))/(I*c + d + (1 + I)*Sqrt[2]*Sqrt[d]*Sq
rt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + (c + I*d)*Tan[(e + f
*x)/2]) + (d^(3/2)*(-3*c + d)*(-I + Tan[(e + f*x)/2])*((-2*(((I*c + d)*Sec[
(e + f*x)/2]^2)/2 + ((1 + I)*d^(3/2)*Cos[e + f*x]*Sqrt[(1 + Cos[e + f*x])^(
-1)])/(Sqrt[2]*Sqrt[c + d*Sin[e + f*x]]) + ((1 + I)*Sqrt[d]*((1 + Cos[e + f
*x])^(-1))^(3/2)*Sin[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/Sqrt[2]))/(d^(3/2)*
(-3*c + d)*(-I + Tan[(e + f*x)/2])) + (Sec[(e + f*x)/2]^2*(c + I*d + (1 + I
)*Sqrt[2]*Sqrt[d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] +
(I*c + d)*Tan[(e + f*x)/2]))/(d^(3/2)*(-3*c + d)*(-I + Tan[(e + f*x)/2])^2)
))/(2*(c + I*d + (1 + I)*Sqrt[2]*Sqrt[d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt
[c + d*Sin[e + f*x]] + (I*c + d)*Tan[(e + f*x)/2])))))
```

Maple [F] time = 0.328, size = 0, normalized size = 0.

$$\int \left(c+d\sin\left(fx+e\right)\right)^{\frac{3}{2}}\frac{1}{\sqrt{a+a\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] int((c+d*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}{\sqrt{a\sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(3/2)/sqrt(a*sin(f*x + e) + a), x)
```

Fricas [B] time = 6.35524, size = 6151, normalized size = 32.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] [-1/8*(4*sqrt(2)*(a*c - a*d + (a*c - a*d)*cos(f*x + e) + (a*c - a*d)*sin(f*
x + e))*sqrt((c - d)/a)*log(-(2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin
(f*x + e) + c)*sqrt((c - d)/a)*(cos(f*x + e) - sin(f*x + e) + 1) + (c - 3*d
)*cos(f*x + e)^2 + (3*c - d)*cos(f*x + e) - ((c - 3*d)*cos(f*x + e) - 2*c -
 2*d)*sin(f*x + e) + 2*c + 2*d)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*
x + e) - cos(f*x + e) - 2)) + (3*a*c - a*d + (3*a*c - a*d)*cos(f*x + e) + (
3*a*c - a*d)*sin(f*x + e))*sqrt(-d/a)*log((128*d^4*cos(f*x + e)^5 + 128*(2*
c*d^3 - d^4)*cos(f*x + e)^4 + c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^3 + d^4 - 3
2*(5*c^2*d^2 - 14*c*d^3 + 13*d^4)*cos(f*x + e)^3 - 32*(c^3*d - 2*c^2*d^2 +
9*c*d^3 - 4*d^4)*cos(f*x + e)^2 + 8*(16*d^3*cos(f*x + e)^4 + 24*(c*d^2 - d^
3)*cos(f*x + e)^3 - c^3 + 17*c^2*d - 59*c*d^2 + 51*d^3 - 2*(5*c^2*d - 26*c*
d^2 + 33*d^3)*cos(f*x + e)^2 - (c^3 - 7*c^2*d + 31*c*d^2 - 25*d^3)*cos(f*x
+ e) + (16*d^3*cos(f*x + e)^3 + c^3 - 17*c^2*d + 59*c*d^2 - 51*d^3 - 8*(3*c
*d^2 - 5*d^3)*cos(f*x + e)^2 - 2*(5*c^2*d - 14*c*d^2 + 13*d^3)*cos(f*x + e)
)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt(-d/a
) + (c^4 - 28*c^3*d + 230*c^2*d^2 - 476*c*d^3 + 289*d^4)*cos(f*x + e) + (12
8*d^4*cos(f*x + e)^4 + c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^3 + d^4 - 256*(c*d
^3 - d^4)*cos(f*x + e)^3 - 32*(5*c^2*d^2 - 6*c*d^3 + 5*d^4)*cos(f*x + e)^2
+ 32*(c^3*d - 7*c^2*d^2 + 15*c*d^3 - 9*d^4)*cos(f*x + e))*sin(f*x + e))/(co
s(f*x + e) + sin(f*x + e) + 1)) + 8*(d*cos(f*x + e) - d*sin(f*x + e) + d)*s
qrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/(a*f*cos(f*x + e) + a*f*s
in(f*x + e) + a*f), -1/4*(2*sqrt(2)*(a*c - a*d + (a*c - a*d)*cos(f*x + e) +
 (a*c - a*d)*sin(f*x + e))*sqrt((c - d)/a)*log(-(2*sqrt(2)*sqrt(a*sin(f*x +
 e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt((c - d)/a)*(cos(f*x + e) - sin(f*x +
 e) + 1) + (c - 3*d)*cos(f*x + e)^2 + (3*c - d)*cos(f*x + e) - ((c - 3*d)*c
os(f*x + e) - 2*c - 2*d)*sin(f*x + e) + 2*c + 2*d)/(cos(f*x + e)^2 - (cos(f
*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) - (3*a*c - a*d + (3*a*c - a*
d)*cos(f*x + e) + (3*a*c - a*d)*sin(f*x + e))*sqrt(d/a)*arctan(1/4*(8*d^2*c
os(f*x + e)^2 - c^2 + 6*c*d - 9*d^2 - 8*(c*d - d^2)*sin(f*x + e))*sqrt(a*si
n(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt(d/a)/(2*d^3*cos(f*x + e)^3 -
(3*c*d^2 - d^3)*cos(f*x + e)*sin(f*x + e) - (c^2*d - c*d^2 + 2*d^3)*cos(f*x
 + e))) + 4*(d*cos(f*x + e) - d*sin(f*x + e) + d)*sqrt(a*sin(f*x + e) + a)*
sqrt(d*sin(f*x + e) + c))/(a*f*cos(f*x + e) + a*f*sin(f*x + e) + a*f), -1/8
*(8*sqrt(2)*(a*c - a*d + (a*c - a*d)*cos(f*x + e) + (a*c - a*d)*sin(f*x + e
))*sqrt(-(c - d)/a)*arctan(-sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x
 + e) + c)*sqrt(-(c - d)/a)/((c - d)*cos(f*x + e))) + (3*a*c - a*d + (3*a*c
 - a*d)*cos(f*x + e) + (3*a*c - a*d)*sin(f*x + e))*sqrt(-d/a)*log((128*d^4*
cos(f*x + e)^5 + 128*(2*c*d^3 - d^4)*cos(f*x + e)^4 + c^4 + 4*c^3*d + 6*c^2
*d^2 + 4*c*d^3 + d^4 - 32*(5*c^2*d^2 - 14*c*d^3 + 13*d^4)*cos(f*x + e)^3 -
32*(c^3*d - 2*c^2*d^2 + 9*c*d^3 - 4*d^4)*cos(f*x + e)^2 + 8*(16*d^3*cos(f*x
 + e)^4 + 24*(c*d^2 - d^3)*cos(f*x + e)^3 - c^3 + 17*c^2*d - 59*c*d^2 + 51*
d^3 - 2*(5*c^2*d - 26*c*d^2 + 33*d^3)*cos(f*x + e)^2 - (c^3 - 7*c^2*d + 31*
c*d^2 - 25*d^3)*cos(f*x + e) + (16*d^3*cos(f*x + e)^3 + c^3 - 17*c^2*d + 59
*c*d^2 - 51*d^3 - 8*(3*c*d^2 - 5*d^3)*cos(f*x + e)^2 - 2*(5*c^2*d - 14*c*d^
2 + 13*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin
(f*x + e) + c)*sqrt(-d/a) + (c^4 - 28*c^3*d + 230*c^2*d^2 - 476*c*d^3 + 289
*d^4)*cos(f*x + e) + (128*d^4*cos(f*x + e)^4 + c^4 + 4*c^3*d + 6*c^2*d^2 +
4*c*d^3 + d^4 - 256*(c*d^3 - d^4)*cos(f*x + e)^3 - 32*(5*c^2*d^2 - 6*c*d^3
+ 5*d^4)*cos(f*x + e)^2 + 32*(c^3*d - 7*c^2*d^2 + 15*c*d^3 - 9*d^4)*cos(f*x
```

```
 + e))*sin(f*x + e))/(cos(f*x + e) + sin(f*x + e) + 1)) + 8*(d*cos(f*x + e)
 - d*sin(f*x + e) + d)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/(
a*f*cos(f*x + e) + a*f*sin(f*x + e) + a*f), -1/4*(4*sqrt(2)*(a*c - a*d + (a
*c - a*d)*cos(f*x + e) + (a*c - a*d)*sin(f*x + e))*sqrt(-(c - d)/a)*arctan(
-sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt(-(c - d)/a)
/((c - d)*cos(f*x + e))) - (3*a*c - a*d + (3*a*c - a*d)*cos(f*x + e) + (3*a
*c - a*d)*sin(f*x + e))*sqrt(d/a)*arctan(1/4*(8*d^2*cos(f*x + e)^2 - c^2 +
6*c*d - 9*d^2 - 8*(c*d - d^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d
*sin(f*x + e) + c)*sqrt(d/a)/(2*d^3*cos(f*x + e)^3 - (3*c*d^2 - d^3)*cos(f*
x + e)*sin(f*x + e) - (c^2*d - c*d^2 + 2*d^3)*cos(f*x + e))) + 4*(d*cos(f*x
 + e) - d*sin(f*x + e) + d)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) +
c))/(a*f*cos(f*x + e) + a*f*sin(f*x + e) + a*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c+d\sin\left(e+fx\right)\right)^{\frac{3}{2}}}{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(3/2)/(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((c + d*sin(e + f*x))**(3/2)/sqrt(a*(sin(e + f*x) + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}{\sqrt{a\sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate((d*sin(f*x + e) + c)^(3/2)/sqrt(a*sin(f*x + e) + a), x)
```

3.590 $\int \frac{\sqrt{c+d\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}}\,dx$

Optimal. Leaf size=141

$$-\frac{2\sqrt{d}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{\sqrt{a}f}-\frac{\sqrt{2}\sqrt{c-d}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{\sqrt{a}f}$$

```
[Out] (-2*Sqrt[d]*ArcTan[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]
*Sqrt[c + d*Sin[e + f*x]])])/(Sqrt[a]*f) - (Sqrt[2]*Sqrt[c - d]*ArcTanh[(Sq
rt[a]*Sqrt[c - d]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c +
d*Sin[e + f*x]])])/(Sqrt[a]*f)
```

Rubi [A] time = 0.296327, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2777, 2775, 205, 2782, 208}

$$-\frac{2\sqrt{d}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{\sqrt{a}f}-\frac{\sqrt{2}\sqrt{c-d}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{\sqrt{a}f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] (-2*Sqrt[d]*ArcTan[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]
*Sqrt[c + d*Sin[e + f*x]])])/(Sqrt[a]*f) - (Sqrt[2]*Sqrt[c - d]*ArcTanh[(Sq
rt[a]*Sqrt[c - d]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c +
d*Sin[e + f*x]])])/(Sqrt[a]*f)
```

Rule 2777

```
Int[Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(a_) + (b_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c
 + d*Sin[e + f*x]], x], x] + Dist[(b*c - a*d)/b, Int[1/(Sqrt[a + b*Sin[e +
f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
```

```
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{c+d\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}}\,dx = (c-d)\int \frac{1}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\,dx + \frac{d\int \frac{\sqrt{a+a\sin(e+fx)}}{\sqrt{c+d\sin(e+fx)}}\,dx}{a}$$

$$= -\frac{(2a(c-d))\,\mathrm{Subst}\left(\int \frac{1}{2a^2-(ac-ad)x^2}\,dx, x, \frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{f} - \frac{(2d)\,\mathrm{Subst}\left(\int \frac{1}{a+dx^2}\right.}{}$$

$$= -\frac{2\sqrt{d}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{\sqrt{a}f} - \frac{\sqrt{2}\sqrt{c-d}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right.}{\sqrt{a}f}$$

Mathematica [C] time = 15.016, size = 1251, normalized size = 8.87

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] ((Sqrt[2]*Sqrt[c - d]*Log[1 + Tan[(e + f*x)/2]] - Sqrt[2]*Sqrt[c - d]*Log[c
 - d + 2*Sqrt[c - d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]]
 + (-c + d)*Tan[(e + f*x)/2]] - I*Sqrt[d]*(Log[(2*(c - I*d + (1 - I)*Sqrt[2
]*Sqrt[d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + ((-I)*c
+ d)*Tan[(e + f*x)/2]))/(d^(3/2)*(I + Tan[(e + f*x)/2]))] - Log[(2*(c + I*d
 + (1 + I)*Sqrt[2]*Sqrt[d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e +
 f*x]] + (I*c + d)*Tan[(e + f*x)/2]))/(d^(3/2)*(-I + Tan[(e + f*x)/2]))]))*
Sqrt[c + d*Sin[e + f*x]])/(f*Sqrt[a*(1 + Sin[e + f*x])]*((Sqrt[c - d]*Sec[(
e + f*x)/2]^2)/(Sqrt[2]*(1 + Tan[(e + f*x)/2])) - (Sqrt[2]*Sqrt[c - d]*(((-
c + d)*Sec[(e + f*x)/2]^2)/2 + (Sqrt[c - d]*d*Cos[e + f*x]*Sqrt[(1 + Cos[e
+ f*x])^(-1)])/Sqrt[c + d*Sin[e + f*x]] + Sqrt[c - d]*((1 + Cos[e + f*x])^(
-1))^(3/2)*Sin[e + f*x]*Sqrt[c + d*Sin[e + f*x]]))/(c - d + 2*Sqrt[c - d]*S
qrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + (-c + d)*Tan[(e + f
*x)/2]) - I*Sqrt[d]*((d^(3/2)*(I + Tan[(e + f*x)/2])*((2*((((-I)*c + d)*Sec
[(e + f*x)/2]^2)/2 + ((1 - I)*d^(3/2)*Cos[e + f*x]*Sqrt[(1 + Cos[e + f*x])^
(-1)])/(Sqrt[2]*Sqrt[c + d*Sin[e + f*x]]) + ((1 - I)*Sqrt[d]*((1 + Cos[e +
f*x])^(-1))^(3/2)*Sin[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/Sqrt[2]))/(d^(3/2)
*(I + Tan[(e + f*x)/2])) - (Sec[(e + f*x)/2]^2*(c - I*d + (1 - I)*Sqrt[2]*S
qrt[d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + ((-I)*c + d
)*Tan[(e + f*x)/2]))/(d^(3/2)*(I + Tan[(e + f*x)/2])^2)))/(2*(c - I*d + (1
- I)*Sqrt[2]*Sqrt[d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]]
 + ((-I)*c + d)*Tan[(e + f*x)/2])) - (d^(3/2)*(-I + Tan[(e + f*x)/2])*((2*(
((I*c + d)*Sec[(e + f*x)/2]^2)/2 + ((1 + I)*d^(3/2)*Cos[e + f*x]*Sqrt[(1 +
Cos[e + f*x])^(-1)])/(Sqrt[2]*Sqrt[c + d*Sin[e + f*x]]) + ((1 + I)*Sqrt[d]*
((1 + Cos[e + f*x])^(-1))^(3/2)*Sin[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/Sqrt
[2]))/(d^(3/2)*(-I + Tan[(e + f*x)/2])) - (Sec[(e + f*x)/2]^2*(c + I*d + (1
 + I)*Sqrt[2]*Sqrt[d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]
```

```
] + (I*c + d)*Tan[(e + f*x)/2]))/(d^(3/2)*(-I + Tan[(e + f*x)/2])^2)))/(2*(
c + I*d + (1 + I)*Sqrt[2]*Sqrt[d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*
Sin[e + f*x]] + (I*c + d)*Tan[(e + f*x)/2])))))
```

Maple [B] time = 0.273, size = 3359, normalized size = 23.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] 1/2/f/d/(c^2-2*c*d+d^2)/(-(d^2/c^2)^(1/2)*c)^(1/2)*(-2^(1/2)*((c+d*sin(f*x+
e))/(cos(f*x+e)+1))^(1/2)*(-(d^2/c^2)^(1/2)*c)^(1/2)*ln(2*((2*c-2*d)^(1/2)*
2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*s
in(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*(2*c-2*
d)^(1/2)*c^2*d*sin(f*x+e)+2*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)
*(-(d^2/c^2)^(1/2)*c)^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))
/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*
cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*(2*c-2*d)^(1/2)*c*d^2*sin(f*x+e)
-2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*(-(d^2/c^2)^(1/2)*c)^(1/2)
*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(
f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+
e)+sin(f*x+e)))*(2*c-2*d)^(1/2)*d^3*sin(f*x+e)-cos(f*x+e)*ln(2*((2*c-2*d)^(
1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e
)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*(-
(d^2/c^2)^(1/2)*c)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*(2
*c-2*d)^(1/2)*c^2*d+2*cos(f*x+e)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*
x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+
e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*(-(d^2/c^2)^(1/2)*c)^(1/2)*
2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*(2*c-2*d)^(1/2)*c*d^2-cos(f
*x+e)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)
*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos
(f*x+e)+sin(f*x+e)))*(-(d^2/c^2)^(1/2)*c)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(
cos(f*x+e)+1))^(1/2)*(2*c-2*d)^(1/2)*d^3-2*(d^2/c^2)^(1/2)*(d*(c+d*sin(f*x+
e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d))^(1/2)*arctan(1/(-(d^2/c^2)^(1/2)*c)^(
1/2)*(d*(c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d))^(1/2))*c^3*d*cos
(f*x+e)+4*(d^2/c^2)^(1/2)*(d*(c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)
+d))^(1/2)*arctan(1/(-(d^2/c^2)^(1/2)*c)^(1/2)*(d*(c+d*sin(f*x+e))/((d^2/c^
2)^(1/2)*c*sin(f*x+e)+d))^(1/2))*c^2*d^2*cos(f*x+e)-2*(d^2/c^2)^(1/2)*(d*(c
+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d))^(1/2)*arctan(1/(-(d^2/c^2)
^(1/2)*c)^(1/2)*(d*(c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d))^(1/2)
)*c*d^3*cos(f*x+e)-ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x
+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e
)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*(-(d^2/c^2)^(1/2)*c)^(1/2)*2^(1/2)*((c+d*
sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*(2*c-2*d)^(1/2)*c^2*d+2*ln(2*((2*c-2*d)^(
1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e
)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*(-
(d^2/c^2)^(1/2)*c)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*(2
*c-2*d)^(1/2)*c*d^2-ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*
x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+
e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*(-(d^2/c^2)^(1/2)*c)^(1/2)*2^(1/2)*((c+d
*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*(2*c-2*d)^(1/2)*d^3+2*(d^2/c^2)^(1/2)*(-
(d^2/c^2)^(1/2)*c)^(1/2)*(((d^2/c^2)^(1/2)*c^4+6*(d^2/c^2)^(1/2)*d^2*c^2+d^
4*(d^2/c^2)^(1/2)-4*c^2*d^2-4*d^4)*c)^(1/2)*(d*(c+d*sin(f*x+e))/((d^2/c^2)^
(1/2)*c*sin(f*x+e)+d))^(1/2)*arctan(((d^2/c^2)^(1/2)*c^2-d^2)*c*((d^2/c^2)^
(1/2)-1)/(((d^2/c^2)^(1/2)*c^4+6*(d^2/c^2)^(1/2)*d^2*c^2+d^4*(d^2/c^2)^(1/2
```

```
)-4*c^2*d^2-4*d^4)*c)^(1/2)*((d^2/c^2)^(1/2)*c*sin(f*x+e)+d*cos(f*x+e)-d)/(
d*(c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d))^(1/2)/((d^2/c^2)^(1/2)
*c*sin(f*x+e)-d*cos(f*x+e)+d))*c*sin(f*x+e)+2*(d*(c+d*sin(f*x+e))/((d^2/c^2
)^(1/2)*c*sin(f*x+e)+d))^(1/2)*arctan(1/(-(d^2/c^2)^(1/2)*c)^(1/2)*(d*(c+d*
sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d))^(1/2))*c^2*d^2*cos(f*x+e)-4*(
d*(c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d))^(1/2)*arctan(1/(-(d^2/
c^2)^(1/2)*c)^(1/2)*(d*(c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d))^(
1/2))*c*d^3*cos(f*x+e)+2*(d*(c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+
d))^(1/2)*arctan(1/(-(d^2/c^2)^(1/2)*c)^(1/2)*(d*(c+d*sin(f*x+e))/((d^2/c^2
)^(1/2)*c*sin(f*x+e)+d))^(1/2))*d^4*cos(f*x+e)+2*(-(d^2/c^2)^(1/2)*c)^(1/2)
*(((d^2/c^2)^(1/2)*c^4+6*(d^2/c^2)^(1/2)*d^2*c^2+d^4*(d^2/c^2)^(1/2)-4*c^2*
d^2-4*d^4)*c)^(1/2)*(d*(c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d))^(
1/2)*arctan(((d^2/c^2)^(1/2)*c^2-d^2)*c*((d^2/c^2)^(1/2)-1)/(((d^2/c^2)^(1/
2)*c^4+6*(d^2/c^2)^(1/2)*d^2*c^2+d^4*(d^2/c^2)^(1/2)-4*c^2*d^2-4*d^4)*c)^(1
/2)*((d^2/c^2)^(1/2)*c*sin(f*x+e)+d*cos(f*x+e)-d)/(d*(c+d*sin(f*x+e))/((d^2
/c^2)^(1/2)*c*sin(f*x+e)+d))^(1/2)/((d^2/c^2)^(1/2)*c*sin(f*x+e)-d*cos(f*x+
e)+d))*d*sin(f*x+e)+2*(d^2/c^2)^(1/2)*(-(d^2/c^2)^(1/2)*c)^(1/2)*(((d^2/c^2
)^(1/2)*c^4+6*(d^2/c^2)^(1/2)*d^2*c^2+d^4*(d^2/c^2)^(1/2)-4*c^2*d^2-4*d^4)*
c)^(1/2)*(d*(c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d))^(1/2)*arctan
(((d^2/c^2)^(1/2)*c^2-d^2)*c*((d^2/c^2)^(1/2)-1)/(((d^2/c^2)^(1/2)*c^4+6*(d
^2/c^2)^(1/2)*d^2*c^2+d^4*(d^2/c^2)^(1/2)-4*c^2*d^2-4*d^4)*c)^(1/2)*((d^2/c
^2)^(1/2)*c*sin(f*x+e)+d*cos(f*x+e)-d)/(d*(c+d*sin(f*x+e))/((d^2/c^2)^(1/2)
*c*sin(f*x+e)+d))^(1/2)/((d^2/c^2)^(1/2)*c*sin(f*x+e)-d*cos(f*x+e)+d))*c+2*
(-(d^2/c^2)^(1/2)*c)^(1/2)*(((d^2/c^2)^(1/2)*c^4+6*(d^2/c^2)^(1/2)*d^2*c^2+
d^4*(d^2/c^2)^(1/2)-4*c^2*d^2-4*d^4)*c)^(1/2)*(d*(c+d*sin(f*x+e))/((d^2/c^2
)^(1/2)*c*sin(f*x+e)+d))^(1/2)*arctan(((d^2/c^2)^(1/2)*c^2-d^2)*c*((d^2/c^2
)^(1/2)-1)/(((d^2/c^2)^(1/2)*c^4+6*(d^2/c^2)^(1/2)*d^2*c^2+d^4*(d^2/c^2)^(1
/2)-4*c^2*d^2-4*d^4)*c)^(1/2)*((d^2/c^2)^(1/2)*c*sin(f*x+e)+d*cos(f*x+e)-d)
/(d*(c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d))^(1/2)/((d^2/c^2)^(1/
2)*c*sin(f*x+e)-d*cos(f*x+e)+d))*d)/(c+d*sin(f*x+e))^(1/2)/(a*(1+sin(f*x+e)
))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d \sin\left(fx+e\right)+c}}{\sqrt{a \sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(d*sin(f*x + e) + c)/sqrt(a*sin(f*x + e) + a), x)
```

Fricas [B] time = 4.08283, size = 4674, normalized size = 33.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] [1/4*(2*sqrt(2)*sqrt((c - d)/a)*log((2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqr
t(d*sin(f*x + e) + c)*sqrt((c - d)/a)*(cos(f*x + e) - sin(f*x + e) + 1) - (
c - 3*d)*cos(f*x + e)^2 - (3*c - d)*cos(f*x + e) + ((c - 3*d)*cos(f*x + e)
- 2*c - 2*d)*sin(f*x + e) - 2*c - 2*d)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)
*sin(f*x + e) - cos(f*x + e) - 2)) + sqrt(-d/a)*log((128*d^4*cos(f*x + e)^5
 + 128*(2*c*d^3 - d^4)*cos(f*x + e)^4 + c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^3
 + d^4 - 32*(5*c^2*d^2 - 14*c*d^3 + 13*d^4)*cos(f*x + e)^3 - 32*(c^3*d - 2*
c^2*d^2 + 9*c*d^3 - 4*d^4)*cos(f*x + e)^2 - 8*(16*d^3*cos(f*x + e)^4 + 24*(
c*d^2 - d^3)*cos(f*x + e)^3 - c^3 + 17*c^2*d - 59*c*d^2 + 51*d^3 - 2*(5*c^2
*d - 26*c*d^2 + 33*d^3)*cos(f*x + e)^2 - (c^3 - 7*c^2*d + 31*c*d^2 - 25*d^3
)*cos(f*x + e) + (16*d^3*cos(f*x + e)^3 + c^3 - 17*c^2*d + 59*c*d^2 - 51*d^
3 - 8*(3*c*d^2 - 5*d^3)*cos(f*x + e)^2 - 2*(5*c^2*d - 14*c*d^2 + 13*d^3)*co
s(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)
*sqrt(-d/a) + (c^4 - 28*c^3*d + 230*c^2*d^2 - 476*c*d^3 + 289*d^4)*cos(f*x
+ e) + (128*d^4*cos(f*x + e)^4 + c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^3 + d^4
- 256*(c*d^3 - d^4)*cos(f*x + e)^3 - 32*(5*c^2*d^2 - 6*c*d^3 + 5*d^4)*cos(f
*x + e)^2 + 32*(c^3*d - 7*c^2*d^2 + 15*c*d^3 - 9*d^4)*cos(f*x + e))*sin(f*x
 + e))/(cos(f*x + e) + sin(f*x + e) + 1)))/f, 1/2*(sqrt(2)*sqrt((c - d)/a)*
log((2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt((c -
d)/a)*(cos(f*x + e) - sin(f*x + e) + 1) - (c - 3*d)*cos(f*x + e)^2 - (3*c -
 d)*cos(f*x + e) + ((c - 3*d)*cos(f*x + e) - 2*c - 2*d)*sin(f*x + e) - 2*c
- 2*d)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2
)) + sqrt(d/a)*arctan(1/4*(8*d^2*cos(f*x + e)^2 - c^2 + 6*c*d - 9*d^2 - 8*(
c*d - d^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*
sqrt(d/a)/(2*d^3*cos(f*x + e)^3 - (3*c*d^2 - d^3)*cos(f*x + e)*sin(f*x + e)
 - (c^2*d - c*d^2 + 2*d^3)*cos(f*x + e))))/f, -1/4*(4*sqrt(2)*sqrt(-(c - d)
/a)*arctan(-sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt(
-(c - d)/a)/((c - d)*cos(f*x + e))) - sqrt(-d/a)*log((128*d^4*cos(f*x + e)^
5 + 128*(2*c*d^3 - d^4)*cos(f*x + e)^4 + c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^
3 + d^4 - 32*(5*c^2*d^2 - 14*c*d^3 + 13*d^4)*cos(f*x + e)^3 - 32*(c^3*d - 2
*c^2*d^2 + 9*c*d^3 - 4*d^4)*cos(f*x + e)^2 - 8*(16*d^3*cos(f*x + e)^4 + 24*
(c*d^2 - d^3)*cos(f*x + e)^3 - c^3 + 17*c^2*d - 59*c*d^2 + 51*d^3 - 2*(5*c^
2*d - 26*c*d^2 + 33*d^3)*cos(f*x + e)^2 - (c^3 - 7*c^2*d + 31*c*d^2 - 25*d^
3)*cos(f*x + e) + (16*d^3*cos(f*x + e)^3 + c^3 - 17*c^2*d + 59*c*d^2 - 51*d
^3 - 8*(3*c*d^2 - 5*d^3)*cos(f*x + e)^2 - 2*(5*c^2*d - 14*c*d^2 + 13*d^3)*c
os(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c
)*sqrt(-d/a) + (c^4 - 28*c^3*d + 230*c^2*d^2 - 476*c*d^3 + 289*d^4)*cos(f*x
 + e) + (128*d^4*cos(f*x + e)^4 + c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^3 + d^4
 - 256*(c*d^3 - d^4)*cos(f*x + e)^3 - 32*(5*c^2*d^2 - 6*c*d^3 + 5*d^4)*cos(
f*x + e)^2 + 32*(c^3*d - 7*c^2*d^2 + 15*c*d^3 - 9*d^4)*cos(f*x + e))*sin(f*
x + e))/(cos(f*x + e) + sin(f*x + e) + 1)))/f, -1/2*(2*sqrt(2)*sqrt(-(c - d
)/a)*arctan(-sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt
(-(c - d)/a)/((c - d)*cos(f*x + e))) - sqrt(d/a)*arctan(1/4*(8*d^2*cos(f*x
+ e)^2 - c^2 + 6*c*d - 9*d^2 - 8*(c*d - d^2)*sin(f*x + e))*sqrt(a*sin(f*x +
 e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt(d/a)/(2*d^3*cos(f*x + e)^3 - (3*c*d^
2 - d^3)*cos(f*x + e)*sin(f*x + e) - (c^2*d - c*d^2 + 2*d^3)*cos(f*x + e)))
)/f]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c + d\sin\left(e + fx\right)}}{\sqrt{a\left(\sin\left(e + fx\right) + 1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(1/2)/(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(c + d*sin(e + f*x))/sqrt(a*(sin(e + f*x) + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d\sin\left(fx+e\right)+c}}{\sqrt{a\sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(d*sin(f*x + e) + c)/sqrt(a*sin(f*x + e) + a), x)
```

3.591 $\int \frac{1}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\,dx$

Optimal. Leaf size=79

$$-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{\sqrt{a}f\sqrt{c-d}}$$

```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[a]*Sqrt[c - d]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*S
in[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])])/(Sqrt[a]*Sqrt[c - d]*f))
```

Rubi [A] time = 0.107281, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2782, 208}

$$-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{\sqrt{a}f\sqrt{c-d}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + a*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),x]
```

```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[a]*Sqrt[c - d]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*S
in[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])])/(Sqrt[a]*Sqrt[c - d]*f))
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\,dx &= -\frac{(2a)\,\text{Subst}\left(\int \frac{1}{2a^2-(ac-ad)x^2}\,dx, x, \frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{f}\\ &= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{\sqrt{a}\sqrt{c-d}f}\end{aligned}$$

Mathematica [B] time = 4.07253, size = 283, normalized size = 3.58

$$\frac{\log\left(\tan\left(\frac{1}{2}(e+fx)\right)+1\right)-\log\left((d-c)\tan\left(\frac{1}{2}(e+fx)\right)+2\sqrt{c-d}\sqrt{\frac{1}{\cos(e+fx)+1}}\sqrt{c+d\sin(e+fx)}+c-d\right)}{f\sqrt{a(\sin(e+fx)+1)}\sqrt{c+d\sin(e+fx)}\left(\frac{\sec^2\left(\frac{1}{2}(e+fx)\right)}{2\tan\left(\frac{1}{2}(e+fx)\right)+2}-\frac{\frac{\sqrt{c-d}\left(\frac{1}{\cos(e+fx)+1}\right)^{3/2}(c\sin(e+fx)+d\cos(e+fx)+d)}{\sqrt{c+d\sin(e+fx)}}-\frac{1}{2}(c-d)\sec^2\left(\frac{1}{2}(e+fx)\right)}{(d-c)\tan\left(\frac{1}{2}(e+fx)\right)+2\sqrt{c-d}\sqrt{\frac{1}{\cos(e+fx)+1}}\sqrt{c+d\sin(e+fx)}+c-d}\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[a + a*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),x]
```

```
[Out] (Log[1 + Tan[(e + f*x)/2]] - Log[c - d + 2*Sqrt[c - d]*Sqrt[(1 + Cos[e + f*
x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + (-c + d)*Tan[(e + f*x)/2]])/(f*Sqrt[a*
(1 + Sin[e + f*x])]*Sqrt[c + d*Sin[e + f*x]]*(Sec[(e + f*x)/2]^2/(2 + 2*Tan
[(e + f*x)/2]) - (-((c - d)*Sec[(e + f*x)/2]^2)/2 + (Sqrt[c - d]*((1 + Cos[
e + f*x])^(-1))^(3/2)*(d + d*Cos[e + f*x] + c*Sin[e + f*x]))/Sqrt[c + d*Sin
[e + f*x]])/(c - d + 2*Sqrt[c - d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d
*Sin[e + f*x]] + (-c + d)*Tan[(e + f*x)/2])))
```

Maple [B] time = 0.139, size = 191, normalized size = 2.4

$$-\frac{\left(1-\cos\left(fx+e\right)+\sin\left(fx+e\right)\right)\sqrt{2}}{f\sin\left(fx+e\right)}\sqrt{c+d\sin\left(fx+e\right)}\ln\left(2\frac{1}{1-\cos\left(fx+e\right)+\sin\left(fx+e\right)}\left(\sqrt{2c-2d}\sqrt{2}\sqrt{\frac{c+d}{\cos}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2),x)
```

```
[Out] -1/f/(2*c-2*d)^(1/2)*(1-cos(f*x+e)+sin(f*x+e))*(c+d*sin(f*x+e))^(1/2)*ln(2*
((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)
+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin
(f*x+e)))/(a*(1+sin(f*x+e)))^(1/2)/sin(f*x+e)*2^(1/2)/((c+d*sin(f*x+e))/(co
s(f*x+e)+1))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{a\sin\left(fx+e\right)+a}\sqrt{d\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate(1/(sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)), x)
```

Fricas [A] time = 2.59621, size = 1199, normalized size = 15.18

$$\left[\sqrt{2}\log\left(\frac{\left(c^2-14\,cd+17\,d^2\right)\cos\left(fx+e\right)^3-\left(13\,c^2-22\,cd-3\,d^2\right)\cos\left(fx+e\right)^2-\frac{4\sqrt{2}\left(\left(c^2-4\,cd+3\,d^2\right)\cos\left(fx+e\right)^2-4\,c^2+8\,cd-4\,d^2-\left(3\,c^2-4\,cd+d^2\right)\cos\left(fx+e\right)+\left(4\,c^2-8\,cd+4\,d^2+\left(c^2-\right.\right.\right.}{\sqrt{ac-ad}}}{\cos\left(fx+e\right)^3+3\cos}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="fri
cas")
```

```
[Out] [1/4*sqrt(2)*log(((c^2 - 14*c*d + 17*d^2)*cos(f*x + e)^3 - (13*c^2 - 22*c*d
 - 3*d^2)*cos(f*x + e)^2 - 4*sqrt(2)*((c^2 - 4*c*d + 3*d^2)*cos(f*x + e)^2
- 4*c^2 + 8*c*d - 4*d^2 - (3*c^2 - 4*c*d + d^2)*cos(f*x + e) + (4*c^2 - 8*c
*d + 4*d^2 + (c^2 - 4*c*d + 3*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f
*x + e) + a)*sqrt(d*sin(f*x + e) + c)/sqrt(a*c - a*d) - 4*c^2 - 8*c*d - 4*d
^2 - 2*(9*c^2 - 14*c*d + 9*d^2)*cos(f*x + e) + ((c^2 - 14*c*d + 17*d^2)*cos
(f*x + e)^2 - 4*c^2 - 8*c*d - 4*d^2 + 2*(7*c^2 - 18*c*d + 7*d^2)*cos(f*x +
e))*sin(f*x + e))/(cos(f*x + e)^3 + 3*cos(f*x + e)^2 + (cos(f*x + e)^2 - 2*
cos(f*x + e) - 4)*sin(f*x + e) - 2*cos(f*x + e) - 4))/(sqrt(a*c - a*d)*f),
1/2*sqrt(2)*sqrt(-1/(a*c - a*d))*arctan(-1/4*sqrt(2)*sqrt(a*sin(f*x + e) +
a)*((c - 3*d)*sin(f*x + e) - 3*c + d)*sqrt(d*sin(f*x + e) + c)*sqrt(-1/(a*c
 - a*d))/(d*cos(f*x + e)*sin(f*x + e) + c*cos(f*x + e)))/f]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}\sqrt{c+d\sin\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**(1/2)/(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a*(sin(e + f*x) + 1))*sqrt(c + d*sin(e + f*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\sin\left(fx+e\right)+a}\sqrt{d\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate(1/(sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)), x)
```

3.592 $\int \frac{1}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=131

$$\frac{2d\cos(e+fx)}{f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{\sqrt{a}f(c-d)^{3/2}}$$

```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[a]*Sqrt[c - d]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*S
in[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])])/(Sqrt[a]*(c - d)^(3/2)*f)) + (2*d*
Cos[e + f*x])/((c^2 - d^2)*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]])
```

Rubi [A] time = 0.242868, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2779, 12, 2782, 208}

$$\frac{2d\cos(e+fx)}{f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{\sqrt{a}f(c-d)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(3/2)),x]
```

```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[a]*Sqrt[c - d]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*S
in[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])])/(Sqrt[a]*(c - d)^(3/2)*f)) + (2*d*
Cos[e + f*x])/((c^2 - d^2)*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]])
```

Rule 2779

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> -Simp[(d*Cos[e + f*x]*(c + d*Sin[e + f*x])^(
n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(2*b*
(n + 1)*(c^2 - d^2)), Int[((c + d*Sin[e + f*x])^(n + 1)*Simp[a*d - 2*b*c*(n
 + 1) + b*d*(2*n + 3)*Sin[e + f*x], x])/Sqrt[a + b*Sin[e + f*x]], x], x] /;
 FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^{3/2}}\,dx = \frac{2d\cos(e+fx)}{\left(c^2-d^2\right)f\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}} + \frac{\int \frac{a(c+d)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\,\mathrm{s}}}}{a\left(c^2-d^2\right)}$$

$$= \frac{2d\cos(e+fx)}{\left(c^2-d^2\right)f\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}} + \frac{\int \frac{1}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\,\mathrm{s}}}}{c-d}$$

$$= \frac{2d\cos(e+fx)}{\left(c^2-d^2\right)f\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}} - \frac{(2a)\,\mathrm{Subst}\left(\int \frac{}{2a^2-(a}\right.}{}$$

$$= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{\sqrt{a}(c-d)^{3/2}f} + \frac{2d\cos(e}{\left(c^2-d^2\right)f\sqrt{a+a\sin(e+}}$$

Mathematica [B] time = 6.35406, size = 306, normalized size = 2.34

$$\frac{\frac{2d\cos(e+fx)}{c+d}+\frac{\log\left(\tan\left(\frac{1}{2}(e+fx)\right)+1\right)-\log\left((d-c)\tan\left(\frac{1}{2}(e+fx)\right)+2\sqrt{c-d}\sqrt{\frac{1}{\cos(e+fx)+1}}\sqrt{c+d\sin(e+fx)}+c-d\right)}{\frac{\sec^2\left(\frac{1}{2}(e+fx)\right)}{2\tan\left(\frac{1}{2}(e+fx)\right)+2}-\frac{\frac{\sqrt{c-d}\left(\frac{1}{\cos(e+fx)+1}\right)^{3/2}(c\sin(e+fx)+d\cos(e+fx)+d)}{\sqrt{c+d\sin(e+fx)}}-\frac{1}{2}(c-d)\sec^2\left(\frac{1}{2}(e+fx)\right)}{(d-c)\tan\left(\frac{1}{2}(e+fx)\right)+2\sqrt{c-d}\sqrt{\frac{1}{\cos(e+fx)+1}}\sqrt{c+d\sin(e+fx)}+c-d}}}{f(c-d)\sqrt{a(\sin(e+fx)+1)}\sqrt{c+d\sin(e+fx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(3/2)),x]
```

```
[Out] ((2*d*Cos[e + f*x])/(c + d) + (Log[1 + Tan[(e + f*x)/2]] - Log[c - d + 2*Sq
rt[c - d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + (-c + d)
*Tan[(e + f*x)/2]])/(Sec[(e + f*x)/2]^2/(2 + 2*Tan[(e + f*x)/2]) - (-((c -
d)*Sec[(e + f*x)/2]^2)/2 + (Sqrt[c - d]*((1 + Cos[e + f*x])^(-1))^(3/2)*(d
+ d*Cos[e + f*x] + c*Sin[e + f*x]))/Sqrt[c + d*Sin[e + f*x]])/(c - d + 2*Sq
rt[c - d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + (-c + d)
*Tan[(e + f*x)/2]])))/((c - d)*f*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c + d*Sin[e
 + f*x]])
```

Maple [B] time = 0.224, size = 879, normalized size = 6.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(3/2),x)
```

```
[Out] 1/f/(c+d)/(2*c-2*d)^(1/2)/(c-d)*(-ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f
*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x
+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1/2)*((c+d*sin(f*x+e))/
(cos(f*x+e)+1))^(1/2)*c*sin(f*x+e)-ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(
```

```
f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*
x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1/2)*((c+d*sin(f*x+e))
/(cos(f*x+e)+1))^(1/2)*d*sin(f*x+e)-cos(f*x+e)*2^(1/2)*ln(2*((2*c-2*d)^(1/2
)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d
*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*((c+d
*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*c-cos(f*x+e)*2^(1/2)*ln(2*((2*c-2*d)^(1/
2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-
d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*((c+
d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*d-2^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)
*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+
e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*((c+d*sin(f*x+
e))/(cos(f*x+e)+1))^(1/2)*c-2^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin
(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f
*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*((c+d*sin(f*x+e))/(cos(f
*x+e)+1))^(1/2)*d+2*d*(2*c-2*d)^(1/2)*cos(f*x+e))/(c+d*sin(f*x+e))^(1/2)/(a
*(1+sin(f*x+e)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\sin\left(fx+e\right)+a}\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="max
ima")
```

```
[Out] integrate(1/(sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^(3/2)), x)
```

Fricas [B] time = 3.16088, size = 2430, normalized size = 18.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="fri
cas")
```

```
[Out] [-1/4*(8*(d*cos(f*x + e) - d*sin(f*x + e) + d)*sqrt(a*sin(f*x + e) + a)*sqr
t(d*sin(f*x + e) + c) - sqrt(2)*(a*c^2 + 2*a*c*d + a*d^2 - (a*c*d + a*d^2)*
cos(f*x + e)^2 + (a*c^2 + a*c*d)*cos(f*x + e) + (a*c^2 + 2*a*c*d + a*d^2 +
(a*c*d + a*d^2)*cos(f*x + e))*sin(f*x + e))*log(((c^2 - 14*c*d + 17*d^2)*co
s(f*x + e)^3 - (13*c^2 - 22*c*d - 3*d^2)*cos(f*x + e)^2 + 4*sqrt(2)*((c^2 -
 4*c*d + 3*d^2)*cos(f*x + e)^2 - 4*c^2 + 8*c*d - 4*d^2 - (3*c^2 - 4*c*d + d
^2)*cos(f*x + e) + (4*c^2 - 8*c*d + 4*d^2 + (c^2 - 4*c*d + 3*d^2)*cos(f*x +
 e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)/sqrt(a
*c - a*d) - 4*c^2 - 8*c*d - 4*d^2 - 2*(9*c^2 - 14*c*d + 9*d^2)*cos(f*x + e)
 + ((c^2 - 14*c*d + 17*d^2)*cos(f*x + e)^2 - 4*c^2 - 8*c*d - 4*d^2 + 2*(7*c
^2 - 18*c*d + 7*d^2)*cos(f*x + e))*sin(f*x + e))/(cos(f*x + e)^3 + 3*cos(f*
x + e)^2 + (cos(f*x + e)^2 - 2*cos(f*x + e) - 4)*sin(f*x + e) - 2*cos(f*x +
 e) - 4))/sqrt(a*c - a*d))/((a*c^2*d - a*d^3)*f*cos(f*x + e)^2 - (a*c^3 - a
*c*d^2)*f*cos(f*x + e) - (a*c^3 + a*c^2*d - a*c*d^2 - a*d^3)*f - ((a*c^2*d
- a*d^3)*f*cos(f*x + e) + (a*c^3 + a*c^2*d - a*c*d^2 - a*d^3)*f)*sin(f*x +
e)), -1/2*(sqrt(2)*(a*c^2 + 2*a*c*d + a*d^2 - (a*c*d + a*d^2)*cos(f*x + e)^
```

```
2 + (a*c^2 + a*c*d)*cos(f*x + e) + (a*c^2 + 2*a*c*d + a*d^2 + (a*c*d + a*d^
2)*cos(f*x + e))*sin(f*x + e))*sqrt(-1/(a*c - a*d))*arctan(-1/4*sqrt(2)*sqr
t(a*sin(f*x + e) + a)*((c - 3*d)*sin(f*x + e) - 3*c + d)*sqrt(d*sin(f*x + e
) + c)*sqrt(-1/(a*c - a*d))/(d*cos(f*x + e)*sin(f*x + e) + c*cos(f*x + e)))
 + 4*(d*cos(f*x + e) - d*sin(f*x + e) + d)*sqrt(a*sin(f*x + e) + a)*sqrt(d*
sin(f*x + e) + c))/((a*c^2*d - a*d^3)*f*cos(f*x + e)^2 - (a*c^3 - a*c*d^2)*
f*cos(f*x + e) - (a*c^3 + a*c^2*d - a*c*d^2 - a*d^3)*f - ((a*c^2*d - a*d^3)
*f*cos(f*x + e) + (a*c^3 + a*c^2*d - a*c*d^2 - a*d^3)*f)*sin(f*x + e))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}\left(c+d\sin\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**(1/2)/(c+d*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral(1/(sqrt(a*(sin(e + f*x) + 1))*(c + d*sin(e + f*x))**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\sin\left(fx+e\right)+a}\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="gia
c")
```

```
[Out] integrate(1/(sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^(3/2)), x)
```

3.593 $\int \frac{1}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=191

$$\frac{2d(5c+d)\cos(e+fx)}{3f\left(c^2-d^2\right)^2\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}+\frac{2d\cos(e+fx)}{3f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^{3/2}}-\frac{\sqrt{2}\tanh$$

```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[a]*Sqrt[c - d]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*S
in[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])]])/(Sqrt[a]*(c - d)^(5/2)*f)) + (2*d*
Cos[e + f*x])/(3*(c^2 - d^2)*f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x]
)^(3/2)) + (2*d*(5*c + d)*Cos[e + f*x])/(3*(c^2 - d^2)^2*f*Sqrt[a + a*Sin[e
 + f*x]]*Sqrt[c + d*Sin[e + f*x]])
```

Rubi [A] time = 0.475902, antiderivative size = 191, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2779, 2984, 12, 2782, 208}

$$\frac{2d(5c+d)\cos(e+fx)}{3f\left(c^2-d^2\right)^2\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}+\frac{2d\cos(e+fx)}{3f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^{3/2}}-\frac{\sqrt{2}\tanh$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(5/2)),x]
```

```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[a]*Sqrt[c - d]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*S
in[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])]])/(Sqrt[a]*(c - d)^(5/2)*f)) + (2*d*
Cos[e + f*x])/(3*(c^2 - d^2)*f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x]
)^(3/2)) + (2*d*(5*c + d)*Cos[e + f*x])/(3*(c^2 - d^2)^2*f*Sqrt[a + a*Sin[e
 + f*x]]*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2779

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> -Simp[(d*Cos[e + f*x]*(c + d*Sin[e + f*x])^(
n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(2*b*
(n + 1)*(c^2 - d^2)), Int[((c + d*Sin[e + f*x])^(n + 1)*Simp[a*d - 2*b*c*(n
 + 1) + b*d*(2*n + 3)*Sin[e + f*x], x])/Sqrt[a + b*Sin[e + f*x]], x], x] /;
 FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^{5/2}}\,dx = \frac{2d\cos(e+fx)}{3\left(c^2-d^2\right)f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^{3/2}} + \frac{\int \frac{a(3c+d)-2ad}{\sqrt{a+a\sin(e+fx)}(c\cdots}}{3a\left(c^2\cdots\right.}$$

$$= \frac{2d\cos(e+fx)}{3\left(c^2-d^2\right)f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^{3/2}} + \frac{2a\cdots}{3\left(c^2-d^2\right)^2 f\sqrt{a\cdots}}$$

$$= \frac{2d\cos(e+fx)}{3\left(c^2-d^2\right)f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^{3/2}} + \frac{2a\cdots}{3\left(c^2-d^2\right)^2 f\sqrt{a\cdots}}$$

$$= \frac{2d\cos(e+fx)}{3\left(c^2-d^2\right)f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^{3/2}} + \frac{2a\cdots}{3\left(c^2-d^2\right)^2 f\sqrt{a\cdots}}$$

$$= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{\sqrt{a}(c-d)^{5/2}f} + \frac{2d\cos(\cdots}{3\left(c^2-d^2\right)f\sqrt{a+a\sin(e\cdots}}$$

Mathematica [B] time = 6.19057, size = 387, normalized size = 2.03

$$\frac{\frac{2d\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(6c^2+d(5c+d)\sin(e+fx)+cd-d^2\right)}{(c+d)^2(c+d\sin(e+fx))} + \frac{3\left(\log\left(\tan\left(\frac{1}{2}(e+fx)\right)+1\right)-\log\left((d-c)\tan\left(\frac{1}{2}(e+fx)\right)+2\sqrt{c-\cdots}\right.\right.}{\frac{\sec^2\left(\frac{1}{2}(e+fx)\right)}{2\tan\left(\frac{1}{2}(e+fx)\right)+2}-\frac{\frac{\sqrt{c-d}\left(\frac{1}{\cos(e+fx)+1}\right)^{3/2}(c\sin(e+fx)+\cdots}{\sqrt{c+d\sin(e+fx)}}}{(d-c)\tan\left(\frac{1}{2}(e+fx)\right)+2\sqrt{c-d}\sqrt{\cdots}}}}{3f(c-d)^2\sqrt{a(\sin(e+fx)+1)}\sqrt{c+d\sin(e+fx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(5/2)),x]
```

```
[Out] ((2*d*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*
x)/2])*(6*c^2 + c*d - d^2 + d*(5*c + d)*Sin[e + f*x]))/((c + d)^2*(c + d*Si
n[e + f*x])) + (3*(Log[1 + Tan[(e + f*x)/2]] - Log[c - d + 2*Sqrt[c - d]*Sq
rt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + (-c + d)*Tan[(e + f*
x)/2]]))/(Sec[(e + f*x)/2]^2/(2 + 2*Tan[(e + f*x)/2]) - (-((c - d)*Sec[(e +
 f*x)/2]^2)/2 + (Sqrt[c - d]*((1 + Cos[e + f*x])^(-1))^(3/2)*(d + d*Cos[e +
```

```
f*x] + c*Sin[e + f*x]))/Sqrt[c + d*Sin[e + f*x]])/(c - d + 2*Sqrt[c - d]*S
qrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + (-c + d)*Tan[(e + f
*x)/2])))/(3*(c - d)^2*f*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c + d*Sin[e + f*x]
])
```

Maple [B] time = 0.278, size = 2575, normalized size = 13.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(5/2),x)
```

```
[Out] -1/3/f/(c+d)^2/(2*c-2*d)^(1/2)/(c-d)^2*(3*((c+d*sin(f*x+e))/(cos(f*x+e)+1))
^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2
)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-co
s(f*x+e)+sin(f*x+e)))*cos(f*x+e)^2*2^(1/2)*c^2*d+6*((c+d*sin(f*x+e))/(cos(f
*x+e)+1))^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)
+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c
+d)/(1-cos(f*x+e)+sin(f*x+e)))*cos(f*x+e)^2*2^(1/2)*c*d^2+3*((c+d*sin(f*x+e
))/(cos(f*x+e)+1))^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(c
os(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos
(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*cos(f*x+e)^2*2^(1/2)*d^3-3*cos(f*x+
e)*sin(f*x+e)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1
))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d
)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2
)*c^2*d-6*cos(f*x+e)*sin(f*x+e)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x
+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e
)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1/2)*((c+d*sin(f*x+e))/(c
os(f*x+e)+1))^(1/2)*c*d^2-3*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*ln(2*((
2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c
*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f
*x+e)))*cos(f*x+e)*sin(f*x+e)*2^(1/2)*d^3-3*((c+d*sin(f*x+e))/(cos(f*x+e)+1
))^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1
/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-
cos(f*x+e)+sin(f*x+e)))*cos(f*x+e)*2^(1/2)*c^3-6*ln(2*((2*c-2*d)^(1/2)*2^(1
/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f
*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*c^2*d*2^(1/
2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)-3*ln(2*((2*c-2*d)^(1/
2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-
d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*c*d^
2*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)-3*ln(2*((2*c-2
*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(
f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)
))*c^3*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)-9*ln(2*((
2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c
*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f
*x+e)))*c^2*d*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)-9*
ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f
*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e
)+sin(f*x+e)))*c*d^2*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*
x+e)-3*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2
)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x
+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*sin(f*x+e)*2^
(1/2)*d^3-3*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*ln(2*((2*c-2*d)^(1/2)*2
^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*si
n(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1/2)*
```

```
c^3-9*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)
*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+
e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1/2)*c^2*d-
9*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c
+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c
*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1/2)*c*d^2-3*ln
(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x
+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+
sin(f*x+e)))*d^3*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)+10*cos(f*x
+e)*sin(f*x+e)*(2*c-2*d)^(1/2)*c*d^2+2*cos(f*x+e)*sin(f*x+e)*(2*c-2*d)^(1/2
)*d^3+12*cos(f*x+e)*(2*c-2*d)^(1/2)*c^2*d+2*cos(f*x+e)*(2*c-2*d)^(1/2)*c*d^
2-2*cos(f*x+e)*(2*c-2*d)^(1/2)*d^3)*(c+d*sin(f*x+e))^(1/2)/(cos(f*x+e)^2*d^
2-2*sin(f*x+e)*c*d-c^2-d^2)/(a*(1+sin(f*x+e)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\sin\left(fx+e\right)+a}\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(5/2),x, algorithm="max
ima")
```

```
[Out] integrate(1/(sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^(5/2)), x)
```

Fricas [B] time = 5.0642, size = 4224, normalized size = 22.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(5/2),x, algorithm="fri
cas")
```

```
[Out] [-1/12*(8*(6*c^2*d - 4*c*d^2 - 2*d^3 + (5*c*d^2 + d^3)*cos(f*x + e)^2 + (6*
c^2*d + c*d^2 - d^3)*cos(f*x + e) - (6*c^2*d - 4*c*d^2 - 2*d^3 - (5*c*d^2 +
 d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x +
 e) + c) + 3*sqrt(2)*(a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 + a*d^4 -
 (a*c^2*d^2 + 2*a*c*d^3 + a*d^4)*cos(f*x + e)^3 - (2*a*c^3*d + 5*a*c^2*d^2
+ 4*a*c*d^3 + a*d^4)*cos(f*x + e)^2 + (a*c^4 + 2*a*c^3*d + 2*a*c^2*d^2 + 2*
a*c*d^3 + a*d^4)*cos(f*x + e) + (a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^
3 + a*d^4 - (a*c^2*d^2 + 2*a*c*d^3 + a*d^4)*cos(f*x + e)^2 + 2*(a*c^3*d + 2
*a*c^2*d^2 + a*c*d^3)*cos(f*x + e))*sin(f*x + e))*log(((c^2 - 14*c*d + 17*d
^2)*cos(f*x + e)^3 - (13*c^2 - 22*c*d - 3*d^2)*cos(f*x + e)^2 - 4*sqrt(2)*(
(c^2 - 4*c*d + 3*d^2)*cos(f*x + e)^2 - 4*c^2 + 8*c*d - 4*d^2 - (3*c^2 - 4*c
*d + d^2)*cos(f*x + e) + (4*c^2 - 8*c*d + 4*d^2 + (c^2 - 4*c*d + 3*d^2)*cos
(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)/
sqrt(a*c - a*d) - 4*c^2 - 8*c*d - 4*d^2 - 2*(9*c^2 - 14*c*d + 9*d^2)*cos(f*
x + e) + ((c^2 - 14*c*d + 17*d^2)*cos(f*x + e)^2 - 4*c^2 - 8*c*d - 4*d^2 +
2*(7*c^2 - 18*c*d + 7*d^2)*cos(f*x + e))*sin(f*x + e))/(cos(f*x + e)^3 + 3*
cos(f*x + e)^2 + (cos(f*x + e)^2 - 2*cos(f*x + e) - 4)*sin(f*x + e) - 2*cos
(f*x + e) - 4))/sqrt(a*c - a*d))/((a*c^4*d^2 - 2*a*c^2*d^4 + a*d^6)*f*cos(f
*x + e)^3 + (2*a*c^5*d + a*c^4*d^2 - 4*a*c^3*d^3 - 2*a*c^2*d^4 + 2*a*c*d^5
```

```
+ a*d^6)*f*cos(f*x + e)^2 - (a*c^6 - a*c^4*d^2 - a*c^2*d^4 + a*d^6)*f*cos(f
*x + e) - (a*c^6 + 2*a*c^5*d - a*c^4*d^2 - 4*a*c^3*d^3 - a*c^2*d^4 + 2*a*c*
d^5 + a*d^6)*f + ((a*c^4*d^2 - 2*a*c^2*d^4 + a*d^6)*f*cos(f*x + e)^2 - 2*(a
*c^5*d - 2*a*c^3*d^3 + a*c*d^5)*f*cos(f*x + e) - (a*c^6 + 2*a*c^5*d - a*c^4
*d^2 - 4*a*c^3*d^3 - a*c^2*d^4 + 2*a*c*d^5 + a*d^6)*f)*sin(f*x + e)), -1/6*
(3*sqrt(2)*(a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 + a*d^4 - (a*c^2*d^
2 + 2*a*c*d^3 + a*d^4)*cos(f*x + e)^3 - (2*a*c^3*d + 5*a*c^2*d^2 + 4*a*c*d^
3 + a*d^4)*cos(f*x + e)^2 + (a*c^4 + 2*a*c^3*d + 2*a*c^2*d^2 + 2*a*c*d^3 +
a*d^4)*cos(f*x + e) + (a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 + a*d^4
- (a*c^2*d^2 + 2*a*c*d^3 + a*d^4)*cos(f*x + e)^2 + 2*(a*c^3*d + 2*a*c^2*d^2
 + a*c*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(-1/(a*c - a*d))*arctan(-1/4*sq
rt(2)*sqrt(a*sin(f*x + e) + a)*((c - 3*d)*sin(f*x + e) - 3*c + d)*sqrt(d*si
n(f*x + e) + c)*sqrt(-1/(a*c - a*d))/(d*cos(f*x + e)*sin(f*x + e) + c*cos(f
*x + e))) + 4*(6*c^2*d - 4*c*d^2 - 2*d^3 + (5*c*d^2 + d^3)*cos(f*x + e)^2 +
 (6*c^2*d + c*d^2 - d^3)*cos(f*x + e) - (6*c^2*d - 4*c*d^2 - 2*d^3 - (5*c*d
^2 + d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f
*x + e) + c))/((a*c^4*d^2 - 2*a*c^2*d^4 + a*d^6)*f*cos(f*x + e)^3 + (2*a*c^
5*d + a*c^4*d^2 - 4*a*c^3*d^3 - 2*a*c^2*d^4 + 2*a*c*d^5 + a*d^6)*f*cos(f*x
+ e)^2 - (a*c^6 - a*c^4*d^2 - a*c^2*d^4 + a*d^6)*f*cos(f*x + e) - (a*c^6 +
2*a*c^5*d - a*c^4*d^2 - 4*a*c^3*d^3 - a*c^2*d^4 + 2*a*c*d^5 + a*d^6)*f + ((
a*c^4*d^2 - 2*a*c^2*d^4 + a*d^6)*f*cos(f*x + e)^2 - 2*(a*c^5*d - 2*a*c^3*d^
3 + a*c*d^5)*f*cos(f*x + e) - (a*c^6 + 2*a*c^5*d - a*c^4*d^2 - 4*a*c^3*d^3
- a*c^2*d^4 + 2*a*c*d^5 + a*d^6)*f)*sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**(1/2)/(c+d*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\sin\left(fx+e\right)+a}\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(5/2),x, algorithm="gia
c")
```

```
[Out] integrate(1/(sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^(5/2)), x)
```

3.594 $\int \frac{(c+d\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=251

$$-\frac{d^{3/2}(5c-3d)\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{a^{3/2}f}-\frac{(c+9d)(c-d)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{2\sqrt{2}a^{3/2}f}-\frac{(c-d)\cos(e}{2f(a}$$

```
[Out] -(((5*c - 3*d)*d^(3/2)*ArcTan[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[a + a*Si
n[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])])/(a^(3/2)*f)) - ((c - d)^(3/2)*(c +
9*d)*ArcTanh[(Sqrt[a]*Sqrt[c - d]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e +
 f*x]]*Sqrt[c + d*Sin[e + f*x]])])/(2*Sqrt[2]*a^(3/2)*f) + ((c - 3*d)*d*Cos
[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(2*a*f*Sqrt[a + a*Sin[e + f*x]]) - ((c
- d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(3/2))/(2*f*(a + a*Sin[e + f*x])^(3/
2))
```

Rubi [A] time = 0.902715, antiderivative size = 251, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2765, 2983, 2982, 2782, 208, 2775, 205}

$$-\frac{d^{3/2}(5c-3d)\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{a^{3/2}f}-\frac{(c+9d)(c-d)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{2\sqrt{2}a^{3/2}f}-\frac{(c-d)\cos(e}{2f(a}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^(5/2)/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] -(((5*c - 3*d)*d^(3/2)*ArcTan[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[a + a*Si
n[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])])/(a^(3/2)*f)) - ((c - d)^(3/2)*(c +
9*d)*ArcTanh[(Sqrt[a]*Sqrt[c - d]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e +
 f*x]]*Sqrt[c + d*Sin[e + f*x]])])/(2*Sqrt[2]*a^(3/2)*f) + ((c - 3*d)*d*Cos
[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(2*a*f*Sqrt[a + a*Sin[e + f*x]]) - ((c
- d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(3/2))/(2*f*(a + a*Sin[e + f*x])^(3/
2))
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
```

```
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{3/2}}\,dx = -\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{2f(a+a\sin(e+fx))^{3/2}} - \frac{\int \frac{\sqrt{c+d\sin(e+fx)}\left(-\frac{1}{2}a\left(c^2+6cd-3d^2\right)+a(c-3d)d\sin(e+f\cdots\right.}{\sqrt{a+a\sin(e+fx)}}}{2a^2}$$

$$= \frac{(c-3d)d\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2af\sqrt{a+a\sin(e+fx)}} - \frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{2f(a+a\sin(e+fx))^{3/2}} - \cdots$$

$$= \frac{(c-3d)d\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2af\sqrt{a+a\sin(e+fx)}} - \frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{2f(a+a\sin(e+fx))^{3/2}} + \cdots$$

$$= \frac{(c-3d)d\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2af\sqrt{a+a\sin(e+fx)}} - \frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{2f(a+a\sin(e+fx))^{3/2}} - \cdots$$

$$= -\frac{(5c-3d)d^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{a^{3/2}f} - \frac{(c-d)^{3/2}(c+9d)\tanh^{-1}\left(\frac{\sqrt{a}\cdots}{\sqrt{2}\sqrt{a+a\sin\cdots}}\right.}{2\sqrt{2}a^{3/2}f}$$

Mathematica [C] time = 17.1685, size = 1844, normalized size = 7.35

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*Sin[e + f*x])^(5/2)/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*(-(d^2*Cos[(e + f*x)/2]) + d^2*Sin
[(e + f*x)/2] - (c - d)^2/(2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])) + (c^2*
Sin[(e + f*x)/2] - 2*c*d*Sin[(e + f*x)/2] + d^2*Sin[(e + f*x)/2])/(Cos[(e +
 f*x)/2] + Sin[(e + f*x)/2])^2)*Sqrt[c + d*Sin[e + f*x]])/(f*(a*(1 + Sin[e
+ f*x]))^(3/2)) + ((((c - d)^(3/2)*(c + 9*d)*Log[1 + Tan[(e + f*x)/2]])/Sqr
t[2] + I*(5*c - 3*d)*d^(3/2)*Log[((-I)*((-I)*c + d + (1 - I)*Sqrt[2]*Sqrt[d
]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + (c - I*d)*Tan[(e
 + f*x)/2]))/(d^(5/2)*(-5*c + 3*d)*(-I + Tan[(e + f*x)/2]))] + I*d^(3/2)*(-
5*c + 3*d)*Log[(I*(I*c + d + (1 + I)*Sqrt[2]*Sqrt[d]*Sqrt[(1 + Cos[e + f*x]
)^(-1)]*Sqrt[c + d*Sin[e + f*x]] + (c + I*d)*Tan[(e + f*x)/2]))/(d^(5/2)*(-
5*c + 3*d)*(I + Tan[(e + f*x)/2]))] - ((c - d)^(3/2)*(c + 9*d)*Log[c - d +
2*Sqrt[c - d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + (-c
+ d)*Tan[(e + f*x)/2]])/Sqrt[2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*(c
^3/(4*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c + d*Sin[e + f*x]]) + (7*
c^2*d)/(4*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c + d*Sin[e + f*x]]) -
 (7*c*d^2)/(4*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c + d*Sin[e + f*x]
]) + (3*d^3)/(4*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c + d*Sin[e + f*
x]]) + (5*c*d^2*Sin[e + f*x])/(2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt
[c + d*Sin[e + f*x]]) - (3*d^3*Sin[e + f*x])/(2*(Cos[(e + f*x)/2] + Sin[(e
+ f*x)/2])*Sqrt[c + d*Sin[e + f*x]])))/(f*(a*(1 + Sin[e + f*x]))^(3/2)*(((c
 - d)^(3/2)*(c + 9*d)*Sec[(e + f*x)/2]^2)/(2*Sqrt[2]*(1 + Tan[(e + f*x)/2])
) - ((c - d)^(3/2)*(c + 9*d)*(((-c + d)*Sec[(e + f*x)/2]^2)/2 + (Sqrt[c - d
]*d*Cos[e + f*x]*Sqrt[(1 + Cos[e + f*x])^(-1)])/Sqrt[c + d*Sin[e + f*x]] +
Sqrt[c - d]*((1 + Cos[e + f*x])^(-1))^(3/2)*Sin[e + f*x]*Sqrt[c + d*Sin[e +
 f*x]]))/(Sqrt[2]*(c - d + 2*Sqrt[c - d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt
[c + d*Sin[e + f*x]] + (-c + d)*Tan[(e + f*x)/2])) - ((5*c - 3*d)*d^4*(-5*c
 + 3*d)*(-I + Tan[(e + f*x)/2])*(((-I)*(((c - I*d)*Sec[(e + f*x)/2]^2)/2 +
((1 - I)*d^(3/2)*Cos[e + f*x]*Sqrt[(1 + Cos[e + f*x])^(-1)])/(Sqrt[2]*Sqrt[
c + d*Sin[e + f*x]]) + ((1 - I)*Sqrt[d]*((1 + Cos[e + f*x])^(-1))^(3/2)*Sin
[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/Sqrt[2]))/(d^(5/2)*(-5*c + 3*d)*(-I + T
an[(e + f*x)/2])) + ((I/2)*Sec[(e + f*x)/2]^2*((-I)*c + d + (1 - I)*Sqrt[2]
*Sqrt[d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + (c - I*d)
*Tan[(e + f*x)/2]))/(d^(5/2)*(-5*c + 3*d)*(-I + Tan[(e + f*x)/2])^2)))/((-I
)*c + d + (1 - I)*Sqrt[2]*Sqrt[d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*
Sin[e + f*x]] + (c - I*d)*Tan[(e + f*x)/2]) + (d^4*(-5*c + 3*d)^2*(I + Tan[
(e + f*x)/2])*((I*(((c + I*d)*Sec[(e + f*x)/2]^2)/2 + ((1 + I)*d^(3/2)*Cos[
e + f*x]*Sqrt[(1 + Cos[e + f*x])^(-1)])/(Sqrt[2]*Sqrt[c + d*Sin[e + f*x]])
+ ((1 + I)*Sqrt[d]*((1 + Cos[e + f*x])^(-1))^(3/2)*Sin[e + f*x]*Sqrt[c + d*
Sin[e + f*x]])/Sqrt[2]))/(d^(5/2)*(-5*c + 3*d)*(I + Tan[(e + f*x)/2])) - ((
I/2)*Sec[(e + f*x)/2]^2*(I*c + d + (1 + I)*Sqrt[2]*Sqrt[d]*Sqrt[(1 + Cos[e
+ f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + (c + I*d)*Tan[(e + f*x)/2]))/(d^(5
/2)*(-5*c + 3*d)*(I + Tan[(e + f*x)/2])^2)))/(I*c + d + (1 + I)*Sqrt[2]*Sqr
t[d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + (c + I*d)*Tan
[(e + f*x)/2]))
```

Maple [F] time = 0.362, size = 0, normalized size = 0.

$$\int \left(c+d\sin\left(fx+e\right)\right)^{\frac{5}{2}}\left(a+a\sin\left(fx+e\right)\right)^{-\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(3/2),x)
```

```
[Out] int((c+d*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(5/2)/(a*sin(f*x + e) + a)^(3/2), x)
```

Fricas [B] time = 5.99168, size = 8276, normalized size = 32.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] [1/8*(2*sqrt(1/2)*(2*a*c^2 + 16*a*c*d - 18*a*d^2 - (a*c^2 + 8*a*c*d - 9*a*d
^2)*cos(f*x + e)^2 + (a*c^2 + 8*a*c*d - 9*a*d^2)*cos(f*x + e) + (2*a*c^2 +
16*a*c*d - 18*a*d^2 + (a*c^2 + 8*a*c*d - 9*a*d^2)*cos(f*x + e))*sin(f*x + e
))*sqrt((c - d)/a)*log(-(4*sqrt(1/2)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*
x + e) + c)*sqrt((c - d)/a)*(cos(f*x + e) - sin(f*x + e) + 1) + (c - 3*d)*c
os(f*x + e)^2 + (3*c - d)*cos(f*x + e) - ((c - 3*d)*cos(f*x + e) - 2*c - 2*
d)*sin(f*x + e) + 2*c + 2*d)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x +
 e) - cos(f*x + e) - 2)) + (10*a*c*d - 6*a*d^2 - (5*a*c*d - 3*a*d^2)*cos(f*
x + e)^2 + (5*a*c*d - 3*a*d^2)*cos(f*x + e) + (10*a*c*d - 6*a*d^2 + (5*a*c*
d - 3*a*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(-d/a)*log((128*d^4*cos(f*x +
e)^5 + 128*(2*c*d^3 - d^4)*cos(f*x + e)^4 + c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c
*d^3 + d^4 - 32*(5*c^2*d^2 - 14*c*d^3 + 13*d^4)*cos(f*x + e)^3 - 32*(c^3*d
- 2*c^2*d^2 + 9*c*d^3 - 4*d^4)*cos(f*x + e)^2 + 8*(16*d^3*cos(f*x + e)^4 +
24*(c*d^2 - d^3)*cos(f*x + e)^3 - c^3 + 17*c^2*d - 59*c*d^2 + 51*d^3 - 2*(5
*c^2*d - 26*c*d^2 + 33*d^3)*cos(f*x + e)^2 - (c^3 - 7*c^2*d + 31*c*d^2 - 25
*d^3)*cos(f*x + e) + (16*d^3*cos(f*x + e)^3 + c^3 - 17*c^2*d + 59*c*d^2 - 5
1*d^3 - 8*(3*c*d^2 - 5*d^3)*cos(f*x + e)^2 - 2*(5*c^2*d - 14*c*d^2 + 13*d^3
)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e)
+ c)*sqrt(-d/a) + (c^4 - 28*c^3*d + 230*c^2*d^2 - 476*c*d^3 + 289*d^4)*cos(
f*x + e) + (128*d^4*cos(f*x + e)^4 + c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^3 +
d^4 - 256*(c*d^3 - d^4)*cos(f*x + e)^3 - 32*(5*c^2*d^2 - 6*c*d^3 + 5*d^4)*c
os(f*x + e)^2 + 32*(c^3*d - 7*c^2*d^2 + 15*c*d^3 - 9*d^4)*cos(f*x + e))*sin
(f*x + e))/(cos(f*x + e) + sin(f*x + e) + 1)) + 4*(2*d^2*cos(f*x + e)^2 + c
^2 - 2*c*d + d^2 + (c^2 - 2*c*d + 3*d^2)*cos(f*x + e) + (2*d^2*cos(f*x + e)
 - c^2 + 2*c*d - d^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x
 + e) + c))/(a^2*f*cos(f*x + e)^2 - a^2*f*cos(f*x + e) - 2*a^2*f - (a^2*f*c
```

```
os(f*x + e) + 2*a^2*f)*sin(f*x + e)), 1/4*(sqrt(1/2)*(2*a*c^2 + 16*a*c*d -
18*a*d^2 - (a*c^2 + 8*a*c*d - 9*a*d^2)*cos(f*x + e)^2 + (a*c^2 + 8*a*c*d -
9*a*d^2)*cos(f*x + e) + (2*a*c^2 + 16*a*c*d - 18*a*d^2 + (a*c^2 + 8*a*c*d -
 9*a*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt((c - d)/a)*log(-(4*sqrt(1/2)*sqr
t(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt((c - d)/a)*(cos(f*x + e
) - sin(f*x + e) + 1) + (c - 3*d)*cos(f*x + e)^2 + (3*c - d)*cos(f*x + e) -
 ((c - 3*d)*cos(f*x + e) - 2*c - 2*d)*sin(f*x + e) + 2*c + 2*d)/(cos(f*x +
e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) - (10*a*c*d - 6
*a*d^2 - (5*a*c*d - 3*a*d^2)*cos(f*x + e)^2 + (5*a*c*d - 3*a*d^2)*cos(f*x +
 e) + (10*a*c*d - 6*a*d^2 + (5*a*c*d - 3*a*d^2)*cos(f*x + e))*sin(f*x + e))
*sqrt(d/a)*arctan(1/4*(8*d^2*cos(f*x + e)^2 - c^2 + 6*c*d - 9*d^2 - 8*(c*d
- d^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt
(d/a)/(2*d^3*cos(f*x + e)^3 - (3*c*d^2 - d^3)*cos(f*x + e)*sin(f*x + e) - (
c^2*d - c*d^2 + 2*d^3)*cos(f*x + e))) + 2*(2*d^2*cos(f*x + e)^2 + c^2 - 2*c
*d + d^2 + (c^2 - 2*c*d + 3*d^2)*cos(f*x + e) + (2*d^2*cos(f*x + e) - c^2 +
 2*c*d - d^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) +
c))/(a^2*f*cos(f*x + e)^2 - a^2*f*cos(f*x + e) - 2*a^2*f - (a^2*f*cos(f*x +
 e) + 2*a^2*f)*sin(f*x + e)), 1/8*(4*sqrt(1/2)*(2*a*c^2 + 16*a*c*d - 18*a*d
^2 - (a*c^2 + 8*a*c*d - 9*a*d^2)*cos(f*x + e)^2 + (a*c^2 + 8*a*c*d - 9*a*d^
2)*cos(f*x + e) + (2*a*c^2 + 16*a*c*d - 18*a*d^2 + (a*c^2 + 8*a*c*d - 9*a*d
^2)*cos(f*x + e))*sin(f*x + e))*sqrt(-(c - d)/a)*arctan(-2*sqrt(1/2)*sqrt(a
*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt(-(c - d)/a)/((c - d)*cos(f
*x + e))) + (10*a*c*d - 6*a*d^2 - (5*a*c*d - 3*a*d^2)*cos(f*x + e)^2 + (5*a
*c*d - 3*a*d^2)*cos(f*x + e) + (10*a*c*d - 6*a*d^2 + (5*a*c*d - 3*a*d^2)*co
s(f*x + e))*sin(f*x + e))*sqrt(-d/a)*log((128*d^4*cos(f*x + e)^5 + 128*(2*c
*d^3 - d^4)*cos(f*x + e)^4 + c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^3 + d^4 - 32
*(5*c^2*d^2 - 14*c*d^3 + 13*d^4)*cos(f*x + e)^3 - 32*(c^3*d - 2*c^2*d^2 + 9
*c*d^3 - 4*d^4)*cos(f*x + e)^2 + 8*(16*d^3*cos(f*x + e)^4 + 24*(c*d^2 - d^3
)*cos(f*x + e)^3 - c^3 + 17*c^2*d - 59*c*d^2 + 51*d^3 - 2*(5*c^2*d - 26*c*d
^2 + 33*d^3)*cos(f*x + e)^2 - (c^3 - 7*c^2*d + 31*c*d^2 - 25*d^3)*cos(f*x +
 e) + (16*d^3*cos(f*x + e)^3 + c^3 - 17*c^2*d + 59*c*d^2 - 51*d^3 - 8*(3*c*
d^2 - 5*d^3)*cos(f*x + e)^2 - 2*(5*c^2*d - 14*c*d^2 + 13*d^3)*cos(f*x + e))
*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt(-d/a)
 + (c^4 - 28*c^3*d + 230*c^2*d^2 - 476*c*d^3 + 289*d^4)*cos(f*x + e) + (128
*d^4*cos(f*x + e)^4 + c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^3 + d^4 - 256*(c*d^
3 - d^4)*cos(f*x + e)^3 - 32*(5*c^2*d^2 - 6*c*d^3 + 5*d^4)*cos(f*x + e)^2 +
 32*(c^3*d - 7*c^2*d^2 + 15*c*d^3 - 9*d^4)*cos(f*x + e))*sin(f*x + e))/(cos
(f*x + e) + sin(f*x + e) + 1)) + 4*(2*d^2*cos(f*x + e)^2 + c^2 - 2*c*d + d^
2 + (c^2 - 2*c*d + 3*d^2)*cos(f*x + e) + (2*d^2*cos(f*x + e) - c^2 + 2*c*d
- d^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/(a^
2*f*cos(f*x + e)^2 - a^2*f*cos(f*x + e) - 2*a^2*f - (a^2*f*cos(f*x + e) + 2
*a^2*f)*sin(f*x + e)), 1/4*(2*sqrt(1/2)*(2*a*c^2 + 16*a*c*d - 18*a*d^2 - (a
*c^2 + 8*a*c*d - 9*a*d^2)*cos(f*x + e)^2 + (a*c^2 + 8*a*c*d - 9*a*d^2)*cos(
f*x + e) + (2*a*c^2 + 16*a*c*d - 18*a*d^2 + (a*c^2 + 8*a*c*d - 9*a*d^2)*cos
(f*x + e))*sin(f*x + e))*sqrt(-(c - d)/a)*arctan(-2*sqrt(1/2)*sqrt(a*sin(f*
x + e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt(-(c - d)/a)/((c - d)*cos(f*x + e)
)) - (10*a*c*d - 6*a*d^2 - (5*a*c*d - 3*a*d^2)*cos(f*x + e)^2 + (5*a*c*d -
3*a*d^2)*cos(f*x + e) + (10*a*c*d - 6*a*d^2 + (5*a*c*d - 3*a*d^2)*cos(f*x +
 e))*sin(f*x + e))*sqrt(d/a)*arctan(1/4*(8*d^2*cos(f*x + e)^2 - c^2 + 6*c*d
 - 9*d^2 - 8*(c*d - d^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(
f*x + e) + c)*sqrt(d/a)/(2*d^3*cos(f*x + e)^3 - (3*c*d^2 - d^3)*cos(f*x + e
)*sin(f*x + e) - (c^2*d - c*d^2 + 2*d^3)*cos(f*x + e))) + 2*(2*d^2*cos(f*x
+ e)^2 + c^2 - 2*c*d + d^2 + (c^2 - 2*c*d + 3*d^2)*cos(f*x + e) + (2*d^2*co
s(f*x + e) - c^2 + 2*c*d - d^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt
(d*sin(f*x + e) + c))/(a^2*f*cos(f*x + e)^2 - a^2*f*cos(f*x + e) - 2*a^2*f
- (a^2*f*cos(f*x + e) + 2*a^2*f)*sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(5/2)/(a+a*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate((d*sin(f*x + e) + c)^(5/2)/(a*sin(f*x + e) + a)^(3/2), x)
```

3.595 $\int \frac{(c+d\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=194

$$-\frac{2d^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{a^{3/2}f}-\frac{\sqrt{c-d}(c+5d)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{2\sqrt{2}a^{3/2}f}-\frac{(c-d)\cos(e+fx)\sqrt{c}}{2f(a\sin(e+fx}$$

```
[Out] (-2*d^(3/2)*ArcTan[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]
*Sqrt[c + d*Sin[e + f*x]])])/(a^(3/2)*f) - (Sqrt[c - d]*(c + 5*d)*ArcTanh[(
Sqrt[a]*Sqrt[c - d]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c
+ d*Sin[e + f*x]])])/(2*Sqrt[2]*a^(3/2)*f) - ((c - d)*Cos[e + f*x]*Sqrt[c +
 d*Sin[e + f*x]])/(2*f*(a + a*Sin[e + f*x])^(3/2))
```

Rubi [A] time = 0.564186, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2765, 2982, 2782, 208, 2775, 205}

$$-\frac{2d^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{a^{3/2}f}-\frac{\sqrt{c-d}(c+5d)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{2\sqrt{2}a^{3/2}f}-\frac{(c-d)\cos(e+fx)\sqrt{c}}{2f(a\sin(e+fx}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^(3/2)/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-2*d^(3/2)*ArcTan[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]
*Sqrt[c + d*Sin[e + f*x]])])/(a^(3/2)*f) - (Sqrt[c - d]*(c + 5*d)*ArcTanh[(
Sqrt[a]*Sqrt[c - d]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c
+ d*Sin[e + f*x]])])/(2*Sqrt[2]*a^(3/2)*f) - ((c - d)*Cos[e + f*x]*Sqrt[c +
 d*Sin[e + f*x]])/(2*f*(a + a*Sin[e + f*x])^(3/2))
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
```

```
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(c+d\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{3/2}}\,dx &= -\frac{(c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2f(a+a\sin(e+fx))^{3/2}} - \frac{\int \frac{-\frac{1}{2}a\left(c^2+4cd-d^2\right)-2ad^2\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\,dx}{2a^2}\\
&= -\frac{(c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2f(a+a\sin(e+fx))^{3/2}} + \frac{d^2\int \frac{\sqrt{a+a\sin(e+fx)}}{\sqrt{c+d\sin(e+fx)}}\,dx}{a^2} + \frac{((c-d)(c+5d))\int \cdots}{\cdots}\\
&= -\frac{(c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2f(a+a\sin(e+fx))^{3/2}} - \frac{\left(2d^2\right)\operatorname{Subst}\left(\int \frac{1}{a+dx^2}\,dx, x, \frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\cdots}}\cdots\right)}{af}\\
&= -\frac{2d^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{a^{3/2}f} - \frac{\sqrt{c-d}(c+5d)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+f\cdots}{\sqrt{2}\sqrt{a+a\sin(e+fx)}\sqrt{c+d\cdots}}\cdots\right)}{2\sqrt{2}a^{3/2}f}
\end{aligned}
$$

Mathematica [C] time = 16.9558, size = 1625, normalized size = 8.38

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*Sin[e + f*x])^(3/2)/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*((-c + d)/(2*(Cos[(e + f*x)/2] + S
in[(e + f*x)/2])) + (c*Sin[(e + f*x)/2] - d*Sin[(e + f*x)/2])/(Cos[(e + f*x
)/2] + Sin[(e + f*x)/2])^2)*Sqrt[c + d*Sin[e + f*x]])/(f*(a*(1 + Sin[e + f*
x]))^(3/2)) + ((Sqrt[2]*(c^2 + 4*c*d - 5*d^2)*Log[1 + Tan[(e + f*x)/2]] - S
qrt[2]*(c^2 + 4*c*d - 5*d^2)*Log[c - d + 2*Sqrt[c - d]*Sqrt[(1 + Cos[e + f*
x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + (-c + d)*Tan[(e + f*x)/2]] - (4*I)*Sqr
t[c - d]*d^(3/2)*(Log[(c - I*(d + (1 + I)*Sqrt[2]*Sqrt[d]*Sqrt[(1 + Cos[e +
 f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]]) + ((-I)*c + d)*Tan[(e + f*x)/2])/(2*
d^(5/2)*(I + Tan[(e + f*x)/2]))] - Log[(c + I*d + (1 + I)*Sqrt[2]*Sqrt[d]*S
qrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + (I*c + d)*Tan[(e +
```

```
f*x)/2])/(2*d^(5/2)*(-I + Tan[(e + f*x)/2]))]))*(Cos[(e + f*x)/2] + Sin[(e
+ f*x)/2])^3*(c^2/(4*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c + d*Sin[e
 + f*x]]) + (c*d)/((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c + d*Sin[e +
 f*x]]) - d^2/(4*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c + d*Sin[e + f
*x]]) + (d^2*Sin[e + f*x])/((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c +
d*Sin[e + f*x]])))/(f*(a*(1 + Sin[e + f*x]))^(3/2)*(((c^2 + 4*c*d - 5*d^2)*
Sec[(e + f*x)/2]^2)/(Sqrt[2]*(1 + Tan[(e + f*x)/2])) - (Sqrt[2]*(c^2 + 4*c*
d - 5*d^2)*(((-c + d)*Sec[(e + f*x)/2]^2)/2 + (Sqrt[c - d]*d*Cos[e + f*x]*S
qrt[(1 + Cos[e + f*x])^(-1)])/Sqrt[c + d*Sin[e + f*x]] + Sqrt[c - d]*((1 +
Cos[e + f*x])^(-1))^(3/2)*Sin[e + f*x]*Sqrt[c + d*Sin[e + f*x]]))/(c - d +
2*Sqrt[c - d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + (-c
+ d)*Tan[(e + f*x)/2]) - (4*I)*Sqrt[c - d]*d^(3/2)*((2*d^(5/2)*(I + Tan[(e
+ f*x)/2])*(((((-I)*c + d)*Sec[(e + f*x)/2]^2)/2 - I*(((1 + I)*d^(3/2)*Cos[
e + f*x]*Sqrt[(1 + Cos[e + f*x])^(-1)])/(Sqrt[2]*Sqrt[c + d*Sin[e + f*x]])
+ ((1 + I)*Sqrt[d]*((1 + Cos[e + f*x])^(-1))^(3/2)*Sin[e + f*x]*Sqrt[c + d*
Sin[e + f*x]])/Sqrt[2]))/(2*d^(5/2)*(I + Tan[(e + f*x)/2])) - (Sec[(e + f*x
)/2]^2*(c - I*(d + (1 + I)*Sqrt[2]*Sqrt[d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sq
rt[c + d*Sin[e + f*x]]) + ((-I)*c + d)*Tan[(e + f*x)/2]))/(4*d^(5/2)*(I + T
an[(e + f*x)/2])^2)))/(c - I*(d + (1 + I)*Sqrt[2]*Sqrt[d]*Sqrt[(1 + Cos[e +
 f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]]) + ((-I)*c + d)*Tan[(e + f*x)/2]) - (
2*d^(5/2)*(-I + Tan[(e + f*x)/2])*((((I*c + d)*Sec[(e + f*x)/2]^2)/2 + ((1
+ I)*d^(3/2)*Cos[e + f*x]*Sqrt[(1 + Cos[e + f*x])^(-1)])/(Sqrt[2]*Sqrt[c +
d*Sin[e + f*x]]) + ((1 + I)*Sqrt[d]*((1 + Cos[e + f*x])^(-1))^(3/2)*Sin[e +
 f*x]*Sqrt[c + d*Sin[e + f*x]])/Sqrt[2])/(2*d^(5/2)*(-I + Tan[(e + f*x)/2])
) - (Sec[(e + f*x)/2]^2*(c + I*d + (1 + I)*Sqrt[2]*Sqrt[d]*Sqrt[(1 + Cos[e
+ f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + (I*c + d)*Tan[(e + f*x)/2]))/(4*d^
(5/2)*(-I + Tan[(e + f*x)/2])^2)))/(c + I*d + (1 + I)*Sqrt[2]*Sqrt[d]*Sqrt[
(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + (I*c + d)*Tan[(e + f*x)
/2]))))
```

Maple [B] time = 0.299, size = 6675, normalized size = 34.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(3/2)/(a*sin(f*x + e) + a)^(3/2), x)
```

Fricas [B] time = 4.28923, size = 7039, normalized size = 36.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] [1/4*(sqrt(1/2)*((a*c + 5*a*d)*cos(f*x + e)^2 - 2*a*c - 10*a*d - (a*c + 5*a
*d)*cos(f*x + e) - (2*a*c + 10*a*d + (a*c + 5*a*d)*cos(f*x + e))*sin(f*x +
e))*sqrt((c - d)/a)*log((4*sqrt(1/2)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*
x + e) + c)*sqrt((c - d)/a)*(cos(f*x + e) - sin(f*x + e) + 1) - (c - 3*d)*c
os(f*x + e)^2 - (3*c - d)*cos(f*x + e) + ((c - 3*d)*cos(f*x + e) - 2*c - 2*
d)*sin(f*x + e) - 2*c - 2*d)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x +
 e) - cos(f*x + e) - 2)) + (a*d*cos(f*x + e)^2 - a*d*cos(f*x + e) - 2*a*d -
 (a*d*cos(f*x + e) + 2*a*d)*sin(f*x + e))*sqrt(-d/a)*log((128*d^4*cos(f*x +
 e)^5 + 128*(2*c*d^3 - d^4)*cos(f*x + e)^4 + c^4 + 4*c^3*d + 6*c^2*d^2 + 4*
c*d^3 + d^4 - 32*(5*c^2*d^2 - 14*c*d^3 + 13*d^4)*cos(f*x + e)^3 - 32*(c^3*d
 - 2*c^2*d^2 + 9*c*d^3 - 4*d^4)*cos(f*x + e)^2 - 8*(16*d^3*cos(f*x + e)^4 +
 24*(c*d^2 - d^3)*cos(f*x + e)^3 - c^3 + 17*c^2*d - 59*c*d^2 + 51*d^3 - 2*(
5*c^2*d - 26*c*d^2 + 33*d^3)*cos(f*x + e)^2 - (c^3 - 7*c^2*d + 31*c*d^2 - 2
5*d^3)*cos(f*x + e) + (16*d^3*cos(f*x + e)^3 + c^3 - 17*c^2*d + 59*c*d^2 -
51*d^3 - 8*(3*c*d^2 - 5*d^3)*cos(f*x + e)^2 - 2*(5*c^2*d - 14*c*d^2 + 13*d^
3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e)
 + c)*sqrt(-d/a) + (c^4 - 28*c^3*d + 230*c^2*d^2 - 476*c*d^3 + 289*d^4)*cos
(f*x + e) + (128*d^4*cos(f*x + e)^4 + c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^3 +
 d^4 - 256*(c*d^3 - d^4)*cos(f*x + e)^3 - 32*(5*c^2*d^2 - 6*c*d^3 + 5*d^4)*
cos(f*x + e)^2 + 32*(c^3*d - 7*c^2*d^2 + 15*c*d^3 - 9*d^4)*cos(f*x + e))*si
n(f*x + e))/(cos(f*x + e) + sin(f*x + e) + 1)) + 2*((c - d)*cos(f*x + e) -
(c - d)*sin(f*x + e) + c - d)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e)
+ c))/(a^2*f*cos(f*x + e)^2 - a^2*f*cos(f*x + e) - 2*a^2*f - (a^2*f*cos(f*x
 + e) + 2*a^2*f)*sin(f*x + e)), 1/4*(sqrt(1/2)*((a*c + 5*a*d)*cos(f*x + e)^
2 - 2*a*c - 10*a*d - (a*c + 5*a*d)*cos(f*x + e) - (2*a*c + 10*a*d + (a*c +
5*a*d)*cos(f*x + e))*sin(f*x + e))*sqrt((c - d)/a)*log((4*sqrt(1/2)*sqrt(a*
sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt((c - d)/a)*(cos(f*x + e) -
sin(f*x + e) + 1) - (c - 3*d)*cos(f*x + e)^2 - (3*c - d)*cos(f*x + e) + ((c
 - 3*d)*cos(f*x + e) - 2*c - 2*d)*sin(f*x + e) - 2*c - 2*d)/(cos(f*x + e)^2
 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) + 2*(a*d*cos(f*x +
e)^2 - a*d*cos(f*x + e) - 2*a*d - (a*d*cos(f*x + e) + 2*a*d)*sin(f*x + e))*
sqrt(d/a)*arctan(1/4*(8*d^2*cos(f*x + e)^2 - c^2 + 6*c*d - 9*d^2 - 8*(c*d -
 d^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt(
d/a)/(2*d^3*cos(f*x + e)^3 - (3*c*d^2 - d^3)*cos(f*x + e)*sin(f*x + e) - (c
^2*d - c*d^2 + 2*d^3)*cos(f*x + e))) + 2*((c - d)*cos(f*x + e) - (c - d)*si
n(f*x + e) + c - d)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/(a^2
*f*cos(f*x + e)^2 - a^2*f*cos(f*x + e) - 2*a^2*f - (a^2*f*cos(f*x + e) + 2*
a^2*f)*sin(f*x + e)), -1/4*(2*sqrt(1/2)*((a*c + 5*a*d)*cos(f*x + e)^2 - 2*a
*c - 10*a*d - (a*c + 5*a*d)*cos(f*x + e) - (2*a*c + 10*a*d + (a*c + 5*a*d)*
cos(f*x + e))*sin(f*x + e))*sqrt(-(c - d)/a)*arctan(-2*sqrt(1/2)*sqrt(a*sin
(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt(-(c - d)/a)/((c - d)*cos(f*x +
 e))) - (a*d*cos(f*x + e)^2 - a*d*cos(f*x + e) - 2*a*d - (a*d*cos(f*x + e)
+ 2*a*d)*sin(f*x + e))*sqrt(-d/a)*log((128*d^4*cos(f*x + e)^5 + 128*(2*c*d^
3 - d^4)*cos(f*x + e)^4 + c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^3 + d^4 - 32*(5
*c^2*d^2 - 14*c*d^3 + 13*d^4)*cos(f*x + e)^3 - 32*(c^3*d - 2*c^2*d^2 + 9*c*
d^3 - 4*d^4)*cos(f*x + e)^2 - 8*(16*d^3*cos(f*x + e)^4 + 24*(c*d^2 - d^3)*c
os(f*x + e)^3 - c^3 + 17*c^2*d - 59*c*d^2 + 51*d^3 - 2*(5*c^2*d - 26*c*d^2
+ 33*d^3)*cos(f*x + e)^2 - (c^3 - 7*c^2*d + 31*c*d^2 - 25*d^3)*cos(f*x + e)
 + (16*d^3*cos(f*x + e)^3 + c^3 - 17*c^2*d + 59*c*d^2 - 51*d^3 - 8*(3*c*d^2
 - 5*d^3)*cos(f*x + e)^2 - 2*(5*c^2*d - 14*c*d^2 + 13*d^3)*cos(f*x + e))*si
```

```
n(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt(-d/a) +
(c^4 - 28*c^3*d + 230*c^2*d^2 - 476*c*d^3 + 289*d^4)*cos(f*x + e) + (128*d^
4*cos(f*x + e)^4 + c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^3 + d^4 - 256*(c*d^3 -
 d^4)*cos(f*x + e)^3 - 32*(5*c^2*d^2 - 6*c*d^3 + 5*d^4)*cos(f*x + e)^2 + 32
*(c^3*d - 7*c^2*d^2 + 15*c*d^3 - 9*d^4)*cos(f*x + e))*sin(f*x + e))/(cos(f*
x + e) + sin(f*x + e) + 1)) - 2*((c - d)*cos(f*x + e) - (c - d)*sin(f*x + e
) + c - d)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/(a^2*f*cos(f*
x + e)^2 - a^2*f*cos(f*x + e) - 2*a^2*f - (a^2*f*cos(f*x + e) + 2*a^2*f)*si
n(f*x + e)), -1/2*(sqrt(1/2)*((a*c + 5*a*d)*cos(f*x + e)^2 - 2*a*c - 10*a*d
 - (a*c + 5*a*d)*cos(f*x + e) - (2*a*c + 10*a*d + (a*c + 5*a*d)*cos(f*x + e
))*sin(f*x + e))*sqrt(-(c - d)/a)*arctan(-2*sqrt(1/2)*sqrt(a*sin(f*x + e) +
 a)*sqrt(d*sin(f*x + e) + c)*sqrt(-(c - d)/a)/((c - d)*cos(f*x + e))) - (a*
d*cos(f*x + e)^2 - a*d*cos(f*x + e) - 2*a*d - (a*d*cos(f*x + e) + 2*a*d)*si
n(f*x + e))*sqrt(d/a)*arctan(1/4*(8*d^2*cos(f*x + e)^2 - c^2 + 6*c*d - 9*d^
2 - 8*(c*d - d^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e
) + c)*sqrt(d/a)/(2*d^3*cos(f*x + e)^3 - (3*c*d^2 - d^3)*cos(f*x + e)*sin(f
*x + e) - (c^2*d - c*d^2 + 2*d^3)*cos(f*x + e))) - ((c - d)*cos(f*x + e) -
(c - d)*sin(f*x + e) + c - d)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e)
+ c))/(a^2*f*cos(f*x + e)^2 - a^2*f*cos(f*x + e) - 2*a^2*f - (a^2*f*cos(f*x
 + e) + 2*a^2*f)*sin(f*x + e))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c+d\sin\left(e+fx\right)\right)^{\frac{3}{2}}}{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(3/2)/(a+a*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral((c + d*sin(e + f*x))**(3/2)/(a*(sin(e + f*x) + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate((d*sin(f*x + e) + c)^(3/2)/(a*sin(f*x + e) + a)^(3/2), x)
```

3.596 $\int \frac{\sqrt{c+d\sin(e+fx)}}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=126

$$-\frac{(c+d)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{2\sqrt{2}a^{3/2}f\sqrt{c-d}}-\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2f(a\sin(e+fx)+a)^{3/2}}$$

```
[Out] -((c + d)*ArcTanh[(Sqrt[a]*Sqrt[c - d]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Si
n[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])])/(2*Sqrt[2]*a^(3/2)*Sqrt[c - d]*f) -
 (Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(2*f*(a + a*Sin[e + f*x])^(3/2))
```

Rubi [A] time = 0.220565, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2764, 12, 2782, 208}

$$-\frac{(c+d)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{2\sqrt{2}a^{3/2}f\sqrt{c-d}}-\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2f(a\sin(e+fx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*Sin[e + f*x]]/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] -((c + d)*ArcTanh[(Sqrt[a]*Sqrt[c - d]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Si
n[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])])/(2*Sqrt[2]*a^(3/2)*Sqrt[c - d]*f) -
 (Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(2*f*(a + a*Sin[e + f*x])^(3/2))
```

Rule 2764

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m
*(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a*d*n - b*c*
(m + 1) - b*d*(m + n + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && L
tQ[m, -1] && LtQ[0, n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c,
 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{c+d\sin(e+fx)}}{(a+a\sin(e+fx))^{3/2}}\,dx &= -\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2f(a+a\sin(e+fx))^{3/2}} + \frac{\int \frac{a(c+d)}{2\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\,dx}{2a^2} \\
&= -\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2f(a+a\sin(e+fx))^{3/2}} + \frac{(c+d)\int \frac{1}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\,dx}{4a} \\
&= -\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2f(a+a\sin(e+fx))^{3/2}} - \frac{(c+d)\,\mathrm{Subst}\left(\int \frac{1}{2a^2-(ac-ad)x^2}\,dx, x, \frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{2f} \\
&= -\frac{(c+d)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{2\sqrt{2}a^{3/2}\sqrt{c-d}f} - \frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2f(a+a\sin(e+fx))^{3/2}}
\end{aligned}
$$

Mathematica [B] time = 5.24486, size = 372, normalized size = 2.95

$$
\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^2\left(\frac{(c+d)\left(\log\left(\tan\left(\frac{1}{2}(e+fx)\right)+1\right)-\log\left((d-c)\tan\left(\frac{1}{2}(e+fx)\right)+2\sqrt{c-d}\sqrt{\frac{1}{\cos(e+fx)+1}}\sqrt{c+d\sin(e+fx)}+c-d\right)\right)}{\frac{\sec^2\left(\frac{1}{2}(e+fx)\right)}{2\tan\left(\frac{1}{2}(e+fx)\right)+2}-\frac{\frac{\sqrt{c-d}\left(\frac{1}{\cos(e+fx)+1}\right)^{3/2}(c\sin(e+fx)+d\cos(e+fx)+d)}{\sqrt{c+d\sin(e+fx)}}-\frac{1}{2}(c-d)\sec^2\left(\frac{1}{2}(e+fx)\right)}{(d-c)\tan\left(\frac{1}{2}(e+fx)\right)+2\sqrt{c-d}\sqrt{\frac{1}{\cos(e+fx)+1}}\sqrt{c+d\sin(e+fx)}+c-d}} - \cdots\right)}{4f(a(\sin(e+fx)+1))^{3/2}\sqrt{c+d\sin(e+fx)}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c + d*Sin[e + f*x]]/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2*((-2*(Cos[(e + f*x)/2] - Sin[(e +
f*x)/2])*(c + d*Sin[e + f*x]))/(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + ((c
+ d)*(Log[1 + Tan[(e + f*x)/2]] - Log[c - d + 2*Sqrt[c - d]*Sqrt[(1 + Cos[e
 + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + (-c + d)*Tan[(e + f*x)/2]]))/(Sec
[(e + f*x)/2]^2/(2 + 2*Tan[(e + f*x)/2]) - (-((c - d)*Sec[(e + f*x)/2]^2)/2
 + (Sqrt[c - d]*((1 + Cos[e + f*x])^(-1))^(3/2)*(d + d*Cos[e + f*x] + c*Sin
[e + f*x]))/Sqrt[c + d*Sin[e + f*x]])/(c - d + 2*Sqrt[c - d]*Sqrt[(1 + Cos[
e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + (-c + d)*Tan[(e + f*x)/2]))))/(4
*f*(a*(1 + Sin[e + f*x]))^(3/2)*Sqrt[c + d*Sin[e + f*x]])
```

Maple [B] time = 0.229, size = 1373, normalized size = 10.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(3/2),x)
```

```
[Out] 1/8/f/(c-d)*(sin(f*x+e)*cos(f*x+e)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(
f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*
```

```
x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1/2)*(2*c-2*d)^(1/2)*c
+sin(f*x+e)*cos(f*x+e)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos
(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f
*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1/2)*(2*c-2*d)^(1/2)*d+cos(f*x+e)^
2*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin
(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x
+e)+sin(f*x+e)))*2^(1/2)*(2*c-2*d)^(1/2)*c+cos(f*x+e)^2*ln(2*((2*c-2*d)^(1/
2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-
d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1
/2)*(2*c-2*d)^(1/2)*d-2*sin(f*x+e)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(
f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*
x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1/2)*(2*c-2*d)^(1/2)*c
-2*sin(f*x+e)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1
))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d
)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1/2)*(2*c-2*d)^(1/2)*d+cos(f*x+e)*ln(2*((2*
c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*s
in(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x
+e)))*2^(1/2)*(2*c-2*d)^(1/2)*c+cos(f*x+e)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((
c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+
c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1/2)*(2*c-2*d)
^(1/2)*d-4*sin(f*x+e)*cos(f*x+e)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*c+
4*sin(f*x+e)*cos(f*x+e)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*d-2*ln(2*((
2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c
*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f
*x+e)))*2^(1/2)*(2*c-2*d)^(1/2)*c-2*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin
(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f
*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1/2)*(2*c-2*d)^(1/2)*
d)*(c+d*sin(f*x+e))^(1/2)/sin(f*x+e)/(a*(1+sin(f*x+e)))^(3/2)/((c+d*sin(f*x
+e))/(cos(f*x+e)+1))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d\sin\left(fx+e\right)+c}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(d*sin(f*x + e) + c)/(a*sin(f*x + e) + a)^(3/2), x)
```

Fricas [B] time = 2.44585, size = 2199, normalized size = 17.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] [1/16*(((c + d)*cos(f*x + e)^2 - (c + d)*cos(f*x + e) - ((c + d)*cos(f*x +
e) + 2*c + 2*d)*sin(f*x + e) - 2*c - 2*d)*sqrt(2*a*c - 2*a*d)*log(((a*c^2 -
```

```
 14*a*c*d + 17*a*d^2)*cos(f*x + e)^3 - 4*a*c^2 - 8*a*c*d - 4*a*d^2 - (13*a*
c^2 - 22*a*c*d - 3*a*d^2)*cos(f*x + e)^2 - 4*((c - 3*d)*cos(f*x + e)^2 - (3
*c - d)*cos(f*x + e) + ((c - 3*d)*cos(f*x + e) + 4*c - 4*d)*sin(f*x + e) -
4*c + 4*d)*sqrt(2*a*c - 2*a*d)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e)
 + c) - 2*(9*a*c^2 - 14*a*c*d + 9*a*d^2)*cos(f*x + e) - (4*a*c^2 + 8*a*c*d
+ 4*a*d^2 - (a*c^2 - 14*a*c*d + 17*a*d^2)*cos(f*x + e)^2 - 2*(7*a*c^2 - 18*
a*c*d + 7*a*d^2)*cos(f*x + e))*sin(f*x + e))/(cos(f*x + e)^3 + 3*cos(f*x +
e)^2 + (cos(f*x + e)^2 - 2*cos(f*x + e) - 4)*sin(f*x + e) - 2*cos(f*x + e)
- 4)) + 8*((c - d)*cos(f*x + e) - (c - d)*sin(f*x + e) + c - d)*sqrt(a*sin(
f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/((a^2*c - a^2*d)*f*cos(f*x + e)^2 -
 (a^2*c - a^2*d)*f*cos(f*x + e) - 2*(a^2*c - a^2*d)*f - ((a^2*c - a^2*d)*f*
cos(f*x + e) + 2*(a^2*c - a^2*d)*f)*sin(f*x + e)), -1/8*(((c + d)*cos(f*x +
 e)^2 - (c + d)*cos(f*x + e) - ((c + d)*cos(f*x + e) + 2*c + 2*d)*sin(f*x +
 e) - 2*c - 2*d)*sqrt(-2*a*c + 2*a*d)*arctan(1/4*sqrt(-2*a*c + 2*a*d)*sqrt(
a*sin(f*x + e) + a)*((c - 3*d)*sin(f*x + e) - 3*c + d)*sqrt(d*sin(f*x + e)
+ c)/((a*c*d - a*d^2)*cos(f*x + e)*sin(f*x + e) + (a*c^2 - a*c*d)*cos(f*x +
 e))) - 4*((c - d)*cos(f*x + e) - (c - d)*sin(f*x + e) + c - d)*sqrt(a*sin(
f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/((a^2*c - a^2*d)*f*cos(f*x + e)^2 -
 (a^2*c - a^2*d)*f*cos(f*x + e) - 2*(a^2*c - a^2*d)*f - ((a^2*c - a^2*d)*f*
cos(f*x + e) + 2*(a^2*c - a^2*d)*f)*sin(f*x + e))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c + d \sin\left(e + fx\right)}}{\left(a \left(\sin\left(e + fx\right) + 1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(1/2)/(a+a*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral(sqrt(c + d*sin(e + f*x))/(a*(sin(e + f*x) + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d \sin\left(fx + e\right) + c}}{\left(a \sin\left(fx + e\right) + a\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(d*sin(f*x + e) + c)/(a*sin(f*x + e) + a)^(3/2), x)
```

3.597 $\int \frac{1}{(a+a\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}\,dx$

Optimal. Leaf size=135

$$-\frac{(c-3d)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{2\sqrt{2}a^{3/2}f(c-d)^{3/2}}-\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2f(c-d)(a\sin(e+fx)+a)^{3/2}}$$

```
[Out] -((c - 3*d)*ArcTanh[(Sqrt[a]*Sqrt[c - d]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*
Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])])/(2*Sqrt[2]*a^(3/2)*(c - d)^(3/2)*
f) - (Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(2*(c - d)*f*(a + a*Sin[e + f*
x])^(3/2))
```

Rubi [A] time = 0.236837, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2766, 12, 2782, 208}

$$-\frac{(c-3d)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{2\sqrt{2}a^{3/2}f(c-d)^{3/2}}-\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2f(c-d)(a\sin(e+fx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]),x]
```


```
[Out] -((c - 3*d)*ArcTanh[(Sqrt[a]*Sqrt[c - d]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*
Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])])/(2*Sqrt[2]*a^(3/2)*(c - d)^(3/2)*
f) - (Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(2*(c - d)*f*(a + a*Sin[e + f*
x])^(3/2))
```


Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}\,dx = -\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2(c-d)f(a+a\sin(e+fx))^{3/2}} - \frac{\int -\frac{a(c-3d)}{2\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\,dx}{2a^2(c-d)}$$

$$= -\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2(c-d)f(a+a\sin(e+fx))^{3/2}} + \frac{(c-3d)\int \frac{1}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\,d}{4a(c-d)}$$

$$= -\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2(c-d)f(a+a\sin(e+fx))^{3/2}} - \frac{(c-3d)\,\text{Subst}\left(\int \frac{1}{2a^2-(ac-ad)x^2}\,dx, x,\right.}{2(c-d)f}$$

$$= -\frac{(c-3d)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{2\sqrt{2}a^{3/2}(c-d)^{3/2}f} - \frac{\cos(e+fx)\sqrt{c+d\sin}}{2(c-d)f(a+a\sin(e}$$

Mathematica [B] time = 5.53513, size = 381, normalized size = 2.82

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^2\left(\frac{(c-3d)\left(\log\left(\tan\left(\frac{1}{2}(e+fx)\right)+1\right)-\log\left((d-c)\tan\left(\frac{1}{2}(e+fx)\right)+2\sqrt{c-d}\sqrt{\frac{1}{\cos(e+fx)+1}}\sqrt{c+d\sin(e+fx)}+c-d\right)\right)}{\frac{\sec^2\left(\frac{1}{2}(e+fx)\right)}{2\tan\left(\frac{1}{2}(e+fx)\right)+2}-\frac{\frac{\sqrt{c-d}\left(\frac{1}{\cos(e+fx)+1}\right)^{3/2}(c\sin(e+fx)+d\cos(e+fx)+d)}{\sqrt{c+d\sin(e+fx)}}-\frac{1}{2}(c-d)\sec^2\left(\frac{1}{2}(e+fx)\right)}{(d-c)\tan\left(\frac{1}{2}(e+fx)\right)+2\sqrt{c-d}\sqrt{\frac{1}{\cos(e+fx)+1}}\sqrt{c+d\sin(e+fx)}+c-d}}-\frac{2(}{}\right.}{4f(c-d)(a(\sin(e+fx)+1))^{3/2}\sqrt{c+d\sin(e+fx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2*((-2*(Cos[(e + f*x)/2] - Sin[(e +
f*x)/2])*(c + d*Sin[e + f*x]))/(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + ((c
- 3*d)*(Log[1 + Tan[(e + f*x)/2]] - Log[c - d + 2*Sqrt[c - d]*Sqrt[(1 + Cos
[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + (-c + d)*Tan[(e + f*x)/2]]))/(S
ec[(e + f*x)/2]^2/(2 + 2*Tan[(e + f*x)/2]) - (-((c - d)*Sec[(e + f*x)/2]^2)
/2 + (Sqrt[c - d]*((1 + Cos[e + f*x])^(-1))^(3/2)*(d + d*Cos[e + f*x] + c*S
in[e + f*x]))/Sqrt[c + d*Sin[e + f*x]])/(c - d + 2*Sqrt[c - d]*Sqrt[(1 + Co
s[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + (-c + d)*Tan[(e + f*x)/2]))))/
(4*(c - d)*f*(a*(1 + Sin[e + f*x]))^(3/2)*Sqrt[c + d*Sin[e + f*x]])
```

Maple [B] time = 0.224, size = 1268, normalized size = 9.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(1/2),x)
```

```
[Out] -1/4/f/(2*c-2*d)^(1/2)/(c-d)*(-sin(f*x+e)*cos(f*x+e)*2^(1/2)*ln(2*((2*c-2*d
)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*
```

```
x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))
*c+3*sin(f*x+e)*cos(f*x+e)*2^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(
f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*
x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*d-cos(f*x+e)^2*2^(1/2)*ln
(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x
+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+
sin(f*x+e)))*c+3*cos(f*x+e)^2*2^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*s
in(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos
(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*d+2*sin(f*x+e)*cos(f*x
+e)*(2*c-2*d)^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)+2*sin(f*x+e)*2^
(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)
*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos
(f*x+e)+sin(f*x+e)))*c-6*sin(f*x+e)*2^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*(
(c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)
+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*d-cos(f*x+e)*2^(
1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*
sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(
f*x+e)+sin(f*x+e)))*c+3*cos(f*x+e)*2^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((
c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+
c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*d+2*2^(1/2)*ln(2*
((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)
+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin
(f*x+e)))*c-6*2^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(
f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*
x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*d)*(c+d*sin(f*x+e))^(1/2)/(a*(1+sin(f*
x+e)))^(3/2)/sin(f*x+e)/((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a \sin\left(fx+e\right)+a\right)^{\frac{3}{2}} \sqrt{d \sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)^(3/2)*sqrt(d*sin(f*x + e) + c)), x)
```

Fricas [B] time = 2.81277, size = 2431, normalized size = 18.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="fri
cas")
```

```
[Out] [1/16*(((c - 3*d)*cos(f*x + e)^2 - (c - 3*d)*cos(f*x + e) - ((c - 3*d)*cos(
f*x + e) + 2*c - 6*d)*sin(f*x + e) - 2*c + 6*d)*sqrt(2*a*c - 2*a*d)*log(((a
*c^2 - 14*a*c*d + 17*a*d^2)*cos(f*x + e)^3 - 4*a*c^2 - 8*a*c*d - 4*a*d^2 -
(13*a*c^2 - 22*a*c*d - 3*a*d^2)*cos(f*x + e)^2 - 4*((c - 3*d)*cos(f*x + e)^
2 - (3*c - d)*cos(f*x + e) + ((c - 3*d)*cos(f*x + e) + 4*c - 4*d)*sin(f*x +
 e) - 4*c + 4*d)*sqrt(2*a*c - 2*a*d)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*
```

```
x + e) + c) - 2*(9*a*c^2 - 14*a*c*d + 9*a*d^2)*cos(f*x + e) - (4*a*c^2 + 8*
a*c*d + 4*a*d^2 - (a*c^2 - 14*a*c*d + 17*a*d^2)*cos(f*x + e)^2 - 2*(7*a*c^2
 - 18*a*c*d + 7*a*d^2)*cos(f*x + e))*sin(f*x + e))/(cos(f*x + e)^3 + 3*cos(
f*x + e)^2 + (cos(f*x + e)^2 - 2*cos(f*x + e) - 4)*sin(f*x + e) - 2*cos(f*x
 + e) - 4)) + 8*((c - d)*cos(f*x + e) - (c - d)*sin(f*x + e) + c - d)*sqrt(
a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/((a^2*c^2 - 2*a^2*c*d + a^2*d
^2)*f*cos(f*x + e)^2 - (a^2*c^2 - 2*a^2*c*d + a^2*d^2)*f*cos(f*x + e) - 2*(
a^2*c^2 - 2*a^2*c*d + a^2*d^2)*f - ((a^2*c^2 - 2*a^2*c*d + a^2*d^2)*f*cos(f
*x + e) + 2*(a^2*c^2 - 2*a^2*c*d + a^2*d^2)*f)*sin(f*x + e)), -1/8*(((c - 3
*d)*cos(f*x + e)^2 - (c - 3*d)*cos(f*x + e) - ((c - 3*d)*cos(f*x + e) + 2*c
 - 6*d)*sin(f*x + e) - 2*c + 6*d)*sqrt(-2*a*c + 2*a*d)*arctan(1/4*sqrt(-2*a
*c + 2*a*d)*sqrt(a*sin(f*x + e) + a)*((c - 3*d)*sin(f*x + e) - 3*c + d)*sqr
t(d*sin(f*x + e) + c)/((a*c*d - a*d^2)*cos(f*x + e)*sin(f*x + e) + (a*c^2 -
 a*c*d)*cos(f*x + e))) - 4*((c - d)*cos(f*x + e) - (c - d)*sin(f*x + e) + c
 - d)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/((a^2*c^2 - 2*a^2*
c*d + a^2*d^2)*f*cos(f*x + e)^2 - (a^2*c^2 - 2*a^2*c*d + a^2*d^2)*f*cos(f*x
 + e) - 2*(a^2*c^2 - 2*a^2*c*d + a^2*d^2)*f - ((a^2*c^2 - 2*a^2*c*d + a^2*d
^2)*f*cos(f*x + e) + 2*(a^2*c^2 - 2*a^2*c*d + a^2*d^2)*f)*sin(f*x + e))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}\sqrt{c+d\sin\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**(3/2)/(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/((a*(sin(e + f*x) + 1))**(3/2)*sqrt(c + d*sin(e + f*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\sqrt{d\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)^(3/2)*sqrt(d*sin(f*x + e) + c)), x)
```

3.598 $\int \frac{1}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=197

$$-\frac{(c-7d)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{2\sqrt{2}a^{3/2}f(c-d)^{5/2}}-\frac{d(c+5d)\cos(e+fx)}{2af(c-d)^2(c+d)\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}-\frac{}{2f(c-d}$$

```
[Out] -((c - 7*d)*ArcTanh[(Sqrt[a]*Sqrt[c - d]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*
Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])])/(2*Sqrt[2]*a^(3/2)*(c - d)^(5/2)*
f) - Cos[e + f*x]/(2*(c - d)*f*(a + a*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]) - (d*(c + 5*d)*Cos[e + f*x])/(2*a*(c - d)^2*(c + d)*f*Sqrt[a + a*S
in[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])
```

Rubi [A] time = 0.494296, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2766, 2984, 12, 2782, 208}

$$-\frac{(c-7d)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{2\sqrt{2}a^{3/2}f(c-d)^{5/2}}-\frac{d(c+5d)\cos(e+fx)}{2af(c-d)^2(c+d)\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}-\frac{}{2f(c-d}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^(3/2)),x]
```

```
[Out] -((c - 7*d)*ArcTanh[(Sqrt[a]*Sqrt[c - d]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*
Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])])/(2*Sqrt[2]*a^(3/2)*(c - d)^(5/2)*
f) - Cos[e + f*x]/(2*(c - d)*f*(a + a*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]) - (d*(c + 5*d)*Cos[e + f*x])/(2*a*(c - d)^2*(c + d)*f*Sqrt[a + a*S
in[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
 + 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{3/2}}\,dx = -\frac{\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}} - \frac{\int \frac{-\frac{1}{2}a(c-5d)-a}{\sqrt{a+a\sin(e+fx)}(c}}{2a^2(c}$$

$$= -\frac{\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}} - \frac{}{2a(c-d)^2(c+d}$$

$$= -\frac{\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}} - \frac{}{2a(c-d)^2(c+d}$$

$$= -\frac{\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}} - \frac{}{2a(c-d)^2(c+d}$$

$$= -\frac{(c-7d)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{2\sqrt{2}a^{3/2}(c-d)^{5/2}f} - \frac{c}{2(c-d)f(a+a\sin(}$$

Mathematica [B] time = 5.35975, size = 401, normalized size = 2.04

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^2\left(\frac{(c-7d)\left(\log\left(\tan\left(\frac{1}{2}(e+fx)\right)+1\right)-\log\left((d-c)\tan\left(\frac{1}{2}(e+fx)\right)+2\sqrt{c-d}\sqrt{\frac{1}{\cos(e+fx)+1}}\sqrt{c+d\sin(e+fx)}+c-d\right)\right)}{\frac{\sec^2\left(\frac{1}{2}(e+fx)\right)}{2\tan\left(\frac{1}{2}(e+fx)\right)+2}-\frac{\frac{\sqrt{c-d}\left(\frac{1}{\cos(e+fx)+1}\right)^{3/2}(c\sin(e+fx)+d\cos(e+fx)+d)}{\sqrt{c+d\sin(e+fx)}}-\frac{1}{2}(c-d)\sec^2\left(\frac{1}{2}(e+fx)\right)}{(d-c)\tan\left(\frac{1}{2}(e+fx)\right)+2\sqrt{c-d}\sqrt{\frac{1}{\cos(e+fx)+1}}\sqrt{c+d\sin(e+fx)}+c-d}} - \frac{2(}{}\right.}{4f(c-d)^2(a(\sin(e+fx)+1))^{3/2}\sqrt{c+d\sin(e+fx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^(3/2)),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2*((-2*(Cos[(e + f*x)/2] - Sin[(e +
f*x)/2])*(c^2 + c*d + 4*d^2 + d*(c + 5*d)*Sin[e + f*x]))/((c + d)*(Cos[(e +
 f*x)/2] + Sin[(e + f*x)/2])) + ((c - 7*d)*(Log[1 + Tan[(e + f*x)/2]] - Log
[c - d + 2*Sqrt[c - d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x
```

```
]] + (-c + d)*Tan[(e + f*x)/2]]))/(Sec[(e + f*x)/2]^2/(2 + 2*Tan[(e + f*x)/
2]) - (-((c - d)*Sec[(e + f*x)/2]^2)/2 + (Sqrt[c - d]*((1 + Cos[e + f*x])^(
-1))^(3/2)*(d + d*Cos[e + f*x] + c*Sin[e + f*x]))/Sqrt[c + d*Sin[e + f*x]])
/(c - d + 2*Sqrt[c - d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*
x]] + (-c + d)*Tan[(e + f*x)/2]))))/(4*(c - d)^2*f*(a*(1 + Sin[e + f*x]))^(
3/2)*Sqrt[c + d*Sin[e + f*x]])
```

Maple [B] time = 0.273, size = 2246, normalized size = 11.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(3/2),x)
```

```
[Out] -1/4/f/(c+d)/(2*c-2*d)^(1/2)/(c-d)^2*(sin(f*x+e)*cos(f*x+e)*ln(2*((2*c-2*d)
^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x
+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*
2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*c^2-6*sin(f*x+e)*cos(f*x+e)
*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(
f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+
e)+sin(f*x+e)))*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*c*d-7*sin(f
*x+e)*cos(f*x+e)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e
)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-
c+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(
1/2)*d^2-cos(f*x+e)^2*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(
f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*
x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+
1))^(1/2)*c^2+6*cos(f*x+e)^2*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e)
)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d
*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1/2)*((c+d*sin(f*x+e))/(cos(
f*x+e)+1))^(1/2)*c*d+7*cos(f*x+e)^2*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin
(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f
*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1/2)*((c+d*sin(f*x+e)
)/(cos(f*x+e)+1))^(1/2)*d^2+2*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e
))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-
d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*c^2*2^(1/2)*((c+d*sin(f*x+e))/
(cos(f*x+e)+1))^(1/2)*sin(f*x+e)-12*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin
(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f
*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*c*d*2^(1/2)*((c+d*sin(f*
x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)-14*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c
+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c
*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*d^2*2^(1/2)*((c+d*
sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+ln(2*((2*c-2*d)^(1/2)*2^(1/2)*
((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e
)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*c^2*2^(1/2)*((c
+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)-6*ln(2*((2*c-2*d)^(1/2)*2^(
1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(
f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*c*d*2^(1/2
)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)-7*ln(2*((2*c-2*d)^(1/2
)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d
*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*d^2*2
^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)+2*sin(f*x+e)*cos(
f*x+e)*(2*c-2*d)^(1/2)*c*d+10*sin(f*x+e)*cos(f*x+e)*(2*c-2*d)^(1/2)*d^2+2*l
n(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*
x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)
+sin(f*x+e)))*c^2*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)-12*ln(2*(
```

```
(2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+
c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(
f*x+e)))*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*c*d-14*ln(2*((2*c-
2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin
(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e
)))*d^2*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)+2*cos(f*x+e)*(2*c-2
*d)^(1/2)*c^2+2*cos(f*x+e)*(2*c-2*d)^(1/2)*c*d+8*cos(f*x+e)*(2*c-2*d)^(1/2)
*d^2)/(c+d*sin(f*x+e))^(1/2)/(a*(1+sin(f*x+e)))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="max
ima")
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e) + c)^(3/2)), x)
```

Fricas [B] time = 4.23902, size = 4350, normalized size = 22.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="fri
cas")
```

```
[Out] [-1/16*(((c^2*d - 6*c*d^2 - 7*d^3)*cos(f*x + e)^3 - 2*c^3 + 10*c^2*d + 26*c
*d^2 + 14*d^3 + (c^3 - 4*c^2*d - 19*c*d^2 - 14*d^3)*cos(f*x + e)^2 - (c^3 -
 5*c^2*d - 13*c*d^2 - 7*d^3)*cos(f*x + e) - (2*c^3 - 10*c^2*d - 26*c*d^2 -
14*d^3 - (c^2*d - 6*c*d^2 - 7*d^3)*cos(f*x + e)^2 + (c^3 - 5*c^2*d - 13*c*d
^2 - 7*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(2*a*c - 2*a*d)*log(((a*c^2 - 1
4*a*c*d + 17*a*d^2)*cos(f*x + e)^3 - 4*a*c^2 - 8*a*c*d - 4*a*d^2 - (13*a*c^
2 - 22*a*c*d - 3*a*d^2)*cos(f*x + e)^2 + 4*((c - 3*d)*cos(f*x + e)^2 - (3*c
 - d)*cos(f*x + e) + ((c - 3*d)*cos(f*x + e) + 4*c - 4*d)*sin(f*x + e) - 4*
c + 4*d)*sqrt(2*a*c - 2*a*d)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) +
 c) - 2*(9*a*c^2 - 14*a*c*d + 9*a*d^2)*cos(f*x + e) - (4*a*c^2 + 8*a*c*d +
4*a*d^2 - (a*c^2 - 14*a*c*d + 17*a*d^2)*cos(f*x + e)^2 - 2*(7*a*c^2 - 18*a*
c*d + 7*a*d^2)*cos(f*x + e))*sin(f*x + e))/(cos(f*x + e)^3 + 3*cos(f*x + e)
^2 + (cos(f*x + e)^2 - 2*cos(f*x + e) - 4)*sin(f*x + e) - 2*cos(f*x + e) -
4)) - 8*(c^3 - c^2*d - c*d^2 + d^3 + (c^2*d + 4*c*d^2 - 5*d^3)*cos(f*x + e)
^2 + (c^3 + 3*c*d^2 - 4*d^3)*cos(f*x + e) - (c^3 - c^2*d - c*d^2 + d^3 - (c
^2*d + 4*c*d^2 - 5*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a
)*sqrt(d*sin(f*x + e) + c))/((a^2*c^4*d - 2*a^2*c^3*d^2 + 2*a^2*c*d^4 - a^2
*d^5)*f*cos(f*x + e)^3 + (a^2*c^5 - 4*a^2*c^3*d^2 + 2*a^2*c^2*d^3 + 3*a^2*c
*d^4 - 2*a^2*d^5)*f*cos(f*x + e)^2 - (a^2*c^5 - a^2*c^4*d - 2*a^2*c^3*d^2 +
 2*a^2*c^2*d^3 + a^2*c*d^4 - a^2*d^5)*f*cos(f*x + e) - 2*(a^2*c^5 - a^2*c^4
*d - 2*a^2*c^3*d^2 + 2*a^2*c^2*d^3 + a^2*c*d^4 - a^2*d^5)*f + ((a^2*c^4*d -
 2*a^2*c^3*d^2 + 2*a^2*c*d^4 - a^2*d^5)*f*cos(f*x + e)^2 - (a^2*c^5 - a^2*c
^4*d - 2*a^2*c^3*d^2 + 2*a^2*c^2*d^3 + a^2*c*d^4 - a^2*d^5)*f*cos(f*x + e)
- 2*(a^2*c^5 - a^2*c^4*d - 2*a^2*c^3*d^2 + 2*a^2*c^2*d^3 + a^2*c*d^4 - a^2*
```

```
d^5)*f)*sin(f*x + e)), -1/8*(((c^2*d - 6*c*d^2 - 7*d^3)*cos(f*x + e)^3 - 2*
c^3 + 10*c^2*d + 26*c*d^2 + 14*d^3 + (c^3 - 4*c^2*d - 19*c*d^2 - 14*d^3)*co
s(f*x + e)^2 - (c^3 - 5*c^2*d - 13*c*d^2 - 7*d^3)*cos(f*x + e) - (2*c^3 - 1
0*c^2*d - 26*c*d^2 - 14*d^3 - (c^2*d - 6*c*d^2 - 7*d^3)*cos(f*x + e)^2 + (c
^3 - 5*c^2*d - 13*c*d^2 - 7*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(-2*a*c +
2*a*d)*arctan(1/4*sqrt(-2*a*c + 2*a*d)*sqrt(a*sin(f*x + e) + a)*((c - 3*d)*
sin(f*x + e) - 3*c + d)*sqrt(d*sin(f*x + e) + c)/((a*c*d - a*d^2)*cos(f*x +
 e)*sin(f*x + e) + (a*c^2 - a*c*d)*cos(f*x + e))) - 4*(c^3 - c^2*d - c*d^2
+ d^3 + (c^2*d + 4*c*d^2 - 5*d^3)*cos(f*x + e)^2 + (c^3 + 3*c*d^2 - 4*d^3)*
cos(f*x + e) - (c^3 - c^2*d - c*d^2 + d^3 - (c^2*d + 4*c*d^2 - 5*d^3)*cos(f
*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/(
(a^2*c^4*d - 2*a^2*c^3*d^2 + 2*a^2*c*d^4 - a^2*d^5)*f*cos(f*x + e)^3 + (a^2
*c^5 - 4*a^2*c^3*d^2 + 2*a^2*c^2*d^3 + 3*a^2*c*d^4 - 2*a^2*d^5)*f*cos(f*x +
 e)^2 - (a^2*c^5 - a^2*c^4*d - 2*a^2*c^3*d^2 + 2*a^2*c^2*d^3 + a^2*c*d^4 -
a^2*d^5)*f*cos(f*x + e) - 2*(a^2*c^5 - a^2*c^4*d - 2*a^2*c^3*d^2 + 2*a^2*c^
2*d^3 + a^2*c*d^4 - a^2*d^5)*f + ((a^2*c^4*d - 2*a^2*c^3*d^2 + 2*a^2*c*d^4
- a^2*d^5)*f*cos(f*x + e)^2 - (a^2*c^5 - a^2*c^4*d - 2*a^2*c^3*d^2 + 2*a^2*
c^2*d^3 + a^2*c*d^4 - a^2*d^5)*f*cos(f*x + e) - 2*(a^2*c^5 - a^2*c^4*d - 2*
a^2*c^3*d^2 + 2*a^2*c^2*d^3 + a^2*c*d^4 - a^2*d^5)*f)*sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**(3/2)/(c+d*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="gia
c")
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e) + c)^(3/2)), x)
```

3.599 $\int \frac{1}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=271

$$-\frac{(c-11d)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{2\sqrt{2}a^{3/2}f(c-d)^{7/2}}-\frac{d\left(3c^2+38cd+19d^2\right)\cos(e+fx)}{6af(c-d)^3(c+d)^2\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}-\frac{}{6af(c-d}$$

```
[Out] -((c - 11*d)*ArcTanh[(Sqrt[a]*Sqrt[c - d]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a
*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])])/(2*Sqrt[2]*a^(3/2)*(c - d)^(7/2)
*f) - Cos[e + f*x]/(2*(c - d)*f*(a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f
*x])^(3/2)) - (d*(3*c + 7*d)*Cos[e + f*x])/(6*a*(c - d)^2*(c + d)*f*Sqrt[a
+ a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(3/2)) - (d*(3*c^2 + 38*c*d + 19*d^2
)*Cos[e + f*x])/(6*a*(c - d)^3*(c + d)^2*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c
+ d*Sin[e + f*x]])
```

Rubi [A] time = 0.854785, antiderivative size = 271, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2766, 2984, 12, 2782, 208}

$$-\frac{(c-11d)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{2\sqrt{2}a^{3/2}f(c-d)^{7/2}}-\frac{d\left(3c^2+38cd+19d^2\right)\cos(e+fx)}{6af(c-d)^3(c+d)^2\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}-\frac{}{6af(c-d}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^(5/2)),x]
```

```
[Out] -((c - 11*d)*ArcTanh[(Sqrt[a]*Sqrt[c - d]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a
*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])])/(2*Sqrt[2]*a^(3/2)*(c - d)^(7/2)
*f) - Cos[e + f*x]/(2*(c - d)*f*(a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f
*x])^(3/2)) - (d*(3*c + 7*d)*Cos[e + f*x])/(6*a*(c - d)^2*(c + d)*f*Sqrt[a
+ a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(3/2)) - (d*(3*c^2 + 38*c*d + 19*d^2
)*Cos[e + f*x])/(6*a*(c - d)^3*(c + d)^2*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c
+ d*Sin[e + f*x]])
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
```

```
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{5/2}}\,dx = -\frac{\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{3/2}} - \frac{\int \frac{-\frac{1}{2}a(c\cdot}{\sqrt{a+a\sin(e}}}{}$$

$$= -\frac{\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{3/2}} - \frac{}{6a(c-d)^2(}$$

$$= -\frac{\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{3/2}} - \frac{}{6a(c-d)^2(}$$

$$= -\frac{\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{3/2}} - \frac{}{6a(c-d)^2(}$$

$$= -\frac{\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{3/2}} - \frac{}{6a(c-d)^2(}$$

$$= -\frac{(c-11d)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{2\sqrt{2}a^{3/2}(c-d)^{7/2}f} - \frac{}{2(c-d)f(a+a}$$

Mathematica [A] time = 8.99197, size = 478, normalized size = 1.76

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^2\left(\frac{3(c-11d)\left(\log\left(\tan\left(\frac{1}{2}(e+fx)\right)+1\right)-\log\left((d-c)\tan\left(\frac{1}{2}(e+fx)\right)+2\sqrt{c-d}\sqrt{\frac{1}{\cos(e+fx)+1}}\sqrt{c+d\sin(e+fx)}+c-d\right)\right)}{\frac{\sec^2\left(\frac{1}{2}(e+fx)\right)}{2\tan\left(\frac{1}{2}(e+fx)\right)+2}-\frac{\frac{\sqrt{c-d}\left(\frac{1}{\cos(e+fx)+1}\right)^{3/2}(c\sin(e+fx)+d\cos(e+fx)+d)}{\sqrt{c+d\sin(e+fx)}}-\frac{1}{2}(c-d)\sec^2\left(\frac{1}{2}(e+fx)\right)}{(d-c)\tan\left(\frac{1}{2}(e+fx)\right)+2\sqrt{c-d}\sqrt{\frac{1}{\cos(e+fx)+1}}\sqrt{c+d\sin(e+fx)}+c-d}}\right.}{12f(c-d)^3(a(\sin(e+fx)+1))}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^(5/2)),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2*(-(((Cos[(e + f*x)/2] - Sin[(e + f
*x)/2])*(6*c^4 + 12*c^3*d + 81*c^2*d^2 + 70*c*d^3 + 11*d^4 - d^2*(3*c^2 + 3
8*c*d + 19*d^2)*Cos[2*(e + f*x)] + 12*d*(c^3 + 8*c^2*d + 9*c*d^2 + 2*d^3)*S
in[e + f*x]))/((c + d)^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(c + d*Sin[e
 + f*x]))) + (3*(c - 11*d)*(Log[1 + Tan[(e + f*x)/2]] - Log[c - d + 2*Sqrt[
c - d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + (-c + d)*Ta
n[(e + f*x)/2]]))/(Sec[(e + f*x)/2]^2/(2 + 2*Tan[(e + f*x)/2]) - (-((c - d)
*Sec[(e + f*x)/2]^2)/2 + (Sqrt[c - d]*((1 + Cos[e + f*x])^(-1))^(3/2)*(d +
d*Cos[e + f*x] + c*Sin[e + f*x]))/Sqrt[c + d*Sin[e + f*x]])/(c - d + 2*Sqrt
[c - d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + (-c + d)*T
an[(e + f*x)/2]))))/(12*(c - d)^3*f*(a*(1 + Sin[e + f*x]))^(3/2)*Sqrt[c + d
*Sin[e + f*x]])
```

Maple [B] time = 0.286, size = 5040, normalized size = 18.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(5/2),x, algorithm="max
ima")
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e) + c)^(5/2)), x)
```

Fricas [B] time = 5.95996, size = 6974, normalized size = 25.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(5/2),x, algorithm="fri
cas")
```

```
[Out] [1/48*(3*(2*c^5 - 14*c^4*d - 76*c^3*d^2 - 124*c^2*d^3 - 86*c*d^4 - 22*d^5 +
 (c^3*d^2 - 9*c^2*d^3 - 21*c*d^4 - 11*d^5)*cos(f*x + e)^4 - (2*c^4*d - 17*c
^3*d^2 - 51*c^2*d^3 - 43*c*d^4 - 11*d^5)*cos(f*x + e)^3 - (c^5 - 5*c^4*d -
54*c^3*d^2 - 122*c^2*d^3 - 107*c*d^4 - 33*d^5)*cos(f*x + e)^2 + (c^5 - 7*c^
4*d - 38*c^3*d^2 - 62*c^2*d^3 - 43*c*d^4 - 11*d^5)*cos(f*x + e) + (2*c^5 -
```

```
14*c^4*d - 76*c^3*d^2 - 124*c^2*d^3 - 86*c*d^4 - 22*d^5 - (c^3*d^2 - 9*c^2*
d^3 - 21*c*d^4 - 11*d^5)*cos(f*x + e)^3 - 2*(c^4*d - 8*c^3*d^2 - 30*c^2*d^3
 - 32*c*d^4 - 11*d^5)*cos(f*x + e)^2 + (c^5 - 7*c^4*d - 38*c^3*d^2 - 62*c^2
*d^3 - 43*c*d^4 - 11*d^5)*cos(f*x + e))*sin(f*x + e))*sqrt(2*a*c - 2*a*d)*l
og(((a*c^2 - 14*a*c*d + 17*a*d^2)*cos(f*x + e)^3 - 4*a*c^2 - 8*a*c*d - 4*a*
d^2 - (13*a*c^2 - 22*a*c*d - 3*a*d^2)*cos(f*x + e)^2 - 4*((c - 3*d)*cos(f*x
 + e)^2 - (3*c - d)*cos(f*x + e) + ((c - 3*d)*cos(f*x + e) + 4*c - 4*d)*sin
(f*x + e) - 4*c + 4*d)*sqrt(2*a*c - 2*a*d)*sqrt(a*sin(f*x + e) + a)*sqrt(d*
sin(f*x + e) + c) - 2*(9*a*c^2 - 14*a*c*d + 9*a*d^2)*cos(f*x + e) - (4*a*c^
2 + 8*a*c*d + 4*a*d^2 - (a*c^2 - 14*a*c*d + 17*a*d^2)*cos(f*x + e)^2 - 2*(7
*a*c^2 - 18*a*c*d + 7*a*d^2)*cos(f*x + e))*sin(f*x + e))/(cos(f*x + e)^3 +
3*cos(f*x + e)^2 + (cos(f*x + e)^2 - 2*cos(f*x + e) - 4)*sin(f*x + e) - 2*c
os(f*x + e) - 4)) - 8*(3*c^5 - 3*c^4*d - 6*c^3*d^2 + 6*c^2*d^3 + 3*c*d^4 -
3*d^5 - (3*c^3*d^2 + 35*c^2*d^3 - 19*c*d^4 - 19*d^5)*cos(f*x + e)^3 + (6*c^
4*d + 39*c^3*d^2 - 29*c^2*d^3 - 23*c*d^4 + 7*d^5)*cos(f*x + e)^2 + 3*(c^5 +
 c^4*d + 12*c^3*d^2 + 4*c^2*d^3 - 13*c*d^4 - 5*d^5)*cos(f*x + e) - (3*c^5 -
 3*c^4*d - 6*c^3*d^2 + 6*c^2*d^3 + 3*c*d^4 - 3*d^5 - (3*c^3*d^2 + 35*c^2*d^
3 - 19*c*d^4 - 19*d^5)*cos(f*x + e)^2 - 6*(c^4*d + 7*c^3*d^2 + c^2*d^3 - 7*
c*d^4 - 2*d^5)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*
sin(f*x + e) + c))/((a^2*c^6*d^2 - 2*a^2*c^5*d^3 - a^2*c^4*d^4 + 4*a^2*c^3*
d^5 - a^2*c^2*d^6 - 2*a^2*c*d^7 + a^2*d^8)*f*cos(f*x + e)^4 - (2*a^2*c^7*d
 - 3*a^2*c^6*d^2 - 4*a^2*c^5*d^3 + 7*a^2*c^4*d^4 + 2*a^2*c^3*d^5 - 5*a^2*c^2
*d^6 + a^2*d^8)*f*cos(f*x + e)^3 - (a^2*c^8 + 2*a^2*c^7*d - 6*a^2*c^6*d^2 -
 6*a^2*c^5*d^3 + 12*a^2*c^4*d^4 + 6*a^2*c^3*d^5 - 10*a^2*c^2*d^6 - 2*a^2*c*
d^7 + 3*a^2*d^8)*f*cos(f*x + e)^2 + (a^2*c^8 - 4*a^2*c^6*d^2 + 6*a^2*c^4*d^
4 - 4*a^2*c^2*d^6 + a^2*d^8)*f*cos(f*x + e) + 2*(a^2*c^8 - 4*a^2*c^6*d^2 +
6*a^2*c^4*d^4 - 4*a^2*c^2*d^6 + a^2*d^8)*f - ((a^2*c^6*d^2 - 2*a^2*c^5*d^3
 - a^2*c^4*d^4 + 4*a^2*c^3*d^5 - a^2*c^2*d^6 - 2*a^2*c*d^7 + a^2*d^8)*f*cos(
f*x + e)^3 + 2*(a^2*c^7*d - a^2*c^6*d^2 - 3*a^2*c^5*d^3 + 3*a^2*c^4*d^4 + 3
*a^2*c^3*d^5 - 3*a^2*c^2*d^6 - a^2*c*d^7 + a^2*d^8)*f*cos(f*x + e)^2 - (a^2
*c^8 - 4*a^2*c^6*d^2 + 6*a^2*c^4*d^4 - 4*a^2*c^2*d^6 + a^2*d^8)*f*cos(f*x +
 e) - 2*(a^2*c^8 - 4*a^2*c^6*d^2 + 6*a^2*c^4*d^4 - 4*a^2*c^2*d^6 + a^2*d^8)
*f)*sin(f*x + e)), -1/24*(3*(2*c^5 - 14*c^4*d - 76*c^3*d^2 - 124*c^2*d^3 -
86*c*d^4 - 22*d^5 + (c^3*d^2 - 9*c^2*d^3 - 21*c*d^4 - 11*d^5)*cos(f*x + e)^
4 - (2*c^4*d - 17*c^3*d^2 - 51*c^2*d^3 - 43*c*d^4 - 11*d^5)*cos(f*x + e)^3
 - (c^5 - 5*c^4*d - 54*c^3*d^2 - 122*c^2*d^3 - 107*c*d^4 - 33*d^5)*cos(f*x +
 e)^2 + (c^5 - 7*c^4*d - 38*c^3*d^2 - 62*c^2*d^3 - 43*c*d^4 - 11*d^5)*cos(f
*x + e) + (2*c^5 - 14*c^4*d - 76*c^3*d^2 - 124*c^2*d^3 - 86*c*d^4 - 22*d^5
 - (c^3*d^2 - 9*c^2*d^3 - 21*c*d^4 - 11*d^5)*cos(f*x + e)^3 - 2*(c^4*d - 8*c
^3*d^2 - 30*c^2*d^3 - 32*c*d^4 - 11*d^5)*cos(f*x + e)^2 + (c^5 - 7*c^4*d -
38*c^3*d^2 - 62*c^2*d^3 - 43*c*d^4 - 11*d^5)*cos(f*x + e))*sin(f*x + e))*sq
rt(-2*a*c + 2*a*d)*arctan(1/4*sqrt(-2*a*c + 2*a*d)*sqrt(a*sin(f*x + e) + a)
*((c - 3*d)*sin(f*x + e) - 3*c + d)*sqrt(d*sin(f*x + e) + c)/((a*c*d - a*d^
2)*cos(f*x + e)*sin(f*x + e) + (a*c^2 - a*c*d)*cos(f*x + e))) + 4*(3*c^5 -
3*c^4*d - 6*c^3*d^2 + 6*c^2*d^3 + 3*c*d^4 - 3*d^5 - (3*c^3*d^2 + 35*c^2*d^3
 - 19*c*d^4 - 19*d^5)*cos(f*x + e)^3 + (6*c^4*d + 39*c^3*d^2 - 29*c^2*d^3 -
 23*c*d^4 + 7*d^5)*cos(f*x + e)^2 + 3*(c^5 + c^4*d + 12*c^3*d^2 + 4*c^2*d^3
 - 13*c*d^4 - 5*d^5)*cos(f*x + e) - (3*c^5 - 3*c^4*d - 6*c^3*d^2 + 6*c^2*d^
3 + 3*c*d^4 - 3*d^5 - (3*c^3*d^2 + 35*c^2*d^3 - 19*c*d^4 - 19*d^5)*cos(f*x
+ e)^2 - 6*(c^4*d + 7*c^3*d^2 + c^2*d^3 - 7*c*d^4 - 2*d^5)*cos(f*x + e))*si
n(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/((a^2*c^6*d^
2 - 2*a^2*c^5*d^3 - a^2*c^4*d^4 + 4*a^2*c^3*d^5 - a^2*c^2*d^6 - 2*a^2*c*d^7
 + a^2*d^8)*f*cos(f*x + e)^4 - (2*a^2*c^7*d - 3*a^2*c^6*d^2 - 4*a^2*c^5*d^3
 + 7*a^2*c^4*d^4 + 2*a^2*c^3*d^5 - 5*a^2*c^2*d^6 + a^2*d^8)*f*cos(f*x + e)^
3 - (a^2*c^8 + 2*a^2*c^7*d - 6*a^2*c^6*d^2 - 6*a^2*c^5*d^3 + 12*a^2*c^4*d^4
 + 6*a^2*c^3*d^5 - 10*a^2*c^2*d^6 - 2*a^2*c*d^7 + 3*a^2*d^8)*f*cos(f*x + e)
^2 + (a^2*c^8 - 4*a^2*c^6*d^2 + 6*a^2*c^4*d^4 - 4*a^2*c^2*d^6 + a^2*d^8)*f*
cos(f*x + e) + 2*(a^2*c^8 - 4*a^2*c^6*d^2 + 6*a^2*c^4*d^4 - 4*a^2*c^2*d^6 +
 a^2*d^8)*f - ((a^2*c^6*d^2 - 2*a^2*c^5*d^3 - a^2*c^4*d^4 + 4*a^2*c^3*d^5 -
```

```
 a^2*c^2*d^6 - 2*a^2*c*d^7 + a^2*d^8)*f*cos(f*x + e)^3 + 2*(a^2*c^7*d - a^2
*c^6*d^2 - 3*a^2*c^5*d^3 + 3*a^2*c^4*d^4 + 3*a^2*c^3*d^5 - 3*a^2*c^2*d^6 -
a^2*c*d^7 + a^2*d^8)*f*cos(f*x + e)^2 - (a^2*c^8 - 4*a^2*c^6*d^2 + 6*a^2*c^
4*d^4 - 4*a^2*c^2*d^6 + a^2*d^8)*f*cos(f*x + e) - 2*(a^2*c^8 - 4*a^2*c^6*d^
2 + 6*a^2*c^4*d^4 - 4*a^2*c^2*d^6 + a^2*d^8)*f)*sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**(3/2)/(c+d*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(5/2),x, algorithm="gia
c")
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e) + c)^(5/2)), x)
```

3.600 $\int \frac{(c+d\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=260

$$-\frac{\sqrt{c-d}\left(3c^2+14cd+43d^2\right)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{16\sqrt{2}a^{5/2}f}-\frac{2d^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{a^{5/2}f}-\frac{(c-d}{}$$

```
[Out] (-2*d^(5/2)*ArcTan[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]
*Sqrt[c + d*Sin[e + f*x]])])/(a^(5/2)*f) - (Sqrt[c - d]*(3*c^2 + 14*c*d + 4
3*d^2)*ArcTanh[(Sqrt[a]*Sqrt[c - d]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e
 + f*x]]*Sqrt[c + d*Sin[e + f*x]])])/(16*Sqrt[2]*a^(5/2)*f) - ((c - d)*(3*c
 + 11*d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(16*a*f*(a + a*Sin[e + f*x]
)^(3/2)) - ((c - d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(3/2))/(4*f*(a + a*Si
n[e + f*x])^(5/2))
```

Rubi [A] time = 0.864062, antiderivative size = 260, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2765, 2977, 2982, 2782, 208, 2775, 205}

$$-\frac{\sqrt{c-d}\left(3c^2+14cd+43d^2\right)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{16\sqrt{2}a^{5/2}f}-\frac{2d^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{a^{5/2}f}-\frac{(c-d}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^(5/2)/(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] (-2*d^(5/2)*ArcTan[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]
*Sqrt[c + d*Sin[e + f*x]])])/(a^(5/2)*f) - (Sqrt[c - d]*(3*c^2 + 14*c*d + 4
3*d^2)*ArcTanh[(Sqrt[a]*Sqrt[c - d]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e
 + f*x]]*Sqrt[c + d*Sin[e + f*x]])])/(16*Sqrt[2]*a^(5/2)*f) - ((c - d)*(3*c
 + 11*d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(16*a*f*(a + a*Sin[e + f*x]
)^(3/2)) - ((c - d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(3/2))/(4*f*(a + a*Si
n[e + f*x])^(5/2))
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
```

```
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = -\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{4f(a+a\sin(e+fx))^{5/2}} - \frac{\int \frac{\sqrt{c+d\sin(e+fx)}\left(-\frac{1}{2}a(3c-d)(c+3d)-4ad^2\sin(e+fx)\right)}{(a+a\sin(e+fx))^{3/2}}\,dx}{4a^2}$$

$$= -\frac{(c-d)(3c+11d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{16af(a+a\sin(e+fx))^{3/2}} - \frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^{3/}}{4f(a+a\sin(e+fx))^{5/2}}$$

$$= -\frac{(c-d)(3c+11d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{16af(a+a\sin(e+fx))^{3/2}} - \frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^{3/}}{4f(a+a\sin(e+fx))^{5/2}}$$

$$= -\frac{(c-d)(3c+11d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{16af(a+a\sin(e+fx))^{3/2}} - \frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^{3/}}{4f(a+a\sin(e+fx))^{5/2}}$$

$$= -\frac{2d^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{a^{5/2}f} - \frac{\sqrt{c-d}\left(3c^2+14cd+43d^2\right)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{}}{\sqrt{2}\sqrt{a+a\sin(}}\right.}{16\sqrt{2}a^{5/2}f}$$

Mathematica [C] time = 17.1589, size = 1845, normalized size = 7.1

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*Sin[e + f*x])^(5/2)/(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5*(-(c - d)^2/(4*(Cos[(e + f*x)/2] +
 Sin[(e + f*x)/2])^3) - (3*(c - d)*(c + 5*d))/(16*(Cos[(e + f*x)/2] + Sin[(
e + f*x)/2])) + (3*(c^2*Sin[(e + f*x)/2] + 4*c*d*Sin[(e + f*x)/2] - 5*d^2*S
in[(e + f*x)/2]))/(8*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2) + (c^2*Sin[(e
 + f*x)/2] - 2*c*d*Sin[(e + f*x)/2] + d^2*Sin[(e + f*x)/2])/(2*(Cos[(e + f*
x)/2] + Sin[(e + f*x)/2])^4))*Sqrt[c + d*Sin[e + f*x]])/(f*(a*(1 + Sin[e +
f*x]))^(5/2)) + ((Sqrt[2]*(3*c^3 + 11*c^2*d + 29*c*d^2 - 43*d^3)*Log[1 + Ta
n[(e + f*x)/2]] - Sqrt[2]*(3*c^3 + 11*c^2*d + 29*c*d^2 - 43*d^3)*Log[c - d
+ 2*Sqrt[c - d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + (-
c + d)*Tan[(e + f*x)/2]] - (32*I)*Sqrt[c - d]*d^(5/2)*(Log[(c - I*(d + (1 +
 I)*Sqrt[2]*Sqrt[d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]])
 + ((-I)*c + d)*Tan[(e + f*x)/2])/(16*d^(7/2)*(I + Tan[(e + f*x)/2]))] - Lo
g[(c + I*d + (1 + I)*Sqrt[2]*Sqrt[d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c +
 d*Sin[e + f*x]] + (I*c + d)*Tan[(e + f*x)/2])/(16*d^(7/2)*(-I + Tan[(e + f
*x)/2]))]))*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5*((3*c^3)/(32*(Cos[(e +
f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c + d*Sin[e + f*x]]) + (11*c^2*d)/(32*(Cos
[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c + d*Sin[e + f*x]]) + (29*c*d^2)/(3
2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c + d*Sin[e + f*x]]) - (11*d^3
)/(32*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c + d*Sin[e + f*x]]) + (d^
3*Sin[e + f*x])/((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c + d*Sin[e + f
*x]])))/(f*(a*(1 + Sin[e + f*x]))^(5/2)*(((3*c^3 + 11*c^2*d + 29*c*d^2 - 43
*d^3)*Sec[(e + f*x)/2]^2)/(Sqrt[2]*(1 + Tan[(e + f*x)/2])) - (Sqrt[2]*(3*c^
3 + 11*c^2*d + 29*c*d^2 - 43*d^3)*(((-c + d)*Sec[(e + f*x)/2]^2)/2 + (Sqrt[
c - d]*d*Cos[e + f*x]*Sqrt[(1 + Cos[e + f*x])^(-1)])/Sqrt[c + d*Sin[e + f*x
]] + Sqrt[c - d]*((1 + Cos[e + f*x])^(-1))^(3/2)*Sin[e + f*x]*Sqrt[c + d*Si
n[e + f*x]]))/(c - d + 2*Sqrt[c - d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c +
 d*Sin[e + f*x]] + (-c + d)*Tan[(e + f*x)/2]) - (32*I)*Sqrt[c - d]*d^(5/2)*
((16*d^(7/2)*(I + Tan[(e + f*x)/2])*(((((-I)*c + d)*Sec[(e + f*x)/2]^2)/2 -
 I*(((1 + I)*d^(3/2)*Cos[e + f*x]*Sqrt[(1 + Cos[e + f*x])^(-1)])/(Sqrt[2]*S
qrt[c + d*Sin[e + f*x]]) + ((1 + I)*Sqrt[d]*((1 + Cos[e + f*x])^(-1))^(3/2)
*Sin[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/Sqrt[2]))/(16*d^(7/2)*(I + Tan[(e +
 f*x)/2])) - (Sec[(e + f*x)/2]^2*(c - I*(d + (1 + I)*Sqrt[2]*Sqrt[d]*Sqrt[(
1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]]) + ((-I)*c + d)*Tan[(e + f
*x)/2]))/(32*d^(7/2)*(I + Tan[(e + f*x)/2])^2)))/(c - I*(d + (1 + I)*Sqrt[2
]*Sqrt[d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]]) + ((-I)*c
 + d)*Tan[(e + f*x)/2]) - (16*d^(7/2)*(-I + Tan[(e + f*x)/2])*((((I*c + d)*
Sec[(e + f*x)/2]^2)/2 + ((1 + I)*d^(3/2)*Cos[e + f*x]*Sqrt[(1 + Cos[e + f*x
])^(-1)])/(Sqrt[2]*Sqrt[c + d*Sin[e + f*x]]) + ((1 + I)*Sqrt[d]*((1 + Cos[e
 + f*x])^(-1))^(3/2)*Sin[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/Sqrt[2])/(16*d^
(7/2)*(-I + Tan[(e + f*x)/2])) - (Sec[(e + f*x)/2]^2*(c + I*d + (1 + I)*Sqr
t[2]*Sqrt[d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + (I*c
+ d)*Tan[(e + f*x)/2]))/(32*d^(7/2)*(-I + Tan[(e + f*x)/2])^2)))/(c + I*d +
 (1 + I)*Sqrt[2]*Sqrt[d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f
*x]] + (I*c + d)*Tan[(e + f*x)/2]))))
```

Maple [B] time = 0.398, size = 10738, normalized size = 41.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(5/2)/(a*sin(f*x + e) + a)^(5/2), x)
```

Fricas [B] time = 5.10096, size = 9397, normalized size = 36.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] [1/32*(sqrt(1/2)*((3*a*c^2 + 14*a*c*d + 43*a*d^2)*cos(f*x + e)^3 - 12*a*c^2
 - 56*a*c*d - 172*a*d^2 + 3*(3*a*c^2 + 14*a*c*d + 43*a*d^2)*cos(f*x + e)^2
- 2*(3*a*c^2 + 14*a*c*d + 43*a*d^2)*cos(f*x + e) - (12*a*c^2 + 56*a*c*d + 1
72*a*d^2 - (3*a*c^2 + 14*a*c*d + 43*a*d^2)*cos(f*x + e)^2 + 2*(3*a*c^2 + 14
*a*c*d + 43*a*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt((c - d)/a)*log((4*sqrt(
1/2)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt((c - d)/a)*(cos
(f*x + e) - sin(f*x + e) + 1) - (c - 3*d)*cos(f*x + e)^2 - (3*c - d)*cos(f*
x + e) + ((c - 3*d)*cos(f*x + e) - 2*c - 2*d)*sin(f*x + e) - 2*c - 2*d)/(co
s(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) + 8*(a*
d^2*cos(f*x + e)^3 + 3*a*d^2*cos(f*x + e)^2 - 2*a*d^2*cos(f*x + e) - 4*a*d^
2 + (a*d^2*cos(f*x + e)^2 - 2*a*d^2*cos(f*x + e) - 4*a*d^2)*sin(f*x + e))*s
qrt(-d/a)*log((128*d^4*cos(f*x + e)^5 + 128*(2*c*d^3 - d^4)*cos(f*x + e)^4
+ c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^3 + d^4 - 32*(5*c^2*d^2 - 14*c*d^3 + 13
*d^4)*cos(f*x + e)^3 - 32*(c^3*d - 2*c^2*d^2 + 9*c*d^3 - 4*d^4)*cos(f*x + e
)^2 - 8*(16*d^3*cos(f*x + e)^4 + 24*(c*d^2 - d^3)*cos(f*x + e)^3 - c^3 + 17
*c^2*d - 59*c*d^2 + 51*d^3 - 2*(5*c^2*d - 26*c*d^2 + 33*d^3)*cos(f*x + e)^2
 - (c^3 - 7*c^2*d + 31*c*d^2 - 25*d^3)*cos(f*x + e) + (16*d^3*cos(f*x + e)^
3 + c^3 - 17*c^2*d + 59*c*d^2 - 51*d^3 - 8*(3*c*d^2 - 5*d^3)*cos(f*x + e)^2
 - 2*(5*c^2*d - 14*c*d^2 + 13*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f
*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt(-d/a) + (c^4 - 28*c^3*d + 230*c^
2*d^2 - 476*c*d^3 + 289*d^4)*cos(f*x + e) + (128*d^4*cos(f*x + e)^4 + c^4 +
 4*c^3*d + 6*c^2*d^2 + 4*c*d^3 + d^4 - 256*(c*d^3 - d^4)*cos(f*x + e)^3 - 3
2*(5*c^2*d^2 - 6*c*d^3 + 5*d^4)*cos(f*x + e)^2 + 32*(c^3*d - 7*c^2*d^2 + 15
*c*d^3 - 9*d^4)*cos(f*x + e))*sin(f*x + e))/(cos(f*x + e) + sin(f*x + e) +
1)) + 2*(3*(c^2 + 4*c*d - 5*d^2)*cos(f*x + e)^2 + 4*c^2 - 8*c*d + 4*d^2 + (
7*c^2 + 4*c*d - 11*d^2)*cos(f*x + e) - (4*c^2 - 8*c*d + 4*d^2 - 3*(c^2 + 4*
c*d - 5*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*si
n(f*x + e) + c))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 - 2*a^3*f*c
```

```
os(f*x + e) - 4*a^3*f + (a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^
3*f)*sin(f*x + e)), 1/32*(sqrt(1/2)*((3*a*c^2 + 14*a*c*d + 43*a*d^2)*cos(f*
x + e)^3 - 12*a*c^2 - 56*a*c*d - 172*a*d^2 + 3*(3*a*c^2 + 14*a*c*d + 43*a*d
^2)*cos(f*x + e)^2 - 2*(3*a*c^2 + 14*a*c*d + 43*a*d^2)*cos(f*x + e) - (12*a
*c^2 + 56*a*c*d + 172*a*d^2 - (3*a*c^2 + 14*a*c*d + 43*a*d^2)*cos(f*x + e)^
2 + 2*(3*a*c^2 + 14*a*c*d + 43*a*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt((c -
 d)/a)*log((4*sqrt(1/2)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*s
qrt((c - d)/a)*(cos(f*x + e) - sin(f*x + e) + 1) - (c - 3*d)*cos(f*x + e)^2
 - (3*c - d)*cos(f*x + e) + ((c - 3*d)*cos(f*x + e) - 2*c - 2*d)*sin(f*x +
e) - 2*c - 2*d)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x
 + e) - 2)) + 16*(a*d^2*cos(f*x + e)^3 + 3*a*d^2*cos(f*x + e)^2 - 2*a*d^2*c
os(f*x + e) - 4*a*d^2 + (a*d^2*cos(f*x + e)^2 - 2*a*d^2*cos(f*x + e) - 4*a*
d^2)*sin(f*x + e))*sqrt(d/a)*arctan(1/4*(8*d^2*cos(f*x + e)^2 - c^2 + 6*c*d
 - 9*d^2 - 8*(c*d - d^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(
f*x + e) + c)*sqrt(d/a)/(2*d^3*cos(f*x + e)^3 - (3*c*d^2 - d^3)*cos(f*x + e
)*sin(f*x + e) - (c^2*d - c*d^2 + 2*d^3)*cos(f*x + e))) + 2*(3*(c^2 + 4*c*d
 - 5*d^2)*cos(f*x + e)^2 + 4*c^2 - 8*c*d + 4*d^2 + (7*c^2 + 4*c*d - 11*d^2)
*cos(f*x + e) - (4*c^2 - 8*c*d + 4*d^2 - 3*(c^2 + 4*c*d - 5*d^2)*cos(f*x +
e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/(a^3*f
*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f +
 (a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f)*sin(f*x + e)), -1/
16*(sqrt(1/2)*((3*a*c^2 + 14*a*c*d + 43*a*d^2)*cos(f*x + e)^3 - 12*a*c^2 -
56*a*c*d - 172*a*d^2 + 3*(3*a*c^2 + 14*a*c*d + 43*a*d^2)*cos(f*x + e)^2 - 2
*(3*a*c^2 + 14*a*c*d + 43*a*d^2)*cos(f*x + e) - (12*a*c^2 + 56*a*c*d + 172*
a*d^2 - (3*a*c^2 + 14*a*c*d + 43*a*d^2)*cos(f*x + e)^2 + 2*(3*a*c^2 + 14*a*
c*d + 43*a*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(-(c - d)/a)*arctan(-2*sqrt
(1/2)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt(-(c - d)/a)/((
c - d)*cos(f*x + e))) - 4*(a*d^2*cos(f*x + e)^3 + 3*a*d^2*cos(f*x + e)^2 -
2*a*d^2*cos(f*x + e) - 4*a*d^2 + (a*d^2*cos(f*x + e)^2 - 2*a*d^2*cos(f*x +
e) - 4*a*d^2)*sin(f*x + e))*sqrt(-d/a)*log((128*d^4*cos(f*x + e)^5 + 128*(2
*c*d^3 - d^4)*cos(f*x + e)^4 + c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^3 + d^4 -
32*(5*c^2*d^2 - 14*c*d^3 + 13*d^4)*cos(f*x + e)^3 - 32*(c^3*d - 2*c^2*d^2 +
 9*c*d^3 - 4*d^4)*cos(f*x + e)^2 - 8*(16*d^3*cos(f*x + e)^4 + 24*(c*d^2 - d
^3)*cos(f*x + e)^3 - c^3 + 17*c^2*d - 59*c*d^2 + 51*d^3 - 2*(5*c^2*d - 26*c
*d^2 + 33*d^3)*cos(f*x + e)^2 - (c^3 - 7*c^2*d + 31*c*d^2 - 25*d^3)*cos(f*x
 + e) + (16*d^3*cos(f*x + e)^3 + c^3 - 17*c^2*d + 59*c*d^2 - 51*d^3 - 8*(3*
c*d^2 - 5*d^3)*cos(f*x + e)^2 - 2*(5*c^2*d - 14*c*d^2 + 13*d^3)*cos(f*x + e
))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt(-d/
a) + (c^4 - 28*c^3*d + 230*c^2*d^2 - 476*c*d^3 + 289*d^4)*cos(f*x + e) + (1
28*d^4*cos(f*x + e)^4 + c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^3 + d^4 - 256*(c*
d^3 - d^4)*cos(f*x + e)^3 - 32*(5*c^2*d^2 - 6*c*d^3 + 5*d^4)*cos(f*x + e)^2
 + 32*(c^3*d - 7*c^2*d^2 + 15*c*d^3 - 9*d^4)*cos(f*x + e))*sin(f*x + e))/(c
os(f*x + e) + sin(f*x + e) + 1)) - (3*(c^2 + 4*c*d - 5*d^2)*cos(f*x + e)^2
+ 4*c^2 - 8*c*d + 4*d^2 + (7*c^2 + 4*c*d - 11*d^2)*cos(f*x + e) - (4*c^2 -
8*c*d + 4*d^2 - 3*(c^2 + 4*c*d - 5*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*
sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f
*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f + (a^3*f*cos(f*x + e)^2 -
2*a^3*f*cos(f*x + e) - 4*a^3*f)*sin(f*x + e)), -1/16*(sqrt(1/2)*((3*a*c^2 +
 14*a*c*d + 43*a*d^2)*cos(f*x + e)^3 - 12*a*c^2 - 56*a*c*d - 172*a*d^2 + 3*
(3*a*c^2 + 14*a*c*d + 43*a*d^2)*cos(f*x + e)^2 - 2*(3*a*c^2 + 14*a*c*d + 43
*a*d^2)*cos(f*x + e) - (12*a*c^2 + 56*a*c*d + 172*a*d^2 - (3*a*c^2 + 14*a*c
*d + 43*a*d^2)*cos(f*x + e)^2 + 2*(3*a*c^2 + 14*a*c*d + 43*a*d^2)*cos(f*x +
 e))*sin(f*x + e))*sqrt(-(c - d)/a)*arctan(-2*sqrt(1/2)*sqrt(a*sin(f*x + e)
 + a)*sqrt(d*sin(f*x + e) + c)*sqrt(-(c - d)/a)/((c - d)*cos(f*x + e))) - 8
*(a*d^2*cos(f*x + e)^3 + 3*a*d^2*cos(f*x + e)^2 - 2*a*d^2*cos(f*x + e) - 4*
a*d^2 + (a*d^2*cos(f*x + e)^2 - 2*a*d^2*cos(f*x + e) - 4*a*d^2)*sin(f*x + e
))*sqrt(d/a)*arctan(1/4*(8*d^2*cos(f*x + e)^2 - c^2 + 6*c*d - 9*d^2 - 8*(c*
d - d^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sq
rt(d/a)/(2*d^3*cos(f*x + e)^3 - (3*c*d^2 - d^3)*cos(f*x + e)*sin(f*x + e) -
```

```
(c^2*d - c*d^2 + 2*d^3)*cos(f*x + e))) - (3*(c^2 + 4*c*d - 5*d^2)*cos(f*x
+ e)^2 + 4*c^2 - 8*c*d + 4*d^2 + (7*c^2 + 4*c*d - 11*d^2)*cos(f*x + e) - (4
*c^2 - 8*c*d + 4*d^2 - 3*(c^2 + 4*c*d - 5*d^2)*cos(f*x + e))*sin(f*x + e))*
sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/(a^3*f*cos(f*x + e)^3 +
3*a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f + (a^3*f*cos(f*x +
e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f)*sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(5/2)/(a+a*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d \sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}{\left(a \sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate((d*sin(f*x + e) + c)^(5/2)/(a*sin(f*x + e) + a)^(5/2), x)
```

3.601 $\int \frac{(c+d\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=184

$$-\frac{3(c+d)^2\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{16\sqrt{2}a^{5/2}f\sqrt{c-d}}-\frac{(3c+7d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{16af(a\sin(e+fx)+a)^{3/2}}-\frac{(c-d)\cos(e+fx)\sqrt{c+}}{4f(a\sin(e+fx)}$$

```
[Out] (-3*(c + d)^2*ArcTanh[(Sqrt[a]*Sqrt[c - d]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a +
a*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])])/(16*Sqrt[2]*a^(5/2)*Sqrt[c - d]
*f) - ((c - d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(4*f*(a + a*Sin[e + f
*x])^(5/2)) - ((3*c + 7*d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(16*a*f*(
a + a*Sin[e + f*x])^(3/2))
```

Rubi [A] time = 0.538081, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2765, 2978, 12, 2782, 208}

$$-\frac{3(c+d)^2\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{16\sqrt{2}a^{5/2}f\sqrt{c-d}}-\frac{(3c+7d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{16af(a\sin(e+fx)+a)^{3/2}}-\frac{(c-d)\cos(e+fx)\sqrt{c+}}{4f(a\sin(e+fx)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^(3/2)/(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] (-3*(c + d)^2*ArcTanh[(Sqrt[a]*Sqrt[c - d]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a +
a*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])])/(16*Sqrt[2]*a^(5/2)*Sqrt[c - d]
*f) - ((c - d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(4*f*(a + a*Sin[e + f
*x])^(5/2)) - ((3*c + 7*d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(16*a*f*(
a + a*Sin[e + f*x])^(3/2))
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = -\frac{(c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4f(a+a\sin(e+fx))^{5/2}} - \frac{\int \frac{-\frac{1}{2}a\left(3c^2+6cd-d^2\right)-ad(c+3d)\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}\,dx}{4a^2}$$

$$= -\frac{(c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4f(a+a\sin(e+fx))^{5/2}} - \frac{(3c+7d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{16af(a+a\sin(e+fx))^{3/2}} + \frac{\int}{4\sqrt{a}\cdots}$$

$$= -\frac{(c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4f(a+a\sin(e+fx))^{5/2}} - \frac{(3c+7d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{16af(a+a\sin(e+fx))^{3/2}} + \frac{\left(3(c\cdots\right.}{\cdots}$$

$$= -\frac{(c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4f(a+a\sin(e+fx))^{5/2}} - \frac{(3c+7d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{16af(a+a\sin(e+fx))^{3/2}} - \frac{\left(3(c\cdots\right.}{\cdots}$$

$$= -\frac{3(c+d)^2\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{16\sqrt{2}a^{5/2}\sqrt{c-d}f} - \frac{(c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4f(a+a\sin(e+fx))^{5/2}} - \cdots$$

Mathematica [B] time = 7.1875, size = 396, normalized size = 2.15

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^4\left(\frac{3(c+d)^2\left(\log\left(\tan\left(\frac{1}{2}(e+fx)\right)+1\right)-\log\left((d-c)\tan\left(\frac{1}{2}(e+fx)\right)+2\sqrt{c-d}\sqrt{\frac{1}{\cos(e+fx)+1}}\sqrt{c+d\sin(e+fx)}+c-d\right)\right)}{\frac{\sec^2\left(\frac{1}{2}(e+fx)\right)}{2\tan\left(\frac{1}{2}(e+fx)\right)+2}-\frac{\frac{\sqrt{c-d}\left(\frac{1}{\cos(e+fx)+1}\right)^{3/2}(c\sin(e+fx)+d\cos(e+fx)+d)}{\sqrt{c+d\sin(e+fx)}}-\frac{1}{2}(c-d)\sec^2\left(\frac{1}{2}(e+fx)\right)}{(d-c)\tan\left(\frac{1}{2}(e+fx)\right)+2\sqrt{c-d}\sqrt{\frac{1}{\cos(e+fx)+1}}\sqrt{c+d\sin(e+fx)}+c-d}}-\frac{2\cdots}{\cdots}\right.}{32f(a(\sin(e+fx)+1))^{5/2}\sqrt{c+d\sin(e+fx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*Sin[e + f*x])^(3/2)/(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*((-2*(Cos[(e + f*x)/2] - Sin[(e +
f*x)/2])*(c + d*Sin[e + f*x])*(7*c + 3*d + (3*c + 7*d)*Sin[e + f*x]))/(Cos[
(e + f*x)/2] + Sin[(e + f*x)/2])^3 + (3*(c + d)^2*(Log[1 + Tan[(e + f*x)/2]
] - Log[c - d + 2*Sqrt[c - d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[
e + f*x]] + (-c + d)*Tan[(e + f*x)/2]]))/(Sec[(e + f*x)/2]^2/(2 + 2*Tan[(e
```

```
+ f*x)/2]) - (-((c - d)*Sec[(e + f*x)/2]^2)/2 + (Sqrt[c - d]*((1 + Cos[e +
f*x])^(-1))^(3/2)*(d + d*Cos[e + f*x] + c*Sin[e + f*x]))/Sqrt[c + d*Sin[e +
 f*x]])/(c - d + 2*Sqrt[c - d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin
[e + f*x]] + (-c + d)*Tan[(e + f*x)/2]))))/(32*f*(a*(1 + Sin[e + f*x]))^(5/
2)*Sqrt[c + d*Sin[e + f*x]])
```

Maple [B] time = 0.23, size = 3050, normalized size = 16.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2),x)
```

```
[Out] 1/64/f/(c-d)*(-12*cos(f*x+e)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*c^2+12
*sin(f*x+e)*cos(f*x+e)*(2*c-2*d)^(1/2)*2^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2
)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x
+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*c*d+6*sin(f*x
+e)*cos(f*x+e)^2*(2*c-2*d)^(1/2)*2^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+
d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*
cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*c*d+3*sin(f*x+e)*co
s(f*x+e)^2*(2*c-2*d)^(1/2)*2^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(
f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*
x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*d^2+28*cos(f*x+e)*((c+d*s
in(f*x+e))/(cos(f*x+e)+1))^(1/2)*d^2+12*cos(f*x+e)^3*((c+d*sin(f*x+e))/(cos
(f*x+e)+1))^(1/2)*c^2-28*cos(f*x+e)^3*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/
2)*d^2-12*(2*c-2*d)^(1/2)*2^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f
*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x
+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*d^2+3*sin(f*x+e)*cos(f*x+e
)^2*(2*c-2*d)^(1/2)*2^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))
/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*
cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*c^2-16*cos(f*x+e)*((c+d*sin(f*x+
e))/(cos(f*x+e)+1))^(1/2)*c*d+16*cos(f*x+e)^3*((c+d*sin(f*x+e))/(cos(f*x+e)
+1))^(1/2)*c*d-28*sin(f*x+e)*cos(f*x+e)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(
1/2)*c^2+12*sin(f*x+e)*cos(f*x+e)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*d
^2-12*(2*c-2*d)^(1/2)*2^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e
))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-
d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*c^2+9*cos(f*x+e)^2*(2*c-2*d)^(
1/2)*2^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1)
)^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)
/(1-cos(f*x+e)+sin(f*x+e)))*c^2+9*cos(f*x+e)^2*(2*c-2*d)^(1/2)*2^(1/2)*ln(2
*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e
)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+si
n(f*x+e)))*d^2-12*sin(f*x+e)*(2*c-2*d)^(1/2)*2^(1/2)*ln(2*((2*c-2*d)^(1/2)*
2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*s
in(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*c^2-12*
sin(f*x+e)*(2*c-2*d)^(1/2)*2^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(
f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*
x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*d^2+6*cos(f*x+e)*(2*c-2*d
)^(1/2)*2^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)
+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c
+d)/(1-cos(f*x+e)+sin(f*x+e)))*c^2+6*cos(f*x+e)*(2*c-2*d)^(1/2)*2^(1/2)*ln(
2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+
e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+s
in(f*x+e)))*d^2+16*sin(f*x+e)*cos(f*x+e)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^
(1/2)*c*d-24*(2*c-2*d)^(1/2)*2^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*si
n(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(
```

```
f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*c*d-3*cos(f*x+e)^3*(2*c
-2*d)^(1/2)*2^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*
x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+
e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*c^2-3*cos(f*x+e)^3*(2*c-2*d)^(1/2)*2^(1/
2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*si
n(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*
x+e)+sin(f*x+e)))*d^2-6*cos(f*x+e)^3*(2*c-2*d)^(1/2)*2^(1/2)*ln(2*((2*c-2*d
)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*
x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))
*c*d+6*sin(f*x+e)*cos(f*x+e)*(2*c-2*d)^(1/2)*2^(1/2)*ln(2*((2*c-2*d)^(1/2)*
2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*s
in(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*c^2+6*s
in(f*x+e)*cos(f*x+e)*(2*c-2*d)^(1/2)*2^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*
((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e
)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*d^2+18*cos(f*x+
e)^2*(2*c-2*d)^(1/2)*2^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e)
)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d
*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*c*d-24*sin(f*x+e)*(2*c-2*d)^(1/
2)*2^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^
(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(
1-cos(f*x+e)+sin(f*x+e)))*c*d+12*cos(f*x+e)*(2*c-2*d)^(1/2)*2^(1/2)*ln(2*((
2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c
*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f
*x+e)))*c*d)*(c+d*sin(f*x+e))^(1/2)/sin(f*x+e)/(a*(1+sin(f*x+e)))^(5/2)/((c
+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(3/2)/(a*sin(f*x + e) + a)^(5/2), x)
```

Fricas [B] time = 2.7812, size = 3136, normalized size = 17.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] [1/128*(3*((c^2 + 2*c*d + d^2)*cos(f*x + e)^3 + 3*(c^2 + 2*c*d + d^2)*cos(f
*x + e)^2 - 4*c^2 - 8*c*d - 4*d^2 - 2*(c^2 + 2*c*d + d^2)*cos(f*x + e) + ((
c^2 + 2*c*d + d^2)*cos(f*x + e)^2 - 4*c^2 - 8*c*d - 4*d^2 - 2*(c^2 + 2*c*d
+ d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(2*a*c - 2*a*d)*log(((a*c^2 - 14*a*c
*d + 17*a*d^2)*cos(f*x + e)^3 - 4*a*c^2 - 8*a*c*d - 4*a*d^2 - (13*a*c^2 - 2
2*a*c*d - 3*a*d^2)*cos(f*x + e)^2 - 4*((c - 3*d)*cos(f*x + e)^2 - (3*c - d)
```

```
*cos(f*x + e) + ((c - 3*d)*cos(f*x + e) + 4*c - 4*d)*sin(f*x + e) - 4*c + 4
*d)*sqrt(2*a*c - 2*a*d)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c) -
 2*(9*a*c^2 - 14*a*c*d + 9*a*d^2)*cos(f*x + e) - (4*a*c^2 + 8*a*c*d + 4*a*d
^2 - (a*c^2 - 14*a*c*d + 17*a*d^2)*cos(f*x + e)^2 - 2*(7*a*c^2 - 18*a*c*d +
 7*a*d^2)*cos(f*x + e))*sin(f*x + e))/(cos(f*x + e)^3 + 3*cos(f*x + e)^2 +
(cos(f*x + e)^2 - 2*cos(f*x + e) - 4)*sin(f*x + e) - 2*cos(f*x + e) - 4)) +
 8*((3*c^2 + 4*c*d - 7*d^2)*cos(f*x + e)^2 + 4*c^2 - 8*c*d + 4*d^2 + (7*c^2
 - 4*c*d - 3*d^2)*cos(f*x + e) - (4*c^2 - 8*c*d + 4*d^2 - (3*c^2 + 4*c*d -
7*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x
+ e) + c))/((a^3*c - a^3*d)*f*cos(f*x + e)^3 + 3*(a^3*c - a^3*d)*f*cos(f*x
+ e)^2 - 2*(a^3*c - a^3*d)*f*cos(f*x + e) - 4*(a^3*c - a^3*d)*f + ((a^3*c -
 a^3*d)*f*cos(f*x + e)^2 - 2*(a^3*c - a^3*d)*f*cos(f*x + e) - 4*(a^3*c - a^
3*d)*f)*sin(f*x + e)), -1/64*(3*((c^2 + 2*c*d + d^2)*cos(f*x + e)^3 + 3*(c^
2 + 2*c*d + d^2)*cos(f*x + e)^2 - 4*c^2 - 8*c*d - 4*d^2 - 2*(c^2 + 2*c*d +
d^2)*cos(f*x + e) + ((c^2 + 2*c*d + d^2)*cos(f*x + e)^2 - 4*c^2 - 8*c*d - 4
*d^2 - 2*(c^2 + 2*c*d + d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(-2*a*c + 2*a*
d)*arctan(1/4*sqrt(-2*a*c + 2*a*d)*sqrt(a*sin(f*x + e) + a)*((c - 3*d)*sin(
f*x + e) - 3*c + d)*sqrt(d*sin(f*x + e) + c)/((a*c*d - a*d^2)*cos(f*x + e)*
sin(f*x + e) + (a*c^2 - a*c*d)*cos(f*x + e))) - 4*((3*c^2 + 4*c*d - 7*d^2)*
cos(f*x + e)^2 + 4*c^2 - 8*c*d + 4*d^2 + (7*c^2 - 4*c*d - 3*d^2)*cos(f*x +
e) - (4*c^2 - 8*c*d + 4*d^2 - (3*c^2 + 4*c*d - 7*d^2)*cos(f*x + e))*sin(f*x
 + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/((a^3*c - a^3*d)*
f*cos(f*x + e)^3 + 3*(a^3*c - a^3*d)*f*cos(f*x + e)^2 - 2*(a^3*c - a^3*d)*f
*cos(f*x + e) - 4*(a^3*c - a^3*d)*f + ((a^3*c - a^3*d)*f*cos(f*x + e)^2 - 2
*(a^3*c - a^3*d)*f*cos(f*x + e) - 4*(a^3*c - a^3*d)*f)*sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(3/2)/(a+a*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate((d*sin(f*x + e) + c)^(3/2)/(a*sin(f*x + e) + a)^(5/2), x)
```

3.602 $\int \frac{\sqrt{c+d\sin(e+fx)}}{(a+a\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=191

$$-\frac{(3c-5d)(c+d)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{16\sqrt{2}a^{5/2}f(c-d)^{3/2}}-\frac{(3c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{16af(c-d)(a\sin(e+fx)+a)^{3/2}}-\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4f(a\sin(e+fx)+a)^{5/2}}$$

```
[Out] -((3*c - 5*d)*(c + d)*ArcTanh[(Sqrt[a]*Sqrt[c - d]*Cos[e + f*x])/(Sqrt[2]*S
qrt[a + a*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])])/(16*Sqrt[2]*a^(5/2)*(c
- d)^(3/2)*f) - (Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(4*f*(a + a*Sin[e +
 f*x])^(5/2)) - ((3*c - d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(16*a*(c
- d)*f*(a + a*Sin[e + f*x])^(3/2))
```

Rubi [A] time = 0.488392, antiderivative size = 191, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2764, 2978, 12, 2782, 208}

$$-\frac{(3c-5d)(c+d)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{16\sqrt{2}a^{5/2}f(c-d)^{3/2}}-\frac{(3c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{16af(c-d)(a\sin(e+fx)+a)^{3/2}}-\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4f(a\sin(e+fx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*Sin[e + f*x]]/(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] -((3*c - 5*d)*(c + d)*ArcTanh[(Sqrt[a]*Sqrt[c - d]*Cos[e + f*x])/(Sqrt[2]*S
qrt[a + a*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])])/(16*Sqrt[2]*a^(5/2)*(c
- d)^(3/2)*f) - (Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(4*f*(a + a*Sin[e +
 f*x])^(5/2)) - ((3*c - d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(16*a*(c
- d)*f*(a + a*Sin[e + f*x])^(3/2))
```

Rule 2764

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m
*(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a*d*n - b*c*
(m + 1) - b*d*(m + n + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && L
tQ[m, -1] && LtQ[0, n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c,
 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{c+d\sin(e+fx)}}{(a+a\sin(e+fx))^{5/2}}\,dx = -\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4f(a+a\sin(e+fx))^{5/2}} + \frac{\int \frac{\frac{1}{2}a(3c+d)+ad\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}\,dx}{4a^2}$$

$$= -\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4f(a+a\sin(e+fx))^{5/2}} - \frac{(3c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{16a(c-d)f(a+a\sin(e+fx))^{3/2}} - \frac{\int -\frac{\ldots}{4\sqrt{a+a\,\mathrm{s}\ldots}}}{\ldots}$$

$$= -\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4f(a+a\sin(e+fx))^{5/2}} - \frac{(3c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{16a(c-d)f(a+a\sin(e+fx))^{3/2}} + \frac{((3c-5d)(\ldots}{\ldots}$$

$$= -\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4f(a+a\sin(e+fx))^{5/2}} - \frac{(3c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{16a(c-d)f(a+a\sin(e+fx))^{3/2}} - \frac{((3c-5d)(\ldots}{\ldots}$$

$$= -\frac{(3c-5d)(c+d)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{16\sqrt{2}a^{5/2}(c-d)^{3/2}f} - \frac{\cos(e+fx)\sqrt{c+d\sin(e+f\ldots}}{4f(a+a\sin(e+fx))^{5/2}}$$

Mathematica [B] time = 6.80194, size = 412, normalized size = 2.16

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^4 \left(\frac{(3c^2-2cd-5d^2)\left(\log\left(\tan\left(\frac{1}{2}(e+fx)\right)+1\right)-\log\left((d-c)\tan\left(\frac{1}{2}(e+fx)\right)+2\sqrt{c-d}\sqrt{\frac{1}{\cos(e+fx)+1}}\sqrt{c+d\sin(e+fx)}+\ldots\right.\right.}{\frac{\sec^2\left(\frac{1}{2}(e+fx)\right)}{2\tan\left(\frac{1}{2}(e+fx)\right)+2}-\frac{\frac{\sqrt{c-d}\left(\frac{1}{\cos(e+fx)+1}\right)^{3/2}(c\sin(e+fx)+d\cos(e+fx)+d)}{\sqrt{c+d\sin(e+fx)}}-\frac{1}{2}(c-d)\sec^2\left(\frac{1}{2}(e+fx)\right)}{(d-c)\tan\left(\frac{1}{2}(e+fx)\right)+2\sqrt{c-d}\sqrt{\frac{1}{\cos(e+fx)+1}}\sqrt{c+d\sin(e+fx)}+c-d}}\ldots\right.}{32f(c-d)(a(\sin(e+fx)+1))^{5/2}\sqrt{c+d\sin(e+fx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c + d*Sin[e + f*x]]/(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*((-2*(Cos[(e + f*x)/2] - Sin[(e +
f*x)/2])*(7*c - 5*d + (3*c - d)*Sin[e + f*x])*(c + d*Sin[e + f*x]))/(Cos[(e
 + f*x)/2] + Sin[(e + f*x)/2])^3 + ((3*c^2 - 2*c*d - 5*d^2)*(Log[1 + Tan[(e
 + f*x)/2]] - Log[c - d + 2*Sqrt[c - d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[
c + d*Sin[e + f*x]] + (-c + d)*Tan[(e + f*x)/2]]))/(Sec[(e + f*x)/2]^2/(2 +
```

```
 2*Tan[(e + f*x)/2]) - (-((c - d)*Sec[(e + f*x)/2]^2)/2 + (Sqrt[c - d]*((1
+ Cos[e + f*x])^(-1))^(3/2)*(d + d*Cos[e + f*x] + c*Sin[e + f*x]))/Sqrt[c +
 d*Sin[e + f*x]])/(c - d + 2*Sqrt[c - d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt
[c + d*Sin[e + f*x]] + (-c + d)*Tan[(e + f*x)/2]))))/(32*(c - d)*f*(a*(1 +
Sin[e + f*x]))^(5/2)*Sqrt[c + d*Sin[e + f*x]])
```

Maple [B] time = 0.234, size = 3050, normalized size = 16.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(5/2),x)
```

```
[Out] 1/64/f/(c-d)^2*(-12*cos(f*x+e)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*c^2-
4*sin(f*x+e)*cos(f*x+e)*(2*c-2*d)^(1/2)*2^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/
2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*
x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*c*d-2*sin(f*
x+e)*cos(f*x+e)^2*(2*c-2*d)^(1/2)*2^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c
+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c
*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*c*d-5*sin(f*x+e)*c
os(f*x+e)^2*(2*c-2*d)^(1/2)*2^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin
(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f
*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*d^2-4*cos(f*x+e)*((c+d*s
in(f*x+e))/(cos(f*x+e)+1))^(1/2)*d^2+12*cos(f*x+e)^3*((c+d*sin(f*x+e))/(cos
(f*x+e)+1))^(1/2)*c^2+4*cos(f*x+e)^3*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2
)*d^2+20*(2*c-2*d)^(1/2)*2^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*
x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+
e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*d^2+3*sin(f*x+e)*cos(f*x+e)
^2*(2*c-2*d)^(1/2)*2^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/
(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*c
os(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*c^2+16*cos(f*x+e)*((c+d*sin(f*x+e
))/(cos(f*x+e)+1))^(1/2)*c*d-16*cos(f*x+e)^3*((c+d*sin(f*x+e))/(cos(f*x+e)+
1))^(1/2)*c*d-28*sin(f*x+e)*cos(f*x+e)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1
/2)*c^2-20*sin(f*x+e)*cos(f*x+e)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*d^
2-12*(2*c-2*d)^(1/2)*2^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e)
)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d
*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*c^2+9*cos(f*x+e)^2*(2*c-2*d)^(1
/2)*2^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))
^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/
(1-cos(f*x+e)+sin(f*x+e)))*c^2-15*cos(f*x+e)^2*(2*c-2*d)^(1/2)*2^(1/2)*ln(2
*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e
)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+si
n(f*x+e)))*d^2-12*sin(f*x+e)*(2*c-2*d)^(1/2)*2^(1/2)*ln(2*((2*c-2*d)^(1/2)*
2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*s
in(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*c^2+20*
sin(f*x+e)*(2*c-2*d)^(1/2)*2^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(
f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*
x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*d^2+6*cos(f*x+e)*(2*c-2*d
)^(1/2)*2^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)
+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c
+d)/(1-cos(f*x+e)+sin(f*x+e)))*c^2-10*cos(f*x+e)*(2*c-2*d)^(1/2)*2^(1/2)*ln
(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x
+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+
sin(f*x+e)))*d^2+48*sin(f*x+e)*cos(f*x+e)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))
^(1/2)*c*d+8*(2*c-2*d)^(1/2)*2^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*si
n(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(
```

```
f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*c*d-3*cos(f*x+e)^3*(2*c
-2*d)^(1/2)*2^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*
x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+
e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*c^2+5*cos(f*x+e)^3*(2*c-2*d)^(1/2)*2^(1/
2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*si
n(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*
x+e)+sin(f*x+e)))*d^2+2*cos(f*x+e)^3*(2*c-2*d)^(1/2)*2^(1/2)*ln(2*((2*c-2*d
)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*
x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))
*c*d+6*sin(f*x+e)*cos(f*x+e)*(2*c-2*d)^(1/2)*2^(1/2)*ln(2*((2*c-2*d)^(1/2)*
2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*s
in(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*c^2-10*
sin(f*x+e)*cos(f*x+e)*(2*c-2*d)^(1/2)*2^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)
*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+
e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*d^2-6*cos(f*x+
e)^2*(2*c-2*d)^(1/2)*2^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e)
)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d
*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*c*d+8*sin(f*x+e)*(2*c-2*d)^(1/2
)*2^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(
1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1
-cos(f*x+e)+sin(f*x+e)))*c*d-4*cos(f*x+e)*(2*c-2*d)^(1/2)*2^(1/2)*ln(2*((2*
c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*s
in(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x
+e)))*c*d)*(c+d*sin(f*x+e))^(1/2)/(a*(1+sin(f*x+e)))^(5/2)/sin(f*x+e)/((c+d
*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d\sin\left(fx+e\right)+c}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(d*sin(f*x + e) + c)/(a*sin(f*x + e) + a)^(5/2), x)
```

Fricas [B] time = 3.20349, size = 3490, normalized size = 18.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] [1/128*(((3*c^2 - 2*c*d - 5*d^2)*cos(f*x + e)^3 + 3*(3*c^2 - 2*c*d - 5*d^2)
*cos(f*x + e)^2 - 12*c^2 + 8*c*d + 20*d^2 - 2*(3*c^2 - 2*c*d - 5*d^2)*cos(f
*x + e) + ((3*c^2 - 2*c*d - 5*d^2)*cos(f*x + e)^2 - 12*c^2 + 8*c*d + 20*d^2
 - 2*(3*c^2 - 2*c*d - 5*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(2*a*c - 2*a*d
)*log(((a*c^2 - 14*a*c*d + 17*a*d^2)*cos(f*x + e)^3 - 4*a*c^2 - 8*a*c*d - 4
*a*d^2 - (13*a*c^2 - 22*a*c*d - 3*a*d^2)*cos(f*x + e)^2 - 4*((c - 3*d)*cos(
```

```
f*x + e)^2 - (3*c - d)*cos(f*x + e) + ((c - 3*d)*cos(f*x + e) + 4*c - 4*d)*
sin(f*x + e) - 4*c + 4*d)*sqrt(2*a*c - 2*a*d)*sqrt(a*sin(f*x + e) + a)*sqrt
(d*sin(f*x + e) + c) - 2*(9*a*c^2 - 14*a*c*d + 9*a*d^2)*cos(f*x + e) - (4*a
*c^2 + 8*a*c*d + 4*a*d^2 - (a*c^2 - 14*a*c*d + 17*a*d^2)*cos(f*x + e)^2 - 2
*(7*a*c^2 - 18*a*c*d + 7*a*d^2)*cos(f*x + e))*sin(f*x + e))/(cos(f*x + e)^3
 + 3*cos(f*x + e)^2 + (cos(f*x + e)^2 - 2*cos(f*x + e) - 4)*sin(f*x + e) -
2*cos(f*x + e) - 4)) + 8*((3*c^2 - 4*c*d + d^2)*cos(f*x + e)^2 + 4*c^2 - 8*
c*d + 4*d^2 + (7*c^2 - 12*c*d + 5*d^2)*cos(f*x + e) - (4*c^2 - 8*c*d + 4*d^
2 - (3*c^2 - 4*c*d + d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) +
 a)*sqrt(d*sin(f*x + e) + c))/((a^3*c^2 - 2*a^3*c*d + a^3*d^2)*f*cos(f*x +
e)^3 + 3*(a^3*c^2 - 2*a^3*c*d + a^3*d^2)*f*cos(f*x + e)^2 - 2*(a^3*c^2 - 2*
a^3*c*d + a^3*d^2)*f*cos(f*x + e) - 4*(a^3*c^2 - 2*a^3*c*d + a^3*d^2)*f + (
(a^3*c^2 - 2*a^3*c*d + a^3*d^2)*f*cos(f*x + e)^2 - 2*(a^3*c^2 - 2*a^3*c*d +
 a^3*d^2)*f*cos(f*x + e) - 4*(a^3*c^2 - 2*a^3*c*d + a^3*d^2)*f)*sin(f*x + e
)), -1/64*(((3*c^2 - 2*c*d - 5*d^2)*cos(f*x + e)^3 + 3*(3*c^2 - 2*c*d - 5*d
^2)*cos(f*x + e)^2 - 12*c^2 + 8*c*d + 20*d^2 - 2*(3*c^2 - 2*c*d - 5*d^2)*co
s(f*x + e) + ((3*c^2 - 2*c*d - 5*d^2)*cos(f*x + e)^2 - 12*c^2 + 8*c*d + 20*
d^2 - 2*(3*c^2 - 2*c*d - 5*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(-2*a*c + 2
*a*d)*arctan(1/4*sqrt(-2*a*c + 2*a*d)*sqrt(a*sin(f*x + e) + a)*((c - 3*d)*s
in(f*x + e) - 3*c + d)*sqrt(d*sin(f*x + e) + c)/((a*c*d - a*d^2)*cos(f*x +
e)*sin(f*x + e) + (a*c^2 - a*c*d)*cos(f*x + e))) - 4*((3*c^2 - 4*c*d + d^2)
*cos(f*x + e)^2 + 4*c^2 - 8*c*d + 4*d^2 + (7*c^2 - 12*c*d + 5*d^2)*cos(f*x
+ e) - (4*c^2 - 8*c*d + 4*d^2 - (3*c^2 - 4*c*d + d^2)*cos(f*x + e))*sin(f*x
 + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/((a^3*c^2 - 2*a^3
*c*d + a^3*d^2)*f*cos(f*x + e)^3 + 3*(a^3*c^2 - 2*a^3*c*d + a^3*d^2)*f*cos(
f*x + e)^2 - 2*(a^3*c^2 - 2*a^3*c*d + a^3*d^2)*f*cos(f*x + e) - 4*(a^3*c^2
- 2*a^3*c*d + a^3*d^2)*f + ((a^3*c^2 - 2*a^3*c*d + a^3*d^2)*f*cos(f*x + e)^
2 - 2*(a^3*c^2 - 2*a^3*c*d + a^3*d^2)*f*cos(f*x + e) - 4*(a^3*c^2 - 2*a^3*c
*d + a^3*d^2)*f)*sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(1/2)/(a+a*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d \sin \left(fx+e\right)+c}}{\left(a \sin \left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(d*sin(f*x + e) + c)/(a*sin(f*x + e) + a)^(5/2), x)
```

3.603 $\int \frac{1}{(a+a\sin(e+fx))^{5/2}\sqrt{c+d\sin(e+fx)}}\,dx$

Optimal. Leaf size=201

$$-\frac{\left(3c^2-10cd+19d^2\right)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{16\sqrt{2}a^{5/2}f(c-d)^{5/2}}-\frac{3(c-3d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{16af(c-d)^2(a\sin(e+fx)+a)^{3/2}}-\frac{\cos(e+fx)\sqrt{}}{4f(c-d)(a\,s}$$

```
[Out] -((3*c^2 - 10*c*d + 19*d^2)*ArcTanh[(Sqrt[a]*Sqrt[c - d]*Cos[e + f*x])/(Sqr
t[2]*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])])/(16*Sqrt[2]*a^(5/
2)*(c - d)^(5/2)*f) - (Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(4*(c - d)*f*
(a + a*Sin[e + f*x])^(5/2)) - (3*(c - 3*d)*Cos[e + f*x]*Sqrt[c + d*Sin[e +
f*x]])/(16*a*(c - d)^2*f*(a + a*Sin[e + f*x])^(3/2))
```

Rubi [A] time = 0.494197, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2766, 2978, 12, 2782, 208}

$$-\frac{\left(3c^2-10cd+19d^2\right)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{16\sqrt{2}a^{5/2}f(c-d)^{5/2}}-\frac{3(c-3d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{16af(c-d)^2(a\sin(e+fx)+a)^{3/2}}-\frac{\cos(e+fx)\sqrt{}}{4f(c-d)(a\,s}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^(5/2)*Sqrt[c + d*Sin[e + f*x]]),x]
```

```
[Out] -((3*c^2 - 10*c*d + 19*d^2)*ArcTanh[(Sqrt[a]*Sqrt[c - d]*Cos[e + f*x])/(Sqr
t[2]*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])])/(16*Sqrt[2]*a^(5/
2)*(c - d)^(5/2)*f) - (Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(4*(c - d)*f*
(a + a*Sin[e + f*x])^(5/2)) - (3*(c - 3*d)*Cos[e + f*x]*Sqrt[c + d*Sin[e +
f*x]])/(16*a*(c - d)^2*f*(a + a*Sin[e + f*x])^(3/2))
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^{5/2}\sqrt{c+d\sin(e+fx)}}\,dx = -\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4(c-d)f(a+a\sin(e+fx))^{5/2}} - \frac{\int \frac{-\frac{1}{2}a(3c-7d)-ad\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}\,dx}{4a^2(c-d)}$$

$$= -\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4(c-d)f(a+a\sin(e+fx))^{5/2}} - \frac{3(c-3d)\cos(e+fx)\sqrt{c+d\sin(e+}}{16a(c-d)^2f(a+a\sin(e+fx))^{3/2}}$$

$$= -\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4(c-d)f(a+a\sin(e+fx))^{5/2}} - \frac{3(c-3d)\cos(e+fx)\sqrt{c+d\sin(e+}}{16a(c-d)^2f(a+a\sin(e+fx))^{3/2}}$$

$$= -\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4(c-d)f(a+a\sin(e+fx))^{5/2}} - \frac{3(c-3d)\cos(e+fx)\sqrt{c+d\sin(e+}}{16a(c-d)^2f(a+a\sin(e+fx))^{3/2}}$$

$$= -\frac{\left(3c^2-10cd+19d^2\right)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{16\sqrt{2}a^{5/2}(c-d)^{5/2}f} - \frac{\cos(e+f}{4(c-d)f}$$

Mathematica [B] time = 6.55716, size = 411, normalized size = 2.04

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^4\left(\frac{\left(3c^2-10cd+19d^2\right)\left(\log\left(\tan\left(\frac{1}{2}(e+fx)\right)+1\right)-\log\left((d-c)\tan\left(\frac{1}{2}(e+fx)\right)+2\sqrt{c-d}\sqrt{\frac{1}{\cos(e+fx)+1}}\sqrt{c+d\sin(e+fx)}+c\right.\right.}{\frac{\sec^2\left(\frac{1}{2}(e+fx)\right)}{2\tan\left(\frac{1}{2}(e+fx)\right)+2}-\frac{\frac{\sqrt{c-d}\left(\frac{1}{\cos(e+fx)+1}\right)^{3/2}(c\sin(e+fx)+d\cos(e+fx)+d)}{\sqrt{c+d\sin(e+fx)}}-\frac{1}{2}(c-d)\sec^2\left(\frac{1}{2}(e+fx)\right)}{(d-c)\tan\left(\frac{1}{2}(e+fx)\right)+2\sqrt{c-d}\sqrt{\frac{1}{\cos(e+fx)+1}}\sqrt{c+d\sin(e+fx)}+c-d}}\right.}{32f(c-d)^2(a(\sin(e+fx)+1))^{5/2}\sqrt{c+d\sin(e+fx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^(5/2)*Sqrt[c + d*Sin[e + f*x]]),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*((-2*(Cos[(e + f*x)/2] - Sin[(e +
f*x)/2])*(7*c - 13*d + 3*(c - 3*d)*Sin[e + f*x])*(c + d*Sin[e + f*x]))/(Cos
[(e + f*x)/2] + Sin[(e + f*x)/2])^3 + ((3*c^2 - 10*c*d + 19*d^2)*(Log[1 + T
an[(e + f*x)/2]] - Log[c - d + 2*Sqrt[c - d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*
```

```
Sqrt[c + d*Sin[e + f*x]] + (-c + d)*Tan[(e + f*x)/2]]))/(Sec[(e + f*x)/2]^2
/(2 + 2*Tan[(e + f*x)/2]) - (-((c - d)*Sec[(e + f*x)/2]^2)/2 + (Sqrt[c - d]
*((1 + Cos[e + f*x])^(-1))^(3/2)*(d + d*Cos[e + f*x] + c*Sin[e + f*x]))/Sqr
t[c + d*Sin[e + f*x]])/(c - d + 2*Sqrt[c - d]*Sqrt[(1 + Cos[e + f*x])^(-1)]
*Sqrt[c + d*Sin[e + f*x]] + (-c + d)*Tan[(e + f*x)/2]]))))/(32*(c - d)^2*f*(
a*(1 + Sin[e + f*x]))^(5/2)*Sqrt[c + d*Sin[e + f*x]])
```

Maple [B] time = 0.273, size = 2805, normalized size = 14.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(1/2),x)
```

```
[Out] 1/32/f/(2*c-2*d)^(1/2)/(c-d)^2*(-20*sin(f*x+e)*cos(f*x+e)*2^(1/2)*ln(2*((2*
c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*s
in(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x
+e)))*c*d-6*cos(f*x+e)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*(2*c-2*d)^(1
/2)*c+18*cos(f*x+e)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*(2*c-2*d)^(1/2)
*d-3*cos(f*x+e)^3*2^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(
cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*co
s(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*c^2-19*cos(f*x+e)^3*2^(1/2)*ln(2*(
(2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+
c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(
f*x+e)))*d^2+9*cos(f*x+e)^2*2^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin
(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f
*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*c^2+57*cos(f*x+e)^2*2^(1
/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*s
in(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f
*x+e)+sin(f*x+e)))*d^2-12*sin(f*x+e)*2^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*
((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e
)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*c^2-76*sin(f*x+
e)*2^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^
(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(
1-cos(f*x+e)+sin(f*x+e)))*d^2+6*cos(f*x+e)*2^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^
(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin
(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*c^2+38*co
s(f*x+e)*2^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e
)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-
c+d)/(1-cos(f*x+e)+sin(f*x+e)))*d^2+6*cos(f*x+e)^3*((c+d*sin(f*x+e))/(cos(f
*x+e)+1))^(1/2)*(2*c-2*d)^(1/2)*c-18*cos(f*x+e)^3*((c+d*sin(f*x+e))/(cos(f*
x+e)+1))^(1/2)*(2*c-2*d)^(1/2)*d+40*2^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*(
(c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)
+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*c*d-10*sin(f*x+e
)*cos(f*x+e)^2*2^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos
(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f
*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*c*d-12*2^(1/2)*ln(2*((2*c-2*d)^(1/2)*
2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*s
in(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*c^2-76*
2^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/
2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-c
os(f*x+e)+sin(f*x+e)))*d^2-30*cos(f*x+e)^2*2^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^
(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin
(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*c*d-14*si
n(f*x+e)*cos(f*x+e)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*(2*c-2*d)^(1/2)
*c+26*sin(f*x+e)*cos(f*x+e)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*(2*c-2*
```

```
d)^(1/2)*d+40*sin(f*x+e)*2^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*
x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+
e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*c*d-20*cos(f*x+e)*2^(1/2)*l
n(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*
x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)
+sin(f*x+e)))*c*d+3*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*ln(2*((2*c-2*d)^(1/2)*2
^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*si
n(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*c^2+19*s
in(f*x+e)*cos(f*x+e)^2*2^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+
e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)
-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*d^2+10*cos(f*x+e)^3*2^(1/2)*l
n(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*
x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)
+sin(f*x+e)))*c*d+6*sin(f*x+e)*cos(f*x+e)*2^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(
1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(
f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*c^2+38*sin
(f*x+e)*cos(f*x+e)*2^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/
(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*c
os(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*d^2)*(c+d*sin(f*x+e))^(1/2)/(a*(1
+sin(f*x+e)))^(5/2)/sin(f*x+e)/((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\sqrt{d\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)^(5/2)*sqrt(d*sin(f*x + e) + c)), x)
```

Fricas [B] time = 4.5489, size = 3838, normalized size = 19.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="fri
cas")
```

```
[Out] [1/128*(((3*c^2 - 10*c*d + 19*d^2)*cos(f*x + e)^3 + 3*(3*c^2 - 10*c*d + 19*
d^2)*cos(f*x + e)^2 - 12*c^2 + 40*c*d - 76*d^2 - 2*(3*c^2 - 10*c*d + 19*d^2
)*cos(f*x + e) + ((3*c^2 - 10*c*d + 19*d^2)*cos(f*x + e)^2 - 12*c^2 + 40*c*
d - 76*d^2 - 2*(3*c^2 - 10*c*d + 19*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(2
*a*c - 2*a*d)*log(((a*c^2 - 14*a*c*d + 17*a*d^2)*cos(f*x + e)^3 - 4*a*c^2 -
 8*a*c*d - 4*a*d^2 - (13*a*c^2 - 22*a*c*d - 3*a*d^2)*cos(f*x + e)^2 - 4*((c
 - 3*d)*cos(f*x + e)^2 - (3*c - d)*cos(f*x + e) + ((c - 3*d)*cos(f*x + e) +
 4*c - 4*d)*sin(f*x + e) - 4*c + 4*d)*sqrt(2*a*c - 2*a*d)*sqrt(a*sin(f*x +
e) + a)*sqrt(d*sin(f*x + e) + c) - 2*(9*a*c^2 - 14*a*c*d + 9*a*d^2)*cos(f*x
 + e) - (4*a*c^2 + 8*a*c*d + 4*a*d^2 - (a*c^2 - 14*a*c*d + 17*a*d^2)*cos(f*
x + e)^2 - 2*(7*a*c^2 - 18*a*c*d + 7*a*d^2)*cos(f*x + e))*sin(f*x + e))/(co
s(f*x + e)^3 + 3*cos(f*x + e)^2 + (cos(f*x + e)^2 - 2*cos(f*x + e) - 4)*sin
```

```
(f*x + e) - 2*cos(f*x + e) - 4)) + 8*(3*(c^2 - 4*c*d + 3*d^2)*cos(f*x + e)^
2 + 4*c^2 - 8*c*d + 4*d^2 + (7*c^2 - 20*c*d + 13*d^2)*cos(f*x + e) - (4*c^2
 - 8*c*d + 4*d^2 - 3*(c^2 - 4*c*d + 3*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt
(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/((a^3*c^3 - 3*a^3*c^2*d + 3*
a^3*c*d^2 - a^3*d^3)*f*cos(f*x + e)^3 + 3*(a^3*c^3 - 3*a^3*c^2*d + 3*a^3*c*
d^2 - a^3*d^3)*f*cos(f*x + e)^2 - 2*(a^3*c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2 -
a^3*d^3)*f*cos(f*x + e) - 4*(a^3*c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2 - a^3*d^3)
*f + ((a^3*c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2 - a^3*d^3)*f*cos(f*x + e)^2 - 2*
(a^3*c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2 - a^3*d^3)*f*cos(f*x + e) - 4*(a^3*c^3
 - 3*a^3*c^2*d + 3*a^3*c*d^2 - a^3*d^3)*f)*sin(f*x + e)), -1/64*(((3*c^2 -
10*c*d + 19*d^2)*cos(f*x + e)^3 + 3*(3*c^2 - 10*c*d + 19*d^2)*cos(f*x + e)^
2 - 12*c^2 + 40*c*d - 76*d^2 - 2*(3*c^2 - 10*c*d + 19*d^2)*cos(f*x + e) + (
(3*c^2 - 10*c*d + 19*d^2)*cos(f*x + e)^2 - 12*c^2 + 40*c*d - 76*d^2 - 2*(3*
c^2 - 10*c*d + 19*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(-2*a*c + 2*a*d)*arc
tan(1/4*sqrt(-2*a*c + 2*a*d)*sqrt(a*sin(f*x + e) + a)*((c - 3*d)*sin(f*x +
e) - 3*c + d)*sqrt(d*sin(f*x + e) + c)/((a*c*d - a*d^2)*cos(f*x + e)*sin(f*
x + e) + (a*c^2 - a*c*d)*cos(f*x + e))) - 4*(3*(c^2 - 4*c*d + 3*d^2)*cos(f*
x + e)^2 + 4*c^2 - 8*c*d + 4*d^2 + (7*c^2 - 20*c*d + 13*d^2)*cos(f*x + e) -
 (4*c^2 - 8*c*d + 4*d^2 - 3*(c^2 - 4*c*d + 3*d^2)*cos(f*x + e))*sin(f*x + e
))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/((a^3*c^3 - 3*a^3*c^2
*d + 3*a^3*c*d^2 - a^3*d^3)*f*cos(f*x + e)^3 + 3*(a^3*c^3 - 3*a^3*c^2*d + 3
*a^3*c*d^2 - a^3*d^3)*f*cos(f*x + e)^2 - 2*(a^3*c^3 - 3*a^3*c^2*d + 3*a^3*c
*d^2 - a^3*d^3)*f*cos(f*x + e) - 4*(a^3*c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2 - a
^3*d^3)*f + ((a^3*c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2 - a^3*d^3)*f*cos(f*x + e)
^2 - 2*(a^3*c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2 - a^3*d^3)*f*cos(f*x + e) - 4*(
a^3*c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2 - a^3*d^3)*f)*sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**(5/2)/(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\sqrt{d\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)^(5/2)*sqrt(d*sin(f*x + e) + c)), x)
```

3.604 $\int \frac{1}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=270

$$-\frac{3\left(c^2-6cd+25d^2\right)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{16\sqrt{2}a^{5/2}f(c-d)^{7/2}}-\frac{d(c-7d)(3c+7d)\cos(e+fx)}{16a^2f(c-d)^3(c+d)\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}-$$

```
[Out] (-3*(c^2 - 6*c*d + 25*d^2)*ArcTanh[(Sqrt[a]*Sqrt[c - d]*Cos[e + f*x])/(Sqrt
[2]*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])])/(16*Sqrt[2]*a^(5/2
)*(c - d)^(7/2)*f) - Cos[e + f*x]/(4*(c - d)*f*(a + a*Sin[e + f*x])^(5/2)*S
qrt[c + d*Sin[e + f*x]]) - ((3*c - 13*d)*Cos[e + f*x])/(16*a*(c - d)^2*f*(a
 + a*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]) - ((c - 7*d)*d*(3*c + 7*
d)*Cos[e + f*x])/(16*a^2*(c - d)^3*(c + d)*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[
c + d*Sin[e + f*x]])
```

Rubi [A] time = 0.861345, antiderivative size = 270, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2766, 2978, 2984, 12, 2782, 208}

$$-\frac{3\left(c^2-6cd+25d^2\right)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{16\sqrt{2}a^{5/2}f(c-d)^{7/2}}-\frac{d(c-7d)(3c+7d)\cos(e+fx)}{16a^2f(c-d)^3(c+d)\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}-$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])^(3/2)),x]
```


```
[Out] (-3*(c^2 - 6*c*d + 25*d^2)*ArcTanh[(Sqrt[a]*Sqrt[c - d]*Cos[e + f*x])/(Sqrt
[2]*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])])/(16*Sqrt[2]*a^(5/2
)*(c - d)^(7/2)*f) - Cos[e + f*x]/(4*(c - d)*f*(a + a*Sin[e + f*x])^(5/2)*S
qrt[c + d*Sin[e + f*x]]) - ((3*c - 13*d)*Cos[e + f*x])/(16*a*(c - d)^2*f*(a
 + a*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]) - ((c - 7*d)*d*(3*c + 7*
d)*Cos[e + f*x])/(16*a^2*(c - d)^3*(c + d)*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[
c + d*Sin[e + f*x]])
```


Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
```

```
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^{3/2}}\,dx = -\frac{\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}\sqrt{c+d\sin(e+fx)}} - \frac{\int \frac{-\frac{3}{2}a(c-3}{(a+a\sin(e+f}}{4}$$

$$= -\frac{\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}\sqrt{c+d\sin(e+fx)}} - \frac{}{16a(c-d)^2f}$$

$$= -\frac{\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}\sqrt{c+d\sin(e+fx)}} - \frac{}{16a(c-d)^2f}$$

$$= -\frac{\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}\sqrt{c+d\sin(e+fx)}} - \frac{}{16a(c-d)^2f}$$

$$= -\frac{\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}\sqrt{c+d\sin(e+fx)}} - \frac{}{16a(c-d)^2f}$$

$$= -\frac{3\left(c^2-6cd+25d^2\right)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{16\sqrt{2}a^{5/2}(c-d)^{7/2}f} - \frac{}{4(c-}$$

Mathematica [A] time = 8.3941, size = 462, normalized size = 1.71

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^4\left(\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left((14c^2d-6c^3+62cd^2+170d^3)\sin(e+fx)+d(3c^2-14cd-49d^2)\cos(2(e+fx))+25c\right.}{(c+d)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3}\right.}{32f(c-d)^3(a(\sin(e+fx)+1))^{5/}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])^(3/2)),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*(((Cos[(e + f*x)/2] - Sin[(e + f*x
)/2])*(-14*c^3 + 25*c^2*d + 56*c*d^2 + 113*d^3 + d*(3*c^2 - 14*c*d - 49*d^2
)*Cos[2*(e + f*x)] + (-6*c^3 + 14*c^2*d + 62*c*d^2 + 170*d^3)*Sin[e + f*x])
)/((c + d)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3) + (3*(c^2 - 6*c*d + 25*
d^2)*(Log[1 + Tan[(e + f*x)/2]] - Log[c - d + 2*Sqrt[c - d]*Sqrt[(1 + Cos[e
 + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + (-c + d)*Tan[(e + f*x)/2]]))/(Sec
[(e + f*x)/2]^2/(2 + 2*Tan[(e + f*x)/2]) - (-((c - d)*Sec[(e + f*x)/2]^2)/2
 + (Sqrt[c - d]*((1 + Cos[e + f*x])^(-1))^(3/2)*(d + d*Cos[e + f*x] + c*Sin
[e + f*x]))/Sqrt[c + d*Sin[e + f*x]])/(c - d + 2*Sqrt[c - d]*Sqrt[(1 + Cos[
e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + (-c + d)*Tan[(e + f*x)/2]))))/(3
2*(c - d)^3*f*(a*(1 + Sin[e + f*x]))^(5/2)*Sqrt[c + d*Sin[e + f*x]])
```

Maple [B] time = 0.263, size = 4262, normalized size = 15.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(3/2),x)
```

```
[Out] 1/32/f/(c+d)/(2*c-2*d)^(1/2)/(c-d)^3*(162*cos(f*x+e)*(2*c-2*d)^(1/2)*d^3-12
*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+
d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*
cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1/2)*c^3-300*ln(
2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+
e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+s
in(f*x+e)))*d^3*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)+170*cos(f*x
+e)*sin(f*x+e)*(2*c-2*d)^(1/2)*d^3+22*cos(f*x+e)*(2*c-2*d)^(1/2)*c^2*d+70*c
os(f*x+e)*(2*c-2*d)^(1/2)*c*d^2-45*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*
ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f
*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e
)+sin(f*x+e)))*cos(f*x+e)^2*2^(1/2)*c^2*d-14*cos(f*x+e)*(2*c-2*d)^(1/2)*c^3
-98*cos(f*x+e)^3*(2*c-2*d)^(1/2)*d^3+171*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^
(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)
*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos
(f*x+e)+sin(f*x+e)))*cos(f*x+e)^2*2^(1/2)*c*d^2-15*cos(f*x+e)^3*ln(2*((2*c-
2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin
(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e
)))*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*c^2*d+57*cos(f*x+e)^3*l
n(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*
x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)
```

```
+sin(f*x+e)))*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*c*d^2+3*sin(f
*x+e)*cos(f*x+e)^2*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x
+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e
)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))
^(1/2)*c^3+75*sin(f*x+e)*cos(f*x+e)^2*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*s
in(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos
(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1/2)*((c+d*sin(f*x+
e))/(cos(f*x+e)+1))^(1/2)*d^3-6*sin(f*x+e)*cos(f*x+e)*ln(2*((2*c-2*d)^(1/2)
*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*
sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1/2
)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*c^3+6*cos(f*x+e)^3*(2*c-2*d)^(1/2
)*c^2*d-28*cos(f*x+e)^3*(2*c-2*d)^(1/2)*c*d^2-6*sin(f*x+e)*cos(f*x+e)*(2*c-
2*d)^(1/2)*c^3-150*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*ln(2*((2*c-2*d)^
(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+
e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*c
os(f*x+e)*sin(f*x+e)*2^(1/2)*d^3+30*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin
(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f
*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*c^2*d*2^(1/2)*((c+d*sin(
f*x+e))/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)-114*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*
((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e
)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*c*d^2*2^(1/2)*(
(c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)+60*ln(2*((2*c-2*d)^(1/2)*
2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*s
in(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*c^2*d*2
^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)-228*ln(2*((2*c-2*
d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f
*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e))
)*c*d^2*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+30*cos(f
*x+e)*sin(f*x+e)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e
)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-
c+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(
1/2)*c^2*d-114*cos(f*x+e)*sin(f*x+e)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*si
n(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(
f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1/2)*((c+d*sin(f*x+e
))/(cos(f*x+e)+1))^(1/2)*c*d^2+225*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*
ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f
*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e
)+sin(f*x+e)))*cos(f*x+e)^2*2^(1/2)*d^3-6*((c+d*sin(f*x+e))/(cos(f*x+e)+1))
^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2
)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-co
s(f*x+e)+sin(f*x+e)))*cos(f*x+e)*2^(1/2)*c^3-12*ln(2*((2*c-2*d)^(1/2)*2^(1/
2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*
x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*c^3*2^(1/2)*
((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)-300*((c+d*sin(f*x+e))/(c
os(f*x+e)+1))^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*
x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+
e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*sin(f*x+e)*2^(1/2)*d^3+60*((c+d*sin(f*x+
e))/(cos(f*x+e)+1))^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(
cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*co
s(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1/2)*c^2*d-228*((c+d*sin(f*x+e)
)/(cos(f*x+e)+1))^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(co
s(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(
f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1/2)*c*d^2+62*cos(f*x+e)*sin(f*x+
e)*(2*c-2*d)^(1/2)*c*d^2-15*sin(f*x+e)*cos(f*x+e)^2*ln(2*((2*c-2*d)^(1/2)*2
^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*si
n(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1/2)*
((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*c^2*d+57*sin(f*x+e)*cos(f*x+e)^2*ln
(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x
+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+
```

```
sin(f*x+e)))*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*c*d^2+14*sin(f
*x+e)*cos(f*x+e)*(2*c-2*d)^(1/2)*c^2*d+3*cos(f*x+e)^3*ln(2*((2*c-2*d)^(1/2)
*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*
sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1/2
)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*c^3+75*cos(f*x+e)^3*ln(2*((2*c-2*
d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f
*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e))
)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*d^3+9*cos(f*x+e)^2*ln(2*(
(2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+
c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(
f*x+e)))*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*c^3-150*ln(2*((2*c
-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*si
n(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+
e)))*d^3*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*cos(f*x+e))/(c+d*s
in(f*x+e))^(1/2)/(a*(1+sin(f*x+e)))^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="max
ima")
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)^(5/2)*(d*sin(f*x + e) + c)^(3/2)), x)
```

Fricas [B] time = 6.62095, size = 6607, normalized size = 24.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="fri
cas")
```

```
[Out] [-1/128*(3*((c^3*d - 5*c^2*d^2 + 19*c*d^3 + 25*d^4)*cos(f*x + e)^4 + 4*c^4
- 16*c^3*d + 56*c^2*d^2 + 176*c*d^3 + 100*d^4 - (c^4 - 3*c^3*d + 9*c^2*d^2
+ 63*c*d^3 + 50*d^4)*cos(f*x + e)^3 - (3*c^4 - 10*c^3*d + 32*c^2*d^2 + 170*
c*d^3 + 125*d^4)*cos(f*x + e)^2 + 2*(c^4 - 4*c^3*d + 14*c^2*d^2 + 44*c*d^3
+ 25*d^4)*cos(f*x + e) + (4*c^4 - 16*c^3*d + 56*c^2*d^2 + 176*c*d^3 + 100*d
^4 - (c^3*d - 5*c^2*d^2 + 19*c*d^3 + 25*d^4)*cos(f*x + e)^3 - (c^4 - 2*c^3*
d + 4*c^2*d^2 + 82*c*d^3 + 75*d^4)*cos(f*x + e)^2 + 2*(c^4 - 4*c^3*d + 14*c
^2*d^2 + 44*c*d^3 + 25*d^4)*cos(f*x + e))*sin(f*x + e))*sqrt(2*a*c - 2*a*d)
*log(((a*c^2 - 14*a*c*d + 17*a*d^2)*cos(f*x + e)^3 - 4*a*c^2 - 8*a*c*d - 4*
a*d^2 - (13*a*c^2 - 22*a*c*d - 3*a*d^2)*cos(f*x + e)^2 + 4*((c - 3*d)*cos(f
*x + e)^2 - (3*c - d)*cos(f*x + e) + ((c - 3*d)*cos(f*x + e) + 4*c - 4*d)*s
in(f*x + e) - 4*c + 4*d)*sqrt(2*a*c - 2*a*d)*sqrt(a*sin(f*x + e) + a)*sqrt(
d*sin(f*x + e) + c) - 2*(9*a*c^2 - 14*a*c*d + 9*a*d^2)*cos(f*x + e) - (4*a*
c^2 + 8*a*c*d + 4*a*d^2 - (a*c^2 - 14*a*c*d + 17*a*d^2)*cos(f*x + e)^2 - 2*
(7*a*c^2 - 18*a*c*d + 7*a*d^2)*cos(f*x + e))*sin(f*x + e))/(cos(f*x + e)^3
+ 3*cos(f*x + e)^2 + (cos(f*x + e)^2 - 2*cos(f*x + e) - 4)*sin(f*x + e) - 2
*cos(f*x + e) - 4)) + 8*(4*c^4 - 8*c^3*d + 8*c*d^3 - 4*d^4 - (3*c^3*d - 17*
```

```
c^2*d^2 - 35*c*d^3 + 49*d^4)*cos(f*x + e)^3 + (3*c^4 - 13*c^3*d - 7*c^2*d^2
 - 19*c*d^3 + 36*d^4)*cos(f*x + e)^2 + (7*c^4 - 18*c^3*d - 24*c^2*d^2 - 46*
c*d^3 + 81*d^4)*cos(f*x + e) - (4*c^4 - 8*c^3*d + 8*c*d^3 - 4*d^4 - (3*c^3*
d - 17*c^2*d^2 - 35*c*d^3 + 49*d^4)*cos(f*x + e)^2 - (3*c^4 - 10*c^3*d - 24
*c^2*d^2 - 54*c*d^3 + 85*d^4)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x +
e) + a)*sqrt(d*sin(f*x + e) + c))/((a^3*c^5*d - 3*a^3*c^4*d^2 + 2*a^3*c^3*d
^3 + 2*a^3*c^2*d^4 - 3*a^3*c*d^5 + a^3*d^6)*f*cos(f*x + e)^4 - (a^3*c^6 - a
^3*c^5*d - 4*a^3*c^4*d^2 + 6*a^3*c^3*d^3 + a^3*c^2*d^4 - 5*a^3*c*d^5 + 2*a^
3*d^6)*f*cos(f*x + e)^3 - (3*a^3*c^6 - 4*a^3*c^5*d - 9*a^3*c^4*d^2 + 16*a^3
*c^3*d^3 + a^3*c^2*d^4 - 12*a^3*c*d^5 + 5*a^3*d^6)*f*cos(f*x + e)^2 + 2*(a^
3*c^6 - 2*a^3*c^5*d - a^3*c^4*d^2 + 4*a^3*c^3*d^3 - a^3*c^2*d^4 - 2*a^3*c*d
^5 + a^3*d^6)*f*cos(f*x + e) + 4*(a^3*c^6 - 2*a^3*c^5*d - a^3*c^4*d^2 + 4*a
^3*c^3*d^3 - a^3*c^2*d^4 - 2*a^3*c*d^5 + a^3*d^6)*f - ((a^3*c^5*d - 3*a^3*c
^4*d^2 + 2*a^3*c^3*d^3 + 2*a^3*c^2*d^4 - 3*a^3*c*d^5 + a^3*d^6)*f*cos(f*x +
 e)^3 + (a^3*c^6 - 7*a^3*c^4*d^2 + 8*a^3*c^3*d^3 + 3*a^3*c^2*d^4 - 8*a^3*c*
d^5 + 3*a^3*d^6)*f*cos(f*x + e)^2 - 2*(a^3*c^6 - 2*a^3*c^5*d - a^3*c^4*d^2
+ 4*a^3*c^3*d^3 - a^3*c^2*d^4 - 2*a^3*c*d^5 + a^3*d^6)*f*cos(f*x + e) - 4*(
a^3*c^6 - 2*a^3*c^5*d - a^3*c^4*d^2 + 4*a^3*c^3*d^3 - a^3*c^2*d^4 - 2*a^3*c
*d^5 + a^3*d^6)*f)*sin(f*x + e)), -1/64*(3*((c^3*d - 5*c^2*d^2 + 19*c*d^3 +
 25*d^4)*cos(f*x + e)^4 + 4*c^4 - 16*c^3*d + 56*c^2*d^2 + 176*c*d^3 + 100*d
^4 - (c^4 - 3*c^3*d + 9*c^2*d^2 + 63*c*d^3 + 50*d^4)*cos(f*x + e)^3 - (3*c^
4 - 10*c^3*d + 32*c^2*d^2 + 170*c*d^3 + 125*d^4)*cos(f*x + e)^2 + 2*(c^4 -
4*c^3*d + 14*c^2*d^2 + 44*c*d^3 + 25*d^4)*cos(f*x + e) + (4*c^4 - 16*c^3*d
+ 56*c^2*d^2 + 176*c*d^3 + 100*d^4 - (c^3*d - 5*c^2*d^2 + 19*c*d^3 + 25*d^4
)*cos(f*x + e)^3 - (c^4 - 2*c^3*d + 4*c^2*d^2 + 82*c*d^3 + 75*d^4)*cos(f*x
+ e)^2 + 2*(c^4 - 4*c^3*d + 14*c^2*d^2 + 44*c*d^3 + 25*d^4)*cos(f*x + e))*s
in(f*x + e))*sqrt(-2*a*c + 2*a*d)*arctan(1/4*sqrt(-2*a*c + 2*a*d)*sqrt(a*si
n(f*x + e) + a)*((c - 3*d)*sin(f*x + e) - 3*c + d)*sqrt(d*sin(f*x + e) + c)
/((a*c*d - a*d^2)*cos(f*x + e)*sin(f*x + e) + (a*c^2 - a*c*d)*cos(f*x + e))
) + 4*(4*c^4 - 8*c^3*d + 8*c*d^3 - 4*d^4 - (3*c^3*d - 17*c^2*d^2 - 35*c*d^3
 + 49*d^4)*cos(f*x + e)^3 + (3*c^4 - 13*c^3*d - 7*c^2*d^2 - 19*c*d^3 + 36*d
^4)*cos(f*x + e)^2 + (7*c^4 - 18*c^3*d - 24*c^2*d^2 - 46*c*d^3 + 81*d^4)*co
s(f*x + e) - (4*c^4 - 8*c^3*d + 8*c*d^3 - 4*d^4 - (3*c^3*d - 17*c^2*d^2 - 3
5*c*d^3 + 49*d^4)*cos(f*x + e)^2 - (3*c^4 - 10*c^3*d - 24*c^2*d^2 - 54*c*d^
3 + 85*d^4)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin
(f*x + e) + c))/((a^3*c^5*d - 3*a^3*c^4*d^2 + 2*a^3*c^3*d^3 + 2*a^3*c^2*d^4
 - 3*a^3*c*d^5 + a^3*d^6)*f*cos(f*x + e)^4 - (a^3*c^6 - a^3*c^5*d - 4*a^3*c
^4*d^2 + 6*a^3*c^3*d^3 + a^3*c^2*d^4 - 5*a^3*c*d^5 + 2*a^3*d^6)*f*cos(f*x +
 e)^3 - (3*a^3*c^6 - 4*a^3*c^5*d - 9*a^3*c^4*d^2 + 16*a^3*c^3*d^3 + a^3*c^2
*d^4 - 12*a^3*c*d^5 + 5*a^3*d^6)*f*cos(f*x + e)^2 + 2*(a^3*c^6 - 2*a^3*c^5*
d - a^3*c^4*d^2 + 4*a^3*c^3*d^3 - a^3*c^2*d^4 - 2*a^3*c*d^5 + a^3*d^6)*f*co
s(f*x + e) + 4*(a^3*c^6 - 2*a^3*c^5*d - a^3*c^4*d^2 + 4*a^3*c^3*d^3 - a^3*c
^2*d^4 - 2*a^3*c*d^5 + a^3*d^6)*f - ((a^3*c^5*d - 3*a^3*c^4*d^2 + 2*a^3*c^3
*d^3 + 2*a^3*c^2*d^4 - 3*a^3*c*d^5 + a^3*d^6)*f*cos(f*x + e)^3 + (a^3*c^6 -
 7*a^3*c^4*d^2 + 8*a^3*c^3*d^3 + 3*a^3*c^2*d^4 - 8*a^3*c*d^5 + 3*a^3*d^6)*f
*cos(f*x + e)^2 - 2*(a^3*c^6 - 2*a^3*c^5*d - a^3*c^4*d^2 + 4*a^3*c^3*d^3 -
a^3*c^2*d^4 - 2*a^3*c*d^5 + a^3*d^6)*f*cos(f*x + e) - 4*(a^3*c^6 - 2*a^3*c^
5*d - a^3*c^4*d^2 + 4*a^3*c^3*d^3 - a^3*c^2*d^4 - 2*a^3*c*d^5 + a^3*d^6)*f)
*sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**(5/2)/(c+d*sin(f*x+e))**(3/2),x)
```

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="gia
c")
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)^(5/2)*(d*sin(f*x + e) + c)^(3/2)), x)
```

3.605 $\int \frac{1}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=355

$$-\frac{d\left(-57c^2d+9c^3-493cd^2-299d^3\right)\cos(e+fx)}{48a^2f(c-d)^4(c+d)^2\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}-\frac{d\left(9c^2-54cd-95d^2\right)\cos(e+fx)}{48a^2f(c-d)^3(c+d)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+}$$

```
[Out] -((3*c^2 - 26*c*d + 163*d^2)*ArcTanh[(Sqrt[a]*Sqrt[c - d]*Cos[e + f*x])/(Sq
rt[2]*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])])/(16*Sqrt[2]*a^(5
/2)*(c - d)^(9/2)*f) - Cos[e + f*x]/(4*(c - d)*f*(a + a*Sin[e + f*x])^(5/2)
*(c + d*Sin[e + f*x])^(3/2)) - ((3*c - 17*d)*Cos[e + f*x])/(16*a*(c - d)^2*
f*(a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^(3/2)) - (d*(9*c^2 - 54*c
*d - 95*d^2)*Cos[e + f*x])/(48*a^2*(c - d)^3*(c + d)*f*Sqrt[a + a*Sin[e + f
*x]]*(c + d*Sin[e + f*x])^(3/2)) - (d*(9*c^3 - 57*c^2*d - 493*c*d^2 - 299*d
^3)*Cos[e + f*x])/(48*a^2*(c - d)^4*(c + d)^2*f*Sqrt[a + a*Sin[e + f*x]]*Sq
rt[c + d*Sin[e + f*x]])
```

Rubi [A] time = 1.26403, antiderivative size = 355, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2766, 2978, 2984, 12, 2782, 208}

$$-\frac{d\left(-57c^2d+9c^3-493cd^2-299d^3\right)\cos(e+fx)}{48a^2f(c-d)^4(c+d)^2\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}-\frac{d\left(9c^2-54cd-95d^2\right)\cos(e+fx)}{48a^2f(c-d)^3(c+d)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])^(5/2)),x]
```


```
[Out] -((3*c^2 - 26*c*d + 163*d^2)*ArcTanh[(Sqrt[a]*Sqrt[c - d]*Cos[e + f*x])/(Sq
rt[2]*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])])/(16*Sqrt[2]*a^(5
/2)*(c - d)^(9/2)*f) - Cos[e + f*x]/(4*(c - d)*f*(a + a*Sin[e + f*x])^(5/2)
*(c + d*Sin[e + f*x])^(3/2)) - ((3*c - 17*d)*Cos[e + f*x])/(16*a*(c - d)^2*
f*(a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^(3/2)) - (d*(9*c^2 - 54*c
*d - 95*d^2)*Cos[e + f*x])/(48*a^2*(c - d)^3*(c + d)*f*Sqrt[a + a*Sin[e + f
*x]]*(c + d*Sin[e + f*x])^(3/2)) - (d*(9*c^3 - 57*c^2*d - 493*c*d^2 - 299*d
^3)*Cos[e + f*x])/(48*a^2*(c - d)^4*(c + d)^2*f*Sqrt[a + a*Sin[e + f*x]]*Sq
rt[c + d*Sin[e + f*x]])
```


Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
```

```
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^{5/2}}\,dx = -\frac{\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^{3/2}} - \int \frac{-\frac{1}{2}a(3\cdots}{(a+a\sin(e\cdots}$$

$$= -\frac{\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^{3/2}} - \frac{\cdots}{16a(c-d)^{\cdots}}$$

$$= -\frac{\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^{3/2}} - \frac{\cdots}{16a(c-d)^{\cdots}}$$

$$= -\frac{\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^{3/2}} - \frac{\cdots}{16a(c-d)^{\cdots}}$$

$$= -\frac{\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^{3/2}} - \frac{\cdots}{16a(c-d)^{\cdots}}$$

$$= -\frac{\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^{3/2}} - \frac{\cdots}{16a(c-d)^{\cdots}}$$

$$= -\frac{(3c^2-26cd+163d^2)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{16\sqrt{2}a^{5/2}(c-d)^{9/2}f} - \frac{\cdots}{4(c\cdots}$$

Mathematica [B] time = 9.8525, size = 717, normalized size = 2.02

$$\frac{(3c^2-26cd+163d^2)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^4\left(\log\left(\tan\left(\frac{1}{2}(e+fx)\right)+1\right)-\log\left((d-c)\tan\left(\frac{1}{2}(e+fx)\right)+\cdots\right.\right.}{32f(c-d)^4(a(\sin(e+fx)+1))^{5/2}\sqrt{c+d\sin(e+fx)}\left(\frac{\sec^2\left(\frac{1}{2}(e+fx)\right)}{2\tan\left(\frac{1}{2}(e+fx)\right)+2}-\frac{\frac{\sqrt{c-d}\left(\frac{1}{\cos(e+fx)+1}\right)^{3/2}(c\sin(e+fx)+\cdots}{\sqrt{c+d\sin(e+fx)}}\cdots}{(d-c)\tan\left(\frac{1}{2}(e+fx)\right)+2\sqrt{c-d}\sqrt{\cdots}}\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])^(5/2)),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5*Sqrt[c + d*Sin[e + f*x]]*(Sin[(e +
 f*x)/2]/(2*(c - d)^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4) - 1/(4*(c -
d)^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3) + (-3*c + 25*d)/(16*(c - d)^4
*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])) + (3*c*Sin[(e + f*x)/2] - 25*d*Sin[
(e + f*x)/2])/(8*(c - d)^4*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2) + (2*(d
^3*Cos[(e + f*x)/2] - d^3*Sin[(e + f*x)/2]))/(3*(c - d)^3*(c + d)*(c + d*Si
n[e + f*x])^2) + (2*(11*c*d^3*Cos[(e + f*x)/2] + 7*d^4*Cos[(e + f*x)/2] - 1
1*c*d^3*Sin[(e + f*x)/2] - 7*d^4*Sin[(e + f*x)/2]))/(3*(c - d)^4*(c + d)^2*
(c + d*Sin[e + f*x]))))/(f*(a*(1 + Sin[e + f*x]))^(5/2)) + ((3*c^2 - 26*c*d
 + 163*d^2)*(Log[1 + Tan[(e + f*x)/2]] - Log[c - d + 2*Sqrt[c - d]*Sqrt[(1
+ Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + (-c + d)*Tan[(e + f*x)/2]]
)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4)/(32*(c - d)^4*f*(a*(1 + Sin[e +
f*x]))^(5/2)*Sqrt[c + d*Sin[e + f*x]]*(Sec[(e + f*x)/2]^2/(2 + 2*Tan[(e + f
*x)/2]) - (-((c - d)*Sec[(e + f*x)/2]^2)/2 + (Sqrt[c - d]*((1 + Cos[e + f*x
])^(-1))^(3/2)*(d + d*Cos[e + f*x] + c*Sin[e + f*x]))/Sqrt[c + d*Sin[e + f*
x]])/(c - d + 2*Sqrt[c - d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e
+ f*x]] + (-c + d)*Tan[(e + f*x)/2])))
```

Maple [B] time = 0.279, size = 8041, normalized size = 22.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(5/2),x, algorithm="max
ima")
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)^(5/2)*(d*sin(f*x + e) + c)^(5/2)), x)
```

Fricas [B] time = 15.5932, size = 10797, normalized size = 30.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(5/2),x, algorithm="fri
cas")
```

```
[Out] [1/384*(3*(12*c^6 - 56*c^5*d + 308*c^4*d^2 + 2032*c^3*d^3 + 3508*c^2*d^4 +
2504*c*d^5 + 652*d^6 + (3*c^4*d^2 - 20*c^3*d^3 + 114*c^2*d^4 + 300*c*d^5 +
163*d^6)*cos(f*x + e)^5 + (6*c^5*d - 31*c^4*d^2 + 168*c^3*d^3 + 942*c^2*d^4
 + 1226*c*d^5 + 489*d^6)*cos(f*x + e)^4 - (3*c^6 - 8*c^5*d + 43*c^4*d^2 + 6
96*c^3*d^3 + 1705*c^2*d^4 + 1552*c*d^5 + 489*d^6)*cos(f*x + e)^3 - (9*c^6 -
 30*c^5*d + 163*c^4*d^2 + 1900*c^3*d^3 + 4287*c^2*d^4 + 3730*c*d^5 + 1141*d
^6)*cos(f*x + e)^2 + 2*(3*c^6 - 14*c^5*d + 77*c^4*d^2 + 508*c^3*d^3 + 877*c
^2*d^4 + 626*c*d^5 + 163*d^6)*cos(f*x + e) + (12*c^6 - 56*c^5*d + 308*c^4*d
^2 + 2032*c^3*d^3 + 3508*c^2*d^4 + 2504*c*d^5 + 652*d^6 + (3*c^4*d^2 - 20*c
^3*d^3 + 114*c^2*d^4 + 300*c*d^5 + 163*d^6)*cos(f*x + e)^4 - 2*(3*c^5*d - 1
7*c^4*d^2 + 94*c^3*d^3 + 414*c^2*d^4 + 463*c*d^5 + 163*d^6)*cos(f*x + e)^3
- (3*c^6 - 2*c^5*d + 9*c^4*d^2 + 884*c^3*d^3 + 2533*c^2*d^4 + 2478*c*d^5 +
815*d^6)*cos(f*x + e)^2 + 2*(3*c^6 - 14*c^5*d + 77*c^4*d^2 + 508*c^3*d^3 +
877*c^2*d^4 + 626*c*d^5 + 163*d^6)*cos(f*x + e))*sin(f*x + e))*sqrt(2*a*c -
 2*a*d)*log(((a*c^2 - 14*a*c*d + 17*a*d^2)*cos(f*x + e)^3 - 4*a*c^2 - 8*a*c
*d - 4*a*d^2 - (13*a*c^2 - 22*a*c*d - 3*a*d^2)*cos(f*x + e)^2 - 4*((c - 3*d
)*cos(f*x + e)^2 - (3*c - d)*cos(f*x + e) + ((c - 3*d)*cos(f*x + e) + 4*c -
 4*d)*sin(f*x + e) - 4*c + 4*d)*sqrt(2*a*c - 2*a*d)*sqrt(a*sin(f*x + e) + a
)*sqrt(d*sin(f*x + e) + c) - 2*(9*a*c^2 - 14*a*c*d + 9*a*d^2)*cos(f*x + e)
- (4*a*c^2 + 8*a*c*d + 4*a*d^2 - (a*c^2 - 14*a*c*d + 17*a*d^2)*cos(f*x + e)
^2 - 2*(7*a*c^2 - 18*a*c*d + 7*a*d^2)*cos(f*x + e))*sin(f*x + e))/(cos(f*x
```

+ e)^3 + 3*cos(f*x + e)^2 + (cos(f*x + e)^2 - 2*cos(f*x + e) - 4)*sin(f*x + e) - 2*cos(f*x + e) - 4)) - 8*(12*c^6 - 24*c^5*d - 12*c^4*d^2 + 48*c^3*d^3 - 12*c^2*d^4 - 24*c*d^5 + 12*d^6 - (9*c^4*d^2 - 66*c^3*d^3 - 436*c^2*d^4 + 194*c*d^5 + 299*d^6)*cos(f*x + e)^4 - (18*c^5*d - 111*c^4*d^2 - 618*c^3*d^3 - 520*c^2*d^4 + 728*c*d^5 + 503*d^6)*cos(f*x + e)^3 + 3*(3*c^6 - 14*c^5*d - 29*c^4*d^2 - 144*c^3*d^3 - 59*c^2*d^4 + 158*c*d^5 + 85*d^6)*cos(f*x + e)^2 + 3*(7*c^6 - 16*c^5*d - 73*c^4*d^2 - 312*c^3*d^3 - 91*c^2*d^4 + 328*c*d^5 + 157*d^6)*cos(f*x + e) - (12*c^6 - 24*c^5*d - 12*c^4*d^2 + 48*c^3*d^3 - 12*c^2*d^4 - 24*c*d^5 + 12*d^6 + (9*c^4*d^2 - 66*c^3*d^3 - 436*c^2*d^4 + 194*c*d^5 + 299*d^6)*cos(f*x + e)^3 - 6*(3*c^5*d - 20*c^4*d^2 - 92*c^3*d^3 - 14*c^2*d^4 + 89*c*d^5 + 34*d^6)*cos(f*x + e)^2 - 3*(3*c^6 - 8*c^5*d - 69*c^4*d^2 - 328*c^3*d^3 - 87*c^2*d^4 + 336*c*d^5 + 153*d^6)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/((a^3*c^7*d^2 - 3*a^3*c^6*d^3 + a^3*c^5*d^4 + 5*a^3*c^4*d^5 - 5*a^3*c^3*d^6 - a^3*c^2*d^7 + 3*a^3*c*d^8 - a^3*d^9)*f*cos(f*x + e)^5 + (2*a^3*c^8*d - 3*a^3*c^7*d^2 - 7*a^3*c^6*d^3 + 13*a^3*c^5*d^4 + 5*a^3*c^4*d^5 - 17*a^3*c^3*d^6 + 3*a^3*c^2*d^7 + 7*a^3*c*d^8 - 3*a^3*d^9)*f*cos(f*x + e)^4 - (a^3*c^9 + a^3*c^8*d - 8*a^3*c^7*d^2 + 18*a^3*c^5*d^4 - 6*a^3*c^4*d^5 - 16*a^3*c^3*d^6 + 8*a^3*c^2*d^7 + 5*a^3*c*d^8 - 3*a^3*d^9)*f*cos(f*x + e)^3 - (3*a^3*c^9 + a^3*c^8*d - 20*a^3*c^7*d^2 + 4*a^3*c^6*d^3 + 42*a^3*c^5*d^4 - 18*a^3*c^4*d^5 - 36*a^3*c^3*d^6 + 20*a^3*c^2*d^7 + 11*a^3*c*d^8 - 7*a^3*d^9)*f*cos(f*x + e)^2 + 2*(a^3*c^9 - a^3*c^8*d - 4*a^3*c^7*d^2 + 4*a^3*c^6*d^3 + 6*a^3*c^5*d^4 - 6*a^3*c^4*d^5 - 4*a^3*c^3*d^6 + 4*a^3*c^2*d^7 + a^3*c*d^8 - a^3*d^9)*f*cos(f*x + e) + 4*(a^3*c^9 - a^3*c^8*d - 4*a^3*c^7*d^2 + 4*a^3*c^6*d^3 + 6*a^3*c^5*d^4 - 6*a^3*c^4*d^5 - 4*a^3*c^3*d^6 + 4*a^3*c^2*d^7 + a^3*c*d^8 - a^3*d^9)*f + ((a^3*c^7*d^2 - 3*a^3*c^6*d^3 + a^3*c^5*d^4 + 5*a^3*c^4*d^5 - 5*a^3*c^3*d^6 - a^3*c^2*d^7 + 3*a^3*c*d^8 - a^3*d^9)*f*cos(f*x + e)^4 - 2*(a^3*c^8*d - 2*a^3*c^7*d^2 - 2*a^3*c^6*d^3 + 6*a^3*c^5*d^4 - 6*a^3*c^3*d^6 + 2*a^3*c^2*d^7 + 2*a^3*c*d^8 - a^3*d^9)*f*cos(f*x + e)^3 - (a^3*c^9 + 3*a^3*c^8*d - 12*a^3*c^7*d^2 - 4*a^3*c^6*d^3 + 30*a^3*c^5*d^4 - 6*a^3*c^4*d^5 - 28*a^3*c^3*d^6 + 12*a^3*c^2*d^7 + 9*a^3*c*d^8 - 5*a^3*d^9)*f*cos(f*x + e)^2 + 2*(a^3*c^9 - a^3*c^8*d - 4*a^3*c^7*d^2 + 4*a^3*c^6*d^3 + 6*a^3*c^5*d^4 - 6*a^3*c^4*d^5 - 4*a^3*c^3*d^6 + 4*a^3*c^2*d^7 + a^3*c*d^8 - a^3*d^9)*f*cos(f*x + e) + 4*(a^3*c^9 - a^3*c^8*d - 4*a^3*c^7*d^2 + 4*a^3*c^6*d^3 + 6*a^3*c^5*d^4 - 6*a^3*c^4*d^5 - 4*a^3*c^3*d^6 + 4*a^3*c^2*d^7 + a^3*c*d^8 - a^3*d^9)*f)*sin(f*x + e)), -1/192*(3*(12*c^6 - 56*c^5*d + 308*c^4*d^2 + 2032*c^3*d^3 + 3508*c^2*d^4 + 2504*c*d^5 + 652*d^6 + (3*c^4*d^2 - 20*c^3*d^3 + 114*c^2*d^4 + 300*c*d^5 + 163*d^6)*cos(f*x + e)^5 + (6*c^5*d - 31*c^4*d^2 + 168*c^3*d^3 + 942*c^2*d^4 + 1226*c*d^5 + 489*d^6)*cos(f*x + e)^4 - (3*c^6 - 8*c^5*d + 43*c^4*d^2 + 696*c^3*d^3 + 1705*c^2*d^4 + 1552*c*d^5 + 489*d^6)*cos(f*x + e)^3 - (9*c^6 - 30*c^5*d + 163*c^4*d^2 + 1900*c^3*d^3 + 4287*c^2*d^4 + 3730*c*d^5 + 1141*d^6)*cos(f*x + e)^2 + 2*(3*c^6 - 14*c^5*d + 77*c^4*d^2 + 508*c^3*d^3 + 877*c^2*d^4 + 626*c*d^5 + 163*d^6)*cos(f*x + e) + (12*c^6 - 56*c^5*d + 308*c^4*d^2 + 2032*c^3*d^3 + 3508*c^2*d^4 + 2504*c*d^5 + 652*d^6 + (3*c^4*d^2 - 20*c^3*d^3 + 114*c^2*d^4 + 300*c*d^5 + 163*d^6)*cos(f*x + e)^4 - 2*(3*c^5*d - 17*c^4*d^2 + 94*c^3*d^3 + 414*c^2*d^4 + 463*c*d^5 + 163*d^6)*cos(f*x + e)^3 - (3*c^6 - 2*c^5*d + 9*c^4*d^2 + 884*c^3*d^3 + 2533*c^2*d^4 + 2478*c*d^5 + 815*d^6)*cos(f*x + e)^2 + 2*(3*c^6 - 14*c^5*d + 77*c^4*d^2 + 508*c^3*d^3 + 877*c^2*d^4 + 626*c*d^5 + 163*d^6)*cos(f*x + e))*sin(f*x + e))*sqrt(-2*a*c + 2*a*d)*arctan(1/4*sqrt(-2*a*c + 2*a*d)*sqrt(a*sin(f*x + e) + a)*((c - 3*d)*sin(f*x + e) - 3*c + d)*sqrt(d*sin(f*x + e) + c)/((a*c*d - a*d^2)*cos(f*x + e)*sin(f*x + e) + (a*c^2 - a*c*d)*cos(f*x + e))) + 4*(12*c^6 - 24*c^5*d - 12*c^4*d^2 + 48*c^3*d^3 - 12*c^2*d^4 - 24*c*d^5 + 12*d^6 - (9*c^4*d^2 - 66*c^3*d^3 - 436*c^2*d^4 + 194*c*d^5 + 299*d^6)*cos(f*x + e)^4 - (18*c^5*d - 111*c^4*d^2 - 618*c^3*d^3 - 520*c^2*d^4 + 728*c*d^5 + 503*d^6)*cos(f*x + e)^3 + 3*(3*c^6 - 14*c^5*d - 29*c^4*d^2 - 144*c^3*d^3 - 59*c^2*d^4 + 158*c*d^5 + 85*d^6)*cos(f*x + e)^2 + 3*(7*c^6 - 16*c^5*d - 73*c^4*d^2 - 312*c^3*d^3 - 91*c^2*d^4 + 328*c*d^5 + 157*d^6)*cos(f*x + e) - (12*c^6 - 24*c^5*d - 12*c^4*d^2 + 48*c^3*d^3 - 12*c^2*d^4 - 24*c*d^5 + 12*d^6 + (9*c^4*

```
d^2 - 66*c^3*d^3 - 436*c^2*d^4 + 194*c*d^5 + 299*d^6)*cos(f*x + e)^3 - 6*(3
*c^5*d - 20*c^4*d^2 - 92*c^3*d^3 - 14*c^2*d^4 + 89*c*d^5 + 34*d^6)*cos(f*x
+ e)^2 - 3*(3*c^6 - 8*c^5*d - 69*c^4*d^2 - 328*c^3*d^3 - 87*c^2*d^4 + 336*c
*d^5 + 153*d^6)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d
*sin(f*x + e) + c))/((a^3*c^7*d^2 - 3*a^3*c^6*d^3 + a^3*c^5*d^4 + 5*a^3*c^4
*d^5 - 5*a^3*c^3*d^6 - a^3*c^2*d^7 + 3*a^3*c*d^8 - a^3*d^9)*f*cos(f*x + e)^
5 + (2*a^3*c^8*d - 3*a^3*c^7*d^2 - 7*a^3*c^6*d^3 + 13*a^3*c^5*d^4 + 5*a^3*c
^4*d^5 - 17*a^3*c^3*d^6 + 3*a^3*c^2*d^7 + 7*a^3*c*d^8 - 3*a^3*d^9)*f*cos(f*
x + e)^4 - (a^3*c^9 + a^3*c^8*d - 8*a^3*c^7*d^2 + 18*a^3*c^5*d^4 - 6*a^3*c^
4*d^5 - 16*a^3*c^3*d^6 + 8*a^3*c^2*d^7 + 5*a^3*c*d^8 - 3*a^3*d^9)*f*cos(f*x
 + e)^3 - (3*a^3*c^9 + a^3*c^8*d - 20*a^3*c^7*d^2 + 4*a^3*c^6*d^3 + 42*a^3*
c^5*d^4 - 18*a^3*c^4*d^5 - 36*a^3*c^3*d^6 + 20*a^3*c^2*d^7 + 11*a^3*c*d^8 -
 7*a^3*d^9)*f*cos(f*x + e)^2 + 2*(a^3*c^9 - a^3*c^8*d - 4*a^3*c^7*d^2 + 4*a
^3*c^6*d^3 + 6*a^3*c^5*d^4 - 6*a^3*c^4*d^5 - 4*a^3*c^3*d^6 + 4*a^3*c^2*d^7
+ a^3*c*d^8 - a^3*d^9)*f*cos(f*x + e) + 4*(a^3*c^9 - a^3*c^8*d - 4*a^3*c^7*
d^2 + 4*a^3*c^6*d^3 + 6*a^3*c^5*d^4 - 6*a^3*c^4*d^5 - 4*a^3*c^3*d^6 + 4*a^3
*c^2*d^7 + a^3*c*d^8 - a^3*d^9)*f + ((a^3*c^7*d^2 - 3*a^3*c^6*d^3 + a^3*c^5
*d^4 + 5*a^3*c^4*d^5 - 5*a^3*c^3*d^6 - a^3*c^2*d^7 + 3*a^3*c*d^8 - a^3*d^9)
*f*cos(f*x + e)^4 - 2*(a^3*c^8*d - 2*a^3*c^7*d^2 - 2*a^3*c^6*d^3 + 6*a^3*c^
5*d^4 - 6*a^3*c^3*d^6 + 2*a^3*c^2*d^7 + 2*a^3*c*d^8 - a^3*d^9)*f*cos(f*x +
e)^3 - (a^3*c^9 + 3*a^3*c^8*d - 12*a^3*c^7*d^2 - 4*a^3*c^6*d^3 + 30*a^3*c^5
*d^4 - 6*a^3*c^4*d^5 - 28*a^3*c^3*d^6 + 12*a^3*c^2*d^7 + 9*a^3*c*d^8 - 5*a^
3*d^9)*f*cos(f*x + e)^2 + 2*(a^3*c^9 - a^3*c^8*d - 4*a^3*c^7*d^2 + 4*a^3*c^
6*d^3 + 6*a^3*c^5*d^4 - 6*a^3*c^4*d^5 - 4*a^3*c^3*d^6 + 4*a^3*c^2*d^7 + a^3
*c*d^8 - a^3*d^9)*f*cos(f*x + e) + 4*(a^3*c^9 - a^3*c^8*d - 4*a^3*c^7*d^2 +
 4*a^3*c^6*d^3 + 6*a^3*c^5*d^4 - 6*a^3*c^4*d^5 - 4*a^3*c^3*d^6 + 4*a^3*c^2*
d^7 + a^3*c*d^8 - a^3*d^9)*f)*sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**(5/2)/(c+d*sin(f*x+e))**(5/2),x)

[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(5/2),x, algorithm="gia
c")

[Out] integrate(1/((a*sin(f*x + e) + a)^(5/2)*(d*sin(f*x + e) + c)^(5/2)), x)
```

3.606 $\int (a+a\sin(e+fx))^m(c+d\sin(e+fx))^n\,dx$

Optimal. Leaf size=129

$$\frac{\sqrt{2}\cos(e+fx)(a\sin(e+fx)+a)^m(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c-d}\right)^{-n}F_1\left(m+\frac{1}{2};\frac{1}{2},-n;m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1),-\frac{d}{c-d}\right)}{f(2m+1)\sqrt{1-\sin(e+fx)}}$$

```
[Out] (Sqrt[2]*AppellF1[1/2 + m, 1/2, -n, 3/2 + m, (1 + Sin[e + f*x])/2, -((d*(1
+ Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c + d*Sin[e
 + f*x])^n)/(f*(1 + 2*m)*Sqrt[1 - Sin[e + f*x]]*((c + d*Sin[e + f*x])/(c -
d))^n)
```

Rubi [A] time = 0.168111, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2788, 140, 139, 138}

$$\frac{\sqrt{2}\cos(e+fx)(a\sin(e+fx)+a)^m(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c-d}\right)^{-n}F_1\left(m+\frac{1}{2};\frac{1}{2},-n;m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1),-\frac{d}{c-d}\right)}{f(2m+1)\sqrt{1-\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] (Sqrt[2]*AppellF1[1/2 + m, 1/2, -n, 3/2 + m, (1 + Sin[e + f*x])/2, -((d*(1
+ Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c + d*Sin[e
 + f*x])^n)/(f*(1 + 2*m)*Sqrt[1 - Sin[e + f*x]]*((c + d*Sin[e + f*x])/(c -
d))^n)
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x
)^n)/Sqrt[a - b*x], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rule 140

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && !GtQ[b/
(b*c - a*d), 0] && !SimplerQ[c + d*x, a + b*x] && !SimplerQ[e + f*x, a +
b*x]
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
```

```
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int (a+a\sin(e+fx))^m(c+d\sin(e+fx))^n\,dx = \frac{\left(a^2\cos(e+fx)\right)\operatorname{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}(c+dx)^n}{\sqrt{a-ax}}\,dx,x,\sin(e+fx)\right)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\left(a^2\cos(e+fx)\sqrt{\frac{a-a\sin(e+fx)}{a}}\right)\operatorname{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}(c+dx)^n}{\sqrt{\frac{1}{2}-\frac{x}{2}}}\,dx,x,\sin(e+fx\right.}{\sqrt{2}f(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\left(a^2\cos(e+fx)\sqrt{\frac{a-a\sin(e+fx)}{a}}(c+d\sin(e+fx))^n\left(\frac{a(c+d\sin(e+fx))}{ac-ad}\right)^{-n}\right)\operatorname{Subs}}{\sqrt{2}f(a-a\sin(e+fx))\sqrt{a+a\sin(}}$$

$$= \frac{\sqrt{2}F_1\left(\frac{1}{2}+m;\frac{1}{2},-n;\frac{3}{2}+m;\frac{1}{2}(1+\sin(e+fx)),-\frac{d(1+\sin(e+fx))}{c-d}\right)\cos(e+f}{f(1+2m)\sqrt{1-\sin(e}}$$

Mathematica [B] time = 1.41089, size = 373, normalized size = 2.89

$$\frac{6(c+d)\cot\left(\frac{1}{4}(2e+2fx+\pi)\right)\sin^2\left(\frac{1}{4}(2e+2fx+\pi)\right)^{\frac{1}{2}-m}\cos^2\left(\frac{1}{4}(2e+2fx-\pi)\right)^{m-\frac{1}{2}}(a}{f\left(\sin^2\left(\frac{1}{4}(2e+2fx-\pi)\right)\left(4dnF_1\left(\frac{3}{2};\frac{1}{2}-m,1-n;\frac{5}{2};\cos^2\left(\frac{1}{4}(2e+2fx+\pi)\right),\frac{2d\sin^2\left(\frac{1}{4}(2e+2fx-\pi)\right)}{c+d}\right)+(2m-1)(c+d)F_1\left(\frac{3}{2}\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] (6*(c + d)*AppellF1[1/2, 1/2 - m, -n, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*
d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)]*(Cos[(2*e - Pi + 2*f*x)/4]^2)^(-1/2
 + m)*Cot[(2*e + Pi + 2*f*x)/4]*(a*(1 + Sin[e + f*x]))^m*(c + d*Sin[e + f*x
])^n*(Sin[(2*e + Pi + 2*f*x)/4]^2)^(1/2 - m))/(f*(-3*(c + d)*AppellF1[1/2,
1/2 - m, -n, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/
4]^2)/(c + d)] + (4*d*n*AppellF1[3/2, 1/2 - m, 1 - n, 5/2, Cos[(2*e + Pi +
2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)] + (c + d)*(-1 + 2*m
)*AppellF1[3/2, 3/2 - m, -n, 5/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*
e - Pi + 2*f*x)/4]^2)/(c + d)])*Sin[(2*e - Pi + 2*f*x)/4]^2))
```

Maple [F] time = 1.124, size = 0, normalized size = 0.

$$\int \left(a+a\sin\left(fx+e\right)\right)^{m} \left(c+d\sin\left(fx+e\right)\right)^{n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(fx+e)+a)^{m}(d\sin(fx+e)+c)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((a\sin(fx+e)+a)^{m}(d\sin(fx+e)+c)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(c+d*sin(f*x+e))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(fx+e)+a)^{m}(d\sin(fx+e)+c)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^n, x)
```

3.607 $\int (a+a\sin(e+fx))^m(c+d\sin(e+fx))^3\,dx$

Optimal. Leaf size=320

$$-\frac{2^{m+\frac{1}{2}}\left(3c^2dm\left(m^2+5m+6\right)+c^3\left(m^3+6m^2+11m+6\right)+3cd^2\left(m^3+4m^2+4m+3\right)+d^3m\left(m^2+3m+5\right)\right)\cos(e+f}{f(m+1)(m+2)(m+3)}$$

```
[Out] -((d*(d^2*(4 + m) - c*d*(5 - 3*m - 2*m^2) + 2*c^2*(8 + 6*m + m^2))*Cos[e +
f*x]*(a + a*Sin[e + f*x])^m)/(f*(1 + m)*(2 + m)*(3 + m))) - (2^(1/2 + m)*(d
^3*m*(5 + 3*m + m^2) + 3*c^2*d*m*(6 + 5*m + m^2) + 3*c*d^2*(3 + 4*m + 4*m^2
 + m^3) + c^3*(6 + 11*m + 6*m^2 + m^3))*Cos[e + f*x]*Hypergeometric2F1[1/2,
 1/2 - m, 3/2, (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*S
in[e + f*x])^m)/(f*(1 + m)*(2 + m)*(3 + m)) - (d^2*(d*m + c*(5 + m))*Cos[e
+ f*x]*(a + a*Sin[e + f*x])^(1 + m))/(a*f*(2 + m)*(3 + m)) - (d*Cos[e + f*x
]*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^2)/(f*(3 + m))
```

Rubi [A] time = 0.662531, antiderivative size = 320, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {2783, 2968, 3023, 2751, 2652, 2651}

$$-\frac{2^{m+\frac{1}{2}}\left(3c^2dm\left(m^2+5m+6\right)+c^3\left(m^3+6m^2+11m+6\right)+3cd^2\left(m^3+4m^2+4m+3\right)+d^3m\left(m^2+3m+5\right)\right)\cos(e+f}{f(m+1)(m+2)(m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^3,x]
```


```
[Out] -((d*(d^2*(4 + m) - c*d*(5 - 3*m - 2*m^2) + 2*c^2*(8 + 6*m + m^2))*Cos[e +
f*x]*(a + a*Sin[e + f*x])^m)/(f*(1 + m)*(2 + m)*(3 + m))) - (2^(1/2 + m)*(d
^3*m*(5 + 3*m + m^2) + 3*c^2*d*m*(6 + 5*m + m^2) + 3*c*d^2*(3 + 4*m + 4*m^2
 + m^3) + c^3*(6 + 11*m + 6*m^2 + m^3))*Cos[e + f*x]*Hypergeometric2F1[1/2,
 1/2 - m, 3/2, (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*S
in[e + f*x])^m)/(f*(1 + m)*(2 + m)*(3 + m)) - (d^2*(d*m + c*(5 + m))*Cos[e
+ f*x]*(a + a*Sin[e + f*x])^(1 + m))/(a*f*(2 + m)*(3 + m)) - (d*Cos[e + f*x
]*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^2)/(f*(3 + m))
```


Rule 2783

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*(c + d*Sin[e + f*x])^(n - 1))/(f*(m + n)), x] + Dist[1/(b*(m + n)), Int[(
a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n - 2)*Simp[d*(a*c*m + b*d*(n -
 1)) + b*c^2*(m + n) + d*(a*d*m + b*c*(m + 2*n - 1))*Sin[e + f*x], x], x],
x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && IntegerQ[n]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2652

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Sin[c + d*x])^FracPart[n])/(1 + (b*Sin[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Sin[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2651

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(2^(n +
 1/2)*a^(n - 1/2)*b*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1*(1
 - (b*Sin[c + d*x])/a))/2])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))^m(c+d\sin(e+fx))^3\,dx = -\frac{d\cos(e+fx)(a+a\sin(e+fx))^m(c+d\sin(e+fx))^2}{f(3+m)}+\frac{\int (a+a\sin(}{}$$

$$= -\frac{d\cos(e+fx)(a+a\sin(e+fx))^m(c+d\sin(e+fx))^2}{f(3+m)}+\frac{\int (a+a\sin(}{}$$

$$= -\frac{d^2(dm+c(5+m))\cos(e+fx)(a+a\sin(e+fx))^{1+m}}{af(2+m)(3+m)}-\frac{d\cos(e+fx}{}$$

$$= -\frac{d\left(d^2(4+m)-cd\left(5-3m-2m^2\right)+2c^2\left(8+6m+m^2\right)\right)\cos(e+fx)}{f(1+m)(2+m)(3+m)}$$

$$= -\frac{d\left(d^2(4+m)-cd\left(5-3m-2m^2\right)+2c^2\left(8+6m+m^2\right)\right)\cos(e+fx)}{f(1+m)(2+m)(3+m)}$$

$$= -\frac{d\left(d^2(4+m)-cd\left(5-3m-2m^2\right)+2c^2\left(8+6m+m^2\right)\right)\cos(e+fx)}{f(1+m)(2+m)(3+m)}$$

Mathematica [B] time = 56.073, size = 3599, normalized size = 11.25

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^3,x]
```

```
[Out] (-22*(Cos[(-e + Pi/2 - f*x)/2]^2)^(1/2 - m)*(1 - Sin[(-e + Pi/2 - f*x)/2]^2
)^(-1/2 + m)*(c + d - 2*d*Sin[(-e + Pi/2 - f*x)/2]^2)^3*(945*(c + d)^3*Gamm
```

a[1/2 - m]*Hypergeometric2F1[1/2, 1/2 - m, 9/2, Sin[(-e + Pi/2 - f*x)/2]^2]
 - 1890*d*(c + d)^2*Gamma[1/2 - m]*Hypergeometric2F1[1/2, 1/2 - m, 9/2, Sin
[(-e + Pi/2 - f*x)/2]^2]*Sin[(-e + Pi/2 - f*x)/2]^2 + 142*(c + d)^3*Gamma[3
/2 - m]*Hypergeometric2F1[3/2, 3/2 - m, 11/2, Sin[(-e + Pi/2 - f*x)/2]^2]*S
in[(-e + Pi/2 - f*x)/2]^2 + 60*(c + d)^3*Gamma[3/2 - m]*HypergeometricPFQ[{
3/2, 2, 3/2 - m}, {1, 11/2}, Sin[(-e + Pi/2 - f*x)/2]^2]*Sin[(-e + Pi/2 - f
x)/2]^2 + 8(c + d)^3*Gamma[3/2 - m]*HypergeometricPFQ[{3/2, 2, 2, 3/2 - m
}, {1, 1, 11/2}, Sin[(-e + Pi/2 - f*x)/2]^2]*Sin[(-e + Pi/2 - f*x)/2]^2 + 2
268*d^2*(c + d)*Gamma[1/2 - m]*Hypergeometric2F1[1/2, 1/2 - m, 9/2, Sin[(-e
 + Pi/2 - f*x)/2]^2]*Sin[(-e + Pi/2 - f*x)/2]^4 - 564*d*(c + d)^2*Gamma[3/2
 - m]*Hypergeometric2F1[3/2, 3/2 - m, 11/2, Sin[(-e + Pi/2 - f*x)/2]^2]*Sin
[(-e + Pi/2 - f*x)/2]^4 - 312*d*(c + d)^2*Gamma[3/2 - m]*HypergeometricPFQ[
{3/2, 2, 3/2 - m}, {1, 11/2}, Sin[(-e + Pi/2 - f*x)/2]^2]*Sin[(-e + Pi/2 -
f*x)/2]^4 - 48*d*(c + d)^2*Gamma[3/2 - m]*HypergeometricPFQ[{3/2, 2, 2, 3/2
 - m}, {1, 1, 11/2}, Sin[(-e + Pi/2 - f*x)/2]^2]*Sin[(-e + Pi/2 - f*x)/2]^4
 - 1080*d^3*Gamma[1/2 - m]*Hypergeometric2F1[1/2, 1/2 - m, 9/2, Sin[(-e + P
i/2 - f*x)/2]^2]*Sin[(-e + Pi/2 - f*x)/2]^6 + 744*d^2*(c + d)*Gamma[3/2 - m
]*Hypergeometric2F1[3/2, 3/2 - m, 11/2, Sin[(-e + Pi/2 - f*x)/2]^2]*Sin[(-e
 + Pi/2 - f*x)/2]^6 + 528*d^2*(c + d)*Gamma[3/2 - m]*HypergeometricPFQ[{3/2
, 2, 3/2 - m}, {1, 11/2}, Sin[(-e + Pi/2 - f*x)/2]^2]*Sin[(-e + Pi/2 - f*x)
/2]^6 + 96*d^2*(c + d)*Gamma[3/2 - m]*HypergeometricPFQ[{3/2, 2, 2, 3/2 - m
}, {1, 1, 11/2}, Sin[(-e + Pi/2 - f*x)/2]^2]*Sin[(-e + Pi/2 - f*x)/2]^6 - 3
68*d^3*Gamma[3/2 - m]*Hypergeometric2F1[3/2, 3/2 - m, 11/2, Sin[(-e + Pi/2
 - f*x)/2]^2]*Sin[(-e + Pi/2 - f*x)/2]^8 - 288*d^3*Gamma[3/2 - m]*Hypergeome
tricPFQ[{3/2, 2, 3/2 - m}, {1, 11/2}, Sin[(-e + Pi/2 - f*x)/2]^2]*Sin[(-e +
 Pi/2 - f*x)/2]^8 - 64*d^3*Gamma[3/2 - m]*HypergeometricPFQ[{3/2, 2, 2, 3/2
 - m}, {1, 1, 11/2}, Sin[(-e + Pi/2 - f*x)/2]^2]*Sin[(-e + Pi/2 - f*x)/2]^8
)*(a + a*Sin[e + f*x])^m*Tan[(-e + Pi/2 - f*x)/2])/(3*f*(3465*(c + d)^3*Gam
ma[1/2 - m]*Hypergeometric2F1[1/2, 1/2 - m, 9/2, Sin[(-e + Pi/2 - f*x)/2]^2
] - 20790*d*(c + d)^2*Gamma[1/2 - m]*Hypergeometric2F1[1/2, 1/2 - m, 9/2, S
in[(-e + Pi/2 - f*x)/2]^2]*Sin[(-e + Pi/2 - f*x)/2]^2 - 385*(c + d)^3*(-1 +
 2*m)*Gamma[1/2 - m]*Hypergeometric2F1[3/2, 3/2 - m, 11/2, Sin[(-e + Pi/2 -
 f*x)/2]^2]*Sin[(-e + Pi/2 - f*x)/2]^2 + 1562*(c + d)^3*Gamma[3/2 - m]*Hype
rgeometric2F1[3/2, 3/2 - m, 11/2, Sin[(-e + Pi/2 - f*x)/2]^2]*Sin[(-e + Pi/
2 - f*x)/2]^2 + 660*(c + d)^3*Gamma[3/2 - m]*HypergeometricPFQ[{3/2, 2, 3/2
 - m}, {1, 11/2}, Sin[(-e + Pi/2 - f*x)/2]^2]*Sin[(-e + Pi/2 - f*x)/2]^2 +
88*(c + d)^3*Gamma[3/2 - m]*HypergeometricPFQ[{3/2, 2, 2, 3/2 - m}, {1, 1,
11/2}, Sin[(-e + Pi/2 - f*x)/2]^2]*Sin[(-e + Pi/2 - f*x)/2]^2 + 41580*d^2*(
c + d)*Gamma[1/2 - m]*Hypergeometric2F1[1/2, 1/2 - m, 9/2, Sin[(-e + Pi/2 -
 f*x)/2]^2]*Sin[(-e + Pi/2 - f*x)/2]^4 + 770*d*(c + d)^2*(-1 + 2*m)*Gamma[1
/2 - m]*Hypergeometric2F1[3/2, 3/2 - m, 11/2, Sin[(-e + Pi/2 - f*x)/2]^2]*S
in[(-e + Pi/2 - f*x)/2]^4 - 10340*d*(c + d)^2*Gamma[3/2 - m]*Hypergeometric
2F1[3/2, 3/2 - m, 11/2, Sin[(-e + Pi/2 - f*x)/2]^2]*Sin[(-e + Pi/2 - f*x)/2
]^4 - 142*(c + d)^3*(-3 + 2*m)*Gamma[3/2 - m]*Hypergeometric2F1[5/2, 5/2 -
m, 13/2, Sin[(-e + Pi/2 - f*x)/2]^2]*Sin[(-e + Pi/2 - f*x)/2]^4 - 5720*d*(c
 + d)^2*Gamma[3/2 - m]*HypergeometricPFQ[{3/2, 2, 3/2 - m}, {1, 11/2}, Sin[
(-e + Pi/2 - f*x)/2]^2]*Sin[(-e + Pi/2 - f*x)/2]^4 - 120*(c + d)^3*(-3 + 2*
m)*Gamma[3/2 - m]*HypergeometricPFQ[{5/2, 3, 5/2 - m}, {2, 13/2}, Sin[(-e +
 Pi/2 - f*x)/2]^2]*Sin[(-e + Pi/2 - f*x)/2]^4 - 880*d*(c + d)^2*Gamma[3/2 -
 m]*HypergeometricPFQ[{3/2, 2, 2, 3/2 - m}, {1, 1, 11/2}, Sin[(-e + Pi/2 -
f*x)/2]^2]*Sin[(-e + Pi/2 - f*x)/2]^4 - 32*(c + d)^3*(-3 + 2*m)*Gamma[3/2 -
 m]*HypergeometricPFQ[{5/2, 3, 3, 5/2 - m}, {2, 2, 13/2}, Sin[(-e + Pi/2 -
f*x)/2]^2]*Sin[(-e + Pi/2 - f*x)/2]^4 - 27720*d^3*Gamma[1/2 - m]*Hypergeome
tric2F1[1/2, 1/2 - m, 9/2, Sin[(-e + Pi/2 - f*x)/2]^2]*Sin[(-e + Pi/2 - f*x
)/2]^6 - 924*d^2*(c + d)*(-1 + 2*m)*Gamma[1/2 - m]*Hypergeometric2F1[3/2, 3
/2 - m, 11/2, Sin[(-e + Pi/2 - f*x)/2]^2]*Sin[(-e + Pi/2 - f*x)/2]^6 + 1909
6*d^2*(c + d)*Gamma[3/2 - m]*Hypergeometric2F1[3/2, 3/2 - m, 11/2, Sin[(-e
+ Pi/2 - f*x)/2]^2]*Sin[(-e + Pi/2 - f*x)/2]^6 + 564*d*(c + d)^2*(-3 + 2*m)
*Gamma[3/2 - m]*Hypergeometric2F1[5/2, 5/2 - m, 13/2, Sin[(-e + Pi/2 - f*x)

```
/2]^2]*Sin[(-e + Pi/2 - f*x)/2]^6 + 13552*d^2*(c + d)*Gamma[3/2 - m]*Hyperg
eometricPFQ[{3/2, 2, 3/2 - m}, {1, 11/2}, Sin[(-e + Pi/2 - f*x)/2]^2]*Sin[(
-e + Pi/2 - f*x)/2]^6 + 624*d*(c + d)^2*(-3 + 2*m)*Gamma[3/2 - m]*Hypergeom
etricPFQ[{5/2, 3, 5/2 - m}, {2, 13/2}, Sin[(-e + Pi/2 - f*x)/2]^2]*Sin[(-e
+ Pi/2 - f*x)/2]^6 + 2464*d^2*(c + d)*Gamma[3/2 - m]*HypergeometricPFQ[{3/2
, 2, 2, 3/2 - m}, {1, 1, 11/2}, Sin[(-e + Pi/2 - f*x)/2]^2]*Sin[(-e + Pi/2
- f*x)/2]^6 + 192*d*(c + d)^2*(-3 + 2*m)*Gamma[3/2 - m]*HypergeometricPFQ[{
5/2, 3, 3, 5/2 - m}, {2, 2, 13/2}, Sin[(-e + Pi/2 - f*x)/2]^2]*Sin[(-e + Pi
/2 - f*x)/2]^6 + 440*d^3*(-1 + 2*m)*Gamma[1/2 - m]*Hypergeometric2F1[3/2, 3
/2 - m, 11/2, Sin[(-e + Pi/2 - f*x)/2]^2]*Sin[(-e + Pi/2 - f*x)/2]^8 - 1214
4*d^3*Gamma[3/2 - m]*Hypergeometric2F1[3/2, 3/2 - m, 11/2, Sin[(-e + Pi/2 -
 f*x)/2]^2]*Sin[(-e + Pi/2 - f*x)/2]^8 - 744*d^2*(c + d)*(-3 + 2*m)*Gamma[3
/2 - m]*Hypergeometric2F1[5/2, 5/2 - m, 13/2, Sin[(-e + Pi/2 - f*x)/2]^2]*S
in[(-e + Pi/2 - f*x)/2]^8 - 9504*d^3*Gamma[3/2 - m]*HypergeometricPFQ[{3/2,
 2, 3/2 - m}, {1, 11/2}, Sin[(-e + Pi/2 - f*x)/2]^2]*Sin[(-e + Pi/2 - f*x)/
2]^8 - 1056*d^2*(c + d)*(-3 + 2*m)*Gamma[3/2 - m]*HypergeometricPFQ[{5/2, 3
, 5/2 - m}, {2, 13/2}, Sin[(-e + Pi/2 - f*x)/2]^2]*Sin[(-e + Pi/2 - f*x)/2]
^8 - 2112*d^3*Gamma[3/2 - m]*HypergeometricPFQ[{3/2, 2, 2, 3/2 - m}, {1, 1,
 11/2}, Sin[(-e + Pi/2 - f*x)/2]^2]*Sin[(-e + Pi/2 - f*x)/2]^8 - 384*d^2*(c
 + d)*(-3 + 2*m)*Gamma[3/2 - m]*HypergeometricPFQ[{5/2, 3, 3, 5/2 - m}, {2,
 2, 13/2}, Sin[(-e + Pi/2 - f*x)/2]^2]*Sin[(-e + Pi/2 - f*x)/2]^8 + 368*d^3
*(-3 + 2*m)*Gamma[3/2 - m]*Hypergeometric2F1[5/2, 5/2 - m, 13/2, Sin[(-e +
Pi/2 - f*x)/2]^2]*Sin[(-e + Pi/2 - f*x)/2]^10 + 576*d^3*(-3 + 2*m)*Gamma[3/
2 - m]*HypergeometricPFQ[{5/2, 3, 5/2 - m}, {2, 13/2}, Sin[(-e + Pi/2 - f*x
)/2]^2]*Sin[(-e + Pi/2 - f*x)/2]^10 + 256*d^3*(-3 + 2*m)*Gamma[3/2 - m]*Hyp
ergeometricPFQ[{5/2, 3, 3, 5/2 - m}, {2, 2, 13/2}, Sin[(-e + Pi/2 - f*x)/2]
^2]*Sin[(-e + Pi/2 - f*x)/2]^10))
```

Maple [F] time = 2.727, size = 0, normalized size = 0.

$$\int \left(a + a\sin\left(fx + e\right)\right)^{m} \left(c + d\sin\left(fx + e\right)\right)^{3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^3,x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\sin\left(fx + e\right) + c\right)^{3} \left(a\sin\left(fx + e\right) + a\right)^{m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] integrate((d*sin(f*x + e) + c)^3*(a*sin(f*x + e) + a)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(3\,cd^{2}\cos\left(fx + e\right)^{2} - c^{3} - 3\,cd^{2} + \left(d^{3}\cos\left(fx + e\right)^{2} - 3\,c^{2}d - d^{3}\right)\sin\left(fx + e\right)\right)\left(a\sin\left(fx + e\right) + a\right)^{m}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] integral(-(3*c*d^2*cos(f*x + e)^2 - c^3 - 3*c*d^2 + (d^3*cos(f*x + e)^2 - 3
*c^2*d - d^3)*sin(f*x + e))*(a*sin(f*x + e) + a)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(c+d*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\sin\left(fx+e\right)+c\right)^{3}\left(a\sin\left(fx+e\right)+a\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate((d*sin(f*x + e) + c)^3*(a*sin(f*x + e) + a)^m, x)
```

3.608 $\int (a + a\sin(e + fx))^m (c + d\sin(e + fx))^2 \, dx$

Optimal. Leaf size=193

$$-\frac{2^{m+\frac{1}{2}}\left(c^2\left(m^2+3m+2\right)+2cdm(m+2)+d^2\left(m^2+m+1\right)\right)\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^m\,{}_2F}{f(m+1)(m+2)}$$

```
[Out] (d*(d - 2*c*(2 + m))*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(f*(1 + m)*(2 + m
)) - (2^(1/2 + m)*(2*c*d*m*(2 + m) + d^2*(1 + m + m^2) + c^2*(2 + 3*m + m^2
))*Cos[e + f*x]*Hypergeometric2F1[1/2, 1/2 - m, 3/2, (1 - Sin[e + f*x])/2]*
(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Sin[e + f*x])^m)/(f*(1 + m)*(2 + m)) -
 (d^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(1 + m))/(a*f*(2 + m))
```

Rubi [A] time = 0.268819, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2761, 2751, 2652, 2651}

$$-\frac{2^{m+\frac{1}{2}}\left(c^2\left(m^2+3m+2\right)+2cdm(m+2)+d^2\left(m^2+m+1\right)\right)\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^m\,{}_2F}{f(m+1)(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^2,x]
```

```
[Out] (d*(d - 2*c*(2 + m))*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(f*(1 + m)*(2 + m
)) - (2^(1/2 + m)*(2*c*d*m*(2 + m) + d^2*(1 + m + m^2) + c^2*(2 + 3*m + m^2
))*Cos[e + f*x]*Hypergeometric2F1[1/2, 1/2 - m, 3/2, (1 - Sin[e + f*x])/2]*
(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Sin[e + f*x])^m)/(f*(1 + m)*(2 + m)) -
 (d^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(1 + m))/(a*f*(2 + m))
```

Rule 2761

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^2, x_Symbol] :> -Simp[(d^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m
 + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*S
imp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2652

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Sin[c + d*x])^FracPart[n])/(1 + (b*Sin[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Sin[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2651

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(2^(n +
 1/2)*a^(n - 1/2)*b*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1*(1
 - (b*Sin[c + d*x])/a))/2])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))^m(c+d\sin(e+fx))^2\,dx=-\frac{d^2\cos(e+fx)(a+a\sin(e+fx))^{1+m}}{af(2+m)}+\frac{\int (a+a\sin(e+fx))^m\left(a\left(d^2(1+\right.\right.}{}$$

$$=\frac{d(d-2c(2+m))\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+m)(2+m)}-\frac{d^2\cos(e+fx)(a+a}{af(2+}$$

$$=\frac{d(d-2c(2+m))\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+m)(2+m)}-\frac{d^2\cos(e+fx)(a+a}{af(2+}$$

$$=\frac{d(d-2c(2+m))\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+m)(2+m)}-\frac{2^{\frac{1}{2}+m}\left(2cdm(2+m)\right.}{}$$

Mathematica [B] time = 103.011, size = 1774, normalized size = 9.19

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^2,x]
```

```
[Out] (-2*(Cos[(-e + Pi/2 - f*x)/2]^2)^(1/2 - m)*(1 - Sin[(-e + Pi/2 - f*x)/2]^2)
^(-1/2 + m)*(c + d - 2*d*Sin[(-e + Pi/2 - f*x)/2]^2)^2*(4*Gamma[3/2 - m]*Hy
pergeometricPFQ[{3/2, 2, 3/2 - m}, {1, 9/2}, Sin[(-e + Pi/2 - f*x)/2]^2]*Si
n[(-e + Pi/2 - f*x)/2]^2*(c + d - 2*d*Sin[(-e + Pi/2 - f*x)/2]^2)^2 + 16*Ga
mma[3/2 - m]*Hypergeometric2F1[3/2, 3/2 - m, 9/2, Sin[(-e + Pi/2 - f*x)/2]^
2]*Sin[(-e + Pi/2 - f*x)/2]^2*(c^2 + c*d*(2 - 3*Sin[(-e + Pi/2 - f*x)/2]^2)
 + d^2*(1 - 3*Sin[(-e + Pi/2 - f*x)/2]^2 + 2*Sin[(-e + Pi/2 - f*x)/2]^4)) +
 7*Gamma[1/2 - m]*Hypergeometric2F1[1/2, 1/2 - m, 7/2, Sin[(-e + Pi/2 - f*x
)/2]^2]*(15*c^2 + 10*c*d*(3 - 2*Sin[(-e + Pi/2 - f*x)/2]^2) + d^2*(15 - 20*
Sin[(-e + Pi/2 - f*x)/2]^2 + 12*Sin[(-e + Pi/2 - f*x)/2]^4)))*(a + a*Sin[e
+ f*x])^m*Tan[(-e + Pi/2 - f*x)/2])/(f*(4*Gamma[3/2 - m]*HypergeometricPFQ[
{3/2, 2, 3/2 - m}, {1, 9/2}, Sin[(-e + Pi/2 - f*x)/2]^2]*Sin[(-e + Pi/2 - f
*x)/2]^2*(c + d - 2*d*Sin[(-e + Pi/2 - f*x)/2]^2)^2 + 16*Gamma[3/2 - m]*Hyp
ergeometric2F1[3/2, 3/2 - m, 9/2, Sin[(-e + Pi/2 - f*x)/2]^2]*Sin[(-e + Pi/
2 - f*x)/2]^2*(c^2 + c*d*(2 - 3*Sin[(-e + Pi/2 - f*x)/2]^2) + d^2*(1 - 3*Si
n[(-e + Pi/2 - f*x)/2]^2 + 2*Sin[(-e + Pi/2 - f*x)/2]^4)) + 7*Gamma[1/2 - m
]*Hypergeometric2F1[1/2, 1/2 - m, 7/2, Sin[(-e + Pi/2 - f*x)/2]^2]*(15*c^2
+ 10*c*d*(3 - 2*Sin[(-e + Pi/2 - f*x)/2]^2) + d^2*(15 - 20*Sin[(-e + Pi/2 -
 f*x)/2]^2 + 12*Sin[(-e + Pi/2 - f*x)/2]^4)) + (2*Sin[(-e + Pi/2 - f*x)/2]^
2*(-48*d*Gamma[3/2 - m]*HypergeometricPFQ[{3/2, 2, 3/2 - m}, {1, 9/2}, Sin[
(-e + Pi/2 - f*x)/2]^2]*Sin[(-e + Pi/2 - f*x)/2]^2*(c + d - 2*d*Sin[(-e + P
i/2 - f*x)/2]^2) + 12*Gamma[3/2 - m]*HypergeometricPFQ[{3/2, 2, 3/2 - m}, {
1, 9/2}, Sin[(-e + Pi/2 - f*x)/2]^2]*(c + d - 2*d*Sin[(-e + Pi/2 - f*x)/2]^
2)^2 - 4*(-3 + 2*m)*Gamma[3/2 - m]*HypergeometricPFQ[{5/2, 3, 5/2 - m}, {2,
 11/2}, Sin[(-e + Pi/2 - f*x)/2]^2]*Sin[(-e + Pi/2 - f*x)/2]^2*(c + d - 2*d
*Sin[(-e + Pi/2 - f*x)/2]^2)^2 + 48*d*Gamma[3/2 - m]*Hypergeometric2F1[3/2,
 3/2 - m, 9/2, Sin[(-e + Pi/2 - f*x)/2]^2]*Sin[(-e + Pi/2 - f*x)/2]*(-3*c*S
in[(-e + Pi/2 - f*x)/2] + d*Sin[(-e + Pi/2 - f*x)/2]*(-3 + 4*Sin[(-e + Pi/2
 - f*x)/2]^2)) + 84*d*Gamma[1/2 - m]*Hypergeometric2F1[1/2, 1/2 - m, 7/2, S
in[(-e + Pi/2 - f*x)/2]^2]*(-5*c + d*(-5 + 6*Sin[(-e + Pi/2 - f*x)/2]^2)) +
```

```
 48*Gamma[3/2 - m]*Hypergeometric2F1[3/2, 3/2 - m, 9/2, Sin[(-e + Pi/2 - f*
x)/2]^2]*(c^2 + c*d*(2 - 3*Sin[(-e + Pi/2 - f*x)/2]^2) + d^2*(1 - 3*Sin[(-e
 + Pi/2 - f*x)/2]^2 + 2*Sin[(-e + Pi/2 - f*x)/2]^4)) - 8*(-3 + 2*m)*Gamma[3
/2 - m]*Hypergeometric2F1[5/2, 5/2 - m, 11/2, Sin[(-e + Pi/2 - f*x)/2]^2]*S
in[(-e + Pi/2 - f*x)/2]^2*(c^2 + c*d*(2 - 3*Sin[(-e + Pi/2 - f*x)/2]^2) + d
^2*(1 - 3*Sin[(-e + Pi/2 - f*x)/2]^2 + 2*Sin[(-e + Pi/2 - f*x)/2]^4)) + 3*(
1/2 - m)*Gamma[1/2 - m]*Hypergeometric2F1[3/2, 3/2 - m, 9/2, Sin[(-e + Pi/2
 - f*x)/2]^2]*(15*c^2 + 10*c*d*(3 - 2*Sin[(-e + Pi/2 - f*x)/2]^2) + d^2*(15
 - 20*Sin[(-e + Pi/2 - f*x)/2]^2 + 12*Sin[(-e + Pi/2 - f*x)/2]^4))))/3))
```

Maple [F] time = 2.927, size = 0, normalized size = 0.

$$\int \left(a+a\sin\left(fx+e\right)\right)^{m}\left(c+d\sin\left(fx+e\right)\right)^{2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^2,x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\sin\left(fx+e\right)+c\right)^{2}\left(a\sin\left(fx+e\right)+a\right)^{m}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate((d*sin(f*x + e) + c)^2*(a*sin(f*x + e) + a)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(d^{2}\cos\left(fx+e\right)^{2}-2\,cd\sin\left(fx+e\right)-c^{2}-d^{2}\right)\left(a\sin\left(fx+e\right)+a\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral(-(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)*(a*sin(f*x
+ e) + a)^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\left(\sin\left(e+fx\right)+1\right)\right)^{m}\left(c+d\sin\left(e+fx\right)\right)^{2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(c+d*sin(f*x+e))**2,x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**m*(c + d*sin(e + f*x))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\sin\left(fx+e\right)+c\right)^{2}\left(a\sin\left(fx+e\right)+a\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*sin(f*x + e) + c)^2*(a*sin(f*x + e) + a)^m, x)
```

3.609 $\int (a+a\sin(e+fx))^m(c+d\sin(e+fx))\,dx$

Optimal. Leaf size=117

$$-\frac{2^{m+\frac{1}{2}}(cm+c+dm)\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{f(m+1)}-dc$$

```
[Out] -((d*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(f*(1 + m))) - (2^(1/2 + m)*(c +
c*m + d*m)*Cos[e + f*x]*Hypergeometric2F1[1/2, 1/2 - m, 3/2, (1 - Sin[e + f
*x])/2]*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Sin[e + f*x])^m)/(f*(1 + m))
```

Rubi [A] time = 0.0796591, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {2751, 2652, 2651}

$$-\frac{2^{m+\frac{1}{2}}(cm+c+dm)\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{f(m+1)}-dc$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x]),x]
```


```
[Out] -((d*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(f*(1 + m))) - (2^(1/2 + m)*(c +
c*m + d*m)*Cos[e + f*x]*Hypergeometric2F1[1/2, 1/2 - m, 3/2, (1 - Sin[e + f
*x])/2]*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Sin[e + f*x])^m)/(f*(1 + m))
```


Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2652

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Sin[c + d*x])^FracPart[n])/(1 + (b*Sin[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Sin[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2651

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(2^(n +
 1/2)*a^(n - 1/2)*b*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1*(1
 - (b*Sin[c + d*x])/a))/2])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))^m(c+d\sin(e+fx))\,dx = -\frac{d\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+m)} + \frac{(c+cm+dm)\int (a+a\sin(e+fx))^{\prime}}{1+m}$$
$$= -\frac{d\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+m)} + \frac{\left((c+cm+dm)(1+\sin(e+fx))^{-m}(\right.}{}$$
$$= -\frac{d\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+m)} - \frac{2^{\frac{1}{2}+m}(c+cm+dm)\cos(e+fx)\,{}_2F_1}{}$$

Mathematica [C] time = 1.83272, size = 275, normalized size = 2.35

$$-\frac{\sin^{-2m}\left(\frac{1}{4}(2e+2fx+\pi)\right)(a(\sin(e+fx)+1))^m\left(\frac{2\sqrt{2}c\sin\left(\frac{1}{4}(2e+2fx-\pi)\right)\cos^{2m+1}\left(\frac{1}{4}(2e+2fx-\pi)\right){}_2F_1\left(\frac{1}{2},m+\frac{1}{2};m+\frac{3}{2};\sin^2\left(\frac{1}{4}(2e+2fx+\pi)\right)\right)}{(2m+1)\sqrt{1-\sin(e+fx)}}+\right.}{f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x]),x]
```

```
[Out] -(((a*(1 + Sin[e + f*x]))^m*(((-1)^(1/4)*2^(-1 - 2*m)*d*(-(((-1)^(3/4)*(I +
 E^(I*(e + f*x))))/E^((I/2)*(e + f*x))))^(1 + 2*m)*(E^((2*I)*(e + f*x))*(-1
 + m)*Hypergeometric2F1[1, m, -m, (-I)/E^(I*(e + f*x))] - (1 + m)*Hypergeom
etric2F1[1, 2 + m, 2 - m, (-I)/E^(I*(e + f*x))]))/(E^(((3*I)/2)*(e + f*x))*
(-1 + m^2)) + (2*Sqrt[2]*c*Cos[(2*e - Pi + 2*f*x)/4]^(1 + 2*m)*Hypergeometr
ic2F1[1/2, 1/2 + m, 3/2 + m, Sin[(2*e + Pi + 2*f*x)/4]^2]*Sin[(2*e - Pi + 2
*f*x)/4])/((1 + 2*m)*Sqrt[1 - Sin[e + f*x]])))/(f*Sin[(2*e + Pi + 2*f*x)/4]
^(2*m)))
```

Maple [F] time = 0.976, size = 0, normalized size = 0.

$$\int \left(a+a\sin\left(fx+e\right)\right)^m\left(c+d\sin\left(fx+e\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e)),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\sin\left(fx+e\right)+c\right)\left(a\sin\left(fx+e\right)+a\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((d*sin(f*x + e) + c)*(a*sin(f*x + e) + a)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(d\sin\left(fx+e\right)+c\right)\left(a\sin\left(fx+e\right)+a\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral((d*sin(f*x + e) + c)*(a*sin(f*x + e) + a)^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\left(\sin\left(e+fx\right)+1\right)\right)^{m}\left(c+d\sin\left(e+fx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(c+d*sin(f*x+e)),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**m*(c + d*sin(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\sin\left(fx+e\right)+c\right)\left(a\sin\left(fx+e\right)+a\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*sin(f*x + e) + c)*(a*sin(f*x + e) + a)^m, x)
```

3.610 $\int (a + a\sin(e + fx))^m\,dx$

Optimal. Leaf size=74

$$-\frac{2^{m+\frac{1}{2}}\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{f}$$

```
[Out] -((2^(1/2 + m)*Cos[e + f*x]*Hypergeometric2F1[1/2, 1/2 - m, 3/2, (1 - Sin[e
 + f*x])/2]*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Sin[e + f*x])^m)/f)
```

Rubi [A] time = 0.0299459, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2652, 2651}

$$-\frac{2^{m+\frac{1}{2}}\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m,x]
```


```
[Out] -((2^(1/2 + m)*Cos[e + f*x]*Hypergeometric2F1[1/2, 1/2 - m, 3/2, (1 - Sin[e
 + f*x])/2]*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Sin[e + f*x])^m)/f)
```


Rule 2652

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Sin[c + d*x])^FracPart[n])/(1 + (b*Sin[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Sin[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] && !IntegerQ[2*n] && !GtQ[a, 0]
```

Rule 2651

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(2^(n +
 1/2)*a^(n - 1/2)*b*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1*(1
 - (b*Sin[c + d*x])/a))/2])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[2*n] && GtQ[a, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))^m\,dx = \left((1+\sin(e+fx))^{-m}(a+a\sin(e+fx))^m\right)\int (1+\sin(e+fx))^m\,dx$$

$$= -\frac{2^{\frac{1}{2}+m}\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)(1+\sin(e+fx))^{-\frac{1}{2}-m}(a+a\sin(e+\ldots}{f}$$

Mathematica [A] time = 0.141116, size = 90, normalized size = 1.22

$$\frac{\sqrt{2}\cos(e+fx)(a(\sin(e+fx)+1))^m\,{}_2F_1\left(\frac{1}{2},m+\frac{1}{2};m+\frac{3}{2};\frac{1}{4}\cos^2(e+fx)\csc^2\left(\frac{1}{4}(2e+2fx-\pi)\right)\right)}{(2fm+f)\sqrt{1-\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^m,x]
```

```
[Out] (Sqrt[2]*Cos[e + f*x]*Hypergeometric2F1[1/2, 1/2 + m, 3/2 + m, (Cos[e + f*x
]^2*Csc[(2*e - Pi + 2*f*x)/4]^2)/4]*(a*(1 + Sin[e + f*x]))^m)/((f + 2*f*m)*
Sqrt[1 - Sin[e + f*x]])
```

Maple [F] time = 0.005, size = 0, normalized size = 0.

$$\int \left(a+a\sin\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m,x)
```

```
[Out] int((a+a*sin(f*x+e))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a\sin\left(fx+e\right)+a\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(e+fx\right)+a\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m,x)
```

```
[Out] Integral((a*sin(e + f*x) + a)**m, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m, x)
```

3.611 $\int \frac{(a+a\sin(e+fx))^m}{c+d\sin(e+fx)}\,dx$

Optimal. Leaf size=100

$$\frac{\sqrt{2}\cos(e+fx)(a\sin(e+fx)+a)^m F_1\left(m+\frac{1}{2};\frac{1}{2},1;m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1),-\frac{d(\sin(e+fx)+1)}{c-d}\right)}{f(2m+1)(c-d)\sqrt{1-\sin(e+fx)}}$$

```
[Out] (Sqrt[2]*AppellF1[1/2 + m, 1/2, 1, 3/2 + m, (1 + Sin[e + f*x])/2, -((d*(1 +
 Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/((c - d)*f*(
1 + 2*m)*Sqrt[1 - Sin[e + f*x]])
```

Rubi [A] time = 0.128288, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2788, 137, 136}

$$\frac{\sqrt{2}\cos(e+fx)(a\sin(e+fx)+a)^m F_1\left(m+\frac{1}{2};\frac{1}{2},1;m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1),-\frac{d(\sin(e+fx)+1)}{c-d}\right)}{f(2m+1)(c-d)\sqrt{1-\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m/(c + d*Sin[e + f*x]),x]
```

```
[Out] (Sqrt[2]*AppellF1[1/2 + m, 1/2, 1, 3/2 + m, (1 + Sin[e + f*x])/2, -((d*(1 +
 Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/((c - d)*f*(
1 + 2*m)*Sqrt[1 - Sin[e + f*x]])
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x
)^n)/Sqrt[a - b*x], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rule 137

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !GtQ[b/(b*c
- a*d), 0] &&  !SimplerQ[c + d*x, a + b*x]
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^m}{c+d\sin(e+fx)}\,dx = \frac{\left(a^2\cos(e+fx)\right)\text{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}}{\sqrt{a-ax}(c+dx)}\,dx, x, \sin(e+fx)\right)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\left(a^2\cos(e+fx)\sqrt{\frac{a-a\sin(e+fx)}{a}}\right)\text{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}}{\sqrt{\frac{1}{2}-\frac{x}{2}}(c+dx)}\,dx, x, \sin(e+fx)\right)}{\sqrt{2}f(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\sqrt{2}F_1\left(\frac{1}{2}+m;\frac{1}{2},1;\frac{3}{2}+m;\frac{1}{2}(1+\sin(e+fx)),-\frac{d(1+\sin(e+fx))}{c-d}\right)\cos(e+fx)(a+a\sin(e+fx))^{m}}{(c-d)f(1+2m)\sqrt{1-\sin(e+fx)}}$$

Mathematica [B] time = 1.17552, size = 363, normalized size = 3.63

$$-\frac{6(c+d)\cot\left(\frac{1}{4}(2e+2fx+\pi)\right)\sin^2\left(\frac{1}{4}(2e+2fx+\pi)\right)^{\frac{1}{2}-m}\cos^2\left(\frac{1}{4}(2e+\right.}{f(c+d\sin(e+fx))\left(\sin^2\left(\frac{1}{4}(2e+2fx-\pi)\right)\right)\left(4dF_1\left(\frac{3}{2};\frac{1}{2}-m,2;\frac{5}{2};\cos^2\left(\frac{1}{4}(2e+2fx+\pi)\right),\frac{2d\sin^2\left(\frac{1}{4}(2e+2fx-\pi)\right)}{c+d}\right)-(2m-\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m/(c + d*Sin[e + f*x]),x]
```

```
[Out] (-6*(c + d)*AppellF1[1/2, 1/2 - m, 1, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*
d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)]*(Cos[(2*e - Pi + 2*f*x)/4]^2)^(-1/2
 + m)*Cot[(2*e + Pi + 2*f*x)/4]*(a*(1 + Sin[e + f*x]))^m*(Sin[(2*e + Pi + 2
*f*x)/4]^2)^(1/2 - m))/(f*(c + d*Sin[e + f*x])*(3*(c + d)*AppellF1[1/2, 1/2
 - m, 1, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2
)/(c + d)] + (4*d*AppellF1[3/2, 1/2 - m, 2, 5/2, Cos[(2*e + Pi + 2*f*x)/4]^
2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)] - (c + d)*(-1 + 2*m)*AppellF1
[3/2, 3/2 - m, 1, 5/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*
f*x)/4]^2)/(c + d)])*Sin[(2*e - Pi + 2*f*x)/4]^2))
```

Maple [F] time = 0.819, size = 0, normalized size = 0.

$$\int \frac{\left(a+a\sin\left(fx+e\right)\right)^m}{c+d\sin\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m/(c+d*sin(f*x+e)),x)
```

```
[Out] int((a+a*sin(f*x+e))^m/(c+d*sin(f*x+e)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^m}{d\sin\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c+d*sin(f*x+e)),x, algorithm="maxima")

[Out] integrate((a*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a\sin\left(fx+e\right)+a\right)^{m}}{d\sin\left(fx+e\right)+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c+d*sin(f*x+e)),x, algorithm="fricas")

[Out] integral((a*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m/(c+d*sin(f*x+e)),x)

[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{m}}{d\sin\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c+d*sin(f*x+e)),x, algorithm="giac")

[Out] integrate((a*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c), x)
```

3.612 $\int \frac{(a+a\sin(e+fx))^m}{(c+d\sin(e+fx))^2}\,dx$

Optimal. Leaf size=100

$$\frac{\sqrt{2}\cos(e+fx)(a\sin(e+fx)+a)^m F_1\left(m+\frac{1}{2};\frac{1}{2},2;m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1),-\frac{d(\sin(e+fx)+1)}{c-d}\right)}{f(2m+1)(c-d)^2\sqrt{1-\sin(e+fx)}}$$

```
[Out] (Sqrt[2]*AppellF1[1/2 + m, 1/2, 2, 3/2 + m, (1 + Sin[e + f*x])/2, -((d*(1 +
 Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/((c - d)^2*f
*(1 + 2*m)*Sqrt[1 - Sin[e + f*x]])
```

Rubi [A] time = 0.125784, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2788, 137, 136}

$$\frac{\sqrt{2}\cos(e+fx)(a\sin(e+fx)+a)^m F_1\left(m+\frac{1}{2};\frac{1}{2},2;m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1),-\frac{d(\sin(e+fx)+1)}{c-d}\right)}{f(2m+1)(c-d)^2\sqrt{1-\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m/(c + d*Sin[e + f*x])^2,x]
```


```
[Out] (Sqrt[2]*AppellF1[1/2 + m, 1/2, 2, 3/2 + m, (1 + Sin[e + f*x])/2, -((d*(1 +
 Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/((c - d)^2*f
*(1 + 2*m)*Sqrt[1 - Sin[e + f*x]])
```


Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x
)^n)/Sqrt[a - b*x], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rule 137

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !GtQ[b/(b*c
- a*d), 0] &&  !SimplerQ[c + d*x, a + b*x]
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^m}{(c+d\sin(e+fx))^2}\,dx = \frac{\left(a^2\cos(e+fx)\right)\text{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}}{\sqrt{a-ax}(c+dx)^2}\,dx, x, \sin(e+fx)\right)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\left(a^2\cos(e+fx)\sqrt{\frac{a-a\sin(e+fx)}{a}}\right)\text{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}}{\sqrt{\frac{1}{2}-\frac{x}{2}}(c+dx)^2}\,dx, x, \sin(e+fx)\right)}{\sqrt{2}f(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\sqrt{2}F_1\left(\frac{1}{2}+m;\frac{1}{2},2;\frac{3}{2}+m;\frac{1}{2}(1+\sin(e+fx)),-\frac{d(1+\sin(e+fx))}{c-d}\right)\cos(e+fx)(a+a\sin(e+f}{(c-d)^2 f(1+2m)\sqrt{1-\sin(e+fx)}}$$

Mathematica [B] time = 1.30564, size = 363, normalized size = 3.63

$$-\frac{6(c+d)\cot\left(\frac{1}{4}(2e+2fx+\pi)\right)\sin^2\left(\frac{1}{4}(2e+2fx+\pi)\right)^{\frac{1}{2}-m}\cos^2\left(\frac{1}{4}(2\right.}{f(c+d\sin(e+fx))^2\left(\sin^2\left(\frac{1}{4}(2e+2fx-\pi)\right)\left(8dF_1\left(\frac{3}{2};\frac{1}{2}-m,3;\frac{5}{2};\cos^2\left(\frac{1}{4}(2e+2fx+\pi)\right),\frac{2d\sin^2\left(\frac{1}{4}(2e+2fx-\pi)\right)}{c+d}\right)-(2\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m/(c + d*Sin[e + f*x])^2,x]
```

```
[Out] (-6*(c + d)*AppellF1[1/2, 1/2 - m, 2, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*
d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)]*(Cos[(2*e - Pi + 2*f*x)/4]^2)^(-1/2
 + m)*Cot[(2*e + Pi + 2*f*x)/4]*(a*(1 + Sin[e + f*x]))^m*(Sin[(2*e + Pi + 2
*f*x)/4]^2)^(1/2 - m))/(f*(c + d*Sin[e + f*x])^2*(3*(c + d)*AppellF1[1/2, 1
/2 - m, 2, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]
^2)/(c + d)] + (8*d*AppellF1[3/2, 1/2 - m, 3, 5/2, Cos[(2*e + Pi + 2*f*x)/4
]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)] - (c + d)*(-1 + 2*m)*Appell
F1[3/2, 3/2 - m, 2, 5/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi +
2*f*x)/4]^2)/(c + d)])*Sin[(2*e - Pi + 2*f*x)/4]^2))
```

Maple [F] time = 0.744, size = 0, normalized size = 0.

$$\int \frac{\left(a+a\sin\left(fx+e\right)\right)^m}{\left(c+d\sin\left(fx+e\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m/(c+d*sin(f*x+e))^2,x)
```

```
[Out] int((a+a*sin(f*x+e))^m/(c+d*sin(f*x+e))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^m}{\left(d\sin\left(fx+e\right)+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c+d*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(a\sin\left(fx+e\right)+a\right)^{m}}{d^{2}\cos\left(fx+e\right)^{2}-2\,cd\sin\left(fx+e\right)-c^{2}-d^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c+d*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral(-(a*sin(f*x + e) + a)^m/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) -
 c^2 - d^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m/(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{m}}{\left(d\sin\left(fx+e\right)+c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c+d*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c)^2, x)
```

3.613 $\int \frac{(a+a\sin(e+fx))^m}{(c+d\sin(e+fx))^3} dx$

Optimal. Leaf size=100

$$\frac{\sqrt{2}\cos(e+fx)(a\sin(e+fx)+a)^m F_1\left(m+\frac{1}{2};\frac{1}{2},3;m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1),-\frac{d(\sin(e+fx)+1)}{c-d}\right)}{f(2m+1)(c-d)^3\sqrt{1-\sin(e+fx)}}$$

```
[Out] (Sqrt[2]*AppellF1[1/2 + m, 1/2, 3, 3/2 + m, (1 + Sin[e + f*x])/2, -((d*(1 +
 Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/((c - d)^3*f
*(1 + 2*m)*Sqrt[1 - Sin[e + f*x]])
```

Rubi [A] time = 0.126223, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2788, 137, 136}

$$\frac{\sqrt{2}\cos(e+fx)(a\sin(e+fx)+a)^m F_1\left(m+\frac{1}{2};\frac{1}{2},3;m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1),-\frac{d(\sin(e+fx)+1)}{c-d}\right)}{f(2m+1)(c-d)^3\sqrt{1-\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m/(c + d*Sin[e + f*x])^3,x]
```


```
[Out] (Sqrt[2]*AppellF1[1/2 + m, 1/2, 3, 3/2 + m, (1 + Sin[e + f*x])/2, -((d*(1 +
 Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/((c - d)^3*f
*(1 + 2*m)*Sqrt[1 - Sin[e + f*x]])
```


Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x
)^n)/Sqrt[a - b*x], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rule 137

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !GtQ[b/(b*c
- a*d), 0] &&  !SimplerQ[c + d*x, a + b*x]
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^m}{(c+d\sin(e+fx))^3}\,dx = \frac{\left(a^2\cos(e+fx)\right)\text{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}}{\sqrt{a-ax}(c+dx)^3}\,dx, x, \sin(e+fx)\right)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\left(a^2\cos(e+fx)\sqrt{\frac{a-a\sin(e+fx)}{a}}\right)\text{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}}{\sqrt{\frac{1}{2}-\frac{x}{2}}(c+dx)^3}\,dx, x, \sin(e+fx)\right)}{\sqrt{2}f(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\sqrt{2}F_1\left(\frac{1}{2}+m;\frac{1}{2},3;\frac{3}{2}+m;\frac{1}{2}(1+\sin(e+fx)),-\frac{d(1+\sin(e+fx))}{c-d}\right)\cos(e+fx)(a+a\sin(e+fx))^{}}{(c-d)^3f(1+2m)\sqrt{1-\sin(e+fx)}}$$

Mathematica [B] time = 1.43038, size = 363, normalized size = 3.63

$$-\frac{6(c+d)\cot\left(\frac{1}{4}(2e+2fx+\pi)\right)\sin^2\left(\frac{1}{4}(2e+2fx+\pi)\right)^{\frac{1}{2}-m}\cos^2\left(\frac{1}{4}(2e\cdot\right.}{f(c+d\sin(e+fx))^3\left(\sin^2\left(\frac{1}{4}(2e+2fx-\pi)\right)\right)\left(12dF_1\left(\frac{3}{2};\frac{1}{2}-m,4;\frac{5}{2};\cos^2\left(\frac{1}{4}(2e+2fx+\pi)\right),\frac{2d\sin^2\left(\frac{1}{4}(2e+2fx-\pi)\right)}{c+d}\right)-(2m\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m/(c + d*Sin[e + f*x])^3,x]
```

```
[Out] (-6*(c + d)*AppellF1[1/2, 1/2 - m, 3, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*
d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)]*(Cos[(2*e - Pi + 2*f*x)/4]^2)^(-1/2
 + m)*Cot[(2*e + Pi + 2*f*x)/4]*(a*(1 + Sin[e + f*x]))^m*(Sin[(2*e + Pi + 2
*f*x)/4]^2)^(1/2 - m))/(f*(c + d*Sin[e + f*x])^3*(3*(c + d)*AppellF1[1/2, 1
/2 - m, 3, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]
^2)/(c + d)] + (12*d*AppellF1[3/2, 1/2 - m, 4, 5/2, Cos[(2*e + Pi + 2*f*x)/
4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)] - (c + d)*(-1 + 2*m)*Appel
lF1[3/2, 3/2 - m, 3, 5/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi +
 2*f*x)/4]^2)/(c + d)])*Sin[(2*e - Pi + 2*f*x)/4]^2))
```

Maple [F] time = 0.942, size = 0, normalized size = 0.

$$\int \frac{\left(a+a\sin\left(fx+e\right)\right)^m}{\left(c+d\sin\left(fx+e\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m/(c+d*sin(f*x+e))^3,x)
```

```
[Out] int((a+a*sin(f*x+e))^m/(c+d*sin(f*x+e))^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^m}{\left(d\sin\left(fx+e\right)+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c+d*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(a\sin\left(fx+e\right)+a\right)^{m}}{3\,cd^{2}\cos\left(fx+e\right)^{2}-c^{3}-3\,cd^{2}+\left(d^{3}\cos\left(fx+e\right)^{2}-3\,c^{2}d-d^{3}\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c+d*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] integral(-(a*sin(f*x + e) + a)^m/(3*c*d^2*cos(f*x + e)^2 - c^3 - 3*c*d^2 +
(d^3*cos(f*x + e)^2 - 3*c^2*d - d^3)*sin(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m/(c+d*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a\sin\left(fx+e\right)+a\right)^{m}}{\left(d\sin\left(fx+e\right)+c\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c+d*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c)^3, x)
```

3.614 $\int (a + a\sin(e + fx))^m (c + d\sin(e + fx))^{5/2}\,dx$

Optimal. Leaf size=138

$$\frac{\sqrt{2}(c-d)^2\cos(e+fx)(a\sin(e+fx)+a)^m\sqrt{c+d\sin(e+fx)}F_1\left(m+\frac{1}{2};\frac{1}{2},-\frac{5}{2};m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1),-\frac{d(\sin(e+fx)+1)}{c-d}\right)}{f(2m+1)\sqrt{1-\sin(e+fx)}\sqrt{\frac{c+d\sin(e+fx)}{c-d}}}$$

```
[Out] (Sqrt[2]*(c - d)^2*AppellF1[1/2 + m, 1/2, -5/2, 3/2 + m, (1 + Sin[e + f*x])
/2, -((d*(1 + Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*
Sqrt[c + d*Sin[e + f*x]])/(f*(1 + 2*m)*Sqrt[1 - Sin[e + f*x]]*Sqrt[(c + d*S
in[e + f*x])/(c - d)])
```

Rubi [A] time = 0.186949, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2788, 140, 139, 138}

$$\frac{\sqrt{2}(c-d)^2\cos(e+fx)(a\sin(e+fx)+a)^m\sqrt{c+d\sin(e+fx)}F_1\left(m+\frac{1}{2};\frac{1}{2},-\frac{5}{2};m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1),-\frac{d(\sin(e+fx)+1)}{c-d}\right)}{f(2m+1)\sqrt{1-\sin(e+fx)}\sqrt{\frac{c+d\sin(e+fx)}{c-d}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (Sqrt[2]*(c - d)^2*AppellF1[1/2 + m, 1/2, -5/2, 3/2 + m, (1 + Sin[e + f*x])
/2, -((d*(1 + Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*
Sqrt[c + d*Sin[e + f*x]])/(f*(1 + 2*m)*Sqrt[1 - Sin[e + f*x]]*Sqrt[(c + d*S
in[e + f*x])/(c - d)])
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x
)^n)/Sqrt[a - b*x], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rule 140

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &&  !GtQ[b/
(b*c - a*d), 0] &&  !SimplerQ[c + d*x, a + b*x] &&  !SimplerQ[e + f*x, a +
b*x]
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
```

```
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int (a+a\sin(e+fx))^m(c+d\sin(e+fx))^{5/2}\,dx = \frac{\left(a^2\cos(e+fx)\right)\text{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}(c+dx)^{5/2}}{\sqrt{a-ax}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\left(a^2\cos(e+fx)\sqrt{\frac{a-a\sin(e+fx)}{a}}\right)\text{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}(c+dx)^{5/2}}{\sqrt{\frac{1}{2}-\frac{x}{2}}}\,dx, x, \sin(e\right.}{\sqrt{2}f(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\left((ac-ad)^2\cos(e+fx)\sqrt{\frac{a-a\sin(e+fx)}{a}}\sqrt{c+d\sin(e+fx)}\right)\text{Subst}\left(\int \right.}{\sqrt{2}f(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\sqrt{2}(c-d)^2F_1\left(\frac{1}{2}+m;\frac{1}{2},-\frac{5}{2};\frac{3}{2}+m;\frac{1}{2}(1+\sin(e+fx)),-\frac{d(1+\sin(e+fx))}{c-d}\right.}{f(1+2m)\sqrt{1-\sin(e+fx)}}$$

Mathematica [B] time = 1.86188, size = 365, normalized size = 2.64

$$-\frac{3\sqrt{2}(c+d)\sqrt{\sin(e+fx)+1}\tan\left(\frac{1}{4}(2e+2fx-\pi)\right)(a(\text{s}}{f\sqrt{\cos^2\left(\frac{1}{4}(2e+2fx-\pi)\right)}\left(\sin^2\left(\frac{1}{4}(2e+2fx-\pi)\right)\left(10dF_1\left(\frac{3}{2};\frac{1}{2}-m,-\frac{3}{2};\frac{5}{2};\cos^2\left(\frac{1}{4}(2e+2fx+\pi)\right),\frac{2d\sin^2\left(\frac{1}{4}(2e+2fx\right.}{c+d}\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (-3*Sqrt[2]*(c + d)*AppellF1[1/2, 1/2 - m, -5/2, 3/2, Cos[(2*e + Pi + 2*f*x
)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)]*Sqrt[1 + Sin[e + f*x]]*(
a*(1 + Sin[e + f*x]))^m*(c + d*Sin[e + f*x])^(5/2)*Tan[(2*e - Pi + 2*f*x)/4
])/(f*Sqrt[Cos[(2*e - Pi + 2*f*x)/4]^2]*(-3*(c + d)*AppellF1[1/2, 1/2 - m,
-5/2, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(
c + d)] + (10*d*AppellF1[3/2, 1/2 - m, -3/2, 5/2, Cos[(2*e + Pi + 2*f*x)/4]
^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)] + (c + d)*(-1 + 2*m)*AppellF
1[3/2, 3/2 - m, -5/2, 5/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi
+ 2*f*x)/4]^2)/(c + d)])*Sin[(2*e - Pi + 2*f*x)/4]^2))
```

Maple [F] time = 0.178, size = 0, normalized size = 0.

$$\int \left(a+a\sin\left(fx+e\right)\right)^{m}\left(c+d\sin\left(fx+e\right)\right)^{\frac{5}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(5/2),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}\left(a\sin\left(fx+e\right)+a\right)^{m}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(5/2)*(a*sin(f*x + e) + a)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(d^{2}\cos\left(fx+e\right)^{2}-2\,cd\sin\left(fx+e\right)-c^{2}-d^{2}\right)\sqrt{d\sin\left(fx+e\right)+c}\left(a\sin\left(fx+e\right)+a\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)*sqrt(d*sin(
f*x + e) + c)*(a*sin(f*x + e) + a)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(c+d*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] sage2
```

3.615 $\int (a+a\sin(e+fx))^m(c+d\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=136

$$\frac{\sqrt{2}(c-d)\cos(e+fx)(a\sin(e+fx)+a)^m\sqrt{c+d\sin(e+fx)}F_1\left(m+\frac{1}{2};\frac{1}{2},-\frac{3}{2};m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1),-\frac{d(\sin(e+fx)+1)}{c-d}\right)}{f(2m+1)\sqrt{1-\sin(e+fx)}\sqrt{\frac{c+d\sin(e+fx)}{c-d}}}$$

```
[Out] (Sqrt[2]*(c - d)*AppellF1[1/2 + m, 1/2, -3/2, 3/2 + m, (1 + Sin[e + f*x])/2
, -((d*(1 + Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*Sq
rt[c + d*Sin[e + f*x]])/(f*(1 + 2*m)*Sqrt[1 - Sin[e + f*x]]*Sqrt[(c + d*Sin
[e + f*x])/(c - d)])
```

Rubi [A] time = 0.184006, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2788, 140, 139, 138}

$$\frac{\sqrt{2}(c-d)\cos(e+fx)(a\sin(e+fx)+a)^m\sqrt{c+d\sin(e+fx)}F_1\left(m+\frac{1}{2};\frac{1}{2},-\frac{3}{2};m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1),-\frac{d(\sin(e+fx)+1)}{c-d}\right)}{f(2m+1)\sqrt{1-\sin(e+fx)}\sqrt{\frac{c+d\sin(e+fx)}{c-d}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(3/2),x]
```



```
[Out] (Sqrt[2]*(c - d)*AppellF1[1/2 + m, 1/2, -3/2, 3/2 + m, (1 + Sin[e + f*x])/2
, -((d*(1 + Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*Sq
rt[c + d*Sin[e + f*x]])/(f*(1 + 2*m)*Sqrt[1 - Sin[e + f*x]]*Sqrt[(c + d*Sin
[e + f*x])/(c - d)])
```


Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x
)^n)/Sqrt[a - b*x], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rule 140

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &&  !GtQ[b/
(b*c - a*d), 0] &&  !SimplerQ[c + d*x, a + b*x] &&  !SimplerQ[e + f*x, a +
b*x]
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
```

```
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int (a+a\sin(e+fx))^m(c+d\sin(e+fx))^{3/2}\,dx = \frac{\left(a^2\cos(e+fx)\right)\text{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}(c+dx)^{3/2}}{\sqrt{a-ax}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\left(a^2\cos(e+fx)\sqrt{\frac{a-a\sin(e+fx)}{a}}\right)\text{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}(c+dx)^{3/2}}{\sqrt{\frac{1}{2}-\frac{x}{2}}}\,dx, x, \sin(\right.}{\sqrt{2}f(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\left(a(ac-ad)\cos(e+fx)\sqrt{\frac{a-a\sin(e+fx)}{a}}\sqrt{c+d\sin(e+fx)}\right)\text{Subst}\left(\int\right.}{\sqrt{2}f(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\sqrt{2}(c-d)F_1\left(\frac{1}{2}+m;\frac{1}{2},-\frac{3}{2};\frac{3}{2}+m;\frac{1}{2}(1+\sin(e+fx)),-\frac{d(1+\sin(e+fx))}{c-d}\right.}{f(1+2m)\sqrt{1-\sin(e+fx)}}$$

Mathematica [B] time = 1.29823, size = 365, normalized size = 2.68

$$-\frac{3\sqrt{2}(c+d)\sqrt{\sin(e+fx)+1}\tan\left(\frac{1}{4}(2e+2fx-\pi)\right)(a(\text{s}}{f\sqrt{\cos^2\left(\frac{1}{4}(2e+2fx-\pi)\right)}\left(\sin^2\left(\frac{1}{4}(2e+2fx-\pi)\right)\right)\left(6dF_1\left(\frac{3}{2};\frac{1}{2}-m,-\frac{1}{2};\frac{5}{2};\cos^2\left(\frac{1}{4}(2e+2fx+\pi)\right),\frac{2d\sin^2\left(\frac{1}{4}(2e+2fx-\right.}{c+d}\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-3*Sqrt[2]*(c + d)*AppellF1[1/2, 1/2 - m, -3/2, 3/2, Cos[(2*e + Pi + 2*f*x
)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)]*Sqrt[1 + Sin[e + f*x]]*(
a*(1 + Sin[e + f*x]))^m*(c + d*Sin[e + f*x])^(3/2)*Tan[(2*e - Pi + 2*f*x)/4
])/(f*Sqrt[Cos[(2*e - Pi + 2*f*x)/4]^2]*(-3*(c + d)*AppellF1[1/2, 1/2 - m,
-3/2, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(
c + d)] + (6*d*AppellF1[3/2, 1/2 - m, -1/2, 5/2, Cos[(2*e + Pi + 2*f*x)/4]^
2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)] + (c + d)*(-1 + 2*m)*AppellF1
[3/2, 3/2 - m, -3/2, 5/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi +
 2*f*x)/4]^2)/(c + d)])*Sin[(2*e - Pi + 2*f*x)/4]^2))
```

Maple [F] time = 0.175, size = 0, normalized size = 0.

$$\int \left(a + a\sin\left(fx + e\right)\right)^{m} \left(c + d\sin\left(fx + e\right)\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(3/2),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\sin\left(fx + e\right) + c\right)^{\frac{3}{2}} \left(a\sin\left(fx + e\right) + a\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(3/2)*(a*sin(f*x + e) + a)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(d\sin\left(fx + e\right) + c\right)^{\frac{3}{2}} \left(a\sin\left(fx + e\right) + a\right)^{m}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((d*sin(f*x + e) + c)^(3/2)*(a*sin(f*x + e) + a)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(c+d*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: AttributeError
```

3.616 $\int (a+a\sin(e+fx))^m\sqrt{c+d\sin(e+fx)}\,dx$

Optimal. Leaf size=131

$$\frac{\sqrt{2}\cos(e+fx)(a\sin(e+fx)+a)^m\sqrt{c+d\sin(e+fx)}F_1\left(m+\frac{1}{2};\frac{1}{2},-\frac{1}{2};m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1),-\frac{d(\sin(e+fx)+1)}{c-d}\right)}{f(2m+1)\sqrt{1-\sin(e+fx)}\sqrt{\frac{c+d\sin(e+fx)}{c-d}}}$$

```
[Out] (Sqrt[2]*AppellF1[1/2 + m, 1/2, -1/2, 3/2 + m, (1 + Sin[e + f*x])/2, -((d*(
1 + Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*Sqrt[c + d
*Sin[e + f*x]])/(f*(1 + 2*m)*Sqrt[1 - Sin[e + f*x]]*Sqrt[(c + d*Sin[e + f*x
])/(c - d)])
```

Rubi [A] time = 0.164852, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2788, 140, 139, 138}

$$\frac{\sqrt{2}\cos(e+fx)(a\sin(e+fx)+a)^m\sqrt{c+d\sin(e+fx)}F_1\left(m+\frac{1}{2};\frac{1}{2},-\frac{1}{2};m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1),-\frac{d(\sin(e+fx)+1)}{c-d}\right)}{f(2m+1)\sqrt{1-\sin(e+fx)}\sqrt{\frac{c+d\sin(e+fx)}{c-d}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*Sqrt[c + d*Sin[e + f*x]],x]
```


```
[Out] (Sqrt[2]*AppellF1[1/2 + m, 1/2, -1/2, 3/2 + m, (1 + Sin[e + f*x])/2, -((d*(
1 + Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*Sqrt[c + d
*Sin[e + f*x]])/(f*(1 + 2*m)*Sqrt[1 - Sin[e + f*x]]*Sqrt[(c + d*Sin[e + f*x
])/(c - d)])
```


Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x
)^n)/Sqrt[a - b*x], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rule 140

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &&  !GtQ[b/
(b*c - a*d), 0] &&  !SimplerQ[c + d*x, a + b*x] &&  !SimplerQ[e + f*x, a +
b*x]
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
```

```
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int (a+a\sin(e+fx))^m\sqrt{c+d\sin(e+fx)}\,dx = \frac{\left(a^2\cos(e+fx)\right)\text{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}\sqrt{c+dx}}{\sqrt{a-ax}}\,dx,x,\sin(e+fx)\right)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\left(a^2\cos(e+fx)\sqrt{\frac{a-a\sin(e+fx)}{a}}\right)\text{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}\sqrt{c+dx}}{\sqrt{\frac{1}{2}-\frac{x}{2}}}\,dx,x,\sin(e+fx)\right.}{\sqrt{2}f(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\left(a^2\cos(e+fx)\sqrt{\frac{a-a\sin(e+fx)}{a}}\sqrt{c+d\sin(e+fx)}\right)\text{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}\sqrt{\frac{ac}{ac-a}}}{\sqrt{\frac{1}{2}-\frac{x}{2}}}\right.}{\sqrt{2}f(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}\sqrt{\frac{a(c+d\sin(e}{ac-ad}}}$$

$$= \frac{\sqrt{2}F_1\left(\frac{1}{2}+m;\frac{1}{2},-\frac{1}{2};\frac{3}{2}+m;\frac{1}{2}(1+\sin(e+fx)),-\frac{d(1+\sin(e+fx))}{c-d}\right)\cos(e+fx}{f(1+2m)\sqrt{1-\sin(e+fx)}\sqrt{\frac{c+d\sin(e+}{c-d}}}$$

Mathematica [B] time = 1.09604, size = 365, normalized size = 2.79

$$-\frac{3\sqrt{2}(c+d)\sqrt{\sin(e+fx)+1}\tan\left(\frac{1}{4}(2e+2fx-\pi)\right)(a(\sin(}{f\sqrt{\cos^2\left(\frac{1}{4}(2e+2fx-\pi)\right)}\left(\sin^2\left(\frac{1}{4}(2e+2fx-\pi)\right)\left(2dF_1\left(\frac{3}{2};\frac{1}{2}-m,\frac{1}{2};\frac{5}{2};\cos^2\left(\frac{1}{4}(2e+2fx+\pi)\right),\frac{2d\sin^2\left(\frac{1}{4}(2e+2fx-\pi)\right)}{c+d}\right)\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] (-3*Sqrt[2]*(c + d)*AppellF1[1/2, 1/2 - m, -1/2, 3/2, Cos[(2*e + Pi + 2*f*x
)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)]*Sqrt[1 + Sin[e + f*x]]*(
a*(1 + Sin[e + f*x]))^m*Sqrt[c + d*Sin[e + f*x]]*Tan[(2*e - Pi + 2*f*x)/4])
/(f*Sqrt[Cos[(2*e - Pi + 2*f*x)/4]^2]*(-3*(c + d)*AppellF1[1/2, 1/2 - m, -1
/2, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c
+ d)] + (2*d*AppellF1[3/2, 1/2 - m, 1/2, 5/2, Cos[(2*e + Pi + 2*f*x)/4]^2,
(2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)] + (c + d)*(-1 + 2*m)*AppellF1[3/
2, 3/2 - m, -1/2, 5/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*
f*x)/4]^2)/(c + d)])*Sin[(2*e - Pi + 2*f*x)/4]^2))
```

Maple [F] time = 0.168, size = 0, normalized size = 0.

$$\int \left(a + a\sin\left(fx + e\right)\right)^{m} \sqrt{c + d\sin\left(fx + e\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(1/2),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d\sin\left(fx + e\right) + c}\left(a\sin\left(fx + e\right) + a\right)^{m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*sin(f*x + e) + c)*(a*sin(f*x + e) + a)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{d\sin\left(fx + e\right) + c}\left(a\sin\left(fx + e\right) + a\right)^{m}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*sin(f*x + e) + c)*(a*sin(f*x + e) + a)^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\left(\sin\left(e + fx\right) + 1\right)\right)^{m} \sqrt{c + d\sin\left(e + fx\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**m*sqrt(c + d*sin(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d\sin\left(fx + e\right) + c}\left(a\sin\left(fx + e\right) + a\right)^{m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*sin(f*x + e) + c)*(a*sin(f*x + e) + a)^m, x)
```

3.617 $\int \frac{(a+a\sin(e+fx))^m}{\sqrt{c+d\sin(e+fx)}}\,dx$

Optimal. Leaf size=131

$$\frac{\sqrt{2}\cos(e+fx)(a\sin(e+fx)+a)^m\sqrt{\frac{c+d\sin(e+fx)}{c-d}}F_1\left(m+\frac{1}{2};\frac{1}{2},\frac{1}{2};m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1),-\frac{d(\sin(e+fx)+1)}{c-d}\right)}{f(2m+1)\sqrt{1-\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}$$

```
[Out] (Sqrt[2]*AppellF1[1/2 + m, 1/2, 1/2, 3/2 + m, (1 + Sin[e + f*x])/2, -((d*(1
 + Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*Sqrt[(c + d
*Sin[e + f*x])/(c - d)])/(f*(1 + 2*m)*Sqrt[1 - Sin[e + f*x]]*Sqrt[c + d*Sin
[e + f*x]])
```

Rubi [A] time = 0.163589, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2788, 140, 139, 138}

$$\frac{\sqrt{2}\cos(e+fx)(a\sin(e+fx)+a)^m\sqrt{\frac{c+d\sin(e+fx)}{c-d}}F_1\left(m+\frac{1}{2};\frac{1}{2},\frac{1}{2};m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1),-\frac{d(\sin(e+fx)+1)}{c-d}\right)}{f(2m+1)\sqrt{1-\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m/Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] (Sqrt[2]*AppellF1[1/2 + m, 1/2, 1/2, 3/2 + m, (1 + Sin[e + f*x])/2, -((d*(1
 + Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*Sqrt[(c + d
*Sin[e + f*x])/(c - d)])/(f*(1 + 2*m)*Sqrt[1 - Sin[e + f*x]]*Sqrt[c + d*Sin
[e + f*x]])
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x
)^n)/Sqrt[a - b*x], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rule 140

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &&  !GtQ[b/
(b*c - a*d), 0] &&  !SimplerQ[c + d*x, a + b*x] &&  !SimplerQ[e + f*x, a +
b*x]
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
```

```
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^m}{\sqrt{c+d\sin(e+fx)}}\,dx = \frac{\left(a^2\cos(e+fx)\right)\text{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}}{\sqrt{a-ax}\sqrt{c+dx}}\,dx,x,\sin(e+fx)\right)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\left(a^2\cos(e+fx)\sqrt{\frac{a-a\sin(e+fx)}{a}}\right)\text{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}}{\sqrt{\frac{1}{2}-\frac{x}{2}}\sqrt{c+dx}}\,dx,x,\sin(e+fx)\right)}{\sqrt{2}f(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\left(a^2\cos(e+fx)\sqrt{\frac{a-a\sin(e+fx)}{a}}\sqrt{\frac{a(c+d\sin(e+fx))}{ac-ad}}\right)\text{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}}{\sqrt{\frac{1}{2}-\frac{x}{2}}\sqrt{\frac{ac}{ac-ad}+\frac{adx}{ac-ad}}}\,dx,x,\sin(e+f\right.}{\sqrt{2}f(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}$$

$$= \frac{\sqrt{2}F_1\left(\frac{1}{2}+m;\frac{1}{2},\frac{1}{2};\frac{3}{2}+m;\frac{1}{2}(1+\sin(e+fx)),-\frac{d(1+\sin(e+fx))}{c-d}\right)\cos(e+fx)(a+a\sin(e+f}{f(1+2m)\sqrt{1-\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}$$

Mathematica [B] time = 1.16155, size = 373, normalized size = 2.85

$$-\frac{6(c+d)\cot\left(\frac{1}{4}(2e+2fx+\pi)\right)\sin^2\left(\frac{1}{4}(2e+2fx+\pi)\right)^{\frac{1}{2}-m}\cos^2\left(\frac{1}{4}(2\right.}{f\sqrt{c+d\sin(e+fx)}\left(\sin^2\left(\frac{1}{4}(2e+2fx-\pi)\right)\right)\left(2dF_1\left(\frac{3}{2};\frac{1}{2}-m,\frac{3}{2};\frac{5}{2};\cos^2\left(\frac{1}{4}(2e+2fx+\pi)\right),\frac{2d\sin^2\left(\frac{1}{4}(2e+2fx-\pi)\right)}{c+d}\right)-(2\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m/Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] (-6*(c + d)*AppellF1[1/2, 1/2 - m, 1/2, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (
2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)]*(Cos[(2*e - Pi + 2*f*x)/4]^2)^(-1
/2 + m)*Cot[(2*e + Pi + 2*f*x)/4]*(a*(1 + Sin[e + f*x]))^m*(Sin[(2*e + Pi +
 2*f*x)/4]^2)^(1/2 - m))/(f*Sqrt[c + d*Sin[e + f*x]]*(3*(c + d)*AppellF1[1/
2, 1/2 - m, 1/2, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f
*x)/4]^2)/(c + d)] + (2*d*AppellF1[3/2, 1/2 - m, 3/2, 5/2, Cos[(2*e + Pi +
2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)] - (c + d)*(-1 + 2*m
)*AppellF1[3/2, 3/2 - m, 1/2, 5/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2
*e - Pi + 2*f*x)/4]^2)/(c + d)])*Sin[(2*e - Pi + 2*f*x)/4]^2))
```

Maple [F] time = 0.171, size = 0, normalized size = 0.

$$\int \left(a+a\sin\left(fx+e\right)\right)^{m} \frac{1}{\sqrt{c+d\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m/(c+d*sin(f*x+e))^(1/2),x)
```

```
[Out] int((a+a*sin(f*x+e))^m/(c+d*sin(f*x+e))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{m}}{\sqrt{d\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c+d*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m/sqrt(d*sin(f*x + e) + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(a\sin\left(fx+e\right)+a\right)^{m}}{\sqrt{d\sin\left(fx+e\right)+c}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c+d*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)^m/sqrt(d*sin(f*x + e) + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{m}}{\sqrt{c+d\sin\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m/(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**m/sqrt(c + d*sin(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\sin(fx+e)+a)^m}{\sqrt{d\sin(fx+e)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c+d*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m/sqrt(d*sin(f*x + e) + c), x)
```

3.618 $\int \frac{(a+a\sin(e+fx))^m}{(c+d\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=138

$$\frac{\sqrt{2}\cos(e+fx)(a\sin(e+fx)+a)^m\sqrt{\frac{c+d\sin(e+fx)}{c-d}}F_1\left(m+\frac{1}{2};\frac{1}{2},\frac{3}{2};m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1),-\frac{d(\sin(e+fx)+1)}{c-d}\right)}{f(2m+1)(c-d)\sqrt{1-\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}$$

```
[Out] (Sqrt[2]*AppellF1[1/2 + m, 1/2, 3/2, 3/2 + m, (1 + Sin[e + f*x])/2, -((d*(1
 + Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*Sqrt[(c + d
*Sin[e + f*x])/(c - d)])/((c - d)*f*(1 + 2*m)*Sqrt[1 - Sin[e + f*x]]*Sqrt[c
 + d*Sin[e + f*x]])
```

Rubi [A] time = 0.182239, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2788, 140, 139, 138}

$$\frac{\sqrt{2}\cos(e+fx)(a\sin(e+fx)+a)^m\sqrt{\frac{c+d\sin(e+fx)}{c-d}}F_1\left(m+\frac{1}{2};\frac{1}{2},\frac{3}{2};m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1),-\frac{d(\sin(e+fx)+1)}{c-d}\right)}{f(2m+1)(c-d)\sqrt{1-\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m/(c + d*Sin[e + f*x])^(3/2),x]
```


```
[Out] (Sqrt[2]*AppellF1[1/2 + m, 1/2, 3/2, 3/2 + m, (1 + Sin[e + f*x])/2, -((d*(1
 + Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*Sqrt[(c + d
*Sin[e + f*x])/(c - d)])/((c - d)*f*(1 + 2*m)*Sqrt[1 - Sin[e + f*x]]*Sqrt[c
 + d*Sin[e + f*x]])
```


Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x
)^n)/Sqrt[a - b*x], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rule 140

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && !GtQ[b/
(b*c - a*d), 0] && !SimplerQ[c + d*x, a + b*x] && !SimplerQ[e + f*x, a +
b*x]
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
```

```
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^m}{(c+d\sin(e+fx))^{3/2}}\,dx = \frac{\left(a^2\cos(e+fx)\right)\text{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}}{\sqrt{a-ax}(c+dx)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\left(a^2\cos(e+fx)\sqrt{\frac{a-a\sin(e+fx)}{a}}\right)\text{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}}{\sqrt{\frac{1}{2}-\frac{x}{2}}(c+dx)^{3/2}}\,dx, x, \sin(e+fx)\right)}{\sqrt{2}f(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\left(a^3\cos(e+fx)\sqrt{\frac{a-a\sin(e+fx)}{a}}\sqrt{\frac{a(c+d\sin(e+fx))}{ac-ad}}\right)\text{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}}{\sqrt{\frac{1}{2}-\frac{x}{2}}\left(\frac{ac}{ac-ad}+\frac{adx}{ac-ad}\right)^{3/2}}\,dx, x, \sin(e\right.}{\sqrt{2}(ac-ad)f(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}$$

$$= \frac{\sqrt{2}F_1\left(\frac{1}{2}+m;\frac{1}{2},\frac{3}{2};\frac{3}{2}+m;\frac{1}{2}(1+\sin(e+fx)),-\frac{d(1+\sin(e+fx))}{c-d}\right)\cos(e+fx)(a+a\sin(e+}{(c-d)f(1+2m)\sqrt{1-\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}$$

Mathematica [B] time = 1.38428, size = 373, normalized size = 2.7

$$-\frac{6(c+d)\cot\left(\frac{1}{4}(2e+2fx+\pi)\right)\sin^2\left(\frac{1}{4}(2e+2fx+\pi)\right)^{\frac{1}{2}-m}\cos^2\left(\frac{1}{4}\right.}{f(c+d\sin(e+fx))^{3/2}\left(\sin^2\left(\frac{1}{4}(2e+2fx-\pi)\right)\right)\left(6dF_1\left(\frac{3}{2};\frac{1}{2}-m,\frac{5}{2};\frac{5}{2};\cos^2\left(\frac{1}{4}(2e+2fx+\pi)\right),\frac{2d\sin^2\left(\frac{1}{4}(2e+2fx-\pi)\right)}{c+d}\right)-\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m/(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-6*(c + d)*AppellF1[1/2, 1/2 - m, 3/2, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (
2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)]*(Cos[(2*e - Pi + 2*f*x)/4]^2)^(-1
/2 + m)*Cot[(2*e + Pi + 2*f*x)/4]*(a*(1 + Sin[e + f*x]))^m*(Sin[(2*e + Pi +
 2*f*x)/4]^2)^(1/2 - m))/(f*(c + d*Sin[e + f*x])^(3/2)*(3*(c + d)*AppellF1[
1/2, 1/2 - m, 3/2, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2
*f*x)/4]^2)/(c + d)] + (6*d*AppellF1[3/2, 1/2 - m, 5/2, 5/2, Cos[(2*e + Pi
+ 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)] - (c + d)*(-1 + 2
*m)*AppellF1[3/2, 3/2 - m, 3/2, 5/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[
(2*e - Pi + 2*f*x)/4]^2)/(c + d)])*Sin[(2*e - Pi + 2*f*x)/4]^2))
```

Maple [F] time = 0.169, size = 0, normalized size = 0.

$$\int \left(a+a\sin\left(fx+e\right)\right)^{m}\left(c+d\sin\left(fx+e\right)\right)^{-\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m/(c+d*sin(f*x+e))^(3/2),x)
```

```
[Out] int((a+a*sin(f*x+e))^m/(c+d*sin(f*x+e))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{m}}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c+d*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{d\sin\left(fx+e\right)+c}\left(a\sin\left(fx+e\right)+a\right)^{m}}{d^{2}\cos\left(fx+e\right)^{2}-2\,cd\sin\left(fx+e\right)-c^{2}-d^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c+d*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(d*sin(f*x + e) + c)*(a*sin(f*x + e) + a)^m/(d^2*cos(f*x + e)
^2 - 2*c*d*sin(f*x + e) - c^2 - d^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{m}}{\left(c+d\sin\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m/(c+d*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**m/(c + d*sin(e + f*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\sin(fx+e)+a)^m}{(d\sin(fx+e)+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c+d*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c)^(3/2), x)
```

3.619 $\int \frac{(a+a\sin(e+fx))^m}{(c+d\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=138

$$\frac{\sqrt{2}\cos(e+fx)(a\sin(e+fx)+a)^m\sqrt{\frac{c+d\sin(e+fx)}{c-d}}F_1\left(m+\frac{1}{2};\frac{1}{2},\frac{5}{2};m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1),-\frac{d(\sin(e+fx)+1)}{c-d}\right)}{f(2m+1)(c-d)^2\sqrt{1-\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}$$

```
[Out] (Sqrt[2]*AppellF1[1/2 + m, 1/2, 5/2, 3/2 + m, (1 + Sin[e + f*x])/2, -((d*(1
 + Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*Sqrt[(c + d
*Sin[e + f*x])/(c - d)])/((c - d)^2*f*(1 + 2*m)*Sqrt[1 - Sin[e + f*x]]*Sqrt
[c + d*Sin[e + f*x]])
```

Rubi [A] time = 0.180086, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2788, 140, 139, 138}

$$\frac{\sqrt{2}\cos(e+fx)(a\sin(e+fx)+a)^m\sqrt{\frac{c+d\sin(e+fx)}{c-d}}F_1\left(m+\frac{1}{2};\frac{1}{2},\frac{5}{2};m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1),-\frac{d(\sin(e+fx)+1)}{c-d}\right)}{f(2m+1)(c-d)^2\sqrt{1-\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m/(c + d*Sin[e + f*x])^(5/2),x]
```



```
[Out] (Sqrt[2]*AppellF1[1/2 + m, 1/2, 5/2, 3/2 + m, (1 + Sin[e + f*x])/2, -((d*(1
 + Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*Sqrt[(c + d
*Sin[e + f*x])/(c - d)])/((c - d)^2*f*(1 + 2*m)*Sqrt[1 - Sin[e + f*x]]*Sqrt
[c + d*Sin[e + f*x]])
```


Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x
)^n)/Sqrt[a - b*x], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rule 140

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && !GtQ[b/
(b*c - a*d), 0] && !SimplerQ[c + d*x, a + b*x] && !SimplerQ[e + f*x, a +
b*x]
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
```

```
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^m}{(c+d\sin(e+fx))^{5/2}}\,dx = \frac{\left(a^2\cos(e+fx)\right)\text{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}}{\sqrt{a-ax}(c+dx)^{5/2}}\,dx,x,\sin(e+fx)\right)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\left(a^2\cos(e+fx)\sqrt{\frac{a-a\sin(e+fx)}{a}}\right)\text{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}}{\sqrt{\frac{1}{2}-\frac{x}{2}}(c+dx)^{5/2}}\,dx,x,\sin(e+fx)\right)}{\sqrt{2}f(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\left(a^4\cos(e+fx)\sqrt{\frac{a-a\sin(e+fx)}{a}}\sqrt{\frac{a(c+d\sin(e+fx))}{ac-ad}}\right)\text{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}}{\sqrt{\frac{1}{2}-\frac{x}{2}}\left(\frac{ac}{ac-ad}+\frac{adx}{ac-ad}\right)^{5/2}}\,dx,x,\sin(e\right.}{\sqrt{2}(ac-ad)^2f(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}$$

$$= \frac{\sqrt{2}F_1\left(\frac{1}{2}+m;\frac{1}{2},\frac{5}{2};\frac{3}{2}+m;\frac{1}{2}(1+\sin(e+fx)),-\frac{d(1+\sin(e+fx))}{c-d}\right)\cos(e+fx)(a+a\sin(e+}{(c-d)^2f(1+2m)\sqrt{1-\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}$$

Mathematica [B] time = 1.55715, size = 373, normalized size = 2.7

$$-\frac{6(c+d)\cot\left(\frac{1}{4}(2e+2fx+\pi)\right)\sin^2\left(\frac{1}{4}(2e+2fx+\pi)\right)^{\frac{1}{2}-m}\cos^2\left(\frac{1}{4}\right.}{f(c+d\sin(e+fx))^{5/2}\left(\sin^2\left(\frac{1}{4}(2e+2fx-\pi)\right)\left(10dF_1\left(\frac{3}{2};\frac{1}{2}-m,\frac{7}{2};\frac{5}{2};\cos^2\left(\frac{1}{4}(2e+2fx+\pi)\right),\frac{2d\sin^2\left(\frac{1}{4}(2e+2fx-\pi)\right)}{c+d}\right)-\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m/(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (-6*(c + d)*AppellF1[1/2, 1/2 - m, 5/2, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (
2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)]*(Cos[(2*e - Pi + 2*f*x)/4]^2)^(-1
/2 + m)*Cot[(2*e + Pi + 2*f*x)/4]*(a*(1 + Sin[e + f*x]))^m*(Sin[(2*e + Pi +
 2*f*x)/4]^2)^(1/2 - m))/(f*(c + d*Sin[e + f*x])^(5/2)*(3*(c + d)*AppellF1[
1/2, 1/2 - m, 5/2, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2
*f*x)/4]^2)/(c + d)] + (10*d*AppellF1[3/2, 1/2 - m, 7/2, 5/2, Cos[(2*e + Pi
 + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)] - (c + d)*(-1 +
2*m)*AppellF1[3/2, 3/2 - m, 5/2, 5/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin
[(2*e - Pi + 2*f*x)/4]^2)/(c + d)])*Sin[(2*e - Pi + 2*f*x)/4]^2))
```

Maple [F] time = 0.175, size = 0, normalized size = 0.

$$\int \left(a+a\sin\left(fx+e\right)\right)^{m} \left(c+d\sin\left(fx+e\right)\right)^{-\frac{5}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m/(c+d*sin(f*x+e))^(5/2),x)
```

```
[Out] int((a+a*sin(f*x+e))^m/(c+d*sin(f*x+e))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{m}}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c+d*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{d\sin\left(fx+e\right)+c}\left(a\sin\left(fx+e\right)+a\right)^{m}}{3\,cd^{2}\cos\left(fx+e\right)^{2}-c^{3}-3\,cd^{2}+\left(d^{3}\cos\left(fx+e\right)^{2}-3\,c^{2}d-d^{3}\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c+d*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(d*sin(f*x + e) + c)*(a*sin(f*x + e) + a)^m/(3*c*d^2*cos(f*x
+ e)^2 - c^3 - 3*c*d^2 + (d^3*cos(f*x + e)^2 - 3*c^2*d - d^3)*sin(f*x + e))
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m/(c+d*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a \sin \left(fx+e\right)+a\right)^{m}}{\left(d \sin \left(fx+e\right)+c\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c+d*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c)^(5/2), x)
```

3.620 $\int (1+\sin(e+fx))^m(3+5\sin(e+fx))^{-1-m}\,dx$

Optimal. Leaf size=62

$$-\frac{4^{-m-1}\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},m+1;\frac{3}{2};\frac{1-\sin(e+fx)}{4(\sin(e+fx)+1)}\right)}{f(\sin(e+fx)+1)}$$

```
[Out] -((4^(-1 - m)*Cos[e + f*x]*Hypergeometric2F1[1/2, 1 + m, 3/2, (1 - Sin[e +
f*x])/(4*(1 + Sin[e + f*x]))])/(f*(1 + Sin[e + f*x])))
```

Rubi [A] time = 0.106098, antiderivative size = 111, normalized size of antiderivative = 1.79, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2788, 132}

$$-\frac{2^{-2m-1}\cos(e+fx)(\sin(e+fx)+1)^{m-1}\left(\frac{\sin(e+fx)+1}{5\sin(e+fx)+3}\right)^{\frac{1}{2}-m}(5\sin(e+fx)+3)^{-m}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};-\frac{1-\sin(e+fx)}{5\sin(e+fx)+3}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sin[e + f*x])^m*(3 + 5*Sin[e + f*x])^(-1 - m),x]
```

```
[Out] -((2^(-1 - 2*m)*Cos[e + f*x]*Hypergeometric2F1[1/2, 1/2 - m, 3/2, -((1 - Si
n[e + f*x])/(3 + 5*Sin[e + f*x]))]*(1 + Sin[e + f*x])^(-1 + m)*((1 + Sin[e
+ f*x])/(3 + 5*Sin[e + f*x]))^(1/2 - m))/(f*(3 + 5*Sin[e + f*x])^m))
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x
)^n)/Sqrt[a - b*x], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1)*
Hypergeometric2F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*
(e + f*x)))])/(((b*e - a*f)*(m + 1))*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*
(e + f*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] && !IntegerQ[n]
```

Rubi steps

$$\int (1+\sin(e+fx))^m(3+5\sin(e+fx))^{-1-m}\,dx=\frac{\cos(e+fx)\,\text{Subst}\left(\int\frac{(1+x)^{-\frac{1}{2}+m}(3+5x)^{-1-m}}{\sqrt{1-x}}\,dx,x,\sin(e+fx)\right)}{f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}$$

$$=-\frac{2^{-1-2m}\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};-\frac{1-\sin(e+fx)}{3+5\sin(e+fx)}\right)(1+\sin(e+fx))^{-1}}{f}$$

Mathematica [C] time = 1.41968, size = 238, normalized size = 3.84

$$\frac{4^m(\cosh(m\log(4))-\sinh(m\log(4)))(\sin(e+fx)+1)^m(5\sin(e+fx)+3)^{-m}(\sin(e+fx)+i\cos(e+fx)+1)\left(-\frac{2\cos(\cdots}{\sin\left(\frac{1}{4}(\cdots\right.}\right.}{f(2m+1)((2+i)\sin(e+fx)+(-1+2i)\cos\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + Sin[e + f*x])^m*(3 + 5*Sin[e + f*x])^(-1 - m),x]
```

```
[Out] (4^m*Hypergeometric2F1[1 + m, 1 + 2*m, 2*(1 + m), (4*Cos[(2*e - Pi + 2*f*x)
/4])/(2*Cos[(2*e - Pi + 2*f*x)/4] + Sin[(2*e - Pi + 2*f*x)/4])]*(1 + Sin[e
+ f*x])^m*(1 + I*Cos[e + f*x] + Sin[e + f*x])*(-((2*Cos[(2*e - Pi + 2*f*x)/
4] + Cos[(2*e + Pi + 2*f*x)/4])/(2*Cos[(2*e - Pi + 2*f*x)/4] + Sin[(2*e - P
i + 2*f*x)/4])))^m*(Cosh[m*Log[4]] - Sinh[m*Log[4]]))/(f*(1 + 2*m)*((2 - I)
 - (1 - 2*I)*Cos[e + f*x] + (2 + I)*Sin[e + f*x])*(3 + 5*Sin[e + f*x])^m)
```

Maple [F] time = 0.241, size = 0, normalized size = 0.

$$\int \left(1+\sin\left(fx+e\right)\right)^{m}\left(3+5\sin\left(fx+e\right)\right)^{-1-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+sin(f*x+e))^m*(3+5*sin(f*x+e))^(-1-m),x)
```

```
[Out] int((1+sin(f*x+e))^m*(3+5*sin(f*x+e))^(-1-m),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(5\sin\left(fx+e\right)+3\right)^{-m-1}\left(\sin\left(fx+e\right)+1\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(f*x+e))^m*(3+5*sin(f*x+e))^(-1-m),x, algorithm="maxima")
```

```
[Out] integrate((5*sin(f*x + e) + 3)^(-m - 1)*(sin(f*x + e) + 1)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(5\sin\left(fx+e\right)+3\right)^{-m-1}\left(\sin\left(fx+e\right)+1\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(f*x+e))^m*(3+5*sin(f*x+e))^(-1-m),x, algorithm="fricas")
```

```
[Out] integral((5*sin(f*x + e) + 3)^(-m - 1)*(sin(f*x + e) + 1)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(f*x+e))**m*(3+5*sin(f*x+e))**(-1-m),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(5 \sin \left(fx+e\right)+3\right)^{-m-1} \left(\sin \left(fx+e\right)+1\right)^{m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(f*x+e))^m*(3+5*sin(f*x+e))^(-1-m),x, algorithm="giac")
```

```
[Out] integrate((5*sin(f*x + e) + 3)^(-m - 1)*(sin(f*x + e) + 1)^m, x)
```

3.621 $\int (1+\sin(e+fx))^m(3+4\sin(e+fx))^{-1-m}\,dx$

Optimal. Leaf size=64

$$-\frac{\left(\frac{7}{2}\right)^{-m-1}\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},m+1;\frac{3}{2};\frac{1-\sin(e+fx)}{7(\sin(e+fx)+1)}\right)}{f(\sin(e+fx)+1)}$$

```
[Out] -(((7/2)^(-1 - m)*Cos[e + f*x]*Hypergeometric2F1[1/2, 1 + m, 3/2, (1 - Sin[
e + f*x])/(7*(1 + Sin[e + f*x]))])/(f*(1 + Sin[e + f*x])))
```

Rubi [A] time = 0.115772, antiderivative size = 122, normalized size of antiderivative = 1.91, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2788, 132}

$$-\frac{2^{m+\frac{1}{2}}7^{-m-\frac{1}{2}}\cos(e+fx)(\sin(e+fx)+1)^{m-1}\left(\frac{\sin(e+fx)+1}{4\sin(e+fx)+3}\right)^{\frac{1}{2}-m}(4\sin(e+fx)+3)^{-m}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};-\frac{1-\sin(e+fx)}{2(4\sin(e+fx)+3)}\right.}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sin[e + f*x])^m*(3 + 4*Sin[e + f*x])^(-1 - m),x]
```

```
[Out] -((2^(1/2 + m)*7^(-1/2 - m)*Cos[e + f*x]*Hypergeometric2F1[1/2, 1/2 - m, 3/
2, -(1 - Sin[e + f*x])/(2*(3 + 4*Sin[e + f*x]))]*(1 + Sin[e + f*x])^(-1 + m
)*((1 + Sin[e + f*x])/(3 + 4*Sin[e + f*x]))^(1/2 - m))/(f*(3 + 4*Sin[e + f*
x])^m))
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x
)^n)/Sqrt[a - b*x], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1)*
Hypergeometric2F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*
(e + f*x)))])/(((b*e - a*f)*(m + 1))*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*
(e + f*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] && !IntegerQ[n]
```

Rubi steps

$$\int (1+\sin(e+fx))^m(3+4\sin(e+fx))^{-1-m}\,dx=\frac{\cos(e+fx)\,\text{Subst}\left(\int\frac{(1+x)^{-\frac{1}{2}+m}(3+4x)^{-1-m}}{\sqrt{1-x}}\,dx,x,\sin(e+fx)\right)}{f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}$$

$$=-\frac{2^{\frac{1}{2}+m}7^{-\frac{1}{2}-m}\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};-\frac{1-\sin(e+fx)}{2(3+4\sin(e+fx))}\right)(1+\sin(\right.}{f}$$

Mathematica [A] time = 0.473336, size = 88, normalized size = 1.38

$$-\frac{2\,7^{-m-1}\cot\left(\frac{1}{4}(2e+2fx+\pi)\right)(\sin(e+fx)+1)^m\sin^2\left(\frac{1}{4}(2e+2fx+\pi)\right)^{-m}\,{}_2F_1\left(\frac{1}{2},m+1;\frac{3}{2};\frac{1}{7}\tan^2\left(\frac{1}{4}(2e+2fx-\pi)\right)\right)}{f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + Sin[e + f*x])^m*(3 + 4*Sin[e + f*x])^(-1 - m),x]
```

```
[Out] (-2*7^(-1 - m)*Cot[(2*e + Pi + 2*f*x)/4]*Hypergeometric2F1[1/2, 1 + m, 3/2,
 Tan[(2*e - Pi + 2*f*x)/4]^2/7]*(1 + Sin[e + f*x])^m)/(f*(Sin[(2*e + Pi + 2
*f*x)/4]^2)^m)
```

Maple [F] time = 0.257, size = 0, normalized size = 0.

$$\int \left(1+\sin\left(fx+e\right)\right)^{m} \left(3+4\sin\left(fx+e\right)\right)^{-1-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+sin(f*x+e))^m*(3+4*sin(f*x+e))^(-1-m),x)
```

```
[Out] int((1+sin(f*x+e))^m*(3+4*sin(f*x+e))^(-1-m),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(4\sin\left(fx+e\right)+3\right)^{-m-1}\left(\sin\left(fx+e\right)+1\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(f*x+e))^m*(3+4*sin(f*x+e))^(-1-m),x, algorithm="maxima")
```

```
[Out] integrate((4*sin(f*x + e) + 3)^(-m - 1)*(sin(f*x + e) + 1)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(4\sin\left(fx+e\right)+3\right)^{-m-1}\left(\sin\left(fx+e\right)+1\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(f*x+e))^m*(3+4*sin(f*x+e))^(-1-m),x, algorithm="fricas")
```

```
[Out] integral((4*sin(f*x + e) + 3)^(-m - 1)*(sin(f*x + e) + 1)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(f*x+e))**m*(3+4*sin(f*x+e))**(-1-m),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (4\sin(fx+e)+3)^{-m-1}(\sin(fx+e)+1)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(f*x+e))^m*(3+4*sin(f*x+e))^(-1-m),x, algorithm="giac")
```

```
[Out] integrate((4*sin(f*x + e) + 3)^(-m - 1)*(sin(f*x + e) + 1)^m, x)
```

3.622 $\int (1+\sin(e+fx))^m(3+3\sin(e+fx))^{-1-m}\,dx$

Optimal. Leaf size=28

$$-\frac{3^{-m-1}\cos(e+fx)}{f(\sin(e+fx)+1)}$$

```
[Out] -((3^(-1 - m)*Cos[e + f*x])/(f*(1 + Sin[e + f*x])))
```

Rubi [A] time = 0.0196123, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {22, 2648}

$$-\frac{3^{-m-1}\cos(e+fx)}{f(\sin(e+fx)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sin[e + f*x])^m*(3 + 3*Sin[e + f*x])^(-1 - m),x]
```

```
[Out] -((3^(-1 - m)*Cos[e + f*x])/(f*(1 + Sin[e + f*x])))
```

Rule 22

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}, x
] && EqQ[b*c - a*d, 0] && GtQ[b/d, 0] &&  !(IntegerQ[m] || IntegerQ[n])
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\begin{aligned}\int (1+\sin(e+fx))^m(3+3\sin(e+fx))^{-1-m}\,dx &= 3^{-m}\int \frac{1}{3+3\sin(e+fx)}\,dx\\ &= -\frac{3^{-1-m}\cos(e+fx)}{f(1+\sin(e+fx))}\end{aligned}$$

Mathematica [A] time = 0.037168, size = 45, normalized size = 1.61

$$\frac{2\,3^{-m-1}\sin\left(\frac{1}{2}(e+fx)\right)}{f\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sin[e + f*x])^m*(3 + 3*Sin[e + f*x])^(-1 - m),x]
```

```
[Out] (2*3^(-1 - m)*Sin[(e + f*x)/2])/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))
```

Maple [F] time = 0.22, size = 0, normalized size = 0.

$$\int \left(1+\sin \left(f x +e \right)\right)^{m} \left(3+3 \sin \left(f x +e \right)\right)^{-1-m}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+sin(f*x+e))^m*(3+3*sin(f*x+e))^(-1-m),x)
```

```
[Out] int((1+sin(f*x+e))^m*(3+3*sin(f*x+e))^(-1-m),x)
```

Maxima [A] time = 1.79773, size = 47, normalized size = 1.68

$$-\frac{2}{\left(3^{m+1} + \frac{3^{m+1}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(f*x+e))^m*(3+3*sin(f*x+e))^(-1-m),x, algorithm="maxima")
```

```
[Out] -2/((3^(m + 1) + 3^(m + 1)*sin(f*x + e)/(cos(f*x + e) + 1))*f)
```

Fricas [A] time = 1.00792, size = 132, normalized size = 4.71

$$-\frac{3^{-m-1}\left(\cos\left(fx+e\right)+1\right) - 3^{-m-1}\sin\left(fx+e\right)}{f\cos\left(fx+e\right) + f\sin\left(fx+e\right) + f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(f*x+e))^m*(3+3*sin(f*x+e))^(-1-m),x, algorithm="fricas")
```

```
[Out] -(3^(-m - 1)*(cos(f*x + e) + 1) - 3^(-m - 1)*sin(f*x + e))/(f*cos(f*x + e)
+ f*sin(f*x + e) + f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(f*x+e))**m*(3+3*sin(f*x+e))**(-1-m),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(3\sin\left(fx+e\right)+3\right)^{-m-1}\left(\sin\left(fx+e\right)+1\right)^{m}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(f*x+e))^m*(3+3*sin(f*x+e))^(-1-m),x, algorithm="giac")
```

```
[Out] integrate((3*sin(f*x + e) + 3)^(-m - 1)*(sin(f*x + e) + 1)^m, x)
```

3.623 $\int (1+\sin(e+fx))^m(3+2\sin(e+fx))^{-1-m}\,dx$

Optimal. Leaf size=122

$$-\frac{2^{m+\frac{1}{2}}5^{-m-\frac{1}{2}}\cos(e+fx)(\sin(e+fx)+1)^{m-1}\left(\frac{\sin(e+fx)+1}{2\sin(e+fx)+3}\right)^{\frac{1}{2}-m}(2\sin(e+fx)+3)^{-m}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1-\sin(e+fx)}{2(2\sin(e+fx)+3)}\right)}{f}$$

```
[Out] -((2^(1/2 + m)*5^(-1/2 - m)*Cos[e + f*x]*Hypergeometric2F1[1/2, 1/2 - m, 3/
2, (1 - Sin[e + f*x])/(2*(3 + 2*Sin[e + f*x]))]*(1 + Sin[e + f*x])^(-1 + m)
*((1 + Sin[e + f*x])/(3 + 2*Sin[e + f*x]))^(1/2 - m))/(f*(3 + 2*Sin[e + f*x
])^m))
```

Rubi [A] time = 0.114781, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2788, 132}

$$-\frac{2^{m+\frac{1}{2}}5^{-m-\frac{1}{2}}\cos(e+fx)(\sin(e+fx)+1)^{m-1}\left(\frac{\sin(e+fx)+1}{2\sin(e+fx)+3}\right)^{\frac{1}{2}-m}(2\sin(e+fx)+3)^{-m}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1-\sin(e+fx)}{2(2\sin(e+fx)+3)}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sin[e + f*x])^m*(3 + 2*Sin[e + f*x])^(-1 - m),x]
```

```
[Out] -((2^(1/2 + m)*5^(-1/2 - m)*Cos[e + f*x]*Hypergeometric2F1[1/2, 1/2 - m, 3/
2, (1 - Sin[e + f*x])/(2*(3 + 2*Sin[e + f*x]))]*(1 + Sin[e + f*x])^(-1 + m)
*((1 + Sin[e + f*x])/(3 + 2*Sin[e + f*x]))^(1/2 - m))/(f*(3 + 2*Sin[e + f*x
])^m))
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x
)^n)/Sqrt[a - b*x], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1)*
Hypergeometric2F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*
(e + f*x)))])/(((b*e - a*f)*(m + 1))*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*
(e + f*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int (1+\sin(e+fx))^m(3+2\sin(e+fx))^{-1-m}\,dx = \frac{\cos(e+fx)\,\mathrm{Subst}\left(\int \frac{(1+x)^{-\frac{1}{2}+m}(3+2x)^{-1-m}}{\sqrt{1-x}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}$$

$$= -\frac{2^{\frac{1}{2}+m}5^{-\frac{1}{2}-m}\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1-\sin(e+fx)}{2(3+2\sin(e+fx))}\right)(1+\sin(e+}{f}$$

Mathematica [A] time = 0.530042, size = 131, normalized size = 1.07

$$\frac{2\,5^{-m-1}\tan\left(\frac{1}{4}(2e+2fx-\pi)\right)(\sin(e+fx)+1)^m(2\sin(e+fx)+3)^{-m}\left((2\sin(e+fx)+3)\sec^2\left(\frac{1}{4}(2e+2fx-\pi)\right)\right)^m\ {}_2F}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sin[e + f*x])^m*(3 + 2*Sin[e + f*x])^(-1 - m),x]
```

```
[Out] (2*5^(-1 - m)*Hypergeometric2F1[1/2, 1 + m, 3/2, -(Cos[(2*e + Pi + 2*f*x)/4
]^2*Sec[(2*e - Pi + 2*f*x)/4]^2)/5]*(1 + Sin[e + f*x])^m*(Sec[(2*e - Pi + 2
*f*x)/4]^2*(3 + 2*Sin[e + f*x]))^m*Tan[(2*e - Pi + 2*f*x)/4])/(f*(3 + 2*Sin
[e + f*x])^m)
```

Maple [F] time = 0.25, size = 0, normalized size = 0.

$$\int \left(1+\sin\left(fx+e\right)\right)^{m}\left(3+2\sin\left(fx+e\right)\right)^{-1-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+sin(f*x+e))^m*(3+2*sin(f*x+e))^(-1-m),x)
```

```
[Out] int((1+sin(f*x+e))^m*(3+2*sin(f*x+e))^(-1-m),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(2\sin\left(fx+e\right)+3\right)^{-m-1}\left(\sin\left(fx+e\right)+1\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(f*x+e))^m*(3+2*sin(f*x+e))^(-1-m),x, algorithm="maxima")
```

```
[Out] integrate((2*sin(f*x + e) + 3)^(-m - 1)*(sin(f*x + e) + 1)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(2\sin\left(fx+e\right)+3\right)^{-m-1}\left(\sin\left(fx+e\right)+1\right)^{m}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(f*x+e))^m*(3+2*sin(f*x+e))^(-1-m),x, algorithm="fricas")
```

```
[Out] integral((2*sin(f*x + e) + 3)^(-m - 1)*(sin(f*x + e) + 1)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(f*x+e))**m*(3+2*sin(f*x+e))**(-1-m),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(2 \sin\left(fx+e\right)+3\right)^{-m-1}\left(\sin\left(fx+e\right)+1\right)^{m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(f*x+e))^m*(3+2*sin(f*x+e))^(-1-m),x, algorithm="giac")
```

```
[Out] integrate((2*sin(f*x + e) + 3)^(-m - 1)*(sin(f*x + e) + 1)^m, x)
```

3.624 $\int (1+\sin(e+fx))^m (3+\sin(e+fx))^{-1-m}\,dx$

Optimal. Leaf size=106

$$-\frac{2^{-m-\frac{1}{2}}\cos(e+fx)(\sin(e+fx)+1)^{m-1}\left(\frac{\sin(e+fx)+1}{\sin(e+fx)+3}\right)^{\frac{1}{2}-m}(\sin(e+fx)+3)^{-m}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1-\sin(e+fx)}{\sin(e+fx)+3}\right)}{f}$$

```
[Out] -((2^(-1/2 - m)*Cos[e + f*x]*Hypergeometric2F1[1/2, 1/2 - m, 3/2, (1 - Sin[
e + f*x])/(3 + Sin[e + f*x])]*(1 + Sin[e + f*x])^(-1 + m)*((1 + Sin[e + f*x
])/(3 + Sin[e + f*x]))^(1/2 - m))/(f*(3 + Sin[e + f*x])^m))
```

Rubi [A] time = 0.101139, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {2788, 132}

$$-\frac{2^{-m-\frac{1}{2}}\cos(e+fx)(\sin(e+fx)+1)^{m-1}\left(\frac{\sin(e+fx)+1}{\sin(e+fx)+3}\right)^{\frac{1}{2}-m}(\sin(e+fx)+3)^{-m}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1-\sin(e+fx)}{\sin(e+fx)+3}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sin[e + f*x])^m*(3 + Sin[e + f*x])^(-1 - m),x]
```

```
[Out] -((2^(-1/2 - m)*Cos[e + f*x]*Hypergeometric2F1[1/2, 1/2 - m, 3/2, (1 - Sin[
e + f*x])/(3 + Sin[e + f*x])]*(1 + Sin[e + f*x])^(-1 + m)*((1 + Sin[e + f*x
])/(3 + Sin[e + f*x]))^(1/2 - m))/(f*(3 + Sin[e + f*x])^m))
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x
)^n)/Sqrt[a - b*x], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1)*
Hypergeometric2F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*
(e + f*x)))])/(((b*e - a*f)*(m + 1))*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*
(e + f*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int (1+\sin(e+fx))^m(3+\sin(e+fx))^{-1-m}\,dx=\frac{\cos(e+fx)\,\text{Subst}\left(\int\frac{(1+x)^{-\frac{1}{2}+m}(3+x)^{-1-m}}{\sqrt{1-x}}\,dx,x,\sin(e+fx)\right)}{f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}$$

$$=-\frac{2^{-\frac{1}{2}-m}\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1-\sin(e+fx)}{3+\sin(e+fx)}\right)(1+\sin(e+fx))^{-1+m}\left(\right.}{f}$$

Mathematica [A] time = 0.58035, size = 167, normalized size = 1.58

$$\frac{2^{-2m-1}\tan\left(\frac{1}{4}(2e+2fx-\pi)\right)(\sin(e+fx)+1)^m\cos^2\left(\frac{1}{4}(2e+2fx-\pi)\right)^{-m}\left(\frac{\cos^2\left(\frac{1}{4}(2e+2fx-\pi)\right)}{\sin(e+fx)+3}\right)^m\left((\sin(e+fx)+3)\sec\right.}{f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + Sin[e + f*x])^m*(3 + Sin[e + f*x])^(-1 - m),x]
```

```
[Out] (2^(-1 - 2*m)*Hypergeometric2F1[1/2, 1 + m, 3/2, -(Cos[(2*e + Pi + 2*f*x)/4
]^2*Sec[(2*e - Pi + 2*f*x)/4]^2)/2]*(1 + Sin[e + f*x])^m*(Cos[(2*e - Pi + 2
*f*x)/4]^2/(3 + Sin[e + f*x]))^m*(Sec[(2*e - Pi + 2*f*x)/4]^2*(3 + Sin[e +
f*x]))^m*Tan[(2*e - Pi + 2*f*x)/4])/(f*(Cos[(2*e - Pi + 2*f*x)/4]^2)^m)
```

Maple [F] time = 0.247, size = 0, normalized size = 0.

$$\int \left(1+\sin\left(fx+e\right)\right)^{m}\left(3+\sin\left(fx+e\right)\right)^{-1-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+sin(f*x+e))^m*(3+sin(f*x+e))^(-1-m),x)
```

```
[Out] int((1+sin(f*x+e))^m*(3+sin(f*x+e))^(-1-m),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\sin\left(fx+e\right)+3\right)^{-m-1}\left(\sin\left(fx+e\right)+1\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(f*x+e))^m*(3+sin(f*x+e))^(-1-m),x, algorithm="maxima")
```

```
[Out] integrate((sin(f*x + e) + 3)^(-m - 1)*(sin(f*x + e) + 1)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(\sin\left(fx+e\right)+3\right)^{-m-1}\left(\sin\left(fx+e\right)+1\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(f*x+e))^m*(3+sin(f*x+e))^(-1-m),x, algorithm="fricas")
```

```
[Out] integral((sin(f*x + e) + 3)^(-m - 1)*(sin(f*x + e) + 1)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(f*x+e))**m*(3+sin(f*x+e))**(-1-m),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\sin\left(fx+e\right)+3\right)^{-m-1}\left(\sin\left(fx+e\right)+1\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(f*x+e))^m*(3+sin(f*x+e))^(-1-m),x, algorithm="giac")
```

```
[Out] integrate((sin(f*x + e) + 3)^(-m - 1)*(sin(f*x + e) + 1)^m, x)
```

3.625 $\int 3^{-1-m}(1+\sin(e+fx))^m\,dx$

Optimal. Leaf size=65

$$-\frac{2^{m+\frac{1}{2}}3^{-m-1}\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{f\sqrt{\sin(e+fx)+1}}$$

```
[Out] -((2^(1/2 + m)*3^(-1 - m)*Cos[e + f*x]*Hypergeometric2F1[1/2, 1/2 - m, 3/2,
 (1 - Sin[e + f*x])/2])/(f*Sqrt[1 + Sin[e + f*x]]))
```

Rubi [A] time = 0.0213046, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {12, 2651}

$$-\frac{2^{m+\frac{1}{2}}3^{-m-1}\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{f\sqrt{\sin(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[3^(-1 - m)*(1 + Sin[e + f*x])^m,x]
```

```
[Out] -((2^(1/2 + m)*3^(-1 - m)*Cos[e + f*x]*Hypergeometric2F1[1/2, 1/2 - m, 3/2,
 (1 - Sin[e + f*x])/2])/(f*Sqrt[1 + Sin[e + f*x]]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2651

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(2^(n +
 1/2)*a^(n - 1/2)*b*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1*(1
 - (b*Sin[c + d*x])/a))/2])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int 3^{-1-m}(1+\sin(e+fx))^m\,dx &= 3^{-1-m}\int (1+\sin(e+fx))^m\,dx\\ &= -\frac{2^{\frac{1}{2}+m}3^{-1-m}\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{f\sqrt{1+\sin(e+fx)}}\end{aligned}$$

Mathematica [A] time = 0.145701, size = 95, normalized size = 1.46

$$\frac{\sqrt{2}3^{-m-1}\cos(e+fx)(\sin(e+fx)+1)^m\,{}_2F_1\left(\frac{1}{2},m+\frac{1}{2};m+\frac{3}{2};\frac{1}{4}\cos^2(e+fx)\csc^2\left(\frac{1}{4}(2e+2fx-\pi)\right)\right)}{(2fm+f)\sqrt{1-\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[3^(-1 - m)*(1 + Sin[e + f*x])^m,x]
```

```
[Out] (Sqrt[2]*3^(-1 - m)*Cos[e + f*x]*Hypergeometric2F1[1/2, 1/2 + m, 3/2 + m, (
Cos[e + f*x]^2*Csc[(2*e - Pi + 2*f*x)/4]^2)/4]*(1 + Sin[e + f*x])^m)/((f +
2*f*m)*Sqrt[1 - Sin[e + f*x]])
```

Maple [F] time = 0.297, size = 0, normalized size = 0.

$$\int 3^{-1-m} \left(1+\sin \left(f x+e\right)\right)^{m} d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(3^(-1-m)*(1+sin(f*x+e))^m,x)
```

```
[Out] int(3^(-1-m)*(1+sin(f*x+e))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$3^{-m-1} \int \left(\sin \left(f x+e\right)+1\right)^{m} d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(3^(-1-m)*(1+sin(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] 3^(-m - 1)*integrate((sin(f*x + e) + 1)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(3^{-m-1} \left(\sin \left(f x+e\right)+1\right)^{m}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(3^(-1-m)*(1+sin(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral(3^(-m - 1)*(sin(f*x + e) + 1)^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$3^{-m-1} \int \left(\sin \left(e+f x\right)+1\right)^{m} d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(3**(-1-m)*(1+sin(f*x+e))**m,x)
```

```
[Out] 3**(-m - 1)*Integral((sin(e + f*x) + 1)**m, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int 3^{-m-1}\left(\sin\left(fx+e\right)+1\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(3^(-1-m)*(1+sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate(3^(-m - 1)*(sin(f*x + e) + 1)^m, x)
```

3.626 $\int (3-\sin(e+fx))^{-1-m}(1+\sin(e+fx))^m \, dx$

Optimal. Leaf size=94

$$-\frac{\cos(e+fx)(3-\sin(e+fx))^{-m-1}\left(\frac{3-\sin(e+fx)}{\sin(e+fx)+1}\right)^{m+1}(\sin(e+fx)+1)^m\,{}_2F_1\left(\frac{1}{2},m+1;\frac{3}{2};-\frac{2(1-\sin(e+fx))}{\sin(e+fx)+1}\right)}{f}$$

```
[Out] -((Cos[e + f*x]*Hypergeometric2F1[1/2, 1 + m, 3/2, (-2*(1 - Sin[e + f*x]))/
(1 + Sin[e + f*x])]*(3 - Sin[e + f*x])^(-1 - m)*((3 - Sin[e + f*x])/(1 + Si
n[e + f*x]))^(1 + m)*(1 + Sin[e + f*x])^m)/f)
```

Rubi [A] time = 0.0922346, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2788, 132}

$$-\frac{\cos(e+fx)(3-\sin(e+fx))^{-m-1}\left(\frac{3-\sin(e+fx)}{\sin(e+fx)+1}\right)^{m+1}(\sin(e+fx)+1)^m\,{}_2F_1\left(\frac{1}{2},m+1;\frac{3}{2};-\frac{2(1-\sin(e+fx))}{\sin(e+fx)+1}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - Sin[e + f*x])^(-1 - m)*(1 + Sin[e + f*x])^m,x]
```

```
[Out] -((Cos[e + f*x]*Hypergeometric2F1[1/2, 1 + m, 3/2, (-2*(1 - Sin[e + f*x]))/
(1 + Sin[e + f*x])]*(3 - Sin[e + f*x])^(-1 - m)*((3 - Sin[e + f*x])/(1 + Si
n[e + f*x]))^(1 + m)*(1 + Sin[e + f*x])^m)/f)
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x
)^n)/Sqrt[a - b*x], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1)*
Hypergeometric2F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*
(e + f*x)))])/(((b*e - a*f)*(m + 1))*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*
(e + f*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] && !IntegerQ[n]
```

Rubi steps

$$\int (3-\sin(e+fx))^{-1-m}(1+\sin(e+fx))^m \, dx = \frac{\cos(e+fx)\,\text{Subst}\left(\int \frac{(3-x)^{-1-m}(1+x)^{-\frac{1}{2}+m}}{\sqrt{1-x}} \, dx, x, \sin(e+fx)\right)}{f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}$$

$$= -\frac{\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},1+m;\frac{3}{2};-\frac{2(1-\sin(e+fx))}{1+\sin(e+fx)}\right)(3-\sin(e+fx))^{-1-m}\left(\frac{3-\sin}{1+\sin}\right.}{f}$$

Mathematica [A] time = 1.06608, size = 182, normalized size = 1.94

$$-\frac{2^{\frac{1}{2}-m}\cot\left(\frac{1}{4}(2e+2fx+\pi)\right)(3-\sin(e+fx))^{-m}(\sin(e+fx)+1)^{m}\sin^{2}\left(\frac{1}{4}(2e+2fx+\pi)\right)^{\frac{1}{2}-m}\cos^{2}\left(\frac{1}{4}(2e+2fx-\pi\right.}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - Sin[e + f*x])^(-1 - m)*(1 + Sin[e + f*x])^m,x]
```

```
[Out] -((2^(1/2 - m)*(Cos[(2*e - Pi + 2*f*x)/4]^2)^(-1/2 + m)*Cot[(2*e + Pi + 2*f
*x)/4]*Hypergeometric2F1[1/2, 1/2 - m, 3/2, (-4*Sin[(2*e - Pi + 2*f*x)/4]^2
)/(-3 + Sin[e + f*x])]*(-(Cos[(2*e - Pi + 2*f*x)/4]^2/(-3 + Sin[e + f*x])))
^(1/2 - m)*(1 + Sin[e + f*x])^m*(Sin[(2*e + Pi + 2*f*x)/4]^2)^(1/2 - m))/(f
*(3 - Sin[e + f*x])^m))
```

Maple [F] time = 0.258, size = 0, normalized size = 0.

$$\int \left(3-\sin\left(fx+e\right)\right)^{-1-m}\left(1+\sin\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3-sin(f*x+e))^(-1-m)*(1+sin(f*x+e))^m,x)
```

```
[Out] int((3-sin(f*x+e))^(-1-m)*(1+sin(f*x+e))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\sin\left(fx+e\right)+1\right)^{m}\left(-\sin\left(fx+e\right)+3\right)^{-m-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-sin(f*x+e))^(-1-m)*(1+sin(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((sin(f*x + e) + 1)^m*(-sin(f*x + e) + 3)^(-m - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(\sin\left(fx+e\right)+1\right)^{m}\left(-\sin\left(fx+e\right)+3\right)^{-m-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-sin(f*x+e))^(-1-m)*(1+sin(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((sin(f*x + e) + 1)^m*(-sin(f*x + e) + 3)^(-m - 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-sin(f*x+e))**(-1-m)*(1+sin(f*x+e))**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\sin\left(fx+e\right)+1\right)^{m}\left(-\sin\left(fx+e\right)+3\right)^{-m-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-sin(f*x+e))^(-1-m)*(1+sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((sin(f*x + e) + 1)^m*(-sin(f*x + e) + 3)^(-m - 1), x)
```

3.627 $\int (3-2\sin(e+fx))^{-1-m}(1+\sin(e+fx))^m\,dx$

Optimal. Leaf size=114

$$\frac{\sqrt{-\frac{1-\sin(e+fx)}{\sin(e+fx)+1}}\cos(e+fx)(3-2\sin(e+fx))^{-m}(\sin(e+fx)+1)^m\,{}_2F_1\left(\frac{1}{2},-m;1-m;\frac{2(3-2\sin(e+fx))}{\sin(e+fx)+1}\right)}{\sqrt{5}fm(1-\sin(e+fx))}$$

```
[Out] (Cos[e + f*x]*Hypergeometric2F1[1/2, -m, 1 - m, (2*(3 - 2*Sin[e + f*x]))/(1
 + Sin[e + f*x])]*Sqrt[-((1 - Sin[e + f*x])/(1 + Sin[e + f*x]))]*(1 + Sin[e
 + f*x])^m)/(Sqrt[5]*f*m*(3 - 2*Sin[e + f*x])^m*(1 - Sin[e + f*x]))
```

Rubi [A] time = 0.099271, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2788, 132}

$$\frac{\sqrt{-\frac{1-\sin(e+fx)}{\sin(e+fx)+1}}\cos(e+fx)(3-2\sin(e+fx))^{-m}(\sin(e+fx)+1)^m\,{}_2F_1\left(\frac{1}{2},-m;1-m;\frac{2(3-2\sin(e+fx))}{\sin(e+fx)+1}\right)}{\sqrt{5}fm(1-\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - 2*Sin[e + f*x])^(-1 - m)*(1 + Sin[e + f*x])^m,x]
```

```
[Out] (Cos[e + f*x]*Hypergeometric2F1[1/2, -m, 1 - m, (2*(3 - 2*Sin[e + f*x]))/(1
 + Sin[e + f*x])]*Sqrt[-((1 - Sin[e + f*x])/(1 + Sin[e + f*x]))]*(1 + Sin[e
 + f*x])^m)/(Sqrt[5]*f*m*(3 - 2*Sin[e + f*x])^m*(1 - Sin[e + f*x]))
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x
)^n)/Sqrt[a - b*x], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1)*
Hypergeometric2F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*
(e + f*x)))])/(((b*e - a*f)*(m + 1))*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*
(e + f*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int (3-2\sin(e+fx))^{-1-m}(1+\sin(e+fx))^m\,dx=\frac{\cos(e+fx)\operatorname{Subst}\left(\int\frac{(3-2x)^{-1-m}(1+x)^{-\frac{1}{2}+m}}{\sqrt{1-x}}\,dx,x,\sin(e+fx)\right)}{f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}$$

$$=\frac{\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-m;1-m;\frac{2(3-2\sin(e+fx))}{1+\sin(e+fx)}\right)(3-2\sin(e+fx))^{-m}\ldots}{\sqrt{5}fm(1-\sin(e+fx))}$$

Mathematica [A] time = 0.884338, size = 177, normalized size = 1.55

$$-\frac{2\cot\left(\frac{1}{4}(2e+2fx+\pi)\right)(3-2\sin(e+fx))^{-m}(\sin(e+fx)+1)^m\sin^2\left(\frac{1}{4}(2e+2fx+\pi)\right)^{\frac{1}{2}-m}\cos^2\left(\frac{1}{4}(2e+2fx-\pi)\right)^{m-\frac{1}{2}}}{f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(3 - 2*Sin[e + f*x])^(-1 - m)*(1 + Sin[e + f*x])^m,x]
```

```
[Out] (-2*(Cos[(2*e - Pi + 2*f*x)/4]^2)^(-1/2 + m)*Cot[(2*e + Pi + 2*f*x)/4]*Hype
rgeometric2F1[1/2, 1/2 - m, 3/2, (5*Sin[(2*e - Pi + 2*f*x)/4]^2)/(3 - 2*Sin
[e + f*x])]*(1 + Sin[e + f*x])^m*(-(Cos[(2*e - Pi + 2*f*x)/4]^2/(-3 + 2*Sin
[e + f*x])))^(1/2 - m)*(Sin[(2*e + Pi + 2*f*x)/4]^2)^(1/2 - m))/(f*(3 - 2*S
in[e + f*x])^m)
```

Maple [F] time = 0.256, size = 0, normalized size = 0.

$$\int \left(3-2\sin\left(fx+e\right)\right)^{-1-m}\left(1+\sin\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3-2*sin(f*x+e))^(-1-m)*(1+sin(f*x+e))^m,x)
```

```
[Out] int((3-2*sin(f*x+e))^(-1-m)*(1+sin(f*x+e))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\sin\left(fx+e\right)+1\right)^{m}\left(-2\sin\left(fx+e\right)+3\right)^{-m-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-2*sin(f*x+e))^(-1-m)*(1+sin(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((sin(f*x + e) + 1)^m*(-2*sin(f*x + e) + 3)^(-m - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(\sin\left(fx+e\right)+1\right)^{m}\left(-2\sin\left(fx+e\right)+3\right)^{-m-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-2*sin(f*x+e))^(-1-m)*(1+sin(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((sin(f*x + e) + 1)^m*(-2*sin(f*x + e) + 3)^(-m - 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-2*sin(f*x+e))**(-1-m)*(1+sin(f*x+e))**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (\sin(fx+e)+1)^m(-2\sin(fx+e)+3)^{-m-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-2*sin(f*x+e))^(-1-m)*(1+sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((sin(f*x + e) + 1)^m*(-2*sin(f*x + e) + 3)^(-m - 1), x)
```

3.628 $\int (3-3\sin(e+fx))^{-1-m}(1+\sin(e+fx))^m\,dx$

Optimal. Leaf size=43

$$\frac{\cos(e+fx)(3-3\sin(e+fx))^{-m-1}(\sin(e+fx)+1)^m}{f(2m+1)}$$

```
[Out] (Cos[e + f*x]*(3 - 3*Sin[e + f*x])^(-1 - m)*(1 + Sin[e + f*x])^m)/(f*(1 + 2
*m))
```

Rubi [A] time = 0.0541109, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {2742}

$$\frac{\cos(e+fx)(3-3\sin(e+fx))^{-m-1}(\sin(e+fx)+1)^m}{f(2m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - 3*Sin[e + f*x])^(-1 - m)*(1 + Sin[e + f*x])^m,x]
```

```
[Out] (Cos[e + f*x]*(3 - 3*Sin[e + f*x])^(-1 - m)*(1 + Sin[e + f*x])^m)/(f*(1 + 2
*m))
```

Rule 2742

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rubi steps

$$\int (3-3\sin(e+fx))^{-1-m}(1+\sin(e+fx))^m\,dx=\frac{\cos(e+fx)(3-3\sin(e+fx))^{-1-m}(1+\sin(e+fx))^m}{f(1+2m)}$$

Mathematica [B] time = 0.522703, size = 97, normalized size = 2.26

$$\frac{\sin\left(\frac{1}{4}(2e+2fx+\pi)\right)(6-6\sin(e+fx))^{-m}(\sin(e+fx)+1)^m\cos^{-2m-1}\left(\frac{1}{4}(2e+2fx+\pi)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+\right.\right.}{3(2fm+f)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - 3*Sin[e + f*x])^(-1 - m)*(1 + Sin[e + f*x])^m,x]
```

```
[Out] (Cos[(2*e + Pi + 2*f*x)/4]^(-1 - 2*m)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])
^(2*m)*(1 + Sin[e + f*x])^m*Sin[(2*e + Pi + 2*f*x)/4])/(3*(f + 2*f*m)*(6 -
6*Sin[e + f*x])^m)
```

Maple [F] time = 0.262, size = 0, normalized size = 0.

$$\int \left(3-3 \sin \left(f x+e\right)\right)^{-1-m} \left(1+\sin \left(f x+e\right)\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3-3*sin(f*x+e))^(-1-m)*(1+sin(f*x+e))^m,x)
```

```
[Out] int((3-3*sin(f*x+e))^(-1-m)*(1+sin(f*x+e))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\sin \left(f x+e\right)+1\right)^{m} \left(-3 \sin \left(f x+e\right)+3\right)^{-m-1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-3*sin(f*x+e))^(-1-m)*(1+sin(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((sin(f*x + e) + 1)^m*(-3*sin(f*x + e) + 3)^(-m - 1), x)
```

Fricas [A] time = 1.06304, size = 105, normalized size = 2.44

$$\frac{\left(\sin \left(f x+e\right)+1\right)^{m} \left(-3 \sin \left(f x+e\right)+3\right)^{-m-1} \cos \left(f x+e\right)}{2 f m+f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-3*sin(f*x+e))^(-1-m)*(1+sin(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] (sin(f*x + e) + 1)^m*(-3*sin(f*x + e) + 3)^(-m - 1)*cos(f*x + e)/(2*f*m + f
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-3*sin(f*x+e))**(-1-m)*(1+sin(f*x+e))**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-3*sin(f*x+e))^(-1-m)*(1+sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] sage2
```

3.629 $\int (3-4\sin(e+fx))^{-1-m}(1+\sin(e+fx))^m\,dx$

Optimal. Leaf size=83

$$\frac{2^{m+1}\cos(e+fx)(3-4\sin(e+fx))^{-m}(4\sin(e+fx)-3)^m\,{}_2F_1\left(\frac{1}{2},m+1;\frac{3}{2};\frac{7(1-\sin(e+fx))}{\sin(e+fx)+1}\right)}{f(\sin(e+fx)+1)}$$

```
[Out] (2^(1 + m)*Cos[e + f*x]*Hypergeometric2F1[1/2, 1 + m, 3/2, (7*(1 - Sin[e +
f*x]))/(1 + Sin[e + f*x])]*(-3 + 4*Sin[e + f*x])^m)/(f*(3 - 4*Sin[e + f*x])
^m*(1 + Sin[e + f*x]))
```

Rubi [A] time = 0.0963059, antiderivative size = 113, normalized size of antiderivative = 1.36, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2788, 132}

$$\frac{\sqrt{\frac{1-\sin(e+fx)}{\sin(e+fx)+1}}\cos(e+fx)(3-4\sin(e+fx))^{-m}(\sin(e+fx)+1)^m\,{}_2F_1\left(\frac{1}{2},-m;1-m;-\frac{2(3-4\sin(e+fx))}{\sin(e+fx)+1}\right)}{\sqrt{7}fm(1-\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - 4*Sin[e + f*x])^(-1 - m)*(1 + Sin[e + f*x])^m,x]
```

```
[Out] (Cos[e + f*x]*Hypergeometric2F1[1/2, -m, 1 - m, (-2*(3 - 4*Sin[e + f*x]))/(
1 + Sin[e + f*x])]*Sqrt[(1 - Sin[e + f*x])/(1 + Sin[e + f*x])]*(1 + Sin[e +
 f*x])^m)/(Sqrt[7]*f*m*(3 - 4*Sin[e + f*x])^m*(1 - Sin[e + f*x]))
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x
)^n)/Sqrt[a - b*x], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1)*
Hypergeometric2F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*
(e + f*x)))])/(((b*e - a*f)*(m + 1))*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*
(e + f*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int (3-4\sin(e+fx))^{-1-m}(1+\sin(e+fx))^m\,dx &= \frac{\cos(e+fx)\,\text{Subst}\left(\int \frac{(3-4x)^{-1-m}(1+x)^{-\frac{1}{2}+m}}{\sqrt{1-x}}\,dx,x,\sin(e+fx)\right)}{f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}\\ &= \frac{\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-m;1-m;-\frac{2(3-4\sin(e+fx))}{1+\sin(e+fx)}\right)(3-4\sin(e+fx))^{-m}}{\sqrt{7}fm(1-\sin(e+fx))}\end{aligned}$$

Mathematica [B] time = 0.882133, size = 176, normalized size = 2.12

$$\frac{2\cot\left(\frac{1}{4}(2e+2fx+\pi)\right)(3-4\sin(e+fx))^{-m}(\sin(e+fx)+1)^m\sin^2\left(\frac{1}{4}(2e+2fx+\pi)\right)^{\frac{1}{2}-m}\cos^2\left(\frac{1}{4}(2e+2fx-\pi)\right)^{m-\frac{1}{2}}\ 2}{f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(3 - 4*Sin[e + f*x])^(-1 - m)*(1 + Sin[e + f*x])^m,x]
```

```
[Out] (2*(Cos[(2*e - Pi + 2*f*x)/4]^2)^(-1/2 + m)*Cot[(2*e + Pi + 2*f*x)/4]*Hyper
geometric2F1[1/2, 1/2 - m, 3/2, (7*Sin[(2*e - Pi + 2*f*x)/4]^2)/(3 - 4*Sin[
e + f*x])]*(1 + Sin[e + f*x])^m*(Cos[(2*e - Pi + 2*f*x)/4]^2/(-3 + 4*Sin[e
+ f*x]))^(1/2 - m)*(Sin[(2*e + Pi + 2*f*x)/4]^2)^(1/2 - m))/(f*(3 - 4*Sin[e
 + f*x])^m)
```

Maple [F] time = 0.262, size = 0, normalized size = 0.

$$\int \left(3-4\sin\left(fx+e\right)\right)^{-1-m}\left(1+\sin\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3-4*sin(f*x+e))^(-1-m)*(1+sin(f*x+e))^m,x)
```

```
[Out] int((3-4*sin(f*x+e))^(-1-m)*(1+sin(f*x+e))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\sin\left(fx+e\right)+1\right)^{m}\left(-4\sin\left(fx+e\right)+3\right)^{-m-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-4*sin(f*x+e))^(-1-m)*(1+sin(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((sin(f*x + e) + 1)^m*(-4*sin(f*x + e) + 3)^(-m - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(\sin\left(fx+e\right)+1\right)^{m}\left(-4\sin\left(fx+e\right)+3\right)^{-m-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-4*sin(f*x+e))^(-1-m)*(1+sin(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((sin(f*x + e) + 1)^m*(-4*sin(f*x + e) + 3)^(-m - 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-4*sin(f*x+e))**(-1-m)*(1+sin(f*x+e))**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\sin \left(fx+e\right)+1\right)^{m}\left(-4 \sin \left(fx+e\right)+3\right)^{-m-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-4*sin(f*x+e))^(-1-m)*(1+sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((sin(f*x + e) + 1)^m*(-4*sin(f*x + e) + 3)^(-m - 1), x)
```

3.630 $\int (3-5\sin(e+fx))^{-1-m}(1+\sin(e+fx))^m\,dx$

Optimal. Leaf size=78

$$\frac{\cos(e+fx)(3-5\sin(e+fx))^{-m}(5\sin(e+fx)-3)^m\,{}_2F_1\left(\frac{1}{2},m+1;\frac{3}{2};\frac{4(1-\sin(e+fx))}{\sin(e+fx)+1}\right)}{f(\sin(e+fx)+1)}$$

```
[Out] (Cos[e + f*x]*Hypergeometric2F1[1/2, 1 + m, 3/2, (4*(1 - Sin[e + f*x]))/(1
+ Sin[e + f*x])]*(-3 + 5*Sin[e + f*x])^m)/(f*(3 - 5*Sin[e + f*x])^m*(1 + Si
n[e + f*x]))
```

Rubi [A] time = 0.0933549, antiderivative size = 111, normalized size of antiderivative = 1.42, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2788, 132}

$$\frac{\sqrt{\frac{1-\sin(e+fx)}{\sin(e+fx)+1}}\cos(e+fx)(3-5\sin(e+fx))^{-m}(\sin(e+fx)+1)^m\,{}_2F_1\left(\frac{1}{2},-m;1-m;-\frac{3-5\sin(e+fx)}{\sin(e+fx)+1}\right)}{4fm(1-\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - 5*Sin[e + f*x])^(-1 - m)*(1 + Sin[e + f*x])^m,x]
```

```
[Out] (Cos[e + f*x]*Hypergeometric2F1[1/2, -m, 1 - m, -((3 - 5*Sin[e + f*x])/(1 +
 Sin[e + f*x]))]*Sqrt[(1 - Sin[e + f*x])/(1 + Sin[e + f*x])]*(1 + Sin[e + f
*x])^m)/(4*f*m*(3 - 5*Sin[e + f*x])^m*(1 - Sin[e + f*x]))
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x
)^n)/Sqrt[a - b*x], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1)*
Hypergeometric2F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*
(e + f*x)))])/(((b*e - a*f)*(m + 1))*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*
(e + f*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int (3-5\sin(e+fx))^{-1-m}(1+\sin(e+fx))^m\,dx = \frac{\cos(e+fx)\,\text{Subst}\left(\int \frac{(3-5x)^{-1-m}(1+x)^{-\frac{1}{2}+m}}{\sqrt{1-x}}\,dx,x,\sin(e+fx)\right)}{f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}$$

$$= \frac{\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-m;1-m;-\frac{3-5\sin(e+fx)}{1+\sin(e+fx)}\right)(3-5\sin(e+fx))^{-m}\sqrt{\frac{1-}{1+}}}{4fm(1-\sin(e+fx))}$$

Mathematica [C] time = 1.76284, size = 246, normalized size = 3.15

$$-\frac{2^{2m-1}(\cosh(m\log(4))-\sinh(m\log(4)))(3-5\sin(e+fx))^{-m}(\sin(e+fx)+1)^m(\sin(e+fx)+i\cos(e+fx)+1)\left(\frac{2\mathrm{s}}{2\mathrm{s}}\right.}{f(2m+1)((1+2i)\sin(e+fx)+(-2+i)\mathrm{c}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(3 - 5*Sin[e + f*x])^(-1 - m)*(1 + Sin[e + f*x])^m,x]
```

```
[Out] -((2^(-1 + 2*m)*Hypergeometric2F1[1 + m, 1 + 2*m, 2*(1 + m), (2*Cos[(2*e -
Pi + 2*f*x)/4])/(Cos[(2*e - Pi + 2*f*x)/4] + 2*Sin[(2*e - Pi + 2*f*x)/4])]*
(1 + Sin[e + f*x])^m*(1 + I*Cos[e + f*x] + Sin[e + f*x])*((-Cos[(2*e - Pi +
 2*f*x)/4] + 2*Sin[(2*e - Pi + 2*f*x)/4])/(Cos[(2*e - Pi + 2*f*x)/4] + 2*Si
n[(2*e - Pi + 2*f*x)/4]))^m*(Cosh[m*Log[4]] - Sinh[m*Log[4]]))/(f*(1 + 2*m)
*(3 - 5*Sin[e + f*x])^m*((1 - 2*I) - (2 - I)*Cos[e + f*x] + (1 + 2*I)*Sin[e
 + f*x])))
```

Maple [F] time = 0.243, size = 0, normalized size = 0.

$$\int \left(3-5\sin\left(fx+e\right)\right)^{-1-m}\left(1+\sin\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3-5*sin(f*x+e))^(-1-m)*(1+sin(f*x+e))^m,x)
```

```
[Out] int((3-5*sin(f*x+e))^(-1-m)*(1+sin(f*x+e))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\sin\left(fx+e\right)+1\right)^{m}\left(-5\sin\left(fx+e\right)+3\right)^{-m-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-5*sin(f*x+e))^(-1-m)*(1+sin(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((sin(f*x + e) + 1)^m*(-5*sin(f*x + e) + 3)^(-m - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(\sin\left(fx+e\right)+1\right)^{m}\left(-5\sin\left(fx+e\right)+3\right)^{-m-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-5*sin(f*x+e))^(-1-m)*(1+sin(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((sin(f*x + e) + 1)^m*(-5*sin(f*x + e) + 3)^(-m - 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-5*sin(f*x+e))**(-1-m)*(1+sin(f*x+e))**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (\sin (fx+e)+1)^m(-5 \sin (fx+e)+3)^{-m-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-5*sin(f*x+e))^(-1-m)*(1+sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((sin(f*x + e) + 1)^m*(-5*sin(f*x + e) + 3)^(-m - 1), x)
```

3.631 $\int (3+5\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx$

Optimal. Leaf size=81

$$-\frac{4^{-m-1}\cos(e+fx)(\sin(e+fx)+1)^{-m-1}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},m+1;\frac{3}{2};\frac{a-a\sin(e+fx)}{4(\sin(e+fx)a+a)}\right)}{f}$$

```
[Out] -((4^(-1 - m)*Cos[e + f*x]*Hypergeometric2F1[1/2, 1 + m, 3/2, (a - a*Sin[e
+ f*x])/(4*(a + a*Sin[e + f*x]))]*(1 + Sin[e + f*x])^(-1 - m)*(a + a*Sin[e
+ f*x])^m)/f)
```

Rubi [A] time = 0.105533, antiderivative size = 115, normalized size of antiderivative = 1.42, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2788, 132}

$$-\frac{\sqrt{\frac{1-\sin(e+fx)}{\sin(e+fx)+1}}\cos(e+fx)(5\sin(e+fx)+3)^{-m}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},-m;1-m;\frac{5\sin(e+fx)+3}{4(\sin(e+fx)+1)}\right)}{4fm(1-\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 5*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] -(Cos[e + f*x]*Hypergeometric2F1[1/2, -m, 1 - m, (3 + 5*Sin[e + f*x])/(4*(1
 + Sin[e + f*x]))]*Sqrt[(1 - Sin[e + f*x])/(1 + Sin[e + f*x])]*(a + a*Sin[e
 + f*x])^m)/(4*f*m*(1 - Sin[e + f*x])*(3 + 5*Sin[e + f*x])^m)
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x
)^n)/Sqrt[a - b*x], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1)*
Hypergeometric2F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*
(e + f*x)))])/(((b*e - a*f)*(m + 1))*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*
(e + f*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int (3+5\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx = \frac{\left(a^2\cos(e+fx)\right)\text{Subst}\left(\int \frac{(3+5x)^{-1-m}(a+ax)^{-\frac{1}{2}+m}}{\sqrt{a-ax}}\,dx,x,\sin(e+fx)\right)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-m;1-m;\frac{3+5\sin(e+fx)}{4(1+\sin(e+fx))}\right)\sqrt{\frac{1-\sin(e+fx)}{1+\sin(e+fx)}}(3+5\sin}{4fm(1-\sin(e+fx))}$$

Mathematica [C] time = 0.568873, size = 240, normalized size = 2.96

$$\frac{4^m(\cosh(m\log(4))-\sinh(m\log(4)))(5\sin(e+fx)+3)^{-m}(\sin(e+fx)+i\cos(e+fx)+1)(a(\sin(e+fx)+1))^m\left(-\frac{2\cos\left(\frac{1}{4}\cdots\right.}{\sin\left(\frac{1}{4}\cdots\right.}\right.}{f(2m+1)((2+i)\sin(e+fx)+(-1+2i)\cos\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(3 + 5*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] (4^m*Hypergeometric2F1[1 + m, 1 + 2*m, 2*(1 + m), (4*Cos[(2*e - Pi + 2*f*x)
/4])/(2*Cos[(2*e - Pi + 2*f*x)/4] + Sin[(2*e - Pi + 2*f*x)/4])]*(a*(1 + Sin
[e + f*x]))^m*(1 + I*Cos[e + f*x] + Sin[e + f*x])*(-((2*Cos[(2*e - Pi + 2*f
*x)/4] + Cos[(2*e + Pi + 2*f*x)/4])/(2*Cos[(2*e - Pi + 2*f*x)/4] + Sin[(2*e
 - Pi + 2*f*x)/4])))^m*(Cosh[m*Log[4]] - Sinh[m*Log[4]]))/(f*(1 + 2*m)*((2
- I) - (1 - 2*I)*Cos[e + f*x] + (2 + I)*Sin[e + f*x])*(3 + 5*Sin[e + f*x])^
m)
```

Maple [F] time = 0.246, size = 0, normalized size = 0.

$$\int \left(3+5 \sin \left(fx+e\right)\right)^{-1-m} \left(a+a \sin \left(fx+e\right)\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3+5*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

```
[Out] int((3+5*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a \sin \left(fx+e\right)+a\right)^{m} \left(5 \sin \left(fx+e\right)+3\right)^{-m-1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+5*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(5*sin(f*x + e) + 3)^(-m - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a \sin \left(fx+e\right)+a\right)^{m} \left(5 \sin \left(fx+e\right)+3\right)^{-m-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+5*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)^m*(5*sin(f*x + e) + 3)^(-m - 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+5*sin(f*x+e))**(-1-m)*(a+a*sin(f*x+e))**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(fx+e)+a)^m(5\sin(fx+e)+3)^{-m-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+5*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(5*sin(f*x + e) + 3)^(-m - 1), x)
```

3.632 $\int (3+4\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx$

Optimal. Leaf size=83

$$-\frac{\left(\frac{7}{2}\right)^{-m-1}\cos(e+fx)(\sin(e+fx)+1)^{-m-1}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},m+1;\frac{3}{2};\frac{a-a\sin(e+fx)}{7(\sin(e+fx)a+a)}\right)}{f}$$

```
[Out] -(((7/2)^(-1 - m)*Cos[e + f*x]*Hypergeometric2F1[1/2, 1 + m, 3/2, (a - a*Si
n[e + f*x])/(7*(a + a*Sin[e + f*x]))]*(1 + Sin[e + f*x])^(-1 - m)*(a + a*Si
n[e + f*x])^m)/f)
```

Rubi [A] time = 0.11348, antiderivative size = 118, normalized size of antiderivative = 1.42, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2788, 132}

$$-\frac{\sqrt{\frac{1-\sin(e+fx)}{\sin(e+fx)+1}}\cos(e+fx)(4\sin(e+fx)+3)^{-m}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},-m;1-m;\frac{2(4\sin(e+fx)+3)}{7(\sin(e+fx)+1)}\right)}{\sqrt{7}fm(1-\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 4*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] -((Cos[e + f*x]*Hypergeometric2F1[1/2, -m, 1 - m, (2*(3 + 4*Sin[e + f*x]))/
(7*(1 + Sin[e + f*x]))]*Sqrt[(1 - Sin[e + f*x])/(1 + Sin[e + f*x])]*(a + a*
Sin[e + f*x])^m)/(Sqrt[7]*f*m*(1 - Sin[e + f*x])*(3 + 4*Sin[e + f*x])^m))
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x
)^n)/Sqrt[a - b*x], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1)*
Hypergeometric2F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*
(e + f*x)))])/(((b*e - a*f)*(m + 1))*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*
(e + f*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int (3+4\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx=\frac{\left(a^2\cos(e+fx)\right)\text{Subst}\left(\int\frac{(3+4x)^{-1-m}(a+ax)^{-\frac{1}{2}+m}}{\sqrt{a-ax}}\,dx,x,\sin(e+fx)\right)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}$$

$$=-\frac{\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-m;1-m;\frac{2(3+4\sin(e+fx))}{7(1+\sin(e+fx))}\right)\sqrt{\frac{1-\sin(e+fx)}{1+\sin(e+fx)}}(3+4\sin}{\sqrt{7}fm(1-\sin(e+fx))}$$

Mathematica [A] time = 0.455197, size = 90, normalized size = 1.08

$$-\frac{2\,7^{-m-1}\cot\left(\frac{1}{4}(2e+2fx+\pi)\right)\sin^2\left(\frac{1}{4}(2e+2fx+\pi)\right)^{-m}(a(\sin(e+fx)+1))^m\,{}_2F_1\left(\frac{1}{2},m+1;\frac{3}{2};\frac{1}{7}\tan^2\left(\frac{1}{4}(2e+2fx\right.\right.}{f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(3 + 4*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] (-2*7^(-1 - m)*Cot[(2*e + Pi + 2*f*x)/4]*Hypergeometric2F1[1/2, 1 + m, 3/2,
 Tan[(2*e - Pi + 2*f*x)/4]^2/7]*(a*(1 + Sin[e + f*x]))^m)/(f*(Sin[(2*e + Pi
 + 2*f*x)/4]^2)^m)
```

Maple [F] time = 0.251, size = 0, normalized size = 0.

$$\int \left(3+4\sin\left(fx+e\right)\right)^{-1-m}\left(a+a\sin\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3+4*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

```
[Out] int((3+4*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^{m}\left(4\sin\left(fx+e\right)+3\right)^{-m-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+4*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(4*sin(f*x + e) + 3)^(-m - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a\sin\left(fx+e\right)+a\right)^{m}\left(4\sin\left(fx+e\right)+3\right)^{-m-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+4*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)^m*(4*sin(f*x + e) + 3)^(-m - 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+4*sin(f*x+e))**(-1-m)*(a+a*sin(f*x+e))**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^{m}\left(4\,\sin\left(fx+e\right)+3\right)^{-m-1}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+4*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(4*sin(f*x + e) + 3)^(-m - 1), x)
```

3.633 $\int (3+3\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx$

Optimal. Leaf size=39

$$-\frac{\cos(e+fx)(3\sin(e+fx)+3)^{-m-1}(a\sin(e+fx)+a)^m}{f}$$

```
[Out] -((Cos[e + f*x]*(3 + 3*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m)/f)
```

Rubi [A] time = 0.0170718, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {23, 2648}

$$-\frac{\cos(e+fx)(3\sin(e+fx)+3)^{-m-1}(a\sin(e+fx)+a)^m}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 3*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] -((Cos[e + f*x]*(3 + 3*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m)/f)
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] && !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\begin{aligned}\int (3+3\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx &= \left((3+3\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^{1+m}\right)\int \frac{1}{a+a\sin(e+f\\ &= -\frac{\cos(e+fx)(3+3\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m}{f}\end{aligned}$$

Mathematica [B] time = 5.03509, size = 104, normalized size = 2.67

$$-\frac{2^{-m}3^{-m-1}\cos\left(\frac{1}{4}(2e+2fx+\pi)\right)(\sin(e+fx)+1)^{-m-1}\sin^{-2m-1}\left(\frac{1}{4}(2e+2fx+\pi)\right)(a(\sin(e+fx)+1))^m\left(\sin\left(\frac{1}{2}(e+\right.\right.}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 3*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] -((3^(-1 - m)*Cos[(2*e + Pi + 2*f*x)/4]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2
])^(2*(1 + m))*(1 + Sin[e + f*x])^(-1 - m)*(a*(1 + Sin[e + f*x]))^m*Sin[(2*
```

```
e + Pi + 2*f*x)/4]^(-1 - 2*m))/(2^m*f))
```

Maple [F] time = 0.254, size = 0, normalized size = 0.

$$\int \left(3+3\sin\left(fx+e\right)\right)^{-1-m} \left(a+a\sin\left(fx+e\right)\right)^{m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3+3*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

```
[Out] int((3+3*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

Maxima [A] time = 1.75836, size = 51, normalized size = 1.31

$$-\frac{2\,a^{m}}{\left(3^{m+1}+\frac{3^{m+1}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+3*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] -2*a^m/((3^(m + 1) + 3^(m + 1)*sin(f*x + e)/(cos(f*x + e) + 1))*f)
```

Fricas [A] time = 1.0135, size = 119, normalized size = 3.05

$$-\frac{\left(\frac{1}{3}\,a\right)^{m}\left(\cos\left(fx+e\right)-\sin\left(fx+e\right)+1\right)}{3\left(f\cos\left(fx+e\right)+f\sin\left(fx+e\right)+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+3*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] -1/3*(1/3*a)^m*(cos(f*x + e) - sin(f*x + e) + 1)/(f*cos(f*x + e) + f*sin(f*
x + e) + f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+3*sin(f*x+e))**(-1-m)*(a+a*sin(f*x+e))**m,x)
```

```
[Out] Timed out
```

<u>**Giac**</u> **[F]** time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^{m}\left(3\sin\left(fx+e\right)+3\right)^{-m-1}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+3*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(3*sin(f*x + e) + 3)^(-m - 1), x)
```

3.634 $\int (3+2\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx$

Optimal. Leaf size=83

$$-\frac{\left(\frac{5}{2}\right)^{-m-1}\cos(e+fx)(\sin(e+fx)+1)^{-m-1}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},m+1;\frac{3}{2};-\frac{a-a\sin(e+fx)}{5(\sin(e+fx)a+a)}\right)}{f}$$

```
[Out] -(((5/2)^(-1 - m)*Cos[e + f*x]*Hypergeometric2F1[1/2, 1 + m, 3/2, -(a - a*S
in[e + f*x])/(5*(a + a*Sin[e + f*x]))]*(1 + Sin[e + f*x])^(-1 - m)*(a + a*S
in[e + f*x])^m)/f)
```

Rubi [A] time = 0.112555, antiderivative size = 118, normalized size of antiderivative = 1.42, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2788, 132}

$$\frac{\sqrt{-\frac{1-\sin(e+fx)}{\sin(e+fx)+1}}\cos(e+fx)(2\sin(e+fx)+3)^{-m}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},-m;1-m;\frac{2(2\sin(e+fx)+3)}{5(\sin(e+fx)+1)}\right)}{\sqrt{5}fm(1-\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 2*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] (Cos[e + f*x]*Hypergeometric2F1[1/2, -m, 1 - m, (2*(3 + 2*Sin[e + f*x]))/(5
*(1 + Sin[e + f*x]))]*Sqrt[-((1 - Sin[e + f*x])/(1 + Sin[e + f*x]))]*(a + a
*Sin[e + f*x])^m)/(Sqrt[5]*f*m*(1 - Sin[e + f*x])*(3 + 2*Sin[e + f*x])^m)
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x
)^n)/Sqrt[a - b*x], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1)*
Hypergeometric2F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*
(e + f*x)))])/(((b*e - a*f)*(m + 1))*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*
(e + f*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int (3+2\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx=\frac{\left(a^2\cos(e+fx)\right)\text{Subst}\left(\int\frac{(3+2x)^{-1-m}(a+ax)^{-\frac{1}{2}+m}}{\sqrt{a-ax}}\,dx,x,\sin(e+fx)\right)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}$$

$$=\frac{\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-m;1-m;\frac{2(3+2\sin(e+fx))}{5(1+\sin(e+fx))}\right)\sqrt{-\frac{1-\sin(e+fx)}{1+\sin(e+fx)}}(3+2\sin}{\sqrt{5}fm(1-\sin(e+fx))}$$

Mathematica [B] time = 0.551539, size = 179, normalized size = 2.16

$$-\frac{2\, 5^{-m-1} \cot\left(\frac{1}{4}(2e+2fx+\pi)\right) (2\sin(e+fx)+3)^{-m} \sin^2\left(\frac{1}{4}(2e+2fx+\pi)\right)^{\frac{1}{2}-m} \cos^2\left(\frac{1}{4}(2e+2fx-\pi)\right)^{m-\frac{1}{2}} (a(\sin(e}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 2*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] (-2*5^(-1 - m)*(Cos[(2*e - Pi + 2*f*x)/4]^2)^(-1/2 + m)*Cot[(2*e + Pi + 2*f
*x)/4]*Hypergeometric2F1[1/2, 1 + m, 3/2, -(Cos[(2*e + Pi + 2*f*x)/4]^2*Sec
[(2*e - Pi + 2*f*x)/4]^2)/5]*(a*(1 + Sin[e + f*x]))^m*(Sec[(2*e - Pi + 2*f*
x)/4]^2*(3 + 2*Sin[e + f*x]))^m*(Sin[(2*e + Pi + 2*f*x)/4]^2)^(1/2 - m))/(f
*(3 + 2*Sin[e + f*x])^m)
```

Maple [F] time = 0.237, size = 0, normalized size = 0.

$$\int \left(3+2\sin\left(fx+e\right)\right)^{-1-m} \left(a+a\sin\left(fx+e\right)\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3+2*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

```
[Out] int((3+2*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^{m} \left(2\sin\left(fx+e\right)+3\right)^{-m-1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(2*sin(f*x + e) + 3)^(-m - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a\sin\left(fx+e\right)+a\right)^{m} \left(2\sin\left(fx+e\right)+3\right)^{-m-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)^m*(2*sin(f*x + e) + 3)^(-m - 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*sin(f*x+e))**(-1-m)*(a+a*sin(f*x+e))**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^{m}\left(2\,\sin\left(fx+e\right)+3\right)^{-m-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(2*sin(f*x + e) + 3)^(-m - 1), x)
```

3.635 $\int (3+\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx$

Optimal. Leaf size=81

$$-\frac{2^{-m-1}\cos(e+fx)(\sin(e+fx)+1)^{-m-1}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},m+1;\frac{3}{2};-\frac{a-a\sin(e+fx)}{2(\sin(e+fx)a+a)}\right)}{f}$$

```
[Out] -((2^(-1 - m)*Cos[e + f*x]*Hypergeometric2F1[1/2, 1 + m, 3/2, -(a - a*Sin[e
 + f*x])/(2*(a + a*Sin[e + f*x]))]*(1 + Sin[e + f*x])^(-1 - m)*(a + a*Sin[e
 + f*x])^m)/f)
```

Rubi [A] time = 0.107264, antiderivative size = 117, normalized size of antiderivative = 1.44, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2788, 132}

$$\frac{\sqrt{-\frac{1-\sin(e+fx)}{\sin(e+fx)+1}}\cos(e+fx)(\sin(e+fx)+3)^{-m}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},-m;1-m;\frac{\sin(e+fx)+3}{2(\sin(e+fx)+1)}\right)}{2\sqrt{2}fm(1-\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] (Cos[e + f*x]*Hypergeometric2F1[1/2, -m, 1 - m, (3 + Sin[e + f*x])/(2*(1 +
Sin[e + f*x]))]*Sqrt[-((1 - Sin[e + f*x])/(1 + Sin[e + f*x]))]*(a + a*Sin[e
 + f*x])^m)/(2*Sqrt[2]*f*m*(1 - Sin[e + f*x])*(3 + Sin[e + f*x])^m)
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x
)^n)/Sqrt[a - b*x], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1)*
Hypergeometric2F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*
(e + f*x)))])/(((b*e - a*f)*(m + 1))*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*
(e + f*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int (3+\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx = \frac{\left(a^2\cos(e+fx)\right)\text{Subst}\left(\int\frac{(3+x)^{-1-m}(a+ax)^{-\frac{1}{2}+m}}{\sqrt{a-ax}}\,dx,x,\sin(e+fx)\right)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-m;1-m;\frac{3+\sin(e+fx)}{2(1+\sin(e+fx))}\right)\sqrt{-\frac{1-\sin(e+fx)}{1+\sin(e+fx)}}(3+\sin(}{2\sqrt{2}fm(1-\sin(e+fx))}$$

Mathematica [B] time = 0.651145, size = 166, normalized size = 2.05

$$-\frac{2^{-2m-1}\cot\left(\frac{1}{4}(2e+2fx+\pi)\right)\sin^2\left(\frac{1}{4}(2e+2fx+\pi)\right)^{-m}(a(\sin(e+fx)+1))^m\left(\frac{\cos^2\left(\frac{1}{4}(2e+2fx-\pi)\right)}{\sin(e+fx)+3}\right)^m\left((\sin(e+fx)+3)\sec\right.}{f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(3 + Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] -((2^(-1 - 2*m)*Cot[(2*e + Pi + 2*f*x)/4]*Hypergeometric2F1[1/2, 1 + m, 3/2
, -(Cos[(2*e + Pi + 2*f*x)/4]^2*Sec[(2*e - Pi + 2*f*x)/4]^2)/2]*(a*(1 + Sin
[e + f*x]))^m*(Cos[(2*e - Pi + 2*f*x)/4]^2/(3 + Sin[e + f*x]))^m*(Sec[(2*e
- Pi + 2*f*x)/4]^2*(3 + Sin[e + f*x]))^m)/(f*(Sin[(2*e + Pi + 2*f*x)/4]^2)^
m))
```

Maple [F] time = 0.25, size = 0, normalized size = 0.

$$\int \left(3+\sin\left(fx+e\right)\right)^{-1-m}\left(a+a\sin\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3+sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

```
[Out] int((3+sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^m\left(\sin\left(fx+e\right)+3\right)^{-m-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(sin(f*x + e) + 3)^(-m - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a\sin\left(fx+e\right)+a\right)^m\left(\sin\left(fx+e\right)+3\right)^{-m-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)^m*(sin(f*x + e) + 3)^(-m - 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+sin(f*x+e))**(-1-m)*(a+a*sin(f*x+e))**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^{m}\left(\sin\left(fx+e\right)+3\right)^{-m-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(sin(f*x + e) + 3)^(-m - 1), x)
```

3.636 $\int 3^{-1-m}(a+a\sin(e+fx))^m\,dx$

Optimal. Leaf size=81

$$-\frac{2^{m+\frac{1}{2}}3^{-m-1}\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{f}$$

```
[Out] -((2^(1/2 + m)*3^(-1 - m)*Cos[e + f*x]*Hypergeometric2F1[1/2, 1/2 - m, 3/2,
 (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Sin[e + f*x])^m
)/f)
```

Rubi [A] time = 0.0412067, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {12, 2652, 2651}

$$-\frac{2^{m+\frac{1}{2}}3^{-m-1}\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[3^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] -((2^(1/2 + m)*3^(-1 - m)*Cos[e + f*x]*Hypergeometric2F1[1/2, 1/2 - m, 3/2,
 (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Sin[e + f*x])^m
)/f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2652

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Sin[c + d*x])^FracPart[n])/(1 + (b*Sin[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Sin[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2651

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(2^(n +
 1/2)*a^(n - 1/2)*b*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1*(1
 - (b*Sin[c + d*x])/a))/2])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int 3^{-1-m}(a+a\sin(e+fx))^m\,dx &= 3^{-1-m}\int (a+a\sin(e+fx))^m\,dx\\ &= \left(3^{-1-m}(1+\sin(e+fx))^{-m}(a+a\sin(e+fx))^m\right)\int (1+\sin(e+fx))^m\,dx\\ &= -\frac{2^{\frac{1}{2}+m}3^{-1-m}\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)(1+\sin(e+fx))^{-\frac{1}{2}-m}(a}{f}\end{aligned}$$

Mathematica [A] time = 0.154885, size = 97, normalized size = 1.2

$$\frac{\sqrt{2}3^{-m-1}\cos(e+fx)(a(\sin(e+fx)+1))^m\,{}_2F_1\left(\frac{1}{2},m+\frac{1}{2};m+\frac{3}{2};\frac{1}{4}\cos^2(e+fx)\csc^2\left(\frac{1}{4}(2e+2fx-\pi)\right)\right)}{(2fm+f)\sqrt{1-\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[3^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] (Sqrt[2]*3^(-1 - m)*Cos[e + f*x]*Hypergeometric2F1[1/2, 1/2 + m, 3/2 + m, (
Cos[e + f*x]^2*Csc[(2*e - Pi + 2*f*x)/4]^2)/4]*(a*(1 + Sin[e + f*x]))^m)/((
f + 2*f*m)*Sqrt[1 - Sin[e + f*x]])
```

Maple [F] time = 0.313, size = 0, normalized size = 0.

$$\int 3^{-1-m}\left(a+a\sin\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(3^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

```
[Out] int(3^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$3^{-m-1}\int\left(a\sin\left(fx+e\right)+a\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(3^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] 3^(-m - 1)*integrate((a*sin(f*x + e) + a)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(3^{-m-1}\left(a\sin\left(fx+e\right)+a\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(3^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral(3^(-m - 1)*(a*sin(f*x + e) + a)^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$3^{-m-1}\int\left(a\sin\left(e+fx\right)+a\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(3**(-1-m)*(a+a*sin(f*x+e))**m,x)
```

```
[Out] 3**(-m - 1)*Integral((a*sin(e + f*x) + a)**m, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int 3^{-m-1}\left(a\sin\left(fx+e\right)+a\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(3^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate(3^(-m - 1)*(a*sin(f*x + e) + a)^m, x)
```

3.637 $\int (3-\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx$

Optimal. Leaf size=72

$$-\frac{\cos(e+fx)(\sin(e+fx)+1)^{-m-1}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},m+1;\frac{3}{2};-\frac{2(a-a\sin(e+fx))}{\sin(e+fx)a+a}\right)}{f}$$

```
[Out] -((Cos[e + f*x]*Hypergeometric2F1[1/2, 1 + m, 3/2, (-2*(a - a*Sin[e + f*x])
)/(a + a*Sin[e + f*x])]*(1 + Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m)
/f)
```

Rubi [A] time = 0.108821, antiderivative size = 118, normalized size of antiderivative = 1.64, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2788, 132}

$$\frac{\sqrt{-\frac{1-\sin(e+fx)}{\sin(e+fx)+1}}\cos(e+fx)(3-\sin(e+fx))^{-m}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},-m;1-m;\frac{3-\sin(e+fx)}{\sin(e+fx)+1}\right)}{2\sqrt{2}fm(1-\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] (Cos[e + f*x]*Hypergeometric2F1[1/2, -m, 1 - m, (3 - Sin[e + f*x])/(1 + Sin
[e + f*x])]*Sqrt[-((1 - Sin[e + f*x])/(1 + Sin[e + f*x]))]*(a + a*Sin[e + f
*x])^m)/(2*Sqrt[2]*f*m*(1 - Sin[e + f*x])*(3 - Sin[e + f*x])^m)
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x
)^n)/Sqrt[a - b*x], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1)*
Hypergeometric2F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*
(e + f*x)))])/(((b*e - a*f)*(m + 1))*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*
(e + f*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int (3-\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx = \frac{\left(a^2\cos(e+fx)\right)\text{Subst}\left(\int \frac{(3-x)^{-1-m}(a+ax)^{-\frac{1}{2}+m}}{\sqrt{a-ax}}\,dx,x,\sin(e+fx)\right)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-m;1-m;\frac{3-\sin(e+fx)}{1+\sin(e+fx)}\right)(3-\sin(e+fx))^{-m}\sqrt{-\frac{1-}{1-}}}{2\sqrt{2}fm(1-\sin(e+fx))}$$

Mathematica [B] time = 0.777993, size = 184, normalized size = 2.56

$$-\frac{2^{\frac{1}{2}-m}\cot\left(\frac{1}{4}(2e+2fx+\pi)\right)(3-\sin(e+fx))^{-m}\sin^2\left(\frac{1}{4}(2e+2fx+\pi)\right)^{\frac{1}{2}-m}\cos^2\left(\frac{1}{4}(2e+2fx-\pi)\right)^{m-\frac{1}{2}}(a(\sin(e+fx)+}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] -((2^(1/2 - m)*(Cos[(2*e - Pi + 2*f*x)/4]^2)^(-1/2 + m)*Cot[(2*e + Pi + 2*f
*x)/4]*Hypergeometric2F1[1/2, 1/2 - m, 3/2, (-4*Sin[(2*e - Pi + 2*f*x)/4]^2
)/(-3 + Sin[e + f*x])]*(-(Cos[(2*e - Pi + 2*f*x)/4]^2/(-3 + Sin[e + f*x])))
^(1/2 - m)*(a*(1 + Sin[e + f*x]))^m*(Sin[(2*e + Pi + 2*f*x)/4]^2)^(1/2 - m)
)/(f*(3 - Sin[e + f*x])^m))
```

Maple [F] time = 0.254, size = 0, normalized size = 0.

$$\int\left(3-\sin\left(fx+e\right)\right)^{-1-m}\left(a+a\sin\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3-sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

```
[Out] int((3-sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-\sin\left(fx+e\right)+3\right)^{-m-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(-sin(f*x + e) + 3)^(-m - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-\sin\left(fx+e\right)+3\right)^{-m-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)^m*(-sin(f*x + e) + 3)^(-m - 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-sin(f*x+e))**(-1-m)*(a+a*sin(f*x+e))**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^{m}\left(-\sin\left(fx+e\right)+3\right)^{-m-1}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(-sin(f*x + e) + 3)^(-m - 1), x)
```

3.638 $\int (3-2\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx$

Optimal. Leaf size=77

$$-\frac{2^{m+1}\cos(e+fx)(\sin(e+fx)+1)^{-m-1}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},m+1;\frac{3}{2};-\frac{5(a-a\sin(e+fx))}{\sin(e+fx)a+a}\right)}{f}$$

```
[Out] -((2^(1 + m)*Cos[e + f*x]*Hypergeometric2F1[1/2, 1 + m, 3/2, (-5*(a - a*Sin
[e + f*x]))/(a + a*Sin[e + f*x])]*(1 + Sin[e + f*x])^(-1 - m)*(a + a*Sin[e
+ f*x])^m)/f)
```

Rubi [A] time = 0.104767, antiderivative size = 116, normalized size of antiderivative = 1.51, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2788, 132}

$$\frac{\sqrt{-\frac{1-\sin(e+fx)}{\sin(e+fx)+1}}\cos(e+fx)(3-2\sin(e+fx))^{-m}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},-m;1-m;\frac{2(3-2\sin(e+fx))}{\sin(e+fx)+1}\right)}{\sqrt{5}fm(1-\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - 2*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] (Cos[e + f*x]*Hypergeometric2F1[1/2, -m, 1 - m, (2*(3 - 2*Sin[e + f*x]))/(1
 + Sin[e + f*x])]*Sqrt[-((1 - Sin[e + f*x])/(1 + Sin[e + f*x]))]*(a + a*Sin
[e + f*x])^m)/(Sqrt[5]*f*m*(3 - 2*Sin[e + f*x])^m*(1 - Sin[e + f*x]))
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x
)^n)/Sqrt[a - b*x], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1)*
Hypergeometric2F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*
(e + f*x)))])/(((b*e - a*f)*(m + 1))*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*
(e + f*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int (3-2\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx=\frac{\left(a^2\cos(e+fx)\right)\text{Subst}\left(\int\frac{(3-2x)^{-1-m}(a+ax)^{-\frac{1}{2}+m}}{\sqrt{a-ax}}\,dx,x,\sin(e+fx)\right)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}$$

$$=\frac{\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-m;1-m;\frac{2(3-2\sin(e+fx))}{1+\sin(e+fx)}\right)(3-2\sin(e+fx))^{-m}\sqrt{}}{\sqrt{5}fm(1-\sin(e+fx))}$$

Mathematica [B] time = 0.624312, size = 179, normalized size = 2.32

$$-\frac{2\cot\left(\frac{1}{4}(2e+2fx+\pi)\right)(3-2\sin(e+fx))^{-m}\sin^2\left(\frac{1}{4}(2e+2fx+\pi)\right)^{\frac{1}{2}-m}\cos^2\left(\frac{1}{4}(2e+2fx-\pi)\right)^{m-\frac{1}{2}}(a(\sin(e+fx)}{f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(3 - 2*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] (-2*(Cos[(2*e - Pi + 2*f*x)/4]^2)^(-1/2 + m)*Cot[(2*e + Pi + 2*f*x)/4]*Hype
rgeometric2F1[1/2, 1/2 - m, 3/2, (5*Sin[(2*e - Pi + 2*f*x)/4]^2)/(3 - 2*Sin
[e + f*x])]*(a*(1 + Sin[e + f*x]))^m*(-(Cos[(2*e - Pi + 2*f*x)/4]^2/(-3 + 2
*Sin[e + f*x])))^(1/2 - m)*(Sin[(2*e + Pi + 2*f*x)/4]^2)^(1/2 - m))/(f*(3 -
 2*Sin[e + f*x])^m)
```

Maple [F] time = 0.24, size = 0, normalized size = 0.

$$\int \left(3-2\sin\left(fx+e\right)\right)^{-1-m}\left(a+a\sin\left(fx+e\right)\right)^{m}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3-2*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

```
[Out] int((3-2*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^{m}\left(-2\sin\left(fx+e\right)+3\right)^{-m-1}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-2*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(-2*sin(f*x + e) + 3)^(-m - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-2\sin\left(fx+e\right)+3\right)^{-m-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-2*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)^m*(-2*sin(f*x + e) + 3)^(-m - 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-2*sin(f*x+e))**(-1-m)*(a+a*sin(f*x+e))**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^{m}\left(-2\sin\left(fx+e\right)+3\right)^{-m-1}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-2*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(-2*sin(f*x + e) + 3)^(-m - 1), x)
```

3.639 $\int (3-3\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx$

Optimal. Leaf size=45

$$\frac{\cos(e+fx)(3-3\sin(e+fx))^{-m-1}(a\sin(e+fx)+a)^m}{f(2m+1)}$$

```
[Out] (Cos[e + f*x]*(3 - 3*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m)/(f*(1 +
 2*m))
```

Rubi [A] time = 0.0624405, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {2742}

$$\frac{\cos(e+fx)(3-3\sin(e+fx))^{-m-1}(a\sin(e+fx)+a)^m}{f(2m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - 3*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] (Cos[e + f*x]*(3 - 3*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m)/(f*(1 +
 2*m))
```

Rule 2742

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rubi steps

$$\int (3-3\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx = \frac{\cos(e+fx)(3-3\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m}{f(1+2m)}$$

Mathematica [B] time = 0.601285, size = 99, normalized size = 2.2

$$\frac{\sin\left(\frac{1}{4}(2e+2fx+\pi)\right)(6-6\sin(e+fx))^{-m}\cos^{-2m-1}\left(\frac{1}{4}(2e+2fx+\pi)\right)(a(\sin(e+fx)+1))^m\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\right.}{3(2fm+f)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - 3*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] (Cos[(2*e + Pi + 2*f*x)/4]^(-1 - 2*m)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])
^(2*m)*(a*(1 + Sin[e + f*x]))^m*Sin[(2*e + Pi + 2*f*x)/4])/(3*(f + 2*f*m)*(
6 - 6*Sin[e + f*x])^m)
```

Maple [F] time = 0.254, size = 0, normalized size = 0.

$$\int \left(3-3 \sin \left(f x +e \right)\right)^{-1-m} \left(a +a \sin \left(f x +e \right)\right)^{m}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3-3*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

```
[Out] int((3-3*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a \sin \left(f x + e\right) + a\right)^{m} \left(-3 \, \sin \left(f x + e\right) + 3\right)^{-m - 1}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-3*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(-3*sin(f*x + e) + 3)^(-m - 1), x)
```

Fricas [A] time = 1.30427, size = 108, normalized size = 2.4

$$\frac{{\left(a \sin \left(f x + e\right) + a\right)}^{m} {\left(-3 \, \sin \left(f x + e\right) + 3\right)}^{-m - 1} \cos \left(f x + e\right)}{2 \, f m + f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-3*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] (a*sin(f*x + e) + a)^m*(-3*sin(f*x + e) + 3)^(-m - 1)*cos(f*x + e)/(2*f*m +
 f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-3*sin(f*x+e))**(-1-m)*(a+a*sin(f*x+e))**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a \sin \left(f x + e\right) + a\right)^{m} \left(-3 \, \sin \left(f x + e\right) + 3\right)^{-m - 1}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-3*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(-3*sin(f*x + e) + 3)^(-m - 1), x)
```

3.640 $\int (3-4\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx$

Optimal. Leaf size=115

$$\frac{\sqrt{\frac{1-\sin(e+fx)}{\sin(e+fx)+1}}\cos(e+fx)(3-4\sin(e+fx))^{-m}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},-m;1-m;-\frac{2(3-4\sin(e+fx))}{\sin(e+fx)+1}\right)}{\sqrt{7}fm(1-\sin(e+fx))}$$

```
[Out] (Cos[e + f*x]*Hypergeometric2F1[1/2, -m, 1 - m, (-2*(3 - 4*Sin[e + f*x]))/(
1 + Sin[e + f*x])]*Sqrt[(1 - Sin[e + f*x])/(1 + Sin[e + f*x])]*(a + a*Sin[e
 + f*x])^m)/(Sqrt[7]*f*m*(3 - 4*Sin[e + f*x])^m*(1 - Sin[e + f*x]))
```

Rubi [A] time = 0.105765, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2788, 132}

$$\frac{\sqrt{\frac{1-\sin(e+fx)}{\sin(e+fx)+1}}\cos(e+fx)(3-4\sin(e+fx))^{-m}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},-m;1-m;-\frac{2(3-4\sin(e+fx))}{\sin(e+fx)+1}\right)}{\sqrt{7}fm(1-\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - 4*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] (Cos[e + f*x]*Hypergeometric2F1[1/2, -m, 1 - m, (-2*(3 - 4*Sin[e + f*x]))/(
1 + Sin[e + f*x])]*Sqrt[(1 - Sin[e + f*x])/(1 + Sin[e + f*x])]*(a + a*Sin[e
 + f*x])^m)/(Sqrt[7]*f*m*(3 - 4*Sin[e + f*x])^m*(1 - Sin[e + f*x]))
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x
)^n)/Sqrt[a - b*x], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1)*
Hypergeometric2F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*
(e + f*x)))])/(((b*e - a*f)*(m + 1))*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*
(e + f*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int (3-4\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx &= \frac{\left(a^2\cos(e+fx)\right)\text{Subst}\left(\int \frac{(3-4x)^{-1-m}(a+ax)^{-\frac{1}{2}+m}}{\sqrt{a-ax}}\,dx,x,\sin(e+fx)\right)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}\\ &= \frac{\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-m;1-m;-\frac{2(3-4\sin(e+fx))}{1+\sin(e+fx)}\right)(3-4\sin(e+fx))^{-m}}{\sqrt{7}fm(1-\sin(e+fx))}\end{aligned}$$

Mathematica [A] time = 0.654773, size = 178, normalized size = 1.55

$$\frac{2\cot\left(\frac{1}{4}(2e+2fx+\pi)\right)(3-4\sin(e+fx))^{-m}\sin^2\left(\frac{1}{4}(2e+2fx+\pi)\right)^{\frac{1}{2}-m}\cos^2\left(\frac{1}{4}(2e+2fx-\pi)\right)^{m-\frac{1}{2}}(a(\sin(e+fx)+}{f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(3 - 4*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] (2*(Cos[(2*e - Pi + 2*f*x)/4]^2)^(-1/2 + m)*Cot[(2*e + Pi + 2*f*x)/4]*Hyper
geometric2F1[1/2, 1/2 - m, 3/2, (7*Sin[(2*e - Pi + 2*f*x)/4]^2)/(3 - 4*Sin[
e + f*x])]*(a*(1 + Sin[e + f*x]))^m*(Cos[(2*e - Pi + 2*f*x)/4]^2/(-3 + 4*Si
n[e + f*x]))^(1/2 - m)*(Sin[(2*e + Pi + 2*f*x)/4]^2)^(1/2 - m))/(f*(3 - 4*S
in[e + f*x])^m)
```

Maple [F] time = 0.263, size = 0, normalized size = 0.

$$\int \left(3-4\sin\left(fx+e\right)\right)^{-1-m}\left(a+a\sin\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3-4*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

```
[Out] int((3-4*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^{m}\left(-4\sin\left(fx+e\right)+3\right)^{-m-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-4*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(-4*sin(f*x + e) + 3)^(-m - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-4\sin\left(fx+e\right)+3\right)^{-m-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-4*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)^m*(-4*sin(f*x + e) + 3)^(-m - 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-4*sin(f*x+e))**(-1-m)*(a+a*sin(f*x+e))**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^{m}\left(-4\sin\left(fx+e\right)+3\right)^{-m-1}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-4*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(-4*sin(f*x + e) + 3)^(-m - 1), x)
```

3.641 $\int (3-5\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx$

Optimal. Leaf size=113

$$\frac{\sqrt{\frac{1-\sin(e+fx)}{\sin(e+fx)+1}}\cos(e+fx)(3-5\sin(e+fx))^{-m}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},-m;1-m;-\frac{3-5\sin(e+fx)}{\sin(e+fx)+1}\right)}{4fm(1-\sin(e+fx))}$$

```
[Out] (Cos[e + f*x]*Hypergeometric2F1[1/2, -m, 1 - m, -((3 - 5*Sin[e + f*x])/(1 +
 Sin[e + f*x]))]*Sqrt[(1 - Sin[e + f*x])/(1 + Sin[e + f*x])]*(a + a*Sin[e +
 f*x])^m)/(4*f*m*(3 - 5*Sin[e + f*x])^m*(1 - Sin[e + f*x]))
```

Rubi [A] time = 0.101995, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2788, 132}

$$\frac{\sqrt{\frac{1-\sin(e+fx)}{\sin(e+fx)+1}}\cos(e+fx)(3-5\sin(e+fx))^{-m}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},-m;1-m;-\frac{3-5\sin(e+fx)}{\sin(e+fx)+1}\right)}{4fm(1-\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - 5*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] (Cos[e + f*x]*Hypergeometric2F1[1/2, -m, 1 - m, -((3 - 5*Sin[e + f*x])/(1 +
 Sin[e + f*x]))]*Sqrt[(1 - Sin[e + f*x])/(1 + Sin[e + f*x])]*(a + a*Sin[e +
 f*x])^m)/(4*f*m*(3 - 5*Sin[e + f*x])^m*(1 - Sin[e + f*x]))
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x
)^n)/Sqrt[a - b*x], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1)*
Hypergeometric2F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*
(e + f*x)))])/(((b*e - a*f)*(m + 1))*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*
(e + f*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int (3-5\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx &= \frac{\left(a^2\cos(e+fx)\right)\operatorname{Subst}\left(\int\frac{(3-5x)^{-1-m}(a+ax)^{-\frac{1}{2}+m}}{\sqrt{a-ax}}\,dx,x,\sin(e+fx)\right)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}\\ &= \frac{\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-m;1-m;-\frac{3-5\sin(e+fx)}{1+\sin(e+fx)}\right)(3-5\sin(e+fx))^{-m}}{4fm(1-\sin(e+fx))}\end{aligned}$$

Mathematica [C] time = 0.553362, size = 248, normalized size = 2.19

$$-\frac{2^{2m-1}(\cosh(m\log(4))-\sinh(m\log(4)))(3-5\sin(e+fx))^{-m}(\sin(e+fx)+i\cos(e+fx)+1)(a(\sin(e+fx)+1))^m\left(\frac{2\sin}{2\sin}\right.}{f(2m+1)((1+2i)\sin(e+fx)+(-2+i)\cos}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(3 - 5*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] -((2^(-1 + 2*m)*Hypergeometric2F1[1 + m, 1 + 2*m, 2*(1 + m), (2*Cos[(2*e -
Pi + 2*f*x)/4])/(Cos[(2*e - Pi + 2*f*x)/4] + 2*Sin[(2*e - Pi + 2*f*x)/4])]*
(a*(1 + Sin[e + f*x]))^m*(1 + I*Cos[e + f*x] + Sin[e + f*x])*((-Cos[(2*e -
Pi + 2*f*x)/4] + 2*Sin[(2*e - Pi + 2*f*x)/4])/(Cos[(2*e - Pi + 2*f*x)/4] +
2*Sin[(2*e - Pi + 2*f*x)/4]))^m*(Cosh[m*Log[4]] - Sinh[m*Log[4]]))/(f*(1 +
2*m)*(3 - 5*Sin[e + f*x])^m*((1 - 2*I) - (2 - I)*Cos[e + f*x] + (1 + 2*I)*S
in[e + f*x])))
```

Maple [F] time = 0.261, size = 0, normalized size = 0.

$$\int \left(3-5\sin\left(fx+e\right)\right)^{-1-m}\left(a+a\sin\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3-5*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

```
[Out] int((3-5*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^{m}\left(-5\sin\left(fx+e\right)+3\right)^{-m-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-5*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(-5*sin(f*x + e) + 3)^(-m - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-5\sin\left(fx+e\right)+3\right)^{-m-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-5*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)^m*(-5*sin(f*x + e) + 3)^(-m - 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-5*sin(f*x+e))**(-1-m)*(a+a*sin(f*x+e))**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^{m}\left(-5\sin\left(fx+e\right)+3\right)^{-m-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-5*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(-5*sin(f*x + e) + 3)^(-m - 1), x)
```

3.642 $\int (-3+5\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx$

Optimal. Leaf size=72

$$-\frac{\cos(e+fx)(\sin(e+fx)+1)^{-m-1}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},m+1;\frac{3}{2};\frac{4(a-a\sin(e+fx))}{\sin(e+fx)a+a}\right)}{f}$$

```
[Out] -((Cos[e + f*x]*Hypergeometric2F1[1/2, 1 + m, 3/2, (4*(a - a*Sin[e + f*x]))
/(a + a*Sin[e + f*x])]*(1 + Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m)/
f)
```

Rubi [A] time = 0.0980652, antiderivative size = 113, normalized size of antiderivative = 1.57, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2788, 132}

$$-\frac{\sqrt{\frac{1-\sin(e+fx)}{\sin(e+fx)+1}}\cos(e+fx)(5\sin(e+fx)-3)^{-m}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},-m;1-m;-\frac{3-5\sin(e+fx)}{\sin(e+fx)+1}\right)}{4fm(1-\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(-3 + 5*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] -(Cos[e + f*x]*Hypergeometric2F1[1/2, -m, 1 - m, -((3 - 5*Sin[e + f*x])/(1
+ Sin[e + f*x]))]*Sqrt[(1 - Sin[e + f*x])/(1 + Sin[e + f*x])]*(a + a*Sin[e
+ f*x])^m)/(4*f*m*(1 - Sin[e + f*x])*(-3 + 5*Sin[e + f*x])^m)
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x
)^n)/Sqrt[a - b*x], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1)*
Hypergeometric2F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*
(e + f*x)))])/(((b*e - a*f)*(m + 1))*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*
(e + f*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int (-3+5\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx = \frac{\left(a^2\cos(e+fx)\right)\text{Subst}\left(\int \frac{(-3+5x)^{-1-m}(a+ax)^{-\frac{1}{2}+m}}{\sqrt{a-ax}}\,dx,x,\sin(e+fx)\right)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-m;1-m;-\frac{3-5\sin(e+fx)}{1+\sin(e+fx)}\right)\sqrt{\frac{1-\sin(e+fx)}{1+\sin(e+fx)}}(-3+5\ldots}{4fm(1-\sin(e+fx))}$$

Mathematica [C] time = 1.52856, size = 247, normalized size = 3.43

$$\frac{2^{2m-1}(\cosh(m\log(4))-\sinh(m\log(4)))(5\sin(e+fx)-3)^{-m}(\sin(e+fx)+i\cos(e+fx)+1)(a(\sin(e+fx)+1))^m\left(\begin{matrix}2\\-\\2\end{matrix}\right.}{f(2m+1)((1+2i)\sin(e+fx)+(-2+i)\,c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-3 + 5*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] (2^(-1 + 2*m)*Hypergeometric2F1[1 + m, 1 + 2*m, 2*(1 + m), (2*Cos[(2*e - Pi
 + 2*f*x)/4])/(Cos[(2*e - Pi + 2*f*x)/4] + 2*Sin[(2*e - Pi + 2*f*x)/4])]*(a
*(1 + Sin[e + f*x]))^m*(1 + I*Cos[e + f*x] + Sin[e + f*x])*((-Cos[(2*e - Pi
 + 2*f*x)/4] + 2*Sin[(2*e - Pi + 2*f*x)/4])/(Cos[(2*e - Pi + 2*f*x)/4] + 2*
Sin[(2*e - Pi + 2*f*x)/4]))^m*(Cosh[m*Log[4]] - Sinh[m*Log[4]]))/(f*(1 + 2*
m)*((1 - 2*I) - (2 - I)*Cos[e + f*x] + (1 + 2*I)*Sin[e + f*x])*(-3 + 5*Sin[
e + f*x])^m)
```

Maple [F] time = 0.228, size = 0, normalized size = 0.

$$\int \left(-3+5\,\sin\left(fx+e\right)\right)^{-1-m}\left(a+a\sin\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3+5*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

```
[Out] int((-3+5*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^{m}\left(5\,\sin\left(fx+e\right)-3\right)^{-m-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+5*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="maxima"
)
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(5*sin(f*x + e) - 3)^(-m - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a\sin\left(fx+e\right)+a\right)^{m}\left(5\,\sin\left(fx+e\right)-3\right)^{-m-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+5*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="fricas"
)
```

```
[Out] integral((a*sin(f*x + e) + a)^m*(5*sin(f*x + e) - 3)^(-m - 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+5*sin(f*x+e))**(-1-m)*(a+a*sin(f*x+e))**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^{m}\left(5\,\sin\left(fx+e\right)-3\right)^{-m-1}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+5*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(5*sin(f*x + e) - 3)^(-m - 1), x)
```

3.643 $\int (-3+4\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx$

Optimal. Leaf size=77

$$-\frac{2^{m+1}\cos(e+fx)(\sin(e+fx)+1)^{-m-1}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},m+1;\frac{3}{2};\frac{7(a-a\sin(e+fx))}{\sin(e+fx)a+a}\right)}{f}$$

```
[Out] -((2^(1 + m)*Cos[e + f*x]*Hypergeometric2F1[1/2, 1 + m, 3/2, (7*(a - a*Sin[
e + f*x]))/(a + a*Sin[e + f*x])]*(1 + Sin[e + f*x])^(-1 - m)*(a + a*Sin[e +
 f*x])^m)/f)
```

Rubi [A] time = 0.105207, antiderivative size = 116, normalized size of antiderivative = 1.51, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2788, 132}

$$-\frac{\sqrt{\frac{1-\sin(e+fx)}{\sin(e+fx)+1}}\cos(e+fx)(4\sin(e+fx)-3)^{-m}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},-m;1-m;-\frac{2(3-4\sin(e+fx))}{\sin(e+fx)+1}\right)}{\sqrt{7}fm(1-\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(-3 + 4*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] -((Cos[e + f*x]*Hypergeometric2F1[1/2, -m, 1 - m, (-2*(3 - 4*Sin[e + f*x]))
/(1 + Sin[e + f*x])]*Sqrt[(1 - Sin[e + f*x])/(1 + Sin[e + f*x])]*(a + a*Sin
[e + f*x])^m)/(Sqrt[7]*f*m*(1 - Sin[e + f*x])*(-3 + 4*Sin[e + f*x])^m))
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x
)^n)/Sqrt[a - b*x], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1)*
Hypergeometric2F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*
(e + f*x)))])/(((b*e - a*f)*(m + 1))*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*
(e + f*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int (-3+4\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx = \frac{\left(a^2\cos(e+fx)\right)\text{Subst}\left(\int \frac{(-3+4x)^{-1-m}(a+ax)^{-\frac{1}{2}+m}}{\sqrt{a-ax}}\,dx,x,\sin(e+fx\right.}{f\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-m;1-m;-\frac{2(3-4\sin(e+fx))}{1+\sin(e+fx)}\right)\sqrt{\frac{1-\sin(e+fx)}{1+\sin(e+fx)}}(-3}{\sqrt{7}fm(1-\sin(e+fx))}$$

Mathematica [A] time = 0.612664, size = 154, normalized size = 2.

$$-\frac{2\cot\left(\frac{1}{4}(2e+2fx+\pi)\right)(4\sin(e+fx)-3)^{-m}\sin^2\left(\frac{1}{4}(2e+2fx+\pi)\right)^{\frac{1}{2}-m}\cos^2\left(\frac{1}{4}(2e+2fx-\pi)\right)^{m-\frac{1}{2}}(a(\sin(e+fx)+1}{f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-3 + 4*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] (-2*(Cos[(2*e - Pi + 2*f*x)/4]^2)^(-1/2 + m)*Cot[(2*e + Pi + 2*f*x)/4]*Hype
rgeometric2F1[1/2, 1 + m, 3/2, 7*Tan[(2*e - Pi + 2*f*x)/4]^2]*(a*(1 + Sin[e
 + f*x]))^m*(Sec[(2*e - Pi + 2*f*x)/4]^2*(-3 + 4*Sin[e + f*x]))^m*(Sin[(2*e
 + Pi + 2*f*x)/4]^2)^(1/2 - m))/(f*(-3 + 4*Sin[e + f*x])^m)
```

Maple [F] time = 0.251, size = 0, normalized size = 0.

$$\int \left(-3+4\sin\left(fx+e\right)\right)^{-1-m}\left(a+a\sin\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3+4*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

```
[Out] int((-3+4*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^{m}\left(4\sin\left(fx+e\right)-3\right)^{-m-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+4*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="maxima"
)
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(4*sin(f*x + e) - 3)^(-m - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a\sin\left(fx+e\right)+a\right)^{m}\left(4\sin\left(fx+e\right)-3\right)^{-m-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+4*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="fricas"
)
```

```
[Out] integral((a*sin(f*x + e) + a)^m*(4*sin(f*x + e) - 3)^(-m - 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+4*sin(f*x+e))**(-1-m)*(a+a*sin(f*x+e))**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a \sin (fx+e)+a)^m (4 \sin (fx+e)-3)^{-m-1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+4*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(4*sin(f*x + e) - 3)^(-m - 1), x)
```

3.644 $\int(-3+3\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx$

Optimal. Leaf size=45

$$\frac{\cos(e+fx)(3\sin(e+fx)-3)^{-m-1}(a\sin(e+fx)+a)^m}{f(2m+1)}$$

```
[Out] (Cos[e + f*x]*(-3 + 3*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m)/(f*(1
+ 2*m))
```

Rubi [A] time = 0.0630852, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {2742}

$$\frac{\cos(e+fx)(3\sin(e+fx)-3)^{-m-1}(a\sin(e+fx)+a)^m}{f(2m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(-3 + 3*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] (Cos[e + f*x]*(-3 + 3*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m)/(f*(1
+ 2*m))
```

Rule 2742

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rubi steps

$$\int(-3+3\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx=\frac{\cos(e+fx)(-3+3\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m}{f(1+2m)}$$

Mathematica [B] time = 0.668599, size = 110, normalized size = 2.44

$$\frac{2^{-m}3^{-m-1}\sin\left(\frac{1}{4}(2e+2fx+\pi)\right)(\sin(e+fx)-1)^{-m-1}\cos^{-2m-1}\left(\frac{1}{4}(2e+2fx+\pi)\right)(a(\sin(e+fx)+1))^m\left(\cos\left(\frac{1}{2}(e+fx)\right.\right.}{2fm+f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3 + 3*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] (3^(-1 - m)*Cos[(2*e + Pi + 2*f*x)/4]^(-1 - 2*m)*(Cos[(e + f*x)/2] - Sin[(e
 + f*x)/2])^(2*(1 + m))*(-1 + Sin[e + f*x])^(-1 - m)*(a*(1 + Sin[e + f*x]))
^m*Sin[(2*e + Pi + 2*f*x)/4])/(2^m*(f + 2*f*m))
```

Maple [F] time = 0.261, size = 0, normalized size = 0.

$$\int \left(-3+3 \sin \left(fx+e\right)\right)^{-1-m} \left(a+a \sin \left(fx+e\right)\right)^{m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3+3*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

```
[Out] int((-3+3*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a \sin \left(fx+e\right)+a\right)^{m} \left(3 \sin \left(fx+e\right)-3\right)^{-m-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+3*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="maxima"
)
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(3*sin(f*x + e) - 3)^(-m - 1), x)
```

Fricas [A] time = 1.5001, size = 107, normalized size = 2.38

$$\frac{\left(a \sin \left(fx+e\right)+a\right)^{m} \left(3 \sin \left(fx+e\right)-3\right)^{-m-1} \cos \left(fx+e\right)}{2 fm+f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+3*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="fricas"
)
```

```
[Out] (a*sin(f*x + e) + a)^m*(3*sin(f*x + e) - 3)^(-m - 1)*cos(f*x + e)/(2*f*m +
f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+3*sin(f*x+e))**(-1-m)*(a+a*sin(f*x+e))**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a \sin \left(fx+e\right)+a\right)^{m} \left(3 \sin \left(fx+e\right)-3\right)^{-m-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+3*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(3*sin(f*x + e) - 3)^(-m - 1), x)
```

3.645 $\int (-3+2\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx$

Optimal. Leaf size=117

$$-\frac{\sqrt{-\frac{1-\sin(e+fx)}{\sin(e+fx)+1}}\cos(e+fx)(2\sin(e+fx)-3)^{-m}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},-m;1-m;\frac{2(3-2\sin(e+fx))}{\sin(e+fx)+1}\right)}{\sqrt{5}fm(1-\sin(e+fx))}$$

```
[Out] -((Cos[e + f*x]*Hypergeometric2F1[1/2, -m, 1 - m, (2*(3 - 2*Sin[e + f*x]))/
(1 + Sin[e + f*x])]*Sqrt[-((1 - Sin[e + f*x])/(1 + Sin[e + f*x]))]*(a + a*S
in[e + f*x])^m)/(Sqrt[5]*f*m*(1 - Sin[e + f*x])*(-3 + 2*Sin[e + f*x])^m))
```

Rubi [A] time = 0.100542, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2788, 132}

$$-\frac{\sqrt{-\frac{1-\sin(e+fx)}{\sin(e+fx)+1}}\cos(e+fx)(2\sin(e+fx)-3)^{-m}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},-m;1-m;\frac{2(3-2\sin(e+fx))}{\sin(e+fx)+1}\right)}{\sqrt{5}fm(1-\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(-3 + 2*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```



```
[Out] -((Cos[e + f*x]*Hypergeometric2F1[1/2, -m, 1 - m, (2*(3 - 2*Sin[e + f*x]))/
(1 + Sin[e + f*x])]*Sqrt[-((1 - Sin[e + f*x])/(1 + Sin[e + f*x]))]*(a + a*S
in[e + f*x])^m)/(Sqrt[5]*f*m*(1 - Sin[e + f*x])*(-3 + 2*Sin[e + f*x])^m))
```


Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x
)^n)/Sqrt[a - b*x], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1)*
Hypergeometric2F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*
(e + f*x)))])/(((b*e - a*f)*(m + 1))*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*
(e + f*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] && !IntegerQ[n]
```

Rubi steps

$$\int (-3+2\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx = \frac{\left(a^2\cos(e+fx)\right)\text{Subst}\left(\int \frac{(-3+2x)^{-1-m}(a+ax)^{-\frac{1}{2}+m}}{\sqrt{a-ax}}\,dx,x,\sin(e+fx\right)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-m;1-m;\frac{2(3-2\sin(e+fx))}{1+\sin(e+fx)}\right)\sqrt{-\frac{1-\sin(e+fx)}{1+\sin(e+fx)}}(-3}{\sqrt{5}fm(1-\sin(e+fx))}$$

Mathematica [A] time = 0.626074, size = 155, normalized size = 1.32

$$\frac{2\cot\left(\frac{1}{4}(2e+2fx+\pi)\right)(2\sin(e+fx)-3)^{-m}\sin^2\left(\frac{1}{4}(2e+2fx+\pi)\right)^{\frac{1}{2}-m}\cos^2\left(\frac{1}{4}(2e+2fx-\pi)\right)^{m-\frac{1}{2}}(a(\sin(e+fx)+1))}{f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-3 + 2*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] (2*(Cos[(2*e - Pi + 2*f*x)/4]^2)^(-1/2 + m)*Cot[(2*e + Pi + 2*f*x)/4]*Hyper
geometric2F1[1/2, 1 + m, 3/2, -5*Tan[(2*e - Pi + 2*f*x)/4]^2]*(a*(1 + Sin[e
 + f*x]))^m*(-(Sec[(2*e - Pi + 2*f*x)/4]^2*(-3 + 2*Sin[e + f*x])))^m*(Sin[(
2*e + Pi + 2*f*x)/4]^2)^(1/2 - m))/(f*(-3 + 2*Sin[e + f*x])^m)
```

Maple [F] time = 0.232, size = 0, normalized size = 0.

$$\int \left(-3+2\,\sin\left(fx+e\right)\right)^{-1-m}\left(a+a\sin\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3+2*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

```
[Out] int((-3+2*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^{m}\left(2\,\sin\left(fx+e\right)-3\right)^{-m-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+2*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="maxima"
)
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(2*sin(f*x + e) - 3)^(-m - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a\sin\left(fx+e\right)+a\right)^{m}\left(2\,\sin\left(fx+e\right)-3\right)^{-m-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+2*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="fricas"
)
```

```
[Out] integral((a*sin(f*x + e) + a)^m*(2*sin(f*x + e) - 3)^(-m - 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+2*sin(f*x+e))**(-1-m)*(a+a*sin(f*x+e))**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^m\left(2\,\sin\left(fx+e\right)-3\right)^{-m-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+2*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(2*sin(f*x + e) - 3)^(-m - 1), x)
```

3.646 $\int (-3+\sin (e+f x))^{-1-m}(a+a \sin (e+f x))^m \, dx$

Optimal. Leaf size=116

$$-\frac{\sqrt{-\frac{1-\sin(e+fx)}{\sin(e+fx)+1}}\cos(e+fx)(\sin(e+fx)-3)^{-m}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},-m;1-m;\frac{3-\sin(e+fx)}{\sin(e+fx)+1}\right)}{2\sqrt{2}fm(1-\sin(e+fx))}$$

```
[Out] -(Cos[e + f*x]*Hypergeometric2F1[1/2, -m, 1 - m, (3 - Sin[e + f*x])/(1 + Si
n[e + f*x])]*Sqrt[-((1 - Sin[e + f*x])/(1 + Sin[e + f*x]))]*(a + a*Sin[e +
f*x])^m)/(2*Sqrt[2]*f*m*(1 - Sin[e + f*x])*(-3 + Sin[e + f*x])^m)
```

Rubi [A] time = 0.0967191, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2788, 132}

$$-\frac{\sqrt{-\frac{1-\sin(e+fx)}{\sin(e+fx)+1}}\cos(e+fx)(\sin(e+fx)-3)^{-m}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},-m;1-m;\frac{3-\sin(e+fx)}{\sin(e+fx)+1}\right)}{2\sqrt{2}fm(1-\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(-3 + Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] -(Cos[e + f*x]*Hypergeometric2F1[1/2, -m, 1 - m, (3 - Sin[e + f*x])/(1 + Si
n[e + f*x])]*Sqrt[-((1 - Sin[e + f*x])/(1 + Sin[e + f*x]))]*(a + a*Sin[e +
f*x])^m)/(2*Sqrt[2]*f*m*(1 - Sin[e + f*x])*(-3 + Sin[e + f*x])^m)
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x
)^n)/Sqrt[a - b*x], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1)*
Hypergeometric2F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*
(e + f*x)))])/(((b*e - a*f)*(m + 1))*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*
(e + f*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] && !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int (-3+\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx &= \frac{\left(a^2\cos(e+fx)\right)\text{Subst}\left(\int \frac{(-3+x)^{-1-m}(a+ax)^{-\frac{1}{2}+m}}{\sqrt{a-ax}}\,dx,x,\sin(e+fx)\right)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}\\ &= -\frac{\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-m;1-m;\frac{3-\sin(e+fx)}{1+\sin(e+fx)}\right)(-3+\sin(e+fx))^{-m}\sqrt{-}}{2\sqrt{2}fm(1-\sin(e+fx))}\end{aligned}$$

Mathematica [A] time = 0.725218, size = 155, normalized size = 1.34

$$\frac{2^{-m}\cot\left(\frac{1}{4}(2e+2fx+\pi)\right)(\sin(e+fx)-3)^{-m}\sin^2\left(\frac{1}{4}(2e+2fx+\pi)\right)^{\frac{1}{2}-m}\cos^2\left(\frac{1}{4}(2e+2fx-\pi)\right)^{m-\frac{1}{2}}(a(\sin(e+fx)\cdots}{f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-3 + Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] ((Cos[(2*e - Pi + 2*f*x)/4]^2)^(-1/2 + m)*Cot[(2*e + Pi + 2*f*x)/4]*Hyperge
ometric2F1[1/2, 1 + m, 3/2, -2*Tan[(2*e - Pi + 2*f*x)/4]^2]*(-(Sec[(2*e - P
i + 2*f*x)/4]^2*(-3 + Sin[e + f*x])))^m*(a*(1 + Sin[e + f*x]))^m*(Sin[(2*e
+ Pi + 2*f*x)/4]^2)^(1/2 - m))/(2^m*f*(-3 + Sin[e + f*x])^m)
```

Maple [F] time = 0.241, size = 0, normalized size = 0.

$$\int \left(-3+\sin\left(fx+e\right)\right)^{-1-m}\left(a+a\sin\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3+sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

```
[Out] int((-3+sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^{m}\left(\sin\left(fx+e\right)-3\right)^{-m-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(sin(f*x + e) - 3)^(-m - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a\sin\left(fx+e\right)+a\right)^{m}\left(\sin\left(fx+e\right)-3\right)^{-m-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)^m*(sin(f*x + e) - 3)^(-m - 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+sin(f*x+e))**(-1-m)*(a+a*sin(f*x+e))**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^{m}\left(\sin\left(fx+e\right)-3\right)^{-m-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(sin(f*x + e) - 3)^(-m - 1), x)
```

3.647 $\int(-3)^{-1-m}(a+a\sin(e+fx))^m\,dx$

Optimal. Leaf size=81

$$-\frac{(-3)^{-m-1}2^{m+\frac{1}{2}}\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{f}$$

```
[Out] -(((-3)^(-1 - m)*2^(1/2 + m)*Cos[e + f*x]*Hypergeometric2F1[1/2, 1/2 - m, 3
/2, (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Sin[e + f*x]
)^m)/f)
```

Rubi [A] time = 0.0470816, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {12, 2652, 2651}

$$-\frac{(-3)^{-m-1}2^{m+\frac{1}{2}}\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(-3)^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] -(((-3)^(-1 - m)*2^(1/2 + m)*Cos[e + f*x]*Hypergeometric2F1[1/2, 1/2 - m, 3
/2, (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Sin[e + f*x]
)^m)/f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2652

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Sin[c + d*x])^FracPart[n])/(1 + (b*Sin[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Sin[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2651

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(2^(n +
 1/2)*a^(n - 1/2)*b*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1*(1
 - (b*Sin[c + d*x])/a))/2])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int(-3)^{-1-m}(a+a\sin(e+fx))^m\,dx &= (-3)^{-1-m}\int(a+a\sin(e+fx))^m\,dx\\ &= \left((-3)^{-1-m}(1+\sin(e+fx))^{-m}(a+a\sin(e+fx))^m\right)\int(1+\sin(e+fx))^m\,dx\\ &= -\frac{(-3)^{-1-m}2^{\frac{1}{2}+m}\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)(1+\sin(e+fx}{f}\end{aligned}$$

Mathematica [A] time = 0.156529, size = 97, normalized size = 1.2

$$\frac{\sqrt{2}(-3)^{-m-1}\cos(e+fx)(a(\sin(e+fx)+1))^m\,{}_2F_1\left(\frac{1}{2},m+\frac{1}{2};m+\frac{3}{2};\frac{1}{4}\cos^2(e+fx)\csc^2\left(\frac{1}{4}(2e+2fx-\pi)\right)\right)}{(2fm+f)\sqrt{1-\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3)^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] ((-3)^(-1 - m)*Sqrt[2]*Cos[e + f*x]*Hypergeometric2F1[1/2, 1/2 + m, 3/2 + m
, (Cos[e + f*x]^2*Csc[(2*e - Pi + 2*f*x)/4]^2)/4]*(a*(1 + Sin[e + f*x]))^m)
/((f + 2*f*m)*Sqrt[1 - Sin[e + f*x]])
```

Maple [F] time = 0.332, size = 0, normalized size = 0.

$$\int (-3)^{-1-m}\left(a+a\sin\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3)^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

```
[Out] int((-3)^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$(-3)^{-m-1}\int\left(a\sin\left(fx+e\right)+a\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3)^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] (-3)^(-m - 1)*integrate((a*sin(f*x + e) + a)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((-3)^{-m-1}\left(a\sin\left(fx+e\right)+a\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3)^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((-3)^(-m - 1)*(a*sin(f*x + e) + a)^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$(-3)^{-m-1}\int\left(a\sin\left(e+fx\right)+a\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3)**(-1-m)*(a+a*sin(f*x+e))**m,x)
```

```
[Out] (-3)**(-m - 1)*Integral((a*sin(e + f*x) + a)**m, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (-3)^{-m-1}\left(a\sin\left(fx+e\right)+a\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3)^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((-3)^(-m - 1)*(a*sin(f*x + e) + a)^m, x)
```

3.648 $\int (-3-\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx$

Optimal. Leaf size=119

$$-\frac{\sqrt{-\frac{1-\sin(e+fx)}{\sin(e+fx)+1}}\cos(e+fx)(-\sin(e+fx)-3)^{-m}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},-m;1-m;\frac{\sin(e+fx)+3}{2(\sin(e+fx)+1)}\right)}{2\sqrt{2}fm(1-\sin(e+fx))}$$

```
[Out] -(Cos[e + f*x]*Hypergeometric2F1[1/2, -m, 1 - m, (3 + Sin[e + f*x])/(2*(1 +
 Sin[e + f*x]))]*Sqrt[-((1 - Sin[e + f*x])/(1 + Sin[e + f*x]))]*(a + a*Sin[
e + f*x])^m)/(2*Sqrt[2]*f*m*(-3 - Sin[e + f*x])^m*(1 - Sin[e + f*x]))
```

Rubi [A] time = 0.10243, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2788, 132}

$$-\frac{\sqrt{-\frac{1-\sin(e+fx)}{\sin(e+fx)+1}}\cos(e+fx)(-\sin(e+fx)-3)^{-m}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},-m;1-m;\frac{\sin(e+fx)+3}{2(\sin(e+fx)+1)}\right)}{2\sqrt{2}fm(1-\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(-3 - Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] -(Cos[e + f*x]*Hypergeometric2F1[1/2, -m, 1 - m, (3 + Sin[e + f*x])/(2*(1 +
 Sin[e + f*x]))]*Sqrt[-((1 - Sin[e + f*x])/(1 + Sin[e + f*x]))]*(a + a*Sin[
e + f*x])^m)/(2*Sqrt[2]*f*m*(-3 - Sin[e + f*x])^m*(1 - Sin[e + f*x]))
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x
)^n)/Sqrt[a - b*x], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1)*
Hypergeometric2F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*
(e + f*x)))])/(((b*e - a*f)*(m + 1))*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*
(e + f*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int (-3-\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx=\frac{\left(a^2\cos(e+fx)\right)\text{Subst}\left(\int\frac{(-3-x)^{-1-m}(a+ax)^{-\frac{1}{2}+m}}{\sqrt{a-ax}}\,dx,x,\sin(e+fx)\right)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}$$

$$=-\frac{\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-m;1-m;\frac{3+\sin(e+fx)}{2(1+\sin(e+fx))}\right)(-3-\sin(e+fx))^{-m}\sqrt{}}{2\sqrt{2}fm(1-\sin(e+fx))}$$

Mathematica [A] time = 0.909052, size = 131, normalized size = 1.1

$$\frac{4^{-m}\cot\left(\frac{1}{4}(2e+2fx+\pi)\right)(-\sin(e+fx)-3)^{-m}(\sin(e+fx)+3)^{m-\frac{1}{2}}\sin^2\left(\frac{1}{4}(2e+2fx+\pi)\right)^{\frac{1}{2}-m}(a(\sin(e+fx)+1))^{m}}{f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-3 - Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] (Cot[(2*e + Pi + 2*f*x)/4]*Hypergeometric2F1[1/2, 1/2 - m, 3/2, (2*Sin[(2*e
 - Pi + 2*f*x)/4]^2)/(3 + Sin[e + f*x])]*(a*(1 + Sin[e + f*x]))^m*(3 + Sin[
e + f*x])^(-1/2 + m)*(Sin[(2*e + Pi + 2*f*x)/4]^2)^(1/2 - m))/(4^m*f*(-3 -
Sin[e + f*x])^m)
```

Maple [F] time = 0.26, size = 0, normalized size = 0.

$$\int \left(-3-\sin\left(fx+e\right)\right)^{-1-m}\left(a+a\sin\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3-sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

```
[Out] int((-3-sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^{m}\left(-\sin\left(fx+e\right)-3\right)^{-m-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3-sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(-sin(f*x + e) - 3)^(-m - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-\sin\left(fx+e\right)-3\right)^{-m-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3-sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)^m*(-sin(f*x + e) - 3)^(-m - 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3-sin(f*x+e))**(-1-m)*(a+a*sin(f*x+e))**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^{m}\left(-\sin\left(fx+e\right)-3\right)^{-m-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3-sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(-sin(f*x + e) - 3)^(-m - 1), x)
```

3.649 $\int(-3-2\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx$

Optimal. Leaf size=119

$$-\frac{\sqrt{-\frac{1-\sin(e+fx)}{\sin(e+fx)+1}}\cos(e+fx)(-2\sin(e+fx)-3)^{-m}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},-m;1-m;\frac{2(2\sin(e+fx)+3)}{5(\sin(e+fx)+1)}\right)}{\sqrt{5}fm(1-\sin(e+fx))}$$

```
[Out] -((Cos[e + f*x]*Hypergeometric2F1[1/2, -m, 1 - m, (2*(3 + 2*Sin[e + f*x]))/
(5*(1 + Sin[e + f*x]))]*Sqrt[-((1 - Sin[e + f*x])/(1 + Sin[e + f*x]))]*(a +
 a*Sin[e + f*x])^m)/(Sqrt[5]*f*m*(-3 - 2*Sin[e + f*x])^m*(1 - Sin[e + f*x])
))
```

Rubi [A] time = 0.113902, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2788, 132}

$$-\frac{\sqrt{-\frac{1-\sin(e+fx)}{\sin(e+fx)+1}}\cos(e+fx)(-2\sin(e+fx)-3)^{-m}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},-m;1-m;\frac{2(2\sin(e+fx)+3)}{5(\sin(e+fx)+1)}\right)}{\sqrt{5}fm(1-\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(-3 - 2*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] -((Cos[e + f*x]*Hypergeometric2F1[1/2, -m, 1 - m, (2*(3 + 2*Sin[e + f*x]))/
(5*(1 + Sin[e + f*x]))]*Sqrt[-((1 - Sin[e + f*x])/(1 + Sin[e + f*x]))]*(a +
 a*Sin[e + f*x])^m)/(Sqrt[5]*f*m*(-3 - 2*Sin[e + f*x])^m*(1 - Sin[e + f*x])
))
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x
)^n)/Sqrt[a - b*x], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1)*
Hypergeometric2F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*
(e + f*x)))])/(((b*e - a*f)*(m + 1))*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*
(e + f*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int(-3-2\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx = \frac{\left(a^2\cos(e+fx)\right)\operatorname{Subst}\left(\int\frac{(-3-2x)^{-1-m}(a+ax)^{-\frac{1}{2}+m}}{\sqrt{a-ax}}\,dx,x,\sin(e+fx)\right)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}$$
$$= -\frac{\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-m;1-m;\frac{2(3+2\sin(e+fx))}{5(1+\sin(e+fx))}\right)(-3-2\sin(e+fx))^{\cdot}}{\sqrt{5}fm(1-\sin(e+fx))}$$

Mathematica [A] time = 1.13692, size = 186, normalized size = 1.56

$$\frac{2\,5^{-m-\frac{1}{2}}\cot\left(\frac{1}{4}(2e+2fx+\pi)\right)(-2\sin(e+fx)-3)^{-m}\sin^2\left(\frac{1}{4}(2e+2fx+\pi)\right)^{\frac{1}{2}-m}\cos^2\left(\frac{1}{4}(2e+2fx-\pi)\right)^{m-\frac{1}{2}}(a(\sin(e+\ldots}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3 - 2*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] (2*5^(-1/2 - m)*(Cos[(2*e - Pi + 2*f*x)/4]^2)^(-1/2 + m)*Cot[(2*e + Pi + 2*
f*x)/4]*Hypergeometric2F1[1/2, 1/2 - m, 3/2, Sin[(2*e - Pi + 2*f*x)/4]^2/(3
 + 2*Sin[e + f*x])]*(a*(1 + Sin[e + f*x]))^m*(Cos[(2*e - Pi + 2*f*x)/4]^2/(
3 + 2*Sin[e + f*x]))^(1/2 - m)*(Sin[(2*e + Pi + 2*f*x)/4]^2)^(1/2 - m))/(f*
(-3 - 2*Sin[e + f*x])^m)
```

Maple [F] time = 0.251, size = 0, normalized size = 0.

$$\int\left(-3-2\sin\left(fx+e\right)\right)^{-1-m}\left(a+a\sin\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3-2*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

```
[Out] int((-3-2*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-2\sin\left(fx+e\right)-3\right)^{-m-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3-2*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="maxima"
)
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(-2*sin(f*x + e) - 3)^(-m - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-2\sin\left(fx+e\right)-3\right)^{-m-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3-2*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="fricas"
)
```

```
[Out] integral((a*sin(f*x + e) + a)^m*(-2*sin(f*x + e) - 3)^(-m - 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3-2*sin(f*x+e))**(-1-m)*(a+a*sin(f*x+e))**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^{m}\left(-2\sin\left(fx+e\right)-3\right)^{-m-1}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3-2*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(-2*sin(f*x + e) - 3)^(-m - 1), x)
```

3.650 $\int (-3-3\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx$

Optimal. Leaf size=39

$$-\frac{\cos(e+fx)(-3\sin(e+fx)-3)^{-m-1}(a\sin(e+fx)+a)^m}{f}$$

```
[Out] -((Cos[e + f*x]*(-3 - 3*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m)/f)
```

Rubi [A] time = 0.0163482, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {23, 2648}

$$-\frac{\cos(e+fx)(-3\sin(e+fx)-3)^{-m-1}(a\sin(e+fx)+a)^m}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(-3 - 3*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] -((Cos[e + f*x]*(-3 - 3*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m)/f)
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] &&  !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int (-3-3\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx = \left((-3-3\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^{1+m}\right)\int \frac{1}{a+a\sin(e+fx)}$$

$$= -\frac{\cos(e+fx)(-3-3\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m}{f}$$

Mathematica [B] time = 0.499504, size = 106, normalized size = 2.72

$$-\frac{2^{-m}3^{-m-1}\cos\left(\frac{1}{4}(2e+2fx+\pi)\right)(-\sin(e+fx)-1)^{-m-1}\sin^{-2m-1}\left(\frac{1}{4}(2e+2fx+\pi)\right)(a(\sin(e+fx)+1))^m\left(\sin\left(\frac{1}{2}(e+f\right.\right.}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3 - 3*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] -((3^(-1 - m)*Cos[(2*e + Pi + 2*f*x)/4]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2
])^(2*(1 + m))*(-1 - Sin[e + f*x])^(-1 - m)*(a*(1 + Sin[e + f*x]))^m*Sin[(2
```

```
*e + Pi + 2*f*x)/4]^(-1 - 2*m))/(2^m*f))
```

Maple [F] time = 0.247, size = 0, normalized size = 0.

$$\int \left(-3-3\sin\left(fx+e\right)\right)^{-1-m}\left(a+a\sin\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3-3*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

```
[Out] int((-3-3*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

Maxima [A] time = 1.77564, size = 61, normalized size = 1.56

$$\frac{2\,a^{m}}{\left(3^{m+1}\left(-1\right)^{m}+\frac{3^{m+1}\left(-1\right)^{m}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3-3*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="maxima"
)
```

```
[Out] 2*a^m/((3^(m + 1)*(-1)^m + 3^(m + 1)*(-1)^m*sin(f*x + e)/(cos(f*x + e) + 1)
)*f)
```

Fricas [A] time = 1.54743, size = 119, normalized size = 3.05

$$\frac{\left(-\frac{1}{3}\,a\right)^{m}\left(\cos\left(fx+e\right)-\sin\left(fx+e\right)+1\right)}{3\left(f\cos\left(fx+e\right)+f\sin\left(fx+e\right)+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3-3*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="fricas"
)
```

```
[Out] 1/3*(-1/3*a)^m*(cos(f*x + e) - sin(f*x + e) + 1)/(f*cos(f*x + e) + f*sin(f*
x + e) + f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3-3*sin(f*x+e))**(-1-m)*(a+a*sin(f*x+e))**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^{m}\left(-3\sin\left(fx+e\right)-3\right)^{-m-1}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3-3*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(-3*sin(f*x + e) - 3)^(-m - 1), x)
```

3.651 $\int (-3-4\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx$

Optimal. Leaf size=117

$$\frac{\sqrt{\frac{1-\sin(e+fx)}{\sin(e+fx)+1}}\cos(e+fx)(-4\sin(e+fx)-3)^{-m}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},-m;1-m;\frac{2(4\sin(e+fx)+3)}{7(\sin(e+fx)+1)}\right)}{\sqrt{7}fm(1-\sin(e+fx))}$$

```
[Out] (Cos[e + f*x]*Hypergeometric2F1[1/2, -m, 1 - m, (2*(3 + 4*Sin[e + f*x]))/(7
*(1 + Sin[e + f*x]))]*Sqrt[(1 - Sin[e + f*x])/(1 + Sin[e + f*x])]*(a + a*Si
n[e + f*x])^m)/(Sqrt[7]*f*m*(-3 - 4*Sin[e + f*x])^m*(1 - Sin[e + f*x]))
```

Rubi [A] time = 0.100379, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2788, 132}

$$\frac{\sqrt{\frac{1-\sin(e+fx)}{\sin(e+fx)+1}}\cos(e+fx)(-4\sin(e+fx)-3)^{-m}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},-m;1-m;\frac{2(4\sin(e+fx)+3)}{7(\sin(e+fx)+1)}\right)}{\sqrt{7}fm(1-\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(-3 - 4*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] (Cos[e + f*x]*Hypergeometric2F1[1/2, -m, 1 - m, (2*(3 + 4*Sin[e + f*x]))/(7
*(1 + Sin[e + f*x]))]*Sqrt[(1 - Sin[e + f*x])/(1 + Sin[e + f*x])]*(a + a*Si
n[e + f*x])^m)/(Sqrt[7]*f*m*(-3 - 4*Sin[e + f*x])^m*(1 - Sin[e + f*x]))
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x
)^n)/Sqrt[a - b*x], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1)*
Hypergeometric2F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*
(e + f*x)))])/(((b*e - a*f)*(m + 1))*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*
(e + f*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] && !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int (-3-4\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx &= \frac{\left(a^2\cos(e+fx)\right)\text{Subst}\left(\int \frac{(-3-4x)^{-1-m}(a+ax)^{-\frac{1}{2}+m}}{\sqrt{a-ax}}\,dx,x,\sin(e+fx)\right)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}\\ &= \frac{\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-m;1-m;\frac{2(3+4\sin(e+fx))}{7(1+\sin(e+fx))}\right)(-3-4\sin(e+fx)}{\sqrt{7}fm(1-\sin(e+fx))}\end{aligned}$$

Mathematica [A] time = 1.17361, size = 187, normalized size = 1.6

$$\frac{2\,7^{-m-\frac{1}{2}}\cot\left(\frac{1}{4}(2e+2fx+\pi)\right)(-4\sin(e+fx)-3)^{-m}\sin^2\left(\frac{1}{4}(2e+2fx+\pi)\right)^{\frac{1}{2}-m}\cos^2\left(\frac{1}{4}(2e+2fx-\pi)\right)^{m-\frac{1}{2}}(a(\sin(e+\ldots}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3 - 4*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] (2*7^(-1/2 - m)*(Cos[(2*e - Pi + 2*f*x)/4]^2)^(-1/2 + m)*Cot[(2*e + Pi + 2*
f*x)/4]*Hypergeometric2F1[1/2, 1/2 - m, 3/2, -(Sin[(2*e - Pi + 2*f*x)/4]^2/
(3 + 4*Sin[e + f*x]))]*(a*(1 + Sin[e + f*x]))^m*(Cos[(2*e - Pi + 2*f*x)/4]^
2/(3 + 4*Sin[e + f*x]))^(1/2 - m)*(Sin[(2*e + Pi + 2*f*x)/4]^2)^(1/2 - m))/
(f*(-3 - 4*Sin[e + f*x])^m)
```

Maple [F] time = 0.266, size = 0, normalized size = 0.

$$\int \left(-3-4\sin\left(fx+e\right)\right)^{-1-m}\left(a+a\sin\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3-4*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

```
[Out] int((-3-4*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^{m}\left(-4\sin\left(fx+e\right)-3\right)^{-m-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3-4*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="maxima"
)
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(-4*sin(f*x + e) - 3)^(-m - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-4\sin\left(fx+e\right)-3\right)^{-m-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3-4*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="fricas"
)
```

```
[Out] integral((a*sin(f*x + e) + a)^m*(-4*sin(f*x + e) - 3)^(-m - 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3-4*sin(f*x+e))**(-1-m)*(a+a*sin(f*x+e))**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^{m}\left(-4\,\sin\left(fx+e\right)-3\right)^{-m-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3-4*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(-4*sin(f*x + e) - 3)^(-m - 1), x)
```

3.652 $\int (-3-5\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx$

Optimal. Leaf size=115

$$\frac{\sqrt{\frac{1-\sin(e+fx)}{\sin(e+fx)+1}}\cos(e+fx)(-5\sin(e+fx)-3)^{-m}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},-m;1-m;\frac{5\sin(e+fx)+3}{4(\sin(e+fx)+1)}\right)}{4fm(1-\sin(e+fx))}$$

```
[Out] (Cos[e + f*x]*Hypergeometric2F1[1/2, -m, 1 - m, (3 + 5*Sin[e + f*x])/(4*(1
+ Sin[e + f*x]))]*Sqrt[(1 - Sin[e + f*x])/(1 + Sin[e + f*x])]*(a + a*Sin[e
+ f*x])^m)/(4*f*m*(-3 - 5*Sin[e + f*x])^m*(1 - Sin[e + f*x]))
```

Rubi [A] time = 0.0982541, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2788, 132}

$$\frac{\sqrt{\frac{1-\sin(e+fx)}{\sin(e+fx)+1}}\cos(e+fx)(-5\sin(e+fx)-3)^{-m}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},-m;1-m;\frac{5\sin(e+fx)+3}{4(\sin(e+fx)+1)}\right)}{4fm(1-\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(-3 - 5*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] (Cos[e + f*x]*Hypergeometric2F1[1/2, -m, 1 - m, (3 + 5*Sin[e + f*x])/(4*(1
+ Sin[e + f*x]))]*Sqrt[(1 - Sin[e + f*x])/(1 + Sin[e + f*x])]*(a + a*Sin[e
+ f*x])^m)/(4*f*m*(-3 - 5*Sin[e + f*x])^m*(1 - Sin[e + f*x]))
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x
)^n)/Sqrt[a - b*x], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1)*
Hypergeometric2F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*
(e + f*x)))])/(((b*e - a*f)*(m + 1))*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*
(e + f*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] && !IntegerQ[n]
```

Rubi steps

$$\int (-3-5\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx = \frac{\left(a^2\cos(e+fx)\right)\text{Subst}\left(\int \frac{(-3-5x)^{-1-m}(a+ax)^{-\frac{1}{2}+m}}{\sqrt{a-ax}}\,dx,x,\sin(e+fx)\right)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-m;1-m;\frac{3+5\sin(e+fx)}{4(1+\sin(e+fx))}\right)(-3-5\sin(e+fx))^{-m}}{4fm(1-\sin(e+fx))}$$

Mathematica [C] time = 1.56132, size = 241, normalized size = 2.1

$$-\frac{4^m(\cosh(m\log(4))-\sinh(m\log(4)))(-5\sin(e+fx)-3)^{-m}(\sin(e+fx)+i\cos(e+fx)+1)(a(\sin(e+fx)+1))^m\Big(-}{f(2m+1)((2+i)\sin(e+fx)+(-1+2i}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-3 - 5*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] -((4^m*Hypergeometric2F1[1 + m, 1 + 2*m, 2*(1 + m), (4*Cos[(2*e - Pi + 2*f*
x)/4])/(2*Cos[(2*e - Pi + 2*f*x)/4] + Sin[(2*e - Pi + 2*f*x)/4])]*(a*(1 + S
in[e + f*x]))^m*(1 + I*Cos[e + f*x] + Sin[e + f*x])*(-((2*Cos[(2*e - Pi + 2
*f*x)/4] + Cos[(2*e + Pi + 2*f*x)/4])/(2*Cos[(2*e - Pi + 2*f*x)/4] + Sin[(2
*e - Pi + 2*f*x)/4])))^m*(Cosh[m*Log[4]] - Sinh[m*Log[4]]))/(f*(1 + 2*m)*(-
3 - 5*Sin[e + f*x])^m*((2 - I) - (1 - 2*I)*Cos[e + f*x] + (2 + I)*Sin[e + f
*x])))
```

Maple [F] time = 0.263, size = 0, normalized size = 0.

$$\int\left(-3-5\sin\left(fx+e\right)\right)^{-1-m}\left(a+a\sin\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3-5*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

```
[Out] int((-3-5*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-5\sin\left(fx+e\right)-3\right)^{-m-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3-5*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="maxima"
)
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(-5*sin(f*x + e) - 3)^(-m - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-5\sin\left(fx+e\right)-3\right)^{-m-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3-5*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="fricas"
)
```

```
[Out] integral((a*sin(f*x + e) + a)^m*(-5*sin(f*x + e) - 3)^(-m - 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3-5*sin(f*x+e))**(-1-m)*(a+a*sin(f*x+e))**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^{m}\left(-5\sin\left(fx+e\right)-3\right)^{-m-1}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3-5*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(-5*sin(f*x + e) - 3)^(-m - 1), x)
```

3.653 $\int (d\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx$

Optimal. Leaf size=116

$$-\frac{\cos(e+fx)\left(\frac{\sin(e+fx)+1}{1-\sin(e+fx)}\right)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^m(d\sin(e+fx))^{-m}\,{}_2F_1\left(\frac{1}{2}-m,-m;1-m;-\frac{2\sin(e+fx)}{1-\sin(e+fx)}\right)}{dfm(\sin(e+fx)+1)}$$

```
[Out] -((Cos[e + f*x]*Hypergeometric2F1[1/2 - m, -m, 1 - m, (-2*Sin[e + f*x])/(1
- Sin[e + f*x])]*((1 + Sin[e + f*x])/(1 - Sin[e + f*x]))^(1/2 - m)*(a + a*S
in[e + f*x])^m)/(d*f*m*(d*Sin[e + f*x])^m*(1 + Sin[e + f*x])))
```

Rubi [A] time = 0.189131, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2787, 2786, 2785, 132}

$$-\frac{\cos(e+fx)\left(\frac{\sin(e+fx)+1}{1-\sin(e+fx)}\right)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^m(d\sin(e+fx))^{-m}\,{}_2F_1\left(\frac{1}{2}-m,-m;1-m;-\frac{2\sin(e+fx)}{1-\sin(e+fx)}\right)}{dfm(\sin(e+fx)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] -((Cos[e + f*x]*Hypergeometric2F1[1/2 - m, -m, 1 - m, (-2*Sin[e + f*x])/(1
- Sin[e + f*x])]*((1 + Sin[e + f*x])/(1 - Sin[e + f*x]))^(1/2 - m)*(a + a*S
in[e + f*x])^m)/(d*f*m*(d*Sin[e + f*x])^m*(1 + Sin[e + f*x])))
```

Rule 2787

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m
])/(1 + (b*Sin[e + f*x])/a)^FracPart[m], Int[(1 + (b*Sin[e + f*x])/a)^m*(d*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 2786

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[((d/b)^IntPart[n]*(d*Sin[e + f*x])^FracPart[n
])/(b*Sin[e + f*x])^FracPart[n], Int[(a + b*Sin[e + f*x])^m*(b*Sin[e + f*x]
)^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !In
tegerQ[m] && GtQ[a, 0] &&  !GtQ[d/b, 0]
```

Rule 2785

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Dist[(b*(d/b)^n*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a - x)^n*(2*a - x)^(m - 1/2)
)/Sqrt[x], x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1)*
```

```
Hypergeometric2F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*
(e + f*x)))])/(((b*e - a*f)*(m + 1))*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*
(e + f*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] && !IntegerQ[n]
```

Rubi steps

$$\int (d\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx = \left((1+\sin(e+fx))^{-m}(a+a\sin(e+fx))^m\right)\int (d\sin(e+fx))^{-1-m}(1+\sin(e$$

$$= \frac{\left(\sin^m(e+fx)(d\sin(e+fx))^{-m}(1+\sin(e+fx))^{-m}(a+a\sin(e+fx))^m\right)\int}{d}$$

$$= -\frac{\left(\cos(e+fx)\sin^m(e+fx)(d\sin(e+fx))^{-m}(1+\sin(e+fx))^{-\frac{1}{2}-m}(a+a\right.}{df\sqrt{1-\sin(e}}$$

$$= -\frac{\cos(e+fx)\,{}_2F_1\left(\frac{1}{2}-m,-m;1-m;-\frac{2\sin(e+fx)}{1-\sin(e+fx)}\right)(d\sin(e+fx))^{-m}\left(\frac{1+\sin}{1-\sin}\right.}{dfm(1+\sin(e+fx))}$$

Mathematica [C] time = 1.50715, size = 194, normalized size = 1.67

$$\frac{(1-i)2^m(\cosh(m\log(2))-\sinh(m\log(2)))(\cos(e+fx)-i(\sin(e+fx)+1))(a(\sin(e+fx)+1))^m(d\sin(e+fx))^{-m}((1-}{df(2m}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] ((1 - I)*2^m*Hypergeometric2F1[1 + m, 1 + 2*m, 2*(1 + m), Sqrt[2]*Cos[(2*e
- Pi + 2*f*x)/4]*Csc[(e + f*x)/2]]*((1 - I)*(1 + Cos[e + f*x] - I*Sin[e + f
*x]))^m*(a*(1 + Sin[e + f*x]))^m*(Cos[e + f*x] - I*(1 + Sin[e + f*x]))*(Cos
h[m*Log[2]] - Sinh[m*Log[2]]))/(d*f*(1 + 2*m)*(-1 + Cos[e + f*x] - I*Sin[e
+ f*x])*((1 + I)*(1 - Cos[e + f*x] + I*Sin[e + f*x]))^m*(d*Sin[e + f*x])^m)
```

Maple [F] time = 0.24, size = 0, normalized size = 0.

$$\int \left(d\sin\left(fx+e\right)\right)^{-1-m}\left(a+a\sin\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

```
[Out] int((d*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^m\left(d\sin\left(fx+e\right)\right)^{-m-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(d*sin(f*x + e))^(-m - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a\sin\left(fx+e\right)+a\right)^{m}\left(d\sin\left(fx+e\right)\right)^{-m-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)^m*(d*sin(f*x + e))^(-m - 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**(-1-m)*(a+a*sin(f*x+e))**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^{m}\left(d\sin\left(fx+e\right)\right)^{-m-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(d*sin(f*x + e))^(-m - 1), x)
```

3.654 $\int (a+a\sin(e+fx))^m(c+d\sin(e+fx))^{-1-m}\,dx$

Optimal. Leaf size=129

$$-\frac{a2^{m+\frac{1}{2}}\cos(e+fx)(a\sin(e+fx)+a)^{m-1}\left(\frac{(c+d)(\sin(e+fx)+1)}{c+d\sin(e+fx)}\right)^{\frac{1}{2}-m}(c+d\sin(e+fx))^{-m}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{(c-d)(1-\sin(e+fx))}{2(c+d\sin(e+fx))}\right)}{f(c+d)}$$

```
[Out] -((2^(1/2 + m)*a*Cos[e + f*x]*Hypergeometric2F1[1/2, 1/2 - m, 3/2, ((c - d)
*(1 - Sin[e + f*x]))/(2*(c + d*Sin[e + f*x]))]*(a + a*Sin[e + f*x])^(-1 + m
)*(((c + d)*(1 + Sin[e + f*x]))/(c + d*Sin[e + f*x]))^(1/2 - m))/((c + d)*f
*(c + d*Sin[e + f*x])^m))
```

Rubi [A] time = 0.148658, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2788, 132}

$$-\frac{a2^{m+\frac{1}{2}}\cos(e+fx)(a\sin(e+fx)+a)^{m-1}\left(\frac{(c+d)(\sin(e+fx)+1)}{c+d\sin(e+fx)}\right)^{\frac{1}{2}-m}(c+d\sin(e+fx))^{-m}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{(c-d)(1-\sin(e+fx))}{2(c+d\sin(e+fx))}\right)}{f(c+d)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(-1 - m),x]
```


```
[Out] -((2^(1/2 + m)*a*Cos[e + f*x]*Hypergeometric2F1[1/2, 1/2 - m, 3/2, ((c - d)
*(1 - Sin[e + f*x]))/(2*(c + d*Sin[e + f*x]))]*(a + a*Sin[e + f*x])^(-1 + m
)*(((c + d)*(1 + Sin[e + f*x]))/(c + d*Sin[e + f*x]))^(1/2 - m))/((c + d)*f
*(c + d*Sin[e + f*x])^m))
```


Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x
)^n)/Sqrt[a - b*x], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1)*
Hypergeometric2F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*
(e + f*x)))])/(((b*e - a*f)*(m + 1))*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*
(e + f*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int (a+a\sin(e+fx))^m(c+d\sin(e+fx))^{-1-m}\,dx = \frac{\left(a^2\cos(e+fx)\right)\text{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}(c+dx)^{-1-m}}{\sqrt{a-ax}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{2^{\frac{1}{2}+m}a\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{(c-d)(1-\sin(e+fx))}{2(c+d\sin(e+fx))}\right)(a+a\sin(e}{(c+d)f}$$

Mathematica [A] time = 1.38097, size = 187, normalized size = 1.45

$$-\frac{2\cot\left(\frac{1}{4}(2e+2fx+\pi)\right)\sin^2\left(\frac{1}{4}(2e+2fx+\pi)\right)^{\frac{1}{2}-m}\cos^2\left(\frac{1}{4}(2e+2fx-\pi)\right)^{m+\frac{1}{2}}(a(\sin(e+fx)+1))^m(c+d\sin(e+}{f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(-1 - m),x]
```

```
[Out] (-2*(Cos[(2*e - Pi + 2*f*x)/4]^2)^(1/2 + m)*Cot[(2*e + Pi + 2*f*x)/4]*Hyper
geometric2F1[1/2, 1/2 - m, 3/2, ((c - d)*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c +
d*Sin[e + f*x])]*(a*(1 + Sin[e + f*x]))^m*(((c + d)*Cos[(2*e - Pi + 2*f*x)/
4]^2)/(c + d*Sin[e + f*x]))^(-1/2 - m)*(c + d*Sin[e + f*x])^(-1 - m)*(Sin[(
2*e + Pi + 2*f*x)/4]^2)^(1/2 - m))/f
```

Maple [F] time = 0.25, size = 0, normalized size = 0.

$$\int \left(a+a\sin\left(fx+e\right)\right)^m\left(c+d\sin\left(fx+e\right)\right)^{-1-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-1-m),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-1-m),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^m\left(d\sin\left(fx+e\right)+c\right)^{-m-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-1-m),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^(-m - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a\sin\left(fx+e\right)+a\right)^m\left(d\sin\left(fx+e\right)+c\right)^{-m-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-1-m),x, algorithm="fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^(-m - 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(c+d*sin(f*x+e))**(-1-m),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a \sin \left(fx + e\right) + a\right)^{m} \left(d \sin \left(fx + e\right) + c\right)^{-m-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-1-m),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^(-m - 1), x)
```

3.655 $\int (a+a\sin (e+fx))^3(c+d\sin (e+fx))^n\,dx$

Optimal. Leaf size=107

$$-\frac{8\sqrt{2}a^3\cos (e+fx)(c+d\sin (e+fx))^n\left(\frac{c+d\sin (e+fx)}{c+d}\right)^{-n}F_1\left(\frac{1}{2};-\frac{5}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin (e+fx)),\frac{d(1-\sin (e+fx))}{c+d}\right)}{f\sqrt{\sin (e+fx)+1}}$$

```
[Out] (-8*Sqrt[2]*a^3*AppellF1[1/2, -5/2, -n, 3/2, (1 - Sin[e + f*x])/2, (d*(1 -
Sin[e + f*x]))/(c + d)]*Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(f*Sqrt[1 + Si
n[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n)
```

Rubi [A] time = 0.122509, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2784, 139, 138}

$$-\frac{8\sqrt{2}a^3\cos (e+fx)(c+d\sin (e+fx))^n\left(\frac{c+d\sin (e+fx)}{c+d}\right)^{-n}F_1\left(\frac{1}{2};-\frac{5}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin (e+fx)),\frac{d(1-\sin (e+fx))}{c+d}\right)}{f\sqrt{\sin (e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] (-8*Sqrt[2]*a^3*AppellF1[1/2, -5/2, -n, 3/2, (1 - Sin[e + f*x])/2, (d*(1 -
Sin[e + f*x]))/(c + d)]*Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(f*Sqrt[1 + Si
n[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n)
```

Rule 2784

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^m*Cos[e + f*x])/(f*Sqrt[1 + Sin[e
+ f*x]]*Sqrt[1 - Sin[e + f*x]]), Subst[Int[((1 + (b*x)/a)^(m - 1/2)*(c + d*
x)^n)/Sqrt[1 - (b*x)/a], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 IntegerQ[m]
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int (a+a\sin(e+fx))^3(c+d\sin(e+fx))^n\,dx = \frac{\left(a^3\cos(e+fx)\right)\text{Subst}\left(\int \frac{(1+x)^{5/2}(c+dx)^n}{\sqrt{1-x}}\,dx,x,\sin(e+fx)\right)}{f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}$$

$$= \frac{\left(a^3\cos(e+fx)(c+d\sin(e+fx))^n\left(-\frac{c+d\sin(e+fx)}{-c-d}\right)^{-n}\right)\text{Subst}\left(\int \frac{(1+x)^{5/2}\left(-\right.}{\sqrt{1}}\right.}{f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}$$

$$= -\frac{8\sqrt{2}a^3F_1\left(\frac{1}{2};-\frac{5}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)(c+}{f\sqrt{1+\sin(e+fx)}}$$

Mathematica [F] time = 35.6257, size = 0, normalized size = 0.

$$\int (a+a\sin(e+fx))^3(c+d\sin(e+fx))^n\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] Integrate[(a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^n, x]
```

Maple [F] time = 0.381, size = 0, normalized size = 0.

$$\int \left(a+a\sin\left(fx+e\right)\right)^3\left(c+d\sin\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(c+d*sin(f*x+e))^n,x)
```

```
[Out] int((a+a*sin(f*x+e))^3*(c+d*sin(f*x+e))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^3\left(d\sin\left(fx+e\right)+c\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c+d*sin(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^3*(d*sin(f*x + e) + c)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(3\,a^3\cos\left(fx+e\right)^2-4\,a^3+\left(a^3\cos\left(fx+e\right)^2-4\,a^3\right)\sin\left(fx+e\right)\right)\left(d\sin\left(fx+e\right)+c\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c+d*sin(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral(-(3*a^3*cos(f*x + e)^2 - 4*a^3 + (a^3*cos(f*x + e)^2 - 4*a^3)*sin(
f*x + e))*(d*sin(f*x + e) + c)^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(c+d*sin(f*x+e))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^{3}\left(d\sin\left(fx+e\right)+c\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c+d*sin(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^3*(d*sin(f*x + e) + c)^n, x)
```

3.656 $\int (a+a\sin(e+fx))^2(c+d\sin(e+fx))^n\,dx$

Optimal. Leaf size=107

$$-\frac{4\sqrt{2}a^2\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{1}{2};-\frac{3}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{f\sqrt{\sin(e+fx)+1}}$$

```
[Out] (-4*Sqrt[2]*a^2*AppellF1[1/2, -3/2, -n, 3/2, (1 - Sin[e + f*x])/2, (d*(1 -
Sin[e + f*x]))/(c + d)]*Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(f*Sqrt[1 + Si
n[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n)
```

Rubi [A] time = 0.107054, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2784, 139, 138}

$$-\frac{4\sqrt{2}a^2\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{1}{2};-\frac{3}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{f\sqrt{\sin(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] (-4*Sqrt[2]*a^2*AppellF1[1/2, -3/2, -n, 3/2, (1 - Sin[e + f*x])/2, (d*(1 -
Sin[e + f*x]))/(c + d)]*Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(f*Sqrt[1 + Si
n[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n)
```

Rule 2784

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^m*Cos[e + f*x])/(f*Sqrt[1 + Sin[e
+ f*x]]*Sqrt[1 - Sin[e + f*x]]), Subst[Int[((1 + (b*x)/a)^(m - 1/2)*(c + d*
x)^n)/Sqrt[1 - (b*x)/a], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 IntegerQ[m]
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int (a+a\sin(e+fx))^2(c+d\sin(e+fx))^n\,dx = \frac{\left(a^2\cos(e+fx)\right)\text{Subst}\left(\int \frac{(1+x)^{3/2}(c+dx)^n}{\sqrt{1-x}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}$$

$$= \frac{\left(a^2\cos(e+fx)(c+d\sin(e+fx))^n\left(-\frac{c+d\sin(e+fx)}{-c-d}\right)^{-n}\right)\text{Subst}\left(\int \frac{(1+x)^{3/}}{\ldots}\right.}{f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}$$

$$= -\frac{4\sqrt{2}a^2F_1\left(\frac{1}{2};-\frac{3}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)(\ldots}{f\sqrt{1+\sin(e+fx)}}$$

Mathematica [F] time = 16.6182, size = 0, normalized size = 0.

$$\int (a+a\sin(e+fx))^2(c+d\sin(e+fx))^n\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] Integrate[(a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^n, x]
```

Maple [F] time = 0.357, size = 0, normalized size = 0.

$$\int \left(a+a\sin\left(fx+e\right)\right)^2\left(c+d\sin\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^n,x)
```

```
[Out] int((a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^2\left(d\sin\left(fx+e\right)+c\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^2*(d*sin(f*x + e) + c)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(a^2\cos\left(fx+e\right)^2-2\,a^2\sin\left(fx+e\right)-2\,a^2\right)\left(d\sin\left(fx+e\right)+c\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral(-(a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x + e) - 2*a^2)*(d*sin(f*x + e)
 + c)^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(c+d*sin(f*x+e))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a \sin \left(fx + e\right) + a\right)^{2} \left(d \sin \left(fx + e\right) + c\right)^{n} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^2*(d*sin(f*x + e) + c)^n, x)
```

3.657 $\int (a+a\sin(e+fx))(c+d\sin(e+fx))^n\,dx$

Optimal. Leaf size=105

$$-\frac{2\sqrt{2}a\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{1}{2};-\frac{1}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{f\sqrt{\sin(e+fx)+1}}$$

```
[Out] (-2*Sqrt[2]*a*AppellF1[1/2, -1/2, -n, 3/2, (1 - Sin[e + f*x])/2, (d*(1 - Si
n[e + f*x]))/(c + d)]*Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(f*Sqrt[1 + Sin[
e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n)
```

Rubi [A] time = 0.0813342, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {2755, 139, 138}

$$-\frac{2\sqrt{2}a\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{1}{2};-\frac{1}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{f\sqrt{\sin(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] (-2*Sqrt[2]*a*AppellF1[1/2, -1/2, -n, 3/2, (1 - Sin[e + f*x])/2, (d*(1 - Si
n[e + f*x]))/(c + d)]*Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(f*Sqrt[1 + Sin[
e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n)
```

Rule 2755

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(c*Cos[e + f*x])/(f*Sqrt[1 + Sin[e + f*x]]*S
qrt[1 - Sin[e + f*x]]), Subst[Int[((a + b*x)^m*Sqrt[1 + (d*x)/c])/Sqrt[1 -
(d*x)/c], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Ne
Q[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*m] && EqQ[c^2 - d^2, 0
]
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int (a+a\sin(e+fx))(c+d\sin(e+fx))^n\,dx = \frac{(a\cos(e+fx))\,\mathrm{Subst}\left(\int\frac{\sqrt{1+x}(c+dx)^n}{\sqrt{1-x}}\,dx,x,\sin(e+fx)\right)}{f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}$$

$$= \frac{\left(a\cos(e+fx)(c+d\sin(e+fx))^n\left(-\frac{c+d\sin(e+fx)}{-c-d}\right)^{-n}\right)\mathrm{Subst}\left(\int\frac{\sqrt{1+x}\left(-\frac{c}{-c-d}-\right.}{\sqrt{1-x}}\right.}{f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}$$

$$= -\frac{2\sqrt{2}aF_1\left(\frac{1}{2};-\frac{1}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)(c+d}{f\sqrt{1+\sin(e+fx)}}$$

Mathematica [F] time = 5.62687, size = 0, normalized size = 0.

$$\int (a+a\sin(e+fx))(c+d\sin(e+fx))^n\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] Integrate[(a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^n, x]
```

Maple [F] time = 0.222, size = 0, normalized size = 0.

$$\int \left(a+a\sin\left(fx+e\right)\right)\left(c+d\sin\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

```
[Out] int((a+a*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)+c\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)+c\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)+c\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^n, x)
```

3.658 $\int (c+d\sin(e+fx))^n\,dx$

Optimal. Leaf size=104

$$-\frac{\sqrt{2}\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{1}{2};\frac{1}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{f\sqrt{\sin(e+fx)+1}}$$

```
[Out] -((Sqrt[2]*AppellF1[1/2, 1/2, -n, 3/2, (1 - Sin[e + f*x])/2, (d*(1 - Sin[e
+ f*x]))/(c + d)]*Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(f*Sqrt[1 + Sin[e +
f*x]]*((c + d*Sin[e + f*x])/(c + d))^n))
```

Rubi [A] time = 0.0636288, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2665, 139, 138}

$$-\frac{\sqrt{2}\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{1}{2};\frac{1}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{f\sqrt{\sin(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^n,x]
```


```
[Out] -((Sqrt[2]*AppellF1[1/2, 1/2, -n, 3/2, (1 - Sin[e + f*x])/2, (d*(1 - Sin[e
+ f*x]))/(c + d)]*Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(f*Sqrt[1 + Sin[e +
f*x]]*((c + d*Sin[e + f*x])/(c + d))^n))
```


Rule 2665

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)
^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*n]
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int (c+d\sin(e+fx))^n\,dx = \frac{\cos(e+fx)\,\text{Subst}\left(\int \frac{(c+dx)^n}{\sqrt{1-x}\sqrt{1+x}}\,dx,x,\sin(e+fx)\right)}{f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}$$

$$= \frac{\left(\cos(e+fx)(c+d\sin(e+fx))^n\left(-\frac{c+d\sin(e+fx)}{-c-d}\right)^{-n}\right)\text{Subst}\left(\int \frac{\left(-\frac{c}{-c-d}-\frac{dx}{-c-d}\right)^n}{\sqrt{1-x}\sqrt{1+x}}\,dx,x,\sin(e+f\right.}{f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}$$

$$= -\frac{\sqrt{2}F_1\left(\frac{1}{2};\frac{1}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\,\text{s}}{}\right.}{f\sqrt{1+\sin(e+fx)}}$$

Mathematica [A] time = 0.278314, size = 120, normalized size = 1.15

$$\frac{\sec(e+fx)\sqrt{-\frac{d(\sin(e+fx)-1)}{c+d}}\sqrt{\frac{d(\sin(e+fx)+1)}{d-c}}(c+d\sin(e+fx))^{n+1}F_1\left(n+1;\frac{1}{2},\frac{1}{2};n+2;\frac{c+d\sin(e+fx)}{c-d},\frac{c+d\sin(e+fx)}{c+d}\right)}{df(n+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*Sin[e + f*x])^n,x]
```

```
[Out] (AppellF1[1 + n, 1/2, 1/2, 2 + n, (c + d*Sin[e + f*x])/(c - d), (c + d*Sin[
e + f*x])/(c + d)]*Sec[e + f*x]*Sqrt[-((d*(-1 + Sin[e + f*x]))/(c + d))]*Sq
rt[(d*(1 + Sin[e + f*x]))/(-c + d)]*(c + d*Sin[e + f*x])^(1 + n))/(d*f*(1 +
 n))
```

Maple [F] time = 0.382, size = 0, normalized size = 0.

$$\int \left(c+d\sin\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^n,x)
```

```
[Out] int((c+d*sin(f*x+e))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\sin\left(fx+e\right)+c\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate((d*sin(f*x + e) + c)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(d\sin\left(fx+e\right)+c\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral((d*sin(f*x + e) + c)^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c + d\sin\left(e + fx\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**n,x)
```

```
[Out] Integral((c + d*sin(e + f*x))**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\sin\left(fx + e\right) + c\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((d*sin(f*x + e) + c)^n, x)
```

3.659 $\int \frac{(c+d\sin(e+fx))^n}{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=107

$$-\frac{\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{1}{2};\frac{3}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{\sqrt{2}af\sqrt{\sin(e+fx)+1}}$$

```
[Out] -((AppellF1[1/2, 3/2, -n, 3/2, (1 - Sin[e + f*x])/2, (d*(1 - Sin[e + f*x]))
/(c + d)]*Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(Sqrt[2]*a*f*Sqrt[1 + Sin[e
+ f*x]]*((c + d*Sin[e + f*x])/(c + d))^n))
```

Rubi [A] time = 0.106512, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2784, 139, 138}

$$-\frac{\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{1}{2};\frac{3}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{\sqrt{2}af\sqrt{\sin(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^n/(a + a*Sin[e + f*x]),x]
```


```
[Out] -((AppellF1[1/2, 3/2, -n, 3/2, (1 - Sin[e + f*x])/2, (d*(1 - Sin[e + f*x]))
/(c + d)]*Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(Sqrt[2]*a*f*Sqrt[1 + Sin[e
+ f*x]]*((c + d*Sin[e + f*x])/(c + d))^n))
```


Rule 2784

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^m*Cos[e + f*x])/(f*Sqrt[1 + Sin[e
+ f*x]]*Sqrt[1 - Sin[e + f*x]]), Subst[Int[((1 + (b*x)/a)^(m - 1/2)*(c + d*
x)^n)/Sqrt[1 - (b*x)/a], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 IntegerQ[m]
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^n}{a+a\sin(e+fx)}\,dx = \frac{\cos(e+fx)\,\text{Subst}\left(\int \frac{(c+dx)^n}{\sqrt{1-x}(1+x)^{3/2}}\,dx,x,\sin(e+fx)\right)}{af\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}$$

$$= \frac{\left(\cos(e+fx)(c+d\sin(e+fx))^n\left(-\frac{c+d\sin(e+fx)}{-c-d}\right)^{-n}\right)\text{Subst}\left(\int \frac{\left(-\frac{c}{-c-d}-\frac{dx}{-c-d}\right)^n}{\sqrt{1-x}(1+x)^{3/2}}\,dx,x,\sin(e+fx)\right.}{af\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}$$

$$= -\frac{F_1\left(\frac{1}{2};\frac{3}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+f}{c+d}\right.}{\sqrt{2}af\sqrt{1+\sin(e+fx)}}$$

Mathematica [F] time = 2.61371, size = 0, normalized size = 0.

$$\int \frac{(c+d\sin(e+fx))^n}{a+a\sin(e+fx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*Sin[e + f*x])^n/(a + a*Sin[e + f*x]),x]
```

```
[Out] Integrate[(c + d*Sin[e + f*x])^n/(a + a*Sin[e + f*x]), x]
```

Maple [F] time = 0.395, size = 0, normalized size = 0.

$$\int \frac{\left(c+d\sin\left(fx+e\right)\right)^n}{a+a\sin\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x)
```

```
[Out] int((c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^n}{a\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((d*sin(f*x + e) + c)^n/(a*sin(f*x + e) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(d\sin\left(fx+e\right)+c\right)^{n}}{a\sin\left(fx+e\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral((d*sin(f*x + e) + c)^n/(a*sin(f*x + e) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**n/(a+a*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{n}}{a\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*sin(f*x + e) + c)^n/(a*sin(f*x + e) + a), x)
```

3.660 $\int \frac{(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^2}\,dx$

Optimal. Leaf size=109

$$-\frac{\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{1}{2};\frac{5}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{2\sqrt{2}a^2f\sqrt{\sin(e+fx)+1}}$$

```
[Out] -(AppellF1[1/2, 5/2, -n, 3/2, (1 - Sin[e + f*x])/2, (d*(1 - Sin[e + f*x]))/
(c + d)]*Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(2*Sqrt[2]*a^2*f*Sqrt[1 + Sin
[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n)
```

Rubi [A] time = 0.10642, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2784, 139, 138}

$$-\frac{\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{1}{2};\frac{5}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{2\sqrt{2}a^2f\sqrt{\sin(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^n/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] -(AppellF1[1/2, 5/2, -n, 3/2, (1 - Sin[e + f*x])/2, (d*(1 - Sin[e + f*x]))/
(c + d)]*Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(2*Sqrt[2]*a^2*f*Sqrt[1 + Sin
[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n)
```

Rule 2784

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^m*Cos[e + f*x])/(f*Sqrt[1 + Sin[e
+ f*x]]*Sqrt[1 - Sin[e + f*x]]), Subst[Int[((1 + (b*x)/a)^(m - 1/2)*(c + d*
x)^n)/Sqrt[1 - (b*x)/a], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 IntegerQ[m]
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^2}\,dx = \frac{\cos(e+fx)\,\text{Subst}\left(\int \frac{(c+dx)^n}{\sqrt{1-x}(1+x)^{5/2}}\,dx, x, \sin(e+fx)\right)}{a^2 f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}$$

$$= \frac{\left(\cos(e+fx)(c+d\sin(e+fx))^n\left(-\frac{c+d\sin(e+fx)}{-c-d}\right)^{-n}\right)\text{Subst}\left(\int \frac{\left(-\frac{c}{-c-d}-\frac{dx}{-c-d}\right)^n}{\sqrt{1-x}(1+x)^{5/2}}\,dx, x, \sin(e+\right.}{a^2 f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}$$

$$= -\frac{F_1\left(\frac{1}{2};\frac{5}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(}{c+}\right.}{2\sqrt{2}a^2 f\sqrt{1+\sin(e+fx)}}$$

Mathematica [F] time = 6.05569, size = 0, normalized size = 0.

$$\int \frac{(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*Sin[e + f*x])^n/(a + a*Sin[e + f*x])^2,x]

[Out] Integrate[(c + d*Sin[e + f*x])^n/(a + a*Sin[e + f*x])^2, x]
```

Maple [F] time = 0.379, size = 0, normalized size = 0.

$$\int \frac{\left(c+d\sin\left(fx+e\right)\right)^n}{\left(a+a\sin\left(fx+e\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x)

[Out] int((c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^n}{\left(a\sin\left(fx+e\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x, algorithm="maxima")

[Out] integrate((d*sin(f*x + e) + c)^n/(a*sin(f*x + e) + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(d\sin\left(fx+e\right)+c\right)^{n}}{a^{2}\cos\left(fx+e\right)^{2}-2\,a^{2}\sin\left(fx+e\right)-2\,a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral(-(d*sin(f*x + e) + c)^n/(a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x + e) -
 2*a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**n/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{n}}{\left(a\sin\left(fx+e\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*sin(f*x + e) + c)^n/(a*sin(f*x + e) + a)^2, x)
```

3.661 $\int \frac{(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^3}\,dx$

Optimal. Leaf size=109

$$-\frac{\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{1}{2};\frac{7}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{4\sqrt{2}a^3f\sqrt{\sin(e+fx)+1}}$$

```
[Out] -(AppellF1[1/2, 7/2, -n, 3/2, (1 - Sin[e + f*x])/2, (d*(1 - Sin[e + f*x]))/
(c + d)]*Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(4*Sqrt[2]*a^3*f*Sqrt[1 + Sin
[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n)
```

Rubi [A] time = 0.107732, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2784, 139, 138}

$$-\frac{\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{1}{2};\frac{7}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{4\sqrt{2}a^3f\sqrt{\sin(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^n/(a + a*Sin[e + f*x])^3,x]
```


```
[Out] -(AppellF1[1/2, 7/2, -n, 3/2, (1 - Sin[e + f*x])/2, (d*(1 - Sin[e + f*x]))/
(c + d)]*Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(4*Sqrt[2]*a^3*f*Sqrt[1 + Sin
[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n)
```


Rule 2784

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^m*Cos[e + f*x])/(f*Sqrt[1 + Sin[e
+ f*x]]*Sqrt[1 - Sin[e + f*x]]), Subst[Int[((1 + (b*x)/a)^(m - 1/2)*(c + d*
x)^n)/Sqrt[1 - (b*x)/a], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 IntegerQ[m]
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$
\begin{aligned}
\int \frac{(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^3}\,dx &= \frac{\cos(e+fx)\,\text{Subst}\left(\int \frac{(c+dx)^n}{\sqrt{1-x}(1+x)^{7/2}}\,dx, x, \sin(e+fx)\right)}{a^3 f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}} \\
&= \frac{\left(\cos(e+fx)(c+d\sin(e+fx))^n\left(-\frac{c+d\sin(e+fx)}{-c-d}\right)^{-n}\right)\text{Subst}\left(\int \frac{\left(-\frac{c}{-c-d}-\frac{dx}{-c-d}\right)^n}{\sqrt{1-x}(1+x)^{7/2}}\,dx, x, \sin(e+fx)\right)}{a^3 f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}} \\
&= -\frac{F_1\left(\frac{1}{2};\frac{7}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+f}{c+d}\right.}{4\sqrt{2}a^3 f\sqrt{1+\sin(e+fx)}}
\end{aligned}
$$

Mathematica [F] time = 12.8206, size = 0, normalized size = 0.

$$\int \frac{(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*Sin[e + f*x])^n/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] Integrate[(c + d*Sin[e + f*x])^n/(a + a*Sin[e + f*x])^3, x]
```

Maple [F] time = 0.572, size = 0, normalized size = 0.

$$\int \frac{\left(c+d\sin\left(fx+e\right)\right)^n}{\left(a+a\sin\left(fx+e\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^3,x)
```

```
[Out] int((c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^n}{\left(a\sin\left(fx+e\right)+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] integrate((d*sin(f*x + e) + c)^n/(a*sin(f*x + e) + a)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(d\sin\left(fx+e\right)+c\right)^{n}}{3\,a^{3}\cos\left(fx+e\right)^{2}-4\,a^{3}+\left(a^{3}\cos\left(fx+e\right)^{2}-4\,a^{3}\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] integral(-(d*sin(f*x + e) + c)^n/(3*a^3*cos(f*x + e)^2 - 4*a^3 + (a^3*cos(f
*x + e)^2 - 4*a^3)*sin(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**n/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(d\sin\left(fx+e\right)+c\right)^{n}}{\left(a\sin\left(fx+e\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate((d*sin(f*x + e) + c)^n/(a*sin(f*x + e) + a)^3, x)
```

3.662 $\int (a + a\sin(e + fx))^{5/2}(c + d\sin(e + fx))^n\,dx$

Optimal. Leaf size=257

$$-\frac{2a^3\left(3c^2-2cd(4n+7)+d^2\left(16n^2+56n+43\right)\right)\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};\frac{d(1-\sin(e+fx))}{c+d}\right)}{d^2f(2n+3)(2n+5)\sqrt{a\sin(e+fx)+a}}$$

```
[Out] (2*a^3*(3*c - d*(11 + 4*n))*Cos[e + f*x]*(c + d*Sin[e + f*x])^(1 + n))/(d^2
*f*(3 + 2*n)*(5 + 2*n)*Sqrt[a + a*Sin[e + f*x]]) - (2*a^2*Cos[e + f*x]*Sqrt
[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(1 + n))/(d*f*(5 + 2*n)) - (2*a^3
*(3*c^2 - 2*c*d*(7 + 4*n) + d^2*(43 + 56*n + 16*n^2))*Cos[e + f*x]*Hypergeo
metric2F1[1/2, -n, 3/2, (d*(1 - Sin[e + f*x]))/(c + d)]*(c + d*Sin[e + f*x]
)^n)/(d^2*f*(3 + 2*n)*(5 + 2*n)*Sqrt[a + a*Sin[e + f*x]]*((c + d*Sin[e + f*
x])/(c + d))^n)
```

Rubi [A] time = 0.481308, antiderivative size = 257, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2763, 2981, 2776, 70, 69}

$$-\frac{2a^3\left(3c^2-2cd(4n+7)+d^2\left(16n^2+56n+43\right)\right)\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};\frac{d(1-\sin(e+fx))}{c+d}\right)}{d^2f(2n+3)(2n+5)\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])^n,x]
```


```
[Out] (2*a^3*(3*c - d*(11 + 4*n))*Cos[e + f*x]*(c + d*Sin[e + f*x])^(1 + n))/(d^2
*f*(3 + 2*n)*(5 + 2*n)*Sqrt[a + a*Sin[e + f*x]]) - (2*a^2*Cos[e + f*x]*Sqrt
[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(1 + n))/(d*f*(5 + 2*n)) - (2*a^3
*(3*c^2 - 2*c*d*(7 + 4*n) + d^2*(43 + 56*n + 16*n^2))*Cos[e + f*x]*Hypergeo
metric2F1[1/2, -n, 3/2, (d*(1 - Sin[e + f*x]))/(c + d)]*(c + d*Sin[e + f*x]
)^n)/(d^2*f*(3 + 2*n)*(5 + 2*n)*Sqrt[a + a*Sin[e + f*x]]*((c + d*Sin[e + f*
x])/(c + d))^n)
```


Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
```

```
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2776

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[(c + d*x)^n/Sqrt[a - b*x], x],
 x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !IntegerQ[2*n]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^n\,dx = -\frac{2a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^{1+n}}{df(5+2n)} + \frac{2\int\sqrt{a}\cdots}{\cdots}$$

$$= \frac{2a^3(3c-d(11+4n))\cos(e+fx)(c+d\sin(e+fx))^{1+n}}{d^2f(3+2n)(5+2n)\sqrt{a+a\sin(e+fx)}} - \frac{2a^2\cos(e+\cdots}{\cdots}$$

$$= \frac{2a^3(3c-d(11+4n))\cos(e+fx)(c+d\sin(e+fx))^{1+n}}{d^2f(3+2n)(5+2n)\sqrt{a+a\sin(e+fx)}} - \frac{2a^2\cos(e+\cdots}{\cdots}$$

$$= \frac{2a^3(3c-d(11+4n))\cos(e+fx)(c+d\sin(e+fx))^{1+n}}{d^2f(3+2n)(5+2n)\sqrt{a+a\sin(e+fx)}} - \frac{2a^2\cos(e+\cdots}{\cdots}$$

$$= \frac{2a^3(3c-d(11+4n))\cos(e+fx)(c+d\sin(e+fx))^{1+n}}{d^2f(3+2n)(5+2n)\sqrt{a+a\sin(e+fx)}} - \frac{2a^2\cos(e+\cdots}{\cdots}$$

Mathematica [A] time = 32.5375, size = 190, normalized size = 0.74

$$\frac{a^2(\sin(e+fx)-1)\sec(e+fx)\sqrt{a(\sin(e+fx)+1)}(c+d\sin(e+fx))^n\left(\left(3c^2-2cd(4n+7)+d^2\left(16n^2+56n+43\right)\right)\left(\cdots\right.\right.}{d^2f\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] (a^2*Sec[e + f*x]*(-1 + Sin[e + f*x])*Sqrt[a*(1 + Sin[e + f*x])]*(c + d*Sin
[e + f*x])^n*(-((3*c - d*(11 + 4*n))*(c + d*Sin[e + f*x])) + d*(3 + 2*n)*(1
 + Sin[e + f*x])*(c + d*Sin[e + f*x]) + ((3*c^2 - 2*c*d*(7 + 4*n) + d^2*(43
 + 56*n + 16*n^2))*Hypergeometric2F1[1/2, -n, 3/2, -((d*(-1 + Sin[e + f*x])
)/(c + d))])/((c + d*Sin[e + f*x])/(c + d))^n))/(d^2*f*(5/2 + n)*(3 + 2*n))
```

Maple [F] time = 0.179, size = 0, normalized size = 0.

$$\int \left(a+a\sin\left(fx+e\right)\right)^{\frac{5}{2}} \left(c+d\sin\left(fx+e\right)\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^n,x)
```

```
[Out] int((a+a*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\left(d\sin\left(fx+e\right)+c\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)*(d*sin(f*x + e) + c)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\left(a^{2}\cos\left(fx+e\right)^{2}-2\,a^{2}\sin\left(fx+e\right)-2\,a^{2}\right)\sqrt{a\sin\left(fx+e\right)+a}\left(d\sin\left(fx+e\right)+c\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral(-(a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x + e) - 2*a^2)*sqrt(a*sin(f*x
+ e) + a)*(d*sin(f*x + e) + c)^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*(c+d*sin(f*x+e))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^n,x, algorithm="giac")
```

```
[Out] sage2
```

3.663 $\int (a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^n\,dx$

Optimal. Leaf size=160

$$\frac{2a^2(c-d(4n+5))\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};\frac{d(1-\sin(e+fx))}{c+d}\right)}{df(2n+3)\sqrt{a\sin(e+fx)+a}}-\frac{2a^2\cos(e+fx)(c+d\,\mathrm{si}}{df(2n+3)\sqrt{a\sin(e}}$$

```
[Out] (-2*a^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(1 + n))/(d*f*(3 + 2*n)*Sqrt[a +
a*Sin[e + f*x]]) + (2*a^2*(c - d*(5 + 4*n))*Cos[e + f*x]*Hypergeometric2F1[
1/2, -n, 3/2, (d*(1 - Sin[e + f*x]))/(c + d)]*(c + d*Sin[e + f*x])^n)/(d*f*
(3 + 2*n)*Sqrt[a + a*Sin[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n)
```

Rubi [A] time = 0.223473, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2763, 21, 2776, 70, 69}

$$\frac{2a^2(c-d(4n+5))\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};\frac{d(1-\sin(e+fx))}{c+d}\right)}{df(2n+3)\sqrt{a\sin(e+fx)+a}}-\frac{2a^2\cos(e+fx)(c+d\,\mathrm{si}}{df(2n+3)\sqrt{a\sin(e}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] (-2*a^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(1 + n))/(d*f*(3 + 2*n)*Sqrt[a +
a*Sin[e + f*x]]) + (2*a^2*(c - d*(5 + 4*n))*Cos[e + f*x]*Hypergeometric2F1[
1/2, -n, 3/2, (d*(1 - Sin[e + f*x]))/(c + d)]*(c + d*Sin[e + f*x])^n)/(d*f*
(3 + 2*n)*Sqrt[a + a*Sin[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n)
```

Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2776

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[(c + d*x)^n/Sqrt[a - b*x], x],
 x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !IntegerQ[2*n]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^n\,dx = -\frac{2a^2\cos(e+fx)(c+d\sin(e+fx))^{1+n}}{df(3+2n)\sqrt{a+a\sin(e+fx)}} + \frac{2\int \frac{(c+d\sin(e+fx))^n\left(-\frac{1}{2}a^2(c-5d\ldots\right.}{\sqrt{a+a\ldots}}}{d(3\ldots}$$

$$= -\frac{2a^2\cos(e+fx)(c+d\sin(e+fx))^{1+n}}{df(3+2n)\sqrt{a+a\sin(e+fx)}} - \frac{(a(c-d(5+4n)))\int\sqrt{a+\ldots}}{\ldots}$$

$$= -\frac{2a^2\cos(e+fx)(c+d\sin(e+fx))^{1+n}}{df(3+2n)\sqrt{a+a\sin(e+fx)}} - \frac{\left(a^3(c-d(5+4n))\cos(e+\ldots\right.}{df(3+2n)\sqrt{a-a\ldots}}$$

$$= -\frac{2a^2\cos(e+fx)(c+d\sin(e+fx))^{1+n}}{df(3+2n)\sqrt{a+a\sin(e+fx)}} - \frac{\left(a^3(c-d(5+4n))\cos(e+\ldots\right.}{\ldots}$$

$$= -\frac{2a^2\cos(e+fx)(c+d\sin(e+fx))^{1+n}}{df(3+2n)\sqrt{a+a\sin(e+fx)}} + \frac{2a^2(c-d(5+4n))\cos(e+\ldots}{\ldots}$$

Mathematica [A] time = 7.49051, size = 133, normalized size = 0.83

$$-\frac{2a^2\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}\left((d(4n+5)-c)\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};-\frac{d(\sin(e+fx)-1)}{c+d}\right)+(c+d)\left(\frac{c+d\sin(e+\ldots}{c+d}\right.\right.}{df(2n+3)\sqrt{a(\sin(e+fx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] (-2*a^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^n*((-c + d*(5 + 4*n))*Hypergeomet
ric2F1[1/2, -n, 3/2, -((d*(-1 + Sin[e + f*x]))/(c + d))] + (c + d)*((c + d*
Sin[e + f*x])/(c + d))^(1 + n)))/(d*f*(3 + 2*n)*Sqrt[a*(1 + Sin[e + f*x])]*
((c + d*Sin[e + f*x])/(c + d))^n)
```

Maple [F] time = 0.161, size = 0, normalized size = 0.

$$\int\left(a+a\sin\left(fx+e\right)\right)^{\frac{3}{2}}\left(c+d\sin\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^n,x)
```

```
[Out] int((a+a*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(d\sin\left(fx+e\right)+c\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e) + c)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(d\sin\left(fx+e\right)+c\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e) + c)^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*(c+d*sin(f*x+e))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(d\sin\left(fx+e\right)+c\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e) + c)^n, x)
```

3.664 $\int \sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^n\,dx$

Optimal. Leaf size=85

$$-\frac{2a\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};\frac{d(1-\sin(e+fx))}{c+d}\right)}{f\sqrt{a\sin(e+fx)+a}}$$

```
[Out] (-2*a*Cos[e + f*x]*Hypergeometric2F1[1/2, -n, 3/2, (d*(1 - Sin[e + f*x]))/(
c + d)]*(c + d*Sin[e + f*x])^n)/(f*Sqrt[a + a*Sin[e + f*x]]*((c + d*Sin[e +
 f*x])/(c + d))^n)
```

Rubi [A] time = 0.0991956, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2776, 70, 69}

$$-\frac{2a\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};\frac{d(1-\sin(e+fx))}{c+d}\right)}{f\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n,x]
```


```
[Out] (-2*a*Cos[e + f*x]*Hypergeometric2F1[1/2, -n, 3/2, (d*(1 - Sin[e + f*x]))/(
c + d)]*(c + d*Sin[e + f*x])^n)/(f*Sqrt[a + a*Sin[e + f*x]]*((c + d*Sin[e +
 f*x])/(c + d))^n)
```


Rule 2776

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[(c + d*x)^n/Sqrt[a - b*x], x],
 x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !IntegerQ[2*n]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^n\,dx = \frac{\left(a^2\cos(e+fx)\right)\text{Subst}\left(\int \frac{(c+dx)^n}{\sqrt{a-ax}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\left(a^2\cos(e+fx)(c+d\sin(e+fx))^n\left(-\frac{a(c+d\sin(e+fx))}{-ac-ad}\right)^{-n}\right)\text{Subst}\left(\int \frac{\left(\frac{c}{c+d}+\frac{dx}{c+d}\right.}{\sqrt{a-ax}}\right.}{f\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{2a\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};\frac{d(1-\sin(e+fx))}{c+d}\right)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+\right.}{c+d}}{f\sqrt{a+a\sin(e+fx)}}$$

Mathematica [F] time = 4.51034, size = 0, normalized size = 0.

$$\int \sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^n\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] Integrate[Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x]
```

Maple [F] time = 0.177, size = 0, normalized size = 0.

$$\int \sqrt{a+a\sin\left(fx+e\right)}\left(c+d\sin\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^n,x)
```

```
[Out] int((a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\sin\left(fx+e\right)+a}\left(d\sin\left(fx+e\right)+c\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{a\sin\left(fx+e\right)+a}\left(d\sin\left(fx+e\right)+c\right)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\left(\sin\left(e+fx\right)+1\right)}\left(c+d\sin\left(e+fx\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(1/2)*(c+d*sin(f*x+e))**n,x)
```

```
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))*(c + d*sin(e + f*x))**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\sin\left(fx+e\right)+a}\left(d\sin\left(fx+e\right)+c\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^n, x)
```

3.665 $\int \frac{(c+d\sin(e+fx))^n}{\sqrt{a+a\sin(e+fx)}}\,dx$

Optimal. Leaf size=99

$$-\frac{\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{1}{2};-n,1;\frac{3}{2};\frac{d(1-\sin(e+fx))}{c+d},\frac{1}{2}(1-\sin(e+fx))\right)}{f\sqrt{a\sin(e+fx)+a}}$$

```
[Out] -((AppellF1[1/2, -n, 1, 3/2, (d*(1 - Sin[e + f*x]))/(c + d), (1 - Sin[e + f
*x])/2]*Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(f*Sqrt[a + a*Sin[e + f*x]]*((
c + d*Sin[e + f*x])/(c + d))^n))
```

Rubi [A] time = 0.168786, antiderivative size = 129, normalized size of antiderivative = 1.3, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2788, 137, 136}

$$-\frac{\cos(e+fx)\sqrt{\frac{d(1-\sin(e+fx))}{c+d}}(c+d\sin(e+fx))^{n+1}F_1\left(n+1;\frac{1}{2},1;n+2;\frac{c+d\sin(e+fx)}{c+d},\frac{c+d\sin(e+fx)}{c-d}\right)}{f(n+1)(c-d)(1-\sin(e+fx))\sqrt{a\sin(e+fx)+a}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(c + d*Sin[e + f*x])^n/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] -((AppellF1[1 + n, 1/2, 1, 2 + n, (c + d*Sin[e + f*x])/(c + d), (c + d*Sin[
e + f*x])/(c - d)]*Cos[e + f*x]*Sqrt[(d*(1 - Sin[e + f*x]))/(c + d)]*(c + d
*Sin[e + f*x])^(1 + n))/((c - d)*f*(1 + n)*(1 - Sin[e + f*x])*Sqrt[a + a*Si
n[e + f*x]]))
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x
)^n)/Sqrt[a - b*x], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rule 137

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !GtQ[b/(b*c
- a*d), 0] &&  !SimplerQ[c + d*x, a + b*x]
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^n}{\sqrt{a+a\sin(e+fx)}}\,dx = \frac{\left(a^2\cos(e+fx)\right)\text{Subst}\left(\int \frac{(c+dx)^n}{\sqrt{a-ax}(a+ax)}\,dx, x, \sin(e+fx)\right)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\left(a^2\cos(e+fx)\sqrt{\frac{d(a-a\sin(e+fx))}{ac+ad}}\right)\text{Subst}\left(\int \frac{(c+dx)^n}{(a+ax)\sqrt{\frac{ad}{ac+ad}-\frac{adx}{ac+ad}}}\,dx, x, \sin(e+fx)\right)}{f(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{F_1\left(1+n;\frac{1}{2},1;2+n;\frac{c+d\sin(e+fx)}{c+d},\frac{c+d\sin(e+fx)}{c-d}\right)\cos(e+fx)\sqrt{\frac{d(1-\sin(e+fx))}{c+d}}(c+d\sin(e+$$

$$(c-d)f(1+n)(1-\sin(e+fx))\sqrt{a+a\sin(e+fx)}$$

Mathematica [B] time = 2.90812, size = 236, normalized size = 2.38

$$\frac{\cos(e+fx)\sqrt{a(\sin(e+fx)+1)}(c+d\sin(e+fx))^n\left(\frac{4\sqrt{\frac{\sin(e+fx)-1}{\sin(e+fx)+1}}\left(\frac{c-d}{d\sin(e+fx)+d}+1\right)^{-n}F_1\left(-n-\frac{1}{2};-\frac{1}{2},-n;\frac{1}{2}-n;\frac{2}{\sin(e+fx)+1},\frac{d-c}{\sin(e+fx)d+d}\right)}{2n+1}\right.}{4af(\sin(e+fx)-1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*Sin[e + f*x])^n/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] (Cos[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*(c + d*Sin[e + f*x])^n*(-((AppellF
1[1, 1/2, -n, 2, (1 + Sin[e + f*x])/2, (d*(1 + Sin[e + f*x]))/(-c + d)]*Sqr
t[2 - 2*Sin[e + f*x]])/((c + d*Sin[e + f*x])/(c - d))^n) + (4*AppellF1[-1/2
 - n, -1/2, -n, 1/2 - n, 2/(1 + Sin[e + f*x]), (-c + d)/(d + d*Sin[e + f*x]
)]*Sqrt[(-1 + Sin[e + f*x])/(1 + Sin[e + f*x])])/((1 + 2*n)*(1 + (c - d)/(d
 + d*Sin[e + f*x]))^n)))/(4*a*f*(-1 + Sin[e + f*x]))
```

Maple [F] time = 0.16, size = 0, normalized size = 0.

$$\int \left(c+d\sin\left(fx+e\right)\right)^n \frac{1}{\sqrt{a+a\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] int((c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^n}{\sqrt{a\sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*sin(f*x + e) + c)^n/sqrt(a*sin(f*x + e) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(d\sin\left(fx+e\right)+c\right)^n}{\sqrt{a\sin\left(fx+e\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((d*sin(f*x + e) + c)^n/sqrt(a*sin(f*x + e) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c+d\sin\left(e+fx\right)\right)^n}{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**n/(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((c + d*sin(e + f*x))**n/sqrt(a*(sin(e + f*x) + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^n}{\sqrt{a\sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*sin(f*x + e) + c)^n/sqrt(a*sin(f*x + e) + a), x)
```

3.666 $\int \frac{(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=104

$$-\frac{\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{1}{2};-n,2;\frac{3}{2};\frac{d(1-\sin(e+fx))}{c+d},\frac{1}{2}(1-\sin(e+fx))\right)}{2af\sqrt{a\sin(e+fx)+a}}$$

```
[Out] -(AppellF1[1/2, -n, 2, 3/2, (d*(1 - Sin[e + f*x]))/(c + d), (1 - Sin[e + f*
x])/2]*Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(2*a*f*Sqrt[a + a*Sin[e + f*x]]
*((c + d*Sin[e + f*x])/(c + d))^n)
```

Rubi [A] time = 0.161295, antiderivative size = 130, normalized size of antiderivative = 1.25, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2788, 137, 136}

$$\frac{d\cos(e+fx)\sqrt{\frac{d(1-\sin(e+fx))}{c+d}}(c+d\sin(e+fx))^{n+1}F_1\left(n+1;\frac{1}{2},2;n+2;\frac{c+d\sin(e+fx)}{c+d},\frac{c+d\sin(e+fx)}{c-d}\right)}{f(n+1)(c-d)^2(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(c + d*Sin[e + f*x])^n/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] (d*AppellF1[1 + n, 1/2, 2, 2 + n, (c + d*Sin[e + f*x])/(c + d), (c + d*Sin[
e + f*x])/(c - d)]*Cos[e + f*x]*Sqrt[(d*(1 - Sin[e + f*x]))/(c + d)]*(c + d
*Sin[e + f*x])^(1 + n))/((c - d)^2*f*(1 + n)*(a - a*Sin[e + f*x])*Sqrt[a +
a*Sin[e + f*x]])
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x
)^n)/Sqrt[a - b*x], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rule 137

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !GtQ[b/(b*c
- a*d), 0] &&  !SimplerQ[c + d*x, a + b*x]
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^{3/2}}\,dx = \frac{\left(a^2\cos(e+fx)\right)\text{Subst}\left(\int \frac{(c+dx)^n}{\sqrt{a-ax}(a+ax)^2}\,dx, x, \sin(e+fx)\right)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\left(a^2\cos(e+fx)\sqrt{\frac{d(a-a\sin(e+fx))}{ac+ad}}\right)\text{Subst}\left(\int \frac{(c+dx)^n}{(a+ax)^2\sqrt{\frac{ad}{ac+ad}-\frac{adx}{ac+ad}}}\,dx, x, \sin(e+fx)\right)}{f(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{dF_1\left(1+n;\frac{1}{2},2;2+n;\frac{c+d\sin(e+fx)}{c+d},\frac{c+d\sin(e+fx)}{c-d}\right)\cos(e+fx)\sqrt{\frac{d(1-\sin(e+fx))}{c+d}}(c+d\sin(e+fx}{(c-d)^2f(1+n)(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}}$$

Mathematica [B] time = 4.68965, size = 319, normalized size = 3.07

$$\frac{\sec(e+fx)(c+d\sin(e+fx))^n\left(a^2\sqrt{2-2\sin(e+fx)}(\sin(e+fx)+1)^2\left(\frac{c+d\sin(e+fx)}{c-d}\right)^{-n}F_1\left(1;\frac{1}{2},-n;2;\frac{1}{2}(\sin(e+fx)+1)\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*Sin[e + f*x])^n/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] (Sec[e + f*x]*(c + d*Sin[e + f*x])^n*((a^2*AppellF1[1, 1/2, -n, 2, (1 + Sin
[e + f*x])/2, (d*(1 + Sin[e + f*x]))/(-c + d)]*Sqrt[2 - 2*Sin[e + f*x]]*(1
+ Sin[e + f*x])^2)/((c + d*Sin[e + f*x])/(c - d))^n - (4*a*(1 + Sin[e + f*x
])*Sqrt[1 - 2/(1 + Sin[e + f*x])]*(2*a*(1 + 2*n)*AppellF1[1/2 - n, -1/2, -n
, 3/2 - n, 2/(1 + Sin[e + f*x]), (-c + d)/(d + d*Sin[e + f*x])] + a*(-1 + 2
*n)*AppellF1[-1/2 - n, -1/2, -n, 1/2 - n, 2/(1 + Sin[e + f*x]), (-c + d)/(d
 + d*Sin[e + f*x])]*(1 + Sin[e + f*x])))/((-1 + 4*n^2)*(1 + (c - d)/(d + d*
Sin[e + f*x]))^n)))/(8*a^3*f*Sqrt[a*(1 + Sin[e + f*x])])
```

Maple [F] time = 0.161, size = 0, normalized size = 0.

$$\int \left(c+d\sin\left(fx+e\right)\right)^n \left(a+a\sin\left(fx+e\right)\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(3/2),x)
```

```
[Out] int((c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^n}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*sin(f*x + e) + c)^n/(a*sin(f*x + e) + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{a\sin\left(fx+e\right)+a}\left(d\sin\left(fx+e\right)+c\right)^{n}}{a^{2}\cos\left(fx+e\right)^{2}-2\,a^{2}\sin\left(fx+e\right)-2\,a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^n/(a^2*cos(f*x + e)
^2 - 2*a^2*sin(f*x + e) - 2*a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(c+d\sin\left(e+fx\right)\right)^{n}}{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**n/(a+a*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral((c + d*sin(e + f*x))**n/(a*(sin(e + f*x) + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(d\sin\left(fx+e\right)+c\right)^{n}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*sin(f*x + e) + c)^n/(a*sin(f*x + e) + a)^(3/2), x)
```

3.667 $\int \frac{(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=104

$$-\frac{\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{1}{2};-n,3;\frac{3}{2};\frac{d(1-\sin(e+fx))}{c+d},\frac{1}{2}(1-\sin(e+fx))\right)}{4a^2f\sqrt{a\sin(e+fx)+a}}$$

```
[Out] -(AppellF1[1/2, -n, 3, 3/2, (d*(1 - Sin[e + f*x]))/(c + d), (1 - Sin[e + f*
x])/2]*Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(4*a^2*f*Sqrt[a + a*Sin[e + f*x
]]*((c + d*Sin[e + f*x])/(c + d))^n)
```

Rubi [A] time = 0.177884, antiderivative size = 137, normalized size of antiderivative = 1.32, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2788, 137, 136}

$$-\frac{d^2\cos(e+fx)\sqrt{\frac{d(1-\sin(e+fx))}{c+d}}(c+d\sin(e+fx))^{n+1}F_1\left(n+1;\frac{1}{2},3;n+2;\frac{c+d\sin(e+fx)}{c+d},\frac{c+d\sin(e+fx)}{c-d}\right)}{f(n+1)(c-d)^3\sqrt{a\sin(e+fx)+a}\left(a^2-a^2\sin(e+fx)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(c + d*Sin[e + f*x])^n/(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] -((d^2*AppellF1[1 + n, 1/2, 3, 2 + n, (c + d*Sin[e + f*x])/(c + d), (c + d*
Sin[e + f*x])/(c - d)]*Cos[e + f*x]*Sqrt[(d*(1 - Sin[e + f*x]))/(c + d)]*(c
 + d*Sin[e + f*x])^(1 + n))/((c - d)^3*f*(1 + n)*Sqrt[a + a*Sin[e + f*x]]*(
a^2 - a^2*Sin[e + f*x])))
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x
)^n)/Sqrt[a - b*x], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rule 137

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !GtQ[b/(b*c
- a*d), 0] &&  !SimplerQ[c + d*x, a + b*x]
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^{5/2}}\,dx = \frac{\left(a^2\cos(e+fx)\right)\text{Subst}\left(\int \frac{(c+dx)^n}{\sqrt{a-ax}(a+ax)^3}\,dx,x,\sin(e+fx)\right)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\left(a^2\cos(e+fx)\sqrt{\frac{d(a-a\sin(e+fx))}{ac+ad}}\right)\text{Subst}\left(\int \frac{(c+dx)^n}{(a+ax)^3\sqrt{\frac{ad}{ac+ad}-\frac{adx}{ac+ad}}}\,dx,x,\sin(e+fx)\right)}{f(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{d^2F_1\left(1+n;\frac{1}{2},3;2+n;\frac{c+d\sin(e+fx)}{c+d},\frac{c+d\sin(e+fx)}{c-d}\right)\cos(e+fx)\sqrt{\frac{d(1-\sin(e+fx))}{c+d}}(c+d\sin(e}{(c-d)^3f(1+n)\sqrt{a+a\sin(e+fx)}\left(a^2-a^2\sin(e+fx)\right)}$$

Mathematica [B] time = 9.46624, size = 414, normalized size = 3.98

$$\frac{\sec(e+fx)(c+d\sin(e+fx))^n\left(a^3\sqrt{2-2\sin(e+fx)}(\sin(e+fx)+1)^3\left(\frac{c+d\sin(e+fx)}{c-d}\right)^{-n}F_1\left(1;\frac{1}{2},-n;2;\frac{1}{2}(\sin(e+fx)+\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*Sin[e + f*x])^n/(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] (Sec[e + f*x]*(c + d*Sin[e + f*x])^n*((a^3*AppellF1[1, 1/2, -n, 2, (1 + Sin
[e + f*x])/2, (d*(1 + Sin[e + f*x]))/(-c + d)]*Sqrt[2 - 2*Sin[e + f*x]]*(1
+ Sin[e + f*x])^3)/((c + d*Sin[e + f*x])/(c - d))^n - (4*a^2*(1 + Sin[e + f
*x])*Sqrt[1 - 2/(1 + Sin[e + f*x])]*(a*(3 - 8*n + 4*n^2)*AppellF1[-1/2 - n,
 -1/2, -n, 1/2 - n, 2/(1 + Sin[e + f*x]), (-c + d)/(d + d*Sin[e + f*x])]*(1
 + Sin[e + f*x])^2 + 2*(1 + 2*n)*(2*a*(-1 + 2*n)*AppellF1[3/2 - n, -1/2, -n
, 5/2 - n, 2/(1 + Sin[e + f*x]), (-c + d)/(d + d*Sin[e + f*x])] + a*(-3 + 2
*n)*AppellF1[1/2 - n, -1/2, -n, 3/2 - n, 2/(1 + Sin[e + f*x]), (-c + d)/(d
+ d*Sin[e + f*x])]*(1 + Sin[e + f*x]))))/((-3 + 2*n)*(-1 + 2*n)*(1 + 2*n)*(
1 + (c - d)/(d + d*Sin[e + f*x]))^n)))/(16*a^4*f*(a*(1 + Sin[e + f*x]))^(3/
2))
```

Maple [F] time = 0.158, size = 0, normalized size = 0.

$$\int \left(c+d\sin\left(fx+e\right)\right)^n\left(a+a\sin\left(fx+e\right)\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(5/2),x)
```

```
[Out] int((c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^n}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*sin(f*x + e) + c)^n/(a*sin(f*x + e) + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{a\sin\left(fx+e\right)+a}\left(d\sin\left(fx+e\right)+c\right)^{n}}{3\,a^{3}\cos\left(fx+e\right)^{2}-4\,a^{3}+\left(a^{3}\cos\left(fx+e\right)^{2}-4\,a^{3}\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^n/(3*a^3*cos(f*x +
e)^2 - 4*a^3 + (a^3*cos(f*x + e)^2 - 4*a^3)*sin(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**n/(a+a*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(d\sin\left(fx+e\right)+c\right)^{n}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((d*sin(f*x + e) + c)^n/(a*sin(f*x + e) + a)^(5/2), x)
```

3.668 $\int (a+a\sin(e+fx))\sqrt[3]{c+d\sin(e+fx)}\,dx$

Optimal. Leaf size=107

$$-\frac{2\sqrt{2}a\cos(e+fx)\sqrt[3]{c+d\sin(e+fx)}F_1\left(\frac{1}{2};-\frac{1}{2},-\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{f\sqrt{\sin(e+fx)+1}\sqrt[3]{\frac{c+d\sin(e+fx)}{c+d}}}$$

```
[Out] (-2*Sqrt[2]*a*AppellF1[1/2, -1/2, -1/3, 3/2, (1 - Sin[e + f*x])/2, (d*(1 -
Sin[e + f*x]))/(c + d)]*Cos[e + f*x]*(c + d*Sin[e + f*x])^(1/3))/(f*Sqrt[1
+ Sin[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^(1/3))
```

Rubi [A] time = 0.0913667, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2755, 139, 138}

$$-\frac{2\sqrt{2}a\cos(e+fx)\sqrt[3]{c+d\sin(e+fx)}F_1\left(\frac{1}{2};-\frac{1}{2},-\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{f\sqrt{\sin(e+fx)+1}\sqrt[3]{\frac{c+d\sin(e+fx)}{c+d}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^(1/3),x]
```

```
[Out] (-2*Sqrt[2]*a*AppellF1[1/2, -1/2, -1/3, 3/2, (1 - Sin[e + f*x])/2, (d*(1 -
Sin[e + f*x]))/(c + d)]*Cos[e + f*x]*(c + d*Sin[e + f*x])^(1/3))/(f*Sqrt[1
+ Sin[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^(1/3))
```

Rule 2755

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(c*Cos[e + f*x])/(f*Sqrt[1 + Sin[e + f*x]]*S
qrt[1 - Sin[e + f*x]]), Subst[Int[((a + b*x)^m*Sqrt[1 + (d*x)/c])/Sqrt[1 -
(d*x)/c], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Ne
Q[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*m] && EqQ[c^2 - d^2, 0
]
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
```

```
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int (a+a\sin(e+fx))\sqrt[3]{c+d\sin(e+fx)}\,dx = \frac{(a\cos(e+fx))\,\text{Subst}\left(\int\frac{\sqrt{1+x}\sqrt[3]{c+dx}}{\sqrt{1-x}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}$$

$$= \frac{\left(a\cos(e+fx)\sqrt[3]{c+d\sin(e+fx)}\right)\text{Subst}\left(\int\frac{\sqrt{1+x}\sqrt[3]{-\frac{c}{-c-d}-\frac{dx}{-c-d}}}{\sqrt{1-x}}\,dx, x, \sin(e+ \right.}{f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}\sqrt[3]{-\frac{c+d\sin(e+fx)}{-c-d}}}$$

$$= -\frac{2\sqrt{2}aF_1\left(\frac{1}{2};-\frac{1}{2},-\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)\sqrt[3]{c+d}}{f\sqrt{1+\sin(e+fx)}\sqrt[3]{\frac{c+d\sin(e+fx)}{c+d}}}$$

Mathematica [B] time = 6.37407, size = 1736, normalized size = 16.22

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^(1/3),x]
```

```
[Out] a*((c*Sec[e]*(1 + Sin[e + f*x])*(-((AppellF1[-2/3, -1/2, -1/2, 1/3, -((Csc[
e]*(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d*Sqrt[1 +
 Cot[e]^2]*(1 - (c*Csc[e])/(d*Sqrt[1 + Cot[e]^2])))), -((Csc[e]*(c + d*Cos[
f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d*Sqrt[1 + Cot[e]^2]*(-1
 - (c*Csc[e])/(d*Sqrt[1 + Cot[e]^2]))))]*Cot[e]*Sin[f*x - ArcTan[Cot[e]]])/
(Sqrt[1 + Cot[e]^2]*Sqrt[(d*Sqrt[1 + Cot[e]^2] + d*Cos[f*x - ArcTan[Cot[e]]
]*Sqrt[1 + Cot[e]^2])/(d*Sqrt[1 + Cot[e]^2] - c*Csc[e])]*Sqrt[(d*Sqrt[1 + C
ot[e]^2] - d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2])/(d*Sqrt[1 + Cot[
e]^2] + c*Csc[e])]*(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[
e])^(2/3))) - ((3*d*Sin[e]*(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]
^2]*Sin[e]))/(d^2*Cos[e]^2 + d^2*Sin[e]^2) - (Cot[e]*Sin[f*x - ArcTan[Cot[e
]]])/Sqrt[1 + Cot[e]^2])/(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2
]*Sin[e])^(2/3)))/(4*f*(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^2) + (d*Se
c[e]*(1 + Sin[e + f*x])*(-((AppellF1[-2/3, -1/2, -1/2, 1/3, -((Csc[e]*(c +
d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d*Sqrt[1 + Cot[e]^
2]*(1 - (c*Csc[e])/(d*Sqrt[1 + Cot[e]^2])))), -((Csc[e]*(c + d*Cos[f*x - Ar
cTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d*Sqrt[1 + Cot[e]^2]*(-1 - (c*Cs
c[e])/(d*Sqrt[1 + Cot[e]^2]))))]*Cot[e]*Sin[f*x - ArcTan[Cot[e]]])/(Sqrt[1
+ Cot[e]^2]*Sqrt[(d*Sqrt[1 + Cot[e]^2] + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1
 + Cot[e]^2])/(d*Sqrt[1 + Cot[e]^2] - c*Csc[e])]*Sqrt[(d*Sqrt[1 + Cot[e]^2]
 - d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2])/(d*Sqrt[1 + Cot[e]^2] +
c*Csc[e])]*(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e])^(2/3
))) - ((3*d*Sin[e]*(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[
e]))/(d^2*Cos[e]^2 + d^2*Sin[e]^2) - (Cot[e]*Sin[f*x - ArcTan[Cot[e]]])/Sqr
t[1 + Cot[e]^2])/(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]
)^(2/3)))/(f*(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^2) + ((1 + Sin[e + f
*x])*(c + d*Sin[e + f*x])^(1/3)*((-3*Cos[e]*Cos[f*x])/(4*f) + (3*Sin[e]*Sin
[f*x])/(4*f) + (3*(c + 4*d)*Tan[e])/(4*d*f)))/(Cos[e/2 + (f*x)/2] + Sin[e/2
 + (f*x)/2])^2 + (3*AppellF1[1/3, 1/2, 1/2, 4/3, -((Sec[e]*(c + d*Cos[e]*Si
n[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]))/(d*Sqrt[1 + Tan[e]^2]*(1 - (c*
Sec[e])/(d*Sqrt[1 + Tan[e]^2])))), -((Sec[e]*(c + d*Cos[e]*Sin[f*x + ArcTan
[Tan[e]]]*Sqrt[1 + Tan[e]^2]))/(d*Sqrt[1 + Tan[e]^2]*(-1 - (c*Sec[e])/(d*Sq
```

```
rt[1 + Tan[e]^2]))))]*Sec[e]*Sec[f*x + ArcTan[Tan[e]]]*(1 + Sin[e + f*x])*S
qrt[(d*Sqrt[1 + Tan[e]^2] - d*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2])
/(c*Sec[e] + d*Sqrt[1 + Tan[e]^2])]*Sqrt[(d*Sqrt[1 + Tan[e]^2] + d*Sin[f*x
+ ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2])/(-(c*Sec[e]) + d*Sqrt[1 + Tan[e]^2])]
*(c + d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2])^(1/3))/(4*f*(C
os[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^2*Sqrt[1 + Tan[e]^2]) + (3*c*Appell
F1[1/3, 1/2, 1/2, 4/3, -((Sec[e]*(c + d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*Sq
rt[1 + Tan[e]^2]))/(d*Sqrt[1 + Tan[e]^2]*(1 - (c*Sec[e])/(d*Sqrt[1 + Tan[e]
^2])))), -((Sec[e]*(c + d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^
2]))/(d*Sqrt[1 + Tan[e]^2]*(-1 - (c*Sec[e])/(d*Sqrt[1 + Tan[e]^2]))))]*Sec[
e]*Sec[f*x + ArcTan[Tan[e]]]*(1 + Sin[e + f*x])*Sqrt[(d*Sqrt[1 + Tan[e]^2]
- d*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2])/(c*Sec[e] + d*Sqrt[1 + Ta
n[e]^2])]*Sqrt[(d*Sqrt[1 + Tan[e]^2] + d*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 +
 Tan[e]^2])/(-(c*Sec[e]) + d*Sqrt[1 + Tan[e]^2])]*(c + d*Cos[e]*Sin[f*x + A
rcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2])^(1/3))/(d*f*(Cos[e/2 + (f*x)/2] + Sin[e/
2 + (f*x)/2])^2*Sqrt[1 + Tan[e]^2]))
```

Maple [F] time = 0.147, size = 0, normalized size = 0.

$$\int \left(a + a\sin\left(fx + e\right)\right)\sqrt[3]{c + d\sin\left(fx + e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(c+d*sin(f*x+e))^(1/3),x)
```

```
[Out] int((a+a*sin(f*x+e))*(c+d*sin(f*x+e))^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx + e\right) + a\right)\left(d\sin\left(fx + e\right) + c\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a\sin\left(fx + e\right) + a\right)\left(d\sin\left(fx + e\right) + c\right)^{\frac{1}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e))^(1/3),x, algorithm="fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^(1/3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int \sqrt[3]{c+d\sin\left(e+fx\right)}\sin\left(e+fx\right)dx+\int \sqrt[3]{c+d\sin\left(e+fx\right)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e))**(1/3),x)
```

```
[Out] a*(Integral((c + d*sin(e + f*x))**(1/3)*sin(e + f*x), x) + Integral((c + d*
sin(e + f*x))**(1/3), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)+c\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e))^(1/3),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^(1/3), x)
```

3.669 $\int \frac{a+a\sin(e+fx)}{\sqrt[3]{c+d\sin(e+fx)}}\,dx$

Optimal. Leaf size=107

$$-\frac{2\sqrt{2}a\cos(e+fx)\sqrt[3]{\frac{c+d\sin(e+fx)}{c+d}}F_1\left(\frac{1}{2};-\frac{1}{2},\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{f\sqrt{\sin(e+fx)+1}\sqrt[3]{c+d\sin(e+fx)}}$$

```
[Out] (-2*Sqrt[2]*a*AppellF1[1/2, -1/2, 1/3, 3/2, (1 - Sin[e + f*x])/2, (d*(1 - S
in[e + f*x]))/(c + d)]*Cos[e + f*x]*((c + d*Sin[e + f*x])/(c + d))^(1/3))/(
f*Sqrt[1 + Sin[e + f*x]]*(c + d*Sin[e + f*x])^(1/3))
```

Rubi [A] time = 0.08664, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2755, 139, 138}

$$-\frac{2\sqrt{2}a\cos(e+fx)\sqrt[3]{\frac{c+d\sin(e+fx)}{c+d}}F_1\left(\frac{1}{2};-\frac{1}{2},\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{f\sqrt{\sin(e+fx)+1}\sqrt[3]{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])/(c + d*Sin[e + f*x])^(1/3),x]
```

```
[Out] (-2*Sqrt[2]*a*AppellF1[1/2, -1/2, 1/3, 3/2, (1 - Sin[e + f*x])/2, (d*(1 - S
in[e + f*x]))/(c + d)]*Cos[e + f*x]*((c + d*Sin[e + f*x])/(c + d))^(1/3))/(
f*Sqrt[1 + Sin[e + f*x]]*(c + d*Sin[e + f*x])^(1/3))
```

Rule 2755

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(c*Cos[e + f*x])/(f*Sqrt[1 + Sin[e + f*x]]*S
qrt[1 - Sin[e + f*x]]), Subst[Int[((a + b*x)^m*Sqrt[1 + (d*x)/c])/Sqrt[1 -
(d*x)/c], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Ne
Q[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*m] && EqQ[c^2 - d^2, 0
]
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
```

```
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int \frac{a+a\sin(e+fx)}{\sqrt[3]{c+d\sin(e+fx)}}\,dx = \frac{(a\cos(e+fx))\,\text{Subst}\left(\int \frac{\sqrt{1+x}}{\sqrt{1-x}\sqrt[3]{c+dx}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}$$

$$= \frac{\left(a\cos(e+fx)\sqrt[3]{-\frac{c+d\sin(e+fx)}{-c-d}}\right)\text{Subst}\left(\int \frac{\sqrt{1+x}}{\sqrt{1-x}\sqrt[3]{-\frac{c}{-c-d}-\frac{dx}{-c-d}}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}\sqrt[3]{c+d\sin(e+fx)}}$$

$$= -\frac{2\sqrt{2}aF_1\left(\frac{1}{2};-\frac{1}{2},\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)\sqrt[3]{\frac{c+d\sin(e+fx)}{c+d}}}{f\sqrt{1+\sin(e+fx)}\sqrt[3]{c+d\sin(e+fx)}}$$

Mathematica [B] time = 6.26852, size = 886, normalized size = 8.28

$$a\left(\frac{\sec(e)\left(-\frac{F_1\left(-\frac{1}{3};-\frac{1}{2},-\frac{1}{2};\frac{2}{3};-\frac{\csc(e)\left(c+d\cos\left(fx-\tan^{-1}(\cot(e))\right)\sqrt{\cot^2(e)+1}\sin(e)\right)}{d\sqrt{\cot^2(e)+1}\left(1-\frac{c\csc(e)}{d\sqrt{\cot^2(e)+1}}\right)},-\frac{\csc(e)\left(c+d\cos\left(fx-\tan^{-1}(\cot(e))\right)\sqrt{\cot^2(e)+1}\sin(e)\right)}{d\sqrt{\cot^2(e)+1}\left(-\frac{c\csc(e)}{d\sqrt{\cot^2(e)+1}}-1\right)}\right)\cot(e)\sin\left(fx-\tan^{-1}(\ldots\right.}{\sqrt{\cot^2(e)+1}\sqrt{\frac{\cos\left(fx-\tan^{-1}(\cot(e))\right)\sqrt{\cot^2(e)+1}d+\sqrt{\cot^2(e)+1}d}{d\sqrt{\cot^2(e)+1}-c\csc(e)}}\sqrt{\frac{d\sqrt{\cot^2(e)+1}-d\cos\left(fx-\tan^{-1}(\cot(e))\right)\sqrt{\cot^2(e)+1}}{\sqrt{\cot^2(e)+1}d+c\csc(e)}}\sqrt[3]{c+d\cos\left(fx-\tan^{-1}(\cot(e))\right)\sqrt{\cot\ldots}}}\right.}{f\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)+\sin\left(\frac{e}{2}+\frac{fx}{2}\right)\ldots\right.}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])/(c + d*Sin[e + f*x])^(1/3),x]
```

```
[Out] a*((Sec[e]*(1 + Sin[e + f*x])*(-((AppellF1[-1/3, -1/2, -1/2, 2/3, -((Csc[e]
*(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d*Sqrt[1 + C
ot[e]^2]*(1 - (c*Csc[e])/(d*Sqrt[1 + Cot[e]^2])))), -((Csc[e]*(c + d*Cos[f*
x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d*Sqrt[1 + Cot[e]^2]*(-1 -
 (c*Csc[e])/(d*Sqrt[1 + Cot[e]^2]))))]*Cot[e]*Sin[f*x - ArcTan[Cot[e]]])/(S
qrt[1 + Cot[e]^2]*Sqrt[(d*Sqrt[1 + Cot[e]^2] + d*Cos[f*x - ArcTan[Cot[e]]]*
Sqrt[1 + Cot[e]^2])/(d*Sqrt[1 + Cot[e]^2] - c*Csc[e])]*Sqrt[(d*Sqrt[1 + Cot
[e]^2] - d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2])/(d*Sqrt[1 + Cot[e]
^2] + c*Csc[e])]*(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]
)^(1/3))) - ((3*d*Sin[e]*(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2
]*Sin[e]))/(2*(d^2*Cos[e]^2 + d^2*Sin[e]^2)) - (Cot[e]*Sin[f*x - ArcTan[Cot
[e]]])/Sqrt[1 + Cot[e]^2])/(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]
^2]*Sin[e])^(1/3)))/(f*(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^2) + (3*(1
 + Sin[e + f*x])*(c + d*Sin[e + f*x])^(2/3)*Tan[e])/(2*d*f*(Cos[e/2 + (f*x)
/2] + Sin[e/2 + (f*x)/2])^2) + (3*AppellF1[2/3, 1/2, 1/2, 5/3, -((Sec[e]*(c
 + d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]))/(d*Sqrt[1 + Tan[
e]^2]*(1 - (c*Sec[e])/(d*Sqrt[1 + Tan[e]^2])))), -((Sec[e]*(c + d*Cos[e]*Si
n[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]))/(d*Sqrt[1 + Tan[e]^2]*(-1 - (c
*Sec[e])/(d*Sqrt[1 + Tan[e]^2]))))]*Sec[e]*Sec[f*x + ArcTan[Tan[e]]]*(1 + S
```

```
in[e + f*x])*Sqrt[(d*Sqrt[1 + Tan[e]^2] - d*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[
1 + Tan[e]^2])/(c*Sec[e] + d*Sqrt[1 + Tan[e]^2])]*Sqrt[(d*Sqrt[1 + Tan[e]^2
] + d*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2])/(-(c*Sec[e]) + d*Sqrt[1
 + Tan[e]^2])]*(c + d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2])^
(2/3))/(2*d*f*(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^2*Sqrt[1 + Tan[e]^2
]))
```

Maple [F] time = 0.331, size = 0, normalized size = 0.

$$\int (a + a\sin(fx+e))\frac{1}{\sqrt[3]{c+d\sin(fx+e)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(1/3),x)

[Out] int((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a\sin(fx+e)+a}{(d\sin(fx+e)+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(1/3),x, algorithm="maxima")

[Out] integrate((a*sin(f*x + e) + a)/(d*sin(f*x + e) + c)^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{a\sin(fx+e)+a}{(d\sin(fx+e)+c)^{\frac{1}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(1/3),x, algorithm="fricas")

[Out] integral((a*sin(f*x + e) + a)/(d*sin(f*x + e) + c)^(1/3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int \frac{\sin(e+fx)}{\sqrt[3]{c+d\sin(e+fx)}}\,dx + \int \frac{1}{\sqrt[3]{c+d\sin(e+fx)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))**(1/3),x)
```

```
[Out] a*(Integral(sin(e + f*x)/(c + d*sin(e + f*x))**(1/3), x) + Integral((c + d*
sin(e + f*x))**(-1/3), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a\sin\left(fx+e\right)+a}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(1/3),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)/(d*sin(f*x + e) + c)^(1/3), x)
```

3.670 $\int \frac{a+a\sin(e+fx)}{(c+d\sin(e+fx))^{4/3}}\,dx$

Optimal. Leaf size=112

$$-\frac{2\sqrt{2}a\cos(e+fx)\sqrt[3]{\frac{c+d\sin(e+fx)}{c+d}}F_1\left(\frac{1}{2};-\frac{1}{2},\frac{4}{3};\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{f(c+d)\sqrt{\sin(e+fx)+1}\sqrt[3]{c+d\sin(e+fx)}}$$

```
[Out] (-2*Sqrt[2]*a*AppellF1[1/2, -1/2, 4/3, 3/2, (1 - Sin[e + f*x])/2, (d*(1 - S
in[e + f*x]))/(c + d)]*Cos[e + f*x]*((c + d*Sin[e + f*x])/(c + d))^(1/3))/(
(c + d)*f*Sqrt[1 + Sin[e + f*x]]*(c + d*Sin[e + f*x])^(1/3))
```

Rubi [A] time = 0.0963662, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2755, 139, 138}

$$-\frac{2\sqrt{2}a\cos(e+fx)\sqrt[3]{\frac{c+d\sin(e+fx)}{c+d}}F_1\left(\frac{1}{2};-\frac{1}{2},\frac{4}{3};\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{f(c+d)\sqrt{\sin(e+fx)+1}\sqrt[3]{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])/(c + d*Sin[e + f*x])^(4/3),x]
```


```
[Out] (-2*Sqrt[2]*a*AppellF1[1/2, -1/2, 4/3, 3/2, (1 - Sin[e + f*x])/2, (d*(1 - S
in[e + f*x]))/(c + d)]*Cos[e + f*x]*((c + d*Sin[e + f*x])/(c + d))^(1/3))/(
(c + d)*f*Sqrt[1 + Sin[e + f*x]]*(c + d*Sin[e + f*x])^(1/3))
```


Rule 2755

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(c*Cos[e + f*x])/(f*Sqrt[1 + Sin[e + f*x]]*S
qrt[1 - Sin[e + f*x]]), Subst[Int[((a + b*x)^m*Sqrt[1 + (d*x)/c])/Sqrt[1 -
(d*x)/c], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Ne
Q[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*m] && EqQ[c^2 - d^2, 0
]
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int \frac{a+a\sin(e+fx)}{(c+d\sin(e+fx))^{4/3}}\,dx = \frac{(a\cos(e+fx))\,\text{Subst}\left(\int \frac{\sqrt{1+x}}{\sqrt{1-x}(c+dx)^{4/3}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}$$

$$= \frac{\left(a\cos(e+fx)\sqrt[3]{-\frac{c+d\sin(e+fx)}{-c-d}}\right)\text{Subst}\left(\int \frac{\sqrt{1+x}}{\sqrt{1-x}\left(-\frac{c}{-c-d}-\frac{dx}{-c-d}\right)^{4/3}}\,dx, x, \sin(e+fx)\right)}{(c+d)f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}\sqrt[3]{c+d\sin(e+fx)}}$$

$$= -\frac{2\sqrt{2}aF_1\left(\frac{1}{2};-\frac{1}{2},\frac{4}{3};\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)\sqrt[3]{\frac{c+d\sin(e+fx)}{c+d}}}{(c+d)f\sqrt{1+\sin(e+fx)}\sqrt[3]{c+d\sin(e+fx)}}$$

Mathematica [B] time = 6.40232, size = 942, normalized size = 8.41

$$a\left(\frac{(c+d\sin(e+fx))^{2/3}\left(\frac{3\csc(e)(c\cos(e)+d\sin(fx))}{d(c+d)f(c+d\sin(e+fx))}-\frac{3\csc(e)\sec(e)}{d(c+d)f}\right)(\sin(e+fx)+1)}{\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)+\sin\left(\frac{e}{2}+\frac{fx}{2}\right)\right)^2} - \frac{2\sec(e)\left(-\frac{F_1\left(-\frac{1}{3};-\frac{1}{2},-\frac{1}{2};\frac{2}{3};-\frac{\csc(e)\left(c+d\cos\left(\ldots\right.\right.}{d\sqrt{c\ldots}}\ldots\right)}{\sqrt{\cot^2(e)+1}\sqrt{\frac{\cos\left(fx-\tan^{-1}(\cot(e))\right)\sqrt{\ldots}}{d\sqrt{\cot^2(e)+\ldots}}}}\ldots\right)}{\ldots}\ldots\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])/(c + d*Sin[e + f*x])^(4/3),x]
```

```
[Out] a*(((1 + Sin[e + f*x])*(c + d*Sin[e + f*x])^(2/3)*((-3*Csc[e]*Sec[e])/(d*(c
 + d)*f) + (3*Csc[e]*(c*Cos[e] + d*Sin[f*x]))/(d*(c + d)*f*(c + d*Sin[e + f
*x]))))/(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^2 - (2*Sec[e]*(1 + Sin[e
+ f*x])*(-((AppellF1[-1/3, -1/2, -1/2, 2/3, -((Csc[e]*(c + d*Cos[f*x - ArcT
an[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d*Sqrt[1 + Cot[e]^2]*(1 - (c*Csc[e
])/(d*Sqrt[1 + Cot[e]^2])))), -((Csc[e]*(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sq
rt[1 + Cot[e]^2]*Sin[e]))/(d*Sqrt[1 + Cot[e]^2]*(-1 - (c*Csc[e])/(d*Sqrt[1
+ Cot[e]^2]))))]*Cot[e]*Sin[f*x - ArcTan[Cot[e]]])/(Sqrt[1 + Cot[e]^2]*Sqrt
[(d*Sqrt[1 + Cot[e]^2] + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2])/(d
*Sqrt[1 + Cot[e]^2] - c*Csc[e])]*Sqrt[(d*Sqrt[1 + Cot[e]^2] - d*Cos[f*x - A
rcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2])/(d*Sqrt[1 + Cot[e]^2] + c*Csc[e])]*(c +
d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e])^(1/3))) - ((3*d*Sin[
e]*(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(2*(d^2*Cos
[e]^2 + d^2*Sin[e]^2)) - (Cot[e]*Sin[f*x - ArcTan[Cot[e]]])/Sqrt[1 + Cot[e]
^2])/(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e])^(1/3)))/((
c + d)*f*(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^2) + (3*AppellF1[2/3, 1/
2, 1/2, 5/3, -((Sec[e]*(c + d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan
[e]^2]))/(d*Sqrt[1 + Tan[e]^2]*(1 - (c*Sec[e])/(d*Sqrt[1 + Tan[e]^2])))), -
((Sec[e]*(c + d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]))/(d*Sq
rt[1 + Tan[e]^2]*(-1 - (c*Sec[e])/(d*Sqrt[1 + Tan[e]^2]))))]*Sec[e]*Sec[f*x
 + ArcTan[Tan[e]]]*(1 + Sin[e + f*x])*Sqrt[(d*Sqrt[1 + Tan[e]^2] - d*Sin[f*
```

```
x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2])/(c*Sec[e] + d*Sqrt[1 + Tan[e]^2])]*
Sqrt[(d*Sqrt[1 + Tan[e]^2] + d*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]
)/(-(c*Sec[e]) + d*Sqrt[1 + Tan[e]^2])]*(c + d*Cos[e]*Sin[f*x + ArcTan[Tan[
e]]]*Sqrt[1 + Tan[e]^2])^(2/3))/(2*d*(c + d)*f*(Cos[e/2 + (f*x)/2] + Sin[e/
2 + (f*x)/2])^2*Sqrt[1 + Tan[e]^2]))
```

Maple [F] time = 0.151, size = 0, normalized size = 0.

$$\int (a + a\sin(fx+e))\left(c + d\sin(fx+e)\right)^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(4/3),x)

[Out] int((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a\sin(fx+e) + a}{\left(d\sin(fx+e) + c\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(4/3),x, algorithm="maxima")

[Out] integrate((a*sin(f*x + e) + a)/(d*sin(f*x + e) + c)^(4/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(a\sin(fx+e) + a\right)\left(d\sin(fx+e) + c\right)^{\frac{2}{3}}}{d^2\cos(fx+e)^2 - 2\,cd\sin(fx+e) - c^2 - d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(4/3),x, algorithm="fricas")

[Out] integral(-(a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^(2/3)/(d^2*cos(f*x + e)
^2 - 2*c*d*sin(f*x + e) - c^2 - d^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))**(4/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a\sin\left(fx+e\right)+a}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(4/3),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)/(d*sin(f*x + e) + c)^(4/3), x)
```

3.671 $\int (a+b\sin(e+fx))(c+d\sin(e+fx))^3\,dx$

Optimal. Leaf size=171

$$-\frac{\left(4ad\left(4c^2+d^2\right)+3b\left(c^3+4cd^2\right)\right)\cos(e+fx)}{6f}-\frac{d\left(20acd+6bc^2+9bd^2\right)\sin(e+fx)\cos(e+fx)}{24f}+\frac{1}{8}x\left(8ac^3+12ac\right.$$

```
[Out] ((8*a*c^3 + 12*b*c^2*d + 12*a*c*d^2 + 3*b*d^3)*x)/8 - ((4*a*d*(4*c^2 + d^2)
 + 3*b*(c^3 + 4*c*d^2))*Cos[e + f*x])/(6*f) - (d*(6*b*c^2 + 20*a*c*d + 9*b*
d^2)*Cos[e + f*x]*Sin[e + f*x])/(24*f) - ((3*b*c + 4*a*d)*Cos[e + f*x]*(c +
 d*Sin[e + f*x])^2)/(12*f) - (b*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(4*f)
```

Rubi [A] time = 0.211378, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2753, 2734}

$$-\frac{\left(4ad\left(4c^2+d^2\right)+3b\left(c^3+4cd^2\right)\right)\cos(e+fx)}{6f}-\frac{d\left(20acd+6bc^2+9bd^2\right)\sin(e+fx)\cos(e+fx)}{24f}+\frac{1}{8}x\left(8ac^3+12ac\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])*(c + d*Sin[e + f*x])^3,x]
```

```
[Out] ((8*a*c^3 + 12*b*c^2*d + 12*a*c*d^2 + 3*b*d^3)*x)/8 - ((4*a*d*(4*c^2 + d^2)
 + 3*b*(c^3 + 4*c*d^2))*Cos[e + f*x])/(6*f) - (d*(6*b*c^2 + 20*a*c*d + 9*b*
d^2)*Cos[e + f*x]*Sin[e + f*x])/(24*f) - ((3*b*c + 4*a*d)*Cos[e + f*x]*(c +
 d*Sin[e + f*x])^2)/(12*f) - (b*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(4*f)
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int (a+b\sin(e+fx))(c+d\sin(e+fx))^3\,dx=-\frac{b\cos(e+fx)(c+d\sin(e+fx))^3}{4f}+\frac{1}{4}\int (c+d\sin(e+fx))^2(4ac+3b\ldots$$

$$=-\frac{(3bc+4ad)\cos(e+fx)(c+d\sin(e+fx))^2}{12f}-\frac{b\cos(e+fx)(c+d\sin(\ldots}{4f}$$

$$=\frac{1}{8}\left(8ac^3+12bc^2d+12acd^2+3bd^3\right)x-\frac{\left(4ad\left(4c^2+d^2\right)+3b\left(c^3+4cd^2\ldots\right.\right.}{6f}$$

Mathematica [A] time = 0.669611, size = 143, normalized size = 0.84

$$\frac{3\left(4(e+fx)\left(8ac^3+12acd^2+12bc^2d+3bd^3\right)-8d\left(3acd+b\left(3c^2+d^2\right)\right)\sin(2(e+fx))+bd^3\sin(4(e+fx))\right)-24\left(3ad\left(\right.\right.}{96f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])*(c + d*Sin[e + f*x])^3,x]
```

```
[Out] (-24*(3*a*d*(4*c^2 + d^2) + b*(4*c^3 + 9*c*d^2))*Cos[e + f*x] + 8*d^2*(3*b*
c + a*d)*Cos[3*(e + f*x)] + 3*(4*(8*a*c^3 + 12*b*c^2*d + 12*a*c*d^2 + 3*b*d
^3)*(e + f*x) - 8*d*(3*a*c*d + b*(3*c^2 + d^2))*Sin[2*(e + f*x)] + b*d^3*Si
n[4*(e + f*x)]))/(96*f)
```

Maple [A] time = 0.029, size = 182, normalized size = 1.1

$$\frac{1}{f}\left(ac^3\left(fx+e\right)-3c^2da\cos\left(fx+e\right)+3acd^2\left(-1/2\sin\left(fx+e\right)\cos\left(fx+e\right)+1/2fx+e/2\right)-\frac{d^3a\left(2+\left(\sin\left(fx+e\right)\right)^2\right)}{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))*(c+d*sin(f*x+e))^3,x)
```

```
[Out] 1/f*(a*c^3*(f*x+e)-3*c^2*d*a*cos(f*x+e)+3*a*c*d^2*(-1/2*sin(f*x+e)*cos(f*x+
e)+1/2*f*x+1/2*e)-1/3*d^3*a*(2+sin(f*x+e)^2)*cos(f*x+e)-c^3*b*cos(f*x+e)+3*
b*c^2*d*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-c*d^2*b*(2+sin(f*x+e)^2)
*cos(f*x+e)+d^3*b*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/
8*e))
```

Maxima [A] time = 1.11255, size = 236, normalized size = 1.38

$$\frac{96\left(fx+e\right)ac^3+72\left(2fx+2e-\sin\left(2fx+2e\right)\right)bc^2d+72\left(2fx+2e-\sin\left(2fx+2e\right)\right)acd^2+96\left(\cos\left(fx+e\right)^3-3\cos\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 1/96*(96*(f*x + e)*a*c^3 + 72*(2*f*x + 2*e - sin(2*f*x + 2*e))*b*c^2*d + 72
*(2*f*x + 2*e - sin(2*f*x + 2*e))*a*c*d^2 + 96*(cos(f*x + e)^3 - 3*cos(f*x
+ e))*b*c*d^2 + 32*(cos(f*x + e)^3 - 3*cos(f*x + e))*a*d^3 + 3*(12*f*x + 12
*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*b*d^3 - 96*b*c^3*cos(f*x + e) -
 288*a*c^2*d*cos(f*x + e))/f
```

Fricas [A] time = 1.65525, size = 340, normalized size = 1.99

$$\frac{8\left(3bcd^2+ad^3\right)\cos\left(fx+e\right)^3+3\left(8ac^3+12bc^2d+12acd^2+3bd^3\right)fx-24\left(bc^3+3ac^2d+3bcd^2+ad^3\right)\cos\left(fx+e\right)+}{24f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/24*(8*(3*b*c*d^2 + a*d^3)*cos(f*x + e)^3 + 3*(8*a*c^3 + 12*b*c^2*d + 12*a
*c*d^2 + 3*b*d^3)*f*x - 24*(b*c^3 + 3*a*c^2*d + 3*b*c*d^2 + a*d^3)*cos(f*x
+ e) + 3*(2*b*d^3*cos(f*x + e)^3 - (12*b*c^2*d + 12*a*c*d^2 + 5*b*d^3)*cos(
f*x + e))*sin(f*x + e))/f
```

Sympy [A] time = 1.7594, size = 386, normalized size = 2.26

$$\begin{cases} ac^3x - \frac{3ac^2d\cos(e+fx)}{f} + \frac{3acd^2x\sin^2(e+fx)}{2} + \frac{3acd^2x\cos^2(e+fx)}{2} - \frac{3acd^2\sin(e+fx)\cos(e+fx)}{2f} - \frac{ad^3\sin^2(e+fx)\cos(e+fx)}{f} - \frac{2ad^3\cos^3(e \\ x(a+b\sin(e))(c+d\sin(e))^3 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))*(c+d*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((a*c**3*x - 3*a*c**2*d*cos(e + f*x)/f + 3*a*c*d**2*x*sin(e + f*x)
**2/2 + 3*a*c*d**2*x*cos(e + f*x)**2/2 - 3*a*c*d**2*sin(e + f*x)*cos(e + f*
x)/(2*f) - a*d**3*sin(e + f*x)**2*cos(e + f*x)/f - 2*a*d**3*cos(e + f*x)**3
/(3*f) - b*c**3*cos(e + f*x)/f + 3*b*c**2*d*x*sin(e + f*x)**2/2 + 3*b*c**2*
d*x*cos(e + f*x)**2/2 - 3*b*c**2*d*sin(e + f*x)*cos(e + f*x)/(2*f) - 3*b*c*
d**2*sin(e + f*x)**2*cos(e + f*x)/f - 2*b*c*d**2*cos(e + f*x)**3/f + 3*b*d*
*3*x*sin(e + f*x)**4/8 + 3*b*d**3*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 3*b
*d**3*x*cos(e + f*x)**4/8 - 5*b*d**3*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 3
*b*d**3*sin(e + f*x)*cos(e + f*x)**3/(8*f), Ne(f, 0)), (x*(a + b*sin(e))*(c
 + d*sin(e))**3, True))
```

Giac [A] time = 1.29152, size = 205, normalized size = 1.2

$$\frac{bd^3\sin(4fx+4e)}{32f} + \frac{1}{8}\left(8ac^3 + 12bc^2d + 12acd^2 + 3bd^3\right)x + \frac{\left(3bcd^2 + ad^3\right)\cos(3fx+3e)}{12f} - \frac{\left(4bc^3 + 12ac^2d + 9\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/32*b*d^3*sin(4*f*x + 4*e)/f + 1/8*(8*a*c^3 + 12*b*c^2*d + 12*a*c*d^2 + 3*
b*d^3)*x + 1/12*(3*b*c*d^2 + a*d^3)*cos(3*f*x + 3*e)/f - 1/4*(4*b*c^3 + 12*
a*c^2*d + 9*b*c*d^2 + 3*a*d^3)*cos(f*x + e)/f - 1/4*(3*b*c^2*d + 3*a*c*d^2
+ b*d^3)*sin(2*f*x + 2*e)/f
```

3.672 $\int (a+b\sin(e+fx))(c+d\sin(e+fx))^2\,dx$

Optimal. Leaf size=106

$$-\frac{2\left(3acd+b\left(c^2+d^2\right)\right)\cos(e+fx)}{3f}+\frac{1}{2}x\left(a\left(2c^2+d^2\right)+2bcd\right)-\frac{d(3ad+2bc)\sin(e+fx)\cos(e+fx)}{6f}-\frac{b\cos(e+fx)(c$$

```
[Out] ((2*b*c*d + a*(2*c^2 + d^2))*x)/2 - (2*(3*a*c*d + b*(c^2 + d^2))*Cos[e + f*
x])/(3*f) - (d*(2*b*c + 3*a*d)*Cos[e + f*x]*Sin[e + f*x])/(6*f) - (b*Cos[e
+ f*x]*(c + d*Sin[e + f*x])^2)/(3*f)
```

Rubi [A] time = 0.100707, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2753, 2734}

$$-\frac{2\left(3acd+b\left(c^2+d^2\right)\right)\cos(e+fx)}{3f}+\frac{1}{2}x\left(a\left(2c^2+d^2\right)+2bcd\right)-\frac{d(3ad+2bc)\sin(e+fx)\cos(e+fx)}{6f}-\frac{b\cos(e+fx)(c$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])*(c + d*Sin[e + f*x])^2,x]
```

```
[Out] ((2*b*c*d + a*(2*c^2 + d^2))*x)/2 - (2*(3*a*c*d + b*(c^2 + d^2))*Cos[e + f*
x])/(3*f) - (d*(2*b*c + 3*a*d)*Cos[e + f*x]*Sin[e + f*x])/(6*f) - (b*Cos[e
+ f*x]*(c + d*Sin[e + f*x])^2)/(3*f)
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int (a+b\sin(e+fx))(c+d\sin(e+fx))^2\,dx &= -\frac{b\cos(e+fx)(c+d\sin(e+fx))^2}{3f}+\frac{1}{3}\int (c+d\sin(e+fx))(3ac+2bd+(\\ &= \frac{1}{2}\left(2bcd+a\left(2c^2+d^2\right)\right)x-\frac{2\left(3acd+b\left(c^2+d^2\right)\right)\cos(e+fx)}{3f}-\frac{d(2bc+3\end{aligned}$$

Mathematica [A] time = 0.297141, size = 90, normalized size = 0.85

$$\frac{6(e+fx)\left(a\left(2c^2+d^2\right)+2bcd\right)-3\left(8acd+4bc^2+3bd^2\right)\cos(e+fx)-3d(ad+2bc)\sin(2(e+fx))+bd^2\cos(3(e+fx))}{12f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])*(c + d*Sin[e + f*x])^2,x]
```

```
[Out] (6*(2*b*c*d + a*(2*c^2 + d^2))*(e + f*x) - 3*(4*b*c^2 + 8*a*c*d + 3*b*d^2)*
Cos[e + f*x] + b*d^2*Cos[3*(e + f*x)] - 3*d*(2*b*c + a*d)*Sin[2*(e + f*x)])
/(12*f)
```

Maple [A] time = 0.026, size = 115, normalized size = 1.1

$$\frac{1}{f}\left(ac^2\left(fx+e\right)-2\,acd\cos\left(fx+e\right)+ad^2\left(-\frac{\sin\left(fx+e\right)\cos\left(fx+e\right)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-c^2b\cos\left(fx+e\right)+2\,bcd\left(-1/2\ \mathrm{si}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))*(c+d*sin(f*x+e))^2,x)
```

```
[Out] 1/f*(a*c^2*(f*x+e)-2*a*c*d*cos(f*x+e)+a*d^2*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2
*f*x+1/2*e)-c^2*b*cos(f*x+e)+2*b*c*d*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/
2*e)-1/3*d^2*b*(2+sin(f*x+e)^2)*cos(f*x+e))
```

Maxima [A] time = 1.14654, size = 151, normalized size = 1.42

$$\frac{12\left(fx+e\right)ac^2+6\left(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right)\right)bcd+3\left(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right)\right)ad^2+4\left(\cos\left(fx+e\right)^3-3\,\cos\right.}{12\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/12*(12*(f*x + e)*a*c^2 + 6*(2*f*x + 2*e - sin(2*f*x + 2*e))*b*c*d + 3*(2*
f*x + 2*e - sin(2*f*x + 2*e))*a*d^2 + 4*(cos(f*x + e)^3 - 3*cos(f*x + e))*b
*d^2 - 12*b*c^2*cos(f*x + e) - 24*a*c*d*cos(f*x + e))/f
```

Fricas [A] time = 1.62584, size = 215, normalized size = 2.03

$$\frac{2\,bd^2\cos\left(fx+e\right)^3+3\left(2\,ac^2+2\,bcd+ad^2\right)fx-3\left(2\,bcd+ad^2\right)\cos\left(fx+e\right)\sin\left(fx+e\right)-6\left(bc^2+2\,acd+bd^2\right)\cos}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/6*(2*b*d^2*cos(f*x + e)^3 + 3*(2*a*c^2 + 2*b*c*d + a*d^2)*f*x - 3*(2*b*c*
d + a*d^2)*cos(f*x + e)*sin(f*x + e) - 6*(b*c^2 + 2*a*c*d + b*d^2)*cos(f*x
+ e))/f
```

Sympy [A] time = 0.790579, size = 199, normalized size = 1.88

$$\begin{cases} ac^2x - \frac{2acd\cos(e+fx)}{f} + \frac{ad^2x\sin^2(e+fx)}{2} + \frac{ad^2x\cos^2(e+fx)}{2} - \frac{ad^2\sin(e+fx)\cos(e+fx)}{2f} - \frac{bc^2\cos(e+fx)}{f} + bcdx\sin^2(e+fx) + bcdx\,\mathrm{co} \\ x(a+b\sin(e))(c+d\sin(e))^2 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))*(c+d*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((a*c**2*x - 2*a*c*d*cos(e + f*x)/f + a*d**2*x*sin(e + f*x)**2/2 +
 a*d**2*x*cos(e + f*x)**2/2 - a*d**2*sin(e + f*x)*cos(e + f*x)/(2*f) - b*c*
*2*cos(e + f*x)/f + b*c*d*x*sin(e + f*x)**2 + b*c*d*x*cos(e + f*x)**2 - b*c
*d*sin(e + f*x)*cos(e + f*x)/f - b*d**2*sin(e + f*x)**2*cos(e + f*x)/f - 2*
b*d**2*cos(e + f*x)**3/(3*f), Ne(f, 0)), (x*(a + b*sin(e))*(c + d*sin(e))**
2, True))
```

Giac [A] time = 1.34707, size = 130, normalized size = 1.23

$$\frac{bd^2\cos(3fx+3e)}{12f} + \frac{1}{2}(2ac^2+2bcd+ad^2)x - \frac{(4bc^2+8acd+3bd^2)\cos(fx+e)}{4f} - \frac{(2bcd+ad^2)\sin(2fx+2e)}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/12*b*d^2*cos(3*f*x + 3*e)/f + 1/2*(2*a*c^2 + 2*b*c*d + a*d^2)*x - 1/4*(4*
b*c^2 + 8*a*c*d + 3*b*d^2)*cos(f*x + e)/f - 1/4*(2*b*c*d + a*d^2)*sin(2*f*x
 + 2*e)/f
```

3.673 $\int (a+b\sin(e+fx))(c+d\sin(e+fx))\,dx$

Optimal. Leaf size=53

$$-\frac{(ad+bc)\cos(e+fx)}{f}+\frac{1}{2}x(2ac+bd)-\frac{bd\sin(e+fx)\cos(e+fx)}{2f}$$

```
[Out] ((2*a*c + b*d)*x)/2 - ((b*c + a*d)*Cos[e + f*x])/f - (b*d*Cos[e + f*x]*Sin[
e + f*x])/(2*f)
```

Rubi [A] time = 0.022245, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2734}

$$-\frac{(ad+bc)\cos(e+fx)}{f}+\frac{1}{2}x(2ac+bd)-\frac{bd\sin(e+fx)\cos(e+fx)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])*(c + d*Sin[e + f*x]),x]
```

```
[Out] ((2*a*c + b*d)*x)/2 - ((b*c + a*d)*Cos[e + f*x])/f - (b*d*Cos[e + f*x]*Sin[
e + f*x])/(2*f)
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int (a+b\sin(e+fx))(c+d\sin(e+fx))\,dx=\frac{1}{2}(2ac+bd)x-\frac{(bc+ad)\cos(e+fx)}{f}-\frac{bd\cos(e+fx)\sin(e+fx)}{2f}$$

Mathematica [A] time = 0.0907934, size = 52, normalized size = 0.98

$$\frac{-4(ad+bc)\cos(e+fx)+4acfx-bd\sin(2(e+fx))+2bde+2bdfx}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])*(c + d*Sin[e + f*x]),x]
```

```
[Out] (2*b*d*e + 4*a*c*f*x + 2*b*d*f*x - 4*(b*c + a*d)*Cos[e + f*x] - b*d*Sin[2*(
e + f*x)])/(4*f)
```

Maple [A] time = 0.022, size = 59, normalized size = 1.1

$$\frac{1}{f}\left(bd\left(-\frac{\sin(fx+e)\cos(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-da\cos(fx+e)-cb\cos(fx+e)+ca(fx+e)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))*(c+d*sin(f*x+e)),x)
```

```
[Out] 1/f*(b*d*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-d*a*cos(f*x+e)-c*b*cos(
f*x+e)+c*a*(f*x+e))
```

Maxima [A] time = 1.03329, size = 77, normalized size = 1.45

$$\frac{4\left(fx+e\right)ac+\left(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right)\right)bd-4\,bc\cos\left(fx+e\right)-4\,ad\cos\left(fx+e\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/4*(4*(f*x + e)*a*c + (2*f*x + 2*e - sin(2*f*x + 2*e))*b*d - 4*b*c*cos(f*x
 + e) - 4*a*d*cos(f*x + e))/f
```

Fricas [A] time = 1.59674, size = 120, normalized size = 2.26

$$-\frac{bd\cos\left(fx+e\right)\sin\left(fx+e\right)-(2\,ac+bd)fx+2\,(bc+ad)\cos\left(fx+e\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] -1/2*(b*d*cos(f*x + e)*sin(f*x + e) - (2*a*c + b*d)*f*x + 2*(b*c + a*d)*cos
(f*x + e))/f
```

Sympy [A] time = 0.337631, size = 94, normalized size = 1.77

$$\begin{cases} acx-\frac{ad\cos\left(e+fx\right)}{f}-\frac{bc\cos\left(e+fx\right)}{f}+\frac{bdx\sin^2\left(e+fx\right)}{2}+\frac{bdx\cos^2\left(e+fx\right)}{2}-\frac{bd\sin\left(e+fx\right)\cos\left(e+fx\right)}{2f} & \text{for } f\neq 0 \\ x\,(a+b\sin{(e)})\,(c+d\sin{(e)}) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))*(c+d*sin(f*x+e)),x)
```

```
[Out] Piecewise((a*c*x - a*d*cos(e + f*x)/f - b*c*cos(e + f*x)/f + b*d*x*sin(e +
f*x)**2/2 + b*d*x*cos(e + f*x)**2/2 - b*d*sin(e + f*x)*cos(e + f*x)/(2*f),
Ne(f, 0)), (x*(a + b*sin(e))*(c + d*sin(e)), True))
```

Giac [A] time = 1.15234, size = 65, normalized size = 1.23

$$\frac{1}{2}\,(2\,ac+bd)x-\frac{bd\sin\left(2\,fx+2\,e\right)}{4\,f}-\frac{(bc+ad)\cos\left(fx+e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/2*(2*a*c + b*d)*x - 1/4*b*d*sin(2*f*x + 2*e)/f - (b*c + a*d)*cos(f*x + e)
/f
```

3.674 $\int (a+b\sin(e+fx))\,dx$

Optimal. Leaf size=16

$$ax-\frac{b\cos(e+fx)}{f}$$

[Out] a*x - (b*Cos[e + f*x])/f

Rubi [A] time = 0.0080893, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {2638}

$$ax-\frac{b\cos(e+fx)}{f}$$

Antiderivative was successfully verified.

[In] Int[a + b*Sin[e + f*x],x]

[Out] a*x - (b*Cos[e + f*x])/f

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a+b\sin(e+fx))\,dx &= ax+b\int \sin(e+fx)\,dx\\ &= ax-\frac{b\cos(e+fx)}{f}\end{aligned}$$

Mathematica [A] time = 0.0061895, size = 27, normalized size = 1.69

$$ax+\frac{b\sin(e)\sin(fx)}{f}-\frac{b\cos(e)\cos(fx)}{f}$$

Antiderivative was successfully verified.

[In] Integrate[a + b*Sin[e + f*x],x]

[Out] a*x - (b*Cos[e]*Cos[f*x])/f + (b*Sin[e]*Sin[f*x])/f

Maple [A] time = 0.009, size = 17, normalized size = 1.1

$$ax-\frac{b\cos\left(fx+e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*sin(f*x+e),x)

[Out] a*x-b*cos(f*x+e)/f
```

Maxima [A] time = 1.20874, size = 22, normalized size = 1.38

$$ax - \frac{b\cos\left(fx + e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sin(f*x+e),x, algorithm="maxima")

[Out] a*x - b*cos(f*x + e)/f
```

Fricas [A] time = 1.55251, size = 38, normalized size = 2.38

$$\frac{afx - b\cos\left(fx + e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sin(f*x+e),x, algorithm="fricas")

[Out] (a*f*x - b*cos(f*x + e))/f
```

Sympy [A] time = 0.139284, size = 19, normalized size = 1.19

$$ax + b\left(\begin{cases} -\frac{\cos\left(e+fx\right)}{f} & \text{for}\ f \neq 0 \\ x\sin\left(e\right) & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sin(f*x+e),x)

[Out] a*x + b*Piecewise((-cos(e + f*x)/f, Ne(f, 0)), (x*sin(e), True))
```

Giac [A] time = 1.24348, size = 23, normalized size = 1.44

$$ax - \frac{b\cos\left(fx + e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sin(f*x+e),x, algorithm="giac")

[Out] a*x - b*cos(f*x + e)/f
```

3.675 $\int \frac{a+b\sin(e+fx)}{c+d\sin(e+fx)}\,dx$

Optimal. Leaf size=65

$$\frac{bx}{d}-\frac{2(bc-ad)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{df\sqrt{c^2-d^2}}$$

```
[Out] (b*x)/d - (2*(b*c - a*d)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/
(d*Sqrt[c^2 - d^2]*f)
```

Rubi [A] time = 0.0882793, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2735, 2660, 618, 204}

$$\frac{bx}{d}-\frac{2(bc-ad)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{df\sqrt{c^2-d^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])/(c + d*Sin[e + f*x]),x]
```

```
[Out] (b*x)/d - (2*(b*c - a*d)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/
(d*Sqrt[c^2 - d^2]*f)
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{a+b\sin(e+fx)}{c+d\sin(e+fx)}\,dx = \frac{bx}{d}-\frac{(bc-ad)\int \frac{1}{c+d\sin(e+fx)}\,dx}{d}$$
$$= \frac{bx}{d}-\frac{(2(bc-ad))\,\text{Subst}\left(\int \frac{1}{c+2dx+cx^2}\,dx,x,\tan\left(\frac{1}{2}(e+fx)\right)\right)}{df}$$
$$= \frac{bx}{d}+\frac{(4(bc-ad))\,\text{Subst}\left(\int \frac{1}{-4(c^2-d^2)-x^2}\,dx,x,2d+2c\tan\left(\frac{1}{2}(e+fx)\right)\right)}{df}$$
$$= \frac{bx}{d}-\frac{2(bc-ad)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{d\sqrt{c^2-d^2}f}$$

Mathematica [A] time = 0.13013, size = 67, normalized size = 1.03

$$\frac{\frac{(2ad-2bc)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{\sqrt{c^2-d^2}}+b(e+fx)}{df}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])/(c + d*Sin[e + f*x]),x]
```

```
[Out] (b*(e + f*x) + ((-2*b*c + 2*a*d)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 -
 d^2]])/Sqrt[c^2 - d^2])/(d*f)
```

Maple [A] time = 0.043, size = 119, normalized size = 1.8

$$2\,\frac{a}{f\sqrt{c^2-d^2}}\arctan\left(1/2\,\frac{2\,c\tan\left(1/2\,fx+e/2\right)+2\,d}{\sqrt{c^2-d^2}}\right)-2\,\frac{cb}{df\sqrt{c^2-d^2}}\arctan\left(1/2\,\frac{2\,c\tan\left(1/2\,fx+e/2\right)+2\,d}{\sqrt{c^2-d^2}}\right)+2\,\frac{b\,a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

```
[Out] 2/f/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2)
)*a-2/f/d/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)
^(1/2))*c*b+2/f*b/d*arctan(tan(1/2*f*x+1/2*e))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.71969, size = 547, normalized size = 8.42

$$\left[\frac{2\left(bc^2-bd^2\right)fx+(bc-ad)\sqrt{-c^2+d^2}\log\left(\frac{\left(2c^2-d^2\right)\cos\left(fx+e\right)^2-2cd\sin\left(fx+e\right)-c^2-d^2+2\left(c\cos\left(fx+e\right)\sin\left(fx+e\right)+d\cos\left(fx+e\right)\right)\sqrt{-c^2+d^2}}{d^2\cos\left(fx+e\right)^2-2cd\sin\left(fx+e\right)-c^2-d^2}\right)}{2\left(c^2d-d^3\right)f}, \ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] [1/2*(2*(b*c^2 - b*d^2)*f*x + (b*c - a*d)*sqrt(-c^2 + d^2)*log(((2*c^2 - d^
2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 + 2*(c*cos(f*x + e)*sin(
f*x + e) + d*cos(f*x + e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*si
n(f*x + e) - c^2 - d^2)))/((c^2*d - d^3)*f), ((b*c^2 - b*d^2)*f*x + (b*c -
a*d)*sqrt(c^2 - d^2)*arctan(-(c*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x
+ e))))/((c^2*d - d^3)*f)]
```

Sympy [A] time = 151.013, size = 502, normalized size = 7.72

$$\begin{cases} \frac{\tilde{\infty}x(a+b\sin(e))}{\sin(e)} \\ \frac{\frac{a\log\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{f}+bx}{d} \\ \frac{2a}{df\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-df}+\frac{bfx\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{df\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-df}-\frac{bfx}{df\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-df}+\frac{2b}{df\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-df} \\ -\frac{2a}{df\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+df}+\frac{bfx\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{df\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+df}+\frac{bfx}{df\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+df}+\frac{2b}{df\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+df} \\ \frac{ax-\frac{b\cos(e+fx)}{f}}{c} \\ \frac{x(a+b\sin(e))}{c+d\sin(e)} \\ -\frac{ad\sqrt{-c^2+d^2}\log\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+\frac{d}{c}-\frac{\sqrt{-c^2+d^2}}{c}\right)}{c^2df-d^3f}+\frac{ad\sqrt{-c^2+d^2}\log\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+\frac{d}{c}+\frac{\sqrt{-c^2+d^2}}{c}\right)}{c^2df-d^3f}+\frac{bc^2fx}{c^2df-d^3f}+\frac{bc\sqrt{-c^2+d^2}\log\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+\frac{d}{c}-\frac{\sqrt{-c^2+d^2}}{c}\right)}{c^2df-d^3f} \ldots \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

```
[Out] Piecewise((zoo*x*(a + b*sin(e))/sin(e), Eq(c, 0) & Eq(d, 0) & Eq(f, 0)), ((
a*log(tan(e/2 + f*x/2))/f + b*x)/d, Eq(c, 0)), (2*a/(d*f*tan(e/2 + f*x/2) -
 d*f) + b*f*x*tan(e/2 + f*x/2)/(d*f*tan(e/2 + f*x/2) - d*f) - b*f*x/(d*f*ta
n(e/2 + f*x/2) - d*f) + 2*b/(d*f*tan(e/2 + f*x/2) - d*f), Eq(c, -d)), (-2*a
/(d*f*tan(e/2 + f*x/2) + d*f) + b*f*x*tan(e/2 + f*x/2)/(d*f*tan(e/2 + f*x/2
) + d*f) + b*f*x/(d*f*tan(e/2 + f*x/2) + d*f) + 2*b/(d*f*tan(e/2 + f*x/2) +
 d*f), Eq(c, d)), ((a*x - b*cos(e + f*x)/f)/c, Eq(d, 0)), (x*(a + b*sin(e))
/(c + d*sin(e)), Eq(f, 0)), (-a*d*sqrt(-c**2 + d**2)*log(tan(e/2 + f*x/2) +
 d/c - sqrt(-c**2 + d**2)/c)/(c**2*d*f - d**3*f) + a*d*sqrt(-c**2 + d**2)*l
og(tan(e/2 + f*x/2) + d/c + sqrt(-c**2 + d**2)/c)/(c**2*d*f - d**3*f) + b*c
**2*f*x/(c**2*d*f - d**3*f) + b*c*sqrt(-c**2 + d**2)*log(tan(e/2 + f*x/2) +
 d/c - sqrt(-c**2 + d**2)/c)/(c**2*d*f - d**3*f) - b*c*sqrt(-c**2 + d**2)*l
og(tan(e/2 + f*x/2) + d/c + sqrt(-c**2 + d**2)/c)/(c**2*d*f - d**3*f) - b*d
**2*f*x/(c**2*d*f - d**3*f), True))
```

Giac [A] time = 1.32551, size = 116, normalized size = 1.78

$$\frac{\frac{(fx+e)b}{d} - \frac{2\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+d}{\sqrt{c^2-d^2}}\right)\right)(bc-ad)}{\sqrt{c^2-d^2}\,d}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] ((f*x + e)*b/d - 2*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan
(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))*(b*c - a*d)/(sqrt(c^2 - d^2)*d))/f
```

3.676 $\int \frac{a+b\sin(e+fx)}{(c+d\sin(e+fx))^2}\,dx$

Optimal. Leaf size=98

$$\frac{2(ac-bd)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{f\left(c^2-d^2\right)^{3/2}}-\frac{(bc-ad)\cos(e+fx)}{f\left(c^2-d^2\right)(c+d\sin(e+fx))}$$

```
[Out] (2*(a*c - b*d)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/((c^2 - d^
2)^(3/2)*f) - ((b*c - a*d)*Cos[e + f*x])/((c^2 - d^2)*f*(c + d*Sin[e + f*x]
))
```

Rubi [A] time = 0.0964343, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2754, 12, 2660, 618, 204}

$$\frac{2(ac-bd)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{f\left(c^2-d^2\right)^{3/2}}-\frac{(bc-ad)\cos(e+fx)}{f\left(c^2-d^2\right)(c+d\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])/(c + d*Sin[e + f*x])^2,x]
```

```
[Out] (2*(a*c - b*d)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/((c^2 - d^
2)^(3/2)*f) - ((b*c - a*d)*Cos[e + f*x])/((c^2 - d^2)*f*(c + d*Sin[e + f*x]
))
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{a+b\sin(e+fx)}{(c+d\sin(e+fx))^2}\,dx = -\frac{(bc-ad)\cos(e+fx)}{\left(c^2-d^2\right)f(c+d\sin(e+fx))} + \frac{\int \frac{-ac+bd}{c+d\sin(e+fx)}\,dx}{-c^2+d^2}$$

$$= -\frac{(bc-ad)\cos(e+fx)}{\left(c^2-d^2\right)f(c+d\sin(e+fx))} + \frac{(ac-bd)\int \frac{1}{c+d\sin(e+fx)}\,dx}{c^2-d^2}$$

$$= -\frac{(bc-ad)\cos(e+fx)}{\left(c^2-d^2\right)f(c+d\sin(e+fx))} + \frac{(2(ac-bd))\,\mathrm{Subst}\left(\int \frac{1}{c+2dx+cx^2}\,dx, x, \tan\left(\frac{1}{2}(e+fx)\right)\right)}{\left(c^2-d^2\right)f}$$

$$= -\frac{(bc-ad)\cos(e+fx)}{\left(c^2-d^2\right)f(c+d\sin(e+fx))} - \frac{(4(ac-bd))\,\mathrm{Subst}\left(\int \frac{1}{-4\left(c^2-d^2\right)-x^2}\,dx, x, 2d+2c\tan\left(\frac{1}{2}(\right.\right.}{\left(c^2-d^2\right)f}$$

$$= \frac{2(ac-bd)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{\left(c^2-d^2\right)^{3/2}f} - \frac{(bc-ad)\cos(e+fx)}{\left(c^2-d^2\right)f(c+d\sin(e+fx))}$$

Mathematica [A] time = 0.288538, size = 96, normalized size = 0.98

$$\frac{\frac{2(ac-bd)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{\left(c^2-d^2\right)^{3/2}} + \frac{(ad-bc)\cos(e+fx)}{(c-d)(c+d)(c+d\sin(e+fx))}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])/(c + d*Sin[e + f*x])^2,x]
```

```
[Out] ((2*(a*c - b*d)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(c^2 - d^
2)^(3/2) + ((-(b*c) + a*d)*Cos[e + f*x])/((c - d)*(c + d)*(c + d*Sin[e + f*
x])))/f
```

Maple [B] time = 0.073, size = 309, normalized size = 3.2

$$2\,\frac{d^2\tan\left(1/2\,fx+e/2\right)a}{f\left(c\left(\tan\left(1/2\,fx+e/2\right)\right)^2+2\,\tan\left(1/2\,fx+e/2\right)d+c\right)\left(c^2-d^2\right)c} - 2\,\frac{\tan\left(1/2\,fx+e/2\right)db}{f\left(c\left(\tan\left(1/2\,fx+e/2\right)\right)^2+2\,\tan\left(1/2\,fx+e\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))/(c+d*sin(f*x+e))^2,x)
```

```
[Out] 2/f/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)*d^2/(c^2-d^2)/c*tan(1
/2*f*x+1/2*e)*a-2/f/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)*d/(c^
2-d^2)*tan(1/2*f*x+1/2*e)*b+2/f/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e
)*d+c)/(c^2-d^2)*d*a-2/f/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)/
(c^2-d^2)*c*b+2/f/(c^2-d^2)^(3/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(
c^2-d^2)^(1/2))*c*a-2/f/(c^2-d^2)^(3/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+
2*d)/(c^2-d^2)^(1/2))*b*d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.74316, size = 871, normalized size = 8.89

$$\left[-\frac{\left(ac^2 - bcd + \left(acd - bd^2\right)\sin\left(fx+e\right)\right)\sqrt{-c^2+d^2}\,\log\left(\frac{\left(2\,c^2-d^2\right)\cos\left(fx+e\right)^2 - 2\,cd\sin\left(fx+e\right) - c^2 - d^2 + 2\left(c\cos\left(fx+e\right)\sin\left(fx+e\right) + d\cos\left(fx+e\right)\cdots\right.\right.}{d^2\cos\left(fx+e\right)^2 - 2\,cd\sin\left(fx+e\right) - c^2 - d^2}\cdots}{2\left(\left(c^4d - 2\,c^2d^3 + d^5\right)f\sin\left(fx+e\right) + \left(c^5 - 2\,c^3d^2 + cd^4\right)f\right)} \cdots \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] [-1/2*((a*c^2 - b*c*d + (a*c*d - b*d^2)*sin(f*x + e))*sqrt(-c^2 + d^2)*log(
((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 + 2*(c*cos(f
*x + e)*sin(f*x + e) + d*cos(f*x + e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^
2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) + 2*(b*c^3 - a*c^2*d - b*c*d^2 + a*d^3
)*cos(f*x + e))/((c^4*d - 2*c^2*d^3 + d^5)*f*sin(f*x + e) + (c^5 - 2*c^3*d^
2 + c*d^4)*f), -((a*c^2 - b*c*d + (a*c*d - b*d^2)*sin(f*x + e))*sqrt(c^2 -
d^2)*arctan(-(c*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) + (b*c^3
- a*c^2*d - b*c*d^2 + a*d^3)*cos(f*x + e))/((c^4*d - 2*c^2*d^3 + d^5)*f*sin
(f*x + e) + (c^5 - 2*c^3*d^2 + c*d^4)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))/(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.22791, size = 213, normalized size = 2.17

$$\frac{2\left(\frac{\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+d}{\sqrt{c^2-d^2}}\right)\right)(ac-bd)}{\left(c^2-d^2\right)^{\frac{3}{2}}}-\frac{bcd\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-ad^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+bc^2-acd}{\left(c^3-cd^2\right)\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+c\right)}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 2*((pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e
) + d)/sqrt(c^2 - d^2)))*(a*c - b*d)/(c^2 - d^2)^(3/2) - (b*c*d*tan(1/2*f*x
 + 1/2*e) - a*d^2*tan(1/2*f*x + 1/2*e) + b*c^2 - a*c*d)/((c^3 - c*d^2)*(c*t
an(1/2*f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e) + c)))/f
```

3.677 $\int \frac{a+b\sin(e+fx)}{(c+d\sin(e+fx))^3}\,dx$

Optimal. Leaf size=164

$$-\frac{\left(3bcd-a\left(2c^2+d^2\right)\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{f\left(c^2-d^2\right)^{5/2}}+\frac{\left(3acd-b\left(c^2+2d^2\right)\right)\cos(e+fx)}{2f\left(c^2-d^2\right)^2(c+d\sin(e+fx))}-\frac{(bc-ad)\cos(e+fx)}{2f\left(c^2-d^2\right)(c+d\sin(e+fx))^2}$$

```
[Out] -(((3*b*c*d - a*(2*c^2 + d^2))*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d
^2]])/((c^2 - d^2)^(5/2)*f)) - ((b*c - a*d)*Cos[e + f*x])/(2*(c^2 - d^2)*f*
(c + d*Sin[e + f*x])^2) + ((3*a*c*d - b*(c^2 + 2*d^2))*Cos[e + f*x])/(2*(c^
2 - d^2)^2*f*(c + d*Sin[e + f*x]))
```

Rubi [A] time = 0.197544, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2754, 12, 2660, 618, 204}

$$-\frac{\left(3bcd-a\left(2c^2+d^2\right)\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{f\left(c^2-d^2\right)^{5/2}}+\frac{\left(3acd-b\left(c^2+2d^2\right)\right)\cos(e+fx)}{2f\left(c^2-d^2\right)^2(c+d\sin(e+fx))}-\frac{(bc-ad)\cos(e+fx)}{2f\left(c^2-d^2\right)(c+d\sin(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])/(c + d*Sin[e + f*x])^3,x]
```

```
[Out] -(((3*b*c*d - a*(2*c^2 + d^2))*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d
^2]])/((c^2 - d^2)^(5/2)*f)) - ((b*c - a*d)*Cos[e + f*x])/(2*(c^2 - d^2)*f*
(c + d*Sin[e + f*x])^2) + ((3*a*c*d - b*(c^2 + 2*d^2))*Cos[e + f*x])/(2*(c^
2 - d^2)^2*f*(c + d*Sin[e + f*x]))
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{a+b\sin(e+fx)}{(c+d\sin(e+fx))^3}\,dx = -\frac{(bc-ad)\cos(e+fx)}{2(c^2-d^2)f(c+d\sin(e+fx))^2} - \frac{\int \frac{-2(ac-bd)-(bc-ad)\sin(e+fx)}{(c+d\sin(e+fx))^2}\,dx}{2(c^2-d^2)}$$

$$= -\frac{(bc-ad)\cos(e+fx)}{2(c^2-d^2)f(c+d\sin(e+fx))^2} + \frac{(3acd-b(c^2+2d^2))\cos(e+fx)}{2(c^2-d^2)^2 f(c+d\sin(e+fx))} + \frac{\int \frac{-3bcd+a(2c^2+d^2}{c+d\sin(e+fx)}}{2(c^2-d^2)^2}$$

$$= -\frac{(bc-ad)\cos(e+fx)}{2(c^2-d^2)f(c+d\sin(e+fx))^2} + \frac{(3acd-b(c^2+2d^2))\cos(e+fx)}{2(c^2-d^2)^2 f(c+d\sin(e+fx))} - \frac{(3bcd-a(2c^2+}{2(}$$

$$= -\frac{(bc-ad)\cos(e+fx)}{2(c^2-d^2)f(c+d\sin(e+fx))^2} + \frac{(3acd-b(c^2+2d^2))\cos(e+fx)}{2(c^2-d^2)^2 f(c+d\sin(e+fx))} - \frac{(3bcd-a(2c^2+}{}$$

$$= -\frac{(bc-ad)\cos(e+fx)}{2(c^2-d^2)f(c+d\sin(e+fx))^2} + \frac{(3acd-b(c^2+2d^2))\cos(e+fx)}{2(c^2-d^2)^2 f(c+d\sin(e+fx))} + \frac{(2(3bcd-a(2c}{}$$

$$= -\frac{(3bcd-a(2c^2+d^2))\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{(c^2-d^2)^{5/2} f} - \frac{(bc-ad)\cos(e+fx)}{2(c^2-d^2)f(c+d\sin(e+fx))^2} + \frac{(3ac}{2(c}$$

Mathematica [A] time = 0.60345, size = 157, normalized size = 0.96

$$\frac{\frac{2(a(2c^2+d^2)-3bcd)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{(c^2-d^2)^{5/2}} - \frac{(b(c^2+2d^2)-3acd)\cos(e+fx)}{(c-d)^2(c+d)^2(c+d\sin(e+fx))} + \frac{(ad-bc)\cos(e+fx)}{(c-d)(c+d)(c+d\sin(e+fx))^2}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])/(c + d*Sin[e + f*x])^3,x]
```

```
[Out] ((2*(-3*b*c*d + a*(2*c^2 + d^2))*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 -
 d^2]])/(c^2 - d^2)^(5/2) + ((-(b*c) + a*d)*Cos[e + f*x])/((c - d)*(c + d)*
(c + d*Sin[e + f*x])^2) - ((-3*a*c*d + b*(c^2 + 2*d^2))*Cos[e + f*x])/((c -
 d)^2*(c + d)^2*(c + d*Sin[e + f*x]))))/(2*f)
```

Maple [B] time = 0.085, size = 1291, normalized size = 7.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))/(c+d*sin(f*x+e))^3,x)
```

```
[Out] 5/f/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2*d^2*c/(c^4-2*c^2*d^
2+d^4)*tan(1/2*f*x+1/2*e)^3*a-2/f/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2
*e)*d+c)^2*d^4/c/(c^4-2*c^2*d^2+d^4)*tan(1/2*f*x+1/2*e)^3*a-3/f/(c*tan(1/2*
f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2*d*c^2/(c^4-2*c^2*d^2+d^4)*tan(1/2*
f*x+1/2*e)^3*b+4/f/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4
-2*c^2*d^2+d^4)*c^2*tan(1/2*f*x+1/2*e)^2*a*d+7/f/(c*tan(1/2*f*x+1/2*e)^2+2*
tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)*tan(1/2*f*x+1/2*e)^2*a*d^3-2/
f/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)/c
^2*tan(1/2*f*x+1/2*e)^2*a*d^5-2/f/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2
*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)*c^3*tan(1/2*f*x+1/2*e)^2*b-5/f/(c*tan(1/2*f*
x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)*c*tan(1/2*f*x+1/
2*e)^2*b*d^2-2/f/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2
*c^2*d^2+d^4)/c*tan(1/2*f*x+1/2*e)^2*b*d^4+11/f/(c*tan(1/2*f*x+1/2*e)^2+2*t
an(1/2*f*x+1/2*e)*d+c)^2*d^2*c/(c^4-2*c^2*d^2+d^4)*tan(1/2*f*x+1/2*e)*a-2/f
/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2*d^4/c/(c^4-2*c^2*d^2+d
^4)*tan(1/2*f*x+1/2*e)*a-5/f/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d
+c)^2*d*c^2/(c^4-2*c^2*d^2+d^4)*tan(1/2*f*x+1/2*e)*b-4/f/(c*tan(1/2*f*x+1/2
*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2*d^3/(c^4-2*c^2*d^2+d^4)*tan(1/2*f*x+1/2*e
)*b+4/f/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+
d^4)*a*c^2*d-1/f/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2
*c^2*d^2+d^4)*a*d^3-2/f/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2
/(c^4-2*c^2*d^2+d^4)*c^3*b-1/f/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)
*d+c)^2/(c^4-2*c^2*d^2+d^4)*c*d^2*b+2/f/(c^4-2*c^2*d^2+d^4)/(c^2-d^2)^(1/2)
*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*c^2*a+1/f/(c^4-2*
c^2*d^2+d^4)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d
^2)^(1/2))*a*d^2-3/f/(c^4-2*c^2*d^2+d^4)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*ta
n(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*b*c*d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.00247, size = 1724, normalized size = 10.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] [1/4*(2*(b*c^4*d - 3*a*c^3*d^2 + b*c^2*d^3 + 3*a*c*d^4 - 2*b*d^5)*cos(f*x +
 e)*sin(f*x + e) + (2*a*c^4 - 3*b*c^3*d + 3*a*c^2*d^2 - 3*b*c*d^3 + a*d^4 -
 (2*a*c^2*d^2 - 3*b*c*d^3 + a*d^4)*cos(f*x + e)^2 + 2*(2*a*c^3*d - 3*b*c^2*
d^2 + a*c*d^3)*sin(f*x + e))*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(f*x +
e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x + e) + d*
cos(f*x + e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) -
```

```
c^2 - d^2)) + 2*(2*b*c^5 - 4*a*c^4*d - b*c^3*d^2 + 5*a*c^2*d^3 - b*c*d^4 -
a*d^5)*cos(f*x + e))/((c^6*d^2 - 3*c^4*d^4 + 3*c^2*d^6 - d^8)*f*cos(f*x + e
)^2 - 2*(c^7*d - 3*c^5*d^3 + 3*c^3*d^5 - c*d^7)*f*sin(f*x + e) - (c^8 - 2*c
^6*d^2 + 2*c^2*d^6 - d^8)*f), 1/2*((b*c^4*d - 3*a*c^3*d^2 + b*c^2*d^3 + 3*a
*c*d^4 - 2*b*d^5)*cos(f*x + e)*sin(f*x + e) + (2*a*c^4 - 3*b*c^3*d + 3*a*c^
2*d^2 - 3*b*c*d^3 + a*d^4 - (2*a*c^2*d^2 - 3*b*c*d^3 + a*d^4)*cos(f*x + e)^
2 + 2*(2*a*c^3*d - 3*b*c^2*d^2 + a*c*d^3)*sin(f*x + e))*sqrt(c^2 - d^2)*arc
tan(-(c*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) + (2*b*c^5 - 4*a*
c^4*d - b*c^3*d^2 + 5*a*c^2*d^3 - b*c*d^4 - a*d^5)*cos(f*x + e))/((c^6*d^2
- 3*c^4*d^4 + 3*c^2*d^6 - d^8)*f*cos(f*x + e)^2 - 2*(c^7*d - 3*c^5*d^3 + 3*
c^3*d^5 - c*d^7)*f*sin(f*x + e) - (c^8 - 2*c^6*d^2 + 2*c^2*d^6 - d^8)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))/(c+d*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.44484, size = 578, normalized size = 3.52

$$\frac{\frac{\left(2\,ac^{2}-3\,bcd+ad^{2}\right)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+d}{\sqrt{c^{2}-d^{2}}}\right)\right)}{\left(c^{4}-2\,c^{2}d^{2}+d^{4}\right)\sqrt{c^{2}-d^{2}}}-\frac{3\,bc^{4}d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}-5\,ac^{3}d^{2}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+2\,acd^{4}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+2\,bc^{5}\tan\left(\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] ((2*a*c^2 - 3*b*c*d + a*d^2)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arc
tan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/((c^4 - 2*c^2*d^2 + d^4)
*sqrt(c^2 - d^2)) - (3*b*c^4*d*tan(1/2*f*x + 1/2*e)^3 - 5*a*c^3*d^2*tan(1/2
*f*x + 1/2*e)^3 + 2*a*c*d^4*tan(1/2*f*x + 1/2*e)^3 + 2*b*c^5*tan(1/2*f*x +
1/2*e)^2 - 4*a*c^4*d*tan(1/2*f*x + 1/2*e)^2 + 5*b*c^3*d^2*tan(1/2*f*x + 1/2
*e)^2 - 7*a*c^2*d^3*tan(1/2*f*x + 1/2*e)^2 + 2*b*c*d^4*tan(1/2*f*x + 1/2*e)
^2 + 2*a*d^5*tan(1/2*f*x + 1/2*e)^2 + 5*b*c^4*d*tan(1/2*f*x + 1/2*e) - 11*a
*c^3*d^2*tan(1/2*f*x + 1/2*e) + 4*b*c^2*d^3*tan(1/2*f*x + 1/2*e) + 2*a*c*d^
4*tan(1/2*f*x + 1/2*e) + 2*b*c^5 - 4*a*c^4*d + b*c^3*d^2 + a*c^2*d^3)/((c^6
 - 2*c^4*d^2 + c^2*d^4)*(c*tan(1/2*f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e
) + c)^2))/f
```

3.678 $\int (a+b\sin(e+fx))^2(c+d\sin(e+fx))^3\,dx$

Optimal. Leaf size=314

$$-\frac{\left(20a^2d^2\left(4c^2+d^2\right)+30abcd\left(c^2+4d^2\right)+b^2\left(-\left(-52c^2d^2+3c^4-16d^4\right)\right)\right)\cos(e+fx)}{30df}-\frac{\left(100a^2cd^2+30abd\left(2c^2+3d^2\right)\right.}{}$$

```
[Out] ((6*a*b*d*(4*c^2 + d^2) + b^2*c*(4*c^2 + 9*d^2) + 4*a^2*(2*c^3 + 3*c*d^2))*
x)/8 - ((20*a^2*d^2*(4*c^2 + d^2) + 30*a*b*c*d*(c^2 + 4*d^2) - b^2*(3*c^4 -
 52*c^2*d^2 - 16*d^4))*Cos[e + f*x])/(30*d*f) - ((100*a^2*c*d^2 + 30*a*b*d*
(2*c^2 + 3*d^2) - b^2*(6*c^3 - 71*c*d^2))*Cos[e + f*x]*Sin[e + f*x])/(120*f
) - ((4*(5*a^2 + 4*b^2)*d^2 - 3*b*c*(b*c - 10*a*d))*Cos[e + f*x]*(c + d*Sin
[e + f*x])^2)/(60*d*f) + (b*(b*c - 10*a*d)*Cos[e + f*x]*(c + d*Sin[e + f*x]
)^3)/(20*d*f) - (b^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^4)/(5*d*f)
```

Rubi [A] time = 0.546176, antiderivative size = 314, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2791, 2753, 2734}

$$-\frac{\left(20a^2d^2\left(4c^2+d^2\right)+30abcd\left(c^2+4d^2\right)+b^2\left(-\left(-52c^2d^2+3c^4-16d^4\right)\right)\right)\cos(e+fx)}{30df}-\frac{\left(100a^2cd^2+30abd\left(2c^2+3d^2\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^3,x]
```


```
[Out] ((6*a*b*d*(4*c^2 + d^2) + b^2*c*(4*c^2 + 9*d^2) + 4*a^2*(2*c^3 + 3*c*d^2))*
x)/8 - ((20*a^2*d^2*(4*c^2 + d^2) + 30*a*b*c*d*(c^2 + 4*d^2) - b^2*(3*c^4 -
 52*c^2*d^2 - 16*d^4))*Cos[e + f*x])/(30*d*f) - ((100*a^2*c*d^2 + 30*a*b*d*
(2*c^2 + 3*d^2) - b^2*(6*c^3 - 71*c*d^2))*Cos[e + f*x]*Sin[e + f*x])/(120*f
) - ((4*(5*a^2 + 4*b^2)*d^2 - 3*b*c*(b*c - 10*a*d))*Cos[e + f*x]*(c + d*Sin
[e + f*x])^2)/(60*d*f) + (b*(b*c - 10*a*d)*Cos[e + f*x]*(c + d*Sin[e + f*x]
)^3)/(20*d*f) - (b^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^4)/(5*d*f)
```


Rule 2791

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> -Simp[(d^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*
Simp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x],
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a
^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
```

```
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int (a+b\sin(e+fx))^2(c+d\sin(e+fx))^3\,dx = -\frac{b^2\cos(e+fx)(c+d\sin(e+fx))^4}{5df} + \frac{\int (c+d\sin(e+fx))^3\left(\left(5a^2+4b\right.\right.}{}$$

$$= \frac{b(bc-10ad)\cos(e+fx)(c+d\sin(e+fx))^3}{20df} - \frac{b^2\cos(e+fx)(c+d\sin}{5df}$$

$$= -\frac{\left(4\left(5a^2+4b^2\right)d^2-3bc(bc-10ad)\right)\cos(e+fx)(c+d\sin(e+fx))^2}{60df} +$$

$$= \frac{1}{8}\left(6abd\left(4c^2+d^2\right)+b^2c\left(4c^2+9d^2\right)+4a^2\left(2c^3+3cd^2\right)\right)x - \frac{\left(20a^2d^2\right.}{}$$

Mathematica [A] time = 1.37333, size = 249, normalized size = 0.79

$$\frac{15\left(4(e+fx)\left(4a^2\left(2c^3+3cd^2\right)+6abd\left(4c^2+d^2\right)+b^2c\left(4c^2+9d^2\right)\right)-8\left(3a^2cd^2+2abd\left(3c^2+d^2\right)+b^2\left(c^3+3cd^2\right)\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^3,x]
```

```
[Out] (-60*(b^2*d*(18*c^2 + 5*d^2) + 4*a*b*c*(4*c^2 + 9*d^2) + 6*a^2*(4*c^2*d + d
^3))*Cos[e + f*x] + 10*d*(24*a*b*c*d + 4*a^2*d^2 + b^2*(12*c^2 + 5*d^2))*Co
s[3*(e + f*x)] - 6*b^2*d^3*Cos[5*(e + f*x)] + 15*(4*(6*a*b*d*(4*c^2 + d^2)
+ b^2*c*(4*c^2 + 9*d^2) + 4*a^2*(2*c^3 + 3*c*d^2))*(e + f*x) - 8*(3*a^2*c*d
^2 + 2*a*b*d*(3*c^2 + d^2) + b^2*(c^3 + 3*c*d^2))*Sin[2*(e + f*x)] + b*d^2*
(3*b*c + 2*a*d)*Sin[4*(e + f*x)]))/(480*f)
```

Maple [A] time = 0.036, size = 325, normalized size = 1.

$$\frac{1}{f}\left(a^2c^3\left(fx+e\right)-3\,a^2c^2d\cos\left(fx+e\right)+3\,a^2cd^2\left(-1/2\,\sin\left(fx+e\right)\cos\left(fx+e\right)+1/2\,fx+e/2\right)-\frac{a^2d^3\left(2+\left(\sin\left(fx\right.\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^2*(c+d*sin(f*x+e))^3,x)
```

```
[Out] 1/f*(a^2*c^3*(f*x+e)-3*a^2*c^2*d*cos(f*x+e)+3*a^2*c*d^2*(-1/2*sin(f*x+e)*co
s(f*x+e)+1/2*f*x+1/2*e)-1/3*a^2*d^3*(2+sin(f*x+e)^2)*cos(f*x+e)-2*a*b*c^3*c
os(f*x+e)+6*a*b*c^2*d*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-2*a*b*c*d^
2*(2+sin(f*x+e)^2)*cos(f*x+e)+2*a*b*d^3*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))
*cos(f*x+e)+3/8*f*x+3/8*e)+b^2*c^3*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*
e)-b^2*c^2*d*(2+sin(f*x+e)^2)*cos(f*x+e)+3*b^2*c*d^2*(-1/4*(sin(f*x+e)^3+3/
2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-1/5*b^2*d^3*(8/3+sin(f*x+e)^4+4/3*s
in(f*x+e)^2)*cos(f*x+e))
```

Maxima [A] time = 1.33854, size = 424, normalized size = 1.35

$$\frac{480\left(fx+e\right)a^2c^3+120\left(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right)\right)b^2c^3+720\left(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right)\right)abc^2d+480\left(\cos\left(fx+e\right)^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2*(c+d*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 1/480*(480*(f*x + e)*a^2*c^3 + 120*(2*f*x + 2*e - sin(2*f*x + 2*e))*b^2*c^3
 + 720*(2*f*x + 2*e - sin(2*f*x + 2*e))*a*b*c^2*d + 480*(cos(f*x + e)^3 - 3
*cos(f*x + e))*b^2*c^2*d + 360*(2*f*x + 2*e - sin(2*f*x + 2*e))*a^2*c*d^2 +
 960*(cos(f*x + e)^3 - 3*cos(f*x + e))*a*b*c*d^2 + 45*(12*f*x + 12*e + sin(
4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*b^2*c*d^2 + 160*(cos(f*x + e)^3 - 3*cos(
f*x + e))*a^2*d^3 + 30*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*
e))*a*b*d^3 - 32*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*b
^2*d^3 - 960*a*b*c^3*cos(f*x + e) - 1440*a^2*c^2*d*cos(f*x + e))/f
```

Fricas [A] time = 1.74943, size = 566, normalized size = 1.8

$$-\frac{24\,b^2d^3\cos\left(fx+e\right)^5-40\left(3\,b^2c^2d+6\,abcd^2+\left(a^2+2\,b^2\right)d^3\right)\cos\left(fx+e\right)^3-15\left(24\,abc^2d+6\,abd^3+4\left(2\,a^2+b^2\right)c^3+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2*(c+d*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -1/120*(24*b^2*d^3*cos(f*x + e)^5 - 40*(3*b^2*c^2*d + 6*a*b*c*d^2 + (a^2 +
2*b^2)*d^3)*cos(f*x + e)^3 - 15*(24*a*b*c^2*d + 6*a*b*d^3 + 4*(2*a^2 + b^2)
*c^3 + 3*(4*a^2 + 3*b^2)*c*d^2)*f*x + 120*(2*a*b*c^3 + 6*a*b*c*d^2 + 3*(a^2
 + b^2)*c^2*d + (a^2 + b^2)*d^3)*cos(f*x + e) - 15*(2*(3*b^2*c*d^2 + 2*a*b*
d^3)*cos(f*x + e)^3 - (4*b^2*c^3 + 24*a*b*c^2*d + 10*a*b*d^3 + 3*(4*a^2 + 5
*b^2)*c*d^2)*cos(f*x + e))*sin(f*x + e))/f
```

Sympy [A] time = 4.18739, size = 729, normalized size = 2.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**2*(c+d*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((a**2*c**3*x - 3*a**2*c**2*d*cos(e + f*x)/f + 3*a**2*c*d**2*x*sin
(e + f*x)**2/2 + 3*a**2*c*d**2*x*cos(e + f*x)**2/2 - 3*a**2*c*d**2*sin(e +
f*x)*cos(e + f*x)/(2*f) - a**2*d**3*sin(e + f*x)**2*cos(e + f*x)/f - 2*a**2
*d**3*cos(e + f*x)**3/(3*f) - 2*a*b*c**3*cos(e + f*x)/f + 3*a*b*c**2*d*x*si
n(e + f*x)**2 + 3*a*b*c**2*d*x*cos(e + f*x)**2 - 3*a*b*c**2*d*sin(e + f*x)*
cos(e + f*x)/f - 6*a*b*c*d**2*sin(e + f*x)**2*cos(e + f*x)/f - 4*a*b*c*d**2
*cos(e + f*x)**3/f + 3*a*b*d**3*x*sin(e + f*x)**4/4 + 3*a*b*d**3*x*sin(e +
f*x)**2*cos(e + f*x)**2/2 + 3*a*b*d**3*x*cos(e + f*x)**4/4 - 5*a*b*d**3*sin
(e + f*x)**3*cos(e + f*x)/(4*f) - 3*a*b*d**3*sin(e + f*x)*cos(e + f*x)**3/(
4*f) + b**2*c**3*x*sin(e + f*x)**2/2 + b**2*c**3*x*cos(e + f*x)**2/2 - b**2
*c**3*sin(e + f*x)*cos(e + f*x)/(2*f) - 3*b**2*c**2*d*sin(e + f*x)**2*cos(e
```

```
 + f*x)/f - 2*b**2*c**2*d*cos(e + f*x)**3/f + 9*b**2*c*d**2*x*sin(e + f*x)*
*4/8 + 9*b**2*c*d**2*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 9*b**2*c*d**2*x*
cos(e + f*x)**4/8 - 15*b**2*c*d**2*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 9*b
**2*c*d**2*sin(e + f*x)*cos(e + f*x)**3/(8*f) - b**2*d**3*sin(e + f*x)**4*c
os(e + f*x)/f - 4*b**2*d**3*sin(e + f*x)**2*cos(e + f*x)**3/(3*f) - 8*b**2*
d**3*cos(e + f*x)**5/(15*f), Ne(f, 0)), (x*(a + b*sin(e))**2*(c + d*sin(e))
**3, True))
```

Giac [A] time = 1.4246, size = 370, normalized size = 1.18

$$-\frac{b^2d^3\cos\left(5\,fx+5\,e\right)}{80\,f}+\frac{1}{8}\left(8\,a^2c^3+4\,b^2c^3+24\,abc^2d+12\,a^2cd^2+9\,b^2cd^2+6\,abd^3\right)x+\frac{\left(12\,b^2c^2d+24\,abcd^2+4\,a^2d\right.}{48}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2*(c+d*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -1/80*b^2*d^3*cos(5*f*x + 5*e)/f + 1/8*(8*a^2*c^3 + 4*b^2*c^3 + 24*a*b*c^2*
d + 12*a^2*c*d^2 + 9*b^2*c*d^2 + 6*a*b*d^3)*x + 1/48*(12*b^2*c^2*d + 24*a*b
*c*d^2 + 4*a^2*d^3 + 5*b^2*d^3)*cos(3*f*x + 3*e)/f - 1/8*(16*a*b*c^3 + 24*a
^2*c^2*d + 18*b^2*c^2*d + 36*a*b*c*d^2 + 6*a^2*d^3 + 5*b^2*d^3)*cos(f*x + e
)/f + 1/32*(3*b^2*c*d^2 + 2*a*b*d^3)*sin(4*f*x + 4*e)/f - 1/4*(b^2*c^3 + 6*
a*b*c^2*d + 3*a^2*c*d^2 + 3*b^2*c*d^2 + 2*a*b*d^3)*sin(2*f*x + 2*e)/f
```

3.679 $\int (a+b\sin(e+fx))^2(c+d\sin(e+fx))^2\,dx$

Optimal. Leaf size=217

$$-\frac{\left(8a^2bcd+a^3\left(-d^2\right)+4ab^2\left(3c^2+2d^2\right)+8b^3cd\right)\cos(e+fx)}{6bf}+\frac{1}{8}x\left(4a^2\left(2c^2+d^2\right)+16abcd+b^2\left(4c^2+3d^2\right)\right)-\frac{(2ad(8b$$

```
[Out] ((16*a*b*c*d + 4*a^2*(2*c^2 + d^2) + b^2*(4*c^2 + 3*d^2))*x)/8 - ((8*a^2*b*
c*d + 8*b^3*c*d - a^3*d^2 + 4*a*b^2*(3*c^2 + 2*d^2))*Cos[e + f*x])/(6*b*f)
- ((2*a*d*(8*b*c - a*d) + 3*b^2*(4*c^2 + 3*d^2))*Cos[e + f*x]*Sin[e + f*x])
/(24*f) - (d*(8*b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^2)/(12*b*f) -
(d^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^3)/(4*b*f)
```

Rubi [A] time = 0.281144, antiderivative size = 217, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2791, 2753, 2734}

$$-\frac{\left(8a^2bcd+a^3\left(-d^2\right)+4ab^2\left(3c^2+2d^2\right)+8b^3cd\right)\cos(e+fx)}{6bf}+\frac{1}{8}x\left(4a^2\left(2c^2+d^2\right)+16abcd+b^2\left(4c^2+3d^2\right)\right)-\frac{(2ad(8b$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^2,x]
```


```
[Out] ((16*a*b*c*d + 4*a^2*(2*c^2 + d^2) + b^2*(4*c^2 + 3*d^2))*x)/8 - ((8*a^2*b*
c*d + 8*b^3*c*d - a^3*d^2 + 4*a*b^2*(3*c^2 + 2*d^2))*Cos[e + f*x])/(6*b*f)
- ((2*a*d*(8*b*c - a*d) + 3*b^2*(4*c^2 + 3*d^2))*Cos[e + f*x]*Sin[e + f*x])
/(24*f) - (d*(8*b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^2)/(12*b*f) -
(d^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^3)/(4*b*f)
```


Rule 2791

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> -Simp[(d^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*
Simp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x],
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a
^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int (a+b\sin(e+fx))^2(c+d\sin(e+fx))^2\,dx = -\frac{d^2\cos(e+fx)(a+b\sin(e+fx))^3}{4bf} + \frac{\int (a+b\sin(e+fx))^2\left(b\left(4c^2+\right.\right.}{}$$
$$= -\frac{d(8bc-ad)\cos(e+fx)(a+b\sin(e+fx))^2}{12bf} - \frac{d^2\cos(e+fx)(a+b\sin}{4bf}$$
$$= \frac{1}{8}\left(16abcd+4a^2\left(2c^2+d^2\right)+b^2\left(4c^2+3d^2\right)\right)x - \frac{\left(8a^2bcd+8b^3cd-a^3\right.}{}$$

Mathematica [A] time = 0.762033, size = 160, normalized size = 0.74

$$\frac{3\left(4(e+fx)\left(4a^2\left(2c^2+d^2\right)+16abcd+b^2\left(4c^2+3d^2\right)\right)-8\left(a^2d^2+4abcd+b^2\left(c^2+d^2\right)\right)\sin(2(e+fx))+b^2d^2\sin(4(\right.}{96f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^2,x]
```

```
[Out] (-48*(4*a^2*c*d + 3*b^2*c*d + a*b*(4*c^2 + 3*d^2))*Cos[e + f*x] + 16*b*d*(b
*c + a*d)*Cos[3*(e + f*x)] + 3*(4*(16*a*b*c*d + 4*a^2*(2*c^2 + d^2) + b^2*(
4*c^2 + 3*d^2))*(e + f*x) - 8*(4*a*b*c*d + a^2*d^2 + b^2*(c^2 + d^2))*Sin[2
*(e + f*x)] + b^2*d^2*Sin[4*(e + f*x)]))/(96*f)
```

Maple [A] time = 0.03, size = 216, normalized size = 1.

$$\frac{1}{f}\left(a^2c^2\left(fx+e\right)-2\,a^2cd\cos\left(fx+e\right)+a^2d^2\left(-\frac{\sin\left(fx+e\right)\cos\left(fx+e\right)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-2\,abc^2\cos\left(fx+e\right)+4\,abcd\,(-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^2*(c+d*sin(f*x+e))^2,x)
```

```
[Out] 1/f*(a^2*c^2*(f*x+e)-2*a^2*c*d*cos(f*x+e)+a^2*d^2*(-1/2*sin(f*x+e)*cos(f*x+
e)+1/2*f*x+1/2*e)-2*a*b*c^2*cos(f*x+e)+4*a*b*c*d*(-1/2*sin(f*x+e)*cos(f*x+e
)+1/2*f*x+1/2*e)-2/3*a*b*d^2*(2+sin(f*x+e)^2)*cos(f*x+e)+c^2*b^2*(-1/2*sin(
f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-2/3*b^2*c*d*(2+sin(f*x+e)^2)*cos(f*x+e)+b^
2*d^2*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e))
```

Maxima [A] time = 1.17893, size = 281, normalized size = 1.29

$$\frac{96\left(fx+e\right)a^2c^2+24\left(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right)\right)b^2c^2+96\left(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right)\right)abcd+64\left(\cos\left(fx+e\right)^3-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2*(c+d*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/96*(96*(f*x + e)*a^2*c^2 + 24*(2*f*x + 2*e - sin(2*f*x + 2*e))*b^2*c^2 +
96*(2*f*x + 2*e - sin(2*f*x + 2*e))*a*b*c*d + 64*(cos(f*x + e)^3 - 3*cos(f*
```

```
x + e))*b^2*c*d + 24*(2*f*x + 2*e - sin(2*f*x + 2*e))*a^2*d^2 + 64*(cos(f*x
 + e)^3 - 3*cos(f*x + e))*a*b*d^2 + 3*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8
*sin(2*f*x + 2*e))*b^2*d^2 - 192*a*b*c^2*cos(f*x + e) - 192*a^2*c*d*cos(f*x
 + e))/f
```

Fricas [A] time = 1.70664, size = 371, normalized size = 1.71

$$\frac{16\left(b^2cd+abd^2\right)\cos\left(fx+e\right)^3+3\left(16\,abcd+4\left(2\,a^2+b^2\right)c^2+\left(4\,a^2+3\,b^2\right)d^2\right)fx-48\left(abc^2+abd^2+\left(a^2+b^2\right)cd\right)\cos\left(\right.}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2*(c+d*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/24*(16*(b^2*c*d + a*b*d^2)*cos(f*x + e)^3 + 3*(16*a*b*c*d + 4*(2*a^2 + b^
2)*c^2 + (4*a^2 + 3*b^2)*d^2)*f*x - 48*(a*b*c^2 + a*b*d^2 + (a^2 + b^2)*c*d
)*cos(f*x + e) + 3*(2*b^2*d^2*cos(f*x + e)^3 - (4*b^2*c^2 + 16*a*b*c*d + (4
*a^2 + 5*b^2)*d^2)*cos(f*x + e))*sin(f*x + e))/f
```

Sympy [A] time = 1.80745, size = 459, normalized size = 2.12

$$\begin{cases} a^2c^2x-\frac{2a^2cd\cos\left(e+fx\right)}{f}+\frac{a^2d^2x\sin^2\left(e+fx\right)}{2}+\frac{a^2d^2x\cos^2\left(e+fx\right)}{2}-\frac{a^2d^2\sin\left(e+fx\right)\cos\left(e+fx\right)}{2f}-\frac{2abc^2\cos\left(e+fx\right)}{f}+2abcdx\sin^2\left(e+fx\right) \\ x\left(a+b\sin\left(e\right)\right)^2\left(c+d\sin\left(e\right)\right)^2 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**2*(c+d*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((a**2*c**2*x - 2*a**2*c*d*cos(e + f*x)/f + a**2*d**2*x*sin(e + f*
x)**2/2 + a**2*d**2*x*cos(e + f*x)**2/2 - a**2*d**2*sin(e + f*x)*cos(e + f*
x)/(2*f) - 2*a*b*c**2*cos(e + f*x)/f + 2*a*b*c*d*x*sin(e + f*x)**2 + 2*a*b*
c*d*x*cos(e + f*x)**2 - 2*a*b*c*d*sin(e + f*x)*cos(e + f*x)/f - 2*a*b*d**2*
sin(e + f*x)**2*cos(e + f*x)/f - 4*a*b*d**2*cos(e + f*x)**3/(3*f) + b**2*c*
*2*x*sin(e + f*x)**2/2 + b**2*c**2*x*cos(e + f*x)**2/2 - b**2*c**2*sin(e +
f*x)*cos(e + f*x)/(2*f) - 2*b**2*c*d*sin(e + f*x)**2*cos(e + f*x)/f - 4*b**
2*c*d*cos(e + f*x)**3/(3*f) + 3*b**2*d**2*x*sin(e + f*x)**4/8 + 3*b**2*d**2
*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 3*b**2*d**2*x*cos(e + f*x)**4/8 - 5*
b**2*d**2*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 3*b**2*d**2*sin(e + f*x)*cos
(e + f*x)**3/(8*f), Ne(f, 0)), (x*(a + b*sin(e))**2*(c + d*sin(e))**2, True
))
```

Giac [A] time = 1.23986, size = 238, normalized size = 1.1

$$\frac{b^2d^2\sin\left(4\,fx+4\,e\right)}{32\,f}+\frac{1}{8}\left(8\,a^2c^2+4\,b^2c^2+16\,abcd+4\,a^2d^2+3\,b^2d^2\right)x+\frac{\left(b^2cd+abd^2\right)\cos\left(3\,fx+3\,e\right)}{6\,f}-\frac{\left(4\,abc^2+4\,a^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2*(c+d*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/32*b^2*d^2*sin(4*f*x + 4*e)/f + 1/8*(8*a^2*c^2 + 4*b^2*c^2 + 16*a*b*c*d +
 4*a^2*d^2 + 3*b^2*d^2)*x + 1/6*(b^2*c*d + a*b*d^2)*cos(3*f*x + 3*e)/f - 1/
2*(4*a*b*c^2 + 4*a^2*c*d + 3*b^2*c*d + 3*a*b*d^2)*cos(f*x + e)/f - 1/4*(b^2
*c^2 + 4*a*b*c*d + a^2*d^2 + b^2*d^2)*sin(2*f*x + 2*e)/f
```

3.680 $\int (a+b\sin(e+fx))^2(c+d\sin(e+fx))\,dx$

Optimal. Leaf size=107

$$-\frac{2\left(a^2d+3abc+b^2d\right)\cos(e+fx)}{3f}+\frac{1}{2}x\left(2a^2c+2abd+b^2c\right)-\frac{b(2ad+3bc)\sin(e+fx)\cos(e+fx)}{6f}-\frac{d\cos(e+fx)(a+}{3f}$$

```
[Out] ((2*a^2*c + b^2*c + 2*a*b*d)*x)/2 - (2*(3*a*b*c + a^2*d + b^2*d)*Cos[e + f*
x])/(3*f) - (b*(3*b*c + 2*a*d)*Cos[e + f*x]*Sin[e + f*x])/(6*f) - (d*Cos[e
+ f*x]*(a + b*Sin[e + f*x])^2)/(3*f)
```

Rubi [A] time = 0.0936561, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2753, 2734}

$$-\frac{2\left(a^2d+3abc+b^2d\right)\cos(e+fx)}{3f}+\frac{1}{2}x\left(2a^2c+2abd+b^2c\right)-\frac{b(2ad+3bc)\sin(e+fx)\cos(e+fx)}{6f}-\frac{d\cos(e+fx)(a+}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^2*(c + d*Sin[e + f*x]),x]
```

```
[Out] ((2*a^2*c + b^2*c + 2*a*b*d)*x)/2 - (2*(3*a*b*c + a^2*d + b^2*d)*Cos[e + f*
x])/(3*f) - (b*(3*b*c + 2*a*d)*Cos[e + f*x]*Sin[e + f*x])/(6*f) - (d*Cos[e
+ f*x]*(a + b*Sin[e + f*x])^2)/(3*f)
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int (a+b\sin(e+fx))^2(c+d\sin(e+fx))\,dx=-\frac{d\cos(e+fx)(a+b\sin(e+fx))^2}{3f}+\frac{1}{3}\int (a+b\sin(e+fx))(3ac+2bd+$$

$$=\frac{1}{2}\left(2a^2c+b^2c+2abd\right)x-\frac{2\left(3abc+a^2d+b^2d\right)\cos(e+fx)}{3f}-\frac{b(3bc+2ad}{}$$

Mathematica [A] time = 0.294564, size = 90, normalized size = 0.84

$$\frac{6(e+fx)\left(2a^2c+2abd+b^2c\right)-3\left(4a^2d+8abc+3b^2d\right)\cos(e+fx)-3b(2ad+bc)\sin(2(e+fx))+b^2d\cos(3(e+fx))}{12f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^2*(c + d*Sin[e + f*x]),x]
```

```
[Out] (6*(2*a^2*c + b^2*c + 2*a*b*d)*(e + f*x) - 3*(8*a*b*c + 4*a^2*d + 3*b^2*d)*
Cos[e + f*x] + b^2*d*Cos[3*(e + f*x)] - 3*b*(b*c + 2*a*d)*Sin[2*(e + f*x)])
/(12*f)
```

Maple [A] time = 0.026, size = 115, normalized size = 1.1

$$\frac{1}{f}\left(a^{2}c\left(fx+e\right)-a^{2}d\cos\left(fx+e\right)-2\,abc\cos\left(fx+e\right)+2\,abd\left(-1/2\,\sin\left(fx+e\right)\cos\left(fx+e\right)+1/2\,fx+e/2\right)+b^{2}c\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^2*(c+d*sin(f*x+e)),x)
```

```
[Out] 1/f*(a^2*c*(f*x+e)-a^2*d*cos(f*x+e)-2*a*b*c*cos(f*x+e)+2*a*b*d*(-1/2*sin(f*
x+e)*cos(f*x+e)+1/2*f*x+1/2*e)+b^2*c*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/
2*e)-1/3*b^2*d*(2+sin(f*x+e)^2)*cos(f*x+e))
```

Maxima [A] time = 1.12328, size = 151, normalized size = 1.41

$$\frac{12\left(fx+e\right)a^{2}c+3\left(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right)\right)b^{2}c+6\left(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right)\right)abd+4\left(\cos\left(fx+e\right)^{3}-3\,\cos\left(\right.\right.}{12\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2*(c+d*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/12*(12*(f*x + e)*a^2*c + 3*(2*f*x + 2*e - sin(2*f*x + 2*e))*b^2*c + 6*(2*
f*x + 2*e - sin(2*f*x + 2*e))*a*b*d + 4*(cos(f*x + e)^3 - 3*cos(f*x + e))*b
^2*d - 24*a*b*c*cos(f*x + e) - 12*a^2*d*cos(f*x + e))/f
```

Fricas [A] time = 1.59181, size = 215, normalized size = 2.01

$$\frac{2\,b^{2}d\cos\left(fx+e\right)^{3}+3\left(2\,abd+\left(2\,a^{2}+b^{2}\right)c\right)fx-3\left(b^{2}c+2\,abd\right)\cos\left(fx+e\right)\sin\left(fx+e\right)-6\left(2\,abc+\left(a^{2}+b^{2}\right)d\right)\mathrm{co}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2*(c+d*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^2*d*cos(f*x + e)^3 + 3*(2*a*b*d + (2*a^2 + b^2)*c)*f*x - 3*(b^2*c
+ 2*a*b*d)*cos(f*x + e)*sin(f*x + e) - 6*(2*a*b*c + (a^2 + b^2)*d)*cos(f*x
+ e))/f
```

Sympy [A] time = 0.786323, size = 199, normalized size = 1.86

$$\begin{cases} a^2cx - \frac{a^2d\cos(e+fx)}{f} - \frac{2abc\cos(e+fx)}{f} + abdx\sin^2(e+fx) + abdx\cos^2(e+fx) - \frac{abd\sin(e+fx)\cos(e+fx)}{f} + \frac{b^2cx\sin^2(e+fx)}{2} + \frac{b^2 \ldots}{\ldots} \\ x(a+b\sin(e))^2(c+d\sin(e)) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**2*(c+d*sin(f*x+e)),x)
```

```
[Out] Piecewise((a**2*c*x - a**2*d*cos(e + f*x)/f - 2*a*b*c*cos(e + f*x)/f + a*b*
d*x*sin(e + f*x)**2 + a*b*d*x*cos(e + f*x)**2 - a*b*d*sin(e + f*x)*cos(e +
f*x)/f + b**2*c*x*sin(e + f*x)**2/2 + b**2*c*x*cos(e + f*x)**2/2 - b**2*c*s
in(e + f*x)*cos(e + f*x)/(2*f) - b**2*d*sin(e + f*x)**2*cos(e + f*x)/f - 2*
b**2*d*cos(e + f*x)**3/(3*f), Ne(f, 0)), (x*(a + b*sin(e))**2*(c + d*sin(e)
), True))
```

Giac [A] time = 1.33933, size = 130, normalized size = 1.21

$$\frac{b^2d\cos(3fx+3e)}{12f} + \frac{1}{2}(2a^2c + b^2c + 2abd)x - \frac{(8abc + 4a^2d + 3b^2d)\cos(fx+e)}{4f} - \frac{(b^2c + 2abd)\sin(2fx+2e)}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2*(c+d*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/12*b^2*d*cos(3*f*x + 3*e)/f + 1/2*(2*a^2*c + b^2*c + 2*a*b*d)*x - 1/4*(8*
a*b*c + 4*a^2*d + 3*b^2*d)*cos(f*x + e)/f - 1/4*(b^2*c + 2*a*b*d)*sin(2*f*x
 + 2*e)/f
```

3.681 $\int (a+b\sin(e+fx))^2\,dx$

Optimal. Leaf size=50

$$\frac{1}{2}x\left(2a^2+b^2\right)-\frac{2ab\cos(e+fx)}{f}-\frac{b^2\sin(e+fx)\cos(e+fx)}{2f}$$

```
[Out] ((2*a^2 + b^2)*x)/2 - (2*a*b*Cos[e + f*x])/f - (b^2*Cos[e + f*x]*Sin[e + f*
x])/(2*f)
```

Rubi [A] time = 0.0161034, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2644}

$$\frac{1}{2}x\left(2a^2+b^2\right)-\frac{2ab\cos(e+fx)}{f}-\frac{b^2\sin(e+fx)\cos(e+fx)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^2,x]
```

```
[Out] ((2*a^2 + b^2)*x)/2 - (2*a*b*Cos[e + f*x])/f - (b^2*Cos[e + f*x]*Sin[e + f*
x])/(2*f)
```

Rule 2644

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[((2*a^2 + b
^2)*x)/2, x] + (-Simp[(2*a*b*Cos[c + d*x])/d, x] - Simp[(b^2*Cos[c + d*x]*S
in[c + d*x])/(2*d), x]) /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$\int (a+b\sin(e+fx))^2\,dx=\frac{1}{2}\left(2a^2+b^2\right)x-\frac{2ab\cos(e+fx)}{f}-\frac{b^2\cos(e+fx)\sin(e+fx)}{2f}$$

Mathematica [A] time = 0.0970928, size = 46, normalized size = 0.92

$$-\frac{-2\left(2a^2+b^2\right)(e+fx)+8ab\cos(e+fx)+b^2\sin(2(e+fx))}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^2,x]
```

```
[Out] -(-2*(2*a^2 + b^2)*(e + f*x) + 8*a*b*Cos[e + f*x] + b^2*Sin[2*(e + f*x)])/(
4*f)
```

Maple [A] time = 0.018, size = 51, normalized size = 1.

$$\frac{1}{f}\left(b^2\left(-\frac{\sin\left(fx+e\right)\cos\left(fx+e\right)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-2\,ab\cos\left(fx+e\right)+a^2\left(fx+e\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^2,x)
```

```
[Out] 1/f*(b^2*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-2*a*b*cos(f*x+e)+a^2*(f
*x+e))
```

Maxima [A] time = 1.01385, size = 62, normalized size = 1.24

$$a^2x + \frac{\left(2\,fx + 2\,e - \sin\left(2\,fx + 2\,e\right)\right)b^2}{4\,f} - \frac{2\,ab\cos\left(fx + e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] a^2*x + 1/4*(2*f*x + 2*e - sin(2*f*x + 2*e))*b^2/f - 2*a*b*cos(f*x + e)/f
```

Fricas [A] time = 1.53957, size = 109, normalized size = 2.18

$$-\frac{b^2\cos\left(fx + e\right)\sin\left(fx + e\right) - \left(2\,a^2 + b^2\right)fx + 4\,ab\cos\left(fx + e\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(b^2*cos(f*x + e)*sin(f*x + e) - (2*a^2 + b^2)*f*x + 4*a*b*cos(f*x + e
))/f
```

Sympy [A] time = 0.308759, size = 78, normalized size = 1.56

$$\begin{cases} a^2x - \frac{2ab\cos\left(e+fx\right)}{f} + \frac{b^2x\sin^2\left(e+fx\right)}{2} + \frac{b^2x\cos^2\left(e+fx\right)}{2} - \frac{b^2\sin\left(e+fx\right)\cos\left(e+fx\right)}{2f} & \text{for } f \neq 0 \\ x\left(a + b\sin\left(e\right)\right)^2 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((a**2*x - 2*a*b*cos(e + f*x)/f + b**2*x*sin(e + f*x)**2/2 + b**2*
x*cos(e + f*x)**2/2 - b**2*sin(e + f*x)*cos(e + f*x)/(2*f), Ne(f, 0)), (x*(
a + b*sin(e))**2, True))
```

Giac [A] time = 1.24285, size = 61, normalized size = 1.22

$$\frac{1}{2}\left(2\,a^2 + b^2\right)x - \frac{2\,ab\cos\left(fx + e\right)}{f} - \frac{b^2\sin\left(2\,fx + 2\,e\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*a^2 + b^2)*x - 2*a*b*cos(f*x + e)/f - 1/4*b^2*sin(2*f*x + 2*e)/f
```

3.682 $\int \frac{(a+b\sin(e+fx))^2}{c+d\sin(e+fx)}\,dx$

Optimal. Leaf size=93

$$\frac{2(bc-ad)^2\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{d^2f\sqrt{c^2-d^2}}-\frac{bx(bc-2ad)}{d^2}-\frac{b^2\cos(e+fx)}{df}$$

```
[Out] -((b*(b*c - 2*a*d)*x)/d^2) + (2*(b*c - a*d)^2*ArcTan[(d + c*Tan[(e + f*x)/2
])/Sqrt[c^2 - d^2]])/(d^2*Sqrt[c^2 - d^2]*f) - (b^2*Cos[e + f*x])/(d*f)
```

Rubi [A] time = 0.181709, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2746, 2735, 2660, 618, 204}

$$\frac{2(bc-ad)^2\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{d^2f\sqrt{c^2-d^2}}-\frac{bx(bc-2ad)}{d^2}-\frac{b^2\cos(e+fx)}{df}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^2/(c + d*Sin[e + f*x]),x]
```

```
[Out] -((b*(b*c - 2*a*d)*x)/d^2) + (2*(b*c - a*d)^2*ArcTan[(d + c*Tan[(e + f*x)/2
])/Sqrt[c^2 - d^2]])/(d^2*Sqrt[c^2 - d^2]*f) - (b^2*Cos[e + f*x])/(d*f)
```

Rule 2746

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2/((c_.) + (d_.)*sin[(e_.) + (f
_.)*(x_)]), x_Symbol] :> -Simp[(b^2*Cos[e + f*x])/(d*f), x] + Dist[1/d, Int
[Simp[a^2*d - b*(b*c - 2*a*d)*Sin[e + f*x], x]/(c + d*Sin[e + f*x]), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(a+b\sin(e+fx))^2}{c+d\sin(e+fx)}\,dx = -\frac{b^2\cos(e+fx)}{df}+\frac{\int \frac{a^2d-b(bc-2ad)\sin(e+fx)}{c+d\sin(e+fx)}\,dx}{d}$$

$$= -\frac{b(bc-2ad)x}{d^2}-\frac{b^2\cos(e+fx)}{df}+\frac{(bc-ad)^2\int \frac{1}{c+d\sin(e+fx)}\,dx}{d^2}$$

$$= -\frac{b(bc-2ad)x}{d^2}-\frac{b^2\cos(e+fx)}{df}+\frac{\left(2(bc-ad)^2\right)\mathrm{Subst}\left(\int \frac{1}{c+2dx+cx^2}\,dx,x,\tan\left(\frac{1}{2}(e+fx)\right.\right.}{d^2f}$$

$$= -\frac{b(bc-2ad)x}{d^2}-\frac{b^2\cos(e+fx)}{df}-\frac{\left(4(bc-ad)^2\right)\mathrm{Subst}\left(\int \frac{1}{-4(c^2-d^2)-x^2}\,dx,x,2d+2c\tan\left(\right.\right.}{d^2f}$$

$$= -\frac{b(bc-2ad)x}{d^2}+\frac{2(bc-ad)^2\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{d^2\sqrt{c^2-d^2}f}-\frac{b^2\cos(e+fx)}{df}$$

Mathematica [A] time = 0.211975, size = 89, normalized size = 0.96

$$-\frac{-\frac{2(bc-ad)^2\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{\sqrt{c^2-d^2}}+b(e+fx)(bc-2ad)+b^2d\cos(e+fx)}{d^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^2/(c + d*Sin[e + f*x]),x]
```

```
[Out] -((b*(b*c - 2*a*d)*(e + f*x) - (2*(b*c - a*d)^2*ArcTan[(d + c*Tan[(e + f*x)
/2])/Sqrt[c^2 - d^2]])/Sqrt[c^2 - d^2] + b^2*d*Cos[e + f*x])/(d^2*f))
```

Maple [B] time = 0.062, size = 226, normalized size = 2.4

$$2\,\frac{a^2}{f\sqrt{c^2-d^2}}\arctan\left(1/2\,\frac{2\,c\tan\left(1/2\,fx+e/2\right)+2\,d}{\sqrt{c^2-d^2}}\right)-4\,\frac{abc}{df\sqrt{c^2-d^2}}\arctan\left(1/2\,\frac{2\,c\tan\left(1/2\,fx+e/2\right)+2\,d}{\sqrt{c^2-d^2}}\right)+2\,\frac{}{fa}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^2/(c+d*sin(f*x+e)),x)
```

```
[Out] 2/f/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2)
)*a^2-4/f/d/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^
2)^(1/2))*a*b*c+2/f/d^2/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+
2*d)/(c^2-d^2)^(1/2))*c^2*b^2-2/f*b^2/d/(1+tan(1/2*f*x+1/2*e)^2)+4/f*b/d*ar
ctan(tan(1/2*f*x+1/2*e))*a-2/f*b^2/d^2*arctan(tan(1/2*f*x+1/2*e))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2/(c+d*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.77071, size = 798, normalized size = 8.58

$$\left[-\frac{2\left(b^2c^3 - 2\,abc^2d - b^2cd^2 + 2\,abd^3\right)fx + \left(b^2c^2 - 2\,abcd + a^2d^2\right)\sqrt{-c^2 + d^2}\log\left(\frac{\left(2\,c^2-d^2\right)\cos\left(fx+e\right)^2 - 2\,cd\sin\left(fx+e\right) - c^2 - d^2 + 2\left(c\cos\cdots\right.}{d^2\cos\left(fx+e\right)^2 - 2\,cd\sin\cdots}\right.}{2\left(c^2d^2 - d^4\right)f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2/(c+d*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*(b^2*c^3 - 2*a*b*c^2*d - b^2*c*d^2 + 2*a*b*d^3)*f*x + (b^2*c^2 - 2
*a*b*c*d + a^2*d^2)*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*
c*d*sin(f*x + e) - c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x + e) + d*cos(f*x +
 e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2
)) + 2*(b^2*c^2*d - b^2*d^3)*cos(f*x + e))/((c^2*d^2 - d^4)*f), -((b^2*c^3
- 2*a*b*c^2*d - b^2*c*d^2 + 2*a*b*d^3)*f*x + (b^2*c^2 - 2*a*b*c*d + a^2*d^2
)*sqrt(c^2 - d^2)*arctan(-(c*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e
))) + (b^2*c^2*d - b^2*d^3)*cos(f*x + e))/((c^2*d^2 - d^4)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**2/(c+d*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.42492, size = 181, normalized size = 1.95

$$-\frac{\frac{\left(b^2c-2\,abd\right)\left(fx+e\right)}{d^2} + \frac{2\,b^2}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)d} - \frac{2\left(b^2c^2-2\,abcd+a^2d^2\right)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{\sqrt{c^2-d^2}\,d^2}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2/(c+d*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] -((b^2*c - 2*a*b*d)*(f*x + e)/d^2 + 2*b^2/((tan(1/2*f*x + 1/2*e)^2 + 1)*d)
- 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c
) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/(sqrt(c^2 - d^2)*
d^2))/f
```

3.683 $\int \frac{(a+b\sin(e+fx))^2}{(c+d\sin(e+fx))^2}\,dx$

Optimal. Leaf size=129

$$-\frac{2(bc-ad)\left(acd+b\left(c^2-2d^2\right)\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{d^2f\left(c^2-d^2\right)^{3/2}}+\frac{(bc-ad)^2\cos(e+fx)}{df\left(c^2-d^2\right)(c+d\sin(e+fx))}+\frac{b^2x}{d^2}$$

```
[Out] (b^2*x)/d^2 - (2*(b*c - a*d)*(a*c*d + b*(c^2 - 2*d^2))*ArcTan[(d + c*Tan[(e
 + f*x)/2])/Sqrt[c^2 - d^2]])/(d^2*(c^2 - d^2)^(3/2)*f) + ((b*c - a*d)^2*Co
s[e + f*x])/(d*(c^2 - d^2)*f*(c + d*Sin[e + f*x]))
```

Rubi [A] time = 0.23082, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2790, 2735, 2660, 618, 204}

$$-\frac{2(bc-ad)\left(acd+b\left(c^2-2d^2\right)\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{d^2f\left(c^2-d^2\right)^{3/2}}+\frac{(bc-ad)^2\cos(e+fx)}{df\left(c^2-d^2\right)(c+d\sin(e+fx))}+\frac{b^2x}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^2/(c + d*Sin[e + f*x])^2,x]
```

```
[Out] (b^2*x)/d^2 - (2*(b*c - a*d)*(a*c*d + b*(c^2 - 2*d^2))*ArcTan[(d + c*Tan[(e
 + f*x)/2])/Sqrt[c^2 - d^2]])/(d^2*(c^2 - d^2)^(3/2)*f) + ((b*c - a*d)^2*Co
s[e + f*x])/(d*(c^2 - d^2)*f*(c + d*Sin[e + f*x]))
```

Rule 2790

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos[e +
 f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] - Dist[1/
(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m + 1)*(2
*b*c*d - a*(c^2 + d^2)) + (a^2*d^2 - 2*a*b*c*d*(m + 2) + b^2*(d^2*(m + 1) +
 c^2*(m + 2)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(a+b\sin(e+fx))^2}{(c+d\sin(e+fx))^2}\,dx = \frac{(bc-ad)^2\cos(e+fx)}{d\left(c^2-d^2\right)f(c+d\sin(e+fx))}+\frac{\int \frac{d\left(\left(a^2+b^2\right)c-2abd\right)+b^2\left(c^2-d^2\right)\sin(e+fx)}{c+d\sin(e+fx)}\,dx}{d\left(c^2-d^2\right)}$$

$$=\frac{b^2x}{d^2}+\frac{(bc-ad)^2\cos(e+fx)}{d\left(c^2-d^2\right)f(c+d\sin(e+fx))}-\frac{\left(-d^2\left(\left(a^2+b^2\right)c-2abd\right)+b^2c\left(c^2-d^2\right)\right)\int \frac{}{c+d\,\mathrm{s}}}{d^2\left(c^2-d^2\right)}$$

$$=\frac{b^2x}{d^2}+\frac{(bc-ad)^2\cos(e+fx)}{d\left(c^2-d^2\right)f(c+d\sin(e+fx))}-\frac{\left(2\left(-d^2\left(\left(a^2+b^2\right)c-2abd\right)+b^2c\left(c^2-d^2\right)\right)\right)\mathrm{Su}}{d^2\left(c^2-d^2\right.}$$

$$=\frac{b^2x}{d^2}+\frac{(bc-ad)^2\cos(e+fx)}{d\left(c^2-d^2\right)f(c+d\sin(e+fx))}+\frac{\left(4\left(-d^2\left(\left(a^2+b^2\right)c-2abd\right)+b^2c\left(c^2-d^2\right)\right)\right)\mathrm{Su}}{d^2\left(c^2\right.}$$

$$=\frac{b^2x}{d^2}-\frac{2(bc-ad)\left(bc^2+acd-2bd^2\right)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{d^2\left(c^2-d^2\right)^{3/2}f}+\frac{(bc-ad)^2\cos(e+fx)}{d\left(c^2-d^2\right)f(c+d\sin(e+}$$

Mathematica [A] time = 0.52621, size = 134, normalized size = 1.04

$$\frac{-\frac{2\left(-a^2cd^2+2abd^3+b^2\left(c^3-2cd^2\right)\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{\left(c^2-d^2\right)^{3/2}}+\frac{d(bc-ad)^2\cos(e+fx)}{(c-d)(c+d)(c+d\sin(e+fx))}+b^2(e+fx)}{d^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^2/(c + d*Sin[e + f*x])^2,x]
```

```
[Out] (b^2*(e + f*x) - (2*(-(a^2*c*d^2) + 2*a*b*d^3 + b^2*(c^3 - 2*c*d^2))*ArcTan
[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(c^2 - d^2)^(3/2) + (d*(b*c - a
*d)^2*Cos[e + f*x])/((c - d)*(c + d)*(c + d*Sin[e + f*x])))/(d^2*f)
```

Maple [B] time = 0.082, size = 556, normalized size = 4.3

$$2\,\frac{d^2\tan\left(1/2\,fx+e/2\right)a^2}{f\left(c\left(\tan\left(1/2\,fx+e/2\right)\right)^2+2\,\tan\left(1/2\,fx+e/2\right)d+c\right)\left(c^2-d^2\right)c}-4\,\frac{\tan\left(1/2\,fx+e/2\right)dab}{f\left(c\left(\tan\left(1/2\,fx+e/2\right)\right)^2+2\,\tan\left(1/2\,fx+e\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^2,x)
```

```
[Out] 2/f*d^2/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)/(c^2-d^2)/c*tan(1
/2*f*x+1/2*e)*a^2-4/f*d/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)/(
c^2-d^2)*tan(1/2*f*x+1/2*e)*a*b+2/f/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1
/2*e)*d+c)/(c^2-d^2)*c*tan(1/2*f*x+1/2*e)*b^2+2/f*d/(c*tan(1/2*f*x+1/2*e)^2
+2*tan(1/2*f*x+1/2*e)*d+c)/(c^2-d^2)*a^2-4/f/(c*tan(1/2*f*x+1/2*e)^2+2*tan(
1/2*f*x+1/2*e)*d+c)/(c^2-d^2)*a*b*c+2/f/d/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2
*f*x+1/2*e)*d+c)/(c^2-d^2)*c^2*b^2+2/f/(c^2-d^2)^(3/2)*arctan(1/2*(2*c*tan(
1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*a^2*c-4/f*d/(c^2-d^2)^(3/2)*arctan(1/2
*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*a*b-2/f/d^2/(c^2-d^2)^(3/2)*
arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*b^2*c^3+4/f/(c^2-d
^2)^(3/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*b^2*c+2/
f*b^2/d^2*arctan(tan(1/2*f*x+1/2*e))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.99023, size = 1434, normalized size = 11.12

$$\left[\frac{2\left(b^2c^4d - 2b^2c^2d^3 + b^2d^5\right)fx\sin\left(fx+e\right) + 2\left(b^2c^5 - 2b^2c^3d^2 + b^2cd^4\right)fx - \left(b^2c^4 + 2abcd^3 - \left(a^2+2b^2\right)c^2d^2 + \left(b^2c^3d\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*(b^2*c^4*d - 2*b^2*c^2*d^3 + b^2*d^5)*f*x*sin(f*x + e) + 2*(b^2*c^5
 - 2*b^2*c^3*d^2 + b^2*c*d^4)*f*x - (b^2*c^4 + 2*a*b*c*d^3 - (a^2 + 2*b^2)*
c^2*d^2 + (b^2*c^3*d + 2*a*b*d^4 - (a^2 + 2*b^2)*c*d^3)*sin(f*x + e))*sqrt(
-c^2 + d^2)*log(-((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 -
 d^2 - 2*(c*cos(f*x + e)*sin(f*x + e) + d*cos(f*x + e))*sqrt(-c^2 + d^2))/(
d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) + 2*(b^2*c^4*d - 2*a*
b*c^3*d^2 + 2*a*b*c*d^4 - a^2*d^5 + (a^2 - b^2)*c^2*d^3)*cos(f*x + e))/((c^
4*d^3 - 2*c^2*d^5 + d^7)*f*sin(f*x + e) + (c^5*d^2 - 2*c^3*d^4 + c*d^6)*f),
 ((b^2*c^4*d - 2*b^2*c^2*d^3 + b^2*d^5)*f*x*sin(f*x + e) + (b^2*c^5 - 2*b^2
*c^3*d^2 + b^2*c*d^4)*f*x + (b^2*c^4 + 2*a*b*c*d^3 - (a^2 + 2*b^2)*c^2*d^2
+ (b^2*c^3*d + 2*a*b*d^4 - (a^2 + 2*b^2)*c*d^3)*sin(f*x + e))*sqrt(c^2 - d^
2)*arctan(-(c*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) + (b^2*c^4*
d - 2*a*b*c^3*d^2 + 2*a*b*c*d^4 - a^2*d^5 + (a^2 - b^2)*c^2*d^3)*cos(f*x +
e))/((c^4*d^3 - 2*c^2*d^5 + d^7)*f*sin(f*x + e) + (c^5*d^2 - 2*c^3*d^4 + c*
d^6)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**2/(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.38909, size = 336, normalized size = 2.6

$$\frac{\frac{(fx+e)b^2}{d^2} - \frac{2\left(b^2c^3 - a^2cd^2 - 2\,b^2cd^2 + 2\,abd^3\right)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi} + \frac{1}{2}\right\rfloor\operatorname{sgn}(c) + \arctan\left(\frac{c\tan\left(\frac{1}{2}\,fx + \frac{1}{2}\,e\right) + d}{\sqrt{c^2-d^2}}\right)\right)}{\left(c^2d^2 - d^4\right)\sqrt{c^2-d^2}} + \frac{2\left(b^2c^2d\tan\left(\frac{1}{2}\,fx + \frac{1}{2}\,e\right) - 2\,abcd^2\tan\left(\frac{1}{2}\,fx + \frac{1}{2}\,e\right) + a^2d^3\tan\left(\frac{1}{2}\,fx + \frac{1}{2}\,e\right) + b^2c^3 - 2\,abc^2d + a^2cd^2\right)}{\left(c^3d - cd^3\right)\left(c\tan\left(\frac{1}{2}\,fx + \frac{1}{2}\,e\right)^2 + 2\,d\tan\left(\frac{1}{2}\,fx + \frac{1}{2}\,e\right) + c\right)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] ((f*x + e)*b^2/d^2 - 2*(b^2*c^3 - a^2*c*d^2 - 2*b^2*c*d^2 + 2*a*b*d^3)*(pi*
floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/
sqrt(c^2 - d^2)))/((c^2*d^2 - d^4)*sqrt(c^2 - d^2)) + 2*(b^2*c^2*d*tan(1/2*
f*x + 1/2*e) - 2*a*b*c*d^2*tan(1/2*f*x + 1/2*e) + a^2*d^3*tan(1/2*f*x + 1/2
*e) + b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)/((c^3*d - c*d^3)*(c*tan(1/2*f*x +
1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e) + c)))/f
```

3.684 $\int \frac{(a+b\sin(e+fx))^2}{(c+d\sin(e+fx))^3}\,dx$

Optimal. Leaf size=196

$$-\frac{\left(a^2\left(-\left(2c^2+d^2\right)\right)+6abcd-b^2\left(c^2+2d^2\right)\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{f\left(c^2-d^2\right)^{5/2}}+\frac{(bc-ad)^2\cos(e+fx)}{2df\left(c^2-d^2\right)(c+d\sin(e+fx))^2}-\frac{\left(3acd+b\left(c^2\right.\right.}{2df\left(c^2-\right.}$$

```
[Out] -(((6*a*b*c*d - a^2*(2*c^2 + d^2) - b^2*(c^2 + 2*d^2))*ArcTan[(d + c*Tan[(e
 + f*x)/2])/Sqrt[c^2 - d^2]])/((c^2 - d^2)^(5/2)*f)) + ((b*c - a*d)^2*Cos[e
 + f*x])/(2*d*(c^2 - d^2)*f*(c + d*Sin[e + f*x])^2) - ((b*c - a*d)*(3*a*c*d
 + b*(c^2 - 4*d^2))*Cos[e + f*x])/(2*d*(c^2 - d^2)^2*f*(c + d*Sin[e + f*x])
)
```

Rubi [A] time = 0.279383, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {2790, 2754, 12, 2660, 618, 204}

$$-\frac{\left(a^2\left(-\left(2c^2+d^2\right)\right)+6abcd-b^2\left(c^2+2d^2\right)\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{f\left(c^2-d^2\right)^{5/2}}+\frac{(bc-ad)^2\cos(e+fx)}{2df\left(c^2-d^2\right)(c+d\sin(e+fx))^2}-\frac{\left(3acd+b\left(c^2\right.\right.}{2df\left(c^2-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^2/(c + d*Sin[e + f*x])^3,x]
```

```
[Out] -(((6*a*b*c*d - a^2*(2*c^2 + d^2) - b^2*(c^2 + 2*d^2))*ArcTan[(d + c*Tan[(e
 + f*x)/2])/Sqrt[c^2 - d^2]])/((c^2 - d^2)^(5/2)*f)) + ((b*c - a*d)^2*Cos[e
 + f*x])/(2*d*(c^2 - d^2)*f*(c + d*Sin[e + f*x])^2) - ((b*c - a*d)*(3*a*c*d
 + b*(c^2 - 4*d^2))*Cos[e + f*x])/(2*d*(c^2 - d^2)^2*f*(c + d*Sin[e + f*x])
)
```

Rule 2790

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos[e +
 f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] - Dist[1/
(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m + 1)*(2
*b*c*d - a*(c^2 + d^2)) + (a^2*d^2 - 2*a*b*c*d*(m + 2) + b^2*(d^2*(m + 1) +
 c^2*(m + 2)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(a+b\sin(e+fx))^2}{(c+d\sin(e+fx))^3}\,dx = \frac{(bc-ad)^2\cos(e+fx)}{2d\left(c^2-d^2\right)f(c+d\sin(e+fx))^2} + \frac{\int \frac{2d\left(\left(a^2+b^2\right)c-2abd\right)+\left(b^2c^2+2abcd-\left(a^2+2b^2\right)d^2\right)\sin(e+fx)}{(c+d\sin(e+fx))^2}\,dx}{2d\left(c^2-d^2\right)}$$

$$= \frac{(bc-ad)^2\cos(e+fx)}{2d\left(c^2-d^2\right)f(c+d\sin(e+fx))^2} - \frac{(bc-ad)\left(3acd+b\left(c^2-4d^2\right)\right)\cos(e+fx)}{2d\left(c^2-d^2\right)^2 f(c+d\sin(e+fx))} - \frac{\int \frac{d(6ab\cdots}{\cdots}}{\cdots}$$

$$= \frac{(bc-ad)^2\cos(e+fx)}{2d\left(c^2-d^2\right)f(c+d\sin(e+fx))^2} - \frac{(bc-ad)\left(3acd+b\left(c^2-4d^2\right)\right)\cos(e+fx)}{2d\left(c^2-d^2\right)^2 f(c+d\sin(e+fx))} - \frac{\left(6abcd\cdots\right.}{\cdots}$$

$$= \frac{(bc-ad)^2\cos(e+fx)}{2d\left(c^2-d^2\right)f(c+d\sin(e+fx))^2} - \frac{(bc-ad)\left(3acd+b\left(c^2-4d^2\right)\right)\cos(e+fx)}{2d\left(c^2-d^2\right)^2 f(c+d\sin(e+fx))} - \frac{\left(6abcd\cdots\right.}{\cdots}$$

$$= \frac{(bc-ad)^2\cos(e+fx)}{2d\left(c^2-d^2\right)f(c+d\sin(e+fx))^2} - \frac{(bc-ad)\left(3acd+b\left(c^2-4d^2\right)\right)\cos(e+fx)}{2d\left(c^2-d^2\right)^2 f(c+d\sin(e+fx))} + \frac{\left(2\left(6ab\cdots\right.\right.}{\cdots}$$

$$= -\frac{\left(6abcd-a^2\left(2c^2+d^2\right)-b^2\left(c^2+2d^2\right)\right)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{\left(c^2-d^2\right)^{5/2}f} + \frac{(bc-ad)^2\cos(e\cdots}{2d\left(c^2-d^2\right)f(c+d\sin\cdots}$$

Mathematica [A] time = 0.922089, size = 202, normalized size = 1.03

$$\frac{\frac{2\left(a^2\left(2c^2+d^2\right)-6abcd+b^2\left(c^2+2d^2\right)\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{\left(c^2-d^2\right)^{5/2}} - \frac{\left(-3a^2cd^2+2abd\left(c^2+2d^2\right)+b^2\left(c^3-4cd^2\right)\right)\cos(e+fx)}{d(c-d)^2(c+d)^2(c+d\sin(e+fx))} + \frac{(bc-ad)^2\cos(e+fx)}{d(c-d)(c+d)(c+d\sin(e+fx))^2}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^2/(c + d*Sin[e + f*x])^3,x]
```

```
[Out] ((2*(-6*a*b*c*d + a^2*(2*c^2 + d^2) + b^2*(c^2 + 2*d^2))*ArcTan[(d + c*Tan[
(e + f*x)/2])/Sqrt[c^2 - d^2]])/(c^2 - d^2)^(5/2) + ((b*c - a*d)^2*Cos[e +
f*x])/((c - d)*d*(c + d)*(c + d*Sin[e + f*x])^2) - ((-3*a^2*c*d^2 + 2*a*b*d
*(c^2 + 2*d^2) + b^2*(c^3 - 4*c*d^2))*Cos[e + f*x])/((c - d)^2*d*(c + d)^2*
(c + d*Sin[e + f*x])))/(2*f)
```

Maple [B] time = 0.088, size = 1923, normalized size = 9.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^3,x)
```

```
[Out] -6/f/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4
)*c^2*tan(1/2*f*x+1/2*e)^3*a*b*d-10/f/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x
+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)*c*tan(1/2*f*x+1/2*e)^2*a*b*d^2-4/f/(c*ta
n(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)/c*tan(1/
2*f*x+1/2*e)^2*a*b*d^4-10/f/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+
c)^2*c^2/(c^4-2*c^2*d^2+d^4)*tan(1/2*f*x+1/2*e)*a*b*d-6/f/(c^4-2*c^2*d^2+d^
4)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))
*a*b*c*d+1/f/(c^4-2*c^2*d^2+d^4)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*
x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*c^2*b^2+2/f/(c^4-2*c^2*d^2+d^4)/(c^2-d^2)^(1
/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*b^2*d^2+3/f/(c
*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)*b^2*c
^2*d+4/f/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2
+d^4)*a^2*c^2*d-4/f/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^
4-2*c^2*d^2+d^4)*a*b*c^3+1/f/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d
+c)^2/(c^4-2*c^2*d^2+d^4)*c^3*tan(1/2*f*x+1/2*e)^3*b^2+7/f/(c*tan(1/2*f*x+1
/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)*tan(1/2*f*x+1/2*e)^
2*a^2*d^3+6/f/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^
2*d^2+d^4)*tan(1/2*f*x+1/2*e)^2*b^2*d^3-1/f/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1
/2*f*x+1/2*e)*d+c)^2*c^3/(c^4-2*c^2*d^2+d^4)*tan(1/2*f*x+1/2*e)*b^2-2/f/(c*
tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)/c*tan(
1/2*f*x+1/2*e)^3*a^2*d^4+2/f/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d
+c)^2/(c^4-2*c^2*d^2+d^4)*c*tan(1/2*f*x+1/2*e)^3*b^2*d^2+4/f/(c*tan(1/2*f*x
+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)*c^2*tan(1/2*f*x+1
/2*e)^2*a^2*d-1/f/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-
2*c^2*d^2+d^4)*a^2*d^3-2/f/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c
)^2/(c^4-2*c^2*d^2+d^4)/c^2*tan(1/2*f*x+1/2*e)^2*a^2*d^5-4/f/(c*tan(1/2*f*x
+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)*c^3*tan(1/2*f*x+1
/2*e)^2*a*b+3/f/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*
c^2*d^2+d^4)*c^2*tan(1/2*f*x+1/2*e)^2*b^2*d+11/f/(c*tan(1/2*f*x+1/2*e)^2+2*
tan(1/2*f*x+1/2*e)*d+c)^2*c/(c^4-2*c^2*d^2+d^4)*tan(1/2*f*x+1/2*e)*a^2*d^2-
2/f/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/c/(c^4-2*c^2*d^2+d^
4)*tan(1/2*f*x+1/2*e)*a^2*d^4-8/f/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2
*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)*tan(1/2*f*x+1/2*e)*a*b*d^3+10/f/(c*tan(1/2*f
*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2*c/(c^4-2*c^2*d^2+d^4)*tan(1/2*f*x+1
/2*e)*b^2*d^2-2/f/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-
2*c^2*d^2+d^4)*a*b*c*d^2+5/f/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d
+c)^2/(c^4-2*c^2*d^2+d^4)*c*tan(1/2*f*x+1/2*e)^3*a^2*d^2+2/f/(c^4-2*c^2*d^2
+d^4)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/
2))*a^2*c^2+1/f/(c^4-2*c^2*d^2+d^4)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2
*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*a^2*d^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.1914, size = 2167, normalized size = 11.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] [1/4*(2*(b^2*c^5 + 2*a*b*c^4*d + 2*a*b*c^2*d^3 - 4*a*b*d^5 - (3*a^2 + 5*b^2
)*c^3*d^2 + (3*a^2 + 4*b^2)*c*d^4)*cos(f*x + e)*sin(f*x + e) - (6*a*b*c^3*d
 + 6*a*b*c*d^3 - (2*a^2 + b^2)*c^4 - 3*(a^2 + b^2)*c^2*d^2 - (a^2 + 2*b^2)*
d^4 - (6*a*b*c*d^3 - (2*a^2 + b^2)*c^2*d^2 - (a^2 + 2*b^2)*d^4)*cos(f*x + e
)^2 + 2*(6*a*b*c^2*d^2 - (2*a^2 + b^2)*c^3*d - (a^2 + 2*b^2)*c*d^3)*sin(f*x
 + e))*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x +
 e) - c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x + e) + d*cos(f*x + e))*sqrt(-c^
2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) + 2*(4*a*b
*c^5 - 2*a*b*c^3*d^2 - 2*a*b*c*d^4 - a^2*d^5 - (4*a^2 + 3*b^2)*c^4*d + (5*a
^2 + 3*b^2)*c^2*d^3)*cos(f*x + e))/((c^6*d^2 - 3*c^4*d^4 + 3*c^2*d^6 - d^8)
*f*cos(f*x + e)^2 - 2*(c^7*d - 3*c^5*d^3 + 3*c^3*d^5 - c*d^7)*f*sin(f*x + e
) - (c^8 - 2*c^6*d^2 + 2*c^2*d^6 - d^8)*f), 1/2*((b^2*c^5 + 2*a*b*c^4*d + 2
*a*b*c^2*d^3 - 4*a*b*d^5 - (3*a^2 + 5*b^2)*c^3*d^2 + (3*a^2 + 4*b^2)*c*d^4)
*cos(f*x + e)*sin(f*x + e) - (6*a*b*c^3*d + 6*a*b*c*d^3 - (2*a^2 + b^2)*c^4
 - 3*(a^2 + b^2)*c^2*d^2 - (a^2 + 2*b^2)*d^4 - (6*a*b*c*d^3 - (2*a^2 + b^2)
*c^2*d^2 - (a^2 + 2*b^2)*d^4)*cos(f*x + e)^2 + 2*(6*a*b*c^2*d^2 - (2*a^2 +
b^2)*c^3*d - (a^2 + 2*b^2)*c*d^3)*sin(f*x + e))*sqrt(c^2 - d^2)*arctan(-(c*
sin(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) + (4*a*b*c^5 - 2*a*b*c^3*
d^2 - 2*a*b*c*d^4 - a^2*d^5 - (4*a^2 + 3*b^2)*c^4*d + (5*a^2 + 3*b^2)*c^2*d
^3)*cos(f*x + e))/((c^6*d^2 - 3*c^4*d^4 + 3*c^2*d^6 - d^8)*f*cos(f*x + e)^2
 - 2*(c^7*d - 3*c^5*d^3 + 3*c^3*d^5 - c*d^7)*f*sin(f*x + e) - (c^8 - 2*c^6*
d^2 + 2*c^2*d^6 - d^8)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**2/(c+d*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.34841, size = 822, normalized size = 4.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] ((2*a^2*c^2 + b^2*c^2 - 6*a*b*c*d + a^2*d^2 + 2*b^2*d^2)*(pi*floor(1/2*(f*x
 + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2
)))/((c^4 - 2*c^2*d^2 + d^4)*sqrt(c^2 - d^2)) + (b^2*c^5*tan(1/2*f*x + 1/2*
e)^3 - 6*a*b*c^4*d*tan(1/2*f*x + 1/2*e)^3 + 5*a^2*c^3*d^2*tan(1/2*f*x + 1/2
*e)^3 + 2*b^2*c^3*d^2*tan(1/2*f*x + 1/2*e)^3 - 2*a^2*c*d^4*tan(1/2*f*x + 1/
2*e)^3 - 4*a*b*c^5*tan(1/2*f*x + 1/2*e)^2 + 4*a^2*c^4*d*tan(1/2*f*x + 1/2*e
)^2 + 3*b^2*c^4*d*tan(1/2*f*x + 1/2*e)^2 - 10*a*b*c^3*d^2*tan(1/2*f*x + 1/2
*e)^2 + 7*a^2*c^2*d^3*tan(1/2*f*x + 1/2*e)^2 + 6*b^2*c^2*d^3*tan(1/2*f*x +
1/2*e)^2 - 4*a*b*c*d^4*tan(1/2*f*x + 1/2*e)^2 - 2*a^2*d^5*tan(1/2*f*x + 1/2
*e)^2 - b^2*c^5*tan(1/2*f*x + 1/2*e) - 10*a*b*c^4*d*tan(1/2*f*x + 1/2*e) +
11*a^2*c^3*d^2*tan(1/2*f*x + 1/2*e) + 10*b^2*c^3*d^2*tan(1/2*f*x + 1/2*e) -
 8*a*b*c^2*d^3*tan(1/2*f*x + 1/2*e) - 2*a^2*c*d^4*tan(1/2*f*x + 1/2*e) - 4*
a*b*c^5 + 4*a^2*c^4*d + 3*b^2*c^4*d - 2*a*b*c^3*d^2 - a^2*c^2*d^3)/((c^6 -
2*c^4*d^2 + c^2*d^4)*(c*tan(1/2*f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e) +
 c)^2))/f
```

3.685 $\int \frac{(a+b\sin(e+fx))^2}{(c+d\sin(e+fx))^4}\,dx$

Optimal. Leaf size=305

$$-\frac{\left(a^2\left(-\left(2c^3+3cd^2\right)\right)+2abd\left(4c^2+d^2\right)-b^2c\left(c^2+4d^2\right)\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{f\left(c^2-d^2\right)^{7/2}}+\frac{\left(a^2d^2\left(11c^2+4d^2\right)-ab\left(4c^3d+2\ldots\right.\right.}{6df\left(c^2-\ldots\right.}$$

```
[Out] -(((2*a*b*d*(4*c^2 + d^2) - b^2*c*(c^2 + 4*d^2) - a^2*(2*c^3 + 3*c*d^2))*Ar
cTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/((c^2 - d^2)^(7/2)*f)) + ((
b*c - a*d)^2*Cos[e + f*x])/(3*d*(c^2 - d^2)*f*(c + d*Sin[e + f*x])^3) - ((b
*c - a*d)*(5*a*c*d + b*(c^2 - 6*d^2))*Cos[e + f*x])/(6*d*(c^2 - d^2)^2*f*(c
 + d*Sin[e + f*x])^2) + ((a^2*d^2*(11*c^2 + 4*d^2) - a*b*(4*c^3*d + 26*c*d^
3) - b^2*(c^4 - 10*c^2*d^2 - 6*d^4))*Cos[e + f*x])/(6*d*(c^2 - d^2)^3*f*(c
+ d*Sin[e + f*x]))
```

Rubi [A] time = 0.563126, antiderivative size = 305, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {2790, 2754, 12, 2660, 618, 204}

$$-\frac{\left(a^2\left(-\left(2c^3+3cd^2\right)\right)+2abd\left(4c^2+d^2\right)-b^2c\left(c^2+4d^2\right)\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{f\left(c^2-d^2\right)^{7/2}}+\frac{\left(a^2d^2\left(11c^2+4d^2\right)-ab\left(4c^3d+2\ldots\right.\right.}{6df\left(c^2-\ldots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^2/(c + d*Sin[e + f*x])^4,x]
```

```
[Out] -(((2*a*b*d*(4*c^2 + d^2) - b^2*c*(c^2 + 4*d^2) - a^2*(2*c^3 + 3*c*d^2))*Ar
cTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/((c^2 - d^2)^(7/2)*f)) + ((
b*c - a*d)^2*Cos[e + f*x])/(3*d*(c^2 - d^2)*f*(c + d*Sin[e + f*x])^3) - ((b
*c - a*d)*(5*a*c*d + b*(c^2 - 6*d^2))*Cos[e + f*x])/(6*d*(c^2 - d^2)^2*f*(c
 + d*Sin[e + f*x])^2) + ((a^2*d^2*(11*c^2 + 4*d^2) - a*b*(4*c^3*d + 26*c*d^
3) - b^2*(c^4 - 10*c^2*d^2 - 6*d^4))*Cos[e + f*x])/(6*d*(c^2 - d^2)^3*f*(c
+ d*Sin[e + f*x]))
```

Rule 2790

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos[e +
 f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] - Dist[1/
(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m + 1)*(2
*b*c*d - a*(c^2 + d^2)) + (a^2*d^2 - 2*a*b*c*d*(m + 2) + b^2*(d^2*(m + 1) +
 c^2*(m + 2)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
```

```
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(a+b\sin(e+fx))^2}{(c+d\sin(e+fx))^4}\,dx = \frac{(bc-ad)^2\cos(e+fx)}{3d\left(c^2-d^2\right)f(c+d\sin(e+fx))^3} + \frac{\int \frac{3d\left(\left(a^2+b^2\right)c-2abd\right)+\left(4abcd-2a^2d^2+b^2\left(c^2-3d^2\right)\right)\sin(e+fx)}{(c+d\sin(e+fx))^3}\,dx}{3d\left(c^2-d^2\right)}$$

$$= \frac{(bc-ad)^2\cos(e+fx)}{3d\left(c^2-d^2\right)f(c+d\sin(e+fx))^3} - \frac{(bc-ad)\left(5acd+b\left(c^2-6d^2\right)\right)\cos(e+fx)}{6d\left(c^2-d^2\right)^2 f(c+d\sin(e+fx))^2} - \frac{\int \frac{2d(10abcd \cdots}{\cdots}}{\cdots}$$

$$= \frac{(bc-ad)^2\cos(e+fx)}{3d\left(c^2-d^2\right)f(c+d\sin(e+fx))^3} - \frac{(bc-ad)\left(5acd+b\left(c^2-6d^2\right)\right)\cos(e+fx)}{6d\left(c^2-d^2\right)^2 f(c+d\sin(e+fx))^2} + \frac{\left(a^2d^2\left(11c \cdots\right.\right.}{\cdots}$$

$$= \frac{(bc-ad)^2\cos(e+fx)}{3d\left(c^2-d^2\right)f(c+d\sin(e+fx))^3} - \frac{(bc-ad)\left(5acd+b\left(c^2-6d^2\right)\right)\cos(e+fx)}{6d\left(c^2-d^2\right)^2 f(c+d\sin(e+fx))^2} + \frac{\left(a^2d^2\left(11c \cdots\right.\right.}{\cdots}$$

$$= \frac{(bc-ad)^2\cos(e+fx)}{3d\left(c^2-d^2\right)f(c+d\sin(e+fx))^3} - \frac{(bc-ad)\left(5acd+b\left(c^2-6d^2\right)\right)\cos(e+fx)}{6d\left(c^2-d^2\right)^2 f(c+d\sin(e+fx))^2} + \frac{\left(a^2d^2\left(11c \cdots\right.\right.}{\cdots}$$

$$= \frac{(bc-ad)^2\cos(e+fx)}{3d\left(c^2-d^2\right)f(c+d\sin(e+fx))^3} - \frac{(bc-ad)\left(5acd+b\left(c^2-6d^2\right)\right)\cos(e+fx)}{6d\left(c^2-d^2\right)^2 f(c+d\sin(e+fx))^2} + \frac{\left(a^2d^2\left(11c \cdots\right.\right.}{\cdots}$$

$$= -\frac{\left(2abd\left(4c^2+d^2\right)-b^2c\left(c^2+4d^2\right)-a^2\left(2c^3+3cd^2\right)\right)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{\left(c^2-d^2\right)^{7/2}f} + \frac{(bc-\cdots}{3d\left(c^2-d^2\right)\cdots}$$

Mathematica [A] time = 1.38977, size = 346, normalized size = 1.13

$$\frac{\frac{12\left(a^2\left(2c^3+3cd^2\right)-2abd\left(4c^2+d^2\right)+b^2c\left(c^2+4d^2\right)\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{\left(c^2-d^2\right)^{7/2}}+\frac{\cos(e+fx)\left(-6\left(-a^2cd^2\left(9c^2+d^2\right)-2abd\left(-9c^2d^2-2c^4+d^4\right)+b^2\left(-9c^3d^2+c^5-2cd^4\right)\right)}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^2/(c + d*Sin[e + f*x])^4,x]
```

```
[Out] ((12*(-2*a*b*d*(4*c^2 + d^2) + b^2*c*(c^2 + 4*d^2) + a^2*(2*c^3 + 3*c*d^2))
*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(c^2 - d^2)^(7/2) + (Cos
[e + f*x]*(-24*a*b*c^5 + 36*a^2*c^4*d + 25*b^2*c^4*d - 44*a*b*c^3*d^2 + a^2
*c^2*d^3 + 14*b^2*c^2*d^3 - 22*a*b*c*d^4 + 8*a^2*d^5 + 6*b^2*d^5 + d*(-(a^2
*d^2*(11*c^2 + 4*d^2)) + a*b*(4*c^3*d + 26*c*d^3) + b^2*(c^4 - 10*c^2*d^2 -
 6*d^4))*Cos[2*(e + f*x)] - 6*(-(a^2*c*d^2*(9*c^2 + d^2)) - 2*a*b*d*(-2*c^4
 - 9*c^2*d^2 + d^4) + b^2*(c^5 - 9*c^3*d^2 - 2*c*d^4))*Sin[e + f*x]))/((c^2
 - d^2)^3*(c + d*Sin[e + f*x])^3))/(12*f)
```

Maple [B] time = 0.105, size = 4818, normalized size = 15.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^4,x)
```

```
[Out] 8/f/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^3/(c^6-3*c^4*d^2+3*c^
2*d^4-d^6)*tan(1/2*f*x+1/2*e)^2*b^2*d^5+1/f/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1
/2*f*x+1/2*e)*d+c)^3*c^5/(c^6-3*c^4*d^2+3*c^2*d^4-d^6)*tan(1/2*f*x+1/2*e)^5
*b^2-1/f/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^3*c^5/(c^6-3*c^4
*d^2+3*c^2*d^4-d^6)*tan(1/2*f*x+1/2*e)*b^2+6/f/(c*tan(1/2*f*x+1/2*e)^2+2*ta
n(1/2*f*x+1/2*e)*d+c)^3/(c^6-3*c^4*d^2+3*c^2*d^4-d^6)*a^2*c^4*d-5/3/f/(c*ta
n(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^3/(c^6-3*c^4*d^2+3*c^2*d^4-d^6
)*a^2*c^2*d^3-4/f/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^3/(c^6-
3*c^4*d^2+3*c^2*d^4-d^6)*a*b*c^5+13/3/f/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f
*x+1/2*e)*d+c)^3/(c^6-3*c^4*d^2+3*c^2*d^4-d^6)*b^2*c^4*d+2/3/f/(c*tan(1/2*f
*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^3/(c^6-3*c^4*d^2+3*c^2*d^4-d^6)*a^2*d
^5-56/f/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^3*c/(c^6-3*c^4*d^
2+3*c^2*d^4-d^6)*tan(1/2*f*x+1/2*e)^2*a*b*d^4+4/f/(c*tan(1/2*f*x+1/2*e)^2+2
*tan(1/2*f*x+1/2*e)*d+c)^3/c/(c^6-3*c^4*d^2+3*c^2*d^4-d^6)*tan(1/2*f*x+1/2*
e)^2*a*b*d^6-16/f/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^3*c^4/(
c^6-3*c^4*d^2+3*c^2*d^4-d^6)*tan(1/2*f*x+1/2*e)*a*b*d-38/f/(c*tan(1/2*f*x+1
/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^3*c^2/(c^6-3*c^4*d^2+3*c^2*d^4-d^6)*tan(1
/2*f*x+1/2*e)*a*b*d^3-8/f/(c^6-3*c^4*d^2+3*c^2*d^4-d^6)/(c^2-d^2)^(1/2)*arc
tan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*a*b*c^2*d-8/f/(c*tan(
1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^3*c^4/(c^6-3*c^4*d^2+3*c^2*d^4-d
^6)*tan(1/2*f*x+1/2*e)^5*a*b*d-2/f/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/
2*e)*d+c)^3*c^2/(c^6-3*c^4*d^2+3*c^2*d^4-d^6)*tan(1/2*f*x+1/2*e)^5*a*b*d^3-
28/f/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^3/(c^6-3*c^4*d^2+3*c
^2*d^4-d^6)*c^3*tan(1/2*f*x+1/2*e)^4*a*b*d^2-22/f/(c*tan(1/2*f*x+1/2*e)^2+2
*tan(1/2*f*x+1/2*e)*d+c)^3/(c^6-3*c^4*d^2+3*c^2*d^4-d^6)*c*tan(1/2*f*x+1/2*
e)^4*a*b*d^4+4/f/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^3/(c^6-3
*c^4*d^2+3*c^2*d^4-d^6)/c*tan(1/2*f*x+1/2*e)^4*a*b*d^6-24/f/(c*tan(1/2*f*x+
1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^3*c^4*d/(c^6-3*c^4*d^2+3*c^2*d^4-d^6)*ta
```

n(1/2*f*x+1/2*e)^3*a*b-56/f/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+
c)^3*c^2*d^3/(c^6-3*c^4*d^2+3*c^2*d^4-d^6)*tan(1/2*f*x+1/2*e)^3*a*b+8/3/f/(
c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^3/c^2*d^7/(c^6-3*c^4*d^2+3
*c^2*d^4-d^6)*tan(1/2*f*x+1/2*e)^3*a*b-40/f/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1
/2*f*x+1/2*e)*d+c)^3*c^3/(c^6-3*c^4*d^2+3*c^2*d^4-d^6)*tan(1/2*f*x+1/2*e)^2
*a*b*d^2+36/f/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^3*c^3*d^2/(
c^6-3*c^4*d^2+3*c^2*d^4-d^6)*tan(1/2*f*x+1/2*e)^3*a^2+14/f/(c*tan(1/2*f*x+1
/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^3*c*d^4/(c^6-3*c^4*d^2+3*c^2*d^4-d^6)*tan
(1/2*f*x+1/2*e)^3*a^2-8/3/f/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+
c)^3/c*d^6/(c^6-3*c^4*d^2+3*c^2*d^4-d^6)*tan(1/2*f*x+1/2*e)^3*a^2+8/3/f/(c*
tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^3/c^3*d^8/(c^6-3*c^4*d^2+3*c
^2*d^4-d^6)*tan(1/2*f*x+1/2*e)^3*a^2+26/f/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2
*f*x+1/2*e)*d+c)^3*c^3*d^2/(c^6-3*c^4*d^2+3*c^2*d^4-d^6)*tan(1/2*f*x+1/2*e)
^3*b^2+64/3/f/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^3*c*d^4/(c^
6-3*c^4*d^2+3*c^2*d^4-d^6)*tan(1/2*f*x+1/2*e)^3*b^2+2/3/f/(c*tan(1/2*f*x+1/
2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^3/(c^6-3*c^4*d^2+3*c^2*d^4-d^6)*a*b*c*d^4+
2/f/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^3/c/(c^6-3*c^4*d^2+3*
c^2*d^4-d^6)*tan(1/2*f*x+1/2*e)^5*a^2*d^6+4/f/(c*tan(1/2*f*x+1/2*e)^2+2*tan
(1/2*f*x+1/2*e)*d+c)^3*c^3/(c^6-3*c^4*d^2+3*c^2*d^4-d^6)*tan(1/2*f*x+1/2*e)
^5*b^2*d^2+6/f/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^3/(c^6-3*c
^4*d^2+3*c^2*d^4-d^6)*c^4*tan(1/2*f*x+1/2*e)^4*a^2*d+27/f/(c*tan(1/2*f*x+1/
2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^3/(c^6-3*c^4*d^2+3*c^2*d^4-d^6)*c^2*tan(1/
2*f*x+1/2*e)^4*a^2*d^3+4/f/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c
)^3/(c^6-3*c^4*d^2+3*c^2*d^4-d^6)/c^2*tan(1/2*f*x+1/2*e)^4*a^2*d^7-4/f/(c*t
an(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^3/(c^6-3*c^4*d^2+3*c^2*d^4-d^
6)*c^5*tan(1/2*f*x+1/2*e)^4*a*b+2/3/f/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x
+1/2*e)*d+c)^3/(c^6-3*c^4*d^2+3*c^2*d^4-d^6)*b^2*c^2*d^3-12/f/(c*tan(1/2*f*
x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^3/(c^6-3*c^4*d^2+3*c^2*d^4-d^6)*tan(1/
2*f*x+1/2*e)^4*a^2*d^5-6/f/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c
)^3/(c^6-3*c^4*d^2+3*c^2*d^4-d^6)*tan(1/2*f*x+1/2*e)^2*a^2*d^5+2/f/(c^6-3*c
^4*d^2+3*c^2*d^4-d^6)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*
d)/(c^2-d^2)^(1/2))*a^2*c^3+1/f/(c^6-3*c^4*d^2+3*c^2*d^4-d^6)/(c^2-d^2)^(1/
2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*b^2*c^3+4/f/(c*
tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^3/(c^6-3*c^4*d^2+3*c^2*d^4-d
^6)*tan(1/2*f*x+1/2*e)*a*b*d^5+8/f/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/
2*e)*d+c)^3*c^4/(c^6-3*c^4*d^2+3*c^2*d^4-d^6)*tan(1/2*f*x+1/2*e)^2*b^2*d+34
/f/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^3*c^2/(c^6-3*c^4*d^2+3
*c^2*d^4-d^6)*tan(1/2*f*x+1/2*e)^2*b^2*d^3+27/f/(c*tan(1/2*f*x+1/2*e)^2+2*t
an(1/2*f*x+1/2*e)*d+c)^3*c^3/(c^6-3*c^4*d^2+3*c^2*d^4-d^6)*tan(1/2*f*x+1/2*
e)*a^2*d^2-4/f/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^3*c/(c^6-3
*c^4*d^2+3*c^2*d^4-d^6)*tan(1/2*f*x+1/2*e)*a^2*d^4+2/f/(c*tan(1/2*f*x+1/2*e
)^2+2*tan(1/2*f*x+1/2*e)*d+c)^3/c/(c^6-3*c^4*d^2+3*c^2*d^4-d^6)*tan(1/2*f*x
+1/2*e)*a^2*d^6+22/f/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^3*c^
3/(c^6-3*c^4*d^2+3*c^2*d^4-d^6)*tan(1/2*f*x+1/2*e)*b^2*d^2+4/f/(c*tan(1/2*f
*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^3*c/(c^6-3*c^4*d^2+3*c^2*d^4-d^6)*tan
(1/2*f*x+1/2*e)*b^2*d^4-20/3/f/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)
*d+c)^3/(c^6-3*c^4*d^2+3*c^2*d^4-d^6)*a*b*c^3*d^2+9/f/(c*tan(1/2*f*x+1/2*e)
^2+2*tan(1/2*f*x+1/2*e)*d+c)^3*c^3/(c^6-3*c^4*d^2+3*c^2*d^4-d^6)*tan(1/2*f*
x+1/2*e)^5*a^2*d^2-6/f/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^3*
c/(c^6-3*c^4*d^2+3*c^2*d^4-d^6)*tan(1/2*f*x+1/2*e)^5*a^2*d^4-68/3/f/(c*tan(
1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^3*d^5/(c^6-3*c^4*d^2+3*c^2*d^4-d
^6)*tan(1/2*f*x+1/2*e)^3*a*b+8/3/f/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/
2*e)*d+c)^3/c*d^6/(c^6-3*c^4*d^2+3*c^2*d^4-d^6)*tan(1/2*f*x+1/2*e)^3*b^2+12
/f/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^3*c^4/(c^6-3*c^4*d^2+3
*c^2*d^4-d^6)*tan(1/2*f*x+1/2*e)^2*a^2*d+40/f/(c*tan(1/2*f*x+1/2*e)^2+2*tan
(1/2*f*x+1/2*e)*d+c)^3*c^2/(c^6-3*c^4*d^2+3*c^2*d^4-d^6)*tan(1/2*f*x+1/2*e)
^2*a^2*d^3+4/f/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^3/c^2/(c^6
-3*c^4*d^2+3*c^2*d^4-d^6)*tan(1/2*f*x+1/2*e)^2*a^2*d^7-8/f/(c*tan(1/2*f*x+1
/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^3*c^5/(c^6-3*c^4*d^2+3*c^2*d^4-d^6)*tan(1

```
/2*f*x+1/2*e)^2*a*b+3/f/(c^6-3*c^4*d^2+3*c^2*d^4-d^6)/(c^2-d^2)^(1/2)*arcta
n(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*a^2*c*d^2-2/f/(c^6-3*c^
4*d^2+3*c^2*d^4-d^6)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d
)/(c^2-d^2)^(1/2))*a*b*d^3+4/f/(c^6-3*c^4*d^2+3*c^2*d^4-d^6)/(c^2-d^2)^(1/2
)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*b^2*c*d^2+5/f/(c
*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^3/(c^6-3*c^4*d^2+3*c^2*d^4-
d^6)*c^4*tan(1/2*f*x+1/2*e)^4*b^2*d+20/f/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*
f*x+1/2*e)*d+c)^3/(c^6-3*c^4*d^2+3*c^2*d^4-d^6)*c^2*tan(1/2*f*x+1/2*e)^4*b^
2*d^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.5338, size = 3687, normalized size = 12.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^4,x, algorithm="fricas")
```

```
[Out] [-1/12*(2*(b^2*c^6*d + 4*a*b*c^5*d^2 + 22*a*b*c^3*d^4 - 26*a*b*c*d^6 - 11*(
a^2 + b^2)*c^4*d^3 + (7*a^2 + 4*b^2)*c^2*d^5 + 2*(2*a^2 + 3*b^2)*d^7)*cos(f
*x + e)^3 - 6*(b^2*c^7 + 4*a*b*c^6*d + 14*a*b*c^4*d^3 - 20*a*b*c^2*d^5 + 2*
a*b*d^7 - (9*a^2 + 10*b^2)*c^5*d^2 + (8*a^2 + 7*b^2)*c^3*d^4 + (a^2 + 2*b^2
)*c*d^6)*cos(f*x + e)*sin(f*x + e) + 3*(8*a*b*c^5*d + 26*a*b*c^3*d^3 + 6*a*
b*c*d^5 - (2*a^2 + b^2)*c^6 - (9*a^2 + 7*b^2)*c^4*d^2 - 3*(3*a^2 + 4*b^2)*c
^2*d^4 - 3*(8*a*b*c^3*d^3 + 2*a*b*c*d^5 - (2*a^2 + b^2)*c^4*d^2 - (3*a^2 +
4*b^2)*c^2*d^4)*cos(f*x + e)^2 + (24*a*b*c^4*d^2 + 14*a*b*c^2*d^4 + 2*a*b*d
^6 - 3*(2*a^2 + b^2)*c^5*d - (11*a^2 + 13*b^2)*c^3*d^3 - (3*a^2 + 4*b^2)*c*
d^5 - (8*a*b*c^2*d^4 + 2*a*b*d^6 - (2*a^2 + b^2)*c^3*d^3 - (3*a^2 + 4*b^2)*
c*d^5)*cos(f*x + e)^2)*sin(f*x + e))*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*co
s(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x +
 e) + d*cos(f*x + e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x
 + e) - c^2 - d^2)) - 12*(2*a*b*c^7 + 2*a*b*c^5*d^2 + 2*a^2*c^4*d^3 + b^2*c
^2*d^5 - 4*a*b*c*d^6 - (3*a^2 + 2*b^2)*c^6*d + (a^2 + b^2)*d^7)*cos(f*x + e
))/(3*(c^9*d^2 - 4*c^7*d^4 + 6*c^5*d^6 - 4*c^3*d^8 + c*d^10)*f*cos(f*x + e)
^2 - (c^11 - c^9*d^2 - 6*c^7*d^4 + 14*c^5*d^6 - 11*c^3*d^8 + 3*c*d^10)*f +
((c^8*d^3 - 4*c^6*d^5 + 6*c^4*d^7 - 4*c^2*d^9 + d^11)*f*cos(f*x + e)^2 - (3
*c^10*d - 11*c^8*d^3 + 14*c^6*d^5 - 6*c^4*d^7 - c^2*d^9 + d^11)*f)*sin(f*x
+ e)), -1/6*((b^2*c^6*d + 4*a*b*c^5*d^2 + 22*a*b*c^3*d^4 - 26*a*b*c*d^6 - 1
1*(a^2 + b^2)*c^4*d^3 + (7*a^2 + 4*b^2)*c^2*d^5 + 2*(2*a^2 + 3*b^2)*d^7)*co
s(f*x + e)^3 - 3*(b^2*c^7 + 4*a*b*c^6*d + 14*a*b*c^4*d^3 - 20*a*b*c^2*d^5 +
 2*a*b*d^7 - (9*a^2 + 10*b^2)*c^5*d^2 + (8*a^2 + 7*b^2)*c^3*d^4 + (a^2 + 2*
b^2)*c*d^6)*cos(f*x + e)*sin(f*x + e) + 3*(8*a*b*c^5*d + 26*a*b*c^3*d^3 + 6
*a*b*c*d^5 - (2*a^2 + b^2)*c^6 - (9*a^2 + 7*b^2)*c^4*d^2 - 3*(3*a^2 + 4*b^2
)*c^2*d^4 - 3*(8*a*b*c^3*d^3 + 2*a*b*c*d^5 - (2*a^2 + b^2)*c^4*d^2 - (3*a^2
 + 4*b^2)*c^2*d^4)*cos(f*x + e)^2 + (24*a*b*c^4*d^2 + 14*a*b*c^2*d^4 + 2*a*
```

```
b*d^6 - 3*(2*a^2 + b^2)*c^5*d - (11*a^2 + 13*b^2)*c^3*d^3 - (3*a^2 + 4*b^2)
*c*d^5 - (8*a*b*c^2*d^4 + 2*a*b*d^6 - (2*a^2 + b^2)*c^3*d^3 - (3*a^2 + 4*b^
2)*c*d^5)*cos(f*x + e)^2)*sin(f*x + e))*sqrt(c^2 - d^2)*arctan(-(c*sin(f*x
+ e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) - 6*(2*a*b*c^7 + 2*a*b*c^5*d^2 +
2*a^2*c^4*d^3 + b^2*c^2*d^5 - 4*a*b*c*d^6 - (3*a^2 + 2*b^2)*c^6*d + (a^2 +
b^2)*d^7)*cos(f*x + e))/(3*(c^9*d^2 - 4*c^7*d^4 + 6*c^5*d^6 - 4*c^3*d^8 + c
*d^10)*f*cos(f*x + e)^2 - (c^11 - c^9*d^2 - 6*c^7*d^4 + 14*c^5*d^6 - 11*c^3
*d^8 + 3*c*d^10)*f + ((c^8*d^3 - 4*c^6*d^5 + 6*c^4*d^7 - 4*c^2*d^9 + d^11)*
f*cos(f*x + e)^2 - (3*c^10*d - 11*c^8*d^3 + 14*c^6*d^5 - 6*c^4*d^7 - c^2*d^
9 + d^11)*f)*sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**2/(c+d*sin(f*x+e))**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.49687, size = 1777, normalized size = 5.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^4,x, algorithm="giac")
```

```
[Out] 1/3*(3*(2*a^2*c^3 + b^2*c^3 - 8*a*b*c^2*d + 3*a^2*c*d^2 + 4*b^2*c*d^2 - 2*a
*b*d^3)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x +
1/2*e) + d)/sqrt(c^2 - d^2)))/((c^6 - 3*c^4*d^2 + 3*c^2*d^4 - d^6)*sqrt(c^2
 - d^2)) + (3*b^2*c^8*tan(1/2*f*x + 1/2*e)^5 - 24*a*b*c^7*d*tan(1/2*f*x + 1
/2*e)^5 + 27*a^2*c^6*d^2*tan(1/2*f*x + 1/2*e)^5 + 12*b^2*c^6*d^2*tan(1/2*f*
x + 1/2*e)^5 - 6*a*b*c^5*d^3*tan(1/2*f*x + 1/2*e)^5 - 18*a^2*c^4*d^4*tan(1/
2*f*x + 1/2*e)^5 + 6*a^2*c^2*d^6*tan(1/2*f*x + 1/2*e)^5 - 12*a*b*c^8*tan(1/
2*f*x + 1/2*e)^4 + 18*a^2*c^7*d*tan(1/2*f*x + 1/2*e)^4 + 15*b^2*c^7*d*tan(1
/2*f*x + 1/2*e)^4 - 84*a*b*c^6*d^2*tan(1/2*f*x + 1/2*e)^4 + 81*a^2*c^5*d^3*
tan(1/2*f*x + 1/2*e)^4 + 60*b^2*c^5*d^3*tan(1/2*f*x + 1/2*e)^4 - 66*a*b*c^4
*d^4*tan(1/2*f*x + 1/2*e)^4 - 36*a^2*c^3*d^5*tan(1/2*f*x + 1/2*e)^4 + 12*a*
b*c^2*d^6*tan(1/2*f*x + 1/2*e)^4 + 12*a^2*c*d^7*tan(1/2*f*x + 1/2*e)^4 - 72
*a*b*c^7*d*tan(1/2*f*x + 1/2*e)^3 + 108*a^2*c^6*d^2*tan(1/2*f*x + 1/2*e)^3
+ 78*b^2*c^6*d^2*tan(1/2*f*x + 1/2*e)^3 - 168*a*b*c^5*d^3*tan(1/2*f*x + 1/2
*e)^3 + 42*a^2*c^4*d^4*tan(1/2*f*x + 1/2*e)^3 + 64*b^2*c^4*d^4*tan(1/2*f*x
+ 1/2*e)^3 - 68*a*b*c^3*d^5*tan(1/2*f*x + 1/2*e)^3 - 8*a^2*c^2*d^6*tan(1/2*
f*x + 1/2*e)^3 + 8*b^2*c^2*d^6*tan(1/2*f*x + 1/2*e)^3 + 8*a*b*c*d^7*tan(1/2
*f*x + 1/2*e)^3 + 8*a^2*d^8*tan(1/2*f*x + 1/2*e)^3 - 24*a*b*c^8*tan(1/2*f*x
 + 1/2*e)^2 + 36*a^2*c^7*d*tan(1/2*f*x + 1/2*e)^2 + 24*b^2*c^7*d*tan(1/2*f*
x + 1/2*e)^2 - 120*a*b*c^6*d^2*tan(1/2*f*x + 1/2*e)^2 + 120*a^2*c^5*d^3*tan
(1/2*f*x + 1/2*e)^2 + 102*b^2*c^5*d^3*tan(1/2*f*x + 1/2*e)^2 - 168*a*b*c^4*
d^4*tan(1/2*f*x + 1/2*e)^2 - 18*a^2*c^3*d^5*tan(1/2*f*x + 1/2*e)^2 + 24*b^2
*c^3*d^5*tan(1/2*f*x + 1/2*e)^2 + 12*a*b*c^2*d^6*tan(1/2*f*x + 1/2*e)^2 + 1
2*a^2*c*d^7*tan(1/2*f*x + 1/2*e)^2 - 3*b^2*c^8*tan(1/2*f*x + 1/2*e) - 48*a*
b*c^7*d*tan(1/2*f*x + 1/2*e) + 81*a^2*c^6*d^2*tan(1/2*f*x + 1/2*e) + 66*b^2
```

```
*c^6*d^2*tan(1/2*f*x + 1/2*e) - 114*a*b*c^5*d^3*tan(1/2*f*x + 1/2*e) - 12*a
^2*c^4*d^4*tan(1/2*f*x + 1/2*e) + 12*b^2*c^4*d^4*tan(1/2*f*x + 1/2*e) + 12*
a*b*c^3*d^5*tan(1/2*f*x + 1/2*e) + 6*a^2*c^2*d^6*tan(1/2*f*x + 1/2*e) - 12*
a*b*c^8 + 18*a^2*c^7*d + 13*b^2*c^7*d - 20*a*b*c^6*d^2 - 5*a^2*c^5*d^3 + 2*
b^2*c^5*d^3 + 2*a*b*c^4*d^4 + 2*a^2*c^3*d^5)/((c^9 - 3*c^7*d^2 + 3*c^5*d^4
- c^3*d^6)*(c*tan(1/2*f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e) + c)^3))/f
```

3.686 $\int (a+b\sin(e+fx))^3(c+d\sin(e+fx))^3\,dx$

Optimal. Leaf size=400

$$\frac{(ad+bc)\left(a^2d^2+8abcd+b^2\left(c^2+6d^2\right)\right)\cos^3(e+fx)}{3f}-\frac{\left(3a^2bc\left(c^2+3d^2\right)+a^3\left(3c^2d+d^3\right)+3ab^2d\left(3c^2+d^2\right)+b^3c\left(c^2\right.\right.}{f}$$

```
[Out] ((18*a^2*b*d*(4*c^2 + d^2) + b^3*d*(18*c^2 + 5*d^2) + 6*a*b^2*c*(4*c^2 + 9*
d^2) + 8*a^3*(2*c^3 + 3*c*d^2))*x)/16 - ((3*a*b^2*d*(3*c^2 + d^2) + 3*a^2*b
*c*(c^2 + 3*d^2) + b^3*c*(c^2 + 3*d^2) + a^3*(3*c^2*d + d^3))*Cos[e + f*x])
/f + ((b*c + a*d)*(8*a*b*c*d + a^2*d^2 + b^2*(c^2 + 6*d^2))*Cos[e + f*x]^3)
/(3*f) - (3*b^2*d^2*(b*c + a*d)*Cos[e + f*x]^5)/(5*f) - ((24*a^3*c*d^2 + 18
*a^2*b*d*(4*c^2 + d^2) + b^3*d*(18*c^2 + 5*d^2) + 6*a*b^2*c*(4*c^2 + 9*d^2)
)*Cos[e + f*x]*Sin[e + f*x])/(16*f) - (5*b^3*d^3*Cos[e + f*x]*Sin[e + f*x]^
3)/(24*f) - (3*b*d*(b^2*c^2 + 3*a*b*c*d + a^2*d^2)*Cos[e + f*x]*Sin[e + f*x
]^3)/(4*f) - (b^3*d^3*Cos[e + f*x]*Sin[e + f*x]^5)/(6*f)
```

Rubi [A] time = 0.947514, antiderivative size = 493, normalized size of antiderivative = 1.23, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2793, 3023, 2753, 2734}

$$-\frac{\left(90a^2bcd^2\left(c^2+4d^2\right)+40a^3d^3\left(4c^2+d^2\right)-6ab^2d\left(-52c^2d^2+3c^4-16d^4\right)+b^3\left(17c^3d^2+2c^5+96cd^4\right)\right)\cos(e+fx)}{60d^2f}+$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^3,x]
```

```
[Out] ((18*a^2*b*d*(4*c^2 + d^2) + b^3*d*(18*c^2 + 5*d^2) + 6*a*b^2*c*(4*c^2 + 9*
d^2) + 8*a^3*(2*c^3 + 3*c*d^2))*x)/16 - ((40*a^3*d^3*(4*c^2 + d^2) + 90*a^2
*b*c*d^2*(c^2 + 4*d^2) - 6*a*b^2*d*(3*c^4 - 52*c^2*d^2 - 16*d^4) + b^3*(2*c
^5 + 17*c^3*d^2 + 96*c*d^4))*Cos[e + f*x])/(60*d^2*f) - ((200*a^3*c*d^3 + 9
0*a^2*b*d^2*(2*c^2 + 3*d^2) - 6*a*b^2*d*(6*c^3 - 71*c*d^2) + b^3*(4*c^4 + 3
6*c^2*d^2 + 75*d^4))*Cos[e + f*x]*Sin[e + f*x])/(240*d*f) - ((90*a^2*b*c*d^
2 + 40*a^3*d^3 + b^3*(2*c^3 + 21*c*d^2) - a*b^2*(18*c^2*d - 96*d^3))*Cos[e
+ f*x]*(c + d*Sin[e + f*x])^2)/(120*d^2*f) + (b*(18*a*b*c*d - 90*a^2*d^2 -
b^2*(2*c^2 + 25*d^2))*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(120*d^2*f) + (b
^2*(2*b*c - 13*a*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^4)/(30*d^2*f) - (b^2*
Cos[e + f*x]*(a + b*Sin[e + f*x])*(c + d*Sin[e + f*x])^4)/(6*d*f)
```

Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
```

```
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int (a+b\sin(e+fx))^3(c+d\sin(e+fx))^3\,dx = -\frac{b^2\cos(e+fx)(a+b\sin(e+fx))(c+d\sin(e+fx))^4}{6df} + \frac{\int (c+d\sin(e$$

$$= \frac{b^2(2bc-13ad)\cos(e+fx)(c+d\sin(e+fx))^4}{30d^2f} - \frac{b^2\cos(e+fx)(a+b\,s}{}$$

$$= \frac{b\left(18abcd-90a^2d^2-b^2\left(2c^2+25d^2\right)\right)\cos(e+fx)(c+d\sin(e+fx))^3}{120d^2f}$$

$$= -\frac{\left(90a^2bcd^2+40a^3d^3+b^3\left(2c^3+21cd^2\right)-ab^2\left(18c^2d-96d^3\right)\right)\cos(e+}{120d^2f}$$

$$= \frac{1}{16}\left(18a^2bd\left(4c^2+d^2\right)+b^3d\left(18c^2+5d^2\right)+6ab^2c\left(4c^2+9d^2\right)+8a^3\left(2\right.\right.$$

Mathematica [A] time = 1.15591, size = 552, normalized size = 1.38

$$\frac{-360\left(2a^2bc\left(4c^2+9d^2\right)+2a^3\left(4c^2d+d^3\right)+ab^2d\left(18c^2+5d^2\right)+b^3c\left(2c^2+5d^2\right)\right)\cos(e+fx)+20\left(36a^2bcd^2+4a^3d^3\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^3,x]
```

```
[Out] (960*a^3*c^3*e + 1440*a*b^2*c^3*e + 4320*a^2*b*c^2*d*e + 1080*b^3*c^2*d*e +
 1440*a^3*c*d^2*e + 3240*a*b^2*c*d^2*e + 1080*a^2*b*d^3*e + 300*b^3*d^3*e +
 960*a^3*c^3*f*x + 1440*a*b^2*c^3*f*x + 4320*a^2*b*c^2*d*f*x + 1080*b^3*c^2
*d*f*x + 1440*a^3*c*d^2*f*x + 3240*a*b^2*c*d^2*f*x + 1080*a^2*b*d^3*f*x + 3
00*b^3*d^3*f*x - 360*(b^3*c*(2*c^2 + 5*d^2) + a*b^2*d*(18*c^2 + 5*d^2) + 2*
a^2*b*c*(4*c^2 + 9*d^2) + 2*a^3*(4*c^2*d + d^3))*Cos[e + f*x] + 20*(36*a^2*
b*c*d^2 + 4*a^3*d^3 + 3*a*b^2*d*(12*c^2 + 5*d^2) + b^3*(4*c^3 + 15*c*d^2))*
Cos[3*(e + f*x)] - 36*b^3*c*d^2*Cos[5*(e + f*x)] - 36*a*b^2*d^3*Cos[5*(e +
f*x)] - 720*a*b^2*c^3*Sin[2*(e + f*x)] - 2160*a^2*b*c^2*d*Sin[2*(e + f*x)]
- 720*b^3*c^2*d*Sin[2*(e + f*x)] - 720*a^3*c*d^2*Sin[2*(e + f*x)] - 2160*a*
b^2*c*d^2*Sin[2*(e + f*x)] - 720*a^2*b*d^3*Sin[2*(e + f*x)] - 225*b^3*d^3*S
```

```
in[2*(e + f*x)] + 90*b^3*c^2*d*Sin[4*(e + f*x)] + 270*a*b^2*c*d^2*Sin[4*(e
+ f*x)] + 90*a^2*b*d^3*Sin[4*(e + f*x)] + 45*b^3*d^3*Sin[4*(e + f*x)] - 5*b
^3*d^3*Sin[6*(e + f*x)])/(960*f)
```

Maple [A] time = 0.041, size = 489, normalized size = 1.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^3*(c+d*sin(f*x+e))^3,x)
```

```
[Out] 1/f*(a^3*c^3*(f*x+e)-3*a^3*c^2*d*cos(f*x+e)+3*a^3*c*d^2*(-1/2*sin(f*x+e)*co
s(f*x+e)+1/2*f*x+1/2*e)-1/3*a^3*d^3*(2+sin(f*x+e)^2)*cos(f*x+e)-3*a^2*b*c^3
*cos(f*x+e)+9*a^2*b*c^2*d*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-3*a^2*
b*c*d^2*(2+sin(f*x+e)^2)*cos(f*x+e)+3*a^2*b*d^3*(-1/4*(sin(f*x+e)^3+3/2*sin
(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)+3*a*b^2*c^3*(-1/2*sin(f*x+e)*cos(f*x+e)+
1/2*f*x+1/2*e)-3*a*b^2*c^2*d*(2+sin(f*x+e)^2)*cos(f*x+e)+9*a*b^2*c*d^2*(-1/
4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-3/5*a*b^2*d^3*(8/
3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)-1/3*b^3*c^3*(2+sin(f*x+e)^2)*co
s(f*x+e)+3*b^3*c^2*d*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x
+3/8*e)-3/5*b^3*c*d^2*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)+b^3*d^
3*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+e)+5/16*f*x
+5/16*e))
```

Maxima [A] time = 1.11274, size = 644, normalized size = 1.61

$$\frac{960\left(fx+e\right)a^3c^3+720\left(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right)\right)ab^2c^3+320\left(\cos\left(fx+e\right)^3-3\,\cos\left(fx+e\right)\right)b^3c^3+2160\left(2\,fx+2\,e\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3*(c+d*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 1/960*(960*(f*x + e)*a^3*c^3 + 720*(2*f*x + 2*e - sin(2*f*x + 2*e))*a*b^2*c
^3 + 320*(cos(f*x + e)^3 - 3*cos(f*x + e))*b^3*c^3 + 2160*(2*f*x + 2*e - si
n(2*f*x + 2*e))*a^2*b*c^2*d + 2880*(cos(f*x + e)^3 - 3*cos(f*x + e))*a*b^2*
c^2*d + 90*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*b^3*c^2*
d + 720*(2*f*x + 2*e - sin(2*f*x + 2*e))*a^3*c*d^2 + 2880*(cos(f*x + e)^3 -
 3*cos(f*x + e))*a^2*b*c*d^2 + 270*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*si
n(2*f*x + 2*e))*a*b^2*c*d^2 - 192*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 1
5*cos(f*x + e))*b^3*c*d^2 + 320*(cos(f*x + e)^3 - 3*cos(f*x + e))*a^3*d^3 +
 90*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*a^2*b*d^3 - 192
*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*a*b^2*d^3 + 5*(4*
sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*sin(4*f*x + 4*e) - 48*sin(2*f*x + 2*
e))*b^3*d^3 - 2880*a^2*b*c^3*cos(f*x + e) - 2880*a^3*c^2*d*cos(f*x + e))/f
```

Fricas [A] time = 1.92437, size = 834, normalized size = 2.08

$$-\frac{144\left(b^3cd^2+ab^2d^3\right)\cos\left(fx+e\right)^5-80\left(b^3c^3+9\,ab^2c^2d+3\left(3\,a^2b+2\,b^3\right)cd^2+\left(a^3+6\,ab^2\right)d^3\right)\cos\left(fx+e\right)^3-15\left(8\left(2\cdots\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3*(c+d*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -1/240*(144*(b^3*c*d^2 + a*b^2*d^3)*cos(f*x + e)^5 - 80*(b^3*c^3 + 9*a*b^2*
c^2*d + 3*(3*a^2*b + 2*b^3)*c*d^2 + (a^3 + 6*a*b^2)*d^3)*cos(f*x + e)^3 - 1
5*(8*(2*a^3 + 3*a*b^2)*c^3 + 18*(4*a^2*b + b^3)*c^2*d + 6*(4*a^3 + 9*a*b^2)
*c*d^2 + (18*a^2*b + 5*b^3)*d^3)*f*x + 240*((3*a^2*b + b^3)*c^3 + 3*(a^3 +
3*a*b^2)*c^2*d + 3*(3*a^2*b + b^3)*c*d^2 + (a^3 + 3*a*b^2)*d^3)*cos(f*x + e
) + 5*(8*b^3*d^3*cos(f*x + e)^5 - 2*(18*b^3*c^2*d + 54*a*b^2*c*d^2 + (18*a^
2*b + 13*b^3)*d^3)*cos(f*x + e)^3 + 3*(24*a*b^2*c^3 + 6*(12*a^2*b + 5*b^3)*
c^2*d + 6*(4*a^3 + 15*a*b^2)*c*d^2 + (30*a^2*b + 11*b^3)*d^3)*cos(f*x + e))
*sin(f*x + e))/f
```

Sympy [A] time = 8.55227, size = 1217, normalized size = 3.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**3*(c+d*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((a**3*c**3*x - 3*a**3*c**2*d*cos(e + f*x)/f + 3*a**3*c*d**2*x*sin
(e + f*x)**2/2 + 3*a**3*c*d**2*x*cos(e + f*x)**2/2 - 3*a**3*c*d**2*sin(e +
f*x)*cos(e + f*x)/(2*f) - a**3*d**3*sin(e + f*x)**2*cos(e + f*x)/f - 2*a**3
*d**3*cos(e + f*x)**3/(3*f) - 3*a**2*b*c**3*cos(e + f*x)/f + 9*a**2*b*c**2*
d*x*sin(e + f*x)**2/2 + 9*a**2*b*c**2*d*x*cos(e + f*x)**2/2 - 9*a**2*b*c**2
*d*sin(e + f*x)*cos(e + f*x)/(2*f) - 9*a**2*b*c*d**2*sin(e + f*x)**2*cos(e
+ f*x)/f - 6*a**2*b*c*d**2*cos(e + f*x)**3/f + 9*a**2*b*d**3*x*sin(e + f*x)
**4/8 + 9*a**2*b*d**3*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 9*a**2*b*d**3*x
*cos(e + f*x)**4/8 - 15*a**2*b*d**3*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 9*
a**2*b*d**3*sin(e + f*x)*cos(e + f*x)**3/(8*f) + 3*a*b**2*c**3*x*sin(e + f*
x)**2/2 + 3*a*b**2*c**3*x*cos(e + f*x)**2/2 - 3*a*b**2*c**3*sin(e + f*x)*co
s(e + f*x)/(2*f) - 9*a*b**2*c**2*d*sin(e + f*x)**2*cos(e + f*x)/f - 6*a*b**
2*c**2*d*cos(e + f*x)**3/f + 27*a*b**2*c*d**2*x*sin(e + f*x)**4/8 + 27*a*b*
*2*c*d**2*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 27*a*b**2*c*d**2*x*cos(e +
f*x)**4/8 - 45*a*b**2*c*d**2*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 27*a*b**2
*c*d**2*sin(e + f*x)*cos(e + f*x)**3/(8*f) - 3*a*b**2*d**3*sin(e + f*x)**4*
cos(e + f*x)/f - 4*a*b**2*d**3*sin(e + f*x)**2*cos(e + f*x)**3/f - 8*a*b**2
*d**3*cos(e + f*x)**5/(5*f) - b**3*c**3*sin(e + f*x)**2*cos(e + f*x)/f - 2*
b**3*c**3*cos(e + f*x)**3/(3*f) + 9*b**3*c**2*d*x*sin(e + f*x)**4/8 + 9*b**
3*c**2*d*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 9*b**3*c**2*d*x*cos(e + f*x)
**4/8 - 15*b**3*c**2*d*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 9*b**3*c**2*d*s
in(e + f*x)*cos(e + f*x)**3/(8*f) - 3*b**3*c*d**2*sin(e + f*x)**4*cos(e + f
*x)/f - 4*b**3*c*d**2*sin(e + f*x)**2*cos(e + f*x)**3/f - 8*b**3*c*d**2*cos
(e + f*x)**5/(5*f) + 5*b**3*d**3*x*sin(e + f*x)**6/16 + 15*b**3*d**3*x*sin(
e + f*x)**4*cos(e + f*x)**2/16 + 15*b**3*d**3*x*sin(e + f*x)**2*cos(e + f*x
)**4/16 + 5*b**3*d**3*x*cos(e + f*x)**6/16 - 11*b**3*d**3*sin(e + f*x)**5*c
os(e + f*x)/(16*f) - 5*b**3*d**3*sin(e + f*x)**3*cos(e + f*x)**3/(6*f) - 5*
b**3*d**3*sin(e + f*x)*cos(e + f*x)**5/(16*f), Ne(f, 0)), (x*(a + b*sin(e))
**3*(c + d*sin(e))**3, True))
```

Giac [A] time = 1.67025, size = 562, normalized size = 1.4

$$-\frac{b^3d^3\sin\left(6\,fx+6\,e\right)}{192\,f}+\frac{1}{16}\left(16\,a^3c^3+24\,ab^2c^3+72\,a^2bc^2d+18\,b^3c^2d+24\,a^3cd^2+54\,ab^2cd^2+18\,a^2bd^3+5\,b^3d^3\right)x-\frac{3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3*(c+d*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -1/192*b^3*d^3*sin(6*f*x + 6*e)/f + 1/16*(16*a^3*c^3 + 24*a*b^2*c^3 + 72*a^
2*b*c^2*d + 18*b^3*c^2*d + 24*a^3*c*d^2 + 54*a*b^2*c*d^2 + 18*a^2*b*d^3 + 5
*b^3*d^3)*x - 3/80*(b^3*c*d^2 + a*b^2*d^3)*cos(5*f*x + 5*e)/f + 1/48*(4*b^3
*c^3 + 36*a*b^2*c^2*d + 36*a^2*b*c*d^2 + 15*b^3*c*d^2 + 4*a^3*d^3 + 15*a*b^
2*d^3)*cos(3*f*x + 3*e)/f - 3/8*(8*a^2*b*c^3 + 2*b^3*c^3 + 8*a^3*c^2*d + 18
*a*b^2*c^2*d + 18*a^2*b*c*d^2 + 5*b^3*c*d^2 + 2*a^3*d^3 + 5*a*b^2*d^3)*cos(
f*x + e)/f + 3/64*(2*b^3*c^2*d + 6*a*b^2*c*d^2 + 2*a^2*b*d^3 + b^3*d^3)*sin
(4*f*x + 4*e)/f - 3/64*(16*a*b^2*c^3 + 48*a^2*b*c^2*d + 16*b^3*c^2*d + 16*a
^3*c*d^2 + 48*a*b^2*c*d^2 + 16*a^2*b*d^3 + 5*b^3*d^3)*sin(2*f*x + 2*e)/f
```

3.687 $\int (a+b\sin(e+fx))^3(c+d\sin(e+fx))^2\,dx$

Optimal. Leaf size=315

$$-\frac{\left(4a^2b^2\left(20c^2+13d^2\right)+30a^3bcd-3a^4d^2+120ab^3cd+4b^4\left(5c^2+4d^2\right)\right)\cos(e+fx)}{30bf}-\frac{\left(60a^2bcd-6a^3d^2+ab^2\left(100c\right.\right.}{}$$

```
[Out] ((24*a^2*b*c*d + 6*b^3*c*d + 4*a^3*(2*c^2 + d^2) + 3*a*b^2*(4*c^2 + 3*d^2))
*x)/8 - ((30*a^3*b*c*d + 120*a*b^3*c*d - 3*a^4*d^2 + 4*b^4*(5*c^2 + 4*d^2)
+ 4*a^2*b^2*(20*c^2 + 13*d^2))*Cos[e + f*x])/(30*b*f) - ((60*a^2*b*c*d + 90
*b^3*c*d - 6*a^3*d^2 + a*b^2*(100*c^2 + 71*d^2))*Cos[e + f*x]*Sin[e + f*x])
/(120*f) - ((3*a*d*(10*b*c - a*d) + 4*b^2*(5*c^2 + 4*d^2))*Cos[e + f*x]*(a
+ b*Sin[e + f*x])^2)/(60*b*f) - (d*(10*b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^3)/(20*b*f) - (d^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^4)/(5*b*f)
```

Rubi [A] time = 0.459742, antiderivative size = 315, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2791, 2753, 2734}

$$-\frac{\left(4a^2b^2\left(20c^2+13d^2\right)+30a^3bcd-3a^4d^2+120ab^3cd+4b^4\left(5c^2+4d^2\right)\right)\cos(e+fx)}{30bf}-\frac{\left(60a^2bcd-6a^3d^2+ab^2\left(100c\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^2,x]
```


```
[Out] ((24*a^2*b*c*d + 6*b^3*c*d + 4*a^3*(2*c^2 + d^2) + 3*a*b^2*(4*c^2 + 3*d^2))
*x)/8 - ((30*a^3*b*c*d + 120*a*b^3*c*d - 3*a^4*d^2 + 4*b^4*(5*c^2 + 4*d^2)
+ 4*a^2*b^2*(20*c^2 + 13*d^2))*Cos[e + f*x])/(30*b*f) - ((60*a^2*b*c*d + 90
*b^3*c*d - 6*a^3*d^2 + a*b^2*(100*c^2 + 71*d^2))*Cos[e + f*x]*Sin[e + f*x])
/(120*f) - ((3*a*d*(10*b*c - a*d) + 4*b^2*(5*c^2 + 4*d^2))*Cos[e + f*x]*(a
+ b*Sin[e + f*x])^2)/(60*b*f) - (d*(10*b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^3)/(20*b*f) - (d^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^4)/(5*b*f)
```


Rule 2791

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> -Simp[(d^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*
Simp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x],
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a
^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
```

```
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int (a+b\sin(e+fx))^3(c+d\sin(e+fx))^2\,dx = -\frac{d^2\cos(e+fx)(a+b\sin(e+fx))^4}{5bf} + \frac{\int (a+b\sin(e+fx))^3\left(b\left(5c^2+4d^2 \ldots\right.\right.}{5b \ldots}$$

$$= -\frac{d(10bc-ad)\cos(e+fx)(a+b\sin(e+fx))^3}{20bf} - \frac{d^2\cos(e+fx)(a+b\sin(\ldots}{5bf}$$

$$= -\frac{\left(3ad(10bc-ad)+4b^2\left(5c^2+4d^2\right)\right)\cos(e+fx)(a+b\sin(e+fx))^2}{60bf} - \frac{d(\ldots}{}$$

$$= \frac{1}{8}\left(24a^2bcd+6b^3cd+4a^3\left(2c^2+d^2\right)+3ab^2\left(4c^2+3d^2\right)\right)x - \frac{\left(30a^3bcd+ \ldots\right.}{}$$

Mathematica [A] time = 1.64838, size = 246, normalized size = 0.78

$$\frac{15\left(4(e+fx)\left(24a^2bcd+4a^3\left(2c^2+d^2\right)+3ab^2\left(4c^2+3d^2\right)+6b^3cd\right)-8\left(6a^2bcd+a^3d^2+3ab^2\left(c^2+d^2\right)+2b^3cd\right)\sin(2(\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^2,x]
```

```
[Out] (-60*(16*a^3*c*d + 36*a*b^2*c*d + 6*a^2*b*(4*c^2 + 3*d^2) + b^3*(6*c^2 + 5*
d^2))*Cos[e + f*x] + 10*b*(24*a*b*c*d + 12*a^2*d^2 + b^2*(4*c^2 + 5*d^2))*C
os[3*(e + f*x)] - 6*b^3*d^2*Cos[5*(e + f*x)] + 15*(4*(24*a^2*b*c*d + 6*b^3*
c*d + 4*a^3*(2*c^2 + d^2) + 3*a*b^2*(4*c^2 + 3*d^2))*(e + f*x) - 8*(6*a^2*b
*c*d + 2*b^3*c*d + a^3*d^2 + 3*a*b^2*(c^2 + d^2))*Sin[2*(e + f*x)] + b^2*d*
(2*b*c + 3*a*d)*Sin[4*(e + f*x)]))/(480*f)
```

Maple [A] time = 0.036, size = 325, normalized size = 1.

$$\frac{1}{f}\left(a^3c^2\left(fx+e\right)-2\,a^3cd\cos\left(fx+e\right)+a^3d^2\left(-\frac{\sin\left(fx+e\right)\cos\left(fx+e\right)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-3\,a^2bc^2\cos\left(fx+e\right)+6\,a^2bcd\left(- \ldots\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^3*(c+d*sin(f*x+e))^2,x)
```

```
[Out] 1/f*(a^3*c^2*(f*x+e)-2*a^3*c*d*cos(f*x+e)+a^3*d^2*(-1/2*sin(f*x+e)*cos(f*x+
e)+1/2*f*x+1/2*e)-3*a^2*b*c^2*cos(f*x+e)+6*a^2*b*c*d*(-1/2*sin(f*x+e)*cos(f
*x+e)+1/2*f*x+1/2*e)-a^2*b*d^2*(2+sin(f*x+e)^2)*cos(f*x+e)+3*a*b^2*c^2*(-1/
2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-2*a*b^2*c*d*(2+sin(f*x+e)^2)*cos(f*x
+e)+3*a*b^2*d^2*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*
e)-1/3*b^3*c^2*(2+sin(f*x+e)^2)*cos(f*x+e)+2*b^3*c*d*(-1/4*(sin(f*x+e)^3+3/
2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-1/5*b^3*d^2*(8/3+sin(f*x+e)^4+4/3*s
in(f*x+e)^2)*cos(f*x+e))
```

Maxima [A] time = 1.0595, size = 424, normalized size = 1.35

$$\frac{480\left(fx+e\right)a^3c^2+360\left(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right)\right)ab^2c^2+160\left(\cos\left(fx+e\right)^3-3\,\cos\left(fx+e\right)\right)b^3c^2+720\left(2\,fx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3*(c+d*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/480*(480*(f*x + e)*a^3*c^2 + 360*(2*f*x + 2*e - sin(2*f*x + 2*e))*a*b^2*c
^2 + 160*(cos(f*x + e)^3 - 3*cos(f*x + e))*b^3*c^2 + 720*(2*f*x + 2*e - sin
(2*f*x + 2*e))*a^2*b*c*d + 960*(cos(f*x + e)^3 - 3*cos(f*x + e))*a*b^2*c*d
+ 30*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*b^3*c*d + 120*
(2*f*x + 2*e - sin(2*f*x + 2*e))*a^3*d^2 + 480*(cos(f*x + e)^3 - 3*cos(f*x
+ e))*a^2*b*d^2 + 45*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e)
)*a*b^2*d^2 - 32*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*b
^3*d^2 - 1440*a^2*b*c^2*cos(f*x + e) - 960*a^3*c*d*cos(f*x + e))/f
```

Fricas [A] time = 1.80173, size = 568, normalized size = 1.8

$$-\frac{24\,b^3d^2\cos\left(fx+e\right)^5-40\left(b^3c^2+6\,ab^2cd+\left(3\,a^2b+2\,b^3\right)d^2\right)\cos\left(fx+e\right)^3-15\left(4\left(2\,a^3+3\,ab^2\right)c^2+6\left(4\,a^2b+b^3\right)c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3*(c+d*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -1/120*(24*b^3*d^2*cos(f*x + e)^5 - 40*(b^3*c^2 + 6*a*b^2*c*d + (3*a^2*b +
2*b^3)*d^2)*cos(f*x + e)^3 - 15*(4*(2*a^3 + 3*a*b^2)*c^2 + 6*(4*a^2*b + b^3
)*c*d + (4*a^3 + 9*a*b^2)*d^2)*f*x + 120*((3*a^2*b + b^3)*c^2 + 2*(a^3 + 3*
a*b^2)*c*d + (3*a^2*b + b^3)*d^2)*cos(f*x + e) - 15*(2*(2*b^3*c*d + 3*a*b^2
*d^2)*cos(f*x + e)^3 - (12*a*b^2*c^2 + 2*(12*a^2*b + 5*b^3)*c*d + (4*a^3 +
15*a*b^2)*d^2)*cos(f*x + e))*sin(f*x + e))/f
```

Sympy [A] time = 4.12948, size = 729, normalized size = 2.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**3*(c+d*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((a**3*c**2*x - 2*a**3*c*d*cos(e + f*x)/f + a**3*d**2*x*sin(e + f*
x)**2/2 + a**3*d**2*x*cos(e + f*x)**2/2 - a**3*d**2*sin(e + f*x)*cos(e + f*
x)/(2*f) - 3*a**2*b*c**2*cos(e + f*x)/f + 3*a**2*b*c*d*x*sin(e + f*x)**2 +
3*a**2*b*c*d*x*cos(e + f*x)**2 - 3*a**2*b*c*d*sin(e + f*x)*cos(e + f*x)/f -
 3*a**2*b*d**2*sin(e + f*x)**2*cos(e + f*x)/f - 2*a**2*b*d**2*cos(e + f*x)*
*3/f + 3*a*b**2*c**2*x*sin(e + f*x)**2/2 + 3*a*b**2*c**2*x*cos(e + f*x)**2/
2 - 3*a*b**2*c**2*sin(e + f*x)*cos(e + f*x)/(2*f) - 6*a*b**2*c*d*sin(e + f*
x)**2*cos(e + f*x)/f - 4*a*b**2*c*d*cos(e + f*x)**3/f + 9*a*b**2*d**2*x*sin
(e + f*x)**4/8 + 9*a*b**2*d**2*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 9*a*b*
*2*d**2*x*cos(e + f*x)**4/8 - 15*a*b**2*d**2*sin(e + f*x)**3*cos(e + f*x)/(
8*f) - 9*a*b**2*d**2*sin(e + f*x)*cos(e + f*x)**3/(8*f) - b**3*c**2*sin(e +
```

```
f*x)**2*cos(e + f*x)/f - 2*b**3*c**2*cos(e + f*x)**3/(3*f) + 3*b**3*c*d*x*
sin(e + f*x)**4/4 + 3*b**3*c*d*x*sin(e + f*x)**2*cos(e + f*x)**2/2 + 3*b**3
*c*d*x*cos(e + f*x)**4/4 - 5*b**3*c*d*sin(e + f*x)**3*cos(e + f*x)/(4*f) -
3*b**3*c*d*sin(e + f*x)*cos(e + f*x)**3/(4*f) - b**3*d**2*sin(e + f*x)**4*c
os(e + f*x)/f - 4*b**3*d**2*sin(e + f*x)**2*cos(e + f*x)**3/(3*f) - 8*b**3*
d**2*cos(e + f*x)**5/(15*f), Ne(f, 0)), (x*(a + b*sin(e))**3*(c + d*sin(e))
**2, True))
```

Giac [A] time = 1.38243, size = 370, normalized size = 1.17

$$-\frac{b^3d^2\cos\left(5\,fx+5\,e\right)}{80\,f}+\frac{1}{8}\left(8\,a^3c^2+12\,ab^2c^2+24\,a^2bcd+6\,b^3cd+4\,a^3d^2+9\,ab^2d^2\right)x+\frac{\left(4\,b^3c^2+24\,ab^2cd+12\,a^2bd^2+\right.}{48\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3*(c+d*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] -1/80*b^3*d^2*cos(5*f*x + 5*e)/f + 1/8*(8*a^3*c^2 + 12*a*b^2*c^2 + 24*a^2*b
*c*d + 6*b^3*c*d + 4*a^3*d^2 + 9*a*b^2*d^2)*x + 1/48*(4*b^3*c^2 + 24*a*b^2*
c*d + 12*a^2*b*d^2 + 5*b^3*d^2)*cos(3*f*x + 3*e)/f - 1/8*(24*a^2*b*c^2 + 6*
b^3*c^2 + 16*a^3*c*d + 36*a*b^2*c*d + 18*a^2*b*d^2 + 5*b^3*d^2)*cos(f*x + e
)/f + 1/32*(2*b^3*c*d + 3*a*b^2*d^2)*sin(4*f*x + 4*e)/f - 1/4*(3*a*b^2*c^2
+ 6*a^2*b*c*d + 2*b^3*c*d + a^3*d^2 + 3*a*b^2*d^2)*sin(2*f*x + 2*e)/f
```

3.688 $\int (a+b\sin(e+fx))^3(c+d\sin(e+fx))\,dx$

Optimal. Leaf size=171

$$-\frac{\left(16a^2bc+3a^3d+12ab^2d+4b^3c\right)\cos(e+fx)}{6f}-\frac{b\left(6a^2d+20abc+9b^2d\right)\sin(e+fx)\cos(e+fx)}{24f}+\frac{1}{8}x\left(12a^2bd+8a\right.$$

```
[Out] ((8*a^3*c + 12*a*b^2*c + 12*a^2*b*d + 3*b^3*d)*x)/8 - ((16*a^2*b*c + 4*b^3*
c + 3*a^3*d + 12*a*b^2*d)*Cos[e + f*x])/(6*f) - (b*(20*a*b*c + 6*a^2*d + 9*
b^2*d)*Cos[e + f*x]*Sin[e + f*x])/(24*f) - ((4*b*c + 3*a*d)*Cos[e + f*x]*(a
 + b*Sin[e + f*x])^2)/(12*f) - (d*Cos[e + f*x]*(a + b*Sin[e + f*x])^3)/(4*f
)
```

Rubi [A] time = 0.197485, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2753, 2734}

$$-\frac{\left(16a^2bc+3a^3d+12ab^2d+4b^3c\right)\cos(e+fx)}{6f}-\frac{b\left(6a^2d+20abc+9b^2d\right)\sin(e+fx)\cos(e+fx)}{24f}+\frac{1}{8}x\left(12a^2bd+8a\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^3*(c + d*Sin[e + f*x]),x]
```

```
[Out] ((8*a^3*c + 12*a*b^2*c + 12*a^2*b*d + 3*b^3*d)*x)/8 - ((16*a^2*b*c + 4*b^3*
c + 3*a^3*d + 12*a*b^2*d)*Cos[e + f*x])/(6*f) - (b*(20*a*b*c + 6*a^2*d + 9*
b^2*d)*Cos[e + f*x]*Sin[e + f*x])/(24*f) - ((4*b*c + 3*a*d)*Cos[e + f*x]*(a
 + b*Sin[e + f*x])^2)/(12*f) - (d*Cos[e + f*x]*(a + b*Sin[e + f*x])^3)/(4*f
)
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int (a+b\sin(e+fx))^3(c+d\sin(e+fx))\,dx=-\frac{d\cos(e+fx)(a+b\sin(e+fx))^3}{4f}+\frac{1}{4}\int (a+b\sin(e+fx))^2(4ac+3b$$

$$=-\frac{(4bc+3ad)\cos(e+fx)(a+b\sin(e+fx))^2}{12f}-\frac{d\cos(e+fx)(a+b\sin(}{4f}$$

$$=\frac{1}{8}\left(8a^3c+12ab^2c+12a^2bd+3b^3d\right)x-\frac{\left(16a^2bc+4b^3c+3a^3d+12ab^2\right.}{6f}$$

Mathematica [A] time = 0.645167, size = 142, normalized size = 0.83

$$\frac{3\left(4(e+fx)\left(12a^2bd+8a^3c+12ab^2c+3b^3d\right)-8b\left(3a^2d+3abc+b^2d\right)\sin(2(e+fx))+b^3d\sin(4(e+fx))\right)-24\left(12a^2bc\right.}{96f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^3*(c + d*Sin[e + f*x]),x]
```

```
[Out] (-24*(12*a^2*b*c + 3*b^3*c + 4*a^3*d + 9*a*b^2*d)*Cos[e + f*x] + 8*b^2*(b*c
 + 3*a*d)*Cos[3*(e + f*x)] + 3*(4*(8*a^3*c + 12*a*b^2*c + 12*a^2*b*d + 3*b^
3*d)*(e + f*x) - 8*b*(3*a*b*c + 3*a^2*d + b^2*d)*Sin[2*(e + f*x)] + b^3*d*S
in[4*(e + f*x)]))/(96*f)
```

Maple [A] time = 0.029, size = 182, normalized size = 1.1

$$\frac{1}{f}\left(a^3c\left(fx+e\right)-a^3d\cos\left(fx+e\right)-3a^2bc\cos\left(fx+e\right)+3a^2bd\left(-1/2\sin\left(fx+e\right)\cos\left(fx+e\right)+1/2fx+e/2\right)+3ab^2c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^3*(c+d*sin(f*x+e)),x)
```

```
[Out] 1/f*(a^3*c*(f*x+e)-a^3*d*cos(f*x+e)-3*a^2*b*c*cos(f*x+e)+3*a^2*b*d*(-1/2*si
n(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)+3*a*b^2*c*(-1/2*sin(f*x+e)*cos(f*x+e)+1/
2*f*x+1/2*e)-a*b^2*d*(2+sin(f*x+e)^2)*cos(f*x+e)-1/3*b^3*c*(2+sin(f*x+e)^2)
*cos(f*x+e)+b^3*d*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/
8*e))
```

Maxima [A] time = 1.08413, size = 236, normalized size = 1.38

$$\frac{96\left(fx+e\right)a^3c+72\left(2fx+2e-\sin\left(2fx+2e\right)\right)ab^2c+32\left(\cos\left(fx+e\right)^3-3\cos\left(fx+e\right)\right)b^3c+72\left(2fx+2e-\sin\left(2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3*(c+d*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/96*(96*(f*x + e)*a^3*c + 72*(2*f*x + 2*e - sin(2*f*x + 2*e))*a*b^2*c + 32
*(cos(f*x + e)^3 - 3*cos(f*x + e))*b^3*c + 72*(2*f*x + 2*e - sin(2*f*x + 2*
e))*a^2*b*d + 96*(cos(f*x + e)^3 - 3*cos(f*x + e))*a*b^2*d + 3*(12*f*x + 12
*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*b^3*d - 288*a^2*b*c*cos(f*x + e
) - 96*a^3*d*cos(f*x + e))/f
```

Fricas [A] time = 1.72184, size = 340, normalized size = 1.99

$$\frac{8\left(b^3c+3ab^2d\right)\cos\left(fx+e\right)^3+3\left(4\left(2a^3+3ab^2\right)c+3\left(4a^2b+b^3\right)d\right)fx-24\left(\left(3a^2b+b^3\right)c+\left(a^3+3ab^2\right)d\right)\cos\left(fx+e\right.}{24f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3*(c+d*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/24*(8*(b^3*c + 3*a*b^2*d)*cos(f*x + e)^3 + 3*(4*(2*a^3 + 3*a*b^2)*c + 3*(
4*a^2*b + b^3)*d)*f*x - 24*((3*a^2*b + b^3)*c + (a^3 + 3*a*b^2)*d)*cos(f*x
+ e) + 3*(2*b^3*d*cos(f*x + e)^3 - (12*a*b^2*c + (12*a^2*b + 5*b^3)*d)*cos(
f*x + e))*sin(f*x + e))/f
```

Sympy [A] time = 1.73381, size = 386, normalized size = 2.26

$$\begin{cases} a^3cx - \frac{a^3d\cos(e+fx)}{f} - \frac{3a^2bc\cos(e+fx)}{f} + \frac{3a^2bdx\sin^2(e+fx)}{2} + \frac{3a^2bdx\cos^2(e+fx)}{2} - \frac{3a^2bd\sin(e+fx)\cos(e+fx)}{2f} + \frac{3ab^2cx\sin^2(e+fx)}{2} + \\ x(a+b\sin(e))^3(c+d\sin(e)) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**3*(c+d*sin(f*x+e)),x)
```

```
[Out] Piecewise((a**3*c*x - a**3*d*cos(e + f*x)/f - 3*a**2*b*c*cos(e + f*x)/f + 3
*a**2*b*d*x*sin(e + f*x)**2/2 + 3*a**2*b*d*x*cos(e + f*x)**2/2 - 3*a**2*b*d
*sin(e + f*x)*cos(e + f*x)/(2*f) + 3*a*b**2*c*x*sin(e + f*x)**2/2 + 3*a*b**
2*c*x*cos(e + f*x)**2/2 - 3*a*b**2*c*sin(e + f*x)*cos(e + f*x)/(2*f) - 3*a*
b**2*d*sin(e + f*x)**2*cos(e + f*x)/f - 2*a*b**2*d*cos(e + f*x)**3/f - b**3
*c*sin(e + f*x)**2*cos(e + f*x)/f - 2*b**3*c*cos(e + f*x)**3/(3*f) + 3*b**3
*d*x*sin(e + f*x)**4/8 + 3*b**3*d*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 3*b
**3*d*x*cos(e + f*x)**4/8 - 5*b**3*d*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 3
*b**3*d*sin(e + f*x)*cos(e + f*x)**3/(8*f), Ne(f, 0)), (x*(a + b*sin(e))**3
*(c + d*sin(e)), True))
```

Giac [A] time = 1.39458, size = 205, normalized size = 1.2

$$\frac{b^3d\sin(4fx+4e)}{32f} + \frac{1}{8}\left(8a^3c + 12ab^2c + 12a^2bd + 3b^3d\right)x + \frac{\left(b^3c + 3ab^2d\right)\cos(3fx+3e)}{12f} - \frac{\left(12a^2bc + 3b^3c + 4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3*(c+d*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/32*b^3*d*sin(4*f*x + 4*e)/f + 1/8*(8*a^3*c + 12*a*b^2*c + 12*a^2*b*d + 3*
b^3*d)*x + 1/12*(b^3*c + 3*a*b^2*d)*cos(3*f*x + 3*e)/f - 1/4*(12*a^2*b*c +
3*b^3*c + 4*a^3*d + 9*a*b^2*d)*cos(f*x + e)/f - 1/4*(3*a*b^2*c + 3*a^2*b*d
+ b^3*d)*sin(2*f*x + 2*e)/f
```

3.689 $\int (a+b\sin(e+fx))^3\,dx$

Optimal. Leaf size=90

$$-\frac{2b(4a^2+b^2)\cos(e+fx)}{3f}+\frac{1}{2}ax(2a^2+3b^2)-\frac{5ab^2\sin(e+fx)\cos(e+fx)}{6f}-\frac{b\cos(e+fx)(a+b\sin(e+fx))^2}{3f}$$

```
[Out] (a*(2*a^2 + 3*b^2)*x)/2 - (2*b*(4*a^2 + b^2)*Cos[e + f*x])/(3*f) - (5*a*b^2
*Cos[e + f*x]*Sin[e + f*x])/(6*f) - (b*Cos[e + f*x]*(a + b*Sin[e + f*x])^2)
/(3*f)
```

Rubi [A] time = 0.0687045, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2656, 2734}

$$-\frac{2b(4a^2+b^2)\cos(e+fx)}{3f}+\frac{1}{2}ax(2a^2+3b^2)-\frac{5ab^2\sin(e+fx)\cos(e+fx)}{6f}-\frac{b\cos(e+fx)(a+b\sin(e+fx))^2}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^3,x]
```

```
[Out] (a*(2*a^2 + 3*b^2)*x)/2 - (2*b*(4*a^2 + b^2)*Cos[e + f*x])/(3*f) - (5*a*b^2
*Cos[e + f*x]*Sin[e + f*x])/(6*f) - (b*Cos[e + f*x]*(a + b*Sin[e + f*x])^2)
/(3*f)
```

Rule 2656

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[1/n, Int[(a + b*Sin
[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x], x
], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] && In
tegerQ[2*n]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int (a+b\sin(e+fx))^3\,dx=-\frac{b\cos(e+fx)(a+b\sin(e+fx))^2}{3f}+\frac{1}{3}\int (a+b\sin(e+fx))\left(3a^2+2b^2+5ab\sin(e+fx)\right)\,d$$
$$=\frac{1}{2}a\left(2a^2+3b^2\right)x-\frac{2b(4a^2+b^2)\cos(e+fx)}{3f}-\frac{5ab^2\cos(e+fx)\sin(e+fx)}{6f}-\frac{b\cos(e+fx)(}{}$$

Mathematica [A] time = 0.169682, size = 71, normalized size = 0.79

$$\frac{6a(2a^2+3b^2)(e+fx)-9b(4a^2+b^2)\cos(e+fx)-9ab^2\sin(2(e+fx))+b^3\cos(3(e+fx))}{12f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^3,x]
```

```
[Out] (6*a*(2*a^2 + 3*b^2)*(e + f*x) - 9*b*(4*a^2 + b^2)*Cos[e + f*x] + b^3*Cos[3
*(e + f*x)] - 9*a*b^2*Sin[2*(e + f*x)])/(12*f)
```

Maple [A] time = 0.023, size = 76, normalized size = 0.8

$$\frac{1}{f}\left(-\frac{b^3\left(2+\left(\sin\left(fx+e\right)\right)^2\right)\cos\left(fx+e\right)}{3}+3\,ab^2\left(-1/2\,\sin\left(fx+e\right)\cos\left(fx+e\right)+1/2\,fx+e/2\right)-3\,a^2b\cos\left(fx+e\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^3,x)
```

```
[Out] 1/f*(-1/3*b^3*(2+sin(f*x+e)^2)*cos(f*x+e)+3*a*b^2*(-1/2*sin(f*x+e)*cos(f*x+
e)+1/2*f*x+1/2*e)-3*a^2*b*cos(f*x+e)+a^3*(f*x+e))
```

Maxima [A] time = 1.038, size = 100, normalized size = 1.11

$$a^3x+\frac{3\left(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right)\right)ab^2}{4\,f}+\frac{\left(\cos\left(fx+e\right)^3-3\,\cos\left(fx+e\right)\right)b^3}{3\,f}-\frac{3\,a^2b\cos\left(fx+e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] a^3*x + 3/4*(2*f*x + 2*e - sin(2*f*x + 2*e))*a*b^2/f + 1/3*(cos(f*x + e)^3
- 3*cos(f*x + e))*b^3/f - 3*a^2*b*cos(f*x + e)/f
```

Fricas [A] time = 1.56898, size = 169, normalized size = 1.88

$$\frac{2\,b^3\cos\left(fx+e\right)^3-9\,ab^2\cos\left(fx+e\right)\sin\left(fx+e\right)+3\left(2\,a^3+3\,ab^2\right)fx-6\left(3\,a^2b+b^3\right)\cos\left(fx+e\right)}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^3*cos(f*x + e)^3 - 9*a*b^2*cos(f*x + e)*sin(f*x + e) + 3*(2*a^3 +
3*a*b^2)*f*x - 6*(3*a^2*b + b^3)*cos(f*x + e))/f
```

Sympy [A] time = 0.654677, size = 128, normalized size = 1.42

$$\begin{cases} a^3x-\frac{3a^2b\cos\left(e+fx\right)}{f}+\frac{3ab^2x\sin^2\left(e+fx\right)}{2}+\frac{3ab^2x\cos^2\left(e+fx\right)}{2}-\frac{3ab^2\sin\left(e+fx\right)\cos\left(e+fx\right)}{2f}-\frac{b^3\sin^2\left(e+fx\right)\cos\left(e+fx\right)}{f}-\frac{2b^3\cos^3\left(e+fx\right)}{3f} \\ x\left(a+b\sin\left(e\right)\right)^3 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((a**3*x - 3*a**2*b*cos(e + f*x)/f + 3*a*b**2*x*sin(e + f*x)**2/2
+ 3*a*b**2*x*cos(e + f*x)**2/2 - 3*a*b**2*sin(e + f*x)*cos(e + f*x)/(2*f) -
 b**3*sin(e + f*x)**2*cos(e + f*x)/f - 2*b**3*cos(e + f*x)**3/(3*f), Ne(f,
0)), (x*(a + b*sin(e))**3, True))
```

Giac [A] time = 1.43832, size = 101, normalized size = 1.12

$$\frac{b^3\cos\left(3\,fx+3\,e\right)}{12\,f}-\frac{3\,ab^2\sin\left(2\,fx+2\,e\right)}{4\,f}+\frac{1}{2}\left(2\,a^3+3\,ab^2\right)x-\frac{3\left(4\,a^2b+b^3\right)\cos\left(fx+e\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/12*b^3*cos(3*f*x + 3*e)/f - 3/4*a*b^2*sin(2*f*x + 2*e)/f + 1/2*(2*a^3 + 3
*a*b^2)*x - 3/4*(4*a^2*b + b^3)*cos(f*x + e)/f
```

3.690 $\int \frac{(a+b\sin(e+fx))^3}{c+d\sin(e+fx)}\,dx$

Optimal. Leaf size=156

$$-\frac{bx\left(-6a^2d^2+6abcd+b^2\left(-\left(2c^2+d^2\right)\right)\right)}{2d^3}+\frac{b^2(2bc-5ad)\cos(e+fx)}{2d^2f}-\frac{b^2\cos(e+fx)(a+b\sin(e+fx))}{2df}-\frac{2(bc-a}{}$$

```
[Out] -(b*(6*a*b*c*d - 6*a^2*d^2 - b^2*(2*c^2 + d^2))*x)/(2*d^3) - (2*(b*c - a*d)
^3*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(d^3*Sqrt[c^2 - d^2]*f
) + (b^2*(2*b*c - 5*a*d)*Cos[e + f*x])/(2*d^2*f) - (b^2*Cos[e + f*x]*(a + b
*Sin[e + f*x]))/(2*d*f)
```

Rubi [A] time = 0.37833, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {2793, 3023, 2735, 2660, 618, 204}

$$-\frac{bx\left(-6a^2d^2+6abcd+b^2\left(-\left(2c^2+d^2\right)\right)\right)}{2d^3}+\frac{b^2(2bc-5ad)\cos(e+fx)}{2d^2f}-\frac{b^2\cos(e+fx)(a+b\sin(e+fx))}{2df}-\frac{2(bc-a}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^3/(c + d*Sin[e + f*x]),x]
```


```
[Out] -(b*(6*a*b*c*d - 6*a^2*d^2 - b^2*(2*c^2 + d^2))*x)/(2*d^3) - (2*(b*c - a*d)
^3*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(d^3*Sqrt[c^2 - d^2]*f
) + (b^2*(2*b*c - 5*a*d)*Cos[e + f*x])/(2*d^2*f) - (b^2*Cos[e + f*x]*(a + b
*Sin[e + f*x]))/(2*d*f)
```


Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(a+b\sin(e+fx))^3}{c+d\sin(e+fx)}\,dx = -\frac{b^2\cos(e+fx)(a+b\sin(e+fx))}{2df} + \frac{\int \frac{b^3c+2a^3d-b\left(abc-6a^2d-b^2d\right)\sin(e+fx)-b^2(2bc-5ad)\sin^2(e+fx)}{c+d\sin(e+fx)}\,dx}{2d}$$

$$= \frac{b^2(2bc-5ad)\cos(e+fx)}{2d^2f} - \frac{b^2\cos(e+fx)(a+b\sin(e+fx))}{2df} + \frac{\int \frac{d\left(b^3c+2a^3d\right)-b\left(6abcd-6a^2d^2-b^2\right.}{c+d\sin(e+fx}}{2d^2}$$

$$= -\frac{b\left(6abcd-6a^2d^2-b^2\left(2c^2+d^2\right)\right)x}{2d^3} + \frac{b^2(2bc-5ad)\cos(e+fx)}{2d^2f} - \frac{b^2\cos(e+fx)(a+b\sin}{2df}$$

$$= -\frac{b\left(6abcd-6a^2d^2-b^2\left(2c^2+d^2\right)\right)x}{2d^3} + \frac{b^2(2bc-5ad)\cos(e+fx)}{2d^2f} - \frac{b^2\cos(e+fx)(a+b\sin}{2df}$$

$$= -\frac{b\left(6abcd-6a^2d^2-b^2\left(2c^2+d^2\right)\right)x}{2d^3} + \frac{b^2(2bc-5ad)\cos(e+fx)}{2d^2f} - \frac{b^2\cos(e+fx)(a+b\sin}{2df}$$

$$= -\frac{b\left(6abcd-6a^2d^2-b^2\left(2c^2+d^2\right)\right)x}{2d^3} - \frac{2(bc-ad)^3\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{d^3\sqrt{c^2-d^2}\,f} + \frac{b^2(2bc-5ad)}{2d^2}$$

Mathematica [A] time = 0.360915, size = 137, normalized size = 0.88

$$\frac{2b(e+fx)\left(6a^2d^2-6abcd+b^2\left(2c^2+d^2\right)\right)+4b^2d(bc-3ad)\cos(e+fx)-\frac{8(bc-ad)^3\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{\sqrt{c^2-d^2}}+b^3\left(-d^2\right)\sin(2(e}{4d^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^3/(c + d*Sin[e + f*x]),x]
```

```
[Out] (2*b*(-6*a*b*c*d + 6*a^2*d^2 + b^2*(2*c^2 + d^2))*(e + f*x) - (8*(b*c - a*d
)^3*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/Sqrt[c^2 - d^2] + 4*b
^2*d*(b*c - 3*a*d)*Cos[e + f*x] - b^3*d^2*Sin[2*(e + f*x)])/(4*d^3*f)
```

Maple [B] time = 0.075, size = 506, normalized size = 3.2

$$2\frac{a^3}{f\sqrt{c^2-d^2}}\arctan\left(1/2\frac{2c\tan\left(1/2fx+e/2\right)+2d}{\sqrt{c^2-d^2}}\right)-6\frac{a^2bc}{df\sqrt{c^2-d^2}}\arctan\left(1/2\frac{2c\tan\left(1/2fx+e/2\right)+2d}{\sqrt{c^2-d^2}}\right)+6\,\frac{}{fa}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e)),x)
```

```
[Out] 2/f/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2)
)*a^3-6/f/d/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^
2)^(1/2))*a^2*b*c+6/f/d^2/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e
)+2*d)/(c^2-d^2)^(1/2))*a*b^2*c^2-2/f/d^3/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*t
an(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*b^3*c^3+1/f*b^3/d/(1+tan(1/2*f*x+1/
2*e)^2)^2*tan(1/2*f*x+1/2*e)^3-6/f*b^2/d/(1+tan(1/2*f*x+1/2*e)^2)^2*tan(1/2
*f*x+1/2*e)^2*a+2/f*b^3/d^2/(1+tan(1/2*f*x+1/2*e)^2)^2*tan(1/2*f*x+1/2*e)^2
*c-1/f*b^3/d/(1+tan(1/2*f*x+1/2*e)^2)^2*tan(1/2*f*x+1/2*e)-6/f*b^2/d/(1+tan
(1/2*f*x+1/2*e)^2)^2*a+2/f*b^3/d^2/(1+tan(1/2*f*x+1/2*e)^2)^2*c+6/f*b/d*arc
tan(tan(1/2*f*x+1/2*e))*a^2-6/f*b^2/d^2*arctan(tan(1/2*f*x+1/2*e))*a*c+2/f*
b^3/d^3*arctan(tan(1/2*f*x+1/2*e))*c^2+1/f*b^3/d*arctan(tan(1/2*f*x+1/2*e))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.87498, size = 1208, normalized size = 7.74

$$\left[\frac{\left(2\,b^3c^4-6\,ab^2c^3d+6\,ab^2cd^3+\left(6\,a^2b-b^3\right)c^2d^2-\left(6\,a^2b+b^3\right)d^4\right)fx-\left(b^3c^2d^2-b^3d^4\right)\cos\left(fx+e\right)\sin\left(fx+e\right)+\left(b\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] [1/2*((2*b^3*c^4 - 6*a*b^2*c^3*d + 6*a*b^2*c*d^3 + (6*a^2*b - b^3)*c^2*d^2
- (6*a^2*b + b^3)*d^4)*f*x - (b^3*c^2*d^2 - b^3*d^4)*cos(f*x + e)*sin(f*x +
 e) + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(-c^2 + d^2)*
log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 + 2*(c*c
```

```
os(f*x + e)*sin(f*x + e) + d*cos(f*x + e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x +
 e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) + 2*(b^3*c^3*d - 3*a*b^2*c^2*d^2 -
 b^3*c*d^3 + 3*a*b^2*d^4)*cos(f*x + e))/((c^2*d^3 - d^5)*f), 1/2*((2*b^3*c^
4 - 6*a*b^2*c^3*d + 6*a*b^2*c*d^3 + (6*a^2*b - b^3)*c^2*d^2 - (6*a^2*b + b^
3)*d^4)*f*x - (b^3*c^2*d^2 - b^3*d^4)*cos(f*x + e)*sin(f*x + e) + 2*(b^3*c^
3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(c^2 - d^2)*arctan(-(c*sin
(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) + 2*(b^3*c^3*d - 3*a*b^2*c^2
*d^2 - b^3*c*d^3 + 3*a*b^2*d^4)*cos(f*x + e))/((c^2*d^3 - d^5)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**3/(c+d*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.57045, size = 340, normalized size = 2.18

$$\frac{\frac{\left(2\,b^3c^2-6\,ab^2cd+6\,a^2bd^2+b^3d^2\right)\left(fx+e\right)}{d^3}-\frac{4\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{\sqrt{c^2-d^2}\,d^3}+\frac{2\left(b^3d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+2\,b^3c\,t\cdots\right.}{\cdots}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/2*((2*b^3*c^2 - 6*a*b^2*c*d + 6*a^2*b*d^2 + b^3*d^2)*(f*x + e)/d^3 - 4*(b
^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*(pi*floor(1/2*(f*x + e)/p
i + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/(sq
rt(c^2 - d^2)*d^3) + 2*(b^3*d*tan(1/2*f*x + 1/2*e)^3 + 2*b^3*c*tan(1/2*f*x
+ 1/2*e)^2 - 6*a*b^2*d*tan(1/2*f*x + 1/2*e)^2 - b^3*d*tan(1/2*f*x + 1/2*e)
+ 2*b^3*c - 6*a*b^2*d)/((tan(1/2*f*x + 1/2*e)^2 + 1)^2*d^2))/f
```

3.691 $\int \frac{(a+b\sin(e+fx))^3}{(c+d\sin(e+fx))^2}\,dx$

Optimal. Leaf size=208

$$\frac{b\left(-a^2d^2+2abcd+b^2\left(-\left(2c^2-d^2\right)\right)\right)\cos(e+fx)}{d^2f\left(c^2-d^2\right)}-\frac{b^2x(2bc-3ad)}{d^3}+\frac{2(bc-ad)^2\left(acd+2bc^2-3bd^2\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e\right.}{\sqrt{c^2}}\right.}{d^3f\left(c^2-d^2\right)^{3/2}}$$

```
[Out] -((b^2*(2*b*c - 3*a*d)*x)/d^3) + (2*(b*c - a*d)^2*(2*b*c^2 + a*c*d - 3*b*d^
2)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(d^3*(c^2 - d^2)^(3/2)
*f) + (b*(2*a*b*c*d - a^2*d^2 - b^2*(2*c^2 - d^2))*Cos[e + f*x])/(d^2*(c^2
- d^2)*f) + ((b*c - a*d)^2*Cos[e + f*x]*(a + b*Sin[e + f*x]))/(d*(c^2 - d^2
)*f*(c + d*Sin[e + f*x]))
```

Rubi [A] time = 0.495365, antiderivative size = 208, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {2792, 3023, 2735, 2660, 618, 204}

$$\frac{b\left(-a^2d^2+2abcd+b^2\left(-\left(2c^2-d^2\right)\right)\right)\cos(e+fx)}{d^2f\left(c^2-d^2\right)}-\frac{b^2x(2bc-3ad)}{d^3}+\frac{2(bc-ad)^2\left(acd+2bc^2-3bd^2\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e\right.}{\sqrt{c^2}}\right.}{d^3f\left(c^2-d^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^3/(c + d*Sin[e + f*x])^2,x]
```

```
[Out] -((b^2*(2*b*c - 3*a*d)*x)/d^3) + (2*(b*c - a*d)^2*(2*b*c^2 + a*c*d - 3*b*d^
2)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(d^3*(c^2 - d^2)^(3/2)
*f) + (b*(2*a*b*c*d - a^2*d^2 - b^2*(2*c^2 - d^2))*Cos[e + f*x])/(d^2*(c^2
- d^2)*f) + ((b*c - a*d)^2*Cos[e + f*x]*(a + b*Sin[e + f*x]))/(d*(c^2 - d^2
)*f*(c + d*Sin[e + f*x]))
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
```

```
!LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(a+b\sin(e+fx))^3}{(c+d\sin(e+fx))^2}\,dx = \frac{(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{d\left(c^2-d^2\right)f(c+d\sin(e+fx))} - \frac{\int \frac{b^3c^2-a^3cd-3ab^2cd+3a^2bd^2-b\left(abc^2+\left(a^2+b^2\right)cd-3abd^2\right)\sin(e\cdots}{c+d\sin(e+fx)}}{d\left(c^2-d^2\right)\cdots}$$

$$= \frac{b\left(2abcd-a^2d^2-b^2\left(2c^2-d^2\right)\right)\cos(e+fx)}{d^2\left(c^2-d^2\right)f} + \frac{(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{d\left(c^2-d^2\right)f(c+d\sin(e+fx))} - \cdots$$

$$= -\frac{b^2(2bc-3ad)x}{d^3} + \frac{b\left(2abcd-a^2d^2-b^2\left(2c^2-d^2\right)\right)\cos(e+fx)}{d^2\left(c^2-d^2\right)f} + \frac{(bc-ad)^2\cos(e+fx)(a\cdots}{d\left(c^2-d^2\right)f(c+d\,\mathrm{si}\cdots}$$

$$= -\frac{b^2(2bc-3ad)x}{d^3} + \frac{b\left(2abcd-a^2d^2-b^2\left(2c^2-d^2\right)\right)\cos(e+fx)}{d^2\left(c^2-d^2\right)f} + \frac{(bc-ad)^2\cos(e+fx)(a\cdots}{d\left(c^2-d^2\right)f(c+d\,\mathrm{si}\cdots}$$

$$= -\frac{b^2(2bc-3ad)x}{d^3} + \frac{b\left(2abcd-a^2d^2-b^2\left(2c^2-d^2\right)\right)\cos(e+fx)}{d^2\left(c^2-d^2\right)f} + \frac{(bc-ad)^2\cos(e+fx)(a\cdots}{d\left(c^2-d^2\right)f(c+d\,\mathrm{si}\cdots}$$

$$= -\frac{b^2(2bc-3ad)x}{d^3} + \frac{2(bc-ad)^2\left(2bc^2+acd-3bd^2\right)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{d^3\left(c^2-d^2\right)^{3/2}f} + \frac{b\left(2abcd-a^2d\cdots\right.}{}$$

Mathematica [A] time = 1.05957, size = 152, normalized size = 0.73

$$\frac{-b^2(e+fx)(2bc-3ad)+\frac{2(bc-ad)^2\left(acd+2bc^2-3bd^2\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{\left(c^2-d^2\right)^{3/2}}+\frac{d(ad-bc)^3\cos(e+fx)}{(c-d)(c+d)(c+d\sin(e+fx))}+b^3(-d)\cos(e+fx)}{d^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^3/(c + d*Sin[e + f*x])^2,x]
```

```
[Out] (-(b^2*(2*b*c - 3*a*d)*(e + f*x)) + (2*(b*c - a*d)^2*(2*b*c^2 + a*c*d - 3*b
*d^2)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(c^2 - d^2)^(3/2) -
 b^3*d*Cos[e + f*x] + (d*(-(b*c) + a*d)^3*Cos[e + f*x])/((c - d)*(c + d)*(c
 + d*Sin[e + f*x])))/(d^3*f)
```

Maple [B] time = 0.096, size = 842, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^2,x)
```

```
[Out] 2/f*d^2/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)/(c^2-d^2)/c*tan(1
/2*f*x+1/2*e)*a^3-6/f*d/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)/(
c^2-d^2)*tan(1/2*f*x+1/2*e)*a^2*b+6/f/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x
+1/2*e)*d+c)/(c^2-d^2)*c*tan(1/2*f*x+1/2*e)*a*b^2-2/f/d/(c*tan(1/2*f*x+1/2*
e)^2+2*tan(1/2*f*x+1/2*e)*d+c)/(c^2-d^2)*c^2*tan(1/2*f*x+1/2*e)*b^3+2/f*d/(
c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)/(c^2-d^2)*a^3-6/f/(c*tan(1
/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)/(c^2-d^2)*a^2*b*c+6/f/d/(c*tan(1/
2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)/(c^2-d^2)*a*b^2*c^2-2/f/d^2/(c*tan
(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)/(c^2-d^2)*c^3*b^3+2/f/(c^2-d^2)
^(3/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*a^3*c-6/f*d
/(c^2-d^2)^(3/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*a
^2*b-6/f/d^2/(c^2-d^2)^(3/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d
^2)^(1/2))*a*b^2*c^3+12/f/(c^2-d^2)^(3/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e
)+2*d)/(c^2-d^2)^(1/2))*a*b^2*c+4/f/d^3/(c^2-d^2)^(3/2)*arctan(1/2*(2*c*tan
(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*b^3*c^4-6/f/d/(c^2-d^2)^(3/2)*arctan(
1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*b^3*c^2-2/f*b^3/d^2/(1+ta
n(1/2*f*x+1/2*e)^2)+6/f*b^2/d^2*arctan(tan(1/2*f*x+1/2*e))*a-4/f*b^3/d^3*ar
ctan(tan(1/2*f*x+1/2*e))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.28077, size = 2221, normalized size = 10.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(2*(2*b^3*c^6 - 3*a*b^2*c^5*d - 4*b^3*c^4*d^2 + 6*a*b^2*c^3*d^3 + 2*b
^3*c^2*d^4 - 3*a*b^2*c*d^5)*f*x + (2*b^3*c^5 - 3*a*b^2*c^4*d - 3*b^3*c^3*d^
2 - 3*a^2*b*c*d^4 + (a^3 + 6*a*b^2)*c^2*d^3 + (2*b^3*c^4*d - 3*a*b^2*c^3*d^
2 - 3*b^3*c^2*d^3 - 3*a^2*b*d^5 + (a^3 + 6*a*b^2)*c*d^4)*sin(f*x + e))*sqrt
(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 -
 d^2 + 2*(c*cos(f*x + e)*sin(f*x + e) + d*cos(f*x + e))*sqrt(-c^2 + d^2))/(
d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) + 2*(2*b^3*c^5*d - 3*
a*b^2*c^4*d^2 + a^3*d^6 + 3*(a^2*b - b^3)*c^3*d^3 - (a^3 - 3*a*b^2)*c^2*d^4
 - (3*a^2*b - b^3)*c*d^5)*cos(f*x + e) + 2*((2*b^3*c^5*d - 3*a*b^2*c^4*d^2
- 4*b^3*c^3*d^3 + 6*a*b^2*c^2*d^4 + 2*b^3*c*d^5 - 3*a*b^2*d^6)*f*x + (b^3*c
^4*d^2 - 2*b^3*c^2*d^4 + b^3*d^6)*cos(f*x + e))*sin(f*x + e))/((c^4*d^4 - 2
*c^2*d^6 + d^8)*f*sin(f*x + e) + (c^5*d^3 - 2*c^3*d^5 + c*d^7)*f), -((2*b^3
*c^6 - 3*a*b^2*c^5*d - 4*b^3*c^4*d^2 + 6*a*b^2*c^3*d^3 + 2*b^3*c^2*d^4 - 3*
a*b^2*c*d^5)*f*x + (2*b^3*c^5 - 3*a*b^2*c^4*d - 3*b^3*c^3*d^2 - 3*a^2*b*c*d
^4 + (a^3 + 6*a*b^2)*c^2*d^3 + (2*b^3*c^4*d - 3*a*b^2*c^3*d^2 - 3*b^3*c^2*d
^3 - 3*a^2*b*d^5 + (a^3 + 6*a*b^2)*c*d^4)*sin(f*x + e))*sqrt(c^2 - d^2)*arc
tan(-(c*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) + (2*b^3*c^5*d -
3*a*b^2*c^4*d^2 + a^3*d^6 + 3*(a^2*b - b^3)*c^3*d^3 - (a^3 - 3*a*b^2)*c^2*d
^4 - (3*a^2*b - b^3)*c*d^5)*cos(f*x + e) + ((2*b^3*c^5*d - 3*a*b^2*c^4*d^2
- 4*b^3*c^3*d^3 + 6*a*b^2*c^2*d^4 + 2*b^3*c*d^5 - 3*a*b^2*d^6)*f*x + (b^3*c
^4*d^2 - 2*b^3*c^2*d^4 + b^3*d^6)*cos(f*x + e))*sin(f*x + e))/((c^4*d^4 - 2
*c^2*d^6 + d^8)*f*sin(f*x + e) + (c^5*d^3 - 2*c^3*d^5 + c*d^7)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**3/(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.40133, size = 790, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] (2*(2*b^3*c^4 - 3*a*b^2*c^3*d - 3*b^3*c^2*d^2 + a^3*c*d^3 + 6*a*b^2*c*d^3 -
 3*a^2*b*d^4)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*
f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/((c^2*d^3 - d^5)*sqrt(c^2 - d^2)) - 2*(
b^3*c^3*d*tan(1/2*f*x + 1/2*e)^3 - 3*a*b^2*c^2*d^2*tan(1/2*f*x + 1/2*e)^3 +
 3*a^2*b*c*d^3*tan(1/2*f*x + 1/2*e)^3 - a^3*d^4*tan(1/2*f*x + 1/2*e)^3 + 2*
b^3*c^4*tan(1/2*f*x + 1/2*e)^2 - 3*a*b^2*c^3*d*tan(1/2*f*x + 1/2*e)^2 + 3*a
^2*b*c^2*d^2*tan(1/2*f*x + 1/2*e)^2 - b^3*c^2*d^2*tan(1/2*f*x + 1/2*e)^2 -
a^3*c*d^3*tan(1/2*f*x + 1/2*e)^2 + 3*b^3*c^3*d*tan(1/2*f*x + 1/2*e) - 3*a*b
^2*c^2*d^2*tan(1/2*f*x + 1/2*e) + 3*a^2*b*c*d^3*tan(1/2*f*x + 1/2*e) - 2*b^
3*c*d^3*tan(1/2*f*x + 1/2*e) - a^3*d^4*tan(1/2*f*x + 1/2*e) + 2*b^3*c^4 - 3
*a*b^2*c^3*d + 3*a^2*b*c^2*d^2 - b^3*c^2*d^2 - a^3*c*d^3)/((c^3*d^2 - c*d^4
```

```
)*(c*tan(1/2*f*x + 1/2*e)^4 + 2*d*tan(1/2*f*x + 1/2*e)^3 + 2*c*tan(1/2*f*x
+ 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e) + c)) - (2*b^3*c - 3*a*b^2*d)*(f*x +
e)/d^3)/f
```

3.692 $\int \frac{(a+b\sin(e+fx))^3}{(c+d\sin(e+fx))^3}\,dx$

Optimal. Leaf size=255

$$-\frac{\left(9a^2bcd^4-a^3d^3\left(2c^2+d^2\right)-3ab^2d^3\left(c^2+2d^2\right)+b^3\left(-5c^3d^2+2c^5+6cd^4\right)\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{d^3f\left(c^2-d^2\right)^{5/2}}+\frac{(bc-ad)^2\left(3acd\right.}{2d^2f\left(c^2-d\right.}$$

```
[Out] (b^3*x)/d^3 - ((9*a^2*b*c*d^4 - a^3*d^3*(2*c^2 + d^2) - 3*a*b^2*d^3*(c^2 +
2*d^2) + b^3*(2*c^5 - 5*c^3*d^2 + 6*c*d^4))*ArcTan[(d + c*Tan[(e + f*x)/2])
/Sqrt[c^2 - d^2]])/(d^3*(c^2 - d^2)^(5/2)*f) + ((b*c - a*d)^2*Cos[e + f*x]*
(a + b*Sin[e + f*x]))/(2*d*(c^2 - d^2)*f*(c + d*Sin[e + f*x])^2) + ((b*c -
a*d)^2*(2*b*c^2 + 3*a*c*d - 5*b*d^2)*Cos[e + f*x])/(2*d^2*(c^2 - d^2)^2*f*(
c + d*Sin[e + f*x]))
```

Rubi [A] time = 0.631105, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {2792, 3021, 2735, 2660, 618, 204}

$$-\frac{\left(9a^2bcd^4-a^3d^3\left(2c^2+d^2\right)-3ab^2d^3\left(c^2+2d^2\right)+b^3\left(-5c^3d^2+2c^5+6cd^4\right)\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{d^3f\left(c^2-d^2\right)^{5/2}}+\frac{(bc-ad)^2\left(3acd\right.}{2d^2f\left(c^2-d\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^3/(c + d*Sin[e + f*x])^3,x]
```

```
[Out] (b^3*x)/d^3 - ((9*a^2*b*c*d^4 - a^3*d^3*(2*c^2 + d^2) - 3*a*b^2*d^3*(c^2 +
2*d^2) + b^3*(2*c^5 - 5*c^3*d^2 + 6*c*d^4))*ArcTan[(d + c*Tan[(e + f*x)/2])
/Sqrt[c^2 - d^2]])/(d^3*(c^2 - d^2)^(5/2)*f) + ((b*c - a*d)^2*Cos[e + f*x]*
(a + b*Sin[e + f*x]))/(2*d*(c^2 - d^2)*f*(c + d*Sin[e + f*x])^2) + ((b*c -
a*d)^2*(2*b*c^2 + 3*a*c*d - 5*b*d^2)*Cos[e + f*x])/(2*d^2*(c^2 - d^2)^2*f*(
c + d*Sin[e + f*x]))
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
```

```
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(a+b\sin(e+fx))^3}{(c+d\sin(e+fx))^3}\,dx = \frac{(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{2d\left(c^2-d^2\right)f(c+d\sin(e+fx))^2} - \frac{\int \frac{b^3c^2-2a^3cd-4ab^2cd+5a^2bd^2-\left(4a^2bcd+2b^3cd-a^3d^2+ab^2(}{(c+d\sin(e+fx)}}{2d\left(c^2-d^2\right.}$$

$$= \frac{(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{2d\left(c^2-d^2\right)f(c+d\sin(e+fx))^2} + \frac{(bc-ad)^2\left(2bc^2+3acd-5bd^2\right)\cos(e+fx)}{2d^2\left(c^2-d^2\right)^2f(c+d\sin(e+fx))}$$

$$= \frac{b^3x}{d^3} + \frac{(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{2d\left(c^2-d^2\right)f(c+d\sin(e+fx))^2} + \frac{(bc-ad)^2\left(2bc^2+3acd-5bd^2\right)\cos(e}{2d^2\left(c^2-d^2\right)^2f(c+d\sin(e+fx)}$$

$$= \frac{b^3x}{d^3} + \frac{(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{2d\left(c^2-d^2\right)f(c+d\sin(e+fx))^2} + \frac{(bc-ad)^2\left(2bc^2+3acd-5bd^2\right)\cos(e}{2d^2\left(c^2-d^2\right)^2f(c+d\sin(e+fx)}$$

$$= \frac{b^3x}{d^3} + \frac{(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{2d\left(c^2-d^2\right)f(c+d\sin(e+fx))^2} + \frac{(bc-ad)^2\left(2bc^2+3acd-5bd^2\right)\cos(e}{2d^2\left(c^2-d^2\right)^2f(c+d\sin(e+fx)}$$

$$= \frac{b^3x}{d^3} - \frac{\left(9a^2bcd^4-a^3d^3\left(2c^2+d^2\right)-3ab^2d^3\left(c^2+2d^2\right)+b^3\left(2c^5-5c^3d^2+6cd^4\right)\right)\tan^{-1}\Big(}{d^3\left(c^2-d^2\right)^{5/2}f}$$

Mathematica [B] time = 2.32157, size = 521, normalized size = 2.04

$$\frac{-3a^2bc^2d^4\sin(2(e+fx))-6a^2bd^6\sin(2(e+fx))+3a^3cd^5\sin(2(e+fx))-3ab^2c^3d^3\sin(2(e+fx))+12ab^2cd^5\sin(2(e+fx))-2d(bc-ad)^2\left(-4ac^2d+ad^3-2bc^3+5bcd^2\right)\cos(e}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^3/(c + d*Sin[e + f*x])^3,x]
```

```
[Out] ((-4*(9*a^2*b*c*d^4 - a^3*d^3*(2*c^2 + d^2) - 3*a*b^2*d^3*(c^2 + 2*d^2) + b
^3*(2*c^5 - 5*c^3*d^2 + 6*c*d^4))*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2
- d^2]])/(c^2 - d^2)^(5/2) + (4*b^3*c^6*e - 6*b^3*c^4*d^2*e + 2*b^3*d^6*e +
 4*b^3*c^6*f*x - 6*b^3*c^4*d^2*f*x + 2*b^3*d^6*f*x - 2*d*(b*c - a*d)^2*(-2*
b*c^3 - 4*a*c^2*d + 5*b*c*d^2 + a*d^3)*Cos[e + f*x] - 2*b^3*(-(c^2*d) + d^3
)^2*(e + f*x)*Cos[2*(e + f*x)] + 8*b^3*c^5*d*e*Sin[e + f*x] - 16*b^3*c^3*d^
3*e*Sin[e + f*x] + 8*b^3*c*d^5*e*Sin[e + f*x] + 8*b^3*c^5*d*f*x*Sin[e + f*x
] - 16*b^3*c^3*d^3*f*x*Sin[e + f*x] + 8*b^3*c*d^5*f*x*Sin[e + f*x] + 3*b^3*
c^4*d^2*Sin[2*(e + f*x)] - 3*a*b^2*c^3*d^3*Sin[2*(e + f*x)] - 3*a^2*b*c^2*d
^4*Sin[2*(e + f*x)] - 6*b^3*c^2*d^4*Sin[2*(e + f*x)] + 3*a^3*c*d^5*Sin[2*(e
 + f*x)] + 12*a*b^2*c*d^5*Sin[2*(e + f*x)] - 6*a^2*b*d^6*Sin[2*(e + f*x)])/
((c^2 - d^2)^2*(c + d*Sin[e + f*x])^2))/(4*d^3*f)
```

Maple [B] time = 0.108, size = 2785, normalized size = 10.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^3,x)
```

```
[Out] -1/f*d^3/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2
+d^4)*a^3+3/f/(c^4-2*c^2*d^2+d^4)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f
*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*a*b^2*c^2-5/f/(c*tan(1/2*f*x+1/2*e)^2+2*tan
(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)*b^3*c^3+2/f*b^3/d^3*arctan(tan(1
/2*f*x+1/2*e))+2/f/(c^4-2*c^2*d^2+d^4)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(
1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*a^3*c^2-6/f/(c*tan(1/2*f*x+1/2*e)^2+2*
tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)*a^2*b*c^3+4/f*d/(c*tan(1/2*f*
x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)*a^3*c^2+2/f/d^2/
(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)*b^3
*c^5+7/f*d^3/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2
*d^2+d^4)*tan(1/2*f*x+1/2*e)^2*a^3+1/f*d^2/(c^4-2*c^2*d^2+d^4)/(c^2-d^2)^(1
/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*a^3-6/f*d^4/(c
*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)/c*tan
(1/2*f*x+1/2*e)^2*a^2*b+9/f*d/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*
d+c)^2/(c^4-2*c^2*d^2+d^4)*c^2*tan(1/2*f*x+1/2*e)^2*a*b^2-15/f*d/(c*tan(1/2
*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2*c^2/(c^4-2*c^2*d^2+d^4)*tan(1/2*f
*x+1/2*e)*a^2*b+30/f*d^2/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^
2*c/(c^4-2*c^2*d^2+d^4)*tan(1/2*f*x+1/2*e)*a*b^2-9/f*d/(c^4-2*c^2*d^2+d^4)/
(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*a^
2*b*c-9/f*d/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*
d^2+d^4)*c^2*tan(1/2*f*x+1/2*e)^3*a^2*b+6/f*d^2/(c*tan(1/2*f*x+1/2*e)^2+2*t
an(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)*c*tan(1/2*f*x+1/2*e)^3*a*b^2-1
5/f*d^2/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+
d^4)*c*tan(1/2*f*x+1/2*e)^2*a^2*b-2/f*d^4/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2
```

```
*f*x+1/2*e)*d+c)^2/c/(c^4-2*c^2*d^2+d^4)*tan(1/2*f*x+1/2*e)*a^3+5/f/d/(c^4-
2*c^2*d^2+d^4)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2
-d^2)^(1/2))*b^3*c^3-6/f*d/(c^4-2*c^2*d^2+d^4)/(c^2-d^2)^(1/2)*arctan(1/2*(
2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*b^3*c+3/f/(c*tan(1/2*f*x+1/2*e
)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)*c^3*tan(1/2*f*x+1/2*e)^
3*a*b^2-6/f/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*
d^2+d^4)*c^3*tan(1/2*f*x+1/2*e)^2*a^2*b-3/f/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1
/2*f*x+1/2*e)*d+c)^2*c^3/(c^4-2*c^2*d^2+d^4)*tan(1/2*f*x+1/2*e)*a*b^2+7/f/d
/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2*c^4/(c^4-2*c^2*d^2+d^4
)*tan(1/2*f*x+1/2*e)*b^3-16/f*d/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e
)*d+c)^2*c^2/(c^4-2*c^2*d^2+d^4)*tan(1/2*f*x+1/2*e)*b^3-3/f*d^2/(c*tan(1/2*
f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)*a^2*b*c+9/f*d/
(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)*a*b
^2*c^2+5/f*d^2/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c
^2*d^2+d^4)*c*tan(1/2*f*x+1/2*e)^3*a^3-2/f*d^4/(c*tan(1/2*f*x+1/2*e)^2+2*ta
n(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)/c*tan(1/2*f*x+1/2*e)^3*a^3+1/f/
d/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)*c
^4*tan(1/2*f*x+1/2*e)^3*b^3-4/f*d/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2
*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)*c^2*tan(1/2*f*x+1/2*e)^3*b^3+4/f*d/(c*tan(1/
2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)*c^2*tan(1/2*
f*x+1/2*e)^2*a^3-2/f*d^5/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^
2/(c^4-2*c^2*d^2+d^4)/c^2*tan(1/2*f*x+1/2*e)^2*a^3+2/f/d^2/(c*tan(1/2*f*x+1
/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)*c^5*tan(1/2*f*x+1/2
*e)^2*b^3-10/f*d^2/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4
-2*c^2*d^2+d^4)*c*tan(1/2*f*x+1/2*e)^2*b^3+11/f*d^2/(c*tan(1/2*f*x+1/2*e)^2
+2*tan(1/2*f*x+1/2*e)*d+c)^2*c/(c^4-2*c^2*d^2+d^4)*tan(1/2*f*x+1/2*e)*a^3+1
8/f*d^3/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+
d^4)*tan(1/2*f*x+1/2*e)^2*a*b^2-12/f*d^3/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*
f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)*tan(1/2*f*x+1/2*e)*a^2*b-2/f/d^3/(c^4
-2*c^2*d^2+d^4)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^
2-d^2)^(1/2))*b^3*c^5+6/f*d^2/(c^4-2*c^2*d^2+d^4)/(c^2-d^2)^(1/2)*arctan(1/
2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*a*b^2-1/f/(c*tan(1/2*f*x+1/
2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)*c^3*tan(1/2*f*x+1/2*
e)^2*b^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^3,x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [B] time = 2.51203, size = 3468, normalized size = 13.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^3,x, algorithm="fricas")

[Out] [1/4*(4*(b^3*c^6*d^2 - 3*b^3*c^4*d^4 + 3*b^3*c^2*d^6 - b^3*d^8)*f*x*cos(f*x
 + e)^2 - 4*(b^3*c^8 - 2*b^3*c^6*d^2 + 2*b^3*c^2*d^6 - b^3*d^8)*f*x - (2*b^
```

```
3*c^7 - 3*b^3*c^5*d^2 - (2*a^3 + 3*a*b^2)*c^4*d^3 + (9*a^2*b + b^3)*c^3*d^4
 - 3*(a^3 + 3*a*b^2)*c^2*d^5 + 3*(3*a^2*b + 2*b^3)*c*d^6 - (a^3 + 6*a*b^2)*
d^7 - (2*b^3*c^5*d^2 - 5*b^3*c^3*d^4 - (2*a^3 + 3*a*b^2)*c^2*d^5 + 3*(3*a^2
*b + 2*b^3)*c*d^6 - (a^3 + 6*a*b^2)*d^7)*cos(f*x + e)^2 + 2*(2*b^3*c^6*d -
5*b^3*c^4*d^3 - (2*a^3 + 3*a*b^2)*c^3*d^4 + 3*(3*a^2*b + 2*b^3)*c^2*d^5 - (
a^3 + 6*a*b^2)*c*d^6)*sin(f*x + e))*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos
(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x +
e) + d*cos(f*x + e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x
+ e) - c^2 - d^2)) - 2*(2*b^3*c^7*d + 3*a^2*b*c*d^7 + a^3*d^8 - (6*a^2*b +
7*b^3)*c^5*d^3 + (4*a^3 + 9*a*b^2)*c^4*d^4 + (3*a^2*b + 5*b^3)*c^3*d^5 - (5
*a^3 + 9*a*b^2)*c^2*d^6)*cos(f*x + e) - 2*(4*(b^3*c^7*d - 3*b^3*c^5*d^3 + 3
*b^3*c^3*d^5 - b^3*c*d^7)*f*x + 3*(b^3*c^6*d^2 - a*b^2*c^5*d^3 + 2*a^2*b*d^
8 - (a^2*b + 3*b^3)*c^4*d^4 + (a^3 + 5*a*b^2)*c^3*d^5 - (a^2*b - 2*b^3)*c^2
*d^6 - (a^3 + 4*a*b^2)*c*d^7)*cos(f*x + e))*sin(f*x + e))/((c^6*d^5 - 3*c^4
*d^7 + 3*c^2*d^9 - d^11)*f*cos(f*x + e)^2 - 2*(c^7*d^4 - 3*c^5*d^6 + 3*c^3*
d^8 - c*d^10)*f*sin(f*x + e) - (c^8*d^3 - 2*c^6*d^5 + 2*c^2*d^9 - d^11)*f),
 1/2*(2*(b^3*c^6*d^2 - 3*b^3*c^4*d^4 + 3*b^3*c^2*d^6 - b^3*d^8)*f*x*cos(f*x
 + e)^2 - 2*(b^3*c^8 - 2*b^3*c^6*d^2 + 2*b^3*c^2*d^6 - b^3*d^8)*f*x - (2*b^
3*c^7 - 3*b^3*c^5*d^2 - (2*a^3 + 3*a*b^2)*c^4*d^3 + (9*a^2*b + b^3)*c^3*d^4
 - 3*(a^3 + 3*a*b^2)*c^2*d^5 + 3*(3*a^2*b + 2*b^3)*c*d^6 - (a^3 + 6*a*b^2)*
d^7 - (2*b^3*c^5*d^2 - 5*b^3*c^3*d^4 - (2*a^3 + 3*a*b^2)*c^2*d^5 + 3*(3*a^2
*b + 2*b^3)*c*d^6 - (a^3 + 6*a*b^2)*d^7)*cos(f*x + e)^2 + 2*(2*b^3*c^6*d -
5*b^3*c^4*d^3 - (2*a^3 + 3*a*b^2)*c^3*d^4 + 3*(3*a^2*b + 2*b^3)*c^2*d^5 - (
a^3 + 6*a*b^2)*c*d^6)*sin(f*x + e))*sqrt(c^2 - d^2)*arctan(-(c*sin(f*x + e)
 + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) - (2*b^3*c^7*d + 3*a^2*b*c*d^7 + a^3*
d^8 - (6*a^2*b + 7*b^3)*c^5*d^3 + (4*a^3 + 9*a*b^2)*c^4*d^4 + (3*a^2*b + 5*
b^3)*c^3*d^5 - (5*a^3 + 9*a*b^2)*c^2*d^6)*cos(f*x + e) - (4*(b^3*c^7*d - 3*
b^3*c^5*d^3 + 3*b^3*c^3*d^5 - b^3*c*d^7)*f*x + 3*(b^3*c^6*d^2 - a*b^2*c^5*d
^3 + 2*a^2*b*d^8 - (a^2*b + 3*b^3)*c^4*d^4 + (a^3 + 5*a*b^2)*c^3*d^5 - (a^2
*b - 2*b^3)*c^2*d^6 - (a^3 + 4*a*b^2)*c*d^7)*cos(f*x + e))*sin(f*x + e))/((
c^6*d^5 - 3*c^4*d^7 + 3*c^2*d^9 - d^11)*f*cos(f*x + e)^2 - 2*(c^7*d^4 - 3*c
^5*d^6 + 3*c^3*d^8 - c*d^10)*f*sin(f*x + e) - (c^8*d^3 - 2*c^6*d^5 + 2*c^2*
d^9 - d^11)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**3/(c+d*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.45529, size = 1200, normalized size = 4.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] ((f*x + e)*b^3/d^3 - (2*b^3*c^5 - 5*b^3*c^3*d^2 - 2*a^3*c^2*d^3 - 3*a*b^2*c
^2*d^3 + 9*a^2*b*c*d^4 + 6*b^3*c*d^4 - a^3*d^5 - 6*a*b^2*d^5)*(pi*floor(1/2
```

```
*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2
- d^2)))/((c^4*d^3 - 2*c^2*d^5 + d^7)*sqrt(c^2 - d^2)) + (b^3*c^6*d*tan(1/2
*f*x + 1/2*e)^3 + 3*a*b^2*c^5*d^2*tan(1/2*f*x + 1/2*e)^3 - 9*a^2*b*c^4*d^3*
tan(1/2*f*x + 1/2*e)^3 - 4*b^3*c^4*d^3*tan(1/2*f*x + 1/2*e)^3 + 5*a^3*c^3*d
^4*tan(1/2*f*x + 1/2*e)^3 + 6*a*b^2*c^3*d^4*tan(1/2*f*x + 1/2*e)^3 - 2*a^3*
c*d^6*tan(1/2*f*x + 1/2*e)^3 + 2*b^3*c^7*tan(1/2*f*x + 1/2*e)^2 - 6*a^2*b*c
^5*d^2*tan(1/2*f*x + 1/2*e)^2 - b^3*c^5*d^2*tan(1/2*f*x + 1/2*e)^2 + 4*a^3*
c^4*d^3*tan(1/2*f*x + 1/2*e)^2 + 9*a*b^2*c^4*d^3*tan(1/2*f*x + 1/2*e)^2 - 1
5*a^2*b*c^3*d^4*tan(1/2*f*x + 1/2*e)^2 - 10*b^3*c^3*d^4*tan(1/2*f*x + 1/2*e
)^2 + 7*a^3*c^2*d^5*tan(1/2*f*x + 1/2*e)^2 + 18*a*b^2*c^2*d^5*tan(1/2*f*x +
 1/2*e)^2 - 6*a^2*b*c*d^6*tan(1/2*f*x + 1/2*e)^2 - 2*a^3*d^7*tan(1/2*f*x +
1/2*e)^2 + 7*b^3*c^6*d*tan(1/2*f*x + 1/2*e) - 3*a*b^2*c^5*d^2*tan(1/2*f*x +
 1/2*e) - 15*a^2*b*c^4*d^3*tan(1/2*f*x + 1/2*e) - 16*b^3*c^4*d^3*tan(1/2*f*
x + 1/2*e) + 11*a^3*c^3*d^4*tan(1/2*f*x + 1/2*e) + 30*a*b^2*c^3*d^4*tan(1/2
*f*x + 1/2*e) - 12*a^2*b*c^2*d^5*tan(1/2*f*x + 1/2*e) - 2*a^3*c*d^6*tan(1/2
*f*x + 1/2*e) + 2*b^3*c^7 - 6*a^2*b*c^5*d^2 - 5*b^3*c^5*d^2 + 4*a^3*c^4*d^3
 + 9*a*b^2*c^4*d^3 - 3*a^2*b*c^3*d^4 - a^3*c^2*d^5)/((c^6*d^2 - 2*c^4*d^4 +
 c^2*d^6)*(c*tan(1/2*f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e) + c)^2))/f
```

3.693 $\int \frac{(a+b\sin(e+fx))^3}{(c+d\sin(e+fx))^4}\,dx$

Optimal. Leaf size=325

$$-\frac{(ac-bd)\left(a^2\left(-\left(2c^2+3d^2\right)\right)+10abcd-b^2\left(3c^2+2d^2\right)\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{f\left(c^2-d^2\right)^{7/2}}-\frac{\left(a^2d^2\left(11c^2+4d^2\right)+5abcd\left(c^2-7d^2\right.\right.}{6d^2f\left(c^2-\right.}$$

```
[Out] -(((a*c - b*d)*(10*a*b*c*d - b^2*(3*c^2 + 2*d^2) - a^2*(2*c^2 + 3*d^2))*Arc
Tan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/((c^2 - d^2)^(7/2)*f)) + ((b
*c - a*d)^2*Cos[e + f*x]*(a + b*Sin[e + f*x]))/(3*d*(c^2 - d^2)*f*(c + d*Si
n[e + f*x])^3) + ((b*c - a*d)^2*(2*b*c^2 + 5*a*c*d - 7*b*d^2)*Cos[e + f*x])
/(6*d^2*(c^2 - d^2)^2*f*(c + d*Sin[e + f*x])^2) - ((b*c - a*d)*(5*a*b*c*d*(
c^2 - 7*d^2) + a^2*d^2*(11*c^2 + 4*d^2) + b^2*(2*c^4 - 5*c^2*d^2 + 18*d^4))
*Cos[e + f*x])/(6*d^2*(c^2 - d^2)^3*f*(c + d*Sin[e + f*x]))
```

Rubi [A] time = 0.724919, antiderivative size = 325, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {2792, 3021, 2754, 12, 2660, 618, 204}

$$-\frac{(ac-bd)\left(a^2\left(-\left(2c^2+3d^2\right)\right)+10abcd-b^2\left(3c^2+2d^2\right)\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{f\left(c^2-d^2\right)^{7/2}}-\frac{\left(a^2d^2\left(11c^2+4d^2\right)+5abcd\left(c^2-7d^2\right.\right.}{6d^2f\left(c^2-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^3/(c + d*Sin[e + f*x])^4,x]
```

```
[Out] -(((a*c - b*d)*(10*a*b*c*d - b^2*(3*c^2 + 2*d^2) - a^2*(2*c^2 + 3*d^2))*Arc
Tan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/((c^2 - d^2)^(7/2)*f)) + ((b
*c - a*d)^2*Cos[e + f*x]*(a + b*Sin[e + f*x]))/(3*d*(c^2 - d^2)*f*(c + d*Si
n[e + f*x])^3) + ((b*c - a*d)^2*(2*b*c^2 + 5*a*c*d - 7*b*d^2)*Cos[e + f*x])
/(6*d^2*(c^2 - d^2)^2*f*(c + d*Sin[e + f*x])^2) - ((b*c - a*d)*(5*a*b*c*d*(
c^2 - 7*d^2) + a^2*d^2*(11*c^2 + 4*d^2) + b^2*(2*c^4 - 5*c^2*d^2 + 18*d^4))
*Cos[e + f*x])/(6*d^2*(c^2 - d^2)^3*f*(c + d*Sin[e + f*x]))
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
```

```
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(a+b\sin(e+fx))^3}{(c+d\sin(e+fx))^4}dx = \frac{(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{3d(c^2-d^2)f(c+d\sin(e+fx))^3} - \frac{\int \frac{b^3c^2-3a^3cd-5ab^2cd+7a^2bd^2-(7a^2bcd+3b^3cd-2a^3d^2+ab^2(c^2-}{(c+d\sin(e}}{3d(c^2}$$

$$= \frac{(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{3d(c^2-d^2)f(c+d\sin(e+fx))^3} + \frac{(bc-ad)^2(2bc^2+5acd-7bd^2)\cos(e+fx)}{6d^2(c^2-d^2)^2 f(c+d\sin(e+fx))^2} + \int$$

$$= \frac{(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{3d(c^2-d^2)f(c+d\sin(e+fx))^3} + \frac{(bc-ad)^2(2bc^2+5acd-7bd^2)\cos(e+fx)}{6d^2(c^2-d^2)^2 f(c+d\sin(e+fx))^2} - (b$$

$$= \frac{(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{3d(c^2-d^2)f(c+d\sin(e+fx))^3} + \frac{(bc-ad)^2(2bc^2+5acd-7bd^2)\cos(e+fx)}{6d^2(c^2-d^2)^2 f(c+d\sin(e+fx))^2} - (b$$

$$= \frac{(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{3d(c^2-d^2)f(c+d\sin(e+fx))^3} + \frac{(bc-ad)^2(2bc^2+5acd-7bd^2)\cos(e+fx)}{6d^2(c^2-d^2)^2 f(c+d\sin(e+fx))^2} - (b$$

$$= \frac{(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{3d(c^2-d^2)f(c+d\sin(e+fx))^3} + \frac{(bc-ad)^2(2bc^2+5acd-7bd^2)\cos(e+fx)}{6d^2(c^2-d^2)^2 f(c+d\sin(e+fx))^2} - (b$$

$$= -\frac{(ac-bd)\left(10abcd-b^2(3c^2+2d^2)-a^2(2c^2+3d^2)\right)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{(c^2-d^2)^{7/2}f} + \frac{(bc-ad)^2 c}{3d(c^2-}$$

Mathematica [A] time = 5.15598, size = 345, normalized size = 1.06

$$\frac{\frac{6\left(-3a^2bd(4c^2+d^2)+a^3(2c^3+3cd^2)+3ab^2c(c^2+4d^2)-b^3d(3c^2+2d^2)\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{(c^2-d^2)^{7/2}} + \frac{\left(3a^2bcd^2(2c^2+13d^2)-a^3d^3(11c^2+4d^2)+3ab^2d(-10c^2d^2+c^4-6d^4)\right.}{d^2(d^2-c^2)^3(c+d\sin(e+fx))}}{6f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^3/(c + d*Sin[e + f*x])^4,x]
```

```
[Out] ((6*(-3*a^2*b*d*(4*c^2 + d^2) - b^3*d*(3*c^2 + 2*d^2) + 3*a*b^2*c*(c^2 + 4*
d^2) + a^3*(2*c^3 + 3*c*d^2))*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^
2]])/(c^2 - d^2)^(7/2) + (2*(b*c - a*d)^3*Cos[e + f*x])/(d^2*(-c^2 + d^2)*(
c + d*Sin[e + f*x])^3) + ((b*c - a*d)^2*(4*b*c^2 + 5*a*c*d - 9*b*d^2)*Cos[e
 + f*x])/(d^2*(c^2 - d^2)^2*(c + d*Sin[e + f*x])^2) + ((-(a^3*d^3*(11*c^2 +
 4*d^2)) + 3*a^2*b*c*d^2*(2*c^2 + 13*d^2) + 3*a*b^2*d*(c^4 - 10*c^2*d^2 - 6
*d^4) + b^3*(2*c^5 - 5*c^3*d^2 + 18*c*d^4))*Cos[e + f*x])/(d^2*(-c^2 + d^2)
^3*(c + d*Sin[e + f*x])))/(6*f)
```

Maple [B] time = 0.125, size = 6128, normalized size = 18.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^4,x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.10513, size = 4540, normalized size = 13.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^4,x, algorithm="fricas")
```

```
[Out] [-1/12*(2*(2*b^3*c^7 + 3*a*b^2*c^6*d + (6*a^2*b - 7*b^3)*c^5*d^2 - 11*(a^3
+ 3*a*b^2)*c^4*d^3 + (33*a^2*b + 23*b^3)*c^3*d^4 + (7*a^3 + 12*a*b^2)*c^2*d
^5 - 3*(13*a^2*b + 6*b^3)*c*d^6 + 2*(2*a^3 + 9*a*b^2)*d^7)*cos(f*x + e)^3 -
 6*(3*a*b^2*c^7 + 3*a^2*b*d^7 + (6*a^2*b + b^3)*c^6*d - 3*(3*a^3 + 10*a*b^2
)*c^5*d^2 + (21*a^2*b + 8*b^3)*c^4*d^3 + (8*a^3 + 21*a*b^2)*c^3*d^4 - 3*(10
*a^2*b + 3*b^3)*c^2*d^5 + (a^3 + 6*a*b^2)*c*d^6)*cos(f*x + e)*sin(f*x + e)
- 3*((2*a^3 + 3*a*b^2)*c^6 - 3*(4*a^2*b + b^3)*c^5*d + 3*(3*a^3 + 7*a*b^2)*
c^4*d^2 - (39*a^2*b + 11*b^3)*c^3*d^3 + 9*(a^3 + 4*a*b^2)*c^2*d^4 - 3*(3*a^
2*b + 2*b^3)*c*d^5 - 3*((2*a^3 + 3*a*b^2)*c^4*d^2 - 3*(4*a^2*b + b^3)*c^3*d
^3 + 3*(a^3 + 4*a*b^2)*c^2*d^4 - (3*a^2*b + 2*b^3)*c*d^5)*cos(f*x + e)^2 +
(3*(2*a^3 + 3*a*b^2)*c^5*d - 9*(4*a^2*b + b^3)*c^4*d^2 + (11*a^3 + 39*a*b^2
)*c^3*d^3 - 3*(7*a^2*b + 3*b^3)*c^2*d^4 + 3*(a^3 + 4*a*b^2)*c*d^5 - (3*a^2*
b + 2*b^3)*d^6 - ((2*a^3 + 3*a*b^2)*c^3*d^3 - 3*(4*a^2*b + b^3)*c^2*d^4 + 3
*(a^3 + 4*a*b^2)*c*d^5 - (3*a^2*b + 2*b^3)*d^6)*cos(f*x + e)^2)*sin(f*x + e
))*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e)
- c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x + e) + d*cos(f*x + e))*sqrt(-c^2 +
d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) - 12*(3*a^2*b*
c^5*d^2 + 2*a^3*c^4*d^3 + 2*b^3*c^3*d^4 + 3*a*b^2*c^2*d^5 + (3*a^2*b + b^3)
*c^7 - 3*(a^3 + 2*a*b^2)*c^6*d - 3*(2*a^2*b + b^3)*c*d^6 + (a^3 + 3*a*b^2)*
d^7)*cos(f*x + e))/(3*(c^9*d^2 - 4*c^7*d^4 + 6*c^5*d^6 - 4*c^3*d^8 + c*d^10
)*f*cos(f*x + e)^2 - (c^11 - c^9*d^2 - 6*c^7*d^4 + 14*c^5*d^6 - 11*c^3*d^8
+ 3*c*d^10)*f + ((c^8*d^3 - 4*c^6*d^5 + 6*c^4*d^7 - 4*c^2*d^9 + d^11)*f*cos
(f*x + e)^2 - (3*c^10*d - 11*c^8*d^3 + 14*c^6*d^5 - 6*c^4*d^7 - c^2*d^9 + d
^11)*f)*sin(f*x + e)), -1/6*((2*b^3*c^7 + 3*a*b^2*c^6*d + (6*a^2*b - 7*b^3)
*c^5*d^2 - 11*(a^3 + 3*a*b^2)*c^4*d^3 + (33*a^2*b + 23*b^3)*c^3*d^4 + (7*a^
3 + 12*a*b^2)*c^2*d^5 - 3*(13*a^2*b + 6*b^3)*c*d^6 + 2*(2*a^3 + 9*a*b^2)*d^
7)*cos(f*x + e)^3 - 3*(3*a*b^2*c^7 + 3*a^2*b*d^7 + (6*a^2*b + b^3)*c^6*d -
3*(3*a^3 + 10*a*b^2)*c^5*d^2 + (21*a^2*b + 8*b^3)*c^4*d^3 + (8*a^3 + 21*a*b
^2)*c^3*d^4 - 3*(10*a^2*b + 3*b^3)*c^2*d^5 + (a^3 + 6*a*b^2)*c*d^6)*cos(f*x
 + e)*sin(f*x + e) - 3*((2*a^3 + 3*a*b^2)*c^6 - 3*(4*a^2*b + b^3)*c^5*d + 3
*(3*a^3 + 7*a*b^2)*c^4*d^2 - (39*a^2*b + 11*b^3)*c^3*d^3 + 9*(a^3 + 4*a*b^2
)*c^2*d^4 - 3*(3*a^2*b + 2*b^3)*c*d^5 - 3*((2*a^3 + 3*a*b^2)*c^4*d^2 - 3*(4
*a^2*b + b^3)*c^3*d^3 + 3*(a^3 + 4*a*b^2)*c^2*d^4 - (3*a^2*b + 2*b^3)*c*d^5
)*cos(f*x + e)^2 + (3*(2*a^3 + 3*a*b^2)*c^5*d - 9*(4*a^2*b + b^3)*c^4*d^2 +
 (11*a^3 + 39*a*b^2)*c^3*d^3 - 3*(7*a^2*b + 3*b^3)*c^2*d^4 + 3*(a^3 + 4*a*b
```

```
^2)*c*d^5 - (3*a^2*b + 2*b^3)*d^6 - ((2*a^3 + 3*a*b^2)*c^3*d^3 - 3*(4*a^2*b
 + b^3)*c^2*d^4 + 3*(a^3 + 4*a*b^2)*c*d^5 - (3*a^2*b + 2*b^3)*d^6)*cos(f*x
+ e)^2)*sin(f*x + e))*sqrt(c^2 - d^2)*arctan(-(c*sin(f*x + e) + d)/(sqrt(c^
2 - d^2)*cos(f*x + e))) - 6*(3*a^2*b*c^5*d^2 + 2*a^3*c^4*d^3 + 2*b^3*c^3*d^
4 + 3*a*b^2*c^2*d^5 + (3*a^2*b + b^3)*c^7 - 3*(a^3 + 2*a*b^2)*c^6*d - 3*(2*
a^2*b + b^3)*c*d^6 + (a^3 + 3*a*b^2)*d^7)*cos(f*x + e))/(3*(c^9*d^2 - 4*c^7
*d^4 + 6*c^5*d^6 - 4*c^3*d^8 + c*d^10)*f*cos(f*x + e)^2 - (c^11 - c^9*d^2 -
 6*c^7*d^4 + 14*c^5*d^6 - 11*c^3*d^8 + 3*c*d^10)*f + ((c^8*d^3 - 4*c^6*d^5
+ 6*c^4*d^7 - 4*c^2*d^9 + d^11)*f*cos(f*x + e)^2 - (3*c^10*d - 11*c^8*d^3 +
 14*c^6*d^5 - 6*c^4*d^7 - c^2*d^9 + d^11)*f)*sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**3/(c+d*sin(f*x+e))**4,x)

[Out] Timed out
```

Giac [B] time = 1.48679, size = 2264, normalized size = 6.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^4,x, algorithm="giac")

[Out] 1/3*(3*(2*a^3*c^3 + 3*a*b^2*c^3 - 12*a^2*b*c^2*d - 3*b^3*c^2*d + 3*a^3*c*d^
2 + 12*a*b^2*c*d^2 - 3*a^2*b*d^3 - 2*b^3*d^3)*(pi*floor(1/2*(f*x + e)/pi +
1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/((c^6 -
 3*c^4*d^2 + 3*c^2*d^4 - d^6)*sqrt(c^2 - d^2)) + (9*a*b^2*c^8*tan(1/2*f*x +
 1/2*e)^5 - 36*a^2*b*c^7*d*tan(1/2*f*x + 1/2*e)^5 - 9*b^3*c^7*d*tan(1/2*f*x
 + 1/2*e)^5 + 27*a^3*c^6*d^2*tan(1/2*f*x + 1/2*e)^5 + 36*a*b^2*c^6*d^2*tan(
1/2*f*x + 1/2*e)^5 - 9*a^2*b*c^5*d^3*tan(1/2*f*x + 1/2*e)^5 - 6*b^3*c^5*d^3
*tan(1/2*f*x + 1/2*e)^5 - 18*a^3*c^4*d^4*tan(1/2*f*x + 1/2*e)^5 + 6*a^3*c^2
*d^6*tan(1/2*f*x + 1/2*e)^5 - 18*a^2*b*c^8*tan(1/2*f*x + 1/2*e)^4 + 18*a^3*
c^7*d*tan(1/2*f*x + 1/2*e)^4 + 45*a*b^2*c^7*d*tan(1/2*f*x + 1/2*e)^4 - 126*
a^2*b*c^6*d^2*tan(1/2*f*x + 1/2*e)^4 - 45*b^3*c^6*d^2*tan(1/2*f*x + 1/2*e)^
4 + 81*a^3*c^5*d^3*tan(1/2*f*x + 1/2*e)^4 + 180*a*b^2*c^5*d^3*tan(1/2*f*x +
 1/2*e)^4 - 99*a^2*b*c^4*d^4*tan(1/2*f*x + 1/2*e)^4 - 30*b^3*c^4*d^4*tan(1/
2*f*x + 1/2*e)^4 - 36*a^3*c^3*d^5*tan(1/2*f*x + 1/2*e)^4 + 18*a^2*b*c^2*d^6
*tan(1/2*f*x + 1/2*e)^4 + 12*a^3*c*d^7*tan(1/2*f*x + 1/2*e)^4 - 108*a^2*b*c
^7*d*tan(1/2*f*x + 1/2*e)^3 - 24*b^3*c^7*d*tan(1/2*f*x + 1/2*e)^3 + 108*a^3
*c^6*d^2*tan(1/2*f*x + 1/2*e)^3 + 234*a*b^2*c^6*d^2*tan(1/2*f*x + 1/2*e)^3
- 252*a^2*b*c^5*d^3*tan(1/2*f*x + 1/2*e)^3 - 82*b^3*c^5*d^3*tan(1/2*f*x + 1
/2*e)^3 + 42*a^3*c^4*d^4*tan(1/2*f*x + 1/2*e)^3 + 192*a*b^2*c^4*d^4*tan(1/2
*f*x + 1/2*e)^3 - 102*a^2*b*c^3*d^5*tan(1/2*f*x + 1/2*e)^3 - 44*b^3*c^3*d^5
*tan(1/2*f*x + 1/2*e)^3 - 8*a^3*c^2*d^6*tan(1/2*f*x + 1/2*e)^3 + 24*a*b^2*c
^2*d^6*tan(1/2*f*x + 1/2*e)^3 + 12*a^2*b*c*d^7*tan(1/2*f*x + 1/2*e)^3 + 8*a
^3*d^8*tan(1/2*f*x + 1/2*e)^3 - 36*a^2*b*c^8*tan(1/2*f*x + 1/2*e)^2 - 12*b^
3*c^8*tan(1/2*f*x + 1/2*e)^2 + 36*a^3*c^7*d*tan(1/2*f*x + 1/2*e)^2 + 72*a*b
^2*c^7*d*tan(1/2*f*x + 1/2*e)^2 - 180*a^2*b*c^6*d^2*tan(1/2*f*x + 1/2*e)^2
```

```
- 36*b^3*c^6*d^2*tan(1/2*f*x + 1/2*e)^2 + 120*a^3*c^5*d^3*tan(1/2*f*x + 1/2
*e)^2 + 306*a*b^2*c^5*d^3*tan(1/2*f*x + 1/2*e)^2 - 252*a^2*b*c^4*d^4*tan(1/
2*f*x + 1/2*e)^2 - 102*b^3*c^4*d^4*tan(1/2*f*x + 1/2*e)^2 - 18*a^3*c^3*d^5*
tan(1/2*f*x + 1/2*e)^2 + 72*a*b^2*c^3*d^5*tan(1/2*f*x + 1/2*e)^2 + 18*a^2*b
*c^2*d^6*tan(1/2*f*x + 1/2*e)^2 + 12*a^3*c*d^7*tan(1/2*f*x + 1/2*e)^2 - 9*a
*b^2*c^8*tan(1/2*f*x + 1/2*e) - 72*a^2*b*c^7*d*tan(1/2*f*x + 1/2*e) - 15*b^
3*c^7*d*tan(1/2*f*x + 1/2*e) + 81*a^3*c^6*d^2*tan(1/2*f*x + 1/2*e) + 198*a*
b^2*c^6*d^2*tan(1/2*f*x + 1/2*e) - 171*a^2*b*c^5*d^3*tan(1/2*f*x + 1/2*e) -
 60*b^3*c^5*d^3*tan(1/2*f*x + 1/2*e) - 12*a^3*c^4*d^4*tan(1/2*f*x + 1/2*e)
+ 36*a*b^2*c^4*d^4*tan(1/2*f*x + 1/2*e) + 18*a^2*b*c^3*d^5*tan(1/2*f*x + 1/
2*e) + 6*a^3*c^2*d^6*tan(1/2*f*x + 1/2*e) - 18*a^2*b*c^8 - 4*b^3*c^8 + 18*a
^3*c^7*d + 39*a*b^2*c^7*d - 30*a^2*b*c^6*d^2 - 11*b^3*c^6*d^2 - 5*a^3*c^5*d
^3 + 6*a*b^2*c^5*d^3 + 3*a^2*b*c^4*d^4 + 2*a^3*c^3*d^5)/((c^9 - 3*c^7*d^2 +
 3*c^5*d^4 - c^3*d^6)*(c*tan(1/2*f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e)
+ c)^3))/f
```

3.694 $\int \frac{\frac{bB}{a}+B\sin(x)}{a+b\sin(x)}\,dx$

Optimal. Leaf size=54

$$\frac{Bx}{b}-\frac{2B\sqrt{a^2-b^2}\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{ab}$$

```
[Out] (B*x)/b - (2*Sqrt[a^2 - b^2]*B*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(a
*b)
```

Rubi [A] time = 0.0816141, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2735, 2660, 618, 204}

$$\frac{Bx}{b}-\frac{2B\sqrt{a^2-b^2}\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{ab}$$

Antiderivative was successfully verified.

```
[In] Int[((b*B)/a + B*Sin[x])/(a + b*Sin[x]),x]
```

```
[Out] (B*x)/b - (2*Sqrt[a^2 - b^2]*B*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(a
*b)
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\frac{bB}{a}+B\sin(x)}{a+b\sin(x)}\,dx = \frac{Bx}{b}-\frac{\left(aB-\frac{b^2B}{a}\right)\int \frac{1}{a+b\sin(x)}\,dx}{b}$$
$$=\frac{Bx}{b}-\frac{\left(2\left(aB-\frac{b^2B}{a}\right)\right)\text{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx,x,\tan\left(\frac{x}{2}\right)\right)}{b}$$
$$=\frac{Bx}{b}+\frac{\left(4\left(aB-\frac{b^2B}{a}\right)\right)\text{Subst}\left(\int \frac{1}{-4(a^2-b^2)-x^2}\,dx,x,2b+2a\tan\left(\frac{x}{2}\right)\right)}{b}$$
$$=\frac{Bx}{b}-\frac{2\sqrt{a^2-b^2}B\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{ab}$$

Mathematica [A] time = 0.0748122, size = 52, normalized size = 0.96

$$\frac{B\left(ax-2\sqrt{a^2-b^2}\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{ab}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*B)/a + B*Sin[x])/(a + b*Sin[x]),x]
```

```
[Out] (B*(a*x - 2*Sqrt[a^2 - b^2]*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]]))/(a*b
)
```

Maple [B] time = 0.037, size = 99, normalized size = 1.8

$$2\,\frac{B\arctan(\tan(x/2))}{b}-2\,\frac{Ba}{b\sqrt{a^2-b^2}}\arctan\left(1/2\,\frac{2\,a\tan(x/2)+2\,b}{\sqrt{a^2-b^2}}\right)+2\,\frac{bB}{a\sqrt{a^2-b^2}}\arctan\left(1/2\,\frac{2\,a\tan(x/2)+2\,b}{\sqrt{a^2-b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*B/a+B*sin(x))/(a+b*sin(x)),x)
```

```
[Out] 2*B/b*arctan(tan(1/2*x))-2*B*a/b/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*x)
+2*b)/(a^2-b^2)^(1/2))+2*B/a*b/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*x)+2
*b)/(a^2-b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*B/a+B*sin(x))/(a+b*sin(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.57714, size = 377, normalized size = 6.98

$$\left[\frac{2\,Bax+\sqrt{-a^2+b^2}B\log\left(\frac{\left(2\,a^2-b^2\right)\cos(x)^2-2\,ab\sin(x)-a^2-b^2+2\,(a\cos(x)\sin(x)+b\cos(x))\sqrt{-a^2+b^2}}{b^2\cos(x)^2-2\,ab\sin(x)-a^2-b^2}\right)}{2\,ab},\ \frac{Bax+\sqrt{a^2-b^2}B\arctan\left(-\frac{a\sin(x}{\sqrt{a^2-b^2}}\right.}{ab}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*B/a+B*sin(x))/(a+b*sin(x)),x, algorithm="fricas")
```

```
[Out] [1/2*(2*B*a*x + sqrt(-a^2 + b^2)*B*log(((2*a^2 - b^2)*cos(x)^2 - 2*a*b*sin(
x) - a^2 - b^2 + 2*(a*cos(x)*sin(x) + b*cos(x))*sqrt(-a^2 + b^2))/(b^2*cos(
x)^2 - 2*a*b*sin(x) - a^2 - b^2)))/(a*b), (B*a*x + sqrt(a^2 - b^2)*B*arctan
(-(a*sin(x) + b)/(sqrt(a^2 - b^2)*cos(x))))/(a*b)]
```

Sympy [A] time = 126.164, size = 87, normalized size = 1.61

$$\begin{cases} \frac{Bx}{b}+\frac{B\sqrt{-a^2+b^2}\log\left(\tan\left(\frac{x}{2}\right)+\frac{b}{a}-\frac{\sqrt{-a^2+b^2}}{a}\right)}{ab}-\frac{B\sqrt{-a^2+b^2}\log\left(\tan\left(\frac{x}{2}\right)+\frac{b}{a}+\frac{\sqrt{-a^2+b^2}}{a}\right)}{ab} & \text{for } b\neq 0 \\ -\frac{B\cos(x)}{a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*B/a+B*sin(x))/(a+b*sin(x)),x)
```

```
[Out] Piecewise((B*x/b + B*sqrt(-a**2 + b**2)*log(tan(x/2) + b/a - sqrt(-a**2 + b
**2)/a)/(a*b) - B*sqrt(-a**2 + b**2)*log(tan(x/2) + b/a + sqrt(-a**2 + b**2
)/a)/(a*b), Ne(b, 0)), (-B*cos(x)/a, True))
```

Giac [A] time = 1.27009, size = 99, normalized size = 1.83

$$\frac{Bx}{b}-\frac{2\left(Ba^2-Bb^2\right)\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\,(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,x\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*B/a+B*sin(x))/(a+b*sin(x)),x, algorithm="giac")
```

```
[Out] B*x/b - 2*(B*a^2 - B*b^2)*(pi*floor(1/2*x/pi + 1/2)*sgn(a) + arctan((a*tan(
1/2*x) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*a*b)
```

3.695 $\int \frac{\frac{aB}{b}+B\sin(x)}{a+b\sin(x)}\,dx$

Optimal. Leaf size=6

$$\frac{Bx}{b}$$

```
[Out] (B*x)/b
```

Rubi [A] time = 0.0013596, antiderivative size = 6, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {21, 8}

$$\frac{Bx}{b}$$

Antiderivative was successfully verified.

```
[In] Int[((a*B)/b + B*Sin[x])/(a + b*Sin[x]),x]
```

```
[Out] (B*x)/b
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\frac{aB}{b}+B\sin(x)}{a+b\sin(x)}\,dx &= \frac{B\int 1\,dx}{b}\\ &= \frac{Bx}{b}\end{aligned}$$

Mathematica [A] time = 0.0004145, size = 6, normalized size = 1.

$$\frac{Bx}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a*B)/b + B*Sin[x])/(a + b*Sin[x]),x]
```

```
[Out] (B*x)/b
```

Maple [A] time = 0.006, size = 7, normalized size = 1.2

$$\frac{Bx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B/b+B*sin(x))/(a+b*sin(x)),x)
```

```
[Out] B*x/b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B/b+B*sin(x))/(a+b*sin(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.11702, size = 9, normalized size = 1.5

$$\frac{Bx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B/b+B*sin(x))/(a+b*sin(x)),x, algorithm="fricas")
```

```
[Out] B*x/b
```

Sympy [A] time = 0.332326, size = 3, normalized size = 0.5

$$\frac{Bx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B/b+B*sin(x))/(a+b*sin(x)),x)
```

```
[Out] B*x/b
```

Giac [A] time = 1.28898, size = 8, normalized size = 1.33

$$\frac{Bx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B/b+B*sin(x))/(a+b*sin(x)),x, algorithm="giac")
```

```
[Out] B*x/b
```

3.696 $\int \frac{a+b\sin(x)}{(b+a\sin(x))^2}\,dx$

Optimal. Leaf size=12

$$-\frac{\cos(x)}{a\sin(x)+b}$$

```
[Out] -(Cos[x]/(b + a*Sin[x]))
```

Rubi [A] time = 0.0288894, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2754, 8}

$$-\frac{\cos(x)}{a\sin(x)+b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[x])/(b + a*Sin[x])^2,x]

[Out] -(Cos[x]/(b + a*Sin[x]))
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\sin(x)}{(b+a\sin(x))^2}\,dx &= -\frac{\cos(x)}{b+a\sin(x)}+\frac{\int 0\,dx}{a^2-b^2}\\ &= -\frac{\cos(x)}{b+a\sin(x)}\end{aligned}$$

Mathematica [A] time = 0.0300018, size = 12, normalized size = 1.

$$-\frac{\cos(x)}{a\sin(x)+b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[x])/(b + a*Sin[x])^2,x]

[Out] -(Cos[x]/(b + a*Sin[x]))
```

Maple [B] time = 0.04, size = 34, normalized size = 2.8

$$2\frac{1}{\left(\tan\left(x/2\right)\right)^{2}b+2\,a\tan\left(x/2\right)+b}\left(-\frac{a\tan\left(x/2\right)}{b}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(x))/(b+a*sin(x))^2,x)
```

```
[Out] 2*(-a/b*tan(1/2*x)-1)/(tan(1/2*x)^2*b+2*a*tan(1/2*x)+b)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(x))/(b+a*sin(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.36419, size = 32, normalized size = 2.67

$$-\frac{\cos\left(x\right)}{a\sin\left(x\right)+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(x))/(b+a*sin(x))^2,x, algorithm="fricas")
```

```
[Out] -cos(x)/(a*sin(x) + b)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(x))/(b+a*sin(x))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.25038, size = 43, normalized size = 3.58

$$-\frac{2\left(a\tan\left(\frac{1}{2}\,x\right)+b\right)}{\left(b\tan\left(\frac{1}{2}\,x\right)^{2}+2\,a\tan\left(\frac{1}{2}\,x\right)+b\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(x))/(b+a*sin(x))^2,x, algorithm="giac")
```

```
[Out] -2*(a*tan(1/2*x) + b)/((b*tan(1/2*x)^2 + 2*a*tan(1/2*x) + b)*b)
```

3.697 $\int \frac{2-\sin(x)}{2+\sin(x)}\,dx$

Optimal. Leaf size=34

$$\frac{4x}{\sqrt{3}}-x+\frac{8\tan^{-1}\left(\frac{\cos(x)}{\sin(x)+\sqrt{3}+2}\right)}{\sqrt{3}}$$

```
[Out] -x + (4*x)/Sqrt[3] + (8*ArcTan[Cos[x]/(2 + Sqrt[3] + Sin[x])])/Sqrt[3]
```

Rubi [A] time = 0.0327097, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2735, 2657}

$$\frac{4x}{\sqrt{3}}-x+\frac{8\tan^{-1}\left(\frac{\cos(x)}{\sin(x)+\sqrt{3}+2}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 - Sin[x])/(2 + Sin[x]),x]
```

```
[Out] -x + (4*x)/Sqrt[3] + (8*ArcTan[Cos[x]/(2 + Sqrt[3] + Sin[x])])/Sqrt[3]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2657

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2*ArcTan[(b*Cos[c + d*x])/(a + q + b*S
in[c + d*x])])/(d*q), x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

Rubi steps

$$\begin{aligned}\int \frac{2-\sin(x)}{2+\sin(x)}\,dx &= -x+4\int \frac{1}{2+\sin(x)}\,dx\\ &= -x+\frac{4x}{\sqrt{3}}+\frac{8\tan^{-1}\left(\frac{\cos(x)}{2+\sqrt{3}+\sin(x)}\right)}{\sqrt{3}}\end{aligned}$$

Mathematica [A] time = 0.0282276, size = 28, normalized size = 0.82

$$\frac{8\tan^{-1}\left(\frac{2\tan\left(\frac{x}{2}\right)+1}{\sqrt{3}}\right)}{\sqrt{3}}-x$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - Sin[x])/(2 + Sin[x]),x]
```

```
[Out] -x + (8*ArcTan[(1 + 2*Tan[x/2])/Sqrt[3]])/Sqrt[3]
```

Maple [A] time = 0.036, size = 24, normalized size = 0.7

$$\frac{8\sqrt{3}}{3}\arctan\left(\frac{\sqrt{3}}{3}\left(1+2\tan\left(x/2\right)\right)\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2-sin(x))/(2+sin(x)),x)
```

```
[Out] 8/3*3^(1/2)*arctan(1/3*(1+2*tan(1/2*x))*3^(1/2))-x
```

Maxima [A] time = 1.70339, size = 49, normalized size = 1.44

$$\frac{8}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2\sin\left(x\right)}{\cos\left(x\right)+1}+1\right)\right)-2\arctan\left(\frac{\sin\left(x\right)}{\cos\left(x\right)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-sin(x))/(2+sin(x)),x, algorithm="maxima")
```

```
[Out] 8/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*sin(x)/(cos(x) + 1) + 1)) - 2*arctan(sin(
x)/(cos(x) + 1))
```

Fricas [A] time = 1.32982, size = 88, normalized size = 2.59

$$\frac{4}{3}\sqrt{3}\arctan\left(\frac{2\sqrt{3}\sin\left(x\right)+\sqrt{3}}{3\cos\left(x\right)}\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-sin(x))/(2+sin(x)),x, algorithm="fricas")
```

```
[Out] 4/3*sqrt(3)*arctan(1/3*(2*sqrt(3)*sin(x) + sqrt(3))/cos(x)) - x
```

Sympy [A] time = 1.08916, size = 42, normalized size = 1.24

$$-x+\frac{8\sqrt{3}\left(\operatorname{atan}\left(\frac{2\sqrt{3}\tan\left(\frac{x}{2}\right)}{3}+\frac{\sqrt{3}}{3}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-sin(x))/(2+sin(x)),x)
```

```
[Out] -x + 8*sqrt(3)*(atan(2*sqrt(3)*tan(x/2)/3 + sqrt(3)/3) + pi*floor((x/2 - pi
/2)/pi))/3
```

Giac [A] time = 1.30076, size = 69, normalized size = 2.03

$$\frac{4}{3}\sqrt{3}\left(x+2\arctan\left(-\frac{\sqrt{3}\sin(x)-\cos(x)-2\sin(x)-1}{\sqrt{3}\cos(x)+\sqrt{3}-2\cos(x)+\sin(x)+2}\right)\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-sin(x))/(2+sin(x)),x, algorithm="giac")
```

```
[Out] 4/3*sqrt(3)*(x + 2*arctan(-(sqrt(3)*sin(x) - cos(x) - 2*sin(x) - 1)/(sqrt(3
)*cos(x) + sqrt(3) - 2*cos(x) + sin(x) + 2))) - x
```

3.698 $\int \frac{(c+d\sin(e+fx))^4}{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=235

$$\frac{d^2\left(-3a^2d^2+12abcd+b^2\left(-\left(17c^2+2d^2\right)\right)\right)\cos(e+fx)}{3b^3f}+\frac{dx\left(8a^2bcd^2-2a^3d^3-ab^2d\left(12c^2+d^2\right)+4b^3c\left(2c^2+d^2\right)\right)}{2b^4}$$

```
[Out] (d*(8*a^2*b*c*d^2 - 2*a^3*d^3 + 4*b^3*c*(2*c^2 + d^2) - a*b^2*d*(12*c^2 + d
^2))*x)/(2*b^4) + (2*(b*c - a*d)^4*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2
 - b^2]])/(b^4*Sqrt[a^2 - b^2]*f) + (d^2*(12*a*b*c*d - 3*a^2*d^2 - b^2*(17*
c^2 + 2*d^2))*Cos[e + f*x])/(3*b^3*f) - (d^3*(8*b*c - 3*a*d)*Cos[e + f*x]*S
in[e + f*x])/(6*b^2*f) - (d^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(3*b*f)
```

Rubi [A] time = 0.647816, antiderivative size = 235, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {2793, 3033, 3023, 2735, 2660, 618, 204}

$$\frac{d^2\left(-3a^2d^2+12abcd+b^2\left(-\left(17c^2+2d^2\right)\right)\right)\cos(e+fx)}{3b^3f}+\frac{dx\left(8a^2bcd^2-2a^3d^3-ab^2d\left(12c^2+d^2\right)+4b^3c\left(2c^2+d^2\right)\right)}{2b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^4/(a + b*Sin[e + f*x]),x]
```


```
[Out] (d*(8*a^2*b*c*d^2 - 2*a^3*d^3 + 4*b^3*c*(2*c^2 + d^2) - a*b^2*d*(12*c^2 + d
^2))*x)/(2*b^4) + (2*(b*c - a*d)^4*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2
 - b^2]])/(b^4*Sqrt[a^2 - b^2]*f) + (d^2*(12*a*b*c*d - 3*a^2*d^2 - b^2*(17*
c^2 + 2*d^2))*Cos[e + f*x])/(3*b^3*f) - (d^3*(8*b*c - 3*a*d)*Cos[e + f*x]*S
in[e + f*x])/(6*b^2*f) - (d^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(3*b*f)
```


Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
```

```
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^4}{a+b\sin(e+fx)}\,dx = -\frac{d^2\cos(e+fx)(c+d\sin(e+fx))^2}{3bf} + \frac{\int \frac{(c+d\sin(e+fx))\left(3bc^3+2ad^3+d\left(9bc^2-acd+2bd^2\right)\sin(e+fx)+d^2(\ldots}{a+b\sin(e+fx)}}{3b}$$

$$= -\frac{d^3(8bc-3ad)\cos(e+fx)\sin(e+fx)}{6b^2f} - \frac{d^2\cos(e+fx)(c+d\sin(e+fx))^2}{3bf} + \frac{\int \frac{3\left(2b^2c^4+\ldots\right.}{\ldots}}{\ldots}$$

$$= \frac{d^2\left(12abcd-3a^2d^2-b^2\left(17c^2+2d^2\right)\right)\cos(e+fx)}{3b^3f} - \frac{d^3(8bc-3ad)\cos(e+fx)\sin(e+f\ldots}{6b^2f}$$

$$= \frac{d\left(8a^2bcd^2-2a^3d^3+4b^3c\left(2c^2+d^2\right)-ab^2d\left(12c^2+d^2\right)\right)x}{2b^4} + \frac{d^2\left(12abcd-3a^2d^2-b^2\left(1\ldots\right.\right.}{3b^3f}$$

$$= \frac{d\left(8a^2bcd^2-2a^3d^3+4b^3c\left(2c^2+d^2\right)-ab^2d\left(12c^2+d^2\right)\right)x}{2b^4} + \frac{d^2\left(12abcd-3a^2d^2-b^2\left(1\ldots\right.\right.}{3b^3f}$$

$$= \frac{d\left(8a^2bcd^2-2a^3d^3+4b^3c\left(2c^2+d^2\right)-ab^2d\left(12c^2+d^2\right)\right)x}{2b^4} + \frac{d^2\left(12abcd-3a^2d^2-b^2\left(1\ldots\right.\right.}{3b^3f}$$

$$= \frac{d\left(8a^2bcd^2-2a^3d^3+4b^3c\left(2c^2+d^2\right)-ab^2d\left(12c^2+d^2\right)\right)x}{2b^4} + \frac{2(bc-ad)^4\tan^{-1}\left(\frac{b+a\tan\left(\ldots\right)}{\sqrt{a^2-\ldots}}\right.}{b^4\sqrt{a^2-b^2}f}$$

Mathematica [A] time = 0.567641, size = 203, normalized size = 0.86

$$\frac{-6d(e+fx)\left(-8a^2bcd^2+2a^3d^3+ab^2d\left(12c^2+d^2\right)-4b^3c\left(2c^2+d^2\right)\right)-3bd^2\left(4a^2d^2-16abcd+3b^2\left(8c^2+d^2\right)\right)\cos(e\ldots}{12b^4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])^4/(a + b*Sin[e + f*x]),x]
```

```
[Out] (-6*d*(-8*a^2*b*c*d^2 + 2*a^3*d^3 - 4*b^3*c*(2*c^2 + d^2) + a*b^2*d*(12*c^2
 + d^2))*(e + f*x) + (24*(b*c - a*d)^4*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt
[a^2 - b^2]])/Sqrt[a^2 - b^2] - 3*b*d^2*(-16*a*b*c*d + 4*a^2*d^2 + 3*b^2*(8
*c^2 + d^2))*Cos[e + f*x] + b^3*d^4*Cos[3*(e + f*x)] - 3*b^2*d^3*(4*b*c - a
*d)*Sin[2*(e + f*x)])/(12*b^4*f)
```

Maple [B] time = 0.084, size = 948, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^4/(a+b*sin(f*x+e)),x)
```

```
[Out] -4/f*d^4/b/(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)^2-2/f*d^4/b^3/(1+t
an(1/2*f*x+1/2*e)^2)^3*a^2+4/f*d^3/b/(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x
+1/2*e)^5*c+8/f*d^3/b^2/(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)^4*a*c
-8/f/b^3/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^
(1/2))*a^3*c*d^3+16/f*d^3/b^2/(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)
```

```
^2*a*c-12/f*d^2/b/(1+tan(1/2*f*x+1/2*e)^2)^3*c^2+8/f*d/b*arctan(tan(1/2*f*x
+1/2*e))*c^3-1/f*d^4/b^2*arctan(tan(1/2*f*x+1/2*e))*a+4/f*d^3/b*arctan(tan(
1/2*f*x+1/2*e))*c-2/f*d^4/b^4*arctan(tan(1/2*f*x+1/2*e))*a^3+2/f/(a^2-b^2)^
(1/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*c^4-4/3/f*d^
4/b/(1+tan(1/2*f*x+1/2*e)^2)^3+12/f/b^2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan
(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*a^2*c^2*d^2-8/f/b/(a^2-b^2)^(1/2)*arc
tan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*a*c^3*d-2/f*d^4/b^3/(
1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)^4*a^2-12/f*d^2/b/(1+tan(1/2*f*
x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)^4*c^2+8/f*d^3/b^3*arctan(tan(1/2*f*x+1/2*e
))*a^2*c-12/f*d^2/b^2*arctan(tan(1/2*f*x+1/2*e))*a*c^2+2/f/b^4/(a^2-b^2)^(1
/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*a^4*d^4+1/f*d^
4/b^2/(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)*a-4/f*d^3/b/(1+tan(1/2*
f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)*c-1/f*d^4/b^2/(1+tan(1/2*f*x+1/2*e)^2)^3
*tan(1/2*f*x+1/2*e)^5*a+8/f*d^3/b^2/(1+tan(1/2*f*x+1/2*e)^2)^3*a*c-4/f*d^4/
b^3/(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)^2*a^2-24/f*d^2/b/(1+tan(1
/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)^2*c^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^4/(a+b*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.74416, size = 1594, normalized size = 6.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^4/(a+b*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] [1/6*(2*(a^2*b^3 - b^5)*d^4*cos(f*x + e)^3 + 3*(8*(a^2*b^3 - b^5)*c^3*d - 1
2*(a^3*b^2 - a*b^4)*c^2*d^2 + 4*(2*a^4*b - a^2*b^3 - b^5)*c*d^3 - (2*a^5 -
a^3*b^2 - a*b^4)*d^4)*f*x - 3*(4*(a^2*b^3 - b^5)*c*d^3 - (a^3*b^2 - a*b^4)*
d^4)*cos(f*x + e)*sin(f*x + e) - 3*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2
*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(f*x
 + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2 + 2*(a*cos(f*x + e)*sin(f*x + e) +
 b*cos(f*x + e))*sqrt(-a^2 + b^2))/(b^2*cos(f*x + e)^2 - 2*a*b*sin(f*x + e)
 - a^2 - b^2)) - 6*(6*(a^2*b^3 - b^5)*c^2*d^2 - 4*(a^3*b^2 - a*b^4)*c*d^3 +
 (a^4*b - b^5)*d^4)*cos(f*x + e))/((a^2*b^4 - b^6)*f), 1/6*(2*(a^2*b^3 - b^
5)*d^4*cos(f*x + e)^3 + 3*(8*(a^2*b^3 - b^5)*c^3*d - 12*(a^3*b^2 - a*b^4)*c
^2*d^2 + 4*(2*a^4*b - a^2*b^3 - b^5)*c*d^3 - (2*a^5 - a^3*b^2 - a*b^4)*d^4)
*f*x - 3*(4*(a^2*b^3 - b^5)*c*d^3 - (a^3*b^2 - a*b^4)*d^4)*cos(f*x + e)*sin
(f*x + e) - 6*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3
+ a^4*d^4)*sqrt(a^2 - b^2)*arctan(-(a*sin(f*x + e) + b)/(sqrt(a^2 - b^2)*co
s(f*x + e))) - 6*(6*(a^2*b^3 - b^5)*c^2*d^2 - 4*(a^3*b^2 - a*b^4)*c*d^3 + (
a^4*b - b^5)*d^4)*cos(f*x + e))/((a^2*b^4 - b^6)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**4/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.37152, size = 628, normalized size = 2.67

$$\frac{3\left(8b^3c^3d-12ab^2c^2d^2+8a^2bcd^3+4b^3cd^3-2a^3d^4-ab^2d^4\right)(fx+e)}{b^4}+\frac{12\left(b^4c^4-4ab^3c^3d+6a^2b^2c^2d^2-4a^3bcd^3+a^4d^4\right)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}fx+\cdots\right)}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^4/(a+b*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/6*(3*(8*b^3*c^3*d - 12*a*b^2*c^2*d^2 + 8*a^2*b*c*d^3 + 4*b^3*c*d^3 - 2*a^
3*d^4 - a*b^2*d^4)*(f*x + e)/b^4 + 12*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*
c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(a)
 + arctan((a*tan(1/2*f*x + 1/2*e) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*b
^4) + 2*(12*b^2*c*d^3*tan(1/2*f*x + 1/2*e)^5 - 3*a*b*d^4*tan(1/2*f*x + 1/2*
e)^5 - 36*b^2*c^2*d^2*tan(1/2*f*x + 1/2*e)^4 + 24*a*b*c*d^3*tan(1/2*f*x + 1
/2*e)^4 - 6*a^2*d^4*tan(1/2*f*x + 1/2*e)^4 - 72*b^2*c^2*d^2*tan(1/2*f*x + 1
/2*e)^2 + 48*a*b*c*d^3*tan(1/2*f*x + 1/2*e)^2 - 12*a^2*d^4*tan(1/2*f*x + 1/
2*e)^2 - 12*b^2*d^4*tan(1/2*f*x + 1/2*e)^2 - 12*b^2*c*d^3*tan(1/2*f*x + 1/2
*e) + 3*a*b*d^4*tan(1/2*f*x + 1/2*e) - 36*b^2*c^2*d^2 + 24*a*b*c*d^3 - 6*a^
2*d^4 - 4*b^2*d^4)/((tan(1/2*f*x + 1/2*e)^2 + 1)^3*b^3))/f
```

3.699 $\int \frac{(c+d\sin(e+fx))^3}{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=156

$$-\frac{dx\left(-2a^2d^2+6abcd+b^2\left(-\left(6c^2+d^2\right)\right)\right)}{2b^3}+\frac{2(bc-ad)^3\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^3f\sqrt{a^2-b^2}}-\frac{d^2(5bc-2ad)\cos(e+fx)}{2b^2f}-\frac{d^2\cos(e\cdot$$

```
[Out] -(d*(6*a*b*c*d - 2*a^2*d^2 - b^2*(6*c^2 + d^2))*x)/(2*b^3) + (2*(b*c - a*d)
^3*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/(b^3*Sqrt[a^2 - b^2]*f
) - (d^2*(5*b*c - 2*a*d)*Cos[e + f*x])/(2*b^2*f) - (d^2*Cos[e + f*x]*(c + d
*Sin[e + f*x]))/(2*b*f)
```

Rubi [A] time = 0.363093, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {2793, 3023, 2735, 2660, 618, 204}

$$-\frac{dx\left(-2a^2d^2+6abcd+b^2\left(-\left(6c^2+d^2\right)\right)\right)}{2b^3}+\frac{2(bc-ad)^3\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^3f\sqrt{a^2-b^2}}-\frac{d^2(5bc-2ad)\cos(e+fx)}{2b^2f}-\frac{d^2\cos(e\cdot$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^3/(a + b*Sin[e + f*x]),x]
```


```
[Out] -(d*(6*a*b*c*d - 2*a^2*d^2 - b^2*(6*c^2 + d^2))*x)/(2*b^3) + (2*(b*c - a*d)
^3*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/(b^3*Sqrt[a^2 - b^2]*f
) - (d^2*(5*b*c - 2*a*d)*Cos[e + f*x])/(2*b^2*f) - (d^2*Cos[e + f*x]*(c + d
*Sin[e + f*x]))/(2*b*f)
```


Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^3}{a+b\sin(e+fx)}\,dx = -\frac{d^2\cos(e+fx)(c+d\sin(e+fx))}{2bf} + \frac{\int \frac{2bc^3+ad^3-d\left(acd-b\left(6c^2+d^2\right)\right)\sin(e+fx)+d^2(5bc-2ad)\sin^2(e+fx)}{a+b\sin(e+fx)}}{2b}$$

$$= -\frac{d^2(5bc-2ad)\cos(e+fx)}{2b^2f} - \frac{d^2\cos(e+fx)(c+d\sin(e+fx))}{2bf} + \frac{\int \frac{b\left(2bc^3+ad^3\right)-d\left(6abcd-2a^2\right.}{a+b\sin}}{2}$$

$$= -\frac{d\left(6abcd-2a^2d^2-b^2\left(6c^2+d^2\right)\right)x}{2b^3} - \frac{d^2(5bc-2ad)\cos(e+fx)}{2b^2f} - \frac{d^2\cos(e+fx)(c+d}{2bf}$$

$$= -\frac{d\left(6abcd-2a^2d^2-b^2\left(6c^2+d^2\right)\right)x}{2b^3} - \frac{d^2(5bc-2ad)\cos(e+fx)}{2b^2f} - \frac{d^2\cos(e+fx)(c+d}{2bf}$$

$$= -\frac{d\left(6abcd-2a^2d^2-b^2\left(6c^2+d^2\right)\right)x}{2b^3} - \frac{d^2(5bc-2ad)\cos(e+fx)}{2b^2f} - \frac{d^2\cos(e+fx)(c+d}{2bf}$$

$$= -\frac{d\left(6abcd-2a^2d^2-b^2\left(6c^2+d^2\right)\right)x}{2b^3} + \frac{2(bc-ad)^3\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{b^3\sqrt{a^2-b^2}f} - \frac{d^2(5bc-2a}{2}$$

Mathematica [A] time = 0.33562, size = 138, normalized size = 0.88

$$\frac{2d(e+fx)\left(2a^2d^2-6abcd+b^2\left(6c^2+d^2\right)\right)+\frac{8(bc-ad)^3\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}-4bd^2(3bc-ad)\cos(e+fx)-b^2d^3\sin(2(e}{4b^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])^3/(a + b*Sin[e + f*x]),x]
```

```
[Out] (2*d*(-6*a*b*c*d + 2*a^2*d^2 + b^2*(6*c^2 + d^2))*(e + f*x) + (8*(b*c - a*d
)^3*ArcTan[(b + a*Tan[(e + f*x)/2]])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] - 4*b
*d^2*(3*b*c - a*d)*Cos[e + f*x] - b^2*d^3*Sin[2*(e + f*x)])/(4*b^3*f)
```

Maple [B] time = 0.076, size = 506, normalized size = 3.2

$$\frac{d^3}{bf}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3\left(1+\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2\right)^{-2}+2\,\frac{d^3\left(\tan\left(1/2\,fx+e/2\right)\right)^2a}{b^2f\left(1+\left(\tan\left(1/2\,fx+e/2\right)\right)^2\right)^2}-6\,\frac{d^2\left(\tan\left(1/2\,fx+e/2\right)\right)^2c}{bf\left(1+\left(\tan\left(1/2\,fx+e/2\right)\right)^2\right)^2}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^3/(a+b*sin(f*x+e)),x)
```

```
[Out] 1/f*d^3/b/(1+tan(1/2*f*x+1/2*e)^2)^2*tan(1/2*f*x+1/2*e)^3+2/f*d^3/b^2/(1+ta
n(1/2*f*x+1/2*e)^2)^2*tan(1/2*f*x+1/2*e)^2*a-6/f*d^2/b/(1+tan(1/2*f*x+1/2*e
)^2)^2*tan(1/2*f*x+1/2*e)^2*c-1/f*d^3/b/(1+tan(1/2*f*x+1/2*e)^2)^2*tan(1/2*
f*x+1/2*e)+2/f*d^3/b^2/(1+tan(1/2*f*x+1/2*e)^2)^2*a-6/f*d^2/b/(1+tan(1/2*f*
x+1/2*e)^2)^2*c+2/f*d^3/b^3*arctan(tan(1/2*f*x+1/2*e))*a^2-6/f*d^2/b^2*arct
an(tan(1/2*f*x+1/2*e))*a*c+6/f*d/b*arctan(tan(1/2*f*x+1/2*e))*c^2+1/f*d^3/b
*arctan(tan(1/2*f*x+1/2*e))-2/f/b^3/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2
*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*a^3*d^3+6/f/b^2/(a^2-b^2)^(1/2)*arctan(1/
2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*a^2*c*d^2-6/f/b/(a^2-b^2)^(
1/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*a*c^2*d+2/f/(
a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*c^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^3/(a+b*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.59816, size = 1153, normalized size = 7.39

$$\left[-\frac{\left(a^2b^2-b^4\right)d^3\cos\left(fx+e\right)\sin\left(fx+e\right)-\left(6\left(a^2b^2-b^4\right)c^2d-6\left(a^3b-ab^3\right)cd^2+\left(2\,a^4-a^2b^2-b^4\right)d^3\right)fx-\left(b^3c^3-3\,ab\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^3/(a+b*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] [-1/2*((a^2*b^2 - b^4)*d^3*cos(f*x + e)*sin(f*x + e) - (6*(a^2*b^2 - b^4)*c
^2*d - 6*(a^3*b - a*b^3)*c*d^2 + (2*a^4 - a^2*b^2 - b^4)*d^3)*f*x - (b^3*c^
3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(-a^2 + b^2)*log(-((2*a^2
- b^2)*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2 - 2*(a*cos(f*x + e)*
```

```
sin(f*x + e) + b*cos(f*x + e))*sqrt(-a^2 + b^2))/(b^2*cos(f*x + e)^2 - 2*a*
b*sin(f*x + e) - a^2 - b^2)) + 2*(3*(a^2*b^2 - b^4)*c*d^2 - (a^3*b - a*b^3)
*d^3)*cos(f*x + e))/((a^2*b^3 - b^5)*f), -1/2*((a^2*b^2 - b^4)*d^3*cos(f*x
+ e)*sin(f*x + e) - (6*(a^2*b^2 - b^4)*c^2*d - 6*(a^3*b - a*b^3)*c*d^2 + (2
*a^4 - a^2*b^2 - b^4)*d^3)*f*x + 2*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2
 - a^3*d^3)*sqrt(a^2 - b^2)*arctan(-(a*sin(f*x + e) + b)/(sqrt(a^2 - b^2)*c
os(f*x + e))) + 2*(3*(a^2*b^2 - b^4)*c*d^2 - (a^3*b - a*b^3)*d^3)*cos(f*x +
 e))/((a^2*b^3 - b^5)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**3/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.37567, size = 340, normalized size = 2.18

$$\frac{\frac{\left(6\,b^2c^2d-6\,abcd^2+2\,a^2d^3+b^2d^3\right)\left(fx+e\right)}{b^3}+\frac{4\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,b^3}+\frac{2\left(bd^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-6\right.}{}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^3/(a+b*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/2*((6*b^2*c^2*d - 6*a*b*c*d^2 + 2*a^2*d^3 + b^2*d^3)*(f*x + e)/b^3 + 4*(b
^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*(pi*floor(1/2*(f*x + e)/p
i + 1/2)*sgn(a) + arctan((a*tan(1/2*f*x + 1/2*e) + b)/sqrt(a^2 - b^2)))/(sq
rt(a^2 - b^2)*b^3) + 2*(b*d^3*tan(1/2*f*x + 1/2*e)^3 - 6*b*c*d^2*tan(1/2*f*
x + 1/2*e)^2 + 2*a*d^3*tan(1/2*f*x + 1/2*e)^2 - b*d^3*tan(1/2*f*x + 1/2*e)
- 6*b*c*d^2 + 2*a*d^3)/((tan(1/2*f*x + 1/2*e)^2 + 1)^2*b^2))/f
```

3.700 $\int \frac{(c+d\sin(e+fx))^2}{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=93

$$\frac{2(bc-ad)^2\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^2f\sqrt{a^2-b^2}}+\frac{dx(2bc-ad)}{b^2}-\frac{d^2\cos(e+fx)}{bf}$$

```
[Out] (d*(2*b*c - a*d)*x)/b^2 + (2*(b*c - a*d)^2*ArcTan[(b + a*Tan[(e + f*x)/2])/
Sqrt[a^2 - b^2]])/(b^2*Sqrt[a^2 - b^2]*f) - (d^2*Cos[e + f*x])/(b*f)
```

Rubi [A] time = 0.167061, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2746, 2735, 2660, 618, 204}

$$\frac{2(bc-ad)^2\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^2f\sqrt{a^2-b^2}}+\frac{dx(2bc-ad)}{b^2}-\frac{d^2\cos(e+fx)}{bf}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^2/(a + b*Sin[e + f*x]),x]
```

```
[Out] (d*(2*b*c - a*d)*x)/b^2 + (2*(b*c - a*d)^2*ArcTan[(b + a*Tan[(e + f*x)/2])/
Sqrt[a^2 - b^2]])/(b^2*Sqrt[a^2 - b^2]*f) - (d^2*Cos[e + f*x])/(b*f)
```

Rule 2746

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2/((c_.) + (d_.)*sin[(e_.) + (f
_.)*(x_)]), x_Symbol] :> -Simp[(b^2*Cos[e + f*x])/(d*f), x] + Dist[1/d, Int
[Simp[a^2*d - b*(b*c - 2*a*d)*Sin[e + f*x], x]/(c + d*Sin[e + f*x]), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^2}{a+b\sin(e+fx)}\,dx = -\frac{d^2\cos(e+fx)}{bf}+\frac{\int \frac{bc^2+d(2bc-ad)\sin(e+fx)}{a+b\sin(e+fx)}\,dx}{b}$$

$$=\frac{d(2bc-ad)x}{b^2}-\frac{d^2\cos(e+fx)}{bf}+\frac{(bc-ad)^2\int \frac{1}{a+b\sin(e+fx)}\,dx}{b^2}$$

$$=\frac{d(2bc-ad)x}{b^2}-\frac{d^2\cos(e+fx)}{bf}+\frac{\left(2(bc-ad)^2\right)\operatorname{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx,x,\tan\left(\frac{1}{2}(e+fx)\right)\right)}{b^2f}$$

$$=\frac{d(2bc-ad)x}{b^2}-\frac{d^2\cos(e+fx)}{bf}-\frac{\left(4(bc-ad)^2\right)\operatorname{Subst}\left(\int \frac{1}{-4(a^2-b^2)-x^2}\,dx,x,2b+2a\tan\left(\frac{1}{2}\right.\right.}{b^2f}$$

$$=\frac{d(2bc-ad)x}{b^2}+\frac{2(bc-ad)^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{b^2\sqrt{a^2-b^2}\,f}-\frac{d^2\cos(e+fx)}{bf}$$

Mathematica [A] time = 0.159349, size = 90, normalized size = 0.97

$$\frac{\frac{2(bc-ad)^2\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+d(e+fx)(2bc-ad)-bd^2\cos(e+fx)}{b^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])^2/(a + b*Sin[e + f*x]),x]
```

```
[Out] (d*(2*b*c - a*d)*(e + f*x) + (2*(b*c - a*d)^2*ArcTan[(b + a*Tan[(e + f*x)/2
])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] - b*d^2*Cos[e + f*x])/(b^2*f)
```

Maple [B] time = 0.065, size = 226, normalized size = 2.4

$$-2\frac{d^2}{bf\left(1+\left(\tan\left(1/2\,fx+e/2\right)\right)^2\right)}-2\frac{d^2\arctan\left(\tan\left(1/2\,fx+e/2\right)\right)a}{b^2f}+4\frac{d\arctan\left(\tan\left(1/2\,fx+e/2\right)\right)c}{bf}+2\frac{a^2}{b^2f\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^2/(a+b*sin(f*x+e)),x)
```

```
[Out] -2/f*d^2/b/(1+tan(1/2*f*x+1/2*e)^2)-2/f*d^2/b^2*arctan(tan(1/2*f*x+1/2*e))*
a+4/f*d/b*arctan(tan(1/2*f*x+1/2*e))*c+2/f/b^2/(a^2-b^2)^(1/2)*arctan(1/2*(
2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*a^2*d^2-4/f/b/(a^2-b^2)^(1/2)*
arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*a*c*d+2/f/(a^2-b^2
)^(1/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*c^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^2/(a+b*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.59945, size = 771, normalized size = 8.29

$$\left[-\frac{2\left(a^2b-b^3\right)d^2\cos\left(fx+e\right)-2\left(2\left(a^2b-b^3\right)cd-\left(a^3-ab^2\right)d^2\right)fx+\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{-a^2+b^2}\log\left(\frac{\left(2\,a^2-b^2\right)\cos(f}{}\right.}{2\left(a^2b^2-b^4\right)f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^2/(a+b*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*(a^2*b - b^3)*d^2*cos(f*x + e) - 2*(2*(a^2*b - b^3)*c*d - (a^3 - a
*b^2)*d^2)*f*x + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-a^2 + b^2)*log(((2*a
^2 - b^2)*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2 + 2*(a*cos(f*x +
e)*sin(f*x + e) + b*cos(f*x + e))*sqrt(-a^2 + b^2))/(b^2*cos(f*x + e)^2 - 2
*a*b*sin(f*x + e) - a^2 - b^2)))/((a^2*b^2 - b^4)*f), -((a^2*b - b^3)*d^2*c
os(f*x + e) - (2*(a^2*b - b^3)*c*d - (a^3 - a*b^2)*d^2)*f*x + (b^2*c^2 - 2*
a*b*c*d + a^2*d^2)*sqrt(a^2 - b^2)*arctan(-(a*sin(f*x + e) + b)/(sqrt(a^2 -
 b^2)*cos(f*x + e))))/((a^2*b^2 - b^4)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**2/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.35917, size = 181, normalized size = 1.95

$$\frac{\frac{\left(2\,bcd-ad^2\right)\left(fx+e\right)}{b^2}-\frac{2\,d^2}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)b}+\frac{2\left(b^2c^2-2\,abcd+a^2d^2\right)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,b^2}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^2/(a+b*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] ((2*b*c*d - a*d^2)*(f*x + e)/b^2 - 2*d^2/((tan(1/2*f*x + 1/2*e)^2 + 1)*b) +
 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(a)
 + arctan((a*tan(1/2*f*x + 1/2*e) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*b
^2))/f
```

3.701 $\int \frac{c+d\sin(e+fx)}{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=65

$$\frac{2(bc-ad)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{bf\sqrt{a^2-b^2}}+\frac{dx}{b}$$

```
[Out] (d*x)/b + (2*(b*c - a*d)*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/
(b*Sqrt[a^2 - b^2]*f)
```

Rubi [A] time = 0.0719839, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2735, 2660, 618, 204}

$$\frac{2(bc-ad)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{bf\sqrt{a^2-b^2}}+\frac{dx}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])/(a + b*Sin[e + f*x]),x]
```

```
[Out] (d*x)/b + (2*(b*c - a*d)*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/
(b*Sqrt[a^2 - b^2]*f)
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{c+d\sin(e+fx)}{a+b\sin(e+fx)}\,dx = \frac{dx}{b} - \frac{(-bc+ad)\int \frac{1}{a+b\sin(e+fx)}\,dx}{b}$$
$$= \frac{dx}{b} + \frac{(2(bc-ad))\,\text{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx, x, \tan\left(\frac{1}{2}(e+fx)\right)\right)}{bf}$$
$$= \frac{dx}{b} - \frac{(4(bc-ad))\,\text{Subst}\left(\int \frac{1}{-4(a^2-b^2)-x^2}\,dx, x, 2b+2a\tan\left(\frac{1}{2}(e+fx)\right)\right)}{bf}$$
$$= \frac{dx}{b} + \frac{2(bc-ad)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}f}$$

Mathematica [A] time = 0.0980203, size = 67, normalized size = 1.03

$$\frac{\frac{2(bc-ad)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + d(e+fx)}{bf}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])/(a + b*Sin[e + f*x]),x]
```

```
[Out] (d*(e + f*x) + (2*(b*c - a*d)*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^
2]])/Sqrt[a^2 - b^2])/(b*f)
```

Maple [A] time = 0.042, size = 119, normalized size = 1.8

$$2\,\frac{d\arctan\left(\tan\left(1/2\,fx+e/2\right)\right)}{bf} - 2\,\frac{da}{bf\sqrt{a^2-b^2}}\arctan\left(1/2\,\frac{2\,a\tan\left(1/2\,fx+e/2\right)+2\,b}{\sqrt{a^2-b^2}}\right) + 2\,\frac{c}{f\sqrt{a^2-b^2}}\arctan\left(1/2\,\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))/(a+b*sin(f*x+e)),x)
```

```
[Out] 2/f*d/b*arctan(tan(1/2*f*x+1/2*e))-2/f/b/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*ta
n(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*d*a+2/f/(a^2-b^2)^(1/2)*arctan(1/2*(
2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))/(a+b*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.42105, size = 543, normalized size = 8.35

$$\left[\frac{2\left(a^2-b^2\right)dfx+\sqrt{-a^2+b^2}(bc-ad)\log\left(-\frac{\left(2a^2-b^2\right)\cos\left(fx+e\right)^2-2ab\sin\left(fx+e\right)-a^2-b^2-2\left(a\cos\left(fx+e\right)\sin\left(fx+e\right)+b\cos\left(fx+e\right)\right)\sqrt{-a^2+b^2}}{b^2\cos\left(fx+e\right)^2-2ab\sin\left(fx+e\right)-a^2-b^2}\right)}{2\left(a^2b-b^3\right)f},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))/(a+b*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] [1/2*(2*(a^2 - b^2)*d*f*x + sqrt(-a^2 + b^2)*(b*c - a*d)*log(-((2*a^2 - b^2
)*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2 - 2*(a*cos(f*x + e)*sin(f
*x + e) + b*cos(f*x + e))*sqrt(-a^2 + b^2))/(b^2*cos(f*x + e)^2 - 2*a*b*sin
(f*x + e) - a^2 - b^2)))/((a^2*b - b^3)*f), ((a^2 - b^2)*d*f*x - sqrt(a^2 -
 b^2)*(b*c - a*d)*arctan(-(a*sin(f*x + e) + b)/(sqrt(a^2 - b^2)*cos(f*x + e
))))/((a^2*b - b^3)*f)]
```

Sympy [A] time = 142.884, size = 502, normalized size = 7.72

$$\begin{cases} \frac{\tilde{\infty}x(c+d\sin(e))}{\sin(e)} \\ \frac{2c}{bf\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-bf}+\frac{dfx\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{bf\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-bf}-\frac{dfx}{bf\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-bf}+\frac{2d}{bf\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-bf} \\ -\frac{2c}{bf\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+bf}+\frac{dfx\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{bf\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+bf}+\frac{dfx}{bf\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+bf}+\frac{2d}{bf\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+bf} \\ \frac{cx-\frac{d\cos(e+fx)}{f}}{a} \\ \frac{x(c+d\sin(e))}{a+b\sin(e)} \\ \frac{\frac{c\log\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{f}+dx}{b} \\ \frac{a^2dfx}{a^2bf-b^3f}+\frac{ad\sqrt{-a^2+b^2}\log\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+\frac{b}{a}-\frac{\sqrt{-a^2+b^2}}{a}\right)}{a^2bf-b^3f}-\frac{ad\sqrt{-a^2+b^2}\log\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+\frac{b}{a}+\frac{\sqrt{-a^2+b^2}}{a}\right)}{a^2bf-b^3f}-\frac{b^2dfx}{a^2bf-b^3f}-\frac{bc\sqrt{-a^2+b^2}\log\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+\frac{b}{a}-\right.}{a^2bf-b^3f} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))/(a+b*sin(f*x+e)),x)
```

```
[Out] Piecewise((zoo*x*(c + d*sin(e))/sin(e), Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), (2
*c/(b*f*tan(e/2 + f*x/2) - b*f) + d*f*x*tan(e/2 + f*x/2)/(b*f*tan(e/2 + f*x
/2) - b*f) - d*f*x/(b*f*tan(e/2 + f*x/2) - b*f) + 2*d/(b*f*tan(e/2 + f*x/2)
 - b*f), Eq(a, -b)), (-2*c/(b*f*tan(e/2 + f*x/2) + b*f) + d*f*x*tan(e/2 + f
*x/2)/(b*f*tan(e/2 + f*x/2) + b*f) + d*f*x/(b*f*tan(e/2 + f*x/2) + b*f) + 2
*d/(b*f*tan(e/2 + f*x/2) + b*f), Eq(a, b)), ((c*x - d*cos(e + f*x)/f)/a, Eq
(b, 0)), (x*(c + d*sin(e))/(a + b*sin(e)), Eq(f, 0)), ((c*log(tan(e/2 + f*x
/2))/f + d*x)/b, Eq(a, 0)), (a**2*d*f*x/(a**2*b*f - b**3*f) + a*d*sqrt(-a**
2 + b**2)*log(tan(e/2 + f*x/2) + b/a - sqrt(-a**2 + b**2)/a)/(a**2*b*f - b*
*3*f) - a*d*sqrt(-a**2 + b**2)*log(tan(e/2 + f*x/2) + b/a + sqrt(-a**2 + b*
*2)/a)/(a**2*b*f - b**3*f) - b**2*d*f*x/(a**2*b*f - b**3*f) - b*c*sqrt(-a**
2 + b**2)*log(tan(e/2 + f*x/2) + b/a - sqrt(-a**2 + b**2)/a)/(a**2*b*f - b*
*3*f) + b*c*sqrt(-a**2 + b**2)*log(tan(e/2 + f*x/2) + b/a + sqrt(-a**2 + b*
*2)/a)/(a**2*b*f - b**3*f), True))
```

Giac [A] time = 1.34768, size = 116, normalized size = 1.78

$$\frac{\frac{(fx+e)d}{b}+\frac{2\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+b}{\sqrt{a^2-b^2}}\right)\right)(bc-ad)}{\sqrt{a^2-b^2}\,b}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))/(a+b*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] ((f*x + e)*d/b + 2*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(a) + arctan((a*tan
(1/2*f*x + 1/2*e) + b)/sqrt(a^2 - b^2)))*(b*c - a*d)/(sqrt(a^2 - b^2)*b))/f
```

3.702 $\int \frac{1}{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=47

$$\frac{2\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{f\sqrt{a^2-b^2}}$$

```
[Out] (2*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/(Sqrt[a^2 - b^2]*f)
```

Rubi [A] time = 0.0341297, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2660, 618, 204}

$$\frac{2\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{f\sqrt{a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^(-1),x]
```

```
[Out] (2*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/(Sqrt[a^2 - b^2]*f)
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{a+b\sin(e+fx)}\,dx = \frac{2\,\text{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx, x, \tan\left(\frac{1}{2}(e+fx)\right)\right)}{f}$$
$$= -\frac{4\,\text{Subst}\left(\int \frac{1}{-4(a^2-b^2)-x^2}\,dx, x, 2b+2a\tan\left(\frac{1}{2}(e+fx)\right)\right)}{f}$$
$$= \frac{2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}f}$$

Mathematica [A] time = 0.031157, size = 47, normalized size = 1.

$$\frac{2\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{f\sqrt{a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^(-1),x]
```

```
[Out] (2*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/(Sqrt[a^2 - b^2]*f)
```

Maple [A] time = 0.033, size = 47, normalized size = 1.

$$2\,\frac{1}{f\sqrt{a^2-b^2}}\arctan\left(1/2\,\frac{2\,a\tan\left(1/2\,fx+e/2\right)+2\,b}{\sqrt{a^2-b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e)),x)
```

```
[Out] 2/f/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2)
)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.3701, size = 428, normalized size = 9.11

$$\left[-\frac{\sqrt{-a^2+b^2}\log\left(\frac{\left(2a^2-b^2\right)\cos\left(fx+e\right)^2-2ab\sin\left(fx+e\right)-a^2-b^2+2\left(a\cos\left(fx+e\right)\sin\left(fx+e\right)+b\cos\left(fx+e\right)\right)\sqrt{-a^2+b^2}}{b^2\cos\left(fx+e\right)^2-2ab\sin\left(fx+e\right)-a^2-b^2}\right)}{2\left(a^2-b^2\right)f},-\frac{\arctan\left(-\frac{a\sin\left(fx+e\right)+b}{\sqrt{a^2-b^2}\cos\left(fx+e\right)}\right)}{\sqrt{a^2-b^2}f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(f*x + e)^2 - 2*a*b*sin(f*x +
e) - a^2 - b^2 + 2*(a*cos(f*x + e)*sin(f*x + e) + b*cos(f*x + e))*sqrt(-a^2
 + b^2))/(b^2*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2))/((a^2 - b^2
)*f), -arctan(-(a*sin(f*x + e) + b)/(sqrt(a^2 - b^2)*cos(f*x + e)))/(sqrt(a
^2 - b^2)*f)]
```

Sympy [A] time = 7.72019, size = 162, normalized size = 3.45

$$\begin{cases} \frac{\tilde{\infty}x}{\sin(e)} & \text{for } a=0\wedge b=0\wedge f=0 \\ \frac{2}{bf\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-bf} & \text{for } a=-b \\ -\frac{2}{bf\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+bf} & \text{for } a=b \\ \frac{x}{a+b\sin(e)} & \text{for } f=0 \\ \frac{\log\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{bf} & \text{for } a=0 \\ -\frac{\sqrt{-a^2+b^2}\log\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+\frac{b}{a}-\frac{\sqrt{-a^2+b^2}}{a}\right)}{a^2f-b^2f}+\frac{\sqrt{-a^2+b^2}\log\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+\frac{b}{a}+\frac{\sqrt{-a^2+b^2}}{a}\right)}{a^2f-b^2f} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)),x)
```

```
[Out] Piecewise((zoo*x/sin(e), Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), (2/(b*f*tan(e/2 +
 f*x/2) - b*f), Eq(a, -b)), (-2/(b*f*tan(e/2 + f*x/2) + b*f), Eq(a, b)), (x
/(a + b*sin(e)), Eq(f, 0)), (log(tan(e/2 + f*x/2))/(b*f), Eq(a, 0)), (-sqrt
(-a**2 + b**2)*log(tan(e/2 + f*x/2) + b/a - sqrt(-a**2 + b**2)/a)/(a**2*f -
 b**2*f) + sqrt(-a**2 + b**2)*log(tan(e/2 + f*x/2) + b/a + sqrt(-a**2 + b**
2)/a)/(a**2*f - b**2*f), True))
```

Giac [A] time = 1.27913, size = 84, normalized size = 1.79

$$\frac{2\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] 2*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*f*x + 1/2*e)
 + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*f)
```

3.703 $\int \frac{1}{(a+b\sin(e+fx))(c+d\sin(e+fx))}\,dx$

Optimal. Leaf size=117

$$\frac{2b\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{f\sqrt{a^2-b^2}(bc-ad)}-\frac{2d\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{f\sqrt{c^2-d^2}(bc-ad)}$$

```
[Out] (2*b*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/(Sqrt[a^2 - b^2]*(b*
c - a*d)*f) - (2*d*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/((b*c
- a*d)*Sqrt[c^2 - d^2]*f)
```

Rubi [A] time = 0.153368, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2747, 2660, 618, 204}

$$\frac{2b\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{f\sqrt{a^2-b^2}(bc-ad)}-\frac{2d\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{f\sqrt{c^2-d^2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sin[e + f*x])*(c + d*Sin[e + f*x])),x]
```

```
[Out] (2*b*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/(Sqrt[a^2 - b^2]*(b*
c - a*d)*f) - (2*d*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/((b*c
- a*d)*Sqrt[c^2 - d^2]*f)
```

Rule 2747

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]),
x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(a+b\sin(e+fx))(c+d\sin(e+fx))}\,dx = \frac{b\int \frac{1}{a+b\sin(e+fx)}\,dx}{bc-ad} - \frac{d\int \frac{1}{c+d\sin(e+fx)}\,dx}{bc-ad}$$
$$= \frac{(2b)\,\text{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx, x, \tan\left(\frac{1}{2}(e+fx)\right)\right)}{(bc-ad)f} - \frac{(2d)\,\text{Subst}\left(\int \frac{1}{c+2dx+cx^2}\,\right.}{(bc-a}$$
$$= -\frac{(4b)\,\text{Subst}\left(\int \frac{1}{-4(a^2-b^2)-x^2}\,dx, x, 2b+2a\tan\left(\frac{1}{2}(e+fx)\right)\right)}{(bc-ad)f} + \frac{(4d)\,\text{Subst}\left(\right.}{}$$
$$= \frac{2b\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}(bc-ad)f} - \frac{2d\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{(bc-ad)\sqrt{c^2-d^2}f}$$

Mathematica [A] time = 0.165576, size = 104, normalized size = 0.89

$$\frac{\frac{2b\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} - \frac{2d\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{\sqrt{c^2-d^2}}}{bcf-adf}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*Sin[e + f*x])*(c + d*Sin[e + f*x])),x]
```

```
[Out] ((2*b*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] - (
2*d*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/Sqrt[c^2 - d^2])/(b*c
*f - a*d*f)
```

Maple [A] time = 0.09, size = 116, normalized size = 1.

$$2\,\frac{d}{f\,(da-cb)\,\sqrt{c^2-d^2}}\arctan\left(1/2\,\frac{2\,c\tan\left(1/2\,fx+e/2\right)+2\,d}{\sqrt{c^2-d^2}}\right)-2\,\frac{b}{f\,(da-cb)\,\sqrt{a^2-b^2}}\arctan\left(1/2\,\frac{2\,a\tan\left(1/2\,fx+e/2\right.}{\sqrt{a^2-b^2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

```
[Out] 2/f/(a*d-b*c)*d/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^
2-d^2)^(1/2))-2/f*b/(a*d-b*c)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*f*x+1
/2*e)+2*b)/(a^2-b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 10.3949, size = 2238, normalized size = 19.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] [1/2*((a^2 - b^2)*sqrt(-c^2 + d^2)*d*log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*
c*d*sin(f*x + e) - c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x + e) + d*cos(f*x +
 e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2
)) + (b*c^2 - b*d^2)*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(f*x + e)^2 -
2*a*b*sin(f*x + e) - a^2 - b^2 - 2*(a*cos(f*x + e)*sin(f*x + e) + b*cos(f*x
 + e))*sqrt(-a^2 + b^2))/(b^2*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b
^2)))/(((a^2*b - b^3)*c^3 - (a^3 - a*b^2)*c^2*d - (a^2*b - b^3)*c*d^2 + (a^
3 - a*b^2)*d^3)*f), 1/2*(2*(a^2 - b^2)*sqrt(c^2 - d^2)*d*arctan(-(c*sin(f*x
 + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) + (b*c^2 - b*d^2)*sqrt(-a^2 + b^
2)*log(-((2*a^2 - b^2)*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2 - 2*
(a*cos(f*x + e)*sin(f*x + e) + b*cos(f*x + e))*sqrt(-a^2 + b^2))/(b^2*cos(f
*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2)))/(((a^2*b - b^3)*c^3 - (a^3 -
a*b^2)*c^2*d - (a^2*b - b^3)*c*d^2 + (a^3 - a*b^2)*d^3)*f), 1/2*((a^2 - b^2
)*sqrt(-c^2 + d^2)*d*log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e)
 - c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x + e) + d*cos(f*x + e))*sqrt(-c^2 +
 d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) - 2*(b*c^2 -
b*d^2)*sqrt(a^2 - b^2)*arctan(-(a*sin(f*x + e) + b)/(sqrt(a^2 - b^2)*cos(f*
x + e))))/(((a^2*b - b^3)*c^3 - (a^3 - a*b^2)*c^2*d - (a^2*b - b^3)*c*d^2 +
 (a^3 - a*b^2)*d^3)*f), ((a^2 - b^2)*sqrt(c^2 - d^2)*d*arctan(-(c*sin(f*x +
 e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) - (b*c^2 - b*d^2)*sqrt(a^2 - b^2)*
arctan(-(a*sin(f*x + e) + b)/(sqrt(a^2 - b^2)*cos(f*x + e))))/(((a^2*b - b^
3)*c^3 - (a^3 - a*b^2)*c^2*d - (a^2*b - b^3)*c*d^2 + (a^3 - a*b^2)*d^3)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.704 $\int \frac{1}{(a+b\sin(e+fx))(c+d\sin(e+fx))^2}\,dx$

Optimal. Leaf size=185

$$\frac{2b^2\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{f\sqrt{a^2-b^2}(bc-ad)^2}+\frac{2d\left(acd-b\left(2c^2-d^2\right)\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{f\left(c^2-d^2\right)^{3/2}(bc-ad)^2}-\frac{d^2\cos(e+fx)}{f\left(c^2-d^2\right)(bc-ad)(c+d\sin(e+fx))}$$

```
[Out] (2*b^2*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/(Sqrt[a^2 - b^2]*(
b*c - a*d)^2*f) + (2*d*(a*c*d - b*(2*c^2 - d^2))*ArcTan[(d + c*Tan[(e + f*x
)/2])/Sqrt[c^2 - d^2]])/((b*c - a*d)^2*(c^2 - d^2)^(3/2)*f) - (d^2*Cos[e +
f*x])/((b*c - a*d)*(c^2 - d^2)*f*(c + d*Sin[e + f*x]))
```

Rubi [A] time = 0.468232, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2802, 3001, 2660, 618, 204}

$$\frac{2b^2\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{f\sqrt{a^2-b^2}(bc-ad)^2}+\frac{2d\left(acd-b\left(2c^2-d^2\right)\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{f\left(c^2-d^2\right)^{3/2}(bc-ad)^2}-\frac{d^2\cos(e+fx)}{f\left(c^2-d^2\right)(bc-ad)(c+d\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sin[e + f*x])*(c + d*Sin[e + f*x])^2),x]
```

```
[Out] (2*b^2*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/(Sqrt[a^2 - b^2]*(
b*c - a*d)^2*f) + (2*d*(a*c*d - b*(2*c^2 - d^2))*ArcTan[(d + c*Tan[(e + f*x
)/2])/Sqrt[c^2 - d^2]])/((b*c - a*d)^2*(c^2 - d^2)^(3/2)*f) - (d^2*Cos[e +
f*x])/((b*c - a*d)*(c^2 - d^2)*f*(c + d*Sin[e + f*x]))
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
```

```
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(a+b\sin(e+fx))(c+d\sin(e+fx))^2}\,dx = -\frac{d^2\cos(e+fx)}{(bc-ad)\left(c^2-d^2\right)f(c+d\sin(e+fx))} + \frac{\int \frac{-acd+b\left(c^2-d^2\right)-bcd\sin(e+fx)}{(a+b\sin(e+fx))(c+d\sin(e+fx))}}{(bc-ad)\left(c^2-d^2\right)}$$

$$= -\frac{d^2\cos(e+fx)}{(bc-ad)\left(c^2-d^2\right)f(c+d\sin(e+fx))} + \frac{b^2\int \frac{1}{a+b\sin(e+fx)}\,dx}{(bc-ad)^2} + \frac{\left(d\left(a\right.\right.}{}$$

$$= -\frac{d^2\cos(e+fx)}{(bc-ad)\left(c^2-d^2\right)f(c+d\sin(e+fx))} + \frac{\left(2b^2\right)\operatorname{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,d\right.}{(bc-ad}$$

$$= -\frac{d^2\cos(e+fx)}{(bc-ad)\left(c^2-d^2\right)f(c+d\sin(e+fx))} - \frac{\left(4b^2\right)\operatorname{Subst}\left(\int \frac{1}{-4\left(a^2-b^2\right)-x^2}\right.}{(bc}$$

$$= \frac{2b^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}(bc-ad)^2 f} + \frac{2d\left(acd-b\left(2c^2-d^2\right)\right)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e\right.}{\sqrt{c^2-d}}\right.}{(bc-ad)^2\left(c^2-d^2\right)^{3/2} f}$$

Mathematica [A] time = 0.883794, size = 165, normalized size = 0.89

$$\frac{\frac{2b^2\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + \frac{2d\left(acd+b\left(d^2-2c^2\right)\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{\left(c^2-d^2\right)^{3/2}} + \frac{d^2(ad-bc)\cos(e+fx)}{(c-d)(c+d)(c+d\sin(e+fx))}}{f(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*Sin[e + f*x])*(c + d*Sin[e + f*x])^2),x]

[Out] ((2*b^2*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] +
 (2*d*(a*c*d + b*(-2*c^2 + d^2))*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 -
 d^2]])/(c^2 - d^2)^(3/2) + (d^2*(-(b*c) + a*d)*Cos[e + f*x])/((c - d)*(c +
 d)*(c + d*Sin[e + f*x])))/((b*c - a*d)^2*f)
```

Maple [B] time = 0.121, size = 514, normalized size = 2.8

$$2\frac{d^4\tan\left(1/2\,fx+e/2\right)a}{f\left(da-cb\right)^2\left(c\left(\tan\left(1/2\,fx+e/2\right)\right)^2+2\,\tan\left(1/2\,fx+e/2\right)d+c\right)c\left(c^2-d^2\right)}-2\frac{d^3\tan\left(1/2\,f\right.}{f\left(da-cb\right)^2\left(c\left(\tan\left(1/2\,fx+e/2\right)\right)^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))^2,x)
```

```
[Out] 2/f*d^4/(a*d-b*c)^2/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)/c/(c^
2-d^2)*tan(1/2*f*x+1/2*e)*a-2/f*d^3/(a*d-b*c)^2/(c*tan(1/2*f*x+1/2*e)^2+2*t
an(1/2*f*x+1/2*e)*d+c)/(c^2-d^2)*tan(1/2*f*x+1/2*e)*b+2/f*d^3/(a*d-b*c)^2/(
c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)/(c^2-d^2)*a-2/f*d^2/(a*d-b
*c)^2/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)/(c^2-d^2)*c*b+2/f*d
^2/(a*d-b*c)^2/(c^2-d^2)^(3/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2
-d^2)^(1/2))*a*c-4/f*d/(a*d-b*c)^2/(c^2-d^2)^(3/2)*arctan(1/2*(2*c*tan(1/2*
f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*c^2*b+2/f*d^3/(a*d-b*c)^2/(c^2-d^2)^(3/2)*
arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*b+2/f*b^2/(a^2*d^2
-2*a*b*c*d+b^2*c^2)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)
/(a^2-b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.28399, size = 416, normalized size = 2.25

$$2\left(\frac{\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+b}{\sqrt{a^2-b^2}}\right)\right)b^2}{\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{a^2-b^2}}-\frac{\left(2\,bc^2d-acd^2-bd^3\right)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{\left(b^2c^4-2\,abc^3d+a^2c^2d^2-b^2c^2d^2+2\,abcd^3-a^2d^4\right)\sqrt{c^2-d^2}}-\frac{d^3\cdots}{\left(bc^4-ac^3d-bc^2d^2+acd^3\right)\cdots}\right.$$
$$\overline{\qquad\qquad f\qquad\qquad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 2*((pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*f*x + 1/2*e
) + b)/sqrt(a^2 - b^2)))*b^2/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(a^2 - b^
2)) - (2*b*c^2*d - a*c*d^2 - b*d^3)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c
) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/((b^2*c^4 - 2*a*b
*c^3*d + a^2*c^2*d^2 - b^2*c^2*d^2 + 2*a*b*c*d^3 - a^2*d^4)*sqrt(c^2 - d^2)
) - (d^3*tan(1/2*f*x + 1/2*e) + c*d^2)/((b*c^4 - a*c^3*d - b*c^2*d^2 + a*c*
d^3)*(c*tan(1/2*f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e) + c)))/f
```

3.705 $\int \frac{1}{(a+b\sin(e+fx))(c+d\sin(e+fx))^3}\,dx$

Optimal. Leaf size=284

$$\frac{d\left(-a^2d^2\left(2c^2+d^2\right)+6abc^3d-b^2\left(-5c^2d^2+6c^4+2d^4\right)\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{f\left(c^2-d^2\right)^{5/2}(bc-ad)^3}+\frac{2b^3\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{f\sqrt{a^2-b^2}(bc-ad)^3}-\frac{d^2\left(-\right.}{2f\left(c^2-\right.}$$

```
[Out] (2*b^3*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/(Sqrt[a^2 - b^2]*(
b*c - a*d)^3*f) + (d*(6*a*b*c^3*d - a^2*d^2*(2*c^2 + d^2) - b^2*(6*c^4 - 5*
c^2*d^2 + 2*d^4))*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/((b*c -
 a*d)^3*(c^2 - d^2)^(5/2)*f) - (d^2*Cos[e + f*x])/(2*(b*c - a*d)*(c^2 - d^2
)*f*(c + d*Sin[e + f*x])^2) - (d^2*(5*b*c^2 - 3*a*c*d - 2*b*d^2)*Cos[e + f*
x])/(2*(b*c - a*d)^2*(c^2 - d^2)^2*f*(c + d*Sin[e + f*x]))
```

Rubi [A] time = 1.07979, antiderivative size = 284, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {2802, 3055, 3001, 2660, 618, 204}

$$\frac{d\left(-a^2d^2\left(2c^2+d^2\right)+6abc^3d-b^2\left(-5c^2d^2+6c^4+2d^4\right)\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{f\left(c^2-d^2\right)^{5/2}(bc-ad)^3}+\frac{2b^3\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{f\sqrt{a^2-b^2}(bc-ad)^3}-\frac{d^2\left(-\right.}{2f\left(c^2-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sin[e + f*x])*(c + d*Sin[e + f*x])^3),x]
```


```
[Out] (2*b^3*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/(Sqrt[a^2 - b^2]*(
b*c - a*d)^3*f) + (d*(6*a*b*c^3*d - a^2*d^2*(2*c^2 + d^2) - b^2*(6*c^4 - 5*
c^2*d^2 + 2*d^4))*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/((b*c -
 a*d)^3*(c^2 - d^2)^(5/2)*f) - (d^2*Cos[e + f*x])/(2*(b*c - a*d)*(c^2 - d^2
)*f*(c + d*Sin[e + f*x])^2) - (d^2*(5*b*c^2 - 3*a*c*d - 2*b*d^2)*Cos[e + f*
x])/(2*(b*c - a*d)^2*(c^2 - d^2)^2*f*(c + d*Sin[e + f*x]))
```


Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
```

```
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(a+b\sin(e+fx))(c+d\sin(e+fx))^3}\,dx = -\frac{d^2\cos(e+fx)}{2(bc-ad)\left(c^2-d^2\right)f(c+d\sin(e+fx))^2} + \frac{\int \frac{-2\left(acd-b\left(c^2-d^2\right)\right)-d(2bc-ad)\sin}{(a+b\sin(e+fx))(c+d\sin}}{2(bc-ad)\left(c^2\right.}$$

$$= -\frac{d^2\cos(e+fx)}{2(bc-ad)\left(c^2-d^2\right)f(c+d\sin(e+fx))^2} - \frac{d^2\left(5bc^2-3acd-2bd^2\right)c}{2(bc-ad)^2\left(c^2-d^2\right)^2 f(c+}$$

$$= -\frac{d^2\cos(e+fx)}{2(bc-ad)\left(c^2-d^2\right)f(c+d\sin(e+fx))^2} - \frac{d^2\left(5bc^2-3acd-2bd^2\right)c}{2(bc-ad)^2\left(c^2-d^2\right)^2 f(c+}$$

$$= -\frac{d^2\cos(e+fx)}{2(bc-ad)\left(c^2-d^2\right)f(c+d\sin(e+fx))^2} - \frac{d^2\left(5bc^2-3acd-2bd^2\right)c}{2(bc-ad)^2\left(c^2-d^2\right)^2 f(c+}$$

$$= -\frac{d^2\cos(e+fx)}{2(bc-ad)\left(c^2-d^2\right)f(c+d\sin(e+fx))^2} - \frac{d^2\left(5bc^2-3acd-2bd^2\right)c}{2(bc-ad)^2\left(c^2-d^2\right)^2 f(c+}$$

$$= \frac{2b^3\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}(bc-ad)^3 f} + \frac{d\left(6abc^3d-a^2d^2\left(2c^2+d^2\right)-b^2\left(6c^4-5c^2\right.\right.}{(bc-ad)^3\left(c^2-\right.}$$

Mathematica [A] time = 2.13891, size = 263, normalized size = 0.93

$$\frac{-\frac{2d\left(a^2d^2\left(2c^2+d^2\right)-6abc^3d+b^2\left(-5c^2d^2+6c^4+2d^4\right)\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{\left(c^2-d^2\right)^{5/2}} + \frac{4b^3\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + \frac{d^2(bc-ad)\left(3acd-5bc^2+2bd^2\right)\cos(e+fx)}{(c-d)^2(c+d)^2(c+d\sin(e+fx))} - \frac{}{(c-}}{2f(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*Sin[e + f*x])*(c + d*Sin[e + f*x])^3),x]
```

```
[Out] ((4*b^3*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] -
 (2*d*(-6*a*b*c^3*d + a^2*d^2*(2*c^2 + d^2) + b^2*(6*c^4 - 5*c^2*d^2 + 2*d^
4))*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(c^2 - d^2)^(5/2) - (
d^2*(b*c - a*d)^2*Cos[e + f*x])/((c - d)*(c + d)*(c + d*Sin[e + f*x])^2) +
(d^2*(b*c - a*d)*(-5*b*c^2 + 3*a*c*d + 2*b*d^2)*Cos[e + f*x])/((c - d)^2*(c
 + d)^2*(c + d*Sin[e + f*x])))/(2*(b*c - a*d)^3*f)
```

Maple [B] time = 0.136, size = 2644, normalized size = 9.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))^3,x)
```

```
[Out] 6/f*d^6/(a*d-b*c)^3/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^
4-2*c^2*d^2+d^4)*tan(1/2*f*x+1/2*e)^3*a*b-2/f*b^3/(a^3*d^3-3*a^2*b*c*d^2+3*
a*b^2*c^2*d-b^3*c^3)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b
)/(a^2-b^2)^(1/2))+7/f*d^3/(a*d-b*c)^3/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*
```

```
x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)*c^3*tan(1/2*f*x+1/2*e)^3*b^2-4/f*d^5/(a
*d-b*c)^3/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^
2+d^4)*c*tan(1/2*f*x+1/2*e)^3*b^2+4/f*d^4/(a*d-b*c)^3/(c*tan(1/2*f*x+1/2*e)
^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)*c^2*tan(1/2*f*x+1/2*e)^2
*a^2-2/f*d^8/(a*d-b*c)^3/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^
2/(c^4-2*c^2*d^2+d^4)/c^2*tan(1/2*f*x+1/2*e)^2*a^2+6/f*d^2/(a*d-b*c)^3/(c*t
an(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)*c^4*tan
(1/2*f*x+1/2*e)^2*b^2+9/f*d^4/(a*d-b*c)^3/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2
*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)*c^2*tan(1/2*f*x+1/2*e)^2*b^2+11/f*d^
5/(a*d-b*c)^3/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2*c/(c^4-2*
c^2*d^2+d^4)*tan(1/2*f*x+1/2*e)*a^2-2/f*d^7/(a*d-b*c)^3/(c*tan(1/2*f*x+1/2*
e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/c/(c^4-2*c^2*d^2+d^4)*tan(1/2*f*x+1/2*e)*a
^2+17/f*d^3/(a*d-b*c)^3/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2
*c^3/(c^4-2*c^2*d^2+d^4)*tan(1/2*f*x+1/2*e)*b^2-8/f*d^5/(a*d-b*c)^3/(c*tan(
1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2*c/(c^4-2*c^2*d^2+d^4)*tan(1/2*
f*x+1/2*e)*b^2-10/f*d^3/(a*d-b*c)^3/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1
/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)*a*b*c^3+4/f*d^5/(a*d-b*c)^3/(c*tan(1/2*f*x
+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)*a*b*c+10/f*d^6/(a
*d-b*c)^3/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^
2+d^4)*tan(1/2*f*x+1/2*e)*a*b+5/f*d^5/(a*d-b*c)^3/(c*tan(1/2*f*x+1/2*e)^2+2
*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)*c*tan(1/2*f*x+1/2*e)^3*a^2-5
/f*d^3/(a*d-b*c)^3/(c^4-2*c^2*d^2+d^4)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(
1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*b^2*c^2+6/f*d/(a*d-b*c)^3/(c^4-2*c^2*d
^2+d^4)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(
1/2))*b^2*c^4+2/f*d^3/(a*d-b*c)^3/(c^4-2*c^2*d^2+d^4)/(c^2-d^2)^(1/2)*arcta
n(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*a^2*c^2-2/f*d^7/(a*d-b*
c)^3/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4
)/c*tan(1/2*f*x+1/2*e)^3*a^2+7/f*d^6/(a*d-b*c)^3/(c*tan(1/2*f*x+1/2*e)^2+2*
tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)*tan(1/2*f*x+1/2*e)^2*a^2-6/f*
d^6/(a*d-b*c)^3/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*
c^2*d^2+d^4)*tan(1/2*f*x+1/2*e)^2*b^2+4/f*d^4/(a*d-b*c)^3/(c*tan(1/2*f*x+1/
2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)*a^2*c^2+6/f*d^2/(a*d
-b*c)^3/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+
d^4)*b^2*c^4-3/f*d^4/(a*d-b*c)^3/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*
e)*d+c)^2/(c^4-2*c^2*d^2+d^4)*b^2*c^2+1/f*d^5/(a*d-b*c)^3/(c^4-2*c^2*d^2+d^
4)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))
*a^2+2/f*d^5/(a*d-b*c)^3/(c^4-2*c^2*d^2+d^4)/(c^2-d^2)^(1/2)*arctan(1/2*(2*
c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*b^2-12/f*d^4/(a*d-b*c)^3/(c*tan(
1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)*c^2*tan(1/
2*f*x+1/2*e)^3*a*b-10/f*d^3/(a*d-b*c)^3/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f
*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)*c^3*tan(1/2*f*x+1/2*e)^2*a*b-16/f*d^5/
(a*d-b*c)^3/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*
d^2+d^4)*c*tan(1/2*f*x+1/2*e)^2*a*b+8/f*d^7/(a*d-b*c)^3/(c*tan(1/2*f*x+1/2*
e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)/c*tan(1/2*f*x+1/2*e)^2
*a*b-28/f*d^4/(a*d-b*c)^3/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)
^2*c^2/(c^4-2*c^2*d^2+d^4)*tan(1/2*f*x+1/2*e)*a*b-6/f*d^2/(a*d-b*c)^3/(c^4-
2*c^2*d^2+d^4)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2
-d^2)^(1/2))*a*b*c^3-1/f*d^6/(a*d-b*c)^3/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*
f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)*a^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.34678, size = 1061, normalized size = 3.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] (2*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*f*x + 1/2*e
) + b)/sqrt(a^2 - b^2)))*b^3/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^
3*d^3)*sqrt(a^2 - b^2)) - (6*b^2*c^4*d - 6*a*b*c^3*d^2 + 2*a^2*c^2*d^3 - 5*
b^2*c^2*d^3 + a^2*d^5 + 2*b^2*d^5)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c)
 + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/((b^3*c^7 - 3*a*b^
2*c^6*d + 3*a^2*b*c^5*d^2 - 2*b^3*c^5*d^2 - a^3*c^4*d^3 + 6*a*b^2*c^4*d^3 -
 6*a^2*b*c^3*d^4 + b^3*c^3*d^4 + 2*a^3*c^2*d^5 - 3*a*b^2*c^2*d^5 + 3*a^2*b*
c*d^6 - a^3*d^7)*sqrt(c^2 - d^2)) - (7*b*c^4*d^3*tan(1/2*f*x + 1/2*e)^3 - 5
*a*c^3*d^4*tan(1/2*f*x + 1/2*e)^3 - 4*b*c^2*d^5*tan(1/2*f*x + 1/2*e)^3 + 2*
a*c*d^6*tan(1/2*f*x + 1/2*e)^3 + 6*b*c^5*d^2*tan(1/2*f*x + 1/2*e)^2 - 4*a*c
^4*d^3*tan(1/2*f*x + 1/2*e)^2 + 9*b*c^3*d^4*tan(1/2*f*x + 1/2*e)^2 - 7*a*c^
2*d^5*tan(1/2*f*x + 1/2*e)^2 - 6*b*c*d^6*tan(1/2*f*x + 1/2*e)^2 + 2*a*d^7*t
an(1/2*f*x + 1/2*e)^2 + 17*b*c^4*d^3*tan(1/2*f*x + 1/2*e) - 11*a*c^3*d^4*ta
n(1/2*f*x + 1/2*e) - 8*b*c^2*d^5*tan(1/2*f*x + 1/2*e) + 2*a*c*d^6*tan(1/2*f
*x + 1/2*e) + 6*b*c^5*d^2 - 4*a*c^4*d^3 - 3*b*c^3*d^4 + a*c^2*d^5)/((b^2*c^
8 - 2*a*b*c^7*d + a^2*c^6*d^2 - 2*b^2*c^6*d^2 + 4*a*b*c^5*d^3 - 2*a^2*c^4*d
^4 + b^2*c^4*d^4 - 2*a*b*c^3*d^5 + a^2*c^2*d^6)*(c*tan(1/2*f*x + 1/2*e)^2 +
 2*d*tan(1/2*f*x + 1/2*e) + c)^2))/f
```

3.706 $\int \frac{(c+d\sin(e+fx))^4}{(a+b\sin(e+fx))^2}\,dx$

Optimal. Leaf size=306

$$\frac{d(2bc-ad)\left(-3a^2d^2+2abcd+b^2\left(-\left(c^2-2d^2\right)\right)\right)\cos(e+fx)}{b^3f\left(a^2-b^2\right)}+\frac{d^2\left(-3a^2d^2+4abcd+b^2\left(-\left(2c^2-d^2\right)\right)\right)\sin(e+fx)\,c}{2b^2f\left(a^2-b^2\right)}$$

```
[Out] -(d^2*(16*a*b*c*d - 6*a^2*d^2 - b^2*(12*c^2 + d^2))*x)/(2*b^4) + (2*(b*c -
a*d)^3*(a*b*c + 3*a^2*d - 4*b^2*d)*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2
 - b^2]])/(b^4*(a^2 - b^2)^(3/2)*f) + (d*(2*b*c - a*d)*(2*a*b*c*d - 3*a^2*d
^2 - b^2*(c^2 - 2*d^2))*Cos[e + f*x])/(b^3*(a^2 - b^2)*f) + (d^2*(4*a*b*c*d
 - 3*a^2*d^2 - b^2*(2*c^2 - d^2))*Cos[e + f*x]*Sin[e + f*x])/(2*b^2*(a^2 -
b^2)*f) + ((b*c - a*d)^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(b*(a^2 - b^2
)*f*(a + b*Sin[e + f*x]))
```

Rubi [A] time = 0.940713, antiderivative size = 306, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {2792, 3033, 3023, 2735, 2660, 618, 204}

$$\frac{d(2bc-ad)\left(-3a^2d^2+2abcd+b^2\left(-\left(c^2-2d^2\right)\right)\right)\cos(e+fx)}{b^3f\left(a^2-b^2\right)}+\frac{d^2\left(-3a^2d^2+4abcd+b^2\left(-\left(2c^2-d^2\right)\right)\right)\sin(e+fx)\,c}{2b^2f\left(a^2-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^4/(a + b*Sin[e + f*x])^2,x]
```


```
[Out] -(d^2*(16*a*b*c*d - 6*a^2*d^2 - b^2*(12*c^2 + d^2))*x)/(2*b^4) + (2*(b*c -
a*d)^3*(a*b*c + 3*a^2*d - 4*b^2*d)*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2
 - b^2]])/(b^4*(a^2 - b^2)^(3/2)*f) + (d*(2*b*c - a*d)*(2*a*b*c*d - 3*a^2*d
^2 - b^2*(c^2 - 2*d^2))*Cos[e + f*x])/(b^3*(a^2 - b^2)*f) + (d^2*(4*a*b*c*d
 - 3*a^2*d^2 - b^2*(2*c^2 - d^2))*Cos[e + f*x]*Sin[e + f*x])/(2*b^2*(a^2 -
b^2)*f) + ((b*c - a*d)^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(b*(a^2 - b^2
)*f*(a + b*Sin[e + f*x]))
```


Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
```

```
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^4}{(a+b\sin(e+fx))^2}\,dx = \frac{(bc-ad)^2\cos(e+fx)(c+d\sin(e+fx))^2}{b\left(a^2-b^2\right)f(a+b\sin(e+fx))} - \frac{\int \frac{(c+d\sin(e+fx))\left(4b^2c^2d+2a^2d^3-abc\left(c^2+5d^2\right)-d\left(a^2cd-3\right.\right.}{}}{}$$

$$= \frac{d^2\left(4abcd-3a^2d^2-b^2\left(2c^2-d^2\right)\right)\cos(e+fx)\sin(e+fx)}{2b^2\left(a^2-b^2\right)f} + \frac{(bc-ad)^2\cos(e+fx)(c+d}{b\left(a^2-b^2\right)f(a+b\sin}$$

$$= \frac{d(2bc-ad)\left(2abcd-3a^2d^2-b^2\left(c^2-2d^2\right)\right)\cos(e+fx)}{b^3\left(a^2-b^2\right)f} + \frac{d^2\left(4abcd-3a^2d^2-b^2\left(2c^2-\right.\right.}{2b^2\left(a^2-\right.}$$

$$= -\frac{d^2\left(16abcd-6a^2d^2-b^2\left(12c^2+d^2\right)\right)x}{2b^4} + \frac{d(2bc-ad)\left(2abcd-3a^2d^2-b^2\left(c^2-2d^2\right)\right)\text{co}}{b^3\left(a^2-b^2\right)f}$$

$$= -\frac{d^2\left(16abcd-6a^2d^2-b^2\left(12c^2+d^2\right)\right)x}{2b^4} + \frac{d(2bc-ad)\left(2abcd-3a^2d^2-b^2\left(c^2-2d^2\right)\right)\text{co}}{b^3\left(a^2-b^2\right)f}$$

$$= -\frac{d^2\left(16abcd-6a^2d^2-b^2\left(12c^2+d^2\right)\right)x}{2b^4} + \frac{d(2bc-ad)\left(2abcd-3a^2d^2-b^2\left(c^2-2d^2\right)\right)\text{co}}{b^3\left(a^2-b^2\right)f}$$

$$= -\frac{d^2\left(16abcd-6a^2d^2-b^2\left(12c^2+d^2\right)\right)x}{2b^4} + \frac{2(bc-ad)^3\left(abc+3a^2d-4b^2d\right)\tan^{-1}\left(\frac{b+a\tan}{\sqrt{a}}\right.}{b^4\left(a^2-b^2\right)^{3/2}f}$$

Mathematica [A] time = 2.01492, size = 199, normalized size = 0.65

$$-\frac{-2d^2(e+fx)\left(6a^2d^2-16abcd+b^2\left(12c^2+d^2\right)\right)+\frac{8(ad-bc)^3\left(3a^2d+abc-4b^2d\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}}+8bd^3(2bc-ad)\cos(e+}{4b^4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])^4/(a + b*Sin[e + f*x])^2,x]
```

```
[Out] -(-2*d^2*(-16*a*b*c*d + 6*a^2*d^2 + b^2*(12*c^2 + d^2))*(e + f*x) + (8*(-(b
*c) + a*d)^3*(a*b*c + 3*a^2*d - 4*b^2*d)*ArcTan[(b + a*Tan[(e + f*x)/2])/Sq
rt[a^2 - b^2]])/(a^2 - b^2)^(3/2) + 8*b*d^3*(2*b*c - a*d)*Cos[e + f*x] - (4
*b*(b*c - a*d)^4*Cos[e + f*x])/((a - b)*(a + b)*(a + b*Sin[e + f*x])) + b^2
*d^4*Sin[2*(e + f*x)])/(4*b^4*f)
```

Maple [B] time = 0.109, size = 1303, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^4/(a+b*sin(f*x+e))^2,x)
```

```
[Out] 6/f*d^4/b^4*arctan(tan(1/2*f*x+1/2*e))*a^2+12/f*d^2/b^2*arctan(tan(1/2*f*x+
1/2*e))*c^2+2/f*b/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)/(a^2-b^
2)*c^4+2/f/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2
```

```
)^(1/2))*a*c^4+1/f*d^4/b^2/(1+tan(1/2*f*x+1/2*e)^2)^2*tan(1/2*f*x+1/2*e)^3-
1/f*d^4/b^2/(1+tan(1/2*f*x+1/2*e)^2)^2*tan(1/2*f*x+1/2*e)+4/f*d^4/b^3/(1+ta
n(1/2*f*x+1/2*e)^2)^2*a-8/f*d^3/b^2/(1+tan(1/2*f*x+1/2*e)^2)^2*c-16/f*d^3/b
^3*arctan(tan(1/2*f*x+1/2*e))*a*c+2/f/b^3/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2
*f*x+1/2*e)*b+a)/(a^2-b^2)*a^4*d^4-6/f/b^4/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*
tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*a^5*d^4+8/f/b^2/(a^2-b^2)^(3/2)*ar
ctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*a^3*d^4-8/f*b/(a^2-b
^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*c^3*d-8/
f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)/(a^2-b^2)*a*c^3*d+24/f/
(a^2-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*a*
c^2*d^2+4/f*d^4/b^3/(1+tan(1/2*f*x+1/2*e)^2)^2*tan(1/2*f*x+1/2*e)^2*a-8/f*d
^3/b^2/(1+tan(1/2*f*x+1/2*e)^2)^2*tan(1/2*f*x+1/2*e)^2*c-8/f/b/(tan(1/2*f*x
+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)/(a^2-b^2)*a^2*tan(1/2*f*x+1/2*e)*c*d^
3+12/f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)/(a^2-b^2)*a*tan(1/
2*f*x+1/2*e)*c^2*d^2+2/f/b^2/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b
+a)/(a^2-b^2)*a^3*tan(1/2*f*x+1/2*e)*d^4-8/f*b/(tan(1/2*f*x+1/2*e)^2*a+2*ta
n(1/2*f*x+1/2*e)*b+a)/(a^2-b^2)*tan(1/2*f*x+1/2*e)*c^3*d+2/f*b^2/(tan(1/2*f
*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)/(a^2-b^2)/a*tan(1/2*f*x+1/2*e)*c^4-
8/f/b^2/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)/(a^2-b^2)*a^3*c*d
^3+1/f*d^4/b^2*arctan(tan(1/2*f*x+1/2*e))+12/f/b/(tan(1/2*f*x+1/2*e)^2*a+2*
tan(1/2*f*x+1/2*e)*b+a)/(a^2-b^2)*a^2*c^2*d^2+16/f/b^3/(a^2-b^2)^(3/2)*arct
an(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*a^4*c*d^3-12/f/b^2/(a^
2-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*a^3*c
^2*d^2-24/f/b/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-
b^2)^(1/2))*a^2*c*d^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^4/(a+b*sin(f*x+e))^2,x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [B] time = 2.96677, size = 2984, normalized size = 9.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^4/(a+b*sin(f*x+e))^2,x, algorithm="fricas")

[Out] [1/2*((a^4*b^3 - 2*a^2*b^5 + b^7)*d^4*cos(f*x + e)^3 + (12*(a^5*b^2 - 2*a^3
*b^4 + a*b^6)*c^2*d^2 - 16*(a^6*b - 2*a^4*b^3 + a^2*b^5)*c*d^3 + (6*a^7 - 1
1*a^5*b^2 + 4*a^3*b^4 + a*b^6)*d^4)*f*x + (a^2*b^4*c^4 - 4*a*b^5*c^3*d - 6*
(a^4*b^2 - 2*a^2*b^4)*c^2*d^2 + 4*(2*a^5*b - 3*a^3*b^3)*c*d^3 - (3*a^6 - 4*
a^4*b^2)*d^4 + (a*b^5*c^4 - 4*b^6*c^3*d - 6*(a^3*b^3 - 2*a*b^5)*c^2*d^2 + 4
*(2*a^4*b^2 - 3*a^2*b^4)*c*d^3 - (3*a^5*b - 4*a^3*b^3)*d^4)*sin(f*x + e))*s
qrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a
^2 - b^2 - 2*(a*cos(f*x + e)*sin(f*x + e) + b*cos(f*x + e))*sqrt(-a^2 + b^2
))/(b^2*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2)) + (2*(a^2*b^5 - b
^7)*c^4 - 8*(a^3*b^4 - a*b^6)*c^3*d + 12*(a^4*b^3 - a^2*b^5)*c^2*d^2 - 8*(2
*a^5*b^2 - 3*a^3*b^4 + a*b^6)*c*d^3 + (6*a^6*b - 11*a^4*b^3 + 6*a^2*b^5 - b
```

```
^7)*d^4)*cos(f*x + e) + ((12*(a^4*b^3 - 2*a^2*b^5 + b^7)*c^2*d^2 - 16*(a^5*
b^2 - 2*a^3*b^4 + a*b^6)*c*d^3 + (6*a^6*b - 11*a^4*b^3 + 4*a^2*b^5 + b^7)*d
^4)*f*x - (8*(a^4*b^3 - 2*a^2*b^5 + b^7)*c*d^3 - 3*(a^5*b^2 - 2*a^3*b^4 + a
*b^6)*d^4)*cos(f*x + e))*sin(f*x + e))/((a^4*b^5 - 2*a^2*b^7 + b^9)*f*sin(f
*x + e) + (a^5*b^4 - 2*a^3*b^6 + a*b^8)*f), 1/2*((a^4*b^3 - 2*a^2*b^5 + b^7
)*d^4*cos(f*x + e)^3 + (12*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*c^2*d^2 - 16*(a^6*
b - 2*a^4*b^3 + a^2*b^5)*c*d^3 + (6*a^7 - 11*a^5*b^2 + 4*a^3*b^4 + a*b^6)*d
^4)*f*x - 2*(a^2*b^4*c^4 - 4*a*b^5*c^3*d - 6*(a^4*b^2 - 2*a^2*b^4)*c^2*d^2
+ 4*(2*a^5*b - 3*a^3*b^3)*c*d^3 - (3*a^6 - 4*a^4*b^2)*d^4 + (a*b^5*c^4 - 4*
b^6*c^3*d - 6*(a^3*b^3 - 2*a*b^5)*c^2*d^2 + 4*(2*a^4*b^2 - 3*a^2*b^4)*c*d^3
 - (3*a^5*b - 4*a^3*b^3)*d^4)*sin(f*x + e))*sqrt(a^2 - b^2)*arctan(-(a*sin(
f*x + e) + b)/(sqrt(a^2 - b^2)*cos(f*x + e))) + (2*(a^2*b^5 - b^7)*c^4 - 8*
(a^3*b^4 - a*b^6)*c^3*d + 12*(a^4*b^3 - a^2*b^5)*c^2*d^2 - 8*(2*a^5*b^2 - 3
*a^3*b^4 + a*b^6)*c*d^3 + (6*a^6*b - 11*a^4*b^3 + 6*a^2*b^5 - b^7)*d^4)*cos
(f*x + e) + ((12*(a^4*b^3 - 2*a^2*b^5 + b^7)*c^2*d^2 - 16*(a^5*b^2 - 2*a^3*
b^4 + a*b^6)*c*d^3 + (6*a^6*b - 11*a^4*b^3 + 4*a^2*b^5 + b^7)*d^4)*f*x - (8
*(a^4*b^3 - 2*a^2*b^5 + b^7)*c*d^3 - 3*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*d^4)*c
os(f*x + e))*sin(f*x + e))/((a^4*b^5 - 2*a^2*b^7 + b^9)*f*sin(f*x + e) + (a
^5*b^4 - 2*a^3*b^6 + a*b^8)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**4/(a+b*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.52387, size = 698, normalized size = 2.28

$$\frac{4\left(ab^4c^4-4b^5c^3d-6a^3b^2c^2d^2+12ab^4c^2d^2+8a^4bcd^3-12a^2b^3cd^3-3a^5d^4+4a^3b^2d^4\right)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^2b^4-b^6\right)\sqrt{a^2-b^2}}+\frac{4\left(b^5c^4\tan\left(\frac{1}{2}fx+\frac{1}{2}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^4/(a+b*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/2*(4*(a*b^4*c^4 - 4*b^5*c^3*d - 6*a^3*b^2*c^2*d^2 + 12*a*b^4*c^2*d^2 + 8*
a^4*b*c*d^3 - 12*a^2*b^3*c*d^3 - 3*a^5*d^4 + 4*a^3*b^2*d^4)*(pi*floor(1/2*(
f*x + e)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*f*x + 1/2*e) + b)/sqrt(a^2 -
b^2)))/((a^2*b^4 - b^6)*sqrt(a^2 - b^2)) + 4*(b^5*c^4*tan(1/2*f*x + 1/2*e)
- 4*a*b^4*c^3*d*tan(1/2*f*x + 1/2*e) + 6*a^2*b^3*c^2*d^2*tan(1/2*f*x + 1/2*
e) - 4*a^3*b^2*c*d^3*tan(1/2*f*x + 1/2*e) + a^4*b*d^4*tan(1/2*f*x + 1/2*e)
+ a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 + a^5*d^4
)/((a^3*b^3 - a*b^5)*(a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e) +
 a)) + (12*b^2*c^2*d^2 - 16*a*b*c*d^3 + 6*a^2*d^4 + b^2*d^4)*(f*x + e)/b^4
+ 2*(b*d^4*tan(1/2*f*x + 1/2*e)^3 - 8*b*c*d^3*tan(1/2*f*x + 1/2*e)^2 + 4*a*
d^4*tan(1/2*f*x + 1/2*e)^2 - b*d^4*tan(1/2*f*x + 1/2*e) - 8*b*c*d^3 + 4*a*d
^4)/((tan(1/2*f*x + 1/2*e)^2 + 1)^2*b^3))/f
```

3.707 $\int \frac{(c+d\sin(e+fx))^3}{(a+b\sin(e+fx))^2}\,dx$

Optimal. Leaf size=205

$$\frac{d\left(-2a^2d^2+2abcd+b^2\left(-\left(c^2-d^2\right)\right)\right)\cos(e+fx)}{b^2f\left(a^2-b^2\right)}+\frac{2(bc-ad)^2\left(2a^2d+abc-3b^2d\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^3f\left(a^2-b^2\right)^{3/2}}+\frac{(bc-ad)^2}{bf(a}$$

```
[Out] (d^2*(3*b*c - 2*a*d)*x)/b^3 + (2*(b*c - a*d)^2*(a*b*c + 2*a^2*d - 3*b^2*d)*
ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/(b^3*(a^2 - b^2)^(3/2)*f)
 + (d*(2*a*b*c*d - 2*a^2*d^2 - b^2*(c^2 - d^2))*Cos[e + f*x])/(b^2*(a^2 - b
^2)*f) + ((b*c - a*d)^2*Cos[e + f*x]*(c + d*Sin[e + f*x]))/(b*(a^2 - b^2)*f
*(a + b*Sin[e + f*x]))
```

Rubi [A] time = 0.459016, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {2792, 3023, 2735, 2660, 618, 204}

$$\frac{d\left(-2a^2d^2+2abcd+b^2\left(-\left(c^2-d^2\right)\right)\right)\cos(e+fx)}{b^2f\left(a^2-b^2\right)}+\frac{2(bc-ad)^2\left(2a^2d+abc-3b^2d\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^3f\left(a^2-b^2\right)^{3/2}}+\frac{(bc-ad)^2}{bf(a}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^3/(a + b*Sin[e + f*x])^2,x]
```

```
[Out] (d^2*(3*b*c - 2*a*d)*x)/b^3 + (2*(b*c - a*d)^2*(a*b*c + 2*a^2*d - 3*b^2*d)*
ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/(b^3*(a^2 - b^2)^(3/2)*f)
 + (d*(2*a*b*c*d - 2*a^2*d^2 - b^2*(c^2 - d^2))*Cos[e + f*x])/(b^2*(a^2 - b
^2)*f) + ((b*c - a*d)^2*Cos[e + f*x]*(c + d*Sin[e + f*x]))/(b*(a^2 - b^2)*f
*(a + b*Sin[e + f*x]))
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
```

```
!LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^3}{(a+b\sin(e+fx))^2}\,dx = \frac{(bc-ad)^2\cos(e+fx)(c+d\sin(e+fx))}{b\left(a^2-b^2\right)f(a+b\sin(e+fx))} - \frac{\int \frac{3b^2c^2d+a^2d^3-abc\left(c^2+3d^2\right)-d\left(a^2cd-3b^2cd+ab\left(c^2+d^2\right)\right)\sin}{a+b\sin(e+}}{b\left(a^2-\right.}$$

$$= \frac{d\left(2abcd-2a^2d^2-b^2\left(c^2-d^2\right)\right)\cos(e+fx)}{b^2\left(a^2-b^2\right)f} + \frac{(bc-ad)^2\cos(e+fx)(c+d\sin(e+fx))}{b\left(a^2-b^2\right)f(a+b\sin(e+fx))}$$

$$= \frac{d^2(3bc-2ad)x}{b^3} + \frac{d\left(2abcd-2a^2d^2-b^2\left(c^2-d^2\right)\right)\cos(e+fx)}{b^2\left(a^2-b^2\right)f} + \frac{(bc-ad)^2\cos(e+fx)(}{b\left(a^2-b^2\right)f(a+b}$$

$$= \frac{d^2(3bc-2ad)x}{b^3} + \frac{d\left(2abcd-2a^2d^2-b^2\left(c^2-d^2\right)\right)\cos(e+fx)}{b^2\left(a^2-b^2\right)f} + \frac{(bc-ad)^2\cos(e+fx)(}{b\left(a^2-b^2\right)f(a+b}$$

$$= \frac{d^2(3bc-2ad)x}{b^3} + \frac{d\left(2abcd-2a^2d^2-b^2\left(c^2-d^2\right)\right)\cos(e+fx)}{b^2\left(a^2-b^2\right)f} + \frac{(bc-ad)^2\cos(e+fx)(}{b\left(a^2-b^2\right)f(a+b}$$

$$= \frac{d^2(3bc-2ad)x}{b^3} + \frac{2(bc-ad)^2\left(abc+2a^2d-3b^2d\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{b^3\left(a^2-b^2\right)^{3/2}f} + \frac{d\left(2abcd-2\right.}{}$$

Mathematica [A] time = 1.09986, size = 151, normalized size = 0.74

$$\frac{\frac{2(bc-ad)^2\left(2a^2d+abc-3b^2d\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}} + d^2(e+fx)(3bc-2ad) + \frac{b(bc-ad)^3\cos(e+fx)}{(a-b)(a+b)(a+b\sin(e+fx))} - bd^3\cos(e+fx)}{b^3 f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])^3/(a + b*Sin[e + f*x])^2,x]
```

```
[Out] (d^2*(3*b*c - 2*a*d)*(e + f*x) + (2*(b*c - a*d)^2*(a*b*c + 2*a^2*d - 3*b^2*
d)*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2) - b*
d^3*Cos[e + f*x] + (b*(b*c - a*d)^3*Cos[e + f*x])/((a - b)*(a + b)*(a + b*S
in[e + f*x])))/(b^3*f)
```

Maple [B] time = 0.101, size = 842, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^3/(a+b*sin(f*x+e))^2,x)
```

```
[Out] -2/f*d^3/b^2/(1+tan(1/2*f*x+1/2*e)^2)-4/f*d^3/b^3*arctan(tan(1/2*f*x+1/2*e)
)*a+6/f*d^2/b^2*arctan(tan(1/2*f*x+1/2*e))*c-2/f/b/(tan(1/2*f*x+1/2*e)^2*a+
2*tan(1/2*f*x+1/2*e)*b+a)/(a^2-b^2)*a^2*tan(1/2*f*x+1/2*e)*d^3+6/f/(tan(1/2
*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)/(a^2-b^2)*a*tan(1/2*f*x+1/2*e)*c*
d^2-6/f*b/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)/(a^2-b^2)*tan(1
/2*f*x+1/2*e)*c^2*d+2/f*b^2/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+
a)/(a^2-b^2)/a*tan(1/2*f*x+1/2*e)*c^3-2/f/b^2/(tan(1/2*f*x+1/2*e)^2*a+2*tan
(1/2*f*x+1/2*e)*b+a)/(a^2-b^2)*a^3*d^3+6/f/b/(tan(1/2*f*x+1/2*e)^2*a+2*tan(
1/2*f*x+1/2*e)*b+a)/(a^2-b^2)*a^2*c*d^2-6/f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1
/2*f*x+1/2*e)*b+a)/(a^2-b^2)*a*c^2*d+2/f*b/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/
2*f*x+1/2*e)*b+a)/(a^2-b^2)*c^3+4/f/b^3/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*tan
(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*a^4*d^3-6/f/b^2/(a^2-b^2)^(3/2)*arcta
n(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*a^3*c*d^2-6/f/b/(a^2-b^
2)^(3/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*a^2*d^3+2
/f/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))
*a*c^3+12/f/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^
2)^(1/2))*a*c*d^2-6/f*b/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+
2*b)/(a^2-b^2)^(1/2))*c^2*d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^3/(a+b*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.54654, size = 2078, normalized size = 10.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^3/(a+b*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*(3*(a^5*b - 2*a^3*b^3 + a*b^5)*c*d^2 - 2*(a^6 - 2*a^4*b^2 + a^2*b^4
)*d^3)*f*x - (a^2*b^3*c^3 - 3*a*b^4*c^2*d - 3*(a^4*b - 2*a^2*b^3)*c*d^2 + (
2*a^5 - 3*a^3*b^2)*d^3 + (a*b^4*c^3 - 3*b^5*c^2*d - 3*(a^3*b^2 - 2*a*b^4)*c
*d^2 + (2*a^4*b - 3*a^2*b^3)*d^3)*sin(f*x + e))*sqrt(-a^2 + b^2)*log(((2*a^
2 - b^2)*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2 + 2*(a*cos(f*x + e
)*sin(f*x + e) + b*cos(f*x + e))*sqrt(-a^2 + b^2))/(b^2*cos(f*x + e)^2 - 2*
a*b*sin(f*x + e) - a^2 - b^2)) + 2*((a^2*b^4 - b^6)*c^3 - 3*(a^3*b^3 - a*b^
5)*c^2*d + 3*(a^4*b^2 - a^2*b^4)*c*d^2 - (2*a^5*b - 3*a^3*b^3 + a*b^5)*d^3)
*cos(f*x + e) - 2*((a^4*b^2 - 2*a^2*b^4 + b^6)*d^3*cos(f*x + e) - (3*(a^4*b
^2 - 2*a^2*b^4 + b^6)*c*d^2 - 2*(a^5*b - 2*a^3*b^3 + a*b^5)*d^3)*f*x)*sin(f
*x + e))/((a^4*b^4 - 2*a^2*b^6 + b^8)*f*sin(f*x + e) + (a^5*b^3 - 2*a^3*b^5
 + a*b^7)*f), ((3*(a^5*b - 2*a^3*b^3 + a*b^5)*c*d^2 - 2*(a^6 - 2*a^4*b^2 +
a^2*b^4)*d^3)*f*x - (a^2*b^3*c^3 - 3*a*b^4*c^2*d - 3*(a^4*b - 2*a^2*b^3)*c*
d^2 + (2*a^5 - 3*a^3*b^2)*d^3 + (a*b^4*c^3 - 3*b^5*c^2*d - 3*(a^3*b^2 - 2*a
*b^4)*c*d^2 + (2*a^4*b - 3*a^2*b^3)*d^3)*sin(f*x + e))*sqrt(a^2 - b^2)*arct
an(-(a*sin(f*x + e) + b)/(sqrt(a^2 - b^2)*cos(f*x + e))) + ((a^2*b^4 - b^6)
*c^3 - 3*(a^3*b^3 - a*b^5)*c^2*d + 3*(a^4*b^2 - a^2*b^4)*c*d^2 - (2*a^5*b -
 3*a^3*b^3 + a*b^5)*d^3)*cos(f*x + e) - ((a^4*b^2 - 2*a^2*b^4 + b^6)*d^3*co
s(f*x + e) - (3*(a^4*b^2 - 2*a^2*b^4 + b^6)*c*d^2 - 2*(a^5*b - 2*a^3*b^3 +
a*b^5)*d^3)*f*x)*sin(f*x + e))/((a^4*b^4 - 2*a^2*b^6 + b^8)*f*sin(f*x + e)
+ (a^5*b^3 - 2*a^3*b^5 + a*b^7)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**3/(a+b*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.50792, size = 782, normalized size = 3.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^3/(a+b*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] (2*(a*b^3*c^3 - 3*b^4*c^2*d - 3*a^3*b*c*d^2 + 6*a*b^3*c*d^2 + 2*a^4*d^3 - 3
*a^2*b^2*d^3)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*
f*x + 1/2*e) + b)/sqrt(a^2 - b^2)))/((a^2*b^3 - b^5)*sqrt(a^2 - b^2)) + 2*(
b^4*c^3*tan(1/2*f*x + 1/2*e)^3 - 3*a*b^3*c^2*d*tan(1/2*f*x + 1/2*e)^3 + 3*a
^2*b^2*c*d^2*tan(1/2*f*x + 1/2*e)^3 - a^3*b*d^3*tan(1/2*f*x + 1/2*e)^3 + a*
b^3*c^3*tan(1/2*f*x + 1/2*e)^2 - 3*a^2*b^2*c^2*d*tan(1/2*f*x + 1/2*e)^2 + 3
*a^3*b*c*d^2*tan(1/2*f*x + 1/2*e)^2 - 2*a^4*d^3*tan(1/2*f*x + 1/2*e)^2 + a^
2*b^2*d^3*tan(1/2*f*x + 1/2*e)^2 + b^4*c^3*tan(1/2*f*x + 1/2*e) - 3*a*b^3*c
^2*d*tan(1/2*f*x + 1/2*e) + 3*a^2*b^2*c*d^2*tan(1/2*f*x + 1/2*e) - 3*a^3*b*
d^3*tan(1/2*f*x + 1/2*e) + 2*a*b^3*d^3*tan(1/2*f*x + 1/2*e) + a*b^3*c^3 - 3
*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - 2*a^4*d^3 + a^2*b^2*d^3)/((a^3*b^2 - a*b^4
)*(a*tan(1/2*f*x + 1/2*e)^4 + 2*b*tan(1/2*f*x + 1/2*e)^3 + 2*a*tan(1/2*f*x
```

```
+ 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e) + a)) + (3*b*c*d^2 - 2*a*d^3)*(f*x +
e)/b^3)/f
```

3.708 $\int \frac{(c+d\sin(e+fx))^2}{(a+b\sin(e+fx))^2}\,dx$

Optimal. Leaf size=129

$$\frac{2(bc-ad)\left(a^2d+abc-2b^2d\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^2f\left(a^2-b^2\right)^{3/2}}+\frac{(bc-ad)^2\cos(e+fx)}{bf\left(a^2-b^2\right)(a+b\sin(e+fx))}+\frac{d^2x}{b^2}$$

```
[Out] (d^2*x)/b^2 + (2*(b*c - a*d)*(a*b*c + a^2*d - 2*b^2*d)*ArcTan[(b + a*Tan[(e
 + f*x)/2])/Sqrt[a^2 - b^2]])/(b^2*(a^2 - b^2)^(3/2)*f) + ((b*c - a*d)^2*Co
s[e + f*x])/(b*(a^2 - b^2)*f*(a + b*Sin[e + f*x]))
```

Rubi [A] time = 0.217906, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2790, 2735, 2660, 618, 204}

$$\frac{2(bc-ad)\left(a^2d+abc-2b^2d\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^2f\left(a^2-b^2\right)^{3/2}}+\frac{(bc-ad)^2\cos(e+fx)}{bf\left(a^2-b^2\right)(a+b\sin(e+fx))}+\frac{d^2x}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^2/(a + b*Sin[e + f*x])^2,x]
```

```
[Out] (d^2*x)/b^2 + (2*(b*c - a*d)*(a*b*c + a^2*d - 2*b^2*d)*ArcTan[(b + a*Tan[(e
 + f*x)/2])/Sqrt[a^2 - b^2]])/(b^2*(a^2 - b^2)^(3/2)*f) + ((b*c - a*d)^2*Co
s[e + f*x])/(b*(a^2 - b^2)*f*(a + b*Sin[e + f*x]))
```

Rule 2790

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos[e +
 f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] - Dist[1/
(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m + 1)*(2
*b*c*d - a*(c^2 + d^2)) + (a^2*d^2 - 2*a*b*c*d*(m + 2) + b^2*(d^2*(m + 1) +
 c^2*(m + 2)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^2}{(a+b\sin(e+fx))^2}\,dx = \frac{(bc-ad)^2\cos(e+fx)}{b\left(a^2-b^2\right)f(a+b\sin(e+fx))}+\frac{\int \frac{-b\left(2bcd-a\left(c^2+d^2\right)\right)+\left(a^2-b^2\right)d^2\sin(e+fx)}{a+b\sin(e+fx)}\,dx}{b\left(a^2-b^2\right)}$$

$$=\frac{d^2x}{b^2}+\frac{(bc-ad)^2\cos(e+fx)}{b\left(a^2-b^2\right)f(a+b\sin(e+fx))}+\frac{\left((bc-ad)\left(abc+a^2d-2b^2d\right)\right)\int \frac{1}{a+b\sin(e+fx)}\,dx}{b^2\left(a^2-b^2\right)}$$

$$=\frac{d^2x}{b^2}+\frac{(bc-ad)^2\cos(e+fx)}{b\left(a^2-b^2\right)f(a+b\sin(e+fx))}+\frac{\left(2(bc-ad)\left(abc+a^2d-2b^2d\right)\right)\operatorname{Subst}\left(\int \frac{1}{a+2bx+ax^2}\right.}{b^2\left(a^2-b^2\right)f}$$

$$=\frac{d^2x}{b^2}+\frac{(bc-ad)^2\cos(e+fx)}{b\left(a^2-b^2\right)f(a+b\sin(e+fx))}-\frac{\left(4(bc-ad)\left(abc+a^2d-2b^2d\right)\right)\operatorname{Subst}\left(\int \frac{1}{-4\left(a^2-b^2\right)-}\right.}{b^2\left(a^2-b^2\right)f}$$

$$=\frac{d^2x}{b^2}+\frac{2(bc-ad)\left(abc+a^2d-2b^2d\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{b^2\left(a^2-b^2\right)^{3/2}f}+\frac{(bc-ad)^2\cos(e+fx)}{b\left(a^2-b^2\right)f(a+b\sin(e+fx\right.}$$

Mathematica [A] time = 0.549897, size = 133, normalized size = 1.03

$$\frac{-\frac{2\left(a^3d^2-ab^2\left(c^2+2d^2\right)+2b^3cd\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}}+\frac{b(bc-ad)^2\cos(e+fx)}{(a-b)(a+b)(a+b\sin(e+fx))}+d^2(e+fx)}{b^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])^2/(a + b*Sin[e + f*x])^2,x]
```

```
[Out] (d^2*(e + f*x) - (2*(2*b^3*c*d + a^3*d^2 - a*b^2*(c^2 + 2*d^2))*ArcTan[(b +
 a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2) + (b*(b*c - a*d)^2
*Cos[e + f*x])/((a - b)*(a + b)*(a + b*Sin[e + f*x])))/(b^2*f)
```

Maple [B] time = 0.079, size = 556, normalized size = 4.3

$$2\,\frac{d^2\arctan\left(\tan\left(1/2\,fx+e/2\right)\right)}{b^2f}+2\,\frac{a\tan\left(1/2\,fx+e/2\right)d^2}{f\left(\left(\tan\left(1/2\,fx+e/2\right)\right)^2a+2\,\tan\left(1/2\,fx+e/2\right)b+a\right)\left(a^2-b^2\right)}-4\,\frac{}{f\left(\left(\tan\left(1/2\,f\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^2/(a+b*sin(f*x+e))^2,x)
```

```
[Out] 2/f*d^2/b^2*arctan(tan(1/2*f*x+1/2*e))+2/f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/
2*f*x+1/2*e)*b+a)/(a^2-b^2)*a*tan(1/2*f*x+1/2*e)*d^2-4/f*b/(tan(1/2*f*x+1/2
*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)/(a^2-b^2)*tan(1/2*f*x+1/2*e)*c*d+2/f*b^2/
(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)/(a^2-b^2)/a*tan(1/2*f*x+1
/2*e)*c^2+2/f/b/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)/(a^2-b^2)
*a^2*d^2-4/f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)/(a^2-b^2)*a*
c*d+2/f*b/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)/(a^2-b^2)*c^2-2
/f/b^2/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1
/2))*a^3*d^2+2/f/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a
^2-b^2)^(1/2))*a*c^2+4/f/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)
+2*b)/(a^2-b^2)^(1/2))*a*d^2-4/f*b/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*
f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*c*d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^2/(a+b*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.27134, size = 1401, normalized size = 10.86

$$\left[\frac{2\left(a^4b - 2a^2b^3 + b^5\right)d^2fx\sin\left(fx+e\right) + 2\left(a^5 - 2a^3b^2 + ab^4\right)d^2fx + \left(a^2b^2c^2 - 2ab^3cd - \left(a^4 - 2a^2b^2\right)d^2 + \left(ab^3c^2 - 2\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^2/(a+b*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*(a^4*b - 2*a^2*b^3 + b^5)*d^2*f*x*sin(f*x + e) + 2*(a^5 - 2*a^3*b^2
 + a*b^4)*d^2*f*x + (a^2*b^2*c^2 - 2*a*b^3*c*d - (a^4 - 2*a^2*b^2)*d^2 + (a
*b^3*c^2 - 2*b^4*c*d - (a^3*b - 2*a*b^3)*d^2)*sin(f*x + e))*sqrt(-a^2 + b^2
)*log(-((2*a^2 - b^2)*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2 - 2*(
a*cos(f*x + e)*sin(f*x + e) + b*cos(f*x + e))*sqrt(-a^2 + b^2))/(b^2*cos(f*
x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2)) + 2*((a^2*b^3 - b^5)*c^2 - 2*(a
^3*b^2 - a*b^4)*c*d + (a^4*b - a^2*b^3)*d^2)*cos(f*x + e))/((a^4*b^3 - 2*a^
2*b^5 + b^7)*f*sin(f*x + e) + (a^5*b^2 - 2*a^3*b^4 + a*b^6)*f), ((a^4*b - 2
*a^2*b^3 + b^5)*d^2*f*x*sin(f*x + e) + (a^5 - 2*a^3*b^2 + a*b^4)*d^2*f*x -
(a^2*b^2*c^2 - 2*a*b^3*c*d - (a^4 - 2*a^2*b^2)*d^2 + (a*b^3*c^2 - 2*b^4*c*d
 - (a^3*b - 2*a*b^3)*d^2)*sin(f*x + e))*sqrt(a^2 - b^2)*arctan(-(a*sin(f*x
+ e) + b)/(sqrt(a^2 - b^2)*cos(f*x + e))) + ((a^2*b^3 - b^5)*c^2 - 2*(a^3*b
^2 - a*b^4)*c*d + (a^4*b - a^2*b^3)*d^2)*cos(f*x + e))/((a^4*b^3 - 2*a^2*b^
5 + b^7)*f*sin(f*x + e) + (a^5*b^2 - 2*a^3*b^4 + a*b^6)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**2/(a+b*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.39138, size = 336, normalized size = 2.6

$$\frac{\frac{(fx+e)d^2}{b^2} + \frac{2\left(ab^2c^2-2\,b^3cd-a^3d^2+2\,ab^2d^2\right)\left(\pi\left\lfloor \frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor \mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^2b^2-b^4\right)\sqrt{a^2-b^2}} + \frac{2\left(b^3c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-2\,ab^2cd\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+a^2bd^2\tan\left(\frac{1}{2}\ldots\right.\right.}{\left(a^3b-ab^3\right)\left(a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+2\,b\tan\left(\frac{1}{2}\ldots\right.\right.}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^2/(a+b*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] ((f*x + e)*d^2/b^2 + 2*(a*b^2*c^2 - 2*b^3*c*d - a^3*d^2 + 2*a*b^2*d^2)*(pi*
floor(1/2*(f*x + e)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*f*x + 1/2*e) + b)/
sqrt(a^2 - b^2)))/((a^2*b^2 - b^4)*sqrt(a^2 - b^2)) + 2*(b^3*c^2*tan(1/2*f*
x + 1/2*e) - 2*a*b^2*c*d*tan(1/2*f*x + 1/2*e) + a^2*b*d^2*tan(1/2*f*x + 1/2
*e) + a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)/((a^3*b - a*b^3)*(a*tan(1/2*f*x +
1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e) + a)))/f
```

3.709 $\int \frac{c+d\sin(e+fx)}{(a+b\sin(e+fx))^2}\,dx$

Optimal. Leaf size=97

$$\frac{2(ac-bd)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{f\left(a^2-b^2\right)^{3/2}}+\frac{(bc-ad)\cos(e+fx)}{f\left(a^2-b^2\right)(a+b\sin(e+fx))}$$

```
[Out] (2*(a*c - b*d)*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^
2)^(3/2)*f) + ((b*c - a*d)*Cos[e + f*x])/((a^2 - b^2)*f*(a + b*Sin[e + f*x]
))
```

Rubi [A] time = 0.0923569, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2754, 12, 2660, 618, 204}

$$\frac{2(ac-bd)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{f\left(a^2-b^2\right)^{3/2}}+\frac{(bc-ad)\cos(e+fx)}{f\left(a^2-b^2\right)(a+b\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])/(a + b*Sin[e + f*x])^2,x]
```

```
[Out] (2*(a*c - b*d)*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^
2)^(3/2)*f) + ((b*c - a*d)*Cos[e + f*x])/((a^2 - b^2)*f*(a + b*Sin[e + f*x]
))
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{c+d\sin(e+fx)}{(a+b\sin(e+fx))^2}\,dx = \frac{(bc-ad)\cos(e+fx)}{\left(a^2-b^2\right)f(a+b\sin(e+fx))} + \frac{\int \frac{-ac+bd}{a+b\sin(e+fx)}\,dx}{-a^2+b^2}$$

$$= \frac{(bc-ad)\cos(e+fx)}{\left(a^2-b^2\right)f(a+b\sin(e+fx))} + \frac{(ac-bd)\int \frac{1}{a+b\sin(e+fx)}\,dx}{a^2-b^2}$$

$$= \frac{(bc-ad)\cos(e+fx)}{\left(a^2-b^2\right)f(a+b\sin(e+fx))} + \frac{(2(ac-bd))\,\text{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx, x, \tan\left(\frac{1}{2}(e+fx)\right)\right)}{\left(a^2-b^2\right)f}$$

$$= \frac{(bc-ad)\cos(e+fx)}{\left(a^2-b^2\right)f(a+b\sin(e+fx))} - \frac{(4(ac-bd))\,\text{Subst}\left(\int \frac{1}{-4\left(a^2-b^2\right)-x^2}\,dx, x, 2b+2a\tan\left(\frac{1}{2}(e+\right.\right.}{\left(a^2-b^2\right)f}$$

$$= \frac{2(ac-bd)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}f} + \frac{(bc-ad)\cos(e+fx)}{\left(a^2-b^2\right)f(a+b\sin(e+fx))}$$

Mathematica [A] time = 0.303997, size = 96, normalized size = 0.99

$$\frac{\frac{2(ac-bd)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}} + \frac{(bc-ad)\cos(e+fx)}{(a-b)(a+b)(a+b\sin(e+fx))}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])/(a + b*Sin[e + f*x])^2,x]
```

```
[Out] ((2*(a*c - b*d)*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^
2)^(3/2) + ((b*c - a*d)*Cos[e + f*x])/((a - b)*(a + b)*(a + b*Sin[e + f*x])
))/f
```

Maple [B] time = 0.072, size = 309, normalized size = 3.2

$$-2\,\frac{\tan\left(1/2\,fx+e/2\right)bd}{f\left(\left(\tan\left(1/2\,fx+e/2\right)\right)^2a+2\,\tan\left(1/2\,fx+e/2\right)b+a\right)\left(a^2-b^2\right)}+2\,\frac{b^2\tan\left(1/2\,fx+e/2\right)c}{f\left(\left(\tan\left(1/2\,fx+e/2\right)\right)^2a+2\,\tan\left(1/2\,fx+e/2\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))/(a+b*sin(f*x+e))^2,x)
```

```
[Out] -2/f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)*b/(a^2-b^2)*tan(1/2*
f*x+1/2*e)*d+2/f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)*b^2/(a^2
-b^2)/a*tan(1/2*f*x+1/2*e)*c-2/f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*
e)*b+a)/(a^2-b^2)*d*a+2/f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)
/(a^2-b^2)*c*b+2/f/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/
(a^2-b^2)^(1/2))*c*a-2/f/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)
+2*b)/(a^2-b^2)^(1/2))*b*d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))/(a+b*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.06916, size = 871, normalized size = 8.98

$$\left[-\frac{\left(a^2c - abd + \left(abc - b^2d\right)\sin\left(fx+e\right)\right)\sqrt{-a^2+b^2}\log\left(\frac{\left(2a^2-b^2\right)\cos\left(fx+e\right)^2 - 2ab\sin\left(fx+e\right) - a^2 - b^2 + 2\left(a\cos\left(fx+e\right)\sin\left(fx+e\right) + b\cos\left(f\right.\right.}{b^2\cos\left(fx+e\right)^2 - 2ab\sin\left(fx+e\right) - a^2 - b^2}\right.}{2\left(\left(a^4b - 2a^2b^3 + b^5\right)f\sin\left(fx+e\right) + \left(a^5 - 2a^3b^2 + ab^4\right)f\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))/(a+b*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] [-1/2*((a^2*c - a*b*d + (a*b*c - b^2*d)*sin(f*x + e))*sqrt(-a^2 + b^2)*log(
((2*a^2 - b^2)*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2 + 2*(a*cos(f
*x + e)*sin(f*x + e) + b*cos(f*x + e))*sqrt(-a^2 + b^2))/(b^2*cos(f*x + e)^
2 - 2*a*b*sin(f*x + e) - a^2 - b^2)) - 2*((a^2*b - b^3)*c - (a^3 - a*b^2)*d
)*cos(f*x + e))/((a^4*b - 2*a^2*b^3 + b^5)*f*sin(f*x + e) + (a^5 - 2*a^3*b^
2 + a*b^4)*f), -((a^2*c - a*b*d + (a*b*c - b^2*d)*sin(f*x + e))*sqrt(a^2 -
b^2)*arctan(-(a*sin(f*x + e) + b)/(sqrt(a^2 - b^2)*cos(f*x + e))) - ((a^2*b
 - b^3)*c - (a^3 - a*b^2)*d)*cos(f*x + e))/((a^4*b - 2*a^2*b^3 + b^5)*f*sin
(f*x + e) + (a^5 - 2*a^3*b^2 + a*b^4)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))/(a+b*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.43228, size = 212, normalized size = 2.19

$$\frac{2\left(\frac{\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+b}{\sqrt{a^2-b^2}}\right)\right)(ac-bd)}{\left(a^2-b^2\right)^{\frac{3}{2}}}+\frac{b^2c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-abd\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+abc-a^2d}{\left(a^3-ab^2\right)\left(a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+a\right)}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))/(a+b*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 2*((pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*f*x + 1/2*e
) + b)/sqrt(a^2 - b^2)))*(a*c - b*d)/(a^2 - b^2)^(3/2) + (b^2*c*tan(1/2*f*x
 + 1/2*e) - a*b*d*tan(1/2*f*x + 1/2*e) + a*b*c - a^2*d)/((a^3 - a*b^2)*(a*t
an(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e) + a)))/f
```

3.710 $\int \frac{1}{(a+b\sin(e+fx))^2}\,dx$

Optimal. Leaf size=83

$$\frac{2a\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{f\left(a^2-b^2\right)^{3/2}}+\frac{b\cos(e+fx)}{f\left(a^2-b^2\right)(a+b\sin(e+fx))}$$

```
[Out] (2*a*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(3/2)*f
) + (b*Cos[e + f*x])/((a^2 - b^2)*f*(a + b*Sin[e + f*x]))
```

Rubi [A] time = 0.0595759, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {2664, 12, 2660, 618, 204}

$$\frac{2a\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{f\left(a^2-b^2\right)^{3/2}}+\frac{b\cos(e+fx)}{f\left(a^2-b^2\right)(a+b\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^(-2),x]
```

```
[Out] (2*a*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(3/2)*f
) + (b*Cos[e + f*x])/((a^2 - b^2)*f*(a + b*Sin[e + f*x]))
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+b\sin(e+fx))^2}\,dx &= \frac{b\cos(e+fx)}{\left(a^2-b^2\right)f(a+b\sin(e+fx))} - \frac{\int \frac{a}{a+b\sin(e+fx)}\,dx}{-a^2+b^2}\\
&= \frac{b\cos(e+fx)}{\left(a^2-b^2\right)f(a+b\sin(e+fx))} + \frac{a\int \frac{1}{a+b\sin(e+fx)}\,dx}{a^2-b^2}\\
&= \frac{b\cos(e+fx)}{\left(a^2-b^2\right)f(a+b\sin(e+fx))} + \frac{(2a)\,\mathrm{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx, x, \tan\left(\frac{1}{2}(e+fx)\right)\right)}{\left(a^2-b^2\right)f}\\
&= \frac{b\cos(e+fx)}{\left(a^2-b^2\right)f(a+b\sin(e+fx))} - \frac{(4a)\,\mathrm{Subst}\left(\int \frac{1}{-4\left(a^2-b^2\right)-x^2}\,dx, x, 2b+2a\tan\left(\frac{1}{2}(e+fx)\right)\right)}{\left(a^2-b^2\right)f}\\
&= \frac{2a\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}f} + \frac{b\cos(e+fx)}{\left(a^2-b^2\right)f(a+b\sin(e+fx))}
\end{aligned}$$

Mathematica [A] time = 0.183385, size = 82, normalized size = 0.99

$$\frac{\frac{2a\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}} + \frac{b\cos(e+fx)}{(a-b)(a+b)(a+b\sin(e+fx))}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^(-2),x]
```

```
[Out] ((2*a*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2) +
 (b*Cos[e + f*x])/((a - b)*(a + b)*(a + b*Sin[e + f*x])))/f
```

Maple [A] time = 0.056, size = 155, normalized size = 1.9

$$2\,\frac{b^2\tan\left(1/2\,fx+e/2\right)}{f\left(\left(\tan\left(1/2\,fx+e/2\right)\right)^2a+2\,\tan\left(1/2\,fx+e/2\right)b+a\right)a\left(a^2-b^2\right)}+2\,\frac{b}{f\left(\left(\tan\left(1/2\,fx+e/2\right)\right)^2a+2\,\tan\left(1/2\,fx+e/2\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e))^2,x)
```

```
[Out] 2/f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)*b^2/a/(a^2-b^2)*tan(1
/2*f*x+1/2*e)+2/f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)/(a^2-b^
2)*b+2/f*a/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2
)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.77273, size = 752, normalized size = 9.06

$$\left[\frac{\left(ab\sin\left(fx+e\right)+a^2\right)\sqrt{-a^2+b^2}\log\left(-\frac{\left(2a^2-b^2\right)\cos\left(fx+e\right)^2-2ab\sin\left(fx+e\right)-a^2-b^2-2\left(a\cos\left(fx+e\right)\sin\left(fx+e\right)+b\cos\left(fx+e\right)\right)\sqrt{-a^2+b^2}}{b^2\cos\left(fx+e\right)^2-2ab\sin\left(fx+e\right)-a^2-b^2}\right)+2}{2\left(\left(a^4b-2a^2b^3+b^5\right)f\sin\left(fx+e\right)+\left(a^5-2a^3b^2+ab^4\right)f\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] [1/2*((a*b*sin(f*x + e) + a^2)*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(f*x
 + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2 - 2*(a*cos(f*x + e)*sin(f*x + e) +
 b*cos(f*x + e))*sqrt(-a^2 + b^2))/(b^2*cos(f*x + e)^2 - 2*a*b*sin(f*x + e)
 - a^2 - b^2)) + 2*(a^2*b - b^3)*cos(f*x + e))/((a^4*b - 2*a^2*b^3 + b^5)*f
*sin(f*x + e) + (a^5 - 2*a^3*b^2 + a*b^4)*f), -((a*b*sin(f*x + e) + a^2)*sq
rt(a^2 - b^2)*arctan(-(a*sin(f*x + e) + b)/(sqrt(a^2 - b^2)*cos(f*x + e)))
- (a^2*b - b^3)*cos(f*x + e))/((a^4*b - 2*a^2*b^3 + b^5)*f*sin(f*x + e) + (
a^5 - 2*a^3*b^2 + a*b^4)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.37075, size = 171, normalized size = 2.06

$$\frac{2\left(\frac{\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+b}{\sqrt{a^2-b^2}}\right)\right)a}{\left(a^2-b^2\right)^{\frac{3}{2}}}+\frac{b^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+ab}{\left(a^3-ab^2\right)\left(a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+a\right)}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 2*((pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*f*x + 1/2*e
) + b)/sqrt(a^2 - b^2)))*a/(a^2 - b^2)^(3/2) + (b^2*tan(1/2*f*x + 1/2*e) +
a*b)/((a^3 - a*b^2)*(a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e) +
a)))/f
```

3.711 $\int \frac{1}{(a+b\sin(e+fx))^2(c+d\sin(e+fx))} dx$

Optimal. Leaf size=181

$$\frac{2b\left(-2a^2d+abc+b^2d\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{f\left(a^2-b^2\right)^{3/2}(bc-ad)^2}+\frac{b^2\cos(e+fx)}{f\left(a^2-b^2\right)(bc-ad)(a+b\sin(e+fx))}+\frac{2d^2\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{f\sqrt{c^2-d^2}(bc-ad)^2}$$

```
[Out] (2*b*(a*b*c - 2*a^2*d + b^2*d)*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b
^2]])/((a^2 - b^2)^(3/2)*(b*c - a*d)^2*f) + (2*d^2*ArcTan[(d + c*Tan[(e + f
*x)/2])/Sqrt[c^2 - d^2]])/((b*c - a*d)^2*Sqrt[c^2 - d^2]*f) + (b^2*Cos[e +
f*x])/((a^2 - b^2)*(b*c - a*d)*f*(a + b*Sin[e + f*x]))
```

Rubi [A] time = 0.437859, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2802, 3001, 2660, 618, 204}

$$\frac{2b\left(-2a^2d+abc+b^2d\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{f\left(a^2-b^2\right)^{3/2}(bc-ad)^2}+\frac{b^2\cos(e+fx)}{f\left(a^2-b^2\right)(bc-ad)(a+b\sin(e+fx))}+\frac{2d^2\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{f\sqrt{c^2-d^2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sin[e + f*x])^2*(c + d*Sin[e + f*x])),x]
```


```
[Out] (2*b*(a*b*c - 2*a^2*d + b^2*d)*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b
^2]])/((a^2 - b^2)^(3/2)*(b*c - a*d)^2*f) + (2*d^2*ArcTan[(d + c*Tan[(e + f
*x)/2])/Sqrt[c^2 - d^2]])/((b*c - a*d)^2*Sqrt[c^2 - d^2]*f) + (b^2*Cos[e +
f*x])/((a^2 - b^2)*(b*c - a*d)*f*(a + b*Sin[e + f*x]))
```


Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
```

```
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(a+b\sin(e+fx))^2(c+d\sin(e+fx))}\,dx = \frac{b^2\cos(e+fx)}{\left(a^2-b^2\right)(bc-ad)f(a+b\sin(e+fx))} - \frac{\int \frac{-abc+a^2d-b^2d-abd\sin(e+fx)}{(a+b\sin(e+fx))(c+d\sin(e+fx))}\,dx}{\left(a^2-b^2\right)(bc-ad)}$$

$$= \frac{b^2\cos(e+fx)}{\left(a^2-b^2\right)(bc-ad)f(a+b\sin(e+fx))} + \frac{d^2\int \frac{1}{c+d\sin(e+fx)}\,dx}{(bc-ad)^2} + \frac{\left(b\left(abc-\right.\right.}{}$$

$$= \frac{b^2\cos(e+fx)}{\left(a^2-b^2\right)(bc-ad)f(a+b\sin(e+fx))} + \frac{\left(2d^2\right)\text{Subst}\left(\int \frac{1}{c+2dx+cx^2}\,dx,x,\text{t}\right.}{(bc-ad)^2 f}$$

$$= \frac{b^2\cos(e+fx)}{\left(a^2-b^2\right)(bc-ad)f(a+b\sin(e+fx))} - \frac{\left(4d^2\right)\text{Subst}\left(\int \frac{1}{-4\left(c^2-d^2\right)-x^2}\,dx,x\right.}{(bc-ad}$$

$$= \frac{2b\left(abc-2a^2d+b^2d\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}(bc-ad)^2 f} + \frac{2d^2\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right.}{\sqrt{c^2-d^2}}\right.}{(bc-ad)^2\sqrt{c^2-d^2}f}$$

Mathematica [A] time = 0.851623, size = 178, normalized size = 0.98

$$\frac{\frac{2b\left(-2a^2d+abc+b^2d\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}(bc-ad)^2} - \frac{b^2\cos(e+fx)}{(a-b)(a+b)(ad-bc)(a+b\sin(e+fx))} + \frac{2d^2\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{\sqrt{c^2-d^2}(bc-ad)^2}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*Sin[e + f*x])^2*(c + d*Sin[e + f*x])),x]
```

```
[Out] ((2*b*(a*b*c - 2*a^2*d + b^2*d)*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 -
b^2]])/((a^2 - b^2)^(3/2)*(b*c - a*d)^2) + (2*d^2*ArcTan[(d + c*Tan[(e + f*
x)/2])/Sqrt[c^2 - d^2]])/((b*c - a*d)^2*Sqrt[c^2 - d^2]) - (b^2*Cos[e + f*x
])/((a - b)*(a + b)*(-(b*c) + a*d)*(a + b*Sin[e + f*x])))/f
```

Maple [B] time = 0.119, size = 514, normalized size = 2.8

$$2\,\frac{d^2}{f\left(a^2d^2-2\,abcd+c^2b^2\right)\sqrt{c^2-d^2}}\arctan\left(1/2\,\frac{2\,c\tan\left(1/2\,fx+e/2\right)+2\,d}{\sqrt{c^2-d^2}}\right)-2\,\frac{b^3\tan\left(1/2\,fx\right.}{f\,(da-cb)^2\left(\left(\tan\left(1/2\,fx+e/2\right)\right)^2a+2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e))^2/(c+d*sin(f*x+e)),x)
```

```
[Out] 2/f*d^2/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2
*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))-2/f*b^3/(a*d-b*c)^2/(tan(1/2*f*x+1/2*e)^2
*a+2*tan(1/2*f*x+1/2*e)*b+a)/(a^2-b^2)*tan(1/2*f*x+1/2*e)*d+2/f*b^4/(a*d-b*
c)^2/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)/a/(a^2-b^2)*tan(1/2*
f*x+1/2*e)*c-2/f*b^2/(a*d-b*c)^2/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*
e)*b+a)/(a^2-b^2)*d*a+2/f*b^3/(a*d-b*c)^2/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2
*f*x+1/2*e)*b+a)/(a^2-b^2)*c-4/f*b/(a*d-b*c)^2/(a^2-b^2)^(3/2)*arctan(1/2*(
2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*a^2*d+2/f*b^2/(a*d-b*c)^2/(a^2
-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*a*c+2/
f*b^3/(a*d-b*c)^2/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(
a^2-b^2)^(1/2))*d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^2/(c+d*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^2/(c+d*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))**2/(c+d*sin(f*x+e)),x)
```

```
[Out] Timed out
```

<u>Giac</u> [A] time = 1.42915, size = 410, normalized size = 2.27

$$\frac{2\left(\frac{\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+d}{\sqrt{c^2-d^2}}\right)\right)d^2}{\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{c^2-d^2}}+\frac{\left(ab^2c-2\,a^2bd+b^3d\right)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^2b^2c^2-b^4c^2-2\,a^3bcd+2\,ab^3cd+a^4d^2-a^2b^2d^2\right)\sqrt{a^2-b^2}}+\frac{b^3\tan}{\left(a^3bc-ab^3c-a^4d+a^2b^2d\right)\left(a\,\mathrm{t}\right.}\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^2/(c+d*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] 2*((pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e
) + d)/sqrt(c^2 - d^2)))*d^2/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(c^2 - d^
2)) + (a*b^2*c - 2*a^2*b*d + b^3*d)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(a
) + arctan((a*tan(1/2*f*x + 1/2*e) + b)/sqrt(a^2 - b^2)))/((a^2*b^2*c^2 - b
^4*c^2 - 2*a^3*b*c*d + 2*a*b^3*c*d + a^4*d^2 - a^2*b^2*d^2)*sqrt(a^2 - b^2)
) + (b^3*tan(1/2*f*x + 1/2*e) + a*b^2)/((a^3*b*c - a*b^3*c - a^4*d + a^2*b^
2*d)*(a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e) + a)))/f
```

3.712 $\int \frac{1}{(a+b\sin(e+fx))^2(c+d\sin(e+fx))^2}\,dx$

Optimal. Leaf size=290

$$\frac{d\left(a^2d^2+b^2\left(c^2-2d^2\right)\right)\cos(e+fx)}{f\left(a^2-b^2\right)\left(c^2-d^2\right)(bc-ad)^2(c+d\sin(e+fx))}+\frac{2b^2\left(-3a^2d+abc+2b^2d\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{f\left(a^2-b^2\right)^{3/2}(bc-ad)^3}+\frac{}{f\left(a^2-b^2\right)(b}$$

```
[Out] (2*b^2*(a*b*c - 3*a^2*d + 2*b^2*d)*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2
 - b^2]])/((a^2 - b^2)^(3/2)*(b*c - a*d)^3*f) + (2*d^2*(3*b*c^2 - a*c*d - 2
*b*d^2)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/((b*c - a*d)^3*(c
^2 - d^2)^(3/2)*f) + (d*(a^2*d^2 + b^2*(c^2 - 2*d^2))*Cos[e + f*x])/((a^2 -
 b^2)*(b*c - a*d)^2*(c^2 - d^2)*f*(c + d*Sin[e + f*x])) + (b^2*Cos[e + f*x]
)/((a^2 - b^2)*(b*c - a*d)*f*(a + b*Sin[e + f*x])*(c + d*Sin[e + f*x]))
```

Rubi [A] time = 1.19715, antiderivative size = 290, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {2802, 3055, 3001, 2660, 618, 204}

$$\frac{d\left(a^2d^2+b^2\left(c^2-2d^2\right)\right)\cos(e+fx)}{f\left(a^2-b^2\right)\left(c^2-d^2\right)(bc-ad)^2(c+d\sin(e+fx))}+\frac{2b^2\left(-3a^2d+abc+2b^2d\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{f\left(a^2-b^2\right)^{3/2}(bc-ad)^3}+\frac{}{f\left(a^2-b^2\right)(b}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^2),x]
```



```
[Out] (2*b^2*(a*b*c - 3*a^2*d + 2*b^2*d)*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2
 - b^2]])/((a^2 - b^2)^(3/2)*(b*c - a*d)^3*f) + (2*d^2*(3*b*c^2 - a*c*d - 2
*b*d^2)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/((b*c - a*d)^3*(c
^2 - d^2)^(3/2)*f) + (d*(a^2*d^2 + b^2*(c^2 - 2*d^2))*Cos[e + f*x])/((a^2 -
 b^2)*(b*c - a*d)^2*(c^2 - d^2)*f*(c + d*Sin[e + f*x])) + (b^2*Cos[e + f*x]
)/((a^2 - b^2)*(b*c - a*d)*f*(a + b*Sin[e + f*x])*(c + d*Sin[e + f*x]))
```


Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
```

```
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(a+b\sin(e+fx))^2(c+d\sin(e+fx))^2}\,dx = \frac{b^2\cos(e+fx)}{\left(a^2-b^2\right)(bc-ad)f(a+b\sin(e+fx))(c+d\sin(e+fx))} - \int \frac{-abc+a^2\ldots}{(a\ldots}$$

$$= \frac{d\left(a^2d^2+b^2\left(c^2-2d^2\right)\right)\cos(e+fx)}{\left(a^2-b^2\right)(bc-ad)^2\left(c^2-d^2\right)f(c+d\sin(e+fx))} + \frac{\ldots}{\left(a^2-b^2\right)(bc-ad\ldots}$$

$$= \frac{d\left(a^2d^2+b^2\left(c^2-2d^2\right)\right)\cos(e+fx)}{\left(a^2-b^2\right)(bc-ad)^2\left(c^2-d^2\right)f(c+d\sin(e+fx))} + \frac{\ldots}{\left(a^2-b^2\right)(bc-ad\ldots}$$

$$= \frac{d\left(a^2d^2+b^2\left(c^2-2d^2\right)\right)\cos(e+fx)}{\left(a^2-b^2\right)(bc-ad)^2\left(c^2-d^2\right)f(c+d\sin(e+fx))} + \frac{\ldots}{\left(a^2-b^2\right)(bc-ad\ldots}$$

$$= \frac{d\left(a^2d^2+b^2\left(c^2-2d^2\right)\right)\cos(e+fx)}{\left(a^2-b^2\right)(bc-ad)^2\left(c^2-d^2\right)f(c+d\sin(e+fx))} + \frac{\ldots}{\left(a^2-b^2\right)(bc-ad\ldots}$$

$$= \frac{2b^2\left(abc-3a^2d+2b^2d\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}(bc-ad)^3f} + \frac{2d^2\left(3bc^2-acd-\ldots\right.}{(bc-a\ldots}$$

Mathematica [A] time = 2.84357, size = 227, normalized size = 0.78

$$\frac{\frac{2b^2\left(-3a^2d+abc+2b^2d\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}} + \frac{b^3(bc-ad)\cos(e+fx)}{(a-b)(a+b)(a+b\sin(e+fx))} - \frac{2d^2\left(acd-3bc^2+2bd^2\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{\left(c^2-d^2\right)^{3/2}} + \frac{d^3(bc-ad)\cos(e+f\ldots}{(c-d)(c+d)(c+d\sin(e+\ldots}}{f(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^2),x]
```

```
[Out] ((2*b^2*(a*b*c - 3*a^2*d + 2*b^2*d)*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^
2 - b^2]])/(a^2 - b^2)^(3/2) - (2*d^2*(-3*b*c^2 + a*c*d + 2*b*d^2)*ArcTan[(
d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(c^2 - d^2)^(3/2) + (b^3*(b*c - a
*d)*Cos[e + f*x])/((a - b)*(a + b)*(a + b*Sin[e + f*x])) + (d^3*(b*c - a*d)
*Cos[e + f*x])/((c - d)*(c + d)*(c + d*Sin[e + f*x])))/((b*c - a*d)^3*f)
```

Maple [B] time = 0.138, size = 886, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^2,x)
```

```
[Out] 2/f*d^5/(a*d-b*c)^3/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)/c/(c^
2-d^2)*tan(1/2*f*x+1/2*e)*a-2/f*d^4/(a*d-b*c)^3/(c*tan(1/2*f*x+1/2*e)^2+2*t
an(1/2*f*x+1/2*e)*d+c)/(c^2-d^2)*tan(1/2*f*x+1/2*e)*b+2/f*d^4/(a*d-b*c)^3/(
c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)/(c^2-d^2)*a-2/f*d^3/(a*d-b
*c)^3/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)/(c^2-d^2)*c*b+2/f*d
^3/(a*d-b*c)^3/(c^2-d^2)^(3/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2
-d^2)^(1/2))*a*c-6/f*d^2/(a*d-b*c)^3/(c^2-d^2)^(3/2)*arctan(1/2*(2*c*tan(1/
```

```
2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*c^2*b+4/f*d^4/(a*d-b*c)^3/(c^2-d^2)^(3/2
)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*b+2/f*b^4/(a*d-b
*c)^3/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)/(a^2-b^2)*tan(1/2*f
*x+1/2*e)*d-2/f*b^5/(a*d-b*c)^3/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e
)*b+a)/a/(a^2-b^2)*tan(1/2*f*x+1/2*e)*c+2/f*b^3/(a*d-b*c)^3/(tan(1/2*f*x+1/
2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)/(a^2-b^2)*d*a-2/f*b^4/(a*d-b*c)^3/(tan(1
/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)/(a^2-b^2)*c+6/f*b^2/(a*d-b*c)^3
/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*a
^2*d-2/f*b^3/(a*d-b*c)^3/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)
+2*b)/(a^2-b^2)^(1/2))*a*c-4/f*b^4/(a*d-b*c)^3/(a^2-b^2)^(3/2)*arctan(1/2*(
2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^2,x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^2,x, algorithm="fricas")

[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))**2/(c+d*sin(f*x+e))**2,x)

[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^2,x, algorithm="giac")

[Out] Exception raised: NotImplementedError
```

3.713 $\int \frac{1}{(a+b\sin(e+fx))^2(c+d\sin(e+fx))^3}\,dx$

Optimal. Leaf size=458

$$-\frac{d^2\left(-a^2d^2\left(2c^2+d^2\right)+2abcd\left(4c^2-d^2\right)-3b^2\left(-5c^2d^2+4c^4+2d^4\right)\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{f\left(c^2-d^2\right)^{5/2}(bc-ad)^4}-\frac{\left(-a^2bd^3\left(7c^2-4d^2\right)\right.}{2f\left(a^2\right.}$$

```
[Out] (2*b^3*(a*b*c - 4*a^2*d + 3*b^2*d)*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2
 - b^2]])/((a^2 - b^2)^(3/2)*(b*c - a*d)^4*f) - (d^2*(2*a*b*c*d*(4*c^2 - d^
2) - a^2*d^2*(2*c^2 + d^2) - 3*b^2*(4*c^4 - 5*c^2*d^2 + 2*d^4))*ArcTan[(d +
 c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/((b*c - a*d)^4*(c^2 - d^2)^(5/2)*f)
+ (d*(a^2*d^2 + b^2*(2*c^2 - 3*d^2))*Cos[e + f*x])/(2*(a^2 - b^2)*(b*c - a*
d)^2*(c^2 - d^2)*f*(c + d*Sin[e + f*x])^2) + (b^2*Cos[e + f*x])/((a^2 - b^2
)*(b*c - a*d)*f*(a + b*Sin[e + f*x])*(c + d*Sin[e + f*x])^2) - ((3*a^3*c*d^
4 - 3*a*b^2*c*d^4 - a^2*b*d^3*(7*c^2 - 4*d^2) - b^3*(2*c^4*d - 11*c^2*d^3 +
 6*d^5))*Cos[e + f*x])/(2*(a^2 - b^2)*(b*c - a*d)^3*(c^2 - d^2)^2*f*(c + d*
Sin[e + f*x]))
```

Rubi [A] time = 2.43866, antiderivative size = 458, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {2802, 3055, 3001, 2660, 618, 204}

$$-\frac{d^2\left(-a^2d^2\left(2c^2+d^2\right)+2abcd\left(4c^2-d^2\right)-3b^2\left(-5c^2d^2+4c^4+2d^4\right)\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{f\left(c^2-d^2\right)^{5/2}(bc-ad)^4}-\frac{\left(-a^2bd^3\left(7c^2-4d^2\right)\right.}{2f\left(a^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^3),x]
```

```
[Out] (2*b^3*(a*b*c - 4*a^2*d + 3*b^2*d)*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2
 - b^2]])/((a^2 - b^2)^(3/2)*(b*c - a*d)^4*f) - (d^2*(2*a*b*c*d*(4*c^2 - d^
2) - a^2*d^2*(2*c^2 + d^2) - 3*b^2*(4*c^4 - 5*c^2*d^2 + 2*d^4))*ArcTan[(d +
 c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/((b*c - a*d)^4*(c^2 - d^2)^(5/2)*f)
+ (d*(a^2*d^2 + b^2*(2*c^2 - 3*d^2))*Cos[e + f*x])/(2*(a^2 - b^2)*(b*c - a*
d)^2*(c^2 - d^2)*f*(c + d*Sin[e + f*x])^2) + (b^2*Cos[e + f*x])/((a^2 - b^2
)*(b*c - a*d)*f*(a + b*Sin[e + f*x])*(c + d*Sin[e + f*x])^2) - ((3*a^3*c*d^
4 - 3*a*b^2*c*d^4 - a^2*b*d^3*(7*c^2 - 4*d^2) - b^3*(2*c^4*d - 11*c^2*d^3 +
 6*d^5))*Cos[e + f*x])/(2*(a^2 - b^2)*(b*c - a*d)^3*(c^2 - d^2)^2*f*(c + d*
Sin[e + f*x]))
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(a+b\sin(e+fx))^2(c+d\sin(e+fx))^3}\,dx = \frac{b^2\cos(e+fx)}{\left(a^2-b^2\right)(bc-ad)f(a+b\sin(e+fx))(c+d\sin(e+fx))^2} - \int \frac{-abc+a\cdots}{\cdots}$$

$$= \frac{d\left(a^2d^2+b^2\left(2c^2-3d^2\right)\right)\cos(e+fx)}{2\left(a^2-b^2\right)(bc-ad)^2\left(c^2-d^2\right)f(c+d\sin(e+fx))^2} + \frac{\cdots}{\left(a^2-b^2\right)(bc-\cdots}$$

$$= \frac{d\left(a^2d^2+b^2\left(2c^2-3d^2\right)\right)\cos(e+fx)}{2\left(a^2-b^2\right)(bc-ad)^2\left(c^2-d^2\right)f(c+d\sin(e+fx))^2} + \frac{\cdots}{\left(a^2-b^2\right)(bc-\cdots}$$

$$= \frac{d\left(a^2d^2+b^2\left(2c^2-3d^2\right)\right)\cos(e+fx)}{2\left(a^2-b^2\right)(bc-ad)^2\left(c^2-d^2\right)f(c+d\sin(e+fx))^2} + \frac{\cdots}{\left(a^2-b^2\right)(bc-\cdots}$$

$$= \frac{d\left(a^2d^2+b^2\left(2c^2-3d^2\right)\right)\cos(e+fx)}{2\left(a^2-b^2\right)(bc-ad)^2\left(c^2-d^2\right)f(c+d\sin(e+fx))^2} + \frac{\cdots}{\left(a^2-b^2\right)(bc-\cdots}$$

$$= \frac{d\left(a^2d^2+b^2\left(2c^2-3d^2\right)\right)\cos(e+fx)}{2\left(a^2-b^2\right)(bc-ad)^2\left(c^2-d^2\right)f(c+d\sin(e+fx))^2} + \frac{\cdots}{\left(a^2-b^2\right)(bc-\cdots}$$

$$= \frac{2b^3\left(abc-4a^2d+3b^2d\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}(bc-ad)^4f} - \frac{d^2\left(2abcd\left(4c^2-d^{\cdots}\right.\right.}{\cdots}$$

Mathematica [A] time = 6.44137, size = 346, normalized size = 0.76

$$\frac{\frac{2d^2\left(a^2d^2\left(2c^2+d^2\right)+2abcd\left(d^2-4c^2\right)+3b^2\left(-5c^2d^2+4c^4+2d^4\right)\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{\left(c^2-d^2\right)^{5/2}(bc-ad)^4}+\frac{4b^3\left(-4a^2d+abc+3b^2d\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}(bc-ad)^4}-\frac{2b^4\cdots}{(a-b)(a+b)(ad\cdots}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^3),x]
```

```
[Out] ((4*b^3*(a*b*c - 4*a^2*d + 3*b^2*d)*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^
2 - b^2]])/((a^2 - b^2)^(3/2)*(b*c - a*d)^4) + (2*d^2*(2*a*b*c*d*(-4*c^2 +
d^2) + a^2*d^2*(2*c^2 + d^2) + 3*b^2*(4*c^4 - 5*c^2*d^2 + 2*d^4))*ArcTan[(d
 + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/((b*c - a*d)^4*(c^2 - d^2)^(5/2))
- (2*b^4*Cos[e + f*x])/((a - b)*(a + b)*(-(b*c) + a*d)^3*(a + b*Sin[e + f*x
])) + (d^3*Cos[e + f*x])/((c - d)*(c + d)*(b*c - a*d)^2*(c + d*Sin[e + f*x]
)^2) + (d^3*(7*b*c^2 - 3*a*c*d - 4*b*d^2)*Cos[e + f*x])/((c - d)^2*(c + d)^
2*(b*c - a*d)^3*(c + d*Sin[e + f*x])))/(2*f)
```

Maple [B] time = 0.174, size = 4023, normalized size = 8.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^3,x)
```

[Out] -1/f*d^7/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(a*d-b*c)^2/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)*a^2+2/f*b^5/(a*d-b*c)^2/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)/(a^2-b^2)*c+6/f*b^5/(a*d-b*c)^2/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*d+23/f*d^4/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(a*d-b*c)^2/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2*c^3/(c^4-2*c^2*d^2+d^4)*tan(1/2*f*x+1/2*e)*b^2-14/f*d^6/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(a*d-b*c)^2/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2*c/(c^4-2*c^2*d^2+d^4)*tan(1/2*f*x+1/2*e)*b^2-12/f*d^4/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(a*d-b*c)^2/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)*a*b*c^3+6/f*d^6/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(a*d-b*c)^2/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)*a*b*c+12/f*d^2/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(a*d-b*c)^2/(c^4-2*c^2*d^2+d^4)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*b^2*c^4-15/f*d^4/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(a*d-b*c)^2/(c^4-2*c^2*d^2+d^4)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*b^2*c^2+2/f*d^4/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(a*d-b*c)^2/(c^4-2*c^2*d^2+d^4)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*a^2*c^2+8/f*d^7/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(a*d-b*c)^2/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)*tan(1/2*f*x+1/2*e)^3*a*b+16/f*d^7/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(a*d-b*c)^2/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)*tan(1/2*f*x+1/2*e)*a*b+5/f*d^6/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(a*d-b*c)^2/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)*c*tan(1/2*f*x+1/2*e)^3*a^2-2/f*d^8/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(a*d-b*c)^2/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)/c*tan(1/2*f*x+1/2*e)^3*a^2+1/f*d^6/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(a*d-b*c)^2/(c^4-2*c^2*d^2+d^4)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*a^2+6/f*d^6/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(a*d-b*c)^2/(c^4-2*c^2*d^2+d^4)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*b^2+4/f*d^5/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(a*d-b*c)^2/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)*a^2*c^2+8/f*d^3/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(a*d-b*c)^2/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)*b^2*c^4-5/f*d^5/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(a*d-b*c)^2/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)*b^2*c^2+7/f*d^7/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(a*d-b*c)^2/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)*tan(1/2*f*x+1/2*e)^2*a^2-2/f*b^5/(a*d-b*c)^2/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)/(a^2-b^2)*tan(1/2*f*x+1/2*e)*d-2/f*b^4/(a*d-b*c)^2/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)/(a^2-b^2)*d*a-8/f*b^3/(a*d-b*c)^2/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*a^2*d+2/f*b^4/(a*d-b*c)^2/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*a*c-10/f*d^7/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(a*d-b*c)^2/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)*tan(1/2*f*x+1/2*e)^2*b^2+11/f*d^5/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(a*d-b*c)^2/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)*c^2*tan(1/2*f*x+1/2*e)^2*b^2+11/f*d^6/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(a*d-b*c)^2/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2*c/(c^4-2*c^2*d^2+d^4)*tan(1/2*f*x+1/2*e)*a^2+2/f*b^6/(a*d-b*c)^2/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)/a/(a^2-b^2)*tan(1/2*f*x+1/2*e)*c-2/f*d^8/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(a*d-b*c)^2/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/c/(c^4-2*c^2*d^2+d^4)*tan(1/2*f*x+1/2*e)*a^2-8/f*d^3/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(a*d-b*c)^2/(c^4-2*c^2*d^2+d^4)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*a*b*c^3+2/f*d^5/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(a*d-b*c)^2/(c^4-2*c^2*d^2+d^4)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*a*b*c-14/f*d^5/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(a*d-b*c)^2/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)*c^2*tan(1/2*f*x+1/2*e)^3*a*b-12/f*d^4/(

```
a^2*d^2-2*a*b*c*d+b^2*c^2)/(a*d-b*c)^2/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*
x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)*c^3*tan(1/2*f*x+1/2*e)^2*a*b-18/f*d^6/(
a^2*d^2-2*a*b*c*d+b^2*c^2)/(a*d-b*c)^2/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*
x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)*c*tan(1/2*f*x+1/2*e)^2*a*b+12/f*d^8/(a^
2*d^2-2*a*b*c*d+b^2*c^2)/(a*d-b*c)^2/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+
1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)/c*tan(1/2*f*x+1/2*e)^2*a*b-34/f*d^5/(a^2*
d^2-2*a*b*c*d+b^2*c^2)/(a*d-b*c)^2/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/
2*e)*d+c)^2*c^2/(c^4-2*c^2*d^2+d^4)*tan(1/2*f*x+1/2*e)*a*b+9/f*d^4/(a^2*d^2
-2*a*b*c*d+b^2*c^2)/(a*d-b*c)^2/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e
)*d+c)^2/(c^4-2*c^2*d^2+d^4)*c^3*tan(1/2*f*x+1/2*e)^3*b^2-6/f*d^6/(a^2*d^2-
2*a*b*c*d+b^2*c^2)/(a*d-b*c)^2/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)
*d+c)^2/(c^4-2*c^2*d^2+d^4)*c*tan(1/2*f*x+1/2*e)^3*b^2+4/f*d^5/(a^2*d^2-2*a
*b*c*d+b^2*c^2)/(a*d-b*c)^2/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+
c)^2/(c^4-2*c^2*d^2+d^4)*c^2*tan(1/2*f*x+1/2*e)^2*a^2-2/f*d^9/(a^2*d^2-2*a*
b*c*d+b^2*c^2)/(a*d-b*c)^2/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c
)^2/(c^4-2*c^2*d^2+d^4)/c^2*tan(1/2*f*x+1/2*e)^2*a^2+8/f*d^3/(a^2*d^2-2*a*b
*c*d+b^2*c^2)/(a*d-b*c)^2/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)
^2/(c^4-2*c^2*d^2+d^4)*c^4*tan(1/2*f*x+1/2*e)^2*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))**2/(c+d*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.46116, size = 1497, normalized size = 3.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] (2*(a*b^4*c - 4*a^2*b^3*d + 3*b^5*d)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(
a) + arctan((a*tan(1/2*f*x + 1/2*e) + b)/sqrt(a^2 - b^2)))/((a^2*b^4*c^4 -
b^6*c^4 - 4*a^3*b^3*c^3*d + 4*a*b^5*c^3*d + 6*a^4*b^2*c^2*d^2 - 6*a^2*b^4*c
^2*d^2 - 4*a^5*b*c*d^3 + 4*a^3*b^3*c*d^3 + a^6*d^4 - a^4*b^2*d^4)*sqrt(a^2
- b^2)) + (12*b^2*c^4*d^2 - 8*a*b*c^3*d^3 + 2*a^2*c^2*d^4 - 15*b^2*c^2*d^4
+ 2*a*b*c*d^5 + a^2*d^6 + 6*b^2*d^6)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(
c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/((b^4*c^8 - 4*a*
b^3*c^7*d + 6*a^2*b^2*c^6*d^2 - 2*b^4*c^6*d^2 - 4*a^3*b*c^5*d^3 + 8*a*b^3*c
^5*d^3 + a^4*c^4*d^4 - 12*a^2*b^2*c^4*d^4 + b^4*c^4*d^4 + 8*a^3*b*c^3*d^5 -
 4*a*b^3*c^3*d^5 - 2*a^4*c^2*d^6 + 6*a^2*b^2*c^2*d^6 - 4*a^3*b*c*d^7 + a^4*
d^8)*sqrt(c^2 - d^2)) + 2*(b^5*tan(1/2*f*x + 1/2*e) + a*b^4)/((a^3*b^3*c^3
- a*b^5*c^3 - 3*a^4*b^2*c^2*d + 3*a^2*b^4*c^2*d + 3*a^5*b*c*d^2 - 3*a^3*b^3
*c*d^2 - a^6*d^3 + a^4*b^2*d^3)*(a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x
 + 1/2*e) + a)) + (9*b*c^4*d^4*tan(1/2*f*x + 1/2*e)^3 - 5*a*c^3*d^5*tan(1/2
*f*x + 1/2*e)^3 - 6*b*c^2*d^6*tan(1/2*f*x + 1/2*e)^3 + 2*a*c*d^7*tan(1/2*f*
x + 1/2*e)^3 + 8*b*c^5*d^3*tan(1/2*f*x + 1/2*e)^2 - 4*a*c^4*d^4*tan(1/2*f*x
 + 1/2*e)^2 + 11*b*c^3*d^5*tan(1/2*f*x + 1/2*e)^2 - 7*a*c^2*d^6*tan(1/2*f*x
 + 1/2*e)^2 - 10*b*c*d^7*tan(1/2*f*x + 1/2*e)^2 + 2*a*d^8*tan(1/2*f*x + 1/2
*e)^2 + 23*b*c^4*d^4*tan(1/2*f*x + 1/2*e) - 11*a*c^3*d^5*tan(1/2*f*x + 1/2*
e) - 14*b*c^2*d^6*tan(1/2*f*x + 1/2*e) + 2*a*c*d^7*tan(1/2*f*x + 1/2*e) + 8
*b*c^5*d^3 - 4*a*c^4*d^4 - 5*b*c^3*d^5 + a*c^2*d^6)/((b^3*c^9 - 3*a*b^2*c^8
*d + 3*a^2*b*c^7*d^2 - 2*b^3*c^7*d^2 - a^3*c^6*d^3 + 6*a*b^2*c^6*d^3 - 6*a^
2*b*c^5*d^4 + b^3*c^5*d^4 + 2*a^3*c^4*d^5 - 3*a*b^2*c^4*d^5 + 3*a^2*b*c^3*d
^6 - a^3*c^2*d^7)*(c*tan(1/2*f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e) + c)
^2))/f
```

3.714 $\int \frac{(c+d\sin(e+fx))^5}{(a+b\sin(e+fx))^3} dx$

Optimal. Leaf size=534

$$-\frac{d\left(-a^3b^2d^2\left(16c^2-21d^2\right)-a^2b^3cd\left(4c^2+55d^2\right)+30a^4bcd^3-12a^5d^4+ab^4\left(43c^2d^2+6c^4-6d^4\right)-b^5cd\left(17c^2-10d^2\right.}{2b^4f\left(a^2-b^2\right)^2}$$

```
[Out] -(d^3*(30*a*b*c*d - 12*a^2*d^2 - b^2*(20*c^2 + d^2))*x)/(2*b^5) + ((b*c - a
*d)^3*(6*a^3*b*c*d - 12*a*b^3*c*d + 12*a^4*d^2 + a^2*b^2*(2*c^2 - 29*d^2) +
 b^4*(c^2 + 20*d^2))*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/(b^5
*(a^2 - b^2)^(5/2)*f) - (d*(30*a^4*b*c*d^3 - 12*a^5*d^4 - a^3*b^2*d^2*(16*c
^2 - 21*d^2) - b^5*c*d*(17*c^2 - 10*d^2) - a^2*b^3*c*d*(4*c^2 + 55*d^2) + a
*b^4*(6*c^4 + 43*c^2*d^2 - 6*d^4))*Cos[e + f*x])/(2*b^4*(a^2 - b^2)^2*f) +
(d^2*(7*a^3*b*c*d^2 - 6*a^4*d^3 + b^4*d*(8*c^2 - d^2) + a^2*b^2*d*(c^2 + 10
*d^2) - a*b^3*c*(3*c^2 + 16*d^2))*Cos[e + f*x]*Sin[e + f*x])/(2*b^3*(a^2 -
b^2)^2*f) + ((b*c - a*d)^2*(3*a*b*c + 4*a^2*d - 7*b^2*d)*Cos[e + f*x]*(c +
d*Sin[e + f*x])^2)/(2*b^2*(a^2 - b^2)^2*f*(a + b*Sin[e + f*x])) + ((b*c - a
*d)^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(2*b*(a^2 - b^2)*f*(a + b*Sin[e
+ f*x])^2)
```

Rubi [A] time = 2.15518, antiderivative size = 534, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {2792, 3047, 3033, 3023, 2735, 2660, 618, 204}

$$-\frac{d\left(-a^3b^2d^2\left(16c^2-21d^2\right)-a^2b^3cd\left(4c^2+55d^2\right)+30a^4bcd^3-12a^5d^4+ab^4\left(43c^2d^2+6c^4-6d^4\right)-b^5cd\left(17c^2-10d^2\right.}{2b^4f\left(a^2-b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^5/(a + b*Sin[e + f*x])^3,x]
```


```
[Out] -(d^3*(30*a*b*c*d - 12*a^2*d^2 - b^2*(20*c^2 + d^2))*x)/(2*b^5) + ((b*c - a
*d)^3*(6*a^3*b*c*d - 12*a*b^3*c*d + 12*a^4*d^2 + a^2*b^2*(2*c^2 - 29*d^2) +
 b^4*(c^2 + 20*d^2))*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/(b^5
*(a^2 - b^2)^(5/2)*f) - (d*(30*a^4*b*c*d^3 - 12*a^5*d^4 - a^3*b^2*d^2*(16*c
^2 - 21*d^2) - b^5*c*d*(17*c^2 - 10*d^2) - a^2*b^3*c*d*(4*c^2 + 55*d^2) + a
*b^4*(6*c^4 + 43*c^2*d^2 - 6*d^4))*Cos[e + f*x])/(2*b^4*(a^2 - b^2)^2*f) +
(d^2*(7*a^3*b*c*d^2 - 6*a^4*d^3 + b^4*d*(8*c^2 - d^2) + a^2*b^2*d*(c^2 + 10
*d^2) - a*b^3*c*(3*c^2 + 16*d^2))*Cos[e + f*x]*Sin[e + f*x])/(2*b^3*(a^2 -
b^2)^2*f) + ((b*c - a*d)^2*(3*a*b*c + 4*a^2*d - 7*b^2*d)*Cos[e + f*x]*(c +
d*Sin[e + f*x])^2)/(2*b^2*(a^2 - b^2)^2*f*(a + b*Sin[e + f*x])) + ((b*c - a
*d)^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(2*b*(a^2 - b^2)*f*(a + b*Sin[e
+ f*x])^2)
```


Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
```

```
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^5}{(a+b\sin(e+fx))^3}\,dx = \frac{(bc-ad)^2\cos(e+fx)(c+d\sin(e+fx))^3}{2b\left(a^2-b^2\right)f(a+b\sin(e+fx))^2} - \frac{\int \frac{(c+d\sin(e+fx))^2\left(7b^2c^2d+3a^2d^3-2abc\left(c^2+4d^2\right)-\left(a^2cd^2 \cdots\right.\right.}{\cdots}}{\cdots}$$

$$= \frac{(bc-ad)^2\left(3abc+4a^2d-7b^2d\right)\cos(e+fx)(c+d\sin(e+fx))^2}{2b^2\left(a^2-b^2\right)^2 f(a+b\sin(e+fx))} + \frac{(bc-ad)^2\cos(e+fx)\cdots}{2b\left(a^2-b^2\right)f(a+\cdots}$$

$$= \frac{d^2\left(7a^3bcd^2-6a^4d^3+b^4d\left(8c^2-d^2\right)+a^2b^2d\left(c^2+10d^2\right)-ab^3c\left(3c^2+16d^2\right)\right)\cos(e+fx\cdots}{2b^3\left(a^2-b^2\right)^2 f}$$

$$= -\frac{d\left(30a^4bcd^3-12a^5d^4-a^3b^2d^2\left(16c^2-21d^2\right)-b^5cd\left(17c^2-10d^2\right)-a^2b^3cd\left(4c^2+55d^2\cdots\right.\right.}{2b^4\left(a^2-b^2\right)^2 f}$$

$$= -\frac{d^3\left(30abcd-12a^2d^2-b^2\left(20c^2+d^2\right)\right)x}{2b^5} - \frac{d\left(30a^4bcd^3-12a^5d^4-a^3b^2d^2\left(16c^2-21d^2\right)\cdots\right.}{\cdots}$$

$$= -\frac{d^3\left(30abcd-12a^2d^2-b^2\left(20c^2+d^2\right)\right)x}{2b^5} - \frac{d\left(30a^4bcd^3-12a^5d^4-a^3b^2d^2\left(16c^2-21d^2\right)\cdots\right.}{\cdots}$$

$$= -\frac{d^3\left(30abcd-12a^2d^2-b^2\left(20c^2+d^2\right)\right)x}{2b^5} - \frac{d\left(30a^4bcd^3-12a^5d^4-a^3b^2d^2\left(16c^2-21d^2\right)\cdots\right.}{\cdots}$$

$$= -\frac{d^3\left(30abcd-12a^2d^2-b^2\left(20c^2+d^2\right)\right)x}{2b^5} + \frac{(bc-ad)^3\left(6a^3bcd-12ab^3cd+12a^4d^2+a^2b\cdots\right.}{b^5\cdots}$$

Mathematica [C] time = 3.80647, size = 341, normalized size = 0.64

$$\frac{2d^3(e+fx)\left(12a^2d^2-30abcd+b^2\left(20c^2+d^2\right)\right)+\frac{4(bc-ad)^3\left(a^2b^2\left(2c^2-29d^2\right)+6a^3bcd+12a^4d^2-12ab^3cd+b^4\left(c^2+20d^2\right)\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)\cdots}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}}\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])^5/(a + b*Sin[e + f*x])^3,x]
```

```
[Out] (2*d^3*(-30*a*b*c*d + 12*a^2*d^2 + b^2*(20*c^2 + d^2))*(e + f*x) + (4*(b*c
- a*d)^3*(6*a^3*b*c*d - 12*a*b^3*c*d + 12*a^4*d^2 + a^2*b^2*(2*c^2 - 29*d^2
) + b^4*(c^2 + 20*d^2))*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/(
a^2 - b^2)^(5/2) + 2*b*d^4*(-5*b*c + 3*a*d)*(Cos[e + f*x] - I*Sin[e + f*x])
 + 2*b*d^4*(-5*b*c + 3*a*d)*(Cos[e + f*x] + I*Sin[e + f*x]) - (2*b*(b*c - a
*d)^5*Cos[e + f*x])/((-a^2 + b^2)*(a + b*Sin[e + f*x])^2) + (2*b*(b*c - a*d
)^4*(3*a*b*c + 7*a^2*d - 10*b^2*d)*Cos[e + f*x])/((a^2 - b^2)^2*(a + b*Sin[
```

```
e + f*x])) - b^2*d^5*Sin[2*(e + f*x)])/(4*b^5*f)
```

Maple [B] time = 0.139, size = 4767, normalized size = 8.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^5/(a+b*sin(f*x+e))^3,x)
```

```
[Out] 6/f*d^5/b^4/(1+tan(1/2*f*x+1/2*e)^2)^2*tan(1/2*f*x+1/2*e)^2*a-10/f*d^4/b^3/
(1+tan(1/2*f*x+1/2*e)^2)^2*tan(1/2*f*x+1/2*e)^2*c+7/f*b^3/(tan(1/2*f*x+1/2*
e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*tan(1/2*f*x+1/2*e)^2
*c^5+6/f/b^4/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2
*b^2+b^4)*a^7*d^5-9/f/b^2/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)
^2/(a^4-2*a^2*b^2+b^4)*a^5*d^5+4/f*b/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+
1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^2*c^5+1/f*b^2/(a^4-2*a^2*b^2+b^4)/(a^2-
b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*c^5-18/
f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a
^3*tan(1/2*f*x+1/2*e)^2*d^5-10/f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*
e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^3*c^4*d-50/f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(
1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^3*c^2*d^3+2/f/(a^4-2*a^2*b^2+b^
4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))
*a^2*c^5-8/f/b/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a
^2*b^2+b^4)*a^4*tan(1/2*f*x+1/2*e)^3*d^5+5/f*b^2/(tan(1/2*f*x+1/2*e)^2*a+2*
tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a*tan(1/2*f*x+1/2*e)^3*c^5-2/
f*b^4/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^
4)/a*tan(1/2*f*x+1/2*e)^3*c^5+6/f/b^4/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x
+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^7*tan(1/2*f*x+1/2*e)^2*d^5+3/f/b^2/(ta
n(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^5*ta
n(1/2*f*x+1/2*e)^2*d^5+4/f*b/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b
+a)^2/(a^4-2*a^2*b^2+b^4)*a^2*tan(1/2*f*x+1/2*e)^2*c^5-2/f*b^5/(tan(1/2*f*x
+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)/a^2*tan(1/2*f*x
+1/2*e)^2*c^5+19/f/b^3/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2*
a^6/(a^4-2*a^2*b^2+b^4)*tan(1/2*f*x+1/2*e)*d^5-28/f/b/(tan(1/2*f*x+1/2*e)^2
*a+2*tan(1/2*f*x+1/2*e)*b+a)^2*a^4/(a^4-2*a^2*b^2+b^4)*tan(1/2*f*x+1/2*e)*d
^5+11/f*b^2/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2*a/(a^4-2*a^
2*b^2+b^4)*tan(1/2*f*x+1/2*e)*c^5-2/f*b^4/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2
*f*x+1/2*e)*b+a)^2/a/(a^4-2*a^2*b^2+b^4)*tan(1/2*f*x+1/2*e)*c^5-20/f/b^3/(t
an(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^6*c
*d^4+20/f/b^2/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^
2*b^2+b^4)*a^5*c^2*d^3+35/f/b/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*
b+a)^2/(a^4-2*a^2*b^2+b^4)*a^4*c*d^4+30/f*b/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1
/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^2*c^3*d^2+20/f*b^2/(a^4-2*a^2*b^
2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1
/2))*c^3*d^2+29/f/b^3/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*t
an(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*a^5*d^5-20/f/b/(a^4-2*a^2*b^2+b^4)/
(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*a^
3*d^5+10/f/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*f*x+
1/2*e)+2*b)/(a^2-b^2)^(1/2))*a^2*c^3*d^2+60/f/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)
^(1/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*a^2*c*d^4-1
0/f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2*a^3/(a^4-2*a^2*b^2+
b^4)*tan(1/2*f*x+1/2*e)*c^3*d^2+110/f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x
+1/2*e)*b+a)^2*a^3/(a^4-2*a^2*b^2+b^4)*tan(1/2*f*x+1/2*e)*c*d^4+10/f/(tan(1
/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^3*tan(1
/2*f*x+1/2*e)^3*c^3*d^2+30/f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b
+a)^2/(a^4-2*a^2*b^2+b^4)*a^3*tan(1/2*f*x+1/2*e)^3*c*d^4-10/f/(tan(1/2*f*x+
```

```
1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^3*tan(1/2*f*x+
1/2*e)^2*c^4*d-10/f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^
4-2*a^2*b^2+b^4)*a^3*tan(1/2*f*x+1/2*e)^2*c^2*d^3+100/f*b^2/(tan(1/2*f*x+1/
2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2*a/(a^4-2*a^2*b^2+b^4)*tan(1/2*f*x+1/2*
e)*c^3*d^2+10/f/b/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-
2*a^2*b^2+b^4)*a^4*tan(1/2*f*x+1/2*e)^3*c^2*d^3-15/f*b/(tan(1/2*f*x+1/2*e)^
2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^2*tan(1/2*f*x+1/2*e)^
3*c^4*d-40/f*b/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a
^2*b^2+b^4)*a^2*tan(1/2*f*x+1/2*e)^3*c^2*d^3+20/f*b^2/(tan(1/2*f*x+1/2*e)^2
*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a*tan(1/2*f*x+1/2*e)^3*c
^3*d^2-20/f/b^3/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*
a^2*b^2+b^4)*a^6*tan(1/2*f*x+1/2*e)^2*c*d^4+20/f/b^2/(tan(1/2*f*x+1/2*e)^2*
a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^5*tan(1/2*f*x+1/2*e)^2*
c^2*d^3-5/f/b/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^
2*b^2+b^4)*a^4*tan(1/2*f*x+1/2*e)^2*c*d^4-25/f*b^2/(tan(1/2*f*x+1/2*e)^2*a+
2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a*tan(1/2*f*x+1/2*e)^2*c^4*
d-100/f*b^2/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*
b^2+b^4)*a*tan(1/2*f*x+1/2*e)^2*c^2*d^3+30/f*b/(tan(1/2*f*x+1/2*e)^2*a+2*ta
n(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^2*tan(1/2*f*x+1/2*e)^2*c^3*d^
2+30/f/b^4/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*f*x+
1/2*e)+2*b)/(a^2-b^2)^(1/2))*a^6*c*d^4-20/f/b^3/(a^4-2*a^2*b^2+b^4)/(a^2-b^
2)^(1/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*a^5*c^2*d
^3-75/f/b^2/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*f*x
+1/2*e)+2*b)/(a^2-b^2)^(1/2))*a^4*c*d^4+1/f*d^5/b^3*arctan(tan(1/2*f*x+1/2*
e))+50/f/b/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*f*x+
1/2*e)+2*b)/(a^2-b^2)^(1/2))*a^3*c^2*d^3-15/f*b/(a^4-2*a^2*b^2+b^4)/(a^2-b^
2)^(1/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*a*c^4*d-6
0/f*b/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e
)+2*b)/(a^2-b^2)^(1/2))*a*c^2*d^3+70/f*b/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*
f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^2*tan(1/2*f*x+1/2*e)^2*c*d^4-10/f*b
^4/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)/
a*tan(1/2*f*x+1/2*e)^2*c^4*d-65/f/b^2/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x
+1/2*e)*b+a)^2*a^5/(a^4-2*a^2*b^2+b^4)*tan(1/2*f*x+1/2*e)*c*d^4+70/f/b/(tan
(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2*a^4/(a^4-2*a^2*b^2+b^4)*tan
(1/2*f*x+1/2*e)*c^2*d^3-25/f*b/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)
*b+a)^2*a^2/(a^4-2*a^2*b^2+b^4)*tan(1/2*f*x+1/2*e)*c^4*d-160/f*b/(tan(1/2*f
*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2*a^2/(a^4-2*a^2*b^2+b^4)*tan(1/2*f
*x+1/2*e)*c^2*d^3-15/f/b^2/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a
)^2/(a^4-2*a^2*b^2+b^4)*a^5*tan(1/2*f*x+1/2*e)^3*c*d^4-12/f/b^5/(a^4-2*a^2*
b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^
(1/2))*a^7*d^5-5/f*b^2/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/
(a^4-2*a^2*b^2+b^4)*a*c^4*d+5/f/b^3/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1
/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^6*tan(1/2*f*x+1/2*e)^3*d^5+60/f*b^3/(tan
(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*tan(1/2
*f*x+1/2*e)^2*c^3*d^2-20/f*b^3/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)
*b+a)^2/(a^4-2*a^2*b^2+b^4)*tan(1/2*f*x+1/2*e)*c^4*d-1/f*b^3/(tan(1/2*f*x+1
/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*c^5+1/f*d^5/b^3/(
1+tan(1/2*f*x+1/2*e)^2)^2*tan(1/2*f*x+1/2*e)^3-1/f*d^5/b^3/(1+tan(1/2*f*x+1
/2*e)^2)^2*tan(1/2*f*x+1/2*e)+6/f*d^5/b^4/(1+tan(1/2*f*x+1/2*e)^2)^2*a-10/f
*d^4/b^3/(1+tan(1/2*f*x+1/2*e)^2)^2*c+12/f*d^5/b^5*arctan(tan(1/2*f*x+1/2*e
))*a^2+20/f*d^3/b^3*arctan(tan(1/2*f*x+1/2*e))*c^2-30/f*d^4/b^4*arctan(tan(
1/2*f*x+1/2*e))*a*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^5/(a+b*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 4.28043, size = 6529, normalized size = 12.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^5/(a+b*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] [1/4*(2*(20*(a^6*b^4 - 3*a^4*b^6 + 3*a^2*b^8 - b^10)*c^2*d^3 - 30*(a^7*b^3
- 3*a^5*b^5 + 3*a^3*b^7 - a*b^9)*c*d^4 + (12*a^8*b^2 - 35*a^6*b^4 + 33*a^4*
b^6 - 9*a^2*b^8 - b^10)*d^5)*f*x*cos(f*x + e)^2 - 4*(5*(a^6*b^4 - 3*a^4*b^6
 + 3*a^2*b^8 - b^10)*c*d^4 - 2*(a^7*b^3 - 3*a^5*b^5 + 3*a^3*b^7 - a*b^9)*d^
5)*cos(f*x + e)^3 - 2*(20*(a^8*b^2 - 2*a^6*b^4 + 2*a^2*b^8 - b^10)*c^2*d^3
- 30*(a^9*b - 2*a^7*b^3 + 2*a^3*b^7 - a*b^9)*c*d^4 + (12*a^10 - 23*a^8*b^2
- 2*a^6*b^4 + 24*a^4*b^6 - 10*a^2*b^8 - b^10)*d^5)*f*x - ((2*a^4*b^5 + 3*a^
2*b^7 + b^9)*c^5 - 15*(a^3*b^6 + a*b^8)*c^4*d + 10*(a^4*b^5 + 3*a^2*b^7 + 2
*b^9)*c^3*d^2 - 10*(2*a^7*b^2 - 3*a^5*b^4 + a^3*b^6 + 6*a*b^8)*c^2*d^3 + 15
*(2*a^8*b - 3*a^6*b^3 - a^4*b^5 + 4*a^2*b^7)*c*d^4 - (12*a^9 - 17*a^7*b^2 -
 9*a^5*b^4 + 20*a^3*b^6)*d^5 + (15*a*b^8*c^4*d - (2*a^2*b^7 + b^9)*c^5 - 10
*(a^2*b^7 + 2*b^9)*c^3*d^2 + 10*(2*a^5*b^4 - 5*a^3*b^6 + 6*a*b^8)*c^2*d^3 -
 15*(2*a^6*b^3 - 5*a^4*b^5 + 4*a^2*b^7)*c*d^4 + (12*a^7*b^2 - 29*a^5*b^4 +
20*a^3*b^6)*d^5)*cos(f*x + e)^2 - 2*(15*a^2*b^7*c^4*d - (2*a^3*b^6 + a*b^8)
*c^5 - 10*(a^3*b^6 + 2*a*b^8)*c^3*d^2 + 10*(2*a^6*b^3 - 5*a^4*b^5 + 6*a^2*b
^7)*c^2*d^3 - 15*(2*a^7*b^2 - 5*a^5*b^4 + 4*a^3*b^6)*c*d^4 + (12*a^8*b - 29
*a^6*b^3 + 20*a^4*b^5)*d^5)*sin(f*x + e))*sqrt(-a^2 + b^2)*log(-((2*a^2 - b
^2)*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2 - 2*(a*cos(f*x + e)*sin
(f*x + e) + b*cos(f*x + e))*sqrt(-a^2 + b^2))/(b^2*cos(f*x + e)^2 - 2*a*b*s
in(f*x + e) - a^2 - b^2)) - 2*((4*a^4*b^6 - 5*a^2*b^8 + b^10)*c^5 - 5*(2*a^
5*b^5 - a^3*b^7 - a*b^9)*c^4*d + 30*(a^4*b^6 - a^2*b^8)*c^3*d^2 + 10*(2*a^7
*b^3 - 7*a^5*b^5 + 5*a^3*b^7)*c^2*d^3 - 5*(6*a^8*b^2 - 15*a^6*b^4 + 7*a^4*b
^6 + 4*a^2*b^8 - 2*b^10)*c*d^4 + (12*a^9*b - 29*a^7*b^3 + 15*a^5*b^5 + 6*a^
3*b^7 - 4*a*b^9)*d^5)*cos(f*x + e) - 2*((a^6*b^4 - 3*a^4*b^6 + 3*a^2*b^8 -
b^10)*d^5*cos(f*x + e)^3 + 2*(20*(a^7*b^3 - 3*a^5*b^5 + 3*a^3*b^7 - a*b^9)*
c^2*d^3 - 30*(a^8*b^2 - 3*a^6*b^4 + 3*a^4*b^6 - a^2*b^8)*c*d^4 + (12*a^9*b
- 35*a^7*b^3 + 33*a^5*b^5 - 9*a^3*b^7 - a*b^9)*d^5)*f*x + (3*(a^3*b^7 - a*b
^9)*c^5 - 5*(a^4*b^6 + a^2*b^8 - 2*b^10)*c^4*d - 10*(a^5*b^5 - 5*a^3*b^7 +
4*a*b^9)*c^3*d^2 + 30*(a^6*b^4 - 3*a^4*b^6 + 2*a^2*b^8)*c^2*d^3 - 5*(9*a^7*
b^3 - 25*a^5*b^5 + 20*a^3*b^7 - 4*a*b^9)*c*d^4 + (18*a^8*b^2 - 51*a^6*b^4 +
 46*a^4*b^6 - 14*a^2*b^8 + b^10)*d^5)*cos(f*x + e))*sin(f*x + e))/((a^6*b^7
 - 3*a^4*b^9 + 3*a^2*b^11 - b^13)*f*cos(f*x + e)^2 - 2*(a^7*b^6 - 3*a^5*b^8
 + 3*a^3*b^10 - a*b^12)*f*sin(f*x + e) - (a^8*b^5 - 2*a^6*b^7 + 2*a^2*b^11
- b^13)*f), 1/2*((20*(a^6*b^4 - 3*a^4*b^6 + 3*a^2*b^8 - b^10)*c^2*d^3 - 30*
(a^7*b^3 - 3*a^5*b^5 + 3*a^3*b^7 - a*b^9)*c*d^4 + (12*a^8*b^2 - 35*a^6*b^4
+ 33*a^4*b^6 - 9*a^2*b^8 - b^10)*d^5)*f*x*cos(f*x + e)^2 - 2*(5*(a^6*b^4 -
3*a^4*b^6 + 3*a^2*b^8 - b^10)*c*d^4 - 2*(a^7*b^3 - 3*a^5*b^5 + 3*a^3*b^7 -
a*b^9)*d^5)*cos(f*x + e)^3 - (20*(a^8*b^2 - 2*a^6*b^4 + 2*a^2*b^8 - b^10)*c
^2*d^3 - 30*(a^9*b - 2*a^7*b^3 + 2*a^3*b^7 - a*b^9)*c*d^4 + (12*a^10 - 23*a
^8*b^2 - 2*a^6*b^4 + 24*a^4*b^6 - 10*a^2*b^8 - b^10)*d^5)*f*x + ((2*a^4*b^5
 + 3*a^2*b^7 + b^9)*c^5 - 15*(a^3*b^6 + a*b^8)*c^4*d + 10*(a^4*b^5 + 3*a^2*
b^7 + 2*b^9)*c^3*d^2 - 10*(2*a^7*b^2 - 3*a^5*b^4 + a^3*b^6 + 6*a*b^8)*c^2*d
^3 + 15*(2*a^8*b - 3*a^6*b^3 - a^4*b^5 + 4*a^2*b^7)*c*d^4 - (12*a^9 - 17*a^
7*b^2 - 9*a^5*b^4 + 20*a^3*b^6)*d^5 + (15*a*b^8*c^4*d - (2*a^2*b^7 + b^9)*c
^5 - 10*(a^2*b^7 + 2*b^9)*c^3*d^2 + 10*(2*a^5*b^4 - 5*a^3*b^6 + 6*a*b^8)*c^
```

```
2*d^3 - 15*(2*a^6*b^3 - 5*a^4*b^5 + 4*a^2*b^7)*c*d^4 + (12*a^7*b^2 - 29*a^5
*b^4 + 20*a^3*b^6)*d^5)*cos(f*x + e)^2 - 2*(15*a^2*b^7*c^4*d - (2*a^3*b^6 +
 a*b^8)*c^5 - 10*(a^3*b^6 + 2*a*b^8)*c^3*d^2 + 10*(2*a^6*b^3 - 5*a^4*b^5 +
6*a^2*b^7)*c^2*d^3 - 15*(2*a^7*b^2 - 5*a^5*b^4 + 4*a^3*b^6)*c*d^4 + (12*a^8
*b - 29*a^6*b^3 + 20*a^4*b^5)*d^5)*sin(f*x + e))*sqrt(a^2 - b^2)*arctan(-(a
*sin(f*x + e) + b)/(sqrt(a^2 - b^2)*cos(f*x + e))) - ((4*a^4*b^6 - 5*a^2*b^
8 + b^10)*c^5 - 5*(2*a^5*b^5 - a^3*b^7 - a*b^9)*c^4*d + 30*(a^4*b^6 - a^2*b
^8)*c^3*d^2 + 10*(2*a^7*b^3 - 7*a^5*b^5 + 5*a^3*b^7)*c^2*d^3 - 5*(6*a^8*b^2
 - 15*a^6*b^4 + 7*a^4*b^6 + 4*a^2*b^8 - 2*b^10)*c*d^4 + (12*a^9*b - 29*a^7*
b^3 + 15*a^5*b^5 + 6*a^3*b^7 - 4*a*b^9)*d^5)*cos(f*x + e) - ((a^6*b^4 - 3*a
^4*b^6 + 3*a^2*b^8 - b^10)*d^5*cos(f*x + e)^3 + 2*(20*(a^7*b^3 - 3*a^5*b^5
+ 3*a^3*b^7 - a*b^9)*c^2*d^3 - 30*(a^8*b^2 - 3*a^6*b^4 + 3*a^4*b^6 - a^2*b^
8)*c*d^4 + (12*a^9*b - 35*a^7*b^3 + 33*a^5*b^5 - 9*a^3*b^7 - a*b^9)*d^5)*f*
x + (3*(a^3*b^7 - a*b^9)*c^5 - 5*(a^4*b^6 + a^2*b^8 - 2*b^10)*c^4*d - 10*(a
^5*b^5 - 5*a^3*b^7 + 4*a*b^9)*c^3*d^2 + 30*(a^6*b^4 - 3*a^4*b^6 + 2*a^2*b^8
)*c^2*d^3 - 5*(9*a^7*b^3 - 25*a^5*b^5 + 20*a^3*b^7 - 4*a*b^9)*c*d^4 + (18*a
^8*b^2 - 51*a^6*b^4 + 46*a^4*b^6 - 14*a^2*b^8 + b^10)*d^5)*cos(f*x + e))*si
n(f*x + e))/((a^6*b^7 - 3*a^4*b^9 + 3*a^2*b^11 - b^13)*f*cos(f*x + e)^2 - 2
*(a^7*b^6 - 3*a^5*b^8 + 3*a^3*b^10 - a*b^12)*f*sin(f*x + e) - (a^8*b^5 - 2*
a^6*b^7 + 2*a^2*b^11 - b^13)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**5/(a+b*sin(f*x+e))**3,x)

[Out] Timed out
```

Giac [B] time = 1.52729, size = 4269, normalized size = 7.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^5/(a+b*sin(f*x+e))^3,x, algorithm="giac")

[Out] 1/2*(2*(2*a^2*b^5*c^5 + b^7*c^5 - 15*a*b^6*c^4*d + 10*a^2*b^5*c^3*d^2 + 20*
b^7*c^3*d^2 - 20*a^5*b^2*c^2*d^3 + 50*a^3*b^4*c^2*d^3 - 60*a*b^6*c^2*d^3 +
30*a^6*b*c*d^4 - 75*a^4*b^3*c*d^4 + 60*a^2*b^5*c*d^4 - 12*a^7*d^5 + 29*a^5*
b^2*d^5 - 20*a^3*b^4*d^5)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(a) + arctan
((a*tan(1/2*f*x + 1/2*e) + b)/sqrt(a^2 - b^2)))/((a^4*b^5 - 2*a^2*b^7 + b^9
)*sqrt(a^2 - b^2)) + 2*(5*a^3*b^6*c^5*tan(1/2*f*x + 1/2*e)^7 - 2*a*b^8*c^5*
tan(1/2*f*x + 1/2*e)^7 - 15*a^4*b^5*c^4*d*tan(1/2*f*x + 1/2*e)^7 + 10*a^5*b
^4*c^3*d^2*tan(1/2*f*x + 1/2*e)^7 + 20*a^3*b^6*c^3*d^2*tan(1/2*f*x + 1/2*e)
^7 + 10*a^6*b^3*c^2*d^3*tan(1/2*f*x + 1/2*e)^7 - 40*a^4*b^5*c^2*d^3*tan(1/2
*f*x + 1/2*e)^7 - 15*a^7*b^2*c*d^4*tan(1/2*f*x + 1/2*e)^7 + 30*a^5*b^4*c*d^
4*tan(1/2*f*x + 1/2*e)^7 + 6*a^8*b*d^5*tan(1/2*f*x + 1/2*e)^7 - 10*a^6*b^3*
d^5*tan(1/2*f*x + 1/2*e)^7 + a^4*b^5*d^5*tan(1/2*f*x + 1/2*e)^7 + 4*a^4*b^5
*c^5*tan(1/2*f*x + 1/2*e)^6 + 7*a^2*b^7*c^5*tan(1/2*f*x + 1/2*e)^6 - 2*b^9*
c^5*tan(1/2*f*x + 1/2*e)^6 - 10*a^5*b^4*c^4*d*tan(1/2*f*x + 1/2*e)^6 - 25*a
^3*b^6*c^4*d*tan(1/2*f*x + 1/2*e)^6 - 10*a*b^8*c^4*d*tan(1/2*f*x + 1/2*e)^6
```

```
 + 30*a^4*b^5*c^3*d^2*tan(1/2*f*x + 1/2*e)^6 + 60*a^2*b^7*c^3*d^2*tan(1/2*f
*x + 1/2*e)^6 + 20*a^7*b^2*c^2*d^3*tan(1/2*f*x + 1/2*e)^6 - 10*a^5*b^4*c^2*
d^3*tan(1/2*f*x + 1/2*e)^6 - 100*a^3*b^6*c^2*d^3*tan(1/2*f*x + 1/2*e)^6 - 3
0*a^8*b*c*d^4*tan(1/2*f*x + 1/2*e)^6 + 15*a^6*b^3*c*d^4*tan(1/2*f*x + 1/2*e
)^6 + 60*a^4*b^5*c*d^4*tan(1/2*f*x + 1/2*e)^6 + 12*a^9*d^5*tan(1/2*f*x + 1/
2*e)^6 - 5*a^7*b^2*d^5*tan(1/2*f*x + 1/2*e)^6 - 20*a^5*b^4*d^5*tan(1/2*f*x
+ 1/2*e)^6 + 4*a^3*b^6*d^5*tan(1/2*f*x + 1/2*e)^6 + 21*a^3*b^6*c^5*tan(1/2*
f*x + 1/2*e)^5 - 6*a*b^8*c^5*tan(1/2*f*x + 1/2*e)^5 - 55*a^4*b^5*c^4*d*tan(
1/2*f*x + 1/2*e)^5 - 20*a^2*b^7*c^4*d*tan(1/2*f*x + 1/2*e)^5 + 10*a^5*b^4*c
^3*d^2*tan(1/2*f*x + 1/2*e)^5 + 140*a^3*b^6*c^3*d^2*tan(1/2*f*x + 1/2*e)^5
+ 90*a^6*b^3*c^2*d^3*tan(1/2*f*x + 1/2*e)^5 - 240*a^4*b^5*c^2*d^3*tan(1/2*f
*x + 1/2*e)^5 - 135*a^7*b^2*c*d^4*tan(1/2*f*x + 1/2*e)^5 + 250*a^5*b^4*c*d^
4*tan(1/2*f*x + 1/2*e)^5 - 40*a^3*b^6*c*d^4*tan(1/2*f*x + 1/2*e)^5 + 54*a^8
*b*d^5*tan(1/2*f*x + 1/2*e)^5 - 90*a^6*b^3*d^5*tan(1/2*f*x + 1/2*e)^5 + 17*
a^4*b^5*d^5*tan(1/2*f*x + 1/2*e)^5 + 4*a^2*b^7*d^5*tan(1/2*f*x + 1/2*e)^5 +
 12*a^4*b^5*c^5*tan(1/2*f*x + 1/2*e)^4 + 13*a^2*b^7*c^5*tan(1/2*f*x + 1/2*e
)^4 - 4*b^9*c^5*tan(1/2*f*x + 1/2*e)^4 - 30*a^5*b^4*c^4*d*tan(1/2*f*x + 1/2
*e)^4 - 55*a^3*b^6*c^4*d*tan(1/2*f*x + 1/2*e)^4 - 20*a*b^8*c^4*d*tan(1/2*f*
x + 1/2*e)^4 + 90*a^4*b^5*c^3*d^2*tan(1/2*f*x + 1/2*e)^4 + 120*a^2*b^7*c^3*
d^2*tan(1/2*f*x + 1/2*e)^4 + 60*a^7*b^2*c^2*d^3*tan(1/2*f*x + 1/2*e)^4 - 70
*a^5*b^4*c^2*d^3*tan(1/2*f*x + 1/2*e)^4 - 200*a^3*b^6*c^2*d^3*tan(1/2*f*x +
 1/2*e)^4 - 90*a^8*b*c*d^4*tan(1/2*f*x + 1/2*e)^4 + 45*a^6*b^3*c*d^4*tan(1/
2*f*x + 1/2*e)^4 + 190*a^4*b^5*c*d^4*tan(1/2*f*x + 1/2*e)^4 - 40*a^2*b^7*c*
d^4*tan(1/2*f*x + 1/2*e)^4 + 36*a^9*d^5*tan(1/2*f*x + 1/2*e)^4 - 15*a^7*b^2
*d^5*tan(1/2*f*x + 1/2*e)^4 - 66*a^5*b^4*d^5*tan(1/2*f*x + 1/2*e)^4 + 24*a^
3*b^6*d^5*tan(1/2*f*x + 1/2*e)^4 + 27*a^3*b^6*c^5*tan(1/2*f*x + 1/2*e)^3 -
6*a*b^8*c^5*tan(1/2*f*x + 1/2*e)^3 - 65*a^4*b^5*c^4*d*tan(1/2*f*x + 1/2*e)^
3 - 40*a^2*b^7*c^4*d*tan(1/2*f*x + 1/2*e)^3 - 10*a^5*b^4*c^3*d^2*tan(1/2*f*
x + 1/2*e)^3 + 220*a^3*b^6*c^3*d^2*tan(1/2*f*x + 1/2*e)^3 + 150*a^6*b^3*c^2
*d^3*tan(1/2*f*x + 1/2*e)^3 - 360*a^4*b^5*c^2*d^3*tan(1/2*f*x + 1/2*e)^3 -
225*a^7*b^2*c*d^4*tan(1/2*f*x + 1/2*e)^3 + 410*a^5*b^4*c*d^4*tan(1/2*f*x +
1/2*e)^3 - 80*a^3*b^6*c*d^4*tan(1/2*f*x + 1/2*e)^3 + 90*a^8*b*d^5*tan(1/2*f
*x + 1/2*e)^3 - 162*a^6*b^3*d^5*tan(1/2*f*x + 1/2*e)^3 + 55*a^4*b^5*d^5*tan
(1/2*f*x + 1/2*e)^3 - 4*a^2*b^7*d^5*tan(1/2*f*x + 1/2*e)^3 + 12*a^4*b^5*c^5
*tan(1/2*f*x + 1/2*e)^2 + 5*a^2*b^7*c^5*tan(1/2*f*x + 1/2*e)^2 - 2*b^9*c^5*
tan(1/2*f*x + 1/2*e)^2 - 30*a^5*b^4*c^4*d*tan(1/2*f*x + 1/2*e)^2 - 35*a^3*b
^6*c^4*d*tan(1/2*f*x + 1/2*e)^2 - 10*a*b^8*c^4*d*tan(1/2*f*x + 1/2*e)^2 + 9
0*a^4*b^5*c^3*d^2*tan(1/2*f*x + 1/2*e)^2 + 60*a^2*b^7*c^3*d^2*tan(1/2*f*x +
 1/2*e)^2 + 60*a^7*b^2*c^2*d^3*tan(1/2*f*x + 1/2*e)^2 - 110*a^5*b^4*c^2*d^3
*tan(1/2*f*x + 1/2*e)^2 - 100*a^3*b^6*c^2*d^3*tan(1/2*f*x + 1/2*e)^2 - 90*a
^8*b*c*d^4*tan(1/2*f*x + 1/2*e)^2 + 85*a^6*b^3*c*d^4*tan(1/2*f*x + 1/2*e)^2
 + 120*a^4*b^5*c*d^4*tan(1/2*f*x + 1/2*e)^2 - 40*a^2*b^7*c*d^4*tan(1/2*f*x
+ 1/2*e)^2 + 36*a^9*d^5*tan(1/2*f*x + 1/2*e)^2 - 31*a^7*b^2*d^5*tan(1/2*f*x
 + 1/2*e)^2 - 40*a^5*b^4*d^5*tan(1/2*f*x + 1/2*e)^2 + 20*a^3*b^6*d^5*tan(1/
2*f*x + 1/2*e)^2 + 11*a^3*b^6*c^5*tan(1/2*f*x + 1/2*e) - 2*a*b^8*c^5*tan(1/
2*f*x + 1/2*e) - 25*a^4*b^5*c^4*d*tan(1/2*f*x + 1/2*e) - 20*a^2*b^7*c^4*d*t
an(1/2*f*x + 1/2*e) - 10*a^5*b^4*c^3*d^2*tan(1/2*f*x + 1/2*e) + 100*a^3*b^6
*c^3*d^2*tan(1/2*f*x + 1/2*e) + 70*a^6*b^3*c^2*d^3*tan(1/2*f*x + 1/2*e) - 1
60*a^4*b^5*c^2*d^3*tan(1/2*f*x + 1/2*e) - 105*a^7*b^2*c*d^4*tan(1/2*f*x + 1
/2*e) + 190*a^5*b^4*c*d^4*tan(1/2*f*x + 1/2*e) - 40*a^3*b^6*c*d^4*tan(1/2*f
*x + 1/2*e) + 42*a^8*b*d^5*tan(1/2*f*x + 1/2*e) - 74*a^6*b^3*d^5*tan(1/2*f*
x + 1/2*e) + 23*a^4*b^5*d^5*tan(1/2*f*x + 1/2*e) + 4*a^4*b^5*c^5 - a^2*b^7*
c^5 - 10*a^5*b^4*c^4*d - 5*a^3*b^6*c^4*d + 30*a^4*b^5*c^3*d^2 + 20*a^7*b^2*
c^2*d^3 - 50*a^5*b^4*c^2*d^3 - 30*a^8*b*c*d^4 + 55*a^6*b^3*c*d^4 - 10*a^4*b
^5*c*d^4 + 12*a^9*d^5 - 21*a^7*b^2*d^5 + 6*a^5*b^4*d^5)/((a^6*b^4 - 2*a^4*b
^6 + a^2*b^8)*(a*tan(1/2*f*x + 1/2*e)^4 + 2*b*tan(1/2*f*x + 1/2*e)^3 + 2*a*
tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e) + a)^2) + (20*b^2*c^2*d^3
 - 30*a*b*c*d^4 + 12*a^2*d^5 + b^2*d^5)*(f*x + e)/b^5)/f
```

3.715 $\int \frac{(c+d\sin(e+fx))^4}{(a+b\sin(e+fx))^3}\,dx$

Optimal. Leaf size=318

$$\frac{d^2\left(-3a^2d^2+2abcd+b^2\left(-\left(c^2-2d^2\right)\right)\right)\cos(e+fx)}{2b^3f\left(a^2-b^2\right)}+\frac{(bc-ad)^2\left(a^2b^2\left(2c^2-15d^2\right)+4a^3bcd+6a^4d^2-10ab^3cd+b^4\right.}{b^4f\left(a^2-b^2\right)^{5/2}}$$

```
[Out] (d^3*(4*b*c - 3*a*d)*x)/b^4 + ((b*c - a*d)^2*(4*a^3*b*c*d - 10*a*b^3*c*d +
6*a^4*d^2 + a^2*b^2*(2*c^2 - 15*d^2) + b^4*(c^2 + 12*d^2))*ArcTan[(b + a*Ta
n[(e + f*x)/2])/Sqrt[a^2 - b^2]])/(b^4*(a^2 - b^2)^(5/2)*f) + (d^2*(2*a*b*c
*d - 3*a^2*d^2 - b^2*(c^2 - 2*d^2))*Cos[e + f*x])/(2*b^3*(a^2 - b^2)*f) + (
3*(b*c - a*d)^3*(a*b*c + a^2*d - 2*b^2*d)*Cos[e + f*x])/(2*b^3*(a^2 - b^2)^
2*f*(a + b*Sin[e + f*x])) + ((b*c - a*d)^2*Cos[e + f*x]*(c + d*Sin[e + f*x]
)^2)/(2*b*(a^2 - b^2)*f*(a + b*Sin[e + f*x])^2)
```

Rubi [A] time = 0.971869, antiderivative size = 318, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {2792, 3031, 3023, 2735, 2660, 618, 204}

$$\frac{d^2\left(-3a^2d^2+2abcd+b^2\left(-\left(c^2-2d^2\right)\right)\right)\cos(e+fx)}{2b^3f\left(a^2-b^2\right)}+\frac{(bc-ad)^2\left(a^2b^2\left(2c^2-15d^2\right)+4a^3bcd+6a^4d^2-10ab^3cd+b^4\right.}{b^4f\left(a^2-b^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^4/(a + b*Sin[e + f*x])^3,x]
```



```
[Out] (d^3*(4*b*c - 3*a*d)*x)/b^4 + ((b*c - a*d)^2*(4*a^3*b*c*d - 10*a*b^3*c*d +
6*a^4*d^2 + a^2*b^2*(2*c^2 - 15*d^2) + b^4*(c^2 + 12*d^2))*ArcTan[(b + a*Ta
n[(e + f*x)/2])/Sqrt[a^2 - b^2]])/(b^4*(a^2 - b^2)^(5/2)*f) + (d^2*(2*a*b*c
*d - 3*a^2*d^2 - b^2*(c^2 - 2*d^2))*Cos[e + f*x])/(2*b^3*(a^2 - b^2)*f) + (
3*(b*c - a*d)^3*(a*b*c + a^2*d - 2*b^2*d)*Cos[e + f*x])/(2*b^3*(a^2 - b^2)^
2*f*(a + b*Sin[e + f*x])) + ((b*c - a*d)^2*Cos[e + f*x]*(c + d*Sin[e + f*x]
)^2)/(2*b*(a^2 - b^2)*f*(a + b*Sin[e + f*x])^2)
```


Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
```

```
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^4}{(a+b\sin(e+fx))^3}\,dx = \frac{(bc-ad)^2\cos(e+fx)(c+d\sin(e+fx))^2}{2b\left(a^2-b^2\right)f(a+b\sin(e+fx))^2} - \frac{\int \frac{(c+d\sin(e+fx))\left(2\left(3b^2c^2d+a^2d^3-abc\left(c^2+3d^2\right)\right)-\left(a^2cd^2 \cdots\right.\right.}{\cdots}}{\cdots}$$

$$= \frac{3(bc-ad)^3\left(abc+a^2d-2b^2d\right)\cos(e+fx)}{2b^3\left(a^2-b^2\right)^2 f(a+b\sin(e+fx))} + \frac{(bc-ad)^2\cos(e+fx)(c+d\sin(e+fx))^2}{2b\left(a^2-b^2\right)f(a+b\sin(e+fx))^2} - \cdots$$

$$= \frac{d^2\left(2abcd-3a^2d^2-b^2\left(c^2-2d^2\right)\right)\cos(e+fx)}{2b^3\left(a^2-b^2\right)f} + \frac{3(bc-ad)^3\left(abc+a^2d-2b^2d\right)\cos(e+\cdots}{2b^3\left(a^2-b^2\right)^2 f(a+b\sin(e+fx))} \cdots$$

$$= \frac{d^3(4bc-3ad)x}{b^4} + \frac{d^2\left(2abcd-3a^2d^2-b^2\left(c^2-2d^2\right)\right)\cos(e+fx)}{2b^3\left(a^2-b^2\right)f} + \frac{3(bc-ad)^3\left(abc+a^2 \cdots\right.}{2b^3\left(a^2-b^2\right)^2 f(\cdots} \cdots$$

$$= \frac{d^3(4bc-3ad)x}{b^4} + \frac{d^2\left(2abcd-3a^2d^2-b^2\left(c^2-2d^2\right)\right)\cos(e+fx)}{2b^3\left(a^2-b^2\right)f} + \frac{3(bc-ad)^3\left(abc+a^2 \cdots\right.}{2b^3\left(a^2-b^2\right)^2 f(\cdots} \cdots$$

$$= \frac{d^3(4bc-3ad)x}{b^4} + \frac{d^2\left(2abcd-3a^2d^2-b^2\left(c^2-2d^2\right)\right)\cos(e+fx)}{2b^3\left(a^2-b^2\right)f} + \frac{3(bc-ad)^3\left(abc+a^2 \cdots\right.}{2b^3\left(a^2-b^2\right)^2 f(\cdots} \cdots$$

$$= \frac{d^3(4bc-3ad)x}{b^4} + \frac{(bc-ad)^2\left(4a^3bcd-10ab^3cd+6a^4d^2+a^2b^2\left(2c^2-15d^2\right)+b^4\left(c^2+12 \cdots\right.\right.}{b^4\left(a^2-b^2\right)^{5/2} f} \cdots$$

Mathematica [B] time = 4.24157, size = 894, normalized size = 2.81

$$\frac{\frac{4\left(6d^2a^4+4bcda^3+b^2\left(2c^2-15d^2\right)a^2-10b^3cda+b^4\left(c^2+12d^2\right)\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)(bc-ad)^2}{\left(a^2-b^2\right)^{5/2}} + \frac{-12d^4ea^7-12d^4fxa^7+16bcd^3ea^6+16bcd^3fxa^6-24bd^4e\sin(e \cdots}{\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])^4/(a + b*Sin[e + f*x])^3,x]
```

```
[Out] ((4*(b*c - a*d)^2*(4*a^3*b*c*d - 10*a*b^3*c*d + 6*a^4*d^2 + a^2*b^2*(2*c^2
- 15*d^2) + b^4*(c^2 + 12*d^2))*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 -
b^2]])/(a^2 - b^2)^(5/2) + (16*a^6*b*c*d^3*e - 24*a^4*b^3*c*d^3*e + 8*b^7*c
*d^3*e - 12*a^7*d^4*e + 18*a^5*b^2*d^4*e - 6*a*b^6*d^4*e + 16*a^6*b*c*d^3*f
*x - 24*a^4*b^3*c*d^3*f*x + 8*b^7*c*d^3*f*x - 12*a^7*d^4*f*x + 18*a^5*b^2*d
^4*f*x - 6*a*b^6*d^4*f*x - b*(8*a*b^5*c^3*d - 16*a^5*b*c*d^3 + 12*a^6*d^4 -
 21*a^4*b^2*d^4 + 8*a^3*b^3*c*d*(2*c^2 + 5*d^2) + b^6*(2*c^4 + d^4) + 2*a^2
*b^4*(-4*c^4 - 18*c^2*d^2 + d^4))*Cos[e + f*x] + 2*b^2*(a^2 - b^2)^2*d^3*(-
4*b*c + 3*a*d)*(e + f*x)*Cos[2*(e + f*x)] + a^4*b^3*d^4*Cos[3*(e + f*x)] -
2*a^2*b^5*d^4*Cos[3*(e + f*x)] + b^7*d^4*Cos[3*(e + f*x)] + 32*a^5*b^2*c*d^
3*e*Sin[e + f*x] - 64*a^3*b^4*c*d^3*e*Sin[e + f*x] + 32*a*b^6*c*d^3*e*Sin[e
 + f*x] - 24*a^6*b*d^4*e*Sin[e + f*x] + 48*a^4*b^3*d^4*e*Sin[e + f*x] - 24*
a^2*b^5*d^4*e*Sin[e + f*x] + 32*a^5*b^2*c*d^3*f*x*Sin[e + f*x] - 64*a^3*b^4
*c*d^3*f*x*Sin[e + f*x] + 32*a*b^6*c*d^3*f*x*Sin[e + f*x] - 24*a^6*b*d^4*f*
x*Sin[e + f*x] + 48*a^4*b^3*d^4*f*x*Sin[e + f*x] - 24*a^2*b^5*d^4*f*x*Sin[e
 + f*x] + 3*a*b^6*c^4*Sin[2*(e + f*x)] - 4*a^2*b^5*c^3*d*Sin[2*(e + f*x)] -
 8*b^7*c^3*d*Sin[2*(e + f*x)] - 6*a^3*b^4*c^2*d^2*Sin[2*(e + f*x)] + 24*a*b
^6*c^2*d^2*Sin[2*(e + f*x)] + 12*a^4*b^3*c*d^3*Sin[2*(e + f*x)] - 24*a^2*b^
5*c*d^3*Sin[2*(e + f*x)] - 9*a^5*b^2*d^4*Sin[2*(e + f*x)] + 16*a^3*b^4*d^4*
```

```
Sin[2*(e + f*x)] - 4*a*b^6*d^4*Sin[2*(e + f*x)])/((a^2 - b^2)^2*(a + b*Sin[
e + f*x])^2))/(4*b^4*f)
```

Maple [B] time = 0.123, size = 3683, normalized size = 11.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^4/(a+b*sin(f*x+e))^3,x)
```

```
[Out] -2/f*d^4/b^3/(1+tan(1/2*f*x+1/2*e)^2)-6/f*d^4/b^4*arctan(tan(1/2*f*x+1/2*e)
)*a+8/f*d^3/b^3*arctan(tan(1/2*f*x+1/2*e))*c-1/f/b/(tan(1/2*f*x+1/2*e)^2*a+
2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^4*tan(1/2*f*x+1/2*e)^2*d^
4+4/f*b/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+
b^4)*a^2*tan(1/2*f*x+1/2*e)^2*c^4+14/f*b/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*
f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^2*tan(1/2*f*x+1/2*e)^2*d^4+12/f*b^2
/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b
)/(a^2-b^2)^(1/2))*c^2*d^2+5/f*b^2/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/
2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a*tan(1/2*f*x+1/2*e)^3*c^4-2/f*b^4/(tan(1/2
*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)/a*tan(1/2*f
*x+1/2*e)^3*c^4+36/f*b^3/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^
2/(a^4-2*a^2*b^2+b^4)*tan(1/2*f*x+1/2*e)^2*c^2*d^2-16/f*b^3/(tan(1/2*f*x+1/
2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*tan(1/2*f*x+1/2*e)
*c^3*d+6/f/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*f*x+
1/2*e)+2*b)/(a^2-b^2)^(1/2))*a^2*c^2*d^2+6/f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(
1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^3*tan(1/2*f*x+1/2*e)^3*c^2*d^2-
8/f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)
*a^3*tan(1/2*f*x+1/2*e)^2*c^3*d-4/f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1
/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^3*tan(1/2*f*x+1/2*e)^2*c*d^3-6/f/(tan(1/
2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2*a^3/(a^4-2*a^2*b^2+b^4)*tan(1/
2*f*x+1/2*e)*c^2*d^2-13/f/b^2/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*
b+a)^2*a^5/(a^4-2*a^2*b^2+b^4)*tan(1/2*f*x+1/2*e)*d^4+11/f*b^2/(tan(1/2*f*x
+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2*a/(a^4-2*a^2*b^2+b^4)*tan(1/2*f*x+1
/2*e)*c^4-2/f*b^4/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/a/(a^
4-2*a^2*b^2+b^4)*tan(1/2*f*x+1/2*e)*c^4+8/f/b^2/(tan(1/2*f*x+1/2*e)^2*a+2*t
an(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^5*c*d^3-8/f/b^3/(a^4-2*a^2*b
^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(
1/2))*a^5*c*d^3+20/f/b/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*
tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*a^3*c*d^3-12/f*b/(a^4-2*a^2*b^2+b^
4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))
*a*c^3*d-24/f*b/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2
*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*a*c*d^3+4/f/b/(tan(1/2*f*x+1/2*e)^2*a+2*t
an(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^4*tan(1/2*f*x+1/2*e)^3*c*d^3
-12/f*b/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+
b^4)*a^2*tan(1/2*f*x+1/2*e)^3*c^3*d-16/f*b/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/
2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^2*tan(1/2*f*x+1/2*e)^3*c*d^3+12/f
*b^2/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4
)*a*tan(1/2*f*x+1/2*e)^3*c^2*d^2+8/f/b^2/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*
f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^5*tan(1/2*f*x+1/2*e)^2*c*d^3+18/f*b
/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^
2*tan(1/2*f*x+1/2*e)^2*c^2*d^2-20/f*b^2/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f
*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a*tan(1/2*f*x+1/2*e)^2*c^3*d+7/f*b^3/(
tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*tan(
1/2*f*x+1/2*e)^2*c^4+22/f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)
^2*a^3/(a^4-2*a^2*b^2+b^4)*tan(1/2*f*x+1/2*e)*d^4-8/f/(tan(1/2*f*x+1/2*e)^2
*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^3*c^3*d-20/f/(tan(1/2*
```

```
f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^3*c*d^3-40
/f*b^2/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b
^4)*a*tan(1/2*f*x+1/2*e)^2*c*d^3-8/f*b^4/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*
f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)/a*tan(1/2*f*x+1/2*e)^2*c^3*d+28/f/b/(
tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2*a^4/(a^4-2*a^2*b^2+b^4)*
tan(1/2*f*x+1/2*e)*c*d^3-20/f*b/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e
)*b+a)^2*a^2/(a^4-2*a^2*b^2+b^4)*tan(1/2*f*x+1/2*e)*c^3*d-64/f*b/(tan(1/2*f
*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2*a^2/(a^4-2*a^2*b^2+b^4)*tan(1/2*f
*x+1/2*e)*c*d^3+60/f*b^2/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^
2*a/(a^4-2*a^2*b^2+b^4)*tan(1/2*f*x+1/2*e)*c^2*d^2-1/f*b^3/(tan(1/2*f*x+1/2
*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*c^4+2/f/(a^4-2*a^2*
b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^
(1/2))*a^2*c^4+6/f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4
-2*a^2*b^2+b^4)*a^3*tan(1/2*f*x+1/2*e)^3*d^4+12/f/(a^4-2*a^2*b^2+b^4)/(a^2-
b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*a^2*d^4
+1/f*b^2/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/
2*e)+2*b)/(a^2-b^2)^(1/2))*c^4+7/f/b/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+
1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^4*d^4+4/f*b/(tan(1/2*f*x+1/2*e)^2*a+2*t
an(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^2*c^4-4/f/b^3/(tan(1/2*f*x+1
/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^6*d^4+18/f*b/(t
an(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^2*c
^2*d^2-4/f*b^2/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a
^2*b^2+b^4)*a*c^3*d-15/f/b^2/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2
*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*a^4*d^4+6/f/b^4/(a^4-2*a^2*b
^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(
1/2))*a^6*d^4-3/f/b^2/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(
a^4-2*a^2*b^2+b^4)*a^5*tan(1/2*f*x+1/2*e)^3*d^4-4/f/b^3/(tan(1/2*f*x+1/2*e)
^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^6*tan(1/2*f*x+1/2*e)
^2*d^4-2/f*b^5/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a
^2*b^2+b^4)/a^2*tan(1/2*f*x+1/2*e)^2*c^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^4/(a+b*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.4938, size = 4743, normalized size = 14.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^4/(a+b*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*(a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*d^4*cos(f*x + e)^3 - 4*(4*
(a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*c*d^3 - 3*(a^7*b^2 - 3*a^5*b^4 + 3*
a^3*b^6 - a*b^8)*d^4)*f*x*cos(f*x + e)^2 + 4*(4*(a^8*b - 2*a^6*b^3 + 2*a^2*
b^7 - b^9)*c*d^3 - 3*(a^9 - 2*a^7*b^2 + 2*a^3*b^6 - a*b^8)*d^4)*f*x - ((2*a
^4*b^4 + 3*a^2*b^6 + b^8)*c^4 - 12*(a^3*b^5 + a*b^7)*c^3*d + 6*(a^4*b^4 + 3
*a^2*b^6 + 2*b^8)*c^2*d^2 - 4*(2*a^7*b - 3*a^5*b^3 + a^3*b^5 + 6*a*b^7)*c*d
```

```
^3 + 3*(2*a^8 - 3*a^6*b^2 - a^4*b^4 + 4*a^2*b^6)*d^4 + (12*a*b^7*c^3*d - (2
*a^2*b^6 + b^8)*c^4 - 6*(a^2*b^6 + 2*b^8)*c^2*d^2 + 4*(2*a^5*b^3 - 5*a^3*b^
5 + 6*a*b^7)*c*d^3 - 3*(2*a^6*b^2 - 5*a^4*b^4 + 4*a^2*b^6)*d^4)*cos(f*x + e
)^2 - 2*(12*a^2*b^6*c^3*d - (2*a^3*b^5 + a*b^7)*c^4 - 6*(a^3*b^5 + 2*a*b^7)
*c^2*d^2 + 4*(2*a^6*b^2 - 5*a^4*b^4 + 6*a^2*b^6)*c*d^3 - 3*(2*a^7*b - 5*a^5
*b^3 + 4*a^3*b^5)*d^4)*sin(f*x + e))*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*co
s(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2 + 2*(a*cos(f*x + e)*sin(f*x +
 e) + b*cos(f*x + e))*sqrt(-a^2 + b^2))/(b^2*cos(f*x + e)^2 - 2*a*b*sin(f*x
 + e) - a^2 - b^2)) + 2*((4*a^4*b^5 - 5*a^2*b^7 + b^9)*c^4 - 4*(2*a^5*b^4 -
 a^3*b^6 - a*b^8)*c^3*d + 18*(a^4*b^5 - a^2*b^7)*c^2*d^2 + 4*(2*a^7*b^2 - 7
*a^5*b^4 + 5*a^3*b^6)*c*d^3 - (6*a^8*b - 15*a^6*b^3 + 7*a^4*b^5 + 4*a^2*b^7
 - 2*b^9)*d^4)*cos(f*x + e) + 2*(4*(4*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*
b^8)*c*d^3 - 3*(a^8*b - 3*a^6*b^3 + 3*a^4*b^5 - a^2*b^7)*d^4)*f*x + (3*(a^3
*b^6 - a*b^8)*c^4 - 4*(a^4*b^5 + a^2*b^7 - 2*b^9)*c^3*d - 6*(a^5*b^4 - 5*a^
3*b^6 + 4*a*b^8)*c^2*d^2 + 12*(a^6*b^3 - 3*a^4*b^5 + 2*a^2*b^7)*c*d^3 - (9*
a^7*b^2 - 25*a^5*b^4 + 20*a^3*b^6 - 4*a*b^8)*d^4)*cos(f*x + e))*sin(f*x + e
))/((a^6*b^6 - 3*a^4*b^8 + 3*a^2*b^10 - b^12)*f*cos(f*x + e)^2 - 2*(a^7*b^5
 - 3*a^5*b^7 + 3*a^3*b^9 - a*b^11)*f*sin(f*x + e) - (a^8*b^4 - 2*a^6*b^6 +
2*a^2*b^10 - b^12)*f), -1/2*(2*(a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*d^4*
cos(f*x + e)^3 - 2*(4*(a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*c*d^3 - 3*(a^
7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*d^4)*f*x*cos(f*x + e)^2 + 2*(4*(a^8*
b - 2*a^6*b^3 + 2*a^2*b^7 - b^9)*c*d^3 - 3*(a^9 - 2*a^7*b^2 + 2*a^3*b^6 - a
*b^8)*d^4)*f*x - ((2*a^4*b^4 + 3*a^2*b^6 + b^8)*c^4 - 12*(a^3*b^5 + a*b^7)*
c^3*d + 6*(a^4*b^4 + 3*a^2*b^6 + 2*b^8)*c^2*d^2 - 4*(2*a^7*b - 3*a^5*b^3 +
a^3*b^5 + 6*a*b^7)*c*d^3 + 3*(2*a^8 - 3*a^6*b^2 - a^4*b^4 + 4*a^2*b^6)*d^4
+ (12*a*b^7*c^3*d - (2*a^2*b^6 + b^8)*c^4 - 6*(a^2*b^6 + 2*b^8)*c^2*d^2 + 4
*(2*a^5*b^3 - 5*a^3*b^5 + 6*a*b^7)*c*d^3 - 3*(2*a^6*b^2 - 5*a^4*b^4 + 4*a^2
*b^6)*d^4)*cos(f*x + e)^2 - 2*(12*a^2*b^6*c^3*d - (2*a^3*b^5 + a*b^7)*c^4 -
 6*(a^3*b^5 + 2*a*b^7)*c^2*d^2 + 4*(2*a^6*b^2 - 5*a^4*b^4 + 6*a^2*b^6)*c*d^
3 - 3*(2*a^7*b - 5*a^5*b^3 + 4*a^3*b^5)*d^4)*sin(f*x + e))*sqrt(a^2 - b^2)*
arctan(-(a*sin(f*x + e) + b)/(sqrt(a^2 - b^2)*cos(f*x + e))) + ((4*a^4*b^5
- 5*a^2*b^7 + b^9)*c^4 - 4*(2*a^5*b^4 - a^3*b^6 - a*b^8)*c^3*d + 18*(a^4*b^
5 - a^2*b^7)*c^2*d^2 + 4*(2*a^7*b^2 - 7*a^5*b^4 + 5*a^3*b^6)*c*d^3 - (6*a^8
*b - 15*a^6*b^3 + 7*a^4*b^5 + 4*a^2*b^7 - 2*b^9)*d^4)*cos(f*x + e) + (4*(4*
(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*c*d^3 - 3*(a^8*b - 3*a^6*b^3 + 3*
a^4*b^5 - a^2*b^7)*d^4)*f*x + (3*(a^3*b^6 - a*b^8)*c^4 - 4*(a^4*b^5 + a^2*b
^7 - 2*b^9)*c^3*d - 6*(a^5*b^4 - 5*a^3*b^6 + 4*a*b^8)*c^2*d^2 + 12*(a^6*b^3
 - 3*a^4*b^5 + 2*a^2*b^7)*c*d^3 - (9*a^7*b^2 - 25*a^5*b^4 + 20*a^3*b^6 - 4*
a*b^8)*d^4)*cos(f*x + e))*sin(f*x + e))/((a^6*b^6 - 3*a^4*b^8 + 3*a^2*b^10
- b^12)*f*cos(f*x + e)^2 - 2*(a^7*b^5 - 3*a^5*b^7 + 3*a^3*b^9 - a*b^11)*f*s
in(f*x + e) - (a^8*b^4 - 2*a^6*b^6 + 2*a^2*b^10 - b^12)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**4/(a+b*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.38063, size = 1565, normalized size = 4.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^4/(a+b*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] ((2*a^2*b^4*c^4 + b^6*c^4 - 12*a*b^5*c^3*d + 6*a^2*b^4*c^2*d^2 + 12*b^6*c^2
*d^2 - 8*a^5*b*c*d^3 + 20*a^3*b^3*c*d^3 - 24*a*b^5*c*d^3 + 6*a^6*d^4 - 15*a
^4*b^2*d^4 + 12*a^2*b^4*d^4)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(a) + arc
tan((a*tan(1/2*f*x + 1/2*e) + b)/sqrt(a^2 - b^2)))/((a^4*b^4 - 2*a^2*b^6 +
b^8)*sqrt(a^2 - b^2)) - 2*d^4/((tan(1/2*f*x + 1/2*e)^2 + 1)*b^3) + (5*a^3*b
^5*c^4*tan(1/2*f*x + 1/2*e)^3 - 2*a*b^7*c^4*tan(1/2*f*x + 1/2*e)^3 - 12*a^4
*b^4*c^3*d*tan(1/2*f*x + 1/2*e)^3 + 6*a^5*b^3*c^2*d^2*tan(1/2*f*x + 1/2*e)^
3 + 12*a^3*b^5*c^2*d^2*tan(1/2*f*x + 1/2*e)^3 + 4*a^6*b^2*c*d^3*tan(1/2*f*x
 + 1/2*e)^3 - 16*a^4*b^4*c*d^3*tan(1/2*f*x + 1/2*e)^3 - 3*a^7*b*d^4*tan(1/2
*f*x + 1/2*e)^3 + 6*a^5*b^3*d^4*tan(1/2*f*x + 1/2*e)^3 + 4*a^4*b^4*c^4*tan(
1/2*f*x + 1/2*e)^2 + 7*a^2*b^6*c^4*tan(1/2*f*x + 1/2*e)^2 - 2*b^8*c^4*tan(1
/2*f*x + 1/2*e)^2 - 8*a^5*b^3*c^3*d*tan(1/2*f*x + 1/2*e)^2 - 20*a^3*b^5*c^3
*d*tan(1/2*f*x + 1/2*e)^2 - 8*a*b^7*c^3*d*tan(1/2*f*x + 1/2*e)^2 + 18*a^4*b
^4*c^2*d^2*tan(1/2*f*x + 1/2*e)^2 + 36*a^2*b^6*c^2*d^2*tan(1/2*f*x + 1/2*e)
^2 + 8*a^7*b*c*d^3*tan(1/2*f*x + 1/2*e)^2 - 4*a^5*b^3*c*d^3*tan(1/2*f*x + 1
/2*e)^2 - 40*a^3*b^5*c*d^3*tan(1/2*f*x + 1/2*e)^2 - 4*a^8*d^4*tan(1/2*f*x +
 1/2*e)^2 - a^6*b^2*d^4*tan(1/2*f*x + 1/2*e)^2 + 14*a^4*b^4*d^4*tan(1/2*f*x
 + 1/2*e)^2 + 11*a^3*b^5*c^4*tan(1/2*f*x + 1/2*e) - 2*a*b^7*c^4*tan(1/2*f*x
 + 1/2*e) - 20*a^4*b^4*c^3*d*tan(1/2*f*x + 1/2*e) - 16*a^2*b^6*c^3*d*tan(1/
2*f*x + 1/2*e) - 6*a^5*b^3*c^2*d^2*tan(1/2*f*x + 1/2*e) + 60*a^3*b^5*c^2*d^
2*tan(1/2*f*x + 1/2*e) + 28*a^6*b^2*c*d^3*tan(1/2*f*x + 1/2*e) - 64*a^4*b^4
*c*d^3*tan(1/2*f*x + 1/2*e) - 13*a^7*b*d^4*tan(1/2*f*x + 1/2*e) + 22*a^5*b^
3*d^4*tan(1/2*f*x + 1/2*e) + 4*a^4*b^4*c^4 - a^2*b^6*c^4 - 8*a^5*b^3*c^3*d
- 4*a^3*b^5*c^3*d + 18*a^4*b^4*c^2*d^2 + 8*a^7*b*c*d^3 - 20*a^5*b^3*c*d^3 -
 4*a^8*d^4 + 7*a^6*b^2*d^4)/((a^6*b^3 - 2*a^4*b^5 + a^2*b^7)*(a*tan(1/2*f*x
 + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e) + a)^2) + (4*b*c*d^3 - 3*a*d^4)*(f*x
 + e)/b^4)/f
```

3.716 $\int \frac{(c+d\sin(e+fx))^3}{(a+b\sin(e+fx))^3}\,dx$

Optimal. Leaf size=248

$$\frac{(bc-ad)\left(a^2b^2\left(2c^2-5d^2\right)+2a^3bcd+2a^4d^2-8ab^3cd+b^4\left(c^2+6d^2\right)\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^3f\left(a^2-b^2\right)^{5/2}}+\frac{(bc-ad)^2\cos(e+fx)(}{2bf\left(a^2-b^2\right)(a+b}$$

```
[Out] (d^3*x)/b^3 + ((b*c - a*d)*(2*a^3*b*c*d - 8*a*b^3*c*d + 2*a^4*d^2 + a^2*b^2
*(2*c^2 - 5*d^2) + b^4*(c^2 + 6*d^2))*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[
a^2 - b^2]])/(b^3*(a^2 - b^2)^(5/2)*f) + ((b*c - a*d)^2*(3*a*b*c + 2*a^2*d
- 5*b^2*d)*Cos[e + f*x])/(2*b^2*(a^2 - b^2)^2*f*(a + b*Sin[e + f*x])) + ((b
*c - a*d)^2*Cos[e + f*x]*(c + d*Sin[e + f*x]))/(2*b*(a^2 - b^2)*f*(a + b*Si
n[e + f*x])^2)
```

Rubi [A] time = 0.806464, antiderivative size = 248, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {2792, 3021, 2735, 2660, 618, 204}

$$\frac{(bc-ad)\left(a^2b^2\left(2c^2-5d^2\right)+2a^3bcd+2a^4d^2-8ab^3cd+b^4\left(c^2+6d^2\right)\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^3f\left(a^2-b^2\right)^{5/2}}+\frac{(bc-ad)^2\cos(e+fx)(}{2bf\left(a^2-b^2\right)(a+b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^3/(a + b*Sin[e + f*x])^3,x]
```

```
[Out] (d^3*x)/b^3 + ((b*c - a*d)*(2*a^3*b*c*d - 8*a*b^3*c*d + 2*a^4*d^2 + a^2*b^2
*(2*c^2 - 5*d^2) + b^4*(c^2 + 6*d^2))*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[
a^2 - b^2]])/(b^3*(a^2 - b^2)^(5/2)*f) + ((b*c - a*d)^2*(3*a*b*c + 2*a^2*d
- 5*b^2*d)*Cos[e + f*x])/(2*b^2*(a^2 - b^2)^2*f*(a + b*Sin[e + f*x])) + ((b
*c - a*d)^2*Cos[e + f*x]*(c + d*Sin[e + f*x]))/(2*b*(a^2 - b^2)*f*(a + b*Si
n[e + f*x])^2)
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
```

```
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^3}{(a+b\sin(e+fx))^3}\,dx = \frac{(bc-ad)^2\cos(e+fx)(c+d\sin(e+fx))}{2b\left(a^2-b^2\right)f(a+b\sin(e+fx))^2} - \frac{\int \frac{5b^2c^2d+a^2d^3-2abc\left(c^2+2d^2\right)-\left(a^2cd^2+2abd\left(2c^2+d^2\right)-b^2\left(c \cdots\right.\right.}{(a+b\sin(e+fx)\cdots}}{2b\left(a^2-b^2 \cdots\right.}$$

$$= \frac{(bc-ad)^2\left(3abc+2a^2d-5b^2d\right)\cos(e+fx)}{2b^2\left(a^2-b^2\right)^2 f(a+b\sin(e+fx))} + \frac{(bc-ad)^2\cos(e+fx)(c+d\sin(e+fx))}{2b\left(a^2-b^2\right)f(a+b\sin(e+fx))^2} \cdots$$

$$= \frac{d^3x}{b^3} + \frac{(bc-ad)^2\left(3abc+2a^2d-5b^2d\right)\cos(e+fx)}{2b^2\left(a^2-b^2\right)^2 f(a+b\sin(e+fx))} + \frac{(bc-ad)^2\cos(e+fx)(c+d\sin(e+ \cdots}{2b\left(a^2-b^2\right)f(a+b\sin(e+fx \cdots}$$

$$= \frac{d^3x}{b^3} + \frac{(bc-ad)^2\left(3abc+2a^2d-5b^2d\right)\cos(e+fx)}{2b^2\left(a^2-b^2\right)^2 f(a+b\sin(e+fx))} + \frac{(bc-ad)^2\cos(e+fx)(c+d\sin(e+ \cdots}{2b\left(a^2-b^2\right)f(a+b\sin(e+fx \cdots}$$

$$= \frac{d^3x}{b^3} + \frac{(bc-ad)^2\left(3abc+2a^2d-5b^2d\right)\cos(e+fx)}{2b^2\left(a^2-b^2\right)^2 f(a+b\sin(e+fx))} + \frac{(bc-ad)^2\cos(e+fx)(c+d\sin(e+ \cdots}{2b\left(a^2-b^2\right)f(a+b\sin(e+fx \cdots}$$

$$= \frac{d^3x}{b^3} + \frac{(bc-ad)\left(2a^2b^2c^2+b^4c^2+2a^3bcd-8ab^3cd+2a^4d^2-5a^2b^2d^2+6b^4d^2\right)\tan^{-1}\left(\frac{b \cdots}{- \cdots}\right.}{b^3\left(a^2-b^2\right)^{5/2} f}$$

Mathematica [B] time = 2.3726, size = 524, normalized size = 2.11

$$\frac{-3a^2b^4c^2d\sin(2(e+fx))-3a^3b^3cd^2\sin(2(e+fx))-2b(bc-ad)^2\left(-4a^2bc-2a^3d+5ab^2d+b^3c\right)\cos(e+fx)+3a^4b^2d^3\sin(2(e+fx))-16a^3b^3d^3e\sin(e+fx)-16a^3b^3d^3fx\sin}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])^3/(a + b*Sin[e + f*x])^3,x]
```

```
[Out] ((-4*(2*a^5*d^3 - 5*a^3*b^2*d^3 + 3*a*b^4*d*(3*c^2 + 2*d^2) - a^2*b^3*c*(2*
c^2 + 3*d^2) - b^5*c*(c^2 + 6*d^2))*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^
2 - b^2]])/(a^2 - b^2)^(5/2) + (4*a^6*d^3*e - 6*a^4*b^2*d^3*e + 2*b^6*d^3*e
 + 4*a^6*d^3*f*x - 6*a^4*b^2*d^3*f*x + 2*b^6*d^3*f*x - 2*b*(b*c - a*d)^2*(-
4*a^2*b*c + b^3*c - 2*a^3*d + 5*a*b^2*d)*Cos[e + f*x] - 2*(-(a^2*b) + b^3)^
2*d^3*(e + f*x)*Cos[2*(e + f*x)] + 8*a^5*b*d^3*e*Sin[e + f*x] - 16*a^3*b^3*
d^3*e*Sin[e + f*x] + 8*a*b^5*d^3*e*Sin[e + f*x] + 8*a^5*b*d^3*f*x*Sin[e + f
*x] - 16*a^3*b^3*d^3*f*x*Sin[e + f*x] + 8*a*b^5*d^3*f*x*Sin[e + f*x] + 3*a*
b^5*c^3*Sin[2*(e + f*x)] - 3*a^2*b^4*c^2*d*Sin[2*(e + f*x)] - 6*b^6*c^2*d*S
in[2*(e + f*x)] - 3*a^3*b^3*c*d^2*Sin[2*(e + f*x)] + 12*a*b^5*c*d^2*Sin[2*(
e + f*x)] + 3*a^4*b^2*d^3*Sin[2*(e + f*x)] - 6*a^2*b^4*d^3*Sin[2*(e + f*x)]
)/((a^2 - b^2)^2*(a + b*Sin[e + f*x])^2))/(4*b^3*f)
```

Maple [B] time = 0.109, size = 2785, normalized size = 11.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^3/(a+b*sin(f*x+e))^3,x)
```

```
[Out] -1/f*b^3/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2
+b^4)*c^3+7/f/b/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2*a^4/(a^
4-2*a^2*b^2+b^4)*tan(1/2*f*x+1/2*e)*d^3-16/f*b/(tan(1/2*f*x+1/2*e)^2*a+2*ta
n(1/2*f*x+1/2*e)*b+a)^2*a^2/(a^4-2*a^2*b^2+b^4)*tan(1/2*f*x+1/2*e)*d^3+11/f
*b^2/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2*a/(a^4-2*a^2*b^2+b
^4)*tan(1/2*f*x+1/2*e)*c^3-2/f*b^4/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/
2*e)*b+a)^2/a/(a^4-2*a^2*b^2+b^4)*tan(1/2*f*x+1/2*e)*c^3+9/f*b/(tan(1/2*f*x
+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^2*c*d^2-3/f*b
^2/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*
a*c^2*d+2/f*d^3/b^3*arctan(tan(1/2*f*x+1/2*e))+6/f*b^2/(tan(1/2*f*x+1/2*e)^
2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a*tan(1/2*f*x+1/2*e)^3*
c*d^2+9/f*b/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*
b^2+b^4)*a^2*tan(1/2*f*x+1/2*e)^2*c*d^2-15/f*b^2/(tan(1/2*f*x+1/2*e)^2*a+2*
tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a*tan(1/2*f*x+1/2*e)^2*c^2*d-
6/f*b^4/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+
b^4)/a*tan(1/2*f*x+1/2*e)^2*c^2*d-15/f*b/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*
f*x+1/2*e)*b+a)^2*a^2/(a^4-2*a^2*b^2+b^4)*tan(1/2*f*x+1/2*e)*c^2*d+30/f*b^2
/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2*a/(a^4-2*a^2*b^2+b^4)*
tan(1/2*f*x+1/2*e)*c*d^2+18/f*b^3/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2
*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*tan(1/2*f*x+1/2*e)^2*c*d^2-12/f*b^3/(tan(1/2
*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*tan(1/2*f*x
+1/2*e)*c^2*d+3/f/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1
/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*a^2*c*d^2+3/f/(tan(1/2*f*x+1/2*e)^2*a+2
*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^3*tan(1/2*f*x+1/2*e)^3*c*d
```

```
^2-6/f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b
^4)*a^3*tan(1/2*f*x+1/2*e)^2*c^2*d-3/f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*
x+1/2*e)*b+a)^2*a^3/(a^4-2*a^2*b^2+b^4)*tan(1/2*f*x+1/2*e)*c*d^2+5/f/b/(a^4
-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^
2-b^2)^(1/2))*a^3*d^3-2/f/b^3/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/
2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*a^5*d^3-6/f*b/(a^4-2*a^2*b^
2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1
/2))*a*d^3+6/f*b^2/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(
1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*c*d^2+1/f/b/(tan(1/2*f*x+1/2*e)^2*a+2*
tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^4*tan(1/2*f*x+1/2*e)^3*d^3-
4/f*b/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^
4)*a^2*tan(1/2*f*x+1/2*e)^3*d^3+5/f*b^2/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f
*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a*tan(1/2*f*x+1/2*e)^3*c^3-2/f*b^4/(ta
n(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)/a*tan(
1/2*f*x+1/2*e)^3*c^3+2/f/b^2/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b
+a)^2/(a^4-2*a^2*b^2+b^4)*a^5*tan(1/2*f*x+1/2*e)^2*d^3+4/f*b/(tan(1/2*f*x+1
/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^2*tan(1/2*f*x+1
/2*e)^2*c^3-10/f*b^2/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a
^4-2*a^2*b^2+b^4)*a*tan(1/2*f*x+1/2*e)^2*d^3-2/f*b^5/(tan(1/2*f*x+1/2*e)^2*
a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)/a^2*tan(1/2*f*x+1/2*e)^2*
c^3+2/f/b^2/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*
b^2+b^4)*a^5*d^3-1/f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a
^4-2*a^2*b^2+b^4)*a^3*tan(1/2*f*x+1/2*e)^2*d^3-6/f/(tan(1/2*f*x+1/2*e)^2*a+
2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^3*c^2*d+2/f/(a^4-2*a^2*b^
2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1
/2))*a^2*c^3+4/f*b/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4
-2*a^2*b^2+b^4)*a^2*c^3+1/f*b^2/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(
1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*c^3+7/f*b^3/(tan(1/2*f*x+
1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*tan(1/2*f*x+1/2*
e)^2*c^3-9/f*b/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*
f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*a*c^2*d-9/f*b/(tan(1/2*f*x+1/2*e)^2*a+2*ta
n(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^2*tan(1/2*f*x+1/2*e)^3*c^2*d-
5/f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)
*a^3*d^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^3/(a+b*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.17332, size = 3313, normalized size = 13.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^3/(a+b*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] [1/4*(4*(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d^3*f*x*cos(f*x + e)^2 - 4*
(a^8 - 2*a^6*b^2 + 2*a^2*b^6 - b^8)*d^3*f*x - ((2*a^4*b^3 + 3*a^2*b^5 + b^7
```

```
)*c^3 - 9*(a^3*b^4 + a*b^6)*c^2*d + 3*(a^4*b^3 + 3*a^2*b^5 + 2*b^7)*c*d^2 -
 (2*a^7 - 3*a^5*b^2 + a^3*b^4 + 6*a*b^6)*d^3 + (9*a*b^6*c^2*d - (2*a^2*b^5
+ b^7)*c^3 - 3*(a^2*b^5 + 2*b^7)*c*d^2 + (2*a^5*b^2 - 5*a^3*b^4 + 6*a*b^6)*
d^3)*cos(f*x + e)^2 - 2*(9*a^2*b^5*c^2*d - (2*a^3*b^4 + a*b^6)*c^3 - 3*(a^3
*b^4 + 2*a*b^6)*c*d^2 + (2*a^6*b - 5*a^4*b^3 + 6*a^2*b^5)*d^3)*sin(f*x + e)
)*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(f*x + e)^2 - 2*a*b*sin(f*x + e)
- a^2 - b^2 - 2*(a*cos(f*x + e)*sin(f*x + e) + b*cos(f*x + e))*sqrt(-a^2 +
b^2))/(b^2*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2)) - 2*((4*a^4*b^
4 - 5*a^2*b^6 + b^8)*c^3 - 3*(2*a^5*b^3 - a^3*b^5 - a*b^7)*c^2*d + 9*(a^4*b
^4 - a^2*b^6)*c*d^2 + (2*a^7*b - 7*a^5*b^3 + 5*a^3*b^5)*d^3)*cos(f*x + e) -
 2*(4*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d^3*f*x + 3*((a^3*b^5 - a*b^7
)*c^3 - (a^4*b^4 + a^2*b^6 - 2*b^8)*c^2*d - (a^5*b^3 - 5*a^3*b^5 + 4*a*b^7)
*c*d^2 + (a^6*b^2 - 3*a^4*b^4 + 2*a^2*b^6)*d^3)*cos(f*x + e))*sin(f*x + e))
/((a^6*b^5 - 3*a^4*b^7 + 3*a^2*b^9 - b^11)*f*cos(f*x + e)^2 - 2*(a^7*b^4 -
3*a^5*b^6 + 3*a^3*b^8 - a*b^10)*f*sin(f*x + e) - (a^8*b^3 - 2*a^6*b^5 + 2*a
^2*b^9 - b^11)*f), 1/2*(2*(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d^3*f*x*c
os(f*x + e)^2 - 2*(a^8 - 2*a^6*b^2 + 2*a^2*b^6 - b^8)*d^3*f*x + ((2*a^4*b^3
 + 3*a^2*b^5 + b^7)*c^3 - 9*(a^3*b^4 + a*b^6)*c^2*d + 3*(a^4*b^3 + 3*a^2*b^
5 + 2*b^7)*c*d^2 - (2*a^7 - 3*a^5*b^2 + a^3*b^4 + 6*a*b^6)*d^3 + (9*a*b^6*c
^2*d - (2*a^2*b^5 + b^7)*c^3 - 3*(a^2*b^5 + 2*b^7)*c*d^2 + (2*a^5*b^2 - 5*a
^3*b^4 + 6*a*b^6)*d^3)*cos(f*x + e)^2 - 2*(9*a^2*b^5*c^2*d - (2*a^3*b^4 + a
*b^6)*c^3 - 3*(a^3*b^4 + 2*a*b^6)*c*d^2 + (2*a^6*b - 5*a^4*b^3 + 6*a^2*b^5)
*d^3)*sin(f*x + e))*sqrt(a^2 - b^2)*arctan(-(a*sin(f*x + e) + b)/(sqrt(a^2
- b^2)*cos(f*x + e))) - ((4*a^4*b^4 - 5*a^2*b^6 + b^8)*c^3 - 3*(2*a^5*b^3 -
 a^3*b^5 - a*b^7)*c^2*d + 9*(a^4*b^4 - a^2*b^6)*c*d^2 + (2*a^7*b - 7*a^5*b^
3 + 5*a^3*b^5)*d^3)*cos(f*x + e) - (4*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^
7)*d^3*f*x + 3*((a^3*b^5 - a*b^7)*c^3 - (a^4*b^4 + a^2*b^6 - 2*b^8)*c^2*d -
 (a^5*b^3 - 5*a^3*b^5 + 4*a*b^7)*c*d^2 + (a^6*b^2 - 3*a^4*b^4 + 2*a^2*b^6)*
d^3)*cos(f*x + e))*sin(f*x + e))/((a^6*b^5 - 3*a^4*b^7 + 3*a^2*b^9 - b^11)*
f*cos(f*x + e)^2 - 2*(a^7*b^4 - 3*a^5*b^6 + 3*a^3*b^8 - a*b^10)*f*sin(f*x +
 e) - (a^8*b^3 - 2*a^6*b^5 + 2*a^2*b^9 - b^11)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**3/(a+b*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.34669, size = 1197, normalized size = 4.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^3/(a+b*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] ((f*x + e)*d^3/b^3 + (2*a^2*b^3*c^3 + b^5*c^3 - 9*a*b^4*c^2*d + 3*a^2*b^3*c
*d^2 + 6*b^5*c*d^2 - 2*a^5*d^3 + 5*a^3*b^2*d^3 - 6*a*b^4*d^3)*(pi*floor(1/2
*(f*x + e)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*f*x + 1/2*e) + b)/sqrt(a^2
- b^2)))/((a^4*b^3 - 2*a^2*b^5 + b^7)*sqrt(a^2 - b^2)) + (5*a^3*b^4*c^3*tan
```

(1/2*f*x + 1/2*e)^3 - 2*a*b^6*c^3*tan(1/2*f*x + 1/2*e)^3 - 9*a^4*b^3*c^2*d*tan(1/2*f*x + 1/2*e)^3 + 3*a^5*b^2*c*d^2*tan(1/2*f*x + 1/2*e)^3 + 6*a^3*b^4*c*d^2*tan(1/2*f*x + 1/2*e)^3 + a^6*b*d^3*tan(1/2*f*x + 1/2*e)^3 - 4*a^4*b^3*d^3*tan(1/2*f*x + 1/2*e)^3 + 4*a^4*b^3*c^3*tan(1/2*f*x + 1/2*e)^2 + 7*a^2*b^5*c^3*tan(1/2*f*x + 1/2*e)^2 - 2*b^7*c^3*tan(1/2*f*x + 1/2*e)^2 - 6*a^5*b^2*c^2*d*tan(1/2*f*x + 1/2*e)^2 - 15*a^3*b^4*c^2*d*tan(1/2*f*x + 1/2*e)^2 - 6*a*b^6*c^2*d*tan(1/2*f*x + 1/2*e)^2 + 9*a^4*b^3*c*d^2*tan(1/2*f*x + 1/2*e)^2 + 18*a^2*b^5*c*d^2*tan(1/2*f*x + 1/2*e)^2 + 2*a^7*d^3*tan(1/2*f*x + 1/2*e)^2 - a^5*b^2*d^3*tan(1/2*f*x + 1/2*e)^2 - 10*a^3*b^4*d^3*tan(1/2*f*x + 1/2*e)^2 + 11*a^3*b^4*c^3*tan(1/2*f*x + 1/2*e) - 2*a*b^6*c^3*tan(1/2*f*x + 1/2*e) - 15*a^4*b^3*c^2*d*tan(1/2*f*x + 1/2*e) - 12*a^2*b^5*c^2*d*tan(1/2*f*x + 1/2*e) - 3*a^5*b^2*c*d^2*tan(1/2*f*x + 1/2*e) + 30*a^3*b^4*c*d^2*tan(1/2*f*x + 1/2*e) + 7*a^6*b*d^3*tan(1/2*f*x + 1/2*e) - 16*a^4*b^3*d^3*tan(1/2*f*x + 1/2*e) + 4*a^4*b^3*c^3 - a^2*b^5*c^3 - 6*a^5*b^2*c^2*d - 3*a^3*b^4*c^2*d + 9*a^4*b^3*c*d^2 + 2*a^7*d^3 - 5*a^5*b^2*d^3)/((a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*(a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e) + a)^2))/f

3.717 $\int \frac{(c+d\sin(e+fx))^2}{(a+b\sin(e+fx))^3}\,dx$

Optimal. Leaf size=196

$$-\frac{\left(a^2\left(-\left(2c^2+d^2\right)\right)+6abcd-b^2\left(c^2+2d^2\right)\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{f\left(a^2-b^2\right)^{5/2}}+\frac{(bc-ad)^2\cos(e+fx)}{2bf\left(a^2-b^2\right)(a+b\sin(e+fx))^2}+\frac{\left(a^2d+3abc-\right.}{2bf\left(a^2-\right.}$$

```
[Out] -(((6*a*b*c*d - a^2*(2*c^2 + d^2) - b^2*(c^2 + 2*d^2))*ArcTan[(b + a*Tan[(e
 + f*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(5/2)*f)) + ((b*c - a*d)^2*Cos[e
 + f*x])/(2*b*(a^2 - b^2)*f*(a + b*Sin[e + f*x])^2) + ((b*c - a*d)*(3*a*b*c
 + a^2*d - 4*b^2*d)*Cos[e + f*x])/(2*b*(a^2 - b^2)^2*f*(a + b*Sin[e + f*x])
)
```

Rubi [A] time = 0.285129, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {2790, 2754, 12, 2660, 618, 204}

$$-\frac{\left(a^2\left(-\left(2c^2+d^2\right)\right)+6abcd-b^2\left(c^2+2d^2\right)\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{f\left(a^2-b^2\right)^{5/2}}+\frac{(bc-ad)^2\cos(e+fx)}{2bf\left(a^2-b^2\right)(a+b\sin(e+fx))^2}+\frac{\left(a^2d+3abc-\right.}{2bf\left(a^2-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^2/(a + b*Sin[e + f*x])^3,x]
```

```
[Out] -(((6*a*b*c*d - a^2*(2*c^2 + d^2) - b^2*(c^2 + 2*d^2))*ArcTan[(b + a*Tan[(e
 + f*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(5/2)*f)) + ((b*c - a*d)^2*Cos[e
 + f*x])/(2*b*(a^2 - b^2)*f*(a + b*Sin[e + f*x])^2) + ((b*c - a*d)*(3*a*b*c
 + a^2*d - 4*b^2*d)*Cos[e + f*x])/(2*b*(a^2 - b^2)^2*f*(a + b*Sin[e + f*x])
)
```

Rule 2790

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos[e +
 f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] - Dist[1/
(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m + 1)*(2
*b*c*d - a*(c^2 + d^2)) + (a^2*d^2 - 2*a*b*c*d*(m + 2) + b^2*(d^2*(m + 1) +
 c^2*(m + 2)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^2}{(a+b\sin(e+fx))^3}\,dx = \frac{(bc-ad)^2\cos(e+fx)}{2b\left(a^2-b^2\right)f(a+b\sin(e+fx))^2} + \frac{\int \frac{-2b\left(2bcd-a\left(c^2+d^2\right)\right)+\left(2abcd+a^2d^2-b^2\left(c^2+2d^2\right)\right)\sin(e+fx)}{(a+b\sin(e+fx))^2}\,dx}{2b\left(a^2-b^2\right)}$$

$$= \frac{(bc-ad)^2\cos(e+fx)}{2b\left(a^2-b^2\right)f(a+b\sin(e+fx))^2} + \frac{(bc-ad)\left(3abc+a^2d-4b^2d\right)\cos(e+fx)}{2b\left(a^2-b^2\right)^2 f(a+b\sin(e+fx))} - \frac{\int \frac{b(6abc}{}}{}$$

$$= \frac{(bc-ad)^2\cos(e+fx)}{2b\left(a^2-b^2\right)f(a+b\sin(e+fx))^2} + \frac{(bc-ad)\left(3abc+a^2d-4b^2d\right)\cos(e+fx)}{2b\left(a^2-b^2\right)^2 f(a+b\sin(e+fx))} - \frac{(6abcd}{}$$

$$= \frac{(bc-ad)^2\cos(e+fx)}{2b\left(a^2-b^2\right)f(a+b\sin(e+fx))^2} + \frac{(bc-ad)\left(3abc+a^2d-4b^2d\right)\cos(e+fx)}{2b\left(a^2-b^2\right)^2 f(a+b\sin(e+fx))} - \frac{(6abcd}{}$$

$$= \frac{(bc-ad)^2\cos(e+fx)}{2b\left(a^2-b^2\right)f(a+b\sin(e+fx))^2} + \frac{(bc-ad)\left(3abc+a^2d-4b^2d\right)\cos(e+fx)}{2b\left(a^2-b^2\right)^2 f(a+b\sin(e+fx))} + \frac{(2(6ab}{}$$

$$= -\frac{\left(6abcd-a^2\left(2c^2+d^2\right)-b^2\left(c^2+2d^2\right)\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}f} + \frac{(bc-ad)^2\cos(e}{2b\left(a^2-b^2\right)f(a+b\sin}$$

Mathematica [A] time = 0.922206, size = 204, normalized size = 1.04

$$\frac{\frac{2\left(a^2\left(2c^2+d^2\right)-6abcd+b^2\left(c^2+2d^2\right)\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}} - \frac{\left(2a^2bcd+a^3d^2-ab^2\left(3c^2+4d^2\right)+4b^3cd\right)\cos(e+fx)}{b(a-b)^2(a+b)^2(a+b\sin(e+fx))} + \frac{(bc-ad)^2\cos(e+fx)}{b(a-b)(a+b)(a+b\sin(e+fx))^2}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])^2/(a + b*Sin[e + f*x])^3,x]
```

```
[Out] ((2*(-6*a*b*c*d + a^2*(2*c^2 + d^2) + b^2*(c^2 + 2*d^2))*ArcTan[(b + a*Tan[
(e + f*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2) + ((b*c - a*d)^2*Cos[e +
f*x])/((a - b)*b*(a + b)*(a + b*Sin[e + f*x])^2) - ((2*a^2*b*c*d + 4*b^3*c*
d + a^3*d^2 - a*b^2*(3*c^2 + 4*d^2))*Cos[e + f*x])/((a - b)^2*b*(a + b)^2*(
a + b*Sin[e + f*x])))/(2*f)
```

Maple [B] time = 0.096, size = 1923, normalized size = 9.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^2/(a+b*sin(f*x+e))^3,x)
```

```
[Out] -1/f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4
)*b^3*c^2-4/f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^
2*b^2+b^4)/a*tan(1/2*f*x+1/2*e)^2*b^4*c*d-10/f/(tan(1/2*f*x+1/2*e)^2*a+2*ta
n(1/2*f*x+1/2*e)*b+a)^2*a^2/(a^4-2*a^2*b^2+b^4)*tan(1/2*f*x+1/2*e)*b*c*d-6/
f/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*
b)/(a^2-b^2)^(1/2))*a*b*c*d-6/f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e
)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^2*tan(1/2*f*x+1/2*e)^3*b*c*d-10/f/(tan(1/2*f
*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a*tan(1/2*f*x
+1/2*e)^2*b^2*c*d-1/f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2*a
^3/(a^4-2*a^2*b^2+b^4)*tan(1/2*f*x+1/2*e)*d^2-4/f/(tan(1/2*f*x+1/2*e)^2*a+2
*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^3*c*d+4/f/(tan(1/2*f*x+1/2
*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^2*b*c^2+3/f/(tan(
1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^2*b*d^
2-4/f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^
4)*a^3*tan(1/2*f*x+1/2*e)^2*c*d+4/f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1
/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^2*tan(1/2*f*x+1/2*e)^2*b*c^2+3/f/(tan(1/
2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^2*tan(1/
2*f*x+1/2*e)^2*b*d^2-2/f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^
2/(a^4-2*a^2*b^2+b^4)/a^2*tan(1/2*f*x+1/2*e)^2*b^5*c^2+2/f/(a^4-2*a^2*b^2+b
^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2)
)*c^2*a^2+1/f/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*f
*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*a^2*d^2+1/f/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(
1/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*c^2*b^2+2/f/(
a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/
(a^2-b^2)^(1/2))*b^2*d^2+1/f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b
+a)^2/(a^4-2*a^2*b^2+b^4)*a^3*tan(1/2*f*x+1/2*e)^3*d^2+7/f/(tan(1/2*f*x+1/2
*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*tan(1/2*f*x+1/2*e)^
2*b^3*c^2+6/f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^
2*b^2+b^4)*tan(1/2*f*x+1/2*e)^2*b^3*d^2+11/f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(
1/2*f*x+1/2*e)*b+a)^2*a/(a^4-2*a^2*b^2+b^4)*tan(1/2*f*x+1/2*e)*b^2*c^2+10/f
/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2*a/(a^4-2*a^2*b^2+b^4)*
tan(1/2*f*x+1/2*e)*b^2*d^2-8/f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)
*b+a)^2/(a^4-2*a^2*b^2+b^4)*tan(1/2*f*x+1/2*e)*b^3*c*d-2/f/(tan(1/2*f*x+1/2
*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/a/(a^4-2*a^2*b^2+b^4)*tan(1/2*f*x+1/2*e
)*b^4*c^2-2/f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^
2*b^2+b^4)*a*b^2*c*d+5/f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^
2/(a^4-2*a^2*b^2+b^4)*a*tan(1/2*f*x+1/2*e)^3*b^2*c^2+2/f/(tan(1/2*f*x+1/2*e
)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a*tan(1/2*f*x+1/2*e)^
3*b^2*d^2-2/f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^
2*b^2+b^4)/a*tan(1/2*f*x+1/2*e)^3*b^4*c^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^2/(a+b*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.75024, size = 2137, normalized size = 10.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^2/(a+b*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*(3*(a^3*b^2 - a*b^4)*c^2 - 2*(a^4*b + a^2*b^3 - 2*b^5)*c*d - (a^5
- 5*a^3*b^2 + 4*a*b^4)*d^2)*cos(f*x + e)*sin(f*x + e) - ((2*a^4 + 3*a^2*b^2
 + b^4)*c^2 - 6*(a^3*b + a*b^3)*c*d + (a^4 + 3*a^2*b^2 + 2*b^4)*d^2 + (6*a*
b^3*c*d - (2*a^2*b^2 + b^4)*c^2 - (a^2*b^2 + 2*b^4)*d^2)*cos(f*x + e)^2 - 2
*(6*a^2*b^2*c*d - (2*a^3*b + a*b^3)*c^2 - (a^3*b + 2*a*b^3)*d^2)*sin(f*x +
e))*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(f*x + e)^2 - 2*a*b*sin(f*x + e)
 - a^2 - b^2 + 2*(a*cos(f*x + e)*sin(f*x + e) + b*cos(f*x + e))*sqrt(-a^2 +
 b^2))/(b^2*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2)) + 2*((4*a^4*b
 - 5*a^2*b^3 + b^5)*c^2 - 2*(2*a^5 - a^3*b^2 - a*b^4)*c*d + 3*(a^4*b - a^2*
b^3)*d^2)*cos(f*x + e))/((a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*f*cos(f*x
+ e)^2 - 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*f*sin(f*x + e) - (a^8 -
2*a^6*b^2 + 2*a^2*b^6 - b^8)*f), -1/2*((3*(a^3*b^2 - a*b^4)*c^2 - 2*(a^4*b
+ a^2*b^3 - 2*b^5)*c*d - (a^5 - 5*a^3*b^2 + 4*a*b^4)*d^2)*cos(f*x + e)*sin(
f*x + e) - ((2*a^4 + 3*a^2*b^2 + b^4)*c^2 - 6*(a^3*b + a*b^3)*c*d + (a^4 +
3*a^2*b^2 + 2*b^4)*d^2 + (6*a*b^3*c*d - (2*a^2*b^2 + b^4)*c^2 - (a^2*b^2 +
2*b^4)*d^2)*cos(f*x + e)^2 - 2*(6*a^2*b^2*c*d - (2*a^3*b + a*b^3)*c^2 - (a^
3*b + 2*a*b^3)*d^2)*sin(f*x + e))*sqrt(a^2 - b^2)*arctan(-(a*sin(f*x + e) +
 b)/(sqrt(a^2 - b^2)*cos(f*x + e))) + ((4*a^4*b - 5*a^2*b^3 + b^5)*c^2 - 2*
(2*a^5 - a^3*b^2 - a*b^4)*c*d + 3*(a^4*b - a^2*b^3)*d^2)*cos(f*x + e))/((a^
6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*f*cos(f*x + e)^2 - 2*(a^7*b - 3*a^5*b^
3 + 3*a^3*b^5 - a*b^7)*f*sin(f*x + e) - (a^8 - 2*a^6*b^2 + 2*a^2*b^6 - b^8)
*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**2/(a+b*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.39004, size = 822, normalized size = 4.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^2/(a+b*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] ((2*a^2*c^2 + b^2*c^2 - 6*a*b*c*d + a^2*d^2 + 2*b^2*d^2)*(pi*floor(1/2*(f*x
 + e)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*f*x + 1/2*e) + b)/sqrt(a^2 - b^2
)))/((a^4 - 2*a^2*b^2 + b^4)*sqrt(a^2 - b^2)) + (5*a^3*b^2*c^2*tan(1/2*f*x
+ 1/2*e)^3 - 2*a*b^4*c^2*tan(1/2*f*x + 1/2*e)^3 - 6*a^4*b*c*d*tan(1/2*f*x +
 1/2*e)^3 + a^5*d^2*tan(1/2*f*x + 1/2*e)^3 + 2*a^3*b^2*d^2*tan(1/2*f*x + 1/
2*e)^3 + 4*a^4*b*c^2*tan(1/2*f*x + 1/2*e)^2 + 7*a^2*b^3*c^2*tan(1/2*f*x + 1
/2*e)^2 - 2*b^5*c^2*tan(1/2*f*x + 1/2*e)^2 - 4*a^5*c*d*tan(1/2*f*x + 1/2*e)
^2 - 10*a^3*b^2*c*d*tan(1/2*f*x + 1/2*e)^2 - 4*a*b^4*c*d*tan(1/2*f*x + 1/2*
e)^2 + 3*a^4*b*d^2*tan(1/2*f*x + 1/2*e)^2 + 6*a^2*b^3*d^2*tan(1/2*f*x + 1/2
*e)^2 + 11*a^3*b^2*c^2*tan(1/2*f*x + 1/2*e) - 2*a*b^4*c^2*tan(1/2*f*x + 1/2
*e) - 10*a^4*b*c*d*tan(1/2*f*x + 1/2*e) - 8*a^2*b^3*c*d*tan(1/2*f*x + 1/2*e
) - a^5*d^2*tan(1/2*f*x + 1/2*e) + 10*a^3*b^2*d^2*tan(1/2*f*x + 1/2*e) + 4*
a^4*b*c^2 - a^2*b^3*c^2 - 4*a^5*c*d - 2*a^3*b^2*c*d + 3*a^4*b*d^2)/((a^6 -
2*a^4*b^2 + a^2*b^4)*(a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e) +
 a)^2))/f
```

3.718 $\int \frac{c+d\sin(e+fx)}{(a+b\sin(e+fx))^3}\,dx$

Optimal. Leaf size=162

$$\frac{\left(2a^2c-3abd+b^2c\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{f\left(a^2-b^2\right)^{5/2}}+\frac{\left(a^2(-d)+3abc-2b^2d\right)\cos(e+fx)}{2f\left(a^2-b^2\right)^2(a+b\sin(e+fx))}+\frac{(bc-ad)\cos(e+fx)}{2f\left(a^2-b^2\right)(a+b\sin(e+fx))^2}$$

```
[Out] ((2*a^2*c + b^2*c - 3*a*b*d)*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2
]])/((a^2 - b^2)^(5/2)*f) + ((b*c - a*d)*Cos[e + f*x])/(2*(a^2 - b^2)*f*(a
+ b*Sin[e + f*x])^2) + ((3*a*b*c - a^2*d - 2*b^2*d)*Cos[e + f*x])/(2*(a^2 -
 b^2)^2*f*(a + b*Sin[e + f*x]))
```

Rubi [A] time = 0.174177, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2754, 12, 2660, 618, 204}

$$\frac{\left(2a^2c-3abd+b^2c\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{f\left(a^2-b^2\right)^{5/2}}+\frac{\left(a^2(-d)+3abc-2b^2d\right)\cos(e+fx)}{2f\left(a^2-b^2\right)^2(a+b\sin(e+fx))}+\frac{(bc-ad)\cos(e+fx)}{2f\left(a^2-b^2\right)(a+b\sin(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])/(a + b*Sin[e + f*x])^3,x]
```

```
[Out] ((2*a^2*c + b^2*c - 3*a*b*d)*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2
]])/((a^2 - b^2)^(5/2)*f) + ((b*c - a*d)*Cos[e + f*x])/(2*(a^2 - b^2)*f*(a
+ b*Sin[e + f*x])^2) + ((3*a*b*c - a^2*d - 2*b^2*d)*Cos[e + f*x])/(2*(a^2 -
 b^2)^2*f*(a + b*Sin[e + f*x]))
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{c+d\sin(e+fx)}{(a+b\sin(e+fx))^3}\,dx = \frac{(bc-ad)\cos(e+fx)}{2\left(a^2-b^2\right)f(a+b\sin(e+fx))^2} - \frac{\int \frac{-2(ac-bd)+(bc-ad)\sin(e+fx)}{(a+b\sin(e+fx))^2}\,dx}{2\left(a^2-b^2\right)}$$

$$= \frac{(bc-ad)\cos(e+fx)}{2\left(a^2-b^2\right)f(a+b\sin(e+fx))^2} + \frac{\left(3abc-a^2d-2b^2d\right)\cos(e+fx)}{2\left(a^2-b^2\right)^2 f(a+b\sin(e+fx))} + \frac{\int \frac{2a^2c+b^2c-3abd}{a+b\sin(e+fx)}\,dx}{2\left(a^2-b^2\right)^2}$$

$$= \frac{(bc-ad)\cos(e+fx)}{2\left(a^2-b^2\right)f(a+b\sin(e+fx))^2} + \frac{\left(3abc-a^2d-2b^2d\right)\cos(e+fx)}{2\left(a^2-b^2\right)^2 f(a+b\sin(e+fx))} + \frac{\left(2a^2c+b^2c-3abd\right)\int}{2\left(a^2-b\right.}$$

$$= \frac{(bc-ad)\cos(e+fx)}{2\left(a^2-b^2\right)f(a+b\sin(e+fx))^2} + \frac{\left(3abc-a^2d-2b^2d\right)\cos(e+fx)}{2\left(a^2-b^2\right)^2 f(a+b\sin(e+fx))} + \frac{\left(2a^2c+b^2c-3abd\right)S}{}$$

$$= \frac{(bc-ad)\cos(e+fx)}{2\left(a^2-b^2\right)f(a+b\sin(e+fx))^2} + \frac{\left(3abc-a^2d-2b^2d\right)\cos(e+fx)}{2\left(a^2-b^2\right)^2 f(a+b\sin(e+fx))} - \frac{\left(2\left(2a^2c+b^2c-3abd\right.\right.}{}$$

$$= \frac{\left(2a^2c+b^2c-3abd\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}f} + \frac{(bc-ad)\cos(e+fx)}{2\left(a^2-b^2\right)f(a+b\sin(e+fx))^2} + \frac{\left(3abc-a^2\right.}{2\left(a^2-b^2\right.}$$

Mathematica [A] time = 0.619793, size = 157, normalized size = 0.97

$$\frac{\frac{2\left(2a^2c-3abd+b^2c\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}} - \frac{\left(a^2d-3abc+2b^2d\right)\cos(e+fx)}{(a-b)^2(a+b)^2(a+b\sin(e+fx))} + \frac{(bc-ad)\cos(e+fx)}{(a-b)(a+b)(a+b\sin(e+fx))^2}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])/(a + b*Sin[e + f*x])^3,x]
```

```
[Out] ((2*(2*a^2*c + b^2*c - 3*a*b*d)*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 -
b^2]])/(a^2 - b^2)^(5/2) + ((b*c - a*d)*Cos[e + f*x])/((a - b)*(a + b)*(a +
 b*Sin[e + f*x])^2) - ((-3*a*b*c + a^2*d + 2*b^2*d)*Cos[e + f*x])/((a - b)^
2*(a + b)^2*(a + b*Sin[e + f*x])))/(2*f)
```

Maple [B] time = 0.087, size = 1291, normalized size = 8.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))/(a+b*sin(f*x+e))^3,x)
```

```
[Out] -3/f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2*b*a^2/(a^4-2*a^2*b
^2+b^4)*tan(1/2*f*x+1/2*e)^3*d+5/f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/
2*e)*b+a)^2*b^2*a/(a^4-2*a^2*b^2+b^4)*tan(1/2*f*x+1/2*e)^3*c-2/f/(tan(1/2*f
*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2*b^4/a/(a^4-2*a^2*b^2+b^4)*tan(1/2
*f*x+1/2*e)^3*c-2/f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^
4-2*a^2*b^2+b^4)*a^3*tan(1/2*f*x+1/2*e)^2*d+4/f/(tan(1/2*f*x+1/2*e)^2*a+2*t
an(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^2*tan(1/2*f*x+1/2*e)^2*b*c-5
/f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*
a*tan(1/2*f*x+1/2*e)^2*b^2*d+7/f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*
e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*tan(1/2*f*x+1/2*e)^2*b^3*c-2/f/(tan(1/2*f*x+1
/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)/a*tan(1/2*f*x+1/2
*e)^2*b^4*d-2/f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*
a^2*b^2+b^4)/a^2*tan(1/2*f*x+1/2*e)^2*b^5*c-5/f/(tan(1/2*f*x+1/2*e)^2*a+2*t
an(1/2*f*x+1/2*e)*b+a)^2*b*a^2/(a^4-2*a^2*b^2+b^4)*tan(1/2*f*x+1/2*e)*d+11/
f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2*b^2*a/(a^4-2*a^2*b^2+
b^4)*tan(1/2*f*x+1/2*e)*c-4/f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*
b+a)^2*b^3/(a^4-2*a^2*b^2+b^4)*tan(1/2*f*x+1/2*e)*d-2/f/(tan(1/2*f*x+1/2*e)
^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2*b^4/a/(a^4-2*a^2*b^2+b^4)*tan(1/2*f*x+1/2*
e)*c-2/f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2
+b^4)*a^3*d+4/f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*
a^2*b^2+b^4)*a^2*b*c-1/f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^
2/(a^4-2*a^2*b^2+b^4)*a*b^2*d-1/f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2
*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*b^3*c+2/f/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2
)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*a^2*c-3/f/(a^4-2
*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-
b^2)^(1/2))*a*b*d+1/f/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*t
an(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*b^2*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))/(a+b*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.84008, size = 1696, normalized size = 10.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))/(a+b*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*(3*(a^3*b^2 - a*b^4)*c - (a^4*b + a^2*b^3 - 2*b^5)*d)*cos(f*x + e)
*sin(f*x + e) + ((3*a*b^3*d - (2*a^2*b^2 + b^4)*c)*cos(f*x + e)^2 + (2*a^4
+ 3*a^2*b^2 + b^4)*c - 3*(a^3*b + a*b^3)*d - 2*(3*a^2*b^2*d - (2*a^3*b + a*
b^3)*c)*sin(f*x + e))*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(f*x + e)^2 -
 2*a*b*sin(f*x + e) - a^2 - b^2 - 2*(a*cos(f*x + e)*sin(f*x + e) + b*cos(f*
x + e))*sqrt(-a^2 + b^2))/(b^2*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 -
```

```
b^2)) + 2*((4*a^4*b - 5*a^2*b^3 + b^5)*c - (2*a^5 - a^3*b^2 - a*b^4)*d)*cos
(f*x + e))/((a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*f*cos(f*x + e)^2 - 2*(a
^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*f*sin(f*x + e) - (a^8 - 2*a^6*b^2 + 2
*a^2*b^6 - b^8)*f), -1/2*((3*(a^3*b^2 - a*b^4)*c - (a^4*b + a^2*b^3 - 2*b^5
)*d)*cos(f*x + e)*sin(f*x + e) - ((3*a*b^3*d - (2*a^2*b^2 + b^4)*c)*cos(f*x
 + e)^2 + (2*a^4 + 3*a^2*b^2 + b^4)*c - 3*(a^3*b + a*b^3)*d - 2*(3*a^2*b^2*
d - (2*a^3*b + a*b^3)*c)*sin(f*x + e))*sqrt(a^2 - b^2)*arctan(-(a*sin(f*x +
 e) + b)/(sqrt(a^2 - b^2)*cos(f*x + e))) + ((4*a^4*b - 5*a^2*b^3 + b^5)*c -
 (2*a^5 - a^3*b^2 - a*b^4)*d)*cos(f*x + e))/((a^6*b^2 - 3*a^4*b^4 + 3*a^2*b
^6 - b^8)*f*cos(f*x + e)^2 - 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*f*si
n(f*x + e) - (a^8 - 2*a^6*b^2 + 2*a^2*b^6 - b^8)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))/(a+b*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.37491, size = 579, normalized size = 3.57

$$\frac{\left(2a^2c+b^2c-3abd\right)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^4-2a^2b^2+b^4\right)\sqrt{a^2-b^2}}+\frac{5a^3b^2c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-2ab^4c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-3a^4bd\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+4a^4bc\tan\left(\frac{1}{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))/(a+b*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] ((2*a^2*c + b^2*c - 3*a*b*d)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(a) + arc
tan((a*tan(1/2*f*x + 1/2*e) + b)/sqrt(a^2 - b^2)))/((a^4 - 2*a^2*b^2 + b^4)
*sqrt(a^2 - b^2)) + (5*a^3*b^2*c*tan(1/2*f*x + 1/2*e)^3 - 2*a*b^4*c*tan(1/2
*f*x + 1/2*e)^3 - 3*a^4*b*d*tan(1/2*f*x + 1/2*e)^3 + 4*a^4*b*c*tan(1/2*f*x
+ 1/2*e)^2 + 7*a^2*b^3*c*tan(1/2*f*x + 1/2*e)^2 - 2*b^5*c*tan(1/2*f*x + 1/2
*e)^2 - 2*a^5*d*tan(1/2*f*x + 1/2*e)^2 - 5*a^3*b^2*d*tan(1/2*f*x + 1/2*e)^2
 - 2*a*b^4*d*tan(1/2*f*x + 1/2*e)^2 + 11*a^3*b^2*c*tan(1/2*f*x + 1/2*e) - 2
*a*b^4*c*tan(1/2*f*x + 1/2*e) - 5*a^4*b*d*tan(1/2*f*x + 1/2*e) - 4*a^2*b^3*
d*tan(1/2*f*x + 1/2*e) + 4*a^4*b*c - a^2*b^3*c - 2*a^5*d - a^3*b^2*d)/((a^6
 - 2*a^4*b^2 + a^2*b^4)*(a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e
) + a)^2))/f
```

3.719 $\int \frac{1}{(a+b\sin(e+fx))^3}\,dx$

Optimal. Leaf size=131

$$\frac{\left(2a^2+b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{f\left(a^2-b^2\right)^{5/2}}+\frac{3ab\cos(e+fx)}{2f\left(a^2-b^2\right)^2(a+b\sin(e+fx))}+\frac{b\cos(e+fx)}{2f\left(a^2-b^2\right)(a+b\sin(e+fx))^2}$$

```
[Out] ((2*a^2 + b^2)*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^
2)^(5/2)*f) + (b*Cos[e + f*x])/(2*(a^2 - b^2)*f*(a + b*Sin[e + f*x])^2) + (
3*a*b*Cos[e + f*x])/(2*(a^2 - b^2)^2*f*(a + b*Sin[e + f*x]))
```

Rubi [A] time = 0.113618, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {2664, 2754, 12, 2660, 618, 204}

$$\frac{\left(2a^2+b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{f\left(a^2-b^2\right)^{5/2}}+\frac{3ab\cos(e+fx)}{2f\left(a^2-b^2\right)^2(a+b\sin(e+fx))}+\frac{b\cos(e+fx)}{2f\left(a^2-b^2\right)(a+b\sin(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^(-3),x]
```

```
[Out] ((2*a^2 + b^2)*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^
2)^(5/2)*f) + (b*Cos[e + f*x])/(2*(a^2 - b^2)*f*(a + b*Sin[e + f*x])^2) + (
3*a*b*Cos[e + f*x])/(2*(a^2 - b^2)^2*f*(a + b*Sin[e + f*x]))
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
```

```
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(a+b\sin(e+fx))^3}\,dx = \frac{b\cos(e+fx)}{2\left(a^2-b^2\right)f(a+b\sin(e+fx))^2} - \frac{\int \frac{-2a+b\sin(e+fx)}{(a+b\sin(e+fx))^2}\,dx}{2\left(a^2-b^2\right)}$$

$$= \frac{b\cos(e+fx)}{2\left(a^2-b^2\right)f(a+b\sin(e+fx))^2} + \frac{3ab\cos(e+fx)}{2\left(a^2-b^2\right)^2 f(a+b\sin(e+fx))} + \frac{\int \frac{2a^2+b^2}{a+b\sin(e+fx)}\,dx}{2\left(a^2-b^2\right)^2}$$

$$= \frac{b\cos(e+fx)}{2\left(a^2-b^2\right)f(a+b\sin(e+fx))^2} + \frac{3ab\cos(e+fx)}{2\left(a^2-b^2\right)^2 f(a+b\sin(e+fx))} + \frac{\left(2a^2+b^2\right)\int \frac{1}{a+b\sin(e+}}{2\left(a^2-b^2\right)^2}$$

$$= \frac{b\cos(e+fx)}{2\left(a^2-b^2\right)f(a+b\sin(e+fx))^2} + \frac{3ab\cos(e+fx)}{2\left(a^2-b^2\right)^2 f(a+b\sin(e+fx))} + \frac{\left(2a^2+b^2\right)\mathrm{Subst}\left(\int\right.}{}$$

$$= \frac{b\cos(e+fx)}{2\left(a^2-b^2\right)f(a+b\sin(e+fx))^2} + \frac{3ab\cos(e+fx)}{2\left(a^2-b^2\right)^2 f(a+b\sin(e+fx))} - \frac{\left(2\left(2a^2+b^2\right)\right)\mathrm{Subst}}{}$$

$$= \frac{\left(2a^2+b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2} f} + \frac{b\cos(e+fx)}{2\left(a^2-b^2\right)f(a+b\sin(e+fx))^2} + \frac{3ab\cos(e+}{2\left(a^2-b^2\right)^2 f(a+b}$$

Mathematica [A] time = 0.382713, size = 114, normalized size = 0.87

$$\frac{\frac{2\left(2a^2+b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}} + \frac{b\cos(e+fx)\left(4a^2+3ab\sin(e+fx)-b^2\right)}{(a-b)^2(a+b)^2(a+b\sin(e+fx))^2}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^(-3),x]
```

```
[Out] ((2*(2*a^2 + b^2)*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/(a^2 -
b^2)^(5/2) + (b*Cos[e + f*x]*(4*a^2 - b^2 + 3*a*b*Sin[e + f*x]))/((a - b)^2
*(a + b)^2*(a + b*Sin[e + f*x])^2))/(2*f)
```

Maple [B] time = 0.075, size = 705, normalized size = 5.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e))^3,x)
```

```
[Out] 5/f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2*b^2/(a^4-2*a^2*b^2+
b^4)*a*tan(1/2*f*x+1/2*e)^3-2/f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e
)*b+a)^2*b^4/(a^4-2*a^2*b^2+b^4)/a*tan(1/2*f*x+1/2*e)^3+4/f/(tan(1/2*f*x+1/
2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2*b/(a^4-2*a^2*b^2+b^4)*a^2*tan(1/2*f*x+
1/2*e)^2+7/f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2*b^3/(a^4-2
*a^2*b^2+b^4)*tan(1/2*f*x+1/2*e)^2-2/f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*
x+1/2*e)*b+a)^2*b^5/(a^4-2*a^2*b^2+b^4)/a^2*tan(1/2*f*x+1/2*e)^2+11/f/(tan(
1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2*b^2*a/(a^4-2*a^2*b^2+b^4)*ta
n(1/2*f*x+1/2*e)-2/f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2*b^
4/a/(a^4-2*a^2*b^2+b^4)*tan(1/2*f*x+1/2*e)+4/f/(tan(1/2*f*x+1/2*e)^2*a+2*ta
n(1/2*f*x+1/2*e)*b+a)^2*b/(a^4-2*a^2*b^2+b^4)*a^2-1/f/(tan(1/2*f*x+1/2*e)^2
*a+2*tan(1/2*f*x+1/2*e)*b+a)^2*b^3/(a^4-2*a^2*b^2+b^4)+2/f/(a^4-2*a^2*b^2+b
^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2)
)*a^2+1/f/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*f*x+1
/2*e)+2*b)/(a^2-b^2)^(1/2))*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.65302, size = 1346, normalized size = 10.27

$$\left[-\frac{6\left(a^{3}b^{2}-ab^{4}\right)\cos\left(fx+e\right)\sin\left(fx+e\right)-\left(2\,a^{4}+3\,a^{2}b^{2}+b^{4}-\left(2\,a^{2}b^{2}+b^{4}\right)\cos\left(fx+e\right)^{2}+2\left(2\,a^{3}b+ab^{3}\right)\sin\left(fx\cdots\right.\right.}{4\left(\left(a^{6}b^{2}-3\,a^{4}b^{4}+3\,a^{2}b^{6}-b^{8}\right)f\cos\left(fx+e\right)^{2}-2\left(a\cdots\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(6*(a^3*b^2 - a*b^4)*cos(f*x + e)*sin(f*x + e) - (2*a^4 + 3*a^2*b^2 +
 b^4 - (2*a^2*b^2 + b^4)*cos(f*x + e)^2 + 2*(2*a^3*b + a*b^3)*sin(f*x + e))
*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) -
a^2 - b^2 + 2*(a*cos(f*x + e)*sin(f*x + e) + b*cos(f*x + e))*sqrt(-a^2 + b^
2))/(b^2*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2)) + 2*(4*a^4*b - 5
*a^2*b^3 + b^5)*cos(f*x + e))/((a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*f*co
s(f*x + e)^2 - 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*f*sin(f*x + e) - (
a^8 - 2*a^6*b^2 + 2*a^2*b^6 - b^8)*f), -1/2*(3*(a^3*b^2 - a*b^4)*cos(f*x +
```

```
e)*sin(f*x + e) - (2*a^4 + 3*a^2*b^2 + b^4 - (2*a^2*b^2 + b^4)*cos(f*x + e)
^2 + 2*(2*a^3*b + a*b^3)*sin(f*x + e))*sqrt(a^2 - b^2)*arctan(-(a*sin(f*x +
 e) + b)/(sqrt(a^2 - b^2)*cos(f*x + e))) + (4*a^4*b - 5*a^2*b^3 + b^5)*cos(
f*x + e))/((a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*f*cos(f*x + e)^2 - 2*(a^
7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*f*sin(f*x + e) - (a^8 - 2*a^6*b^2 + 2*
a^2*b^6 - b^8)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.2564, size = 383, normalized size = 2.92

$$\frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+b}{\sqrt{a^2-b^2}}\right)\right)(2a^2+b^2)}{(a^4-2a^2b^2+b^4)\sqrt{a^2-b^2}}+\frac{5a^3b^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-2ab^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+4a^4b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+7a^2b^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-2}{(a^6-2a^4b^2+a^2b^4)\left(a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2\right.}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] ((pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*f*x + 1/2*e)
+ b)/sqrt(a^2 - b^2)))*(2*a^2 + b^2)/((a^4 - 2*a^2*b^2 + b^4)*sqrt(a^2 - b^
2)) + (5*a^3*b^2*tan(1/2*f*x + 1/2*e)^3 - 2*a*b^4*tan(1/2*f*x + 1/2*e)^3 +
4*a^4*b*tan(1/2*f*x + 1/2*e)^2 + 7*a^2*b^3*tan(1/2*f*x + 1/2*e)^2 - 2*b^5*t
an(1/2*f*x + 1/2*e)^2 + 11*a^3*b^2*tan(1/2*f*x + 1/2*e) - 2*a*b^4*tan(1/2*f
*x + 1/2*e) + 4*a^4*b - a^2*b^3)/((a^6 - 2*a^4*b^2 + a^2*b^4)*(a*tan(1/2*f*
x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e) + a)^2))/f
```

3.720 $\int \frac{1}{(a+b\sin(e+fx))^3(c+d\sin(e+fx))}\,dx$

Optimal. Leaf size=285

$$-\frac{b\left(-a^2b^2\left(2c^2-5d^2\right)+6a^3bcd-6a^4d^2-b^4\left(c^2+2d^2\right)\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{f\left(a^2-b^2\right)^{5/2}(bc-ad)^3}+\frac{b^2\left(-5a^2d+3abc+2b^2d\right)\cos(e}{2f\left(a^2-b^2\right)^2(bc-ad)^2(a+b\sin(e}$$

```
[Out] -((b*(6*a^3*b*c*d - 6*a^4*d^2 - a^2*b^2*(2*c^2 - 5*d^2) - b^4*(c^2 + 2*d^2)
)*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(5/2)*(b*c
 - a*d)^3*f)) - (2*d^3*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/((
b*c - a*d)^3*Sqrt[c^2 - d^2]*f) + (b^2*Cos[e + f*x])/(2*(a^2 - b^2)*(b*c -
a*d)*f*(a + b*Sin[e + f*x])^2) + (b^2*(3*a*b*c - 5*a^2*d + 2*b^2*d)*Cos[e +
 f*x])/(2*(a^2 - b^2)^2*(b*c - a*d)^2*f*(a + b*Sin[e + f*x]))
```

Rubi [A] time = 1.03696, antiderivative size = 285, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {2802, 3055, 3001, 2660, 618, 204}

$$-\frac{b\left(-a^2b^2\left(2c^2-5d^2\right)+6a^3bcd-6a^4d^2-b^4\left(c^2+2d^2\right)\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{f\left(a^2-b^2\right)^{5/2}(bc-ad)^3}+\frac{b^2\left(-5a^2d+3abc+2b^2d\right)\cos(e}{2f\left(a^2-b^2\right)^2(bc-ad)^2(a+b\sin(e}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sin[e + f*x])^3*(c + d*Sin[e + f*x])),x]
```


```
[Out] -((b*(6*a^3*b*c*d - 6*a^4*d^2 - a^2*b^2*(2*c^2 - 5*d^2) - b^4*(c^2 + 2*d^2)
)*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(5/2)*(b*c
 - a*d)^3*f)) - (2*d^3*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/((
b*c - a*d)^3*Sqrt[c^2 - d^2]*f) + (b^2*Cos[e + f*x])/(2*(a^2 - b^2)*(b*c -
a*d)*f*(a + b*Sin[e + f*x])^2) + (b^2*(3*a*b*c - 5*a^2*d + 2*b^2*d)*Cos[e +
 f*x])/(2*(a^2 - b^2)^2*(b*c - a*d)^2*f*(a + b*Sin[e + f*x]))
```


Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0]))))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
```

```
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(a+b\sin(e+fx))^3(c+d\sin(e+fx))}\,dx = \frac{b^2\cos(e+fx)}{2\left(a^2-b^2\right)(bc-ad)f(a+b\sin(e+fx))^2} - \frac{\int \frac{-2\left(abc-a^2d+b^2d\right)+b(bc-2ad)\,\mathrm{si}}{(a+b\sin(e+fx))^2(c+}}{2\left(a^2-b^2\right)(}$$

$$= \frac{b^2\cos(e+fx)}{2\left(a^2-b^2\right)(bc-ad)f(a+b\sin(e+fx))^2} + \frac{b^2\left(3abc-5a^2d+2b^2d\right.}{2\left(a^2-b^2\right)^2(bc-ad)^2f(a}$$

$$= \frac{b^2\cos(e+fx)}{2\left(a^2-b^2\right)(bc-ad)f(a+b\sin(e+fx))^2} + \frac{b^2\left(3abc-5a^2d+2b^2d\right.}{2\left(a^2-b^2\right)^2(bc-ad)^2f(a}$$

$$= \frac{b^2\cos(e+fx)}{2\left(a^2-b^2\right)(bc-ad)f(a+b\sin(e+fx))^2} + \frac{b^2\left(3abc-5a^2d+2b^2d\right.}{2\left(a^2-b^2\right)^2(bc-ad)^2f(a}$$

$$= \frac{b^2\cos(e+fx)}{2\left(a^2-b^2\right)(bc-ad)f(a+b\sin(e+fx))^2} + \frac{b^2\left(3abc-5a^2d+2b^2d\right.}{2\left(a^2-b^2\right)^2(bc-ad)^2f(a}$$

$$= -\frac{b\left(6a^3bcd-6a^4d^2-a^2b^2\left(2c^2-5d^2\right)-b^4\left(c^2+2d^2\right)\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}\right.}{\sqrt{a^2}}\right.}{\left(a^2-b^2\right)^{5/2}(bc-ad)^3f}$$

Mathematica [A] time = 2.2313, size = 275, normalized size = 0.96

$$\frac{-\frac{2b\left(a^2b^2\left(2c^2-5d^2\right)-6a^3bcd+6a^4d^2+b^4\left(c^2+2d^2\right)\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}(ad-bc)^3} + \frac{b^2\left(-5a^2d+3abc+2b^2d\right)\cos(e+fx)}{(a-b)^2(a+b)^2(bc-ad)^2(a+b\sin(e+fx))} - \frac{b^2\cos(e+fx)}{(a-b)(a+b)(ad-bc)(a+b\sin(e+fx))^2}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*Sin[e + f*x])^3*(c + d*Sin[e + f*x])),x]
```

```
[Out] ((-2*b*(-6*a^3*b*c*d + 6*a^4*d^2 + a^2*b^2*(2*c^2 - 5*d^2) + b^4*(c^2 + 2*d
^2))*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(5/2)*(
-(b*c) + a*d)^3) + (4*d^3*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])
/((-(b*c) + a*d)^3*Sqrt[c^2 - d^2]) - (b^2*Cos[e + f*x])/((a - b)*(a + b)*(
-(b*c) + a*d)*(a + b*Sin[e + f*x])^2) + (b^2*(3*a*b*c - 5*a^2*d + 2*b^2*d)*
Cos[e + f*x])/((a - b)^2*(a + b)^2*(b*c - a*d)^2*(a + b*Sin[e + f*x])))/(2*
f)
```

Maple [B] time = 0.136, size = 2644, normalized size = 9.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e))^3/(c+d*sin(f*x+e)),x)
```

```
[Out] 2/f*d^3/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/(c^2-d^2)^(1/2)*arcta
n(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))+1/f*b^6/(a*d-b*c)^3/(ta
n(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*c^2-2/
f*b^5/(a*d-b*c)^3/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1
```

```
/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*d^2-6/f*b^2/(a*d-b*c)^3/(tan(1/2*f*x+1/
2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^4*d^2-4/f*b^4/(a
*d-b*c)^3/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^
2+b^4)*c^2*a^2+3/f*b^4/(a*d-b*c)^3/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/
2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^2*d^2-7/f*b^6/(a*d-b*c)^3/(tan(1/2*f*x+1/
2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*tan(1/2*f*x+1/2*e)
^2*c^2+6/f*b^6/(a*d-b*c)^3/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a
)^2/(a^4-2*a^2*b^2+b^4)*tan(1/2*f*x+1/2*e)^2*d^2-1/f*b^5/(a*d-b*c)^3/(a^4-2
*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-
b^2)^(1/2))*c^2+4/f*b^5/(a*d-b*c)^3/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1
/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a*tan(1/2*f*x+1/2*e)^3*d^2+2/f*b^7/(a*d-b*
c)^3/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4
)/a*tan(1/2*f*x+1/2*e)^3*c^2-6/f*b^2/(a*d-b*c)^3/(tan(1/2*f*x+1/2*e)^2*a+2*
tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^4*tan(1/2*f*x+1/2*e)^2*d^2-
4/f*b^4/(a*d-b*c)^3/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^
4-2*a^2*b^2+b^4)*a^2*tan(1/2*f*x+1/2*e)^2*c^2-9/f*b^4/(a*d-b*c)^3/(tan(1/2*
f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^2*tan(1/2*
f*x+1/2*e)^2*d^2+2/f*b^8/(a*d-b*c)^3/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+
1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)/a^2*tan(1/2*f*x+1/2*e)^2*c^2-17/f*b^3/(a*
d-b*c)^3/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2*a^3/(a^4-2*a^2
*b^2+b^4)*tan(1/2*f*x+1/2*e)*d^2-11/f*b^5/(a*d-b*c)^3/(tan(1/2*f*x+1/2*e)^2
*a+2*tan(1/2*f*x+1/2*e)*b+a)^2*a/(a^4-2*a^2*b^2+b^4)*tan(1/2*f*x+1/2*e)*c^2
+8/f*b^5/(a*d-b*c)^3/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2*a/
(a^4-2*a^2*b^2+b^4)*tan(1/2*f*x+1/2*e)*d^2+2/f*b^7/(a*d-b*c)^3/(tan(1/2*f*x
+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/a/(a^4-2*a^2*b^2+b^4)*tan(1/2*f*x+1
/2*e)*c^2+10/f*b^3/(a*d-b*c)^3/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)
*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^3*c*d-6/f*b/(a*d-b*c)^3/(a^4-2*a^2*b^2+b^4)/(
a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*a^4
*d^2-2/f*b^3/(a*d-b*c)^3/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*
a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*c^2*a^2-4/f*b^5/(a*d-b*c)^3/(tan
(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a*c*d-6
/f*b^6/(a*d-b*c)^3/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4
-2*a^2*b^2+b^4)*tan(1/2*f*x+1/2*e)^3*c*d-10/f*b^6/(a*d-b*c)^3/(tan(1/2*f*x+
1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*tan(1/2*f*x+1/2*
e)*c*d-7/f*b^3/(a*d-b*c)^3/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a
)^2/(a^4-2*a^2*b^2+b^4)*a^3*tan(1/2*f*x+1/2*e)^3*d^2-5/f*b^5/(a*d-b*c)^3/(t
an(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a*tan
(1/2*f*x+1/2*e)^3*c^2+5/f*b^3/(a*d-b*c)^3/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/
2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*a^2*d^2+12/f*b^
4/(a*d-b*c)^3/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^
2*b^2+b^4)*a^2*tan(1/2*f*x+1/2*e)^3*c*d+6/f*b^2/(a*d-b*c)^3/(a^4-2*a^2*b^2+
b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2
))*a^3*c*d+10/f*b^3/(a*d-b*c)^3/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e
)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^3*tan(1/2*f*x+1/2*e)^2*c*d+16/f*b^5/(a*d-b*c
)^3/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)
*a*tan(1/2*f*x+1/2*e)^2*c*d-8/f*b^7/(a*d-b*c)^3/(tan(1/2*f*x+1/2*e)^2*a+2*t
an(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)/a*tan(1/2*f*x+1/2*e)^2*c*d+28/
f*b^4/(a*d-b*c)^3/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2*a^2/(
a^4-2*a^2*b^2+b^4)*tan(1/2*f*x+1/2*e)*c*d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^3/(c+d*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^3/(c+d*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))**3/(c+d*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.5606, size = 1064, normalized size = 3.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^3/(c+d*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] -(2*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*
e) + d)/sqrt(c^2 - d^2)))*d^3/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a
^3*d^3)*sqrt(c^2 - d^2)) - (2*a^2*b^3*c^2 + b^5*c^2 - 6*a^3*b^2*c*d + 6*a^4
*b*d^2 - 5*a^2*b^3*d^2 + 2*b^5*d^2)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(a
) + arctan((a*tan(1/2*f*x + 1/2*e) + b)/sqrt(a^2 - b^2)))/((a^4*b^3*c^3 - 2
*a^2*b^5*c^3 + b^7*c^3 - 3*a^5*b^2*c^2*d + 6*a^3*b^4*c^2*d - 3*a*b^6*c^2*d
+ 3*a^6*b*c*d^2 - 6*a^4*b^3*c*d^2 + 3*a^2*b^5*c*d^2 - a^7*d^3 + 2*a^5*b^2*d
^3 - a^3*b^4*d^3)*sqrt(a^2 - b^2)) - (5*a^3*b^4*c*tan(1/2*f*x + 1/2*e)^3 -
2*a*b^6*c*tan(1/2*f*x + 1/2*e)^3 - 7*a^4*b^3*d*tan(1/2*f*x + 1/2*e)^3 + 4*a
^2*b^5*d*tan(1/2*f*x + 1/2*e)^3 + 4*a^4*b^3*c*tan(1/2*f*x + 1/2*e)^2 + 7*a^
2*b^5*c*tan(1/2*f*x + 1/2*e)^2 - 2*b^7*c*tan(1/2*f*x + 1/2*e)^2 - 6*a^5*b^2
*d*tan(1/2*f*x + 1/2*e)^2 - 9*a^3*b^4*d*tan(1/2*f*x + 1/2*e)^2 + 6*a*b^6*d*
tan(1/2*f*x + 1/2*e)^2 + 11*a^3*b^4*c*tan(1/2*f*x + 1/2*e) - 2*a*b^6*c*tan(
1/2*f*x + 1/2*e) - 17*a^4*b^3*d*tan(1/2*f*x + 1/2*e) + 8*a^2*b^5*d*tan(1/2*
f*x + 1/2*e) + 4*a^4*b^3*c - a^2*b^5*c - 6*a^5*b^2*d + 3*a^3*b^4*d)/((a^6*b
^2*c^2 - 2*a^4*b^4*c^2 + a^2*b^6*c^2 - 2*a^7*b*c*d + 4*a^5*b^3*c*d - 2*a^3*
b^5*c*d + a^8*d^2 - 2*a^6*b^2*d^2 + a^4*b^4*d^2)*(a*tan(1/2*f*x + 1/2*e)^2
+ 2*b*tan(1/2*f*x + 1/2*e) + a)^2))/f
```

3.721 $\int \frac{1}{(a+b\sin(e+fx))^3(c+d\sin(e+fx))^2}\,dx$

Optimal. Leaf size=454

$$-\frac{b^2\left(-a^2b^2\left(2c^2-15d^2\right)+8a^3bcd-12a^4d^2-2ab^3cd-b^4\left(c^2+6d^2\right)\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{f\left(a^2-b^2\right)^{5/2}(bc-ad)^4}-\frac{d\left(a^2b^2d\left(7c^2-11d^2\right)+2\right.}{2f\left(a^2-b^2\right.}$$

```
[Out] -((b^2*(8*a^3*b*c*d - 2*a*b^3*c*d - 12*a^4*d^2 - a^2*b^2*(2*c^2 - 15*d^2) -
 b^4*(c^2 + 6*d^2))*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/((a^2
 - b^2)^(5/2)*(b*c - a*d)^4*f)) - (2*d^3*(4*b*c^2 - a*c*d - 3*b*d^2)*ArcTan
[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/((b*c - a*d)^4*(c^2 - d^2)^(3/2
)*f) - (d*(2*a^4*d^3 + a^2*b^2*d*(7*c^2 - 11*d^2) - 2*b^4*d*(2*c^2 - 3*d^2)
 - 3*a*b^3*c*(c^2 - d^2))*Cos[e + f*x])/(2*(a^2 - b^2)^2*(b*c - a*d)^3*(c^2
 - d^2)*f*(c + d*Sin[e + f*x])) + (b^2*Cos[e + f*x])/(2*(a^2 - b^2)*(b*c -
a*d)*f*(a + b*Sin[e + f*x])^2*(c + d*Sin[e + f*x])) + (3*b^2*(a*b*c - 2*a^2
*d + b^2*d)*Cos[e + f*x])/(2*(a^2 - b^2)^2*(b*c - a*d)^2*f*(a + b*Sin[e + f
*x])*(c + d*Sin[e + f*x]))
```

Rubi [A] time = 2.43873, antiderivative size = 454, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {2802, 3055, 3001, 2660, 618, 204}

$$-\frac{b^2\left(-a^2b^2\left(2c^2-15d^2\right)+8a^3bcd-12a^4d^2-2ab^3cd-b^4\left(c^2+6d^2\right)\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{f\left(a^2-b^2\right)^{5/2}(bc-ad)^4}-\frac{d\left(a^2b^2d\left(7c^2-11d^2\right)+2\right.}{2f\left(a^2-b^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^2),x]
```


```
[Out] -((b^2*(8*a^3*b*c*d - 2*a*b^3*c*d - 12*a^4*d^2 - a^2*b^2*(2*c^2 - 15*d^2) -
 b^4*(c^2 + 6*d^2))*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/((a^2
 - b^2)^(5/2)*(b*c - a*d)^4*f)) - (2*d^3*(4*b*c^2 - a*c*d - 3*b*d^2)*ArcTan
[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/((b*c - a*d)^4*(c^2 - d^2)^(3/2
)*f) - (d*(2*a^4*d^3 + a^2*b^2*d*(7*c^2 - 11*d^2) - 2*b^4*d*(2*c^2 - 3*d^2)
 - 3*a*b^3*c*(c^2 - d^2))*Cos[e + f*x])/(2*(a^2 - b^2)^2*(b*c - a*d)^3*(c^2
 - d^2)*f*(c + d*Sin[e + f*x])) + (b^2*Cos[e + f*x])/(2*(a^2 - b^2)*(b*c -
a*d)*f*(a + b*Sin[e + f*x])^2*(c + d*Sin[e + f*x])) + (3*b^2*(a*b*c - 2*a^2
*d + b^2*d)*Cos[e + f*x])/(2*(a^2 - b^2)^2*(b*c - a*d)^2*f*(a + b*Sin[e + f
*x])*(c + d*Sin[e + f*x]))
```


Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(a+b\sin(e+fx))^3(c+d\sin(e+fx))^2}\,dx = \frac{b^2\cos(e+fx)}{2\left(a^2-b^2\right)(bc-ad)f(a+b\sin(e+fx))^2(c+d\sin(e+fx))} - \frac{\int \frac{-2abc+2a}{}}{}$$

$$= \frac{b^2\cos(e+fx)}{2\left(a^2-b^2\right)(bc-ad)f(a+b\sin(e+fx))^2(c+d\sin(e+fx))} + \frac{}{2\left(a^2-b^2\right.}$$

$$= -\frac{d\left(2a^4d^3+a^2b^2d\left(7c^2-11d^2\right)-2b^4d\left(2c^2-3d^2\right)-3ab^3c\left(c^2-d^2\right)\right)\cos}{2\left(a^2-b^2\right)^2(bc-ad)^3\left(c^2-d^2\right)f(c+d\sin(e+fx))}$$

$$= -\frac{d\left(2a^4d^3+a^2b^2d\left(7c^2-11d^2\right)-2b^4d\left(2c^2-3d^2\right)-3ab^3c\left(c^2-d^2\right)\right)\cos}{2\left(a^2-b^2\right)^2(bc-ad)^3\left(c^2-d^2\right)f(c+d\sin(e+fx))}$$

$$= -\frac{d\left(2a^4d^3+a^2b^2d\left(7c^2-11d^2\right)-2b^4d\left(2c^2-3d^2\right)-3ab^3c\left(c^2-d^2\right)\right)\cos}{2\left(a^2-b^2\right)^2(bc-ad)^3\left(c^2-d^2\right)f(c+d\sin(e+fx))}$$

$$= -\frac{d\left(2a^4d^3+a^2b^2d\left(7c^2-11d^2\right)-2b^4d\left(2c^2-3d^2\right)-3ab^3c\left(c^2-d^2\right)\right)\cos}{2\left(a^2-b^2\right)^2(bc-ad)^3\left(c^2-d^2\right)f(c+d\sin(e+fx))}$$

$$= -\frac{b^2\left(8a^3bcd-2ab^3cd-12a^4d^2-a^2b^2\left(2c^2-15d^2\right)-b^4\left(c^2+6d^2\right)\right)\tan^{-}}{\left(a^2-b^2\right)^{5/2}(bc-ad)^4f}$$

Mathematica [A] time = 5.31827, size = 346, normalized size = 0.76

$$\frac{\frac{2b^2\left(a^2b^2\left(2c^2-15d^2\right)-8a^3bcd+12a^4d^2+2ab^3cd+b^4\left(c^2+6d^2\right)\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}(bc-ad)^4}+\frac{b^3\left(7a^2d-3abc-4b^2d\right)\cos(e+fx)}{(a-b)^2(a+b)^2(ad-bc)^3(a+b\sin(e+fx))}+\frac{b^3\cos(e+fx)}{(a-b)(a+b)(bc-ad)^2(a+b\sin(}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^2),x]
```

```
[Out] ((2*b^2*(-8*a^3*b*c*d + 2*a*b^3*c*d + 12*a^4*d^2 + a^2*b^2*(2*c^2 - 15*d^2)
 + b^4*(c^2 + 6*d^2))*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/((a
^2 - b^2)^(5/2)*(b*c - a*d)^4) + (4*d^3*(-4*b*c^2 + a*c*d + 3*b*d^2)*ArcTan
[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/((b*c - a*d)^4*(c^2 - d^2)^(3/2
)) + (b^3*Cos[e + f*x])/((a - b)*(a + b)*(b*c - a*d)^2*(a + b*Sin[e + f*x])
^2) + (b^3*(-3*a*b*c + 7*a^2*d - 4*b^2*d)*Cos[e + f*x])/((a - b)^2*(a + b)^
2*(-(b*c) + a*d)^3*(a + b*Sin[e + f*x])) - (2*d^4*Cos[e + f*x])/((c - d)*(c
 + d)*(b*c - a*d)^3*(c + d*Sin[e + f*x])))/(2*f)
```

Maple [B] time = 0.186, size = 3241, normalized size = 7.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^2,x)
```

```
[Out] -1/f*b^7/(a*d-b*c)^4/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a
^4-2*a^2*b^2+b^4)*c^2-2/f*b^8/(a*d-b*c)^4/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2
*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)/a*tan(1/2*f*x+1/2*e)^3*c^2+8/f*b^3/(
a*d-b*c)^4/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b
^2+b^4)*a^4*tan(1/2*f*x+1/2*e)^2*d^2+4/f*b^5/(a*d-b*c)^4/(tan(1/2*f*x+1/2*e
)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^2*tan(1/2*f*x+1/2*e
)^2*c^2+11/f*b^5/(a*d-b*c)^4/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b
+a)^2/(a^4-2*a^2*b^2+b^4)*a^2*tan(1/2*f*x+1/2*e)^2*d^2+5/f*b^6/(a*d-b*c)^4/
(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a*t
an(1/2*f*x+1/2*e)^3*c^2+2/f*b^4/(a*d-b*c)^4/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(
1/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*c^2*a^2-15/f*
b^4/(a*d-b*c)^4/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2
*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*a^2*d^2-2/f*b^9/(a*d-b*c)^4/(tan(1/2*f*x+
1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)/a^2*tan(1/2*f*x+
1/2*e)^2*c^2+23/f*b^4/(a*d-b*c)^4/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2
*e)*b+a)^2*a^3/(a^4-2*a^2*b^2+b^4)*tan(1/2*f*x+1/2*e)*d^2-2/f*d^5/(a^2*d^2-
2*a*b*c*d+b^2*c^2)/(a*d-b*c)^2/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)
*d+c)/(c^2-d^2)*tan(1/2*f*x+1/2*e)*b-2/f*d^4/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(a
*d-b*c)^2/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)/(c^2-d^2)*c*b+2
/f*d^4/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(a*d-b*c)^2/(c^2-d^2)^(3/2)*arctan(1/2*(
2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*a*c-8/f*d^3/(a^2*d^2-2*a*b*c*d
+b^2*c^2)/(a*d-b*c)^2/(c^2-d^2)^(3/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*
d)/(c^2-d^2)^(1/2))*c^2*b+11/f*b^6/(a*d-b*c)^4/(tan(1/2*f*x+1/2*e)^2*a+2*ta
n(1/2*f*x+1/2*e)*b+a)^2*a/(a^4-2*a^2*b^2+b^4)*tan(1/2*f*x+1/2*e)*c^2-14/f*b
^6/(a*d-b*c)^4/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2*a/(a^4-2
*a^2*b^2+b^4)*tan(1/2*f*x+1/2*e)*d^2-2/f*b^8/(a*d-b*c)^4/(tan(1/2*f*x+1/2*e
)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/a/(a^4-2*a^2*b^2+b^4)*tan(1/2*f*x+1/2*e)*
c^2-12/f*b^4/(a*d-b*c)^4/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^
2/(a^4-2*a^2*b^2+b^4)*a^3*c*d+6/f*b^6/(a*d-b*c)^4/(tan(1/2*f*x+1/2*e)^2*a+2
*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a*c*d+12/f*b^2/(a*d-b*c)^4/(
a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/
(a^2-b^2)^(1/2))*a^4*d^2+9/f*b^4/(a*d-b*c)^4/(tan(1/2*f*x+1/2*e)^2*a+2*tan(
1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^3*tan(1/2*f*x+1/2*e)^3*d^2+8/f*
b^7/(a*d-b*c)^4/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*
a^2*b^2+b^4)*tan(1/2*f*x+1/2*e)^3*c*d+16/f*b^7/(a*d-b*c)^4/(tan(1/2*f*x+1/2
*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*tan(1/2*f*x+1/2*e)*
c*d-6/f*b^6/(a*d-b*c)^4/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2
/(a^4-2*a^2*b^2+b^4)*a*tan(1/2*f*x+1/2*e)^3*d^2-18/f*b^6/(a*d-b*c)^4/(tan(1
/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a*tan(1/2
*f*x+1/2*e)^2*c*d+12/f*b^8/(a*d-b*c)^4/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*
x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)/a*tan(1/2*f*x+1/2*e)^2*c*d-34/f*b^5/(a*
d-b*c)^4/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2*a^2/(a^4-2*a^2
*b^2+b^4)*tan(1/2*f*x+1/2*e)*c*d-8/f*b^3/(a*d-b*c)^4/(a^4-2*a^2*b^2+b^4)/(a
^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*a^3*
c*d+2/f*b^5/(a*d-b*c)^4/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a
*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*a*c*d-14/f*b^5/(a*d-b*c)^4/(tan(1
/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^2*tan(1
/2*f*x+1/2*e)^3*c*d+7/f*b^7/(a*d-b*c)^4/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f
*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*tan(1/2*f*x+1/2*e)^2*c^2+2/f*d^6/(a^2*
d^2-2*a*b*c*d+b^2*c^2)/(a*d-b*c)^2/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/
2*e)*d+c)/c/(c^2-d^2)*tan(1/2*f*x+1/2*e)*a-12/f*b^4/(a*d-b*c)^4/(tan(1/2*f*
x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^3*tan(1/2*f*
x+1/2*e)^2*c*d-10/f*b^7/(a*d-b*c)^4/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1
/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*tan(1/2*f*x+1/2*e)^2*d^2+6/f*b^6/(a*d-b*c)
^4/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2
*b)/(a^2-b^2)^(1/2))*d^2+1/f*b^6/(a*d-b*c)^4/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^
(1/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*c^2+2/f*d^5/
(a^2*d^2-2*a*b*c*d+b^2*c^2)/(a*d-b*c)^2/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f
*x+1/2*e)*d+c)/(c^2-d^2)*a+6/f*d^5/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(a*d-b*c)^2/
```

```
(c^2-d^2)^(3/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*b+
8/f*b^3/(a*d-b*c)^4/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^
4-2*a^2*b^2+b^4)*a^4*d^2+4/f*b^5/(a*d-b*c)^4/(tan(1/2*f*x+1/2*e)^2*a+2*tan(
1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*c^2*a^2-5/f*b^5/(a*d-b*c)^4/(tan(
1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^2*d^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^2,x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^2,x, algorithm="fricas")

[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))**3/(c+d*sin(f*x+e))**2,x)

[Out] Timed out
```

Giac [B] time = 1.50065, size = 1500, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^2,x, algorithm="giac")

[Out] ((2*a^2*b^4*c^2 + b^6*c^2 - 8*a^3*b^3*c*d + 2*a*b^5*c*d + 12*a^4*b^2*d^2 -
15*a^2*b^4*d^2 + 6*b^6*d^2)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(a) + arct
an((a*tan(1/2*f*x + 1/2*e) + b)/sqrt(a^2 - b^2)))/((a^4*b^4*c^4 - 2*a^2*b^6
*c^4 + b^8*c^4 - 4*a^5*b^3*c^3*d + 8*a^3*b^5*c^3*d - 4*a*b^7*c^3*d + 6*a^6*
b^2*c^2*d^2 - 12*a^4*b^4*c^2*d^2 + 6*a^2*b^6*c^2*d^2 - 4*a^7*b*c*d^3 + 8*a^
```

```
5*b^3*c*d^3 - 4*a^3*b^5*c*d^3 + a^8*d^4 - 2*a^6*b^2*d^4 + a^4*b^4*d^4)*sqrt
(a^2 - b^2)) - 2*(4*b*c^2*d^3 - a*c*d^4 - 3*b*d^5)*(pi*floor(1/2*(f*x + e)/
pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/((
b^4*c^6 - 4*a*b^3*c^5*d + 6*a^2*b^2*c^4*d^2 - b^4*c^4*d^2 - 4*a^3*b*c^3*d^3
 + 4*a*b^3*c^3*d^3 + a^4*c^2*d^4 - 6*a^2*b^2*c^2*d^4 + 4*a^3*b*c*d^5 - a^4*
d^6)*sqrt(c^2 - d^2)) - 2*(d^5*tan(1/2*f*x + 1/2*e) + c*d^4)/((b^3*c^6 - 3*
a*b^2*c^5*d + 3*a^2*b*c^4*d^2 - b^3*c^4*d^2 - a^3*c^3*d^3 + 3*a*b^2*c^3*d^3
 - 3*a^2*b*c^2*d^4 + a^3*c*d^5)*(c*tan(1/2*f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x
 + 1/2*e) + c)) + (5*a^3*b^5*c*tan(1/2*f*x + 1/2*e)^3 - 2*a*b^7*c*tan(1/2*f
*x + 1/2*e)^3 - 9*a^4*b^4*d*tan(1/2*f*x + 1/2*e)^3 + 6*a^2*b^6*d*tan(1/2*f*
x + 1/2*e)^3 + 4*a^4*b^4*c*tan(1/2*f*x + 1/2*e)^2 + 7*a^2*b^6*c*tan(1/2*f*x
 + 1/2*e)^2 - 2*b^8*c*tan(1/2*f*x + 1/2*e)^2 - 8*a^5*b^3*d*tan(1/2*f*x + 1/
2*e)^2 - 11*a^3*b^5*d*tan(1/2*f*x + 1/2*e)^2 + 10*a*b^7*d*tan(1/2*f*x + 1/2
*e)^2 + 11*a^3*b^5*c*tan(1/2*f*x + 1/2*e) - 2*a*b^7*c*tan(1/2*f*x + 1/2*e)
- 23*a^4*b^4*d*tan(1/2*f*x + 1/2*e) + 14*a^2*b^6*d*tan(1/2*f*x + 1/2*e) + 4
*a^4*b^4*c - a^2*b^6*c - 8*a^5*b^3*d + 5*a^3*b^5*d)/((a^6*b^3*c^3 - 2*a^4*b
^5*c^3 + a^2*b^7*c^3 - 3*a^7*b^2*c^2*d + 6*a^5*b^4*c^2*d - 3*a^3*b^6*c^2*d
+ 3*a^8*b*c*d^2 - 6*a^6*b^3*c*d^2 + 3*a^4*b^5*c*d^2 - a^9*d^3 + 2*a^7*b^2*d
^3 - a^5*b^4*d^3)*(a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e) + a)
^2))/f
```

3.722 $\int \frac{1}{(a+b\sin(e+fx))^3(c+d\sin(e+fx))^3}\,dx$

Optimal. Leaf size=669

$$-\frac{b^3\left(-a^2b^2\left(2c^2-29d^2\right)+10a^3bcd-20a^4d^2-4ab^3cd-b^4\left(c^2+12d^2\right)\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{f\left(a^2-b^2\right)^{5/2}(bc-ad)^5}-\frac{d^3\left(a^2d^2\left(2c^2+d^2\right)-a\right.}{}$$

```
[Out] -((b^3*(10*a^3*b*c*d - 4*a*b^3*c*d - 20*a^4*d^2 - a^2*b^2*(2*c^2 - 29*d^2)
- b^4*(c^2 + 12*d^2))*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/((a
^2 - b^2)^(5/2)*(b*c - a*d)^5*f)) - (d^3*(a^2*d^2*(2*c^2 + d^2) - a*b*(10*c
^3*d - 4*c*d^3) + b^2*(20*c^4 - 29*c^2*d^2 + 12*d^4))*ArcTan[(d + c*Tan[(e
+ f*x)/2])/Sqrt[c^2 - d^2]])/((b*c - a*d)^5*(c^2 - d^2)^(5/2)*f) - (d*(a^4*
d^3 - b^4*d*(5*c^2 - 6*d^2) + 2*a^2*b^2*d*(4*c^2 - 5*d^2) - 3*a*b^3*c*(c^2
- d^2))*Cos[e + f*x])/(2*(a^2 - b^2)^2*(b*c - a*d)^3*(c^2 - d^2)*f*(c + d*S
in[e + f*x])^2) + (b^2*Cos[e + f*x])/(2*(a^2 - b^2)*(b*c - a*d)*f*(a + b*Si
n[e + f*x])^2*(c + d*Sin[e + f*x])^2) + (b^2*(3*a*b*c - 7*a^2*d + 4*b^2*d)*
Cos[e + f*x])/(2*(a^2 - b^2)^2*(b*c - a*d)^2*f*(a + b*Sin[e + f*x])*(c + d*
Sin[e + f*x])^2) + (3*d*(a^5*c*d^4 - 2*a^3*b^2*c*d^4 + a*b^4*c*(c^4 - 2*c^2
*d^2 + 2*d^4) + b^5*d*(2*c^4 - 7*c^2*d^2 + 4*d^4) - a^2*b^3*d*(3*c^4 - 12*c
^2*d^2 + 7*d^4) - a^4*b*(3*c^2*d^3 - 2*d^5))*Cos[e + f*x])/(2*(a^2 - b^2)^2
*(b*c - a*d)^4*(c^2 - d^2)^2*f*(c + d*Sin[e + f*x]))
```

Rubi [A] time = 3.30899, antiderivative size = 669, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {2802, 3055, 3001, 2660, 618, 204}

$$-\frac{b^3\left(-a^2b^2\left(2c^2-29d^2\right)+10a^3bcd-20a^4d^2-4ab^3cd-b^4\left(c^2+12d^2\right)\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{f\left(a^2-b^2\right)^{5/2}(bc-ad)^5}-\frac{d^3\left(a^2d^2\left(2c^2+d^2\right)-a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^3),x]
```

```
[Out] -((b^3*(10*a^3*b*c*d - 4*a*b^3*c*d - 20*a^4*d^2 - a^2*b^2*(2*c^2 - 29*d^2)
- b^4*(c^2 + 12*d^2))*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/((a
^2 - b^2)^(5/2)*(b*c - a*d)^5*f)) - (d^3*(a^2*d^2*(2*c^2 + d^2) - a*b*(10*c
^3*d - 4*c*d^3) + b^2*(20*c^4 - 29*c^2*d^2 + 12*d^4))*ArcTan[(d + c*Tan[(e
+ f*x)/2])/Sqrt[c^2 - d^2]])/((b*c - a*d)^5*(c^2 - d^2)^(5/2)*f) - (d*(a^4*
d^3 - b^4*d*(5*c^2 - 6*d^2) + 2*a^2*b^2*d*(4*c^2 - 5*d^2) - 3*a*b^3*c*(c^2
- d^2))*Cos[e + f*x])/(2*(a^2 - b^2)^2*(b*c - a*d)^3*(c^2 - d^2)*f*(c + d*S
in[e + f*x])^2) + (b^2*Cos[e + f*x])/(2*(a^2 - b^2)*(b*c - a*d)*f*(a + b*Si
n[e + f*x])^2*(c + d*Sin[e + f*x])^2) + (b^2*(3*a*b*c - 7*a^2*d + 4*b^2*d)*
Cos[e + f*x])/(2*(a^2 - b^2)^2*(b*c - a*d)^2*f*(a + b*Sin[e + f*x])*(c + d*
Sin[e + f*x])^2) + (3*d*(a^5*c*d^4 - 2*a^3*b^2*c*d^4 + a*b^4*c*(c^4 - 2*c^2
*d^2 + 2*d^4) + b^5*d*(2*c^4 - 7*c^2*d^2 + 4*d^4) - a^2*b^3*d*(3*c^4 - 12*c
^2*d^2 + 7*d^4) - a^4*b*(3*c^2*d^3 - 2*d^5))*Cos[e + f*x])/(2*(a^2 - b^2)^2
*(b*c - a*d)^4*(c^2 - d^2)^2*f*(c + d*Sin[e + f*x]))
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
```

```
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(a+b\sin(e+fx))^3(c+d\sin(e+fx))^3}\,dx = \frac{b^2\cos(e+fx)}{2\left(a^2-b^2\right)(bc-ad)f(a+b\sin(e+fx))^2(c+d\sin(e+fx))^2} - \frac{\int \frac{-2(abc-}{}}{}$$

$$= \frac{b^2\cos(e+fx)}{2\left(a^2-b^2\right)(bc-ad)f(a+b\sin(e+fx))^2(c+d\sin(e+fx))^2} + \frac{}{2\left(a^2-b\right.}$$

$$= -\frac{d\left(a^4d^3-b^4d\left(5c^2-6d^2\right)+2a^2b^2d\left(4c^2-5d^2\right)-3ab^3c\left(c^2-d^2\right)\right)\cos(e}{2\left(a^2-b^2\right)^2(bc-ad)^3\left(c^2-d^2\right)f(c+d\sin(e+fx))^2}$$

$$= -\frac{d\left(a^4d^3-b^4d\left(5c^2-6d^2\right)+2a^2b^2d\left(4c^2-5d^2\right)-3ab^3c\left(c^2-d^2\right)\right)\cos(e}{2\left(a^2-b^2\right)^2(bc-ad)^3\left(c^2-d^2\right)f(c+d\sin(e+fx))^2}$$

$$= -\frac{d\left(a^4d^3-b^4d\left(5c^2-6d^2\right)+2a^2b^2d\left(4c^2-5d^2\right)-3ab^3c\left(c^2-d^2\right)\right)\cos(e}{2\left(a^2-b^2\right)^2(bc-ad)^3\left(c^2-d^2\right)f(c+d\sin(e+fx))^2}$$

$$= -\frac{d\left(a^4d^3-b^4d\left(5c^2-6d^2\right)+2a^2b^2d\left(4c^2-5d^2\right)-3ab^3c\left(c^2-d^2\right)\right)\cos(e}{2\left(a^2-b^2\right)^2(bc-ad)^3\left(c^2-d^2\right)f(c+d\sin(e+fx))^2}$$

$$= -\frac{d\left(a^4d^3-b^4d\left(5c^2-6d^2\right)+2a^2b^2d\left(4c^2-5d^2\right)-3ab^3c\left(c^2-d^2\right)\right)\cos(e}{2\left(a^2-b^2\right)^2(bc-ad)^3\left(c^2-d^2\right)f(c+d\sin(e+fx))^2}$$

$$= -\frac{b^3\left(10a^3bcd-4ab^3cd-20a^4d^2-a^2b^2\left(2c^2-29d^2\right)-b^4\left(c^2+12d^2\right)\right)\mathrm{ta}}{\left(a^2-b^2\right)^{5/2}(bc-ad)^5f}$$

<u>Mathematica</u> [B] time = 8.3794, size = 1815, normalized size = 2.71

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^3),x]
```

```
[Out] -((b^3*(2*a^2*b^2*c^2 + b^4*c^2 - 10*a^3*b*c*d + 4*a*b^3*c*d + 20*a^4*d^2 -
 29*a^2*b^2*d^2 + 12*b^4*d^2)*ArcTan[(Sec[(e + f*x)/2]*(b*Cos[(e + f*x)/2]
+ a*Sin[(e + f*x)/2]))/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(5/2)*(-(b*c) + a*d)^
5*f)) - (d^3*(20*b^2*c^4 - 10*a*b*c^3*d + 2*a^2*c^2*d^2 - 29*b^2*c^2*d^2 +
4*a*b*c*d^3 + a^2*d^4 + 12*b^2*d^4)*ArcTan[(Sec[(e + f*x)/2]*(d*Cos[(e + f*
x)/2] + c*Sin[(e + f*x)/2]))/Sqrt[c^2 - d^2]])/((b*c - a*d)^5*(c^2 - d^2)^(
5/2)*f) + (32*a^2*b^5*c^7*Cos[e + f*x] - 8*b^7*c^7*Cos[e + f*x] - 80*a^3*b^
4*c^6*d*Cos[e + f*x] + 68*a*b^6*c^6*d*Cos[e + f*x] - 92*a^2*b^5*c^5*d^2*Cos
[e + f*x] + 38*b^7*c^5*d^2*Cos[e + f*x] + 140*a^3*b^4*c^4*d^3*Cos[e + f*x]
- 122*a*b^6*c^4*d^3*Cos[e + f*x] - 80*a^6*b*c^3*d^4*Cos[e + f*x] + 140*a^4*
b^3*c^3*d^4*Cos[e + f*x] + 48*a^2*b^5*c^3*d^4*Cos[e + f*x] - 72*b^7*c^3*d^4
*Cos[e + f*x] + 32*a^7*c^2*d^5*Cos[e + f*x] - 92*a^5*b^2*c^2*d^5*Cos[e + f*
x] + 48*a^3*b^4*c^2*d^5*Cos[e + f*x] + 12*a*b^6*c^2*d^5*Cos[e + f*x] + 68*a
^6*b*c*d^6*Cos[e + f*x] - 122*a^4*b^3*c*d^6*Cos[e + f*x] + 12*a^2*b^5*c*d^6
*Cos[e + f*x] + 36*b^7*c*d^6*Cos[e + f*x] - 8*a^7*d^7*Cos[e + f*x] + 38*a^5
*b^2*d^7*Cos[e + f*x] - 72*a^3*b^4*d^7*Cos[e + f*x] + 36*a*b^6*d^7*Cos[e +
f*x] - 12*a*b^6*c^6*d*Cos[3*(e + f*x)] + 28*a^2*b^5*c^5*d^2*Cos[3*(e + f*x)
] - 22*b^7*c^5*d^2*Cos[3*(e + f*x)] + 20*a^3*b^4*c^4*d^3*Cos[3*(e + f*x)] +
```

```
10*a*b^6*c^4*d^3*Cos[3*(e + f*x)] + 20*a^4*b^3*c^3*d^4*Cos[3*(e + f*x)] -
96*a^2*b^5*c^3*d^4*Cos[3*(e + f*x)] + 64*b^7*c^3*d^4*Cos[3*(e + f*x)] + 28*
a^5*b^2*c^2*d^5*Cos[3*(e + f*x)] - 96*a^3*b^4*c^2*d^5*Cos[3*(e + f*x)] + 44
*a*b^6*c^2*d^5*Cos[3*(e + f*x)] - 12*a^6*b*c*d^6*Cos[3*(e + f*x)] + 10*a^4*
b^3*c*d^6*Cos[3*(e + f*x)] + 44*a^2*b^5*c*d^6*Cos[3*(e + f*x)] - 36*b^7*c*d
^6*Cos[3*(e + f*x)] - 22*a^5*b^2*d^7*Cos[3*(e + f*x)] + 64*a^3*b^4*d^7*Cos[
3*(e + f*x)] - 36*a*b^6*d^7*Cos[3*(e + f*x)] + 12*a*b^6*c^7*Sin[2*(e + f*x)
] - 4*a^2*b^5*c^6*d*Sin[2*(e + f*x)] + 16*b^7*c^6*d*Sin[2*(e + f*x)] - 80*a
^3*b^4*c^5*d^2*Sin[2*(e + f*x)] + 38*a*b^6*c^5*d^2*Sin[2*(e + f*x)] - 10*a^
2*b^5*c^4*d^3*Sin[2*(e + f*x)] - 20*b^7*c^4*d^3*Sin[2*(e + f*x)] - 80*a^5*b
^2*c^3*d^4*Sin[2*(e + f*x)] + 320*a^3*b^4*c^3*d^4*Sin[2*(e + f*x)] - 192*a*
b^6*c^3*d^4*Sin[2*(e + f*x)] - 4*a^6*b*c^2*d^5*Sin[2*(e + f*x)] - 10*a^4*b^
3*c^2*d^5*Sin[2*(e + f*x)] + 64*a^2*b^5*c^2*d^5*Sin[2*(e + f*x)] - 26*b^7*c
^2*d^5*Sin[2*(e + f*x)] + 12*a^7*c*d^6*Sin[2*(e + f*x)] + 38*a^5*b^2*c*d^6*
Sin[2*(e + f*x)] - 192*a^3*b^4*c*d^6*Sin[2*(e + f*x)] + 124*a*b^6*c*d^6*Sin
[2*(e + f*x)] + 16*a^6*b*d^7*Sin[2*(e + f*x)] - 20*a^4*b^3*d^7*Sin[2*(e + f
*x)] - 26*a^2*b^5*d^7*Sin[2*(e + f*x)] + 24*b^7*d^7*Sin[2*(e + f*x)] - 3*a*
b^6*c^5*d^2*Sin[4*(e + f*x)] + 9*a^2*b^5*c^4*d^3*Sin[4*(e + f*x)] - 6*b^7*c
^4*d^3*Sin[4*(e + f*x)] + 6*a*b^6*c^3*d^4*Sin[4*(e + f*x)] + 9*a^4*b^3*c^2*
d^5*Sin[4*(e + f*x)] - 36*a^2*b^5*c^2*d^5*Sin[4*(e + f*x)] + 21*b^7*c^2*d^5
*Sin[4*(e + f*x)] - 3*a^5*b^2*c*d^6*Sin[4*(e + f*x)] + 6*a^3*b^4*c*d^6*Sin[
4*(e + f*x)] - 6*a*b^6*c*d^6*Sin[4*(e + f*x)] - 6*a^4*b^3*d^7*Sin[4*(e + f*
x)] + 21*a^2*b^5*d^7*Sin[4*(e + f*x)] - 12*b^7*d^7*Sin[4*(e + f*x)])/(16*(a
^2 - b^2)^2*(-(b*c) + a*d)^4*(c^2 - d^2)^2*f*(a + b*Sin[e + f*x])^2*(c + d*
Sin[e + f*x])^2)
```

Maple [B] time = 0.234, size = 7348, normalized size = 11.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^3,x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))**3/(c+d*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] Timed out
```

3.723 $\int (a+b\sin(e+fx))(c+d\sin(e+fx))^{5/2}\,dx$

Optimal. Leaf size=298

$$-\frac{2\left(56acd+15bc^2+25bd^2\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{105f}-\frac{2\left(c^2-d^2\right)\left(56acd+15bc^2+25bd^2\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(\right.\right.}{105df\sqrt{c+d\sin(e+fx)}}$$

```
[Out] (-2*(15*b*c^2 + 56*a*c*d + 25*b*d^2)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])
/(105*f) - (2*(5*b*c + 7*a*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(3/2))/(35*
f) - (2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^(5/2))/(7*f) + (2*(15*b*c^3 + 1
61*a*c^2*d + 145*b*c*d^2 + 63*a*d^3)*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c
 + d)]*Sqrt[c + d*Sin[e + f*x]])/(105*d*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)
]) - (2*(c^2 - d^2)*(15*b*c^2 + 56*a*c*d + 25*b*d^2)*EllipticF[(e - Pi/2 +
f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(105*d*f*Sqrt[c
+ d*Sin[e + f*x]])
```

Rubi [A] time = 0.480587, antiderivative size = 298, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {2753, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2\left(56acd+15bc^2+25bd^2\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{105f}-\frac{2\left(c^2-d^2\right)\left(56acd+15bc^2+25bd^2\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(\right.\right.}{105df\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])*(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (-2*(15*b*c^2 + 56*a*c*d + 25*b*d^2)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])
/(105*f) - (2*(5*b*c + 7*a*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(3/2))/(35*
f) - (2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^(5/2))/(7*f) + (2*(15*b*c^3 + 1
61*a*c^2*d + 145*b*c*d^2 + 63*a*d^3)*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c
 + d)]*Sqrt[c + d*Sin[e + f*x]])/(105*d*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)
]) - (2*(c^2 - d^2)*(15*b*c^2 + 56*a*c*d + 25*b*d^2)*EllipticF[(e - Pi/2 +
f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(105*d*f*Sqrt[c
+ d*Sin[e + f*x]])
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int (a+b\sin(e+fx))(c+d\sin(e+fx))^{5/2}\,dx = -\frac{2b\cos(e+fx)(c+d\sin(e+fx))^{5/2}}{7f} + \frac{2}{7}\int (c+d\sin(e+fx))^{3/2}\left(\frac{1}{2}(7ac\right.$$

$$= -\frac{2(5bc+7ad)\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{35f} - \frac{2b\cos(e+fx)(c+d\,\mathrm{si}}{7f}$$

$$= -\frac{2\left(15bc^2+56acd+25bd^2\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{105f} - \frac{2(5bc+7a}{}$$

$$= -\frac{2\left(15bc^2+56acd+25bd^2\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{105f} - \frac{2(5bc+7a}{}$$

$$= -\frac{2\left(15bc^2+56acd+25bd^2\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{105f} - \frac{2(5bc+7a}{}$$

$$= -\frac{2\left(15bc^2+56acd+25bd^2\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{105f} - \frac{2(5bc+7a}{}$$

Mathematica [A] time = 1.07843, size = 275, normalized size = 0.92

$$\frac{-d\cos(e+fx)(c+d\sin(e+fx))\left(6d(7ad+15bc)\sin(e+fx)+154acd+90bc^2-15bd^2\cos(2(e+fx))+65bd^2\right)-2d\left(7\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])*(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (-2*d*(5*b*d*(27*c^2 + 5*d^2) + 7*a*(15*c^3 + 17*c*d^2))*EllipticF[(-2*e +
Pi - 2*f*x)/4, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)] - 2*(7*a*d
```

```
*(23*c^2 + 9*d^2) + 5*b*(3*c^3 + 29*c*d^2))*((c + d)*EllipticE[(-2*e + Pi -
 2*f*x)/4, (2*d)/(c + d)] - c*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d
)])*Sqrt[(c + d*Sin[e + f*x])/(c + d)] - d*Cos[e + f*x]*(c + d*Sin[e + f*x]
)*(90*b*c^2 + 154*a*c*d + 65*b*d^2 - 15*b*d^2*Cos[2*(e + f*x)] + 6*d*(15*b*
c + 7*a*d)*Sin[e + f*x]))/(105*d*f*Sqrt[c + d*Sin[e + f*x]])
```

Maple [B] time = 1.23, size = 1839, normalized size = 6.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))*(c+d*sin(f*x+e))^(5/2),x)
```

```
[Out] 2/105*(-45*b*c^3*d^2-25*b*c*d^4-77*a*c^2*d^3-130*((c+d*sin(f*x+e))/(c-d))^(
1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*Ellip
ticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*b*c^3*d^2+145*((c+
d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+
e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/
2))*b*c*d^4+56*a*c^3*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+
d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d)
)^(1/2),((c-d)/(c+d))^(1/2))*d^2-42*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+si
n(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*si
n(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*a*c^2*d^3-56*((c+d*sin(f*x+e))/
(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/
2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*a*c*d^4+15
*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin
(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d)
)^(1/2))*b*c^4*d+120*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+
d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d)
)^(1/2),((c-d)/(c+d))^(1/2))*b*c^3*d^2+10*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-
(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((
c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*b*c^2*d^3-120*((c+d*sin(f
*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-
d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*b*c
*d^4-161*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-
d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-
d)/(c+d))^(1/2))*a*c^4*d+105*a*c^4*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin
(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin
(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*d+98*((c+d*sin(f*x+e))/(c-d))^(1
/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*Ellipt
icE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*a*c^2*d^3+63*((c+d*
sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e)
)/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)
)*a*d^5+10*b*d^5*sin(f*x+e)^3-21*a*d^5*sin(f*x+e)^2-25*b*d^5*sin(f*x+e)+15*
b*d^5*sin(f*x+e)^5+21*a*d^5*sin(f*x+e)^4+60*b*c*d^4*sin(f*x+e)^4+98*a*c*d^4
*sin(f*x+e)^3+90*b*c^2*d^3*sin(f*x+e)^3+77*a*c^2*d^3*sin(f*x+e)^2+45*b*c^3*
d^2*sin(f*x+e)^2-35*b*c*d^4*sin(f*x+e)^2-98*a*c*d^4*sin(f*x+e)-90*b*c^2*d^3
*sin(f*x+e)-25*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1
/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2
),((c-d)/(c+d))^(1/2))*b*d^5-63*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*
x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*
x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*a*d^5-15*((c+d*sin(f*x+e))/(c-d))^(
1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*Ellip
ticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*b*c^5)/d^2/cos(f*x
+e)/(c+d*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))*(c+d*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(ac^2+2\,bcd+ad^2-\left(2\,bcd+ad^2\right)\cos\left(fx+e\right)^2-\left(bd^2\cos\left(fx+e\right)^2-bc^2-2\,acd-bd^2\right)\sin\left(fx+e\right)\right)\sqrt{d\sin$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))*(c+d*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((a*c^2 + 2*b*c*d + a*d^2 - (2*b*c*d + a*d^2)*cos(f*x + e)^2 - (b*d
^2*cos(f*x + e)^2 - b*c^2 - 2*a*c*d - b*d^2)*sin(f*x + e))*sqrt(d*sin(f*x +
 e) + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))*(c+d*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))*(c+d*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.724 $\int (a+b\sin(e+fx))(c+d\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=235

$$-\frac{2\left(c^2-d^2\right)(5ad+3bc)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{15df\sqrt{c+d\sin(e+fx)}}+\frac{2\left(20acd+3b\left(c^2+3d^2\right)\right)\sqrt{c+d\sin(e+fx)}E\left(\frac{1}{2}\left(e+\cdots\right.\right.}{15df\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

```
[Out] (-2*(3*b*c + 5*a*d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(15*f) - (2*b*Co
s[e + f*x]*(c + d*Sin[e + f*x])^(3/2))/(5*f) + (2*(20*a*c*d + 3*b*(c^2 + 3*
d^2))*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]]
)/(15*d*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) - (2*(3*b*c + 5*a*d)*(c^2 - d
^2)*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/
(c + d)])/(15*d*f*Sqrt[c + d*Sin[e + f*x]])
```

Rubi [A] time = 0.358819, antiderivative size = 235, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {2753, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2\left(c^2-d^2\right)(5ad+3bc)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{15df\sqrt{c+d\sin(e+fx)}}+\frac{2\left(20acd+3b\left(c^2+3d^2\right)\right)\sqrt{c+d\sin(e+fx)}E\left(\frac{1}{2}\left(e+\cdots\right.\right.}{15df\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])*(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-2*(3*b*c + 5*a*d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(15*f) - (2*b*Co
s[e + f*x]*(c + d*Sin[e + f*x])^(3/2))/(5*f) + (2*(20*a*c*d + 3*b*(c^2 + 3*
d^2))*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]]
)/(15*d*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) - (2*(3*b*c + 5*a*d)*(c^2 - d
^2)*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/
(c + d)])/(15*d*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
```

```
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int (a+b\sin(e+fx))(c+d\sin(e+fx))^{3/2}\,dx = -\frac{2b\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{5f}+\frac{2}{5}\int\sqrt{c+d\sin(e+fx)}\left(\frac{1}{2}(5ac+\right.$$

$$= -\frac{2(3bc+5ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15f}-\frac{2b\cos(e+fx)(c+d\sin(}{5f}$$

$$= -\frac{2(3bc+5ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15f}-\frac{2b\cos(e+fx)(c+d\sin(}{5f}$$

$$= -\frac{2(3bc+5ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15f}-\frac{2b\cos(e+fx)(c+d\sin(}{5f}$$

$$= -\frac{2(3bc+5ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15f}-\frac{2b\cos(e+fx)(c+d\sin(}{5f}$$

Mathematica [A] time = 0.728198, size = 218, normalized size = 0.93

$$\frac{-2d\left(5a\left(3c^2+d^2\right)+12bcd\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{4}(-2e-2fx+\pi)|\frac{2d}{c+d}\right)-2\left(20acd+3b\left(c^2+3d^2\right)\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\left((c+d)E\left(\right.\right.}{15df\sqrt{c+d\,s}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])*(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-2*d*(12*b*c*d + 5*a*(3*c^2 + d^2))*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)
/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)] - 2*(20*a*c*d + 3*b*(c^2 + 3*d
^2))*((c + d)*EllipticE[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)] - c*EllipticF
[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)])*Sqrt[(c + d*Sin[e + f*x])/(c + d)]
- 2*d*Cos[e + f*x]*(c + d*Sin[e + f*x])*(6*b*c + 5*a*d + 3*b*d*Sin[e + f*x]
))/(15*d*f*Sqrt[c + d*Sin[e + f*x]])
```

Maple [B] time = 1.217, size = 1449, normalized size = 6.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))*(c+d*sin(f*x+e))^(3/2),x)
```

```
[Out] 2/15*(15*c^3*a*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1
/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2
),((c-d)/(c+d))^(1/2))*d+5*c^2*a*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f
*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f
*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*d^2-15*((c+d*sin(f*x+e))/(c-d))^(1
/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*Ellipt
icF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*a*c*d^3-5*((c+d*sin
(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(
c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*a
*d^4+3*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*
(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)
/(c+d))^(1/2))*b*c^3*d+9*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d
/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(
c-d))^(1/2),((c-d)/(c+d))^(1/2))*b*c^2*d^2-3*((c+d*sin(f*x+e))/(c-d))^(1/2)
*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF
(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*b*c*d^3-9*((c+d*sin(f*
x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d
))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*b*d^
4-20*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1
+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(
c+d))^(1/2))*a*c^3*d+20*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/
(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c
-d))^(1/2),((c-d)/(c+d))^(1/2))*a*c*d^3-3*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-
(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((
c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*b*c^4-6*((c+d*sin(f*x+e))
/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1
/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*b*c^2*d^2
+9*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+s
in(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+
d))^(1/2))*b*d^4+3*b*d^4*sin(f*x+e)^4+5*a*d^4*sin(f*x+e)^3+9*b*c*d^3*sin(f*
x+e)^3+5*a*c*d^3*sin(f*x+e)^2+6*b*c^2*d^2*sin(f*x+e)^2-3*b*d^4*sin(f*x+e)^2
-5*a*d^4*sin(f*x+e)-9*b*c*d^3*sin(f*x+e)-5*a*c*d^3-6*b*c^2*d^2)/d^2/cos(f*x
+e)/(c+d*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))*(c+d*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(bd\cos\left(fx+e\right)^{2}-ac-bd-(bc+ad)\sin\left(fx+e\right)\right)\sqrt{d\sin\left(fx+e\right)+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))*(c+d*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-(b*d*cos(f*x + e)^2 - a*c - b*d - (b*c + a*d)*sin(f*x + e))*sqrt(
d*sin(f*x + e) + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\sin\left(e+fx\right)\right)\left(c+d\sin\left(e+fx\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))*(c+d*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral((a + b*sin(e + f*x))*(c + d*sin(e + f*x))**(3/2), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))*(c+d*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.725 $\int (a+b\sin(e+fx))\sqrt{c+d\sin(e+fx)}\,dx$

Optimal. Leaf size=181

$$\frac{2(3ad+bc)\sqrt{c+d\sin(e+fx)}E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3df\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}-\frac{2b\left(c^2-d^2\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3df\sqrt{c+d\sin(e+fx)}}-\frac{2b\cos(e-}{}$$

```
[Out] (-2*b*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(3*f) + (2*(b*c + 3*a*d)*Ellip
ticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(3*d*f*Sq
rt[(c + d*Sin[e + f*x])/(c + d)]) - (2*b*(c^2 - d^2)*EllipticF[(e - Pi/2 +
f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(3*d*f*Sqrt[c +
d*Sin[e + f*x]])
```

Rubi [A] time = 0.213058, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {2753, 2752, 2663, 2661, 2655, 2653}

$$\frac{2(3ad+bc)\sqrt{c+d\sin(e+fx)}E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3df\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}-\frac{2b\left(c^2-d^2\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3df\sqrt{c+d\sin(e+fx)}}-\frac{2b\cos(e-}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]],x]
```


```
[Out] (-2*b*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(3*f) + (2*(b*c + 3*a*d)*Ellip
ticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(3*d*f*Sq
rt[(c + d*Sin[e + f*x])/(c + d)]) - (2*b*(c^2 - d^2)*EllipticF[(e - Pi/2 +
f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(3*d*f*Sqrt[c +
d*Sin[e + f*x]])
```


Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int (a+b\sin(e+fx))\sqrt{c+d\sin(e+fx)}\,dx = -\frac{2b\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3f}+\frac{2}{3}\int\frac{\frac{1}{2}(3ac+bd)+\frac{1}{2}(bc+3ad)\sin(e$$

$$= -\frac{2b\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3f}+\frac{(bc+3ad)\int\sqrt{c+d\sin(e+fx)}\,dx}{3d} -$$

$$= -\frac{2b\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3f}+\frac{\left((bc+3ad)\sqrt{c+d\sin(e+fx)}\right)\int\sqrt{\frac{}{c}}}{3d\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

$$= -\frac{2b\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3f}+\frac{2(bc+3ad)E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2d}{c+d}\right)\sqrt{}}{3df\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

Mathematica [A] time = 0.629658, size = 152, normalized size = 0.84

$$-\frac{2\left((c+d)(3ad+bc)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}E\left(\frac{1}{4}(-2e-2fx+\pi)|\frac{2d}{c+d}\right)-b\left(c^2-d^2\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{4}(-2e-2fx+\pi)|\frac{2d}{c+d}\right)+bd\,\mathrm{co}\right.}{3df\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] (-2*(b*d*Cos[e + f*x]*(c + d*Sin[e + f*x]) + (c + d)*(b*c + 3*a*d)*Elliptic
E[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)]
- b*(c^2 - d^2)*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]*Sqrt[(c + d
*Sin[e + f*x])/(c + d)]))/(3*d*f*Sqrt[c + d*Sin[e + f*x]])
```

Maple [B] time = 1.213, size = 862, normalized size = 4.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))*(c+d*sin(f*x+e))^(1/2),x)
```

```
[Out] 2/3*(3*c^2*a*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2
)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),
((c-d)/(c+d))^(1/2))*d-3*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d
/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(
c-d))^(1/2),((c-d)/(c+d))^(1/2))*a*d^3+((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1
+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d
*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*b*c^2*d-((c+d*sin(f*x+e))/(c
-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)
*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*b*d^3-3*((c+
d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+
e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/
2))*a*c^2*d+3*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/
2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2)
,((c-d)/(c+d))^(1/2))*a*d^3-((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e)
)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e)
)/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*b*c^3+((c+d*sin(f*x+e))/(c-d))^(1/2)*(-
(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((
c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*b*c*d^2+b*d^3*sin(f*x+e)^
3+b*c*d^2*sin(f*x+e)^2-b*d^3*sin(f*x+e)-c*d^2*b)/d^2/cos(f*x+e)/(c+d*sin(f*
x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)+a\right)\sqrt{d\sin\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))*(c+d*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b\sin\left(fx+e\right)+a\right)\sqrt{d\sin\left(fx+e\right)+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))*(c+d*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a+b\sin\left(e+fx\right)\right)\sqrt{c+d\sin\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))*(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((a + b*sin(e + f*x))*sqrt(c + d*sin(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)+a\right)\sqrt{d\sin\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))*(c+d*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c), x)
```

3.726 $\int \frac{a+b\sin(e+fx)}{\sqrt{c+d\sin(e+fx)}}\,dx$

Optimal. Leaf size=140

$$\frac{2b\sqrt{c+d\sin(e+fx)}E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{df\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}-\frac{2(bc-ad)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{df\sqrt{c+d\sin(e+fx)}}$$

```
[Out] (2*b*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])
/(d*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) - (2*(b*c - a*d)*EllipticF[(e - P
i/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(d*f*Sqrt[
c + d*Sin[e + f*x]])
```

Rubi [A] time = 0.124015, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2752, 2663, 2661, 2655, 2653}

$$\frac{2b\sqrt{c+d\sin(e+fx)}E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{df\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}-\frac{2(bc-ad)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{df\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])/Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] (2*b*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])
/(d*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) - (2*(b*c - a*d)*EllipticF[(e - P
i/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(d*f*Sqrt[
c + d*Sin[e + f*x]])
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
```

```
0] && !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{a+b\sin(e+fx)}{\sqrt{c+d\sin(e+fx)}}\,dx = \frac{b\int\sqrt{c+d\sin(e+fx)}\,dx}{d} + \frac{(-bc+ad)\int\frac{1}{\sqrt{c+d\sin(e+fx)}}\,dx}{d}$$
$$= \frac{\left(b\sqrt{c+d\sin(e+fx)}\right)\int\sqrt{\frac{c}{c+d}+\frac{d\sin(e+fx)}{c+d}}\,dx}{d\sqrt{\frac{c+d\sin(e+fx)}{c+d}}} + \frac{\left((-bc+ad)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\right)\int\frac{1}{\sqrt{\frac{c}{c+d}+\frac{d\sin(e+fx)}{c+d}}}\,d}{d\sqrt{c+d\sin(e+fx)}}$$
$$= \frac{2bE\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2d}{c+d}\right)\sqrt{c+d\sin(e+fx)}}{df\sqrt{\frac{c+d\sin(e+fx)}{c+d}}} - \frac{2(bc-ad)F\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2d}{c+d}\right)\sqrt{\frac{c+d\sin(e+f}{c+d}}}{df\sqrt{c+d\sin(e+fx)}}$$

Mathematica [A] time = 2.55958, size = 101, normalized size = 0.72

$$-\frac{2\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\left((ad-bc)F\left(\frac{1}{4}(-2e-2fx+\pi)|\frac{2d}{c+d}\right)+b(c+d)E\left(\frac{1}{4}(-2e-2fx+\pi)|\frac{2d}{c+d}\right)\right)}{df\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])/Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] (-2*(b*(c + d)*EllipticE[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)] + (-(b*c) +
a*d)*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)])*Sqrt[(c + d*Sin[e + f
*x])/(c + d)])/(d*f*Sqrt[c + d*Sin[e + f*x]])
```

Maple [A] time = 1.069, size = 243, normalized size = 1.7

$$-2\,\frac{c-d}{d^2\cos\left(fx+e\right)\sqrt{c+d\sin\left(fx+e\right)}f}\sqrt{\frac{c+d\sin\left(fx+e\right)}{c-d}}\sqrt{-\frac{\left(-1+\sin\left(fx+e\right)\right)d}{c+d}}\sqrt{-\frac{d\left(1+\sin\left(fx+e\right)\right)}{c-d}}\Bigg(\mathrm{EllipticE}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))/(c+d*sin(f*x+e))^(1/2),x)
```

```
[Out] -2*(c-d)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-
d*(1+sin(f*x+e))/(c-d))^(1/2)*(EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c
-d)/(c+d))^(1/2))*b*c+EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d)
)^(1/2))*b*d-a*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)
)*d-EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*b*d)/d^2/
cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\sin\left(fx+e\right)+a}{\sqrt{d\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))/(c+d*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)/sqrt(d*sin(f*x + e) + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b\sin\left(fx+e\right)+a}{\sqrt{d\sin\left(fx+e\right)+c}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))/(c+d*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*sin(f*x + e) + a)/sqrt(d*sin(f*x + e) + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a+b\sin\left(e+fx\right)}{\sqrt{c+d\sin\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))/(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((a + b*sin(e + f*x))/sqrt(c + d*sin(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\sin\left(fx+e\right)+a}{\sqrt{d\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))/(c+d*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)/sqrt(d*sin(f*x + e) + c), x)
```

3.727 $\int \frac{a+b\sin(e+fx)}{(c+d\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=195

$$-\frac{2(bc-ad)\cos(e+fx)}{f\left(c^2-d^2\right)\sqrt{c+d\sin(e+fx)}}-\frac{2(bc-ad)\sqrt{c+d\sin(e+fx)}E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{df\left(c^2-d^2\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}+\frac{2b\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{df\sqrt{c+d\sin(e+fx)}}$$

```
[Out] (-2*(b*c - a*d)*Cos[e + f*x])/((c^2 - d^2)*f*Sqrt[c + d*Sin[e + f*x]]) - (2
*(b*c - a*d)*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e
+ f*x]])/(d*(c^2 - d^2)*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + (2*b*Ellipt
icF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/
(d*f*Sqrt[c + d*Sin[e + f*x]])
```

Rubi [A] time = 0.215671, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {2754, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2(bc-ad)\cos(e+fx)}{f\left(c^2-d^2\right)\sqrt{c+d\sin(e+fx)}}-\frac{2(bc-ad)\sqrt{c+d\sin(e+fx)}E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{df\left(c^2-d^2\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}+\frac{2b\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{df\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])/(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-2*(b*c - a*d)*Cos[e + f*x])/((c^2 - d^2)*f*Sqrt[c + d*Sin[e + f*x]]) - (2
*(b*c - a*d)*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e
+ f*x]])/(d*(c^2 - d^2)*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + (2*b*Ellipt
icF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/
(d*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+b\sin(e+fx)}{(c+d\sin(e+fx))^{3/2}}\,dx &= -\frac{2(bc-ad)\cos(e+fx)}{\left(c^2-d^2\right)f\sqrt{c+d\sin(e+fx)}}-\frac{2\int \frac{\frac{1}{2}(-ac+bd)+\frac{1}{2}(bc-ad)\sin(e+fx)}{\sqrt{c+d\sin(e+fx)}}\,dx}{c^2-d^2}\\
&= -\frac{2(bc-ad)\cos(e+fx)}{\left(c^2-d^2\right)f\sqrt{c+d\sin(e+fx)}}+\frac{b\int \frac{1}{\sqrt{c+d\sin(e+fx)}}\,dx}{d}-\frac{(bc-ad)\int \sqrt{c+d\sin(e+fx)}}{d\left(c^2-d^2\right)}\\
&= -\frac{2(bc-ad)\cos(e+fx)}{\left(c^2-d^2\right)f\sqrt{c+d\sin(e+fx)}}-\frac{\left((bc-ad)\sqrt{c+d\sin(e+fx)}\right)\int \sqrt{\frac{c}{c+d}+\frac{d\sin(e+fx)}{c+d}}\,dx}{d\left(c^2-d^2\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}\\
&= -\frac{2(bc-ad)\cos(e+fx)}{\left(c^2-d^2\right)f\sqrt{c+d\sin(e+fx)}}-\frac{2(bc-ad)E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2d}{c+d}\right)\sqrt{c+d\sin(e+fx)}}{d\left(c^2-d^2\right)f\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}
\end{aligned}
$$

Mathematica [A] time = 0.573545, size = 159, normalized size = 0.82

$$
\frac{2\left(d(ad-bc)\cos(e+fx)+(c+d)(bc-ad)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}E\left(\frac{1}{4}(-2e-2fx+\pi)|\frac{2d}{c+d}\right)-b\left(c^2-d^2\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{4}(-2\right.\right.}{df(c-d)(c+d)\sqrt{c+d\sin(e+fx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])/(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] (2*(d*(-(b*c) + a*d)*Cos[e + f*x] + (c + d)*(b*c - a*d)*EllipticE[(-2*e + P
i - 2*f*x)/4, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)] - b*(c^2 -
d^2)*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*
x])/(c + d)]))/((c - d)*d*(c + d)*f*Sqrt[c + d*Sin[e + f*x]])
```

Maple [B] time = 2.57, size = 567, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))/(c+d*sin(f*x+e))^(3/2),x)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*(2*b/d*(c/d-1)*((c+d*sin(f*x+e))/(c
-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(
-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(
1/2),((c-d)/(c+d))^(1/2))+(a*d-b*c)/d*(2*d*cos(f*x+e)^2/(c^2-d^2)/(-(-d*sin
(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2*c/(c^2-d^2)*(c/d-1)*((c+d*sin(f*x+e))/(c-d
))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(
-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/
2),((c-d)/(c+d))^(1/2))+2/(c^2-d^2)*d*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2
)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(
f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^
(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/
(c+d))^(1/2)))))/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\sin\left(fx+e\right)+a}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))/(c+d*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)/(d*sin(f*x + e) + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-\frac{\left(b\sin\left(fx+e\right)+a\right)\sqrt{d\sin\left(fx+e\right)+c}}{d^2\cos\left(fx+e\right)^2-2\,cd\sin\left(fx+e\right)-c^2-d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))/(c+d*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)/(d^2*cos(f*x + e)^2
 - 2*c*d*sin(f*x + e) - c^2 - d^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))/(c+d*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\sin\left(fx+e\right)+a}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))/(c+d*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)/(d*sin(f*x + e) + c)^(3/2), x)
```

3.728 $\int \frac{a+b\sin(e+fx)}{(c+d\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=285

$$\frac{2\left(4acd-b\left(c^2+3d^2\right)\right)\cos(e+fx)}{3f\left(c^2-d^2\right)^2\sqrt{c+d\sin(e+fx)}}-\frac{2(bc-ad)\cos(e+fx)}{3f\left(c^2-d^2\right)(c+d\sin(e+fx))^{3/2}}+\frac{2(bc-ad)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3df\left(c^2-d^2\right)\sqrt{c+d\sin(e+fx)}}$$

```
[Out] (-2*(b*c - a*d)*Cos[e + f*x])/(3*(c^2 - d^2)*f*(c + d*Sin[e + f*x])^(3/2))
+ (2*(4*a*c*d - b*(c^2 + 3*d^2))*Cos[e + f*x])/(3*(c^2 - d^2)^2*f*Sqrt[c +
d*Sin[e + f*x]]) + (2*(4*a*c*d - b*(c^2 + 3*d^2))*EllipticE[(e - Pi/2 + f*x
)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(3*d*(c^2 - d^2)^2*f*Sqrt[(c
+ d*Sin[e + f*x])/(c + d)]) + (2*(b*c - a*d)*EllipticF[(e - Pi/2 + f*x)/2,
(2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(3*d*(c^2 - d^2)*f*Sqrt[
c + d*Sin[e + f*x]])
```

Rubi [A] time = 0.394533, antiderivative size = 285, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {2754, 2752, 2663, 2661, 2655, 2653}

$$\frac{2\left(4acd-b\left(c^2+3d^2\right)\right)\cos(e+fx)}{3f\left(c^2-d^2\right)^2\sqrt{c+d\sin(e+fx)}}-\frac{2(bc-ad)\cos(e+fx)}{3f\left(c^2-d^2\right)(c+d\sin(e+fx))^{3/2}}+\frac{2(bc-ad)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3df\left(c^2-d^2\right)\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])/(c + d*Sin[e + f*x])^(5/2),x]
```


```
[Out] (-2*(b*c - a*d)*Cos[e + f*x])/(3*(c^2 - d^2)*f*(c + d*Sin[e + f*x])^(3/2))
+ (2*(4*a*c*d - b*(c^2 + 3*d^2))*Cos[e + f*x])/(3*(c^2 - d^2)^2*f*Sqrt[c +
d*Sin[e + f*x]]) + (2*(4*a*c*d - b*(c^2 + 3*d^2))*EllipticE[(e - Pi/2 + f*x
)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(3*d*(c^2 - d^2)^2*f*Sqrt[(c
+ d*Sin[e + f*x])/(c + d)]) + (2*(b*c - a*d)*EllipticF[(e - Pi/2 + f*x)/2,
(2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(3*d*(c^2 - d^2)*f*Sqrt[
c + d*Sin[e + f*x]])
```


Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
```

```
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{a+b\sin(e+fx)}{(c+d\sin(e+fx))^{5/2}}\,dx = -\frac{2(bc-ad)\cos(e+fx)}{3\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}} - \frac{2\int \frac{-\frac{3}{2}(ac-bd)-\frac{1}{2}(bc-ad)\sin(e+fx)}{(c+d\sin(e+fx))^{3/2}}\,dx}{3\left(c^2-d^2\right)}$$

$$= -\frac{2(bc-ad)\cos(e+fx)}{3\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}} + \frac{2\left(4acd-b\left(c^2+3d^2\right)\right)\cos(e+fx)}{3\left(c^2-d^2\right)^2 f\sqrt{c+d\sin(e+fx)}} + \frac{4\int \frac{1}{4}(-4bcd\cdots}{\cdots}$$

$$= -\frac{2(bc-ad)\cos(e+fx)}{3\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}} + \frac{2\left(4acd-b\left(c^2+3d^2\right)\right)\cos(e+fx)}{3\left(c^2-d^2\right)^2 f\sqrt{c+d\sin(e+fx)}} + \frac{(bc-ad)\int\cdots}{3d\cdots}$$

$$= -\frac{2(bc-ad)\cos(e+fx)}{3\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}} + \frac{2\left(4acd-b\left(c^2+3d^2\right)\right)\cos(e+fx)}{3\left(c^2-d^2\right)^2 f\sqrt{c+d\sin(e+fx)}} + \frac{\left(\left(4acd-b\cdots\right.\right.}{\cdots}$$

$$= -\frac{2(bc-ad)\cos(e+fx)}{3\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}} + \frac{2\left(4acd-b\left(c^2+3d^2\right)\right)\cos(e+fx)}{3\left(c^2-d^2\right)^2 f\sqrt{c+d\sin(e+fx)}} + \frac{2\left(4acd-b\cdots\right.}{\cdots}$$

Mathematica [A] time = 1.48343, size = 199, normalized size = 0.7

$$\frac{2\left(\frac{\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{3/2}\left(\left(b\left(c^2+3d^2\right)-4acd\right)E\left(\frac{1}{4}(-2e-2fx+\pi)|\frac{2d}{c+d}\right)-(c-d)(bc-ad)F\left(\frac{1}{4}(-2e-2fx+\pi)|\frac{2d}{c+d}\right)\right)}{d(c-d)^2} - \frac{\cos(e+fx)\left(d\left(b\left(c^2+3d^2\right)-4acd\right)\sin(e+fx)+ad\left(d^2-\cdots\right.\right.}{\left(c^2-d^2\right)^2}\cdots\right.}{3f(c+d\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])/(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (2*((((-4*a*c*d + b*(c^2 + 3*d^2))*EllipticE[(-2*e + Pi - 2*f*x)/4, (2*d)/(
c + d)] - (c - d)*(b*c - a*d)*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d
```

```
)])*((c + d*Sin[e + f*x])/(c + d))^(3/2))/((c - d)^2*d) - (Cos[e + f*x]*(a*
d*(-5*c^2 + d^2) + 2*b*c*(c^2 + d^2) + d*(-4*a*c*d + b*(c^2 + 3*d^2))*Sin[e
 + f*x]))/(c^2 - d^2)^2))/(3*f*(c + d*Sin[e + f*x])^(3/2))
```

Maple [B] time = 4.556, size = 887, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))/(c+d*sin(f*x+e))^(5/2),x)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((a*d-b*c)/d*(2/3/(c^2-d^2)/d*(-(-d
*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(sin(f*x+e)+c/d)^2+8/3*d*cos(f*x+e)^2/(c
^2-d^2)^2*c/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2*(3*c^2+d^2)/(3*c^4-6*
c^2*d^2+3*d^4)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+
d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)
^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+8/3*c*
d/(c^2-d^2)^2*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d
))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^
(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2
))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+b/d*(2*d
*cos(f*x+e)^2/(c^2-d^2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2*c/(c^2-d^
2)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((
-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*Ellip
ticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+2/(c^2-d^2)*d*(c/d
-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*
x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*El
lipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d
*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))))/cos(f*x+e)/(c+d*sin(f*x+e
))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\sin\left(fx+e\right)+a}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))/(c+d*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)/(d*sin(f*x + e) + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(b\sin\left(fx+e\right)+a\right)\sqrt{d\sin\left(fx+e\right)+c}}{3\,cd^2\cos\left(fx+e\right)^2-c^3-3\,cd^2+\left(d^3\cos\left(fx+e\right)^2-3\,c^2d-d^3\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))/(c+d*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)/(3*c*d^2*cos(f*x +
e)^2 - c^3 - 3*c*d^2 + (d^3*cos(f*x + e)^2 - 3*c^2*d - d^3)*sin(f*x + e)),
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))/(c+d*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \sin\left(fx + e\right) + a}{\left(d \sin\left(fx + e\right) + c\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))/(c+d*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)/(d*sin(f*x + e) + c)^(5/2), x)
```

3.729 $\int \frac{a+b\sin(e+fx)}{(c+d\sin(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=369

$$-\frac{2\left(-23ac^2d-9ad^3+3bc^3+29bcd^2\right)\cos(e+fx)}{15f\left(c^2-d^2\right)^3\sqrt{c+d\sin(e+fx)}}-\frac{2\left(-8acd+3bc^2+5bd^2\right)\cos(e+fx)}{15f\left(c^2-d^2\right)^2(c+d\sin(e+fx))^{3/2}}-\frac{2(bc-ad)\cos(e+fx)}{5f\left(c^2-d^2\right)(c+d\sin(e+f$$

```
[Out] (-2*(b*c - a*d)*Cos[e + f*x])/(5*(c^2 - d^2)*f*(c + d*Sin[e + f*x])^(5/2))
- (2*(3*b*c^2 - 8*a*c*d + 5*b*d^2)*Cos[e + f*x])/(15*(c^2 - d^2)^2*f*(c + d
*Sin[e + f*x])^(3/2)) - (2*(3*b*c^3 - 23*a*c^2*d + 29*b*c*d^2 - 9*a*d^3)*Co
s[e + f*x])/(15*(c^2 - d^2)^3*f*Sqrt[c + d*Sin[e + f*x]]) - (2*(3*b*c^3 - 2
3*a*c^2*d + 29*b*c*d^2 - 9*a*d^3)*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c +
d)]*Sqrt[c + d*Sin[e + f*x]])/(15*d*(c^2 - d^2)^3*f*Sqrt[(c + d*Sin[e + f*x
])/(c + d)]) + (2*(3*b*c^2 - 8*a*c*d + 5*b*d^2)*EllipticF[(e - Pi/2 + f*x)/
2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(15*d*(c^2 - d^2)^2*f
*Sqrt[c + d*Sin[e + f*x]])
```

Rubi [A] time = 0.526389, antiderivative size = 369, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {2754, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2\left(-23ac^2d-9ad^3+3bc^3+29bcd^2\right)\cos(e+fx)}{15f\left(c^2-d^2\right)^3\sqrt{c+d\sin(e+fx)}}-\frac{2\left(-8acd+3bc^2+5bd^2\right)\cos(e+fx)}{15f\left(c^2-d^2\right)^2(c+d\sin(e+fx))^{3/2}}-\frac{2(bc-ad)\cos(e+fx)}{5f\left(c^2-d^2\right)(c+d\sin(e+f$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])/(c + d*Sin[e + f*x])^(7/2),x]
```


```
[Out] (-2*(b*c - a*d)*Cos[e + f*x])/(5*(c^2 - d^2)*f*(c + d*Sin[e + f*x])^(5/2))
- (2*(3*b*c^2 - 8*a*c*d + 5*b*d^2)*Cos[e + f*x])/(15*(c^2 - d^2)^2*f*(c + d
*Sin[e + f*x])^(3/2)) - (2*(3*b*c^3 - 23*a*c^2*d + 29*b*c*d^2 - 9*a*d^3)*Co
s[e + f*x])/(15*(c^2 - d^2)^3*f*Sqrt[c + d*Sin[e + f*x]]) - (2*(3*b*c^3 - 2
3*a*c^2*d + 29*b*c*d^2 - 9*a*d^3)*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c +
d)]*Sqrt[c + d*Sin[e + f*x]])/(15*d*(c^2 - d^2)^3*f*Sqrt[(c + d*Sin[e + f*x
])/(c + d)]) + (2*(3*b*c^2 - 8*a*c*d + 5*b*d^2)*EllipticF[(e - Pi/2 + f*x)/
2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(15*d*(c^2 - d^2)^2*f
*Sqrt[c + d*Sin[e + f*x]])
```


Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{a+b\sin(e+fx)}{(c+d\sin(e+fx))^{7/2}}\,dx = -\frac{2(bc-ad)\cos(e+fx)}{5\left(c^2-d^2\right)f(c+d\sin(e+fx))^{5/2}} - \frac{2\int \frac{-\frac{5}{2}(ac-bd)-\frac{3}{2}(bc-ad)\sin(e+fx)}{(c+d\sin(e+fx))^{5/2}}\,dx}{5\left(c^2-d^2\right)}$$

$$= -\frac{2(bc-ad)\cos(e+fx)}{5\left(c^2-d^2\right)f(c+d\sin(e+fx))^{5/2}} - \frac{2\left(3bc^2-8acd+5bd^2\right)\cos(e+fx)}{15\left(c^2-d^2\right)^2 f(c+d\sin(e+fx))^{3/2}} + \frac{4\int \frac{\frac{3}{4}\left(5ac^2-\cdots\right.}{\cdots}}{\cdots}$$

$$= -\frac{2(bc-ad)\cos(e+fx)}{5\left(c^2-d^2\right)f(c+d\sin(e+fx))^{5/2}} - \frac{2\left(3bc^2-8acd+5bd^2\right)\cos(e+fx)}{15\left(c^2-d^2\right)^2 f(c+d\sin(e+fx))^{3/2}} - \frac{2\left(3bc^3-2\cdots\right.}{15\left(\cdots\right.}$$

$$= -\frac{2(bc-ad)\cos(e+fx)}{5\left(c^2-d^2\right)f(c+d\sin(e+fx))^{5/2}} - \frac{2\left(3bc^2-8acd+5bd^2\right)\cos(e+fx)}{15\left(c^2-d^2\right)^2 f(c+d\sin(e+fx))^{3/2}} - \frac{2\left(3bc^3-2\cdots\right.}{15\left(\cdots\right.}$$

$$= -\frac{2(bc-ad)\cos(e+fx)}{5\left(c^2-d^2\right)f(c+d\sin(e+fx))^{5/2}} - \frac{2\left(3bc^2-8acd+5bd^2\right)\cos(e+fx)}{15\left(c^2-d^2\right)^2 f(c+d\sin(e+fx))^{3/2}} - \frac{2\left(3bc^3-2\cdots\right.}{15\left(\cdots\right.}$$

$$= -\frac{2(bc-ad)\cos(e+fx)}{5\left(c^2-d^2\right)f(c+d\sin(e+fx))^{5/2}} - \frac{2\left(3bc^2-8acd+5bd^2\right)\cos(e+fx)}{15\left(c^2-d^2\right)^2 f(c+d\sin(e+fx))^{3/2}} - \frac{2\left(3bc^3-2\cdots\right.}{15\left(\cdots\right.}$$

Mathematica [A] time = 2.9782, size = 297, normalized size = 0.8

$$\frac{2\left(\frac{\cos(e+fx)\left(d^2\left(23ac^2d+9ad^3-3bc^3-29bcd^2\right)\sin^2(e+fx)+d\left(54ac^3d+10acd^3-60bc^2d^2-9bc^4+5bd^4\right)\sin(e+fx)+ad\left(-5c^2d^2+34c^4+3d^4\right)+b\left(-25c^3d^2-9c^5+2cd\cdots\right.\right.}{\left(c^2-d^2\right)^3}\right.}{15f(c+d\sin(e+fx)\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])/(c + d*Sin[e + f*x])^(7/2),x]
```

```
[Out] (2*((((3*b*c^3 - 23*a*c^2*d + 29*b*c*d^2 - 9*a*d^3)*EllipticE[(-2*e + Pi -
2*f*x)/4, (2*d)/(c + d)] - (c - d)*(3*b*c^2 - 8*a*c*d + 5*b*d^2)*EllipticF[
(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)])*((c + d*Sin[e + f*x])/(c + d))^(5/2)
)/((c - d)^3*d) + (Cos[e + f*x]*(a*d*(34*c^4 - 5*c^2*d^2 + 3*d^4) + b*(-9*c
^5 - 25*c^3*d^2 + 2*c*d^4) + d*(-9*b*c^4 + 54*a*c^3*d - 60*b*c^2*d^2 + 10*a
*c*d^3 + 5*b*d^4)*Sin[e + f*x] + d^2*(-3*b*c^3 + 23*a*c^2*d - 29*b*c*d^2 +
9*a*d^3)*Sin[e + f*x]^2))/(c^2 - d^2)^3))/(15*f*(c + d*Sin[e + f*x])^(5/2))
```

Maple [B] time = 6.319, size = 1049, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))/(c+d*sin(f*x+e))^(7/2),x)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((a*d-b*c)/d*(2/5/(c^2-d^2)/d^2*(-(
-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(sin(f*x+e)+c/d)^3+16/15*c/(c^2-d^2)^2
/d*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(sin(f*x+e)+c/d)^2+2/15*d*cos(f*
x+e)^2/(c^2-d^2)^3*(23*c^2+9*d^2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2
*(15*c^3+17*c*d^2)/(15*c^6-45*c^4*d^2+45*c^2*d^4-15*d^6)*(c/d-1)*((c+d*sin(
f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d
))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e)
)/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+2/15*d*(23*c^2+9*d^2)/(c^2-d^2)^3*(c/d-
1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x
+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*Ell
ipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*
sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+b/d*(2/3/(c^2-d^2)/d*(-(-d*
sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(sin(f*x+e)+c/d)^2+8/3*d*cos(f*x+e)^2/(c^
2-d^2)^2*c/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2*(3*c^2+d^2)/(3*c^4-6*c
^2*d^2+3*d^4)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d
))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^
(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+8/3*c*d
/(c^2-d^2)^2*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d)
)^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(
1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)
)+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))))/cos(f*x+
e)/(c+d*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\sin\left(fx+e\right)+a}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))/(c+d*sin(f*x+e))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)/(d*sin(f*x + e) + c)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b\sin\left(fx+e\right)+a\right)\sqrt{d\sin\left(fx+e\right)+c}}{d^4\cos\left(fx+e\right)^4+c^4+6\,c^2d^2+d^4-2\left(3\,c^2d^2+d^4\right)\cos\left(fx+e\right)^2-4\left(cd^3\cos\left(fx+e\right)^2-c^3d-cd^3\right)\sin\left(f\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))/(c+d*sin(f*x+e))^(7/2),x, algorithm="fricas")
```

```
[Out] integral((b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)/(d^4*cos(f*x + e)^4
+ c^4 + 6*c^2*d^2 + d^4 - 2*(3*c^2*d^2 + d^4)*cos(f*x + e)^2 - 4*(c*d^3*cos
(f*x + e)^2 - c^3*d - c*d^3)*sin(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))/(c+d*sin(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\sin\left(fx+e\right)+a}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))/(c+d*sin(f*x+e))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)/(d*sin(f*x + e) + c)^(7/2), x)
```

3.730 $\int (a+b\sin(e+fx))^2(c+d\sin(e+fx))^{5/2}\,dx$

Optimal. Leaf size=451

$$-\frac{4\left(84a^2cd^2+15abd\left(3c^2+5d^2\right)+b^2\left(-\left(5c^3-57cd^2\right)\right)\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{315df}+\frac{4\left(c^2-d^2\right)\left(-84a^2cd^2-45abc\right.}{}$$

```
[Out] (-4*(84*a^2*c*d^2 + 15*a*b*d*(3*c^2 + 5*d^2) - b^2*(5*c^3 - 57*c*d^2))*Cos[
e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(315*d*f) - (2*(7*(9*a^2 + 7*b^2)*d^2 -
10*b*c*(b*c - 9*a*d))*Cos[e + f*x]*(c + d*Sin[e + f*x])^(3/2))/(315*d*f) +
(4*b*(b*c - 9*a*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(5/2))/(63*d*f) - (2*b
^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(7/2))/(9*d*f) + (2*(21*a^2*d^2*(23*c^
2 + 9*d^2) + 30*a*b*d*(3*c^3 + 29*c*d^2) - b^2*(10*c^4 - 279*c^2*d^2 - 147*
d^4))*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]]
)/(315*d^2*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + (4*(c^2 - d^2)*(5*b^2*c^
3 - 45*a*b*c^2*d - 84*a^2*c*d^2 - 57*b^2*c*d^2 - 75*a*b*d^3)*EllipticF[(e -
 Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(315*d^2
*f*Sqrt[c + d*Sin[e + f*x]])
```

Rubi [A] time = 0.94531, antiderivative size = 451, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2791, 2753, 2752, 2663, 2661, 2655, 2653}

$$-\frac{4\left(84a^2cd^2+15abd\left(3c^2+5d^2\right)+b^2\left(-\left(5c^3-57cd^2\right)\right)\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{315df}+\frac{4\left(c^2-d^2\right)\left(-84a^2cd^2-45abc\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^(5/2),x]
```


```
[Out] (-4*(84*a^2*c*d^2 + 15*a*b*d*(3*c^2 + 5*d^2) - b^2*(5*c^3 - 57*c*d^2))*Cos[
e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(315*d*f) - (2*(7*(9*a^2 + 7*b^2)*d^2 -
10*b*c*(b*c - 9*a*d))*Cos[e + f*x]*(c + d*Sin[e + f*x])^(3/2))/(315*d*f) +
(4*b*(b*c - 9*a*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(5/2))/(63*d*f) - (2*b
^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(7/2))/(9*d*f) + (2*(21*a^2*d^2*(23*c^
2 + 9*d^2) + 30*a*b*d*(3*c^3 + 29*c*d^2) - b^2*(10*c^4 - 279*c^2*d^2 - 147*
d^4))*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]]
)/(315*d^2*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + (4*(c^2 - d^2)*(5*b^2*c^
3 - 45*a*b*c^2*d - 84*a^2*c*d^2 - 57*b^2*c*d^2 - 75*a*b*d^3)*EllipticF[(e -
 Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(315*d^2
*f*Sqrt[c + d*Sin[e + f*x]])
```


Rule 2791

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> -Simp[(d^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*
Simp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x],
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a
^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
```

```
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int (a+b\sin(e+fx))^2(c+d\sin(e+fx))^{5/2}\,dx = -\frac{2b^2\cos(e+fx)(c+d\sin(e+fx))^{7/2}}{9df} + \frac{2\int (c+d\sin(e+fx))^{5/2}\left(\frac{1}{2}(9a\right.}{}$$

$$= \frac{4b(bc-9ad)\cos(e+fx)(c+d\sin(e+fx))^{5/2}}{63df} - \frac{2b^2\cos(e+fx)(c+d\,\mathrm{si}}{9df}$$

$$= -\frac{2\left(7\left(9a^2+7b^2\right)d^2-10bc(bc-9ad)\right)\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{315df}$$

$$= -\frac{4\left(84a^2cd^2+15abd\left(3c^2+5d^2\right)-b^2\left(5c^3-57cd^2\right)\right)\cos(e+fx)\sqrt{c+d}}{315df}$$

$$= -\frac{4\left(84a^2cd^2+15abd\left(3c^2+5d^2\right)-b^2\left(5c^3-57cd^2\right)\right)\cos(e+fx)\sqrt{c+d}}{315df}$$

$$= -\frac{4\left(84a^2cd^2+15abd\left(3c^2+5d^2\right)-b^2\left(5c^3-57cd^2\right)\right)\cos(e+fx)\sqrt{c+d}}{315df}$$

$$= -\frac{4\left(84a^2cd^2+15abd\left(3c^2+5d^2\right)-b^2\left(5c^3-57cd^2\right)\right)\cos(e+fx)\sqrt{c+d}}{315df}$$

Mathematica [A] time = 1.74823, size = 382, normalized size = 0.85

$$\frac{8\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\left(\left(-21a^2d^2\left(23c^2+9d^2\right)-30abd\left(3c^3+29cd^2\right)+b^2\left(-279c^2d^2+10c^4-147d^4\right)\right)\left((c+d)E\left(\frac{1}{4}(-2e-2fx+\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (8*(-(d^2*(30*a*b*d*(27*c^2 + 5*d^2) + b^2*c*(155*c^2 + 261*d^2) + 21*a^2*(
15*c^3 + 17*c*d^2))*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]) + (-21
*a^2*d^2*(23*c^2 + 9*d^2) - 30*a*b*d*(3*c^3 + 29*c*d^2) + b^2*(10*c^4 - 279
*c^2*d^2 - 147*d^4))*((c + d)*EllipticE[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d
)] - c*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]))*Sqrt[(c + d*Sin[e
+ f*x])/(c + d)] - d*(c + d*Sin[e + f*x])*(2*(924*a^2*c*d^2 + 30*a*b*d*(36*
c^2 + 23*d^2) + b^2*(20*c^3 + 747*c*d^2))*Cos[e + f*x] - 10*b*d^2*(19*b*c +
 18*a*d)*Cos[3*(e + f*x)] + 2*d*(540*a*b*c*d + 126*a^2*d^2 + b^2*(150*c^2 +
 133*d^2))*Sin[2*(e + f*x)] - 35*b^2*d^3*Sin[4*(e + f*x)]))/(1260*d^2*f*Sqr
t[c + d*Sin[e + f*x]])
```

Maple [B] time = 5.909, size = 2112, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^2*(c+d*sin(f*x+e))^(5/2),x)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*(b^2*d^3*(-2/9/d*sin(f*x+e)^3*(-(-d
*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+16/63*c/d^2*sin(f*x+e)^2*(-(-d*sin(f*x+e
```

```
)-c)*cos(f*x+e)^2)^(1/2)-2/5*(7/9+16/21*c^2/d^2)/d*sin(f*x+e)*(-(-d*sin(f*x
+e)-c)*cos(f*x+e)^2)^(1/2)-2/315*(-64*c^3-62*c*d^2)/d^4*(-(-d*sin(f*x+e)-c)
*cos(f*x+e)^2)^(1/2)+2/315*(32*c^3+36*c*d^2)/d^3*(c/d-1)*((c+d*sin(f*x+e))/
(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)
/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))
^(1/2),((c-d)/(c+d))^(1/2))+2/315*(128*c^4+108*c^2*d^2+147*d^4)/d^4*(c/d-1)
*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e
)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*Ellip
ticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*si
n(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+(2*a*b*d^3+3*b^2*c*d^2)*(-2/7
/d*sin(f*x+e)^2*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+12/35*c/d^2*sin(f*x
+e)*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)-2/3*(5/7+24/35*c^2/d^2)/d*(-(-d
*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2*(-4/35*c^2/d^2+5/21)*(c/d-1)*((c+d*sin
(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-
d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e
))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+2/105*(-48*c^3-44*c*d^2)/d^3*(c/d-1)*(
(c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-
1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*Ellipti
cE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(
f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+(a^2*d^3+6*a*b*c*d^2+3*b^2*c^2*
d)*(-2/5/d*sin(f*x+e)*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+8/15*c/d^2*(-
(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+4/15*c/d*(c/d-1)*((c+d*sin(f*x+e))/(c
-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(
-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(
1/2),((c-d)/(c+d))^(1/2))+2*(3/5+8/15*c^2/d^2)*(c/d-1)*((c+d*sin(f*x+e))/(c
-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(
-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e)
)/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2
),((c-d)/(c+d))^(1/2))))+(3*a^2*c*d^2+6*a*b*c^2*d+b^2*c^3)*(-2/3/d*(-(-d*si
n(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2/3*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*
(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*
x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)
/(c+d))^(1/2))-4/3*c/d*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x
+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f
*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c
+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))
+2*(3*a^2*c^2*d+2*a*b*c^3)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin
(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*c
os(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d
)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2
)))+2*a^2*c^3*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d
))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^
(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))/cos(f*
x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)+a\right)^{2}\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2*(c+d*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)^2*(d*sin(f*x + e) + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b^2 d^2 \cos\left(fx+e\right)^4 + 4\,abcd + \left(a^2+b^2\right)c^2 + \left(a^2+b^2\right)d^2 - \left(b^2c^2 + 4\,abcd + \left(a^2+2\,b^2\right)d^2\right)\cos\left(fx+e\right)^2 + 2\left(a\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2*(c+d*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*d^2*cos(f*x + e)^4 + 4*a*b*c*d + (a^2 + b^2)*c^2 + (a^2 + b^2
)*d^2 - (b^2*c^2 + 4*a*b*c*d + (a^2 + 2*b^2)*d^2)*cos(f*x + e)^2 + 2*(a*b*c
^2 + a*b*d^2 + (a^2 + b^2)*c*d - (b^2*c*d + a*b*d^2)*cos(f*x + e)^2)*sin(f*
x + e))*sqrt(d*sin(f*x + e) + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**2*(c+d*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2*(c+d*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.731 $\int (a+b\sin(e+fx))^2(c+d\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=347

$$-\frac{2\left(c^2-d^2\right)\left(35a^2d^2+42abcd+b^2\left(-\left(6c^2-25d^2\right)\right)\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{105d^2f\sqrt{c+d\sin(e+fx)}}+\frac{4\left(70a^2cd^2+21abd\left(c^2+\right.\right.}{}$$

```
[Out] (-2*(5*(7*a^2 + 5*b^2)*d^2 - 6*b*c*(b*c - 7*a*d))*Cos[e + f*x]*Sqrt[c + d*S
in[e + f*x]])/(105*d*f) + (4*b*(b*c - 7*a*d)*Cos[e + f*x]*(c + d*Sin[e + f*
x])^(3/2))/(35*d*f) - (2*b^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(5/2))/(7*d*
f) + (4*(70*a^2*c*d^2 + 21*a*b*d*(c^2 + 3*d^2) - b^2*(3*c^3 - 41*c*d^2))*El
lipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(105*d
^2*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) - (2*(c^2 - d^2)*(42*a*b*c*d + 35*
a^2*d^2 - b^2*(6*c^2 - 25*d^2))*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)
]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(105*d^2*f*Sqrt[c + d*Sin[e + f*x]])
```

Rubi [A] time = 0.633734, antiderivative size = 347, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2791, 2753, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2\left(c^2-d^2\right)\left(35a^2d^2+42abcd+b^2\left(-\left(6c^2-25d^2\right)\right)\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{105d^2f\sqrt{c+d\sin(e+fx)}}+\frac{4\left(70a^2cd^2+21abd\left(c^2+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^(3/2),x]
```


```
[Out] (-2*(5*(7*a^2 + 5*b^2)*d^2 - 6*b*c*(b*c - 7*a*d))*Cos[e + f*x]*Sqrt[c + d*S
in[e + f*x]])/(105*d*f) + (4*b*(b*c - 7*a*d)*Cos[e + f*x]*(c + d*Sin[e + f*
x])^(3/2))/(35*d*f) - (2*b^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(5/2))/(7*d*
f) + (4*(70*a^2*c*d^2 + 21*a*b*d*(c^2 + 3*d^2) - b^2*(3*c^3 - 41*c*d^2))*El
lipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(105*d
^2*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) - (2*(c^2 - d^2)*(42*a*b*c*d + 35*
a^2*d^2 - b^2*(6*c^2 - 25*d^2))*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)
]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(105*d^2*f*Sqrt[c + d*Sin[e + f*x]])
```


Rule 2791

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> -Simp[(d^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*
Simp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x],
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a
^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int (a+b\sin(e+fx))^2(c+d\sin(e+fx))^{3/2}\,dx = -\frac{2b^2\cos(e+fx)(c+d\sin(e+fx))^{5/2}}{7df} + \frac{2\int (c+d\sin(e+fx))^{3/2}\left(\frac{1}{2}\left(7a\right.\right.}{}$$

$$= \frac{4b(bc-7ad)\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{35df} - \frac{2b^2\cos(e+fx)(c+d\,\mathrm{si}}{7df}$$

$$= -\frac{2\left(5\left(7a^2+5b^2\right)d^2-6bc(bc-7ad)\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{105df} + $$

$$= -\frac{2\left(5\left(7a^2+5b^2\right)d^2-6bc(bc-7ad)\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{105df} + $$

$$= -\frac{2\left(5\left(7a^2+5b^2\right)d^2-6bc(bc-7ad)\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{105df} + $$

$$= -\frac{2\left(5\left(7a^2+5b^2\right)d^2-6bc(bc-7ad)\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{105df} + $$

Mathematica [A] time = 1.17567, size = 292, normalized size = 0.84

$$4\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\left(d^2\left(-\left(35a^2\left(3c^2+d^2\right)+168abcd+b^2\left(51c^2+25d^2\right)\right)\right)F\left(\frac{1}{4}(-2e-2fx+\pi)|\frac{2d}{c+d}\right)-2\left(70a^2cd^2+21ab\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] (4*(-(d^2*(168*a*b*c*d + 35*a^2*(3*c^2 + d^2) + b^2*(51*c^2 + 25*d^2))*Elli
pticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]) - 2*(70*a^2*c*d^2 + 21*a*b*d*(
c^2 + 3*d^2) + b^2*(-3*c^3 + 41*c*d^2))*((c + d)*EllipticE[(-2*e + Pi - 2*f
*x)/4, (2*d)/(c + d)] - c*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]))
*Sqrt[(c + d*Sin[e + f*x])/(c + d)] - d*(c + d*Sin[e + f*x])*((336*a*b*c*d
+ 140*a^2*d^2 + b^2*(12*c^2 + 115*d^2))*Cos[e + f*x] + 3*b*d*(-5*b*d*Cos[3*
(e + f*x)] + 4*(4*b*c + 7*a*d)*Sin[2*(e + f*x)])))/(210*d^2*f*Sqrt[c + d*Si
n[e + f*x]])
```

Maple [B] time = 4.46, size = 1575, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^2*(c+d*sin(f*x+e))^(3/2),x)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*(b^2*d^2*(-2/7/d*sin(f*x+e)^2*(-(-d
*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+12/35*c/d^2*sin(f*x+e)*(-(-d*sin(f*x+e)-
c)*cos(f*x+e)^2)^(1/2)-2/3*(5/7+24/35*c^2/d^2)/d*(-(-d*sin(f*x+e)-c)*cos(f*
x+e)^2)^(1/2)+2*(-4/35*c^2/d^2+5/21)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)
*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f
*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d
)/(c+d))^(1/2))+2/105*(-48*c^3-44*c*d^2)/d^3*(c/d-1)*((c+d*sin(f*x+e))/(c-d
))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(
-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/
(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),
((c-d)/(c+d))^(1/2))))+(2*a*b*d^2+2*b^2*c*d)*(-2/5/d*sin(f*x+e)*(-(-d*sin(f
*x+e)-c)*cos(f*x+e)^2)^(1/2)+8/15*c/d^2*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(
1/2)+4/15*c/d*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d
))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^
(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+2*(3/5+
8/15*c^2/d^2)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d
))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^
(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2
))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+(a^2*d^2
+4*a*b*c*d+b^2*c^2)*(-2/3/d*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2/3*(c/
d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f
*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF((
(c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))-4/3*c/d*(c/d-1)*((c+d*si
n(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c
-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+
d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))
/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+2*(2*a^2*c*d+2*a*b*c^2)*(c/d-1)*((c+d*
sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/
(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((
c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e
```

```
))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))+2*c^2*a^2*(c/d-1)*((c+d*sin(f*x+e))/(
c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/
(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^
(1/2),((c-d)/(c+d))^(1/2)))/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)+a\right)^2\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2*(c+d*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)^2*(d*sin(f*x + e) + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(2\,abd-\left(b^2c+2\,abd\right)\cos\left(fx+e\right)^2+\left(a^2+b^2\right)c-\left(b^2d\cos\left(fx+e\right)^2-2\,abc-\left(a^2+b^2\right)d\right)\sin\left(fx+e\right)\right)\sqrt{d\sin\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2*(c+d*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((2*a*b*d - (b^2*c + 2*a*b*d)*cos(f*x + e)^2 + (a^2 + b^2)*c - (b^2
*d*cos(f*x + e)^2 - 2*a*b*c - (a^2 + b^2)*d)*sin(f*x + e))*sqrt(d*sin(f*x +
 e) + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a+b\sin\left(e+fx\right)\right)^2\left(c+d\sin\left(e+fx\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**2*(c+d*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral((a + b*sin(e + f*x))**2*(c + d*sin(e + f*x))**(3/2), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2*(c+d*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.732 $\int (a+b\sin(e+fx))^2\sqrt{c+d\sin(e+fx)}\,dx$

Optimal. Leaf size=254

$$\frac{2\left(3d^2\left(5a^2+3b^2\right)-2bc(bc-5ad)\right)\sqrt{c+d\sin(e+fx)}E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{15d^2f\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}+\frac{4b\left(c^2-d^2\right)(bc-5ad)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F}{15d^2f\sqrt{c+d\sin(e}}$$

```
[Out] (4*b*(b*c - 5*a*d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(15*d*f) - (2*b^2
*Cos[e + f*x]*(c + d*Sin[e + f*x])^(3/2))/(5*d*f) + (2*(3*(5*a^2 + 3*b^2)*d
^2 - 2*b*c*(b*c - 5*a*d))*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt
[c + d*Sin[e + f*x]])/(15*d^2*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + (4*b*
(b*c - 5*a*d)*(c^2 - d^2)*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt
[(c + d*Sin[e + f*x])/(c + d)])/(15*d^2*f*Sqrt[c + d*Sin[e + f*x]])
```

Rubi [A] time = 0.411492, antiderivative size = 254, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2791, 2753, 2752, 2663, 2661, 2655, 2653}

$$\frac{2\left(3d^2\left(5a^2+3b^2\right)-2bc(bc-5ad)\right)\sqrt{c+d\sin(e+fx)}E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{15d^2f\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}+\frac{4b\left(c^2-d^2\right)(bc-5ad)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F}{15d^2f\sqrt{c+d\sin(e}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^2*Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] (4*b*(b*c - 5*a*d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(15*d*f) - (2*b^2
*Cos[e + f*x]*(c + d*Sin[e + f*x])^(3/2))/(5*d*f) + (2*(3*(5*a^2 + 3*b^2)*d
^2 - 2*b*c*(b*c - 5*a*d))*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt
[c + d*Sin[e + f*x]])/(15*d^2*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + (4*b*
(b*c - 5*a*d)*(c^2 - d^2)*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt
[(c + d*Sin[e + f*x])/(c + d)])/(15*d^2*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2791

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> -Simp[(d^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*
Simp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x],
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a
^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
```

```
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int (a+b\sin(e+fx))^2\sqrt{c+d\sin(e+fx)}\,dx = -\frac{2b^2\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{5df} + \frac{2\int\sqrt{c+d\sin(e+fx)}\left(\frac{1}{2}\left(5a^2+\right.\right.}{}$$

$$= \frac{4b(bc-5ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15df} - \frac{2b^2\cos(e+fx)(c+d\sin(e}{5df}$$

$$= \frac{4b(bc-5ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15df} - \frac{2b^2\cos(e+fx)(c+d\sin(e}{5df}$$

$$= \frac{4b(bc-5ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15df} - \frac{2b^2\cos(e+fx)(c+d\sin(e}{5df}$$

$$= \frac{4b(bc-5ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15df} - \frac{2b^2\cos(e+fx)(c+d\sin(e}{5df}$$

Mathematica [A] time = 0.895404, size = 214, normalized size = 0.84

$$\frac{2\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\left(\left(-15a^2d^2-10abcd+b^2\left(2c^2-9d^2\right)\right)\left((c+d)E\left(\frac{1}{4}(-2e-2fx+\pi)|\frac{2d}{c+d}\right)-cF\left(\frac{1}{4}(-2e-2fx+\pi)|\frac{2d}{c+d}\right)\right)-a}{15d^2f\sqrt{c+d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^2*Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] (2*(-(d^2*(15*a^2*c + 7*b^2*c + 10*a*b*d)*EllipticF[(-2*e + Pi - 2*f*x)/4,
(2*d)/(c + d)]) + (-10*a*b*c*d - 15*a^2*d^2 + b^2*(2*c^2 - 9*d^2))*((c + d)
*EllipticE[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)] - c*EllipticF[(-2*e + Pi -
 2*f*x)/4, (2*d)/(c + d)]))*Sqrt[(c + d*Sin[e + f*x])/(c + d)] - 2*b*d*Cos[
e + f*x]*(c + d*Sin[e + f*x])*(b*c + 10*a*d + 3*b*d*Sin[e + f*x]))/(15*d^2*
f*Sqrt[c + d*Sin[e + f*x]])
```

Maple [B] time = 3.353, size = 1100, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^2*(c+d*sin(f*x+e))^(1/2),x)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*(b^2*d*(-2/5/d*sin(f*x+e)*(-(-d*sin
(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+8/15*c/d^2*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)
^(1/2)+4/15*c/d*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c
+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2
)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+2*(3/
5+8/15*c^2/d^2)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c
+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2
)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1
/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+(2*a*b
*d+b^2*c)*(-2/3/d*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2/3*(c/d-1)*((c+d
*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d
/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f
*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))-4/3*c/d*(c/d-1)*((c+d*sin(f*x+e))/
(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)
/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+
e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1
/2),((c-d)/(c+d))^(1/2))))+2*(a^2*d+2*a*b*c)*(c/d-1)*((c+d*sin(f*x+e))/(c-d
))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(
-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/
(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),
((c-d)/(c+d))^(1/2)))+2*a^2*c*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-
sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c
)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d)
)^(1/2)))/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)+a\right)^2\sqrt{d\sin\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2*(c+d*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)^2*sqrt(d*sin(f*x + e) + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(b^2\cos\left(fx+e\right)^2-2\,ab\sin\left(fx+e\right)-a^2-b^2\right)\sqrt{d\sin\left(fx+e\right)+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2*(c+d*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-(b^2*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2)*sqrt(d*sin(
f*x + e) + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\sin\left(e+fx\right)\right)^2\sqrt{c+d\sin\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**2*(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((a + b*sin(e + f*x))**2*sqrt(c + d*sin(e + f*x)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2*(c+d*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.733 $\int \frac{(a+b\sin(e+fx))^2}{\sqrt{c+d\sin(e+fx)}}\,dx$

Optimal. Leaf size=203

$$\frac{2\left(d^2\left(3a^2+b^2\right)+2bc(bc-3ad)\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3d^2f\sqrt{c+d\sin(e+fx)}}-\frac{4b(bc-3ad)\sqrt{c+d\sin(e+fx)}E\left(\frac{1}{2}\left(e+fx\right.\right.}{3d^2f\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

```
[Out] (-2*b^2*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(3*d*f) - (4*b*(b*c - 3*a*d)
*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(3*
d^2*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + (2*((3*a^2 + b^2)*d^2 + 2*b*c*(
b*c - 3*a*d))*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[
e + f*x])/(c + d)])/(3*d^2*f*Sqrt[c + d*Sin[e + f*x]])
```

Rubi [A] time = 0.28484, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2791, 2752, 2663, 2661, 2655, 2653}

$$\frac{2\left(d^2\left(3a^2+b^2\right)+2bc(bc-3ad)\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3d^2f\sqrt{c+d\sin(e+fx)}}-\frac{4b(bc-3ad)\sqrt{c+d\sin(e+fx)}E\left(\frac{1}{2}\left(e+fx\right.\right.}{3d^2f\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^2/Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] (-2*b^2*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(3*d*f) - (4*b*(b*c - 3*a*d)
*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(3*
d^2*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + (2*((3*a^2 + b^2)*d^2 + 2*b*c*(
b*c - 3*a*d))*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[
e + f*x])/(c + d)])/(3*d^2*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2791

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> -Simp[(d^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*
Simp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x],
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a
^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{(a+b\sin(e+fx))^2}{\sqrt{c+d\sin(e+fx)}}\,dx = -\frac{2b^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3df} + \frac{2\int \frac{\frac{1}{2}\left(3a^2+b^2\right)d-b(bc-3ad)\sin(e+fx)}{\sqrt{c+d\sin(e+fx)}}\,dx}{3d}$$

$$= -\frac{2b^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3df} - \frac{(2b(bc-3ad))\int \sqrt{c+d\sin(e+fx)}\,dx}{3d^2} + \frac{1}{3}\left(3a^2+b^2\right.$$

$$= -\frac{2b^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3df} - \frac{\left(2b(bc-3ad)\sqrt{c+d\sin(e+fx)}\right)\int \sqrt{\frac{c}{c+d}+\frac{d\sin(e+fx)}{c+d}}}{3d^2\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

$$= -\frac{2b^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3df} - \frac{4b(bc-3ad)E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2d}{c+d}\right)\sqrt{c+d\sin(e+f}}{3d^2f\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

Mathematica [A] time = 0.905828, size = 173, normalized size = 0.85

$$-\frac{2\left(\left(3a^2d^2-6abcd+b^2\left(2c^2+d^2\right)\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{4}(-2e-2fx+\pi)|\frac{2d}{c+d}\right)-2b(c+d)(bc-3ad)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}E\left(\frac{1}{4}(-2e\right.\right.}{3d^2f\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^2/Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] (-2*(b^2*d*Cos[e + f*x]*(c + d*Sin[e + f*x]) - 2*b*(c + d)*(b*c - 3*a*d)*El
lipticE[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c
+ d)] + (-6*a*b*c*d + 3*a^2*d^2 + b^2*(2*c^2 + d^2))*EllipticF[(-2*e + Pi -
 2*f*x)/4, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)]))/(3*d^2*f*Sqr
t[c + d*Sin[e + f*x]])
```

Maple [B] time = 2.608, size = 695, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^(1/2),x)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*(b^2*(-2/3/d*(-(-d*sin(f*x+e)-c)*co
s(f*x+e)^2)^(1/2)+2/3*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+
e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*
x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))
-4/3*c/d*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1
/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)
*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+El
lipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+4*a*b*(c/d-1)
*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e
)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*Ellip
ticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*si
n(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))+2*a^2*(c/d-1)*((c+d*sin(f*x+e)
)/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/
2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d
))^(1/2),((c-d)/(c+d))^(1/2)))/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\sin\left(fx+e\right)+a\right)^2}{\sqrt{d\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)^2/sqrt(d*sin(f*x + e) + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{b^2\cos\left(fx+e\right)^2-2\,ab\sin\left(fx+e\right)-a^2-b^2}{\sqrt{d\sin\left(fx+e\right)+c}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-(b^2*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2)/sqrt(d*sin(
f*x + e) + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+b\sin\left(e+fx\right)\right)^2}{\sqrt{c+d\sin\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**2/(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((a + b*sin(e + f*x))**2/sqrt(c + d*sin(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\sin\left(fx+e\right)+a\right)^2}{\sqrt{d\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)^2/sqrt(d*sin(f*x + e) + c), x)
```

3.734 $\int \frac{(a+b\sin(e+fx))^2}{(c+d\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=228

$$\frac{2\left(d^2\left(a^2-b^2\right)-2abcd+2b^2c^2\right)\sqrt{c+d\sin(e+fx)}E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{d^2f\left(c^2-d^2\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}+\frac{2(bc-ad)^2\cos(e+fx)}{df\left(c^2-d^2\right)\sqrt{c+d\sin(e+fx)}}-\frac{4b(bc$$

```
[Out] (2*(b*c - a*d)^2*Cos[e + f*x])/(d*(c^2 - d^2)*f*Sqrt[c + d*Sin[e + f*x]]) +
 (2*(2*b^2*c^2 - 2*a*b*c*d + (a^2 - b^2)*d^2)*EllipticE[(e - Pi/2 + f*x)/2,
 (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(d^2*(c^2 - d^2)*f*Sqrt[(c + d*Si
n[e + f*x])/(c + d)]) - (4*b*(b*c - a*d)*EllipticF[(e - Pi/2 + f*x)/2, (2*d
)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(d^2*f*Sqrt[c + d*Sin[e + f*
x]])
```

Rubi [A] time = 0.307907, antiderivative size = 228, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2790, 2752, 2663, 2661, 2655, 2653}

$$\frac{2\left(d^2\left(a^2-b^2\right)-2abcd+2b^2c^2\right)\sqrt{c+d\sin(e+fx)}E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{d^2f\left(c^2-d^2\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}+\frac{2(bc-ad)^2\cos(e+fx)}{df\left(c^2-d^2\right)\sqrt{c+d\sin(e+fx)}}-\frac{4b(bc$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^2/(c + d*Sin[e + f*x])^(3/2),x]
```


```
[Out] (2*(b*c - a*d)^2*Cos[e + f*x])/(d*(c^2 - d^2)*f*Sqrt[c + d*Sin[e + f*x]]) +
 (2*(2*b^2*c^2 - 2*a*b*c*d + (a^2 - b^2)*d^2)*EllipticE[(e - Pi/2 + f*x)/2,
 (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(d^2*(c^2 - d^2)*f*Sqrt[(c + d*Si
n[e + f*x])/(c + d)]) - (4*b*(b*c - a*d)*EllipticF[(e - Pi/2 + f*x)/2, (2*d
)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(d^2*f*Sqrt[c + d*Sin[e + f*
x]])
```


Rule 2790

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos[e +
 f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] - Dist[1/
(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m + 1)*(2
*b*c*d - a*(c^2 + d^2)) + (a^2*d^2 - 2*a*b*c*d*(m + 2) + b^2*(d^2*(m + 1) +
 c^2*(m + 2)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
```

```
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] && !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{(a+b\sin(e+fx))^2}{(c+d\sin(e+fx))^{3/2}}\,dx = \frac{2(bc-ad)^2\cos(e+fx)}{d\left(c^2-d^2\right)f\sqrt{c+d\sin(e+fx)}} + \frac{2\int \frac{\frac{1}{2}d\left(a^2c+b^2c-2abd\right)+\frac{1}{2}\left(2b^2c^2-2abcd+\left(a^2-b^2\right)d^2\right)\sin(e+fx)}{\sqrt{c+d\sin(e+fx)}}\,dx}{d\left(c^2-d^2\right)}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)}{d\left(c^2-d^2\right)f\sqrt{c+d\sin(e+fx)}} - \frac{(2b(bc-ad))\int \frac{1}{\sqrt{c+d\sin(e+fx)}}\,dx}{d^2} - \frac{\left(2abcd-a^2d^2-b^2\left(2c\right.\right.}{d}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)}{d\left(c^2-d^2\right)f\sqrt{c+d\sin(e+fx)}} - \frac{\left(\left(2abcd-a^2d^2-b^2\left(2c^2-d^2\right)\right)\sqrt{c+d\sin(e+fx)}\right)\int \sqrt{}}{d^2\left(c^2-d^2\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)}{d\left(c^2-d^2\right)f\sqrt{c+d\sin(e+fx)}} - \frac{2\left(2abcd-a^2d^2-b^2\left(2c^2-d^2\right)\right)E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2d}{c+d}\right)}{d^2\left(c^2-d^2\right)f\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

Mathematica [A] time = 0.865774, size = 172, normalized size = 0.75

$$\frac{2\left(\frac{\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\left(\left(-a^2d^2+2abcd+b^2\left(d^2-2c^2\right)\right)E\left(\frac{1}{4}(-2e-2fx+\pi)|\frac{2d}{c+d}\right)+2b(c-d)(bc-ad)F\left(\frac{1}{4}(-2e-2fx+\pi)|\frac{2d}{c+d}\right)\right)}{d(c-d)}+\frac{(bc-ad)^2\cos(e+fx)}{c^2-d^2}\right)}{df\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^2/(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] (2*(((b*c - a*d)^2*Cos[e + f*x])/(c^2 - d^2) + (((2*a*b*c*d - a^2*d^2 + b^2
*(-2*c^2 + d^2))*EllipticE[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)] + 2*b*(c -
 d)*(b*c - a*d)*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)])*Sqrt[(c +
d*Sin[e + f*x])/(c + d)])/((c - d)*d)))/(d*f*Sqrt[c + d*Sin[e + f*x]])
```

Maple [B] time = 3.56, size = 888, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^(3/2),x)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*(b/d^2*(2*b*d*(c/d-1)*((c+d*sin(f*x
+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^
(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin
(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d
))^(1/2),((c-d)/(c+d))^(1/2)))+4*d*a*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)
*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f
*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d
)/(c+d))^(1/2))-2*c*b*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+
e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*
x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))
)+(a^2*d^2-2*a*b*c*d+b^2*c^2)/d^2*(2*d*cos(f*x+e)^2/(c^2-d^2)/(-(-d*sin(f*x
+e)-c)*cos(f*x+e)^2)^(1/2)+2*c/(c^2-d^2)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(
1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*s
in(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),(
(c-d)/(c+d))^(1/2))+2/(c^2-d^2)*d*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d
*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+
e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2
),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d
))^(1/2)))))/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\sin\left(fx+e\right)+a\right)^2}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)^2/(d*sin(f*x + e) + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2\cos\left(fx+e\right)^2-2\,ab\sin\left(fx+e\right)-a^2-b^2\right)\sqrt{d\sin\left(fx+e\right)+c}}{d^2\cos\left(fx+e\right)^2-2\,cd\sin\left(fx+e\right)-c^2-d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2)*sqrt(d*sin(f
*x + e) + c)/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**2/(c+d*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\sin\left(fx+e\right)+a\right)^2}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)^2/(d*sin(f*x + e) + c)^(3/2), x)
```

3.735 $\int \frac{(a+b\sin(e+fx))^2}{(c+d\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=329

$$\frac{2\left(-a^2d^2+2abcd+b^2\left(2c^2-3d^2\right)\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3d^2f\left(c^2-d^2\right)\sqrt{c+d\sin(e+fx)}}+\frac{2(bc-ad)^2\cos(e+fx)}{3df\left(c^2-d^2\right)(c+d\sin(e+fx))^{3/2}}-\frac{4(2a}{}$$

```
[Out] (2*(b*c - a*d)^2*Cos[e + f*x])/(3*d*(c^2 - d^2)*f*(c + d*Sin[e + f*x])^(3/2
)) - (4*(b*c - a*d)*(2*a*c*d + b*(c^2 - 3*d^2))*Cos[e + f*x])/(3*d*(c^2 - d
^2)^2*f*Sqrt[c + d*Sin[e + f*x]]) - (4*(b*c - a*d)*(2*a*c*d + b*(c^2 - 3*d^
2))*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/
(3*d^2*(c^2 - d^2)^2*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + (2*(2*a*b*c*d
- a^2*d^2 + b^2*(2*c^2 - 3*d^2))*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d
)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(3*d^2*(c^2 - d^2)*f*Sqrt[c + d*Sin[
e + f*x]])
```

Rubi [A] time = 0.499152, antiderivative size = 329, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2790, 2754, 2752, 2663, 2661, 2655, 2653}

$$\frac{2\left(-a^2d^2+2abcd+b^2\left(2c^2-3d^2\right)\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3d^2f\left(c^2-d^2\right)\sqrt{c+d\sin(e+fx)}}+\frac{2(bc-ad)^2\cos(e+fx)}{3df\left(c^2-d^2\right)(c+d\sin(e+fx))^{3/2}}-\frac{4(2a}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^2/(c + d*Sin[e + f*x])^(5/2),x]
```


```
[Out] (2*(b*c - a*d)^2*Cos[e + f*x])/(3*d*(c^2 - d^2)*f*(c + d*Sin[e + f*x])^(3/2
)) - (4*(b*c - a*d)*(2*a*c*d + b*(c^2 - 3*d^2))*Cos[e + f*x])/(3*d*(c^2 - d
^2)^2*f*Sqrt[c + d*Sin[e + f*x]]) - (4*(b*c - a*d)*(2*a*c*d + b*(c^2 - 3*d^
2))*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/
(3*d^2*(c^2 - d^2)^2*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + (2*(2*a*b*c*d
- a^2*d^2 + b^2*(2*c^2 - 3*d^2))*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d
)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(3*d^2*(c^2 - d^2)*f*Sqrt[c + d*Sin[
e + f*x]])
```


Rule 2790

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos[e +
 f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] - Dist[1/
(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m + 1)*(2
*b*c*d - a*(c^2 + d^2)) + (a^2*d^2 - 2*a*b*c*d*(m + 2) + b^2*(d^2*(m + 1) +
 c^2*(m + 2)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
```

```
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{(a+b\sin(e+fx))^2}{(c+d\sin(e+fx))^{5/2}}\,dx = \frac{2(bc-ad)^2\cos(e+fx)}{3d\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}} + \frac{2\int \frac{\frac{3}{2}d\left(a^2c+b^2c-2abd\right)+\frac{1}{2}\left(2b^2c^2+2abcd-\left(a^2+3b^2\right)d^2\right)\sin(e+fx)}{(c+d\sin(e+fx))^{3/2}}\,dx}{3d\left(c^2-d^2\right)\cdots}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)}{3d\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}} - \frac{4(bc-ad)\left(2acd+b\left(c^2-3d^2\right)\right)\cos(e+fx)}{3d\left(c^2-d^2\right)^2 f\sqrt{c+d\sin(e+fx)}} - \frac{4\int \frac{\frac{1}{4}\cdots}{\cdots}}{\cdots}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)}{3d\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}} - \frac{4(bc-ad)\left(2acd+b\left(c^2-3d^2\right)\right)\cos(e+fx)}{3d\left(c^2-d^2\right)^2 f\sqrt{c+d\sin(e+fx)}} - \frac{\left(2(bc\cdots\right.}{\cdots}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)}{3d\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}} - \frac{4(bc-ad)\left(2acd+b\left(c^2-3d^2\right)\right)\cos(e+fx)}{3d\left(c^2-d^2\right)^2 f\sqrt{c+d\sin(e+fx)}} - \frac{\left(2(bc\cdots\right.}{\cdots}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)}{3d\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}} - \frac{4(bc-ad)\left(2acd+b\left(c^2-3d^2\right)\right)\cos(e+fx)}{3d\left(c^2-d^2\right)^2 f\sqrt{c+d\sin(e+fx)}} - \frac{4(bc-\cdots}{\cdots}$$

Mathematica [A] time = 2.40562, size = 302, normalized size = 0.92

$$\frac{2\left(\frac{(-c-d\sin(e+fx))\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\left(d^2\left(a^2\left(3c^2+d^2\right)-8abcd+b^2\left(c^2+3d^2\right)\right)F\left(\frac{1}{4}(-2e-2fx+\pi)|\frac{2d}{c+d}\right)-2\left(-2a^2cd^2+abd\left(c^2+3d^2\right)+b^2\left(c^3-3cd^2\right)\right)\left((c+d)E\left(\frac{1}{4}(-2e-2f\right.\right.\right.}{(c-d)^2(c+d)^2}}{3d^2f(c+d\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^2/(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (2*(((d^2*(-8*a*b*c*d + a^2*(3*c^2 + d^2) + b^2*(c^2 + 3*d^2))*EllipticF[(-
2*e + Pi - 2*f*x)/4, (2*d)/(c + d)] - 2*(-2*a^2*c*d^2 + a*b*d*(c^2 + 3*d^2)
 + b^2*(c^3 - 3*c*d^2))*((c + d)*EllipticE[(-2*e + Pi - 2*f*x)/4, (2*d)/(c
+ d)] - c*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]))*(-c - d*Sin[e +
 f*x])*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/((c - d)^2*(c + d)^2) - (d*(-(b*
c) + a*d)*Cos[e + f*x]*(-(b*c^3) - 5*a*c^2*d + 5*b*c*d^2 + a*d^3 - 2*d*(2*a
*c*d + b*(c^2 - 3*d^2))*Sin[e + f*x]))/(c^2 - d^2)^2))/(3*d^2*f*(c + d*Sin[
e + f*x])^(3/2))
```

Maple [B] time = 4.746, size = 1043, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^(5/2),x)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*(2*b^2/d^2*(c/d-1)*((c+d*sin(f*x+e)
)/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/
2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d
))^(1/2),((c-d)/(c+d))^(1/2))+2*b/d^2*(a*d-b*c)*(2*d*cos(f*x+e)^2/(c^2-d^2)
/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2*c/(c^2-d^2)*(c/d-1)*((c+d*sin(f*
x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))
^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/
(c-d))^(1/2),((c-d)/(c+d))^(1/2))+2/(c^2-d^2)*d*(c/d-1)*((c+d*sin(f*x+e))/(
c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/
(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e
))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/
2),((c-d)/(c+d))^(1/2))))+1/d^2*(a^2*d^2-2*a*b*c*d+b^2*c^2)*(2/3/(c^2-d^2)/
d*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(sin(f*x+e)+c/d)^2+8/3*d*cos(f*x+
e)^2/(c^2-d^2)^2*c/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2*(3*c^2+d^2)/(3
*c^4-6*c^2*d^2+3*d^4)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+
e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*
x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))
+8/3*c*d/(c^2-d^2)^2*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e
))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x
+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d
))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))))/
cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\sin\left(fx+e\right)+a\right)^2}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)^2/(d*sin(f*x + e) + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2\cos\left(fx+e\right)^2-2\,ab\sin\left(fx+e\right)-a^2-b^2\right)\sqrt{d\sin\left(fx+e\right)+c}}{3\,cd^2\cos\left(fx+e\right)^2-c^3-3\,cd^2+\left(d^3\cos\left(fx+e\right)^2-3\,c^2d-d^3\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2)*sqrt(d*sin(f
*x + e) + c)/(3*c*d^2*cos(f*x + e)^2 - c^3 - 3*c*d^2 + (d^3*cos(f*x + e)^2
- 3*c^2*d - d^3)*sin(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**2/(c+d*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\sin\left(fx+e\right)+a\right)^2}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)^2/(d*sin(f*x + e) + c)^(5/2), x)
```

3.736 $\int \frac{(a+b\sin(e+fx))^2}{(c+d\sin(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=460

$$\frac{2\left(a^2d^2\left(23c^2+9d^2\right)-ab\left(6c^3d+58cd^3\right)+b^2\left(-\left(-19c^2d^2+2c^4-15d^4\right)\right)\right)\cos(e+fx)}{15df\left(c^2-d^2\right)^3\sqrt{c+d\sin(e+fx)}}+\frac{2\left(a^2d^2\left(23c^2+9d^2\right)-ab\left(6\right.\right.}{}$$

```
[Out] (2*(b*c - a*d)^2*Cos[e + f*x])/(5*d*(c^2 - d^2)*f*(c + d*Sin[e + f*x])^(5/2
)) - (4*(b*c - a*d)*(4*a*c*d + b*(c^2 - 5*d^2))*Cos[e + f*x])/(15*d*(c^2 -
d^2)^2*f*(c + d*Sin[e + f*x])^(3/2)) + (2*(a^2*d^2*(23*c^2 + 9*d^2) - a*b*(
6*c^3*d + 58*c*d^3) - b^2*(2*c^4 - 19*c^2*d^2 - 15*d^4))*Cos[e + f*x])/(15*
d*(c^2 - d^2)^3*f*Sqrt[c + d*Sin[e + f*x]]) + (2*(a^2*d^2*(23*c^2 + 9*d^2)
- a*b*(6*c^3*d + 58*c*d^3) - b^2*(2*c^4 - 19*c^2*d^2 - 15*d^4))*EllipticE[(
e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(15*d^2*(c^2 -
d^2)^3*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + (4*(b*c - a*d)*(4*a*c*d + b*
(c^2 - 5*d^2))*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin
[e + f*x])/(c + d)])/(15*d^2*(c^2 - d^2)^2*f*Sqrt[c + d*Sin[e + f*x]])
```

Rubi [A] time = 0.862603, antiderivative size = 460, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2790, 2754, 2752, 2663, 2661, 2655, 2653}

$$\frac{2\left(a^2d^2\left(23c^2+9d^2\right)-ab\left(6c^3d+58cd^3\right)+b^2\left(-\left(-19c^2d^2+2c^4-15d^4\right)\right)\right)\cos(e+fx)}{15df\left(c^2-d^2\right)^3\sqrt{c+d\sin(e+fx)}}+\frac{2\left(a^2d^2\left(23c^2+9d^2\right)-ab\left(6\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^2/(c + d*Sin[e + f*x])^(7/2),x]
```


```
[Out] (2*(b*c - a*d)^2*Cos[e + f*x])/(5*d*(c^2 - d^2)*f*(c + d*Sin[e + f*x])^(5/2
)) - (4*(b*c - a*d)*(4*a*c*d + b*(c^2 - 5*d^2))*Cos[e + f*x])/(15*d*(c^2 -
d^2)^2*f*(c + d*Sin[e + f*x])^(3/2)) + (2*(a^2*d^2*(23*c^2 + 9*d^2) - a*b*(
6*c^3*d + 58*c*d^3) - b^2*(2*c^4 - 19*c^2*d^2 - 15*d^4))*Cos[e + f*x])/(15*
d*(c^2 - d^2)^3*f*Sqrt[c + d*Sin[e + f*x]]) + (2*(a^2*d^2*(23*c^2 + 9*d^2)
- a*b*(6*c^3*d + 58*c*d^3) - b^2*(2*c^4 - 19*c^2*d^2 - 15*d^4))*EllipticE[(
e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(15*d^2*(c^2 -
d^2)^3*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + (4*(b*c - a*d)*(4*a*c*d + b*
(c^2 - 5*d^2))*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin
[e + f*x])/(c + d)])/(15*d^2*(c^2 - d^2)^2*f*Sqrt[c + d*Sin[e + f*x]])
```


Rule 2790

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos[e +
 f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] - Dist[1/
(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m + 1)*(2
*b*c*d - a*(c^2 + d^2)) + (a^2*d^2 - 2*a*b*c*d*(m + 2) + b^2*(d^2*(m + 1) +
 c^2*(m + 2)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
```

```
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{(a+b\sin(e+fx))^2}{(c+d\sin(e+fx))^{7/2}}\,dx = \frac{2(bc-ad)^2\cos(e+fx)}{5d\left(c^2-d^2\right)f(c+d\sin(e+fx))^{5/2}} + \frac{2\int \frac{\frac{5}{2}d\left(\left(a^2+b^2\right)c-2abd\right)+\frac{1}{2}\left(6abcd-3a^2d^2+b^2\left(2c^2-5d^2\right)\right)\sin(e+f}{(c+d\sin(e+fx))^{5/2}}}{5d\left(c^2-d^2\right)}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)}{5d\left(c^2-d^2\right)f(c+d\sin(e+fx))^{5/2}} - \frac{4(bc-ad)\left(4acd+b\left(c^2-5d^2\right)\right)\cos(e+fx)}{15d\left(c^2-d^2\right)^2 f(c+d\sin(e+fx))^{3/2}} - \frac{4\int}{}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)}{5d\left(c^2-d^2\right)f(c+d\sin(e+fx))^{5/2}} - \frac{4(bc-ad)\left(4acd+b\left(c^2-5d^2\right)\right)\cos(e+fx)}{15d\left(c^2-d^2\right)^2 f(c+d\sin(e+fx))^{3/2}} + \frac{2(}{}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)}{5d\left(c^2-d^2\right)f(c+d\sin(e+fx))^{5/2}} - \frac{4(bc-ad)\left(4acd+b\left(c^2-5d^2\right)\right)\cos(e+fx)}{15d\left(c^2-d^2\right)^2 f(c+d\sin(e+fx))^{3/2}} + \frac{2(}{}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)}{5d\left(c^2-d^2\right)f(c+d\sin(e+fx))^{5/2}} - \frac{4(bc-ad)\left(4acd+b\left(c^2-5d^2\right)\right)\cos(e+fx)}{15d\left(c^2-d^2\right)^2 f(c+d\sin(e+fx))^{3/2}} + \frac{2(}{}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)}{5d\left(c^2-d^2\right)f(c+d\sin(e+fx))^{5/2}} - \frac{4(bc-ad)\left(4acd+b\left(c^2-5d^2\right)\right)\cos(e+fx)}{15d\left(c^2-d^2\right)^2 f(c+d\sin(e+fx))^{3/2}} + \frac{2(}{}$$

Mathematica [A] time = 4.96598, size = 424, normalized size = 0.92

$$2\left(\frac{d\cos(e+fx)\left(-2\left(c^2-d^2\right)\left(-4a^2cd^2+abd\left(3c^2+5d^2\right)+b^2\left(c^3-5cd^2\right)\right)(c+d\sin(e+fx))-\left(-a^2d^2\left(23c^2+9d^2\right)+ab\left(6c^3d+58cd^3\right)+b^2\left(-19c^2d^2+2c^4-15d^4\right)\right)(c+d\sin(e\right.}{\left(c^2-d^2\right)^3}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^2/(c + d*Sin[e + f*x])^(7/2),x]
```

```
[Out] (2*(-(((d^2*(-2*a*b*d*(27*c^2 + 5*d^2) + b^2*c*(7*c^2 + 25*d^2) + a^2*(15*c
^3 + 17*c*d^2))*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)] - (-(a^2*d^
2*(23*c^2 + 9*d^2)) + a*b*(6*c^3*d + 58*c*d^3) + b^2*(2*c^4 - 19*c^2*d^2 -
15*d^4))*((c + d)*EllipticE[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)] - c*Ellip
ticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]))*((c + d*Sin[e + f*x])/(c + d))
^(5/2))/((c - d)^3*(c + d))) + (d*Cos[e + f*x]*(3*(b*c - a*d)^2*(c^2 - d^2)
^2 - 2*(c^2 - d^2)*(-4*a^2*c*d^2 + a*b*d*(3*c^2 + 5*d^2) + b^2*(c^3 - 5*c*d
^2))*(c + d*Sin[e + f*x]) - (-(a^2*d^2*(23*c^2 + 9*d^2)) + a*b*(6*c^3*d + 5
8*c*d^3) + b^2*(2*c^4 - 19*c^2*d^2 - 15*d^4))*(c + d*Sin[e + f*x])^2))/(c^2
 - d^2)^3))/(15*d^2*f*(c + d*Sin[e + f*x])^(5/2))
```

Maple [B] time = 7.268, size = 1450, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^(7/2),x)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((a^2*d^2-2*a*b*c*d+b^2*c^2)/d^2*(2
/5/(c^2-d^2)/d^2*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(sin(f*x+e)+c/d)^3
+16/15*c/(c^2-d^2)^2/d*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(sin(f*x+e)+
c/d)^2+2/15*d*cos(f*x+e)^2/(c^2-d^2)^3*(23*c^2+9*d^2)/(-(-d*sin(f*x+e)-c)*c
os(f*x+e)^2)^(1/2)+2*(15*c^3+17*c*d^2)/(15*c^6-45*c^4*d^2+45*c^2*d^4-15*d^6
)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-
sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*Ellipt
icF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+2/15*d*(23*c^2+9*d^
2)/(c^2-d^2)^3*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+
d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)
^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/
2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+b^2/d^2
*(2*d*cos(f*x+e)^2/(c^2-d^2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2*c/(c
^2-d^2)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/
2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*
EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+2/(c^2-d^2)*d
*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-s
in(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-
1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(
((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+2*b*(a*d-b*c)/d^2*(2/
3/(c^2-d^2)/d*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(sin(f*x+e)+c/d)^2+8/
3*d*cos(f*x+e)^2/(c^2-d^2)^2*c/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2*(3
*c^2+d^2)/(3*c^4-6*c^2*d^2+3*d^4)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d
*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+
e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(
c+d))^(1/2))+8/3*c*d/(c^2-d^2)^2*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*
(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e
)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2)
,((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d)
)^(1/2)))))/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\sin\left(fx+e\right)+a\right)^2}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)^2/(d*sin(f*x + e) + c)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(b^2\cos\left(fx+e\right)^2-2\,ab\sin\left(fx+e\right)-a^2-b^2\right)\sqrt{d\sin\left(fx+e\right)+c}}{d^4\cos\left(fx+e\right)^4+c^4+6\,c^2d^2+d^4-2\left(3\,c^2d^2+d^4\right)\cos\left(fx+e\right)^2-4\left(cd^3\cos\left(fx+e\right)^2-c^3d-cd^3\right)\sin\left(fx\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^(7/2),x, algorithm="fricas")
```

```
[Out] integral(-(b^2*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2)*sqrt(d*sin(
f*x + e) + c)/(d^4*cos(f*x + e)^4 + c^4 + 6*c^2*d^2 + d^4 - 2*(3*c^2*d^2 +
```

```
d^4)*cos(f*x + e)^2 - 4*(c*d^3*cos(f*x + e)^2 - c^3*d - c*d^3)*sin(f*x + e)
), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**2/(c+d*sin(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\sin\left(fx+e\right)+a\right)^2}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)^2/(d*sin(f*x + e) + c)^(7/2), x)
```

3.737 $\int (a+b\sin(e+fx))^3(c+d\sin(e+fx))^{5/2}\,dx$

Optimal. Leaf size=642

$$\frac{2b\left(-297a^2d^2+66abcd+b^2\left(-\left(8c^2+81d^2\right)\right)\right)\cos(e+fx)(c+d\sin(e+fx))^{5/2}}{693d^2f}-\frac{2\left(1485a^2bcd^2+693a^3d^3-33ab^2d\left(10\right.\right.}{}$$

```
[Out] (-2*(1848*a^3*c*d^3 + 495*a^2*b*d^2*(3*c^2 + 5*d^2) - 66*a*b^2*d*(5*c^3 - 5
7*c*d^2) + 5*b^3*(8*c^4 + 57*c^2*d^2 + 135*d^4))*Cos[e + f*x]*Sqrt[c + d*Si
n[e + f*x]])/(3465*d^2*f) - (2*(1485*a^2*b*c*d^2 + 693*a^3*d^3 - 33*a*b^2*d
*(10*c^2 - 49*d^2) + 5*b^3*(8*c^3 + 67*c*d^2))*Cos[e + f*x]*(c + d*Sin[e +
f*x])^(3/2))/(3465*d^2*f) + (2*b*(66*a*b*c*d - 297*a^2*d^2 - b^2*(8*c^2 + 8
1*d^2))*Cos[e + f*x]*(c + d*Sin[e + f*x])^(5/2))/(693*d^2*f) + (8*b^2*(b*c
- 6*a*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(7/2))/(99*d^2*f) - (2*b^2*Cos[e
 + f*x]*(a + b*Sin[e + f*x])*(c + d*Sin[e + f*x])^(7/2))/(11*d*f) + (2*(231
*a^3*d^3*(23*c^2 + 9*d^2) + 495*a^2*b*c*d^2*(3*c^2 + 29*d^2) - 33*a*b^2*d*(
10*c^4 - 279*c^2*d^2 - 147*d^4) + 5*b^3*(8*c^5 + 51*c^3*d^2 + 741*c*d^4))*E
llipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(3465
*d^3*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) - (2*(c^2 - d^2)*(1848*a^3*c*d^3
 + 495*a^2*b*d^2*(3*c^2 + 5*d^2) - 66*a*b^2*d*(5*c^3 - 57*c*d^2) + 5*b^3*(8
*c^4 + 57*c^2*d^2 + 135*d^4))*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*
Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(3465*d^3*f*Sqrt[c + d*Sin[e + f*x]])
```

Rubi [A] time = 1.40057, antiderivative size = 642, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2793, 3023, 2753, 2752, 2663, 2661, 2655, 2653}

$$\frac{2b\left(-297a^2d^2+66abcd+b^2\left(-\left(8c^2+81d^2\right)\right)\right)\cos(e+fx)(c+d\sin(e+fx))^{5/2}}{693d^2f}-\frac{2\left(1485a^2bcd^2+693a^3d^3-33ab^2d\left(10\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^(5/2),x]
```


```
[Out] (-2*(1848*a^3*c*d^3 + 495*a^2*b*d^2*(3*c^2 + 5*d^2) - 66*a*b^2*d*(5*c^3 - 5
7*c*d^2) + 5*b^3*(8*c^4 + 57*c^2*d^2 + 135*d^4))*Cos[e + f*x]*Sqrt[c + d*Si
n[e + f*x]])/(3465*d^2*f) - (2*(1485*a^2*b*c*d^2 + 693*a^3*d^3 - 33*a*b^2*d
*(10*c^2 - 49*d^2) + 5*b^3*(8*c^3 + 67*c*d^2))*Cos[e + f*x]*(c + d*Sin[e +
f*x])^(3/2))/(3465*d^2*f) + (2*b*(66*a*b*c*d - 297*a^2*d^2 - b^2*(8*c^2 + 8
1*d^2))*Cos[e + f*x]*(c + d*Sin[e + f*x])^(5/2))/(693*d^2*f) + (8*b^2*(b*c
- 6*a*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(7/2))/(99*d^2*f) - (2*b^2*Cos[e
 + f*x]*(a + b*Sin[e + f*x])*(c + d*Sin[e + f*x])^(7/2))/(11*d*f) + (2*(231
*a^3*d^3*(23*c^2 + 9*d^2) + 495*a^2*b*c*d^2*(3*c^2 + 29*d^2) - 33*a*b^2*d*(
10*c^4 - 279*c^2*d^2 - 147*d^4) + 5*b^3*(8*c^5 + 51*c^3*d^2 + 741*c*d^4))*E
llipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(3465
*d^3*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) - (2*(c^2 - d^2)*(1848*a^3*c*d^3
 + 495*a^2*b*d^2*(3*c^2 + 5*d^2) - 66*a*b^2*d*(5*c^3 - 57*c*d^2) + 5*b^3*(8
*c^4 + 57*c^2*d^2 + 135*d^4))*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*
Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(3465*d^3*f*Sqrt[c + d*Sin[e + f*x]])
```


Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
```

```
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int (a+b\sin(e+fx))^3(c+d\sin(e+fx))^{5/2}\,dx = -\frac{2b^2\cos(e+fx)(a+b\sin(e+fx))(c+d\sin(e+fx))^{7/2}}{11df} + \frac{2\int(c+d\sin}{}$$

$$= \frac{8b^2(bc-6ad)\cos(e+fx)(c+d\sin(e+fx))^{7/2}}{99d^2f} - \frac{2b^2\cos(e+fx)(a+b\sin}{}$$

$$= \frac{2b\left(66abcd-297a^2d^2-b^2\left(8c^2+81d^2\right)\right)\cos(e+fx)(c+d\sin(e+fx))^{5}}{693d^2f}$$

$$= -\frac{2\left(1485a^2bcd^2+693a^3d^3-33ab^2d\left(10c^2-49d^2\right)+5b^3\left(8c^3+67cd^2\right)\right)}{3465d^2f}$$

$$= -\frac{2\left(1848a^3cd^3+495a^2bd^2\left(3c^2+5d^2\right)-66ab^2d\left(5c^3-57cd^2\right)+5b^3\left(8c\right.\right.}{3465d^2f}$$

$$= -\frac{2\left(1848a^3cd^3+495a^2bd^2\left(3c^2+5d^2\right)-66ab^2d\left(5c^3-57cd^2\right)+5b^3\left(8c\right.\right.}{3465d^2f}$$

$$= -\frac{2\left(1848a^3cd^3+495a^2bd^2\left(3c^2+5d^2\right)-66ab^2d\left(5c^3-57cd^2\right)+5b^3\left(8c\right.\right.}{3465d^2f}$$

$$= -\frac{2\left(1848a^3cd^3+495a^2bd^2\left(3c^2+5d^2\right)-66ab^2d\left(5c^3-57cd^2\right)+5b^3\left(8c\right.\right.}{3465d^2f}$$

Mathematica [A] time = 2.62406, size = 545, normalized size = 0.85

$$\frac{d(c+d\sin(e+fx))\left(-4d\left(8910a^2bcd^2+1386a^3d^3+33ab^2d\left(150c^2+133d^2\right)+5b^3\left(6c^3+619cd^2\right)\right)\sin(2(e+fx))+5bd^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (-16*(d^2*(495*a^2*b*d^2*(27*c^2 + 5*d^2) + 231*a^3*c*d*(15*c^2 + 17*d^2) +
 33*a*b^2*d*(155*c^3 + 261*c*d^2) + 5*b^3*(2*c^4 + 663*c^2*d^2 + 135*d^4))*
EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)] + (231*a^3*d^3*(23*c^2 + 9*
d^2) + 495*a^2*b*c*d^2*(3*c^2 + 29*d^2) + 33*a*b^2*d*(-10*c^4 + 279*c^2*d^2
 + 147*d^4) + 5*b^3*(8*c^5 + 51*c^3*d^2 + 741*c*d^4))*((c + d)*EllipticE[(-
2*e + Pi - 2*f*x)/4, (2*d)/(c + d)] - c*EllipticF[(-2*e + Pi - 2*f*x)/4, (2
*d)/(c + d)]))*Sqrt[(c + d*Sin[e + f*x])/(c + d)] + d*(c + d*Sin[e + f*x])*
(2*(-20328*a^3*c*d^3 - 990*a^2*b*d^2*(36*c^2 + 23*d^2) - 66*a*b^2*d*(20*c^3
 + 747*c*d^2) + 5*b^3*(32*c^4 - 1866*c^2*d^2 - 1305*d^4))*Cos[e + f*x] + 5*
b*d^2*(2508*a*b*c*d + 1188*a^2*d^2 + b^2*(452*c^2 + 513*d^2))*Cos[3*(e + f*
x)] - 315*b^3*d^4*Cos[5*(e + f*x)] - 4*d*(8910*a^2*b*c*d^2 + 1386*a^3*d^3 +
 33*a*b^2*d*(150*c^2 + 133*d^2) + 5*b^3*(6*c^3 + 619*c*d^2))*Sin[2*(e + f*x
)] + 70*b^2*d^3*(23*b*c + 33*a*d)*Sin[4*(e + f*x)]))/(27720*d^3*f*Sqrt[c +
d*Sin[e + f*x]])
```

Maple [B] time = 7.577, size = 2728, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^3*(c+d*sin(f*x+e))^(5/2),x)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*(b^3*d^3*(-2/11/d*sin(f*x+e)^4*(-(-
d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+20/99*c/d^2*sin(f*x+e)^3*(-(-d*sin(f*x+
e)-c)*cos(f*x+e)^2)^(1/2)-2/7*(9/11+80/99*c^2/d^2)/d*sin(f*x+e)^2*(-(-d*sin
(f*x+e)-c)*cos(f*x+e)^2)^(1/2)-2/3465*(-480*c^3-472*c*d^2)/d^4*sin(f*x+e)*(
-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)-2/3465*(640*c^4+596*c^2*d^2+675*d^4)
/d^5*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2/3465*(-320*c^4-348*c^2*d^2+6
75*d^4)/d^4*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))
^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1
/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+2/3465*(-
1280*c^5-1032*c^3*d^2-1146*c*d^4)/d^5*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2
)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(
f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^
(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/
(c+d))^(1/2))))+(3*a*b^2*d^3+3*b^3*c*d^2)*(-2/9/d*sin(f*x+e)^3*(-(-d*sin(f*
x+e)-c)*cos(f*x+e)^2)^(1/2)+16/63*c/d^2*sin(f*x+e)^2*(-(-d*sin(f*x+e)-c)*co
s(f*x+e)^2)^(1/2)-2/5*(7/9+16/21*c^2/d^2)/d*sin(f*x+e)*(-(-d*sin(f*x+e)-c)*
cos(f*x+e)^2)^(1/2)-2/315*(-64*c^3-62*c*d^2)/d^4*(-(-d*sin(f*x+e)-c)*cos(f*
x+e)^2)^(1/2)+2/315*(32*c^3+36*c*d^2)/d^3*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^
(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*
sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),
((c-d)/(c+d))^(1/2))+2/315*(128*c^4+108*c^2*d^2+147*d^4)/d^4*(c/d-1)*((c+d*
sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/
(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((
c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e
))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+(3*a^2*b*d^3+9*a*b^2*c*d^2+3*b^3*c^2
*d)*(-2/7/d*sin(f*x+e)^2*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+12/35*c/d^
2*sin(f*x+e)*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)-2/3*(5/7+24/35*c^2/d^2
)/d*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2*(-4/35*c^2/d^2+5/21)*(c/d-1)*
((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)
-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*
sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+2/105*(-48*c^3-44*c*d^2)/d^3*
(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-si
n(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1
)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF((
(c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+(a^3*d^3+9*a^2*b*c*d^2
+9*a*b^2*c^2*d+b^3*c^3)*(-2/5/d*sin(f*x+e)*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2
)^(1/2)+8/15*c/d^2*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+4/15*c/d*(c/d-1)
*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e
)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d
*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+2*(3/5+8/15*c^2/d^2)*(c/d-1)
*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e
)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*Ellip
ticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*si
n(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+(3*a^3*c*d^2+9*a^2*b*c^2*d+3*
a*b^2*c^3)*(-2/3/d*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2/3*(c/d-1)*((c+
d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*
d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(
f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))-4/3*c/d*(c/d-1)*((c+d*sin(f*x+e))
/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2
)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x
```

```
+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(
1/2),((c-d)/(c+d))^(1/2))))+2*(3*a^3*c^2*d+3*a^2*b*c^3)*(c/d-1)*((c+d*sin(f
*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d)
)^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*s
in(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c
-d))^(1/2),((c-d)/(c+d))^(1/2)))+2*a^3*c^3*(c/d-1)*((c+d*sin(f*x+e))/(c-d))
^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d
*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2)
,((c-d)/(c+d))^(1/2)))/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)+a\right)^{3}\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3*(c+d*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)^3*(d*sin(f*x + e) + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(\left(\left(2\,b^{3}cd+3\,ab^{2}d^{2}\right)\cos\left(fx+e\right)^{4}+\left(a^{3}+3\,ab^{2}\right)c^{2}+2\left(3\,a^{2}b+b^{3}\right)cd+\left(a^{3}+3\,ab^{2}\right)d^{2}-\left(3\,ab^{2}c^{2}+2\left(3\,a^{2}b+2\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3*(c+d*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((((2*b^3*c*d + 3*a*b^2*d^2)*cos(f*x + e)^4 + (a^3 + 3*a*b^2)*c^2 +
2*(3*a^2*b + b^3)*c*d + (a^3 + 3*a*b^2)*d^2 - (3*a*b^2*c^2 + 2*(3*a^2*b + 2
*b^3)*c*d + (a^3 + 6*a*b^2)*d^2)*cos(f*x + e)^2 + (b^3*d^2*cos(f*x + e)^4 +
 (3*a^2*b + b^3)*c^2 + 2*(a^3 + 3*a*b^2)*c*d + (3*a^2*b + b^3)*d^2 - (b^3*c
^2 + 6*a*b^2*c*d + (3*a^2*b + 2*b^3)*d^2)*cos(f*x + e)^2)*sin(f*x + e))*sqr
t(d*sin(f*x + e) + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**3*(c+d*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3*(c+d*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.738 $\int (a+b\sin(e+fx))^3(c+d\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=496

$$\frac{2b\left(-189a^2d^2+54abcd+b^2\left(-\left(8c^2+49d^2\right)\right)\right)\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{315d^2f}-\frac{2\left(189a^2bcd^2+105a^3d^3-9ab^2d\left(6c^2-\right.\right.}{}$$

```
[Out] (-2*(189*a^2*b*c*d^2 + 105*a^3*d^3 - 9*a*b^2*d*(6*c^2 - 25*d^2) + b^3*(8*c^
3 + 39*c*d^2))*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(315*d^2*f) + (2*b*(5
4*a*b*c*d - 189*a^2*d^2 - b^2*(8*c^2 + 49*d^2))*Cos[e + f*x]*(c + d*Sin[e +
 f*x])^(3/2))/(315*d^2*f) + (8*b^2*(b*c - 5*a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(5/2))/(63*d^2*f) - (2*b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])*(c + d
*Sin[e + f*x])^(5/2))/(9*d*f) + (2*(420*a^3*c*d^3 + 189*a^2*b*d^2*(c^2 + 3*
d^2) - a*b^2*(54*c^3*d - 738*c*d^3) + b^3*(8*c^4 + 33*c^2*d^2 + 147*d^4))*E
llipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(315*
d^3*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) - (2*(c^2 - d^2)*(189*a^2*b*c*d^2
 + 105*a^3*d^3 - 9*a*b^2*d*(6*c^2 - 25*d^2) + b^3*(8*c^3 + 39*c*d^2))*Ellip
ticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])
/(315*d^3*f*Sqrt[c + d*Sin[e + f*x]])
```

Rubi [A] time = 1.02559, antiderivative size = 496, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2793, 3023, 2753, 2752, 2663, 2661, 2655, 2653}

$$\frac{2b\left(-189a^2d^2+54abcd+b^2\left(-\left(8c^2+49d^2\right)\right)\right)\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{315d^2f}-\frac{2\left(189a^2bcd^2+105a^3d^3-9ab^2d\left(6c^2-\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^(3/2),x]
```


```
[Out] (-2*(189*a^2*b*c*d^2 + 105*a^3*d^3 - 9*a*b^2*d*(6*c^2 - 25*d^2) + b^3*(8*c^
3 + 39*c*d^2))*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(315*d^2*f) + (2*b*(5
4*a*b*c*d - 189*a^2*d^2 - b^2*(8*c^2 + 49*d^2))*Cos[e + f*x]*(c + d*Sin[e +
 f*x])^(3/2))/(315*d^2*f) + (8*b^2*(b*c - 5*a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(5/2))/(63*d^2*f) - (2*b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])*(c + d
*Sin[e + f*x])^(5/2))/(9*d*f) + (2*(420*a^3*c*d^3 + 189*a^2*b*d^2*(c^2 + 3*
d^2) - a*b^2*(54*c^3*d - 738*c*d^3) + b^3*(8*c^4 + 33*c^2*d^2 + 147*d^4))*E
llipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(315*
d^3*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) - (2*(c^2 - d^2)*(189*a^2*b*c*d^2
 + 105*a^3*d^3 - 9*a*b^2*d*(6*c^2 - 25*d^2) + b^3*(8*c^3 + 39*c*d^2))*Ellip
ticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])
/(315*d^3*f*Sqrt[c + d*Sin[e + f*x]])
```


Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
```

```
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int (a+b\sin(e+fx))^3(c+d\sin(e+fx))^{3/2}\,dx = -\frac{2b^2\cos(e+fx)(a+b\sin(e+fx))(c+d\sin(e+fx))^{5/2}}{9df} + \frac{2\int (c+d\sin}{}$$

$$= \frac{8b^2(bc-5ad)\cos(e+fx)(c+d\sin(e+fx))^{5/2}}{63d^2f} - \frac{2b^2\cos(e+fx)(a+b\sin}{}$$

$$= \frac{2b\left(54abcd-189a^2d^2-b^2\left(8c^2+49d^2\right)\right)\cos(e+fx)(c+d\sin(e+fx))^3}{315d^2f}$$

$$= -\frac{2\left(189a^2bcd^2+105a^3d^3-9ab^2d\left(6c^2-25d^2\right)+b^3\left(8c^3+39cd^2\right)\right)\cos(}{315d^2f}$$

$$= -\frac{2\left(189a^2bcd^2+105a^3d^3-9ab^2d\left(6c^2-25d^2\right)+b^3\left(8c^3+39cd^2\right)\right)\cos(}{315d^2f}$$

$$= -\frac{2\left(189a^2bcd^2+105a^3d^3-9ab^2d\left(6c^2-25d^2\right)+b^3\left(8c^3+39cd^2\right)\right)\cos(}{315d^2f}$$

$$= -\frac{2\left(189a^2bcd^2+105a^3d^3-9ab^2d\left(6c^2-25d^2\right)+b^3\left(8c^3+39cd^2\right)\right)\cos(}{315d^2f}$$

Mathematica [A] time = 2.36794, size = 410, normalized size = 0.83

$$\frac{d(c+d\sin(e+fx))\left(bd\left(10bd(27ad+10bc)\cos(3(e+fx))-2\sin(2(e+fx))\left(378a^2d^2+432abcd+b^2\left(6c^2+133d^2\right)-3\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-8*(d^2*(756*a^2*b*c*d^2 + 105*a^3*d*(3*c^2 + d^2) + 9*a*b^2*d*(51*c^2 + 2
5*d^2) + 2*b^3*(c^3 + 93*c*d^2))*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c
+ d)] + (420*a^3*c*d^3 + 189*a^2*b*d^2*(c^2 + 3*d^2) + a*b^2*(-54*c^3*d + 7
38*c*d^3) + b^3*(8*c^4 + 33*c^2*d^2 + 147*d^4))*((c + d)*EllipticE[(-2*e +
Pi - 2*f*x)/4, (2*d)/(c + d)] - c*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c
 + d)]))*Sqrt[(c + d*Sin[e + f*x])/(c + d)] + d*(c + d*Sin[e + f*x])*(-2*(1
512*a^2*b*c*d^2 + 420*a^3*d^3 + 9*a*b^2*d*(12*c^2 + 115*d^2) + b^3*(-16*c^3
 + 402*c*d^2))*Cos[e + f*x] + b*d*(10*b*d*(10*b*c + 27*a*d)*Cos[3*(e + f*x)
] - 2*(432*a*b*c*d + 378*a^2*d^2 + b^2*(6*c^2 + 133*d^2) - 35*b^2*d^2*Cos[2
*(e + f*x)])*Sin[2*(e + f*x)])))/(1260*d^3*f*Sqrt[c + d*Sin[e + f*x]])
```

Maple [B] time = 5.964, size = 2112, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^3*(c+d*sin(f*x+e))^(3/2),x)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*(b^3*d^2*(-2/9/d*sin(f*x+e)^3*(-(-d
*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+16/63*c/d^2*sin(f*x+e)^2*(-(-d*sin(f*x+e
```

```
)-c)*cos(f*x+e)^2)^(1/2)-2/5*(7/9+16/21*c^2/d^2)/d*sin(f*x+e)*(-(-d*sin(f*x
+e)-c)*cos(f*x+e)^2)^(1/2)-2/315*(-64*c^3-62*c*d^2)/d^4*(-(-d*sin(f*x+e)-c)
*cos(f*x+e)^2)^(1/2)+2/315*(32*c^3+36*c*d^2)/d^3*(c/d-1)*((c+d*sin(f*x+e))/
(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)
/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))
^(1/2),((c-d)/(c+d))^(1/2))+2/315*(128*c^4+108*c^2*d^2+147*d^4)/d^4*(c/d-1)
*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e
)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*Ellip
ticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*si
n(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+(3*a*b^2*d^2+2*b^3*c*d)*(-2/7
/d*sin(f*x+e)^2*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+12/35*c/d^2*sin(f*x
+e)*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)-2/3*(5/7+24/35*c^2/d^2)/d*(-(-d
*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2*(-4/35*c^2/d^2+5/21)*(c/d-1)*((c+d*sin
(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-
d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e
))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+2/105*(-48*c^3-44*c*d^2)/d^3*(c/d-1)*(
(c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-
1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*Ellipti
cE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(
f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+(3*a^2*b*d^2+6*a*b^2*c*d+b^3*c^
2)*(-2/5/d*sin(f*x+e)*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+8/15*c/d^2*(-
(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+4/15*c/d*(c/d-1)*((c+d*sin(f*x+e))/(c
-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(
-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(
1/2),((c-d)/(c+d))^(1/2))+2*(3/5+8/15*c^2/d^2)*(c/d-1)*((c+d*sin(f*x+e))/(c
-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(
-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e)
)/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2
),((c-d)/(c+d))^(1/2))))+(a^3*d^2+6*a^2*b*c*d+3*a*b^2*c^2)*(-2/3/d*(-(-d*si
n(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2/3*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*
(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*
x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)
/(c+d))^(1/2))-4/3*c/d*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x
+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f
*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c
+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))
+2*(2*a^3*c*d+3*a^2*b*c^2)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin
(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*c
os(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d
)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2
)))+2*a^3*c^2*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d
))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^
(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))/cos(f*
x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)+a\right)^{3}\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3*(c+d*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)^3*(d*sin(f*x + e) + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^3 d\cos\left(fx+e\right)^4-\left(3\,ab^2c+\left(3\,a^2b+2\,b^3\right)d\right)\cos\left(fx+e\right)^2+\left(a^3+3\,ab^2\right)c+\left(3\,a^2b+b^3\right)d-\left(\left(b^3c+3\,ab^2d\right)\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3*(c+d*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b^3*d*cos(f*x + e)^4 - (3*a*b^2*c + (3*a^2*b + 2*b^3)*d)*cos(f*x
+ e)^2 + (a^3 + 3*a*b^2)*c + (3*a^2*b + b^3)*d - ((b^3*c + 3*a*b^2*d)*cos(f
*x + e)^2 - (3*a^2*b + b^3)*c - (a^3 + 3*a*b^2)*d)*sin(f*x + e))*sqrt(d*sin
(f*x + e) + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**3*(c+d*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3*(c+d*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.739 $\int (a+b\sin(e+fx))^3\sqrt{c+d\sin(e+fx)}\,dx$

Optimal. Leaf size=375

$$\frac{2b\left(-105a^2d^2+42abcd+b^2\left(-\left(8c^2+25d^2\right)\right)\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{105d^2f}+\frac{2b\left(c^2-d^2\right)\left(-105a^2d^2+42abcd+b^2\right.}{105\!\!\!\!}$$

```
[Out] (2*b*(42*a*b*c*d - 105*a^2*d^2 - b^2*(8*c^2 + 25*d^2))*Cos[e + f*x]*Sqrt[c
+ d*Sin[e + f*x]])/(105*d^2*f) + (8*b^2*(b*c - 4*a*d)*Cos[e + f*x]*(c + d*S
in[e + f*x])^(3/2))/(35*d^2*f) - (2*b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])*(
c + d*Sin[e + f*x])^(3/2))/(7*d*f) + (2*(105*a^2*b*c*d^2 + 105*a^3*d^3 - 21
*a*b^2*d*(2*c^2 - 9*d^2) + b^3*(8*c^3 + 19*c*d^2))*EllipticE[(e - Pi/2 + f*
x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(105*d^3*f*Sqrt[(c + d*Sin[e
 + f*x])/(c + d)]) + (2*b*(c^2 - d^2)*(42*a*b*c*d - 105*a^2*d^2 - b^2*(8*c^
2 + 25*d^2))*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e
 + f*x])/(c + d)])/(105*d^3*f*Sqrt[c + d*Sin[e + f*x]])
```

Rubi [A] time = 0.677772, antiderivative size = 375, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2793, 3023, 2753, 2752, 2663, 2661, 2655, 2653}

$$\frac{2b\left(-105a^2d^2+42abcd+b^2\left(-\left(8c^2+25d^2\right)\right)\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{105d^2f}+\frac{2b\left(c^2-d^2\right)\left(-105a^2d^2+42abcd+b^2\right.}{105\!\!\!\!}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^3*Sqrt[c + d*Sin[e + f*x]],x]
```


```
[Out] (2*b*(42*a*b*c*d - 105*a^2*d^2 - b^2*(8*c^2 + 25*d^2))*Cos[e + f*x]*Sqrt[c
+ d*Sin[e + f*x]])/(105*d^2*f) + (8*b^2*(b*c - 4*a*d)*Cos[e + f*x]*(c + d*S
in[e + f*x])^(3/2))/(35*d^2*f) - (2*b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])*(
c + d*Sin[e + f*x])^(3/2))/(7*d*f) + (2*(105*a^2*b*c*d^2 + 105*a^3*d^3 - 21
*a*b^2*d*(2*c^2 - 9*d^2) + b^3*(8*c^3 + 19*c*d^2))*EllipticE[(e - Pi/2 + f*
x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(105*d^3*f*Sqrt[(c + d*Sin[e
 + f*x])/(c + d)]) + (2*b*(c^2 - d^2)*(42*a*b*c*d - 105*a^2*d^2 - b^2*(8*c^
2 + 25*d^2))*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e
 + f*x])/(c + d)])/(105*d^3*f*Sqrt[c + d*Sin[e + f*x]])
```


Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
```

```
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 !LtQ[m, -1]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int (a+b\sin(e+fx))^3\sqrt{c+d\sin(e+fx)}\,dx = -\frac{2b^2\cos(e+fx)(a+b\sin(e+fx))(c+d\sin(e+fx))^{3/2}}{7df} + \frac{2\int\sqrt{c+d}}{}$$

$$= \frac{8b^2(bc-4ad)\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{35d^2f} - \frac{2b^2\cos(e+fx)(a+b}{}$$

$$= \frac{2b\left(42abcd-105a^2d^2-b^2\left(8c^2+25d^2\right)\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{105d^2f}$$

$$= \frac{2b\left(42abcd-105a^2d^2-b^2\left(8c^2+25d^2\right)\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{105d^2f}$$

$$= \frac{2b\left(42abcd-105a^2d^2-b^2\left(8c^2+25d^2\right)\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{105d^2f}$$

$$= \frac{2b\left(42abcd-105a^2d^2-b^2\left(8c^2+25d^2\right)\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{105d^2f}$$

Mathematica [A] time = 1.41046, size = 306, normalized size = 0.82

$$\frac{bd(c+d\sin(e+fx))\left(\left(-420a^2d^2-84abcd+b^2\left(16c^2-115d^2\right)\right)\cos(e+fx)+3bd(5bd\cos(3(e+fx))-2(21ad+bc)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^3*Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] (-4*(d^2*(105*a^3*c*d + 147*a*b^2*c*d + 105*a^2*b*d^2 + b^3*(2*c^2 + 25*d^2
))*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)] + (105*a^2*b*c*d^2 + 105
*a^3*d^3 + 21*a*b^2*d*(-2*c^2 + 9*d^2) + b^3*(8*c^3 + 19*c*d^2))*((c + d)*E
llipticE[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)] - c*EllipticF[(-2*e + Pi - 2
*f*x)/4, (2*d)/(c + d)]))*Sqrt[(c + d*Sin[e + f*x])/(c + d)] + b*d*(c + d*S
in[e + f*x])*((-84*a*b*c*d - 420*a^2*d^2 + b^2*(16*c^2 - 115*d^2))*Cos[e +
f*x] + 3*b*d*(5*b*d*Cos[3*(e + f*x)] - 2*(b*c + 21*a*d)*Sin[2*(e + f*x)])))
/(210*d^3*f*Sqrt[c + d*Sin[e + f*x]])
```

Maple [B] time = 4.738, size = 1561, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^3*(c+d*sin(f*x+e))^(1/2),x)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*(b^3*d*(-2/7/d*sin(f*x+e)^2*(-(-d*s
in(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+12/35*c/d^2*sin(f*x+e)*(-(-d*sin(f*x+e)-c)
*cos(f*x+e)^2)^(1/2)-2/3*(5/7+24/35*c^2/d^2)/d*(-(-d*sin(f*x+e)-c)*cos(f*x+
e)^2)^(1/2)+2*(-4/35*c^2/d^2+5/21)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(
d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x
+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/
```

```
(c+d))^(1/2))+2/105*(-48*c^3-44*c*d^2)/d^3*(c/d-1)*((c+d*sin(f*x+e))/(c-d))
^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d
*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c
-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((
c-d)/(c+d))^(1/2))))+(3*a*b^2*d+b^3*c)*(-2/5/d*sin(f*x+e)*(-(-d*sin(f*x+e)-
c)*cos(f*x+e)^2)^(1/2)+8/15*c/d^2*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+4
/15*c/d*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/
2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*
EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+2*(3/5+8/15*c
^2/d^2)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/
2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*
((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+Ell
ipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+(3*a^2*b*d+3*a
*b^2*c)*(-2/3/d*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2/3*(c/d-1)*((c+d*s
in(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(
c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x
+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))-4/3*c/d*(c/d-1)*((c+d*sin(f*x+e))/(c
-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(
-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e)
)/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2
),((c-d)/(c+d))^(1/2))))+2*(a^3*d+3*a^2*b*c)*(c/d-1)*((c+d*sin(f*x+e))/(c-d
))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(
-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/
(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),
((c-d)/(c+d))^(1/2)))+2*a^3*c*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-
sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c
)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d)
)^(1/2)))/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)+a\right)^{3}\sqrt{d\sin\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3*(c+d*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)^3*sqrt(d*sin(f*x + e) + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-\left(3\,ab^{2}\cos\left(fx+e\right)^{2}-a^{3}-3\,ab^{2}+\left(b^{3}\cos\left(fx+e\right)^{2}-3\,a^{2}b-b^{3}\right)\sin\left(fx+e\right)\right)\sqrt{d\sin\left(fx+e\right)+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3*(c+d*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-(3*a*b^2*cos(f*x + e)^2 - a^3 - 3*a*b^2 + (b^3*cos(f*x + e)^2 - 3
*a^2*b - b^3)*sin(f*x + e))*sqrt(d*sin(f*x + e) + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a+b\sin\left(e+fx\right)\right)^3\sqrt{c+d\sin\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**3*(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((a + b*sin(e + f*x))**3*sqrt(c + d*sin(e + f*x)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3*(c+d*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.740 $\int \frac{(a+b\sin(e+fx))^3}{\sqrt{c+d\sin(e+fx)}}\,dx$

Optimal. Leaf size=302

$$-\frac{2\left(45a^2bcd^2-15a^3d^3-15ab^2d\left(2c^2+d^2\right)+b^3\left(8c^3+7cd^2\right)\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{15d^3f\sqrt{c+d\sin(e+fx)}}-\frac{2b\left(-45a^2d^2+30a\right.}{}$$

```
[Out] (8*b^2*(b*c - 3*a*d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(15*d^2*f) - (2
*b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]])/(5*d*f) -
(2*b*(30*a*b*c*d - 45*a^2*d^2 - b^2*(8*c^2 + 9*d^2))*EllipticE[(e - Pi/2 +
f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(15*d^3*f*Sqrt[(c + d*Sin[
e + f*x])/(c + d)]) - (2*(45*a^2*b*c*d^2 - 15*a^3*d^3 - 15*a*b^2*d*(2*c^2 +
 d^2) + b^3*(8*c^3 + 7*c*d^2))*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]
*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(15*d^3*f*Sqrt[c + d*Sin[e + f*x]])
```

Rubi [A] time = 0.479075, antiderivative size = 302, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2793, 3023, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2\left(45a^2bcd^2-15a^3d^3-15ab^2d\left(2c^2+d^2\right)+b^3\left(8c^3+7cd^2\right)\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{15d^3f\sqrt{c+d\sin(e+fx)}}-\frac{2b\left(-45a^2d^2+30a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^3/Sqrt[c + d*Sin[e + f*x]],x]
```


```
[Out] (8*b^2*(b*c - 3*a*d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(15*d^2*f) - (2
*b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]])/(5*d*f) -
(2*b*(30*a*b*c*d - 45*a^2*d^2 - b^2*(8*c^2 + 9*d^2))*EllipticE[(e - Pi/2 +
f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(15*d^3*f*Sqrt[(c + d*Sin[
e + f*x])/(c + d)]) - (2*(45*a^2*b*c*d^2 - 15*a^3*d^3 - 15*a*b^2*d*(2*c^2 +
 d^2) + b^3*(8*c^3 + 7*c*d^2))*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]
*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(15*d^3*f*Sqrt[c + d*Sin[e + f*x]])
```


Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
```

```
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{(a+b\sin(e+fx))^3}{\sqrt{c+d\sin(e+fx)}}\,dx = -\frac{2b^2\cos(e+fx)(a+b\sin(e+fx))\sqrt{c+d\sin(e+fx)}}{5df} + \frac{2\int \frac{\frac{1}{2}\left(2b^3c+5a^3d+ab^2d\right)-\frac{1}{2}b\left(2abc-15a^2\right.}{\sqrt{c+}}}{}$$

$$= \frac{8b^2(bc-3ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15d^2f} - \frac{2b^2\cos(e+fx)(a+b\sin(e+fx))\sqrt{c+d}}{5df}$$

$$= \frac{8b^2(bc-3ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15d^2f} - \frac{2b^2\cos(e+fx)(a+b\sin(e+fx))\sqrt{c+d}}{5df}$$

$$= \frac{8b^2(bc-3ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15d^2f} - \frac{2b^2\cos(e+fx)(a+b\sin(e+fx))\sqrt{c+d}}{5df}$$

$$= \frac{8b^2(bc-3ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15d^2f} - \frac{2b^2\cos(e+fx)(a+b\sin(e+fx))\sqrt{c+d}}{5df}$$

Mathematica [A] time = 1.18337, size = 219, normalized size = 0.73

$$\frac{-2\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\left(b\left(45a^2d^2-30abcd+b^2\left(8c^2+9d^2\right)\right)\left((c+d)E\left(\frac{1}{4}(-2e-2fx+\pi)|\frac{2d}{c+d}\right)-cF\left(\frac{1}{4}(-2e-2fx+\pi)|\frac{2d}{c+d}\right)\right)+}{15d^3f\sqrt{c+}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^3/Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] (-2*(d^2*(2*b^3*c + 15*a^3*d + 15*a*b^2*d)*EllipticF[(-2*e + Pi - 2*f*x)/4,
 (2*d)/(c + d)] + b*(-30*a*b*c*d + 45*a^2*d^2 + b^2*(8*c^2 + 9*d^2))*((c +
d)*EllipticE[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)] - c*EllipticF[(-2*e + Pi
 - 2*f*x)/4, (2*d)/(c + d)]))*Sqrt[(c + d*Sin[e + f*x])/(c + d)] - 2*b^2*d*
Cos[e + f*x]*(c + d*Sin[e + f*x])*(-4*b*c + 15*a*d + 3*b*d*Sin[e + f*x]))/(
15*d^3*f*Sqrt[c + d*Sin[e + f*x]])
```

Maple [B] time = 3.21, size = 1085, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^(1/2),x)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*(b^3*(-2/5/d*sin(f*x+e)*(-(-d*sin(f
*x+e)-c)*cos(f*x+e)^2)^(1/2)+8/15*c/d^2*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(
1/2)+4/15*c/d*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d
))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^
(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+2*(3/5+
8/15*c^2/d^2)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d
))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^
(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2
))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+3*a*b^2*
(-2/3/d*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2/3*(c/d-1)*((c+d*sin(f*x+e
))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1
/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-
d))^(1/2),((c-d)/(c+d))^(1/2))-4/3*c/d*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/
2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin
(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))
^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)
/(c+d))^(1/2))))+6*a^2*b*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f
*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos
(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/
(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))
)+2*a^3*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/
2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*
EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))/cos(f*x+e)/(
c+d*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\sin\left(fx+e\right)+a\right)^3}{\sqrt{d\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)^3/sqrt(d*sin(f*x + e) + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{3\,ab^2\cos\left(fx+e\right)^2 - a^3 - 3\,ab^2 + \left(b^3\cos\left(fx+e\right)^2 - 3\,a^2b - b^3\right)\sin\left(fx+e\right)}{\sqrt{d\sin\left(fx+e\right)+c}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-(3*a*b^2*cos(f*x + e)^2 - a^3 - 3*a*b^2 + (b^3*cos(f*x + e)^2 - 3
*a^2*b - b^3)*sin(f*x + e))/sqrt(d*sin(f*x + e) + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+b\sin\left(e+fx\right)\right)^3}{\sqrt{c+d\sin\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**3/(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((a + b*sin(e + f*x))**3/sqrt(c + d*sin(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\sin\left(fx+e\right)+a\right)^3}{\sqrt{d\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)^3/sqrt(d*sin(f*x + e) + c), x)
```

3.741 $\int \frac{(a+b\sin(e+fx))^3}{(c+d\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=361

$$\frac{2b\left(-3a^2d^2+6abcd+b^2\left(-\left(4c^2-d^2\right)\right)\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3d^2f\left(c^2-d^2\right)}-\frac{2b\left(-9a^2d^2+18abcd+b^2\left(-\left(8c^2+d^2\right)\right)\right)\sqrt{\frac{c+a}{}}}{3d^3f\sqrt{c+d\sin(e+}}$$

```
[Out] (2*(b*c - a*d)^2*Cos[e + f*x]*(a + b*Sin[e + f*x]))/(d*(c^2 - d^2)*f*Sqrt[c
 + d*Sin[e + f*x]]) + (2*b*(6*a*b*c*d - 3*a^2*d^2 - b^2*(4*c^2 - d^2))*Cos[
e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(3*d^2*(c^2 - d^2)*f) - (2*(9*a^2*b*c*d^
2 - 3*a^3*d^3 - 9*a*b^2*d*(2*c^2 - d^2) + b^3*(8*c^3 - 5*c*d^2))*EllipticE[
(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(3*d^3*(c^2 -
d^2)*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) - (2*b*(18*a*b*c*d - 9*a^2*d^2 -
 b^2*(8*c^2 + d^2))*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c +
d*Sin[e + f*x])/(c + d)])/(3*d^3*f*Sqrt[c + d*Sin[e + f*x]])
```

Rubi [A] time = 0.621814, antiderivative size = 361, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2792, 3023, 2752, 2663, 2661, 2655, 2653}

$$\frac{2b\left(-3a^2d^2+6abcd+b^2\left(-\left(4c^2-d^2\right)\right)\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3d^2f\left(c^2-d^2\right)}-\frac{2b\left(-9a^2d^2+18abcd+b^2\left(-\left(8c^2+d^2\right)\right)\right)\sqrt{\frac{c+a}{}}}{3d^3f\sqrt{c+d\sin(e+}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^3/(c + d*Sin[e + f*x])^(3/2),x]
```


```
[Out] (2*(b*c - a*d)^2*Cos[e + f*x]*(a + b*Sin[e + f*x]))/(d*(c^2 - d^2)*f*Sqrt[c
 + d*Sin[e + f*x]]) + (2*b*(6*a*b*c*d - 3*a^2*d^2 - b^2*(4*c^2 - d^2))*Cos[
e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(3*d^2*(c^2 - d^2)*f) - (2*(9*a^2*b*c*d^
2 - 3*a^3*d^3 - 9*a*b^2*d*(2*c^2 - d^2) + b^3*(8*c^3 - 5*c*d^2))*EllipticE[
(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(3*d^3*(c^2 -
d^2)*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) - (2*b*(18*a*b*c*d - 9*a^2*d^2 -
 b^2*(8*c^2 + d^2))*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c +
d*Sin[e + f*x])/(c + d)])/(3*d^3*f*Sqrt[c + d*Sin[e + f*x]])
```


Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
```

```
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{(a+b\sin(e+fx))^3}{(c+d\sin(e+fx))^{3/2}}\,dx = \frac{2(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{d\left(c^2-d^2\right)f\sqrt{c+d\sin(e+fx)}} - \frac{2\int \frac{\frac{1}{2}\left(2b(bc-ad)^2-ad\left(\left(a^2+b^2\right)c-2abd\right)\right)+\frac{1}{2}\left(a^2bcd-b^3cd-a^3\cdots\right.}{\cdots}}{\cdots}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{d\left(c^2-d^2\right)f\sqrt{c+d\sin(e+fx)}} + \frac{2b\left(6abcd-3a^2d^2-b^2\left(4c^2-d^2\right)\right)\cos(e+fx\cdots}{3d^2\left(c^2-d^2\right)f}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{d\left(c^2-d^2\right)f\sqrt{c+d\sin(e+fx)}} + \frac{2b\left(6abcd-3a^2d^2-b^2\left(4c^2-d^2\right)\right)\cos(e+fx\cdots}{3d^2\left(c^2-d^2\right)f}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{d\left(c^2-d^2\right)f\sqrt{c+d\sin(e+fx)}} + \frac{2b\left(6abcd-3a^2d^2-b^2\left(4c^2-d^2\right)\right)\cos(e+fx\cdots}{3d^2\left(c^2-d^2\right)f}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{d\left(c^2-d^2\right)f\sqrt{c+d\sin(e+fx)}} + \frac{2b\left(6abcd-3a^2d^2-b^2\left(4c^2-d^2\right)\right)\cos(e+fx\cdots}{3d^2\left(c^2-d^2\right)f}$$

Mathematica [A] time = 2.05759, size = 311, normalized size = 0.86

$$\frac{2\left(\frac{\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\left(d^2\left(9a^2bd^2-3a^3cd-9ab^2cd+b^3\left(2c^2+d^2\right)\right)F\left(\frac{1}{4}(-2e-2fx+\pi)|\frac{2d}{c+d}\right)+\left(9a^2bcd^2-3a^3d^3+9ab^2d\left(d^2-2c^2\right)+b^3\left(8c^3-5cd^2\right)\right)\left((c+d)E\left(\frac{1}{4}(-2e-2fx+\pi)|\frac{2d}{c+d}\cdots\right.\right.\right.}{(c-d)(c+d)}\cdots\right)}{3d^3f\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^3/(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] (2*(((d^2*(-3*a^3*c*d - 9*a*b^2*c*d + 9*a^2*b*d^2 + b^3*(2*c^2 + d^2))*Elli
pticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)] + (9*a^2*b*c*d^2 - 3*a^3*d^3 +
9*a*b^2*d*(-2*c^2 + d^2) + b^3*(8*c^3 - 5*c*d^2))*((c + d)*EllipticE[(-2*e
+ Pi - 2*f*x)/4, (2*d)/(c + d)] - c*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/
(c + d)]))*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/((c - d)*(c + d)) - (d*Cos[e
 + f*x]*(9*a*b^2*c^2*d - 9*a^2*b*c*d^2 + 3*a^3*d^3 + b^3*(-4*c^3 + c*d^2) +
 b^3*d*(-c^2 + d^2)*Sin[e + f*x]))/(-c^2 + d^2)))/(3*d^3*f*Sqrt[c + d*Sin[e
 + f*x]])
```

Maple [B] time = 4.096, size = 1398, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^(3/2),x)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*(b/d^3*(b^2*d^2*(-2/3/d*(-(-d*sin(f
*x+e)-c)*cos(f*x+e)^2)^(1/2)+2/3*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*
(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e
)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c
+d))^(1/2))-4/3*c/d*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e)
)/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+
```

```
e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d)
)^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+2*
(3*a*b*d^2-b^2*c*d)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e)
)/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+
e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d)
)^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))+6*a
^2*d^2*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2
)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*E
llipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))-6*a*b*c*d*(c/d
-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*
x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((
c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+2*c^2*b^2*(c/d-1)*((c+d*s
in(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(
c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x
+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))+(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*
d-b^3*c^3)/d^3*(2*d*cos(f*x+e)^2/(c^2-d^2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2
)^(1/2)+2*c/(c^2-d^2)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+
e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*
x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))
+2/(c^2-d^2)*d*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+
d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)
^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/
2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))))/cos(f*
x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\sin\left(fx+e\right)+a\right)^3}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)^3/(d*sin(f*x + e) + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(3\,ab^2\cos\left(fx+e\right)^2-a^3-3\,ab^2+\left(b^3\cos\left(fx+e\right)^2-3\,a^2b-b^3\right)\sin\left(fx+e\right)\right)\sqrt{d\sin\left(fx+e\right)+c}}{d^2\cos\left(fx+e\right)^2-2\,cd\sin\left(fx+e\right)-c^2-d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((3*a*b^2*cos(f*x + e)^2 - a^3 - 3*a*b^2 + (b^3*cos(f*x + e)^2 - 3*
a^2*b - b^3)*sin(f*x + e))*sqrt(d*sin(f*x + e) + c)/(d^2*cos(f*x + e)^2 - 2
*c*d*sin(f*x + e) - c^2 - d^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**3/(c+d*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\sin\left(fx+e\right)+a\right)^3}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)^3/(d*sin(f*x + e) + c)^(3/2), x)
```

3.742 $\int \frac{(a+b\sin(e+fx))^3}{(c+d\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=391

$$-\frac{2\left(-a^2d^2+2abcd+b^2\left(8c^2-9d^2\right)\right)(bc-ad)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3d^3f\left(c^2-d^2\right)\sqrt{c+d\sin(e+fx)}}+\frac{2\left(-3a^2bd^2\left(c^2+3d^2\right)+4a^3cd^3\right.}{}$$

```
[Out] (2*(b*c - a*d)^2*Cos[e + f*x]*(a + b*Sin[e + f*x]))/(3*d*(c^2 - d^2)*f*(c +
 d*Sin[e + f*x])^(3/2)) + (8*(b*c - a*d)^2*(a*c*d + b*(c^2 - 2*d^2))*Cos[e
+ f*x])/(3*d^2*(c^2 - d^2)^2*f*Sqrt[c + d*Sin[e + f*x]]) + (2*(4*a^3*c*d^3
- 6*a*b^2*c*d*(c^2 - 3*d^2) - 3*a^2*b*d^2*(c^2 + 3*d^2) + b^3*(8*c^4 - 15*c
^2*d^2 + 3*d^4))*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Si
n[e + f*x]])/(3*d^3*(c^2 - d^2)^2*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) - (
2*(b*c - a*d)*(2*a*b*c*d - a^2*d^2 + b^2*(8*c^2 - 9*d^2))*EllipticF[(e - Pi
/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(3*d^3*(c^2
 - d^2)*f*Sqrt[c + d*Sin[e + f*x]])
```

Rubi [A] time = 0.74793, antiderivative size = 391, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2792, 3021, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2\left(-a^2d^2+2abcd+b^2\left(8c^2-9d^2\right)\right)(bc-ad)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3d^3f\left(c^2-d^2\right)\sqrt{c+d\sin(e+fx)}}+\frac{2\left(-3a^2bd^2\left(c^2+3d^2\right)+4a^3cd^3\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^3/(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (2*(b*c - a*d)^2*Cos[e + f*x]*(a + b*Sin[e + f*x]))/(3*d*(c^2 - d^2)*f*(c +
 d*Sin[e + f*x])^(3/2)) + (8*(b*c - a*d)^2*(a*c*d + b*(c^2 - 2*d^2))*Cos[e
+ f*x])/(3*d^2*(c^2 - d^2)^2*f*Sqrt[c + d*Sin[e + f*x]]) + (2*(4*a^3*c*d^3
- 6*a*b^2*c*d*(c^2 - 3*d^2) - 3*a^2*b*d^2*(c^2 + 3*d^2) + b^3*(8*c^4 - 15*c
^2*d^2 + 3*d^4))*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Si
n[e + f*x]])/(3*d^3*(c^2 - d^2)^2*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) - (
2*(b*c - a*d)*(2*a*b*c*d - a^2*d^2 + b^2*(8*c^2 - 9*d^2))*EllipticF[(e - Pi
/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(3*d^3*(c^2
 - d^2)*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{(a+b\sin(e+fx))^3}{(c+d\sin(e+fx))^{5/2}}\,dx = \frac{2(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{3d\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}} - \frac{2\int \frac{\frac{1}{2}\left(2b(bc-ad)^2-3ad\left(\left(a^2+b^2\right)c-2abd\right)\right)-\frac{1}{2}\left(5a^2bcd+3b\ldots\right.}{\ldots}}{\ldots}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{3d\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}} + \frac{8(bc-ad)^2\left(acd+b\left(c^2-2d^2\right)\right)\cos(e+fx)\ldots}{3d^2\left(c^2-d^2\right)^2 f\sqrt{c+d\sin(e+fx)}}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{3d\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}} + \frac{8(bc-ad)^2\left(acd+b\left(c^2-2d^2\right)\right)\cos(e+fx)\ldots}{3d^2\left(c^2-d^2\right)^2 f\sqrt{c+d\sin(e+fx)}}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{3d\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}} + \frac{8(bc-ad)^2\left(acd+b\left(c^2-2d^2\right)\right)\cos(e+fx)\ldots}{3d^2\left(c^2-d^2\right)^2 f\sqrt{c+d\sin(e+fx)}}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{3d\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}} + \frac{8(bc-ad)^2\left(acd+b\left(c^2-2d^2\right)\right)\cos(e+fx)\ldots}{3d^2\left(c^2-d^2\right)^2 f\sqrt{c+d\sin(e+fx)}}$$

Mathematica [A] time = 3.6357, size = 357, normalized size = 0.91

$$\frac{2\left(\frac{(-c-d\sin(e+fx))\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\left(d^2\left(-12a^2bcd^2+a^3d\left(3c^2+d^2\right)+3ab^2d\left(c^2+3d^2\right)+2b^3\left(c^3-3cd^2\right)\right)F\left(\frac{1}{4}(-2e-2fx+\pi)|\frac{2d}{c+d}\right)+\left(-3a^2bd^2\left(c^2+3d^2\right)+4a^3cd^3-6ab^2c\ldots\right.\right.}{(c-d)^2(c+d)^2}\ldots\right)}{3d^3f\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^3/(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (2*(((d^2*(-12*a^2*b*c*d^2 + a^3*d*(3*c^2 + d^2) + 3*a*b^2*d*(c^2 + 3*d^2)
+ 2*b^3*(c^3 - 3*c*d^2))*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)] +
(4*a^3*c*d^3 - 6*a*b^2*c*d*(c^2 - 3*d^2) - 3*a^2*b*d^2*(c^2 + 3*d^2) + b^3*
(8*c^4 - 15*c^2*d^2 + 3*d^4))*((c + d)*EllipticE[(-2*e + Pi - 2*f*x)/4, (2*
d)/(c + d)] - c*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]))*(-c - d*S
in[e + f*x])*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/((c - d)^2*(c + d)^2) - (d
*(b*c - a*d)^2*Cos[e + f*x]*(-4*b*c^3 - 5*a*c^2*d + 8*b*c*d^2 + a*d^3 + d*(
-5*b*c^2 - 4*a*c*d + 9*b*d^2)*Sin[e + f*x]))/(c^2 - d^2)^2))/(3*d^3*f*(c +
d*Sin[e + f*x])^(3/2))
```

Maple [B] time = 5.682, size = 1379, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^(5/2),x)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*(b^2/d^3*(2*b*d*(c/d-1)*((c+d*sin(f
*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d)
)^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*s
in(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c
-d))^(1/2),((c-d)/(c+d))^(1/2)))+6*d*a*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/
```

```
2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin
(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c
-d)/(c+d))^(1/2))-4*c*b*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*
x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(
f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2
)))+3*b/d^3*(a^2*d^2-2*a*b*c*d+b^2*c^2)*(2*d*cos(f*x+e)^2/(c^2-d^2)/(-(-d*s
in(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2*c/(c^2-d^2)*(c/d-1)*((c+d*sin(f*x+e))/(c
-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(
-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(
1/2),((c-d)/(c+d))^(1/2))+2/(c^2-d^2)*d*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1
/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*si
n(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d)
)^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d
)/(c+d))^(1/2))))+1/d^3*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)*(2/3/
(c^2-d^2)/d*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(sin(f*x+e)+c/d)^2+8/3*
d*cos(f*x+e)^2/(c^2-d^2)^2*c/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2*(3*c
^2+d^2)/(3*c^4-6*c^2*d^2+3*d^4)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(
1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)
-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+
d))^(1/2))+8/3*c*d/(c^2-d^2)^2*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1
-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-
c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),(
(c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^
(1/2)))))/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\sin\left(fx+e\right)+a\right)^{3}}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)^3/(d*sin(f*x + e) + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(3\,ab^{2}\cos\left(fx+e\right)^{2}-a^{3}-3\,ab^{2}+\left(b^{3}\cos\left(fx+e\right)^{2}-3\,a^{2}b-b^{3}\right)\sin\left(fx+e\right)\right)\sqrt{d\sin\left(fx+e\right)+c}}{3\,cd^{2}\cos\left(fx+e\right)^{2}-c^{3}-3\,cd^{2}+\left(d^{3}\cos\left(fx+e\right)^{2}-3\,c^{2}d-d^{3}\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((3*a*b^2*cos(f*x + e)^2 - a^3 - 3*a*b^2 + (b^3*cos(f*x + e)^2 - 3*
a^2*b - b^3)*sin(f*x + e))*sqrt(d*sin(f*x + e) + c)/(3*c*d^2*cos(f*x + e)^2
 - c^3 - 3*c*d^2 + (d^3*cos(f*x + e)^2 - 3*c^2*d - d^3)*sin(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**3/(c+d*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\sin\left(fx+e\right)+a\right)^3}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)^3/(d*sin(f*x + e) + c)^(5/2), x)
```

3.743 $\int \frac{(a+b\sin(e+fx))^3}{(c+d\sin(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=532

$$-\frac{2\left(a^2d^2\left(23c^2+9d^2\right)+2abd\left(7c^3-39cd^2\right)+b^2\left(-21c^2d^2+8c^4+45d^4\right)\right)(bc-ad)\cos(e+fx)}{15d^2f\left(c^2-d^2\right)^3\sqrt{c+d\sin(e+fx)}}-\frac{2\left(-3a^2bd^2\left(3c^2+5d^2\right)\right.}{}$$

```
[Out] (2*(b*c - a*d)^2*Cos[e + f*x]*(a + b*Sin[e + f*x]))/(5*d*(c^2 - d^2)*f*(c +
 d*Sin[e + f*x])^(5/2)) + (8*(b*c - a*d)^2*(2*a*c*d + b*(c^2 - 3*d^2))*Cos[
e + f*x])/(15*d^2*(c^2 - d^2)^2*f*(c + d*Sin[e + f*x])^(3/2)) - (2*(b*c - a
*d)*(a^2*d^2*(23*c^2 + 9*d^2) + 2*a*b*d*(7*c^3 - 39*c*d^2) + b^2*(8*c^4 - 2
1*c^2*d^2 + 45*d^4))*Cos[e + f*x])/(15*d^2*(c^2 - d^2)^3*f*Sqrt[c + d*Sin[e
 + f*x]]) - (2*(b*c - a*d)*(a^2*d^2*(23*c^2 + 9*d^2) + 2*a*b*d*(7*c^3 - 39*
c*d^2) + b^2*(8*c^4 - 21*c^2*d^2 + 45*d^4))*EllipticE[(e - Pi/2 + f*x)/2, (
2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(15*d^3*(c^2 - d^2)^3*f*Sqrt[(c + d
*Sin[e + f*x])/(c + d)]) - (2*(8*a^3*c*d^3 - 6*a*b^2*c*d*(c^2 - 5*d^2) - 3*
a^2*b*d^2*(3*c^2 + 5*d^2) - b^3*(8*c^4 - 15*c^2*d^2 + 15*d^4))*EllipticF[(e
 - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(15*d^
3*(c^2 - d^2)^2*f*Sqrt[c + d*Sin[e + f*x]])
```

Rubi [A] time = 1.10912, antiderivative size = 532, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2792, 3021, 2754, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2\left(a^2d^2\left(23c^2+9d^2\right)+2abd\left(7c^3-39cd^2\right)+b^2\left(-21c^2d^2+8c^4+45d^4\right)\right)(bc-ad)\cos(e+fx)}{15d^2f\left(c^2-d^2\right)^3\sqrt{c+d\sin(e+fx)}}-\frac{2\left(-3a^2bd^2\left(3c^2+5d^2\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^3/(c + d*Sin[e + f*x])^(7/2),x]
```


```
[Out] (2*(b*c - a*d)^2*Cos[e + f*x]*(a + b*Sin[e + f*x]))/(5*d*(c^2 - d^2)*f*(c +
 d*Sin[e + f*x])^(5/2)) + (8*(b*c - a*d)^2*(2*a*c*d + b*(c^2 - 3*d^2))*Cos[
e + f*x])/(15*d^2*(c^2 - d^2)^2*f*(c + d*Sin[e + f*x])^(3/2)) - (2*(b*c - a
*d)*(a^2*d^2*(23*c^2 + 9*d^2) + 2*a*b*d*(7*c^3 - 39*c*d^2) + b^2*(8*c^4 - 2
1*c^2*d^2 + 45*d^4))*Cos[e + f*x])/(15*d^2*(c^2 - d^2)^3*f*Sqrt[c + d*Sin[e
 + f*x]]) - (2*(b*c - a*d)*(a^2*d^2*(23*c^2 + 9*d^2) + 2*a*b*d*(7*c^3 - 39*
c*d^2) + b^2*(8*c^4 - 21*c^2*d^2 + 45*d^4))*EllipticE[(e - Pi/2 + f*x)/2, (
2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(15*d^3*(c^2 - d^2)^3*f*Sqrt[(c + d
*Sin[e + f*x])/(c + d)]) - (2*(8*a^3*c*d^3 - 6*a*b^2*c*d*(c^2 - 5*d^2) - 3*
a^2*b*d^2*(3*c^2 + 5*d^2) - b^3*(8*c^4 - 15*c^2*d^2 + 15*d^4))*EllipticF[(e
 - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(15*d^
3*(c^2 - d^2)^2*f*Sqrt[c + d*Sin[e + f*x]])
```


Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
```

```
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{(a+b\sin(e+fx))^3}{(c+d\sin(e+fx))^{7/2}}\,dx = \frac{2(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{5d\left(c^2-d^2\right)f(c+d\sin(e+fx))^{5/2}} - \frac{2\int \frac{\frac{1}{2}\left(2b(bc-ad)^2-5ad\left(\left(a^2+b^2\right)c-2abd\right)\right)+\frac{1}{2}\left(3a(bc-ad)^2-5b\cdots\right.}{\cdots}}{\cdots}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{5d\left(c^2-d^2\right)f(c+d\sin(e+fx))^{5/2}} + \frac{8(bc-ad)^2\left(2acd+b\left(c^2-3d^2\right)\right)\cos(e+fx)\cdots}{15d^2\left(c^2-d^2\right)^2 f(c+d\sin(e+fx))^{3/2}}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{5d\left(c^2-d^2\right)f(c+d\sin(e+fx))^{5/2}} + \frac{8(bc-ad)^2\left(2acd+b\left(c^2-3d^2\right)\right)\cos(e+fx)\cdots}{15d^2\left(c^2-d^2\right)^2 f(c+d\sin(e+fx))^{3/2}}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{5d\left(c^2-d^2\right)f(c+d\sin(e+fx))^{5/2}} + \frac{8(bc-ad)^2\left(2acd+b\left(c^2-3d^2\right)\right)\cos(e+fx)\cdots}{15d^2\left(c^2-d^2\right)^2 f(c+d\sin(e+fx))^{3/2}}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{5d\left(c^2-d^2\right)f(c+d\sin(e+fx))^{5/2}} + \frac{8(bc-ad)^2\left(2acd+b\left(c^2-3d^2\right)\right)\cos(e+fx)\cdots}{15d^2\left(c^2-d^2\right)^2 f(c+d\sin(e+fx))^{3/2}}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{5d\left(c^2-d^2\right)f(c+d\sin(e+fx))^{5/2}} + \frac{8(bc-ad)^2\left(2acd+b\left(c^2-3d^2\right)\right)\cos(e+fx)\cdots}{15d^2\left(c^2-d^2\right)^2 f(c+d\sin(e+fx))^{3/2}}$$

Mathematica [A] time = 5.30722, size = 584, normalized size = 1.1

$$\frac{2\left(\frac{d(bc-ad)\cos(e+fx)\left(2d\left(2a^2cd^2\left(27c^2+5d^2\right)+abd\left(-170c^2d^2+27c^4+15d^4\right)+b^2\left(-20c^3d^2+9c^5+75cd^4\right)\right)\sin(e+fx)-d^2\left(a^2d^2\left(23c^2+9d^2\right)+2abd\left(7c^3-39cd^2\right)+b^2\left(-21\cdots\right.\right.\right.}{2\left(d^2-c^2\right)^3}\cdots\right)}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^3/(c + d*Sin[e + f*x])^(7/2),x]
```

```
[Out] (2*(((d^2*(3*a^2*b*d^2*(27*c^2 + 5*d^2) - a^3*c*d*(15*c^2 + 17*d^2) - 3*a*b
^2*d*(7*c^3 + 25*c*d^2) + b^3*(2*c^4 + 15*c^2*d^2 + 15*d^4))*EllipticF[(-2*
e + Pi - 2*f*x)/4, (2*d)/(c + d)] + (-(a^3*d^3*(23*c^2 + 9*d^2)) + 3*a^2*b*
c*d^2*(3*c^2 + 29*d^2) - 3*a*b^2*d*(-2*c^4 + 19*c^2*d^2 + 15*d^4) + b^3*(8*
c^5 - 21*c^3*d^2 + 45*c*d^4))*((c + d)*EllipticE[(-2*e + Pi - 2*f*x)/4, (2*
d)/(c + d)] - c*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]))*((c + d*S
in[e + f*x])/(c + d))^(5/2))/((c - d)^3*(c + d)) + (d*(b*c - a*d)*Cos[e + f
*x]*(8*b^2*c^6 + 14*a*b*c^5*d + 68*a^2*c^4*d^2 - 2*b^2*c^4*d^2 - 146*a*b*c^
3*d^3 + 13*a^2*c^2*d^4 + 45*b^2*c^2*d^4 - 60*a*b*c*d^5 + 15*a^2*d^6 + 45*b^
2*d^6 - d^2*(a^2*d^2*(23*c^2 + 9*d^2) + 2*a*b*d*(7*c^3 - 39*c*d^2) + b^2*(8
*c^4 - 21*c^2*d^2 + 45*d^4))*Cos[2*(e + f*x)] + 2*d*(2*a^2*c*d^2*(27*c^2 +
5*d^2) + a*b*d*(27*c^4 - 170*c^2*d^2 + 15*d^4) + b^2*(9*c^5 - 20*c^3*d^2 +
75*c*d^4))*Sin[e + f*x]))/(2*(-c^2 + d^2)^3)))/(15*d^3*f*(c + d*Sin[e + f*x
])^(5/2))
```

Maple [B] time = 8.145, size = 1621, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^(7/2),x)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*(2*b^3/d^3*(c/d-1)*((c+d*sin(f*x+e)
)/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/
2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d
))^(1/2),((c-d)/(c+d))^(1/2))+1/d^3*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^
3*c^3)*(2/5/(c^2-d^2)/d^2*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(sin(f*x+
e)+c/d)^3+16/15*c/(c^2-d^2)^2/d*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(si
n(f*x+e)+c/d)^2+2/15*d*cos(f*x+e)^2/(c^2-d^2)^3*(23*c^2+9*d^2)/(-(-d*sin(f*
x+e)-c)*cos(f*x+e)^2)^(1/2)+2*(15*c^3+17*c*d^2)/(15*c^6-45*c^4*d^2+45*c^2*d
^4-15*d^6)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^
(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/
2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+2/15*d*(23
*c^2+9*d^2)/(c^2-d^2)^3*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*
x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(
f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(
c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))
)+3*b^2/d^3*(a*d-b*c)*(2*d*cos(f*x+e)^2/(c^2-d^2)/(-(-d*sin(f*x+e)-c)*cos(f
*x+e)^2)^(1/2)+2*c/(c^2-d^2)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-s
in(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)
*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))
^(1/2))+2/(c^2-d^2)*d*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+
e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*
x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+
d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+
3*b/d^3*(a^2*d^2-2*a*b*c*d+b^2*c^2)*(2/3/(c^2-d^2)/d*(-(-d*sin(f*x+e)-c)*co
s(f*x+e)^2)^(1/2)/(sin(f*x+e)+c/d)^2+8/3*d*cos(f*x+e)^2/(c^2-d^2)^2*c/(-(-d
*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2*(3*c^2+d^2)/(3*c^4-6*c^2*d^2+3*d^4)*(c
/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(
f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(
((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+8/3*c*d/(c^2-d^2)^2*(c/
d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f
*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*E
llipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+
d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))))/cos(f*x+e)/(c+d*sin(f*x+
e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\sin\left(fx+e\right)+a\right)^3}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)^3/(d*sin(f*x + e) + c)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(3\,ab^2\cos\left(fx+e\right)^2-a^3-3\,ab^2+\left(b^3\cos\left(fx+e\right)^2-3\,a^2b-b^3\right)\sin\left(fx+e\right)\right)\sqrt{d\sin\left(fx+e\right)+c}}{d^4\cos\left(fx+e\right)^4+c^4+6\,c^2d^2+d^4-2\left(3\,c^2d^2+d^4\right)\cos\left(fx+e\right)^2-4\left(cd^3\cos\left(fx+e\right)^2-c^3d-cd^3\right)\sin\left(\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^(7/2),x, algorithm="fricas")
```

```
[Out] integral(-(3*a*b^2*cos(f*x + e)^2 - a^3 - 3*a*b^2 + (b^3*cos(f*x + e)^2 - 3
*a^2*b - b^3)*sin(f*x + e))*sqrt(d*sin(f*x + e) + c)/(d^4*cos(f*x + e)^4 +
c^4 + 6*c^2*d^2 + d^4 - 2*(3*c^2*d^2 + d^4)*cos(f*x + e)^2 - 4*(c*d^3*cos(f
*x + e)^2 - c^3*d - c*d^3)*sin(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**3/(c+d*sin(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b \sin \left(fx+e\right)+a\right)^{3}}{\left(d \sin \left(fx+e\right)+c\right)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)^3/(d*sin(f*x + e) + c)^(7/2), x)
```

3.744 $\int \frac{(a+b\sin(e+fx))^3}{(c+d\sin(e+fx))^{9/2}}\,dx$

Optimal. Leaf size=716

$$-\frac{2\left(a^2d^2\left(71c^2+25d^2\right)+ab\left(26c^3d-218cd^3\right)+b^2\left(-17c^2d^2+8c^4+105d^4\right)\right)(bc-ad)\cos(e+fx)}{105d^2f\left(c^2-d^2\right)^3(c+d\sin(e+fx))^{3/2}}+\frac{2\left(-9a^2bd^2\left(102\right.\right.}{}$$

```
[Out] (2*(b*c - a*d)^2*Cos[e + f*x]*(a + b*Sin[e + f*x]))/(7*d*(c^2 - d^2)*f*(c +
 d*Sin[e + f*x])^(7/2)) + (8*(b*c - a*d)^2*(3*a*c*d + b*(c^2 - 4*d^2))*Cos[
e + f*x])/(35*d^2*(c^2 - d^2)^2*f*(c + d*Sin[e + f*x])^(5/2)) - (2*(b*c - a
*d)*(a^2*d^2*(71*c^2 + 25*d^2) + a*b*(26*c^3*d - 218*c*d^3) + b^2*(8*c^4 -
17*c^2*d^2 + 105*d^4))*Cos[e + f*x])/(105*d^2*(c^2 - d^2)^3*f*(c + d*Sin[e
+ f*x])^(3/2)) + (2*(16*a^3*c*d^3*(11*c^2 + 13*d^2) - 6*a*b^2*c*d*(3*c^4 -
62*c^2*d^2 - 133*d^4) - 9*a^2*b*d^2*(5*c^4 + 102*c^2*d^2 + 21*d^4) - b^3*(8
*c^6 - 23*c^4*d^2 + 294*c^2*d^4 + 105*d^6))*Cos[e + f*x])/(105*d^2*(c^2 - d
^2)^4*f*Sqrt[c + d*Sin[e + f*x]]) + (2*(16*a^3*c*d^3*(11*c^2 + 13*d^2) - 6*
a*b^2*c*d*(3*c^4 - 62*c^2*d^2 - 133*d^4) - 9*a^2*b*d^2*(5*c^4 + 102*c^2*d^2
 + 21*d^4) - b^3*(8*c^6 - 23*c^4*d^2 + 294*c^2*d^4 + 105*d^6))*EllipticE[(e
 - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(105*d^3*(c^2 -
d^2)^4*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + (2*(b*c - a*d)*(a^2*d^2*(71*
c^2 + 25*d^2) + a*b*(26*c^3*d - 218*c*d^3) + b^2*(8*c^4 - 17*c^2*d^2 + 105*
d^4))*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x]
)/(c + d)])/(105*d^3*(c^2 - d^2)^3*f*Sqrt[c + d*Sin[e + f*x]])
```

Rubi [A] time = 1.43605, antiderivative size = 716, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2792, 3021, 2754, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2\left(a^2d^2\left(71c^2+25d^2\right)+ab\left(26c^3d-218cd^3\right)+b^2\left(-17c^2d^2+8c^4+105d^4\right)\right)(bc-ad)\cos(e+fx)}{105d^2f\left(c^2-d^2\right)^3(c+d\sin(e+fx))^{3/2}}+\frac{2\left(-9a^2bd^2\left(102\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^3/(c + d*Sin[e + f*x])^(9/2),x]
```


```
[Out] (2*(b*c - a*d)^2*Cos[e + f*x]*(a + b*Sin[e + f*x]))/(7*d*(c^2 - d^2)*f*(c +
 d*Sin[e + f*x])^(7/2)) + (8*(b*c - a*d)^2*(3*a*c*d + b*(c^2 - 4*d^2))*Cos[
e + f*x])/(35*d^2*(c^2 - d^2)^2*f*(c + d*Sin[e + f*x])^(5/2)) - (2*(b*c - a
*d)*(a^2*d^2*(71*c^2 + 25*d^2) + a*b*(26*c^3*d - 218*c*d^3) + b^2*(8*c^4 -
17*c^2*d^2 + 105*d^4))*Cos[e + f*x])/(105*d^2*(c^2 - d^2)^3*f*(c + d*Sin[e
+ f*x])^(3/2)) + (2*(16*a^3*c*d^3*(11*c^2 + 13*d^2) - 6*a*b^2*c*d*(3*c^4 -
62*c^2*d^2 - 133*d^4) - 9*a^2*b*d^2*(5*c^4 + 102*c^2*d^2 + 21*d^4) - b^3*(8
*c^6 - 23*c^4*d^2 + 294*c^2*d^4 + 105*d^6))*Cos[e + f*x])/(105*d^2*(c^2 - d
^2)^4*f*Sqrt[c + d*Sin[e + f*x]]) + (2*(16*a^3*c*d^3*(11*c^2 + 13*d^2) - 6*
a*b^2*c*d*(3*c^4 - 62*c^2*d^2 - 133*d^4) - 9*a^2*b*d^2*(5*c^4 + 102*c^2*d^2
 + 21*d^4) - b^3*(8*c^6 - 23*c^4*d^2 + 294*c^2*d^4 + 105*d^6))*EllipticE[(e
 - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(105*d^3*(c^2 -
d^2)^4*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + (2*(b*c - a*d)*(a^2*d^2*(71*
c^2 + 25*d^2) + a*b*(26*c^3*d - 218*c*d^3) + b^2*(8*c^4 - 17*c^2*d^2 + 105*
d^4))*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x]
)/(c + d)])/(105*d^3*(c^2 - d^2)^3*f*Sqrt[c + d*Sin[e + f*x]])
```


Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{(a+b\sin(e+fx))^3}{(c+d\sin(e+fx))^{9/2}}\,dx = \frac{2(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{7d\left(c^2-d^2\right)f(c+d\sin(e+fx))^{7/2}} - \frac{2\int \frac{\frac{1}{2}\left(2b(bc-ad)^2-7ad\left(\left(a^2+b^2\right)c-2abd\right)\right)+\frac{1}{2}\left(5a(bc-ad)\right.}{}}{}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{7d\left(c^2-d^2\right)f(c+d\sin(e+fx))^{7/2}} + \frac{8(bc-ad)^2\left(3acd+b\left(c^2-4d^2\right)\right)\cos(e+f}{35d^2\left(c^2-d^2\right)^2f(c+d\sin(e+fx))^{5/2}}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{7d\left(c^2-d^2\right)f(c+d\sin(e+fx))^{7/2}} + \frac{8(bc-ad)^2\left(3acd+b\left(c^2-4d^2\right)\right)\cos(e+f}{35d^2\left(c^2-d^2\right)^2f(c+d\sin(e+fx))^{5/2}}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{7d\left(c^2-d^2\right)f(c+d\sin(e+fx))^{7/2}} + \frac{8(bc-ad)^2\left(3acd+b\left(c^2-4d^2\right)\right)\cos(e+f}{35d^2\left(c^2-d^2\right)^2f(c+d\sin(e+fx))^{5/2}}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{7d\left(c^2-d^2\right)f(c+d\sin(e+fx))^{7/2}} + \frac{8(bc-ad)^2\left(3acd+b\left(c^2-4d^2\right)\right)\cos(e+f}{35d^2\left(c^2-d^2\right)^2f(c+d\sin(e+fx))^{5/2}}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{7d\left(c^2-d^2\right)f(c+d\sin(e+fx))^{7/2}} + \frac{8(bc-ad)^2\left(3acd+b\left(c^2-4d^2\right)\right)\cos(e+f}{35d^2\left(c^2-d^2\right)^2f(c+d\sin(e+fx))^{5/2}}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{7d\left(c^2-d^2\right)f(c+d\sin(e+fx))^{7/2}} + \frac{8(bc-ad)^2\left(3acd+b\left(c^2-4d^2\right)\right)\cos(e+f}{35d^2\left(c^2-d^2\right)^2f(c+d\sin(e+fx))^{5/2}}$$

Mathematica [A] time = 6.94125, size = 1127, normalized size = 1.57

$$\frac{\sqrt{c+d\sin(e+fx)}\left(-\frac{2\left(-b^3\cos(e+fx)c^3+3ab^2d\cos(e+fx)c^2-3a^2bd^2\cos(e+fx)c+a^3d^3\cos(e+fx)\right)}{7d^2\left(d^2-c^2\right)(c+d\sin(e+fx))^4} - \frac{2\left(8b^3\cos(e+fx)c^6+18ab^2d\cos(e+fx)c^5-23b^3\right.}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^3/(c + d*Sin[e + f*x])^(9/2),x]
```

```
[Out] (Sqrt[c + d*Sin[e + f*x]]*((-2*(-(b^3*c^3*Cos[e + f*x]) + 3*a*b^2*c^2*d*Cos
[e + f*x] - 3*a^2*b*c*d^2*Cos[e + f*x] + a^3*d^3*Cos[e + f*x]))/(7*d^2*(-c^
2 + d^2)*(c + d*Sin[e + f*x])^4) - (6*(-3*b^3*c^4*Cos[e + f*x] + 2*a*b^2*c^
3*d*Cos[e + f*x] + 5*a^2*b*c^2*d^2*Cos[e + f*x] + 7*b^3*c^2*d^2*Cos[e + f*x
] - 4*a^3*c*d^3*Cos[e + f*x] - 14*a*b^2*c*d^3*Cos[e + f*x] + 7*a^2*b*d^4*Co
s[e + f*x]))/(35*d^2*(-c^2 + d^2)^2*(c + d*Sin[e + f*x])^3) - (2*(-8*b^3*c^
5*Cos[e + f*x] - 18*a*b^2*c^4*d*Cos[e + f*x] - 45*a^2*b*c^3*d^2*Cos[e + f*x
] + 17*b^3*c^3*d^2*Cos[e + f*x] + 71*a^3*c^2*d^3*Cos[e + f*x] + 201*a*b^2*c
^2*d^3*Cos[e + f*x] - 243*a^2*b*c*d^4*Cos[e + f*x] - 105*b^3*c*d^4*Cos[e +
f*x] + 25*a^3*d^5*Cos[e + f*x] + 105*a*b^2*d^5*Cos[e + f*x]))/(105*d^2*(-c^
2 + d^2)^3*(c + d*Sin[e + f*x])^2) - (2*(8*b^3*c^6*Cos[e + f*x] + 18*a*b^2*
c^5*d*Cos[e + f*x] + 45*a^2*b*c^4*d^2*Cos[e + f*x] - 23*b^3*c^4*d^2*Cos[e +
```

```
 f*x] - 176*a^3*c^3*d^3*Cos[e + f*x] - 372*a*b^2*c^3*d^3*Cos[e + f*x] + 918
*a^2*b*c^2*d^4*Cos[e + f*x] + 294*b^3*c^2*d^4*Cos[e + f*x] - 208*a^3*c*d^5*
Cos[e + f*x] - 798*a*b^2*c*d^5*Cos[e + f*x] + 189*a^2*b*d^6*Cos[e + f*x] +
105*b^3*d^6*Cos[e + f*x]))/(105*d^2*(-c^2 + d^2)^4*(c + d*Sin[e + f*x]))))/
f - ((-2*(2*b^3*c^5*d - 105*a^3*c^4*d^2 - 153*a*b^2*c^4*d^2 + 720*a^2*b*c^3
*d^3 + 172*b^3*c^3*d^3 - 254*a^3*c^2*d^4 - 894*a*b^2*c^2*d^4 + 432*a^2*b*c*
d^5 + 210*b^3*c*d^5 - 25*a^3*d^6 - 105*a*b^2*d^6)*EllipticF[(-e + Pi/2 - f*
x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/Sqrt[c + d*Sin[e +
 f*x]] - ((8*b^3*c^6 + 18*a*b^2*c^5*d + 45*a^2*b*c^4*d^2 - 23*b^3*c^4*d^2 -
 176*a^3*c^3*d^3 - 372*a*b^2*c^3*d^3 + 918*a^2*b*c^2*d^4 + 294*b^3*c^2*d^4
- 208*a^3*c*d^5 - 798*a*b^2*c*d^5 + 189*a^2*b*d^6 + 105*b^3*d^6)*((2*(c + d
)*EllipticE[(-e + Pi/2 - f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(
c + d)])/Sqrt[c + d*Sin[e + f*x]] - (2*c*EllipticF[(-e + Pi/2 - f*x)/2, (2*
d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/Sqrt[c + d*Sin[e + f*x]]))/
d)/(105*(c - d)^4*d^2*(c + d)^4*f)
```

Maple [B] time = 12.731, size = 2111, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^(9/2),x)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*(3*b*(a^2*d^2-2*a*b*c*d+b^2*c^2)/d^
3*(2/5/(c^2-d^2)/d^2*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(sin(f*x+e)+c/
d)^3+16/15*c/(c^2-d^2)^2/d*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(sin(f*x
+e)+c/d)^2+2/15*d*cos(f*x+e)^2/(c^2-d^2)^3*(23*c^2+9*d^2)/(-(-d*sin(f*x+e)-
c)*cos(f*x+e)^2)^(1/2)+2*(15*c^3+17*c*d^2)/(15*c^6-45*c^4*d^2+45*c^2*d^4-15
*d^6)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)
*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*El
lipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+2/15*d*(23*c^2+
9*d^2)/(c^2-d^2)^3*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))
/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e
)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))
^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+(a^
3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/d^3*(2/7/(c^2-d^2)/d^3*(-(-d*sin
(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(sin(f*x+e)+c/d)^4+24/35/(c^2-d^2)^2/d^2*c*(
-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(sin(f*x+e)+c/d)^3+2/105*(71*c^2+25*
d^2)/d/(c^2-d^2)^3*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(sin(f*x+e)+c/d)
^2+32/105*d*cos(f*x+e)^2/(c^2-d^2)^4*c*(11*c^2+13*d^2)/(-(-d*sin(f*x+e)-c)*
cos(f*x+e)^2)^(1/2)+2*(105*c^4+254*c^2*d^2+25*d^4)/(105*c^8-420*c^6*d^2+630
*c^4*d^4-420*c^2*d^6+105*d^8)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-
sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c
)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d)
)^(1/2))+32/105*c*d*(11*c^2+13*d^2)/(c^2-d^2)^4*(c/d-1)*((c+d*sin(f*x+e))/(
c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/
(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e
))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/
2),((c-d)/(c+d))^(1/2))))+b^3/d^3*(2*d*cos(f*x+e)^2/(c^2-d^2)/(-(-d*sin(f*x
+e)-c)*cos(f*x+e)^2)^(1/2)+2*c/(c^2-d^2)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(
1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*s
in(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),(
(c-d)/(c+d))^(1/2))+2/(c^2-d^2)*d*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d
*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+
e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2
),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d
```

```
))^(1/2))))+3*b^2*(a*d-b*c)/d^3*(2/3/(c^2-d^2)/d*(-(-d*sin(f*x+e)-c)*cos(f*
x+e)^2)^(1/2)/(sin(f*x+e)+c/d)^2+8/3*d*cos(f*x+e)^2/(c^2-d^2)^2*c/(-(-d*sin
(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2*(3*c^2+d^2)/(3*c^4-6*c^2*d^2+3*d^4)*(c/d-1
)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+
e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+
d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+8/3*c*d/(c^2-d^2)^2*(c/d-1)
*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e
)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*Ellip
ticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*si
n(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))))/cos(f*x+e)/(c+d*sin(f*x+e))^
(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\sin\left(fx+e\right)+a\right)^{3}}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)^3/(d*sin(f*x + e) + c)^(9/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(3\,ab^{2}\cos\left(fx+e\right)^{2}-a^{3}-3\,ab^{2}+\left(b^{3}\cos\left(fx+e\right)^{2}-3\,a^{2}b-b^{3}\right)\sin\left(fx+\right.}{5\,cd^{4}\cos\left(fx+e\right)^{4}+c^{5}+10\,c^{3}d^{2}+5\,cd^{4}-10\left(c^{3}d^{2}+cd^{4}\right)\cos\left(fx+e\right)^{2}+\left(d^{5}\cos\left(fx+e\right)^{4}+5\,c^{4}d+10\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^(9/2),x, algorithm="fricas")
```

```
[Out] integral(-(3*a*b^2*cos(f*x + e)^2 - a^3 - 3*a*b^2 + (b^3*cos(f*x + e)^2 - 3
*a^2*b - b^3)*sin(f*x + e))*sqrt(d*sin(f*x + e) + c)/(5*c*d^4*cos(f*x + e)^
4 + c^5 + 10*c^3*d^2 + 5*c*d^4 - 10*(c^3*d^2 + c*d^4)*cos(f*x + e)^2 + (d^5
*cos(f*x + e)^4 + 5*c^4*d + 10*c^2*d^3 + d^5 - 2*(5*c^2*d^3 + d^5)*cos(f*x
+ e)^2)*sin(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**3/(c+d*sin(f*x+e))**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b \sin\left(fx+e\right)+a\right)^{3}}{\left(d \sin\left(fx+e\right)+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^(9/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)^3/(d*sin(f*x + e) + c)^(9/2), x)
```

3.745 $\int \frac{(c+d\sin(e+fx))^{5/2}}{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=296

$$-\frac{2d\left(-3a^2d^2+6abcd+b^2\left(-\left(2c^2+d^2\right)\right)\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3b^3f\sqrt{c+d\sin(e+fx)}}+\frac{2d(7bc-3ad)\sqrt{c+d\sin(e+fx)}E\left(\right.}{3b^2f\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

```
[Out] (-2*d^2*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(3*b*f) + (2*d*(7*b*c - 3*a*
d)*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(
3*b^2*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) - (2*d*(6*a*b*c*d - 3*a^2*d^2 -
 b^2*(2*c^2 + d^2))*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c +
d*Sin[e + f*x])/(c + d)])/(3*b^3*f*Sqrt[c + d*Sin[e + f*x]]) + (2*(b*c - a*
d)^3*EllipticPi[(2*b)/(a + b), (e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c +
 d*Sin[e + f*x])/(c + d)])/(b^3*(a + b)*f*Sqrt[c + d*Sin[e + f*x]])
```

Rubi [A] time = 1.08829, antiderivative size = 296, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2793, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$-\frac{2d\left(-3a^2d^2+6abcd+b^2\left(-\left(2c^2+d^2\right)\right)\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3b^3f\sqrt{c+d\sin(e+fx)}}+\frac{2d(7bc-3ad)\sqrt{c+d\sin(e+fx)}E\left(\right.}{3b^2f\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^(5/2)/(a + b*Sin[e + f*x]),x]
```

```
[Out] (-2*d^2*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(3*b*f) + (2*d*(7*b*c - 3*a*
d)*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(
3*b^2*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) - (2*d*(6*a*b*c*d - 3*a^2*d^2 -
 b^2*(2*c^2 + d^2))*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c +
d*Sin[e + f*x])/(c + d)])/(3*b^3*f*Sqrt[c + d*Sin[e + f*x]]) + (2*(b*c - a*
d)^3*EllipticPi[(2*b)/(a + b), (e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c +
 d*Sin[e + f*x])/(c + d)])/(b^3*(a + b)*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
```

```
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^{5/2}}{a+b\sin(e+fx)}\,dx = -\frac{2d^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3bf} + \frac{2\int \frac{\frac{1}{2}(3bc^3+ad^3)-\frac{1}{2}d(2acd-b(9c^2+d^2))\sin(e+fx)+\frac{1}{2}d^2(7bc-3}{(a+b\sin(e+fx))\sqrt{c+d\sin(e+fx)}}}{3b}$$

$$= -\frac{2d^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3bf} - \frac{2\int \frac{\frac{1}{2}d(acd(7bc-3ad)-b(3bc^3+ad^3))+\frac{1}{2}d^2(6abcd-3a^2d^2-b^2(2c}{(a+b\sin(e+fx))\sqrt{c+d\sin(e+fx)}}}{3b^2d}$$

$$= -\frac{2d^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3bf} + \frac{(bc-ad)^3\int \frac{1}{(a+b\sin(e+fx))\sqrt{c+d\sin(e+fx)}}dx}{b^3} - \frac{(d(6}{}$$

$$= -\frac{2d^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3bf} + \frac{2d(7bc-3ad)E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2d}{c+d}\right)\sqrt{c+d\sin}}{3b^2f\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

$$= -\frac{2d^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3bf} + \frac{2d(7bc-3ad)E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2d}{c+d}\right)\sqrt{c+d\sin}}{3b^2f\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

Mathematica [C] time = 5.88064, size = 606, normalized size = 2.05

$$\frac{2i(3ad-7bc)\sec(e+fx)\sqrt{-\frac{d(\sin(e+fx)-1)}{c+d}}\sqrt{\frac{d(\sin(e+fx)+1)}{d-c}}\left(d\left(d(b^2-2a^2)\Pi\left(\frac{b(c+d)}{bc-ad};i\sinh^{-1}\left(\sqrt{-\frac{1}{c+d}}\sqrt{c+d\sin(e+fx)}\right)|\frac{c+d}{c-d}\right)+2(a+b)(ad-bc)F\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{c+d}}\sqrt{c}\right.\right.\right.\right.}{b^2\sqrt{-\frac{1}{c+d}}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])^(5/2)/(a + b*Sin[e + f*x]),x]
```

```
[Out] (((4*I)*(-2*a*c*d + b*(9*c^2 + d^2))*((-(b*c) + a*d)*EllipticF[I*ArcSinh[Sq
rt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)] - a*d*Ellipti
cPi[(b*(c + d))/(b*c - a*d), I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e
 + f*x]]], (c + d)/(c - d)])*Sec[e + f*x]*Sqrt[-((d*(-1 + Sin[e + f*x]))/(c
 + d))]*Sqrt[(d*(1 + Sin[e + f*x]))/(-c + d)])/(b*Sqrt[-(c + d)^(-1)]*(b*c
- a*d)) + ((2*I)*(-7*b*c + 3*a*d)*(-2*b*(c - d)*(b*c - a*d)*EllipticE[I*Arc
Sinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)] + d*(2
*(a + b)*(-(b*c) + a*d)*EllipticF[I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*
Sin[e + f*x]]], (c + d)/(c - d)] + (-2*a^2 + b^2)*d*EllipticPi[(b*(c + d))/
(b*c - a*d), I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c +
d)/(c - d)]))*Sec[e + f*x]*Sqrt[-((d*(-1 + Sin[e + f*x]))/(c + d))]*Sqrt[(d
*(1 + Sin[e + f*x]))/(-c + d)])/(b^2*Sqrt[-(c + d)^(-1)]*(b*c - a*d)) - 4*d
^2*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]] - (2*(6*b*c^3 + 7*b*c*d^2 - a*d^3)
*EllipticPi[(2*b)/(a + b), (-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]*Sqrt[(c +
d*Sin[e + f*x])/(c + d)])/((a + b)*Sqrt[c + d*Sin[e + f*x]]))/(6*b*f)
```

Maple [B] time = 3.692, size = 1190, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*(d/b^3*(b^2*d^2*(-2/3/d*(-(-d*sin(f
*x+e)-c)*cos(f*x+e)^2)^(1/2)+2/3*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*
(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e
)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c
+d))^(1/2))-4/3*c/d*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e)
)/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+
e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d)
)^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+2*
(-a*b*d^2+3*b^2*c*d)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e
))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x
+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d
))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))+2*
a^2*d^2*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/
2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*
EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))-6*a*b*c*d*(c/
d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f
*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF((
(c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+6*c^2*b^2*(c/d-1)*((c+d*
sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/
(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*
x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))+2*(-a^3*d^3+3*a^2*b*c*d^2-3*a*b^2*
c^2*d+b^3*c^3)/b^4*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))
/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e
)^2)^(1/2)/(-c/d+a/b)*EllipticPi(((c+d*sin(f*x+e))/(c-d))^(1/2),(-c/d+1)/(-
c/d+a/b),((c-d)/(c+d))^(1/2)))/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(5/2)/(b*sin(f*x + e) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(5/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

<u>**Giac**</u> **[F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.746 $\int \frac{(c+d\sin(e+fx))^{3/2}}{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=229

$$\frac{2d(bc-ad)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{b^2f\sqrt{c+d\sin(e+fx)}}+\frac{2(bc-ad)^2\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\Pi\left(\frac{2b}{a+b};\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{b^2f(a+b)\sqrt{c+d\sin(e+fx)}}+\frac{2d\sqrt{c+d\sin(}}{}$$

```
[Out] (2*d*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])
/(b*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + (2*d*(b*c - a*d)*EllipticF[(e -
 Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(b^2*f*S
qrt[c + d*Sin[e + f*x]]) + (2*(b*c - a*d)^2*EllipticPi[(2*b)/(a + b), (e -
Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(b^2*(a +
 b)*f*Sqrt[c + d*Sin[e + f*x]])
```

Rubi [A] time = 0.491392, antiderivative size = 229, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2804, 2655, 2653, 2803, 2663, 2661, 2807, 2805}

$$\frac{2d(bc-ad)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{b^2f\sqrt{c+d\sin(e+fx)}}+\frac{2(bc-ad)^2\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\Pi\left(\frac{2b}{a+b};\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{b^2f(a+b)\sqrt{c+d\sin(e+fx)}}+\frac{2d\sqrt{c+d\sin(}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^(3/2)/(a + b*Sin[e + f*x]),x]
```


```
[Out] (2*d*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])
/(b*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + (2*d*(b*c - a*d)*EllipticF[(e -
 Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(b^2*f*S
qrt[c + d*Sin[e + f*x]]) + (2*(b*c - a*d)^2*EllipticPi[(2*b)/(a + b), (e -
Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(b^2*(a +
 b)*f*Sqrt[c + d*Sin[e + f*x]])
```


Rule 2804

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)/((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[b/d, Int[Sqrt[a + b*Sin[e + f*x]], x], x]
 - Dist[(b*c - a*d)/d, Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2803

```
Int[Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_.) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[1/Sqrt[c + d*Sin[e + f*x]], x], x]
 + Dist[(b*c - a*d)/b, Int[1/((a + b*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^{3/2}}{a+b\sin(e+fx)}\,dx = \frac{d\int\sqrt{c+d\sin(e+fx)}\,dx}{b} - \frac{(-bc+ad)\int\frac{\sqrt{c+d\sin(e+fx)}}{a+b\sin(e+fx)}\,dx}{b}$$

$$= \frac{(d(bc-ad))\int\frac{1}{\sqrt{c+d\sin(e+fx)}}\,dx}{b^2} + \frac{(bc-ad)^2\int\frac{1}{(a+b\sin(e+fx))\sqrt{c+d\sin(e+fx)}}\,dx}{b^2} + \frac{\left(d\sqrt{c+d\,\mathrm{s}\right.}{}$$

$$= \frac{2dE\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2d}{c+d}\right)\sqrt{c+d\sin(e+fx)}}{bf\sqrt{\frac{c+d\sin(e+fx)}{c+d}}} + \frac{\left(d(bc-ad)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\right)\int\frac{1}{\sqrt{\frac{c}{c+d}+\frac{d\sin(e}{c+d}}}}{b^2\sqrt{c+d\sin(e+fx)}}$$

$$= \frac{2dE\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2d}{c+d}\right)\sqrt{c+d\sin(e+fx)}}{bf\sqrt{\frac{c+d\sin(e+fx)}{c+d}}} + \frac{2d(bc-ad)F\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2d}{c+d}\right)\sqrt{c+}}{b^2f\sqrt{c+d\sin(e+fx)}}$$

Mathematica [C] time = 3.92112, size = 242, normalized size = 1.06

$$\frac{2i\sec(e+fx)\sqrt{-\frac{d(\sin(e+fx)-1)}{c+d}}\sqrt{-\frac{d(\sin(e+fx)+1)}{c-d}}\left((ad+b(d-2c))F\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{c+d}}\sqrt{c+d\sin(e+fx)}\right)|\frac{c+d}{c-d}\right)+(bc-ad)\Gamma\right.}{b^2f\sqrt{-\frac{1}{c+d}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])^(3/2)/(a + b*Sin[e + f*x]),x]
```

```
[Out] ((2*I)*(b*(c - d)*EllipticE[I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e
+ f*x]]], (c + d)/(c - d)] + (a*d + b*(-2*c + d))*EllipticF[I*ArcSinh[Sqrt[
-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)] + (b*c - a*d)*El
lipticPi[(b*(c + d))/(b*c - a*d), I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*
Sin[e + f*x]]], (c + d)/(c - d)])*Sec[e + f*x]*Sqrt[-((d*(-1 + Sin[e + f*x]
))/(c + d))]*Sqrt[-((d*(1 + Sin[e + f*x]))/(c - d))])/(b^2*Sqrt[-(c + d)^(-
1)]*f)
```

Maple [A] time = 1.343, size = 391, normalized size = 1.7

$$-2\frac{c-d}{b^2\cos\left(fx+e\right)\sqrt{c+d\sin\left(fx+e\right)}f}\left(\text{EllipticE}\left(\sqrt{\frac{c+d\sin\left(fx+e\right)}{c-d}},\sqrt{\frac{c-d}{c+d}}\right)bc+\text{EllipticE}\left(\sqrt{\frac{c+d\sin\left(fx+e\right)}{c-d}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] -2*(EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*b*c+Ellip
ticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*b*d+a*EllipticF(((
c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*d-2*c*b*EllipticF(((c+d*s
in(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))-EllipticF(((c+d*sin(f*x+e))/(c
-d))^(1/2),((c-d)/(c+d))^(1/2))*b*d-EllipticPi(((c+d*sin(f*x+e))/(c-d))^(1/
2),-(c-d)*b/(a*d-b*c),((c-d)/(c+d))^(1/2))*a*d+EllipticPi(((c+d*sin(f*x+e))
/(c-d))^(1/2),-(c-d)*b/(a*d-b*c),((c-d)/(c+d))^(1/2))*b*c)/b^2*(-d*(1+sin(f
*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*((c+d*sin(f*x+e))/(c-d
))^(1/2)*(c-d)/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(3/2)/(b*sin(f*x + e) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}{b\sin\left(fx+e\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral((d*sin(f*x + e) + c)^(3/2)/(b*sin(f*x + e) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(3/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.747 $\int \frac{\sqrt{c+d\sin(e+fx)}}{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=153

$$\frac{2(bc-ad)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\Pi\left(\frac{2b}{a+b};\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{bf(a+b)\sqrt{c+d\sin(e+fx)}}+\frac{2d\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{bf\sqrt{c+d\sin(e+fx)}}$$

```
[Out] (2*d*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])
/(c + d)])/(b*f*Sqrt[c + d*Sin[e + f*x]]) + (2*(b*c - a*d)*EllipticPi[(2*b)
/(a + b), (e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c +
 d)])/(b*(a + b)*f*Sqrt[c + d*Sin[e + f*x]])
```

Rubi [A] time = 0.328846, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2803, 2663, 2661, 2807, 2805}

$$\frac{2(bc-ad)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\Pi\left(\frac{2b}{a+b};\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{bf(a+b)\sqrt{c+d\sin(e+fx)}}+\frac{2d\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{bf\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*Sin[e + f*x]]/(a + b*Sin[e + f*x]),x]
```

```
[Out] (2*d*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])
/(c + d)])/(b*f*Sqrt[c + d*Sin[e + f*x]]) + (2*(b*c - a*d)*EllipticPi[(2*b)
/(a + b), (e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c +
 d)])/(b*(a + b)*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2803

```
Int[Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_.) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[1/Sqrt[c + d*Sin[e + f*x]], x], x]
 + Dist[(b*c - a*d)/b, Int[1/((a + b*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
```

```
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\sqrt{c+d\sin(e+fx)}}{a+b\sin(e+fx)}\,dx = \frac{d\int \frac{1}{\sqrt{c+d\sin(e+fx)}}\,dx}{b} + \frac{(bc-ad)\int \frac{1}{(a+b\sin(e+fx))\sqrt{c+d\sin(e+fx)}}\,dx}{b}$$
$$= \frac{\left(d\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\right)\int \frac{1}{\sqrt{\frac{c}{c+d}+\frac{d\sin(e+fx)}{c+d}}}\,dx}{b\sqrt{c+d\sin(e+fx)}} + \frac{\left((bc-ad)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\right)\int \frac{1}{(a+b\sin(e+fx))\sqrt{\frac{c}{c+d}+\frac{d\sin(e+fx)}{c+d}}}$$
$$= \frac{2dF\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2d}{c+d}\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}{bf\sqrt{c+d\sin(e+fx)}} + \frac{2(bc-ad)\Pi\left(\frac{2b}{a+b};\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2d}{c+d}\right)\sqrt{\frac{c+d\sin}{c+}}}{b(a+b)f\sqrt{c+d\sin(e+fx)}}$$

Mathematica [A] time = 2.8193, size = 114, normalized size = 0.75

$$-\frac{2\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\left(d(a+b)F\left(\frac{1}{4}(-2e-2fx+\pi)|\frac{2d}{c+d}\right)+(bc-ad)\Pi\left(\frac{2b}{a+b};\frac{1}{4}(-2e-2fx+\pi)|\frac{2d}{c+d}\right)\right)}{bf(a+b)\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*Sin[e + f*x]]/(a + b*Sin[e + f*x]),x]
```

```
[Out] (-2*((a + b)*d*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)] + (b*c - a*d
)*EllipticPi[(2*b)/(a + b), (-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)])*Sqrt[(c
+ d*Sin[e + f*x])/(c + d)])/(b*(a + b)*f*Sqrt[c + d*Sin[e + f*x]])
```

Maple [A] time = 1.151, size = 181, normalized size = 1.2

$$2\,\frac{c-d}{b\cos\left(fx+e\right)\sqrt{c+d\sin\left(fx+e\right)}f}\left(\mathrm{EllipticF}\left(\sqrt{\frac{c+d\sin\left(fx+e\right)}{c-d}},\sqrt{\frac{c-d}{c+d}}\right)-\mathrm{EllipticPi}\left(\sqrt{\frac{c+d\sin\left(fx+e\right)}{c-d}},\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] 2*(EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))-EllipticPi
(((c+d*sin(f*x+e))/(c-d))^(1/2),-(c-d)*b/(a*d-b*c),((c-d)/(c+d))^(1/2)))/b*
(-d*(1+sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*((c+d*sin(
f*x+e))/(c-d))^(1/2)*(c-d)/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d\sin\left(fx+e\right)+c}}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*sin(f*x + e) + c)/(b*sin(f*x + e) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c+d\sin\left(e+fx\right)}}{a+b\sin\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(1/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Integral(sqrt(c + d*sin(e + f*x))/(a + b*sin(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d\sin\left(fx+e\right)+c}}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*sin(f*x + e) + c)/(b*sin(f*x + e) + a), x)
```

3.748 $\int \frac{1}{(a+b\sin(e+fx))\sqrt{c+d\sin(e+fx)}}\,dx$

Optimal. Leaf size=75

$$\frac{2\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{f(a+b)\sqrt{c+d\sin(e+fx)}}$$

```
[Out] (2*EllipticPi[(2*b)/(a + b), (e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d
*Sin[e + f*x])/(c + d)])/((a + b)*f*Sqrt[c + d*Sin[e + f*x]])
```

Rubi [A] time = 0.213989, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2807, 2805}

$$\frac{2\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{f(a+b)\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]]),x]
```

```
[Out] (2*EllipticPi[(2*b)/(a + b), (e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d
*Sin[e + f*x])/(c + d)])/((a + b)*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+b\sin(e+fx))\sqrt{c+d\sin(e+fx)}}\,dx &= \frac{\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\int \frac{1}{(a+b\sin(e+fx))\sqrt{\frac{c}{c+d}+\frac{d\sin(e+fx)}{c+d}}}\,dx}{\sqrt{c+d\sin(e+fx)}}\\ &= \frac{2\Pi\left(\frac{2b}{a+b};\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2d}{c+d}\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}{(a+b)f\sqrt{c+d\sin(e+fx)}}\end{aligned}$$

Mathematica [A] time = 0.114261, size = 74, normalized size = 0.99

$$-\frac{2\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\Pi\left(\frac{2b}{a+b};\frac{1}{4}(-2e-2fx+\pi)|\frac{2d}{c+d}\right)}{f(a+b)\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]]),x]
```

```
[Out] (-2*EllipticPi[(2*b)/(a + b), (-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]*Sqrt[(c
 + d*Sin[e + f*x])/(c + d)])/((a + b)*f*Sqrt[c + d*Sin[e + f*x]])
```

Maple [A] time = 0.948, size = 151, normalized size = 2.

$$2\frac{c-d}{(da-cb)\cos\left(fx+e\right)\sqrt{c+d\sin\left(fx+e\right)}f}\sqrt{\frac{c+d\sin\left(fx+e\right)}{c-d}}\sqrt{-\frac{\left(-1+\sin\left(fx+e\right)\right)d}{c+d}}\sqrt{-\frac{d\left(1+\sin\left(fx+e\right)\right)}{c-d}}\mathrm{Ellip}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))^(1/2),x)
```

```
[Out] 2*(c-d)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(-1+sin(f*x+e))*d/(c+d))^(1/2)*(-d
*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticPi(((c+d*sin(f*x+e))/(c-d))^(1/2),-(c-
d)*b/(a*d-b*c),((c-d)/(c+d))^(1/2))/(a*d-b*c)/cos(f*x+e)/(c+d*sin(f*x+e))^(
1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(b\sin\left(fx+e\right)+a\right)\sqrt{d\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\sin\left(fx+e\right)+a\right)\sqrt{d\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)), x)
```

3.749 $\int \frac{1}{(a+b\sin(e+fx))(c+d\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=220

$$-\frac{2d^2\cos(e+fx)}{f(c^2-d^2)(bc-ad)\sqrt{c+d\sin(e+fx)}}-\frac{2d\sqrt{c+d\sin(e+fx)}E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{f(c^2-d^2)(bc-ad)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}+\frac{2b\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\Pi\left(\frac{2b}{a+b};\frac{1}{2}\left(e+\right.\right.}{f(a+b)(bc-ad)\sqrt{c+d\,\mathrm{s}}}$$

```
[Out] (-2*d^2*Cos[e + f*x])/((b*c - a*d)*(c^2 - d^2)*f*Sqrt[c + d*Sin[e + f*x]])
- (2*d*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]
])/((b*c - a*d)*(c^2 - d^2)*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + (2*b*El
lipticPi[(2*b)/(a + b), (e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[
e + f*x])/(c + d)])/((a + b)*(b*c - a*d)*f*Sqrt[c + d*Sin[e + f*x]])
```

Rubi [A] time = 0.630355, antiderivative size = 220, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2802, 3059, 2655, 2653, 12, 2807, 2805}

$$-\frac{2d^2\cos(e+fx)}{f(c^2-d^2)(bc-ad)\sqrt{c+d\sin(e+fx)}}-\frac{2d\sqrt{c+d\sin(e+fx)}E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{f(c^2-d^2)(bc-ad)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}+\frac{2b\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\Pi\left(\frac{2b}{a+b};\frac{1}{2}\left(e+\right.\right.}{f(a+b)(bc-ad)\sqrt{c+d\,\mathrm{s}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sin[e + f*x])*(c + d*Sin[e + f*x])^(3/2)),x]
```


```
[Out] (-2*d^2*Cos[e + f*x])/((b*c - a*d)*(c^2 - d^2)*f*Sqrt[c + d*Sin[e + f*x]])
- (2*d*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]
])/((b*c - a*d)*(c^2 - d^2)*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + (2*b*El
lipticPi[(2*b)/(a + b), (e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[
e + f*x])/(c + d)])/((a + b)*(b*c - a*d)*f*Sqrt[c + d*Sin[e + f*x]])
```


Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\sin(e+fx))(c+d\sin(e+fx))^{3/2}}\,dx = -\frac{2d^2\cos(e+fx)}{(bc-ad)\left(c^2-d^2\right)f\sqrt{c+d\sin(e+fx)}} + \frac{2\int \frac{\frac{1}{2}\left(-acd+b\left(c^2-d^2\right)\right)-\frac{1}{2}d(bc+\ldots}{(a+b\sin(e+fx\ldots}}{(bc-ad\ldots}$$

$$= -\frac{2d^2\cos(e+fx)}{(bc-ad)\left(c^2-d^2\right)f\sqrt{c+d\sin(e+fx)}} - \frac{2\int -\frac{b^2d\left(c^2-d^2\right)}{2(a+b\sin(e+fx))\sqrt{c+d\sin\ldots}}}{bd(bc-ad)\left(c^2-d\ldots\right.}$$

$$= -\frac{2d^2\cos(e+fx)}{(bc-ad)\left(c^2-d^2\right)f\sqrt{c+d\sin(e+fx)}} + \frac{b\int \frac{1}{(a+b\sin(e+fx))\sqrt{c+d\sin(e+\ldots}}}{bc-ad}$$

$$= -\frac{2d^2\cos(e+fx)}{(bc-ad)\left(c^2-d^2\right)f\sqrt{c+d\sin(e+fx)}} - \frac{2dE\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2d}{c+d}\right)\ldots}{(bc-ad)\left(c^2-d^2\right)f\ldots}$$

$$= -\frac{2d^2\cos(e+fx)}{(bc-ad)\left(c^2-d^2\right)f\sqrt{c+d\sin(e+fx)}} - \frac{2dE\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2d}{c+d}\right)\ldots}{(bc-ad)\left(c^2-d^2\right)f\ldots}$$

Mathematica [C] time = 6.95446, size = 617, normalized size = 2.8

$$-\frac{\frac{4d^2\cos(e+fx)}{(c^2-d^2)\sqrt{c+d\sin(e+fx)}}+\frac{-\frac{2i\sec(e+fx)\sqrt{-\frac{d(\sin(e+fx)-1)}{c+d}}\sqrt{\frac{d(\sin(e+fx)+1)}{d-c}}\left(d\left(d(b^2-2a^2)\Pi\left(\frac{b(c+d)}{bc-ad};i\sinh^{-1}\left(\sqrt{-\frac{1}{c+d}}\sqrt{c+d\sin(e+fx)}\right)\middle|\frac{c+d}{c-d}\right)+2(a+b)(ad-bc)F\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{c+}}\right.\right.\right.\right.}{b\sqrt{-\frac{1}{c+d}}(bc-ad)}}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*Sin[e + f*x])*(c + d*Sin[e + f*x])^(3/2)),x]
```

```
[Out] -((4*d^2*Cos[e + f*x])/((c^2 - d^2)*Sqrt[c + d*Sin[e + f*x]]) + (((4*I)*(b*
c + a*d)*((-(b*c) + a*d)*EllipticF[I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d
*Sin[e + f*x]]], (c + d)/(c - d)] - a*d*EllipticPi[(b*(c + d))/(b*c - a*d),
 I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)])
*Sec[e + f*x]*Sqrt[-((d*(-1 + Sin[e + f*x]))/(c + d))]*Sqrt[(d*(1 + Sin[e +
 f*x]))/(-c + d)])/(b*Sqrt[-(c + d)^(-1)]*(b*c - a*d)) - ((2*I)*(-2*b*(c -
d)*(b*c - a*d)*EllipticE[I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f
*x]]], (c + d)/(c - d)] + d*(2*(a + b)*(-(b*c) + a*d)*EllipticF[I*ArcSinh[S
qrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)] + (-2*a^2 +
b^2)*d*EllipticPi[(b*(c + d))/(b*c - a*d), I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sq
rt[c + d*Sin[e + f*x]]], (c + d)/(c - d)]))*Sec[e + f*x]*Sqrt[-((d*(-1 + Si
n[e + f*x]))/(c + d))]*Sqrt[(d*(1 + Sin[e + f*x]))/(-c + d)])/(b*Sqrt[-(c +
 d)^(-1)]*(b*c - a*d)) + (2*(2*b*c^2 - 2*a*c*d - 3*b*d^2)*EllipticPi[(2*b)/
(a + b), (-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c
 + d)])/((a + b)*Sqrt[c + d*Sin[e + f*x]]))/((c - d)*(c + d)))/(2*(b*c - a*
d)*f)
```

Maple [B] time = 3.242, size = 610, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))^(3/2),x)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*(-2/(a*d-b*c)*(c/d-1)*((c+d*sin(f*x
+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^
(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(-c/d+a/b)*EllipticPi(((c+d*s
in(f*x+e))/(c-d))^(1/2),(-c/d+1)/(-c/d+a/b),((c-d)/(c+d))^(1/2))+d/(a*d-b*c
)*(2*d*cos(f*x+e)^2/(c^2-d^2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2*c/(
c^2-d^2)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1
/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)
*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+2/(c^2-d^2)*
d*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-
sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d
-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF
(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))))/cos(f*x+e)/(c+d*sin
(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sin(fx+e)+a)(d\sin(fx+e)+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^(3/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^(3/2)), x)
```

3.750 $\int \frac{1}{(a+b\sin(e+fx))(c+d\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=399

$$\frac{2b^2\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\Pi\left(\frac{2b}{a+b};\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{f(a+b)(bc-ad)^2\sqrt{c+d\sin(e+fx)}}-\frac{2d^2\left(-4acd+7bc^2-3bd^2\right)\cos(e+fx)}{3f\left(c^2-d^2\right)^2(bc-ad)^2\sqrt{c+d\sin(e+fx)}}-\frac{2d^2\cos(e+fx}{3f\left(c^2-d^2\right)(bc-ad)(c+d}$$

```
[Out] (-2*d^2*Cos[e + f*x])/(3*(b*c - a*d)*(c^2 - d^2)*f*(c + d*Sin[e + f*x])^(3/
2)) - (2*d^2*(7*b*c^2 - 4*a*c*d - 3*b*d^2)*Cos[e + f*x])/(3*(b*c - a*d)^2*(
c^2 - d^2)^2*f*Sqrt[c + d*Sin[e + f*x]]) - (2*d*(7*b*c^2 - 4*a*c*d - 3*b*d^
2)*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(
3*(b*c - a*d)^2*(c^2 - d^2)^2*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + (2*d*
EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c +
 d)])/(3*(b*c - a*d)*(c^2 - d^2)*f*Sqrt[c + d*Sin[e + f*x]]) + (2*b^2*Ellip
ticPi[(2*b)/(a + b), (e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e +
 f*x])/(c + d)])/((a + b)*(b*c - a*d)^2*f*Sqrt[c + d*Sin[e + f*x]])
```

Rubi [A] time = 1.65294, antiderivative size = 399, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.37, Rules used = {2802, 3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{2b^2\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\Pi\left(\frac{2b}{a+b};\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{f(a+b)(bc-ad)^2\sqrt{c+d\sin(e+fx)}}-\frac{2d^2\left(-4acd+7bc^2-3bd^2\right)\cos(e+fx)}{3f\left(c^2-d^2\right)^2(bc-ad)^2\sqrt{c+d\sin(e+fx)}}-\frac{2d^2\cos(e+fx}{3f\left(c^2-d^2\right)(bc-ad)(c+d}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sin[e + f*x])*(c + d*Sin[e + f*x])^(5/2)),x]
```



```
[Out] (-2*d^2*Cos[e + f*x])/(3*(b*c - a*d)*(c^2 - d^2)*f*(c + d*Sin[e + f*x])^(3/
2)) - (2*d^2*(7*b*c^2 - 4*a*c*d - 3*b*d^2)*Cos[e + f*x])/(3*(b*c - a*d)^2*(
c^2 - d^2)^2*f*Sqrt[c + d*Sin[e + f*x]]) - (2*d*(7*b*c^2 - 4*a*c*d - 3*b*d^
2)*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(
3*(b*c - a*d)^2*(c^2 - d^2)^2*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + (2*d*
EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c +
 d)])/(3*(b*c - a*d)*(c^2 - d^2)*f*Sqrt[c + d*Sin[e + f*x]]) + (2*b^2*Ellip
ticPi[(2*b)/(a + b), (e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e +
 f*x])/(c + d)])/((a + b)*(b*c - a*d)^2*f*Sqrt[c + d*Sin[e + f*x]])
```


Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\sin(e+fx))(c+d\sin(e+fx))^{5/2}}\,dx = -\frac{2d^2\cos(e+fx)}{3(bc-ad)\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}} + \frac{2\int \frac{-\frac{3}{2}\left(acd-b\left(c^2-d^2\right)\right)-\frac{1}{2}d(3bc\cdots}{(a+b\sin(e+fx)\cdots}}{3(bc-a\cdots}$$

$$= -\frac{2d^2\cos(e+fx)}{3(bc-ad)\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}} - \frac{2d^2\left(7bc^2-4acd-3b\cdots\right.}{3(bc-ad)^2\left(c^2-d^2\right)^2 f\sqrt{\cdots}}$$

$$= -\frac{2d^2\cos(e+fx)}{3(bc-ad)\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}} - \frac{2d^2\left(7bc^2-4acd-3b\cdots\right.}{3(bc-ad)^2\left(c^2-d^2\right)^2 f\sqrt{\cdots}}$$

$$= -\frac{2d^2\cos(e+fx)}{3(bc-ad)\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}} - \frac{2d^2\left(7bc^2-4acd-3b\cdots\right.}{3(bc-ad)^2\left(c^2-d^2\right)^2 f\sqrt{\cdots}}$$

$$= -\frac{2d^2\cos(e+fx)}{3(bc-ad)\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}} - \frac{2d^2\left(7bc^2-4acd-3b\cdots\right.}{3(bc-ad)^2\left(c^2-d^2\right)^2 f\sqrt{\cdots}}$$

$$= -\frac{2d^2\cos(e+fx)}{3(bc-ad)\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}} - \frac{2d^2\left(7bc^2-4acd-3b\cdots\right.}{3(bc-ad)^2\left(c^2-d^2\right)^2 f\sqrt{\cdots}}$$

Mathematica [C] time = 7.17685, size = 1079, normalized size = 2.7

$$\frac{\sqrt{c+d\sin(e+fx)}\left(\frac{2\left(3b\cos(e+fx)d^4+4ac\cos(e+fx)d^3-7bc^2\cos(e+fx)d^2\right)}{3(bc-ad)^2\left(c^2-d^2\right)^2(c+d\sin(e+fx))}-\frac{2d^2\cos(e+fx)}{3(bc-ad)\left(c^2-d^2\right)(c+d\sin(e+fx))^2}\right)}{f}+\frac{-\frac{2\left(6b^2c^4-12abdc^3+6a^2d^2c^2-19b\cdots\right.}{\cdots}}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*Sin[e + f*x])*(c + d*Sin[e + f*x])^(5/2)),x]
```

```
[Out] (Sqrt[c + d*Sin[e + f*x]]*((-2*d^2*Cos[e + f*x])/(3*(b*c - a*d)*(c^2 - d^2)
*(c + d*Sin[e + f*x])^2) + (2*(-7*b*c^2*d^2*Cos[e + f*x] + 4*a*c*d^3*Cos[e
+ f*x] + 3*b*d^4*Cos[e + f*x]))/(3*(b*c - a*d)^2*(c^2 - d^2)^2*(c + d*Sin[e
 + f*x]))))/f + ((-2*(6*b^2*c^4 - 12*a*b*c^3*d + 6*a^2*c^2*d^2 - 19*b^2*c^2
*d^2 + 8*a*b*c*d^3 + 2*a^2*d^4 + 9*b^2*d^4)*EllipticPi[(2*b)/(a + b), (-e +
```

```
 Pi/2 - f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/((a + b)
*Sqrt[c + d*Sin[e + f*x]]) - ((2*I)*(-12*b^2*c^3*d - 8*a*b*c^2*d^2 + 8*a^2*
c*d^3 + 4*b^2*c*d^3 + 8*a*b*d^4)*Cos[e + f*x]*((b*c - a*d)*EllipticF[I*ArcS
inh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)] + a*d*E
llipticPi[(b*(c + d))/(b*c - a*d), I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d
*Sin[e + f*x]]], (c + d)/(c - d)])*Sqrt[(d - d*Sin[e + f*x])/(c + d)]*Sqrt[
-((d + d*Sin[e + f*x])/(c - d))]*(-(b*c) + a*d + b*(c + d*Sin[e + f*x])))/(
b*d^2*Sqrt[-(c + d)^(-1)]*(b*c - a*d)*(a + b*Sin[e + f*x])*Sqrt[1 - Sin[e +
 f*x]^2]*Sqrt[-((c^2 - d^2 - 2*c*(c + d*Sin[e + f*x]) + (c + d*Sin[e + f*x]
)^2)/d^2)]) - ((2*I)*(7*b^2*c^2*d^2 - 4*a*b*c*d^3 - 3*b^2*d^4)*Cos[e + f*x]
*Cos[2*(e + f*x)]*(2*b*(c - d)*(b*c - a*d)*EllipticE[I*ArcSinh[Sqrt[-(c + d
)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)] + d*(-2*(a + b)*(-(b*c)
 + a*d)*EllipticF[I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]],
(c + d)/(c - d)] + (2*a^2 - b^2)*d*EllipticPi[(b*(c + d))/(b*c - a*d), I*Ar
cSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)]))*Sqr
t[(d - d*Sin[e + f*x])/(c + d)]*Sqrt[-((d + d*Sin[e + f*x])/(c - d))]*(-(b*
c) + a*d + b*(c + d*Sin[e + f*x])))/(b^2*d*Sqrt[-(c + d)^(-1)]*(b*c - a*d)*
(a + b*Sin[e + f*x])*Sqrt[1 - Sin[e + f*x]^2]*(-2*c^2 + d^2 + 4*c*(c + d*Si
n[e + f*x]) - 2*(c + d*Sin[e + f*x])^2)*Sqrt[-((c^2 - d^2 - 2*c*(c + d*Sin[
e + f*x]) + (c + d*Sin[e + f*x])^2)/d^2)]))/(6*(c - d)^2*(c + d)^2*(b*c - a
*d)^2*f)
```

Maple [B] time = 5.758, size = 1072, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))^(5/2),x)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*(2*b/(a*d-b*c)^2*(c/d-1)*((c+d*sin(
f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d
))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(-c/d+a/b)*EllipticPi(((c+
d*sin(f*x+e))/(c-d))^(1/2),(-c/d+1)/(-c/d+a/b),((c-d)/(c+d))^(1/2))+d/(a*d-
b*c)*(2/3/(c^2-d^2)/d*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(sin(f*x+e)+c
/d)^2+8/3*d*cos(f*x+e)^2/(c^2-d^2)^2*c/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1
/2)+2*(3*c^2+d^2)/(3*c^4-6*c^2*d^2+3*d^4)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^
(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*
sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),
((c-d)/(c+d))^(1/2))+8/3*c*d/(c^2-d^2)^2*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(
1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*s
in(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d
))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-
d)/(c+d))^(1/2))))-d*b/(a*d-b*c)^2*(2*d*cos(f*x+e)^2/(c^2-d^2)/(-(-d*sin(f*
x+e)-c)*cos(f*x+e)^2)^(1/2)+2*c/(c^2-d^2)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^
(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*
sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),
((c-d)/(c+d))^(1/2))+2/(c^2-d^2)*d*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(
d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x
+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/
2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+
d))^(1/2)))))/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b \sin\left(fx+e\right)+a\right)\left(d \sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^(5/2)), x)
```

3.751 $\int \frac{(c+d\sin(e+fx))^{7/2}}{(a+b\sin(e+fx))^2}\,dx$

Optimal. Leaf size=534

$$\frac{d\left(-5a^2d^2+6abcd+b^2\left(-\left(3c^2-2d^2\right)\right)\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3b^2f\left(a^2-b^2\right)}-\frac{\left(2a^2b^2d^2\left(c^2+8d^2\right)+24a^3bcd^3-15a^4d^4-\right.}{}$$

```
[Out] (d*(6*a*b*c*d - 5*a^2*d^2 - b^2*(3*c^2 - 2*d^2))*Cos[e + f*x]*Sqrt[c + d*Si
n[e + f*x]])/(3*b^2*(a^2 - b^2)*f) + ((b*c - a*d)^2*Cos[e + f*x]*(c + d*Sin
[e + f*x])^(3/2))/(b*(a^2 - b^2)*f*(a + b*Sin[e + f*x])) + ((29*a^2*b*c*d^2
 - 15*a^3*d^3 + b^3*(3*c^3 - 20*c*d^2) - a*b^2*(9*c^2*d - 12*d^3))*Elliptic
E[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(3*b^3*(a^2
- b^2)*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) - ((24*a^3*b*c*d^3 - 15*a^4*d^
4 - 12*a*b^3*c*d*(c^2 + 3*d^2) + 2*a^2*b^2*d^2*(c^2 + 8*d^2) + b^4*(3*c^4 +
 16*c^2*d^2 + 2*d^4))*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c
+ d*Sin[e + f*x])/(c + d)])/(3*b^4*(a^2 - b^2)*f*Sqrt[c + d*Sin[e + f*x]])
+ ((b*c - a*d)^3*(2*a*b*c + 5*a^2*d - 7*b^2*d)*EllipticPi[(2*b)/(a + b), (e
 - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/((a -
b)*b^4*(a + b)^2*f*Sqrt[c + d*Sin[e + f*x]])
```

Rubi [A] time = 2.04004, antiderivative size = 534, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.37, Rules used = {2792, 3049, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{d\left(-5a^2d^2+6abcd+b^2\left(-\left(3c^2-2d^2\right)\right)\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3b^2f\left(a^2-b^2\right)}-\frac{\left(2a^2b^2d^2\left(c^2+8d^2\right)+24a^3bcd^3-15a^4d^4-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^(7/2)/(a + b*Sin[e + f*x])^2,x]
```


```
[Out] (d*(6*a*b*c*d - 5*a^2*d^2 - b^2*(3*c^2 - 2*d^2))*Cos[e + f*x]*Sqrt[c + d*Si
n[e + f*x]])/(3*b^2*(a^2 - b^2)*f) + ((b*c - a*d)^2*Cos[e + f*x]*(c + d*Sin
[e + f*x])^(3/2))/(b*(a^2 - b^2)*f*(a + b*Sin[e + f*x])) + ((29*a^2*b*c*d^2
 - 15*a^3*d^3 + b^3*(3*c^3 - 20*c*d^2) - a*b^2*(9*c^2*d - 12*d^3))*Elliptic
E[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(3*b^3*(a^2
- b^2)*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) - ((24*a^3*b*c*d^3 - 15*a^4*d^
4 - 12*a*b^3*c*d*(c^2 + 3*d^2) + 2*a^2*b^2*d^2*(c^2 + 8*d^2) + b^4*(3*c^4 +
 16*c^2*d^2 + 2*d^4))*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c
+ d*Sin[e + f*x])/(c + d)])/(3*b^4*(a^2 - b^2)*f*Sqrt[c + d*Sin[e + f*x]])
+ ((b*c - a*d)^3*(2*a*b*c + 5*a^2*d - 7*b^2*d)*EllipticPi[(2*b)/(a + b), (e
 - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/((a -
b)*b^4*(a + b)^2*f*Sqrt[c + d*Sin[e + f*x]])
```


Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
```

```
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
```

```
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^{7/2}}{(a+b\sin(e+fx))^2}\,dx = \frac{(bc-ad)^2\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{b\left(a^2-b^2\right)f(a+b\sin(e+fx))} - \int \frac{\sqrt{c+d\sin(e+fx)}\left(\frac{1}{2}\left(7b^2c^2d+3a^2d^3-2abc\left(c^2+4d^2\right)\right)-\ldots\right)}{\ldots}$$

$$= \frac{d\left(6abcd-5a^2d^2-b^2\left(3c^2-2d^2\right)\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3b^2\left(a^2-b^2\right)f} + \frac{(bc-ad)^2\cos(e+\ldots}{b\left(a^2-b^2\right)f(\ldots}$$

$$= \frac{d\left(6abcd-5a^2d^2-b^2\left(3c^2-2d^2\right)\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3b^2\left(a^2-b^2\right)f} + \frac{(bc-ad)^2\cos(e+\ldots}{b\left(a^2-b^2\right)f(\ldots}$$

$$= \frac{d\left(6abcd-5a^2d^2-b^2\left(3c^2-2d^2\right)\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3b^2\left(a^2-b^2\right)f} + \frac{(bc-ad)^2\cos(e+\ldots}{b\left(a^2-b^2\right)f(\ldots}$$

$$= \frac{d\left(6abcd-5a^2d^2-b^2\left(3c^2-2d^2\right)\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3b^2\left(a^2-b^2\right)f} + \frac{(bc-ad)^2\cos(e+\ldots}{b\left(a^2-b^2\right)f(\ldots}$$

$$= \frac{d\left(6abcd-5a^2d^2-b^2\left(3c^2-2d^2\right)\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3b^2\left(a^2-b^2\right)f} + \frac{(bc-ad)^2\cos(e+\ldots}{b\left(a^2-b^2\right)f(\ldots}$$

Mathematica [C] time = 8.1094, size = 1109, normalized size = 2.08

$$\frac{\sqrt{c+d\sin(e+fx)}\left(\frac{-b^3\cos(e+fx)c^3+3ab^2d\cos(e+fx)c^2-3a^2bd^2\cos(e+fx)c+a^3d^3\cos(e+fx)}{b^2\left(b^2-a^2\right)(a+b\sin(e+fx))}-\frac{2d^3\cos(e+fx)}{3b^2}\right)}{f} - \frac{-\frac{2\left(-12ab^2c^4+39b^3dc^3-45ab^2\ldots\right.}{\ldots}}{\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*Sin[e + f*x])^(7/2)/(a + b*Sin[e + f*x])^2,x]
```

```
[Out] (Sqrt[c + d*Sin[e + f*x]]*((-2*d^3*Cos[e + f*x])/(3*b^2) + (-(b^3*c^3*Cos[e
 + f*x]) + 3*a*b^2*c^2*d*Cos[e + f*x] - 3*a^2*b*c*d^2*Cos[e + f*x] + a^3*d^
```

```
3*Cos[e + f*x])/(b^2*(-a^2 + b^2)*(a + b*Sin[e + f*x]))))/f - ((-2*(-12*a*b
^2*c^4 + 39*b^3*c^3*d - 45*a*b^2*c^2*d^2 + a^2*b*c*d^3 + 20*b^3*c*d^3 + 5*a
^3*d^4 - 8*a*b^2*d^4)*EllipticPi[(2*b)/(a + b), (-e + Pi/2 - f*x)/2, (2*d)/
(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/((a + b)*Sqrt[c + d*Sin[e + f*
x]]) - ((2*I)*(-12*a*b^2*c^3*d - 36*a^2*b*c^2*d^2 + 72*b^3*c^2*d^2 + 20*a^3
*c*d^3 - 56*a*b^2*c*d^3 + 8*a^2*b*d^4 + 4*b^3*d^4)*Cos[e + f*x]*((b*c - a*d
)*EllipticF[I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d
)/(c - d)] + a*d*EllipticPi[(b*(c + d))/(b*c - a*d), I*ArcSinh[Sqrt[-(c + d
)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)])*Sqrt[(d - d*Sin[e + f*
x])/(c + d)]*Sqrt[-((d + d*Sin[e + f*x])/(c - d))]*(-(b*c) + a*d + b*(c + d
*Sin[e + f*x])))/(b*d^2*Sqrt[-(c + d)^(-1)]*(b*c - a*d)*(a + b*Sin[e + f*x]
)*Sqrt[1 - Sin[e + f*x]^2]*Sqrt[-((c^2 - d^2 - 2*c*(c + d*Sin[e + f*x]) + (
c + d*Sin[e + f*x])^2)/d^2)]) - ((2*I)*(3*b^3*c^3*d - 9*a*b^2*c^2*d^2 + 29*
a^2*b*c*d^3 - 20*b^3*c*d^3 - 15*a^3*d^4 + 12*a*b^2*d^4)*Cos[e + f*x]*Cos[2*
(e + f*x)]*(2*b*(c - d)*(b*c - a*d)*EllipticE[I*ArcSinh[Sqrt[-(c + d)^(-1)]
*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)] + d*(-2*(a + b)*(-(b*c) + a*d)
*EllipticF[I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)
/(c - d)] + (2*a^2 - b^2)*d*EllipticPi[(b*(c + d))/(b*c - a*d), I*ArcSinh[S
qrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)]))*Sqrt[(d -
d*Sin[e + f*x])/(c + d)]*Sqrt[-((d + d*Sin[e + f*x])/(c - d))]*(-(b*c) + a*
d + b*(c + d*Sin[e + f*x])))/(b^2*d*Sqrt[-(c + d)^(-1)]*(b*c - a*d)*(a + b*
Sin[e + f*x])*Sqrt[1 - Sin[e + f*x]^2]*(-2*c^2 + d^2 + 4*c*(c + d*Sin[e + f
*x]) - 2*(c + d*Sin[e + f*x])^2)*Sqrt[-((c^2 - d^2 - 2*c*(c + d*Sin[e + f*x
]) + (c + d*Sin[e + f*x])^2)/d^2)]))/(12*(a - b)*b^2*(a + b)*f)
```

Maple [B] time = 6.068, size = 1886, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(7/2)/(a+b*sin(f*x+e))^2,x)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*(d^2/b^4*(b^2*d^2*(-2/3/d*(-(-d*sin
(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2/3*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(
d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x
+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/
(c+d))^(1/2))-4/3*c/d*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+
e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*
x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+
d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+
2*(-2*a*b*d^2+4*b^2*c*d)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f
*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos
(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/
(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))
)+6*a^2*d^2*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))
^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1
/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))-16*a*b*c*
d*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-
sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*Ellipt
icF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+12*c^2*b^2*(c/d-1)*
((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)
-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*
sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))-8/b^5*d*(a^3*d^3-3*a^2*b*c*d
^2+3*a*b^2*c^2*d-b^3*c^3)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(
f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*co
s(f*x+e)^2)^(1/2)/(-c/d+a/b)*EllipticPi(((c+d*sin(f*x+e))/(c-d))^(1/2),(-c/
```

```
d+1)/(-c/d+a/b),((c-d)/(c+d))^(1/2))+1/b^4*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2
*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)*(-b^2/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)*(-(-d*s
in(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(a+b*sin(f*x+e))-a*d/(a^3*d-a^2*b*c-a*b^2*
d+b^3*c)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1
/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)
*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))-b*d/(a^3*d-a
^2*b*c-a*b^2*d+b^3*c)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+
e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*
x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+
d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))+(
3*a^2*d-2*a*b*c-b^2*d)/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)/b*(c/d-1)*((c+d*sin(f*
x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))
^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(-c/d+a/b)*EllipticPi(((c+d*
sin(f*x+e))/(c-d))^(1/2),(-c/d+1)/(-c/d+a/b),((c-d)/(c+d))^(1/2))))/cos(f*x
+e)/(c+d*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}}{\left(b\sin\left(fx+e\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(7/2)/(a+b*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(7/2)/(b*sin(f*x + e) + a)^2, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(7/2)/(a+b*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(7/2)/(a+b*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}}{\left(b\sin\left(fx+e\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(7/2)/(a+b*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(7/2)/(b*sin(f*x + e) + a)^2, x)
```

3.752 $\int \frac{(c+d\sin(e+fx))^{5/2}}{(a+b\sin(e+fx))^2}\,dx$

Optimal. Leaf size=390

$$\frac{\left(3a^2d^2+2abcd+b^2\left(-\left(c^2+4d^2\right)\right)\right)(bc-ad)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{b^3f\left(a^2-b^2\right)\sqrt{c+d\sin(e+fx)}}-\frac{\left(-3a^2d^2+2abcd+b^2\left(-\left(c^2-2c\right.\right.\right.}{b^2f\left(a\right.}$$

```
[Out] ((b*c - a*d)^2*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(b*(a^2 - b^2)*f*(a +
 b*Sin[e + f*x])) - ((2*a*b*c*d - 3*a^2*d^2 - b^2*(c^2 - 2*d^2))*EllipticE[
(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(b^2*(a^2 - b^
2)*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + ((b*c - a*d)*(2*a*b*c*d + 3*a^2*
d^2 - b^2*(c^2 + 4*d^2))*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[
(c + d*Sin[e + f*x])/(c + d)])/(b^3*(a^2 - b^2)*f*Sqrt[c + d*Sin[e + f*x]])
 + ((b*c - a*d)^2*(2*a*b*c + 3*a^2*d - 5*b^2*d)*EllipticPi[(2*b)/(a + b), (
e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/((a -
 b)*b^3*(a + b)^2*f*Sqrt[c + d*Sin[e + f*x]])
```

Rubi [A] time = 1.26106, antiderivative size = 390, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2792, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{\left(3a^2d^2+2abcd+b^2\left(-\left(c^2+4d^2\right)\right)\right)(bc-ad)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{b^3f\left(a^2-b^2\right)\sqrt{c+d\sin(e+fx)}}-\frac{\left(-3a^2d^2+2abcd+b^2\left(-\left(c^2-2c\right.\right.\right.}{b^2f\left(a\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^(5/2)/(a + b*Sin[e + f*x])^2,x]
```


```
[Out] ((b*c - a*d)^2*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(b*(a^2 - b^2)*f*(a +
 b*Sin[e + f*x])) - ((2*a*b*c*d - 3*a^2*d^2 - b^2*(c^2 - 2*d^2))*EllipticE[
(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(b^2*(a^2 - b^
2)*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + ((b*c - a*d)*(2*a*b*c*d + 3*a^2*
d^2 - b^2*(c^2 + 4*d^2))*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[
(c + d*Sin[e + f*x])/(c + d)])/(b^3*(a^2 - b^2)*f*Sqrt[c + d*Sin[e + f*x]])
 + ((b*c - a*d)^2*(2*a*b*c + 3*a^2*d - 5*b^2*d)*EllipticPi[(2*b)/(a + b), (
e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/((a -
 b)*b^3*(a + b)^2*f*Sqrt[c + d*Sin[e + f*x]])
```


Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^{5/2}}{(a+b\sin(e+fx))^2}\,dx = \frac{(bc-ad)^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{b\left(a^2-b^2\right)f(a+b\sin(e+fx))} - \frac{\int \frac{\frac{1}{2}\left(5b^2c^2d+a^2d^3-2abc\left(c^2+2d^2\right)\right)-d\left(a^2cd-3b^2cd+ab\left(c^2+\cdots\right.\right.}{(a+b\sin(e+fx\cdots}}{b\left(\cdots\right.}$$

$$= \frac{(bc-ad)^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{b\left(a^2-b^2\right)f(a+b\sin(e+fx))} + \frac{\int \frac{\frac{1}{2}d(bc-ad)\left(abc^2+3a^2cd-5b^2cd+abd^2\right)-\frac{1}{2}d(bc-ad)\left(b^2c^2\cdots\right.}{(a+b\sin(e+fx))\sqrt{c+d\sin(\cdots}}}{b^2\left(a^2-b^2\right)d\cdots}$$

$$= \frac{(bc-ad)^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{b\left(a^2-b^2\right)f(a+b\sin(e+fx))} + \frac{\left((bc-ad)^2\left(2abc+3a^2d-5b^2d\right)\right)\int \frac{\cdots}{(a+b\sin(\cdots}}{2b^3\left(a^2-b^2\right)\cdots}$$

$$= \frac{(bc-ad)^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{b\left(a^2-b^2\right)f(a+b\sin(e+fx))} - \frac{\left(2abcd-3a^2d^2-b^2\left(c^2-2d^2\right)\right)E\left(\frac{1}{2}\left(e-\frac{\pi}{2}\cdots\right.\right.}{b^2\left(a^2-b^2\right)f\sqrt{\frac{c+d\,\mathrm{s}\cdots}{\cdots}}}$$

$$= \frac{(bc-ad)^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{b\left(a^2-b^2\right)f(a+b\sin(e+fx))} - \frac{\left(2abcd-3a^2d^2-b^2\left(c^2-2d^2\right)\right)E\left(\frac{1}{2}\left(e-\frac{\pi}{2}\cdots\right.\right.}{b^2\left(a^2-b^2\right)f\sqrt{\frac{c+d\,\mathrm{s}\cdots}{\cdots}}}$$

Mathematica [C] time = 7.99406, size = 986, normalized size = 2.53

$$\frac{\sqrt{c+d\sin(e+fx)}\left(-b^2\cos(e+fx)c^2+2abd\cos(e+fx)c-a^2d^2\cos(e+fx)\right)}{b\left(b^2-a^2\right)f(a+b\sin(e+fx))} + \frac{-\frac{2\left(4abc^3-9b^2dc^2+6abd^2c+a^2d^3-2b^2d^3\right)\sqrt{\frac{c+d\,\mathrm{s}\cdots}{\cdots}}}{(a+b)\sqrt{c+d\sin(\cdots}}}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*Sin[e + f*x])^(5/2)/(a + b*Sin[e + f*x])^2,x]
```

```
[Out] ((-(b^2*c^2*Cos[e + f*x]) + 2*a*b*c*d*Cos[e + f*x] - a^2*d^2*Cos[e + f*x])*
Sqrt[c + d*Sin[e + f*x]])/(b*(-a^2 + b^2)*f*(a + b*Sin[e + f*x])) + ((-2*(4
*a*b*c^3 - 9*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3 - 2*b^2*d^3)*EllipticPi[(2*b
)/(a + b), (-e + Pi/2 - f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c
 + d)])/((a + b)*Sqrt[c + d*Sin[e + f*x]]) - ((2*I)*(4*a*b*c^2*d + 4*a^2*c*
d^2 - 12*b^2*c*d^2 + 4*a*b*d^3)*Cos[e + f*x]*((b*c - a*d)*EllipticF[I*ArcSi
nh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)] + a*d*El
lipticPi[(b*(c + d))/(b*c - a*d), I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*
Sin[e + f*x]]], (c + d)/(c - d)])*Sqrt[(d - d*Sin[e + f*x])/(c + d)]*Sqrt[-
((d + d*Sin[e + f*x])/(c - d))]*(-(b*c) + a*d + b*(c + d*Sin[e + f*x]))))/(b
*d^2*Sqrt[-(c + d)^(-1)]*(b*c - a*d)*(a + b*Sin[e + f*x])*Sqrt[1 - Sin[e +
f*x]^2]*Sqrt[-((c^2 - d^2 - 2*c*(c + d*Sin[e + f*x]) + (c + d*Sin[e + f*x])
^2)/d^2)]) - ((2*I)*(-(b^2*c^2*d) + 2*a*b*c*d^2 - 3*a^2*d^3 + 2*b^2*d^3)*Co
s[e + f*x]*Cos[2*(e + f*x)]*(2*b*(c - d)*(b*c - a*d)*EllipticE[I*ArcSinh[Sq
rt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)] + d*(-2*(a +
b)*(-(b*c) + a*d)*EllipticF[I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e
+ f*x]]], (c + d)/(c - d)] + (2*a^2 - b^2)*d*EllipticPi[(b*(c + d))/(b*c -
a*d), I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c -
 d)]))*Sqrt[(d - d*Sin[e + f*x])/(c + d)]*Sqrt[-((d + d*Sin[e + f*x])/(c -
d))]*(-(b*c) + a*d + b*(c + d*Sin[e + f*x]))))/(b^2*d*Sqrt[-(c + d)^(-1)]*(b
*c - a*d)*(a + b*Sin[e + f*x])*Sqrt[1 - Sin[e + f*x]^2]*(-2*c^2 + d^2 + 4*c
*(c + d*Sin[e + f*x]) - 2*(c + d*Sin[e + f*x])^2)*Sqrt[-((c^2 - d^2 - 2*c*(
c + d*Sin[e + f*x]) + (c + d*Sin[e + f*x])^2)/d^2)]))/(4*(a - b)*b*(a + b)*
f)
```

Maple [B] time = 5.217, size = 1363, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e))^2,x)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*(d^2/b^3*(2*b*d*(c/d-1)*((c+d*sin(f
*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d)
)^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*s
in(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c
-d))^(1/2),((c-d)/(c+d))^(1/2)))-4*d*a*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/
2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin
(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c
-d)/(c+d))^(1/2))+6*c*b*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*
x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(
f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2
)))+6/b^4*d*(a^2*d^2-2*a*b*c*d+b^2*c^2)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1
/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*si
n(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(-c/d+a/b)*EllipticPi(((c+d*sin(f*x+e))/(c-
d))^(1/2),(-c/d+1)/(-c/d+a/b),((c-d)/(c+d))^(1/2))+1/b^3*(-a^3*d^3+3*a^2*b*
c*d^2-3*a*b^2*c^2*d+b^3*c^3)*(-b^2/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)*(-(-d*sin(
f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(a+b*sin(f*x+e))-a*d/(a^3*d-a^2*b*c-a*b^2*d+b
^3*c)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)
*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*El
lipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))-b*d/(a^3*d-a^2*
b*c-a*b^2*d+b^3*c)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))
/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e
)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))
^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))+(3*a
^2*d-2*a*b*c-b^2*d)/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)/b*(c/d-1)*((c+d*sin(f*x+e
))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1
/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(-c/d+a/b)*EllipticPi(((c+d*sin
(f*x+e))/(c-d))^(1/2),(-c/d+1)/(-c/d+a/b),((c-d)/(c+d))^(1/2))))/cos(f*x+e)
/(c+d*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}{\left(b\sin\left(fx+e\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(5/2)/(b*sin(f*x + e) + a)^2, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(5/2)/(a+b*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}{\left(b\sin\left(fx+e\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(5/2)/(b*sin(f*x + e) + a)^2, x)
```

3.753 $\int \frac{(c+d\sin(e+fx))^{3/2}}{(a+b\sin(e+fx))^2} dx$

Optimal. Leaf size=351

$$\frac{\left(a^2d^2+2abcd+b^2\left(-\left(c^2+2d^2\right)\right)\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{b^2f\left(a^2-b^2\right)\sqrt{c+d\sin(e+fx)}}+\frac{(bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{f\left(a^2-b^2\right)(a+b\sin(e+fx))}+\frac{(bc}{}$$

```
[Out] ((b*c - a*d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/((a^2 - b^2)*f*(a + b*S
in[e + f*x])) + ((b*c - a*d)*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*S
qrt[c + d*Sin[e + f*x]])/(b*(a^2 - b^2)*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)
]) + ((2*a*b*c*d + a^2*d^2 - b^2*(c^2 + 2*d^2))*EllipticF[(e - Pi/2 + f*x)/
2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(b^2*(a^2 - b^2)*f*Sq
rt[c + d*Sin[e + f*x]]) + ((b*c - a*d)*(2*a*b*c + a^2*d - 3*b^2*d)*Elliptic
Pi[(2*b)/(a + b), (e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*
x])/(c + d)])/((a - b)*b^2*(a + b)^2*f*Sqrt[c + d*Sin[e + f*x]])
```

Rubi [A] time = 1.03696, antiderivative size = 351, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2799, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{\left(a^2d^2+2abcd+b^2\left(-\left(c^2+2d^2\right)\right)\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{b^2f\left(a^2-b^2\right)\sqrt{c+d\sin(e+fx)}}+\frac{(bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{f\left(a^2-b^2\right)(a+b\sin(e+fx))}+\frac{(bc}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^(3/2)/(a + b*Sin[e + f*x])^2,x]
```


```
[Out] ((b*c - a*d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/((a^2 - b^2)*f*(a + b*S
in[e + f*x])) + ((b*c - a*d)*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*S
qrt[c + d*Sin[e + f*x]])/(b*(a^2 - b^2)*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)
]) + ((2*a*b*c*d + a^2*d^2 - b^2*(c^2 + 2*d^2))*EllipticF[(e - Pi/2 + f*x)/
2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(b^2*(a^2 - b^2)*f*Sq
rt[c + d*Sin[e + f*x]]) + ((b*c - a*d)*(2*a*b*c + a^2*d - 3*b^2*d)*Elliptic
Pi[(2*b)/(a + b), (e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*
x])/(c + d)])/((a - b)*b^2*(a + b)^2*f*Sqrt[c + d*Sin[e + f*x]])
```


Rule 2799

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin
[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1))/(f*(m + 1)*(a^2 - b^2)), x
] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*S
in[e + f*x])^(n - 2)*Simp[c*(a*c - b*d)*(m + 1) + d*(b*c - a*d)*(n - 1) + (
d*(a*c - b*d)*(m + 1) - c*(b*c - a*d)*(m + 2))*Sin[e + f*x] - d*(b*c - a*d)
*(m + n + 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& LtQ[1, n, 2] && IntegersQ[2*m, 2*n]
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
```

```
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^{3/2}}{(a+b\sin(e+fx))^2}\,dx = \frac{(bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{(a^2-b^2)\,f(a+b\sin(e+fx))} + \frac{\int \frac{\frac{1}{2}\left(3bcd-a\left(2c^2+d^2\right)\right)-d(ac-bd)\sin(e+fx)-\frac{1}{2}d(bc-ad)\sin^2(e+}{(a+b\sin(e+fx))\sqrt{c+d\sin(e+fx)}}}{-a^2+b^2}$$

$$= \frac{(bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{(a^2-b^2)\,f(a+b\sin(e+fx))} + \frac{\int \frac{\frac{1}{2}d\left(a^2cd-3b^2cd+ab\left(c^2+d^2\right)\right)+\frac{1}{2}d\left(2abcd+a^2d^2-b^2\left(c^2+2d^2\right)\right)}{(a+b\sin(e+fx))\sqrt{c+d\sin(e+fx)}}}{b\left(a^2-b^2\right)d}$$

$$= \frac{(bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{(a^2-b^2)\,f(a+b\sin(e+fx))} + \frac{\left((bc-ad)\left(2abc+a^2d-3b^2d\right)\right)\int \frac{1}{(a+b\sin(e+fx))\sqrt{}}}{2b^2\left(a^2-b^2\right)}$$

$$= \frac{(bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{(a^2-b^2)\,f(a+b\sin(e+fx))} + \frac{(bc-ad)E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2d}{c+d}\right)\sqrt{c+d\sin(e+}}{b\left(a^2-b^2\right)f\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

$$= \frac{(bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{(a^2-b^2)\,f(a+b\sin(e+fx))} + \frac{(bc-ad)E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2d}{c+d}\right)\sqrt{c+d\sin(e+}}{b\left(a^2-b^2\right)f\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

Mathematica [C] time = 7.2076, size = 891, normalized size = 2.54

$$\frac{\sqrt{c+d\sin(e+fx)}(bc\cos(e+fx)-ad\cos(e+fx))}{\left(a^2-b^2\right)f(a+b\sin(e+fx))} + \frac{-\frac{2\left(4ac^2-5bdc+ad^2\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\Pi\left(\frac{2b}{a+b};\frac{1}{2}\left(-e-fx+\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{(a+b)\sqrt{c+d\sin(e+fx)}} - \frac{2i\left(4acd-4bd^2\right)\cos(e}{}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*Sin[e + f*x])^(3/2)/(a + b*Sin[e + f*x])^2,x]
```

```
[Out] ((b*c*Cos[e + f*x] - a*d*Cos[e + f*x])*Sqrt[c + d*Sin[e + f*x]])/((a^2 - b^
2)*f*(a + b*Sin[e + f*x])) + ((-2*(4*a*c^2 - 5*b*c*d + a*d^2)*EllipticPi[(2
*b)/(a + b), (-e + Pi/2 - f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/
(c + d)])/((a + b)*Sqrt[c + d*Sin[e + f*x]]) - ((2*I)*(4*a*c*d - 4*b*d^2)*C
os[e + f*x]*((b*c - a*d)*EllipticF[I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d
*Sin[e + f*x]]], (c + d)/(c - d)] + a*d*EllipticPi[(b*(c + d))/(b*c - a*d),
 I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)])
*Sqrt[(d - d*Sin[e + f*x])/(c + d)]*Sqrt[-((d + d*Sin[e + f*x])/(c - d))]*(
-(b*c) + a*d + b*(c + d*Sin[e + f*x])))/(b*d^2*Sqrt[-(c + d)^(-1)]*(b*c - a
*d)*(a + b*Sin[e + f*x])*Sqrt[1 - Sin[e + f*x]^2]*Sqrt[-((c^2 - d^2 - 2*c*(
c + d*Sin[e + f*x]) + (c + d*Sin[e + f*x])^2)/d^2)]) - ((2*I)*(-(b*c*d) + a
*d^2)*Cos[e + f*x]*Cos[2*(e + f*x)]*(2*b*(c - d)*(b*c - a*d)*EllipticE[I*Ar
cSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)] + d*(
-2*(a + b)*(-(b*c) + a*d)*EllipticF[I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c +
d*Sin[e + f*x]]], (c + d)/(c - d)] + (2*a^2 - b^2)*d*EllipticPi[(b*(c + d))
/(b*c - a*d), I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c +
 d)/(c - d)]))*Sqrt[(d - d*Sin[e + f*x])/(c + d)]*Sqrt[-((d + d*Sin[e + f*x
])/(c - d))]*(-(b*c) + a*d + b*(c + d*Sin[e + f*x])))/(b^2*d*Sqrt[-(c + d)^
(-1)]*(b*c - a*d)*(a + b*Sin[e + f*x])*Sqrt[1 - Sin[e + f*x]^2]*(-2*c^2 + d
^2 + 4*c*(c + d*Sin[e + f*x]) - 2*(c + d*Sin[e + f*x])^2)*Sqrt[-((c^2 - d^2
 - 2*c*(c + d*Sin[e + f*x]) + (c + d*Sin[e + f*x])^2)/d^2)]))/(4*(a - b)*(a
 + b)*f)
```

Maple [B] time = 4.618, size = 1027, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))^2,x)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*(2*d^2/b^2*(c/d-1)*((c+d*sin(f*x+e)
)/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/
2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d
))^(1/2),((c-d)/(c+d))^(1/2))-4/b^3*d*(a*d-b*c)*(c/d-1)*((c+d*sin(f*x+e))/(
c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/
(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(-c/d+a/b)*EllipticPi(((c+d*sin(f*x
+e))/(c-d))^(1/2),(-c/d+1)/(-c/d+a/b),((c-d)/(c+d))^(1/2))+1/b^2*(a^2*d^2-2
*a*b*c*d+b^2*c^2)*(-b^2/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)*(-(-d*sin(f*x+e)-c)*c
os(f*x+e)^2)^(1/2)/(a+b*sin(f*x+e))-a*d/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)*(c/d-
1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x
+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c
+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))-b*d/(a^3*d-a^2*b*c-a*b^2*d
+b^3*c)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/
2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*
((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+Ell
ipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))+(3*a^2*d-2*a*b*
c-b^2*d)/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)/b*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(
1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*s
in(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(-c/d+a/b)*EllipticPi(((c+d*sin(f*x+e))/(c
-d))^(1/2),(-c/d+1)/(-c/d+a/b),((c-d)/(c+d))^(1/2))))/cos(f*x+e)/(c+d*sin(f
*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}{\left(b\sin\left(fx+e\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(3/2)/(b*sin(f*x + e) + a)^2, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(3/2)/(a+b*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}{\left(b\sin\left(fx+e\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(3/2)/(b*sin(f*x + e) + a)^2, x)
```

3.754 $\int \frac{\sqrt{c+d\sin(e+fx)}}{(a+b\sin(e+fx))^2}\,dx$

Optimal. Leaf size=307

$$\frac{b\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{f\left(a^2-b^2\right)(a+b\sin(e+fx))}-\frac{(bc-ad)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{bf\left(a^2-b^2\right)\sqrt{c+d\sin(e+fx)}}+\frac{\sqrt{c+d\sin(e+fx)}E\left(\frac{1}{2}\left(e+fx-\right.\right.}{f\left(a^2-b^2\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

```
[Out] (b*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/((a^2 - b^2)*f*(a + b*Sin[e + f*x
])) + (EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]
])/((a^2 - b^2)*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) - ((b*c - a*d)*Ellipt
icF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/
(b*(a^2 - b^2)*f*Sqrt[c + d*Sin[e + f*x]]) + ((2*a*b*c - a^2*d - b^2*d)*Ell
ipticPi[(2*b)/(a + b), (e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e
 + f*x])/(c + d)])/((a - b)*b*(a + b)^2*f*Sqrt[c + d*Sin[e + f*x]])
```

Rubi [A] time = 0.867269, antiderivative size = 307, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2796, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{b\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{f\left(a^2-b^2\right)(a+b\sin(e+fx))}-\frac{(bc-ad)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{bf\left(a^2-b^2\right)\sqrt{c+d\sin(e+fx)}}+\frac{\sqrt{c+d\sin(e+fx)}E\left(\frac{1}{2}\left(e+fx-\right.\right.}{f\left(a^2-b^2\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*Sin[e + f*x]]/(a + b*Sin[e + f*x])^2,x]
```


```
[Out] (b*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/((a^2 - b^2)*f*(a + b*Sin[e + f*x
])) + (EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]
])/((a^2 - b^2)*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) - ((b*c - a*d)*Ellipt
icF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/
(b*(a^2 - b^2)*f*Sqrt[c + d*Sin[e + f*x]]) + ((2*a*b*c - a^2*d - b^2*d)*Ell
ipticPi[(2*b)/(a + b), (e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e
 + f*x])/(c + d)])/((a - b)*b*(a + b)^2*f*Sqrt[c + d*Sin[e + f*x]])
```


Rule 2796

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m + 1)*(c + d*Sin[e + f*x])^n)/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m +
 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n
- 1)*Simp[a*c*(m + 1) + b*d*n + (a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] -
b*d*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1
] && LtQ[0, n, 1] && IntegersQ[2*m, 2*n]
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

```
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\sqrt{c+d\sin(e+fx)}}{(a+b\sin(e+fx))^2}\,dx = \frac{b\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{\left(a^2-b^2\right)f(a+b\sin(e+fx))} + \frac{\int \frac{\frac{1}{2}(-2ac+bd)-ad\sin(e+fx)-\frac{1}{2}bd\sin^2(e+fx)}{(a+b\sin(e+fx))\sqrt{c+d\sin(e+fx)}}\,dx}{-a^2+b^2}$$

$$= \frac{b\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{\left(a^2-b^2\right)f(a+b\sin(e+fx))} + \frac{\int \sqrt{c+d\sin(e+fx)}\,dx}{2\left(a^2-b^2\right)} + \frac{\int \frac{\frac{1}{2}bd(ac-bd)-\frac{1}{2}bd(bc-ad)\sin(e+}{(a+b\sin(e+fx))\sqrt{c+d\sin(e+f}}}{b\left(a^2-b^2\right)d}$$

$$= \frac{b\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{\left(a^2-b^2\right)f(a+b\sin(e+fx))} - \frac{(bc-ad)\int \frac{1}{\sqrt{c+d\sin(e+fx)}}\,dx}{2b\left(a^2-b^2\right)} + \frac{\left(2abc-a^2d-b^2d\right)\int \frac{}{(a}}{2b\,(a}$$

$$= \frac{b\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{\left(a^2-b^2\right)f(a+b\sin(e+fx))} + \frac{E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2d}{c+d}\right)\sqrt{c+d\sin(e+fx)}}{\left(a^2-b^2\right)f\sqrt{\frac{c+d\sin(e+fx)}{c+d}}} - \frac{\left((bc-a\right.}{2}$$

$$= \frac{b\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{\left(a^2-b^2\right)f(a+b\sin(e+fx))} + \frac{E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2d}{c+d}\right)\sqrt{c+d\sin(e+fx)}}{\left(a^2-b^2\right)f\sqrt{\frac{c+d\sin(e+fx)}{c+d}}} - \frac{(bc-ad}{b}$$

Mathematica [C] time = 7.19087, size = 846, normalized size = 2.76

$$-\frac{\frac{2(4ac-bd)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\Pi\left(\frac{2b}{a+b};\frac{1}{2}\left(-e-fx+\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{(a+b)\sqrt{c+d\sin(e+fx)}} - \frac{8ia\cos(e+fx)\left((bc-ad)F\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{c+d}}\sqrt{c+d\sin(e+fx)}\right)|\frac{c+d}{c-d}\right)+ad\Pi\left(\frac{b(c+d)}{bc-ad};i\sinh^{-1}\left(\sqrt{-\frac{1}{c+d}}\sqrt{c+}\right.\right.\right.}{bd\sqrt{-\frac{1}{c+d}}(bc-ad)(a+b\sin(e+fx))\sqrt{1-\sin^2(e+fx)}\sqrt{-\frac{c^2}{}}}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c + d*Sin[e + f*x]]/(a + b*Sin[e + f*x])^2,x]
```

```
[Out] -((b*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/((-a^2 + b^2)*f*(a + b*Sin[e +
f*x]))) + ((-2*(4*a*c - b*d)*EllipticPi[(2*b)/(a + b), (-e + Pi/2 - f*x)/2,
 (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/((a + b)*Sqrt[c + d*Sin
[e + f*x]]) - ((8*I)*a*Cos[e + f*x]*((b*c - a*d)*EllipticF[I*ArcSinh[Sqrt[-
(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)] + a*d*EllipticPi[
(b*(c + d))/(b*c - a*d), I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f
*x]]], (c + d)/(c - d)])*Sqrt[(d - d*Sin[e + f*x])/(c + d)]*Sqrt[-((d + d*S
in[e + f*x])/(c - d))]*(-(b*c) + a*d + b*(c + d*Sin[e + f*x])))/(b*d*Sqrt[-
(c + d)^(-1)]*(b*c - a*d)*(a + b*Sin[e + f*x])*Sqrt[1 - Sin[e + f*x]^2]*Sqr
t[-((c^2 - d^2 - 2*c*(c + d*Sin[e + f*x]) + (c + d*Sin[e + f*x])^2)/d^2)])
+ ((2*I)*Cos[e + f*x]*Cos[2*(e + f*x)]*(2*b*(c - d)*(b*c - a*d)*EllipticE[I
*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)] +
d*(-2*(a + b)*(-(b*c) + a*d)*EllipticF[I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c
 + d*Sin[e + f*x]]], (c + d)/(c - d)] + (2*a^2 - b^2)*d*EllipticPi[(b*(c +
d))/(b*c - a*d), I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (
c + d)/(c - d)]))*Sqrt[(d - d*Sin[e + f*x])/(c + d)]*Sqrt[-((d + d*Sin[e +
f*x])/(c - d))]*(-(b*c) + a*d + b*(c + d*Sin[e + f*x])))/(b*Sqrt[-(c + d)^(
-1)]*(b*c - a*d)*(a + b*Sin[e + f*x])*Sqrt[1 - Sin[e + f*x]^2]*(-2*c^2 + d^
2 + 4*c*(c + d*Sin[e + f*x]) - 2*(c + d*Sin[e + f*x])^2)*Sqrt[-((c^2 - d^2
- 2*c*(c + d*Sin[e + f*x]) + (c + d*Sin[e + f*x])^2)/d^2)]))/(4*(a - b)*(a
+ b)*f)
```

Maple [B] time = 4.293, size = 872, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^2,x)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*(2*d/b^2*(c/d-1)*((c+d*sin(f*x+e))/
(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)
/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(-c/d+a/b)*EllipticPi(((c+d*sin(f*
x+e))/(c-d))^(1/2),(-c/d+1)/(-c/d+a/b),((c-d)/(c+d))^(1/2))+(-a*d+b*c)/b*(-
b^2/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(
a+b*sin(f*x+e))-a*d/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)*(c/d-1)*((c+d*sin(f*x+e))
/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2
)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d)
)^(1/2),((c-d)/(c+d))^(1/2))-b*d/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)*(c/d-1)*((c+
d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*
d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(
((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x
+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))+(3*a^2*d-2*a*b*c-b^2*d)/(a^3*d-a^2*
b*c-a*b^2*d+b^3*c)/b*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e
))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x
+e)^2)^(1/2)/(-c/d+a/b)*EllipticPi(((c+d*sin(f*x+e))/(c-d))^(1/2),(-c/d+1)/
(-c/d+a/b),((c-d)/(c+d))^(1/2))))/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d\sin\left(fx+e\right)+c}}{\left(b\sin\left(fx+e\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*sin(f*x + e) + c)/(b*sin(f*x + e) + a)^2, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(1/2)/(a+b*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d\sin\left(fx+e\right)+c}}{\left(b\sin\left(fx+e\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*sin(f*x + e) + c)/(b*sin(f*x + e) + a)^2, x)
```

3.755 $\int \frac{1}{(a+b\sin(e+fx))^2\sqrt{c+d\sin(e+fx)}}\,dx$

Optimal. Leaf size=325

$$\frac{b^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{f\left(a^2-b^2\right)(bc-ad)(a+b\sin(e+fx))}-\frac{\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{f\left(a^2-b^2\right)\sqrt{c+d\sin(e+fx)}}+\frac{b\sqrt{c+d\sin(e+fx)}E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{f\left(a^2-b^2\right)(bc-ad)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

```
[Out] (b^2*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/((a^2 - b^2)*(b*c - a*d)*f*(a +
 b*Sin[e + f*x])) + (b*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c
+ d*Sin[e + f*x]])/((a^2 - b^2)*(b*c - a*d)*f*Sqrt[(c + d*Sin[e + f*x])/(c
+ d)]) - (EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e +
f*x])/(c + d)])/((a^2 - b^2)*f*Sqrt[c + d*Sin[e + f*x]]) + ((2*a*b*c - 3*a^
2*d + b^2*d)*EllipticPi[(2*b)/(a + b), (e - Pi/2 + f*x)/2, (2*d)/(c + d)]*S
qrt[(c + d*Sin[e + f*x])/(c + d)])/((a - b)*(a + b)^2*(b*c - a*d)*f*Sqrt[c
+ d*Sin[e + f*x]])
```

Rubi [A] time = 0.968121, antiderivative size = 325, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2802, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{b^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{f\left(a^2-b^2\right)(bc-ad)(a+b\sin(e+fx))}-\frac{\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{f\left(a^2-b^2\right)\sqrt{c+d\sin(e+fx)}}+\frac{b\sqrt{c+d\sin(e+fx)}E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{f\left(a^2-b^2\right)(bc-ad)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sin[e + f*x])^2*Sqrt[c + d*Sin[e + f*x]]),x]
```

```
[Out] (b^2*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/((a^2 - b^2)*(b*c - a*d)*f*(a +
 b*Sin[e + f*x])) + (b*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c
+ d*Sin[e + f*x]])/((a^2 - b^2)*(b*c - a*d)*f*Sqrt[(c + d*Sin[e + f*x])/(c
+ d)]) - (EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e +
f*x])/(c + d)])/((a^2 - b^2)*f*Sqrt[c + d*Sin[e + f*x]]) + ((2*a*b*c - 3*a^
2*d + b^2*d)*EllipticPi[(2*b)/(a + b), (e - Pi/2 + f*x)/2, (2*d)/(c + d)]*S
qrt[(c + d*Sin[e + f*x])/(c + d)])/((a - b)*(a + b)^2*(b*c - a*d)*f*Sqrt[c
+ d*Sin[e + f*x]])
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
```

```
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\sin(e+fx))^2\sqrt{c+d\sin(e+fx)}}\,dx = \frac{b^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{(a^2-b^2)(bc-ad)f(a+b\sin(e+fx))} - \frac{\int \frac{\frac{1}{2}(-2abc+2a^2d-b^2d)-abd\sin(e+fx)-\frac{1}{2}\cdots}{(a+b\sin(e+fx))\sqrt{c+d\sin(e+\cdots}}}{(a^2-b^2)(bc-ad\cdots}$$

$$= \frac{b^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{(a^2-b^2)(bc-ad)f(a+b\sin(e+fx))} + \frac{b\int\sqrt{c+d\sin(e+fx)}\,dx}{2(a^2-b^2)(bc-ad)} + \cdots$$

$$= \frac{b^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{(a^2-b^2)(bc-ad)f(a+b\sin(e+fx))} - \frac{\int\frac{1}{\sqrt{c+d\sin(e+fx)}}\,dx}{2(a^2-b^2)} + \frac{(2abc-3a\cdots}{\cdots}$$

$$= \frac{b^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{(a^2-b^2)(bc-ad)f(a+b\sin(e+fx))} + \frac{bE\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2d}{c+d}\right)\sqrt{c+d\cdots}}{(a^2-b^2)(bc-ad)f\sqrt{\frac{c+d\sin\cdots}{c\cdots}}}$$

$$= \frac{b^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{(a^2-b^2)(bc-ad)f(a+b\sin(e+fx))} + \frac{bE\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2d}{c+d}\right)\sqrt{c+d\cdots}}{(a^2-b^2)(bc-ad)f\sqrt{\frac{c+d\sin\cdots}{c\cdots}}}$$

Mathematica [C] time = 7.25027, size = 871, normalized size = 2.68

$$-\frac{2(4da^2-4bca-3b^2d)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\Pi\left(\frac{2b}{a+b};\frac{1}{2}\left(-e-fx+\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{(a+b)\sqrt{c+d\sin(e+fx)}} + \frac{8ia\cos(e+fx)\left((bc-ad)F\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{c+d}}\sqrt{c+d\sin(e+fx)}\right)|\frac{c+d}{c-d}\right)+ad\Pi\left(\frac{b(c+d)}{bc-ad};i\sinh^{-1}\left(\sqrt{-\cdots}\right.\right.\right.}{d\sqrt{-\frac{1}{c+d}}(bc-ad)(a+b\sin(e+fx))\sqrt{1-\sin^2(e+fx)}\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*Sin[e + f*x])^2*Sqrt[c + d*Sin[e + f*x]]),x]
```

```
[Out] -((b^2*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/((a^2 - b^2)*(-(b*c) + a*d)*f
*(a + b*Sin[e + f*x]))) + ((-2*(-4*a*b*c + 4*a^2*d - 3*b^2*d)*EllipticPi[(2
*b)/(a + b), (-e + Pi/2 - f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/
(c + d)])/((a + b)*Sqrt[c + d*Sin[e + f*x]]) + ((8*I)*a*Cos[e + f*x]*((b*c
- a*d)*EllipticF[I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (
c + d)/(c - d)] + a*d*EllipticPi[(b*(c + d))/(b*c - a*d), I*ArcSinh[Sqrt[-(
c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)])*Sqrt[(d - d*Sin[e
 + f*x])/(c + d)]*Sqrt[-((d + d*Sin[e + f*x])/(c - d))]*(-(b*c) + a*d + b*(
c + d*Sin[e + f*x])))/(d*Sqrt[-(c + d)^(-1)]*(b*c - a*d)*(a + b*Sin[e + f*x
])*Sqrt[1 - Sin[e + f*x]^2]*Sqrt[-((c^2 - d^2 - 2*c*(c + d*Sin[e + f*x]) +
(c + d*Sin[e + f*x])^2)/d^2)]) - ((2*I)*Cos[e + f*x]*Cos[2*(e + f*x)]*(2*b*
(c - d)*(b*c - a*d)*EllipticE[I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[
e + f*x]]], (c + d)/(c - d)] + d*(-2*(a + b)*(-(b*c) + a*d)*EllipticF[I*Arc
Sinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)] + (2*a
^2 - b^2)*d*EllipticPi[(b*(c + d))/(b*c - a*d), I*ArcSinh[Sqrt[-(c + d)^(-1
)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)]))*Sqrt[(d - d*Sin[e + f*x])/
(c + d)]*Sqrt[-((d + d*Sin[e + f*x])/(c - d))]*(-(b*c) + a*d + b*(c + d*Sin
[e + f*x])))/(Sqrt[-(c + d)^(-1)]*(b*c - a*d)*(a + b*Sin[e + f*x])*Sqrt[1 -
 Sin[e + f*x]^2]*(-2*c^2 + d^2 + 4*c*(c + d*Sin[e + f*x]) - 2*(c + d*Sin[e
+ f*x])^2)*Sqrt[-((c^2 - d^2 - 2*c*(c + d*Sin[e + f*x]) + (c + d*Sin[e + f*
x])^2)/d^2)]))/(4*(a - b)*(a + b)*(-(b*c) + a*d)*f)
```

Maple [A] time = 3.542, size = 690, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^(1/2),x)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*(-b^2/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)
*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(a+b*sin(f*x+e))-a*d/(a^3*d-a^2*b*
c-a*b^2*d+b^3*c)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(
c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^
2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))-b*d/
(a^3*d-a^2*b*c-a*b^2*d+b^3*c)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-
sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c
)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((
c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(
1/2)))+(3*a^2*d-2*a*b*c-b^2*d)/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)/b*(c/d-1)*((c+
d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*
d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(-c/d+a/b)*EllipticP
i(((c+d*sin(f*x+e))/(c-d))^(1/2),(-c/d+1)/(-c/d+a/b),((c-d)/(c+d))^(1/2)))/
cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^(1/2),x, algorithm="maxima"
)
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^(1/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))**2/(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\sin\left(fx+e\right)+a\right)^{2}\sqrt{d\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sin(f*x + e) + a)^2*sqrt(d*sin(f*x + e) + c)), x)
```

3.756 $\int \frac{1}{(a+b\sin(e+fx))^2(c+d\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=449

$$\frac{d\left(2a^2d^2+b^2\left(c^2-3d^2\right)\right)\cos(e+fx)}{f\left(a^2-b^2\right)\left(c^2-d^2\right)(bc-ad)^2\sqrt{c+d\sin(e+fx)}}+\frac{\left(2a^2d^2+b^2\left(c^2-3d^2\right)\right)\sqrt{c+d\sin(e+fx)}E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{f\left(a^2-b^2\right)\left(c^2-d^2\right)(bc-ad)^2\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

```
[Out] (d*(2*a^2*d^2 + b^2*(c^2 - 3*d^2))*Cos[e + f*x])/((a^2 - b^2)*(b*c - a*d)^2
*(c^2 - d^2)*f*Sqrt[c + d*Sin[e + f*x]]) + (b^2*Cos[e + f*x])/((a^2 - b^2)*
(b*c - a*d)*f*(a + b*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]]) + ((2*a^2*d^2
+ b^2*(c^2 - 3*d^2))*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c +
d*Sin[e + f*x]])/((a^2 - b^2)*(b*c - a*d)^2*(c^2 - d^2)*f*Sqrt[(c + d*Sin[e
 + f*x])/(c + d)]) - (b*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(
c + d*Sin[e + f*x])/(c + d)])/((a^2 - b^2)*(b*c - a*d)*f*Sqrt[c + d*Sin[e +
 f*x]]) + (b*(2*a*b*c - 5*a^2*d + 3*b^2*d)*EllipticPi[(2*b)/(a + b), (e - P
i/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/((a - b)*(
a + b)^2*(b*c - a*d)^2*f*Sqrt[c + d*Sin[e + f*x]])
```

Rubi [A] time = 1.62744, antiderivative size = 449, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.37, Rules used = {2802, 3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{d\left(2a^2d^2+b^2\left(c^2-3d^2\right)\right)\cos(e+fx)}{f\left(a^2-b^2\right)\left(c^2-d^2\right)(bc-ad)^2\sqrt{c+d\sin(e+fx)}}+\frac{\left(2a^2d^2+b^2\left(c^2-3d^2\right)\right)\sqrt{c+d\sin(e+fx)}E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{f\left(a^2-b^2\right)\left(c^2-d^2\right)(bc-ad)^2\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^(3/2)),x]
```

```
[Out] (d*(2*a^2*d^2 + b^2*(c^2 - 3*d^2))*Cos[e + f*x])/((a^2 - b^2)*(b*c - a*d)^2
*(c^2 - d^2)*f*Sqrt[c + d*Sin[e + f*x]]) + (b^2*Cos[e + f*x])/((a^2 - b^2)*
(b*c - a*d)*f*(a + b*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]]) + ((2*a^2*d^2
+ b^2*(c^2 - 3*d^2))*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c +
d*Sin[e + f*x]])/((a^2 - b^2)*(b*c - a*d)^2*(c^2 - d^2)*f*Sqrt[(c + d*Sin[e
 + f*x])/(c + d)]) - (b*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(
c + d*Sin[e + f*x])/(c + d)])/((a^2 - b^2)*(b*c - a*d)*f*Sqrt[c + d*Sin[e +
 f*x]]) + (b*(2*a*b*c - 5*a^2*d + 3*b^2*d)*EllipticPi[(2*b)/(a + b), (e - P
i/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/((a - b)*(
a + b)^2*(b*c - a*d)^2*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\sin(e+fx))^2(c+d\sin(e+fx))^{3/2}}\,dx = \frac{b^2\cos(e+fx)}{\left(a^2-b^2\right)(bc-ad)f(a+b\sin(e+fx))\sqrt{c+d\sin(e+fx)}} - \int \frac{\frac{1}{2}\left(-2ab\right.}{}$$

$$= \frac{d\left(2a^2d^2+b^2\left(c^2-3d^2\right)\right)\cos(e+fx)}{\left(a^2-b^2\right)(bc-ad)^2\left(c^2-d^2\right)f\sqrt{c+d\sin(e+fx)}} + \frac{}{\left(a^2-b^2\right)(bc-}$$

$$= \frac{d\left(2a^2d^2+b^2\left(c^2-3d^2\right)\right)\cos(e+fx)}{\left(a^2-b^2\right)(bc-ad)^2\left(c^2-d^2\right)f\sqrt{c+d\sin(e+fx)}} + \frac{}{\left(a^2-b^2\right)(bc-}$$

$$= \frac{d\left(2a^2d^2+b^2\left(c^2-3d^2\right)\right)\cos(e+fx)}{\left(a^2-b^2\right)(bc-ad)^2\left(c^2-d^2\right)f\sqrt{c+d\sin(e+fx)}} + \frac{}{\left(a^2-b^2\right)(bc-}$$

$$= \frac{d\left(2a^2d^2+b^2\left(c^2-3d^2\right)\right)\cos(e+fx)}{\left(a^2-b^2\right)(bc-ad)^2\left(c^2-d^2\right)f\sqrt{c+d\sin(e+fx)}} + \frac{}{\left(a^2-b^2\right)(bc-}$$

$$= \frac{d\left(2a^2d^2+b^2\left(c^2-3d^2\right)\right)\cos(e+fx)}{\left(a^2-b^2\right)(bc-ad)^2\left(c^2-d^2\right)f\sqrt{c+d\sin(e+fx)}} + \frac{}{\left(a^2-b^2\right)(bc-}$$

Mathematica [C] time = 7.63817, size = 1057, normalized size = 2.35

$$\frac{\sqrt{c+d\sin(e+fx)}\left(\frac{\cos(e+fx)b^3}{\left(a^2-b^2\right)(ad-bc)^2(a+b\sin(e+fx))}+\frac{2d^3\cos(e+fx)}{(bc-ad)^2\left(c^2-d^2\right)(c+d\sin(e+fx))}\right)}{f} + \frac{-\frac{2\left(4cd^2a^3+10bd^3a^2-8bc^2da^2+4b^2c^3a-8b^2cd^2a-9b^3d\right.}{(a+b)\sqrt{c+}}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^(3/2)),x]
```

```
[Out] (Sqrt[c + d*Sin[e + f*x]]*((b^3*Cos[e + f*x])/((a^2 - b^2)*(-(b*c) + a*d)^2
*(a + b*Sin[e + f*x])) + (2*d^3*Cos[e + f*x])/((b*c - a*d)^2*(c^2 - d^2)*(c
 + d*Sin[e + f*x]))))/f + ((-2*(4*a*b^2*c^3 - 8*a^2*b*c^2*d + 7*b^3*c^2*d +
 4*a^3*c*d^2 - 8*a*b^2*c*d^2 + 10*a^2*b*d^3 - 9*b^3*d^3)*EllipticPi[(2*b)/(
a + b), (-e + Pi/2 - f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c +
```

```
d)])/((a + b)*Sqrt[c + d*Sin[e + f*x]]) - ((2*I)*(4*a*b^2*c^2*d + 4*a^2*b*c
*d^2 - 4*b^3*c*d^2 + 4*a^3*d^3 - 8*a*b^2*d^3)*Cos[e + f*x]*((b*c - a*d)*Ell
ipticF[I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c
- d)] + a*d*EllipticPi[(b*(c + d))/(b*c - a*d), I*ArcSinh[Sqrt[-(c + d)^(-1
)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)])*Sqrt[(d - d*Sin[e + f*x])/(
c + d)]*Sqrt[-((d + d*Sin[e + f*x])/(c - d))]*(-(b*c) + a*d + b*(c + d*Sin[
e + f*x])))/(b*d^2*Sqrt[-(c + d)^(-1)]*(b*c - a*d)*(a + b*Sin[e + f*x])*Sqr
t[1 - Sin[e + f*x]^2]*Sqrt[-((c^2 - d^2 - 2*c*(c + d*Sin[e + f*x]) + (c + d
*Sin[e + f*x])^2)/d^2)]) - ((2*I)*(-(b^3*c^2*d) - 2*a^2*b*d^3 + 3*b^3*d^3)*
Cos[e + f*x]*Cos[2*(e + f*x)]*(2*b*(c - d)*(b*c - a*d)*EllipticE[I*ArcSinh[
Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)] + d*(-2*(a
+ b)*(-(b*c) + a*d)*EllipticF[I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[
e + f*x]]], (c + d)/(c - d)] + (2*a^2 - b^2)*d*EllipticPi[(b*(c + d))/(b*c
- a*d), I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c
 - d)]))*Sqrt[(d - d*Sin[e + f*x])/(c + d)]*Sqrt[-((d + d*Sin[e + f*x])/(c
- d))]*(-(b*c) + a*d + b*(c + d*Sin[e + f*x])))/(b^2*d*Sqrt[-(c + d)^(-1)]*
(b*c - a*d)*(a + b*Sin[e + f*x])*Sqrt[1 - Sin[e + f*x]^2]*(-2*c^2 + d^2 + 4
*c*(c + d*Sin[e + f*x]) - 2*(c + d*Sin[e + f*x])^2)*Sqrt[-((c^2 - d^2 - 2*c
*(c + d*Sin[e + f*x]) + (c + d*Sin[e + f*x])^2)/d^2)]))/(4*(a - b)*(a + b)*
(c - d)*(c + d)*(-(b*c) + a*d)^2*f)
```

Maple [B] time = 5.793, size = 1266, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^(3/2),x)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*(-2*d/(a*d-b*c)^2*(c/d-1)*((c+d*sin
(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-
d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(-c/d+a/b)*EllipticPi(((c
+d*sin(f*x+e))/(c-d))^(1/2),(-c/d+1)/(-c/d+a/b),((c-d)/(c+d))^(1/2))+d^2/(a
*d-b*c)^2*(2*d*cos(f*x+e)^2/(c^2-d^2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/
2)+2*c/(c^2-d^2)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(
c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^
2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+2/(c
^2-d^2)*d*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(
1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2
)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+E
llipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))-b/(a*d-b*c)*
(-b^2/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)
/(a+b*sin(f*x+e))-a*d/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)*(c/d-1)*((c+d*sin(f*x+e
))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1
/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-
d))^(1/2),((c-d)/(c+d))^(1/2))-b*d/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)*(c/d-1)*((
c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1
)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*Elliptic
E(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f
*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))+(3*a^2*d-2*a*b*c-b^2*d)/(a^3*d-a^
2*b*c-a*b^2*d+b^3*c)/b*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x
+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f
*x+e)^2)^(1/2)/(-c/d+a/b)*EllipticPi(((c+d*sin(f*x+e))/(c-d))^(1/2),(-c/d+1
)/(-c/d+a/b),((c-d)/(c+d))^(1/2))))/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^(3/2),x, algorithm="maxima"
)
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^(3/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))**2/(c+d*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.757 $\int \frac{1}{(a+b\sin(e+fx))^2(c+d\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=661

$$-\frac{\left(-4a^2bd^3\left(5c^2-3d^2\right)+8a^3cd^4-8ab^2cd^4-b^3\left(-26c^2d^3+3c^4d+15d^5\right)\right)\cos(e+fx)}{3f\left(a^2-b^2\right)\left(c^2-d^2\right)^2(bc-ad)^3\sqrt{c+d\sin(e+fx)}}+\frac{d\left(2a^2d^2+b^2\left(3c^2-5d^2\right.\right.}{3f\left(a^2-b^2\right)\left(c^2-d^2\right)(bc-ad)^2}$$

```
[Out] (d*(2*a^2*d^2 + b^2*(3*c^2 - 5*d^2))*Cos[e + f*x])/(3*(a^2 - b^2)*(b*c - a*
d)^2*(c^2 - d^2)*f*(c + d*Sin[e + f*x])^(3/2)) + (b^2*Cos[e + f*x])/((a^2 -
 b^2)*(b*c - a*d)*f*(a + b*Sin[e + f*x])*(c + d*Sin[e + f*x])^(3/2)) - ((8*
a^3*c*d^4 - 8*a*b^2*c*d^4 - 4*a^2*b*d^3*(5*c^2 - 3*d^2) - b^3*(3*c^4*d - 26
*c^2*d^3 + 15*d^5))*Cos[e + f*x])/(3*(a^2 - b^2)*(b*c - a*d)^3*(c^2 - d^2)^
2*f*Sqrt[c + d*Sin[e + f*x]]) - ((8*a^3*c*d^3 - 8*a*b^2*c*d^3 - 4*a^2*b*d^2
*(5*c^2 - 3*d^2) - b^3*(3*c^4 - 26*c^2*d^2 + 15*d^4))*EllipticE[(e - Pi/2 +
 f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(3*(a^2 - b^2)*(b*c - a*d
)^3*(c^2 - d^2)^2*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) - ((2*a^2*d^2 + b^2
*(3*c^2 - 5*d^2))*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*
Sin[e + f*x])/(c + d)])/(3*(a^2 - b^2)*(b*c - a*d)^2*(c^2 - d^2)*f*Sqrt[c +
 d*Sin[e + f*x]]) + (b^2*(2*a*b*c - 7*a^2*d + 5*b^2*d)*EllipticPi[(2*b)/(a
+ b), (e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)]
)/((a - b)*(a + b)^2*(b*c - a*d)^3*f*Sqrt[c + d*Sin[e + f*x]])
```

Rubi [A] time = 2.7986, antiderivative size = 661, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.37, Rules used = {2802, 3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$-\frac{\left(-4a^2bd^3\left(5c^2-3d^2\right)+8a^3cd^4-8ab^2cd^4-b^3\left(-26c^2d^3+3c^4d+15d^5\right)\right)\cos(e+fx)}{3f\left(a^2-b^2\right)\left(c^2-d^2\right)^2(bc-ad)^3\sqrt{c+d\sin(e+fx)}}+\frac{d\left(2a^2d^2+b^2\left(3c^2-5d^2\right.\right.}{3f\left(a^2-b^2\right)\left(c^2-d^2\right)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^(5/2)),x]
```

```
[Out] (d*(2*a^2*d^2 + b^2*(3*c^2 - 5*d^2))*Cos[e + f*x])/(3*(a^2 - b^2)*(b*c - a*
d)^2*(c^2 - d^2)*f*(c + d*Sin[e + f*x])^(3/2)) + (b^2*Cos[e + f*x])/((a^2 -
 b^2)*(b*c - a*d)*f*(a + b*Sin[e + f*x])*(c + d*Sin[e + f*x])^(3/2)) - ((8*
a^3*c*d^4 - 8*a*b^2*c*d^4 - 4*a^2*b*d^3*(5*c^2 - 3*d^2) - b^3*(3*c^4*d - 26
*c^2*d^3 + 15*d^5))*Cos[e + f*x])/(3*(a^2 - b^2)*(b*c - a*d)^3*(c^2 - d^2)^
2*f*Sqrt[c + d*Sin[e + f*x]]) - ((8*a^3*c*d^3 - 8*a*b^2*c*d^3 - 4*a^2*b*d^2
*(5*c^2 - 3*d^2) - b^3*(3*c^4 - 26*c^2*d^2 + 15*d^4))*EllipticE[(e - Pi/2 +
 f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(3*(a^2 - b^2)*(b*c - a*d
)^3*(c^2 - d^2)^2*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) - ((2*a^2*d^2 + b^2
*(3*c^2 - 5*d^2))*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*
Sin[e + f*x])/(c + d)])/(3*(a^2 - b^2)*(b*c - a*d)^2*(c^2 - d^2)*f*Sqrt[c +
 d*Sin[e + f*x]]) + (b^2*(2*a*b*c - 7*a^2*d + 5*b^2*d)*EllipticPi[(2*b)/(a
+ b), (e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)]
)/((a - b)*(a + b)^2*(b*c - a*d)^3*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
```

```
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
```

```
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\sin(e+fx))^2(c+d\sin(e+fx))^{5/2}}\,dx = \frac{b^2\cos(e+fx)}{\left(a^2-b^2\right)(bc-ad)f(a+b\sin(e+fx))(c+d\sin(e+fx))^{3/2}} - \frac{\int \frac{\frac{1}{2}\left(-2abc\right.}{}}{}$$

$$= \frac{d\left(2a^2d^2+b^2\left(3c^2-5d^2\right)\right)\cos(e+fx)}{3\left(a^2-b^2\right)(bc-ad)^2\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}} + \frac{}{\left(a^2-b^2\right)(bc-}$$

$$= \frac{d\left(2a^2d^2+b^2\left(3c^2-5d^2\right)\right)\cos(e+fx)}{3\left(a^2-b^2\right)(bc-ad)^2\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}} + \frac{}{\left(a^2-b^2\right)(bc-}$$

$$= \frac{d\left(2a^2d^2+b^2\left(3c^2-5d^2\right)\right)\cos(e+fx)}{3\left(a^2-b^2\right)(bc-ad)^2\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}} + \frac{}{\left(a^2-b^2\right)(bc-}$$

$$= \frac{d\left(2a^2d^2+b^2\left(3c^2-5d^2\right)\right)\cos(e+fx)}{3\left(a^2-b^2\right)(bc-ad)^2\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}} + \frac{}{\left(a^2-b^2\right)(bc-}$$

$$= \frac{d\left(2a^2d^2+b^2\left(3c^2-5d^2\right)\right)\cos(e+fx)}{3\left(a^2-b^2\right)(bc-ad)^2\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}} + \frac{}{\left(a^2-b^2\right)(bc-}$$

$$= \frac{d\left(2a^2d^2+b^2\left(3c^2-5d^2\right)\right)\cos(e+fx)}{3\left(a^2-b^2\right)(bc-ad)^2\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}} + \frac{}{\left(a^2-b^2\right)(bc-}$$

Mathematica [C] time = 8.61782, size = 1319, normalized size = 2.

$$\frac{\sqrt{c+d\sin(e+fx)}\left(-\frac{\cos(e+fx)b^4}{(a^2-b^2)(ad-bc)^3(a+b\sin(e+fx))}-\frac{4\left(3b\cos(e+fx)d^5+2ac\cos(e+fx)d^4-5bc^2\cos(e+fx)d^3\right)}{3(bc-ad)^3\left(c^2-d^2\right)^2(c+d\sin(e+fx))}+\frac{2d^3\cos(e+fx)}{3(bc-ad)^2\left(c^2-d^2\right)(c+d\sin(e+f}\right.}{f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^(5/2)),x]
```

```
[Out] (Sqrt[c + d*Sin[e + f*x]]*(-((b^4*Cos[e + f*x])/((a^2 - b^2)*(-(b*c) + a*d)
^3*(a + b*Sin[e + f*x]))) + (2*d^3*Cos[e + f*x])/(3*(b*c - a*d)^2*(c^2 - d^
2)*(c + d*Sin[e + f*x])^2) - (4*(-5*b*c^2*d^3*Cos[e + f*x] + 2*a*c*d^4*Cos[
e + f*x] + 3*b*d^5*Cos[e + f*x]))/(3*(b*c - a*d)^3*(c^2 - d^2)^2*(c + d*Sin
[e + f*x]))))/f + ((-2*(-12*a*b^3*c^5 + 36*a^2*b^2*c^4*d - 33*b^4*c^4*d - 3
6*a^3*b*c^3*d^2 + 60*a*b^3*c^3*d^2 + 12*a^4*c^2*d^3 - 104*a^2*b^2*c^2*d^3 +
 86*b^4*c^2*d^3 + 28*a^3*b*c*d^4 - 40*a*b^3*c*d^4 + 4*a^4*d^5 + 44*a^2*b^2*
d^5 - 45*b^4*d^5)*EllipticPi[(2*b)/(a + b), (-e + Pi/2 - f*x)/2, (2*d)/(c +
 d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/((a + b)*Sqrt[c + d*Sin[e + f*x]])
 - ((2*I)*(-12*a*b^3*c^4*d - 36*a^2*b^2*c^3*d^2 + 36*b^4*c^3*d^2 - 28*a^3*b
*c^2*d^3 + 52*a*b^3*c^2*d^3 + 16*a^4*c*d^4 + 4*a^2*b^2*c*d^4 - 20*b^4*c*d^4
 + 28*a^3*b*d^5 - 40*a*b^3*d^5)*Cos[e + f*x]*((b*c - a*d)*EllipticF[I*ArcSi
nh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)] + a*d*El
lipticPi[(b*(c + d))/(b*c - a*d), I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*
Sin[e + f*x]]], (c + d)/(c - d)])*Sqrt[(d - d*Sin[e + f*x])/(c + d)]*Sqrt[-
((d + d*Sin[e + f*x])/(c - d))]*(-(b*c) + a*d + b*(c + d*Sin[e + f*x])))/(b
*d^2*Sqrt[-(c + d)^(-1)]*(b*c - a*d)*(a + b*Sin[e + f*x])*Sqrt[1 - Sin[e +
f*x]^2]*Sqrt[-((c^2 - d^2 - 2*c*(c + d*Sin[e + f*x]) + (c + d*Sin[e + f*x])
^2)/d^2)]) - ((2*I)*(3*b^4*c^4*d + 20*a^2*b^2*c^2*d^3 - 26*b^4*c^2*d^3 - 8*
a^3*b*c*d^4 + 8*a*b^3*c*d^4 - 12*a^2*b^2*d^5 + 15*b^4*d^5)*Cos[e + f*x]*Cos
[2*(e + f*x)]*(2*b*(c - d)*(b*c - a*d)*EllipticE[I*ArcSinh[Sqrt[-(c + d)^(-
1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)] + d*(-2*(a + b)*(-(b*c) + a
*d)*EllipticF[I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c +
 d)/(c - d)] + (2*a^2 - b^2)*d*EllipticPi[(b*(c + d))/(b*c - a*d), I*ArcSin
h[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)]))*Sqrt[(d
 - d*Sin[e + f*x])/(c + d)]*Sqrt[-((d + d*Sin[e + f*x])/(c - d))]*(-(b*c) +
 a*d + b*(c + d*Sin[e + f*x])))/(b^2*d*Sqrt[-(c + d)^(-1)]*(b*c - a*d)*(a +
 b*Sin[e + f*x])*Sqrt[1 - Sin[e + f*x]^2]*(-2*c^2 + d^2 + 4*c*(c + d*Sin[e
+ f*x]) - 2*(c + d*Sin[e + f*x])^2)*Sqrt[-((c^2 - d^2 - 2*c*(c + d*Sin[e +
f*x]) + (c + d*Sin[e + f*x])^2)/d^2)])))/(12*(a - b)*(a + b)*(c - d)^2*(c +
d)^2*(-(b*c) + a*d)^3*f)
```

Maple [B] time = 10.434, size = 1731, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^(5/2),x)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*(4*b/(a*d-b*c)^3*d*(c/d-1)*((c+d*si
n(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c
-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(-c/d+a/b)*EllipticPi(((
c+d*sin(f*x+e))/(c-d))^(1/2),(-c/d+1)/(-c/d+a/b),((c-d)/(c+d))^(1/2))+d^2/(
a*d-b*c)^2*(2/3/(c^2-d^2)/d*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(sin(f*
```

```
x+e)+c/d)^2+8/3*d*cos(f*x+e)^2/(c^2-d^2)^2*c/(-(-d*sin(f*x+e)-c)*cos(f*x+e)
^2)^(1/2)+2*(3*c^2+d^2)/(3*c^4-6*c^2*d^2+3*d^4)*(c/d-1)*((c+d*sin(f*x+e))/(
c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/
(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^
(1/2),((c-d)/(c+d))^(1/2))+8/3*c*d/(c^2-d^2)^2*(c/d-1)*((c+d*sin(f*x+e))/(c
-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(
-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e)
)/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2
),((c-d)/(c+d))^(1/2))))-2*d^2/(a*d-b*c)^3*b*(2*d*cos(f*x+e)^2/(c^2-d^2)/(-
(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2*c/(c^2-d^2)*(c/d-1)*((c+d*sin(f*x+e
))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1
/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-
d))^(1/2),((c-d)/(c+d))^(1/2))+2/(c^2-d^2)*d*(c/d-1)*((c+d*sin(f*x+e))/(c-d
))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(
-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/
(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),
((c-d)/(c+d))^(1/2))))+b^2/(a*d-b*c)^2*(-b^2/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)*
(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(a+b*sin(f*x+e))-a*d/(a^3*d-a^2*b*c
-a*b^2*d+b^3*c)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c
+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2
)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))-b*d/(
a^3*d-a^2*b*c-a*b^2*d+b^3*c)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-s
in(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)
*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c
-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1
/2)))+(3*a^2*d-2*a*b*c-b^2*d)/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)/b*(c/d-1)*((c+d
*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d
/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(-c/d+a/b)*EllipticPi
(((c+d*sin(f*x+e))/(c-d))^(1/2),(-c/d+1)/(-c/d+a/b),((c-d)/(c+d))^(1/2))))/
cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^(5/2),x, algorithm="maxima"
)
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^(5/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))**2/(c+d*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.758 $\int \frac{(c+d\sin(e+fx))^{9/2}}{(a+b\sin(e+fx))^3}\,dx$

Optimal. Leaf size=816

$$\frac{\left(35d^2a^4+20bcda^3+2b^2\left(4c^2-43d^2\right)a^2-44b^3cda+b^4\left(4c^2+63d^2\right)\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}(bc-ad}{4(a-b)^2b^5(a+b)^3f\sqrt{c+d\sin(e+fx)}}$$

```
[Out] (d*(36*a^3*b*c*d^2 - 35*a^4*d^3 + b^4*d*(45*c^2 - 8*d^2) - 18*a*b^3*c*(c^2
+ 5*d^2) + a^2*b^2*d*(9*c^2 + 61*d^2))*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]
])/(12*b^3*(a^2 - b^2)^2*f) + ((b*c - a*d)^2*(6*a*b*c + 7*a^2*d - 13*b^2*d)
*Cos[e + f*x]*(c + d*Sin[e + f*x])^(3/2))/(4*b^2*(a^2 - b^2)^2*f*(a + b*Sin
[e + f*x])) + ((b*c - a*d)^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(5/2))/(2*b*
(a^2 - b^2)*f*(a + b*Sin[e + f*x])^2) + ((185*a^4*b*c*d^3 - 105*a^5*d^4 - b
^5*c*d*(51*c^2 - 104*d^2) - 15*a^3*b^2*d^2*(3*c^2 - 13*d^2) - a^2*b^3*c*d*(
21*c^2 + 361*d^2) + 9*a*b^4*(2*c^4 + 17*c^2*d^2 - 8*d^4))*EllipticE[(e - Pi
/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(12*b^4*(a^2 - b^2)^2
*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) - ((150*a^5*b*c*d^4 - 105*a^6*d^5 -
12*a^3*b^3*c*d^2*(4*c^2 + 29*d^2) + a^4*b^2*d^3*(26*c^2 + 223*d^2) - b^6*d*
(57*c^4 + 136*c^2*d^2 + 8*d^4) + 6*a*b^5*c*(3*c^4 + 38*c^2*d^2 + 48*d^4) -
a^2*b^4*d*(33*c^4 + 70*c^2*d^2 + 128*d^4))*EllipticF[(e - Pi/2 + f*x)/2, (2
*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(12*b^5*(a^2 - b^2)^2*f*Sq
rt[c + d*Sin[e + f*x]]) + ((b*c - a*d)^3*(20*a^3*b*c*d - 44*a*b^3*c*d + 35*
a^4*d^2 + 2*a^2*b^2*(4*c^2 - 43*d^2) + b^4*(4*c^2 + 63*d^2))*EllipticPi[(2*
b)/(a + b), (e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c
 + d)])/(4*(a - b)^2*b^5*(a + b)^3*f*Sqrt[c + d*Sin[e + f*x]])
```

Rubi [A] time = 3.17912, antiderivative size = 816, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 11, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.407, Rules used = {2792, 3047, 3049, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{\left(35d^2a^4+20bcda^3+2b^2\left(4c^2-43d^2\right)a^2-44b^3cda+b^4\left(4c^2+63d^2\right)\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}(bc-ad}{4(a-b)^2b^5(a+b)^3f\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^(9/2)/(a + b*Sin[e + f*x])^3,x]
```

```
[Out] (d*(36*a^3*b*c*d^2 - 35*a^4*d^3 + b^4*d*(45*c^2 - 8*d^2) - 18*a*b^3*c*(c^2
+ 5*d^2) + a^2*b^2*d*(9*c^2 + 61*d^2))*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]
])/(12*b^3*(a^2 - b^2)^2*f) + ((b*c - a*d)^2*(6*a*b*c + 7*a^2*d - 13*b^2*d)
*Cos[e + f*x]*(c + d*Sin[e + f*x])^(3/2))/(4*b^2*(a^2 - b^2)^2*f*(a + b*Sin
[e + f*x])) + ((b*c - a*d)^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(5/2))/(2*b*
(a^2 - b^2)*f*(a + b*Sin[e + f*x])^2) + ((185*a^4*b*c*d^3 - 105*a^5*d^4 - b
^5*c*d*(51*c^2 - 104*d^2) - 15*a^3*b^2*d^2*(3*c^2 - 13*d^2) - a^2*b^3*c*d*(
21*c^2 + 361*d^2) + 9*a*b^4*(2*c^4 + 17*c^2*d^2 - 8*d^4))*EllipticE[(e - Pi
/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(12*b^4*(a^2 - b^2)^2
*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) - ((150*a^5*b*c*d^4 - 105*a^6*d^5 -
12*a^3*b^3*c*d^2*(4*c^2 + 29*d^2) + a^4*b^2*d^3*(26*c^2 + 223*d^2) - b^6*d*
(57*c^4 + 136*c^2*d^2 + 8*d^4) + 6*a*b^5*c*(3*c^4 + 38*c^2*d^2 + 48*d^4) -
a^2*b^4*d*(33*c^4 + 70*c^2*d^2 + 128*d^4))*EllipticF[(e - Pi/2 + f*x)/2, (2
*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(12*b^5*(a^2 - b^2)^2*f*Sq
rt[c + d*Sin[e + f*x]]) + ((b*c - a*d)^3*(20*a^3*b*c*d - 44*a*b^3*c*d + 35*
a^4*d^2 + 2*a^2*b^2*(4*c^2 - 43*d^2) + b^4*(4*c^2 + 63*d^2))*EllipticPi[(2*
```

```
b)/(a + b), (e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c
 + d)])/(4*(a - b)^2*b^5*(a + b)^3*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
```

```
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^{9/2}}{(a+b\sin(e+fx))^3}\,dx = \frac{(bc-ad)^2\cos(e+fx)(c+d\sin(e+fx))^{5/2}}{2b\left(a^2-b^2\right)f(a+b\sin(e+fx))^2} - \frac{\int \frac{(c+d\sin(e+fx))^{3/2}\left(\frac{1}{2}\left(5d(bc-ad)^2+4bc\left(2bcd-a\left(c^2+\right.\right.\right.\right.}{}}{}$$

$$= \frac{(bc-ad)^2\left(6abc+7a^2d-13b^2d\right)\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{4b^2\left(a^2-b^2\right)^2 f(a+b\sin(e+fx))} + \frac{(bc-ad)^2\cos(e+}{2b\left(a^2-b^2\right)f}$$

$$= \frac{d\left(36a^3bcd^2-35a^4d^3+b^4d\left(45c^2-8d^2\right)-18ab^3c\left(c^2+5d^2\right)+a^2b^2d\left(9c^2+61d^2\right)\right)\cos}{12b^3\left(a^2-b^2\right)^2 f}$$

$$= \frac{d\left(36a^3bcd^2-35a^4d^3+b^4d\left(45c^2-8d^2\right)-18ab^3c\left(c^2+5d^2\right)+a^2b^2d\left(9c^2+61d^2\right)\right)\cos}{12b^3\left(a^2-b^2\right)^2 f}$$

$$= \frac{d\left(36a^3bcd^2-35a^4d^3+b^4d\left(45c^2-8d^2\right)-18ab^3c\left(c^2+5d^2\right)+a^2b^2d\left(9c^2+61d^2\right)\right)\cos}{12b^3\left(a^2-b^2\right)^2 f}$$

$$= \frac{d\left(36a^3bcd^2-35a^4d^3+b^4d\left(45c^2-8d^2\right)-18ab^3c\left(c^2+5d^2\right)+a^2b^2d\left(9c^2+61d^2\right)\right)\cos}{12b^3\left(a^2-b^2\right)^2 f}$$

$$= \frac{d\left(36a^3bcd^2-35a^4d^3+b^4d\left(45c^2-8d^2\right)-18ab^3c\left(c^2+5d^2\right)+a^2b^2d\left(9c^2+61d^2\right)\right)\cos}{12b^3\left(a^2-b^2\right)^2 f}$$

Mathematica [C] time = 9.05907, size = 1526, normalized size = 1.87

$$\frac{\sqrt{c+d\sin(e+fx)}\left(-\frac{2\cos(e+fx)d^4}{3b^3}+\frac{-11d^4\cos(e+fx)a^5+27bcd^3\cos(e+fx)a^4+17b^2d^4\cos(e+fx)a^3-15b^2c^2d^2\cos(e+fx)a^3-51b^3cd^3\cos(e+fx)a^2}{4b^3\left(b^2-a^2\right)^2(a+b\sin(e+fx)}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*Sin[e + f*x])^(9/2)/(a + b*Sin[e + f*x])^3,x]
```

```
[Out] (Sqrt[c + d*Sin[e + f*x]]*((-2*d^4*Cos[e + f*x])/(3*b^3) + (-(b^4*c^4*Cos[e
 + f*x]) + 4*a*b^3*c^3*d*Cos[e + f*x] - 6*a^2*b^2*c^2*d^2*Cos[e + f*x] + 4*
a^3*b*c*d^3*Cos[e + f*x] - a^4*d^4*Cos[e + f*x])/(2*b^3*(-a^2 + b^2)*(a + b
*Sin[e + f*x])^2) + (6*a*b^4*c^4*Cos[e + f*x] - 7*a^2*b^3*c^3*d*Cos[e + f*x
] - 17*b^5*c^3*d*Cos[e + f*x] - 15*a^3*b^2*c^2*d^2*Cos[e + f*x] + 51*a*b^4*
c^2*d^2*Cos[e + f*x] + 27*a^4*b*c*d^3*Cos[e + f*x] - 51*a^2*b^3*c*d^3*Cos[e
 + f*x] - 11*a^5*d^4*Cos[e + f*x] + 17*a^3*b^2*d^4*Cos[e + f*x])/(4*b^3*(-a
^2 + b^2)^2*(a + b*Sin[e + f*x]))))/f - ((-2*(-48*a^2*b^3*c^5 - 24*b^5*c^5
+ 306*a*b^4*c^4*d - 177*a^2*b^3*c^3*d^2 - 327*b^5*c^3*d^2 - 105*a^3*b^2*c^2
*d^3 + 501*a*b^4*c^2*d^3 + 13*a^4*b*c*d^4 - 53*a^2*b^3*c*d^4 - 104*b^5*c*d^
4 + 35*a^5*d^5 - 73*a^3*b^2*d^5 + 56*a*b^4*d^5)*EllipticPi[(2*b)/(a + b), (
-e + Pi/2 - f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/((a
+ b)*Sqrt[c + d*Sin[e + f*x]]) - ((2*I)*(-60*a^2*b^3*c^4*d - 12*b^5*c^4*d +
 36*a^3*b^2*c^3*d^2 + 252*a*b^4*c^3*d^2 - 228*a^4*b*c^2*d^3 + 276*a^2*b^3*c
^2*d^3 - 480*b^5*c^2*d^3 + 140*a^5*c*d^4 - 364*a^3*b^2*c*d^4 + 512*a*b^4*c*
d^4 + 56*a^4*b*d^5 - 112*a^2*b^3*d^5 - 16*b^5*d^5)*Cos[e + f*x]*((b*c - a*d
)*EllipticF[I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d
)/(c - d)] + a*d*EllipticPi[(b*(c + d))/(b*c - a*d), I*ArcSinh[Sqrt[-(c + d
```

```
)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)])*Sqrt[(d - d*Sin[e + f*
x])/(c + d)]*Sqrt[-((d + d*Sin[e + f*x])/(c - d))]*(-(b*c) + a*d + b*(c + d
*Sin[e + f*x])))/(b*d^2*Sqrt[-(c + d)^(-1)]*(b*c - a*d)*(a + b*Sin[e + f*x]
)*Sqrt[1 - Sin[e + f*x]^2]*Sqrt[-((c^2 - d^2 - 2*c*(c + d*Sin[e + f*x]) + (
c + d*Sin[e + f*x])^2)/d^2)]) - ((2*I)*(18*a*b^4*c^4*d - 21*a^2*b^3*c^3*d^2
 - 51*b^5*c^3*d^2 - 45*a^3*b^2*c^2*d^3 + 153*a*b^4*c^2*d^3 + 185*a^4*b*c*d^
4 - 361*a^2*b^3*c*d^4 + 104*b^5*c*d^4 - 105*a^5*d^5 + 195*a^3*b^2*d^5 - 72*
a*b^4*d^5)*Cos[e + f*x]*Cos[2*(e + f*x)]*(2*b*(c - d)*(b*c - a*d)*EllipticE
[I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)]
+ d*(-2*(a + b)*(-(b*c) + a*d)*EllipticF[I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt
[c + d*Sin[e + f*x]]], (c + d)/(c - d)] + (2*a^2 - b^2)*d*EllipticPi[(b*(c
+ d))/(b*c - a*d), I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]],
 (c + d)/(c - d)]))*Sqrt[(d - d*Sin[e + f*x])/(c + d)]*Sqrt[-((d + d*Sin[e
+ f*x])/(c - d))]*(-(b*c) + a*d + b*(c + d*Sin[e + f*x])))/(b^2*d*Sqrt[-(c
+ d)^(-1)]*(b*c - a*d)*(a + b*Sin[e + f*x])*Sqrt[1 - Sin[e + f*x]^2]*(-2*c^
2 + d^2 + 4*c*(c + d*Sin[e + f*x]) - 2*(c + d*Sin[e + f*x])^2)*Sqrt[-((c^2
- d^2 - 2*c*(c + d*Sin[e + f*x]) + (c + d*Sin[e + f*x])^2)/d^2)]))/(48*(a -
 b)^2*b^3*(a + b)^2*f)
```

Maple [B] time = 11.506, size = 2775, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(9/2)/(a+b*sin(f*x+e))^3,x)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*(d^3/b^5*(b^2*d^2*(-2/3/d*(-(-d*sin
(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2/3*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(
d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x
+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/
(c+d))^(1/2))-4/3*c/d*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+
e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*
x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+
d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+
2*(-3*a*b*d^2+5*b^2*c*d)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f
*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos
(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/
(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))
)+12*a^2*d^2*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d)
)^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(
1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))-30*a*b*c
*d*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((
-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*Ellip
ticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+20*c^2*b^2*(c/d-1)
*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e
)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d
*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))-20/b^6*d^2*(a^3*d^3-3*a^2*b
*c*d^2+3*a*b^2*c^2*d-b^3*c^3)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-
sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c
)*cos(f*x+e)^2)^(1/2)/(-c/d+a/b)*EllipticPi(((c+d*sin(f*x+e))/(c-d))^(1/2),
(-c/d+1)/(-c/d+a/b),((c-d)/(c+d))^(1/2))+1/b^5*(-a^5*d^5+5*a^4*b*c*d^4-10*a
^3*b^2*c^2*d^3+10*a^2*b^3*c^3*d^2-5*a*b^4*c^4*d+b^5*c^5)*(-1/2*b^2/(a^3*d-a
^2*b*c-a*b^2*d+b^3*c)*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(a+b*sin(f*x+
e))^2-3/4*b^2*(3*a^2*d-2*a*b*c-b^2*d)/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)^2*(-(-d
*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(a+b*sin(f*x+e))-1/4*d*(7*a^3*d-4*a^2*b*
c-a*b^2*d-2*b^3*c)/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)^2*(c/d-1)*((c+d*sin(f*x+e)
```

```
)/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/
2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d
))^(1/2),((c-d)/(c+d))^(1/2))-3/4*b*d*(3*a^2*d-2*a*b*c-b^2*d)/(a^3*d-a^2*b*
c-a*b^2*d+b^3*c)^2*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))
/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e
)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))
^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))+1/4*
(15*a^4*d^2-20*a^3*b*c*d+8*a^2*b^2*c^2-6*a^2*b^2*d^2-4*a*b^3*c*d+4*b^4*c^2+
3*b^4*d^2)/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)^2/b*(c/d-1)*((c+d*sin(f*x+e))/(c-d
))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(
-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(-c/d+a/b)*EllipticPi(((c+d*sin(f*x+e)
)/(c-d))^(1/2),(-c/d+1)/(-c/d+a/b),((c-d)/(c+d))^(1/2)))+5/b^5*d*(a^4*d^4-4
*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)*(-b^2/(a^3*d-a^2*b*c-
a*b^2*d+b^3*c)*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(a+b*sin(f*x+e))-a*d
/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1
-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-
c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d
))^(1/2))-b*d/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)*(c/d-1)*((c+d*sin(f*x+e))/(c-d)
)^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-
d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(
c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),(
(c-d)/(c+d))^(1/2)))+(3*a^2*d-2*a*b*c-b^2*d)/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)/
b*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-
sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(-c/d+
a/b)*EllipticPi(((c+d*sin(f*x+e))/(c-d))^(1/2),(-c/d+1)/(-c/d+a/b),((c-d)/(
c+d))^(1/2))))/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(9/2)/(a+b*sin(f*x+e))^3,x, algorithm="maxima")

[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(9/2)/(a+b*sin(f*x+e))^3,x, algorithm="fricas")

[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(9/2)/(a+b*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{\frac{9}{2}}}{\left(b\sin\left(fx+e\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(9/2)/(a+b*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(9/2)/(b*sin(f*x + e) + a)^3, x)
```

3.759 $\int \frac{(c+d\sin(e+fx))^{7/2}}{(a+b\sin(e+fx))^3}\,dx$

Optimal. Leaf size=605

$$\frac{3\left(a^2b^2d\left(c^2-11d^2\right)+4a^3bcd^2+5a^4d^3-2ab^3c\left(c^2+5d^2\right)+b^4d\left(5c^2+8d^2\right)\right)(bc-ad)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\right.}{4b^4f\left(a^2-b^2\right)^2\sqrt{c+d\sin(e+fx)}}$$

```
[Out] ((b*c - a*d)^2*(6*a*b*c + 5*a^2*d - 11*b^2*d)*Cos[e + f*x]*Sqrt[c + d*Sin[e
 + f*x]])/(4*b^2*(a^2 - b^2)^2*f*(a + b*Sin[e + f*x])) + ((b*c - a*d)^2*Cos
[e + f*x]*(c + d*Sin[e + f*x])^(3/2))/(2*b*(a^2 - b^2)*f*(a + b*Sin[e + f*x
])^2) - ((8*a^3*b*c*d^2 - 15*a^4*d^3 + b^4*d*(13*c^2 - 8*d^2) - 2*a*b^3*c*(
3*c^2 + 13*d^2) + a^2*b^2*d*(5*c^2 + 29*d^2))*EllipticE[(e - Pi/2 + f*x)/2,
 (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(4*b^3*(a^2 - b^2)^2*f*Sqrt[(c +
d*Sin[e + f*x])/(c + d)]) + (3*(b*c - a*d)*(4*a^3*b*c*d^2 + 5*a^4*d^3 + a^2
*b^2*d*(c^2 - 11*d^2) - 2*a*b^3*c*(c^2 + 5*d^2) + b^4*d*(5*c^2 + 8*d^2))*El
lipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d
)])/(4*b^4*(a^2 - b^2)^2*f*Sqrt[c + d*Sin[e + f*x]]) + ((b*c - a*d)^2*(12*a
^3*b*c*d - 36*a*b^3*c*d + 15*a^4*d^2 + 2*a^2*b^2*(4*c^2 - 19*d^2) + b^4*(4*
c^2 + 35*d^2))*EllipticPi[(2*b)/(a + b), (e - Pi/2 + f*x)/2, (2*d)/(c + d)]
*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(4*(a - b)^2*b^4*(a + b)^3*f*Sqrt[c +
d*Sin[e + f*x]])
```

Rubi [A] time = 2.20118, antiderivative size = 605, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.37, Rules used = {2792, 3047, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{3\left(a^2b^2d\left(c^2-11d^2\right)+4a^3bcd^2+5a^4d^3-2ab^3c\left(c^2+5d^2\right)+b^4d\left(5c^2+8d^2\right)\right)(bc-ad)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\right.}{4b^4f\left(a^2-b^2\right)^2\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^(7/2)/(a + b*Sin[e + f*x])^3,x]
```


```
[Out] ((b*c - a*d)^2*(6*a*b*c + 5*a^2*d - 11*b^2*d)*Cos[e + f*x]*Sqrt[c + d*Sin[e
 + f*x]])/(4*b^2*(a^2 - b^2)^2*f*(a + b*Sin[e + f*x])) + ((b*c - a*d)^2*Cos
[e + f*x]*(c + d*Sin[e + f*x])^(3/2))/(2*b*(a^2 - b^2)*f*(a + b*Sin[e + f*x
])^2) - ((8*a^3*b*c*d^2 - 15*a^4*d^3 + b^4*d*(13*c^2 - 8*d^2) - 2*a*b^3*c*(
3*c^2 + 13*d^2) + a^2*b^2*d*(5*c^2 + 29*d^2))*EllipticE[(e - Pi/2 + f*x)/2,
 (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(4*b^3*(a^2 - b^2)^2*f*Sqrt[(c +
d*Sin[e + f*x])/(c + d)]) + (3*(b*c - a*d)*(4*a^3*b*c*d^2 + 5*a^4*d^3 + a^2
*b^2*d*(c^2 - 11*d^2) - 2*a*b^3*c*(c^2 + 5*d^2) + b^4*d*(5*c^2 + 8*d^2))*El
lipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d
)])/(4*b^4*(a^2 - b^2)^2*f*Sqrt[c + d*Sin[e + f*x]]) + ((b*c - a*d)^2*(12*a
^3*b*c*d - 36*a*b^3*c*d + 15*a^4*d^2 + 2*a^2*b^2*(4*c^2 - 19*d^2) + b^4*(4*
c^2 + 35*d^2))*EllipticPi[(2*b)/(a + b), (e - Pi/2 + f*x)/2, (2*d)/(c + d)]
*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(4*(a - b)^2*b^4*(a + b)^3*f*Sqrt[c +
d*Sin[e + f*x]])
```


Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
```

```
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
```

```
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^{7/2}}{(a+b\sin(e+fx))^3}\,dx = \frac{(bc-ad)^2\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{2b\left(a^2-b^2\right)f(a+b\sin(e+fx))^2} - \int \frac{\sqrt{c+d\sin(e+fx)}\left(\frac{1}{2}\left(3d(bc-ad)^2+4bc\left(2bcd-a\left(c^2+d^2\right)\right.\right.\right.}{}$$

$$= \frac{(bc-ad)^2\left(6abc+5a^2d-11b^2d\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4b^2\left(a^2-b^2\right)^2f(a+b\sin(e+fx))} + \frac{(bc-ad)^2\cos(e+fx}{2b\left(a^2-b^2\right)f(a}$$

$$= \frac{(bc-ad)^2\left(6abc+5a^2d-11b^2d\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4b^2\left(a^2-b^2\right)^2f(a+b\sin(e+fx))} + \frac{(bc-ad)^2\cos(e+fx}{2b\left(a^2-b^2\right)f(a}$$

$$= \frac{(bc-ad)^2\left(6abc+5a^2d-11b^2d\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4b^2\left(a^2-b^2\right)^2f(a+b\sin(e+fx))} + \frac{(bc-ad)^2\cos(e+fx}{2b\left(a^2-b^2\right)f(a}$$

$$= \frac{(bc-ad)^2\left(6abc+5a^2d-11b^2d\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4b^2\left(a^2-b^2\right)^2f(a+b\sin(e+fx))} + \frac{(bc-ad)^2\cos(e+fx}{2b\left(a^2-b^2\right)f(a}$$

$$= \frac{(bc-ad)^2\left(6abc+5a^2d-11b^2d\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4b^2\left(a^2-b^2\right)^2f(a+b\sin(e+fx))} + \frac{(bc-ad)^2\cos(e+fx}{2b\left(a^2-b^2\right)f(a}$$

Mathematica [C] time = 8.31852, size = 1323, normalized size = 2.19

$$\frac{\sqrt{c+d\sin(e+fx)}\left(\frac{-b^3\cos(e+fx)c^3+3ab^2d\cos(e+fx)c^2-3a^2bd^2\cos(e+fx)c+a^3d^3\cos(e+fx)}{2b^2\left(b^2-a^2\right)(a+b\sin(e+fx))^2}+\frac{7d^3\cos(e+fx)a^4-8bcd^2\cos(e+fx)a^3-13b^2d^3\cos(e}{}\right.}{f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*Sin[e + f*x])^(7/2)/(a + b*Sin[e + f*x])^3,x]
```

```
[Out] (Sqrt[c + d*Sin[e + f*x]]*((-(b^3*c^3*Cos[e + f*x]) + 3*a*b^2*c^2*d*Cos[e +
 f*x] - 3*a^2*b*c*d^2*Cos[e + f*x] + a^3*d^3*Cos[e + f*x])/(2*b^2*(-a^2 + b
^2)*(a + b*Sin[e + f*x])^2) + (6*a*b^3*c^3*Cos[e + f*x] - 5*a^2*b^2*c^2*d*C
os[e + f*x] - 13*b^4*c^2*d*Cos[e + f*x] - 8*a^3*b*c*d^2*Cos[e + f*x] + 26*a
*b^3*c*d^2*Cos[e + f*x] + 7*a^4*d^3*Cos[e + f*x] - 13*a^2*b^2*d^3*Cos[e + f
*x])/(4*b^2*(-a^2 + b^2)^2*(a + b*Sin[e + f*x]))))/f + ((-2*(16*a^2*b^2*c^4
 + 8*b^4*c^4 - 78*a*b^3*c^3*d + 33*a^2*b^2*c^2*d^2 + 57*b^4*c^2*d^2 + 8*a^3
*b*c*d^3 - 50*a*b^3*c*d^3 + 5*a^4*d^4 - 7*a^2*b^2*d^4 + 8*b^4*d^4)*Elliptic
Pi[(2*b)/(a + b), (-e + Pi/2 - f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f
*x])/(c + d)])/((a + b)*Sqrt[c + d*Sin[e + f*x]]) - ((2*I)*(20*a^2*b^2*c^3*
d + 4*b^4*c^3*d - 8*a^3*b*c^2*d^2 - 64*a*b^3*c^2*d^2 + 20*a^4*c*d^3 - 12*a^
2*b^2*c*d^3 + 64*b^4*c*d^3 + 8*a^3*b*d^4 - 32*a*b^3*d^4)*Cos[e + f*x]*((b*c
 - a*d)*EllipticF[I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]],
(c + d)/(c - d)] + a*d*EllipticPi[(b*(c + d))/(b*c - a*d), I*ArcSinh[Sqrt[-
(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)])*Sqrt[(d - d*Sin[
e + f*x])/(c + d)]*Sqrt[-((d + d*Sin[e + f*x])/(c - d))]*(-(b*c) + a*d + b*
(c + d*Sin[e + f*x])))/(b*d^2*Sqrt[-(c + d)^(-1)]*(b*c - a*d)*(a + b*Sin[e
+ f*x])*Sqrt[1 - Sin[e + f*x]^2]*Sqrt[-((c^2 - d^2 - 2*c*(c + d*Sin[e + f*x
]) + (c + d*Sin[e + f*x])^2)/d^2)]) - ((2*I)*(-6*a*b^3*c^3*d + 5*a^2*b^2*c^
2*d^2 + 13*b^4*c^2*d^2 + 8*a^3*b*c*d^3 - 26*a*b^3*c*d^3 - 15*a^4*d^4 + 29*a
^2*b^2*d^4 - 8*b^4*d^4)*Cos[e + f*x]*Cos[2*(e + f*x)]*(2*b*(c - d)*(b*c - a
*d)*EllipticE[I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c +
 d)/(c - d)] + d*(-2*(a + b)*(-(b*c) + a*d)*EllipticF[I*ArcSinh[Sqrt[-(c +
d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)] + (2*a^2 - b^2)*d*Elli
pticPi[(b*(c + d))/(b*c - a*d), I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Si
n[e + f*x]]], (c + d)/(c - d)]))*Sqrt[(d - d*Sin[e + f*x])/(c + d)]*Sqrt[-(
(d + d*Sin[e + f*x])/(c - d))]*(-(b*c) + a*d + b*(c + d*Sin[e + f*x])))/(b^
2*d*Sqrt[-(c + d)^(-1)]*(b*c - a*d)*(a + b*Sin[e + f*x])*Sqrt[1 - Sin[e + f
*x]^2]*(-2*c^2 + d^2 + 4*c*(c + d*Sin[e + f*x]) - 2*(c + d*Sin[e + f*x])^2)
*Sqrt[-((c^2 - d^2 - 2*c*(c + d*Sin[e + f*x]) + (c + d*Sin[e + f*x])^2)/d^2
)]))/(16*(a - b)^2*b^2*(a + b)^2*f)
```

Maple [B] time = 9.81, size = 2237, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(7/2)/(a+b*sin(f*x+e))^3,x)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*(d^3/b^4*(2*b*d*(c/d-1)*((c+d*sin(f
*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d)
)^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*s
in(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c
-d))^(1/2),((c-d)/(c+d))^(1/2)))-6*d*a*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/
2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin
(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c
-d)/(c+d))^(1/2))+8*c*b*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*
x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(
f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2
)))+12/b^5*d^2*(a^2*d^2-2*a*b*c*d+b^2*c^2)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))
^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d
*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(-c/d+a/b)*EllipticPi(((c+d*sin(f*x+e))/
(c-d))^(1/2),(-c/d+1)/(-c/d+a/b),((c-d)/(c+d))^(1/2))+1/b^4*(a^4*d^4-4*a^3*
b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)*(-1/2*b^2/(a^3*d-a^2*b*c-a
```

```
*b^2*d+b^3*c)*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(a+b*sin(f*x+e))^2-3/
4*b^2*(3*a^2*d-2*a*b*c-b^2*d)/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)^2*(-(-d*sin(f*x
+e)-c)*cos(f*x+e)^2)^(1/2)/(a+b*sin(f*x+e))-1/4*d*(7*a^3*d-4*a^2*b*c-a*b^2*
d-2*b^3*c)/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)^2*(c/d-1)*((c+d*sin(f*x+e))/(c-d))
^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d
*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2)
,((c-d)/(c+d))^(1/2))-3/4*b*d*(3*a^2*d-2*a*b*c-b^2*d)/(a^3*d-a^2*b*c-a*b^2*
d+b^3*c)^2*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^
(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/
2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+
EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))+1/4*(15*a^4*
d^2-20*a^3*b*c*d+8*a^2*b^2*c^2-6*a^2*b^2*d^2-4*a*b^3*c*d+4*b^4*c^2+3*b^4*d^
2)/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)^2/b*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)
*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f
*x+e)-c)*cos(f*x+e)^2)^(1/2)/(-c/d+a/b)*EllipticPi(((c+d*sin(f*x+e))/(c-d))
^(1/2),(-c/d+1)/(-c/d+a/b),((c-d)/(c+d))^(1/2)))-4/b^4*d*(a^3*d^3-3*a^2*b*c
*d^2+3*a*b^2*c^2*d-b^3*c^3)*(-b^2/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)*(-(-d*sin(f
*x+e)-c)*cos(f*x+e)^2)^(1/2)/(a+b*sin(f*x+e))-a*d/(a^3*d-a^2*b*c-a*b^2*d+b^
3*c)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*
((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*Ell
ipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))-b*d/(a^3*d-a^2*b
*c-a*b^2*d+b^3*c)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/
(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)
^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^
(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))+(3*a^
2*d-2*a*b*c-b^2*d)/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)/b*(c/d-1)*((c+d*sin(f*x+e)
)/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/
2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(-c/d+a/b)*EllipticPi(((c+d*sin(
f*x+e))/(c-d))^(1/2),(-c/d+1)/(-c/d+a/b),((c-d)/(c+d))^(1/2))))/cos(f*x+e)/
(c+d*sin(f*x+e))^(1/2)/f
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(7/2)/(a+b*sin(f*x+e))^3,x, algorithm="maxima")

[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(7/2)/(a+b*sin(f*x+e))^3,x, algorithm="fricas")

[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(7/2)/(a+b*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}}{\left(b\sin\left(fx+e\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(7/2)/(a+b*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(7/2)/(b*sin(f*x + e) + a)^3, x)
```

3.760 $\int \frac{(c+d\sin(e+fx))^{5/2}}{(a+b\sin(e+fx))^3}\,dx$

Optimal. Leaf size=549

$$\frac{\left(a^2b^2d\left(7c^2-5d^2\right)+4a^3bcd^2+3a^4d^3-2ab^3c\left(3c^2+11d^2\right)+b^4d\left(11c^2+8d^2\right)\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{4b^3f\left(a^2-b^2\right)^2\sqrt{c+d\sin(e+fx)}}+$$

```
[Out] ((b*c - a*d)^2*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(2*b*(a^2 - b^2)*f*(a
 + b*Sin[e + f*x])^2) + (3*(b*c - a*d)*(2*a*b*c + a^2*d - 3*b^2*d)*Cos[e +
f*x]*Sqrt[c + d*Sin[e + f*x]])/(4*b*(a^2 - b^2)^2*f*(a + b*Sin[e + f*x])) +
 (3*(b*c - a*d)*(2*a*b*c + a^2*d - 3*b^2*d)*EllipticE[(e - Pi/2 + f*x)/2, (
2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(4*b^2*(a^2 - b^2)^2*f*Sqrt[(c + d*
Sin[e + f*x])/(c + d)]) + ((4*a^3*b*c*d^2 + 3*a^4*d^3 + a^2*b^2*d*(7*c^2 -
5*d^2) + b^4*d*(11*c^2 + 8*d^2) - 2*a*b^3*c*(3*c^2 + 11*d^2))*EllipticF[(e
- Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(4*b^3*
(a^2 - b^2)^2*f*Sqrt[c + d*Sin[e + f*x]]) + ((b*c - a*d)*(4*a^3*b*c*d - 28*
a*b^3*c*d + 3*a^4*d^2 + 2*a^2*b^2*(4*c^2 - 3*d^2) + b^4*(4*c^2 + 15*d^2))*E
llipticPi[(2*b)/(a + b), (e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin
[e + f*x])/(c + d)])/(4*(a - b)^2*b^3*(a + b)^3*f*Sqrt[c + d*Sin[e + f*x]])
```

Rubi [A] time = 2.07565, antiderivative size = 549, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.37, Rules used = {2792, 3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{\left(a^2b^2d\left(7c^2-5d^2\right)+4a^3bcd^2+3a^4d^3-2ab^3c\left(3c^2+11d^2\right)+b^4d\left(11c^2+8d^2\right)\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{4b^3f\left(a^2-b^2\right)^2\sqrt{c+d\sin(e+fx)}}+$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^(5/2)/(a + b*Sin[e + f*x])^3,x]
```


```
[Out] ((b*c - a*d)^2*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(2*b*(a^2 - b^2)*f*(a
 + b*Sin[e + f*x])^2) + (3*(b*c - a*d)*(2*a*b*c + a^2*d - 3*b^2*d)*Cos[e +
f*x]*Sqrt[c + d*Sin[e + f*x]])/(4*b*(a^2 - b^2)^2*f*(a + b*Sin[e + f*x])) +
 (3*(b*c - a*d)*(2*a*b*c + a^2*d - 3*b^2*d)*EllipticE[(e - Pi/2 + f*x)/2, (
2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(4*b^2*(a^2 - b^2)^2*f*Sqrt[(c + d*
Sin[e + f*x])/(c + d)]) + ((4*a^3*b*c*d^2 + 3*a^4*d^3 + a^2*b^2*d*(7*c^2 -
5*d^2) + b^4*d*(11*c^2 + 8*d^2) - 2*a*b^3*c*(3*c^2 + 11*d^2))*EllipticF[(e
- Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(4*b^3*
(a^2 - b^2)^2*f*Sqrt[c + d*Sin[e + f*x]]) + ((b*c - a*d)*(4*a^3*b*c*d - 28*
a*b^3*c*d + 3*a^4*d^2 + 2*a^2*b^2*(4*c^2 - 3*d^2) + b^4*(4*c^2 + 15*d^2))*E
llipticPi[(2*b)/(a + b), (e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin
[e + f*x])/(c + d)])/(4*(a - b)^2*b^3*(a + b)^3*f*Sqrt[c + d*Sin[e + f*x]])
```


Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
```

2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^{5/2}}{(a+b\sin(e+fx))^3}\,dx = \frac{(bc-ad)^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2b\left(a^2-b^2\right)f(a+b\sin(e+fx))^2} - \int \frac{\frac{1}{2}\left(-4abc^3+9b^2c^2d-6abcd^2+a^2d^3\right)-\left(a^2cd^2+2abd\left(2c^2+\right.\right.}{(a+b\sin}$$

$$= \frac{(bc-ad)^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2b\left(a^2-b^2\right)f(a+b\sin(e+fx))^2} + \frac{3(bc-ad)\left(2abc+a^2d-3b^2d\right)\cos(e+fx)}{4b\left(a^2-b^2\right)^2 f(a+b\sin(e+}$$

$$= \frac{(bc-ad)^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2b\left(a^2-b^2\right)f(a+b\sin(e+fx))^2} + \frac{3(bc-ad)\left(2abc+a^2d-3b^2d\right)\cos(e+fx)}{4b\left(a^2-b^2\right)^2 f(a+b\sin(e+}$$

$$= \frac{(bc-ad)^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2b\left(a^2-b^2\right)f(a+b\sin(e+fx))^2} + \frac{3(bc-ad)\left(2abc+a^2d-3b^2d\right)\cos(e+fx)}{4b\left(a^2-b^2\right)^2 f(a+b\sin(e+}$$

$$= \frac{(bc-ad)^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2b\left(a^2-b^2\right)f(a+b\sin(e+fx))^2} + \frac{3(bc-ad)\left(2abc+a^2d-3b^2d\right)\cos(e+fx)}{4b\left(a^2-b^2\right)^2 f(a+b\sin(e+}$$

$$= \frac{(bc-ad)^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2b\left(a^2-b^2\right)f(a+b\sin(e+fx))^2} + \frac{3(bc-ad)\left(2abc+a^2d-3b^2d\right)\cos(e+fx)}{4b\left(a^2-b^2\right)^2 f(a+b\sin(e+}$$

Mathematica [C] time = 7.99975, size = 1149, normalized size = 2.09

$$\frac{\sqrt{c+d\sin(e+fx)}\left(\frac{-b^2\cos(e+fx)c^2+2abd\cos(e+fx)c-a^2d^2\cos(e+fx)}{2b\left(b^2-a^2\right)(a+b\sin(e+fx))^2} - \frac{3\left(d^2\cos(e+fx)a^3+bcd\cos(e+fx)a^2-2b^2c^2\cos(e+fx)a-3b^2d^2\cos(e+fx)a\right.}{4b\left(b^2-a^2\right)^2(a+b\sin(e+fx))}\right.}{f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*Sin[e + f*x])^(5/2)/(a + b*Sin[e + f*x])^3,x]
```

```
[Out] (Sqrt[c + d*Sin[e + f*x]]*((-(b^2*c^2*Cos[e + f*x]) + 2*a*b*c*d*Cos[e + f*x
] - a^2*d^2*Cos[e + f*x])/(2*b*(-a^2 + b^2)*(a + b*Sin[e + f*x])^2) - (3*(-
2*a*b^2*c^2*Cos[e + f*x] + a^2*b*c*d*Cos[e + f*x] + 3*b^3*c*d*Cos[e + f*x]
+ a^3*d^2*Cos[e + f*x] - 3*a*b^2*d^2*Cos[e + f*x]))/(4*b*(-a^2 + b^2)^2*(a
+ b*Sin[e + f*x]))))/f - ((-2*(-16*a^2*b*c^3 - 8*b^3*c^3 + 54*a*b^2*c^2*d -
 15*a^2*b*c*d^2 - 21*b^3*c*d^2 + a^3*d^3 + 5*a*b^2*d^3)*EllipticPi[(2*b)/(a
 + b), (-e + Pi/2 - f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d
)])/((a + b)*Sqrt[c + d*Sin[e + f*x]]) - ((2*I)*(-20*a^2*b*c^2*d - 4*b^3*c^
2*d + 4*a^3*c*d^2 + 44*a*b^2*c*d^2 - 8*a^2*b*d^3 - 16*b^3*d^3)*Cos[e + f*x]
*((b*c - a*d)*EllipticF[I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*
x]]], (c + d)/(c - d)] + a*d*EllipticPi[(b*(c + d))/(b*c - a*d), I*ArcSinh[
Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)])*Sqrt[(d -
d*Sin[e + f*x])/(c + d)]*Sqrt[-((d + d*Sin[e + f*x])/(c - d))]*(-(b*c) + a*
d + b*(c + d*Sin[e + f*x])))/(b*d^2*Sqrt[-(c + d)^(-1)]*(b*c - a*d)*(a + b*
Sin[e + f*x])*Sqrt[1 - Sin[e + f*x]^2]*Sqrt[-((c^2 - d^2 - 2*c*(c + d*Sin[e
 + f*x]) + (c + d*Sin[e + f*x])^2)/d^2)]) - ((2*I)*(6*a*b^2*c^2*d - 3*a^2*b
*c*d^2 - 9*b^3*c*d^2 - 3*a^3*d^3 + 9*a*b^2*d^3)*Cos[e + f*x]*Cos[2*(e + f*x
)]*(2*b*(c - d)*(b*c - a*d)*EllipticE[I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c
+ d*Sin[e + f*x]]], (c + d)/(c - d)] + d*(-2*(a + b)*(-(b*c) + a*d)*Ellipti
cF[I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)
] + (2*a^2 - b^2)*d*EllipticPi[(b*(c + d))/(b*c - a*d), I*ArcSinh[Sqrt[-(c
+ d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)]))*Sqrt[(d - d*Sin[e
+ f*x])/(c + d)]*Sqrt[-((d + d*Sin[e + f*x])/(c - d))]*(-(b*c) + a*d + b*(c
 + d*Sin[e + f*x])))/(b^2*d*Sqrt[-(c + d)^(-1)]*(b*c - a*d)*(a + b*Sin[e +
f*x])*Sqrt[1 - Sin[e + f*x]^2]*(-2*c^2 + d^2 + 4*c*(c + d*Sin[e + f*x]) - 2
*(c + d*Sin[e + f*x])^2)*Sqrt[-((c^2 - d^2 - 2*c*(c + d*Sin[e + f*x]) + (c
+ d*Sin[e + f*x])^2)/d^2)]))/(16*(a - b)^2*b*(a + b)^2*f)
```

Maple [B] time = 8.871, size = 1888, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e))^3,x)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*(2*d^3/b^3*(c/d-1)*((c+d*sin(f*x+e)
)/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/
2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d
))^(1/2),((c-d)/(c+d))^(1/2))-6/b^4*d^2*(a*d-b*c)*(c/d-1)*((c+d*sin(f*x+e))
/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2
)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(-c/d+a/b)*EllipticPi(((c+d*sin(f
*x+e))/(c-d))^(1/2),(-c/d+1)/(-c/d+a/b),((c-d)/(c+d))^(1/2))+1/b^3*(-a^3*d^
3+3*a^2*b*c*d^2-3*a*b^2*c^2*d+b^3*c^3)*(-1/2*b^2/(a^3*d-a^2*b*c-a*b^2*d+b^3
*c)*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(a+b*sin(f*x+e))^2-3/4*b^2*(3*a
^2*d-2*a*b*c-b^2*d)/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)^2*(-(-d*sin(f*x+e)-c)*cos
(f*x+e)^2)^(1/2)/(a+b*sin(f*x+e))-1/4*d*(7*a^3*d-4*a^2*b*c-a*b^2*d-2*b^3*c)
/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)^2*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*
(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e
)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c
+d))^(1/2))-3/4*b*d*(3*a^2*d-2*a*b*c-b^2*d)/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)^2
*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-s
in(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-
1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(
((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))+1/4*(15*a^4*d^2-20*a^3
*b*c*d+8*a^2*b^2*c^2-6*a^2*b^2*d^2-4*a*b^3*c*d+4*b^4*c^2+3*b^4*d^2)/(a^3*d-
a^2*b*c-a*b^2*d+b^3*c)^2/b*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin
```

```
(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*c
os(f*x+e)^2)^(1/2)/(-c/d+a/b)*EllipticPi(((c+d*sin(f*x+e))/(c-d))^(1/2),(-c
/d+1)/(-c/d+a/b),((c-d)/(c+d))^(1/2)))+3/b^3*d*(a^2*d^2-2*a*b*c*d+b^2*c^2)*
(-b^2/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)
/(a+b*sin(f*x+e))-a*d/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)*(c/d-1)*((c+d*sin(f*x+e
))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1
/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-
d))^(1/2),((c-d)/(c+d))^(1/2))-b*d/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)*(c/d-1)*((
c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1
)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*Elliptic
E(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f
*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))+(3*a^2*d-2*a*b*c-b^2*d)/(a^3*d-a^
2*b*c-a*b^2*d+b^3*c)/b*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x
+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f
*x+e)^2)^(1/2)/(-c/d+a/b)*EllipticPi(((c+d*sin(f*x+e))/(c-d))^(1/2),(-c/d+1
)/(-c/d+a/b),((c-d)/(c+d))^(1/2))))/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e))^3,x, algorithm="maxima")

[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e))^3,x, algorithm="fricas")

[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(5/2)/(a+b*sin(f*x+e))**3,x)

[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d \sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}{\left(b \sin\left(fx+e\right)+a\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(5/2)/(b*sin(f*x + e) + a)^3, x)
```

3.761 $\int \frac{(c+d\sin(e+fx))^{3/2}}{(a+b\sin(e+fx))^3}\,dx$

Optimal. Leaf size=472

$$-\frac{\left(-2a^2b^2\left(4c^2+5d^2\right)+4a^3bcd+a^4d^2+20ab^3cd-b^4\left(4c^2+3d^2\right)\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\Pi\left(\frac{2b}{a+b};\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{4b^2f(a-b)^2(a+b)^3\sqrt{c+d\sin(e+fx)}}+\frac{(a^2(-}{}$$

```
[Out] ((b*c - a*d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(2*(a^2 - b^2)*f*(a + b
*Sin[e + f*x])^2) + ((6*a*b*c - a^2*d - 5*b^2*d)*Cos[e + f*x]*Sqrt[c + d*Si
n[e + f*x]])/(4*(a^2 - b^2)^2*f*(a + b*Sin[e + f*x])) + ((6*a*b*c - a^2*d -
 5*b^2*d)*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f
*x]])/(4*b*(a^2 - b^2)^2*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) - ((b*c - a*
d)*(6*a*b*c + a^2*d - 7*b^2*d)*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]
*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(4*b^2*(a^2 - b^2)^2*f*Sqrt[c + d*Sin[
e + f*x]]) - ((4*a^3*b*c*d + 20*a*b^3*c*d + a^4*d^2 - b^4*(4*c^2 + 3*d^2) -
 2*a^2*b^2*(4*c^2 + 5*d^2))*EllipticPi[(2*b)/(a + b), (e - Pi/2 + f*x)/2, (
2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(4*(a - b)^2*b^2*(a + b)^
3*f*Sqrt[c + d*Sin[e + f*x]])
```

Rubi [A] time = 1.80938, antiderivative size = 472, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.37, Rules used = {2799, 3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$-\frac{\left(-2a^2b^2\left(4c^2+5d^2\right)+4a^3bcd+a^4d^2+20ab^3cd-b^4\left(4c^2+3d^2\right)\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\Pi\left(\frac{2b}{a+b};\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{4b^2f(a-b)^2(a+b)^3\sqrt{c+d\sin(e+fx)}}+\frac{(a^2(-}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^(3/2)/(a + b*Sin[e + f*x])^3,x]
```


```
[Out] ((b*c - a*d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(2*(a^2 - b^2)*f*(a + b
*Sin[e + f*x])^2) + ((6*a*b*c - a^2*d - 5*b^2*d)*Cos[e + f*x]*Sqrt[c + d*Si
n[e + f*x]])/(4*(a^2 - b^2)^2*f*(a + b*Sin[e + f*x])) + ((6*a*b*c - a^2*d -
 5*b^2*d)*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f
*x]])/(4*b*(a^2 - b^2)^2*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) - ((b*c - a*
d)*(6*a*b*c + a^2*d - 7*b^2*d)*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]
*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(4*b^2*(a^2 - b^2)^2*f*Sqrt[c + d*Sin[
e + f*x]]) - ((4*a^3*b*c*d + 20*a*b^3*c*d + a^4*d^2 - b^4*(4*c^2 + 3*d^2) -
 2*a^2*b^2*(4*c^2 + 5*d^2))*EllipticPi[(2*b)/(a + b), (e - Pi/2 + f*x)/2, (
2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(4*(a - b)^2*b^2*(a + b)^
3*f*Sqrt[c + d*Sin[e + f*x]])
```


Rule 2799

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin
[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1))/(f*(m + 1)*(a^2 - b^2)), x
] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*S
in[e + f*x])^(n - 2)*Simp[c*(a*c - b*d)*(m + 1) + d*(b*c - a*d)*(n - 1) + (
d*(a*c - b*d)*(m + 1) - c*(b*c - a*d)*(m + 2))*Sin[e + f*x] - d*(b*c - a*d)
*(m + n + 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& LtQ[1, n, 2] && IntegersQ[2*m, 2*n]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^{3/2}}{(a+b\sin(e+fx))^3}\,dx = \frac{(bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2\left(a^2-b^2\right)f(a+b\sin(e+fx))^2} - \frac{\int \frac{\frac{1}{2}\left(5bcd-a\left(4c^2+d^2\right)\right)-\left(3acd-b\left(c^2+2d^2\right)\right)\sin(e+fx)+\frac{1}{2}d(\ldots}{(a+b\sin(e+fx))^2\sqrt{c+d\sin(e+fx)}}}{2\left(a^2-b^2\right)}$$

$$= \frac{(bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2\left(a^2-b^2\right)f(a+b\sin(e+fx))^2} + \frac{\left(6abc-a^2d-5b^2d\right)\cos(e+fx)\sqrt{c+d\sin(e\ldots}}{4\left(a^2-b^2\right)^2 f(a+b\sin(e+fx))}$$

$$= \frac{(bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2\left(a^2-b^2\right)f(a+b\sin(e+fx))^2} + \frac{\left(6abc-a^2d-5b^2d\right)\cos(e+fx)\sqrt{c+d\sin(e\ldots}}{4\left(a^2-b^2\right)^2 f(a+b\sin(e+fx))}$$

$$= \frac{(bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2\left(a^2-b^2\right)f(a+b\sin(e+fx))^2} + \frac{\left(6abc-a^2d-5b^2d\right)\cos(e+fx)\sqrt{c+d\sin(e\ldots}}{4\left(a^2-b^2\right)^2 f(a+b\sin(e+fx))}$$

$$= \frac{(bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2\left(a^2-b^2\right)f(a+b\sin(e+fx))^2} + \frac{\left(6abc-a^2d-5b^2d\right)\cos(e+fx)\sqrt{c+d\sin(e\ldots}}{4\left(a^2-b^2\right)^2 f(a+b\sin(e+fx))}$$

$$= \frac{(bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2\left(a^2-b^2\right)f(a+b\sin(e+fx))^2} + \frac{\left(6abc-a^2d-5b^2d\right)\cos(e+fx)\sqrt{c+d\sin(e\ldots}}{4\left(a^2-b^2\right)^2 f(a+b\sin(e+fx))}$$

Mathematica [C] time = 7.51627, size = 1001, normalized size = 2.12

$$\frac{\sqrt{c+d\sin(e+fx)}\left(\frac{bc\cos(e+fx)-ad\cos(e+fx)}{2\left(a^2-b^2\right)(a+b\sin(e+fx))^2}+\frac{-d\cos(e+fx)a^2+6bc\cos(e+fx)a-5b^2d\cos(e+fx)}{4\left(a^2-b^2\right)^2(a+b\sin(e+fx))}\right)}{f} + \frac{-\frac{2\left(16a^2c^2+8b^2c^2-30abdc+5a^2d^2+b^2d^2\right)\ldots}{(a+b)\sqrt{c+\ldots}}}{\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*Sin[e + f*x])^(3/2)/(a + b*Sin[e + f*x])^3,x]
```

```
[Out] (Sqrt[c + d*Sin[e + f*x]]*((b*c*Cos[e + f*x] - a*d*Cos[e + f*x])/(2*(a^2 -
b^2)*(a + b*Sin[e + f*x])^2) + (6*a*b*c*Cos[e + f*x] - a^2*d*Cos[e + f*x] -
 5*b^2*d*Cos[e + f*x])/(4*(a^2 - b^2)^2*(a + b*Sin[e + f*x]))))/f + ((-2*(1
6*a^2*c^2 + 8*b^2*c^2 - 30*a*b*c*d + 5*a^2*d^2 + b^2*d^2)*EllipticPi[(2*b)/
```

```
(a + b), (-e + Pi/2 - f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c +
 d)])/((a + b)*Sqrt[c + d*Sin[e + f*x]]) - ((2*I)*(20*a^2*c*d + 4*b^2*c*d -
 24*a*b*d^2)*Cos[e + f*x]*((b*c - a*d)*EllipticF[I*ArcSinh[Sqrt[-(c + d)^(-
1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)] + a*d*EllipticPi[(b*(c + d)
)/(b*c - a*d), I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c
+ d)/(c - d)])*Sqrt[(d - d*Sin[e + f*x])/(c + d)]*Sqrt[-((d + d*Sin[e + f*x
])/(c - d))]*(-(b*c) + a*d + b*(c + d*Sin[e + f*x])))/(b*d^2*Sqrt[-(c + d)^
(-1)]*(b*c - a*d)*(a + b*Sin[e + f*x])*Sqrt[1 - Sin[e + f*x]^2]*Sqrt[-((c^2
 - d^2 - 2*c*(c + d*Sin[e + f*x]) + (c + d*Sin[e + f*x])^2)/d^2)]) - ((2*I)
*(-6*a*b*c*d + a^2*d^2 + 5*b^2*d^2)*Cos[e + f*x]*Cos[2*(e + f*x)]*(2*b*(c -
 d)*(b*c - a*d)*EllipticE[I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e +
f*x]]], (c + d)/(c - d)] + d*(-2*(a + b)*(-(b*c) + a*d)*EllipticF[I*ArcSinh
[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)] + (2*a^2 -
 b^2)*d*EllipticPi[(b*(c + d))/(b*c - a*d), I*ArcSinh[Sqrt[-(c + d)^(-1)]*S
qrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)]))*Sqrt[(d - d*Sin[e + f*x])/(c +
 d)]*Sqrt[-((d + d*Sin[e + f*x])/(c - d))]*(-(b*c) + a*d + b*(c + d*Sin[e +
 f*x])))/(b^2*d*Sqrt[-(c + d)^(-1)]*(b*c - a*d)*(a + b*Sin[e + f*x])*Sqrt[1
 - Sin[e + f*x]^2]*(-2*c^2 + d^2 + 4*c*(c + d*Sin[e + f*x]) - 2*(c + d*Sin[
e + f*x])^2)*Sqrt[-((c^2 - d^2 - 2*c*(c + d*Sin[e + f*x]) + (c + d*Sin[e +
f*x])^2)/d^2)]))/(16*(a - b)^2*(a + b)^2*f)
```

Maple [B] time = 8.563, size = 1718, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))^3,x)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*(2*d^2/b^3*(c/d-1)*((c+d*sin(f*x+e)
)/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/
2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(-c/d+a/b)*EllipticPi(((c+d*sin(
f*x+e))/(c-d))^(1/2),(-c/d+1)/(-c/d+a/b),((c-d)/(c+d))^(1/2))+(a^2*d^2-2*a*
b*c*d+b^2*c^2)/b^2*(-1/2*b^2/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)*(-(-d*sin(f*x+e)
-c)*cos(f*x+e)^2)^(1/2)/(a+b*sin(f*x+e))^2-3/4*b^2*(3*a^2*d-2*a*b*c-b^2*d)/
(a^3*d-a^2*b*c-a*b^2*d+b^3*c)^2*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(a+
b*sin(f*x+e))-1/4*d*(7*a^3*d-4*a^2*b*c-a*b^2*d-2*b^3*c)/(a^3*d-a^2*b*c-a*b^
2*d+b^3*c)^2*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d)
)^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(
1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))-3/4*b*d*
(3*a^2*d-2*a*b*c-b^2*d)/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)^2*(c/d-1)*((c+d*sin(f
*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d)
)^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*s
in(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c
-d))^(1/2),((c-d)/(c+d))^(1/2)))+1/4*(15*a^4*d^2-20*a^3*b*c*d+8*a^2*b^2*c^2
-6*a^2*b^2*d^2-4*a*b^3*c*d+4*b^4*c^2+3*b^4*d^2)/(a^3*d-a^2*b*c-a*b^2*d+b^3*
c)^2/b*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2
)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(
-c/d+a/b)*EllipticPi(((c+d*sin(f*x+e))/(c-d))^(1/2),(-c/d+1)/(-c/d+a/b),((c
-d)/(c+d))^(1/2)))-2*d*(a*d-b*c)/b^2*(-b^2/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)*(-
(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(a+b*sin(f*x+e))-a*d/(a^3*d-a^2*b*c-a
*b^2*d+b^3*c)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d
))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^
(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))-b*d/(a^
3*d-a^2*b*c-a*b^2*d+b^3*c)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin
(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*c
os(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d
```

```
)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2
)))+(3*a^2*d-2*a*b*c-b^2*d)/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)/b*(c/d-1)*((c+d*s
in(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(
c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(-c/d+a/b)*EllipticPi((
(c+d*sin(f*x+e))/(c-d))^(1/2),(-c/d+1)/(-c/d+a/b),((c-d)/(c+d))^(1/2))))/co
s(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(3/2)/(a+b*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d \sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}{\left(b \sin\left(fx+e\right)+a\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(3/2)/(b*sin(f*x + e) + a)^3, x)
```

3.762 $\int \frac{\sqrt{c+d\sin(e+fx)}}{(a+b\sin(e+fx))^3}\,dx$

Optimal. Leaf size=487

$$-\frac{\left(-2a^2b^2\left(4c^2+5d^2\right)+12a^3bcd-3a^4d^2+12ab^3cd-b^4\left(4c^2-d^2\right)\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{4bf(a-b)^2(a+b)^3(bc-ad)\sqrt{c+d\sin(e+fx)}}+\frac{b\left(-5a^2\right.}{4}$$

```
[Out] (b*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(2*(a^2 - b^2)*f*(a + b*Sin[e + f
*x])^2) + (b*(6*a*b*c - 5*a^2*d - b^2*d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*
x]])/(4*(a^2 - b^2)^2*(b*c - a*d)*f*(a + b*Sin[e + f*x])) + ((6*a*b*c - 5*a
^2*d - b^2*d)*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e
 + f*x]])/(4*(a^2 - b^2)^2*(b*c - a*d)*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]
) - (3*(2*a*b*c - a^2*d - b^2*d)*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d
)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(4*b*(a^2 - b^2)^2*f*Sqrt[c + d*Sin[
e + f*x]]) - ((12*a^3*b*c*d + 12*a*b^3*c*d - 3*a^4*d^2 - b^4*(4*c^2 - d^2)
- 2*a^2*b^2*(4*c^2 + 5*d^2))*EllipticPi[(2*b)/(a + b), (e - Pi/2 + f*x)/2,
(2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(4*(a - b)^2*b*(a + b)^3
*(b*c - a*d)*f*Sqrt[c + d*Sin[e + f*x]])
```

Rubi [A] time = 1.60472, antiderivative size = 487, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.37, Rules used = {2796, 3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$-\frac{\left(-2a^2b^2\left(4c^2+5d^2\right)+12a^3bcd-3a^4d^2+12ab^3cd-b^4\left(4c^2-d^2\right)\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{4bf(a-b)^2(a+b)^3(bc-ad)\sqrt{c+d\sin(e+fx)}}+\frac{b\left(-5a^2\right.}{4}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*Sin[e + f*x]]/(a + b*Sin[e + f*x])^3,x]
```

```
[Out] (b*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(2*(a^2 - b^2)*f*(a + b*Sin[e + f
*x])^2) + (b*(6*a*b*c - 5*a^2*d - b^2*d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*
x]])/(4*(a^2 - b^2)^2*(b*c - a*d)*f*(a + b*Sin[e + f*x])) + ((6*a*b*c - 5*a
^2*d - b^2*d)*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e
 + f*x]])/(4*(a^2 - b^2)^2*(b*c - a*d)*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]
) - (3*(2*a*b*c - a^2*d - b^2*d)*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d
)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(4*b*(a^2 - b^2)^2*f*Sqrt[c + d*Sin[
e + f*x]]) - ((12*a^3*b*c*d + 12*a*b^3*c*d - 3*a^4*d^2 - b^4*(4*c^2 - d^2)
- 2*a^2*b^2*(4*c^2 + 5*d^2))*EllipticPi[(2*b)/(a + b), (e - Pi/2 + f*x)/2,
(2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(4*(a - b)^2*b*(a + b)^3
*(b*c - a*d)*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2796

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m + 1)*(c + d*Sin[e + f*x])^n)/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m +
 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n
- 1)*Simp[a*c*(m + 1) + b*d*n + (a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] -
b*d*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1
] && LtQ[0, n, 1] && IntegersQ[2*m, 2*n]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\sqrt{c+d\sin(e+fx)}}{(a+b\sin(e+fx))^3}\,dx = \frac{b\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2\left(a^2-b^2\right)f(a+b\sin(e+fx))^2} - \frac{\int \frac{\frac{1}{2}(-4ac+bd)+(bc-2ad)\sin(e+fx)+\frac{1}{2}bd\sin^2(e+fx)}{(a+b\sin(e+fx))^2\sqrt{c+d\sin(e+fx)}}\,dx}{2\left(a^2-b^2\right)}$$

$$= \frac{b\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2\left(a^2-b^2\right)f(a+b\sin(e+fx))^2} + \frac{b\left(6abc-5a^2d-b^2d\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4\left(a^2-b^2\right)^2(bc-ad)f(a+b\sin(e+fx))} + \cdots$$

$$= \frac{b\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2\left(a^2-b^2\right)f(a+b\sin(e+fx))^2} + \frac{b\left(6abc-5a^2d-b^2d\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4\left(a^2-b^2\right)^2(bc-ad)f(a+b\sin(e+fx))} - \cdots$$

$$= \frac{b\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2\left(a^2-b^2\right)f(a+b\sin(e+fx))^2} + \frac{b\left(6abc-5a^2d-b^2d\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4\left(a^2-b^2\right)^2(bc-ad)f(a+b\sin(e+fx))} - \cdots$$

$$= \frac{b\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2\left(a^2-b^2\right)f(a+b\sin(e+fx))^2} + \frac{b\left(6abc-5a^2d-b^2d\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4\left(a^2-b^2\right)^2(bc-ad)f(a+b\sin(e+fx))} + \cdots$$

$$= \frac{b\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2\left(a^2-b^2\right)f(a+b\sin(e+fx))^2} + \frac{b\left(6abc-5a^2d-b^2d\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4\left(a^2-b^2\right)^2(bc-ad)f(a+b\sin(e+fx))} + \cdots$$

Mathematica [C] time = 7.83568, size = 1038, normalized size = 2.13

$$\frac{\sqrt{c+d\sin(e+fx)}\left(\frac{b\cos(e+fx)}{2\left(a^2-b^2\right)(a+b\sin(e+fx))^2} - \frac{-d\cos(e+fx)b^3+6ac\cos(e+fx)b^2-5a^2d\cos(e+fx)b}{4\left(a^2-b^2\right)^2(ad-bc)(a+b\sin(e+fx))}\right)}{f} + \frac{-\frac{2\left(16cda^3-16bc^2a^2-9bd^2a^2+14b^2cda-8b^3c^2\cdots\right.}{(a+b)\sqrt{\cdots}}}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c + d*Sin[e + f*x]]/(a + b*Sin[e + f*x])^3,x]
```

```
[Out] (Sqrt[c + d*Sin[e + f*x]]*((b*Cos[e + f*x])/(2*(a^2 - b^2)*(a + b*Sin[e + f
*x])^2) - (6*a*b^2*c*Cos[e + f*x] - 5*a^2*b*d*Cos[e + f*x] - b^3*d*Cos[e +
f*x])/(4*(a^2 - b^2)^2*(-(b*c) + a*d)*(a + b*Sin[e + f*x]))))/f + ((-2*(-16
*a^2*b*c^2 - 8*b^3*c^2 + 16*a^3*c*d + 14*a*b^2*c*d - 9*a^2*b*d^2 + 3*b^3*d^
```

```
2)*EllipticPi[(2*b)/(a + b), (-e + Pi/2 - f*x)/2, (2*d)/(c + d)]*Sqrt[(c +
d*Sin[e + f*x])/(c + d)])/((a + b)*Sqrt[c + d*Sin[e + f*x]]) - ((2*I)*(-20*
a^2*b*c*d - 4*b^3*c*d + 16*a^3*d^2 + 8*a*b^2*d^2)*Cos[e + f*x]*((b*c - a*d)
*EllipticF[I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)
/(c - d)] + a*d*EllipticPi[(b*(c + d))/(b*c - a*d), I*ArcSinh[Sqrt[-(c + d)
^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)])*Sqrt[(d - d*Sin[e + f*x
])/(c + d)]*Sqrt[-((d + d*Sin[e + f*x])/(c - d))]*(-(b*c) + a*d + b*(c + d*
Sin[e + f*x])))/(b*d^2*Sqrt[-(c + d)^(-1)]*(b*c - a*d)*(a + b*Sin[e + f*x])
*Sqrt[1 - Sin[e + f*x]^2]*Sqrt[-((c^2 - d^2 - 2*c*(c + d*Sin[e + f*x]) + (c
 + d*Sin[e + f*x])^2)/d^2)]) - ((2*I)*(6*a*b^2*c*d - 5*a^2*b*d^2 - b^3*d^2)
*Cos[e + f*x]*Cos[2*(e + f*x)]*(2*b*(c - d)*(b*c - a*d)*EllipticE[I*ArcSinh
[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)] + d*(-2*(a
 + b)*(-(b*c) + a*d)*EllipticF[I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin
[e + f*x]]], (c + d)/(c - d)] + (2*a^2 - b^2)*d*EllipticPi[(b*(c + d))/(b*c
 - a*d), I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(
c - d)]))*Sqrt[(d - d*Sin[e + f*x])/(c + d)]*Sqrt[-((d + d*Sin[e + f*x])/(c
 - d))]*(-(b*c) + a*d + b*(c + d*Sin[e + f*x])))/(b^2*d*Sqrt[-(c + d)^(-1)]
*(b*c - a*d)*(a + b*Sin[e + f*x])*Sqrt[1 - Sin[e + f*x]^2]*(-2*c^2 + d^2 +
4*c*(c + d*Sin[e + f*x]) - 2*(c + d*Sin[e + f*x])^2)*Sqrt[-((c^2 - d^2 - 2*
c*(c + d*Sin[e + f*x]) + (c + d*Sin[e + f*x])^2)/d^2)]))/(16*(a - b)^2*(a +
 b)^2*(-(b*c) + a*d)*f)
```

Maple [B] time = 8.669, size = 1525, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^3,x)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-a*d+b*c)/b*(-1/2*b^2/(a^3*d-a^2*
b*c-a*b^2*d+b^3*c)*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(a+b*sin(f*x+e))
^2-3/4*b^2*(3*a^2*d-2*a*b*c-b^2*d)/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)^2*(-(-d*si
n(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(a+b*sin(f*x+e))-1/4*d*(7*a^3*d-4*a^2*b*c-a
*b^2*d-2*b^3*c)/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)^2*(c/d-1)*((c+d*sin(f*x+e))/(
c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/
(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^
(1/2),((c-d)/(c+d))^(1/2))-3/4*b*d*(3*a^2*d-2*a*b*c-b^2*d)/(a^3*d-a^2*b*c-a
*b^2*d+b^3*c)^2*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c
+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2
)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1
/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))+1/4*(15
*a^4*d^2-20*a^3*b*c*d+8*a^2*b^2*c^2-6*a^2*b^2*d^2-4*a*b^3*c*d+4*b^4*c^2+3*b
^4*d^2)/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)^2/b*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^
(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*
sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(-c/d+a/b)*EllipticPi(((c+d*sin(f*x+e))/(
c-d))^(1/2),(-c/d+1)/(-c/d+a/b),((c-d)/(c+d))^(1/2)))+d/b*(-b^2/(a^3*d-a^2*
b*c-a*b^2*d+b^3*c)*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(a+b*sin(f*x+e))
-a*d/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(
d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x
+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/
(c+d))^(1/2))-b*d/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)*(c/d-1)*((c+d*sin(f*x+e))/(
c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/
(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e
))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/
2),((c-d)/(c+d))^(1/2)))+(3*a^2*d-2*a*b*c-b^2*d)/(a^3*d-a^2*b*c-a*b^2*d+b^3
*c)/b*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)
```

```
*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(-
c/d+a/b)*EllipticPi(((c+d*sin(f*x+e))/(c-d))^(1/2),(-c/d+1)/(-c/d+a/b),((c-
d)/(c+d))^(1/2))))/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(1/2)/(a+b*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d\sin\left(fx+e\right)+c}}{\left(b\sin\left(fx+e\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*sin(f*x + e) + c)/(b*sin(f*x + e) + a)^3, x)
```

3.763 $\int \frac{1}{(a+b\sin(e+fx))^3\sqrt{c+d\sin(e+fx)}}\,dx$

Optimal. Leaf size=503

$$-\frac{\left(-2a^2b^2\left(4c^2-3d^2\right)+20a^3bcd-15a^4d^2+4ab^3cd-b^4\left(4c^2+3d^2\right)\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{4f(a-b)^2(a+b)^3(bc-ad)^2\sqrt{c+d\sin(e+fx)}}+3b^2$$

```
[Out] (b^2*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(2*(a^2 - b^2)*(b*c - a*d)*f*(a
 + b*Sin[e + f*x])^2) + (3*b^2*(2*a*b*c - 3*a^2*d + b^2*d)*Cos[e + f*x]*Sqr
t[c + d*Sin[e + f*x]])/(4*(a^2 - b^2)^2*(b*c - a*d)^2*f*(a + b*Sin[e + f*x]
)) + (3*b*(2*a*b*c - 3*a^2*d + b^2*d)*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(
c + d)]*Sqrt[c + d*Sin[e + f*x]])/(4*(a^2 - b^2)^2*(b*c - a*d)^2*f*Sqrt[(c
+ d*Sin[e + f*x])/(c + d)]) - ((6*a*b*c - 7*a^2*d + b^2*d)*EllipticF[(e - P
i/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(4*(a^2 -
b^2)^2*(b*c - a*d)*f*Sqrt[c + d*Sin[e + f*x]]) - ((20*a^3*b*c*d + 4*a*b^3*c
*d - 15*a^4*d^2 - 2*a^2*b^2*(4*c^2 - 3*d^2) - b^4*(4*c^2 + 3*d^2))*Elliptic
Pi[(2*b)/(a + b), (e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*
x])/(c + d)])/(4*(a - b)^2*(a + b)^3*(b*c - a*d)^2*f*Sqrt[c + d*Sin[e + f*x
]])
```

Rubi [A] time = 1.67523, antiderivative size = 503, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.37, Rules used = {2802, 3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$-\frac{\left(-2a^2b^2\left(4c^2-3d^2\right)+20a^3bcd-15a^4d^2+4ab^3cd-b^4\left(4c^2+3d^2\right)\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{4f(a-b)^2(a+b)^3(bc-ad)^2\sqrt{c+d\sin(e+fx)}}+3b^2$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sin[e + f*x])^3*Sqrt[c + d*Sin[e + f*x]]),x]
```

```
[Out] (b^2*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(2*(a^2 - b^2)*(b*c - a*d)*f*(a
 + b*Sin[e + f*x])^2) + (3*b^2*(2*a*b*c - 3*a^2*d + b^2*d)*Cos[e + f*x]*Sqr
t[c + d*Sin[e + f*x]])/(4*(a^2 - b^2)^2*(b*c - a*d)^2*f*(a + b*Sin[e + f*x]
)) + (3*b*(2*a*b*c - 3*a^2*d + b^2*d)*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(
c + d)]*Sqrt[c + d*Sin[e + f*x]])/(4*(a^2 - b^2)^2*(b*c - a*d)^2*f*Sqrt[(c
+ d*Sin[e + f*x])/(c + d)]) - ((6*a*b*c - 7*a^2*d + b^2*d)*EllipticF[(e - P
i/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(4*(a^2 -
b^2)^2*(b*c - a*d)*f*Sqrt[c + d*Sin[e + f*x]]) - ((20*a^3*b*c*d + 4*a*b^3*c
*d - 15*a^4*d^2 - 2*a^2*b^2*(4*c^2 - 3*d^2) - b^4*(4*c^2 + 3*d^2))*Elliptic
Pi[(2*b)/(a + b), (e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*
x])/(c + d)])/(4*(a - b)^2*(a + b)^3*(b*c - a*d)^2*f*Sqrt[c + d*Sin[e + f*x
]])
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
```

```
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\sin(e+fx))^3\sqrt{c+d\sin(e+fx)}}dx = \frac{b^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2\left(a^2-b^2\right)(bc-ad)f(a+b\sin(e+fx))^2} - \frac{\int \frac{\frac{1}{2}\left(-4abc+4a^2d-3b^2d\right)+b(bc-2a}{(a+b\sin(e+fx))^2}}{2\left(a^2-b^2\right.}$$

$$= \frac{b^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2\left(a^2-b^2\right)(bc-ad)f(a+b\sin(e+fx))^2} + \frac{3b^2\left(2abc-3a^2d+b^2d\right)\mathrm{c}}{4\left(a^2-b^2\right)^2(bc-ad}$$

$$= \frac{b^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2\left(a^2-b^2\right)(bc-ad)f(a+b\sin(e+fx))^2} + \frac{3b^2\left(2abc-3a^2d+b^2d\right)\mathrm{c}}{4\left(a^2-b^2\right)^2(bc-ad}$$

$$= \frac{b^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2\left(a^2-b^2\right)(bc-ad)f(a+b\sin(e+fx))^2} + \frac{3b^2\left(2abc-3a^2d+b^2d\right)\mathrm{c}}{4\left(a^2-b^2\right)^2(bc-ad}$$

$$= \frac{b^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2\left(a^2-b^2\right)(bc-ad)f(a+b\sin(e+fx))^2} + \frac{3b^2\left(2abc-3a^2d+b^2d\right)\mathrm{c}}{4\left(a^2-b^2\right)^2(bc-ad}$$

$$= \frac{b^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2\left(a^2-b^2\right)(bc-ad)f(a+b\sin(e+fx))^2} + \frac{3b^2\left(2abc-3a^2d+b^2d\right)\mathrm{c}}{4\left(a^2-b^2\right)^2(bc-ad}$$

Mathematica [C] time = 7.69447, size = 1069, normalized size = 2.13

$$\frac{\sqrt{c+d\sin(e+fx)}\left(\frac{3\left(d\cos(e+fx)b^4+2ac\cos(e+fx)b^3-3a^2d\cos(e+fx)b^2\right)}{4\left(a^2-b^2\right)^2(ad-bc)^2(a+b\sin(e+fx))}-\frac{b^2\cos(e+fx)}{2\left(a^2-b^2\right)(ad-bc)(a+b\sin(e+fx))^2}\right)}{f} + \frac{-\frac{2\left(16d^2a^4-32bcda^3+16b^2c^2a\right.}{}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*Sin[e + f*x])^3*Sqrt[c + d*Sin[e + f*x]]),x]
```

```
[Out] (Sqrt[c + d*Sin[e + f*x]]*(-(b^2*Cos[e + f*x])/(2*(a^2 - b^2)*(-(b*c) + a*d
)*(a + b*Sin[e + f*x])^2) + (3*(2*a*b^3*c*Cos[e + f*x] - 3*a^2*b^2*d*Cos[e
+ f*x] + b^4*d*Cos[e + f*x]))/(4*(a^2 - b^2)^2*(-(b*c) + a*d)^2*(a + b*Sin[
e + f*x]))))/f + ((-2*(16*a^2*b^2*c^2 + 8*b^4*c^2 - 32*a^3*b*c*d + 2*a*b^3*
c*d + 16*a^4*d^2 - 19*a^2*b^2*d^2 + 9*b^4*d^2)*EllipticPi[(2*b)/(a + b), (-
e + Pi/2 - f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/((a +
 b)*Sqrt[c + d*Sin[e + f*x]]) - ((2*I)*(20*a^2*b^2*c*d + 4*b^4*c*d - 32*a^3
*b*d^2 + 8*a*b^3*d^2)*Cos[e + f*x]*((b*c - a*d)*EllipticF[I*ArcSinh[Sqrt[-(
c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)] + a*d*EllipticPi[(
b*(c + d))/(b*c - a*d), I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*
x]]], (c + d)/(c - d)])*Sqrt[(d - d*Sin[e + f*x])/(c + d)]*Sqrt[-((d + d*Si
n[e + f*x])/(c - d))]*(-(b*c) + a*d + b*(c + d*Sin[e + f*x])))/(b*d^2*Sqrt[
-(c + d)^(-1)]*(b*c - a*d)*(a + b*Sin[e + f*x])*Sqrt[1 - Sin[e + f*x]^2]*Sq
rt[-((c^2 - d^2 - 2*c*(c + d*Sin[e + f*x]) + (c + d*Sin[e + f*x])^2)/d^2)])
 - ((2*I)*(-6*a*b^3*c*d + 9*a^2*b^2*d^2 - 3*b^4*d^2)*Cos[e + f*x]*Cos[2*(e
+ f*x)]*(2*b*(c - d)*(b*c - a*d)*EllipticE[I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sq
rt[c + d*Sin[e + f*x]]], (c + d)/(c - d)] + d*(-2*(a + b)*(-(b*c) + a*d)*El
lipticF[I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c
 - d)] + (2*a^2 - b^2)*d*EllipticPi[(b*(c + d))/(b*c - a*d), I*ArcSinh[Sqrt
[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)]))*Sqrt[(d - d*S
in[e + f*x])/(c + d)]*Sqrt[-((d + d*Sin[e + f*x])/(c - d))]*(-(b*c) + a*d +
 b*(c + d*Sin[e + f*x])))/(b^2*d*Sqrt[-(c + d)^(-1)]*(b*c - a*d)*(a + b*Sin
[e + f*x])*Sqrt[1 - Sin[e + f*x]^2]*(-2*c^2 + d^2 + 4*c*(c + d*Sin[e + f*x]
) - 2*(c + d*Sin[e + f*x])^2)*Sqrt[-((c^2 - d^2 - 2*c*(c + d*Sin[e + f*x])
+ (c + d*Sin[e + f*x])^2)/d^2)]))/(16*(a - b)^2*(a + b)^2*(-(b*c) + a*d)^2*
f)
```

Maple [A] time = 4.98, size = 867, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^(1/2),x)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*(-1/2*b^2/(a^3*d-a^2*b*c-a*b^2*d+b^
3*c)*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(a+b*sin(f*x+e))^2-3/4*b^2*(3*
a^2*d-2*a*b*c-b^2*d)/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)^2*(-(-d*sin(f*x+e)-c)*co
s(f*x+e)^2)^(1/2)/(a+b*sin(f*x+e))-1/4*d*(7*a^3*d-4*a^2*b*c-a*b^2*d-2*b^3*c
)/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)^2*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d
*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+
e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(
c+d))^(1/2))-3/4*b*d*(3*a^2*d-2*a*b*c-b^2*d)/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)^
2*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-
sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d
-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF
(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))+1/4*(15*a^4*d^2-20*a^
3*b*c*d+8*a^2*b^2*c^2-6*a^2*b^2*d^2-4*a*b^3*c*d+4*b^4*c^2+3*b^4*d^2)/(a^3*d
-a^2*b*c-a*b^2*d+b^3*c)^2/b*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-si
n(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*
cos(f*x+e)^2)^(1/2)/(-c/d+a/b)*EllipticPi(((c+d*sin(f*x+e))/(c-d))^(1/2),(-
c/d+1)/(-c/d+a/b),((c-d)/(c+d))^(1/2)))/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^(1/2),x, algorithm="maxima"
)
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^(1/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))**3/(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\sin\left(fx+e\right)+a\right)^{3}\sqrt{d\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sin(f*x + e) + a)^3*sqrt(d*sin(f*x + e) + c)), x)
```

3.764 $\int \frac{1}{(a+b\sin(e+fx))^3(c+d\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=682

$$-\frac{d\left(a^2b^2d\left(13c^2-29d^2\right)+8a^4d^3-6ab^3c\left(c^2-d^2\right)-b^4d\left(7c^2-15d^2\right)\right)\cos(e+fx)}{4f\left(a^2-b^2\right)^2\left(c^2-d^2\right)(bc-ad)^3\sqrt{c+d\sin(e+fx)}}-\frac{\left(a^2b^2d\left(13c^2-29d^2\right)+8a^4d^3-6ab\right.}{4f\left(\right.}$$

```
[Out] -(d*(8*a^4*d^3 + a^2*b^2*d*(13*c^2 - 29*d^2) - b^4*d*(7*c^2 - 15*d^2) - 6*a
*b^3*c*(c^2 - d^2))*Cos[e + f*x])/(4*(a^2 - b^2)^2*(b*c - a*d)^3*(c^2 - d^2
)*f*Sqrt[c + d*Sin[e + f*x]]) + (b^2*Cos[e + f*x])/(2*(a^2 - b^2)*(b*c - a*
d)*f*(a + b*Sin[e + f*x])^2*Sqrt[c + d*Sin[e + f*x]]) + (b^2*(6*a*b*c - 11*
a^2*d + 5*b^2*d)*Cos[e + f*x])/(4*(a^2 - b^2)^2*(b*c - a*d)^2*f*(a + b*Sin[
e + f*x])*Sqrt[c + d*Sin[e + f*x]]) - ((8*a^4*d^3 + a^2*b^2*d*(13*c^2 - 29*
d^2) - b^4*d*(7*c^2 - 15*d^2) - 6*a*b^3*c*(c^2 - d^2))*EllipticE[(e - Pi/2
+ f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(4*(a^2 - b^2)^2*(b*c -
a*d)^3*(c^2 - d^2)*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) - (b*(6*a*b*c - 11
*a^2*d + 5*b^2*d)*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*
Sin[e + f*x])/(c + d)])/(4*(a^2 - b^2)^2*(b*c - a*d)^2*f*Sqrt[c + d*Sin[e +
 f*x]]) - (b*(28*a^3*b*c*d - 4*a*b^3*c*d - 35*a^4*d^2 - 2*a^2*b^2*(4*c^2 -
19*d^2) - b^4*(4*c^2 + 15*d^2))*EllipticPi[(2*b)/(a + b), (e - Pi/2 + f*x)/
2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(4*(a - b)^2*(a + b)^
3*(b*c - a*d)^3*f*Sqrt[c + d*Sin[e + f*x]])
```

Rubi [A] time = 2.76845, antiderivative size = 682, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.37, Rules used = {2802, 3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$-\frac{d\left(a^2b^2d\left(13c^2-29d^2\right)+8a^4d^3-6ab^3c\left(c^2-d^2\right)-b^4d\left(7c^2-15d^2\right)\right)\cos(e+fx)}{4f\left(a^2-b^2\right)^2\left(c^2-d^2\right)(bc-ad)^3\sqrt{c+d\sin(e+fx)}}-\frac{\left(a^2b^2d\left(13c^2-29d^2\right)+8a^4d^3-6ab\right.}{4f\left(\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^(3/2)),x]
```


```
[Out] -(d*(8*a^4*d^3 + a^2*b^2*d*(13*c^2 - 29*d^2) - b^4*d*(7*c^2 - 15*d^2) - 6*a
*b^3*c*(c^2 - d^2))*Cos[e + f*x])/(4*(a^2 - b^2)^2*(b*c - a*d)^3*(c^2 - d^2
)*f*Sqrt[c + d*Sin[e + f*x]]) + (b^2*Cos[e + f*x])/(2*(a^2 - b^2)*(b*c - a*
d)*f*(a + b*Sin[e + f*x])^2*Sqrt[c + d*Sin[e + f*x]]) + (b^2*(6*a*b*c - 11*
a^2*d + 5*b^2*d)*Cos[e + f*x])/(4*(a^2 - b^2)^2*(b*c - a*d)^2*f*(a + b*Sin[
e + f*x])*Sqrt[c + d*Sin[e + f*x]]) - ((8*a^4*d^3 + a^2*b^2*d*(13*c^2 - 29*
d^2) - b^4*d*(7*c^2 - 15*d^2) - 6*a*b^3*c*(c^2 - d^2))*EllipticE[(e - Pi/2
+ f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(4*(a^2 - b^2)^2*(b*c -
a*d)^3*(c^2 - d^2)*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) - (b*(6*a*b*c - 11
*a^2*d + 5*b^2*d)*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*
Sin[e + f*x])/(c + d)])/(4*(a^2 - b^2)^2*(b*c - a*d)^2*f*Sqrt[c + d*Sin[e +
 f*x]]) - (b*(28*a^3*b*c*d - 4*a*b^3*c*d - 35*a^4*d^2 - 2*a^2*b^2*(4*c^2 -
19*d^2) - b^4*(4*c^2 + 15*d^2))*EllipticPi[(2*b)/(a + b), (e - Pi/2 + f*x)/
2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(4*(a - b)^2*(a + b)^
3*(b*c - a*d)^3*f*Sqrt[c + d*Sin[e + f*x]])
```


Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
```

```
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
```

```
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\sin(e+fx))^3(c+d\sin(e+fx))^{3/2}}\,dx = \frac{b^2\cos(e+fx)}{2\left(a^2-b^2\right)(bc-ad)f(a+b\sin(e+fx))^2\sqrt{c+d\sin(e+fx)}} - \frac{\int \frac{1}{2}\left(-4ab \cdots\right.}{\cdots}$$

$$= \frac{b^2\cos(e+fx)}{2\left(a^2-b^2\right)(bc-ad)f(a+b\sin(e+fx))^2\sqrt{c+d\sin(e+fx)}} + \frac{\cdots}{4\left(a^2-\cdots\right.}$$

$$= -\frac{d\left(8a^4d^3+a^2b^2d\left(13c^2-29d^2\right)-b^4d\left(7c^2-15d^2\right)-6ab^3c\left(c^2-d^2\right)\right)\cdots}{4\left(a^2-b^2\right)^2(bc-ad)^3\left(c^2-d^2\right)f\sqrt{c+d\sin(e+fx)}}$$

$$= -\frac{d\left(8a^4d^3+a^2b^2d\left(13c^2-29d^2\right)-b^4d\left(7c^2-15d^2\right)-6ab^3c\left(c^2-d^2\right)\right)\cdots}{4\left(a^2-b^2\right)^2(bc-ad)^3\left(c^2-d^2\right)f\sqrt{c+d\sin(e+fx)}}$$

$$= -\frac{d\left(8a^4d^3+a^2b^2d\left(13c^2-29d^2\right)-b^4d\left(7c^2-15d^2\right)-6ab^3c\left(c^2-d^2\right)\right)\cdots}{4\left(a^2-b^2\right)^2(bc-ad)^3\left(c^2-d^2\right)f\sqrt{c+d\sin(e+fx)}}$$

$$= -\frac{d\left(8a^4d^3+a^2b^2d\left(13c^2-29d^2\right)-b^4d\left(7c^2-15d^2\right)-6ab^3c\left(c^2-d^2\right)\right)\cdots}{4\left(a^2-b^2\right)^2(bc-ad)^3\left(c^2-d^2\right)f\sqrt{c+d\sin(e+fx)}}$$

$$= -\frac{d\left(8a^4d^3+a^2b^2d\left(13c^2-29d^2\right)-b^4d\left(7c^2-15d^2\right)-6ab^3c\left(c^2-d^2\right)\right)\cdots}{4\left(a^2-b^2\right)^2(bc-ad)^3\left(c^2-d^2\right)f\sqrt{c+d\sin(e+fx)}}$$

Mathematica [C] time = 8.70336, size = 1318, normalized size = 1.93

$$\frac{\sqrt{c+d\sin(e+fx)}\left(-\frac{2\cos(e+fx)d^4}{(bc-ad)^3(c^2-d^2)(c+d\sin(e+fx))}-\frac{7d\cos(e+fx)b^5+6ac\cos(e+fx)b^4-13a^2d\cos(e+fx)b^3}{4(a^2-b^2)^2(ad-bc)^3(a+b\sin(e+fx))}+\frac{b^3\cos(e+fx)}{2(a^2-b^2)(ad-bc)^2(a+b\sin(e+fx)}\right.}{f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^(3/2)),x]
```

```
[Out] (Sqrt[c + d*Sin[e + f*x]]*((b^3*Cos[e + f*x])/(2*(a^2 - b^2)*(-(b*c) + a*d)
^2*(a + b*Sin[e + f*x])^2) - (6*a*b^4*c*Cos[e + f*x] - 13*a^2*b^3*d*Cos[e +
 f*x] + 7*b^5*d*Cos[e + f*x])/(4*(a^2 - b^2)^2*(-(b*c) + a*d)^3*(a + b*Sin[
e + f*x])) - (2*d^4*Cos[e + f*x])/((b*c - a*d)^3*(c^2 - d^2)*(c + d*Sin[e +
 f*x]))))/f + ((-2*(-16*a^2*b^3*c^4 - 8*b^5*c^4 + 48*a^3*b^2*c^3*d - 18*a*b
^4*c^3*d - 48*a^4*b*c^2*d^2 + 95*a^2*b^3*c^2*d^2 - 29*b^5*c^2*d^2 + 16*a^5*
c*d^3 - 80*a^3*b^2*c*d^3 + 34*a*b^4*c*d^3 + 56*a^4*b*d^4 - 95*a^2*b^3*d^4 +
 45*b^5*d^4)*EllipticPi[(2*b)/(a + b), (-e + Pi/2 - f*x)/2, (2*d)/(c + d)]*
Sqrt[(c + d*Sin[e + f*x])/(c + d)])/((a + b)*Sqrt[c + d*Sin[e + f*x]]) - ((
2*I)*(-20*a^2*b^3*c^3*d - 4*b^5*c^3*d + 48*a^3*b^2*c^2*d^2 - 24*a*b^4*c^2*d
^2 + 16*a^4*b*c*d^3 - 12*a^2*b^3*c*d^3 + 20*b^5*c*d^3 + 16*a^5*d^4 - 80*a^3
*b^2*d^4 + 40*a*b^4*d^4)*Cos[e + f*x]*((b*c - a*d)*EllipticF[I*ArcSinh[Sqrt
[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)] + a*d*EllipticP
i[(b*(c + d))/(b*c - a*d), I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e +
 f*x]]], (c + d)/(c - d)])*Sqrt[(d - d*Sin[e + f*x])/(c + d)]*Sqrt[-((d + d
*Sin[e + f*x])/(c - d))]*(-(b*c) + a*d + b*(c + d*Sin[e + f*x]))))/(b*d^2*Sq
rt[-(c + d)^(-1)]*(b*c - a*d)*(a + b*Sin[e + f*x])*Sqrt[1 - Sin[e + f*x]^2]
*Sqrt[-((c^2 - d^2 - 2*c*(c + d*Sin[e + f*x]) + (c + d*Sin[e + f*x])^2)/d^2
)]) - ((2*I)*(6*a*b^4*c^3*d - 13*a^2*b^3*c^2*d^2 + 7*b^5*c^2*d^2 - 6*a*b^4*
c*d^3 - 8*a^4*b*d^4 + 29*a^2*b^3*d^4 - 15*b^5*d^4)*Cos[e + f*x]*Cos[2*(e +
f*x)]*(2*b*(c - d)*(b*c - a*d)*EllipticE[I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt
[c + d*Sin[e + f*x]]], (c + d)/(c - d)] + d*(-2*(a + b)*(-(b*c) + a*d)*Elli
pticF[I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c -
 d)] + (2*a^2 - b^2)*d*EllipticPi[(b*(c + d))/(b*c - a*d), I*ArcSinh[Sqrt[-
(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)]))*Sqrt[(d - d*Sin
[e + f*x])/(c + d)]*Sqrt[-((d + d*Sin[e + f*x])/(c - d))]*(-(b*c) + a*d + b
*(c + d*Sin[e + f*x]))))/(b^2*d*Sqrt[-(c + d)^(-1)]*(b*c - a*d)*(a + b*Sin[e
 + f*x])*Sqrt[1 - Sin[e + f*x]^2]*(-2*c^2 + d^2 + 4*c*(c + d*Sin[e + f*x])
- 2*(c + d*Sin[e + f*x])^2)*Sqrt[-((c^2 - d^2 - 2*c*(c + d*Sin[e + f*x]) +
(c + d*Sin[e + f*x])^2)/d^2)]))/(16*(a - b)^2*(a + b)^2*(c - d)*(c + d)*(-(
b*c) + a*d)^3*f)
```

Maple [B] time = 11.483, size = 2099, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^(3/2),x)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*(-2*d^2/(a*d-b*c)^3*(c/d-1)*((c+d*s
in(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(
c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(-c/d+a/b)*EllipticPi((
(c+d*sin(f*x+e))/(c-d))^(1/2),(-c/d+1)/(-c/d+a/b),((c-d)/(c+d))^(1/2))-b*d/
(a*d-b*c)^2*(-b^2/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)*(-(-d*sin(f*x+e)-c)*cos(f*x
```

```
+e)^2)^(1/2)/(a+b*sin(f*x+e))-a*d/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)*(c/d-1)*((c
+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)
*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin
(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))-b*d/(a^3*d-a^2*b*c-a*b^2*d+b^3*c
)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-
sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d
-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF
(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))+(3*a^2*d-2*a*b*c-b^2*
d)/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)/b*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(
d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x
+e)-c)*cos(f*x+e)^2)^(1/2)/(-c/d+a/b)*EllipticPi(((c+d*sin(f*x+e))/(c-d))^(
1/2),(-c/d+1)/(-c/d+a/b),((c-d)/(c+d))^(1/2)))-b/(a*d-b*c)*(-1/2*b^2/(a^3*d
-a^2*b*c-a*b^2*d+b^3*c)*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(a+b*sin(f*
x+e))^2-3/4*b^2*(3*a^2*d-2*a*b*c-b^2*d)/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)^2*(-(
-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(a+b*sin(f*x+e))-1/4*d*(7*a^3*d-4*a^2*
b*c-a*b^2*d-2*b^3*c)/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)^2*(c/d-1)*((c+d*sin(f*x+
e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(
1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c
-d))^(1/2),((c-d)/(c+d))^(1/2))-3/4*b*d*(3*a^2*d-2*a*b*c-b^2*d)/(a^3*d-a^2*
b*c-a*b^2*d+b^3*c)^2*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e
))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x
+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d
))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))+1/
4*(15*a^4*d^2-20*a^3*b*c*d+8*a^2*b^2*c^2-6*a^2*b^2*d^2-4*a*b^3*c*d+4*b^4*c^
2+3*b^4*d^2)/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)^2/b*(c/d-1)*((c+d*sin(f*x+e))/(c
-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(
-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(-c/d+a/b)*EllipticPi(((c+d*sin(f*x+
e))/(c-d))^(1/2),(-c/d+1)/(-c/d+a/b),((c-d)/(c+d))^(1/2)))+d^3/(a*d-b*c)^3*
(2*d*cos(f*x+e)^2/(c^2-d^2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2*c/(c^
2-d^2)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2
)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*E
llipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+2/(c^2-d^2)*d*
(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-si
n(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1
)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF((
(c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))))/cos(f*x+e)/(c+d*sin(f
*x+e))^(1/2)/f
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^(3/2),x, algorithm="maxima"
)

[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^(3/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))**3/(c+d*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\sin\left(fx+e\right)+a\right)^{3}\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sin(f*x + e) + a)^3*(d*sin(f*x + e) + c)^(3/2)), x)
```

3.765 $\int \sqrt{a+b\sin(e+fx)}(c+d\sin(e+fx))^{5/2}\,dx$

Optimal. Leaf size=888

$$-\frac{\cos(e+fx)(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}d^2}{3bf}-\frac{(13bc-3ad)\cos(e+fx)\sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin(e+fx)}d}{12bf}$$

```
[Out] (Sqrt[a + b]*(c - d)*Sqrt[c + d]*(14*a*b*c*d - 3*a^2*d^2 + b^2*(33*c^2 + 16
*d^2))*EllipticE[ArcSin[(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])/(Sqrt[c + d]
*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/((a + b)*(c - d))]*Sec[e + f
*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))
]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*(a
+ b*Sin[e + f*x]))/(24*b^2*(b*c - a*d)*f) - (Sqrt[c + d]*(5*a^2*b*c*d^2 - a
^3*d^3 - a*b^2*d*(15*c^2 + 4*d^2) - 5*b^3*(c^3 + 4*c*d^2))*EllipticPi[(b*(c
 + d))/((a + b)*d), ArcSin[(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])/(Sqrt[c +
 d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/((a + b)*(c - d))]*Sec[e
+ f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x]
)))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*
(a + b*Sin[e + f*x]))/(8*b^3*Sqrt[a + b]*d*f) - ((14*a*b*c*d - 3*a^2*d^2 +
b^2*(33*c^2 + 16*d^2))*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(24*b*f*Sqrt[
a + b*Sin[e + f*x]]) - (d*(13*b*c - 3*a*d)*Cos[e + f*x]*Sqrt[a + b*Sin[e +
f*x]]*Sqrt[c + d*Sin[e + f*x]])/(12*b*f) - (d^2*Cos[e + f*x]*(a + b*Sin[e +
 f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]])/(3*b*f) + ((a + b)^(3/2)*(3*a^2*d^2
- 6*a*b*d*(2*c + d) + b^2*(33*c^2 + 26*c*d + 16*d^2))*EllipticF[ArcSin[(Sqr
t[c + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])],
 ((a + b)*(c - d))/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - S
in[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[
e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/(24*b^3*S
qrt[c + d]*f)
```

Rubi [A] time = 3.38603, antiderivative size = 888, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2793, 3049, 3061, 3053, 2811, 2998, 2818, 2996}

$$-\frac{\cos(e+fx)(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}d^2}{3bf}-\frac{(13bc-3ad)\cos(e+fx)\sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin(e+fx)}d}{12bf}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (Sqrt[a + b]*(c - d)*Sqrt[c + d]*(14*a*b*c*d - 3*a^2*d^2 + b^2*(33*c^2 + 16
*d^2))*EllipticE[ArcSin[(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])/(Sqrt[c + d]
*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/((a + b)*(c - d))]*Sec[e + f
*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))
]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*(a
+ b*Sin[e + f*x]))/(24*b^2*(b*c - a*d)*f) - (Sqrt[c + d]*(5*a^2*b*c*d^2 - a
^3*d^3 - a*b^2*d*(15*c^2 + 4*d^2) - 5*b^3*(c^3 + 4*c*d^2))*EllipticPi[(b*(c
 + d))/((a + b)*d), ArcSin[(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])/(Sqrt[c +
 d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/((a + b)*(c - d))]*Sec[e
+ f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x]
)))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*
(a + b*Sin[e + f*x]))/(8*b^3*Sqrt[a + b]*d*f) - ((14*a*b*c*d - 3*a^2*d^2 +
b^2*(33*c^2 + 16*d^2))*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(24*b*f*Sqrt[
```

```
a + b*Sin[e + f*x]]) - (d*(13*b*c - 3*a*d)*Cos[e + f*x]*Sqrt[a + b*Sin[e +
f*x]]*Sqrt[c + d*Sin[e + f*x]])/(12*b*f) - (d^2*Cos[e + f*x]*(a + b*Sin[e +
 f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]])/(3*b*f) + ((a + b)^(3/2)*(3*a^2*d^2
- 6*a*b*d*(2*c + d) + b^2*(33*c^2 + 26*c*d + 16*d^2))*EllipticF[ArcSin[(Sqr
t[c + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])],
 ((a + b)*(c - d))/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - S
in[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[
e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/(24*b^3*S
qrt[c + d]*f)
```

Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2811

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[(2*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d
)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(
1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*EllipticPi[(b*(c + d))/
(d*(a + b)), ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt[c + d*Sin[e + f*x]])/Sqrt[
a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)*(c - d))])/(d*f*Rt[(a + b)
/(c + d), 2]*Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2818

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*(c + d*Sin[e + f*x])*Sqrt[((b*c -
a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d
)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[
(c + d)/(a + b), 2]*(Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (
(a + b)*(c - d))/((a - b)*(c + d))])/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*
Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 2996

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(-2*A*(c - d)*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/
((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c
 + d)*(a + b*Sin[e + f*x])))]*EllipticE[ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt
[c + d*Sin[e + f*x]])/Sqrt[a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)
*(c - d))])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2]*Cos[e + f*x]), x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rubi steps

$$\int \sqrt{a+b\sin(e+fx)}(c+d\sin(e+fx))^{5/2}\,dx = -\frac{d^2\cos(e+fx)(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}{3bf} + \frac{\int \frac{\sqrt{a+b\sin(}}{}}{}$$

$$= -\frac{d(13bc-3ad)\cos(e+fx)\sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}{12bf} - \frac{d}{}$$

$$= -\frac{\left(14abcd-3a^2d^2+b^2\left(33c^2+16d^2\right)\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{24bf\sqrt{a+b\sin(e+fx)}}$$

$$= -\frac{\left(14abcd-3a^2d^2+b^2\left(33c^2+16d^2\right)\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{24bf\sqrt{a+b\sin(e+fx)}}$$

$$= -\frac{\sqrt{c+d}\left(5a^2bcd^2-a^3d^3-ab^2d\left(15c^2+4d^2\right)-5b^3\left(c^3+4cd^2\right)\right)\Pi\left(\frac{b}{(}\right.}{}$$

$$= \frac{\sqrt{a+b}(c-d)\sqrt{c+d}\left(14abcd-3a^2d^2+b^2\left(33c^2+16d^2\right)\right)E\left(\sin^{-1}\left(\right.\right.}{}$$

<u>**Mathematica**</u> **[B]** time = 7.12279, size = 1948, normalized size = 2.19

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] -((-4*(-(b*c) + a*d)*(-48*a*b*c^3 - 59*b^2*c^2*d - 58*a*b*c*d^2 + a^2*d^3 -
 16*b^2*d^3)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[
ArcSin[Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(
b*c) + a*d))]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]
*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b
*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*
(c + d*Sin[e + f*x]))/(-(b*c) + a*d))])/((a + b)*(c + d)*Sqrt[a + b*Sin[e +
 f*x]]*Sqrt[c + d*Sin[e + f*x]]) - 4*(-(b*c) + a*d)*(-48*b^2*c^3 - 92*a*b*c
^2*d + 4*a^2*c*d^2 - 76*b^2*c*d^2 - 28*a*b*d^3)*((Sqrt[((c + d)*Cot[(-e + P
i/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[-(((a + b)*Csc[(-e + Pi/2
- f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))]/Sqrt[2]], (2*(-(b*c) + a
*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c +
 d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[-
(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))]
)/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - (Sq
rt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c) + a*d)
/((a + b)*d), ArcSin[Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[
e + f*x]))/(-(b*c) + a*d))]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))
]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*
x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[-(((a + b)*Csc[(-e + Pi/
2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))])/((a + b)*d*Sqrt[a + b
*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])) + 2*(33*b^2*c^2*d + 14*a*b*c*d^2
- 3*a^2*d^3 + 16*b^2*d^3)*((Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(d*Sqrt[
a + b*Sin[e + f*x]]) + (Sqrt[(a - b)/(a + b)]*(a + b)*Cos[(-e + Pi/2 - f*x)
/2]*EllipticE[ArcSin[(Sqrt[(a - b)/(a + b)]*Sin[(-e + Pi/2 - f*x)/2])/Sqrt[
(a + b*Sin[e + f*x])/(a + b)]], (2*(-(b*c) + a*d))/((a - b)*(c + d))]*Sqrt[
```

```
c + d*Sin[e + f*x]])/(b*d*Sqrt[((a + b)*Cos[(-e + Pi/2 - f*x)/2]^2)/(a + b*
Sin[e + f*x])]*Sqrt[a + b*Sin[e + f*x]]*Sqrt[(a + b*Sin[e + f*x])/(a + b)]*
Sqrt[((a + b)*(c + d*Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x]))]]) - (2*(
-(b*c) + a*d)*((((a + b)*c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)
/(-c + d)]*EllipticF[ArcSin[Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c +
 d*Sin[e + f*x]))/(-(b*c) + a*d))]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-
c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi
/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[-(((a + b)*Csc[(-
e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))])/((a + b)*(c +
d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - ((b*c + a*d)*Sqrt[(
(c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a
 + b)*d), ArcSin[Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e +
f*x]))/(-(b*c) + a*d))]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Se
c[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2
]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[-(((a + b)*Csc[(-e + Pi/2 -
f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))])/((a + b)*d*Sqrt[a + b*Sin
[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])))/(b*d)))/(48*b*f) + (Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[c + d*Sin[e + f*x]]*(-(d*(13*b*c + a*d)*Cos[e + f*x])/(12*b)
- (d^2*Sin[2*(e + f*x)])/6))/f
```

Maple [C] time = 9.008, size = 404333, normalized size = 455.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^(5/2),x)

[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sin\left(fx+e\right)+a}\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^(5/2),x, algorithm="maxim
a")

[Out] integrate(sqrt(b*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^(5/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^(5/2),x, algorithm="frica
s")

[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**(1/2)*(c+d*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError
```

3.766 $\int \sqrt{a+b\sin(e+fx)}(c+d\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=784

$$\frac{\sqrt{c+d}\left(-a^2d^2+6abcd+b^2\left(3c^2+4d^2\right)\right)\sec(e+fx)(a+b\sin(e+fx))\sqrt{-\frac{(bc-ad)(1-\sin(e+fx))}{(c+d)(a+b\sin(e+fx))}}\sqrt{\frac{(bc-ad)(\sin(e+fx)+1)}{(c-d)(a+b\sin(e+fx))}}\Pi\left(\frac{b(c+d)}{(a+b)d}\right.}{4b^2df\sqrt{a+b}}$$

```
[Out] (Sqrt[a + b]*(c - d)*Sqrt[c + d]*(5*b*c + a*d)*EllipticE[ArcSin[(Sqrt[a + b
]*Sqrt[c + d*Sin[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a -
b)*(c + d))/((a + b)*(c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e
+ f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*
x]))/((c - d)*(a + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/(4*b*(b*c - a*d)
*f) + (Sqrt[c + d]*(6*a*b*c*d - a^2*d^2 + b^2*(3*c^2 + 4*d^2))*EllipticPi[(
b*(c + d))/((a + b)*d), ArcSin[(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])/(Sqrt
[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/((a + b)*(c - d))]*Se
c[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e +
f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]
))]*(a + b*Sin[e + f*x]))/(4*b^2*Sqrt[a + b]*d*f) + ((b*c - a*d)*Cos[e + f*
x]*Sqrt[c + d*Sin[e + f*x]])/(2*f*Sqrt[a + b*Sin[e + f*x]]) - ((5*b*c + a*d
)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(4*f*Sqrt[a + b*Sin[e + f*x]]) + (
(a + b)^(3/2)*(5*b*c - a*d + 2*b*d)*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a +
b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))
/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a
+ b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b
)*(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/(4*b^2*Sqrt[c + d]*f) - (b*
Cos[e + f*x]*(c + d*Sin[e + f*x])^(3/2))/(2*f*Sqrt[a + b*Sin[e + f*x]])
```

Rubi [A] time = 3.51634, antiderivative size = 784, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2821, 3047, 3061, 3053, 2811, 2998, 2818, 2996}

$$\frac{\sqrt{c+d}\left(-a^2d^2+6abcd+b^2\left(3c^2+4d^2\right)\right)\sec(e+fx)(a+b\sin(e+fx))\sqrt{-\frac{(bc-ad)(1-\sin(e+fx))}{(c+d)(a+b\sin(e+fx))}}\sqrt{\frac{(bc-ad)(\sin(e+fx)+1)}{(c-d)(a+b\sin(e+fx))}}\Pi\left(\frac{b(c+d)}{(a+b)d}\right.}{4b^2df\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] (Sqrt[a + b]*(c - d)*Sqrt[c + d]*(5*b*c + a*d)*EllipticE[ArcSin[(Sqrt[a + b
]*Sqrt[c + d*Sin[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a -
b)*(c + d))/((a + b)*(c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e
+ f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*
x]))/((c - d)*(a + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/(4*b*(b*c - a*d)
*f) + (Sqrt[c + d]*(6*a*b*c*d - a^2*d^2 + b^2*(3*c^2 + 4*d^2))*EllipticPi[(
b*(c + d))/((a + b)*d), ArcSin[(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])/(Sqrt
[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/((a + b)*(c - d))]*Se
c[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e +
f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]
))]*(a + b*Sin[e + f*x]))/(4*b^2*Sqrt[a + b]*d*f) + ((b*c - a*d)*Cos[e + f*
x]*Sqrt[c + d*Sin[e + f*x]])/(2*f*Sqrt[a + b*Sin[e + f*x]]) - ((5*b*c + a*d
)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(4*f*Sqrt[a + b*Sin[e + f*x]]) + (
(a + b)^(3/2)*(5*b*c - a*d + 2*b*d)*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a +
b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))
/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a
```

```
+ b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b
)*(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/(4*b^2*Sqrt[c + d]*f) - (b*
Cos[e + f*x]*(c + d*Sin[e + f*x])^(3/2))/(2*f*Sqrt[a + b*Sin[e + f*x]])
```

Rule 2821

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[1/(d*(m + n)), Int[
(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a^2*c*d*(m +
 n) + b*d*(b*c*(m - 1) + a*d*n) + (a*d*(2*b*c + a*d)*(m + n) - b*d*(a*c - b
*d*(m + n - 1)))*Sin[e + f*x] + b*d*(b*c*n + a*d*(2*m + n - 1))*Sin[e + f*x
]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[0, m, 2] && LtQ[-1, n, 2] && NeQ
[m + n, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2811

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[(2*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d
)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(
1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*EllipticPi[(b*(c + d))/
(d*(a + b)), ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt[c + d*Sin[e + f*x]])/Sqrt[
a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)*(c - d))])/(d*f*Rt[(a + b)
```

```
/(c + d), 2]*Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2818

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*(c + d*Sin[e + f*x])*Sqrt[((b*c -
a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d
)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[
(c + d)/(a + b), 2]*(Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (
(a + b)*(c - d))/((a - b)*(c + d))])/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*
Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 2996

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(-2*A*(c - d)*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/
((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c
 + d)*(a + b*Sin[e + f*x])))]*EllipticE[ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt
[c + d*Sin[e + f*x]])/Sqrt[a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)
*(c - d))])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2]*Cos[e + f*x]), x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rubi steps

$$\int \sqrt{a+b\sin(e+fx)}(c+d\sin(e+fx))^{3/2}\,dx = -\frac{b\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{2f\sqrt{a+b\sin(e+fx)}} + \frac{\int \frac{\sqrt{c+d\sin(e+fx)}\left(\frac{1}{2}d\left(4a^2c-b^2c+3abd\right.\right.}{}}{}$$

$$= \frac{(bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2f\sqrt{a+b\sin(e+fx)}} - \frac{b\cos(e+fx)(c+d\sin(e+}{2f\sqrt{a+b\sin(e+f}}$$

$$= \frac{(bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2f\sqrt{a+b\sin(e+fx)}} - \frac{(5bc+ad)\cos(e+fx)\sqrt{c}}{4f\sqrt{a+b\sin(e}}$$

$$= \frac{(bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2f\sqrt{a+b\sin(e+fx)}} - \frac{(5bc+ad)\cos(e+fx)\sqrt{c}}{4f\sqrt{a+b\sin(e}}$$

$$= \frac{\sqrt{c+d}\left(6abcd-a^2d^2+b^2\left(3c^2+4d^2\right)\right)\Pi\left(\frac{b(c+d)}{(a+b)d};\sin^{-1}\left(\frac{\sqrt{a+b}\sqrt{c+d\sin(}}{\sqrt{c+d}\sqrt{a+b\sin(}}\right.\right.}{}$$

$$= \frac{\sqrt{a+b}(c-d)\sqrt{c+d}(5bc+ad)E\left(\sin^{-1}\left(\frac{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)}{4b(bc}$$

Mathematica [B] time = 9.48356, size = 1849, normalized size = 2.36

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] -(d*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])/(2*f) +
 ((-4*(-(b*c) + a*d)*(8*a*c^2 + 7*b*c*d + 3*a*d^2)*Sqrt[((c + d)*Cot[(-e +
Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[-(((a + b)*Csc[(-e + Pi/2
 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))]/Sqrt[2]], (2*(-(b*c) +
a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c
+ d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[
-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))
])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - 4*
(-(b*c) + a*d)*(8*b*c^2 + 12*a*c*d + 4*b*d^2)*((Sqrt[((c + d)*Cot[(-e + Pi/
2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[-(((a + b)*Csc[(-e + Pi/2 -
f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))]/Sqrt[2]], (2*(-(b*c) + a*d
))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d
)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[-((
(a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))])/
((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - (Sqrt
[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c) + a*d)/(
(a + b)*d), ArcSin[Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e
+ f*x]))/(-(b*c) + a*d))]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*
Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)
/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[-(((a + b)*Csc[(-e + Pi/2
- f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))])/((a + b)*d*Sqrt[a + b*S
in[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])) + 2*(-5*b*c*d - a*d^2)*((Cos[e + f*
x]*Sqrt[c + d*Sin[e + f*x]])/(d*Sqrt[a + b*Sin[e + f*x]]) + (Sqrt[(a - b)/(
a + b)]*(a + b)*Cos[(-e + Pi/2 - f*x)/2]*EllipticE[ArcSin[(Sqrt[(a - b)/(a
+ b)]*Sin[(-e + Pi/2 - f*x)/2])/Sqrt[(a + b*Sin[e + f*x])/(a + b)]], (2*(-(
b*c) + a*d))/((a - b)*(c + d))]*Sqrt[c + d*Sin[e + f*x]])/(b*d*Sqrt[((a + b
```

```
)*Cos[(-e + Pi/2 - f*x)/2]^2)/(a + b*Sin[e + f*x])]*Sqrt[a + b*Sin[e + f*x]
]*Sqrt[(a + b*Sin[e + f*x])/(a + b)]*Sqrt[((a + b)*(c + d*Sin[e + f*x]))/((
c + d)*(a + b*Sin[e + f*x]))]]) - (2*(-(b*c) + a*d)*((((a + b)*c + a*d)*Sqrt
[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[-(((a
 + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))]/Sqr
t[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 -
 f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(
-(b*c) + a*d)]*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*
x]))/(-(b*c) + a*d))])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d
*Sin[e + f*x]]) - ((b*c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-
c + d)]*EllipticPi[(-(b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[-(((a + b)*Csc[(
-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))]/Sqrt[2]], (2*(
-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*
Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*
d)]*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c
) + a*d))])/((a + b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])))
/(b*d)))/(8*f)
```

Maple [C] time = 4.23, size = 277165, normalized size = 353.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sin\left(fx+e\right)+a}\left(d \sin\left(fx+e\right)+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(b*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^(3/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**(1/2)*(c+d*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: AttributeError
```

3.767 $\int \sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin(e+fx)}\,dx$

Optimal. Leaf size=628

$$-\frac{b\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{f\sqrt{a+b\sin(e+fx)}}+\frac{(a+b)^{3/2}\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}}F\Bigg(s}{bf\sqrt{c+d}}$$

```
[Out] (Sqrt[a + b]*(c - d)*Sqrt[c + d]*EllipticE[ArcSin[(Sqrt[a + b]*Sqrt[c + d*S
in[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/((
a + b)*(c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c +
 d)*(a + b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d)*
(a + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/((b*c - a*d)*f) + (Sqrt[c + d]
*(b*c + a*d)*EllipticPi[(b*(c + d))/((a + b)*d), ArcSin[(Sqrt[a + b]*Sqrt[c
 + d*Sin[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c +
d))/((a + b)*(c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))
/((c + d)*(a + b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c
 - d)*(a + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/(b*Sqrt[a + b]*d*f) - (b
*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(f*Sqrt[a + b*Sin[e + f*x]]) + ((a
+ b)^(3/2)*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a
+ b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sec[e
 + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x])
)]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]
*(c + d*Sin[e + f*x]))/(b*Sqrt[c + d]*f)
```

Rubi [A] time = 1.32657, antiderivative size = 628, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2821, 3053, 2811, 12, 2801, 2818, 2996}

$$-\frac{b\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{f\sqrt{a+b\sin(e+fx)}}+\frac{(a+b)^{3/2}\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}}F\Bigg(s}{bf\sqrt{c+d}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] (Sqrt[a + b]*(c - d)*Sqrt[c + d]*EllipticE[ArcSin[(Sqrt[a + b]*Sqrt[c + d*S
in[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/((
a + b)*(c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c +
 d)*(a + b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d)*
(a + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/((b*c - a*d)*f) + (Sqrt[c + d]
*(b*c + a*d)*EllipticPi[(b*(c + d))/((a + b)*d), ArcSin[(Sqrt[a + b]*Sqrt[c
 + d*Sin[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c +
d))/((a + b)*(c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))
/((c + d)*(a + b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c
 - d)*(a + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/(b*Sqrt[a + b]*d*f) - (b
*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(f*Sqrt[a + b*Sin[e + f*x]]) + ((a
+ b)^(3/2)*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a
+ b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sec[e
 + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x])
)]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]
*(c + d*Sin[e + f*x]))/(b*Sqrt[c + d]*f)
```

Rule 2821

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[1/(d*(m + n)), Int[
(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a^2*c*d*(m +
 n) + b*d*(b*c*(m - 1) + a*d*n) + (a*d*(2*b*c + a*d)*(m + n) - b*d*(a*c - b
*d*(m + n - 1)))*Sin[e + f*x] + b*d*(b*c*n + a*d*(2*m + n - 1))*Sin[e + f*x
]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[0, m, 2] && LtQ[-1, n, 2] && NeQ
[m + n, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2811

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[(2*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d
)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(
1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*EllipticPi[(b*(c + d))/
(d*(a + b)), ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt[c + d*Sin[e + f*x]])/Sqrt[
a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)*(c - d))])/(d*f*Rt[(a + b)
/(c + d), 2]*Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2801

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin
[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[1/(a - b), Int[1/(Sqrt[a + b*Sin[
e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[b/(a - b), Int[(1 + Sin[
e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0]
```

Rule 2818

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*(c + d*Sin[e + f*x])*Sqrt[((b*c -
a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d
)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[
(c + d)/(a + b), 2]*(Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (
(a + b)*(c - d))/((a - b)*(c + d))])/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*
Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 2996

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
```

```
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(-2*A*(c - d)*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/
((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c
 + d)*(a + b*Sin[e + f*x])))]*EllipticE[ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt
[c + d*Sin[e + f*x]])/Sqrt[a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)
*(c - d))])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2]*Cos[e + f*x]), x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rubi steps

$$\int \sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin(e+fx)}\,dx = -\frac{b\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{f\sqrt{a+b\sin(e+fx)}} + \frac{\int \frac{\frac{1}{2}d\left(2a^2c-b^2c+abd\right)+ad(bc+ad)\sin(e+fx)+\frac{1}{2}bd}{(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+}}}{d}$$

$$= -\frac{b\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{f\sqrt{a+b\sin(e+fx)}} + \frac{\int \frac{-\frac{1}{2}a^2bd(bc+ad)+\frac{1}{2}b^2d\left(2a^2c-b^2c+abd\right)}{(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}\,dx}{b^2d} +$$

$$= \frac{\sqrt{c+d}(bc+ad)\Pi\left(\frac{b(c+d)}{(a+b)d};\sin^{-1}\left(\frac{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sec(e+fx)}{b\sqrt{a+b}df}$$

$$= \frac{\sqrt{c+d}(bc+ad)\Pi\left(\frac{b(c+d)}{(a+b)d};\sin^{-1}\left(\frac{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sec(e+fx)}{b\sqrt{a+b}df}$$

$$= \frac{\sqrt{a+b}(c-d)\sqrt{c+d}E\left(\sin^{-1}\left(\frac{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sec(e+fx)\sqrt{-}}{(bc-ad)f}$$

Mathematica [C] time = 31.6832, size = 228392, normalized size = 363.68

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] Result too large to show
```

Maple [C] time = 1.348, size = 146613, normalized size = 233.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sin\left(fx+e\right)+a}\sqrt{d\sin\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b\sin\left(e + fx\right)}\sqrt{c + d\sin\left(e + fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**(1/2)*(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sin(e + f*x))*sqrt(c + d*sin(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sin\left(fx + e\right) + a}\sqrt{d\sin\left(fx + e\right) + c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c), x)
```

3.768 $\int \frac{\sqrt{a+b\sin(e+fx)}}{\sqrt{c+d\sin(e+fx)}}\,dx$

Optimal. Leaf size=198

$$\frac{2\sqrt{c+d}\sec(e+fx)(a+b\sin(e+fx))\sqrt{-\frac{(bc-ad)(1-\sin(e+fx))}{(c+d)(a+b\sin(e+fx))}}\sqrt{\frac{(bc-ad)(\sin(e+fx)+1)}{(c-d)(a+b\sin(e+fx))}}\Pi\left(\frac{b(c+d)}{(a+b)d};\sin^{-1}\left(\frac{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}\right)|\frac{(a-b)(c}{(a+b)(c}\right.}{df\sqrt{a+b}}$$

```
[Out] (2*Sqrt[c + d]*EllipticPi[(b*(c + d))/((a + b)*d), ArcSin[(Sqrt[a + b]*Sqrt
[c + d*Sin[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c
+ d))/((a + b)*(c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]
))/((c + d)*(a + b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/(
(c - d)*(a + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/(Sqrt[a + b]*d*f)
```

Rubi [A] time = 0.113887, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {2811}

$$\frac{2\sqrt{c+d}\sec(e+fx)(a+b\sin(e+fx))\sqrt{-\frac{(bc-ad)(1-\sin(e+fx))}{(c+d)(a+b\sin(e+fx))}}\sqrt{\frac{(bc-ad)(\sin(e+fx)+1)}{(c-d)(a+b\sin(e+fx))}}\Pi\left(\frac{b(c+d)}{(a+b)d};\sin^{-1}\left(\frac{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}\right)|\frac{(a-b)(c}{(a+b)(c}\right.}{df\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] (2*Sqrt[c + d]*EllipticPi[(b*(c + d))/((a + b)*d), ArcSin[(Sqrt[a + b]*Sqrt
[c + d*Sin[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c
+ d))/((a + b)*(c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]
))/((c + d)*(a + b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/(
(c - d)*(a + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/(Sqrt[a + b]*d*f)
```

Rule 2811

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[(2*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d
)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(
1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*EllipticPi[(b*(c + d))/
(d*(a + b)), ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt[c + d*Sin[e + f*x]])/Sqrt[
a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)*(c - d))])/(d*f*Rt[(a + b)
/(c + d), 2]*Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

Rubi steps

$$\int \frac{\sqrt{a+b\sin(e+fx)}}{\sqrt{c+d\sin(e+fx)}}\,dx = \frac{2\sqrt{c+d}\Pi\left(\frac{b(c+d)}{(a+b)d};\sin^{-1}\left(\frac{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sec(e+fx)\sqrt{-\frac{(bc-ad)(1-\sin(e+fx))}{(c+d)(a+b\sin(e+fx))}}\sqrt{\frac{(bc-}{(c-}}}{\sqrt{a+b}df}$$

Mathematica [A] time = 0.218514, size = 195, normalized size = 0.98

$$\frac{2\sqrt{c+d}\sec(e+fx)(a+b\sin(e+fx))\sqrt{\frac{(bc-ad)(\sin(e+fx)-1)}{(c+d)(a+b\sin(e+fx))}}\sqrt{\frac{(bc-ad)(\sin(e+fx)+1)}{(c-d)(a+b\sin(e+fx))}}\Pi\left(\frac{b(c+d)}{(a+b)d};\sin^{-1}\left(\frac{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}\right)|\frac{(a-b)(c+}{(a+b)(c-}\right.}{df\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] (2*Sqrt[c + d]*EllipticPi[(b*(c + d))/((a + b)*d), ArcSin[(Sqrt[a + b]*Sqrt
[c + d*Sin[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c
+ d))/((a + b)*(c - d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(-1 + Sin[e + f*x])
)/((c + d)*(a + b*Sin[e + f*x]))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c
 - d)*(a + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/(Sqrt[a + b]*d*f)
```

Maple [C] time = 4.787, size = 248028, normalized size = 1252.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b\sin\left(fx+e\right)+a}}{\sqrt{d\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(b*sin(f*x + e) + a)/sqrt(d*sin(f*x + e) + c), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+b\sin\left(e+fx\right)}}{\sqrt{c+d\sin\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**(1/2)/(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sin(e + f*x))/sqrt(c + d*sin(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b\sin\left(fx+e\right)+a}}{\sqrt{d\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(b*sin(f*x + e) + a)/sqrt(d*sin(f*x + e) + c), x)
```

3.769 $\int \frac{\sqrt{a+b\sin(e+fx)}}{(c+d\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=409

$$\frac{2(a-b)\sqrt{a+b}\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}}F\left(\sin^{-1}\left(\frac{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}\right)|\frac{(a+}{(a-}\right.}{f(c-d)\sqrt{c+d}(bc-ad)}$$

```
[Out] (-2*(a - b)*Sqrt[a + b]*EllipticE[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*
x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c
 + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*
Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin
[e + f*x])))]*(c + d*Sin[e + f*x]))/((c - d)*Sqrt[c + d]*(b*c - a*d)*f) + (
2*(a - b)*Sqrt[a + b]*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]
])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c +
 d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Si
n[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e
 + f*x])))]*(c + d*Sin[e + f*x]))/((c - d)*Sqrt[c + d]*(b*c - a*d)*f)
```

Rubi [A] time = 0.494367, antiderivative size = 409, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2795, 2818, 2996}

$$\frac{2(a-b)\sqrt{a+b}\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}}F\left(\sin^{-1}\left(\frac{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}\right)|\frac{(a+}{(a-}\right.}{f(c-d)\sqrt{c+d}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x])^(3/2),x]
```


```
[Out] (-2*(a - b)*Sqrt[a + b]*EllipticE[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*
x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c
 + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*
Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin
[e + f*x])))]*(c + d*Sin[e + f*x]))/((c - d)*Sqrt[c + d]*(b*c - a*d)*f) + (
2*(a - b)*Sqrt[a + b]*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]
])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c +
 d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Si
n[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e
 + f*x])))]*(c + d*Sin[e + f*x]))/((c - d)*Sqrt[c + d]*(b*c - a*d)*f)
```


Rule 2795

```
Int[Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_.) + (b_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Dist[(c - d)/(a - b), Int[1/(Sqrt[a + b*Sin
[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[(b*c - a*d)/(a - b), In
t[(1 + Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2818

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*(c + d*Sin[e + f*x])*Sqrt[((b*c -
```

```
a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d
)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[
(c + d)/(a + b), 2]*(Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (
(a + b)*(c - d))/((a - b)*(c + d))])/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*
Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 2996

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(-2*A*(c - d)*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/
((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c
 + d)*(a + b*Sin[e + f*x])))]*EllipticE[ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt
[c + d*Sin[e + f*x]])/Sqrt[a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)
*(c - d))])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2]*Cos[e + f*x]), x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rubi steps

$$\int \frac{\sqrt{a+b\sin(e+fx)}}{(c+d\sin(e+fx))^{3/2}}\,dx = \frac{(a-b)\int \frac{1}{\sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\,dx}{c-d} + \frac{(bc-ad)\int \frac{1+\sin(e+fx)}{\sqrt{a+b\sin(e+fx)}(c+d\sin(e+fx))^{3/2}}\,dx}{c-d}$$

$$= -\frac{2(a-b)\sqrt{a+b}E\left(\sin^{-1}\left(\frac{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}\right)|\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)\sec(e+fx)\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-}}{(c-d)\sqrt{c+d}(bc-ad)f}$$

Mathematica [A] time = 7.34445, size = 263, normalized size = 0.64

$$\frac{2\left(-(bc-ad)\cos(e+fx)-\frac{\sqrt{2}\sqrt{\frac{a-b}{a+b}}(a+b)(c+d)\cos\left(\frac{1}{4}(2e+2fx-\pi)\right)\sqrt{\frac{a+b\sin(e+fx)}{a+b}}\sqrt{\frac{(a+b)(c+d\sin(e+fx))}{(c+d)(a+b\sin(e+fx))}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{a-b}{a+b}}\cos\left(\frac{1}{4}(2e+2fx+\pi)\right)}{\sqrt{\frac{a+b\sin(e+fx)}{a+b}}}\right)|\frac{2(ad-bc)}{(a-b)(c+d)}\right)}{\sqrt{\frac{(a+b)(\sin(e+fx)+1)}{a+b\sin(e+fx)}}}\right)}{f(c-d)(c+d)\sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] (2*(-((b*c - a*d)*Cos[e + f*x]) - (Sqrt[2]*Sqrt[(a - b)/(a + b)]*(a + b)*(c
 + d)*Cos[(2*e - Pi + 2*f*x)/4]*EllipticE[ArcSin[(Sqrt[(a - b)/(a + b)]*Cos
[(2*e + Pi + 2*f*x)/4])/Sqrt[(a + b*Sin[e + f*x])/(a + b)]], (2*(-(b*c) + a
*d))/((a - b)*(c + d))]*Sqrt[(a + b*Sin[e + f*x])/(a + b)]*Sqrt[((a + b)*(c
 + d*Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x]))])/Sqrt[((a + b)*(1 + Sin
[e + f*x]))/(a + b*Sin[e + f*x])]))/((c - d)*(c + d)*f*Sqrt[a + b*Sin[e + f
*x]]*Sqrt[c + d*Sin[e + f*x]])
```

Maple [B] time = 0.766, size = 46599, normalized size = 113.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b\sin\left(fx+e\right)+a}}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(b*sin(f*x + e) + a)/(d*sin(f*x + e) + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{b\sin\left(fx+e\right)+a}\sqrt{d\sin\left(fx+e\right)+c}}{d^2\cos\left(fx+e\right)^2-2\,cd\sin\left(fx+e\right)-c^2-d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] integral(-sqrt(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)/(d^2*cos(f*x +
e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+b\sin\left(e+fx\right)}}{\left(c+d\sin\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**(1/2)/(c+d*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral(sqrt(a + b*sin(e + f*x))/(c + d*sin(e + f*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b\sin\left(fx+e\right)+a}}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(b*sin(f*x + e) + a)/(d*sin(f*x + e) + c)^(3/2), x)
```

3.770 $\int \frac{\sqrt{a+b\sin(e+fx)}}{(c+d\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=489

$$\frac{2d\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{3f\left(c^2-d^2\right)(c+d\sin(e+fx))^{3/2}}+\frac{2(a-b)\sqrt{a+b}\left(4acd-b\left(3c^2+d^2\right)\right)\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(}{(a+b)(c+d\sin(}}}{3f(c-d)^2(c+d)^{3/2}(bc}$$

```
[Out] (2*d*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]])/(3*(c^2 - d^2)*f*(c + d*Sin[e +
 f*x])^(3/2)) + (2*(a - b)*Sqrt[a + b]*(4*a*c*d - b*(3*c^2 + d^2))*Elliptic
E[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin
[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c
- a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a
*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x
]))/(3*(c - d)^2*(c + d)^(3/2)*(b*c - a*d)^2*f) + (2*(a - b)*Sqrt[a + b]*(3
*c + d)*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b
]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sec[e +
f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*
Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*(c
 + d*Sin[e + f*x]))/(3*(c - d)^2*(c + d)^(3/2)*(b*c - a*d)*f)
```

Rubi [A] time = 0.86501, antiderivative size = 489, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2796, 2998, 2818, 2996}

$$\frac{2d\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{3f\left(c^2-d^2\right)(c+d\sin(e+fx))^{3/2}}+\frac{2(a-b)\sqrt{a+b}\left(4acd-b\left(3c^2+d^2\right)\right)\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(}{(a+b)(c+d\sin(}}}{3f(c-d)^2(c+d)^{3/2}(bc}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (2*d*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]])/(3*(c^2 - d^2)*f*(c + d*Sin[e +
 f*x])^(3/2)) + (2*(a - b)*Sqrt[a + b]*(4*a*c*d - b*(3*c^2 + d^2))*Elliptic
E[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin
[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c
- a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a
*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x
]))/(3*(c - d)^2*(c + d)^(3/2)*(b*c - a*d)^2*f) + (2*(a - b)*Sqrt[a + b]*(3
*c + d)*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b
]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sec[e +
f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*
Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*(c
 + d*Sin[e + f*x]))/(3*(c - d)^2*(c + d)^(3/2)*(b*c - a*d)*f)
```

Rule 2796

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m + 1)*(c + d*Sin[e + f*x])^n)/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m +
 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n
- 1)*Simp[a*c*(m + 1) + b*d*n + (a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] -
b*d*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1
```

```
] && LtQ[0, n, 1] && IntegersQ[2*m, 2*n]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2818

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*(c + d*Sin[e + f*x])*Sqrt[((b*c -
a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d
)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[
(c + d)/(a + b), 2]*(Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (
(a + b)*(c - d))/((a - b)*(c + d))])/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*
Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 2996

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(-2*A*(c - d)*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/
((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c
 + d)*(a + b*Sin[e + f*x])))]*EllipticE[ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt
[c + d*Sin[e + f*x]])/Sqrt[a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)
*(c - d))])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2]*Cos[e + f*x]), x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rubi steps

$$\int \frac{\sqrt{a+b\sin(e+fx)}}{(c+d\sin(e+fx))^{5/2}}\,dx = \frac{2d\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{3\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}} - \frac{2\int \frac{\frac{1}{2}(-3ac+bd)-\frac{1}{2}(3bc-ad)\sin(e+fx)}{\sqrt{a+b\sin(e+fx)}(c+d\sin(e+fx))^{3/2}}\,dx}{3\left(c^2-d^2\right)}$$

$$= \frac{2d\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{3\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}} + \frac{((a-b)(3c+d))\int \frac{1}{\sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\,dx}{3(c-d)^2(c+d)} - \frac{(4ac}{}$$

$$= \frac{2d\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{3\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}} + \frac{2(a-b)\sqrt{a+b}\left(4acd-b\left(3c^2+d^2\right)\right)E\left(\sin^{-1}\left(\frac{\sqrt{c+d}\sqrt{a}}{\sqrt{a+b}\sqrt{c}}\right.\right.}{}$$

Mathematica [B] time = 6.39381, size = 2037, normalized size = 4.17

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]*((2*d*Cos[e + f*x])/(3*(
c^2 - d^2)*(c + d*Sin[e + f*x])^2) + (2*(3*b*c^2*d*Cos[e + f*x] - 4*a*c*d^2
*Cos[e + f*x] + b*d^3*Cos[e + f*x]))/(3*(b*c - a*d)*(c^2 - d^2)^2*(c + d*Si
n[e + f*x]))))/f + ((-4*(-(b*c) + a*d)*(-3*a*b*c^3 + 3*a^2*c^2*d + b^2*c^2*
d - a*b*c*d^2 + a^2*d^3 - b^2*d^3)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2
)/(-c + d)]*EllipticF[ArcSin[Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c
+ d*Sin[e + f*x]))/(-(b*c) + a*d))]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(
-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + P
i/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[-(((a + b)*Csc[(
-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))])/((a + b)*(c +
 d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - 4*(-(b*c) + a*d)*(
-3*b^2*c^3 + a*b*c^2*d + 4*a^2*c*d^2 - b^2*c*d^2 - a*b*d^3)*((Sqrt[((c + d)
*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[-(((a + b)*Csc
[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))]/Sqrt[2]], (2
*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^
4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) +
a*d)]*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b
*c) + a*d))])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e +
f*x]]) - (Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-
(b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2
*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))]/Sqrt[2]], (2*(-(b*c) + a*d))/((a +
b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e
 + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[-(((a + b)*C
sc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))])/((a + b)*
d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])) + 2*(3*b^2*c^2*d - 4*
a*b*c*d^2 + b^2*d^3)*((Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(d*Sqrt[a + b
*Sin[e + f*x]]) + (Sqrt[(a - b)/(a + b)]*(a + b)*Cos[(-e + Pi/2 - f*x)/2]*E
llipticE[ArcSin[(Sqrt[(a - b)/(a + b)]*Sin[(-e + Pi/2 - f*x)/2])/Sqrt[(a +
b*Sin[e + f*x])/(a + b)]], (2*(-(b*c) + a*d))/((a - b)*(c + d))]*Sqrt[c + d
*Sin[e + f*x]])/(b*d*Sqrt[((a + b)*Cos[(-e + Pi/2 - f*x)/2]^2)/(a + b*Sin[e
 + f*x])]*Sqrt[a + b*Sin[e + f*x]]*Sqrt[(a + b*Sin[e + f*x])/(a + b)]*Sqrt[
((a + b)*(c + d*Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x]))]) - (2*(-(b*c
) + a*d)*((((a + b)*c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c
+ d)]*EllipticF[ArcSin[Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Si
n[e + f*x]))/(-(b*c) + a*d))]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d
))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 -
f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[-(((a + b)*Csc[(-e + P
i/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))])/((a + b)*(c + d)*Sq
rt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - ((b*c + a*d)*Sqrt[((c +
d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a + b)
*d), ArcSin[Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x])
)/(-(b*c) + a*d))]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e +
 f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(
a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/
2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))])/((a + b)*d*Sqrt[a + b*Sin[e +
f*x]]*Sqrt[c + d*Sin[e + f*x]])))/(b*d)))/(3*(c - d)^2*(c + d)^2*(-(b*c) +
a*d)*f)
```

Maple [B] time = 2.937, size = 197178, normalized size = 403.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b\sin\left(fx+e\right)+a}}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(b*sin(f*x + e) + a)/(d*sin(f*x + e) + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{b\sin\left(fx+e\right)+a}\sqrt{d\sin\left(fx+e\right)+c}}{3\,cd^2\cos\left(fx+e\right)^2-c^3-3\,cd^2+\left(d^3\cos\left(fx+e\right)^2-3\,c^2d-d^3\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] integral(-sqrt(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)/(3*c*d^2*cos(f*
x + e)^2 - c^3 - 3*c*d^2 + (d^3*cos(f*x + e)^2 - 3*c^2*d - d^3)*sin(f*x + e
)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**(1/2)/(c+d*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b\sin\left(fx+e\right)+a}}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(b*sin(f*x + e) + a)/(d*sin(f*x + e) + c)^(5/2), x)
```

3.771 $\int (a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{5/2}\,dx$

Optimal. Leaf size=1080

result too large to display

```
[Out] (Sqrt[a + b]*(c - d)*Sqrt[c + d]*(57*a^2*b*c*d^2 - 9*a^3*d^3 + a*b^2*d*(337
*c^2 + 156*d^2) + b^3*(15*c^3 + 284*c*d^2))*EllipticE[ArcSin[(Sqrt[a + b]*S
qrt[c + d*Sin[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*
(c + d))/((a + b)*(c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f
*x]))/((c + d)*(a + b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x])
)/((c - d)*(a + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/(192*b^2*d*(b*c - a
*d)*f) - (Sqrt[c + d]*(20*a^3*b*c*d^3 - 3*a^4*d^4 - 60*a*b^3*c*d*(c^2 + 4*d
^2) - 6*a^2*b^2*d^2*(15*c^2 + 4*d^2) + b^4*(5*c^4 - 120*c^2*d^2 - 48*d^4))*
EllipticPi[(b*(c + d))/((a + b)*d), ArcSin[(Sqrt[a + b]*Sqrt[c + d*Sin[e +
f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/((a + b)*
(c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c + d)*(a
+ b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d)*(a + b*
Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/(64*b^3*Sqrt[a + b]*d^2*f) - ((57*a^2
*b*c*d^2 - 9*a^3*d^3 + a*b^2*d*(337*c^2 + 156*d^2) + b^3*(15*c^3 + 284*c*d^
2))*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(192*b*d*f*Sqrt[a + b*Sin[e + f*
x]]) - ((54*a*b*c*d - 9*a^2*d^2 + b^2*(59*c^2 + 36*d^2))*Cos[e + f*x]*Sqrt[
a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])/(96*b*f) - (d*(17*b*c - 3*a*d
)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]])/(24*b*f
) - (d^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^(5/2)*Sqrt[c + d*Sin[e + f*x]])/
(4*b*f) + ((a + b)^(3/2)*(9*a^3*d^3 - 3*a^2*b*d^2*(17*c + 6*d) + 3*a*b^2*d*
(73*c^2 + 36*c*d + 28*d^2) + b^3*(15*c^3 + 118*c^2*d + 284*c*d^2 + 72*d^3))
*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[
c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sec[e + f*x]*Sq
rt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(
((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*(c + d*Si
n[e + f*x]))/(192*b^3*d*Sqrt[c + d]*f)
```

Rubi [A] time = 5.23808, antiderivative size = 1080, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2793, 3049, 3061, 3053, 2811, 2998, 2818, 2996}

$$-\frac{d^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}(a+b\sin(e+fx))^{5/2}}{4bf}-\frac{d(17bc-3ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}(a+b\sin(e+\ldots}{24bf}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^(5/2),x]
```


```
[Out] (Sqrt[a + b]*(c - d)*Sqrt[c + d]*(57*a^2*b*c*d^2 - 9*a^3*d^3 + a*b^2*d*(337
*c^2 + 156*d^2) + b^3*(15*c^3 + 284*c*d^2))*EllipticE[ArcSin[(Sqrt[a + b]*S
qrt[c + d*Sin[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*
(c + d))/((a + b)*(c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f
*x]))/((c + d)*(a + b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x])
)/((c - d)*(a + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/(192*b^2*d*(b*c - a
*d)*f) - (Sqrt[c + d]*(20*a^3*b*c*d^3 - 3*a^4*d^4 - 60*a*b^3*c*d*(c^2 + 4*d
^2) - 6*a^2*b^2*d^2*(15*c^2 + 4*d^2) + b^4*(5*c^4 - 120*c^2*d^2 - 48*d^4))*
EllipticPi[(b*(c + d))/((a + b)*d), ArcSin[(Sqrt[a + b]*Sqrt[c + d*Sin[e +
f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/((a + b)*
(c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c + d)*(a
```

```
+ b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d)*(a + b*
Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/(64*b^3*Sqrt[a + b]*d^2*f) - ((57*a^2
*b*c*d^2 - 9*a^3*d^3 + a*b^2*d*(337*c^2 + 156*d^2) + b^3*(15*c^3 + 284*c*d^
2))*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(192*b*d*f*Sqrt[a + b*Sin[e + f*
x]]) - ((54*a*b*c*d - 9*a^2*d^2 + b^2*(59*c^2 + 36*d^2))*Cos[e + f*x]*Sqrt[
a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])/(96*b*f) - (d*(17*b*c - 3*a*d
)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]])/(24*b*f
) - (d^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^(5/2)*Sqrt[c + d*Sin[e + f*x]])/
(4*b*f) + ((a + b)^(3/2)*(9*a^3*d^3 - 3*a^2*b*d^2*(17*c + 6*d) + 3*a*b^2*d*
(73*c^2 + 36*c*d + 28*d^2) + b^3*(15*c^3 + 118*c^2*d + 284*c*d^2 + 72*d^3))
*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[
c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sec[e + f*x]*Sq
rt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(
((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*(c + d*Si
n[e + f*x]))/(192*b^3*d*Sqrt[c + d]*f)
```

Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
```

```
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2811

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[(2*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d
)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(
1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*EllipticPi[(b*(c + d))/
(d*(a + b)), ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt[c + d*Sin[e + f*x]])/Sqrt[
a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)*(c - d))])/(d*f*Rt[(a + b)
/(c + d), 2]*Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2818

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*(c + d*Sin[e + f*x])*Sqrt[((b*c -
a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d
)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[
(c + d)/(a + b), 2]*(Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (
(a + b)*(c - d))/((a - b)*(c + d))])/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*
Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 2996

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(-2*A*(c - d)*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/
((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c
 + d)*(a + b*Sin[e + f*x])))]*EllipticE[ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt
[c + d*Sin[e + f*x]])/Sqrt[a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)
*(c - d))])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2]*Cos[e + f*x]), x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rubi steps

$$\int (a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{5/2}\,dx = -\frac{d^2\cos(e+fx)(a+b\sin(e+fx))^{5/2}\sqrt{c+d\sin(e+fx)}}{4bf} + \frac{\int \frac{(a+b\sin(e+\ldots}{\ldots}}{\ldots}$$

$$= -\frac{d(17bc-3ad)\cos(e+fx)(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}{24bf} - \ldots$$

$$= -\frac{\left(54abcd-9a^2d^2+b^2\left(59c^2+36d^2\right)\right)\cos(e+fx)\sqrt{a+b\sin(e+fx)}\sqrt{\ldots}}{96bf}$$

$$= -\frac{\left(57a^2bcd^2-9a^3d^3+ab^2d\left(337c^2+156d^2\right)+b^3\left(15c^3+284cd^2\right)\right)\cos\ldots}{192bdf\sqrt{a+b\sin(e+fx)}}$$

$$= -\frac{\left(57a^2bcd^2-9a^3d^3+ab^2d\left(337c^2+156d^2\right)+b^3\left(15c^3+284cd^2\right)\right)\cos\ldots}{192bdf\sqrt{a+b\sin(e+fx)}}$$

$$= -\frac{\sqrt{c+d}\left(20a^3bcd^3-3a^4d^4-60ab^3cd\left(c^2+4d^2\right)-6a^2b^2d^2\left(15c^2+4d\ldots\right.\right.}{\ldots}$$

$$= \frac{\sqrt{a+b}(c-d)\sqrt{c+d}\left(57a^2bcd^2-9a^3d^3+ab^2d\left(337c^2+156d^2\right)+b^3\left(1\ldots\right.\right.}{\ldots}$$

Mathematica [A] time = 7.47473, size = 2061, normalized size = 1.91

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] -((-4*(-(b*c) + a*d)*(-384*a^2*b*c^3 - 133*b^3*c^3 - 971*a*b^2*c^2*d - 451*
a^2*b*c*d^2 - 356*b^3*c*d^2 + 3*a^3*d^3 - 228*a*b^2*d^3)*Sqrt[((c + d)*Cot[
(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[-(((a + b)*Csc[(-e
+ Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))]/Sqrt[2]], (2*(-(b
*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqr
t[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]
*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) +
 a*d))])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]
) - 4*(-(b*c) + a*d)*(-532*a*b^2*c^3 - 664*a^2*b*c^2*d - 644*b^3*c^2*d + 12
*a^3*c*d^2 - 1160*a*b^2*c*d^2 - 228*a^2*b*d^3 - 144*b^3*d^3)*((Sqrt[((c + d
)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[-(((a + b)*Cs
c[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))]/Sqrt[2]], (
2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]
^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) +
 a*d)]*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(
b*c) + a*d))])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e +
 f*x]]) - (Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(
-(b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^
2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))]/Sqrt[2]], (2*(-(b*c) + a*d))/((a +
 b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-
e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[-(((a + b)*
Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))])/((a + b)
*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])) + 2*(15*b^3*c^3 + 33
```

```
7*a*b^2*c^2*d + 57*a^2*b*c*d^2 + 284*b^3*c*d^2 - 9*a^3*d^3 + 156*a*b^2*d^3)
*((Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(d*Sqrt[a + b*Sin[e + f*x]]) + (S
qrt[(a - b)/(a + b)]*(a + b)*Cos[(-e + Pi/2 - f*x)/2]*EllipticE[ArcSin[(Sqr
t[(a - b)/(a + b)]*Sin[(-e + Pi/2 - f*x)/2])/Sqrt[(a + b*Sin[e + f*x])/(a +
 b)]], (2*(-(b*c) + a*d))/((a - b)*(c + d))]*Sqrt[c + d*Sin[e + f*x]])/(b*d
*Sqrt[((a + b)*Cos[(-e + Pi/2 - f*x)/2]^2)/(a + b*Sin[e + f*x])]*Sqrt[a + b
*Sin[e + f*x]]*Sqrt[(a + b*Sin[e + f*x])/(a + b)]*Sqrt[((a + b)*(c + d*Sin[
e + f*x]))/((c + d)*(a + b*Sin[e + f*x]))]) - (2*(-(b*c) + a*d)*((((a + b)*
c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcS
in[Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c)
 + a*d))]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin
[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin
[e + f*x]))/(-(b*c) + a*d)]*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c +
 d*Sin[e + f*x]))/(-(b*c) + a*d))])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]]) - ((b*c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 -
f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[-((
(a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))]/S
qrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2
 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))
/(-(b*c) + a*d)]*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e +
f*x]))/(-(b*c) + a*d))])/((a + b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin
[e + f*x]]))))/(b*d)))/(384*b*f) + (Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[
e + f*x]]*(-((59*b^2*c^2 + 122*a*b*c*d + 3*a^2*d^2 + 42*b^2*d^2)*Cos[e + f*
x])/(96*b) + (b*d^2*Cos[3*(e + f*x)])/16 - (d*(17*b*c + 9*a*d)*Sin[2*(e + f
*x)])/48))/f
```

Maple [B] time = 20.865, size = 577146, normalized size = 534.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^(5/2),x)

[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^(5/2),x, algorithm="maxim
a")

[Out] integrate((b*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e) + c)^(5/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**(3/2)*(c+d*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError
```

3.772 $\int (a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=870

$$-\frac{\left(-\left(3c^2+14dc+16d^2\right)b^2-6ad(4c+d)b+3a^2d^2\right)F\left(\sin^{-1}\left(\frac{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}\right)|\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)\sec(e+fx)\sqrt{\frac{(bc-ad)(1-\sin(e}{(a+b)(c+d\sin(e}}}{24b^2d\sqrt{c+d}f}$$

```
[Out] (Sqrt[a + b]*(c - d)*Sqrt[c + d]*(38*a*b*c*d + 3*a^2*d^2 + b^2*(3*c^2 + 16*
d^2))*EllipticE[ArcSin[(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])/(Sqrt[c + d]*
Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/((a + b)*(c - d))]*Sec[e + f*
x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]
*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*(a +
 b*Sin[e + f*x]))/(24*b*d*(b*c - a*d)*f) + (Sqrt[c + d]*(b*c + a*d)*(10*a*b
*c*d - a^2*d^2 - b^2*(c^2 - 12*d^2))*EllipticPi[(b*(c + d))/((a + b)*d), Ar
cSin[(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e +
 f*x]])], ((a - b)*(c + d))/((a + b)*(c - d))]*Sec[e + f*x]*Sqrt[-(((b*c -
a*d)*(1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)
*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/
(8*b^2*Sqrt[a + b]*d^2*f) - ((38*a*b*c*d + 3*a^2*d^2 + b^2*(3*c^2 + 16*d^2)
)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(24*d*f*Sqrt[a + b*Sin[e + f*x]])
- ((3*b*c + 7*a*d)*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e +
 f*x]])/(12*f) - ((a + b)^(3/2)*(3*a^2*d^2 - 6*a*b*d*(4*c + d) - b^2*(3*c^2
 + 14*c*d + 16*d^2))*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]]
)/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c +
d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin
[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e
+ f*x])))]*(c + d*Sin[e + f*x]))/(24*b^2*d*Sqrt[c + d]*f) - (b*Cos[e + f*x]
*Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(3/2))/(3*f)
```

Rubi [A] time = 3.35895, antiderivative size = 870, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2821, 3049, 3061, 3053, 2811, 2998, 2818, 2996}

$$-\frac{\left(-\left(3c^2+14dc+16d^2\right)b^2-6ad(4c+d)b+3a^2d^2\right)F\left(\sin^{-1}\left(\frac{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}\right)|\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)\sec(e+fx)\sqrt{\frac{(bc-ad)(1-\sin(e}{(a+b)(c+d\sin(e}}}{24b^2d\sqrt{c+d}f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] (Sqrt[a + b]*(c - d)*Sqrt[c + d]*(38*a*b*c*d + 3*a^2*d^2 + b^2*(3*c^2 + 16*
d^2))*EllipticE[ArcSin[(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])/(Sqrt[c + d]*
Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/((a + b)*(c - d))]*Sec[e + f*
x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]
*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*(a +
 b*Sin[e + f*x]))/(24*b*d*(b*c - a*d)*f) + (Sqrt[c + d]*(b*c + a*d)*(10*a*b
*c*d - a^2*d^2 - b^2*(c^2 - 12*d^2))*EllipticPi[(b*(c + d))/((a + b)*d), Ar
cSin[(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e +
 f*x]])], ((a - b)*(c + d))/((a + b)*(c - d))]*Sec[e + f*x]*Sqrt[-(((b*c -
a*d)*(1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)
*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/
(8*b^2*Sqrt[a + b]*d^2*f) - ((38*a*b*c*d + 3*a^2*d^2 + b^2*(3*c^2 + 16*d^2)
)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(24*d*f*Sqrt[a + b*Sin[e + f*x]])
- ((3*b*c + 7*a*d)*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e +
```

```
f*x]])/(12*f) - ((a + b)^(3/2)*(3*a^2*d^2 - 6*a*b*d*(4*c + d) - b^2*(3*c^2
 + 14*c*d + 16*d^2))*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]]
)/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c +
d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin
[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e
+ f*x])))]*(c + d*Sin[e + f*x]))/(24*b^2*d*Sqrt[c + d]*f) - (b*Cos[e + f*x]
*Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(3/2))/(3*f)
```

Rule 2821

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[1/(d*(m + n)), Int[
(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a^2*c*d*(m +
 n) + b*d*(b*c*(m - 1) + a*d*n) + (a*d*(2*b*c + a*d)*(m + n) - b*d*(a*c - b
*d*(m + n - 1)))*Sin[e + f*x] + b*d*(b*c*n + a*d*(2*m + n - 1))*Sin[e + f*x
]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[0, m, 2] && LtQ[-1, n, 2] && NeQ
[m + n, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2811

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[(2*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d
)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(
```

```
1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x]))))]*EllipticPi[(b*(c + d))/
(d*(a + b)), ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt[c + d*Sin[e + f*x]])/Sqrt[
a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)*(c - d))])/(d*f*Rt[(a + b)
/(c + d), 2]*Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2818

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*(c + d*Sin[e + f*x])*Sqrt[((b*c -
a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d
)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[
(c + d)/(a + b), 2]*(Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (
(a + b)*(c - d))/((a - b)*(c + d))])/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*
Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 2996

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(-2*A*(c - d)*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/
((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c
 + d)*(a + b*Sin[e + f*x])))]*EllipticE[ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt
[c + d*Sin[e + f*x]])/Sqrt[a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)
*(c - d))])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2]*Cos[e + f*x]), x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rubi steps

$$\int (a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{3/2}\,dx = -\frac{b\cos(e+fx)\sqrt{a+b\sin(e+fx)}(c+d\sin(e+fx))^{3/2}}{3f} + \frac{\int \frac{\sqrt{c+d\sin(e+f}}{}}{}$$

$$= -\frac{(3bc+7ad)\cos(e+fx)\sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}{12f} - \frac{b\cos}{}$$

$$= -\frac{\left(38abcd+3a^2d^2+b^2\left(3c^2+16d^2\right)\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{24df\sqrt{a+b\sin(e+fx)}} -$$

$$= -\frac{\left(38abcd+3a^2d^2+b^2\left(3c^2+16d^2\right)\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{24df\sqrt{a+b\sin(e+fx)}} -$$

$$= \frac{\sqrt{c+d}(bc+ad)\left(10abcd-a^2d^2-b^2\left(c^2-12d^2\right)\right)\Pi\left(\frac{b(c+d)}{(a+b)d};\sin^{-1}\left(\frac{\sqrt{a+b}}{\sqrt{c+d}}\right.\right.}{}$$

$$= \frac{\sqrt{a+b}(c-d)\sqrt{c+d}\left(38abcd+3a^2d^2+b^2\left(3c^2+16d^2\right)\right)E\left(\sin^{-1}\left(\frac{\sqrt{a+}}{\sqrt{c+}}\right.\right.}{}$$

Mathematica [B] time = 6.2906, size = 1922, normalized size = 2.21

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] ((-4*(-(b*c) + a*d)*(48*a^2*c^2 + 17*b^2*c^2 + 82*a*b*c*d + 17*a^2*d^2 + 16
*b^2*d^2)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[Arc
Sin[Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c
) + a*d))]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Si
n[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Si
n[e + f*x]))/(-(b*c) + a*d)]*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c
+ d*Sin[e + f*x]))/(-(b*c) + a*d))])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]) - 4*(-(b*c) + a*d)*(68*a*b*c^2 + 68*a^2*c*d +
 52*b^2*c*d + 52*a*b*d^2)*((Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c +
 d)]*EllipticF[ArcSin[Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin
[e + f*x]))/(-(b*c) + a*d))]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d)
)]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f
*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[-(((a + b)*Csc[(-e + Pi
/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))])/((a + b)*(c + d)*Sqr
t[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - (Sqrt[((c + d)*Cot[(-e +
Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a + b)*d), ArcSin[S
qrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a
*d))]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e
 + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e +
 f*x]))/(-(b*c) + a*d)]*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*S
in[e + f*x]))/(-(b*c) + a*d))])/((a + b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c
+ d*Sin[e + f*x]])) + 2*(-3*b^2*c^2 - 38*a*b*c*d - 3*a^2*d^2 - 16*b^2*d^2)*
((Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(d*Sqrt[a + b*Sin[e + f*x]]) + (Sq
rt[(a - b)/(a + b)]*(a + b)*Cos[(-e + Pi/2 - f*x)/2]*EllipticE[ArcSin[(Sqrt
[(a - b)/(a + b)]*Sin[(-e + Pi/2 - f*x)/2])/Sqrt[(a + b*Sin[e + f*x])/(a +
b)]], (2*(-(b*c) + a*d))/((a - b)*(c + d))]*Sqrt[c + d*Sin[e + f*x]])/(b*d*
```

```
Sqrt[((a + b)*Cos[(-e + Pi/2 - f*x)/2]^2)/(a + b*Sin[e + f*x])]*Sqrt[a + b*
Sin[e + f*x]]*Sqrt[(a + b*Sin[e + f*x])/(a + b)]*Sqrt[((a + b)*(c + d*Sin[e
 + f*x]))/((c + d)*(a + b*Sin[e + f*x]))]) - (2*(-(b*c) + a*d)*((((a + b)*c
 + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSi
n[Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c)
+ a*d))]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[
(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[
e + f*x]))/(-(b*c) + a*d)]*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c +
d*Sin[e + f*x]))/(-(b*c) + a*d))])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]
]*Sqrt[c + d*Sin[e + f*x]]) - ((b*c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f
*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[-(((
a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))]/Sq
rt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2
- f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/
(-(b*c) + a*d)]*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f
*x]))/(-(b*c) + a*d))])/((a + b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[
e + f*x]])))/(b*d)))/(48*f) + (Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e +
f*x]]*((-7*(b*c + a*d)*Cos[e + f*x])/12 - (b*d*Sin[2*(e + f*x)])/6))/f
```

Maple [C] time = 8.178, size = 409584, normalized size = 470.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^(3/2),x)

[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^(3/2),x, algorithm="maxim
a")

[Out] integrate((b*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e) + c)^(3/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^(3/2),x, algorithm="frica
s")

[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**(3/2)*(c+d*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError
```

3.773 $\int (a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}\,dx$

Optimal. Leaf size=740

$$\frac{\sqrt{c+d}\left(3a^2d^2+6abcd+b^2\left(-\left(c^2-4d^2\right)\right)\right)\sec(e+fx)(a+b\sin(e+fx))\sqrt{-\frac{(bc-ad)(1-\sin(e+fx))}{(c+d)(a+b\sin(e+fx))}}\sqrt{\frac{(bc-ad)(\sin(e+fx)+1)}{(c-d)(a+b\sin(e+fx))}}\Pi\left(\right.}{4bd^2f\sqrt{a+b}}$$

```
[Out] (Sqrt[a + b]*(c - d)*Sqrt[c + d]*(b*c + 5*a*d)*EllipticE[ArcSin[(Sqrt[a + b
]*Sqrt[c + d*Sin[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a -
b)*(c + d))/((a + b)*(c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e
+ f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*
x]))/((c - d)*(a + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/(4*d*(b*c - a*d)
*f) + (Sqrt[c + d]*(6*a*b*c*d + 3*a^2*d^2 - b^2*(c^2 - 4*d^2))*EllipticPi[(
b*(c + d))/((a + b)*d), ArcSin[(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])/(Sqrt
[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/((a + b)*(c - d))]*Se
c[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e +
f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]
))]*(a + b*Sin[e + f*x]))/(4*b*Sqrt[a + b]*d^2*f) - (b*(b*c + 5*a*d)*Cos[e
+ f*x]*Sqrt[c + d*Sin[e + f*x]])/(4*d*f*Sqrt[a + b*Sin[e + f*x]]) - (b*Cos[
e + f*x]*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])/(2*f) + ((a + b
)^(3/2)*(3*a*d + b*(c + 2*d))*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[
e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a -
 b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(
c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c +
 d*Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/(4*b*d*Sqrt[c + d]*f)
```

Rubi [A] time = 2.29439, antiderivative size = 740, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2821, 3061, 3053, 2811, 2998, 2818, 2996}

$$\frac{\sqrt{c+d}\left(3a^2d^2+6abcd+b^2\left(-\left(c^2-4d^2\right)\right)\right)\sec(e+fx)(a+b\sin(e+fx))\sqrt{-\frac{(bc-ad)(1-\sin(e+fx))}{(c+d)(a+b\sin(e+fx))}}\sqrt{\frac{(bc-ad)(\sin(e+fx)+1)}{(c-d)(a+b\sin(e+fx))}}\Pi\left(\right.}{4bd^2f\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] (Sqrt[a + b]*(c - d)*Sqrt[c + d]*(b*c + 5*a*d)*EllipticE[ArcSin[(Sqrt[a + b
]*Sqrt[c + d*Sin[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a -
b)*(c + d))/((a + b)*(c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e
+ f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*
x]))/((c - d)*(a + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/(4*d*(b*c - a*d)
*f) + (Sqrt[c + d]*(6*a*b*c*d + 3*a^2*d^2 - b^2*(c^2 - 4*d^2))*EllipticPi[(
b*(c + d))/((a + b)*d), ArcSin[(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])/(Sqrt
[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/((a + b)*(c - d))]*Se
c[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e +
f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]
))]*(a + b*Sin[e + f*x]))/(4*b*Sqrt[a + b]*d^2*f) - (b*(b*c + 5*a*d)*Cos[e
+ f*x]*Sqrt[c + d*Sin[e + f*x]])/(4*d*f*Sqrt[a + b*Sin[e + f*x]]) - (b*Cos[
e + f*x]*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])/(2*f) + ((a + b
)^(3/2)*(3*a*d + b*(c + 2*d))*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[
e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a -
 b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(
c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c +
```

```
 d*Sin[e + f*x]))))]*(c + d*Sin[e + f*x]))/(4*b*d*Sqrt[c + d]*f)
```

Rule 2821

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[1/(d*(m + n)), Int[
(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a^2*c*d*(m +
 n) + b*d*(b*c*(m - 1) + a*d*n) + (a*d*(2*b*c + a*d)*(m + n) - b*d*(a*c - b
*d*(m + n - 1)))*Sin[e + f*x] + b*d*(b*c*n + a*d*(2*m + n - 1))*Sin[e + f*x
]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[0, m, 2] && LtQ[-1, n, 2] && NeQ
[m + n, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2811

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[(2*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d
)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(
1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*EllipticPi[(b*(c + d))/
(d*(a + b)), ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt[c + d*Sin[e + f*x]])/Sqrt[
a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)*(c - d))])/(d*f*Rt[(a + b)
/(c + d), 2]*Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2818

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
```

```
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*(c + d*Sin[e + f*x])*Sqrt[((b*c -
a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d
)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[
(c + d)/(a + b), 2]*(Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (
(a + b)*(c - d))/((a - b)*(c + d))])/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*
Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 2996

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(-2*A*(c - d)*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/
((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c
 + d)*(a + b*Sin[e + f*x])))]*EllipticE[ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt
[c + d*Sin[e + f*x]])/Sqrt[a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)
*(c - d))])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2]*Cos[e + f*x]), x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rubi steps

$$\int (a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}\,dx = -\frac{b\cos(e+fx)\sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}{2f} + \frac{\int \frac{\frac{1}{2}d\left(4a^2c+b^2c+\ldots\right.}{\ldots}}{\ldots}$$

$$= -\frac{b(bc+5ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4df\sqrt{a+b\sin(e+fx)}} - \frac{b\cos(e+fx)\sqrt{a+b\,\mathrm{si}\ldots}}{\ldots}$$

$$= -\frac{b(bc+5ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4df\sqrt{a+b\sin(e+fx)}} - \frac{b\cos(e+fx)\sqrt{a+b\,\mathrm{si}\ldots}}{\ldots}$$

$$= \frac{\sqrt{c+d}\left(6ac-\frac{bc^2}{d}+\frac{3a^2d}{b}+4bd\right)\Pi\left(\frac{b(c+d)}{(a+b)d};\sin^{-1}\left(\frac{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}\right)|\ldots\right)}{\ldots}$$

$$= \frac{\sqrt{a+b}(c-d)\sqrt{c+d}(bc+5ad)E\left(\sin^{-1}\left(\frac{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)}{4d(bc\ldots}$$

Mathematica [B] time = 9.45004, size = 1849, normalized size = 2.5

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] -(b*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])/(2*f) +
 ((-4*(-(b*c) + a*d)*(8*a^2*c + 3*b^2*c + 7*a*b*d)*Sqrt[((c + d)*Cot[(-e +
Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[-(((a + b)*Csc[(-e + Pi/2
 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))]/Sqrt[2]], (2*(-(b*c) +
a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c
+ d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[
-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))
])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - 4*
```

```
(-(b*c) + a*d)*(12*a*b*c + 8*a^2*d + 4*b^2*d)*((Sqrt[((c + d)*Cot[(-e + Pi/
2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[-(((a + b)*Csc[(-e + Pi/2 -
f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))]/Sqrt[2]], (2*(-(b*c) + a*d
))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d
)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[-((
(a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))])/
((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - (Sqrt
[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c) + a*d)/(
(a + b)*d), ArcSin[Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e
+ f*x]))/(-(b*c) + a*d))]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*
Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)
/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[-(((a + b)*Csc[(-e + Pi/2
- f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))])/((a + b)*d*Sqrt[a + b*S
in[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])) + 2*(-(b^2*c) - 5*a*b*d)*((Cos[e +
f*x]*Sqrt[c + d*Sin[e + f*x]])/(d*Sqrt[a + b*Sin[e + f*x]]) + (Sqrt[(a - b)
/(a + b)]*(a + b)*Cos[(-e + Pi/2 - f*x)/2]*EllipticE[ArcSin[(Sqrt[(a - b)/(
a + b)]*Sin[(-e + Pi/2 - f*x)/2])/Sqrt[(a + b*Sin[e + f*x])/(a + b)]], (2*(
-(b*c) + a*d))/((a - b)*(c + d))]*Sqrt[c + d*Sin[e + f*x]])/(b*d*Sqrt[((a +
 b)*Cos[(-e + Pi/2 - f*x)/2]^2)/(a + b*Sin[e + f*x])]*Sqrt[a + b*Sin[e + f*
x]]*Sqrt[(a + b*Sin[e + f*x])/(a + b)]*Sqrt[((a + b)*(c + d*Sin[e + f*x]))/
((c + d)*(a + b*Sin[e + f*x]))]) - (2*(-(b*c) + a*d)*((((a + b)*c + a*d)*Sq
rt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[-((
(a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))]/S
qrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2
 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))
/(-(b*c) + a*d)]*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e +
f*x]))/(-(b*c) + a*d))])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c +
 d*Sin[e + f*x]]) - ((b*c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/
(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[-(((a + b)*Csc
[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))]/Sqrt[2]], (2
*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^
4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) +
a*d)]*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b
*c) + a*d))])/((a + b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])
))/(b*d)))/(8*f)
```

Maple [C] time = 3.464, size = 278382, normalized size = 376.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\sqrt{d\sin\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate((b*sin(f*x + e) + a)^(3/2)*sqrt(d*sin(f*x + e) + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\sqrt{d\sin\left(fx+e\right)+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] integral((b*sin(f*x + e) + a)^(3/2)*sqrt(d*sin(f*x + e) + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**(3/2)*(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: AttributeError
```

3.774 $\int \frac{(a+b\sin(e+fx))^{3/2}}{\sqrt{c+d\sin(e+fx)}}\,dx$

Optimal. Leaf size=644

$$\frac{\sqrt{a+b}(b(c-d)-2ad)\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}}F\left(\sin^{-1}\left(\frac{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}\right.\right.}{d^2f\sqrt{c+d}}$$

```
[Out] -((b*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]])/(f*Sqrt[c + d*Sin[e + f*x]])) -
 ((a - b)*b*Sqrt[a + b]*Sqrt[c + d]*EllipticE[ArcSin[(Sqrt[c + d]*Sqrt[a +
b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))
/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a
+ b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b
)*(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/(d*(b*c - a*d)*f) + (Sqrt[a
 + b]*(b*(c - d) - 2*a*d)*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e +
f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*
(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c +
d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*S
in[e + f*x])))]*(c + d*Sin[e + f*x]))/(d^2*Sqrt[c + d]*f) - (Sqrt[a + b]*(b
*c - 3*a*d)*EllipticPi[((a + b)*d)/(b*(c + d)), ArcSin[(Sqrt[c + d]*Sqrt[a
+ b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d
))/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((
a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a -
 b)*(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/(d^2*Sqrt[c + d]*f)
```

Rubi [A] time = 1.53792, antiderivative size = 644, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2821, 3053, 2811, 2998, 2818, 2996}

$$\frac{\sqrt{a+b}(b(c-d)-2ad)\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}}F\left(\sin^{-1}\left(\frac{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}\right.\right.}{d^2f\sqrt{c+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^(3/2)/Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] -((b*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]])/(f*Sqrt[c + d*Sin[e + f*x]])) -
 ((a - b)*b*Sqrt[a + b]*Sqrt[c + d]*EllipticE[ArcSin[(Sqrt[c + d]*Sqrt[a +
b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))
/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a
+ b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b
)*(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/(d*(b*c - a*d)*f) + (Sqrt[a
 + b]*(b*(c - d) - 2*a*d)*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e +
f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*
(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c +
d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*S
in[e + f*x])))]*(c + d*Sin[e + f*x]))/(d^2*Sqrt[c + d]*f) - (Sqrt[a + b]*(b
*c - 3*a*d)*EllipticPi[((a + b)*d)/(b*(c + d)), ArcSin[(Sqrt[c + d]*Sqrt[a
+ b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d
))/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((
a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a -
 b)*(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/(d^2*Sqrt[c + d]*f)
```

Rule 2821

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[1/(d*(m + n)), Int[
(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a^2*c*d*(m +
 n) + b*d*(b*c*(m - 1) + a*d*n) + (a*d*(2*b*c + a*d)*(m + n) - b*d*(a*c - b
*d*(m + n - 1)))*Sin[e + f*x] + b*d*(b*c*n + a*d*(2*m + n - 1))*Sin[e + f*x
]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[0, m, 2] && LtQ[-1, n, 2] && NeQ
[m + n, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2811

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[(2*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d
)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(
1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*EllipticPi[(b*(c + d))/
(d*(a + b)), ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt[c + d*Sin[e + f*x]])/Sqrt[
a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)*(c - d))])/(d*f*Rt[(a + b)
/(c + d), 2]*Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2818

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*(c + d*Sin[e + f*x])*Sqrt[((b*c -
a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d
)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[
(c + d)/(a + b), 2]*(Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (
(a + b)*(c - d))/((a - b)*(c + d))])/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*
Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 2996

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(-2*A*(c - d)*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/
((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c
 + d)*(a + b*Sin[e + f*x])))]*EllipticE[ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt
```

```
[c + d*Sin[e + f*x]])/Sqrt[a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)
*(c - d))])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2]*Cos[e + f*x]), x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rubi steps

$$\int \frac{(a+b\sin(e+fx))^{3/2}}{\sqrt{c+d\sin(e+fx)}}\,dx = -\frac{b\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{f\sqrt{c+d\sin(e+fx)}} + \frac{\int \frac{\frac{1}{2}d\left(2a^2c+b^2c-abd\right)+ad(bc+ad)\sin(e+fx)-\frac{1}{2}bd(bc-3ad)\sin^2(e+fx)}{\sqrt{a+b\sin(e+fx)}(c+d\sin(e+fx))^{3/2}}}{d}$$

$$= -\frac{b\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{f\sqrt{c+d\sin(e+fx)}} + \frac{\int \frac{\frac{1}{2}bc^2d(bc-3ad)+\frac{1}{2}d^3\left(2a^2c+b^2c-abd\right)+d\left(bcd(bc-3ad)+ad^2(bc+ad)\right)\mathrm{si}}{\sqrt{a+b\sin(e+fx)}(c+d\sin(e+fx))^{3/2}}}{d^3}$$

$$= -\frac{b\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{f\sqrt{c+d\sin(e+fx)}} - \frac{\sqrt{a+b}(bc-3ad)\Pi\left(\frac{(a+b)d}{b(c+d)};\sin^{-1}\left(\frac{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}\right)|\right.}{}$$

$$= -\frac{b\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{f\sqrt{c+d\sin(e+fx)}} - \frac{(a-b)b\sqrt{a+b}\sqrt{c+d}E\left(\sin^{-1}\left(\frac{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}\right)|\frac{(a+}{(a-}\right.}{}$$

Mathematica [C] time = 32.6955, size = 222963, normalized size = 346.22

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sin[e + f*x])^(3/2)/Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] Result too large to show
```

Maple [B] time = 8.722, size = 529273, normalized size = 821.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}{\sqrt{d\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate((b*sin(f*x + e) + a)^(3/2)/sqrt(d*sin(f*x + e) + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(b\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}{\sqrt{d\sin\left(fx+e\right)+c}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] integral((b*sin(f*x + e) + a)^(3/2)/sqrt(d*sin(f*x + e) + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a+b\sin\left(e+fx\right)\right)^{\frac{3}{2}}}{\sqrt{c+d\sin\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**(3/2)/(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((a + b*sin(e + f*x))**(3/2)/sqrt(c + d*sin(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(b\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}{\sqrt{d\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate((b*sin(f*x + e) + a)^(3/2)/sqrt(d*sin(f*x + e) + c), x)
```

3.775 $\int \frac{(a+b\sin(e+fx))^{3/2}}{(c+d\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=600

$$-\frac{2\sqrt{a+b}(ad+b(c-2d))\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}}F\left(\sin^{-1}\left(\frac{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}\right.\right.}{d^2f(c-d)\sqrt{c+d}}$$

```
[Out] (2*(a - b)*Sqrt[a + b]*EllipticE[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x
]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c
+ d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*S
in[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[
e + f*x])))]*(c + d*Sin[e + f*x]))/((c - d)*d*Sqrt[c + d]*f) - (2*Sqrt[a +
b]*(b*(c - 2*d) + a*d)*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x
]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c
+ d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*S
in[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[
e + f*x])))]*(c + d*Sin[e + f*x]))/((c - d)*d^2*Sqrt[c + d]*f) + (2*b*Sqrt[
a + b]*EllipticPi[((a + b)*d)/(b*(c + d)), ArcSin[(Sqrt[c + d]*Sqrt[a + b*S
in[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((
a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b
)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(
c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/(d^2*Sqrt[c + d]*f)
```

Rubi [A] time = 0.93382, antiderivative size = 600, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2798, 2811, 2998, 2818, 2996}

$$-\frac{2\sqrt{a+b}(ad+b(c-2d))\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}}F\left(\sin^{-1}\left(\frac{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}\right.\right.}{d^2f(c-d)\sqrt{c+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^(3/2)/(c + d*Sin[e + f*x])^(3/2),x]
```


```
[Out] (2*(a - b)*Sqrt[a + b]*EllipticE[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x
]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c
+ d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*S
in[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[
e + f*x])))]*(c + d*Sin[e + f*x]))/((c - d)*d*Sqrt[c + d]*f) - (2*Sqrt[a +
b]*(b*(c - 2*d) + a*d)*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x
]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c
+ d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*S
in[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[
e + f*x])))]*(c + d*Sin[e + f*x]))/((c - d)*d^2*Sqrt[c + d]*f) + (2*b*Sqrt[
a + b]*EllipticPi[((a + b)*d)/(b*(c + d)), ArcSin[(Sqrt[c + d]*Sqrt[a + b*S
in[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((
a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b
)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(
c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/(d^2*Sqrt[c + d]*f)
```


Rule 2798

```
Int[((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)/((a_.) + (b_.)*sin[(e_.) +
 (f_.)*(x_)])^(3/2), x_Symbol] :> Dist[d^2/b^2, Int[Sqrt[a + b*Sin[e + f*x]
```

```
]/Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[(b*c - a*d)/b^2, Int[Simp[b*c + a
*d + 2*b*d*Sin[e + f*x], x]/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e +
f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2811

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[(2*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d
)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(
1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*EllipticPi[(b*(c + d))/
(d*(a + b)), ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt[c + d*Sin[e + f*x]])/Sqrt[
a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)*(c - d))])/(d*f*Rt[(a + b)
/(c + d), 2]*Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2818

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*(c + d*Sin[e + f*x])*Sqrt[((b*c -
a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d
)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[
(c + d)/(a + b), 2]*(Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (
(a + b)*(c - d))/((a - b)*(c + d))])/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*
Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 2996

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(-2*A*(c - d)*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/
((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c
 + d)*(a + b*Sin[e + f*x])))]*EllipticE[ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt
[c + d*Sin[e + f*x]])/Sqrt[a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)
*(c - d))])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2]*Cos[e + f*x]), x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rubi steps

$$\int \frac{(a+b\sin(e+fx))^{3/2}}{(c+d\sin(e+fx))^{3/2}}\,dx = \frac{b^2\int\frac{\sqrt{c+d\sin(e+fx)}}{\sqrt{a+b\sin(e+fx)}}\,dx}{d^2} - \frac{(bc-ad)\int\frac{bc+ad+2bd\sin(e+fx)}{\sqrt{a+b\sin(e+fx)}(c+d\sin(e+fx))^{3/2}}\,dx}{d^2}$$

$$= \frac{2b\sqrt{a+b}\,\Pi\left(\frac{(a+b)d}{b(c+d)};\sin^{-1}\left(\frac{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}\right)|\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)\sec(e+fx)\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\cdots}}{d^2\sqrt{c+d}f}$$

$$= \frac{2(a-b)\sqrt{a+b}E\left(\sin^{-1}\left(\frac{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}\right)|\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)\sec(e+fx)\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\cdots}}{(c-d)d\sqrt{c+d}f}$$

Mathematica [B] time = 9.35715, size = 1866, normalized size = 3.11

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sin[e + f*x])^(3/2)/(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-2*(b*c*Cos[e + f*x] - a*d*Cos[e + f*x])*Sqrt[a + b*Sin[e + f*x]])/((c^2 -
 d^2)*f*Sqrt[c + d*Sin[e + f*x]]) + ((-4*(-(b*c) + a*d)*(a^2*c - a*b*d)*Sqr
t[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[-(((
a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))]/Sq
rt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2
- f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/
(-(b*c) + a*d)]*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f
*x]))/(-(b*c) + a*d))])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c +
d*Sin[e + f*x]]) - 4*(-(b*c) + a*d)*(a^2*d - b^2*d)*((Sqrt[((c + d)*Cot[(-e
 + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[-(((a + b)*Csc[(-e + P
i/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))]/Sqrt[2]], (2*(-(b*c)
 + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[(
(c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sq
rt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*
d))])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) -
 (Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c) +
a*d)/((a + b)*d), ArcSin[Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*
Sin[e + f*x]))/(-(b*c) + a*d))]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c +
 d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2
- f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[-(((a + b)*Csc[(-e +
 Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))])/((a + b)*d*Sqrt[a
 + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])) + 2*(b^2*c - a*b*d)*((Cos[e +
 f*x]*Sqrt[c + d*Sin[e + f*x]])/(d*Sqrt[a + b*Sin[e + f*x]]) + (Sqrt[(a - b
)/(a + b)]*(a + b)*Cos[(-e + Pi/2 - f*x)/2]*EllipticE[ArcSin[(Sqrt[(a - b)/
(a + b)]*Sin[(-e + Pi/2 - f*x)/2])/Sqrt[(a + b*Sin[e + f*x])/(a + b)]], (2*
(-(b*c) + a*d))/((a - b)*(c + d))]*Sqrt[c + d*Sin[e + f*x]])/(b*d*Sqrt[((a
+ b)*Cos[(-e + Pi/2 - f*x)/2]^2)/(a + b*Sin[e + f*x])]*Sqrt[a + b*Sin[e + f
*x]]*Sqrt[(a + b*Sin[e + f*x])/(a + b)]*Sqrt[((a + b)*(c + d*Sin[e + f*x]))
/((c + d)*(a + b*Sin[e + f*x]))]) - (2*(-(b*c) + a*d)*((((a + b)*c + a*d)*S
qrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[-(
((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))]/
Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/
2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x])
)/(-(b*c) + a*d)]*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e +
 f*x]))/(-(b*c) + a*d))])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c
+ d*Sin[e + f*x]]) - ((b*c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)
/(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[-(((a + b)*Cs
```

```
c[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))]/Sqrt[2]], (
2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]
^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) +
 a*d)]*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(
b*c) + a*d))])/((a + b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]
)))/(b*d)))/((c - d)*(c + d)*f)
```

Maple [B] time = 123.669, size = 2626517, normalized size = 4377.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b \sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}{\left(d \sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate((b*sin(f*x + e) + a)^(3/2)/(d*sin(f*x + e) + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\left(b \sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\sqrt{d \sin\left(fx+e\right)+c}}{d^2 \cos\left(fx+e\right)^2-2\,cd \sin\left(fx+e\right)-c^2-d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] integral(-(b*sin(f*x + e) + a)^(3/2)*sqrt(d*sin(f*x + e) + c)/(d^2*cos(f*x
+ e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+b \sin\left(e+fx\right)\right)^{\frac{3}{2}}}{\left(c+d \sin\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**(3/2)/(c+d*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral((a + b*sin(e + f*x))**(3/2)/(c + d*sin(e + f*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate((b*sin(f*x + e) + a)^(3/2)/(d*sin(f*x + e) + c)^(3/2), x)
```

3.776 $\int \frac{(a+b\sin(e+fx))^{3/2}}{(c+d\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=497

$$-\frac{2(bc-ad)\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{3f\left(c^2-d^2\right)(c+d\sin(e+fx))^{3/2}}+\frac{2(a-b)\sqrt{a+b}(a(3c+d)-b(c+3d))\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(b}{(a}}}{3f(c-d)^2(c}$$

```
[Out] (-2*(b*c - a*d)*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]])/(3*(c^2 - d^2)*f*(c
+ d*Sin[e + f*x])^(3/2)) - (8*(a - b)*Sqrt[a + b]*(a*c - b*d)*EllipticE[Arc
Sin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e +
f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d
)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(
1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/(
3*(c - d)^2*(c + d)^(3/2)*(b*c - a*d)*f) + (2*(a - b)*Sqrt[a + b]*(a*(3*c +
 d) - b*(c + 3*d))*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/
(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d)
)]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e
 + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e +
f*x])))]*(c + d*Sin[e + f*x]))/(3*(c - d)^2*(c + d)^(3/2)*(b*c - a*d)*f)
```

Rubi [A] time = 0.953766, antiderivative size = 497, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2799, 2998, 2818, 2996}

$$-\frac{2(bc-ad)\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{3f\left(c^2-d^2\right)(c+d\sin(e+fx))^{3/2}}+\frac{2(a-b)\sqrt{a+b}(a(3c+d)-b(c+3d))\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(b}{(a}}}{3f(c-d)^2(c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^(3/2)/(c + d*Sin[e + f*x])^(5/2),x]
```


```
[Out] (-2*(b*c - a*d)*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]])/(3*(c^2 - d^2)*f*(c
+ d*Sin[e + f*x])^(3/2)) - (8*(a - b)*Sqrt[a + b]*(a*c - b*d)*EllipticE[Arc
Sin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e +
f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d
)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(
1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/(
3*(c - d)^2*(c + d)^(3/2)*(b*c - a*d)*f) + (2*(a - b)*Sqrt[a + b]*(a*(3*c +
 d) - b*(c + 3*d))*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/
(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d)
)]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e
 + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e +
f*x])))]*(c + d*Sin[e + f*x]))/(3*(c - d)^2*(c + d)^(3/2)*(b*c - a*d)*f)
```


Rule 2799

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin
[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1))/(f*(m + 1)*(a^2 - b^2)), x
] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*S
in[e + f*x])^(n - 2)*Simp[c*(a*c - b*d)*(m + 1) + d*(b*c - a*d)*(n - 1) + (
d*(a*c - b*d)*(m + 1) - c*(b*c - a*d)*(m + 2))*Sin[e + f*x] - d*(b*c - a*d)
*(m + n + 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
```

```
NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& LtQ[1, n, 2] && IntegersQ[2*m, 2*n]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2818

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*(c + d*Sin[e + f*x])*Sqrt[((b*c -
a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d
)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[
(c + d)/(a + b), 2]*(Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (
(a + b)*(c - d))/((a - b)*(c + d))])/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*
Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 2996

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(-2*A*(c - d)*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/
((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c
 + d)*(a + b*Sin[e + f*x])))]*EllipticE[ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt
[c + d*Sin[e + f*x]])/Sqrt[a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)
*(c - d))])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2]*Cos[e + f*x]), x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rubi steps

$$\int \frac{(a+b\sin(e+fx))^{3/2}}{(c+d\sin(e+fx))^{5/2}}\,dx = -\frac{2(bc-ad)\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{3\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}} - \frac{2\int \frac{\frac{1}{2}\left(-3a^2c-b^2c+4abd\right)-\frac{1}{2}\left(4abc-a^2d-3b^2d\right)\sin(e+fx)}{\sqrt{a+b\sin(e+fx)}(c+d\sin(e+fx))^{3/2}}\,d}{3\left(c^2-d^2\right)}$$

$$= -\frac{2(bc-ad)\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{3\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}} + \frac{(4(bc-ad)(ac-bd))\int \frac{1+\sin(e+fx)}{\sqrt{a+b\sin(e+fx)}(c+d\sin(e+}}{3(c-d)^2(c+d)}$$

$$= -\frac{2(bc-ad)\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{3\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}} - \frac{8(a-b)\sqrt{a+b}(ac-bd)E\left(\sin^{-1}\left(\frac{\sqrt{c+d}\sqrt{a+b\sin}}{\sqrt{a+b}\sqrt{c+d\sin}}\right.\right.}{}$$

Mathematica [B] time = 6.32168, size = 1982, normalized size = 3.99

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sin[e + f*x])^(3/2)/(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]*((-2*(b*c*Cos[e + f*x] -
 a*d*Cos[e + f*x]))/(3*(c^2 - d^2)*(c + d*Sin[e + f*x])^2) - (8*(-(a*c*d*Co
s[e + f*x]) + b*d^2*Cos[e + f*x]))/(3*(c^2 - d^2)^2*(c + d*Sin[e + f*x]))))
/f + ((-4*(-(b*c) + a*d)*(3*a^2*c^2 + b^2*c^2 - 4*a*b*c*d + a^2*d^2 - b^2*d
^2)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sq
rt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*
d))]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e
+ Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e +
f*x]))/(-(b*c) + a*d)]*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Si
n[e + f*x]))/(-(b*c) + a*d))])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sq
rt[c + d*Sin[e + f*x]]) - 4*(-(b*c) + a*d)*(4*a*b*c^2 + 4*a^2*c*d - 4*b^2*c
*d - 4*a*b*d^2)*((Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*Ellip
ticF[ArcSin[Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x])
)/(-(b*c) + a*d))]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e +
 f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(
a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/
2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))])/((a + b)*(c + d)*Sqrt[a + b*Si
n[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - (Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x
)/2]^2)/(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[-(((a
+ b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))]/Sqrt
[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 -
f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-
(b*c) + a*d)]*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x
]))/(-(b*c) + a*d))])/((a + b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e
+ f*x]])) + 2*(-4*a*b*c*d + 4*b^2*d^2)*((Cos[e + f*x]*Sqrt[c + d*Sin[e + f*
x]])/(d*Sqrt[a + b*Sin[e + f*x]]) + (Sqrt[(a - b)/(a + b)]*(a + b)*Cos[(-e
+ Pi/2 - f*x)/2]*EllipticE[ArcSin[(Sqrt[(a - b)/(a + b)]*Sin[(-e + Pi/2 - f
*x)/2])/Sqrt[(a + b*Sin[e + f*x])/(a + b)]], (2*(-(b*c) + a*d))/((a - b)*(c
 + d))]*Sqrt[c + d*Sin[e + f*x]])/(b*d*Sqrt[((a + b)*Cos[(-e + Pi/2 - f*x)/
2]^2)/(a + b*Sin[e + f*x])]*Sqrt[a + b*Sin[e + f*x]]*Sqrt[(a + b*Sin[e + f*
x])/(a + b)]*Sqrt[((a + b)*(c + d*Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*
x]))]) - (2*(-(b*c) + a*d)*((((a + b)*c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2
 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f
*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))]/Sqrt[2]], (2*(-(b*c) + a*d)
)/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)
*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[-(((
a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))])/(
(a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - ((b*c
+ a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*
c) + a*d)/((a + b)*d), ArcSin[Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c
 + d*Sin[e + f*x]))/(-(b*c) + a*d))]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*
(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e +
Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[-(((a + b)*Csc[
(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))])/((a + b)*d*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])))/(b*d)))/(3*(c - d)^2*(c
 + d)^2*f)
```

Maple [B] time = 3.327, size = 189727, normalized size = 381.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] integrate((b*sin(f*x + e) + a)^(3/2)/(d*sin(f*x + e) + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(b\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\sqrt{d\sin\left(fx+e\right)+c}}{3\,cd^{2}\cos\left(fx+e\right)^{2}-c^{3}-3\,cd^{2}+\left(d^{3}\cos\left(fx+e\right)^{2}-3\,c^{2}d-d^{3}\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] integral(-(b*sin(f*x + e) + a)^(3/2)*sqrt(d*sin(f*x + e) + c)/(3*c*d^2*cos(
f*x + e)^2 - c^3 - 3*c*d^2 + (d^3*cos(f*x + e)^2 - 3*c^2*d - d^3)*sin(f*x +
 e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**(3/2)/(c+d*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate((b*sin(f*x + e) + a)^(3/2)/(d*sin(f*x + e) + c)^(5/2), x)
```

3.777 $\int (a+b\sin(e+fx))^{5/2}(c+d\sin(e+fx))^{5/2}\,dx$

Optimal. Leaf size=1295

result too large to display

```
[Out] (Sqrt[a + b]*(c - d)*Sqrt[c + d]*(360*a^3*b*c*d^3 - 45*a^4*d^4 + 2*a^2*b^2*
d^2*(1877*c^2 + 846*d^2) + 8*a*b^3*d*(45*c^3 + 791*c*d^2) - b^4*(45*c^4 - 1
692*c^2*d^2 - 1024*d^4))*EllipticE[ArcSin[(Sqrt[a + b]*Sqrt[c + d*Sin[e + f
*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/((a + b)*(
c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c + d)*(a +
 b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d)*(a + b*S
in[e + f*x]))]*(a + b*Sin[e + f*x]))/(1920*b^2*d^2*(b*c - a*d)*f) - (Sqrt[c
 + d]*(b*c + a*d)*(28*a^3*b*c*d^3 - 3*a^4*d^4 + 28*a*b^3*c*d*(c^2 - 20*d^2)
 - 2*a^2*b^2*d^2*(89*c^2 + 20*d^2) - b^4*(3*c^4 + 40*c^2*d^2 + 240*d^4))*El
lipticPi[(b*(c + d))/((a + b)*d), ArcSin[(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*
x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/((a + b)*(c
 - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c + d)*(a +
b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Si
n[e + f*x]))]*(a + b*Sin[e + f*x]))/(128*b^3*Sqrt[a + b]*d^3*f) - ((360*a^3
*b*c*d^3 - 45*a^4*d^4 + 2*a^2*b^2*d^2*(1877*c^2 + 846*d^2) + 8*a*b^3*d*(45*
c^3 + 791*c*d^2) - b^4*(45*c^4 - 1692*c^2*d^2 - 1024*d^4))*Cos[e + f*x]*Sqr
t[c + d*Sin[e + f*x]])/(1920*b*d^2*f*Sqrt[a + b*Sin[e + f*x]]) - ((917*a^2*
b*c*d^2 + 15*a^3*d^3 + a*b^2*d*(345*c^2 + 772*d^2) - b^3*(45*c^3 - 516*c*d^
2))*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])/(960*b*
d*f) + ((a + b)^(3/2)*(45*a^4*d^4 - 30*a^3*b*d^3*(11*c + 3*d) + 30*a^2*b^2*
d^2*(64*c^2 + 23*c*d + 22*d^2) + 2*a*b^3*d*(165*c^3 + 917*c^2*d + 2392*c*d^
2 + 516*d^3) - b^4*(45*c^4 - 30*c^3*d - 1692*c^2*d^2 - 1544*c*d^3 - 1024*d^
4))*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sq
rt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sec[e + f*x]
*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt
[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*(c + d
*Sin[e + f*x]))/(1920*b^3*d^2*Sqrt[c + d]*f) - ((110*a*b*c*d + 93*a^2*d^2 -
 b^2*(15*c^2 - 64*d^2))*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e
+ f*x])^(3/2))/(240*d*f) + (3*b*(b*c - 7*a*d)*Cos[e + f*x]*Sqrt[a + b*Sin[e
 + f*x]]*(c + d*Sin[e + f*x])^(5/2))/(40*d*f) - (b^2*Cos[e + f*x]*Sqrt[a +
b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(7/2))/(5*d*f)
```

Rubi [A] time = 7.84428, antiderivative size = 1295, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2793, 3049, 3061, 3053, 2811, 2998, 2818, 2996}

$$-\frac{b^2\cos(e+fx)\sqrt{a+b\sin(e+fx)}(c+d\sin(e+fx))^{7/2}}{5df}+\frac{3b(bc-7ad)\cos(e+fx)\sqrt{a+b\sin(e+fx)}(c+d\sin(e+f}{40df}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])^(5/2),x]
```


```
[Out] (Sqrt[a + b]*(c - d)*Sqrt[c + d]*(360*a^3*b*c*d^3 - 45*a^4*d^4 + 2*a^2*b^2*
d^2*(1877*c^2 + 846*d^2) + 8*a*b^3*d*(45*c^3 + 791*c*d^2) - b^4*(45*c^4 - 1
692*c^2*d^2 - 1024*d^4))*EllipticE[ArcSin[(Sqrt[a + b]*Sqrt[c + d*Sin[e + f
*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/((a + b)*(
c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c + d)*(a +
 b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d)*(a + b*S
```

```
in[e + f*x]))]*(a + b*Sin[e + f*x]))/(1920*b^2*d^2*(b*c - a*d)*f) - (Sqrt[c
 + d]*(b*c + a*d)*(28*a^3*b*c*d^3 - 3*a^4*d^4 + 28*a*b^3*c*d*(c^2 - 20*d^2)
 - 2*a^2*b^2*d^2*(89*c^2 + 20*d^2) - b^4*(3*c^4 + 40*c^2*d^2 + 240*d^4))*El
lipticPi[(b*(c + d))/((a + b)*d), ArcSin[(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*
x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/((a + b)*(c
 - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c + d)*(a +
b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Si
n[e + f*x]))]*(a + b*Sin[e + f*x]))/(128*b^3*Sqrt[a + b]*d^3*f) - ((360*a^3
*b*c*d^3 - 45*a^4*d^4 + 2*a^2*b^2*d^2*(1877*c^2 + 846*d^2) + 8*a*b^3*d*(45*
c^3 + 791*c*d^2) - b^4*(45*c^4 - 1692*c^2*d^2 - 1024*d^4))*Cos[e + f*x]*Sqr
t[c + d*Sin[e + f*x]])/(1920*b*d^2*f*Sqrt[a + b*Sin[e + f*x]]) - ((917*a^2*
b*c*d^2 + 15*a^3*d^3 + a*b^2*d*(345*c^2 + 772*d^2) - b^3*(45*c^3 - 516*c*d^
2))*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])/(960*b*
d*f) + ((a + b)^(3/2)*(45*a^4*d^4 - 30*a^3*b*d^3*(11*c + 3*d) + 30*a^2*b^2*
d^2*(64*c^2 + 23*c*d + 22*d^2) + 2*a*b^3*d*(165*c^3 + 917*c^2*d + 2392*c*d^
2 + 516*d^3) - b^4*(45*c^4 - 30*c^3*d - 1692*c^2*d^2 - 1544*c*d^3 - 1024*d^
4))*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sq
rt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sec[e + f*x]
*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt
[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*(c + d
*Sin[e + f*x]))/(1920*b^3*d^2*Sqrt[c + d]*f) - ((110*a*b*c*d + 93*a^2*d^2 -
 b^2*(15*c^2 - 64*d^2))*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e
+ f*x])^(3/2))/(240*d*f) + (3*b*(b*c - 7*a*d)*Cos[e + f*x]*Sqrt[a + b*Sin[e
 + f*x]]*(c + d*Sin[e + f*x])^(5/2))/(40*d*f) - (b^2*Cos[e + f*x]*Sqrt[a +
b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(7/2))/(5*d*f)
```

Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
```

```
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2811

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[(2*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d
)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(
1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*EllipticPi[(b*(c + d))/
(d*(a + b)), ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt[c + d*Sin[e + f*x]])/Sqrt[
a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)*(c - d))])/(d*f*Rt[(a + b)
/(c + d), 2]*Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2818

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*(c + d*Sin[e + f*x])*Sqrt[((b*c -
a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d
)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[
(c + d)/(a + b), 2]*(Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (
(a + b)*(c - d))/((a - b)*(c + d))])/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*
Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 2996

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(-2*A*(c - d)*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/
((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c
 + d)*(a + b*Sin[e + f*x])))]*EllipticE[ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt
[c + d*Sin[e + f*x]])/Sqrt[a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)
*(c - d))])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2]*Cos[e + f*x]), x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rubi steps

$$\int (a+b\sin(e+fx))^{5/2}(c+d\sin(e+fx))^{5/2}\,dx = -\frac{b^2\cos(e+fx)\sqrt{a+b\sin(e+fx)}(c+d\sin(e+fx))^{7/2}}{5df} + \frac{\int \frac{(c+d\sin(e+f}{}}{}$$

$$= \frac{3b(bc-7ad)\cos(e+fx)\sqrt{a+b\sin(e+fx)}(c+d\sin(e+fx))^{5/2}}{40df} - \frac{b^2}{}$$

$$= -\frac{\left(110abcd+93a^2d^2-b^2\left(15c^2-64d^2\right)\right)\cos(e+fx)\sqrt{a+b\sin(e+fx}}{240df}$$

$$= -\frac{\left(917a^2bcd^2+15a^3d^3+ab^2d\left(345c^2+772d^2\right)-b^3\left(45c^3-516cd^2\right)\right)c}{960bdf}$$

$$= -\frac{\left(360a^3bcd^3-45a^4d^4+2a^2b^2d^2\left(1877c^2+846d^2\right)+8ab^3d\left(45c^3+79\right.\right.}{1920bd^2f\sqrt{a}}$$

$$= -\frac{\left(360a^3bcd^3-45a^4d^4+2a^2b^2d^2\left(1877c^2+846d^2\right)+8ab^3d\left(45c^3+79\right.\right.}{1920bd^2f\sqrt{a}}$$

$$= -\frac{\sqrt{c+d}(bc+ad)\left(28a^3bcd^3-3a^4d^4+28ab^3cd\left(c^2-20d^2\right)-2a^2b^2d^2\right.}{}$$

$$= \frac{\sqrt{a+b}(c-d)\sqrt{c+d}\left(360a^3bcd^3-45a^4d^4+2a^2b^2d^2\left(1877c^2+846d^2\right)\right.}{}$$

Mathematica [A] time = 8.42936, size = 2246, normalized size = 1.73

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] ((-4*(-(b*c) + a*d)*(-15*b^4*c^4 + 3840*a^3*b*c^3*d + 4456*a*b^3*c^3*d + 14
702*a^2*b^2*c^2*d^2 + 3236*b^4*c^2*d^2 + 4456*a^3*b*c*d^3 + 10440*a*b^3*c*d
^3 - 15*a^4*d^4 + 3236*a^2*b^2*d^4 + 1024*b^4*d^4)*Sqrt[((c + d)*Cot[(-e +
Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[-(((a + b)*Csc[(-e + Pi/2
 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))]/Sqrt[2]], (2*(-(b*c) +
a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c
+ d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[
-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))
])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - 4*
(-(b*c) + a*d)*(-60*a*b^3*c^4 + 6364*a^2*b^2*c^3*d + 2292*b^4*c^3*d + 6364*
a^3*b*c^2*d^2 + 17020*a*b^3*c^2*d^2 - 60*a^4*c*d^3 + 17020*a^2*b^2*c*d^3 +
4624*b^4*c*d^3 + 2292*a^3*b*d^4 + 4624*a*b^3*d^4)*((Sqrt[((c + d)*Cot[(-e +
 Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[-(((a + b)*Csc[(-e + Pi/
2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))]/Sqrt[2]], (2*(-(b*c) +
 a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c
 + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt
[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)
)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - (
Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c) + a*
```

```
d)/((a + b)*d), ArcSin[Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Si
n[e + f*x]))/(-(b*c) + a*d))]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d
))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 -
f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[-(((a + b)*Csc[(-e + P
i/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))])/((a + b)*d*Sqrt[a +
 b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])) + 2*(45*b^4*c^4 - 360*a*b^3*c^3
*d - 3754*a^2*b^2*c^2*d^2 - 1692*b^4*c^2*d^2 - 360*a^3*b*c*d^3 - 6328*a*b^3
*c*d^3 + 45*a^4*d^4 - 1692*a^2*b^2*d^4 - 1024*b^4*d^4)*((Cos[e + f*x]*Sqrt[
c + d*Sin[e + f*x]])/(d*Sqrt[a + b*Sin[e + f*x]]) + (Sqrt[(a - b)/(a + b)]*
(a + b)*Cos[(-e + Pi/2 - f*x)/2]*EllipticE[ArcSin[(Sqrt[(a - b)/(a + b)]*Si
n[(-e + Pi/2 - f*x)/2])/Sqrt[(a + b*Sin[e + f*x])/(a + b)]], (2*(-(b*c) + a
*d))/((a - b)*(c + d))]*Sqrt[c + d*Sin[e + f*x]])/(b*d*Sqrt[((a + b)*Cos[(-
e + Pi/2 - f*x)/2]^2)/(a + b*Sin[e + f*x])]*Sqrt[a + b*Sin[e + f*x]]*Sqrt[(
a + b*Sin[e + f*x])/(a + b)]*Sqrt[((a + b)*(c + d*Sin[e + f*x]))/((c + d)*(
a + b*Sin[e + f*x]))]) - (2*(-(b*c) + a*d)*((((a + b)*c + a*d)*Sqrt[((c + d
)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[-(((a + b)*Cs
c[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))]/Sqrt[2]], (
2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]
^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) +
 a*d)]*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(
b*c) + a*d))])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e +
 f*x]]) - ((b*c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*
EllipticPi[(-(b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[-(((a + b)*Csc[(-e + Pi/
2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))]/Sqrt[2]], (2*(-(b*c) +
 a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c
 + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt
[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)
)])/((a + b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])))/(b*d)))
/(3840*b*d*f) + (Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]*(-((15*b
^3*c^3 + 1289*a*b^2*c^2*d + 1289*a^2*b*c*d^2 + 898*b^3*c*d^2 + 15*a^3*d^3 +
 898*a*b^2*d^3)*Cos[e + f*x])/(960*b*d) + (21*b*d*(b*c + a*d)*Cos[3*(e + f*
x)])/160 - ((93*b^2*c^2 + 362*a*b*c*d + 93*a^2*d^2 + 88*b^2*d^2)*Sin[2*(e +
 f*x)])/480 + (b^2*d^2*Sin[4*(e + f*x)])/40))/f
```

Maple [B] time = 38.718, size = 752449, normalized size = 581.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] integrate((b*sin(f*x + e) + a)^(5/2)*(d*sin(f*x + e) + c)^(5/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**(5/2)*(c+d*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

3.778 $\int (a+b\sin(e+fx))^{5/2}(c+d\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=1071

result too large to display

```
[Out] (Sqrt[a + b]*(c - d)*Sqrt[c + d]*(337*a^2*b*c*d^2 + 15*a^3*d^3 + a*b^2*d*(5
7*c^2 + 284*d^2) - b^3*(9*c^3 - 156*c*d^2))*EllipticE[ArcSin[(Sqrt[a + b]*S
qrt[c + d*Sin[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*
(c + d))/((a + b)*(c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f
*x]))/((c + d)*(a + b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x])
)/((c - d)*(a + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/(192*b*d^2*(b*c - a
*d)*f) + (Sqrt[c + d]*(60*a^3*b*c*d^3 - 5*a^4*d^4 - 20*a*b^3*c*d*(c^2 - 12*
d^2) + 3*b^4*(c^2 + 4*d^2)^2 + 30*a^2*b^2*d^2*(3*c^2 + 4*d^2))*EllipticPi[(
b*(c + d))/((a + b)*d), ArcSin[(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])/(Sqrt
[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/((a + b)*(c - d))]*Se
c[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e +
f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]
))]*(a + b*Sin[e + f*x]))/(64*b^2*Sqrt[a + b]*d^3*f) - ((337*a^2*b*c*d^2 +
15*a^3*d^3 + a*b^2*d*(57*c^2 + 284*d^2) - b^3*(9*c^3 - 156*c*d^2))*Cos[e +
f*x]*Sqrt[c + d*Sin[e + f*x]])/(192*d^2*f*Sqrt[a + b*Sin[e + f*x]]) - ((54*
a*b*c*d + 59*a^2*d^2 - 9*b^2*(c^2 - 4*d^2))*Cos[e + f*x]*Sqrt[a + b*Sin[e +
 f*x]]*Sqrt[c + d*Sin[e + f*x]])/(96*d*f) - ((a + b)^(3/2)*(15*a^3*d^3 - 15
*a^2*b*d^2*(11*c + 2*d) - a*b^2*d*(51*c^2 + 172*c*d + 212*d^2) + b^3*(9*c^3
 - 6*c^2*d - 156*c*d^2 - 72*d^3))*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*
Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/(
(a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a +
b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*
(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/(192*b^2*d^2*Sqrt[c + d]*f) +
 (b*(3*b*c - 17*a*d)*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f
*x])^(3/2))/(24*d*f) - (b^2*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]]*(c + d*Si
n[e + f*x])^(5/2))/(4*d*f)
```

Rubi [A] time = 4.99506, antiderivative size = 1071, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2793, 3049, 3061, 3053, 2811, 2998, 2818, 2996}

$$-\frac{b^2\cos(e+fx)\sqrt{a+b\sin(e+fx)}(c+d\sin(e+fx))^{5/2}}{4df}+\frac{b(3bc-17ad)\cos(e+fx)\sqrt{a+b\sin(e+fx)}(c+d\sin(e+\ldots}{24df}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])^(3/2),x]
```


```
[Out] (Sqrt[a + b]*(c - d)*Sqrt[c + d]*(337*a^2*b*c*d^2 + 15*a^3*d^3 + a*b^2*d*(5
7*c^2 + 284*d^2) - b^3*(9*c^3 - 156*c*d^2))*EllipticE[ArcSin[(Sqrt[a + b]*S
qrt[c + d*Sin[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*
(c + d))/((a + b)*(c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f
*x]))/((c + d)*(a + b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x])
)/((c - d)*(a + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/(192*b*d^2*(b*c - a
*d)*f) + (Sqrt[c + d]*(60*a^3*b*c*d^3 - 5*a^4*d^4 - 20*a*b^3*c*d*(c^2 - 12*
d^2) + 3*b^4*(c^2 + 4*d^2)^2 + 30*a^2*b^2*d^2*(3*c^2 + 4*d^2))*EllipticPi[(
b*(c + d))/((a + b)*d), ArcSin[(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])/(Sqrt
[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/((a + b)*(c - d))]*Se
c[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e +
```

```
f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]
))]*(a + b*Sin[e + f*x]))/(64*b^2*Sqrt[a + b]*d^3*f) - ((337*a^2*b*c*d^2 +
15*a^3*d^3 + a*b^2*d*(57*c^2 + 284*d^2) - b^3*(9*c^3 - 156*c*d^2))*Cos[e +
f*x]*Sqrt[c + d*Sin[e + f*x]])/(192*d^2*f*Sqrt[a + b*Sin[e + f*x]]) - ((54*
a*b*c*d + 59*a^2*d^2 - 9*b^2*(c^2 - 4*d^2))*Cos[e + f*x]*Sqrt[a + b*Sin[e +
 f*x]]*Sqrt[c + d*Sin[e + f*x]])/(96*d*f) - ((a + b)^(3/2)*(15*a^3*d^3 - 15
*a^2*b*d^2*(11*c + 2*d) - a*b^2*d*(51*c^2 + 172*c*d + 212*d^2) + b^3*(9*c^3
 - 6*c^2*d - 156*c*d^2 - 72*d^3))*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*
Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/(
(a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a +
b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*
(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/(192*b^2*d^2*Sqrt[c + d]*f) +
 (b*(3*b*c - 17*a*d)*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f
*x])^(3/2))/(24*d*f) - (b^2*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]]*(c + d*Si
n[e + f*x])^(5/2))/(4*d*f)
```

Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
```

```
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2811

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[(2*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d
)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(
1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*EllipticPi[(b*(c + d))/
(d*(a + b)), ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt[c + d*Sin[e + f*x]])/Sqrt[
a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)*(c - d))])/(d*f*Rt[(a + b)
/(c + d), 2]*Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2818

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*(c + d*Sin[e + f*x])*Sqrt[((b*c -
a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d
)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[
(c + d)/(a + b), 2]*(Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (
(a + b)*(c - d))/((a - b)*(c + d))])/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*
Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 2996

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(-2*A*(c - d)*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/
((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c
 + d)*(a + b*Sin[e + f*x])))]*EllipticE[ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt
[c + d*Sin[e + f*x]])/Sqrt[a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)
*(c - d))])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2]*Cos[e + f*x]), x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rubi steps

$$\int (a+b\sin(e+fx))^{5/2}(c+d\sin(e+fx))^{3/2}\,dx = -\frac{b^2\cos(e+fx)\sqrt{a+b\sin(e+fx)}(c+d\sin(e+fx))^{5/2}}{4df} + \frac{\int \frac{(c+d\sin(e+f}{}}{}$$

$$= \frac{b(3bc-17ad)\cos(e+fx)\sqrt{a+b\sin(e+fx)}(c+d\sin(e+fx))^{3/2}}{24df} - \frac{b^2}{}$$

$$= -\frac{\left(54abcd+59a^2d^2-9b^2\left(c^2-4d^2\right)\right)\cos(e+fx)\sqrt{a+b\sin(e+fx)}\sqrt{c}}{96df}$$

$$= -\frac{\left(337a^2bcd^2+15a^3d^3+ab^2d\left(57c^2+284d^2\right)-b^3\left(9c^3-156cd^2\right)\right)\cos}{192d^2f\sqrt{a+b\sin(e+fx)}}$$

$$= -\frac{\left(337a^2bcd^2+15a^3d^3+ab^2d\left(57c^2+284d^2\right)-b^3\left(9c^3-156cd^2\right)\right)\cos}{192d^2f\sqrt{a+b\sin(e+fx)}}$$

$$= \frac{\sqrt{c+d}\left(60a^3bcd^3-5a^4d^4-20ab^3cd\left(c^2-12d^2\right)+3b^4\left(c^2+4d^2\right)^2+3\right.}{}$$

$$= \frac{\sqrt{a+b}(c-d)\sqrt{c+d}\left(337a^2bcd^2+15a^3d^3+ab^2d\left(57c^2+284d^2\right)-b^3\right.}{}$$

Mathematica [A] time = 7.39697, size = 2061, normalized size = 1.92

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] ((-4*(-(b*c) + a*d)*(-3*b^3*c^3 + 384*a^3*c^2*d + 451*a*b^2*c^2*d + 971*a^2
*b*c*d^2 + 228*b^3*c*d^2 + 133*a^3*d^3 + 356*a*b^2*d^3)*Sqrt[((c + d)*Cot[(
-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[-(((a + b)*Csc[(-e +
 Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))]/Sqrt[2]], (2*(-(b*
c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt
[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*
Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) +
a*d))])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])
 - 4*(-(b*c) + a*d)*(-12*a*b^2*c^3 + 664*a^2*b*c^2*d + 228*b^3*c^2*d + 532*
a^3*c*d^2 + 1160*a*b^2*c*d^2 + 644*a^2*b*d^3 + 144*b^3*d^3)*((Sqrt[((c + d)
*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[-(((a + b)*Csc
[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))]/Sqrt[2]], (2
*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^
4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) +
a*d)]*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b
*c) + a*d))])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e +
f*x]]) - (Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-
(b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2
*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))]/Sqrt[2]], (2*(-(b*c) + a*d))/((a +
b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e
 + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[-(((a + b)*C
sc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))])/((a + b)*
d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])) + 2*(9*b^3*c^3 - 57*a
```

```
*b^2*c^2*d - 337*a^2*b*c*d^2 - 156*b^3*c*d^2 - 15*a^3*d^3 - 284*a*b^2*d^3)*
((Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(d*Sqrt[a + b*Sin[e + f*x]]) + (Sq
rt[(a - b)/(a + b)]*(a + b)*Cos[(-e + Pi/2 - f*x)/2]*EllipticE[ArcSin[(Sqrt
[(a - b)/(a + b)]*Sin[(-e + Pi/2 - f*x)/2])/Sqrt[(a + b*Sin[e + f*x])/(a +
b)]], (2*(-(b*c) + a*d))/((a - b)*(c + d))]*Sqrt[c + d*Sin[e + f*x]])/(b*d*
Sqrt[((a + b)*Cos[(-e + Pi/2 - f*x)/2]^2)/(a + b*Sin[e + f*x])]*Sqrt[a + b*
Sin[e + f*x]]*Sqrt[(a + b*Sin[e + f*x])/(a + b)]*Sqrt[((a + b)*(c + d*Sin[e
 + f*x]))/((c + d)*(a + b*Sin[e + f*x]))]) - (2*(-(b*c) + a*d)*((((a + b)*c
 + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSi
n[Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c)
+ a*d))]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[
(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[
e + f*x]))/(-(b*c) + a*d)]*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c +
d*Sin[e + f*x]))/(-(b*c) + a*d))])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]
]*Sqrt[c + d*Sin[e + f*x]]) - ((b*c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f
*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[-(((
a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))]/Sq
rt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2
- f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/
(-(b*c) + a*d)]*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f
*x]))/(-(b*c) + a*d))])/((a + b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[
e + f*x]]))))/(b*d)))/(384*d*f) + (Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e
 + f*x]]*(-((3*b^2*c^2 + 122*a*b*c*d + 59*a^2*d^2 + 42*b^2*d^2)*Cos[e + f*x
])/(96*d) + (b^2*d*Cos[3*(e + f*x)])/16 - (b*(9*b*c + 17*a*d)*Sin[2*(e + f*
x)])/48))/f
```

Maple [B] time = 17.068, size = 577718, normalized size = 539.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^(3/2),x)

[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^(3/2),x, algorithm="maxim
a")

[Out] integrate((b*sin(f*x + e) + a)^(5/2)*(d*sin(f*x + e) + c)^(3/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**(5/2)*(c+d*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

3.779 $\int (a+b\sin(e+fx))^{5/2}\sqrt{c+d\sin(e+fx)}\,dx$

Optimal. Leaf size=894

$$-\frac{\cos(e+fx)\sqrt{a+b\sin(e+fx)}(c+d\sin(e+fx))^{3/2}b^2}{3df}+\frac{(3bc-13ad)\cos(e+fx)\sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin(e+f}}{12df}$$

```
[Out] (Sqrt[a + b]*(c - d)*Sqrt[c + d]*(14*a*b*c*d + 33*a^2*d^2 - b^2*(3*c^2 - 16
*d^2))*EllipticE[ArcSin[(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])/(Sqrt[c + d]
*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/((a + b)*(c - d))]*Sec[e + f
*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))
]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*(a
+ b*Sin[e + f*x]))/(24*d^2*(b*c - a*d)*f) + (Sqrt[c + d]*(15*a^2*b*c*d^2 +
5*a^3*d^3 - 5*a*b^2*d*(c^2 - 4*d^2) + b^3*(c^3 + 4*c*d^2))*EllipticPi[(b*(c
 + d))/((a + b)*d), ArcSin[(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])/(Sqrt[c +
 d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/((a + b)*(c - d))]*Sec[e
+ f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x]
)))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*
(a + b*Sin[e + f*x]))/(8*b*Sqrt[a + b]*d^3*f) - (b*(14*a*b*c*d + 33*a^2*d^2
 - b^2*(3*c^2 - 16*d^2))*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(24*d^2*f*S
qrt[a + b*Sin[e + f*x]]) + (b*(3*b*c - 13*a*d)*Cos[e + f*x]*Sqrt[a + b*Sin[
e + f*x]]*Sqrt[c + d*Sin[e + f*x]])/(12*d*f) + ((a + b)^(3/2)*(15*a^2*d^2 +
 6*a*b*d*(2*c + 3*d) - b^2*(3*c^2 - 2*c*d - 16*d^2))*EllipticF[ArcSin[(Sqrt
[c + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])],
((a + b)*(c - d))/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Si
n[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e
 + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/(24*b*d^2*
Sqrt[c + d]*f) - (b^2*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e +
f*x])^(3/2))/(3*d*f)
```

Rubi [A] time = 3.2685, antiderivative size = 894, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2793, 3049, 3061, 3053, 2811, 2998, 2818, 2996}

$$-\frac{\cos(e+fx)\sqrt{a+b\sin(e+fx)}(c+d\sin(e+fx))^{3/2}b^2}{3df}+\frac{(3bc-13ad)\cos(e+fx)\sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin(e+f}}{12df}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^(5/2)*Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] (Sqrt[a + b]*(c - d)*Sqrt[c + d]*(14*a*b*c*d + 33*a^2*d^2 - b^2*(3*c^2 - 16
*d^2))*EllipticE[ArcSin[(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])/(Sqrt[c + d]
*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/((a + b)*(c - d))]*Sec[e + f
*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))
]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*(a
+ b*Sin[e + f*x]))/(24*d^2*(b*c - a*d)*f) + (Sqrt[c + d]*(15*a^2*b*c*d^2 +
5*a^3*d^3 - 5*a*b^2*d*(c^2 - 4*d^2) + b^3*(c^3 + 4*c*d^2))*EllipticPi[(b*(c
 + d))/((a + b)*d), ArcSin[(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])/(Sqrt[c +
 d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/((a + b)*(c - d))]*Sec[e
+ f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x]
)))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*
(a + b*Sin[e + f*x]))/(8*b*Sqrt[a + b]*d^3*f) - (b*(14*a*b*c*d + 33*a^2*d^2
 - b^2*(3*c^2 - 16*d^2))*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(24*d^2*f*S
```

```
qrt[a + b*Sin[e + f*x]]) + (b*(3*b*c - 13*a*d)*Cos[e + f*x]*Sqrt[a + b*Sin[
e + f*x]]*Sqrt[c + d*Sin[e + f*x]])/(12*d*f) + ((a + b)^(3/2)*(15*a^2*d^2 +
 6*a*b*d*(2*c + 3*d) - b^2*(3*c^2 - 2*c*d - 16*d^2))*EllipticF[ArcSin[(Sqrt
[c + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])],
((a + b)*(c - d))/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Si
n[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e
 + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/(24*b*d^2*
Sqrt[c + d]*f) - (b^2*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e +
f*x])^(3/2))/(3*d*f)
```

Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2811

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[(2*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d
)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(
1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*EllipticPi[(b*(c + d))/
(d*(a + b)), ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt[c + d*Sin[e + f*x]])/Sqrt[
a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)*(c - d))])/(d*f*Rt[(a + b)
/(c + d), 2]*Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2818

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*(c + d*Sin[e + f*x])*Sqrt[((b*c -
a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d
)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[
(c + d)/(a + b), 2]*(Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (
(a + b)*(c - d))/((a - b)*(c + d))])/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*
Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 2996

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(-2*A*(c - d)*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/
((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c
 + d)*(a + b*Sin[e + f*x])))]*EllipticE[ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt
[c + d*Sin[e + f*x]])/Sqrt[a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)
*(c - d))])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2]*Cos[e + f*x]), x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rubi steps

$$\int (a+b\sin(e+fx))^{5/2}\sqrt{c+d\sin(e+fx)}\,dx = -\frac{b^2\cos(e+fx)\sqrt{a+b\sin(e+fx)}(c+d\sin(e+fx))^{3/2}}{3df} + \frac{\int \frac{\sqrt{c+d\sin(e+fx}}{}}{}$$

$$= \frac{b(3bc-13ad)\cos(e+fx)\sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}{12df} - \frac{b^2\cos}{}$$

$$= -\frac{b\left(14abcd+33a^2d^2-b^2\left(3c^2-16d^2\right)\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{24d^2f\sqrt{a+b\sin(e+fx)}} +$$

$$= -\frac{b\left(14abcd+33a^2d^2-b^2\left(3c^2-16d^2\right)\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{24d^2f\sqrt{a+b\sin(e+fx)}} +$$

$$= \frac{\sqrt{c+d}\left(15a^2bcd^2+5a^3d^3-5ab^2d\left(c^2-4d^2\right)+b^3\left(c^3+4cd^2\right)\right)\Pi\left(\frac{b(c+d)}{(a+b)d};\right.}{}$$

$$= \frac{\sqrt{a+b}(c-d)\sqrt{c+d}\left(14abcd+33a^2d^2-b^2\left(3c^2-16d^2\right)\right)E\left(\sin^{-1}\left(\frac{\sqrt{a+b}}{\sqrt{c+d}}\right.\right.}{}$$

Mathematica [B] time = 7.04505, size = 1949, normalized size = 2.18

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sin[e + f*x])^(5/2)*Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] ((-4*(-(b*c) + a*d)*(-(b^3*c^2) + 48*a^3*c*d + 58*a*b^2*c*d + 59*a^2*b*d^2
+ 16*b^3*d^2)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF
[ArcSin[Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-
(b*c) + a*d))]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x
]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a +
b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2
*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))])/((a + b)*(c + d)*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[c + d*Sin[e + f*x]]) - 4*(-(b*c) + a*d)*(-4*a*b^2*c^2 + 92*a^2
*b*c*d + 28*b^3*c*d + 48*a^3*d^2 + 76*a*b^2*d^2)*((Sqrt[((c + d)*Cot[(-e +
Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[-(((a + b)*Csc[(-e + Pi/2
 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))]/Sqrt[2]], (2*(-(b*c) +
a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c
+ d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[
-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))
])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - (S
qrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c) + a*d
)/((a + b)*d), ArcSin[Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin
[e + f*x]))/(-(b*c) + a*d))]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d)
)]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f
*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[-(((a + b)*Csc[(-e + Pi
/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))])/((a + b)*d*Sqrt[a +
b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])) + 2*(3*b^3*c^2 - 14*a*b^2*c*d -
33*a^2*b*d^2 - 16*b^3*d^2)*((Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(d*Sqrt
[a + b*Sin[e + f*x]]) + (Sqrt[(a - b)/(a + b)]*(a + b)*Cos[(-e + Pi/2 - f*x
)/2]*EllipticE[ArcSin[(Sqrt[(a - b)/(a + b)]*Sin[(-e + Pi/2 - f*x)/2])/Sqrt
[(a + b*Sin[e + f*x])/(a + b)]], (2*(-(b*c) + a*d))/((a - b)*(c + d))]*Sqrt
```

```
[c + d*Sin[e + f*x]])/(b*d*Sqrt[((a + b)*Cos[(-e + Pi/2 - f*x)/2]^2)/(a + b
*Sin[e + f*x])]*Sqrt[a + b*Sin[e + f*x]]*Sqrt[(a + b*Sin[e + f*x])/(a + b)]
*Sqrt[((a + b)*(c + d*Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x]))]) - (2*
(-(b*c) + a*d)*((((a + b)*c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2
)/(-c + d)]*EllipticF[ArcSin[Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c
+ d*Sin[e + f*x]))/(-(b*c) + a*d))]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(
-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + P
i/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[-(((a + b)*Csc[(
-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))])/((a + b)*(c +
 d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - ((b*c + a*d)*Sqrt[
((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c) + a*d)/((
a + b)*d), ArcSin[Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e +
 f*x]))/(-(b*c) + a*d))]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*S
ec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/
2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[-(((a + b)*Csc[(-e + Pi/2 -
 f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))])/((a + b)*d*Sqrt[a + b*Si
n[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])))/(b*d)))/(48*d*f) + (Sqrt[a + b*Sin[
e + f*x]]*Sqrt[c + d*Sin[e + f*x]]*(-(b*(b*c + 13*a*d)*Cos[e + f*x])/(12*d)
 - (b^2*Sin[2*(e + f*x)])/6))/f
```

Maple [C] time = 10.796, size = 409146, normalized size = 457.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\sqrt{d\sin\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate((b*sin(f*x + e) + a)^(5/2)*sqrt(d*sin(f*x + e) + c), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**(5/2)*(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError
```

3.780 $\int \frac{(a+b\sin(e+fx))^{5/2}}{\sqrt{c+d\sin(e+fx)}}\,dx$

Optimal. Leaf size=745

$$-\frac{\sqrt{c+d}\left(-15a^2d^2+10abcd+b^2\left(-\left(3c^2+4d^2\right)\right)\right)\sec(e+fx)(a+b\sin(e+fx))\sqrt{-\frac{(bc-ad)(1-\sin(e+fx))}{(c+d)(a+b\sin(e+fx))}}\sqrt{\frac{(bc-ad)(\sin(e+fx)}{(c-d)(a+b\sin(e+}}}{4d^3f\sqrt{a+b}}$$

```
[Out] (-3*b*Sqrt[a + b]*(c - d)*Sqrt[c + d]*(b*c - 3*a*d)*EllipticE[ArcSin[(Sqrt[
a + b]*Sqrt[c + d*Sin[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], (
(a - b)*(c + d))/((a + b)*(c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - S
in[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e
 + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/(4*d^2*(b*c
 - a*d)*f) - (Sqrt[c + d]*(10*a*b*c*d - 15*a^2*d^2 - b^2*(3*c^2 + 4*d^2))*E
llipticPi[(b*(c + d))/((a + b)*d), ArcSin[(Sqrt[a + b]*Sqrt[c + d*Sin[e + f
*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/((a + b)*(
c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c + d)*(a +
 b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d)*(a + b*S
in[e + f*x]))]*(a + b*Sin[e + f*x]))/(4*Sqrt[a + b]*d^3*f) + (3*b^2*(b*c -
3*a*d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(4*d^2*f*Sqrt[a + b*Sin[e + f
*x]]) - (b^2*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]
)/(2*d*f) - ((a + b)^(3/2)*(3*b*c - 7*a*d - 2*b*d)*EllipticF[ArcSin[(Sqrt[c
 + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((
a + b)*(c - d))/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[
e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e +
 f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/(4*d^2*Sqrt[
c + d]*f)
```

Rubi [A] time = 2.18927, antiderivative size = 745, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2793, 3061, 3053, 2811, 2998, 2818, 2996}

$$-\frac{\sqrt{c+d}\left(-15a^2d^2+10abcd+b^2\left(-\left(3c^2+4d^2\right)\right)\right)\sec(e+fx)(a+b\sin(e+fx))\sqrt{-\frac{(bc-ad)(1-\sin(e+fx))}{(c+d)(a+b\sin(e+fx))}}\sqrt{\frac{(bc-ad)(\sin(e+fx)}{(c-d)(a+b\sin(e+}}}{4d^3f\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^(5/2)/Sqrt[c + d*Sin[e + f*x]],x]
```


```
[Out] (-3*b*Sqrt[a + b]*(c - d)*Sqrt[c + d]*(b*c - 3*a*d)*EllipticE[ArcSin[(Sqrt[
a + b]*Sqrt[c + d*Sin[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], (
(a - b)*(c + d))/((a + b)*(c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - S
in[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e
 + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/(4*d^2*(b*c
 - a*d)*f) - (Sqrt[c + d]*(10*a*b*c*d - 15*a^2*d^2 - b^2*(3*c^2 + 4*d^2))*E
llipticPi[(b*(c + d))/((a + b)*d), ArcSin[(Sqrt[a + b]*Sqrt[c + d*Sin[e + f
*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/((a + b)*(
c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c + d)*(a +
 b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d)*(a + b*S
in[e + f*x]))]*(a + b*Sin[e + f*x]))/(4*Sqrt[a + b]*d^3*f) + (3*b^2*(b*c -
3*a*d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(4*d^2*f*Sqrt[a + b*Sin[e + f
*x]]) - (b^2*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]
)/(2*d*f) - ((a + b)^(3/2)*(3*b*c - 7*a*d - 2*b*d)*EllipticF[ArcSin[(Sqrt[c
 + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((
```

```
a + b)*(c - d))/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[
e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e +
 f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/(4*d^2*Sqrt[
c + d]*f)
```

Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2811

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[(2*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d
)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(
1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*EllipticPi[(b*(c + d))/
(d*(a + b)), ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt[c + d*Sin[e + f*x]])/Sqrt[
a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)*(c - d))])/(d*f*Rt[(a + b)
/(c + d), 2]*Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2818

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*(c + d*Sin[e + f*x])*Sqrt[((b*c -
a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d
)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[
(c + d)/(a + b), 2]*(Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (
(a + b)*(c - d))/((a - b)*(c + d))])/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*
Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 2996

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(-2*A*(c - d)*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/
((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c
 + d)*(a + b*Sin[e + f*x])))]*EllipticE[ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt
[c + d*Sin[e + f*x]])/Sqrt[a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)
*(c - d))])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2]*Cos[e + f*x]), x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rubi steps

$$\int \frac{(a+b\sin(e+fx))^{5/2}}{\sqrt{c+d\sin(e+fx)}}\,dx = -\frac{b^2\cos(e+fx)\sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}{2df} + \frac{\int \frac{\frac{1}{2}\left(b^3c+4a^3d+ab^2d\right)-b\left(abc-6a^2d-b^2d\right.}{\sqrt{a+b\sin(e+fx)}}}{2}$$

$$= \frac{3b^2(bc-3ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4d^2f\sqrt{a+b\sin(e+fx)}} - \frac{b^2\cos(e+fx)\sqrt{a+b\sin(e+fx)}\sqrt{c+d}}{2df}$$

$$= \frac{3b^2(bc-3ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4d^2f\sqrt{a+b\sin(e+fx)}} - \frac{b^2\cos(e+fx)\sqrt{a+b\sin(e+fx)}\sqrt{c+d}}{2df}$$

$$= -\frac{\sqrt{c+d}\left(10abcd-15a^2d^2-b^2\left(3c^2+4d^2\right)\right)\Pi\left(\frac{b(c+d)}{(a+b)d};\sin^{-1}\left(\frac{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}\right)|\frac{(a-b)(c+}{(a+b)(c-}}{4\sqrt{a+b}d^3f}$$

$$= -\frac{3b\sqrt{a+b}(c-d)\sqrt{c+d}(bc-3ad)E\left(\sin^{-1}\left(\frac{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sec(e+fx)\sqrt{}}{4d^2(bc-ad)f}$$

Mathematica [B] time = 10.1844, size = 1864, normalized size = 2.5

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sin[e + f*x])^(5/2)/Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] -(b^2*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])/(2*d*
f) + ((-4*(-(b*c) + a*d)*(-(b^3*c) + 8*a^3*d + 11*a*b^2*d)*Sqrt[((c + d)*Co
t[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[-(((a + b)*Csc[(-
e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))]/Sqrt[2]], (2*(-
```

```
(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*S
qrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d
)]*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c)
 + a*d))])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]) - 4*(-(b*c) + a*d)*(-4*a*b^2*c + 24*a^2*b*d + 4*b^3*d)*((Sqrt[((c + d)*
Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[-(((a + b)*Csc[
(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))]/Sqrt[2]], (2*
(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4
*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a
*d)]*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*
c) + a*d))])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f
*x]]) - (Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(
b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*
(c + d*Sin[e + f*x]))/(-(b*c) + a*d))]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b
)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e
+ Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[-(((a + b)*Cs
c[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))])/((a + b)*d
*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])) + 2*(3*b^3*c - 9*a*b^2
*d)*((Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(d*Sqrt[a + b*Sin[e + f*x]]) +
 (Sqrt[(a - b)/(a + b)]*(a + b)*Cos[(-e + Pi/2 - f*x)/2]*EllipticE[ArcSin[(
Sqrt[(a - b)/(a + b)]*Sin[(-e + Pi/2 - f*x)/2])/Sqrt[(a + b*Sin[e + f*x])/(
a + b)]], (2*(-(b*c) + a*d))/((a - b)*(c + d))]*Sqrt[c + d*Sin[e + f*x]])/(
b*d*Sqrt[((a + b)*Cos[(-e + Pi/2 - f*x)/2]^2)/(a + b*Sin[e + f*x])]*Sqrt[a
+ b*Sin[e + f*x]]*Sqrt[(a + b*Sin[e + f*x])/(a + b)]*Sqrt[((a + b)*(c + d*S
in[e + f*x]))/((c + d)*(a + b*Sin[e + f*x]))]) - (2*(-(b*c) + a*d)*((((a +
b)*c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[A
rcSin[Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b
*c) + a*d))]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*
Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*
Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(
c + d*Sin[e + f*x]))/(-(b*c) + a*d))])/((a + b)*(c + d)*Sqrt[a + b*Sin[e +
f*x]]*Sqrt[c + d*Sin[e + f*x]]) - ((b*c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2
 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[
-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))
]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + P
i/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x
]))/(-(b*c) + a*d)]*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e
 + f*x]))/(-(b*c) + a*d))])/((a + b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*
Sin[e + f*x]])))/(b*d)))/(8*d*f)
```

Maple [B] time = 18.651, size = 730173, normalized size = 980.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b \sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}{\sqrt{d \sin\left(fx+e\right)+c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate((b*sin(f*x + e) + a)^(5/2)/sqrt(d*sin(f*x + e) + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(b^{2}\cos\left(fx+e\right)^{2}-2\,ab\sin\left(fx+e\right)-a^{2}-b^{2}\right)\sqrt{b\sin\left(fx+e\right)+a}}{\sqrt{d\sin\left(fx+e\right)+c}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] integral(-(b^2*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2)*sqrt(b*sin(
f*x + e) + a)/sqrt(d*sin(f*x + e) + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**(5/2)/(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}{\sqrt{d\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate((b*sin(f*x + e) + a)^(5/2)/sqrt(d*sin(f*x + e) + c), x)
```

3.781 $\int \frac{(a+b\sin(e+fx))^{5/2}}{(c+d\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=780

$$\frac{b\left(-2a^2d^2+4abcd+b^2\left(-\left(3c^2-d^2\right)\right)\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{d^2f\left(c^2-d^2\right)\sqrt{a+b\sin(e+fx)}}-\frac{\sqrt{a+b}\left(-2a^2d^2+4abcd+b^2\left(-\left(3c^2-d^2\right)\right)\right)\sec}{}$$

```
[Out] -((Sqrt[a + b]*(4*a*b*c*d - 2*a^2*d^2 - b^2*(3*c^2 - d^2))*EllipticE[ArcSin
[(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x
]])], ((a - b)*(c + d))/((a + b)*(c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)
*(1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1
+ Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/(d^2
*Sqrt[c + d]*(b*c - a*d)*f)) - (b*Sqrt[c + d]*(3*b*c - 5*a*d)*EllipticPi[(b
*(c + d))/((a + b)*d), ArcSin[(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])/(Sqrt[
c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/((a + b)*(c - d))]*Sec
[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f
*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x])
)]*(a + b*Sin[e + f*x]))/(Sqrt[a + b]*d^3*f) + (2*(b*c - a*d)^2*Cos[e + f*x
]*Sqrt[a + b*Sin[e + f*x]])/(d*(c^2 - d^2)*f*Sqrt[c + d*Sin[e + f*x]]) + (b
*(4*a*b*c*d - 2*a^2*d^2 - b^2*(3*c^2 - d^2))*Cos[e + f*x]*Sqrt[c + d*Sin[e
+ f*x]])/(d^2*(c^2 - d^2)*f*Sqrt[a + b*Sin[e + f*x]]) - ((a + b)^(3/2)*(2*a
*d - b*(3*c + d))*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(
Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))
]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e
+ f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f
*x])))]*(c + d*Sin[e + f*x]))/(d^2*(c + d)^(3/2)*f)
```

Rubi [A] time = 2.46831, antiderivative size = 780, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2792, 3061, 3053, 2811, 2998, 2818, 2996}

$$\frac{b\left(-2a^2d^2+4abcd+b^2\left(-\left(3c^2-d^2\right)\right)\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{d^2f\left(c^2-d^2\right)\sqrt{a+b\sin(e+fx)}}-\frac{\sqrt{a+b}\left(-2a^2d^2+4abcd+b^2\left(-\left(3c^2-d^2\right)\right)\right)\sec}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^(5/2)/(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] -((Sqrt[a + b]*(4*a*b*c*d - 2*a^2*d^2 - b^2*(3*c^2 - d^2))*EllipticE[ArcSin
[(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x
]])], ((a - b)*(c + d))/((a + b)*(c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)
*(1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1
+ Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/(d^2
*Sqrt[c + d]*(b*c - a*d)*f)) - (b*Sqrt[c + d]*(3*b*c - 5*a*d)*EllipticPi[(b
*(c + d))/((a + b)*d), ArcSin[(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])/(Sqrt[
c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/((a + b)*(c - d))]*Sec
[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f
*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x])
)]*(a + b*Sin[e + f*x]))/(Sqrt[a + b]*d^3*f) + (2*(b*c - a*d)^2*Cos[e + f*x
]*Sqrt[a + b*Sin[e + f*x]])/(d*(c^2 - d^2)*f*Sqrt[c + d*Sin[e + f*x]]) + (b
*(4*a*b*c*d - 2*a^2*d^2 - b^2*(3*c^2 - d^2))*Cos[e + f*x]*Sqrt[c + d*Sin[e
+ f*x]])/(d^2*(c^2 - d^2)*f*Sqrt[a + b*Sin[e + f*x]]) - ((a + b)^(3/2)*(2*a
*d - b*(3*c + d))*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(
```

```
Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))
]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e
+ f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f
*x])))]*(c + d*Sin[e + f*x]))/(d^2*(c + d)^(3/2)*f)
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2811

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[(2*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d
)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(
1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*EllipticPi[(b*(c + d))/
(d*(a + b)), ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt[c + d*Sin[e + f*x]])/Sqrt[
a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)*(c - d))])/(d*f*Rt[(a + b)
/(c + d), 2]*Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

```
&& NeQ[A, B]
```

Rule 2818

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*(c + d*Sin[e + f*x])*Sqrt[((b*c -
a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d
)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[
(c + d)/(a + b), 2]*(Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (
(a + b)*(c - d))/((a - b)*(c + d))])/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*
Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 2996

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(-2*A*(c - d)*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/
((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c
 + d)*(a + b*Sin[e + f*x])))]*EllipticE[ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt
[c + d*Sin[e + f*x]])/Sqrt[a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)
*(c - d))])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2]*Cos[e + f*x]), x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rubi steps

$$\int \frac{(a+b\sin(e+fx))^{5/2}}{(c+d\sin(e+fx))^{3/2}}\,dx = \frac{2(bc-ad)^2\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{d\left(c^2-d^2\right)f\sqrt{c+d\sin(e+fx)}} - \frac{2\int \frac{\frac{1}{2}\left(b^3c^2-a^3cd-3ab^2cd+3a^2bd^2\right)+\frac{1}{2}\left(a^2bcd-b^3cd-a^3d^2-a\right.}{\sqrt{a+b\sin}}}{d}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{d\left(c^2-d^2\right)f\sqrt{c+d\sin(e+fx)}} + \frac{b\left(4abcd-2a^2d^2-b^2\left(3c^2-d^2\right)\right)\cos(e+fx)}{d^2\left(c^2-d^2\right)f\sqrt{a+b\sin(e+}}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{d\left(c^2-d^2\right)f\sqrt{c+d\sin(e+fx)}} + \frac{b\left(4abcd-2a^2d^2-b^2\left(3c^2-d^2\right)\right)\cos(e+fx)}{d^2\left(c^2-d^2\right)f\sqrt{a+b\sin(e+}}$$

$$= -\frac{b\sqrt{c+d}(3bc-5ad)\Pi\left(\frac{b(c+d)}{(a+b)d};\sin^{-1}\left(\frac{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sec(e+fx)\sqrt{-\frac{(bc-ad)(1}{(c+d)(a+b}}}{\sqrt{a+b}d^3f}$$

$$= -\frac{\sqrt{a+b}\left(4abcd-2a^2d^2-b^2\left(3c^2-d^2\right)\right)E\left(\sin^{-1}\left(\frac{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sec(e+fx}{d^2\sqrt{c+d}(bc-ad)f}$$

Mathematica [B] time = 6.80133, size = 1976, normalized size = 2.53

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sin[e + f*x])^(5/2)/(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-2*(b^2*c^2*Cos[e + f*x] - 2*a*b*c*d*Cos[e + f*x] + a^2*d^2*Cos[e + f*x])*
Sqrt[a + b*Sin[e + f*x]])/(d*(-c^2 + d^2)*f*Sqrt[c + d*Sin[e + f*x]]) - ((-
```

```
4*(-(b*c) + a*d)*(-(b^3*c^2) - 2*a^3*c*d - 2*a*b^2*c*d + 4*a^2*b*d^2 + b^3*
d^2)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[S
qrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a
*d))]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e
 + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e +
 f*x]))/(-(b*c) + a*d)]*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*S
in[e + f*x]))/(-(b*c) + a*d))])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*S
qrt[c + d*Sin[e + f*x]]) - 4*(-(b*c) + a*d)*(-4*a*b^2*c^2 + 2*a^2*b*c*d - 2
*b^3*c*d - 2*a^3*d^2 + 6*a*b^2*d^2)*((Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2
]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*
(c + d*Sin[e + f*x]))/(-(b*c) + a*d))]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b
)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e
+ Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[-(((a + b)*Cs
c[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))])/((a + b)*(
c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - (Sqrt[((c + d)*
Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a + b)*d)
, ArcSin[Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(
-(b*c) + a*d))]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*
x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a +
 b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^
2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))])/((a + b)*d*Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]])) + 2*(3*b^3*c^2 - 4*a*b^2*c*d + 2*a^2*b*d^2 -
b^3*d^2)*((Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(d*Sqrt[a + b*Sin[e + f*x
]]) + (Sqrt[(a - b)/(a + b)]*(a + b)*Cos[(-e + Pi/2 - f*x)/2]*EllipticE[Arc
Sin[(Sqrt[(a - b)/(a + b)]*Sin[(-e + Pi/2 - f*x)/2])/Sqrt[(a + b*Sin[e + f*
x])/(a + b)]], (2*(-(b*c) + a*d))/((a - b)*(c + d))]*Sqrt[c + d*Sin[e + f*x
]])/(b*d*Sqrt[((a + b)*Cos[(-e + Pi/2 - f*x)/2]^2)/(a + b*Sin[e + f*x])]*Sq
rt[a + b*Sin[e + f*x]]*Sqrt[(a + b*Sin[e + f*x])/(a + b)]*Sqrt[((a + b)*(c
+ d*Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x]))]) - (2*(-(b*c) + a*d)*(((
(a + b)*c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*Ellipt
icF[ArcSin[Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))
/(-(b*c) + a*d))]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e +
f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a
 + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2
]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))])/((a + b)*(c + d)*Sqrt[a + b*Sin
[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - ((b*c + a*d)*Sqrt[((c + d)*Cot[(-e +
 Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a + b)*d), ArcSin[
Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) +
a*d))]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-
e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e
+ f*x]))/(-(b*c) + a*d)]*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*
Sin[e + f*x]))/(-(b*c) + a*d))])/((a + b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c
 + d*Sin[e + f*x]])))/(b*d)))/(2*(c - d)*d*(c + d)*f)
```

Maple [B] time = 39.216, size = 3434386, normalized size = 4403.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate((b*sin(f*x + e) + a)^(5/2)/(d*sin(f*x + e) + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2\cos\left(fx+e\right)^2-2\,ab\sin\left(fx+e\right)-a^2-b^2\right)\sqrt{b\sin\left(fx+e\right)+a}\sqrt{d\sin\left(fx+e\right)+c}}{d^2\cos\left(fx+e\right)^2-2\,cd\sin\left(fx+e\right)-c^2-d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] integral((b^2*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2)*sqrt(b*sin(f
*x + e) + a)*sqrt(d*sin(f*x + e) + c)/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x +
 e) - c^2 - d^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**(5/2)/(c+d*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate((b*sin(f*x + e) + a)^(5/2)/(d*sin(f*x + e) + c)^(3/2), x)
```

3.782 $\int \frac{(a+b\sin(e+fx))^{5/2}}{(c+d\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=737

$$-\frac{2\sqrt{a+b}\left(a^2d^2(3c+d)+abd\left(3c^2-4cd-7d^2\right)+b^2\left(-6c^2d+3c^3-2cd^2+9d^3\right)\right)\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-}{(a+}}}{3d^3f(c-d)^2(c+d)^{3/2}}$$

```
[Out] (2*(b*c - a*d)^2*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]])/(3*d*(c^2 - d^2)*f*
(c + d*Sin[e + f*x])^(3/2)) + (2*(a - b)*Sqrt[a + b]*(3*b*c^2 + 4*a*c*d - 7
*b*d^2)*EllipticE[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b
]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sec[e +
f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*
Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*(c
 + d*Sin[e + f*x]))/(3*(c - d)^2*d^2*(c + d)^(3/2)*f) - (2*Sqrt[a + b]*(a^2
*d^2*(3*c + d) + a*b*d*(3*c^2 - 4*c*d - 7*d^2) + b^2*(3*c^3 - 6*c^2*d - 2*c
*d^2 + 9*d^3))*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqr
t[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*S
ec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f
*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x]
)))]*(c + d*Sin[e + f*x]))/(3*(c - d)^2*d^3*(c + d)^(3/2)*f) + (2*b^2*Sqrt[
a + b]*EllipticPi[((a + b)*d)/(b*(c + d)), ArcSin[(Sqrt[c + d]*Sqrt[a + b*S
in[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((
a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b
)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(
c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/(d^3*Sqrt[c + d]*f)
```

Rubi [A] time = 1.78075, antiderivative size = 737, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2792, 3053, 2811, 2998, 2818, 2996}

$$-\frac{2\sqrt{a+b}\left(a^2d^2(3c+d)+abd\left(3c^2-4cd-7d^2\right)+b^2\left(-6c^2d+3c^3-2cd^2+9d^3\right)\right)\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-}{(a+}}}{3d^3f(c-d)^2(c+d)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^(5/2)/(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (2*(b*c - a*d)^2*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]])/(3*d*(c^2 - d^2)*f*
(c + d*Sin[e + f*x])^(3/2)) + (2*(a - b)*Sqrt[a + b]*(3*b*c^2 + 4*a*c*d - 7
*b*d^2)*EllipticE[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b
]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sec[e +
f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*
Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*(c
 + d*Sin[e + f*x]))/(3*(c - d)^2*d^2*(c + d)^(3/2)*f) - (2*Sqrt[a + b]*(a^2
*d^2*(3*c + d) + a*b*d*(3*c^2 - 4*c*d - 7*d^2) + b^2*(3*c^3 - 6*c^2*d - 2*c
*d^2 + 9*d^3))*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqr
t[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*S
ec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f
*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x]
)))]*(c + d*Sin[e + f*x]))/(3*(c - d)^2*d^3*(c + d)^(3/2)*f) + (2*b^2*Sqrt[
a + b]*EllipticPi[((a + b)*d)/(b*(c + d)), ArcSin[(Sqrt[c + d]*Sqrt[a + b*S
in[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((
a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b
```

```
)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(
c + d*Sin[e + f*x]))))]*(c + d*Sin[e + f*x]))/(d^3*Sqrt[c + d]*f)
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2811

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[(2*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d
)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(
1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*EllipticPi[(b*(c + d))/
(d*(a + b)), ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt[c + d*Sin[e + f*x]])/Sqrt[
a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)*(c - d))])/(d*f*Rt[(a + b)
/(c + d), 2]*Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2818

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*(c + d*Sin[e + f*x])*Sqrt[((b*c -
a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d
)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[
(c + d)/(a + b), 2]*(Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (
(a + b)*(c - d))/((a - b)*(c + d))])/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*
Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 2996

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(-2*A*(c - d)*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/
((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c
 + d)*(a + b*Sin[e + f*x])))]*EllipticE[ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt
[c + d*Sin[e + f*x]])/Sqrt[a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)
*(c - d))])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2]*Cos[e + f*x]), x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rubi steps

$$\int \frac{(a+b\sin(e+fx))^{5/2}}{(c+d\sin(e+fx))^{5/2}}\,dx = \frac{2(bc-ad)^2\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{3d\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}} - \frac{2\int \frac{\frac{1}{2}\left(b^3c^2-3a^3cd-5ab^2cd+7a^2bd^2\right)-\frac{1}{2}\left(5a^2bcd+3b^3cd \cdots\right.}{\sqrt{a+b\sin(e+f \cdots}}}{3d\left(\cdots \right.}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{3d\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}} + \frac{b^3\int \frac{\sqrt{c+d\sin(e+fx)}}{\sqrt{a+b\sin(e+fx)}}\,dx}{d^3} - \frac{2\int \frac{\frac{3}{2}b^3c^2\left(c^2-d^2\right)+\frac{1}{2}d^2 \cdots}{\cdots}}{\cdots}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{3d\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}} + \frac{2b^2\sqrt{a+b}\,\Pi\left(\frac{(a+b)d}{b(c+d)};\sin^{-1}\left(\frac{\sqrt{c+d}\sqrt{a+b\sin(e+ \cdots}}{\sqrt{a+b}\sqrt{c+d\sin(e+ \cdots}}\right.\right.}{\cdots}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{3d\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}} + \frac{2(a-b)\sqrt{a+b}\left(3bc^2+4acd-7bd^2\right)E\left(\mathrm{si} \cdots\right.}{\cdots}$$

Mathematica [B] time = 6.9234, size = 2139, normalized size = 2.9

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sin[e + f*x])^(5/2)/(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]*((-2*(b^2*c^2*Cos[e + f*
x] - 2*a*b*c*d*Cos[e + f*x] + a^2*d^2*Cos[e + f*x]))/(3*d*(-c^2 + d^2)*(c +
 d*Sin[e + f*x])^2) - (2*(3*b^2*c^3*Cos[e + f*x] + a*b*c^2*d*Cos[e + f*x] -
 4*a^2*c*d^2*Cos[e + f*x] - 7*b^2*c*d^2*Cos[e + f*x] + 7*a*b*d^3*Cos[e + f*
x]))/(3*d*(-c^2 + d^2)^2*(c + d*Sin[e + f*x]))))/f + ((-4*(-(b*c) + a*d)*(-
(b^3*c^3) + 3*a^3*c^2*d + 2*a*b^2*c^2*d - 8*a^2*b*c*d^2 + b^3*c*d^2 + a^3*d
^3 + 2*a*b^2*d^3)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*Ellip
ticF[ArcSin[Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x])
)/(-(b*c) + a*d))]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e +
 f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(
a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/
2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))])/((a + b)*(c + d)*Sqrt[a + b*Si
n[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - 4*(-(b*c) + a*d)*(-4*a*b^2*c^3 + 3*
a^2*b*c^2*d + b^3*c^2*d + 4*a^3*c*d^2 - 7*a^2*b*d^3 + 3*b^3*d^3)*((Sqrt[((c
 + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[-(((a + b
)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))]/Sqrt[2]
], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x
)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*
c) + a*d)]*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))
```

```
/(-(b*c) + a*d))])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin
[e + f*x]]) - (Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*Elliptic
Pi[(-(b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)
/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))]/Sqrt[2]], (2*(-(b*c) + a*d))/(
(a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Cs
c[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[-(((a +
 b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))])/((a
+ b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])) + 2*(3*b^3*c^3 +
 a*b^2*c^2*d - 4*a^2*b*c*d^2 - 7*b^3*c*d^2 + 7*a*b^2*d^3)*((Cos[e + f*x]*Sq
rt[c + d*Sin[e + f*x]])/(d*Sqrt[a + b*Sin[e + f*x]]) + (Sqrt[(a - b)/(a + b
)]*(a + b)*Cos[(-e + Pi/2 - f*x)/2]*EllipticE[ArcSin[(Sqrt[(a - b)/(a + b)]
*Sin[(-e + Pi/2 - f*x)/2])/Sqrt[(a + b*Sin[e + f*x])/(a + b)]], (2*(-(b*c)
+ a*d))/((a - b)*(c + d))]*Sqrt[c + d*Sin[e + f*x]])/(b*d*Sqrt[((a + b)*Cos
[(-e + Pi/2 - f*x)/2]^2)/(a + b*Sin[e + f*x])]*Sqrt[a + b*Sin[e + f*x]]*Sqr
t[(a + b*Sin[e + f*x])/(a + b)]*Sqrt[((a + b)*(c + d*Sin[e + f*x]))/((c + d
)*(a + b*Sin[e + f*x]))]) - (2*(-(b*c) + a*d)*((((a + b)*c + a*d)*Sqrt[((c
+ d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[-(((a + b)
*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))]/Sqrt[2]]
, (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)
/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c
) + a*d)]*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/
(-(b*c) + a*d))])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[
e + f*x]]) - ((b*c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d
)]*EllipticPi[(-(b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[-(((a + b)*Csc[(-e +
Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))]/Sqrt[2]], (2*(-(b*c
) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[
((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*S
qrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a
*d))])/((a + b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])))/(b*d
)))/(3*(c - d)^2*d*(c + d)^2*f)
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int \left(a+b\sin\left(fx+e\right)\right)^{\frac{5}{2}}\left(c+d\sin\left(fx+e\right)\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(5/2),x)
```

```
[Out] int((a+b*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] integrate((b*sin(f*x + e) + a)^(5/2)/(d*sin(f*x + e) + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2\cos\left(fx+e\right)^2-2\,ab\sin\left(fx+e\right)-a^2-b^2\right)\sqrt{b\sin\left(fx+e\right)+a}\sqrt{d\sin\left(fx+e\right)+c}}{3\,cd^2\cos\left(fx+e\right)^2-c^3-3\,cd^2+\left(d^3\cos\left(fx+e\right)^2-3\,c^2d-d^3\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] integral((b^2*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2)*sqrt(b*sin(f
*x + e) + a)*sqrt(d*sin(f*x + e) + c)/(3*c*d^2*cos(f*x + e)^2 - c^3 - 3*c*d
^2 + (d^3*cos(f*x + e)^2 - 3*c^2*d - d^3)*sin(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**(5/2)/(c+d*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate((b*sin(f*x + e) + a)^(5/2)/(d*sin(f*x + e) + c)^(5/2), x)
```

3.783 $\int \frac{(c+d\sin(e+fx))^{5/2}}{\sqrt{a+b\sin(e+fx)}}\,dx$

Optimal. Leaf size=772

$$\frac{\sqrt{a+b}\left(3a^2d^2-abd(7c+3d)+b^2\left(8c^2+9cd+2d^2\right)\right)\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-ad)(\sin(e+fx}{(a-b)(c+d\sin(e+}}}{4b^3f\sqrt{c+d}}$$

```
[Out] (3*Sqrt[a + b]*(c - d)*d*Sqrt[c + d]*(3*b*c - a*d)*EllipticE[ArcSin[(Sqrt[a
 + b]*Sqrt[c + d*Sin[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((
a - b)*(c + d))/((a + b)*(c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Si
n[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e
+ f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/(4*b^2*(b*c
- a*d)*f) - (Sqrt[c + d]*(10*a*b*c*d - 3*a^2*d^2 - b^2*(15*c^2 + 4*d^2))*El
lipticPi[(b*(c + d))/((a + b)*d), ArcSin[(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*
x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/((a + b)*(c
 - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c + d)*(a +
b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Si
n[e + f*x]))]*(a + b*Sin[e + f*x]))/(4*b^3*Sqrt[a + b]*f) - (3*d*(3*b*c - a
*d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(4*b*f*Sqrt[a + b*Sin[e + f*x]])
 - (d^2*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])/(2*
b*f) + (Sqrt[a + b]*(3*a^2*d^2 - a*b*d*(7*c + 3*d) + b^2*(8*c^2 + 9*c*d + 2
*d^2))*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]
*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sec[e + f
*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*S
qrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*(c
+ d*Sin[e + f*x]))/(4*b^3*Sqrt[c + d]*f)
```

Rubi [A] time = 2.30294, antiderivative size = 772, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2793, 3061, 3053, 2811, 2998, 2818, 2996}

$$\frac{\sqrt{a+b}\left(3a^2d^2-abd(7c+3d)+b^2\left(8c^2+9cd+2d^2\right)\right)\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-ad)(\sin(e+fx}{(a-b)(c+d\sin(e+}}}{4b^3f\sqrt{c+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^(5/2)/Sqrt[a + b*Sin[e + f*x]],x]
```

```
[Out] (3*Sqrt[a + b]*(c - d)*d*Sqrt[c + d]*(3*b*c - a*d)*EllipticE[ArcSin[(Sqrt[a
 + b]*Sqrt[c + d*Sin[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((
a - b)*(c + d))/((a + b)*(c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Si
n[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e
+ f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/(4*b^2*(b*c
- a*d)*f) - (Sqrt[c + d]*(10*a*b*c*d - 3*a^2*d^2 - b^2*(15*c^2 + 4*d^2))*El
lipticPi[(b*(c + d))/((a + b)*d), ArcSin[(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*
x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/((a + b)*(c
 - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c + d)*(a +
b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Si
n[e + f*x]))]*(a + b*Sin[e + f*x]))/(4*b^3*Sqrt[a + b]*f) - (3*d*(3*b*c - a
*d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(4*b*f*Sqrt[a + b*Sin[e + f*x]])
 - (d^2*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])/(2*
b*f) + (Sqrt[a + b]*(3*a^2*d^2 - a*b*d*(7*c + 3*d) + b^2*(8*c^2 + 9*c*d + 2
*d^2))*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]
```

```
*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sec[e + f
*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*S
qrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*(c
+ d*Sin[e + f*x]))/(4*b^3*Sqrt[c + d]*f)
```

Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2811

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[(2*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d
)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(
1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*EllipticPi[(b*(c + d))/
(d*(a + b)), ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt[c + d*Sin[e + f*x]])/Sqrt[
a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)*(c - d))])/(d*f*Rt[(a + b)
/(c + d), 2]*Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2818

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*(c + d*Sin[e + f*x])*Sqrt[((b*c -
a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d
)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[
(c + d)/(a + b), 2]*(Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (
(a + b)*(c - d))/((a - b)*(c + d))])/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*
Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 2996

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(-2*A*(c - d)*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/
((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c
 + d)*(a + b*Sin[e + f*x])))]*EllipticE[ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt
[c + d*Sin[e + f*x]])/Sqrt[a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)
*(c - d))])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2]*Cos[e + f*x]), x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^{5/2}}{\sqrt{a+b\sin(e+fx)}}\,dx = -\frac{d^2\cos(e+fx)\sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}{2bf} + \frac{\int \frac{\frac{1}{2}\left(ad^3+bc\left(4c^2+d^2\right)\right)-d\left(acd-b\left(6c^2+d^2\right)\right)\sin}{\sqrt{a+b\sin(e+fx)}\sqrt{c+}}}{2b}$$

$$= -\frac{3d(3bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4bf\sqrt{a+b\sin(e+fx)}} - \frac{d^2\cos(e+fx)\sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin}}{2bf}$$

$$= -\frac{3d(3bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4bf\sqrt{a+b\sin(e+fx)}} - \frac{d^2\cos(e+fx)\sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin}}{2bf}$$

$$= -\frac{\sqrt{c+d}\left(10abcd-3a^2d^2-b^2\left(15c^2+4d^2\right)\right)\Pi\left(\frac{b(c+d)}{(a+b)d};\sin^{-1}\left(\frac{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)}{4b^3\sqrt{a+b}f}$$

$$= \frac{3\sqrt{a+b}(c-d)d\sqrt{c+d}(3bc-ad)E\left(\sin^{-1}\left(\frac{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sec(e+fx)\sqrt{-\frac{(bc}{(c+}}}{4b^2(bc-ad)f}$$

Mathematica [B] time = 10.3014, size = 1864, normalized size = 2.41

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*Sin[e + f*x])^(5/2)/Sqrt[a + b*Sin[e + f*x]],x]
```

```
[Out] -(d^2*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])/(2*b*
f) + ((-4*(-(b*c) + a*d)*(8*b*c^3 + 11*b*c*d^2 - a*d^3)*Sqrt[((c + d)*Cot[(
-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[-(((a + b)*Csc[(-e +
 Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))]/Sqrt[2]], (2*(-(b*
```

```
c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt
[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*
Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) +
a*d))])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])
 - 4*(-(b*c) + a*d)*(24*b*c^2*d - 4*a*c*d^2 + 4*b*d^3)*((Sqrt[((c + d)*Cot[
(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[-(((a + b)*Csc[(-e
+ Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))]/Sqrt[2]], (2*(-(b
*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqr
t[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]
*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) +
 a*d))])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]
) - (Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c)
 + a*d)/((a + b)*d), ArcSin[Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c +
 d*Sin[e + f*x]))/(-(b*c) + a*d))]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-
c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi
/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[-(((a + b)*Csc[(-
e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))])/((a + b)*d*Sqr
t[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])) + 2*(-9*b*c*d^2 + 3*a*d^3)
*((Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(d*Sqrt[a + b*Sin[e + f*x]]) + (S
qrt[(a - b)/(a + b)]*(a + b)*Cos[(-e + Pi/2 - f*x)/2]*EllipticE[ArcSin[(Sqr
t[(a - b)/(a + b)]*Sin[(-e + Pi/2 - f*x)/2])/Sqrt[(a + b*Sin[e + f*x])/(a +
 b)]], (2*(-(b*c) + a*d))/((a - b)*(c + d))]*Sqrt[c + d*Sin[e + f*x]])/(b*d
*Sqrt[((a + b)*Cos[(-e + Pi/2 - f*x)/2]^2)/(a + b*Sin[e + f*x])]*Sqrt[a + b
*Sin[e + f*x]]*Sqrt[(a + b*Sin[e + f*x])/(a + b)]*Sqrt[((a + b)*(c + d*Sin[
e + f*x]))/((c + d)*(a + b*Sin[e + f*x]))]) - (2*(-(b*c) + a*d)*((((a + b)*
c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcS
in[Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c)
 + a*d))]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin
[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin
[e + f*x]))/(-(b*c) + a*d)]*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c +
 d*Sin[e + f*x]))/(-(b*c) + a*d))])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]]) - ((b*c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 -
f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[-((
(a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))]/S
qrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2
 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))
/(-(b*c) + a*d)]*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e +
f*x]))/(-(b*c) + a*d))])/((a + b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin
[e + f*x]])))/(b*d)))/(8*b*f)
```

Maple [B] time = 13.859, size = 731123, normalized size = 947.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e))^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}{\sqrt{b\sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(5/2)/sqrt(b*sin(f*x + e) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(5/2)/(a+b*sin(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d \sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}{\sqrt{b \sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate((d*sin(f*x + e) + c)^(5/2)/sqrt(b*sin(f*x + e) + a), x)
```

3.784 $\int \frac{(c+d\sin(e+fx))^{3/2}}{\sqrt{a+b\sin(e+fx)}}\,dx$

Optimal. Leaf size=644

$$-\frac{\sqrt{a+b}(ad-b(2c+d))\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}}F\left(\sin^{-1}\left(\frac{\sqrt{c+d}\sqrt{a+b\sin(}}{\sqrt{a+b}\sqrt{c+d\sin(}}\right.\right.}{b^2 f\sqrt{c+d}}$$

```
[Out] (Sqrt[a + b]*(c - d)*d*Sqrt[c + d]*EllipticE[ArcSin[(Sqrt[a + b]*Sqrt[c + d
*Sin[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/
((a + b)*(c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c
 + d)*(a + b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d
)*(a + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/(b*(b*c - a*d)*f) + (Sqrt[c
+ d]*(3*b*c - a*d)*EllipticPi[(b*(c + d))/((a + b)*d), ArcSin[(Sqrt[a + b]*
Sqrt[c + d*Sin[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)
*(c + d))/((a + b)*(c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e +
f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]
))/((c - d)*(a + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/(b^2*Sqrt[a + b]*f
) - (d*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(f*Sqrt[a + b*Sin[e + f*x]])
- (Sqrt[a + b]*(a*d - b*(2*c + d))*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b
*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/
((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a +
 b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)
*(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/(b^2*Sqrt[c + d]*f)
```

Rubi [A] time = 1.57965, antiderivative size = 644, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2821, 3053, 2811, 2998, 2818, 2996}

$$-\frac{\sqrt{a+b}(ad-b(2c+d))\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}}F\left(\sin^{-1}\left(\frac{\sqrt{c+d}\sqrt{a+b\sin(}}{\sqrt{a+b}\sqrt{c+d\sin(}}\right.\right.}{b^2 f\sqrt{c+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^(3/2)/Sqrt[a + b*Sin[e + f*x]],x]
```

```
[Out] (Sqrt[a + b]*(c - d)*d*Sqrt[c + d]*EllipticE[ArcSin[(Sqrt[a + b]*Sqrt[c + d
*Sin[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/
((a + b)*(c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c
 + d)*(a + b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d
)*(a + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/(b*(b*c - a*d)*f) + (Sqrt[c
+ d]*(3*b*c - a*d)*EllipticPi[(b*(c + d))/((a + b)*d), ArcSin[(Sqrt[a + b]*
Sqrt[c + d*Sin[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)
*(c + d))/((a + b)*(c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e +
f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]
))/((c - d)*(a + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/(b^2*Sqrt[a + b]*f
) - (d*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(f*Sqrt[a + b*Sin[e + f*x]])
- (Sqrt[a + b]*(a*d - b*(2*c + d))*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b
*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/
((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a +
 b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)
*(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/(b^2*Sqrt[c + d]*f)
```

Rule 2821

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[1/(d*(m + n)), Int[
(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a^2*c*d*(m +
 n) + b*d*(b*c*(m - 1) + a*d*n) + (a*d*(2*b*c + a*d)*(m + n) - b*d*(a*c - b
*d*(m + n - 1)))*Sin[e + f*x] + b*d*(b*c*n + a*d*(2*m + n - 1))*Sin[e + f*x
]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[0, m, 2] && LtQ[-1, n, 2] && NeQ
[m + n, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2811

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[(2*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d
)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(
1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*EllipticPi[(b*(c + d))/
(d*(a + b)), ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt[c + d*Sin[e + f*x]])/Sqrt[
a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)*(c - d))])/(d*f*Rt[(a + b)
/(c + d), 2]*Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2818

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*(c + d*Sin[e + f*x])*Sqrt[((b*c -
a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d
)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[
(c + d)/(a + b), 2]*(Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (
(a + b)*(c - d))/((a - b)*(c + d))])/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*
Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 2996

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(-2*A*(c - d)*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/
((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c
 + d)*(a + b*Sin[e + f*x])))]*EllipticE[ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt
```

```
[c + d*Sin[e + f*x]])/Sqrt[a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)
*(c - d))])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2]*Cos[e + f*x]), x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^{3/2}}{\sqrt{a+b\sin(e+fx)}}\,dx = -\frac{d\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{f\sqrt{a+b\sin(e+fx)}} + \frac{\int \frac{-\frac{1}{2}b\left(bcd-a\left(2c^2+d^2\right)\right)+bc(bc+ad)\sin(e+fx)+\frac{1}{2}bd(3bc-ad)\sin^2(e}{(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}}{b}$$

$$= -\frac{d\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{f\sqrt{a+b\sin(e+fx)}} + \frac{\int \frac{-\frac{1}{2}a^2bd(3bc-ad)-\frac{1}{2}b^3\left(bcd-a\left(2c^2+d^2\right)\right)+b\left(-abd(3bc-ad)+b^2c(bc\right.}{(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}}{b^3}$$

$$= \frac{\sqrt{c+d}(3bc-ad)\Pi\left(\frac{b(c+d)}{(a+b)d};\sin^{-1}\left(\frac{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sec(e+fx)\sqrt{-\frac{(bc-ad)(1-}{(c+d)(a+b}}}{b^2\sqrt{a+b}f}$$

$$= \frac{\sqrt{a+b}(c-d)d\sqrt{c+d}E\left(\sin^{-1}\left(\frac{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sec(e+fx)\sqrt{-\frac{(bc-ad)(1-\sin}{(c+d)(a+b\sin}}}{b(bc-ad)f}$$

Mathematica [C] time = 32.5356, size = 222963, normalized size = 346.22

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*Sin[e + f*x])^(3/2)/Sqrt[a + b*Sin[e + f*x]],x]
```

```
[Out] Result too large to show
```

Maple [B] time = 7.095, size = 544147, normalized size = 845.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}{\sqrt{b\sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(3/2)/sqrt(b*sin(f*x + e) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(c + d\sin(e + fx))^{\frac{3}{2}}}{\sqrt{a + b\sin(e + fx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(3/2)/(a+b*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((c + d*sin(e + f*x))**(3/2)/sqrt(a + b*sin(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d\sin(fx + e) + c)^{\frac{3}{2}}}{\sqrt{b\sin(fx + e) + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate((d*sin(f*x + e) + c)^(3/2)/sqrt(b*sin(f*x + e) + a), x)
```

3.785 $\int \frac{\sqrt{c+d\sin(e+fx)}}{\sqrt{a+b\sin(e+fx)}}\,dx$

Optimal. Leaf size=198

$$\frac{2\sqrt{a+b}\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}}\Pi\left(\frac{(a+b)d}{b(c+d)};\sin^{-1}\left(\frac{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}\right)|\frac{(a-}{(a-}\right.}{bf\sqrt{c+d}}$$

```
[Out] (2*Sqrt[a + b]*EllipticPi[((a + b)*d)/(b*(c + d)), ArcSin[(Sqrt[c + d]*Sqrt
[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c
- d))/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))
/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((
a - b)*(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/(b*Sqrt[c + d]*f)
```

Rubi [A] time = 0.109509, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {2811}

$$\frac{2\sqrt{a+b}\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}}\Pi\left(\frac{(a+b)d}{b(c+d)};\sin^{-1}\left(\frac{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}\right)|\frac{(a-}{(a-}\right.}{bf\sqrt{c+d}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]],x]
```

```
[Out] (2*Sqrt[a + b]*EllipticPi[((a + b)*d)/(b*(c + d)), ArcSin[(Sqrt[c + d]*Sqrt
[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c
- d))/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))
/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((
a - b)*(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/(b*Sqrt[c + d]*f)
```

Rule 2811

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[(2*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d
)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(
1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*EllipticPi[(b*(c + d))/
(d*(a + b)), ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt[c + d*Sin[e + f*x]])/Sqrt[
a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)*(c - d))])/(d*f*Rt[(a + b)
/(c + d), 2]*Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

Rubi steps

$$\int \frac{\sqrt{c+d\sin(e+fx)}}{\sqrt{a+b\sin(e+fx)}}\,dx = \frac{2\sqrt{a+b}\,\Pi\left(\frac{(a+b)d}{b(c+d)};\sin^{-1}\left(\frac{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}\right)|\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)\sec(e+fx)\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-}}{b\sqrt{c+d}f}$$

Mathematica [A] time = 0.265166, size = 195, normalized size = 0.98

$$\frac{2\sqrt{a+b}\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(ad-bc)(\sin(e+fx)-1)}{(a+b)(c+d\sin(e+fx))}}\sqrt{\frac{(ad-bc)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}}\Pi\left(\frac{(a+b)d}{b(c+d)};\sin^{-1}\left(\frac{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}\right)|\frac{(a+b}{(a-b}\right.}{bf\sqrt{c+d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]],x]
```

```
[Out] (2*Sqrt[a + b]*EllipticPi[((a + b)*d)/(b*(c + d)), ArcSin[(Sqrt[c + d]*Sqrt
[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c
- d))/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((-(b*c) + a*d)*(-1 + Sin[e + f*
x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[((-(b*c) + a*d)*(1 + Sin[e + f*x]
))/((a - b)*(c + d*Sin[e + f*x]))]*(c + d*Sin[e + f*x]))/(b*Sqrt[c + d]*f)
```

Maple [C] time = 4.023, size = 248962, normalized size = 1257.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d\sin\left(fx+e\right)+c}}{\sqrt{b\sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(d*sin(f*x + e) + c)/sqrt(b*sin(f*x + e) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{d\sin\left(fx+e\right)+c}}{\sqrt{b\sin\left(fx+e\right)+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] integral(sqrt(d*sin(f*x + e) + c)/sqrt(b*sin(f*x + e) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c+d\sin\left(e+fx\right)}}{\sqrt{a+b\sin\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(1/2)/(a+b*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(c + d*sin(e + f*x))/sqrt(a + b*sin(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d \sin\left(fx+e\right)+c}}{\sqrt{b \sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(d*sin(f*x + e) + c)/sqrt(b*sin(f*x + e) + a), x)
```

3.786 $\int \frac{1}{\sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\,dx$

Optimal. Leaf size=192

$$\frac{2\sqrt{a+b}\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}}F\left(\sin^{-1}\left(\frac{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}\right)|\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)}{f\sqrt{c+d}(bc-ad)}$$

```
[Out] (2*Sqrt[a + b]*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqr
t[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*S
ec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f
*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x]
)))]*(c + d*Sin[e + f*x]))/(Sqrt[c + d]*(b*c - a*d)*f)
```

Rubi [A] time = 0.120762, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {2818}

$$\frac{2\sqrt{a+b}\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}}F\left(\sin^{-1}\left(\frac{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}\right)|\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)}{f\sqrt{c+d}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),x]
```


```
[Out] (2*Sqrt[a + b]*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqr
t[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*S
ec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f
*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x]
)))]*(c + d*Sin[e + f*x]))/(Sqrt[c + d]*(b*c - a*d)*f)
```


Rule 2818

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*(c + d*Sin[e + f*x])*Sqrt[((b*c -
a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d
)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[
(c + d)/(a + b), 2]*(Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (
(a + b)*(c - d))/((a - b)*(c + d))])/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*
Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\,dx = \frac{2\sqrt{a+b}F\left(\sin^{-1}\left(\frac{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}\right)|\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)\sec(e+fx)\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}}{\sqrt{c+d}(bc-ad)f}$$

Mathematica [A] time = 0.217785, size = 189, normalized size = 0.98

$$\frac{2\sqrt{a+b}\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(ad-bc)(\sin(e+fx)-1)}{(a+b)(c+d\sin(e+fx))}}\sqrt{\frac{(ad-bc)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}}F\left(\sin^{-1}\left(\frac{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}\right)|\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)}{f\sqrt{c+d}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),x]
```

```
[Out] (2*Sqrt[a + b]*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqr
t[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*S
ec[e + f*x]*Sqrt[((-(b*c) + a*d)*(-1 + Sin[e + f*x]))/((a + b)*(c + d*Sin[e
 + f*x]))]*Sqrt[((-(b*c) + a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e +
 f*x]))]*(c + d*Sin[e + f*x]))/(Sqrt[c + d]*(b*c - a*d)*f)
```

Maple [B] time = 0.516, size = 1228, normalized size = 6.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2),x)
```

```
[Out] -4/f/(-c^2+d^2)^(1/2)/(-c*(-a^2+b^2)^(1/2)+a*(-c^2+d^2)^(1/2)+d*a-c*b)*Elli
pticF(((c*(-a^2+b^2)^(1/2)-a*(-c^2+d^2)^(1/2)-d*a+c*b)/(c*(-a^2+b^2)^(1/2)+
a*(-c^2+d^2)^(1/2)-d*a+c*b)*(c*sin(f*x+e)-cos(f*x+e)*(-c^2+d^2)^(1/2)+d*cos
(f*x+e)-(-c^2+d^2)^(1/2)+d)/(c*sin(f*x+e)+cos(f*x+e)*(-c^2+d^2)^(1/2)+d*cos
(f*x+e)+(-c^2+d^2)^(1/2)+d))^(1/2),((c*(-a^2+b^2)^(1/2)+a*(-c^2+d^2)^(1/2)+
d*a-c*b)*(c*(-a^2+b^2)^(1/2)+a*(-c^2+d^2)^(1/2)-d*a+c*b)/(a*(-c^2+d^2)^(1/2
)-c*(-a^2+b^2)^(1/2)-d*a+c*b)/(-c*(-a^2+b^2)^(1/2)+a*(-c^2+d^2)^(1/2)+d*a-c
*b))^(1/2))*((c*(-a^2+b^2)^(1/2)-a*(-c^2+d^2)^(1/2)-d*a+c*b)/(c*(-a^2+b^2)^
(1/2)+a*(-c^2+d^2)^(1/2)-d*a+c*b)*(c*sin(f*x+e)-cos(f*x+e)*(-c^2+d^2)^(1/2)
+d*cos(f*x+e)-(-c^2+d^2)^(1/2)+d)/(c*sin(f*x+e)+cos(f*x+e)*(-c^2+d^2)^(1/2)
+d*cos(f*x+e)+(-c^2+d^2)^(1/2)+d))^(1/2)*((-c^2+d^2)^(1/2)*c/(c*(-a^2+b^2)^
(1/2)+a*(-c^2+d^2)^(1/2)-d*a+c*b)*(cos(f*x+e)*(-a^2+b^2)^(1/2)+a*sin(f*x+e)
+b*cos(f*x+e)+(-a^2+b^2)^(1/2)+b)/(c*sin(f*x+e)+cos(f*x+e)*(-c^2+d^2)^(1/2)
+d*cos(f*x+e)+(-c^2+d^2)^(1/2)+d))^(1/2)*((-c^2+d^2)^(1/2)*c*(a*sin(f*x+e)-
cos(f*x+e)*(-a^2+b^2)^(1/2)+b*cos(f*x+e)-(-a^2+b^2)^(1/2)+b)/(a*(-c^2+d^2)^
(1/2)-c*(-a^2+b^2)^(1/2)-d*a+c*b)/(c*sin(f*x+e)+cos(f*x+e)*(-c^2+d^2)^(1/2)
+d*cos(f*x+e)+(-c^2+d^2)^(1/2)+d))^(1/2)*(a+b*sin(f*x+e))^(1/2)*(c+d*sin(f*
x+e))^(1/2)*(cos(f*x+e)+1)^2*(-1+cos(f*x+e))^2*(c*(-c^2+d^2)^(1/2)*(-a^2+b^
2)^(1/2)*sin(f*x+e)+b*c*(-c^2+d^2)^(1/2)*sin(f*x+e)+c*d*(-a^2+b^2)^(1/2)*si
n(f*x+e)-a*c^2*sin(f*x+e)+b*c*d*sin(f*x+e)+cos(f*x+e)*(-c^2+d^2)^(1/2)*(-a^
2+b^2)^(1/2)*d-cos(f*x+e)*(-c^2+d^2)^(1/2)*a*c+cos(f*x+e)*(-c^2+d^2)^(1/2)*
b*d-cos(f*x+e)*(-a^2+b^2)^(1/2)*c^2+cos(f*x+e)*(-a^2+b^2)^(1/2)*d^2-c^2*b*c
os(f*x+e)+cos(f*x+e)*b*d^2+d*(-c^2+d^2)^(1/2)*(-a^2+b^2)^(1/2)+b*d*(-c^2+d^
2)^(1/2)+d^2*(-a^2+b^2)^(1/2)-a*c*d+d^2*b)/sin(f*x+e)^4/(-b*cos(f*x+e)^2*d+
a*d*sin(f*x+e)+b*c*sin(f*x+e)+c*a+b*d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\sin\left(fx+e\right)+a}\sqrt{d\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate(1/(sqrt(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{b\sin\left(fx+e\right)+a}\sqrt{d\sin\left(fx+e\right)+c}}{bd\cos\left(fx+e\right)^2-ac-bd-(bc+ad)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="fri
cas")
```

```
[Out] integral(-sqrt(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)/(b*d*cos(f*x +
e)^2 - a*c - b*d - (b*c + a*d)*sin(f*x + e)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{a+b\sin\left(e+fx\right)}\sqrt{c+d\sin\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))**(1/2)/(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*sin(e + f*x))*sqrt(c + d*sin(e + f*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{b\sin\left(fx+e\right)+a}\sqrt{d\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate(1/(sqrt(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)), x)
```

3.787 $\int \frac{1}{\sqrt{a+b\sin(e+fx)}(c+d\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=405

$$\frac{2\sqrt{a+b}\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}}F\left(\sin^{-1}\left(\frac{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}\right)|\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)}{f(c-d)\sqrt{c+d}(bc-ad)}$$

```
[Out] (2*(a - b)*Sqrt[a + b]*d*EllipticE[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f
*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(
c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d
*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Si
n[e + f*x])))]*(c + d*Sin[e + f*x]))/((c - d)*Sqrt[c + d]*(b*c - a*d)^2*f)
+ (2*Sqrt[a + b]*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(S
qrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]
*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e +
 f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*
x])))]*(c + d*Sin[e + f*x]))/((c - d)*Sqrt[c + d]*(b*c - a*d)*f)
```

Rubi [A] time = 0.459213, antiderivative size = 405, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2801, 2818, 2996}

$$\frac{2\sqrt{a+b}\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}}F\left(\sin^{-1}\left(\frac{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}\right)|\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)}{f(c-d)\sqrt{c+d}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(3/2)),x]
```


```
[Out] (2*(a - b)*Sqrt[a + b]*d*EllipticE[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f
*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(
c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d
*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Si
n[e + f*x])))]*(c + d*Sin[e + f*x]))/((c - d)*Sqrt[c + d]*(b*c - a*d)^2*f)
+ (2*Sqrt[a + b]*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(S
qrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]
*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e +
 f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*
x])))]*(c + d*Sin[e + f*x]))/((c - d)*Sqrt[c + d]*(b*c - a*d)*f)
```


Rule 2801

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin
[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[1/(a - b), Int[1/(Sqrt[a + b*Sin[
e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[b/(a - b), Int[(1 + Sin[
e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0]
```

Rule 2818

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*(c + d*Sin[e + f*x])*Sqrt[((b*c -
```

```
a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d
)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[
(c + d)/(a + b), 2]*(Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (
(a + b)*(c - d))/((a - b)*(c + d))])/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*
Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 2996

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(-2*A*(c - d)*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/
((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c
 + d)*(a + b*Sin[e + f*x])))]*EllipticE[ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt
[c + d*Sin[e + f*x]])/Sqrt[a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)
*(c - d))])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2]*Cos[e + f*x]), x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\sin(e+fx)}(c+d\sin(e+fx))^{3/2}}\,dx = \frac{\int \frac{1}{\sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\,dx}{c-d} - \frac{d\int \frac{1+\sin(e+fx)}{\sqrt{a+b\sin(e+fx)}(c+d\sin(e+fx))^{3/2}}\,dx}{c-d}$$

$$= \frac{2(a-b)\sqrt{a+b}dE\left(\sin^{-1}\left(\frac{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}\right)|\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)\sec(e+fx)\sqrt{\frac{(bc}{(a-}}}{(c-d)\sqrt{c+d}(bc-ad}$$

Mathematica [B] time = 32.4505, size = 90261, normalized size = 222.87

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(3/2)),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.846, size = 41715, normalized size = 103.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\sin\left(fx+e\right)+a}\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="max
ima")
```

```
[Out] integrate(1/(sqrt(b*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sin\left(fx+e\right)+a}\sqrt{d\sin\left(fx+e\right)+c}}{ac^2+2\,bcd+ad^2-\left(2\,bcd+ad^2\right)\cos\left(fx+e\right)^2-\left(bd^2\cos\left(fx+e\right)^2-bc^2-2\,acd-bd^2\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="fri
cas")
```

```
[Out] integral(sqrt(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)/(a*c^2 + 2*b*c*d
 + a*d^2 - (2*b*c*d + a*d^2)*cos(f*x + e)^2 - (b*d^2*cos(f*x + e)^2 - b*c^2
 - 2*a*c*d - b*d^2)*sin(f*x + e)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+b\sin\left(e+fx\right)}\left(c+d\sin\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))**(1/2)/(c+d*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*sin(e + f*x))*(c + d*sin(e + f*x))**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\sin\left(fx+e\right)+a}\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="gia
c")
```

```
[Out] integrate(1/(sqrt(b*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^(3/2)), x)
```

3.788 $\int \frac{1}{\sqrt{a+b\sin(e+fx)}(c+d\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=521

$$-\frac{2d^2\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{3f\left(c^2-d^2\right)(bc-ad)(c+d\sin(e+fx))^{3/2}}-\frac{2\sqrt{a+b}\left(ad(3c+d)-b\left(3c^2+3cd-2d^2\right)\right)\sec(e+fx)(c+d\sin(e+fx))}{3f(c-d)^2(c}$$

```
[Out] (-2*d^2*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]])/(3*(b*c - a*d)*(c^2 - d^2)*f
*(c + d*Sin[e + f*x])^(3/2)) - (4*(a - b)*Sqrt[a + b]*d*(2*a*c*d - b*(3*c^2
 - d^2))*EllipticE[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a +
b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sec[e +
 f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]
*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*(
c + d*Sin[e + f*x]))/(3*(c - d)^2*(c + d)^(3/2)*(b*c - a*d)^3*f) - (2*Sqrt[
a + b]*(a*d*(3*c + d) - b*(3*c^2 + 3*c*d - 2*d^2))*EllipticF[ArcSin[(Sqrt[c
 + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((
a + b)*(c - d))/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[
e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e +
 f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/(3*(c - d)^2
*(c + d)^(3/2)*(b*c - a*d)^2*f)
```

Rubi [A] time = 1.0186, antiderivative size = 521, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2802, 2998, 2818, 2996}

$$-\frac{2d^2\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{3f\left(c^2-d^2\right)(bc-ad)(c+d\sin(e+fx))^{3/2}}-\frac{2\sqrt{a+b}\left(ad(3c+d)-b\left(3c^2+3cd-2d^2\right)\right)\sec(e+fx)(c+d\sin(e+fx))}{3f(c-d)^2(c}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(5/2)),x]
```


```
[Out] (-2*d^2*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]])/(3*(b*c - a*d)*(c^2 - d^2)*f
*(c + d*Sin[e + f*x])^(3/2)) - (4*(a - b)*Sqrt[a + b]*d*(2*a*c*d - b*(3*c^2
 - d^2))*EllipticE[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a +
b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sec[e +
 f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]
*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*(
c + d*Sin[e + f*x]))/(3*(c - d)^2*(c + d)^(3/2)*(b*c - a*d)^3*f) - (2*Sqrt[
a + b]*(a*d*(3*c + d) - b*(3*c^2 + 3*c*d - 2*d^2))*EllipticF[ArcSin[(Sqrt[c
 + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((
a + b)*(c - d))/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[
e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e +
 f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/(3*(c - d)^2
*(c + d)^(3/2)*(b*c - a*d)^2*f)
```


Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
```

```
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2818

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*(c + d*Sin[e + f*x])*Sqrt[((b*c -
a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d
)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[
(c + d)/(a + b), 2]*(Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (
(a + b)*(c - d))/((a - b)*(c + d))])/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*
Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 2996

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(-2*A*(c - d)*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/
((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c
 + d)*(a + b*Sin[e + f*x])))]*EllipticE[ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt
[c + d*Sin[e + f*x]])/Sqrt[a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)
*(c - d))])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2]*Cos[e + f*x]), x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\sin(e+fx)}(c+d\sin(e+fx))^{5/2}}\,dx = -\frac{2d^2\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{3(bc-ad)\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}} + \frac{2\int \frac{\frac{1}{2}\left(-2bd^2+3c(bc-ad)\right)-\cdots}{\sqrt{a+b\sin(e+fx)}(c\cdots}}{3(bc-ad)\cdots}$$

$$= -\frac{2d^2\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{3(bc-ad)\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}} - \frac{\left(ad(3c+d)-b\left(3c^2+\cdots\right.\right.}{3\cdots}$$

$$= -\frac{2d^2\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{3(bc-ad)\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}} - \frac{4(a-b)\sqrt{a+b}d\left(2ac\cdots\right.}{\cdots}$$

Mathematica [B] time = 6.4407, size = 2072, normalized size = 3.98

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(5/2)),x]
```

```
[Out] (Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]*((-2*d^2*Cos[e + f*x])/(
3*(b*c - a*d)*(c^2 - d^2)*(c + d*Sin[e + f*x])^2) + (4*(-3*b*c^2*d^2*Cos[e
+ f*x] + 2*a*c*d^3*Cos[e + f*x] + b*d^4*Cos[e + f*x]))/(3*(b*c - a*d)^2*(c^
2 - d^2)^2*(c + d*Sin[e + f*x]))))/f + ((-4*(-(b*c) + a*d)*(3*b^2*c^4 - 6*a
*b*c^3*d + 3*a^2*c^2*d^2 - 5*b^2*c^2*d^2 + 2*a*b*c*d^3 + a^2*d^4 + 2*b^2*d^
4)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqr
t[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d
))]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e +
 Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f
*x]))/(-(b*c) + a*d)]*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin
[e + f*x]))/(-(b*c) + a*d))])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqr
t[c + d*Sin[e + f*x]]) - 4*(-(b*c) + a*d)*(-6*b^2*c^3*d - 2*a*b*c^2*d^2 + 4
*a^2*c*d^3 + 2*b^2*c*d^3 + 2*a*b*d^4)*((Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)
/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^
2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))]/Sqrt[2]], (2*(-(b*c) + a*d))/((a +
 b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-
e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[-(((a + b)*
Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))])/((a + b)
*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - (Sqrt[((c + d
)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a + b)*
d), ArcSin[Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))
/(-(b*c) + a*d))]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e +
f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a
 + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2
]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))])/((a + b)*d*Sqrt[a + b*Sin[e + f
*x]]*Sqrt[c + d*Sin[e + f*x]])) + 2*(6*b^2*c^2*d^2 - 4*a*b*c*d^3 - 2*b^2*d^
4)*((Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(d*Sqrt[a + b*Sin[e + f*x]]) +
(Sqrt[(a - b)/(a + b)]*(a + b)*Cos[(-e + Pi/2 - f*x)/2]*EllipticE[ArcSin[(S
qrt[(a - b)/(a + b)]*Sin[(-e + Pi/2 - f*x)/2])/Sqrt[(a + b*Sin[e + f*x])/(a
 + b)]], (2*(-(b*c) + a*d))/((a - b)*(c + d))]*Sqrt[c + d*Sin[e + f*x]])/(b
*d*Sqrt[((a + b)*Cos[(-e + Pi/2 - f*x)/2]^2)/(a + b*Sin[e + f*x])]*Sqrt[a +
 b*Sin[e + f*x]]*Sqrt[(a + b*Sin[e + f*x])/(a + b)]*Sqrt[((a + b)*(c + d*Si
n[e + f*x]))/((c + d)*(a + b*Sin[e + f*x]))]) - (2*(-(b*c) + a*d)*((((a + b
)*c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[Ar
cSin[Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*
c) + a*d))]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*S
in[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*S
in[e + f*x]))/(-(b*c) + a*d)]*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c
 + d*Sin[e + f*x]))/(-(b*c) + a*d))])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f
*x]]*Sqrt[c + d*Sin[e + f*x]]) - ((b*c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2
- f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[-
(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))]
/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi
/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]
))/(-(b*c) + a*d)]*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e
+ f*x]))/(-(b*c) + a*d))])/((a + b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])))/(b*d)))/(3*(c - d)^2*(c + d)^2*(b*c - a*d)^2*f)
```

Maple [B] time = 3.897, size = 219162, normalized size = 420.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\sin\left(fx+e\right)+a}\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(5/2),x, algorithm="max
ima")
```

```
[Out] integrate(1/(sqrt(b*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^(5/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sin\left(fx+e\right)+a}\sqrt{d\sin\left(fx+e\right)+c}}{bd^3\cos\left(fx+e\right)^4+ac^3+3\,bc^2d+3\,acd^2+bd^3-\left(3\,bc^2d+3\,acd^2+2\,bd^3\right)\cos\left(fx+e\right)^2+\left(bc^3+3\,ac^2d+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(5/2),x, algorithm="fri
cas")
```

```
[Out] integral(sqrt(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)/(b*d^3*cos(f*x +
 e)^4 + a*c^3 + 3*b*c^2*d + 3*a*c*d^2 + b*d^3 - (3*b*c^2*d + 3*a*c*d^2 + 2*
b*d^3)*cos(f*x + e)^2 + (b*c^3 + 3*a*c^2*d + 3*b*c*d^2 + a*d^3 - (3*b*c*d^2
 + a*d^3)*cos(f*x + e)^2)*sin(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))**(1/2)/(c+d*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\sin\left(fx+e\right)+a}\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(5/2),x, algorithm="gia
c")

[Out] integrate(1/(sqrt(b*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^(5/2)), x)
```

3.789 $\int \frac{(c+d\sin(e+fx))^{5/2}}{(a+b\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=822

$$\frac{2\cos(e+fx)\sqrt{c+d\sin(e+fx)}(bc-ad)^2}{b\left(a^2-b^2\right)f\sqrt{a+b\sin(e+fx)}}+\frac{d\sqrt{c+d}(5bc-3ad)\Pi\left(\frac{b(c+d)}{(a+b)d};\sin^{-1}\left(\frac{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sec(e+}{b^3\sqrt{a+b}}$$

```
[Out] ((c - d)*Sqrt[c + d]*(2*b^2*c^2 - 4*a*b*c*d + 3*a^2*d^2 - b^2*d^2)*Elliptic
E[ArcSin[(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin
[e + f*x]])], ((a - b)*(c + d))/((a + b)*(c - d))]*Sec[e + f*x]*Sqrt[-(((b*
c - a*d)*(1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*Sqrt[((b*c -
a*d)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x
]))/((a - b)*b^2*Sqrt[a + b]*(b*c - a*d)*f) + (d*Sqrt[c + d]*(5*b*c - 3*a*d
)*EllipticPi[(b*(c + d))/((a + b)*d), ArcSin[(Sqrt[a + b]*Sqrt[c + d*Sin[e
+ f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/((a + b
)*(c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c + d)*(
a + b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d)*(a +
b*Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/(b^3*Sqrt[a + b]*f) + (2*(b*c - a*d
)^2*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(b*(a^2 - b^2)*f*Sqrt[a + b*Sin[
e + f*x]]) + ((4*a*b*c*d - 3*a^2*d^2 - b^2*(2*c^2 - d^2))*Cos[e + f*x]*Sqrt
[c + d*Sin[e + f*x]])/(b*(a^2 - b^2)*f*Sqrt[a + b*Sin[e + f*x]]) - (Sqrt[a
+ b]*(3*a^2*d^2 - 2*a*b*d*(c + 3*d) - b^2*(2*c^2 - 6*c*d - d^2))*EllipticF[
ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e
 + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c -
a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d
)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x])
)/((a - b)*b^3*Sqrt[c + d]*f)
```

Rubi [A] time = 2.65171, antiderivative size = 822, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2792, 3061, 3053, 2811, 2998, 2818, 2996}

$$\frac{2\cos(e+fx)\sqrt{c+d\sin(e+fx)}(bc-ad)^2}{b\left(a^2-b^2\right)f\sqrt{a+b\sin(e+fx)}}+\frac{d\sqrt{c+d}(5bc-3ad)\Pi\left(\frac{b(c+d)}{(a+b)d};\sin^{-1}\left(\frac{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sec(e+}{b^3\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^(5/2)/(a + b*Sin[e + f*x])^(3/2),x]
```

```
[Out] ((c - d)*Sqrt[c + d]*(2*b^2*c^2 - 4*a*b*c*d + 3*a^2*d^2 - b^2*d^2)*Elliptic
E[ArcSin[(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin
[e + f*x]])], ((a - b)*(c + d))/((a + b)*(c - d))]*Sec[e + f*x]*Sqrt[-(((b*
c - a*d)*(1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*Sqrt[((b*c -
a*d)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x
]))/((a - b)*b^2*Sqrt[a + b]*(b*c - a*d)*f) + (d*Sqrt[c + d]*(5*b*c - 3*a*d
)*EllipticPi[(b*(c + d))/((a + b)*d), ArcSin[(Sqrt[a + b]*Sqrt[c + d*Sin[e
+ f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/((a + b
)*(c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c + d)*(
a + b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d)*(a +
b*Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/(b^3*Sqrt[a + b]*f) + (2*(b*c - a*d
)^2*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(b*(a^2 - b^2)*f*Sqrt[a + b*Sin[
e + f*x]]) + ((4*a*b*c*d - 3*a^2*d^2 - b^2*(2*c^2 - d^2))*Cos[e + f*x]*Sqrt
[c + d*Sin[e + f*x]])/(b*(a^2 - b^2)*f*Sqrt[a + b*Sin[e + f*x]]) - (Sqrt[a
```

```
+ b]*(3*a^2*d^2 - 2*a*b*d*(c + 3*d) - b^2*(2*c^2 - 6*c*d - d^2))*EllipticF[
ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e
 + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c -
a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d
)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x])
)/((a - b)*b^3*Sqrt[c + d]*f)
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2811

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[(2*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d
)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(
1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*EllipticPi[(b*(c + d))/
(d*(a + b)), ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt[c + d*Sin[e + f*x]])/Sqrt[
a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)*(c - d))])/(d*f*Rt[(a + b)
/(c + d), 2]*Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
```

```
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2818

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*(c + d*Sin[e + f*x])*Sqrt[((b*c -
a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d
)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[
(c + d)/(a + b), 2]*(Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (
(a + b)*(c - d))/((a - b)*(c + d))])/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*
Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 2996

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(-2*A*(c - d)*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/
((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c
 + d)*(a + b*Sin[e + f*x])))]*EllipticE[ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt
[c + d*Sin[e + f*x]])/Sqrt[a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)
*(c - d))])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2]*Cos[e + f*x]), x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^{5/2}}{(a+b\sin(e+fx))^{3/2}}\,dx = \frac{2(bc-ad)^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{b\left(a^2-b^2\right)f\sqrt{a+b\sin(e+fx)}} - \frac{2\int \frac{\frac{1}{2}\left(3b^2c^2d+a^2d^3-abc\left(c^2+3d^2\right)\right)-\frac{1}{2}\left(2a^2cd^2-abd\left(c^2-\cdots\right.\right.}{\sqrt{a+b\cdots}}}{\cdots}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{b\left(a^2-b^2\right)f\sqrt{a+b\sin(e+fx)}} + \frac{\left(4abcd-3a^2d^2-b^2\left(2c^2-d^2\right)\right)\cos(e+f\cdots}{b\left(a^2-b^2\right)f\sqrt{a+b\sin(e+\cdots}}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{b\left(a^2-b^2\right)f\sqrt{a+b\sin(e+fx)}} + \frac{\left(4abcd-3a^2d^2-b^2\left(2c^2-d^2\right)\right)\cos(e+f\cdots}{b\left(a^2-b^2\right)f\sqrt{a+b\sin(e+\cdots}}$$

$$= \frac{d\sqrt{c+d}(5bc-3ad)\Pi\left(\frac{b(c+d)}{(a+b)d};\sin^{-1}\left(\frac{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}\right)\middle|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sec(e+fx)\sqrt{-\frac{(bc-ad)\cdots}{(c+d)(a\cdots}}}{b^3\sqrt{a+b}f\cdots}$$

$$= -\frac{(c-d)\sqrt{c+d}\left(4abcd-3a^2d^2-b^2\left(2c^2-d^2\right)\right)E\left(\sin^{-1}\left(\frac{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}\right)\middle|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\cdots}{(a-b)b^2\sqrt{a+b}(bc-\cdots}$$

Mathematica [B] time = 6.7905, size = 1975, normalized size = 2.4

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*Sin[e + f*x])^(5/2)/(a + b*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-2*(b^2*c^2*Cos[e + f*x] - 2*a*b*c*d*Cos[e + f*x] + a^2*d^2*Cos[e + f*x])*
Sqrt[c + d*Sin[e + f*x]])/(b*(-a^2 + b^2)*f*Sqrt[a + b*Sin[e + f*x]]) + ((-
4*(-(b*c) + a*d)*(2*a*b*c^3 - 4*b^2*c^2*d + 2*a*b*c*d^2 + a^2*d^3 - b^2*d^3
)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt
[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)
)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e +
Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*
x]))/(-(b*c) + a*d)]*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[
e + f*x]))/(-(b*c) + a*d))])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt
[c + d*Sin[e + f*x]]) - 4*(-(b*c) + a*d)*(2*b^2*c^3 - 2*a*b*c^2*d + 4*a^2*c
*d^2 - 6*b^2*c*d^2 + 2*a*b*d^3)*((Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)
/(-c + d)]*EllipticF[ArcSin[Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c +
 d*Sin[e + f*x]))/(-(b*c) + a*d))]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-
c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi
/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[-(((a + b)*Csc[(-
e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))])/((a + b)*(c +
d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - (Sqrt[((c + d)*Cot[
(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a + b)*d), Ar
cSin[Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*
c) + a*d))]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*S
in[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*S
in[e + f*x]))/(-(b*c) + a*d)]*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c
 + d*Sin[e + f*x]))/(-(b*c) + a*d))])/((a + b)*d*Sqrt[a + b*Sin[e + f*x]]*S
qrt[c + d*Sin[e + f*x]])) + 2*(-2*b^2*c^2*d + 4*a*b*c*d^2 - 3*a^2*d^3 + b^2
*d^3)*((Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(d*Sqrt[a + b*Sin[e + f*x]])
 + (Sqrt[(a - b)/(a + b)]*(a + b)*Cos[(-e + Pi/2 - f*x)/2]*EllipticE[ArcSin
[(Sqrt[(a - b)/(a + b)]*Sin[(-e + Pi/2 - f*x)/2])/Sqrt[(a + b*Sin[e + f*x])
/(a + b)]], (2*(-(b*c) + a*d))/((a - b)*(c + d))]*Sqrt[c + d*Sin[e + f*x]])
/(b*d*Sqrt[((a + b)*Cos[(-e + Pi/2 - f*x)/2]^2)/(a + b*Sin[e + f*x])]*Sqrt[
a + b*Sin[e + f*x]]*Sqrt[(a + b*Sin[e + f*x])/(a + b)]*Sqrt[((a + b)*(c + d
*Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x]))]) - (2*(-(b*c) + a*d)*((((a
+ b)*c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF
[ArcSin[Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-
(b*c) + a*d))]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x
]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a +
b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2
*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))])/((a + b)*(c + d)*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[c + d*Sin[e + f*x]]) - ((b*c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi
/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a + b)*d), ArcSin[Sqr
t[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d
))]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e +
 Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f
*x]))/(-(b*c) + a*d)]*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin
[e + f*x]))/(-(b*c) + a*d))])/((a + b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c +
d*Sin[e + f*x]])))/(b*d)))/(2*(a - b)*b*(a + b)*f)
```

Maple [B] time = 51.96, size = 3901706, normalized size = 4746.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e))^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}{\left(b\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(5/2)/(b*sin(f*x + e) + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(d^2\cos\left(fx+e\right)^2-2\,cd\sin\left(fx+e\right)-c^2-d^2\right)\sqrt{b\sin\left(fx+e\right)+a}\sqrt{d\sin\left(fx+e\right)+c}}{b^2\cos\left(fx+e\right)^2-2\,ab\sin\left(fx+e\right)-a^2-b^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] integral((d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)*sqrt(b*sin(f
*x + e) + a)*sqrt(d*sin(f*x + e) + c)/(b^2*cos(f*x + e)^2 - 2*a*b*sin(f*x +
 e) - a^2 - b^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(5/2)/(a+b*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}{\left(b\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate((d*sin(f*x + e) + c)^(5/2)/(b*sin(f*x + e) + a)^(3/2), x)
```

3.790 $\int \frac{(c+d\sin(e+fx))^{3/2}}{(a+b\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=600

$$\frac{2\sqrt{a+b}(ad+b(c-2d))\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}}F\left(\sin^{-1}\left(\frac{\sqrt{c+d}\sqrt{a+b\sin(e+fx}}{\sqrt{a+b}\sqrt{c+d\sin(e+fx}}\right.\right.}{b^2f(a-b)\sqrt{c+d}}$$

```
[Out] (2*(c - d)*Sqrt[c + d]*EllipticE[ArcSin[(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x
]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/((a + b)*(c
- d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c + d)*(a + b
*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin
[e + f*x]))]*(a + b*Sin[e + f*x]))/((a - b)*b*Sqrt[a + b]*f) + (2*d*Sqrt[c
+ d]*EllipticPi[(b*(c + d))/((a + b)*d), ArcSin[(Sqrt[a + b]*Sqrt[c + d*Sin
[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/((a
+ b)*(c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c + d
)*(a + b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d)*(a
 + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/(b^2*Sqrt[a + b]*f) + (2*Sqrt[a
+ b]*(b*(c - 2*d) + a*d)*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f
*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(
c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d
*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Si
n[e + f*x])))]*(c + d*Sin[e + f*x]))/((a - b)*b^2*Sqrt[c + d]*f)
```

Rubi [A] time = 0.928265, antiderivative size = 600, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2798, 2811, 2998, 2818, 2996}

$$\frac{2\sqrt{a+b}(ad+b(c-2d))\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}}F\left(\sin^{-1}\left(\frac{\sqrt{c+d}\sqrt{a+b\sin(e+fx}}{\sqrt{a+b}\sqrt{c+d\sin(e+fx}}\right.\right.}{b^2f(a-b)\sqrt{c+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^(3/2)/(a + b*Sin[e + f*x])^(3/2),x]
```

```
[Out] (2*(c - d)*Sqrt[c + d]*EllipticE[ArcSin[(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x
]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/((a + b)*(c
- d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c + d)*(a + b
*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin
[e + f*x]))]*(a + b*Sin[e + f*x]))/((a - b)*b*Sqrt[a + b]*f) + (2*d*Sqrt[c
+ d]*EllipticPi[(b*(c + d))/((a + b)*d), ArcSin[(Sqrt[a + b]*Sqrt[c + d*Sin
[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/((a
+ b)*(c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c + d
)*(a + b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d)*(a
 + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/(b^2*Sqrt[a + b]*f) + (2*Sqrt[a
+ b]*(b*(c - 2*d) + a*d)*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f
*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(
c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d
*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Si
n[e + f*x])))]*(c + d*Sin[e + f*x]))/((a - b)*b^2*Sqrt[c + d]*f)
```

Rule 2798

```
Int[((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)/((a_.) + (b_.)*sin[(e_.) +
 (f_.)*(x_)])^(3/2), x_Symbol] :> Dist[d^2/b^2, Int[Sqrt[a + b*Sin[e + f*x]
```

```
]/Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[(b*c - a*d)/b^2, Int[Simp[b*c + a
*d + 2*b*d*Sin[e + f*x], x]/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e +
f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2811

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[(2*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d
)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(
1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*EllipticPi[(b*(c + d))/
(d*(a + b)), ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt[c + d*Sin[e + f*x]])/Sqrt[
a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)*(c - d))])/(d*f*Rt[(a + b)
/(c + d), 2]*Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2818

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*(c + d*Sin[e + f*x])*Sqrt[((b*c -
a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d
)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[
(c + d)/(a + b), 2]*(Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (
(a + b)*(c - d))/((a - b)*(c + d))])/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*
Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 2996

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(-2*A*(c - d)*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/
((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c
 + d)*(a + b*Sin[e + f*x])))]*EllipticE[ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt
[c + d*Sin[e + f*x]])/Sqrt[a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)
*(c - d))])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2]*Cos[e + f*x]), x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^{3/2}}{(a+b\sin(e+fx))^{3/2}}\,dx = \frac{d^2\int \frac{\sqrt{a+b\sin(e+fx)}}{\sqrt{c+d\sin(e+fx)}}\,dx}{b^2} + \frac{(bc-ad)\int \frac{bc+ad+2bd\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}\,dx}{b^2}$$

$$= \frac{2d\sqrt{c+d}\,\Pi\left(\frac{b(c+d)}{(a+b)d};\sin^{-1}\left(\frac{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sec(e+fx)\sqrt{-\frac{(bc-ad)(1-\sin(e+fx))}{(c+d)(a+b\sin(e+fx))}}\sqrt{}}{b^2\sqrt{a+b}f}$$

$$= \frac{2(c-d)\sqrt{c+d}E\left(\sin^{-1}\left(\frac{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sec(e+fx)\sqrt{-\frac{(bc-ad)(1-\sin(e+fx))}{(c+d)(a+b\sin(e+fx))}}\sqrt{}}{(a-b)b\sqrt{a+b}f}$$

<u>Mathematica</u> [B] time = 9.39221, size = 1866, normalized size = 3.11

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*Sin[e + f*x])^(3/2)/(a + b*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-2*(-(b*c*Cos[e + f*x]) + a*d*Cos[e + f*x])*Sqrt[c + d*Sin[e + f*x]])/((a^
2 - b^2)*f*Sqrt[a + b*Sin[e + f*x]]) + ((-4*(-(b*c) + a*d)*(a*c^2 - b*c*d)*
Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[-
(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))]
/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi
/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]
))/(-(b*c) + a*d)]*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e
+ f*x]))/(-(b*c) + a*d))])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c
 + d*Sin[e + f*x]]) - 4*(-(b*c) + a*d)*(b*c^2 - b*d^2)*((Sqrt[((c + d)*Cot[
(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[-(((a + b)*Csc[(-e
+ Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))]/Sqrt[2]], (2*(-(b
*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqr
t[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]
*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) +
 a*d))])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]
) - (Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c)
 + a*d)/((a + b)*d), ArcSin[Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c +
 d*Sin[e + f*x]))/(-(b*c) + a*d))]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-
c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi
/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[-(((a + b)*Csc[(-
e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))])/((a + b)*d*Sqr
t[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])) + 2*(-(b*c*d) + a*d^2)*((C
os[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(d*Sqrt[a + b*Sin[e + f*x]]) + (Sqrt[
(a - b)/(a + b)]*(a + b)*Cos[(-e + Pi/2 - f*x)/2]*EllipticE[ArcSin[(Sqrt[(a
 - b)/(a + b)]*Sin[(-e + Pi/2 - f*x)/2])/Sqrt[(a + b*Sin[e + f*x])/(a + b)]
], (2*(-(b*c) + a*d))/((a - b)*(c + d))]*Sqrt[c + d*Sin[e + f*x]])/(b*d*Sqr
t[((a + b)*Cos[(-e + Pi/2 - f*x)/2]^2)/(a + b*Sin[e + f*x])]*Sqrt[a + b*Sin
[e + f*x]]*Sqrt[(a + b*Sin[e + f*x])/(a + b)]*Sqrt[((a + b)*(c + d*Sin[e +
f*x]))/((c + d)*(a + b*Sin[e + f*x]))])) - (2*(-(b*c) + a*d)*((((a + b)*c +
a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[S
qrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a
*d))]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e
 + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e +
 f*x]))/(-(b*c) + a*d)]*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*S
in[e + f*x]))/(-(b*c) + a*d))])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*S
qrt[c + d*Sin[e + f*x]]) - ((b*c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)
/2]^2)/(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[-(((a +
```

```
b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))]/Sqrt[
2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f
*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(
b*c) + a*d)]*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]
))/(-(b*c) + a*d))])/((a + b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e +
f*x]])))/(b*d)))/((a - b)*(a + b)*f)
```

Maple [B] time = 134.514, size = 2946560, normalized size = 4910.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}{\left(b\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(3/2)/(b*sin(f*x + e) + a)^(3/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c+d\sin\left(e+fx\right)\right)^{\frac{3}{2}}}{\left(a+b\sin\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(3/2)/(a+b*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral((c + d*sin(e + f*x))**(3/2)/(a + b*sin(e + f*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}{\left(b\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate((d*sin(f*x + e) + c)^(3/2)/(b*sin(f*x + e) + a)^(3/2), x)
```

3.791 $\int \frac{\sqrt{c+d \sin(e+fx)}}{(a+b \sin(e+fx))^{3/2}} \, dx$

Optimal. Leaf size=409

$$\frac{2\sqrt{a+b}(c-d)\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}}F\left(\sin^{-1}\left(\frac{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}\right)|\frac{(a+}{(a-}\right.}{f(a-b)\sqrt{c+d}(bc-ad)}$$

```
[Out] (2*(c - d)*Sqrt[c + d]*EllipticE[ArcSin[(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x
]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/((a + b)*(c
- d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c + d)*(a + b
*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin
[e + f*x]))]*(a + b*Sin[e + f*x]))/((a - b)*Sqrt[a + b]*(b*c - a*d)*f) + (2
*Sqrt[a + b]*(c - d)*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]]
)/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c +
d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin
[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e
+ f*x])))]*(c + d*Sin[e + f*x]))/((a - b)*Sqrt[c + d]*(b*c - a*d)*f)
```

Rubi [A] time = 0.455118, antiderivative size = 409, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2795, 2818, 2996}

$$\frac{2\sqrt{a+b}(c-d)\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}}F\left(\sin^{-1}\left(\frac{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}\right)|\frac{(a+}{(a-}\right.}{f(a-b)\sqrt{c+d}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*Sin[e + f*x]]/(a + b*Sin[e + f*x])^(3/2),x]
```

```
[Out] (2*(c - d)*Sqrt[c + d]*EllipticE[ArcSin[(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x
]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/((a + b)*(c
- d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c + d)*(a + b
*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin
[e + f*x]))]*(a + b*Sin[e + f*x]))/((a - b)*Sqrt[a + b]*(b*c - a*d)*f) + (2
*Sqrt[a + b]*(c - d)*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]]
)/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c +
d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin
[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e
+ f*x])))]*(c + d*Sin[e + f*x]))/((a - b)*Sqrt[c + d]*(b*c - a*d)*f)
```

Rule 2795

```
Int[Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_.) + (b_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Dist[(c - d)/(a - b), Int[1/(Sqrt[a + b*Sin
[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[(b*c - a*d)/(a - b), In
t[(1 + Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2818

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*(c + d*Sin[e + f*x])*Sqrt[((b*c -
```

```
a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d
)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[
(c + d)/(a + b), 2]*(Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (
(a + b)*(c - d))/((a - b)*(c + d))]/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*
Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 2996

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(-2*A*(c - d)*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/
((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c
 + d)*(a + b*Sin[e + f*x])))]*EllipticE[ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt
[c + d*Sin[e + f*x]])/Sqrt[a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)
*(c - d))]/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2]*Cos[e + f*x]), x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rubi steps

$$\int \frac{\sqrt{c+d\sin(e+fx)}}{(a+b\sin(e+fx))^{3/2}}\,dx = \frac{(c-d)\int \frac{1}{\sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\,dx}{a-b} - \frac{(bc-ad)\int \frac{1+\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}\,dx}{a-b}$$

$$= \frac{2(c-d)\sqrt{c+d}E\left(\sin^{-1}\left(\frac{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}\right)\middle|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sec(e+fx)\sqrt{-\frac{(bc-ad)(1-\sin(e+fx))}{(c+d)(a+b\sin(e+fx))}}\sqrt{\frac{(b\cdots}{(c\cdots}}}{(a-b)\sqrt{a+b}(bc-ad)f}$$

Mathematica [A] time = 4.16573, size = 226, normalized size = 0.55

$$-\frac{2\sqrt{2}\cos\left(\frac{1}{4}(2e+2fx-\pi)\right)\sqrt{\frac{a+b\sin(e+fx)}{a+b}}\sqrt{c+d\sin(e+fx)}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{a-b}{a+b}}\cos\left(\frac{1}{4}(2e+2fx+\pi)\right)}{\sqrt{\frac{a+b\sin(e+fx)}{a+b}}}\right)\middle|\frac{2(ad-bc)}{(a-b)(c+d)}\right)}{f\sqrt{\frac{a-b}{a+b}}\sqrt{\frac{(a+b)(\sin(e+fx)+1)}{a+b\sin(e+fx)}}(a+b\sin(e+fx))^{3/2}\sqrt{\frac{(a+b)(c+d\sin(e+fx))}{(c+d)(a+b\sin(e+fx))}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*Sin[e + f*x]]/(a + b*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-2*Sqrt[2]*Cos[(2*e - Pi + 2*f*x)/4]*EllipticE[ArcSin[(Sqrt[(a - b)/(a + b
)]*Cos[(2*e + Pi + 2*f*x)/4])/Sqrt[(a + b*Sin[e + f*x])/(a + b)]], (2*(-(b*
c) + a*d))/((a - b)*(c + d))]*Sqrt[(a + b*Sin[e + f*x])/(a + b)]*Sqrt[c + d
*Sin[e + f*x]])/(Sqrt[(a - b)/(a + b)]*f*Sqrt[((a + b)*(1 + Sin[e + f*x]))/
(a + b*Sin[e + f*x])]*(a + b*Sin[e + f*x])^(3/2)*Sqrt[((a + b)*(c + d*Sin[e
 + f*x]))/((c + d)*(a + b*Sin[e + f*x]))])
```

Maple [B] time = 0.763, size = 46962, normalized size = 114.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d\sin\left(fx+e\right)+c}}{\left(b\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(d*sin(f*x + e) + c)/(b*sin(f*x + e) + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{b\sin\left(fx+e\right)+a}\sqrt{d\sin\left(fx+e\right)+c}}{b^2\cos\left(fx+e\right)^2-2\,ab\sin\left(fx+e\right)-a^2-b^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] integral(-sqrt(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)/(b^2*cos(f*x +
e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c+d\sin\left(e+fx\right)}}{\left(a+b\sin\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(1/2)/(a+b*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral(sqrt(c + d*sin(e + f*x))/(a + b*sin(e + f*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d\sin\left(fx+e\right)+c}}{\left(b\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(d*sin(f*x + e) + c)/(b*sin(f*x + e) + a)^(3/2), x)
```

3.792 $\int \frac{1}{(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}\,dx$

Optimal. Leaf size=405

$$\frac{2\sqrt{a+b}\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}}F\left(\sin^{-1}\left(\frac{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}\right)|\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)}{f(a-b)\sqrt{c+d}(bc-ad)}$$

```
[Out] (2*b*(c - d)*Sqrt[c + d]*EllipticE[ArcSin[(Sqrt[a + b]*Sqrt[c + d*Sin[e + f
*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/((a + b)*(
c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c + d)*(a +
 b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d)*(a + b*S
in[e + f*x]))]*(a + b*Sin[e + f*x]))/((a - b)*Sqrt[a + b]*(b*c - a*d)^2*f)
+ (2*Sqrt[a + b]*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(S
qrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]
*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e +
 f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*
x])))]*(c + d*Sin[e + f*x]))/((a - b)*Sqrt[c + d]*(b*c - a*d)*f)
```

Rubi [A] time = 0.450766, antiderivative size = 405, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2801, 2818, 2996}

$$\frac{2\sqrt{a+b}\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}}F\left(\sin^{-1}\left(\frac{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}\right)|\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)}{f(a-b)\sqrt{c+d}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]),x]
```

```
[Out] (2*b*(c - d)*Sqrt[c + d]*EllipticE[ArcSin[(Sqrt[a + b]*Sqrt[c + d*Sin[e + f
*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/((a + b)*(
c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c + d)*(a +
 b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d)*(a + b*S
in[e + f*x]))]*(a + b*Sin[e + f*x]))/((a - b)*Sqrt[a + b]*(b*c - a*d)^2*f)
+ (2*Sqrt[a + b]*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(S
qrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]
*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e +
 f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*
x])))]*(c + d*Sin[e + f*x]))/((a - b)*Sqrt[c + d]*(b*c - a*d)*f)
```

Rule 2801

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin
[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[1/(a - b), Int[1/(Sqrt[a + b*Sin[
e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[b/(a - b), Int[(1 + Sin[
e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0]
```

Rule 2818

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*(c + d*Sin[e + f*x])*Sqrt[((b*c -
```

```
a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d
)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[
(c + d)/(a + b), 2]*(Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (
(a + b)*(c - d))/((a - b)*(c + d))])/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*
Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 2996

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(-2*A*(c - d)*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/
((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c
 + d)*(a + b*Sin[e + f*x])))]*EllipticE[ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt
[c + d*Sin[e + f*x]])/Sqrt[a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)
*(c - d))])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2]*Cos[e + f*x]), x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rubi steps

$$\int \frac{1}{(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}\,dx = \frac{\int \frac{1}{\sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\,dx}{a-b} - \frac{b\int \frac{1+\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}\,dx}{a-b}$$

$$= \frac{2b(c-d)\sqrt{c+d}E\left(\sin^{-1}\left(\frac{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sec(e+fx)\sqrt{-\frac{(b}{(}}}{(a-b)\sqrt{a+b}(bc-ad}$$

Mathematica [B] time = 32.7626, size = 90261, normalized size = 222.87

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.822, size = 40572, normalized size = 100.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\sqrt{d\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate(1/((b*sin(f*x + e) + a)^(3/2)*sqrt(d*sin(f*x + e) + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sin\left(fx+e\right)+a}\sqrt{d\sin\left(fx+e\right)+c}}{2\,abd-\left(b^{2}c+2\,abd\right)\cos\left(fx+e\right)^{2}+\left(a^{2}+b^{2}\right)c-\left(b^{2}d\cos\left(fx+e\right)^{2}-2\,abc-\left(a^{2}+b^{2}\right)d\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="fri
cas")
```

```
[Out] integral(sqrt(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)/(2*a*b*d - (b^2*
c + 2*a*b*d)*cos(f*x + e)^2 + (a^2 + b^2)*c - (b^2*d*cos(f*x + e)^2 - 2*a*b
*c - (a^2 + b^2)*d)*sin(f*x + e)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(a+b\sin\left(e+fx\right)\right)^{\frac{3}{2}}\sqrt{c+d\sin\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))**(3/2)/(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/((a + b*sin(e + f*x))**(3/2)*sqrt(c + d*sin(e + f*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(b\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\sqrt{d\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate(1/((b*sin(f*x + e) + a)^(3/2)*sqrt(d*sin(f*x + e) + c)), x)
```

3.793 $\int \frac{1}{(a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=495

$$-\frac{2\left(a^2d^2+b^2\left(c^2-2d^2\right)\right)\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}}E\left(\sin^{-1}\left(\frac{\sqrt{c+d}\sqrt{a+b\sin(e+f}}{\sqrt{a+b}\sqrt{c+d\sin(e+f}}\right.\right.}{f\sqrt{a+b}(c-d)\sqrt{c+d}(bc-ad)^3}$$

```
[Out] (2*b^2*Cos[e + f*x])/((a^2 - b^2)*(b*c - a*d)*f*Sqrt[a + b*Sin[e + f*x]]*Sq
rt[c + d*Sin[e + f*x]]) - (2*(a^2*d^2 + b^2*(c^2 - 2*d^2))*EllipticE[ArcSin
[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x
]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(
1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 +
 Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/(Sqr
t[a + b]*(c - d)*Sqrt[c + d]*(b*c - a*d)^3*f) + (2*(b*(c - 2*d) - a*d)*Elli
pticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d
*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((
b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c
 - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*(c + d*Sin[e +
 f*x]))/(Sqrt[a + b]*(c - d)*Sqrt[c + d]*(b*c - a*d)^2*f)
```

Rubi [A] time = 0.908064, antiderivative size = 495, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2802, 2998, 2818, 2996}

$$-\frac{2\left(a^2d^2+b^2\left(c^2-2d^2\right)\right)\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}}E\left(\sin^{-1}\left(\frac{\sqrt{c+d}\sqrt{a+b\sin(e+f}}{\sqrt{a+b}\sqrt{c+d\sin(e+f}}\right.\right.}{f\sqrt{a+b}(c-d)\sqrt{c+d}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^(3/2)),x]
```


```
[Out] (2*b^2*Cos[e + f*x])/((a^2 - b^2)*(b*c - a*d)*f*Sqrt[a + b*Sin[e + f*x]]*Sq
rt[c + d*Sin[e + f*x]]) - (2*(a^2*d^2 + b^2*(c^2 - 2*d^2))*EllipticE[ArcSin
[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x
]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(
1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 +
 Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/(Sqr
t[a + b]*(c - d)*Sqrt[c + d]*(b*c - a*d)^3*f) + (2*(b*(c - 2*d) - a*d)*Elli
pticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d
*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((
b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c
 - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*(c + d*Sin[e +
 f*x]))/(Sqrt[a + b]*(c - d)*Sqrt[c + d]*(b*c - a*d)^2*f)
```


Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
```

```
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2818

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*(c + d*Sin[e + f*x])*Sqrt[((b*c -
a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d
)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[
(c + d)/(a + b), 2]*(Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (
(a + b)*(c - d))/((a - b)*(c + d))])/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*
Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 2996

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(-2*A*(c - d)*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/
((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c
 + d)*(a + b*Sin[e + f*x])))]*EllipticE[ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt
[c + d*Sin[e + f*x]])/Sqrt[a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)
*(c - d))])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2]*Cos[e + f*x]), x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rubi steps

$$\int \frac{1}{(a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{3/2}}\,dx = \frac{2b^2\cos(e+fx)}{\left(a^2-b^2\right)(bc-ad)f\sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin(e+fx)}} - \frac{2\int \frac{1}{2}(\cdots}{\cdots}$$

$$= \frac{2b^2\cos(e+fx)}{\left(a^2-b^2\right)(bc-ad)f\sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin(e+fx)}} - \frac{\left(2\left(\frac{1}{2}b\cdots\right.\right.}{\cdots}$$

$$= \frac{2b^2\cos(e+fx)}{\left(a^2-b^2\right)(bc-ad)f\sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin(e+fx)}} - \frac{2\left(a^2d\cdots\right.}{\cdots}$$

Mathematica [B] time = 6.73523, size = 2052, normalized size = 4.15

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^(3/2)),x]
```

```
[Out] (Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]*((2*b^3*Cos[e + f*x])/((
a^2 - b^2)*(-(b*c) + a*d)^2*(a + b*Sin[e + f*x])) + (2*d^3*Cos[e + f*x])/((
b*c - a*d)^2*(c^2 - d^2)*(c + d*Sin[e + f*x]))))/f + ((-4*(-(b*c) + a*d)*(a
*b^2*c^3 - 2*a^2*b*c^2*d + 2*b^3*c^2*d + a^3*c*d^2 - 2*a*b^2*c*d^2 + 2*a^2*
b*d^3 - 2*b^3*d^3)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*Elli
pticF[ArcSin[Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]
))/(-(b*c) + a*d))]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e
+ f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*
(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)
/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))])/((a + b)*(c + d)*Sqrt[a + b*S
in[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - 4*(-(b*c) + a*d)*(b^3*c^3 + a*b^2*
c^2*d + a^2*b*c*d^2 - 2*b^3*c*d^2 + a^3*d^3 - 2*a*b^2*d^3)*((Sqrt[((c + d)*
Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[-(((a + b)*Csc[
(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))]/Sqrt[2]], (2*
(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4
*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a
*d)]*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*
c) + a*d))])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f
*x]]) - (Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(
b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*
(c + d*Sin[e + f*x]))/(-(b*c) + a*d))]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b
)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e
+ Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[-(((a + b)*Cs
c[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))])/((a + b)*d
*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])) + 2*(-(b^3*c^2*d) - a^
2*b*d^3 + 2*b^3*d^3)*((Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(d*Sqrt[a + b
*Sin[e + f*x]]) + (Sqrt[(a - b)/(a + b)]*(a + b)*Cos[(-e + Pi/2 - f*x)/2]*E
llipticE[ArcSin[(Sqrt[(a - b)/(a + b)]*Sin[(-e + Pi/2 - f*x)/2])/Sqrt[(a +
b*Sin[e + f*x])/(a + b)]], (2*(-(b*c) + a*d))/((a - b)*(c + d))]*Sqrt[c + d
*Sin[e + f*x]])/(b*d*Sqrt[((a + b)*Cos[(-e + Pi/2 - f*x)/2]^2)/(a + b*Sin[e
 + f*x])]*Sqrt[a + b*Sin[e + f*x]]*Sqrt[(a + b*Sin[e + f*x])/(a + b)]*Sqrt[
((a + b)*(c + d*Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x]))]]) - (2*(-(b*c
) + a*d)*((((a + b)*c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c
+ d)]*EllipticF[ArcSin[Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Si
n[e + f*x]))/(-(b*c) + a*d))]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d
))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 -
f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[-(((a + b)*Csc[(-e + P
i/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))])/((a + b)*(c + d)*Sq
rt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - ((b*c + a*d)*Sqrt[((c +
d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a + b)
*d), ArcSin[Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x])
)/(-(b*c) + a*d))]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e +
 f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(
a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/
2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))])/((a + b)*d*Sqrt[a + b*Sin[e +
f*x]]*Sqrt[c + d*Sin[e + f*x]])))/(b*d)))/((a - b)*(a + b)*(c - d)*(c + d)*
(-(b*c) + a*d)^2*f)
```

Maple [B] time = 1.815, size = 119239, normalized size = 240.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="max
ima")
```

```
[Out] integrate(1/((b*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e) + c)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sin\left(fx+e\right)+a}\sqrt{d\sin\left(fx+e\right)+}}{b^2d^2\cos\left(fx+e\right)^4+4\,abcd+\left(a^2+b^2\right)c^2+\left(a^2+b^2\right)d^2-\left(b^2c^2+4\,abcd+\left(a^2+2\,b^2\right)d^2\right)\cos\left(fx+e\right)^2+2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="fri
cas")
```

```
[Out] integral(sqrt(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)/(b^2*d^2*cos(f*x
 + e)^4 + 4*a*b*c*d + (a^2 + b^2)*c^2 + (a^2 + b^2)*d^2 - (b^2*c^2 + 4*a*b*
c*d + (a^2 + 2*b^2)*d^2)*cos(f*x + e)^2 + 2*(a*b*c^2 + a*b*d^2 + (a^2 + b^2
)*c*d - (b^2*c*d + a*b*d^2)*cos(f*x + e)^2)*sin(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))**(3/2)/(c+d*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="gia
c")
```

```
[Out] integrate(1/((b*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e) + c)^(3/2)), x)
```

3.794 $\int \frac{1}{(a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=681

$$\frac{2d\left(a^2d^2+b^2\left(3c^2-4d^2\right)\right)\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{3f\left(a^2-b^2\right)\left(c^2-d^2\right)(bc-ad)^2(c+d\sin(e+fx))^{3/2}}+\frac{2\left(a^2d^2(3c+d)-6abd\left(c^2-d^2\right)+b^2\left(-9c^2d+3c^3-6cd^2\right.\right.}{}$$

```
[Out] (2*b^2*Cos[e + f*x])/((a^2 - b^2)*(b*c - a*d)*f*Sqrt[a + b*Sin[e + f*x]]*(c
 + d*Sin[e + f*x])^(3/2)) + (2*d*(a^2*d^2 + b^2*(3*c^2 - 4*d^2))*Cos[e + f*
x]*Sqrt[a + b*Sin[e + f*x]])/(3*(a^2 - b^2)*(b*c - a*d)^2*(c^2 - d^2)*f*(c
+ d*Sin[e + f*x])^(3/2)) + (2*(4*a^3*c*d^3 - 4*a*b^2*c*d^3 - a^2*b*d^2*(9*c
^2 - 5*d^2) - b^3*(3*c^4 - 15*c^2*d^2 + 8*d^4))*EllipticE[ArcSin[(Sqrt[c +
d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a +
 b)*(c - d))/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e +
 f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*
x]))/((a - b)*(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/(3*Sqrt[a + b]*
(c - d)^2*(c + d)^(3/2)*(b*c - a*d)^4*f) + (2*(a^2*d^2*(3*c + d) - 6*a*b*d*
(c^2 - d^2) + b^2*(3*c^3 - 9*c^2*d - 6*c*d^2 + 8*d^3))*EllipticF[ArcSin[(Sq
rt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])]
, ((a + b)*(c - d))/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 -
Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin
[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/(3*Sqrt[
a + b]*(c - d)^2*(c + d)^(3/2)*(b*c - a*d)^3*f)
```

Rubi [A] time = 2.67994, antiderivative size = 681, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2802, 3055, 2998, 2818, 2996}

$$\frac{2d\left(a^2d^2+b^2\left(3c^2-4d^2\right)\right)\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{3f\left(a^2-b^2\right)\left(c^2-d^2\right)(bc-ad)^2(c+d\sin(e+fx))^{3/2}}+\frac{2\left(a^2d^2(3c+d)-6abd\left(c^2-d^2\right)+b^2\left(-9c^2d+3c^3-6cd^2\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^(5/2)),x]
```


```
[Out] (2*b^2*Cos[e + f*x])/((a^2 - b^2)*(b*c - a*d)*f*Sqrt[a + b*Sin[e + f*x]]*(c
 + d*Sin[e + f*x])^(3/2)) + (2*d*(a^2*d^2 + b^2*(3*c^2 - 4*d^2))*Cos[e + f*
x]*Sqrt[a + b*Sin[e + f*x]])/(3*(a^2 - b^2)*(b*c - a*d)^2*(c^2 - d^2)*f*(c
+ d*Sin[e + f*x])^(3/2)) + (2*(4*a^3*c*d^3 - 4*a*b^2*c*d^3 - a^2*b*d^2*(9*c
^2 - 5*d^2) - b^3*(3*c^4 - 15*c^2*d^2 + 8*d^4))*EllipticE[ArcSin[(Sqrt[c +
d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a +
 b)*(c - d))/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e +
 f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*
x]))/((a - b)*(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/(3*Sqrt[a + b]*
(c - d)^2*(c + d)^(3/2)*(b*c - a*d)^4*f) + (2*(a^2*d^2*(3*c + d) - 6*a*b*d*
(c^2 - d^2) + b^2*(3*c^3 - 9*c^2*d - 6*c*d^2 + 8*d^3))*EllipticF[ArcSin[(Sq
rt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])]
, ((a + b)*(c - d))/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 -
Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin
[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/(3*Sqrt[
a + b]*(c - d)^2*(c + d)^(3/2)*(b*c - a*d)^3*f)
```


Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2818

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*(c + d*Sin[e + f*x])*Sqrt[((b*c -
a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d
)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[
(c + d)/(a + b), 2]*(Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (
(a + b)*(c - d))/((a - b)*(c + d))])/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*
Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 2996

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(-2*A*(c - d)*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/
((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c
 + d)*(a + b*Sin[e + f*x])))]*EllipticE[ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt
[c + d*Sin[e + f*x]])/Sqrt[a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)
*(c - d))])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2]*Cos[e + f*x]), x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rubi steps

$$\int \frac{1}{(a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{5/2}}\,dx = \frac{2b^2\cos(e+fx)}{\left(a^2-b^2\right)(bc-ad)f\sqrt{a+b\sin(e+fx)}(c+d\sin(e+fx))^{3/2}} - \frac{2\int}{}$$

$$= \frac{2b^2\cos(e+fx)}{\left(a^2-b^2\right)(bc-ad)f\sqrt{a+b\sin(e+fx)}(c+d\sin(e+fx))^{3/2}} + \frac{2d\big(}{3}$$

$$= \frac{2b^2\cos(e+fx)}{\left(a^2-b^2\right)(bc-ad)f\sqrt{a+b\sin(e+fx)}(c+d\sin(e+fx))^{3/2}} + \frac{2d\big(}{3}$$

$$= \frac{2b^2\cos(e+fx)}{\left(a^2-b^2\right)(bc-ad)f\sqrt{a+b\sin(e+fx)}(c+d\sin(e+fx))^{3/2}} + \frac{2d\big(}{3}$$

Mathematica [B] time = 7.39269, size = 2320, normalized size = 3.41

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^(5/2)),x]
```

```
[Out] (Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]*((-2*b^4*Cos[e + f*x])/(
(a^2 - b^2)*(-(b*c) + a*d)^3*(a + b*Sin[e + f*x])) + (2*d^3*Cos[e + f*x])/(
3*(b*c - a*d)^2*(c^2 - d^2)*(c + d*Sin[e + f*x])^2) - (2*(-9*b*c^2*d^3*Cos[
e + f*x] + 4*a*c*d^4*Cos[e + f*x] + 5*b*d^5*Cos[e + f*x]))/(3*(b*c - a*d)^3
*(c^2 - d^2)^2*(c + d*Sin[e + f*x]))))/f + ((-4*(-(b*c) + a*d)*(-3*a*b^3*c^
5 + 9*a^2*b^2*c^4*d - 9*b^4*c^4*d - 9*a^3*b*c^3*d^2 + 15*a*b^3*c^3*d^2 + 3*
a^4*c^2*d^3 - 20*a^2*b^2*c^2*d^3 + 17*b^4*c^2*d^3 + 5*a^3*b*c*d^4 - 8*a*b^3
*c*d^4 + a^4*d^5 + 7*a^2*b^2*d^5 - 8*b^4*d^5)*Sqrt[((c + d)*Cot[(-e + Pi/2
- f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*
x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))]/Sqrt[2]], (2*(-(b*c) + a*d))
/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*
Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[-(((a
 + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))])/((
a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - 4*(-(b*
c) + a*d)*(-3*b^4*c^5 - 3*a*b^3*c^4*d - 9*a^2*b^2*c^3*d^2 + 15*b^4*c^3*d^2
- 5*a^3*b*c^2*d^3 + 11*a*b^3*c^2*d^3 + 4*a^4*c*d^4 + a^2*b^2*c*d^4 - 8*b^4*
c*d^4 + 5*a^3*b*d^5 - 8*a*b^3*d^5)*((Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]
^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(
c + d*Sin[e + f*x]))/(-(b*c) + a*d))]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)
*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e +
 Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[-(((a + b)*Csc
[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))])/((a + b)*(c
 + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - (Sqrt[((c + d)*C
ot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a + b)*d),
 ArcSin[Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-
(b*c) + a*d))]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x
]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a +
b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2
*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))])/((a + b)*d*Sqrt[a + b*Sin[e + f*x]
]*Sqrt[c + d*Sin[e + f*x]])) + 2*(3*b^4*c^4*d + 9*a^2*b^2*c^2*d^3 - 15*b^4*
```

```
c^2*d^3 - 4*a^3*b*c*d^4 + 4*a*b^3*c*d^4 - 5*a^2*b^2*d^5 + 8*b^4*d^5)*((Cos[
e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(d*Sqrt[a + b*Sin[e + f*x]]) + (Sqrt[(a
- b)/(a + b)]*(a + b)*Cos[(-e + Pi/2 - f*x)/2]*EllipticE[ArcSin[(Sqrt[(a -
b)/(a + b)]*Sin[(-e + Pi/2 - f*x)/2])/Sqrt[(a + b*Sin[e + f*x])/(a + b)]],
(2*(-(b*c) + a*d))/((a - b)*(c + d))]*Sqrt[c + d*Sin[e + f*x]])/(b*d*Sqrt[(
(a + b)*Cos[(-e + Pi/2 - f*x)/2]^2)/(a + b*Sin[e + f*x])]*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[(a + b*Sin[e + f*x])/(a + b)]*Sqrt[((a + b)*(c + d*Sin[e + f*x
]))/((c + d)*(a + b*Sin[e + f*x]))]) - (2*(-(b*c) + a*d)*((((a + b)*c + a*d
)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt
[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)
)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e +
Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*
x]))/(-(b*c) + a*d)]*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[
e + f*x]))/(-(b*c) + a*d))])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt
[c + d*Sin[e + f*x]]) - ((b*c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]
^2)/(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[-(((a + b)
*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))]/Sqrt[2]]
, (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)
/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c
) + a*d)]*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/
(-(b*c) + a*d))])/((a + b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]])))/(b*d)))/(3*(a - b)*(a + b)*(c - d)^2*(c + d)^2*(-(b*c) + a*d)^3*f)
```

Maple [B] time = 8.03, size = 415876, normalized size = 610.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(5/2),x, algorithm="max
ima")
```

```
[Out] integrate(1/((b*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e) + c)^(5/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{}{6\,abc^2d+2\,abd^3+\left(3\,b^2cd^2+2\,abd^3\right)\cos\left(fx+e\right)^4+\left(a^2+b^2\right)c^3+3\left(a^2+b^2\right)cd^2-\left(b^2c^3+6\,abc^2d+4\,abd^3+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(5/2),x, algorithm="fri
cas")
```

```
[Out] integral(sqrt(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)/(6*a*b*c^2*d + 2
*a*b*d^3 + (3*b^2*c*d^2 + 2*a*b*d^3)*cos(f*x + e)^4 + (a^2 + b^2)*c^3 + 3*(
a^2 + b^2)*c*d^2 - (b^2*c^3 + 6*a*b*c^2*d + 4*a*b*d^3 + 3*(a^2 + 2*b^2)*c*d
^2)*cos(f*x + e)^2 + (b^2*d^3*cos(f*x + e)^4 + 2*a*b*c^3 + 6*a*b*c*d^2 + 3*
(a^2 + b^2)*c^2*d + (a^2 + b^2)*d^3 - (3*b^2*c^2*d + 6*a*b*c*d^2 + (a^2 + 2
*b^2)*d^3)*cos(f*x + e)^2)*sin(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))**(3/2)/(c+d*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(5/2),x, algorithm="gia
c")
```

```
[Out] integrate(1/((b*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e) + c)^(5/2)), x)
```

3.795 $\int \frac{(c+d\sin(e+fx))^{5/2}}{(a+b\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=736

$$\frac{2\left(3a^2bd(c-2d)+3a^3d^2+ab^2\left(3c^2-4cd-2d^2\right)+b^3\left(c^2-7cd+9d^2\right)\right)\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}}{3b^3f(a-b)^2\sqrt{a+b}\sqrt{c+d}}$$

```
[Out] (2*(c - d)*Sqrt[c + d]*(4*a*b*c + 3*a^2*d - 7*b^2*d)*EllipticE[ArcSin[(Sqrt
[a + b]*Sqrt[c + d*Sin[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])],
((a - b)*(c + d))/((a + b)*(c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 -
Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[
e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/(3*(a - b)
^2*b^2*(a + b)^(3/2)*f) + (2*d^2*Sqrt[c + d]*EllipticPi[(b*(c + d))/((a + b
)*d), ArcSin[(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b
*Sin[e + f*x]])], ((a - b)*(c + d))/((a + b)*(c - d))]*Sec[e + f*x]*Sqrt[-(
((b*c - a*d)*(1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*Sqrt[((b*
c - a*d)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*(a + b*Sin[e +
 f*x]))/(b^3*Sqrt[a + b]*f) + (2*(b*c - a*d)^2*Cos[e + f*x]*Sqrt[c + d*Sin[
e + f*x]])/(3*b*(a^2 - b^2)*f*(a + b*Sin[e + f*x])^(3/2)) + (2*(3*a^2*b*(c
- 2*d)*d + 3*a^3*d^2 + a*b^2*(3*c^2 - 4*c*d - 2*d^2) + b^3*(c^2 - 7*c*d + 9
*d^2))*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]
*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sec[e + f
*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*S
qrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*(c
+ d*Sin[e + f*x]))/(3*(a - b)^2*b^3*Sqrt[a + b]*Sqrt[c + d]*f)
```

Rubi [A] time = 1.84295, antiderivative size = 736, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2792, 3053, 2811, 2998, 2818, 2996}

$$\frac{2\left(3a^2bd(c-2d)+3a^3d^2+ab^2\left(3c^2-4cd-2d^2\right)+b^3\left(c^2-7cd+9d^2\right)\right)\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}}{3b^3f(a-b)^2\sqrt{a+b}\sqrt{c+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^(5/2)/(a + b*Sin[e + f*x])^(5/2),x]
```

```
[Out] (2*(c - d)*Sqrt[c + d]*(4*a*b*c + 3*a^2*d - 7*b^2*d)*EllipticE[ArcSin[(Sqrt
[a + b]*Sqrt[c + d*Sin[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])],
((a - b)*(c + d))/((a + b)*(c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 -
Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[
e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/(3*(a - b)
^2*b^2*(a + b)^(3/2)*f) + (2*d^2*Sqrt[c + d]*EllipticPi[(b*(c + d))/((a + b
)*d), ArcSin[(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b
*Sin[e + f*x]])], ((a - b)*(c + d))/((a + b)*(c - d))]*Sec[e + f*x]*Sqrt[-(
((b*c - a*d)*(1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*Sqrt[((b*
c - a*d)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*(a + b*Sin[e +
 f*x]))/(b^3*Sqrt[a + b]*f) + (2*(b*c - a*d)^2*Cos[e + f*x]*Sqrt[c + d*Sin[
e + f*x]])/(3*b*(a^2 - b^2)*f*(a + b*Sin[e + f*x])^(3/2)) + (2*(3*a^2*b*(c
- 2*d)*d + 3*a^3*d^2 + a*b^2*(3*c^2 - 4*c*d - 2*d^2) + b^3*(c^2 - 7*c*d + 9
*d^2))*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]
*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sec[e + f
*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*S
```

```
qrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*(c
+ d*Sin[e + f*x]))/(3*(a - b)^2*b^3*Sqrt[a + b]*Sqrt[c + d]*f)
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2811

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[(2*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d
)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(
1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*EllipticPi[(b*(c + d))/
(d*(a + b)), ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt[c + d*Sin[e + f*x]])/Sqrt[
a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)*(c - d))])/(d*f*Rt[(a + b)
/(c + d), 2]*Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2818

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*(c + d*Sin[e + f*x])*Sqrt[((b*c -
a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d
)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[
(c + d)/(a + b), 2]*(Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (
(a + b)*(c - d))/((a - b)*(c + d))])/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*
Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 2996

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(-2*A*(c - d)*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/
((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c
 + d)*(a + b*Sin[e + f*x])))]*EllipticE[ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt
[c + d*Sin[e + f*x]])/Sqrt[a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)
*(c - d))])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2]*Cos[e + f*x]), x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^{5/2}}{(a+b\sin(e+fx))^{5/2}}\,dx = \frac{2(bc-ad)^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3b\left(a^2-b^2\right)f(a+b\sin(e+fx))^{3/2}} - \frac{2\int \frac{\frac{1}{2}\left(-3abc^3+7b^2c^2d-5abcd^2+a^2d^3\right)-\frac{1}{2}\left(2a^2cd^2+abd\left(5c^2+\cdots\right.\right.}{(a+b\sin(e+fx))^{3/\cdots}}}{3b\left(a^2\cdots\right.}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3b\left(a^2-b^2\right)f(a+b\sin(e+fx))^{3/2}} - \frac{2\int \frac{\frac{3}{2}a^2\left(a^2-b^2\right)d^3+\frac{1}{2}b^2\left(-3abc^3+7b^2c^2d-5abcd^2+a^2d^3\right)+b\left(\cdots\right.}{(a+b\sin(e\cdots}}{3b\cdots}$$

$$= \frac{2d^2\sqrt{c+d}\,\Pi\left(\frac{b(c+d)}{(a+b)d};\sin^{-1}\left(\frac{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}\right)\middle|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sec(e+fx)\sqrt{-\frac{(bc-ad)(1-\sin(e+fx))}{(c+d)(a+b\sin(e+fx))}}\cdots}{b^3\sqrt{a+b}f}$$

$$= \frac{2(c-d)\sqrt{c+d}\left(4abc+3a^2d-7b^2d\right)E\left(\sin^{-1}\left(\frac{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}\right)\middle|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sec(e+fx)\sqrt{\cdots}}{3(a-b)^2b^2(a+b)^{3/2}f}$$

Mathematica [B] time = 7.01603, size = 2142, normalized size = 2.91

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*Sin[e + f*x])^(5/2)/(a + b*Sin[e + f*x])^(5/2),x]
```

```
[Out] (Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]*((-2*(b^2*c^2*Cos[e + f*
x] - 2*a*b*c*d*Cos[e + f*x] + a^2*d^2*Cos[e + f*x]))/(3*b*(-a^2 + b^2)*(a +
 b*Sin[e + f*x])^2) - (2*(-4*a*b^2*c^2*Cos[e + f*x] + a^2*b*c*d*Cos[e + f*x
] + 7*b^3*c*d*Cos[e + f*x] + 3*a^3*d^2*Cos[e + f*x] - 7*a*b^2*d^2*Cos[e + f
*x]))/(3*b*(-a^2 + b^2)^2*(a + b*Sin[e + f*x]))))/f - ((-4*(-(b*c) + a*d)*(
-3*a^2*b*c^3 - b^3*c^3 + 8*a*b^2*c^2*d - 2*a^2*b*c*d^2 - 2*b^3*c*d^2 + a^3*
d^3 - a*b^2*d^3)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*Ellipt
icF[ArcSin[Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))
/(-(b*c) + a*d))]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e +
f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a
 + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2
]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))])/((a + b)*(c + d)*Sqrt[a + b*Sin
[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - 4*(-(b*c) + a*d)*(-4*a*b^2*c^3 - 3*a
^2*b*c^2*d + 7*b^3*c^2*d + 4*a^3*c*d^2 - a^2*b*d^3 - 3*b^3*d^3)*((Sqrt[((c
+ d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[-(((a + b)
*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))]/Sqrt[2]]
, (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)
/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c
) + a*d)]*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/
```

```
(-(b*c) + a*d))])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[
e + f*x]]) - (Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticP
i[(-(b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/
2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))]/Sqrt[2]], (2*(-(b*c) + a*d))/((
a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc
[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[-(((a +
b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))])/((a +
 b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])) + 2*(4*a*b^2*c^2*
d - a^2*b*c*d^2 - 7*b^3*c*d^2 - 3*a^3*d^3 + 7*a*b^2*d^3)*((Cos[e + f*x]*Sqr
t[c + d*Sin[e + f*x]])/(d*Sqrt[a + b*Sin[e + f*x]]) + (Sqrt[(a - b)/(a + b)
]*(a + b)*Cos[(-e + Pi/2 - f*x)/2]*EllipticE[ArcSin[(Sqrt[(a - b)/(a + b)]*
Sin[(-e + Pi/2 - f*x)/2])/Sqrt[(a + b*Sin[e + f*x])/(a + b)]], (2*(-(b*c) +
 a*d))/((a - b)*(c + d))]*Sqrt[c + d*Sin[e + f*x]])/(b*d*Sqrt[((a + b)*Cos[
(-e + Pi/2 - f*x)/2]^2)/(a + b*Sin[e + f*x])]*Sqrt[a + b*Sin[e + f*x]]*Sqrt
[(a + b*Sin[e + f*x])/(a + b)]*Sqrt[((a + b)*(c + d*Sin[e + f*x]))/((c + d)
*(a + b*Sin[e + f*x]))]) - (2*(-(b*c) + a*d)*((((a + b)*c + a*d)*Sqrt[((c +
 d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[-(((a + b)*
Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))]/Sqrt[2]],
 (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/
2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c)
 + a*d)]*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(
-(b*c) + a*d))])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e
 + f*x]]) - ((b*c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)
]*EllipticPi[(-(b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[-(((a + b)*Csc[(-e + P
i/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))]/Sqrt[2]], (2*(-(b*c)
 + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[(
(c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sq
rt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*
d))])/((a + b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])))/(b*d)
))/(3*(a - b)^2*b*(a + b)^2*f)
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int \left(c+d\sin\left(fx+e\right)\right)^{\frac{5}{2}}\left(a+b\sin\left(fx+e\right)\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e))^(5/2),x)
```

```
[Out] int((c+d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}{\left(b\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(5/2)/(b*sin(f*x + e) + a)^(5/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(5/2)/(a+b*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}{\left(b\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate((d*sin(f*x + e) + c)^(5/2)/(b*sin(f*x + e) + a)^(5/2), x)
```

3.796 $\int \frac{(c+d\sin(e+fx))^{3/2}}{(a+b\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=497

$$\frac{2(bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3f\left(a^2-b^2\right)(a+b\sin(e+fx))^{3/2}}+\frac{2(c-d)(3ac-ad+bc-3bd)\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(}{(a+b)(c+d\sin(}}}{3f(a-b)^2\sqrt{a+b}\sqrt{c}}$$

```
[Out] (8*(c - d)*Sqrt[c + d]*(a*c - b*d)*EllipticE[ArcSin[(Sqrt[a + b]*Sqrt[c + d
*Sin[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/
((a + b)*(c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c
 + d)*(a + b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d
)*(a + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/(3*(a - b)^2*(a + b)^(3/2)*(
b*c - a*d)*f) + (2*(b*c - a*d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(3*(a
^2 - b^2)*f*(a + b*Sin[e + f*x])^(3/2)) + (2*(c - d)*(3*a*c + b*c - a*d - 3
*b*d)*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*
Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sec[e + f*
x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sq
rt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*(c +
 d*Sin[e + f*x]))/(3*(a - b)^2*Sqrt[a + b]*Sqrt[c + d]*(b*c - a*d)*f)
```

Rubi [A] time = 1.00071, antiderivative size = 497, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2799, 2998, 2818, 2996}

$$\frac{2(bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3f\left(a^2-b^2\right)(a+b\sin(e+fx))^{3/2}}+\frac{2(c-d)(3ac-ad+bc-3bd)\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(}{(a+b)(c+d\sin(}}}{3f(a-b)^2\sqrt{a+b}\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^(3/2)/(a + b*Sin[e + f*x])^(5/2),x]
```


```
[Out] (8*(c - d)*Sqrt[c + d]*(a*c - b*d)*EllipticE[ArcSin[(Sqrt[a + b]*Sqrt[c + d
*Sin[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/
((a + b)*(c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c
 + d)*(a + b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d
)*(a + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/(3*(a - b)^2*(a + b)^(3/2)*(
b*c - a*d)*f) + (2*(b*c - a*d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(3*(a
^2 - b^2)*f*(a + b*Sin[e + f*x])^(3/2)) + (2*(c - d)*(3*a*c + b*c - a*d - 3
*b*d)*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*
Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sec[e + f*
x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sq
rt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*(c +
 d*Sin[e + f*x]))/(3*(a - b)^2*Sqrt[a + b]*Sqrt[c + d]*(b*c - a*d)*f)
```


Rule 2799

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin
[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1))/(f*(m + 1)*(a^2 - b^2)), x
] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*S
in[e + f*x])^(n - 2)*Simp[c*(a*c - b*d)*(m + 1) + d*(b*c - a*d)*(n - 1) + (
d*(a*c - b*d)*(m + 1) - c*(b*c - a*d)*(m + 2))*Sin[e + f*x] - d*(b*c - a*d)
*(m + n + 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
```

```
NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& LtQ[1, n, 2] && IntegersQ[2*m, 2*n]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2818

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*(c + d*Sin[e + f*x])*Sqrt[((b*c -
a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d
)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[
(c + d)/(a + b), 2]*(Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (
(a + b)*(c - d))/((a - b)*(c + d))])/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*
Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 2996

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(-2*A*(c - d)*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/
((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c
 + d)*(a + b*Sin[e + f*x])))]*EllipticE[ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt
[c + d*Sin[e + f*x]])/Sqrt[a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)
*(c - d))])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2]*Cos[e + f*x]), x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^{3/2}}{(a+b\sin(e+fx))^{5/2}}\,dx = \frac{2(bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3\left(a^2-b^2\right)f(a+b\sin(e+fx))^{3/2}} - \frac{2\int \frac{\frac{1}{2}\left(4bcd-a\left(3c^2+d^2\right)\right)-\frac{1}{2}\left(4acd-b\left(c^2+3d^2\right)\right)\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}\,dx}{3\left(a^2-b^2\right)}$$

$$= \frac{2(bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3\left(a^2-b^2\right)f(a+b\sin(e+fx))^{3/2}} + \frac{((c-d)(3ac+bc-ad-3bd))\int \frac{1}{\sqrt{a+b\sin(e+fx)}\sqrt{\cdots}}}{3(a-b)^2(a+b)}$$

$$= \frac{8(c-d)\sqrt{c+d}(ac-bd)E\left(\sin^{-1}\left(\frac{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sec(e+fx)\sqrt{-\frac{(bc-ad)(1-\sin(e\cdots}{(c+d)(a+b\sin(e\cdots}}}{3(a-b)^2(a+b)^{3/2}(bc-ad)f}$$

Mathematica [B] time = 6.34027, size = 1982, normalized size = 3.99

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*Sin[e + f*x])^(3/2)/(a + b*Sin[e + f*x])^(5/2),x]
```

```
[Out] (Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]*((-2*(-(b*c*Cos[e + f*x]
) + a*d*Cos[e + f*x]))/(3*(a^2 - b^2)*(a + b*Sin[e + f*x])^2) - (8*(-(a*b*c
*Cos[e + f*x]) + b^2*d*Cos[e + f*x]))/(3*(a^2 - b^2)^2*(a + b*Sin[e + f*x])
)))/f + ((-4*(-(b*c) + a*d)*(3*a^2*c^2 + b^2*c^2 - 4*a*b*c*d + a^2*d^2 - b^
2*d^2)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin
[Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) +
 a*d))]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(
-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e
 + f*x]))/(-(b*c) + a*d)]*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d
*Sin[e + f*x]))/(-(b*c) + a*d))])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]
*Sqrt[c + d*Sin[e + f*x]]) - 4*(-(b*c) + a*d)*(4*a*b*c^2 + 4*a^2*c*d - 4*b^
2*c*d - 4*a*b*d^2)*((Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*El
lipticF[ArcSin[Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*
x]))/(-(b*c) + a*d))]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[
e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^
2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*
x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))])/((a + b)*(c + d)*Sqrt[a + b
*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - (Sqrt[((c + d)*Cot[(-e + Pi/2 -
f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[-((
(a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))]/S
qrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2
 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))
/(-(b*c) + a*d)]*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e +
f*x]))/(-(b*c) + a*d))])/((a + b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin
[e + f*x]])) + 2*(-4*a*b*c*d + 4*b^2*d^2)*((Cos[e + f*x]*Sqrt[c + d*Sin[e +
 f*x]])/(d*Sqrt[a + b*Sin[e + f*x]]) + (Sqrt[(a - b)/(a + b)]*(a + b)*Cos[(
-e + Pi/2 - f*x)/2]*EllipticE[ArcSin[(Sqrt[(a - b)/(a + b)]*Sin[(-e + Pi/2
- f*x)/2])/Sqrt[(a + b*Sin[e + f*x])/(a + b)]], (2*(-(b*c) + a*d))/((a - b)
*(c + d))]*Sqrt[c + d*Sin[e + f*x]])/(b*d*Sqrt[((a + b)*Cos[(-e + Pi/2 - f*
x)/2]^2)/(a + b*Sin[e + f*x])]*Sqrt[a + b*Sin[e + f*x]]*Sqrt[(a + b*Sin[e +
 f*x])/(a + b)]*Sqrt[((a + b)*(c + d*Sin[e + f*x]))/((c + d)*(a + b*Sin[e +
 f*x]))]) - (2*(-(b*c) + a*d)*((((a + b)*c + a*d)*Sqrt[((c + d)*Cot[(-e + P
i/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[-(((a + b)*Csc[(-e + Pi/2
- f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))]/Sqrt[2]], (2*(-(b*c) + a
*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c +
 d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[-
(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))]
)/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - ((b
*c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-
(b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2
*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))]/Sqrt[2]], (2*(-(b*c) + a*d))/((a +
b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e
 + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[-(((a + b)*C
sc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))])/((a + b)*
d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])))/(b*d)))/(3*(a - b)^2
*(a + b)^2*f)
```

Maple [B] time = 3.203, size = 195453, normalized size = 393.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d \sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}{\left(b \sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(3/2)/(b*sin(f*x + e) + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{b \sin\left(fx+e\right)+a}\left(d \sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}{3\,ab^{2} \cos\left(fx+e\right)^{2}-a^{3}-3\,ab^{2}+\left(b^{3} \cos\left(fx+e\right)^{2}-3\,a^{2}b-b^{3}\right)\sin\left(fx+e\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] integral(-sqrt(b*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^(3/2)/(3*a*b^2*cos(
f*x + e)^2 - a^3 - 3*a*b^2 + (b^3*cos(f*x + e)^2 - 3*a^2*b - b^3)*sin(f*x +
 e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(3/2)/(a+b*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d \sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}{\left(b \sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate((d*sin(f*x + e) + c)^(3/2)/(b*sin(f*x + e) + a)^(5/2), x)
```

3.797 $\int \frac{\sqrt{c+d\sin(e+fx)}}{(a+b\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=489

$$\frac{2b\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3f\left(a^2-b^2\right)(a+b\sin(e+fx))^{3/2}}+\frac{2(c-d)\sqrt{c+d}\left(-3a^2d+4abc-b^2d\right)\sec(e+fx)(a+b\sin(e+fx))\sqrt{-\frac{(bc-ad)(1-\sin}{(c+d)(a+b\sin}}}{3f(a-b)^2(a+b)^{3/2}(bc}$$

```
[Out] (2*(c - d)*Sqrt[c + d]*(4*a*b*c - 3*a^2*d - b^2*d)*EllipticE[ArcSin[(Sqrt[a
 + b]*Sqrt[c + d*Sin[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((
a - b)*(c + d))/((a + b)*(c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Si
n[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e
+ f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/(3*(a - b)^2
*(a + b)^(3/2)*(b*c - a*d)^2*f) + (2*b*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]
])/(3*(a^2 - b^2)*f*(a + b*Sin[e + f*x])^(3/2)) + (2*(3*a + b)*(c - d)*Elli
pticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d
*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((
b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c
 - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*(c + d*Sin[e +
 f*x]))/(3*(a - b)^2*Sqrt[a + b]*Sqrt[c + d]*(b*c - a*d)*f)
```

Rubi [A] time = 0.859839, antiderivative size = 489, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2796, 2998, 2818, 2996}

$$\frac{2b\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3f\left(a^2-b^2\right)(a+b\sin(e+fx))^{3/2}}+\frac{2(c-d)\sqrt{c+d}\left(-3a^2d+4abc-b^2d\right)\sec(e+fx)(a+b\sin(e+fx))\sqrt{-\frac{(bc-ad)(1-\sin}{(c+d)(a+b\sin}}}{3f(a-b)^2(a+b)^{3/2}(bc}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*Sin[e + f*x]]/(a + b*Sin[e + f*x])^(5/2),x]
```


```
[Out] (2*(c - d)*Sqrt[c + d]*(4*a*b*c - 3*a^2*d - b^2*d)*EllipticE[ArcSin[(Sqrt[a
 + b]*Sqrt[c + d*Sin[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((
a - b)*(c + d))/((a + b)*(c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Si
n[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e
+ f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/(3*(a - b)^2
*(a + b)^(3/2)*(b*c - a*d)^2*f) + (2*b*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]
])/(3*(a^2 - b^2)*f*(a + b*Sin[e + f*x])^(3/2)) + (2*(3*a + b)*(c - d)*Elli
pticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d
*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((
b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c
 - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*(c + d*Sin[e +
 f*x]))/(3*(a - b)^2*Sqrt[a + b]*Sqrt[c + d]*(b*c - a*d)*f)
```


Rule 2796

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m + 1)*(c + d*Sin[e + f*x])^n)/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m +
 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n
- 1)*Simp[a*c*(m + 1) + b*d*n + (a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] -
b*d*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1
```

```
] && LtQ[0, n, 1] && IntegersQ[2*m, 2*n]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2818

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*(c + d*Sin[e + f*x])*Sqrt[((b*c -
a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d
)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[
(c + d)/(a + b), 2]*(Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (
(a + b)*(c - d))/((a - b)*(c + d))])/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*
Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 2996

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(-2*A*(c - d)*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/
((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c
 + d)*(a + b*Sin[e + f*x])))]*EllipticE[ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt
[c + d*Sin[e + f*x]])/Sqrt[a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)
*(c - d))])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2]*Cos[e + f*x]), x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rubi steps

$$\int \frac{\sqrt{c+d\sin(e+fx)}}{(a+b\sin(e+fx))^{5/2}}\,dx = \frac{2b\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3\left(a^2-b^2\right)f(a+b\sin(e+fx))^{3/2}} - \frac{2\int \frac{\frac{1}{2}(-3ac+bd)+\frac{1}{2}(bc-3ad)\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}\,dx}{3\left(a^2-b^2\right)}$$

$$= \frac{2b\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3\left(a^2-b^2\right)f(a+b\sin(e+fx))^{3/2}} + \frac{((3a+b)(c-d))\int \frac{1}{\sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\,dx}{3(a-b)^2(a+b)} - \frac{(4ab\cdots}{\cdots}$$

$$= \frac{2(c-d)\sqrt{c+d}\left(4abc-3a^2d-b^2d\right)E\left(\sin^{-1}\left(\frac{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}\right)\middle|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sec(e+fx)\sqrt{-\cdots}}{3(a-b)^2(a+b)^{3/2}(bc-ad)^2 f}$$

Mathematica [B] time = 6.41213, size = 2037, normalized size = 4.17

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c + d*Sin[e + f*x]]/(a + b*Sin[e + f*x])^(5/2),x]
```

```
[Out] (Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]*((2*b*Cos[e + f*x])/(3*(
a^2 - b^2)*(a + b*Sin[e + f*x])^2) + (2*(-4*a*b^2*c*Cos[e + f*x] + 3*a^2*b*
d*Cos[e + f*x] + b^3*d*Cos[e + f*x]))/(3*(a^2 - b^2)^2*(-(b*c) + a*d)*(a +
b*Sin[e + f*x]))))/f + ((-4*(-(b*c) + a*d)*(-3*a^2*b*c^2 - b^3*c^2 + 3*a^3*
c*d + a*b^2*c*d - a^2*b*d^2 + b^3*d^2)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/
2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2
*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))]/Sqrt[2]], (2*(-(b*c) + a*d))/((a +
b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e
 + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[-(((a + b)*C
sc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))])/((a + b)*
(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - 4*(-(b*c) + a*
d)*(-4*a*b^2*c^2 - a^2*b*c*d + b^3*c*d + 3*a^3*d^2 + a*b^2*d^2)*((Sqrt[((c
+ d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[-(((a + b)
*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))]/Sqrt[2]]
, (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)
/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c
) + a*d)]*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/
(-(b*c) + a*d))])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[
e + f*x]]) - (Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticP
i[(-(b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/
2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))]/Sqrt[2]], (2*(-(b*c) + a*d))/((
a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc
[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[-(((a +
b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))])/((a +
 b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])) + 2*(4*a*b^2*c*d
- 3*a^2*b*d^2 - b^3*d^2)*((Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(d*Sqrt[a
 + b*Sin[e + f*x]]) + (Sqrt[(a - b)/(a + b)]*(a + b)*Cos[(-e + Pi/2 - f*x)/
2]*EllipticE[ArcSin[(Sqrt[(a - b)/(a + b)]*Sin[(-e + Pi/2 - f*x)/2])/Sqrt[(
a + b*Sin[e + f*x])/(a + b)]], (2*(-(b*c) + a*d))/((a - b)*(c + d))]*Sqrt[c
 + d*Sin[e + f*x]])/(b*d*Sqrt[((a + b)*Cos[(-e + Pi/2 - f*x)/2]^2)/(a + b*S
in[e + f*x])]*Sqrt[a + b*Sin[e + f*x]]*Sqrt[(a + b*Sin[e + f*x])/(a + b)]*S
qrt[((a + b)*(c + d*Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x]))]) - (2*(-
(b*c) + a*d)*((((a + b)*c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/
(-c + d)]*EllipticF[ArcSin[Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c +
d*Sin[e + f*x]))/(-(b*c) + a*d))]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c
 + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/
2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[-(((a + b)*Csc[(-e
 + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))])/((a + b)*(c + d
)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - ((b*c + a*d)*Sqrt[((
c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a
+ b)*d), ArcSin[Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f
*x]))/(-(b*c) + a*d))]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec
[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]
^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f
*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))])/((a + b)*d*Sqrt[a + b*Sin[
e + f*x]]*Sqrt[c + d*Sin[e + f*x]])))/(b*d)))/(3*(a - b)^2*(a + b)^2*(-(b*c
) + a*d)*f)
```

Maple [B] time = 2.955, size = 212512, normalized size = 434.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d\sin\left(fx+e\right)+c}}{\left(b\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(d*sin(f*x + e) + c)/(b*sin(f*x + e) + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{b\sin\left(fx+e\right)+a}\sqrt{d\sin\left(fx+e\right)+c}}{3\,ab^{2}\cos\left(fx+e\right)^{2}-a^{3}-3\,ab^{2}+\left(b^{3}\cos\left(fx+e\right)^{2}-3\,a^{2}b-b^{3}\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] integral(-sqrt(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)/(3*a*b^2*cos(f*
x + e)^2 - a^3 - 3*a*b^2 + (b^3*cos(f*x + e)^2 - 3*a^2*b - b^3)*sin(f*x + e
)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(1/2)/(a+b*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d\sin\left(fx+e\right)+c}}{\left(b\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(d*sin(f*x + e) + c)/(b*sin(f*x + e) + a)^(5/2), x)
```

3.798 $\int \frac{1}{(a+b\sin(e+fx))^{5/2}\sqrt{c+d\sin(e+fx)}}\,dx$

Optimal. Leaf size=516

$$\frac{2b^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3f\left(a^2-b^2\right)(bc-ad)(a+b\sin(e+fx))^{3/2}}+\frac{2\left(-3a^2d+3ab(c-d)+b^2(c+2d)\right)\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-}{(a+}}}{3f(a-b)^2\sqrt{a+}}$$

```
[Out] (4*b*(c - d)*Sqrt[c + d]*(2*a*b*c - 3*a^2*d + b^2*d)*EllipticE[ArcSin[(Sqrt
[a + b]*Sqrt[c + d*Sin[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])],
((a - b)*(c + d))/((a + b)*(c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 -
Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[
e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/(3*(a - b)
^2*(a + b)^(3/2)*(b*c - a*d)^3*f) + (2*b^2*Cos[e + f*x]*Sqrt[c + d*Sin[e +
f*x]])/(3*(a^2 - b^2)*(b*c - a*d)*f*(a + b*Sin[e + f*x])^(3/2)) + (2*(3*a*b
*(c - d) - 3*a^2*d + b^2*(c + 2*d))*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a +
b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))
/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a
+ b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b
)*(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/(3*(a - b)^2*Sqrt[a + b]*Sq
rt[c + d]*(b*c - a*d)^2*f)
```

Rubi [A] time = 0.993577, antiderivative size = 516, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2802, 2998, 2818, 2996}

$$\frac{2b^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3f\left(a^2-b^2\right)(bc-ad)(a+b\sin(e+fx))^{3/2}}+\frac{2\left(-3a^2d+3ab(c-d)+b^2(c+2d)\right)\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-}{(a+}}}{3f(a-b)^2\sqrt{a+}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sin[e + f*x])^(5/2)*Sqrt[c + d*Sin[e + f*x]]),x]
```

```
[Out] (4*b*(c - d)*Sqrt[c + d]*(2*a*b*c - 3*a^2*d + b^2*d)*EllipticE[ArcSin[(Sqrt
[a + b]*Sqrt[c + d*Sin[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])],
((a - b)*(c + d))/((a + b)*(c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 -
Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[
e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/(3*(a - b)
^2*(a + b)^(3/2)*(b*c - a*d)^3*f) + (2*b^2*Cos[e + f*x]*Sqrt[c + d*Sin[e +
f*x]])/(3*(a^2 - b^2)*(b*c - a*d)*f*(a + b*Sin[e + f*x])^(3/2)) + (2*(3*a*b
*(c - d) - 3*a^2*d + b^2*(c + 2*d))*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a +
b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))
/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a
+ b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b
)*(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/(3*(a - b)^2*Sqrt[a + b]*Sq
rt[c + d]*(b*c - a*d)^2*f)
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
```

```
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2818

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*(c + d*Sin[e + f*x])*Sqrt[((b*c -
a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d
)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[
(c + d)/(a + b), 2]*(Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (
(a + b)*(c - d))/((a - b)*(c + d))])/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*
Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 2996

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(-2*A*(c - d)*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/
((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c
 + d)*(a + b*Sin[e + f*x])))]*EllipticE[ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt
[c + d*Sin[e + f*x]])/Sqrt[a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)
*(c - d))])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2]*Cos[e + f*x]), x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rubi steps

$$\int \frac{1}{(a+b\sin(e+fx))^{5/2}\sqrt{c+d\sin(e+fx)}}\,dx = \frac{2b^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3\left(a^2-b^2\right)(bc-ad)f(a+b\sin(e+fx))^{3/2}} - \frac{2\int \frac{\frac{1}{2}\left(-2b^2d-3a(bc-ad)\right)+\frac{1}{2}b(bc-}{(a+b\sin(e+fx))^{3/2}\sqrt{c+d}}}{3\left(a^2-b^2\right)(bc-}$$

$$= \frac{2b^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3\left(a^2-b^2\right)(bc-ad)f(a+b\sin(e+fx))^{3/2}} - \frac{\left(2b\left(2abc-3a^2d+b^2d\right)\right)\int}{3(a-b)^2(}$$

$$= \frac{4b(c-d)\sqrt{c+d}\left(2abc-3a^2d+b^2d\right)E\left(\sin^{-1}\left(\frac{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}\right)|\frac{(a-b)(c}{(a+b)(c}}{3(a-b)^2(a+}$$

Mathematica [B] time = 6.43239, size = 2072, normalized size = 4.02

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*Sin[e + f*x])^(5/2)*Sqrt[c + d*Sin[e + f*x]]),x]
```

```
[Out] (Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]*((-2*b^2*Cos[e + f*x])/(
3*(a^2 - b^2)*(-(b*c) + a*d)*(a + b*Sin[e + f*x])^2) + (4*(2*a*b^3*c*Cos[e
+ f*x] - 3*a^2*b^2*d*Cos[e + f*x] + b^4*d*Cos[e + f*x]))/(3*(a^2 - b^2)^2*(
-(b*c) + a*d)^2*(a + b*Sin[e + f*x]))))/f + ((-4*(-(b*c) + a*d)*(3*a^2*b^2*
c^2 + b^4*c^2 - 6*a^3*b*c*d + 2*a*b^3*c*d + 3*a^4*d^2 - 5*a^2*b^2*d^2 + 2*b
^4*d^2)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSi
n[Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c)
+ a*d))]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[
(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[
e + f*x]))/(-(b*c) + a*d)]*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c +
d*Sin[e + f*x]))/(-(b*c) + a*d))])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]
]*Sqrt[c + d*Sin[e + f*x]]) - 4*(-(b*c) + a*d)*(4*a*b^3*c^2 - 2*a^2*b^2*c*d
 + 2*b^4*c*d - 6*a^3*b*d^2 + 2*a*b^3*d^2)*((Sqrt[((c + d)*Cot[(-e + Pi/2 -
f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)
/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))]/Sqrt[2]], (2*(-(b*c) + a*d))/(
(a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Cs
c[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[-(((a +
 b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))])/((a
+ b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - (Sqrt[((c
 + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a +
 b)*d), ArcSin[Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*
x]))/(-(b*c) + a*d))]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[
e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^
2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*
x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))])/((a + b)*d*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[c + d*Sin[e + f*x]])) + 2*(-4*a*b^3*c*d + 6*a^2*b^2*d^2 - 2*b
^4*d^2)*((Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(d*Sqrt[a + b*Sin[e + f*x]
]) + (Sqrt[(a - b)/(a + b)]*(a + b)*Cos[(-e + Pi/2 - f*x)/2]*EllipticE[ArcS
in[(Sqrt[(a - b)/(a + b)]*Sin[(-e + Pi/2 - f*x)/2])/Sqrt[(a + b*Sin[e + f*x
])/(a + b)]], (2*(-(b*c) + a*d))/((a - b)*(c + d))]*Sqrt[c + d*Sin[e + f*x]
])/(b*d*Sqrt[((a + b)*Cos[(-e + Pi/2 - f*x)/2]^2)/(a + b*Sin[e + f*x])]*Sqr
t[a + b*Sin[e + f*x]]*Sqrt[(a + b*Sin[e + f*x])/(a + b)]*Sqrt[((a + b)*(c +
 d*Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x]))]) - (2*(-(b*c) + a*d)*((((
a + b)*c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*Ellipti
cF[ArcSin[Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/
(-(b*c) + a*d))]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f
*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a
+ b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]
^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))])/((a + b)*(c + d)*Sqrt[a + b*Sin[
e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - ((b*c + a*d)*Sqrt[((c + d)*Cot[(-e +
Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a + b)*d), ArcSin[S
qrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a
*d))]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e
 + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e +
 f*x]))/(-(b*c) + a*d)]*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*S
in[e + f*x]))/(-(b*c) + a*d))])/((a + b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c
+ d*Sin[e + f*x]])))/(b*d)))/(3*(a - b)^2*(a + b)^2*(-(b*c) + a*d)^2*f)
```

Maple [B] time = 4.226, size = 243193, normalized size = 471.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\sqrt{d\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate(1/((b*sin(f*x + e) + a)^(5/2)*sqrt(d*sin(f*x + e) + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sin\left(fx+e\right)+a}\sqrt{d\sin\left(fx+e\right)+c}}{b^3d\cos\left(fx+e\right)^4-\left(3\,ab^2c+\left(3\,a^2b+2\,b^3\right)d\right)\cos\left(fx+e\right)^2+\left(a^3+3\,ab^2\right)c+\left(3\,a^2b+b^3\right)d-\left(\left(b^3c+3\,ab^2d\right)c\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="fri
cas")
```

```
[Out] integral(sqrt(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)/(b^3*d*cos(f*x +
 e)^4 - (3*a*b^2*c + (3*a^2*b + 2*b^3)*d)*cos(f*x + e)^2 + (a^3 + 3*a*b^2)*
c + (3*a^2*b + b^3)*d - ((b^3*c + 3*a*b^2*d)*cos(f*x + e)^2 - (3*a^2*b + b^
3)*c - (a^3 + 3*a*b^2)*d)*sin(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))**(5/2)/(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\sqrt{d\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate(1/((b*sin(f*x + e) + a)^(5/2)*sqrt(d*sin(f*x + e) + c)), x)
```

3.799 $\int \frac{1}{(a+b\sin(e+fx))^{5/2}(c+d\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=688

$$-\frac{2\left(3a^2bd(2c-3d)-3a^3d^2-3ab^2\left(c^2-2d^2\right)+b^3\left(c^2-6cd+8d^2\right)\right)\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-}}{3f\sqrt{a+b}\left(a^2-b^2\right)(c-d)\sqrt{c+d}(bc-ad)^3}$$

```
[Out] (2*b^2*Cos[e + f*x])/(3*(a^2 - b^2)*(b*c - a*d)*f*(a + b*Sin[e + f*x])^(3/2
)*Sqrt[c + d*Sin[e + f*x]]) + (8*b^2*(a*b*c - 2*a^2*d + b^2*d)*Cos[e + f*x]
)/(3*(a^2 - b^2)^2*(b*c - a*d)^2*f*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[
e + f*x]]) + (2*(3*a^4*d^3 - b^4*d*(5*c^2 - 8*d^2) + 3*a^2*b^2*d*(3*c^2 - 5
*d^2) - 4*a*b^3*c*(c^2 - d^2))*EllipticE[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin
[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a
- b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*
(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c
+ d*Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/(3*Sqrt[a + b]*(a^2 - b^2)*(c -
d)*Sqrt[c + d]*(b*c - a*d)^4*f) - (2*(3*a^2*b*(2*c - 3*d)*d - 3*a^3*d^2 - 3
*a*b^2*(c^2 - 2*d^2) + b^3*(c^2 - 6*c*d + 8*d^2))*EllipticF[ArcSin[(Sqrt[c
+ d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a
 + b)*(c - d))/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e
 + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e +
f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/(3*Sqrt[a + b
]*(a^2 - b^2)*(c - d)*Sqrt[c + d]*(b*c - a*d)^3*f)
```

Rubi [A] time = 1.84657, antiderivative size = 688, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2802, 3055, 2998, 2818, 2996}

$$-\frac{2\left(3a^2bd(2c-3d)-3a^3d^2-3ab^2\left(c^2-2d^2\right)+b^3\left(c^2-6cd+8d^2\right)\right)\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-}}{3f\sqrt{a+b}\left(a^2-b^2\right)(c-d)\sqrt{c+d}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])^(3/2)),x]
```

```
[Out] (2*b^2*Cos[e + f*x])/(3*(a^2 - b^2)*(b*c - a*d)*f*(a + b*Sin[e + f*x])^(3/2
)*Sqrt[c + d*Sin[e + f*x]]) + (8*b^2*(a*b*c - 2*a^2*d + b^2*d)*Cos[e + f*x]
)/(3*(a^2 - b^2)^2*(b*c - a*d)^2*f*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[
e + f*x]]) + (2*(3*a^4*d^3 - b^4*d*(5*c^2 - 8*d^2) + 3*a^2*b^2*d*(3*c^2 - 5
*d^2) - 4*a*b^3*c*(c^2 - d^2))*EllipticE[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin
[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a
- b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*
(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c
+ d*Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/(3*Sqrt[a + b]*(a^2 - b^2)*(c -
d)*Sqrt[c + d]*(b*c - a*d)^4*f) - (2*(3*a^2*b*(2*c - 3*d)*d - 3*a^3*d^2 - 3
*a*b^2*(c^2 - 2*d^2) + b^3*(c^2 - 6*c*d + 8*d^2))*EllipticF[ArcSin[(Sqrt[c
+ d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a
 + b)*(c - d))/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e
 + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e +
f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/(3*Sqrt[a + b
]*(a^2 - b^2)*(c - d)*Sqrt[c + d]*(b*c - a*d)^3*f)
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2818

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*(c + d*Sin[e + f*x])*Sqrt[((b*c -
a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d
)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[
(c + d)/(a + b), 2]*(Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (
(a + b)*(c - d))/((a - b)*(c + d))])/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*
Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 2996

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(-2*A*(c - d)*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/
((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c
 + d)*(a + b*Sin[e + f*x])))]*EllipticE[ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt
[c + d*Sin[e + f*x]])/Sqrt[a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)
*(c - d))])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2]*Cos[e + f*x]), x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rubi steps

$$\int \frac{1}{(a+b\sin(e+fx))^{5/2}(c+d\sin(e+fx))^{3/2}}\,dx = \frac{2b^2\cos(e+fx)}{3\left(a^2-b^2\right)(bc-ad)f(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}} - \frac{2\int \frac{1}{2}}{}$$

$$= \frac{2b^2\cos(e+fx)}{3\left(a^2-b^2\right)(bc-ad)f(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}} + \frac{}{3\left(a^2\right.}$$

$$= \frac{2b^2\cos(e+fx)}{3\left(a^2-b^2\right)(bc-ad)f(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}} + \frac{}{3\left(a^2\right.}$$

$$= \frac{2b^2\cos(e+fx)}{3\left(a^2-b^2\right)(bc-ad)f(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}} + \frac{}{3\left(a^2\right.}$$

Mathematica [B] time = 7.37752, size = 2322, normalized size = 3.38

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])^(3/2)),x]
```

```
[Out] (Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]*((2*b^3*Cos[e + f*x])/(3
*(a^2 - b^2)*(-(b*c) + a*d)^2*(a + b*Sin[e + f*x])^2) - (2*(4*a*b^4*c*Cos[e
 + f*x] - 9*a^2*b^3*d*Cos[e + f*x] + 5*b^5*d*Cos[e + f*x]))/(3*(a^2 - b^2)^
2*(-(b*c) + a*d)^3*(a + b*Sin[e + f*x])) - (2*d^4*Cos[e + f*x])/((b*c - a*d
)^3*(c^2 - d^2)*(c + d*Sin[e + f*x]))))/f + ((-4*(-(b*c) + a*d)*(-3*a^2*b^3
*c^4 - b^5*c^4 + 9*a^3*b^2*c^3*d - 5*a*b^4*c^3*d - 9*a^4*b*c^2*d^2 + 20*a^2
*b^3*c^2*d^2 - 7*b^5*c^2*d^2 + 3*a^5*c*d^3 - 15*a^3*b^2*c*d^3 + 8*a*b^4*c*d
^3 + 9*a^4*b*d^4 - 17*a^2*b^3*d^4 + 8*b^5*d^4)*Sqrt[((c + d)*Cot[(-e + Pi/2
 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f
*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))]/Sqrt[2]], (2*(-(b*c) + a*d)
)/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)
*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[-(((
a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))])/(
(a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - 4*(-(b
*c) + a*d)*(-4*a*b^4*c^4 + 5*a^2*b^3*c^3*d - 5*b^5*c^3*d + 9*a^3*b^2*c^2*d^
2 - a*b^4*c^2*d^2 + 3*a^4*b*c*d^3 - 11*a^2*b^3*c*d^3 + 8*b^5*c*d^3 + 3*a^5*
d^4 - 15*a^3*b^2*d^4 + 8*a*b^4*d^4)*((Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2
]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*
(c + d*Sin[e + f*x]))/(-(b*c) + a*d))]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b
)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e
+ Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[-(((a + b)*Cs
c[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))])/((a + b)*(
c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - (Sqrt[((c + d)*
Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a + b)*d)
, ArcSin[Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(
-(b*c) + a*d))]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*
x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a +
 b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^
2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))])/((a + b)*d*Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]])) + 2*(4*a*b^4*c^3*d - 9*a^2*b^3*c^2*d^2 + 5*b^
```

```
5*c^2*d^2 - 4*a*b^4*c*d^3 - 3*a^4*b*d^4 + 15*a^2*b^3*d^4 - 8*b^5*d^4)*((Cos
[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(d*Sqrt[a + b*Sin[e + f*x]]) + (Sqrt[(a
 - b)/(a + b)]*(a + b)*Cos[(-e + Pi/2 - f*x)/2]*EllipticE[ArcSin[(Sqrt[(a -
 b)/(a + b)]*Sin[(-e + Pi/2 - f*x)/2])/Sqrt[(a + b*Sin[e + f*x])/(a + b)]],
 (2*(-(b*c) + a*d))/((a - b)*(c + d))]*Sqrt[c + d*Sin[e + f*x]])/(b*d*Sqrt[
((a + b)*Cos[(-e + Pi/2 - f*x)/2]^2)/(a + b*Sin[e + f*x])]*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[(a + b*Sin[e + f*x])/(a + b)]*Sqrt[((a + b)*(c + d*Sin[e + f*
x]))/((c + d)*(a + b*Sin[e + f*x]))]]) - (2*(-(b*c) + a*d)*((((a + b)*c + a*
d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqr
t[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d
))]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e +
 Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f
*x]))/(-(b*c) + a*d)]*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin
[e + f*x]))/(-(b*c) + a*d))])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqr
t[c + d*Sin[e + f*x]]) - ((b*c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2
]^2)/(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[-(((a + b
)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))]/Sqrt[2]
], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x
)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*
c) + a*d)]*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))
/(-(b*c) + a*d))])/((a + b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f
*x]])))/(b*d)))/(3*(a - b)^2*(a + b)^2*(c - d)*(c + d)*(-(b*c) + a*d)^3*f)
```

Maple [B] time = 8.531, size = 439275, normalized size = 638.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="max
ima")
```

```
[Out] integrate(1/((b*sin(f*x + e) + a)^(5/2)*(d*sin(f*x + e) + c)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{}{\left(2\,b^3cd+3\,ab^2d^2\right)\cos\left(fx+e\right)^4+\left(a^3+3\,ab^2\right)c^2+2\left(3\,a^2b+b^3\right)cd+\left(a^3+3\,ab^2\right)d^2-\left(3\,ab^2c^2+2\left(3\,a^2b+\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="fri
cas")
```

```
[Out] integral(sqrt(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)/((2*b^3*c*d + 3*
a*b^2*d^2)*cos(f*x + e)^4 + (a^3 + 3*a*b^2)*c^2 + 2*(3*a^2*b + b^3)*c*d + (
a^3 + 3*a*b^2)*d^2 - (3*a*b^2*c^2 + 2*(3*a^2*b + 2*b^3)*c*d + (a^3 + 6*a*b^
2)*d^2)*cos(f*x + e)^2 + (b^3*d^2*cos(f*x + e)^4 + (3*a^2*b + b^3)*c^2 + 2*
(a^3 + 3*a*b^2)*c*d + (3*a^2*b + b^3)*d^2 - (b^3*c^2 + 6*a*b^2*c*d + (3*a^2
*b + 2*b^3)*d^2)*cos(f*x + e)^2)*sin(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))**(5/2)/(c+d*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="gia
c")
```

```
[Out] integrate(1/((b*sin(f*x + e) + a)^(5/2)*(d*sin(f*x + e) + c)^(3/2)), x)
```

3.800 $\int \frac{1}{(a+b\sin(e+fx))^{5/2}(c+d\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=941

$$\frac{4\left(-5da^2+2bca+3b^2d\right)\cos(e+fx)b^2}{3\left(a^2-b^2\right)^2(bc-ad)^2f\sqrt{a+b\sin(e+fx)}(c+d\sin(e+fx))^{3/2}}+\frac{2\cos(e+fx)b^2}{3\left(a^2-b^2\right)(bc-ad)f(a+b\sin(e+fx))^{3/2}(c+d\sin}$$

```
[Out] (2*b^2*Cos[e + f*x])/(3*(a^2 - b^2)*(b*c - a*d)*f*(a + b*Sin[e + f*x])^(3/2
)*(c + d*Sin[e + f*x])^(3/2)) + (4*b^2*(2*a*b*c - 5*a^2*d + 3*b^2*d)*Cos[e
+ f*x])/(3*(a^2 - b^2)^2*(b*c - a*d)^2*f*Sqrt[a + b*Sin[e + f*x]]*(c + d*Si
n[e + f*x])^(3/2)) - (2*d*(a^4*d^3 + a^2*b^2*d*(11*c^2 - 13*d^2) - b^4*d*(7
*c^2 - 8*d^2) - 4*a*b^3*c*(c^2 - d^2))*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]
])/(3*(a^2 - b^2)^2*(b*c - a*d)^3*(c^2 - d^2)*f*(c + d*Sin[e + f*x])^(3/2))
 - (8*(a^5*c*d^4 - 2*a^3*b^2*c*d^4 + a*b^4*c*(c^4 - 2*c^2*d^2 + 2*d^4) + b^
5*d*(2*c^4 - 7*c^2*d^2 + 4*d^4) - a^2*b^3*d*(3*c^4 - 12*c^2*d^2 + 7*d^4) -
a^4*b*(3*c^2*d^3 - 2*d^5))*EllipticE[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e +
 f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)
*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c +
 d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*
Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/(3*Sqrt[a + b]*(a^2 - b^2)*(c - d)^2
*(c + d)^(3/2)*(b*c - a*d)^5*f) - (2*(a^4*d^3*(3*c + d) - 9*a^3*b*d^2*(c^2
- d^2) + a^2*b^2*d*(9*c^3 - 18*c^2*d - 15*c*d^2 + 16*d^3) + b^4*(c^4 - 9*c^
3*d + 16*c^2*d^2 + 12*c*d^3 - 16*d^4) - 3*a*b^3*(c^4 - 5*c^2*d^2 + 4*d^4))*
EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c
 + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sec[e + f*x]*Sqr
t[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-((
(b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*(c + d*Sin
[e + f*x]))/(3*Sqrt[a + b]*(a^2 - b^2)*(c - d)^2*(c + d)^(3/2)*(b*c - a*d)^
4*f)
```

Rubi [A] time = 4.47217, antiderivative size = 941, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2802, 3055, 2998, 2818, 2996}

$$\frac{4\left(-5da^2+2bca+3b^2d\right)\cos(e+fx)b^2}{3\left(a^2-b^2\right)^2(bc-ad)^2f\sqrt{a+b\sin(e+fx)}(c+d\sin(e+fx))^{3/2}}+\frac{2\cos(e+fx)b^2}{3\left(a^2-b^2\right)(bc-ad)f(a+b\sin(e+fx))^{3/2}(c+d\sin}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])^(5/2)),x]
```

```
[Out] (2*b^2*Cos[e + f*x])/(3*(a^2 - b^2)*(b*c - a*d)*f*(a + b*Sin[e + f*x])^(3/2
)*(c + d*Sin[e + f*x])^(3/2)) + (4*b^2*(2*a*b*c - 5*a^2*d + 3*b^2*d)*Cos[e
+ f*x])/(3*(a^2 - b^2)^2*(b*c - a*d)^2*f*Sqrt[a + b*Sin[e + f*x]]*(c + d*Si
n[e + f*x])^(3/2)) - (2*d*(a^4*d^3 + a^2*b^2*d*(11*c^2 - 13*d^2) - b^4*d*(7
*c^2 - 8*d^2) - 4*a*b^3*c*(c^2 - d^2))*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]
])/(3*(a^2 - b^2)^2*(b*c - a*d)^3*(c^2 - d^2)*f*(c + d*Sin[e + f*x])^(3/2))
 - (8*(a^5*c*d^4 - 2*a^3*b^2*c*d^4 + a*b^4*c*(c^4 - 2*c^2*d^2 + 2*d^4) + b^
5*d*(2*c^4 - 7*c^2*d^2 + 4*d^4) - a^2*b^3*d*(3*c^4 - 12*c^2*d^2 + 7*d^4) -
a^4*b*(3*c^2*d^3 - 2*d^5))*EllipticE[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e +
 f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)
*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c +
 d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*
```

```
Sin[e + f*x]))))]*(c + d*Sin[e + f*x]))/(3*Sqrt[a + b]*(a^2 - b^2)*(c - d)^2
*(c + d)^(3/2)*(b*c - a*d)^5*f) - (2*(a^4*d^3*(3*c + d) - 9*a^3*b*d^2*(c^2
- d^2) + a^2*b^2*d*(9*c^3 - 18*c^2*d - 15*c*d^2 + 16*d^3) + b^4*(c^4 - 9*c^
3*d + 16*c^2*d^2 + 12*c*d^3 - 16*d^4) - 3*a*b^3*(c^4 - 5*c^2*d^2 + 4*d^4))*
EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c
 + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sec[e + f*x]*Sqr
t[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-((
(b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*(c + d*Sin
[e + f*x]))/(3*Sqrt[a + b]*(a^2 - b^2)*(c - d)^2*(c + d)^(3/2)*(b*c - a*d)^
4*f)
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2818

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*(c + d*Sin[e + f*x])*Sqrt[((b*c -
a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d
)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[
(c + d)/(a + b), 2]*(Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (
(a + b)*(c - d))/((a - b)*(c + d))])/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*
Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
```

```
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 2996

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(-2*A*(c - d)*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/
((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c
 + d)*(a + b*Sin[e + f*x])))]*EllipticE[ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt
[c + d*Sin[e + f*x]])/Sqrt[a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)
*(c - d))])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2]*Cos[e + f*x]), x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rubi steps

$$\int \frac{1}{(a+b\sin(e+fx))^{5/2}(c+d\sin(e+fx))^{5/2}}\,dx = \frac{2b^2\cos(e+fx)}{3\left(a^2-b^2\right)(bc-ad)f(a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{3/2}} -$$

$$= \frac{2b^2\cos(e+fx)}{3\left(a^2-b^2\right)(bc-ad)f(a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{3/2}} +$$

$$= \frac{2b^2\cos(e+fx)}{3\left(a^2-b^2\right)(bc-ad)f(a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{3/2}} +$$

$$= \frac{2b^2\cos(e+fx)}{3\left(a^2-b^2\right)(bc-ad)f(a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{3/2}} +$$

$$= \frac{2b^2\cos(e+fx)}{3\left(a^2-b^2\right)(bc-ad)f(a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{3/2}} +$$

Mathematica [B] time = 8.60211, size = 2639, normalized size = 2.8

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])^(5/2)),x]
```

```
[Out] (Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]*((-2*b^4*Cos[e + f*x])/(
3*(a^2 - b^2)*(-(b*c) + a*d)^3*(a + b*Sin[e + f*x])^2) + (8*(a*b^5*c*Cos[e
+ f*x] - 3*a^2*b^4*d*Cos[e + f*x] + 2*b^6*d*Cos[e + f*x]))/(3*(a^2 - b^2)^2
*(-(b*c) + a*d)^4*(a + b*Sin[e + f*x])) - (2*d^4*Cos[e + f*x])/(3*(b*c - a*
d)^3*(c^2 - d^2)*(c + d*Sin[e + f*x])^2) + (8*(-3*b*c^2*d^4*Cos[e + f*x] +
a*c*d^5*Cos[e + f*x] + 2*b*d^6*Cos[e + f*x]))/(3*(b*c - a*d)^4*(c^2 - d^2)^
2*(c + d*Sin[e + f*x]))))/f + ((-4*(-(b*c) + a*d)*(3*a^2*b^4*c^6 + b^6*c^6
- 12*a^3*b^3*c^5*d + 8*a*b^5*c^5*d + 18*a^4*b^2*c^4*d^2 - 41*a^2*b^4*c^4*d^
2 + 15*b^6*c^4*d^2 - 12*a^5*b*c^3*d^3 + 48*a^3*b^3*c^3*d^3 - 28*a*b^5*c^3*d
^3 + 3*a^6*c^2*d^4 - 41*a^4*b^2*c^2*d^4 + 74*a^2*b^4*c^2*d^4 - 32*b^6*c^2*d
^4 + 8*a^5*b*c*d^5 - 28*a^3*b^3*c*d^5 + 16*a*b^5*c*d^5 + a^6*d^6 + 15*a^4*b
^2*d^6 - 32*a^2*b^4*d^6 + 16*b^6*d^6)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2
]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*
```

```
(c + d*Sin[e + f*x]))/(-(b*c) + a*d))]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b
)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e
+ Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[-(((a + b)*Cs
c[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))])/((a + b)*(
c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - 4*(-(b*c) + a*d
)*(4*a*b^5*c^6 - 8*a^2*b^4*c^5*d + 8*b^6*c^5*d - 12*a^3*b^3*c^4*d^2 - 12*a^
4*b^2*c^3*d^3 + 40*a^2*b^4*c^3*d^3 - 28*b^6*c^3*d^3 - 8*a^5*b*c^2*d^4 + 40*
a^3*b^3*c^2*d^4 - 20*a*b^5*c^2*d^4 + 4*a^6*c*d^5 - 20*a^2*b^4*c*d^5 + 16*b^
6*c*d^5 + 8*a^5*b*d^6 - 28*a^3*b^3*d^6 + 16*a*b^5*d^6)*((Sqrt[((c + d)*Cot[
(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[-(((a + b)*Csc[(-e
+ Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))]/Sqrt[2]], (2*(-(b
*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqr
t[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]
*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) +
 a*d))])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]
) - (Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c)
 + a*d)/((a + b)*d), ArcSin[Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c +
 d*Sin[e + f*x]))/(-(b*c) + a*d))]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-
c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi
/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[-(((a + b)*Csc[(-
e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))])/((a + b)*d*Sqr
t[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])) + 2*(-4*a*b^5*c^5*d + 12*a
^2*b^4*c^4*d^2 - 8*b^6*c^4*d^2 + 8*a*b^5*c^3*d^3 + 12*a^4*b^2*c^2*d^4 - 48*
a^2*b^4*c^2*d^4 + 28*b^6*c^2*d^4 - 4*a^5*b*c*d^5 + 8*a^3*b^3*c*d^5 - 8*a*b^
5*c*d^5 - 8*a^4*b^2*d^6 + 28*a^2*b^4*d^6 - 16*b^6*d^6)*((Cos[e + f*x]*Sqrt[
c + d*Sin[e + f*x]])/(d*Sqrt[a + b*Sin[e + f*x]]) + (Sqrt[(a - b)/(a + b)]*
(a + b)*Cos[(-e + Pi/2 - f*x)/2]*EllipticE[ArcSin[(Sqrt[(a - b)/(a + b)]*Si
n[(-e + Pi/2 - f*x)/2])/Sqrt[(a + b*Sin[e + f*x])/(a + b)]], (2*(-(b*c) + a
*d))/((a - b)*(c + d))]*Sqrt[c + d*Sin[e + f*x]])/(b*d*Sqrt[((a + b)*Cos[(-
e + Pi/2 - f*x)/2]^2)/(a + b*Sin[e + f*x])]*Sqrt[a + b*Sin[e + f*x]]*Sqrt[(
a + b*Sin[e + f*x])/(a + b)]*Sqrt[((a + b)*(c + d*Sin[e + f*x]))/((c + d)*(
a + b*Sin[e + f*x]))]) - (2*(-(b*c) + a*d)*((((a + b)*c + a*d)*Sqrt[((c + d
)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[-(((a + b)*Cs
c[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))]/Sqrt[2]], (
2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]
^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) +
 a*d)]*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(
b*c) + a*d))])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e +
 f*x]]) - ((b*c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*
EllipticPi[(-(b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[-(((a + b)*Csc[(-e + Pi/
2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))]/Sqrt[2]], (2*(-(b*c) +
 a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c
 + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt
[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)
)])/((a + b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])))/(b*d)))
/(3*(a - b)^2*(a + b)^2*(c - d)^2*(c + d)^2*(-(b*c) + a*d)^4*f)
```

Maple [B] time = 18.731, size = 1123207, normalized size = 1193.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(5/2),x, algorithm="max
ima")
```

```
[Out] integrate(1/((b*sin(f*x + e) + a)^(5/2)*(d*sin(f*x + e) + c)^(5/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{}{b^3d^3\cos\left(fx+e\right)^6-3\left(b^3c^2d+3\,ab^2cd^2+\left(a^2b+b^3\right)d^3\right)\cos\left(fx+e\right)^4-\left(a^3+3\,ab^2\right)c^3-3\left(3\,a^2b+b^3\right)c^2d\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(5/2),x, algorithm="fri
cas")
```

```
[Out] integral(-sqrt(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)/(b^3*d^3*cos(f*
x + e)^6 - 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + (a^2*b + b^3)*d^3)*cos(f*x + e)^4
 - (a^3 + 3*a*b^2)*c^3 - 3*(3*a^2*b + b^3)*c^2*d - 3*(a^3 + 3*a*b^2)*c*d^2
- (3*a^2*b + b^3)*d^3 + 3*(a*b^2*c^3 + (3*a^2*b + 2*b^3)*c^2*d + (a^3 + 6*a
*b^2)*c*d^2 + (2*a^2*b + b^3)*d^3)*cos(f*x + e)^2 - (3*(b^3*c*d^2 + a*b^2*d
^3)*cos(f*x + e)^4 + (3*a^2*b + b^3)*c^3 + 3*(a^3 + 3*a*b^2)*c^2*d + 3*(3*a
^2*b + b^3)*c*d^2 + (a^3 + 3*a*b^2)*d^3 - (b^3*c^3 + 9*a*b^2*c^2*d + 3*(3*a
^2*b + 2*b^3)*c*d^2 + (a^3 + 6*a*b^2)*d^3)*cos(f*x + e)^2)*sin(f*x + e)), x
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))**(5/2)/(c+d*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(5/2),x, algorithm="gia
c")
```

```
[Out] integrate(1/((b*sin(f*x + e) + a)^(5/2)*(d*sin(f*x + e) + c)^(5/2)), x)
```

3.801 $\int (a + b\sin(e + fx))^m(c + d\sin(e + fx))^n \, dx$

Optimal. Leaf size=27

$$\text{Unintegrable}\left((a + b\sin(e + fx))^m(c + d\sin(e + fx))^n, x\right)$$

```
[Out] Unintegrable[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x]
```

Rubi [A] time = 0.0497791, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (a + b\sin(e + fx))^m(c + d\sin(e + fx))^n \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] Defer[Int][(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x]
```

Rubi steps

$$\int (a + b\sin(e + fx))^m(c + d\sin(e + fx))^n \, dx = \int (a + b\sin(e + fx))^m(c + d\sin(e + fx))^n \, dx$$

Mathematica [A] time = 3.00906, size = 0, normalized size = 0.

$$\int (a + b\sin(e + fx))^m(c + d\sin(e + fx))^n \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] Integrate[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x]
```

Maple [A] time = 1.065, size = 0, normalized size = 0.

$$\int \left(a + b\sin\left(fx + e\right)\right)^m \left(c + d\sin\left(fx + e\right)\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x)
```

```
[Out] int((a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx + e\right) + a\right)^m \left(d\sin\left(fx + e\right) + c\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^n, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\sin\left(fx+e\right)+a\right)^{m}\left(d\sin\left(fx+e\right)+c\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral((b*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**m*(c+d*sin(f*x+e))**n,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)+a\right)^{m}\left(d\sin\left(fx+e\right)+c\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^n, x)
```

3.802 $\int (a+b\sin(e+fx))^m (c+d\sin(e+fx))^2\,dx$

Optimal. Leaf size=311

$$-\frac{\sqrt{2}\cos(e+fx)\left(ad(ad-2bc(m+2))+b^2\left(c^2(m+2)+d^2(m+1)\right)\right)(a+b\sin(e+fx))^m\left(\frac{a+b\sin(e+fx)}{a+b}\right)^{-m}F_1\left(\frac{1}{2};\frac{1}{2},-m\right.}{b^2f(m+2)\sqrt{\sin(e+fx)+1}}$$

```
[Out] -((d^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^(1 + m))/(b*f*(2 + m))) + (Sqrt[2]
*(a + b)*d*(a*d - 2*b*c*(2 + m))*AppellF1[1/2, 1/2, -1 - m, 3/2, (1 - Sin[e
 + f*x])/2, (b*(1 - Sin[e + f*x]))/(a + b)]*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m)/(b^2*f*(2 + m)*Sqrt[1 + Sin[e + f*x]]*((a + b*Sin[e + f*x])/(a + b))^
m) - (Sqrt[2]*(a*d*(a*d - 2*b*c*(2 + m)) + b^2*(d^2*(1 + m) + c^2*(2 + m)))
*AppellF1[1/2, 1/2, -m, 3/2, (1 - Sin[e + f*x])/2, (b*(1 - Sin[e + f*x]))/(
a + b)]*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(b^2*f*(2 + m)*Sqrt[1 + Sin[e
+ f*x]]*((a + b*Sin[e + f*x])/(a + b))^m)
```

Rubi [A] time = 0.439, antiderivative size = 311, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2791, 2756, 2665, 139, 138}

$$-\frac{\sqrt{2}\cos(e+fx)\left(ad(ad-2bc(m+2))+b^2\left(c^2(m+2)+d^2(m+1)\right)\right)(a+b\sin(e+fx))^m\left(\frac{a+b\sin(e+fx)}{a+b}\right)^{-m}F_1\left(\frac{1}{2};\frac{1}{2},-m\right.}{b^2f(m+2)\sqrt{\sin(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^2,x]
```

```
[Out] -((d^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^(1 + m))/(b*f*(2 + m))) + (Sqrt[2]
*(a + b)*d*(a*d - 2*b*c*(2 + m))*AppellF1[1/2, 1/2, -1 - m, 3/2, (1 - Sin[e
 + f*x])/2, (b*(1 - Sin[e + f*x]))/(a + b)]*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m)/(b^2*f*(2 + m)*Sqrt[1 + Sin[e + f*x]]*((a + b*Sin[e + f*x])/(a + b))^
m) - (Sqrt[2]*(a*d*(a*d - 2*b*c*(2 + m)) + b^2*(d^2*(1 + m) + c^2*(2 + m)))
*AppellF1[1/2, 1/2, -m, 3/2, (1 - Sin[e + f*x])/2, (b*(1 - Sin[e + f*x]))/(
a + b)]*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(b^2*f*(2 + m)*Sqrt[1 + Sin[e
+ f*x]]*((a + b*Sin[e + f*x])/(a + b))^m)
```

Rule 2791

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> -Simp[(d^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*
Simp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x],
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a
^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 2756

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[(b*c - a*d)/b, Int[(a + b*Sin[e + f*x])^m,
x], x] + Dist[d/b, Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2665

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)
^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*n]
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int (a+b\sin(e+fx))^m(c+d\sin(e+fx))^2\,dx = -\frac{d^2\cos(e+fx)(a+b\sin(e+fx))^{1+m}}{bf(2+m)} + \frac{\int (a+b\sin(e+fx))^m \left(b\left(d^2(1+\right.\right.}{}$$

$$= -\frac{d^2\cos(e+fx)(a+b\sin(e+fx))^{1+m}}{bf(2+m)} - \frac{(d(ad-2bc(2+m)))\int (a+b\sin}{b^2(2+m)}$$

$$= -\frac{d^2\cos(e+fx)(a+b\sin(e+fx))^{1+m}}{bf(2+m)} - \frac{(d(ad-2bc(2+m))\cos(e+fx)}{b^2f(2+m)\sqrt{1-\sin}}$$

$$= -\frac{d^2\cos(e+fx)(a+b\sin(e+fx))^{1+m}}{bf(2+m)} + \frac{\left((-a-b)d(ad-2bc(2+m))\cos\right.}{}$$

$$= -\frac{d^2\cos(e+fx)(a+b\sin(e+fx))^{1+m}}{bf(2+m)} + \frac{\sqrt{2}(a+b)d(ad-2bc(2+m))F_1}{}$$

Mathematica [F] time = 17.9353, size = 0, normalized size = 0.

$$\int (a+b\sin(e+fx))^m(c+d\sin(e+fx))^2\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^2,x]
```

```
[Out] Integrate[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^2, x]
```

Maple [F] time = 0.369, size = 0, normalized size = 0.

$$\int \left(a+b\sin\left(fx+e\right)\right)^{m}\left(c+d\sin\left(fx+e\right)\right)^{2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^2,x)
```

```
[Out] int((a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\sin\left(fx+e\right)+c\right)^{2}\left(b\sin\left(fx+e\right)+a\right)^{m}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate((d*sin(f*x + e) + c)^2*(b*sin(f*x + e) + a)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\left(d^{2}\cos\left(fx+e\right)^{2}-2\,cd\sin\left(fx+e\right)-c^{2}-d^{2}\right)\left(b\sin\left(fx+e\right)+a\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral(-(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)*(b*sin(f*x
+ e) + a)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**m*(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\sin\left(fx+e\right)+c\right)^{2}\left(b\sin\left(fx+e\right)+a\right)^{m}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*sin(f*x + e) + c)^2*(b*sin(f*x + e) + a)^m, x)
```

3.803 $\int (a+b\sin(e+fx))^m(c+d\sin(e+fx))\,dx$

Optimal. Leaf size=229

$$-\frac{\sqrt{2}(bc-ad)\cos(e+fx)(a+b\sin(e+fx))^m\left(\frac{a+b\sin(e+fx)}{a+b}\right)^{-m}F_1\left(\frac{1}{2};\frac{1}{2},-m;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{b(1-\sin(e+fx))}{a+b}\right)}{bf\sqrt{\sin(e+fx)+1}}-\frac{\sqrt{2}d(a$$

```
[Out] -((Sqrt[2]*(a + b)*d*AppellF1[1/2, 1/2, -1 - m, 3/2, (1 - Sin[e + f*x])/2,
(b*(1 - Sin[e + f*x]))/(a + b)]*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(b*f*S
qrt[1 + Sin[e + f*x]]*((a + b*Sin[e + f*x])/(a + b))^m)) - (Sqrt[2]*(b*c -
a*d)*AppellF1[1/2, 1/2, -m, 3/2, (1 - Sin[e + f*x])/2, (b*(1 - Sin[e + f*x]
))/(a + b)]*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(b*f*Sqrt[1 + Sin[e + f*x]
]*((a + b*Sin[e + f*x])/(a + b))^m)
```

Rubi [A] time = 0.203456, antiderivative size = 229, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2756, 2665, 139, 138}

$$-\frac{\sqrt{2}(bc-ad)\cos(e+fx)(a+b\sin(e+fx))^m\left(\frac{a+b\sin(e+fx)}{a+b}\right)^{-m}F_1\left(\frac{1}{2};\frac{1}{2},-m;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{b(1-\sin(e+fx))}{a+b}\right)}{bf\sqrt{\sin(e+fx)+1}}-\frac{\sqrt{2}d(a$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x]),x]
```


```
[Out] -((Sqrt[2]*(a + b)*d*AppellF1[1/2, 1/2, -1 - m, 3/2, (1 - Sin[e + f*x])/2,
(b*(1 - Sin[e + f*x]))/(a + b)]*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(b*f*S
qrt[1 + Sin[e + f*x]]*((a + b*Sin[e + f*x])/(a + b))^m)) - (Sqrt[2]*(b*c -
a*d)*AppellF1[1/2, 1/2, -m, 3/2, (1 - Sin[e + f*x])/2, (b*(1 - Sin[e + f*x]
))/(a + b)]*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(b*f*Sqrt[1 + Sin[e + f*x]
]*((a + b*Sin[e + f*x])/(a + b))^m)
```


Rule 2756

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[(b*c - a*d)/b, Int[(a + b*Sin[e + f*x])^m,
x], x] + Dist[d/b, Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2665

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)
^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*n]
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int (a+b\sin(e+fx))^m(c+d\sin(e+fx))\,dx = \frac{d\int (a+b\sin(e+fx))^{1+m}\,dx}{b}+\frac{(bc-ad)\int (a+b\sin(e+fx))^m\,dx}{b}$$

$$=\frac{(d\cos(e+fx))\,\text{Subst}\left(\int \frac{(a+bx)^{1+m}}{\sqrt{1-x}\sqrt{1+x}}\,dx,x,\sin(e+fx)\right)}{bf\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}+\frac{((bc-ad)\cos\text{(}}{bf}$$

$$=-\frac{\left((-a-b)d\cos(e+fx)(a+b\sin(e+fx))^m\left(-\frac{a+b\sin(e+fx)}{-a-b}\right)^{-m}\right)\text{Subst}}{bf\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+f}}$$

$$=-\frac{\sqrt{2}(a+b)dF_1\left(\frac{1}{2};\frac{1}{2},-1-m;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{b(1-\sin(e+fx))}{a+b}\right)\cos(e}{bf\sqrt{1+\sin(e+fx)}}$$

Mathematica [A] time = 0.57007, size = 200, normalized size = 0.87

$$\frac{\sec(e+fx)\sqrt{-\frac{b(\sin(e+fx)-1)}{a+b}}\sqrt{\frac{b(\sin(e+fx)+1)}{b-a}}(a+b\sin(e+fx))^{m+1}\left((m+2)(bc-ad)F_1\left(m+1;\frac{1}{2},\frac{1}{2};m+2;\frac{a+b\sin(e+fx)}{a-b},\right.\right.}{b^2f(m+1)(m+2)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x]),x]
```

```
[Out] (Sec[e + f*x]*Sqrt[-((b*(-1 + Sin[e + f*x]))/(a + b))]*Sqrt[(b*(1 + Sin[e +
 f*x]))/(-a + b)]*(a + b*Sin[e + f*x])^(1 + m)*((b*c - a*d)*(2 + m)*AppellF
1[1 + m, 1/2, 1/2, 2 + m, (a + b*Sin[e + f*x])/(a - b), (a + b*Sin[e + f*x]
)/(a + b)] + d*(1 + m)*AppellF1[2 + m, 1/2, 1/2, 3 + m, (a + b*Sin[e + f*x]
)/(a - b), (a + b*Sin[e + f*x])/(a + b)]*(a + b*Sin[e + f*x])))/(b^2*f*(1 +
 m)*(2 + m))
```

Maple [F] time = 0.202, size = 0, normalized size = 0.

$$\int \left(a+b\sin\left(fx+e\right)\right)^m\left(c+d\sin\left(fx+e\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^m*(c+d*sin(f*x+e)),x)
```

```
[Out] int((a+b*sin(f*x+e))^m*(c+d*sin(f*x+e)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (d\sin(fx+e)+c)(b\sin(fx+e)+a)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m*(c+d*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((d*sin(f*x + e) + c)*(b*sin(f*x + e) + a)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((d\sin(fx+e)+c)(b\sin(fx+e)+a)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m*(c+d*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral((d*sin(f*x + e) + c)*(b*sin(f*x + e) + a)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**m*(c+d*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (d\sin(fx+e)+c)(b\sin(fx+e)+a)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m*(c+d*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*sin(f*x + e) + c)*(b*sin(f*x + e) + a)^m, x)
```

3.804 $\int (a+b\sin(e+fx))^m\,dx$

Optimal. Leaf size=104

$$-\frac{\sqrt{2}\cos(e+fx)(a+b\sin(e+fx))^m\left(\frac{a+b\sin(e+fx)}{a+b}\right)^{-m}F_1\left(\frac{1}{2};\frac{1}{2},-m;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{b(1-\sin(e+fx))}{a+b}\right)}{f\sqrt{\sin(e+fx)+1}}$$

```
[Out] -((Sqrt[2]*AppellF1[1/2, 1/2, -m, 3/2, (1 - Sin[e + f*x])/2, (b*(1 - Sin[e
+ f*x]))/(a + b)]*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f*Sqrt[1 + Sin[e +
f*x]]*((a + b*Sin[e + f*x])/(a + b))^m))
```

Rubi [A] time = 0.0661564, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2665, 139, 138}

$$-\frac{\sqrt{2}\cos(e+fx)(a+b\sin(e+fx))^m\left(\frac{a+b\sin(e+fx)}{a+b}\right)^{-m}F_1\left(\frac{1}{2};\frac{1}{2},-m;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{b(1-\sin(e+fx))}{a+b}\right)}{f\sqrt{\sin(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^m,x]
```

```
[Out] -((Sqrt[2]*AppellF1[1/2, 1/2, -m, 3/2, (1 - Sin[e + f*x])/2, (b*(1 - Sin[e
+ f*x]))/(a + b)]*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f*Sqrt[1 + Sin[e +
f*x]]*((a + b*Sin[e + f*x])/(a + b))^m))
```

Rule 2665

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)
^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*n]
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int (a+b\sin(e+fx))^m\,dx = \frac{\cos(e+fx)\,\text{Subst}\left(\int \frac{(a+bx)^m}{\sqrt{1-x}\sqrt{1+x}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}$$
$$= \frac{\left(\cos(e+fx)(a+b\sin(e+fx))^m\left(-\frac{a+b\sin(e+fx)}{-a-b}\right)^{-m}\right)\text{Subst}\left(\int \frac{\left(-\frac{a}{-a-b}-\frac{bx}{-a-b}\right)^m}{\sqrt{1-x}\sqrt{1+x}}\,dx, x, \sin(e+fx\right.}{f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}$$
$$= -\frac{\sqrt{2}F_1\left(\frac{1}{2}; \frac{1}{2}, -m; \frac{3}{2}; \frac{1}{2}(1-\sin(e+fx)), \frac{b(1-\sin(e+fx))}{a+b}\right)\cos(e+fx)(a+b\sin(e+fx))^m\left(\frac{a+b\sin}{a+\right.}{f\sqrt{1+\sin(e+fx)}}$$

Mathematica [A] time = 0.253665, size = 120, normalized size = 1.15

$$\frac{\sec(e+fx)\sqrt{-\frac{b(\sin(e+fx)-1)}{a+b}}\sqrt{\frac{b(\sin(e+fx)+1)}{b-a}}(a+b\sin(e+fx))^{m+1}F_1\left(m+1; \frac{1}{2}, \frac{1}{2}; m+2; \frac{a+b\sin(e+fx)}{a-b}, \frac{a+b\sin(e+fx)}{a+b}\right)}{bf(m+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sin[e + f*x])^m,x]
```

```
[Out] (AppellF1[1 + m, 1/2, 1/2, 2 + m, (a + b*Sin[e + f*x])/(a - b), (a + b*Sin[
e + f*x])/(a + b)]*Sec[e + f*x]*Sqrt[-((b*(-1 + Sin[e + f*x]))/(a + b))]*Sq
rt[(b*(1 + Sin[e + f*x]))/(-a + b)]*(a + b*Sin[e + f*x])^(1 + m))/(b*f*(1 +
 m))
```

Maple [F] time = 0.393, size = 0, normalized size = 0.

$$\int (a+b\sin(fx+e))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^m,x)
```

```
[Out] int((a+b*sin(f*x+e))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sin(fx+e)+a)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b\sin(fx+e)+a)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((b*sin(f*x + e) + a)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)+a\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)^m, x)
```

3.805 $\int \frac{(a+b\sin(e+fx))^m}{c+d\sin(e+fx)}\,dx$

Optimal. Leaf size=27

$$\text{Unintegrable}\left(\frac{(a+b\sin(e+fx))^m}{c+d\sin(e+fx)},x\right)$$

```
[Out] Unintegrable[(a + b*Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x]
```

Rubi [A] time = 0.057435, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(a+b\sin(e+fx))^m}{c+d\sin(e+fx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sin[e + f*x])^m/(c + d*Sin[e + f*x]),x]

[Out] Defer[Int][(a + b*Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x]
```

Rubi steps

$$\int \frac{(a+b\sin(e+fx))^m}{c+d\sin(e+fx)}\,dx=\int \frac{(a+b\sin(e+fx))^m}{c+d\sin(e+fx)}\,dx$$

Mathematica [A] time = 2.58644, size = 0, normalized size = 0.

$$\int \frac{(a+b\sin(e+fx))^m}{c+d\sin(e+fx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sin[e + f*x])^m/(c + d*Sin[e + f*x]),x]

[Out] Integrate[(a + b*Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x]
```

Maple [A] time = 0.913, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\sin\left(fx+e\right)\right)^m}{c+d\sin\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^m/(c+d*sin(f*x+e)),x)

[Out] int((a+b*sin(f*x+e))^m/(c+d*sin(f*x+e)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\sin\left(fx+e\right)+a\right)^{m}}{d\sin\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m/(c+d*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b\sin\left(fx+e\right)+a\right)^{m}}{d\sin\left(fx+e\right)+c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m/(c+d*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral((b*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**m/(c+d*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\sin\left(fx+e\right)+a\right)^{m}}{d\sin\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m/(c+d*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c), x)
```

3.806 $\int \frac{(a+b\sin(e+fx))^m}{(c+d\sin(e+fx))^2}\,dx$

Optimal. Leaf size=27

$$\text{Unintegrable}\left(\frac{(a+b\sin(e+fx))^m}{(c+d\sin(e+fx))^2},x\right)$$

```
[Out] Unintegrable[(a + b*Sin[e + f*x])^m/(c + d*Sin[e + f*x])^2, x]
```

Rubi [A] time = 0.0563576, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(a+b\sin(e+fx))^m}{(c+d\sin(e+fx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sin[e + f*x])^m/(c + d*Sin[e + f*x])^2,x]
```

```
[Out] Defer[Int][(a + b*Sin[e + f*x])^m/(c + d*Sin[e + f*x])^2, x]
```

Rubi steps

$$\int \frac{(a+b\sin(e+fx))^m}{(c+d\sin(e+fx))^2}\,dx = \int \frac{(a+b\sin(e+fx))^m}{(c+d\sin(e+fx))^2}\,dx$$

Mathematica [A] time = 4.58633, size = 0, normalized size = 0.

$$\int \frac{(a+b\sin(e+fx))^m}{(c+d\sin(e+fx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sin[e + f*x])^m/(c + d*Sin[e + f*x])^2,x]
```

```
[Out] Integrate[(a + b*Sin[e + f*x])^m/(c + d*Sin[e + f*x])^2, x]
```

Maple [A] time = 0.632, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\sin\left(fx+e\right)\right)^m}{\left(c+d\sin\left(fx+e\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^m/(c+d*sin(f*x+e))^2,x)
```

```
[Out] int((a+b*sin(f*x+e))^m/(c+d*sin(f*x+e))^2,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m/(c+d*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(b\sin\left(fx+e\right)+a\right)^{m}}{d^2\cos\left(fx+e\right)^2-2\,cd\sin\left(fx+e\right)-c^2-d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m/(c+d*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral(-(b*sin(f*x + e) + a)^m/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) -
 c^2 - d^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**m/(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\sin\left(fx+e\right)+a\right)^{m}}{\left(d\sin\left(fx+e\right)+c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m/(c+d*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c)^2, x)
```

3.807 $\int \frac{(a+b\sin(e+fx))^m}{(c+d\sin(e+fx))^3}\,dx$

Optimal. Leaf size=27

$$\text{Unintegrable}\left(\frac{(a+b\sin(e+fx))^m}{(c+d\sin(e+fx))^3},x\right)$$

```
[Out] Unintegrable[(a + b*Sin[e + f*x])^m/(c + d*Sin[e + f*x])^3, x]
```

Rubi [A] time = 0.0549524, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(a+b\sin(e+fx))^m}{(c+d\sin(e+fx))^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sin[e + f*x])^m/(c + d*Sin[e + f*x])^3,x]
```

```
[Out] Defer[Int][(a + b*Sin[e + f*x])^m/(c + d*Sin[e + f*x])^3, x]
```

Rubi steps

$$\int \frac{(a+b\sin(e+fx))^m}{(c+d\sin(e+fx))^3}\,dx = \int \frac{(a+b\sin(e+fx))^m}{(c+d\sin(e+fx))^3}\,dx$$

Mathematica [A] time = 13.1328, size = 0, normalized size = 0.

$$\int \frac{(a+b\sin(e+fx))^m}{(c+d\sin(e+fx))^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sin[e + f*x])^m/(c + d*Sin[e + f*x])^3,x]
```

```
[Out] Integrate[(a + b*Sin[e + f*x])^m/(c + d*Sin[e + f*x])^3, x]
```

Maple [A] time = 0.967, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\sin\left(fx+e\right)\right)^m}{\left(c+d\sin\left(fx+e\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^m/(c+d*sin(f*x+e))^3,x)
```

```
[Out] int((a+b*sin(f*x+e))^m/(c+d*sin(f*x+e))^3,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m/(c+d*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(b\sin\left(fx+e\right)+a\right)^m}{3\,cd^2\cos\left(fx+e\right)^2-c^3-3\,cd^2+\left(d^3\cos\left(fx+e\right)^2-3\,c^2d-d^3\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m/(c+d*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] integral(-(b*sin(f*x + e) + a)^m/(3*c*d^2*cos(f*x + e)^2 - c^3 - 3*c*d^2 +
(d^3*cos(f*x + e)^2 - 3*c^2*d - d^3)*sin(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**m/(c+d*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\sin\left(fx+e\right)+a\right)^m}{\left(d\sin\left(fx+e\right)+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m/(c+d*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c)^3, x)
```

3.808 $\int (a+b\sin(e+fx))^m(c+d\sin(e+fx))^{5/2}\,dx$

Optimal. Leaf size=29

$$\text{Unintegrable}\left((c+d\sin(e+fx))^{5/2}(a+b\sin(e+fx))^m,x\right)$$

```
[Out] Unintegrable[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(5/2), x]
```

Rubi [A] time = 0.0776108, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (a+b\sin(e+fx))^m(c+d\sin(e+fx))^{5/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] Defer[Int][(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(5/2), x]
```

Rubi steps

$$\int (a+b\sin(e+fx))^m(c+d\sin(e+fx))^{5/2}\,dx=\int (a+b\sin(e+fx))^m(c+d\sin(e+fx))^{5/2}\,dx$$

Mathematica [A] time = 34.107, size = 0, normalized size = 0.

$$\int (a+b\sin(e+fx))^m(c+d\sin(e+fx))^{5/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] Integrate[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(5/2), x]
```

Maple [A] time = 0.171, size = 0, normalized size = 0.

$$\int \left(a+b\sin\left(fx+e\right)\right)^m\left(c+d\sin\left(fx+e\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^(5/2),x)
```

```
[Out] int((a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^(5/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}\left(b\sin\left(fx+e\right)+a\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(5/2)*(b*sin(f*x + e) + a)^m, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(d^2\cos\left(fx+e\right)^2-2\,cd\sin\left(fx+e\right)-c^2-d^2\right)\sqrt{d\sin\left(fx+e\right)+c}\left(b\sin\left(fx+e\right)+a\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)*sqrt(d*sin(
f*x + e) + c)*(b*sin(f*x + e) + a)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**m*(c+d*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\text{sage}_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] sage2
```

3.809 $\int (a+b\sin(e+fx))^m(c+d\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=29

$$\text{Unintegrable}\left((c+d\sin(e+fx))^{3/2}(a+b\sin(e+fx))^m,x\right)$$

```
[Out] Unintegrable[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(3/2), x]
```

Rubi [A] time = 0.0755849, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (a+b\sin(e+fx))^m(c+d\sin(e+fx))^{3/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] Defer[Int][(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(3/2), x]
```

Rubi steps

$$\int (a+b\sin(e+fx))^m(c+d\sin(e+fx))^{3/2}\,dx=\int (a+b\sin(e+fx))^m(c+d\sin(e+fx))^{3/2}\,dx$$

Mathematica [A] time = 13.6366, size = 0, normalized size = 0.

$$\int (a+b\sin(e+fx))^m(c+d\sin(e+fx))^{3/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] Integrate[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(3/2), x]
```

Maple [A] time = 0.163, size = 0, normalized size = 0.

$$\int \left(a+b\sin\left(fx+e\right)\right)^m\left(c+d\sin\left(fx+e\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^(3/2),x)
```

```
[Out] int((a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^(3/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}\left(b\sin\left(fx+e\right)+a\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(3/2)*(b*sin(f*x + e) + a)^m, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}\left(b\sin\left(fx+e\right)+a\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((d*sin(f*x + e) + c)^(3/2)*(b*sin(f*x + e) + a)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**m*(c+d*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: AttributeError
```

3.810 $\int (a+b\sin(e+fx))^m\sqrt{c+d\sin(e+fx)}\,dx$

Optimal. Leaf size=29

$$\text{Unintegrable}\left(\sqrt{c+d\sin(e+fx)}(a+b\sin(e+fx))^m,x\right)$$

```
[Out] Unintegrable[(a + b*Sin[e + f*x])^m*Sqrt[c + d*Sin[e + f*x]], x]
```

Rubi [A] time = 0.0690777, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (a+b\sin(e+fx))^m\sqrt{c+d\sin(e+fx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sin[e + f*x])^m*Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] Defer[Int][(a + b*Sin[e + f*x])^m*Sqrt[c + d*Sin[e + f*x]], x]
```

Rubi steps

$$\int (a+b\sin(e+fx))^m\sqrt{c+d\sin(e+fx)}\,dx=\int (a+b\sin(e+fx))^m\sqrt{c+d\sin(e+fx)}\,dx$$

Mathematica [A] time = 0.49436, size = 0, normalized size = 0.

$$\int (a+b\sin(e+fx))^m\sqrt{c+d\sin(e+fx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sin[e + f*x])^m*Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] Integrate[(a + b*Sin[e + f*x])^m*Sqrt[c + d*Sin[e + f*x]], x]
```

Maple [A] time = 0.157, size = 0, normalized size = 0.

$$\int \left(a+b\sin\left(fx+e\right)\right)^m\sqrt{c+d\sin\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^(1/2),x)
```

```
[Out] int((a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d\sin\left(fx+e\right)+c}\left(b\sin\left(fx+e\right)+a\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*sin(f*x + e) + c)*(b*sin(f*x + e) + a)^m, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{d\sin\left(fx+e\right)+c}\left(b\sin\left(fx+e\right)+a\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*sin(f*x + e) + c)*(b*sin(f*x + e) + a)^m, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \left(a+b\sin\left(e+fx\right)\right)^m \sqrt{c+d\sin\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**m*(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((a + b*sin(e + f*x))**m*sqrt(c + d*sin(e + f*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d\sin\left(fx+e\right)+c}\left(b\sin\left(fx+e\right)+a\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*sin(f*x + e) + c)*(b*sin(f*x + e) + a)^m, x)
```

3.811 $\int \frac{(a+b\sin(e+fx))^m}{\sqrt{c+d\sin(e+fx)}}\,dx$

Optimal. Leaf size=29

$$\text{Unintegrable}\left(\frac{(a+b\sin(e+fx))^m}{\sqrt{c+d\sin(e+fx)}},x\right)$$

```
[Out] Unintegrable[(a + b*Sin[e + f*x])^m/Sqrt[c + d*Sin[e + f*x]], x]
```

Rubi [A] time = 0.0694018, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(a+b\sin(e+fx))^m}{\sqrt{c+d\sin(e+fx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sin[e + f*x])^m/Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] Defer[Int][(a + b*Sin[e + f*x])^m/Sqrt[c + d*Sin[e + f*x]], x]
```

Rubi steps

$$\int \frac{(a+b\sin(e+fx))^m}{\sqrt{c+d\sin(e+fx)}}\,dx = \int \frac{(a+b\sin(e+fx))^m}{\sqrt{c+d\sin(e+fx)}}\,dx$$

Mathematica [A] time = 3.15539, size = 0, normalized size = 0.

$$\int \frac{(a+b\sin(e+fx))^m}{\sqrt{c+d\sin(e+fx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sin[e + f*x])^m/Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] Integrate[(a + b*Sin[e + f*x])^m/Sqrt[c + d*Sin[e + f*x]], x]
```

Maple [A] time = 0.156, size = 0, normalized size = 0.

$$\int \left(a+b\sin\left(fx+e\right)\right)^m \frac{1}{\sqrt{c+d\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^m/(c+d*sin(f*x+e))^(1/2),x)
```

```
[Out] int((a+b*sin(f*x+e))^m/(c+d*sin(f*x+e))^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b \sin\left(fx+e\right)+a\right)^{m}}{\sqrt{d \sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m/(c+d*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)^m/sqrt(d*sin(f*x + e) + c), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b \sin\left(fx+e\right)+a\right)^{m}}{\sqrt{d \sin\left(fx+e\right)+c}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m/(c+d*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*sin(f*x + e) + a)^m/sqrt(d*sin(f*x + e) + c), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+b \sin\left(e+fx\right)\right)^{m}}{\sqrt{c+d \sin\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**m/(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((a + b*sin(e + f*x))**m/sqrt(c + d*sin(e + f*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b \sin\left(fx+e\right)+a\right)^{m}}{\sqrt{d \sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m/(c+d*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)^m/sqrt(d*sin(f*x + e) + c), x)
```

3.812 $\int \frac{(a+b\sin(e+fx))^m}{(c+d\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=29

$$\text{Unintegrable}\left(\frac{(a+b\sin(e+fx))^m}{(c+d\sin(e+fx))^{3/2}},x\right)$$

```
[Out] Unintegrable[(a + b*Sin[e + f*x])^m/(c + d*Sin[e + f*x])^(3/2), x]
```

Rubi [A] time = 0.0762709, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(a+b\sin(e+fx))^m}{(c+d\sin(e+fx))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sin[e + f*x])^m/(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] Defer[Int][(a + b*Sin[e + f*x])^m/(c + d*Sin[e + f*x])^(3/2), x]
```

Rubi steps

$$\int \frac{(a+b\sin(e+fx))^m}{(c+d\sin(e+fx))^{3/2}}\,dx=\int \frac{(a+b\sin(e+fx))^m}{(c+d\sin(e+fx))^{3/2}}\,dx$$

Mathematica [A] time = 4.57487, size = 0, normalized size = 0.

$$\int \frac{(a+b\sin(e+fx))^m}{(c+d\sin(e+fx))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sin[e + f*x])^m/(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] Integrate[(a + b*Sin[e + f*x])^m/(c + d*Sin[e + f*x])^(3/2), x]
```

Maple [A] time = 0.155, size = 0, normalized size = 0.

$$\int \left(a+b\sin\left(fx+e\right)\right)^{m}\left(c+d\sin\left(fx+e\right)\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^m/(c+d*sin(f*x+e))^(3/2),x)
```

```
[Out] int((a+b*sin(f*x+e))^m/(c+d*sin(f*x+e))^(3/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\sin\left(fx+e\right)+a\right)^m}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m/(c+d*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c)^(3/2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{d\sin\left(fx+e\right)+c}\left(b\sin\left(fx+e\right)+a\right)^m}{d^2\cos\left(fx+e\right)^2-2\,cd\sin\left(fx+e\right)-c^2-d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m/(c+d*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(d*sin(f*x + e) + c)*(b*sin(f*x + e) + a)^m/(d^2*cos(f*x + e)
^2 - 2*c*d*sin(f*x + e) - c^2 - d^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+b\sin\left(e+fx\right)\right)^m}{\left(c+d\sin\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**m/(c+d*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral((a + b*sin(e + f*x))**m/(c + d*sin(e + f*x))**(3/2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\sin\left(fx+e\right)+a\right)^m}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m/(c+d*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c)^(3/2), x)
```

3.813 $\int \frac{(a+b\sin(e+fx))^m}{(c+d\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=29

$$\text{Unintegrable}\left(\frac{(a+b\sin(e+fx))^m}{(c+d\sin(e+fx))^{5/2}},x\right)$$

```
[Out] Unintegrable[(a + b*Sin[e + f*x])^m/(c + d*Sin[e + f*x])^(5/2), x]
```

Rubi [A] time = 0.0777509, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(a+b\sin(e+fx))^m}{(c+d\sin(e+fx))^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sin[e + f*x])^m/(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] Defer[Int][(a + b*Sin[e + f*x])^m/(c + d*Sin[e + f*x])^(5/2), x]
```

Rubi steps

$$\int \frac{(a+b\sin(e+fx))^m}{(c+d\sin(e+fx))^{5/2}}\,dx = \int \frac{(a+b\sin(e+fx))^m}{(c+d\sin(e+fx))^{5/2}}\,dx$$

Mathematica [A] time = 9.8531, size = 0, normalized size = 0.

$$\int \frac{(a+b\sin(e+fx))^m}{(c+d\sin(e+fx))^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sin[e + f*x])^m/(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] Integrate[(a + b*Sin[e + f*x])^m/(c + d*Sin[e + f*x])^(5/2), x]
```

Maple [A] time = 0.155, size = 0, normalized size = 0.

$$\int \left(a+b\sin\left(fx+e\right)\right)^m \left(c+d\sin\left(fx+e\right)\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^m/(c+d*sin(f*x+e))^(5/2),x)
```

```
[Out] int((a+b*sin(f*x+e))^m/(c+d*sin(f*x+e))^(5/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\sin\left(fx+e\right)+a\right)^{m}}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m/(c+d*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c)^(5/2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{d\sin\left(fx+e\right)+c}\left(b\sin\left(fx+e\right)+a\right)^{m}}{3\,cd^{2}\cos\left(fx+e\right)^{2}-c^{3}-3\,cd^{2}+\left(d^{3}\cos\left(fx+e\right)^{2}-3\,c^{2}d-d^{3}\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m/(c+d*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(d*sin(f*x + e) + c)*(b*sin(f*x + e) + a)^m/(3*c*d^2*cos(f*x
+ e)^2 - c^3 - 3*c*d^2 + (d^3*cos(f*x + e)^2 - 3*c^2*d - d^3)*sin(f*x + e))
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**m/(c+d*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\sin\left(fx+e\right)+a\right)^{m}}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m/(c+d*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c)^(5/2), x)
```

3.814 $\int (d\csc(e+fx))^n (a+a\sin(e+fx))^3\,dx$

Optimal. Leaf size=272

$$\frac{a^3d^4(11-4n)\cos(e+fx)(d\csc(e+fx))^{n-4}\,{}_2F_1\left(\frac{1}{2},\frac{4-n}{2};\frac{6-n}{2};\sin^2(e+fx)\right)}{f(2-n)(4-n)\sqrt{\cos^2(e+fx)}}+\frac{a^3d^3(5-4n)\cos(e+fx)(d\csc(e+fx))^{n-3}\,{}_2}{f(1-n)(3-n)\sqrt{\cos^2(}}$$

```
[Out] (a^3*d^3*(1 - 2*n)*Cot[e + f*x]*(d*Csc[e + f*x])^(-3 + n))/(f*(1 - n)*(2 -
n)) + (d^3*Cot[e + f*x]*(d*Csc[e + f*x])^(-3 + n)*(a^3 + a^3*Csc[e + f*x]))
/(f*(1 - n)) + (a^3*d^3*(5 - 4*n)*Cos[e + f*x]*(d*Csc[e + f*x])^(-3 + n)*Hy
pergeometric2F1[1/2, (3 - n)/2, (5 - n)/2, Sin[e + f*x]^2])/(f*(1 - n)*(3 -
 n)*Sqrt[Cos[e + f*x]^2]) + (a^3*d^4*(11 - 4*n)*Cos[e + f*x]*(d*Csc[e + f*x
])^(-4 + n)*Hypergeometric2F1[1/2, (4 - n)/2, (6 - n)/2, Sin[e + f*x]^2])/(
f*(2 - n)*(4 - n)*Sqrt[Cos[e + f*x]^2])
```

Rubi [A] time = 0.458791, antiderivative size = 272, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3238, 3814, 3997, 3787, 3772, 2643}

$$\frac{a^3d^4(11-4n)\cos(e+fx)(d\csc(e+fx))^{n-4}\,{}_2F_1\left(\frac{1}{2},\frac{4-n}{2};\frac{6-n}{2};\sin^2(e+fx)\right)}{f(2-n)(4-n)\sqrt{\cos^2(e+fx)}}+\frac{a^3d^3(5-4n)\cos(e+fx)(d\csc(e+fx))^{n-3}\,{}_2}{f(1-n)(3-n)\sqrt{\cos^2(}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Csc[e + f*x])^n*(a + a*Sin[e + f*x])^3,x]
```


```
[Out] (a^3*d^3*(1 - 2*n)*Cot[e + f*x]*(d*Csc[e + f*x])^(-3 + n))/(f*(1 - n)*(2 -
n)) + (d^3*Cot[e + f*x]*(d*Csc[e + f*x])^(-3 + n)*(a^3 + a^3*Csc[e + f*x]))
/(f*(1 - n)) + (a^3*d^3*(5 - 4*n)*Cos[e + f*x]*(d*Csc[e + f*x])^(-3 + n)*Hy
pergeometric2F1[1/2, (3 - n)/2, (5 - n)/2, Sin[e + f*x]^2])/(f*(1 - n)*(3 -
 n)*Sqrt[Cos[e + f*x]^2]) + (a^3*d^4*(11 - 4*n)*Cos[e + f*x]*(d*Csc[e + f*x
])^(-4 + n)*Hypergeometric2F1[1/2, (4 - n)/2, (6 - n)/2, Sin[e + f*x]^2])/(
f*(2 - n)*(4 - n)*Sqrt[Cos[e + f*x]^2])
```


Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3814

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[b/(m + n - 1), Int[(a + b*
Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n -
4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2,
0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
```

```
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int (d\csc(e+fx))^n(a+a\sin(e+fx))^3\,dx = d^3\int (d\csc(e+fx))^{-3+n}(a+a\csc(e+fx))^3\,dx$$

$$= \frac{d^3\cot(e+fx)(d\csc(e+fx))^{-3+n}\left(a^3+a^3\csc(e+fx)\right)}{f(1-n)} - \frac{\left(ad^3\right)\int (d\csc(e$$

$$= \frac{a^3d^3(1-2n)\cot(e+fx)(d\csc(e+fx))^{-3+n}}{f(1-n)(2-n)} + \frac{d^3\cot(e+fx)(d\csc(e+f}{f(1}$$

$$= \frac{a^3d^3(1-2n)\cot(e+fx)(d\csc(e+fx))^{-3+n}}{f(1-n)(2-n)} + \frac{d^3\cot(e+fx)(d\csc(e+f}{f(1}$$

$$= \frac{a^3d^3(1-2n)\cot(e+fx)(d\csc(e+fx))^{-3+n}}{f(1-n)(2-n)} + \frac{d^3\cot(e+fx)(d\csc(e+f}{f(1}$$

$$= \frac{a^3d^3(1-2n)\cot(e+fx)(d\csc(e+fx))^{-3+n}}{f(1-n)(2-n)} + \frac{d^3\cot(e+fx)(d\csc(e+f}{f(1}$$

Mathematica [A] time = 11.5614, size = 493, normalized size = 1.81

$$2^{1-n}\tan\left(\frac{1}{2}(e+fx)\right)(a\sin(e+fx)+a)^3\left(\tan^2\left(\frac{1}{2}(e+fx)\right)+1\right)^{-n}\csc^{-n}(e+fx)(d\csc(e+fx))^n\left(\frac{\tan^6\left(\frac{1}{2}(e+fx)\right){}_2F_1\left(4-n,\right.}{7}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Csc[e + f*x])^n*(a + a*Sin[e + f*x])^3,x]
```

```
[Out] (2^(1 - n)*(d*Csc[e + f*x])^n*(a + a*Sin[e + f*x])^3*Tan[(e + f*x)/2]*(Cot[
(e + f*x)/2] + Tan[(e + f*x)/2])^n*(Hypergeometric2F1[4 - n, 1/2 - n/2, 3/2
 - n/2, -Tan[(e + f*x)/2]^2]/(1 - n) - (6*Hypergeometric2F1[4 - n, 1 - n/2,
```

```
 2 - n/2, -Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2])/(-2 + n) - (15*Hypergeomet
ric2F1[(3 - n)/2, 4 - n, (5 - n)/2, -Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]^2
)/(-3 + n) - (20*Hypergeometric2F1[4 - n, 2 - n/2, 3 - n/2, -Tan[(e + f*x)/
2]^2]*Tan[(e + f*x)/2]^3)/(-4 + n) - (15*Hypergeometric2F1[4 - n, (5 - n)/2
, (7 - n)/2, -Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]^4)/(-5 + n) - (6*Hyperge
ometric2F1[4 - n, 3 - n/2, 4 - n/2, -Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]^5
)/(-6 + n) + (Hypergeometric2F1[4 - n, 7/2 - n/2, 9/2 - n/2, -Tan[(e + f*x)
/2]^2]*Tan[(e + f*x)/2]^6)/(7 - n)))/(f*Csc[e + f*x]^n*(Cos[(e + f*x)/2] +
Sin[(e + f*x)/2])^6*(1 + Tan[(e + f*x)/2]^2)^n)
```

Maple [F] time = 2.757, size = 0, normalized size = 0.

$$\int \left(d \csc \left(f x+e\right)\right)^{n} \left(a+a \sin \left(f x+e\right)\right)^{3} d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(f*x+e))^n*(a+a*sin(f*x+e))^3,x)
```

```
[Out] int((d*csc(f*x+e))^n*(a+a*sin(f*x+e))^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a \sin \left(f x+e\right)+a\right)^{3} \left(d \csc \left(f x+e\right)\right)^{n} d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^n*(a+a*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^3*(d*csc(f*x + e))^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\left(3 a^{3} \cos \left(f x+e\right)^{2}-4 a^{3}+\left(a^{3} \cos \left(f x+e\right)^{2}-4 a^{3}\right) \sin \left(f x+e\right)\right) \left(d \csc \left(f x+e\right)\right)^{n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^n*(a+a*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] integral(-(3*a^3*cos(f*x + e)^2 - 4*a^3 + (a^3*cos(f*x + e)^2 - 4*a^3)*sin(
f*x + e))*(d*csc(f*x + e))^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^{3} \left(\int \left(d \csc \left(e+f x\right)\right)^{n} d x+\int 3 \left(d \csc \left(e+f x\right)\right)^{n} \sin \left(e+f x\right) d x+\int 3 \left(d \csc \left(e+f x\right)\right)^{n} \sin^{2} \left(e+f x\right) d x+\int \left(d \csc \left(e\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))**n*(a+a*sin(f*x+e))**3,x)
```

```
[Out] a**3*(Integral((d*csc(e + f*x))**n, x) + Integral(3*(d*csc(e + f*x))**n*sin
(e + f*x), x) + Integral(3*(d*csc(e + f*x))**n*sin(e + f*x)**2, x) + Integr
al((d*csc(e + f*x))**n*sin(e + f*x)**3, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^{3}\left(d\csc\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^n*(a+a*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^3*(d*csc(f*x + e))^n, x)
```

3.815 $\int (d\csc(e+fx))^n(a+a\sin(e+fx))^2\,dx$

Optimal. Leaf size=203

$$\frac{a^2d^3(3-2n)\cos(e+fx)(d\csc(e+fx))^{n-3}\,{}_2F_1\left(\frac{1}{2},\frac{3-n}{2};\frac{5-n}{2};\sin^2(e+fx)\right)}{f(1-n)(3-n)\sqrt{\cos^2(e+fx)}}+\frac{2a^2d^2\cos(e+fx)(d\csc(e+fx))^{n-2}\,{}_2F_1\left(\frac{1}{2},\frac{2}{}\right.}{f(2-n)\sqrt{\cos^2(e+fx)}}$$

```
[Out] (a^2*d^2*Cot[e + f*x]*(d*Csc[e + f*x])^(-2 + n))/(f*(1 - n)) + (2*a^2*d^2*C
os[e + f*x]*(d*Csc[e + f*x])^(-2 + n)*Hypergeometric2F1[1/2, (2 - n)/2, (4
- n)/2, Sin[e + f*x]^2])/(f*(2 - n)*Sqrt[Cos[e + f*x]^2]) + (a^2*d^3*(3 - 2
*n)*Cos[e + f*x]*(d*Csc[e + f*x])^(-3 + n)*Hypergeometric2F1[1/2, (3 - n)/2
, (5 - n)/2, Sin[e + f*x]^2])/(f*(1 - n)*(3 - n)*Sqrt[Cos[e + f*x]^2])
```

Rubi [A] time = 0.256929, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3238, 3788, 3772, 2643, 4046}

$$\frac{a^2d^3(3-2n)\cos(e+fx)(d\csc(e+fx))^{n-3}\,{}_2F_1\left(\frac{1}{2},\frac{3-n}{2};\frac{5-n}{2};\sin^2(e+fx)\right)}{f(1-n)(3-n)\sqrt{\cos^2(e+fx)}}+\frac{2a^2d^2\cos(e+fx)(d\csc(e+fx))^{n-2}\,{}_2F_1\left(\frac{1}{2},\frac{2}{}\right.}{f(2-n)\sqrt{\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Csc[e + f*x])^n*(a + a*Sin[e + f*x])^2,x]
```

```
[Out] (a^2*d^2*Cot[e + f*x]*(d*Csc[e + f*x])^(-2 + n))/(f*(1 - n)) + (2*a^2*d^2*C
os[e + f*x]*(d*Csc[e + f*x])^(-2 + n)*Hypergeometric2F1[1/2, (2 - n)/2, (4
- n)/2, Sin[e + f*x]^2])/(f*(2 - n)*Sqrt[Cos[e + f*x]^2]) + (a^2*d^3*(3 - 2
*n)*Cos[e + f*x]*(d*Csc[e + f*x])^(-3 + n)*Hypergeometric2F1[1/2, (3 - n)/2
, (5 - n)/2, Sin[e + f*x]^2])/(f*(1 - n)*(3 - n)*Sqrt[Cos[e + f*x]^2])
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
```

```
&& !IntegerQ[2*n]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Rubi steps

$$\int (d\csc(e+fx))^n(a+a\sin(e+fx))^2\,dx = d^2\int (d\csc(e+fx))^{-2+n}(a+a\csc(e+fx))^2\,dx$$
$$= \left(2a^2d\right)\int (d\csc(e+fx))^{-1+n}\,dx + d^2\int (d\csc(e+fx))^{-2+n}\left(a^2+a^2\csc^2(e$$
$$= \frac{a^2d^2\cot(e+fx)(d\csc(e+fx))^{-2+n}}{f(1-n)} + \frac{\left(a^2d^2(3-2n)\right)\int (d\csc(e+fx))^{-2+}}{1-n}$$
$$= \frac{a^2d^2\cot(e+fx)(d\csc(e+fx))^{-2+n}}{f(1-n)} + \frac{2a^2\cos(e+fx)(d\csc(e+fx))^n\,{}_2F_1}{f(2-n)}$$
$$= \frac{a^2d^2\cot(e+fx)(d\csc(e+fx))^{-2+n}}{f(1-n)} + \frac{2a^2\cos(e+fx)(d\csc(e+fx))^n\,{}_2F_1}{f(2-n)}$$

Mathematica [A] time = 6.11137, size = 342, normalized size = 1.68

$$\frac{2\tan\left(\frac{1}{2}(e+fx)\right)(a\sin(e+fx)+a)^2\sec^2\left(\frac{1}{2}(e+fx)\right)^{-n}(d\csc(e+fx))^n\left(\frac{{}_2F_1\left(3-n,\frac{1}{2}-\frac{n}{2};\frac{3}{2}-\frac{n}{2};-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)}{1-n}+\tan\left(\frac{1}{2}(e+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Csc[e + f*x])^n*(a + a*Sin[e + f*x])^2,x]
```

```
[Out] (2*(d*Csc[e + f*x])^n*(a + a*Sin[e + f*x])^2*Tan[(e + f*x)/2]*(Hypergeometr
ic2F1[3 - n, 1/2 - n/2, 3/2 - n/2, -Tan[(e + f*x)/2]^2]/(1 - n) + Tan[(e +
f*x)/2]*((-4*Hypergeometric2F1[3 - n, 1 - n/2, 2 - n/2, -Tan[(e + f*x)/2]^2
])/(-2 + n) + Tan[(e + f*x)/2]*((-6*Hypergeometric2F1[(3 - n)/2, 3 - n, (5
- n)/2, -Tan[(e + f*x)/2]^2])/(-3 + n) - (4*Hypergeometric2F1[3 - n, 2 - n/
2, 3 - n/2, -Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2])/(-4 + n) + (Hypergeometr
ic2F1[3 - n, 5/2 - n/2, 7/2 - n/2, -Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]^2)
/(5 - n)))))/(f*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)^4)
```

Maple [F] time = 3.602, size = 0, normalized size = 0.

$$\int \left(d\csc\left(fx+e\right)\right)^n\left(a+a\sin\left(fx+e\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(f*x+e))^n*(a+a*sin(f*x+e))^2,x)
```

```
[Out] int((d*csc(f*x+e))^n*(a+a*sin(f*x+e))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^2\left(d\csc\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^n*(a+a*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^2*(d*csc(f*x + e))^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(a^2\cos\left(fx+e\right)^2-2\,a^2\sin\left(fx+e\right)-2\,a^2\right)\left(d\csc\left(fx+e\right)\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^n*(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral(-(a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x + e) - 2*a^2)*(d*csc(f*x + e)
)^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2\left(\int\left(d\csc\left(e+fx\right)\right)^n\,dx+\int 2\left(d\csc\left(e+fx\right)\right)^n\sin\left(e+fx\right)dx+\int\left(d\csc\left(e+fx\right)\right)^n\sin^2\left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))**n*(a+a*sin(f*x+e))**2,x)
```

```
[Out] a**2*(Integral((d*csc(e + f*x))**n, x) + Integral(2*(d*csc(e + f*x))**n*sin
(e + f*x), x) + Integral((d*csc(e + f*x))**n*sin(e + f*x)**2, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^2\left(d\csc\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^n*(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^2*(d*csc(f*x + e))^n, x)
```

3.816 $\int (d\csc(e+fx))^n(a+a\sin(e+fx))\,dx$

Optimal. Leaf size=149

$$\frac{ad^2\cos(e+fx)(d\csc(e+fx))^{n-2}\,{}_2F_1\left(\frac{1}{2},\frac{2-n}{2};\frac{4-n}{2};\sin^2(e+fx)\right)}{f(2-n)\sqrt{\cos^2(e+fx)}}+\frac{ad\cos(e+fx)(d\csc(e+fx))^{n-1}\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\mathrm{s}\right.}{f(1-n)\sqrt{\cos^2(e+fx)}}$$

```
[Out] (a*d*Cos[e + f*x]*(d*Csc[e + f*x])^(-1 + n)*Hypergeometric2F1[1/2, (1 - n)/
2, (3 - n)/2, Sin[e + f*x]^2])/(f*(1 - n)*Sqrt[Cos[e + f*x]^2]) + (a*d^2*Co
s[e + f*x]*(d*Csc[e + f*x])^(-2 + n)*Hypergeometric2F1[1/2, (2 - n)/2, (4 -
 n)/2, Sin[e + f*x]^2])/(f*(2 - n)*Sqrt[Cos[e + f*x]^2])
```

Rubi [A] time = 0.149725, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3238, 3787, 3772, 2643}

$$\frac{ad^2\cos(e+fx)(d\csc(e+fx))^{n-2}\,{}_2F_1\left(\frac{1}{2},\frac{2-n}{2};\frac{4-n}{2};\sin^2(e+fx)\right)}{f(2-n)\sqrt{\cos^2(e+fx)}}+\frac{ad\cos(e+fx)(d\csc(e+fx))^{n-1}\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\mathrm{s}\right.}{f(1-n)\sqrt{\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Csc[e + f*x])^n*(a + a*Sin[e + f*x]),x]
```

```
[Out] (a*d*Cos[e + f*x]*(d*Csc[e + f*x])^(-1 + n)*Hypergeometric2F1[1/2, (1 - n)/
2, (3 - n)/2, Sin[e + f*x]^2])/(f*(1 - n)*Sqrt[Cos[e + f*x]^2]) + (a*d^2*Co
s[e + f*x]*(d*Csc[e + f*x])^(-2 + n)*Hypergeometric2F1[1/2, (2 - n)/2, (4 -
 n)/2, Sin[e + f*x]^2])/(f*(2 - n)*Sqrt[Cos[e + f*x]^2])
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int (d\csc(e+fx))^n(a+a\sin(e+fx))\,dx = d\int (d\csc(e+fx))^{-1+n}(a+a\csc(e+fx))\,dx$$
$$= a\int (d\csc(e+fx))^n\,dx + (ad)\int (d\csc(e+fx))^{-1+n}\,dx$$
$$= \left(a(d\csc(e+fx))^n\left(\frac{\sin(e+fx)}{d}\right)^n\right)\int\left(\frac{\sin(e+fx)}{d}\right)^{-n}dx + \left(ad(d\csc(e+fx)\right.$$
$$= \frac{a\cos(e+fx)(d\csc(e+fx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+fx)\right)\sin(e+fx)}{f(1-n)\sqrt{\cos^2(e+fx)}} + \frac{a\,\mathrm{co}}{}$$

Mathematica [C] time = 1.64396, size = 280, normalized size = 1.88

$$\frac{a2^{n-1}\left(-1+e^{2i(e+fx)}\right)e^{-i(e+fnx)}\left(\frac{ie^{i(e+fx)}}{-1+e^{2i(e+fx)}}\right)^n(\csc(e+fx)+1)\left(e^{ie}(n-1)\left(ne^{i(e+f(n+1)x)}\,{}_2F_1\left(1,\frac{3-n}{2};\frac{n+3}{2};e^{2i(e+fx)}\right)+2i(n+\right.\right.}{f(n-1)n(n+1)\left(\sin\left(\frac{1}{2}(e+fx\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Csc[e + f*x])^n*(a + a*Sin[e + f*x]),x]
```

```
[Out] (2^(-1 + n)*a*((I*E^(I*(e + f*x)))/(-1 + E^((2*I)*(e + f*x))))^n*(-1 + E^((
2*I)*(e + f*x)))*Csc[e + f*x]^(-1 - n)*(d*Csc[e + f*x])^n*(1 + Csc[e + f*x]
)*(-(E^(I*f*(-1 + n)*x)*n*(1 + n)*Hypergeometric2F1[1, (1 - n)/2, (1 + n)/2
, E^((2*I)*(e + f*x))]) + E^(I*e)*(-1 + n)*(E^(I*(e + f*(1 + n)*x))*n*Hyper
geometric2F1[1, (3 - n)/2, (3 + n)/2, E^((2*I)*(e + f*x))] + (2*I)*E^(I*f*n
*x)*(1 + n)*Hypergeometric2F1[1, 1 - n/2, (2 + n)/2, E^((2*I)*(e + f*x))]))
)/(E^(I*(e + f*n*x))*f*(-1 + n)*n*(1 + n)*(Cos[(e + f*x)/2] + Sin[(e + f*x)
/2])^2)
```

Maple [F] time = 1.033, size = 0, normalized size = 0.

$$\int \left(d\csc\left(fx+e\right)\right)^n\left(a+a\sin\left(fx+e\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(f*x+e))^n*(a+a*sin(f*x+e)),x)
```

```
[Out] int((d*csc(f*x+e))^n*(a+a*sin(f*x+e)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)\left(d\csc\left(fx+e\right)\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^n*(a+a*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)*(d*csc(f*x + e))^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a\sin\left(fx+e\right)+a\right)\left(d\csc\left(fx+e\right)\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^n*(a+a*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)*(d*csc(f*x + e))^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int\left(d\csc\left(e+fx\right)\right)^{n}\,dx+\int\left(d\csc\left(e+fx\right)\right)^{n}\sin\left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))**n*(a+a*sin(f*x+e)),x)
```

```
[Out] a*(Integral((d*csc(e + f*x))**n, x) + Integral((d*csc(e + f*x))**n*sin(e +
f*x), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a\sin\left(fx+e\right)+a\right)\left(d\csc\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^n*(a+a*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)*(d*csc(f*x + e))^n, x)
```

3.817 $\int \frac{(d\csc(e+fx))^n}{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=171

$$\frac{dn\cos(e+fx)(d\csc(e+fx))^{n-1}\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+fx)\right)}{af(1-n)\sqrt{\cos^2(e+fx)}}+\frac{\cos(e+fx)(d\csc(e+fx))^n\,{}_2F_1\left(\frac{1}{2},-\frac{n}{2};\frac{2-n}{2};\sin^2(e+f\right.}{af\sqrt{\cos^2(e+fx)}}$$

```
[Out] -((Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(a + a*Csc[e + f*x]))) + (d*n*Cos[e
+ f*x]*(d*Csc[e + f*x])^(-1 + n)*Hypergeometric2F1[1/2, (1 - n)/2, (3 - n)/
2, Sin[e + f*x]^2])/(a*f*(1 - n)*Sqrt[Cos[e + f*x]^2]) + (Cos[e + f*x]*(d*C
sc[e + f*x])^n*Hypergeometric2F1[1/2, -n/2, (2 - n)/2, Sin[e + f*x]^2])/(a*
f*Sqrt[Cos[e + f*x]^2])
```

Rubi [A] time = 0.237542, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3238, 3820, 3787, 3772, 2643}

$$\frac{dn\cos(e+fx)(d\csc(e+fx))^{n-1}\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+fx)\right)}{af(1-n)\sqrt{\cos^2(e+fx)}}+\frac{\cos(e+fx)(d\csc(e+fx))^n\,{}_2F_1\left(\frac{1}{2},-\frac{n}{2};\frac{2-n}{2};\sin^2(e+f\right.}{af\sqrt{\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Csc[e + f*x])^n/(a + a*Sin[e + f*x]),x]
```

```
[Out] -((Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(a + a*Csc[e + f*x]))) + (d*n*Cos[e
+ f*x]*(d*Csc[e + f*x])^(-1 + n)*Hypergeometric2F1[1/2, (1 - n)/2, (3 - n)/
2, Sin[e + f*x]^2])/(a*f*(1 - n)*Sqrt[Cos[e + f*x]^2]) + (Cos[e + f*x]*(d*C
sc[e + f*x])^n*Hypergeometric2F1[1/2, -n/2, (2 - n)/2, Sin[e + f*x]^2])/(a*
f*Sqrt[Cos[e + f*x]^2])
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3820

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(b*d*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 1))/(a*f*(
a + b*Csc[e + f*x])), x] + Dist[(d*(n - 1))/(a*b), Int[(d*Csc[e + f*x])^(n
- 1)*(a - b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^
2 - b^2, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
```

```
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}
\int \frac{(d\csc(e+fx))^n}{a+a\sin(e+fx)}\,dx &= \frac{\int \frac{(d\csc(e+fx))^{1+n}}{a+a\csc(e+fx)}\,dx}{d} \\
&= -\frac{\cot(e+fx)(d\csc(e+fx))^n}{f(a+a\csc(e+fx))} + \frac{n\int (d\csc(e+fx))^n(a-a\csc(e+fx))\,dx}{a^2} \\
&= -\frac{\cot(e+fx)(d\csc(e+fx))^n}{f(a+a\csc(e+fx))} + \frac{n\int (d\csc(e+fx))^n\,dx}{a} - \frac{n\int (d\csc(e+fx))^{1+n}\,dx}{ad} \\
&= -\frac{\cot(e+fx)(d\csc(e+fx))^n}{f(a+a\csc(e+fx))} + \frac{\left(n(d\csc(e+fx))^n\left(\frac{\sin(e+fx)}{d}\right)^n\right)\int \left(\frac{\sin(e+fx)}{d}\right)^{-n}dx}{a} - \frac{\left(n(d\,\mathrm{c}\right.}{} \\
&= -\frac{\cot(e+fx)(d\csc(e+fx))^n}{f(a+a\csc(e+fx))} + \frac{\cos(e+fx)(d\csc(e+fx))^n\,{}_2F_1\left(\frac{1}{2},-\frac{n}{2};\frac{2-n}{2};\sin^2(e+fx)\right)}{af\sqrt{\cos^2(e+fx)}} -
\end{aligned}$$

Mathematica [F] time = 2.80116, size = 0, normalized size = 0.

$$\int \frac{(d\csc(e+fx))^n}{a+a\sin(e+fx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*Csc[e + f*x])^n/(a + a*Sin[e + f*x]),x]

[Out] Integrate[(d*Csc[e + f*x])^n/(a + a*Sin[e + f*x]), x]
```

Maple [F] time = 0.483, size = 0, normalized size = 0.

$$\int \frac{\left(d\csc\left(fx+e\right)\right)^n}{a+a\sin\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(f*x+e))^n/(a+a*sin(f*x+e)),x)

[Out] int((d*csc(f*x+e))^n/(a+a*sin(f*x+e)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\csc\left(fx+e\right)\right)^n}{a\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^n/(a+a*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((d*csc(f*x + e))^n/(a*sin(f*x + e) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(d\csc\left(fx+e\right)\right)^{n}}{a\sin\left(fx+e\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^n/(a+a*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral((d*csc(f*x + e))^n/(a*sin(f*x + e) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\left(d\csc\left(e+fx\right)\right)^{n}}{\sin\left(e+fx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))**n/(a+a*sin(f*x+e)),x)
```

```
[Out] Integral((d*csc(e + f*x))**n/(sin(e + f*x) + 1), x)/a
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\csc\left(fx+e\right)\right)^{n}}{a\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^n/(a+a*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*csc(f*x + e))^n/(a*sin(f*x + e) + a), x)
```

3.818 $\int \frac{(d\csc(e+fx))^n}{(a+a\sin(e+fx))^2}\,dx$

Optimal. Leaf size=231

$$\frac{2n\cos(e+fx)(d\csc(e+fx))^{n+2}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-n-2);-\frac{n}{2};\sin^2(e+fx)\right)}{3a^2d^2f\sqrt{\cos^2(e+fx)}}-\frac{2n\cot(e+fx)(d\csc(e+fx))^{n+2}}{3a^2d^2f(\csc(e+fx)+1)}-\frac{(2n+1)\,\mathrm{c}}{}$$

```
[Out] (-2*n*Cot[e + f*x]*(d*Csc[e + f*x])^(2 + n))/(3*a^2*d^2*f*(1 + Csc[e + f*x]
)) + (Cot[e + f*x]*(d*Csc[e + f*x])^(2 + n))/(3*d^2*f*(a + a*Csc[e + f*x])^
2) + (2*n*Cos[e + f*x]*(d*Csc[e + f*x])^(2 + n)*Hypergeometric2F1[1/2, (-2
- n)/2, -n/2, Sin[e + f*x]^2])/(3*a^2*d^2*f*Sqrt[Cos[e + f*x]^2]) - ((1 + 2
*n)*Cos[e + f*x]*(d*Csc[e + f*x])^(1 + n)*Hypergeometric2F1[1/2, (-1 - n)/2
, (1 - n)/2, Sin[e + f*x]^2])/(3*a^2*d*f*Sqrt[Cos[e + f*x]^2])
```

Rubi [A] time = 0.442028, antiderivative size = 231, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3238, 3817, 4020, 3787, 3772, 2643}

$$\frac{2n\cos(e+fx)(d\csc(e+fx))^{n+2}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-n-2);-\frac{n}{2};\sin^2(e+fx)\right)}{3a^2d^2f\sqrt{\cos^2(e+fx)}}-\frac{2n\cot(e+fx)(d\csc(e+fx))^{n+2}}{3a^2d^2f(\csc(e+fx)+1)}-\frac{(2n+1)\,\mathrm{c}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Csc[e + f*x])^n/(a + a*Sin[e + f*x])^2,x]
```



```
[Out] (-2*n*Cot[e + f*x]*(d*Csc[e + f*x])^(2 + n))/(3*a^2*d^2*f*(1 + Csc[e + f*x]
)) + (Cot[e + f*x]*(d*Csc[e + f*x])^(2 + n))/(3*d^2*f*(a + a*Csc[e + f*x])^
2) + (2*n*Cos[e + f*x]*(d*Csc[e + f*x])^(2 + n)*Hypergeometric2F1[1/2, (-2
- n)/2, -n/2, Sin[e + f*x]^2])/(3*a^2*d^2*f*Sqrt[Cos[e + f*x]^2]) - ((1 + 2
*n)*Cos[e + f*x]*(d*Csc[e + f*x])^(1 + n)*Hypergeometric2F1[1/2, (-1 - n)/2
, (1 - n)/2, Sin[e + f*x]^2])/(3*a^2*d*f*Sqrt[Cos[e + f*x]^2])
```


Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3817

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m,
 -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
```

```
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{(d\csc(e+fx))^n}{(a+a\sin(e+fx))^2}\,dx = \frac{\int \frac{(d\csc(e+fx))^{2+n}}{(a+a\csc(e+fx))^2}\,dx}{d^2}$$

$$= \frac{\cot(e+fx)(d\csc(e+fx))^{2+n}}{3d^2f(a+a\csc(e+fx))^2} - \frac{\int \frac{(d\csc(e+fx))^{2+n}(a(-1+n)-a(1+n)\csc(e+fx))}{a+a\csc(e+fx)}\,dx}{3a^2d^2}$$

$$= -\frac{2n\cot(e+fx)(d\csc(e+fx))^{2+n}}{3a^2d^2f(1+\csc(e+fx))} + \frac{\cot(e+fx)(d\csc(e+fx))^{2+n}}{3d^2f(a+a\csc(e+fx))^2} - \frac{\int (d\csc(e+fx))^{2+n} (\cdots}{\cdots}$$

$$= -\frac{2n\cot(e+fx)(d\csc(e+fx))^{2+n}}{3a^2d^2f(1+\csc(e+fx))} + \frac{\cot(e+fx)(d\csc(e+fx))^{2+n}}{3d^2f(a+a\csc(e+fx))^2} - \frac{(2n(2+n))\int (d\csc(e \cdots}{3a^2d^3}$$

$$= -\frac{2n\cot(e+fx)(d\csc(e+fx))^{2+n}}{3a^2d^2f(1+\csc(e+fx))} + \frac{\cot(e+fx)(d\csc(e+fx))^{2+n}}{3d^2f(a+a\csc(e+fx))^2} - \frac{\left(2n(2+n)(d\csc(e+\cdots\right.}{\cdots}$$

$$= -\frac{2n\cot(e+fx)(d\csc(e+fx))^{2+n}}{3a^2d^2f(1+\csc(e+fx))} + \frac{\cot(e+fx)(d\csc(e+fx))^{2+n}}{3d^2f(a+a\csc(e+fx))^2} + \frac{2n\cot(e+fx)\csc(e\cdots}{\cdots}$$

Mathematica [F] time = 4.64156, size = 0, normalized size = 0.

$$\int \frac{(d\csc(e+fx))^n}{(a+a\sin(e+fx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*Csc[e + f*x])^n/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] Integrate[(d*Csc[e + f*x])^n/(a + a*Sin[e + f*x])^2, x]
```

Maple [F] time = 0.776, size = 0, normalized size = 0.

$$\int \frac{\left(d\csc\left(fx+e\right)\right)^{n}}{\left(a+a\sin\left(fx+e\right)\right)^{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(f*x+e))^n/(a+a*sin(f*x+e))^2,x)
```

```
[Out] int((d*csc(f*x+e))^n/(a+a*sin(f*x+e))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\csc\left(fx+e\right)\right)^{n}}{\left(a\sin\left(fx+e\right)+a\right)^{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^n/(a+a*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate((d*csc(f*x + e))^n/(a*sin(f*x + e) + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(d\csc\left(fx+e\right)\right)^{n}}{a^{2}\cos\left(fx+e\right)^{2}-2\,a^{2}\sin\left(fx+e\right)-2\,a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^n/(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral(-(d*csc(f*x + e))^n/(a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x + e) - 2*a
^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\left(d\csc\left(e+fx\right)\right)^{n}}{\sin^{2}\left(e+fx\right)+2\sin\left(e+fx\right)+1}\,dx}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))**n/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Integral((d*csc(e + f*x))**n/(sin(e + f*x)**2 + 2*sin(e + f*x) + 1), x)/a**
2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\csc\left(fx+e\right)\right)^n}{\left(a\sin\left(fx+e\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^n/(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*csc(f*x + e))^n/(a*sin(f*x + e) + a)^2, x)
```

3.819 $\int \left(c(d\sin(e+fx))^p\right)^n (a+a\sin(e+fx))^m\,dx$

Optimal. Leaf size=113

$$-\frac{2^{m+\frac{1}{2}}\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^m\sin^{-np}(e+fx)F_1\left(\frac{1}{2};-np,\frac{1}{2}-m;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\right.}{f}$$

```
[Out] -((2^(1/2 + m)*AppellF1[1/2, -(n*p), 1/2 - m, 3/2, 1 - Sin[e + f*x], (1 - S
in[e + f*x])/2]*Cos[e + f*x]*(c*(d*Sin[e + f*x])^p)^n*(1 + Sin[e + f*x])^(-
1/2 - m)*(a + a*Sin[e + f*x])^m)/(f*Sin[e + f*x]^(n*p)))
```

Rubi [A] time = 0.232322, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2826, 2787, 2786, 2785, 133}

$$-\frac{2^{m+\frac{1}{2}}\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^m\sin^{-np}(e+fx)F_1\left(\frac{1}{2};-np,\frac{1}{2}-m;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\right.}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(c*(d*Sin[e + f*x])^p)^n*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] -((2^(1/2 + m)*AppellF1[1/2, -(n*p), 1/2 - m, 3/2, 1 - Sin[e + f*x], (1 - S
in[e + f*x])/2]*Cos[e + f*x]*(c*(d*Sin[e + f*x])^p)^n*(1 + Sin[e + f*x])^(-
1/2 - m)*(a + a*Sin[e + f*x])^m)/(f*Sin[e + f*x]^(n*p)))
```

Rule 2826

```
Int[((c_.)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(p_))^(n_)*((a_.) + (b_.)*sin[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c^IntPart[n]*(c*(d*Sin[e + f*x
])^p)^FracPart[n])/(d*Sin[e + f*x])^(p*FracPart[n]), Int[(a + b*Sin[e + f*x
])^m*(d*Sin[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] &&  !IntegerQ[n]
```

Rule 2787

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m
])/(1 + (b*Sin[e + f*x])/a)^FracPart[m], Int[(1 + (b*Sin[e + f*x])/a)^m*(d*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 2786

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[((d/b)^IntPart[n]*(d*Sin[e + f*x])^FracPart[n
])/(b*Sin[e + f*x])^FracPart[n], Int[(a + b*Sin[e + f*x])^m*(b*Sin[e + f*x]
)^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !In
tegerQ[m] && GtQ[a, 0] &&  !GtQ[d/b, 0]
```

Rule 2785

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Dist[(b*(d/b)^n*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a - x)^n*(2*a - x)^(m - 1/2)
```

```
)/Sqrt[x], x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int \left(c(d\sin(e+fx))^p\right)^n (a+a\sin(e+fx))^m\,dx = \left((d\sin(e+fx))^{-np}\left(c(d\sin(e+fx))^p\right)^n\right)\int (d\sin(e+fx))^{np}(a+a\sin(e$$

$$= \left((d\sin(e+fx))^{-np}\left(c(d\sin(e+fx))^p\right)^n(1+\sin(e+fx))^{-m}(a+a\sin(e+$$

$$= \left(\sin^{-np}(e+fx)\left(c(d\sin(e+fx))^p\right)^n(1+\sin(e+fx))^{-m}(a+a\sin(e+f$$

$$= -\frac{\left(\cos(e+fx)\sin^{-np}(e+fx)\left(c(d\sin(e+fx))^p\right)^n(1+\sin(e+fx))^{-\frac{1}{2}-m}\right.}{f\sqrt{1-\sin}}$$

$$= -\frac{2^{\frac{1}{2}+m}F_1\left(\frac{1}{2};-np,\frac{1}{2}-m;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)\cos(e+}{}$$

Mathematica [B] time = 15.1318, size = 2967, normalized size = 26.26

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*(d*Sin[e + f*x])^p)^n*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] (-3*AppellF1[1/2, -(n*p), 1 + m + n*p, 3/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Ta
n[(-e + Pi/2 - f*x)/2]^2]*Cos[e + f*x]*Sin[e + f*x]^(n*p)*(c*(d*Sin[e + f*x
])^p)^n*(a + a*Sin[e + f*x])^m)/(f*(Sec[(-e + Pi/2 - f*x)/2]^2)^m*(3*Appell
F1[1/2, -(n*p), 1 + m + n*p, 3/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi
/2 - f*x)/2]^2] - 2*((1 + m + n*p)*AppellF1[3/2, -(n*p), 2 + m + n*p, 5/2,
Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2] + n*p*AppellF1[3/2
, 1 - n*p, 1 + m + n*p, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 -
f*x)/2]^2])*Tan[(-e + Pi/2 - f*x)/2]^2)*((-3*n*p*AppellF1[1/2, -(n*p), 1 +
m + n*p, 3/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2]*Cos[
e + f*x]^2*Sin[e + f*x]^(-1 + n*p))/((Sec[(-e + Pi/2 - f*x)/2]^2)^m*(3*Appe
llF1[1/2, -(n*p), 1 + m + n*p, 3/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e +
Pi/2 - f*x)/2]^2] - 2*((1 + m + n*p)*AppellF1[3/2, -(n*p), 2 + m + n*p, 5/2
, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2] + n*p*AppellF1[3
/2, 1 - n*p, 1 + m + n*p, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2
- f*x)/2]^2])*Tan[(-e + Pi/2 - f*x)/2]^2)) + (3*AppellF1[1/2, -(n*p), 1 + m
 + n*p, 3/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2]*Sin[e
 + f*x]^(1 + n*p))/((Sec[(-e + Pi/2 - f*x)/2]^2)^m*(3*AppellF1[1/2, -(n*p),
 1 + m + n*p, 3/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2]
 - 2*((1 + m + n*p)*AppellF1[3/2, -(n*p), 2 + m + n*p, 5/2, Tan[(-e + Pi/2
- f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2] + n*p*AppellF1[3/2, 1 - n*p, 1 +
m + n*p, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2])*Tan
[(-e + Pi/2 - f*x)/2]^2)) - (3*m*AppellF1[1/2, -(n*p), 1 + m + n*p, 3/2, Ta
n[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2]*Cos[e + f*x]*Sin[e +
```

```
 f*x]^(n*p)*Tan[(-e + Pi/2 - f*x)/2])/((Sec[(-e + Pi/2 - f*x)/2]^2)^m*(3*Ap
pellF1[1/2, -(n*p), 1 + m + n*p, 3/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e
+ Pi/2 - f*x)/2]^2] - 2*((1 + m + n*p)*AppellF1[3/2, -(n*p), 2 + m + n*p, 5
/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2] + n*p*AppellF1
[3/2, 1 - n*p, 1 + m + n*p, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/
2 - f*x)/2]^2])*Tan[(-e + Pi/2 - f*x)/2]^2)) + (3*Cos[e + f*x]*Sin[e + f*x]
^(n*p)*(-((1 + m + n*p)*AppellF1[3/2, -(n*p), 2 + m + n*p, 5/2, Tan[(-e + P
i/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2]*Sec[(-e + Pi/2 - f*x)/2]^2*Ta
n[(-e + Pi/2 - f*x)/2])/3 - (n*p*AppellF1[3/2, 1 - n*p, 1 + m + n*p, 5/2, T
an[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2]*Sec[(-e + Pi/2 - f*
x)/2]^2*Tan[(-e + Pi/2 - f*x)/2])/3))/((Sec[(-e + Pi/2 - f*x)/2]^2)^m*(3*Ap
pellF1[1/2, -(n*p), 1 + m + n*p, 3/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e
+ Pi/2 - f*x)/2]^2] - 2*((1 + m + n*p)*AppellF1[3/2, -(n*p), 2 + m + n*p, 5
/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2] + n*p*AppellF1
[3/2, 1 - n*p, 1 + m + n*p, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/
2 - f*x)/2]^2])*Tan[(-e + Pi/2 - f*x)/2]^2)) - (3*AppellF1[1/2, -(n*p), 1 +
 m + n*p, 3/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2]*Cos
[e + f*x]*Sin[e + f*x]^(n*p)*(-2*((1 + m + n*p)*AppellF1[3/2, -(n*p), 2 + m
 + n*p, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2] + n*p
*AppellF1[3/2, 1 - n*p, 1 + m + n*p, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[
(-e + Pi/2 - f*x)/2]^2])*Sec[(-e + Pi/2 - f*x)/2]^2*Tan[(-e + Pi/2 - f*x)/2
] + 3*(-((1 + m + n*p)*AppellF1[3/2, -(n*p), 2 + m + n*p, 5/2, Tan[(-e + Pi
/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2]*Sec[(-e + Pi/2 - f*x)/2]^2*Tan
[(-e + Pi/2 - f*x)/2])/3 - (n*p*AppellF1[3/2, 1 - n*p, 1 + m + n*p, 5/2, Ta
n[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2]*Sec[(-e + Pi/2 - f*x
)/2]^2*Tan[(-e + Pi/2 - f*x)/2])/3) - 2*Tan[(-e + Pi/2 - f*x)/2]^2*((1 + m
+ n*p)*((-3*(2 + m + n*p)*AppellF1[5/2, -(n*p), 3 + m + n*p, 7/2, Tan[(-e +
 Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2]*Sec[(-e + Pi/2 - f*x)/2]^2*
Tan[(-e + Pi/2 - f*x)/2])/5 - (3*n*p*AppellF1[5/2, 1 - n*p, 2 + m + n*p, 7/
2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2]*Sec[(-e + Pi/2
- f*x)/2]^2*Tan[(-e + Pi/2 - f*x)/2])/5) + n*p*((-3*(1 + m + n*p)*AppellF1[
5/2, 1 - n*p, 2 + m + n*p, 7/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2
 - f*x)/2]^2]*Sec[(-e + Pi/2 - f*x)/2]^2*Tan[(-e + Pi/2 - f*x)/2])/5 + (3*(
1 - n*p)*AppellF1[5/2, 2 - n*p, 1 + m + n*p, 7/2, Tan[(-e + Pi/2 - f*x)/2]^
2, -Tan[(-e + Pi/2 - f*x)/2]^2]*Sec[(-e + Pi/2 - f*x)/2]^2*Tan[(-e + Pi/2 -
 f*x)/2])/5))))/((Sec[(-e + Pi/2 - f*x)/2]^2)^m*(3*AppellF1[1/2, -(n*p), 1
+ m + n*p, 3/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2] -
2*((1 + m + n*p)*AppellF1[3/2, -(n*p), 2 + m + n*p, 5/2, Tan[(-e + Pi/2 - f
*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2] + n*p*AppellF1[3/2, 1 - n*p, 1 + m +
 n*p, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2])*Tan[(-
e + Pi/2 - f*x)/2]^2)^2)))
```

Maple [F] time = 0.41, size = 0, normalized size = 0.

$$\int \left(c\left(d\sin\left(fx+e\right)\right)^{p}\right)^{n}\left(a+a\sin\left(fx+e\right)\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*sin(f*x+e))^p)^n*(a+a*sin(f*x+e))^m,x)
```

```
[Out] int((c*(d*sin(f*x+e))^p)^n*(a+a*sin(f*x+e))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(d\sin\left(fx+e\right)\right)^{p} c\right)^{n}\left(a\sin\left(fx+e\right)+a\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n*(a+a*sin(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate(((d*sin(f*x + e))^p*c)^n*(a*sin(f*x + e) + a)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(\left(d\sin\left(fx+e\right)\right)^{p}c\right)^{n}\left(a\sin\left(fx+e\right)+a\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n*(a+a*sin(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral(((d*sin(f*x + e))^p*c)^n*(a*sin(f*x + e) + a)^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{m}\left(c\left(d\sin\left(e+fx\right)\right)^{p}\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))**p)**n*(a+a*sin(f*x+e))**m,x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**m*(c*(d*sin(e + f*x))**p)**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(\left(d\sin\left(fx+e\right)\right)^{p}c\right)^{n}\left(a\sin\left(fx+e\right)+a\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n*(a+a*sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate(((d*sin(f*x + e))^p*c)^n*(a*sin(f*x + e) + a)^m, x)
```

3.820 $\int \left(c(d\sin(e+fx))^p\right)^n (a+a\sin(e+fx))^3\,dx$

Optimal. Leaf size=299

$$\frac{a^3(4np+11)\sin^2(e+fx)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(np+2);\frac{1}{2}(np+4);\sin^2(e+fx)\right)\left(c(d\sin(e+fx))^p\right)^n}{f(np+2)(np+3)\sqrt{\cos^2(e+fx)}}+\frac{a^3(4np+5)\,\mathrm{si}}{}$$

```
[Out] -((a^3*(7 + 2*n*p)*Cos[e + f*x]*Sin[e + f*x]*(c*(d*Sin[e + f*x])^p)^n)/(f*(
2 + n*p)*(3 + n*p))) + (a^3*(5 + 4*n*p)*Cos[e + f*x]*Hypergeometric2F1[1/2,
 (1 + n*p)/2, (3 + n*p)/2, Sin[e + f*x]^2]*Sin[e + f*x]*(c*(d*Sin[e + f*x])
^p)^n)/(f*(1 + n*p)*(2 + n*p)*Sqrt[Cos[e + f*x]^2]) + (a^3*(11 + 4*n*p)*Cos
[e + f*x]*Hypergeometric2F1[1/2, (2 + n*p)/2, (4 + n*p)/2, Sin[e + f*x]^2]*
Sin[e + f*x]^2*(c*(d*Sin[e + f*x])^p)^n)/(f*(2 + n*p)*(3 + n*p)*Sqrt[Cos[e
+ f*x]^2]) - (Cos[e + f*x]*Sin[e + f*x]*(c*(d*Sin[e + f*x])^p)^n*(a^3 + a^3
*Sin[e + f*x]))/(f*(3 + n*p))
```

Rubi [A] time = 0.495259, antiderivative size = 299, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2826, 2763, 2968, 3023, 2748, 2643}

$$\frac{a^3(4np+11)\sin^2(e+fx)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(np+2);\frac{1}{2}(np+4);\sin^2(e+fx)\right)\left(c(d\sin(e+fx))^p\right)^n}{f(np+2)(np+3)\sqrt{\cos^2(e+fx)}}+\frac{a^3(4np+5)\,\mathrm{si}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c*(d*Sin[e + f*x])^p)^n*(a + a*Sin[e + f*x])^3,x]
```

```
[Out] -((a^3*(7 + 2*n*p)*Cos[e + f*x]*Sin[e + f*x]*(c*(d*Sin[e + f*x])^p)^n)/(f*(
2 + n*p)*(3 + n*p))) + (a^3*(5 + 4*n*p)*Cos[e + f*x]*Hypergeometric2F1[1/2,
 (1 + n*p)/2, (3 + n*p)/2, Sin[e + f*x]^2]*Sin[e + f*x]*(c*(d*Sin[e + f*x])
^p)^n)/(f*(1 + n*p)*(2 + n*p)*Sqrt[Cos[e + f*x]^2]) + (a^3*(11 + 4*n*p)*Cos
[e + f*x]*Hypergeometric2F1[1/2, (2 + n*p)/2, (4 + n*p)/2, Sin[e + f*x]^2]*
Sin[e + f*x]^2*(c*(d*Sin[e + f*x])^p)^n)/(f*(2 + n*p)*(3 + n*p)*Sqrt[Cos[e
+ f*x]^2]) - (Cos[e + f*x]*Sin[e + f*x]*(c*(d*Sin[e + f*x])^p)^n*(a^3 + a^3
*Sin[e + f*x]))/(f*(3 + n*p))
```

Rule 2826

```
Int[((c_.)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(p_))^(n_)*((a_.) + (b_.)*sin[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c^IntPart[n]*(c*(d*Sin[e + f*x
])^p)^FracPart[n])/(d*Sin[e + f*x])^(p*FracPart[n]), Int[(a + b*Sin[e + f*x
])^m*(d*Sin[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] &&  !IntegerQ[n]
```

Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
```

```
0]))
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int \left(c(d\sin(e+fx))^p\right)^n (a+a\sin(e+fx))^3\,dx = \left((d\sin(e+fx))^{-np}\left(c(d\sin(e+fx))^p\right)^n\right)\int (d\sin(e+fx))^{np}(a+a\sin(e$$

$$= -\frac{\cos(e+fx)\sin(e+fx)\left(c(d\sin(e+fx))^p\right)^n\left(a^3+a^3\sin(e+fx)\right)}{f(3+np)} + \frac{\left((\right.}{}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)\left(c(d\sin(e+fx))^p\right)^n\left(a^3+a^3\sin(e+fx)\right)}{f(3+np)} + \frac{\left((\right.}{}$$

$$= -\frac{a^3(7+2np)\cos(e+fx)\sin(e+fx)\left(c(d\sin(e+fx))^p\right)^n}{f(2+np)(3+np)} - \frac{\cos(e+fx)}{}$$

$$= -\frac{a^3(7+2np)\cos(e+fx)\sin(e+fx)\left(c(d\sin(e+fx))^p\right)^n}{f(2+np)(3+np)} - \frac{\cos(e+fx)}{}$$

$$= -\frac{a^3(7+2np)\cos(e+fx)\sin(e+fx)\left(c(d\sin(e+fx))^p\right)^n}{f(2+np)(3+np)} + \frac{a^3(5+4np)}{}$$

Mathematica [A] time = 1.39676, size = 297, normalized size = 0.99

$$-\frac{a^3\sin(e+fx)\cos(e+fx)\sqrt{\cos^2(e+fx)}\left(\left(n^3p^3+9n^2p^2+26np+24\right){}_2F_1\left(\frac{1}{2},\frac{1}{2}(np+1);\frac{1}{2}(np+3);\sin^2(e+fx)\right)+\frac{1}{2}(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*(d*Sin[e + f*x])^p)^n*(a + a*Sin[e + f*x])^3,x]
```

```
[Out] -((a^3*Cos[e + f*x]*Sqrt[Cos[e + f*x]^2]*Sin[e + f*x]*(c*(d*Sin[e + f*x])^p
)^n*((24 + 26*n*p + 9*n^2*p^2 + n^3*p^3)*Hypergeometric2F1[1/2, (1 + n*p)/2
, (3 + n*p)/2, Sin[e + f*x]^2] + ((1 + n*p)*Sin[e + f*x]*(6*(12 + 7*n*p + n
^2*p^2)*Hypergeometric2F1[1/2, 1 + (n*p)/2, 2 + (n*p)/2, Sin[e + f*x]^2] +
2*(2 + n*p)*Sin[e + f*x]*(3*(4 + n*p)*Hypergeometric2F1[1/2, (3 + n*p)/2, (
5 + n*p)/2, Sin[e + f*x]^2] + (3 + n*p)*Hypergeometric2F1[1/2, 2 + (n*p)/2,
 3 + (n*p)/2, Sin[e + f*x]^2]*Sin[e + f*x])))/2))/(f*(1 + n*p)*(2 + n*p)*(3
 + n*p)*(4 + n*p)*(-1 + Sin[e + f*x])*(1 + Sin[e + f*x])))
```

Maple [F] time = 0.44, size = 0, normalized size = 0.

$$\int \left(c \left(d \sin \left(f x +e \right)\right)^{p}\right)^{n} \left(a +a \sin \left(f x +e \right)\right)^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*sin(f*x+e))^p)^n*(a+a*sin(f*x+e))^3,x)
```

```
[Out] int((c*(d*sin(f*x+e))^p)^n*(a+a*sin(f*x+e))^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int {\left(a \sin \left(f x + e\right) + a\right)}^{3} \left(\left(d \sin \left(f x + e\right)\right)^{p} c\right)^{n}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n*(a+a*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^3*((d*sin(f*x + e))^p*c)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$${\rm integral}\left(-{\left(3 \, a^{3} \cos \left(f x + e\right)^{2} - 4 \, a^{3} + {\left(a^{3} \cos \left(f x + e\right)^{2} - 4 \, a^{3}\right)} \sin \left(f x + e\right)\right)} \left(\left(d \sin \left(f x + e\right)\right)^{p} c\right)^{n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n*(a+a*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] integral(-(3*a^3*cos(f*x + e)^2 - 4*a^3 + (a^3*cos(f*x + e)^2 - 4*a^3)*sin(
f*x + e))*((d*sin(f*x + e))^p*c)^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))**p)**n*(a+a*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^{3}\left(\left(d\sin\left(fx+e\right)\right)^{p}c\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n*(a+a*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^3*((d*sin(f*x + e))^p*c)^n, x)
```

3.821 $\int \left(c(d\sin(e+fx))^p\right)^n (a+a\sin(e+fx))^2\,dx$

Optimal. Leaf size=222

$$\frac{2a^2\sin^2(e+fx)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(np+2);\frac{1}{2}(np+4);\sin^2(e+fx)\right)\left(c(d\sin(e+fx))^p\right)^n}{f(np+2)\sqrt{\cos^2(e+fx)}}+\frac{a^2(2np+3)\sin(e+fx}{}$$

```
[Out] -((a^2*Cos[e + f*x]*Sin[e + f*x]*(c*(d*Sin[e + f*x])^p)^n)/(f*(2 + n*p))) +
 (a^2*(3 + 2*n*p)*Cos[e + f*x]*Hypergeometric2F1[1/2, (1 + n*p)/2, (3 + n*p
)/2, Sin[e + f*x]^2]*Sin[e + f*x]*(c*(d*Sin[e + f*x])^p)^n)/(f*(1 + n*p)*(2
 + n*p)*Sqrt[Cos[e + f*x]^2]) + (2*a^2*Cos[e + f*x]*Hypergeometric2F1[1/2,
(2 + n*p)/2, (4 + n*p)/2, Sin[e + f*x]^2]*Sin[e + f*x]^2*(c*(d*Sin[e + f*x]
)^p)^n)/(f*(2 + n*p)*Sqrt[Cos[e + f*x]^2])
```

Rubi [A] time = 0.249206, antiderivative size = 222, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2826, 2763, 2748, 2643}

$$\frac{2a^2\sin^2(e+fx)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(np+2);\frac{1}{2}(np+4);\sin^2(e+fx)\right)\left(c(d\sin(e+fx))^p\right)^n}{f(np+2)\sqrt{\cos^2(e+fx)}}+\frac{a^2(2np+3)\sin(e+fx}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c*(d*Sin[e + f*x])^p)^n*(a + a*Sin[e + f*x])^2,x]
```

```
[Out] -((a^2*Cos[e + f*x]*Sin[e + f*x]*(c*(d*Sin[e + f*x])^p)^n)/(f*(2 + n*p))) +
 (a^2*(3 + 2*n*p)*Cos[e + f*x]*Hypergeometric2F1[1/2, (1 + n*p)/2, (3 + n*p
)/2, Sin[e + f*x]^2]*Sin[e + f*x]*(c*(d*Sin[e + f*x])^p)^n)/(f*(1 + n*p)*(2
 + n*p)*Sqrt[Cos[e + f*x]^2]) + (2*a^2*Cos[e + f*x]*Hypergeometric2F1[1/2,
(2 + n*p)/2, (4 + n*p)/2, Sin[e + f*x]^2]*Sin[e + f*x]^2*(c*(d*Sin[e + f*x]
)^p)^n)/(f*(2 + n*p)*Sqrt[Cos[e + f*x]^2])
```

Rule 2826

```
Int[((c_.)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(p_))^(n_)*((a_.) + (b_.)*sin[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c^IntPart[n]*(c*(d*Sin[e + f*x
])^p)^FracPart[n])/(d*Sin[e + f*x])^(p*FracPart[n]), Int[(a + b*Sin[e + f*x
])^m*(d*Sin[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] && !IntegerQ[n]
```

Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int \left(c(d\sin(e+fx))^p\right)^n (a+a\sin(e+fx))^2\,dx = \left((d\sin(e+fx))^{-np}\left(c(d\sin(e+fx))^p\right)^n\right)\int (d\sin(e+fx))^{np}(a+a\sin(e-$$

$$= -\frac{a^2\cos(e+fx)\sin(e+fx)\left(c(d\sin(e+fx))^p\right)^n}{f(2+np)} + \frac{\left((d\sin(e+fx))^{-np}\left(c\right.\right.}{}$$

$$= -\frac{a^2\cos(e+fx)\sin(e+fx)\left(c(d\sin(e+fx))^p\right)^n}{f(2+np)} + \frac{\left(2a^2(d\sin(e+fx))^{-n}\right.}{}$$

$$= -\frac{a^2\cos(e+fx)\sin(e+fx)\left(c(d\sin(e+fx))^p\right)^n}{f(2+np)} + \frac{a^2(3+2np)\cos(e+f}{}$$

Mathematica [A] time = 0.676297, size = 222, normalized size = 1.

$$-\frac{a^2\sin(e+fx)\cos(e+fx)\sqrt{\cos^2(e+fx)}\left(\left(n^2p^2+5np+6\right)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(np+1);\frac{1}{2}(np+3);\sin^2(e+fx)\right)+(np+1)\sin(e\right.}{f(np+1)(np+2)(}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*(d*Sin[e + f*x])^p)^n*(a + a*Sin[e + f*x])^2,x]
```

```
[Out] -((a^2*Cos[e + f*x]*Sqrt[Cos[e + f*x]^2]*Sin[e + f*x]*(c*(d*Sin[e + f*x])^p
)^n*((6 + 5*n*p + n^2*p^2)*Hypergeometric2F1[1/2, (1 + n*p)/2, (3 + n*p)/2,
 Sin[e + f*x]^2] + (1 + n*p)*Sin[e + f*x]*(2*(3 + n*p)*Hypergeometric2F1[1/
2, 1 + (n*p)/2, 2 + (n*p)/2, Sin[e + f*x]^2] + (2 + n*p)*Hypergeometric2F1[
1/2, (3 + n*p)/2, (5 + n*p)/2, Sin[e + f*x]^2]*Sin[e + f*x])))/(f*(1 + n*p)
*(2 + n*p)*(3 + n*p)*(-1 + Sin[e + f*x])*(1 + Sin[e + f*x])))
```

Maple [F] time = 0.427, size = 0, normalized size = 0.

$$\int \left(c\left(d\sin\left(fx+e\right)\right)^p\right)^n \left(a+a\sin\left(fx+e\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*sin(f*x+e))^p)^n*(a+a*sin(f*x+e))^2,x)
```

```
[Out] int((c*(d*sin(f*x+e))^p)^n*(a+a*sin(f*x+e))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^{2}\left(\left(d\sin\left(fx+e\right)\right)^{p}c\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n*(a+a*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^2*((d*sin(f*x + e))^p*c)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-\left(a^{2}\cos\left(fx+e\right)^{2}-2\,a^{2}\sin\left(fx+e\right)-2\,a^{2}\right)\left(\left(d\sin\left(fx+e\right)\right)^{p}c\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n*(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral(-(a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x + e) - 2*a^2)*((d*sin(f*x + e
))^p*c)^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^{2}\left(\int \left(c\left(d\sin\left(e+fx\right)\right)^{p}\right)^{n}\,dx+\int 2\left(c\left(d\sin\left(e+fx\right)\right)^{p}\right)^{n}\sin\left(e+fx\right)dx+\int \left(c\left(d\sin\left(e+fx\right)\right)^{p}\right)^{n}\sin^{2}\left(e+fx\right)d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))**p)**n*(a+a*sin(f*x+e))**2,x)
```

```
[Out] a**2*(Integral((c*(d*sin(e + f*x))**p)**n, x) + Integral(2*(c*(d*sin(e + f*
x))**p)**n*sin(e + f*x), x) + Integral((c*(d*sin(e + f*x))**p)**n*sin(e + f
*x)**2, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^{2}\left(\left(d\sin\left(fx+e\right)\right)^{p}c\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n*(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^2*((d*sin(f*x + e))^p*c)^n, x)
```

3.822 $\int \left(c(d\sin(e+fx))^p\right)^n (a+a\sin(e+fx))\,dx$

Optimal. Leaf size=163

$$\frac{a\sin^2(e+fx)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(np+2);\frac{1}{2}(np+4);\sin^2(e+fx)\right)\left(c(d\sin(e+fx))^p\right)^n}{f(np+2)\sqrt{\cos^2(e+fx)}}+\frac{a\sin(e+fx)\cos(e+fx)\,{}_2F_1$$

```
[Out] (a*Cos[e + f*x]*Hypergeometric2F1[1/2, (1 + n*p)/2, (3 + n*p)/2, Sin[e + f*
x]^2]*Sin[e + f*x]*(c*(d*Sin[e + f*x])^p)^n)/(f*(1 + n*p)*Sqrt[Cos[e + f*x]
^2]) + (a*Cos[e + f*x]*Hypergeometric2F1[1/2, (2 + n*p)/2, (4 + n*p)/2, Sin
[e + f*x]^2]*Sin[e + f*x]^2*(c*(d*Sin[e + f*x])^p)^n)/(f*(2 + n*p)*Sqrt[Cos
[e + f*x]^2])
```

Rubi [A] time = 0.118872, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2826, 2748, 2643}

$$\frac{a\sin^2(e+fx)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(np+2);\frac{1}{2}(np+4);\sin^2(e+fx)\right)\left(c(d\sin(e+fx))^p\right)^n}{f(np+2)\sqrt{\cos^2(e+fx)}}+\frac{a\sin(e+fx)\cos(e+fx)\,{}_2F_1$$

Antiderivative was successfully verified.

```
[In] Int[(c*(d*Sin[e + f*x])^p)^n*(a + a*Sin[e + f*x]),x]
```


```
[Out] (a*Cos[e + f*x]*Hypergeometric2F1[1/2, (1 + n*p)/2, (3 + n*p)/2, Sin[e + f*
x]^2]*Sin[e + f*x]*(c*(d*Sin[e + f*x])^p)^n)/(f*(1 + n*p)*Sqrt[Cos[e + f*x]
^2]) + (a*Cos[e + f*x]*Hypergeometric2F1[1/2, (2 + n*p)/2, (4 + n*p)/2, Sin
[e + f*x]^2]*Sin[e + f*x]^2*(c*(d*Sin[e + f*x])^p)^n)/(f*(2 + n*p)*Sqrt[Cos
[e + f*x]^2])
```


Rule 2826

```
Int[((c_.)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(p_))^(n_)*((a_.) + (b_.)*sin[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c^IntPart[n]*(c*(d*Sin[e + f*x
])^p)^FracPart[n])/(d*Sin[e + f*x])^(p*FracPart[n]), Int[(a + b*Sin[e + f*x
])^m*(d*Sin[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] && !IntegerQ[n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \left(c(d\sin(e+fx))^p\right)^n (a+a\sin(e+fx))\,dx = \left((d\sin(e+fx))^{-np}\left(c(d\sin(e+fx))^p\right)^n\right)\int (d\sin(e+fx))^{np}(a+a\sin($$

$$= \left(a(d\sin(e+fx))^{-np}\left(c(d\sin(e+fx))^p\right)^n\right)\int (d\sin(e+fx))^{np}\,dx + \frac{\left(a(\right.}{}$$

$$= \frac{a\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(1+np);\frac{1}{2}(3+np);\sin^2(e+fx)\right)\sin(e+fx)\left(c\right.}{f(1+np)\sqrt{\cos^2(e+fx)}}$$

Mathematica [C] time = 1.53437, size = 270, normalized size = 1.66

$$\frac{a2^{-np-1}(\sin(e+fx)+1)\left(-ie^{-i(e+fx)}\left(-1+e^{2i(e+fx)}\right)\right)^{np+1}\left(2\left(n^2p^2-1\right)e^{i(e+fx)}\,{}_2F_1\left(1,\frac{np}{2}+1;1-\frac{np}{2};e^{2i(e+fx)}\right)+inp\left((n\right.\right.}{fnp(np-1)(np+1}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*(d*Sin[e + f*x])^p)^n*(a + a*Sin[e + f*x]),x]
```

```
[Out] (2^(-1 - n*p)*a*(((-I)*(-1 + E^((2*I)*(e + f*x))))/E^(I*(e + f*x)))^(1 + n*
p)*(2*E^(I*(e + f*x))*(-1 + n^2*p^2)*Hypergeometric2F1[1, 1 + (n*p)/2, 1 -
(n*p)/2, E^((2*I)*(e + f*x))] + I*n*p*((-1 + n*p)*Hypergeometric2F1[1, (1 +
 n*p)/2, (1 - n*p)/2, E^((2*I)*(e + f*x))] - E^((2*I)*(e + f*x))*(1 + n*p)*
Hypergeometric2F1[1, (3 + n*p)/2, (3 - n*p)/2, E^((2*I)*(e + f*x))]))*(c*(d
*Sin[e + f*x])^p)^n*(1 + Sin[e + f*x]))/(f*n*p*(-1 + n*p)*(1 + n*p)*(Cos[(e
 + f*x)/2] + Sin[(e + f*x)/2])^2*Sin[e + f*x]^(n*p))
```

Maple [F] time = 0.19, size = 0, normalized size = 0.

$$\int \left(c\left(d\sin\left(fx+e\right)\right)^p\right)^n\left(a+a\sin\left(fx+e\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*sin(f*x+e))^p)^n*(a+a*sin(f*x+e)),x)
```

```
[Out] int((c*(d*sin(f*x+e))^p)^n*(a+a*sin(f*x+e)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)\left(\left(d\sin\left(fx+e\right)\right)^p c\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n*(a+a*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)*((d*sin(f*x + e))^p*c)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a\sin\left(fx+e\right)+a\right)\left(\left(d\sin\left(fx+e\right)\right)^{p}c\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n*(a+a*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)*((d*sin(f*x + e))^p*c)^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int\left(c\left(d\sin\left(e+fx\right)\right)^{p}\right)^{n}dx+\int\left(c\left(d\sin\left(e+fx\right)\right)^{p}\right)^{n}\sin\left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))**p)**n*(a+a*sin(f*x+e)),x)
```

```
[Out] a*(Integral((c*(d*sin(e + f*x))**p)**n, x) + Integral((c*(d*sin(e + f*x))**
p)**n*sin(e + f*x), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a\sin\left(fx+e\right)+a\right)\left(\left(d\sin\left(fx+e\right)\right)^{p}c\right)^{n}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n*(a+a*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)*((d*sin(f*x + e))^p*c)^n, x)
```

3.823 $\int \frac{(c(d\sin(e+fx))^p)^n}{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=189

$$\frac{\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{np}{2};\frac{1}{2}(np+2);\sin^2(e+fx)\right)\left(c(d\sin(e+fx))^p\right)^n}{af\sqrt{\cos^2(e+fx)}}-\frac{np\sin(e+fx)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(np+1);\frac{1}{2}(\right.}{af(np+1)\sqrt{\cos}}$$

```
[Out] (Cos[e + f*x]*Hypergeometric2F1[1/2, (n*p)/2, (2 + n*p)/2, Sin[e + f*x]^2]*
(c*(d*Sin[e + f*x])^p)^n)/(a*f*Sqrt[Cos[e + f*x]^2]) - (n*p*Cos[e + f*x]*Hy
pergeometric2F1[1/2, (1 + n*p)/2, (3 + n*p)/2, Sin[e + f*x]^2]*Sin[e + f*x]
*(c*(d*Sin[e + f*x])^p)^n)/(a*f*(1 + n*p)*Sqrt[Cos[e + f*x]^2]) - (Cos[e +
f*x]*(c*(d*Sin[e + f*x])^p)^n)/(f*(a + a*Sin[e + f*x]))
```

Rubi [A] time = 0.229935, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2826, 2769, 2748, 2643}

$$\frac{\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{np}{2};\frac{1}{2}(np+2);\sin^2(e+fx)\right)\left(c(d\sin(e+fx))^p\right)^n}{af\sqrt{\cos^2(e+fx)}}-\frac{np\sin(e+fx)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(np+1);\frac{1}{2}(\right.}{af(np+1)\sqrt{\cos}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*(d*Sin[e + f*x])^p)^n/(a + a*Sin[e + f*x]),x]
```


```
[Out] (Cos[e + f*x]*Hypergeometric2F1[1/2, (n*p)/2, (2 + n*p)/2, Sin[e + f*x]^2]*
(c*(d*Sin[e + f*x])^p)^n)/(a*f*Sqrt[Cos[e + f*x]^2]) - (n*p*Cos[e + f*x]*Hy
pergeometric2F1[1/2, (1 + n*p)/2, (3 + n*p)/2, Sin[e + f*x]^2]*Sin[e + f*x]
*(c*(d*Sin[e + f*x])^p)^n)/(a*f*(1 + n*p)*Sqrt[Cos[e + f*x]^2]) - (Cos[e +
f*x]*(c*(d*Sin[e + f*x])^p)^n)/(f*(a + a*Sin[e + f*x]))
```


Rule 2826

```
Int[((c_.)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(p_))^(n_)*((a_.) + (b_.)*sin[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c^IntPart[n]*(c*(d*Sin[e + f*x
])^p)^FracPart[n])/(d*Sin[e + f*x])^(p*FracPart[n]), Int[(a + b*Sin[e + f*x
])^m*(d*Sin[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] && !IntegerQ[n]
```

Rule 2769

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(a
*f*(a + b*Sin[e + f*x])), x] + Dist[(d*n)/(a*b), Int[(c + d*Sin[e + f*x])^(
n - 1)*(a - b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (IntegerQ[2*n
] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{\left(c(d\sin(e+fx))^p\right)^n}{a+a\sin(e+fx)}\,dx = \left((d\sin(e+fx))^{-np}\left(c(d\sin(e+fx))^p\right)^n\right)\int \frac{(d\sin(e+fx))^{np}}{a+a\sin(e+fx)}\,dx$$

$$= -\frac{\cos(e+fx)\left(c(d\sin(e+fx))^p\right)^n}{f(a+a\sin(e+fx))} + \frac{\left(dnp(d\sin(e+fx))^{-np}\left(c(d\sin(e+fx))^p\right)^n\right)\int (d\sin(e\,-}{a^2}$$

$$= -\frac{\cos(e+fx)\left(c(d\sin(e+fx))^p\right)^n}{f(a+a\sin(e+fx))} - \frac{\left(np(d\sin(e+fx))^{-np}\left(c(d\sin(e+fx))^p\right)^n\right)\int (d\sin(e+}{a}$$

$$= \frac{\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{np}{2};\frac{1}{2}(2+np);\sin^2(e+fx)\right)\left(c(d\sin(e+fx))^p\right)^n}{af\sqrt{\cos^2(e+fx)}} - \frac{np\cos(e+fx)\,{}_2F_1\left(\frac{1}{2}\right.}{}$$

Mathematica [A] time = 0.299217, size = 157, normalized size = 0.83

$$\frac{\sin(e+fx)\cos(e+fx)\sqrt{\cos^2(e+fx)}\left((np+1)\sin(e+fx)\,{}_2F_1\left(\frac{3}{2},\frac{np}{2}+1;\frac{np}{2}+2;\sin^2(e+fx)\right)-(np+2)\,{}_2F_1\left(\frac{3}{2},\frac{1}{2}(np+\right.\right.}{af(np+1)(np+2)(\sin(e+fx)-1)(\sin(e+fx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*(d*Sin[e + f*x])^p)^n/(a + a*Sin[e + f*x]),x]
```

```
[Out] (Cos[e + f*x]*Sqrt[Cos[e + f*x]^2]*Sin[e + f*x]*(c*(d*Sin[e + f*x])^p)^n*(-
((2 + n*p)*Hypergeometric2F1[3/2, (1 + n*p)/2, (3 + n*p)/2, Sin[e + f*x]^2]
) + (1 + n*p)*Hypergeometric2F1[3/2, 1 + (n*p)/2, 2 + (n*p)/2, Sin[e + f*x]
^2]*Sin[e + f*x]))/(a*f*(1 + n*p)*(2 + n*p)*(-1 + Sin[e + f*x])*(1 + Sin[e
+ f*x]))
```

Maple [F] time = 0.155, size = 0, normalized size = 0.

$$\int \frac{\left(c\left(d\sin\left(fx+e\right)\right)^p\right)^n}{a+a\sin\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*sin(f*x+e))^p)^n/(a+a*sin(f*x+e)),x)
```

```
[Out] int((c*(d*sin(f*x+e))^p)^n/(a+a*sin(f*x+e)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(d\sin\left(fx+e\right)\right)^p c\right)^n}{a\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n/(a+a*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate(((d*sin(f*x + e))^p*c)^n/(a*sin(f*x + e) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(\left(d\sin\left(fx+e\right)\right)^{p}c\right)^{n}}{a\sin\left(fx+e\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n/(a+a*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral(((d*sin(f*x + e))^p*c)^n/(a*sin(f*x + e) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{\left(c\left(d\sin\left(e+fx\right)\right)^{p}\right)^{n}}{\sin\left(e+fx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))**p)**n/(a+a*sin(f*x+e)),x)
```

```
[Out] Integral((c*(d*sin(e + f*x))**p)**n/(sin(e + f*x) + 1), x)/a
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(\left(d\sin\left(fx+e\right)\right)^{p}c\right)^{n}}{a\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n/(a+a*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate(((d*sin(f*x + e))^p*c)^n/(a*sin(f*x + e) + a), x)
```

3.824 $\int \frac{\left(c(d\sin(e+fx))^p\right)^n}{(a+a\sin(e+fx))^2}\,dx$

Optimal. Leaf size=288

$$\frac{2\left(1-n^2p^2\right)\sin^2(e+fx)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(np+2);\frac{1}{2}(np+4);\sin^2(e+fx)\right)\left(c(d\sin(e+fx))^p\right)^n}{3a^2f(np+2)\sqrt{\cos^2(e+fx)}}-\frac{np(1-2np)\sin(e}{}$$

```
[Out] -(n*p*(1 - 2*n*p)*Cos[e + f*x]*Hypergeometric2F1[1/2, (1 + n*p)/2, (3 + n*p
)/2, Sin[e + f*x]^2]*Sin[e + f*x]*(c*(d*Sin[e + f*x])^p)^n)/(3*a^2*f*(1 + n
*p)*Sqrt[Cos[e + f*x]^2]) + (2*(1 - n^2*p^2)*Cos[e + f*x]*Hypergeometric2F1
[1/2, (2 + n*p)/2, (4 + n*p)/2, Sin[e + f*x]^2]*Sin[e + f*x]^2*(c*(d*Sin[e
+ f*x])^p)^n)/(3*a^2*f*(2 + n*p)*Sqrt[Cos[e + f*x]^2]) + (2*(1 - n*p)*Cos[e
 + f*x]*Sin[e + f*x]*(c*(d*Sin[e + f*x])^p)^n)/(3*a^2*f*(1 + Sin[e + f*x]))
 + (Cos[e + f*x]*Sin[e + f*x]*(c*(d*Sin[e + f*x])^p)^n)/(3*f*(a + a*Sin[e +
 f*x])^2)
```

Rubi [A] time = 0.486528, antiderivative size = 288, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2826, 2766, 2978, 2748, 2643}

$$\frac{2\left(1-n^2p^2\right)\sin^2(e+fx)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(np+2);\frac{1}{2}(np+4);\sin^2(e+fx)\right)\left(c(d\sin(e+fx))^p\right)^n}{3a^2f(np+2)\sqrt{\cos^2(e+fx)}}-\frac{np(1-2np)\sin(e}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c*(d*Sin[e + f*x])^p)^n/(a + a*Sin[e + f*x])^2,x]
```


```
[Out] -(n*p*(1 - 2*n*p)*Cos[e + f*x]*Hypergeometric2F1[1/2, (1 + n*p)/2, (3 + n*p
)/2, Sin[e + f*x]^2]*Sin[e + f*x]*(c*(d*Sin[e + f*x])^p)^n)/(3*a^2*f*(1 + n
*p)*Sqrt[Cos[e + f*x]^2]) + (2*(1 - n^2*p^2)*Cos[e + f*x]*Hypergeometric2F1
[1/2, (2 + n*p)/2, (4 + n*p)/2, Sin[e + f*x]^2]*Sin[e + f*x]^2*(c*(d*Sin[e
+ f*x])^p)^n)/(3*a^2*f*(2 + n*p)*Sqrt[Cos[e + f*x]^2]) + (2*(1 - n*p)*Cos[e
 + f*x]*Sin[e + f*x]*(c*(d*Sin[e + f*x])^p)^n)/(3*a^2*f*(1 + Sin[e + f*x]))
 + (Cos[e + f*x]*Sin[e + f*x]*(c*(d*Sin[e + f*x])^p)^n)/(3*f*(a + a*Sin[e +
 f*x])^2)
```


Rule 2826

```
Int[((c_.)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(p_))^(n_)*((a_.) + (b_.)*sin[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c^IntPart[n]*(c*(d*Sin[e + f*x
])^p)^FracPart[n])/(d*Sin[e + f*x])^(p*FracPart[n]), Int[(a + b*Sin[e + f*x
])^m*(d*Sin[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] &&  !IntegerQ[n]
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
```

```
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{\left(c(d\sin(e+fx))^p\right)^n}{(a+a\sin(e+fx))^2}\,dx = \left((d\sin(e+fx))^{-np}\left(c(d\sin(e+fx))^p\right)^n\right)\int \frac{(d\sin(e+fx))^{np}}{(a+a\sin(e+fx))^2}\,dx$$

$$= \frac{\cos(e+fx)\sin(e+fx)\left(c(d\sin(e+fx))^p\right)^n}{3f(a+a\sin(e+fx))^2} + \frac{\left((d\sin(e+fx))^{-np}\left(c(d\sin(e+fx))^p\right)^n\right)\int}{3a^2d}$$

$$= \frac{2(1-np)\cos(e+fx)\sin(e+fx)\left(c(d\sin(e+fx))^p\right)^n}{3a^2f(1+\sin(e+fx))} + \frac{\cos(e+fx)\sin(e+fx)\left(c(d\sin\right.}{3f(a+a\sin(e+fx))}$$

$$= \frac{2(1-np)\cos(e+fx)\sin(e+fx)\left(c(d\sin(e+fx))^p\right)^n}{3a^2f(1+\sin(e+fx))} + \frac{\cos(e+fx)\sin(e+fx)\left(c(d\sin\right.}{3f(a+a\sin(e+fx))}$$

$$= -\frac{np(1-2np)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(1+np);\frac{1}{2}(3+np);\sin^2(e+fx)\right)\sin(e+fx)\left(c(d\sin\right.}{3a^2f(1+np)\sqrt{\cos^2(e+fx)}}$$

Mathematica [A] time = 2.86747, size = 195, normalized size = 0.68

$$\frac{\sin(e+fx)\cos(e+fx)\left(c(d\sin(e+fx))^p\right)^n\left(-\frac{2\left(n^2p^2-1\right)\sqrt{\cos^2(e+fx)}\tan(e+fx)\sec(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{np}{2}+1;\frac{np}{2}+2;\sin^2(e+fx)\right)}{np+2}+\frac{np(2np-1)\,2}{}\right.}{3a^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*(d*Sin[e + f*x])^p)^n/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] (Cos[e + f*x]*Sin[e + f*x]*(c*(d*Sin[e + f*x])^p)^n*((n*p*(-1 + 2*n*p)*Hype
rgeometric2F1[1/2, (1 + n*p)/2, (3 + n*p)/2, Sin[e + f*x]^2])/((1 + n*p)*Sq
```

```
rt[Cos[e + f*x]^2]) + (3 - 2*n*p + (2 - 2*n*p)*Sin[e + f*x])/(1 + Sin[e + f
*x])^2 - (2*(-1 + n^2*p^2)*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[1/2, 1 +
(n*p)/2, 2 + (n*p)/2, Sin[e + f*x]^2]*Sec[e + f*x]*Tan[e + f*x])/(2 + n*p))
)/(3*a^2*f)
```

Maple [F] time = 0.349, size = 0, normalized size = 0.

$$\int \frac{\left(c\left(d\sin\left(fx+e\right)\right)^{p}\right)^{n}}{\left(a+a\sin\left(fx+e\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*sin(f*x+e))^p)^n/(a+a*sin(f*x+e))^2,x)
```

```
[Out] int((c*(d*sin(f*x+e))^p)^n/(a+a*sin(f*x+e))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(d\sin\left(fx+e\right)\right)^{p}c\right)^{n}}{\left(a\sin\left(fx+e\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n/(a+a*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate(((d*sin(f*x + e))^p*c)^n/(a*sin(f*x + e) + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(\left(d\sin\left(fx+e\right)\right)^{p}c\right)^{n}}{a^{2}\cos\left(fx+e\right)^{2}-2\,a^{2}\sin\left(fx+e\right)-2\,a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n/(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral(-((d*sin(f*x + e))^p*c)^n/(a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x + e)
 - 2*a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\left(c\left(d\sin\left(e+fx\right)\right)^{p}\right)^{n}}{\sin^{2}\left(e+fx\right)+2\sin\left(e+fx\right)+1}\,dx}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))**p)**n/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Integral((c*(d*sin(e + f*x))**p)**n/(sin(e + f*x)**2 + 2*sin(e + f*x) + 1),
 x)/a**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(d\sin\left(fx+e\right)\right)^{p}c\right)^{n}}{\left(a\sin\left(fx+e\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n/(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate(((d*sin(f*x + e))^p*c)^n/(a*sin(f*x + e) + a)^2, x)
```

3.825 $\int (d\csc(e+fx))^n(a+b\sin(e+fx))^3\,dx$

Optimal. Leaf size=298

$$\frac{bd^4\left(3a^2(3-n)+b^2(2-n)\right)\cos(e+fx)(d\csc(e+fx))^{n-4}{}_2F_1\left(\frac{1}{2},\frac{4-n}{2};\frac{6-n}{2};\sin^2(e+fx)\right)}{f(2-n)(4-n)\sqrt{\cos^2(e+fx)}}+\frac{ad^3\left(a^2(2-n)+3b^2(1-n)\right)\cdots}{\cdots}$$

```
[Out] (a^2*b*d^3*(1 - 2*n)*Cot[e + f*x]*(d*Csc[e + f*x])^(-3 + n))/(f*(1 - n)*(2
- n)) + (a^2*d^3*Cot[e + f*x]*(d*Csc[e + f*x])^(-3 + n)*(b + a*Csc[e + f*x]
))/(f*(1 - n)) + (a*d^3*(3*b^2*(1 - n) + a^2*(2 - n))*Cos[e + f*x]*(d*Csc[e
 + f*x])^(-3 + n)*Hypergeometric2F1[1/2, (3 - n)/2, (5 - n)/2, Sin[e + f*x]
^2])/(f*(1 - n)*(3 - n)*Sqrt[Cos[e + f*x]^2]) + (b*d^4*(b^2*(2 - n) + 3*a^2
*(3 - n))*Cos[e + f*x]*(d*Csc[e + f*x])^(-4 + n)*Hypergeometric2F1[1/2, (4
- n)/2, (6 - n)/2, Sin[e + f*x]^2])/(f*(2 - n)*(4 - n)*Sqrt[Cos[e + f*x]^2]
)
```

Rubi [A] time = 0.565934, antiderivative size = 298, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3238, 3842, 4047, 3772, 2643, 4046}

$$\frac{bd^4\left(3a^2(3-n)+b^2(2-n)\right)\cos(e+fx)(d\csc(e+fx))^{n-4}{}_2F_1\left(\frac{1}{2},\frac{4-n}{2};\frac{6-n}{2};\sin^2(e+fx)\right)}{f(2-n)(4-n)\sqrt{\cos^2(e+fx)}}+\frac{ad^3\left(a^2(2-n)+3b^2(1-n)\right)\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Csc[e + f*x])^n*(a + b*Sin[e + f*x])^3,x]
```

```
[Out] (a^2*b*d^3*(1 - 2*n)*Cot[e + f*x]*(d*Csc[e + f*x])^(-3 + n))/(f*(1 - n)*(2
- n)) + (a^2*d^3*Cot[e + f*x]*(d*Csc[e + f*x])^(-3 + n)*(b + a*Csc[e + f*x]
))/(f*(1 - n)) + (a*d^3*(3*b^2*(1 - n) + a^2*(2 - n))*Cos[e + f*x]*(d*Csc[e
 + f*x])^(-3 + n)*Hypergeometric2F1[1/2, (3 - n)/2, (5 - n)/2, Sin[e + f*x]
^2])/(f*(1 - n)*(3 - n)*Sqrt[Cos[e + f*x]^2]) + (b*d^4*(b^2*(2 - n) + 3*a^2
*(3 - n))*Cos[e + f*x]*(d*Csc[e + f*x])^(-4 + n)*Hypergeometric2F1[1/2, (4
- n)/2, (6 - n)/2, Sin[e + f*x]^2])/(f*(2 - n)*(4 - n)*Sqrt[Cos[e + f*x]^2]
)
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3842

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[1/(d*(m + n - 1)), Int[(a
+ b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*b^2
*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*d*(
3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x] &
& NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&
  !(IGtQ[n, 2] &&  !IntegerQ[m])
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int (d\csc(e+fx))^n(a+b\sin(e+fx))^3\,dx = d^3\int (d\csc(e+fx))^{-3+n}(b+a\csc(e+fx))^3\,dx$$

$$= \frac{a^2d^3\cot(e+fx)(d\csc(e+fx))^{-3+n}(b+a\csc(e+fx))}{f(1-n)} - \frac{d^2\int (d\csc(e+f$$

$$= \frac{a^2d^3\cot(e+fx)(d\csc(e+fx))^{-3+n}(b+a\csc(e+fx))}{f(1-n)} - \frac{d^2\int (d\csc(e+f$$

$$= \frac{a^2bd^3(1-2n)\cot(e+fx)(d\csc(e+fx))^{-3+n}}{f(1-n)(2-n)} + \frac{a^2d^3\cot(e+fx)(d\csc(e+}{f(}$$

$$= \frac{a^2bd^3(1-2n)\cot(e+fx)(d\csc(e+fx))^{-3+n}}{f(1-n)(2-n)} + \frac{a^2d^3\cot(e+fx)(d\csc(e+}{f(}$$

$$= \frac{a^2bd^3(1-2n)\cot(e+fx)(d\csc(e+fx))^{-3+n}}{f(1-n)(2-n)} + \frac{a^2d^3\cot(e+fx)(d\csc(e+}{f(}$$

Mathematica [A] time = 0.543765, size = 167, normalized size = 0.56

$$-\frac{d\cos(e+fx)\sin^2(e+fx)^{\frac{n-1}{2}}(d\csc(e+fx))^{n-1}\left(b\sqrt{\sin^2(e+fx)}\csc(e+fx)\left(3a^2\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{3}{2};\cos^2(e+fx)\right)+b^2\,{}_2F_1}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Csc[e + f*x])^n*(a + b*Sin[e + f*x])^3,x]
```

```
[Out] -((d*Cos[e + f*x]*(d*Csc[e + f*x])^(-1 + n)*(Sin[e + f*x]^2)^((-1 + n)/2)*(
3*a*b^2*Hypergeometric2F1[1/2, (-1 + n)/2, 3/2, Cos[e + f*x]^2] + a^3*Hyper
```

```
geometric2F1[1/2, (1 + n)/2, 3/2, Cos[e + f*x]^2] + b*Csc[e + f*x]*(b^2*Hyp
ergeometric2F1[1/2, (-2 + n)/2, 3/2, Cos[e + f*x]^2] + 3*a^2*Hypergeometric
2F1[1/2, n/2, 3/2, Cos[e + f*x]^2])*Sqrt[Sin[e + f*x]^2]))/f)
```

Maple [F] time = 2.937, size = 0, normalized size = 0.

$$\int \left(d\csc\left(fx+e\right)\right)^{n}\left(a+b\sin\left(fx+e\right)\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(f*x+e))^n*(a+b*sin(f*x+e))^3,x)
```

```
[Out] int((d*csc(f*x+e))^n*(a+b*sin(f*x+e))^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)+a\right)^{3}\left(d\csc\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^n*(a+b*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)^3*(d*csc(f*x + e))^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\left(3\,ab^{2}\cos\left(fx+e\right)^{2}-a^{3}-3\,ab^{2}+\left(b^{3}\cos\left(fx+e\right)^{2}-3\,a^{2}b-b^{3}\right)\sin\left(fx+e\right)\right)\left(d\csc\left(fx+e\right)\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^n*(a+b*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] integral(-(3*a*b^2*cos(f*x + e)^2 - a^3 - 3*a*b^2 + (b^3*cos(f*x + e)^2 - 3
*a^2*b - b^3)*sin(f*x + e))*(d*csc(f*x + e))^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\csc\left(e+fx\right)\right)^{n}\left(a+b\sin\left(e+fx\right)\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))**n*(a+b*sin(f*x+e))**3,x)
```

```
[Out] Integral((d*csc(e + f*x))**n*(a + b*sin(e + f*x))**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)+a\right)^{3}\left(d\csc\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^n*(a+b*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)^3*(d*csc(f*x + e))^n, x)
```

3.826 $\int (d\csc(e+fx))^n(a+b\sin(e+fx))^2\,dx$

Optimal. Leaf size=213

$$\frac{d^3\left(a^2(2-n)+b^2(1-n)\right)\cos(e+fx)(d\csc(e+fx))^{n-3}\,{}_2F_1\left(\frac{1}{2},\frac{3-n}{2};\frac{5-n}{2};\sin^2(e+fx)\right)}{f(1-n)(3-n)\sqrt{\cos^2(e+fx)}}+\frac{a^2d^2\cot(e+fx)(d\csc(e+fx))^{n}}{f(1-n)}$$

```
[Out] (a^2*d^2*Cot[e + f*x]*(d*Csc[e + f*x])^(-2 + n))/(f*(1 - n)) + (2*a*b*d^2*C
os[e + f*x]*(d*Csc[e + f*x])^(-2 + n)*Hypergeometric2F1[1/2, (2 - n)/2, (4
- n)/2, Sin[e + f*x]^2])/(f*(2 - n)*Sqrt[Cos[e + f*x]^2]) + (d^3*(b^2*(1 -
n) + a^2*(2 - n))*Cos[e + f*x]*(d*Csc[e + f*x])^(-3 + n)*Hypergeometric2F1[
1/2, (3 - n)/2, (5 - n)/2, Sin[e + f*x]^2])/(f*(1 - n)*(3 - n)*Sqrt[Cos[e +
 f*x]^2])
```

Rubi [A] time = 0.269088, antiderivative size = 213, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3238, 3788, 3772, 2643, 4046}

$$\frac{d^3\left(a^2(2-n)+b^2(1-n)\right)\cos(e+fx)(d\csc(e+fx))^{n-3}\,{}_2F_1\left(\frac{1}{2},\frac{3-n}{2};\frac{5-n}{2};\sin^2(e+fx)\right)}{f(1-n)(3-n)\sqrt{\cos^2(e+fx)}}+\frac{a^2d^2\cot(e+fx)(d\csc(e+fx))^{n}}{f(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Csc[e + f*x])^n*(a + b*Sin[e + f*x])^2,x]
```

```
[Out] (a^2*d^2*Cot[e + f*x]*(d*Csc[e + f*x])^(-2 + n))/(f*(1 - n)) + (2*a*b*d^2*C
os[e + f*x]*(d*Csc[e + f*x])^(-2 + n)*Hypergeometric2F1[1/2, (2 - n)/2, (4
- n)/2, Sin[e + f*x]^2])/(f*(2 - n)*Sqrt[Cos[e + f*x]^2]) + (d^3*(b^2*(1 -
n) + a^2*(2 - n))*Cos[e + f*x]*(d*Csc[e + f*x])^(-3 + n)*Hypergeometric2F1[
1/2, (3 - n)/2, (5 - n)/2, Sin[e + f*x]^2])/(f*(1 - n)*(3 - n)*Sqrt[Cos[e +
 f*x]^2])
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
```

```
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int (d\csc(e+fx))^n(a+b\sin(e+fx))^2\,dx = d^2\int (d\csc(e+fx))^{-2+n}(b+a\csc(e+fx))^2\,dx$$
$$= (2abd)\int (d\csc(e+fx))^{-1+n}\,dx + d^2\int (d\csc(e+fx))^{-2+n}\left(b^2+a^2\csc^2(e\right.$$
$$= \frac{a^2d^2\cot(e+fx)(d\csc(e+fx))^{-2+n}}{f(1-n)} + \left(d^2\left(b^2+\frac{a^2(2-n)}{1-n}\right)\right)\int (d\csc(e+$$
$$= \frac{a^2d^2\cot(e+fx)(d\csc(e+fx))^{-2+n}}{f(1-n)} + \frac{2ab\cos(e+fx)(d\csc(e+fx))^n\,{}_2F_1}{f(2-n)}$$
$$= \frac{a^2d^2\cot(e+fx)(d\csc(e+fx))^{-2+n}}{f(1-n)} + \frac{2ab\cos(e+fx)(d\csc(e+fx))^n\,{}_2F_1}{f(2-n)}$$

Mathematica [A] time = 0.374521, size = 135, normalized size = 0.63

$$-\frac{d\cos(e+fx)\sin^2(e+fx)^{\frac{n-1}{2}}(d\csc(e+fx))^{n-1}\left(a\left(a\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{3}{2};\cos^2(e+fx)\right)+2b\sqrt{\sin^2(e+fx)}\csc(e+fx)\,{}_2F_1\right.\right.}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Csc[e + f*x])^n*(a + b*Sin[e + f*x])^2,x]
```

```
[Out] -((d*Cos[e + f*x]*(d*Csc[e + f*x])^(-1 + n)*(Sin[e + f*x]^2)^((-1 + n)/2)*(
b^2*Hypergeometric2F1[1/2, (-1 + n)/2, 3/2, Cos[e + f*x]^2] + a*(a*Hypergeo
metric2F1[1/2, (1 + n)/2, 3/2, Cos[e + f*x]^2] + 2*b*Csc[e + f*x]*Hypergeom
etric2F1[1/2, n/2, 3/2, Cos[e + f*x]^2]*Sqrt[Sin[e + f*x]^2])))/f)
```

Maple [F] time = 3.652, size = 0, normalized size = 0.

$$\int \left(d\csc\left(fx+e\right)\right)^n\left(a+b\sin\left(fx+e\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(f*x+e))^n*(a+b*sin(f*x+e))^2,x)
```

```
[Out] int((d*csc(f*x+e))^n*(a+b*sin(f*x+e))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)+a\right)^2\left(d\csc\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^n*(a+b*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)^2*(d*csc(f*x + e))^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(b^2\cos\left(fx+e\right)^2-2\,ab\sin\left(fx+e\right)-a^2-b^2\right)\left(d\csc\left(fx+e\right)\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^n*(a+b*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral(-(b^2*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2)*(d*csc(f*x
+ e))^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\csc\left(e+fx\right)\right)^n\left(a+b\sin\left(e+fx\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))**n*(a+b*sin(f*x+e))**2,x)
```

```
[Out] Integral((d*csc(e + f*x))**n*(a + b*sin(e + f*x))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)+a\right)^2\left(d\csc\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^n*(a+b*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)^2*(d*csc(f*x + e))^n, x)
```

3.827 $\int (d\csc(e+fx))^n(a+b\sin(e+fx))\,dx$

Optimal. Leaf size=149

$$\frac{ad\cos(e+fx)(d\csc(e+fx))^{n-1}\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+fx)\right)}{f(1-n)\sqrt{\cos^2(e+fx)}}+\frac{bd^2\cos(e+fx)(d\csc(e+fx))^{n-2}\,{}_2F_1\left(\frac{1}{2},\frac{2-n}{2};\frac{4-n}{2};\text{s}\right.}{f(2-n)\sqrt{\cos^2(e+fx)}}$$

```
[Out] (a*d*Cos[e + f*x]*(d*Csc[e + f*x])^(-1 + n)*Hypergeometric2F1[1/2, (1 - n)/
2, (3 - n)/2, Sin[e + f*x]^2])/(f*(1 - n)*Sqrt[Cos[e + f*x]^2]) + (b*d^2*Co
s[e + f*x]*(d*Csc[e + f*x])^(-2 + n)*Hypergeometric2F1[1/2, (2 - n)/2, (4 -
 n)/2, Sin[e + f*x]^2])/(f*(2 - n)*Sqrt[Cos[e + f*x]^2])
```

Rubi [A] time = 0.148498, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3238, 3787, 3772, 2643}

$$\frac{ad\cos(e+fx)(d\csc(e+fx))^{n-1}\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+fx)\right)}{f(1-n)\sqrt{\cos^2(e+fx)}}+\frac{bd^2\cos(e+fx)(d\csc(e+fx))^{n-2}\,{}_2F_1\left(\frac{1}{2},\frac{2-n}{2};\frac{4-n}{2};\text{s}\right.}{f(2-n)\sqrt{\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Csc[e + f*x])^n*(a + b*Sin[e + f*x]),x]
```


```
[Out] (a*d*Cos[e + f*x]*(d*Csc[e + f*x])^(-1 + n)*Hypergeometric2F1[1/2, (1 - n)/
2, (3 - n)/2, Sin[e + f*x]^2])/(f*(1 - n)*Sqrt[Cos[e + f*x]^2]) + (b*d^2*Co
s[e + f*x]*(d*Csc[e + f*x])^(-2 + n)*Hypergeometric2F1[1/2, (2 - n)/2, (4 -
 n)/2, Sin[e + f*x]^2])/(f*(2 - n)*Sqrt[Cos[e + f*x]^2])
```


Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int (d\csc(e+fx))^n(a+b\sin(e+fx))\,dx = d\int (d\csc(e+fx))^{-1+n}(b+a\csc(e+fx))\,dx$$
$$= a\int (d\csc(e+fx))^n\,dx + (bd)\int (d\csc(e+fx))^{-1+n}\,dx$$
$$= \left(a(d\csc(e+fx))^n\left(\frac{\sin(e+fx)}{d}\right)^n\right)\int\left(\frac{\sin(e+fx)}{d}\right)^{-n}dx + \left(bd(d\csc(e+fx)\right.$$
$$= \frac{a\cos(e+fx)(d\csc(e+fx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+fx)\right)\sin(e+fx)}{f(1-n)\sqrt{\cos^2(e+fx)}} + \frac{b\,\mathrm{cc}}{}$$

Mathematica [A] time = 0.246177, size = 105, normalized size = 0.7

$$-\frac{d\cos(e+fx)\sin^2(e+fx)^{\frac{n-1}{2}}(d\csc(e+fx))^{n-1}\left(a\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{3}{2};\cos^2(e+fx)\right)+b\sqrt{\sin^2(e+fx)}\csc(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\right.\right.}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Csc[e + f*x])^n*(a + b*Sin[e + f*x]),x]
```

```
[Out] -((d*Cos[e + f*x]*(d*Csc[e + f*x])^(-1 + n)*(Sin[e + f*x]^2)^((-1 + n)/2)*(
a*Hypergeometric2F1[1/2, (1 + n)/2, 3/2, Cos[e + f*x]^2] + b*Csc[e + f*x]*H
ypergeometric2F1[1/2, n/2, 3/2, Cos[e + f*x]^2]*Sqrt[Sin[e + f*x]^2]))/f)
```

Maple [F] time = 1.145, size = 0, normalized size = 0.

$$\int \left(d\csc\left(fx+e\right)\right)^n\left(a+b\sin\left(fx+e\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(f*x+e))^n*(a+b*sin(f*x+e)),x)
```

```
[Out] int((d*csc(f*x+e))^n*(a+b*sin(f*x+e)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)+a\right)\left(d\csc\left(fx+e\right)\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^n*(a+b*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)*(d*csc(f*x + e))^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b\sin\left(fx+e\right)+a\right)\left(d\csc\left(fx+e\right)\right)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^n*(a+b*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral((b*sin(f*x + e) + a)*(d*csc(f*x + e))^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\csc\left(e+fx\right)\right)^{n}\left(a+b\sin\left(e+fx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))**n*(a+b*sin(f*x+e)),x)
```

```
[Out] Integral((d*csc(e + f*x))**n*(a + b*sin(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)+a\right)\left(d\csc\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^n*(a+b*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)*(d*csc(f*x + e))^n, x)
```

3.828 $\int \frac{(d\csc(e+fx))^n}{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=204

$$\frac{b\sin(e+fx)\cos(e+fx)\sin^2(e+fx)^{n/2}(d\csc(e+fx))^{n+1}F_1\left(\frac{1}{2};\frac{n}{2},1;\frac{3}{2};\cos^2(e+fx),-\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)}{df\left(a^2-b^2\right)}-\frac{a\cos(e+fx)\sin^2(}{}$$

```
[Out] (b*AppellF1[1/2, n/2, 1, 3/2, Cos[e + f*x]^2, -((b^2*Cos[e + f*x]^2)/(a^2 -
 b^2))]*Cos[e + f*x]*(d*Csc[e + f*x])^(1 + n)*Sin[e + f*x]*(Sin[e + f*x]^2)
^(n/2))/((a^2 - b^2)*d*f) - (a*AppellF1[1/2, (1 + n)/2, 1, 3/2, Cos[e + f*x
]^2, -((b^2*Cos[e + f*x]^2)/(a^2 - b^2))]*Cos[e + f*x]*(d*Csc[e + f*x])^(1
+ n)*(Sin[e + f*x]^2)^((1 + n)/2))/((a^2 - b^2)*d*f)
```

Rubi [A] time = 0.398981, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3238, 3869, 2823, 3189, 429}

$$\frac{b\sin(e+fx)\cos(e+fx)\sin^2(e+fx)^{n/2}(d\csc(e+fx))^{n+1}F_1\left(\frac{1}{2};\frac{n}{2},1;\frac{3}{2};\cos^2(e+fx),-\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)}{df\left(a^2-b^2\right)}-\frac{a\cos(e+fx)\sin^2(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Csc[e + f*x])^n/(a + b*Sin[e + f*x]),x]
```


```
[Out] (b*AppellF1[1/2, n/2, 1, 3/2, Cos[e + f*x]^2, -((b^2*Cos[e + f*x]^2)/(a^2 -
 b^2))]*Cos[e + f*x]*(d*Csc[e + f*x])^(1 + n)*Sin[e + f*x]*(Sin[e + f*x]^2)
^(n/2))/((a^2 - b^2)*d*f) - (a*AppellF1[1/2, (1 + n)/2, 1, 3/2, Cos[e + f*x
]^2, -((b^2*Cos[e + f*x]^2)/(a^2 - b^2))]*Cos[e + f*x]*(d*Csc[e + f*x])^(1
+ n)*(Sin[e + f*x]^2)^((1 + n)/2))/((a^2 - b^2)*d*f)
```


Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3869

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Dist[Sin[e + f*x]^n*(d*Csc[e + f*x])^n, Int[(b +
a*Sin[e + f*x])^m/Sin[e + f*x]^(m + n), x], x] /; FreeQ[{a, b, d, e, f, n},
 x] && NeQ[a^2 - b^2, 0] && IntegerQ[m]
```

Rule 2823

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[a, Int[(d*Sin[e + f*x])^n/(a^2 - b^2*Sin[e + f*x]^
2), x], x] - Dist[b/d, Int[(d*Sin[e + f*x])^(n + 1)/(a^2 - b^2*Sin[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3189

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[
```

```
(ff*d^(2*IntPart[(m - 1)/2] + 1)*(d*Sin[e + f*x])^(2*FracPart[(m - 1)/2]))/
(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2]), Subst[Int[(1 - ff^2*x^2)^((m - 1)
/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, d,
 e, f, m, p}, x] &&  !IntegerQ[m]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{(d\csc(e+fx))^n}{a+b\sin(e+fx)}\,dx = \frac{\int \frac{(d\csc(e+fx))^{1+n}}{b+a\csc(e+fx)}\,dx}{d}$$

$$= \frac{\left((d\csc(e+fx))^{1+n}\sin^{1+n}(e+fx)\right)\int \frac{\sin^{-n}(e+fx)}{a+b\sin(e+fx)}\,dx}{d}$$

$$= \frac{\left(a(d\csc(e+fx))^{1+n}\sin^{1+n}(e+fx)\right)\int \frac{\sin^{-n}(e+fx)}{a^2-b^2\sin^2(e+fx)}\,dx}{d} - \frac{\left(b(d\csc(e+fx))^{1+n}\sin^{1+n}(e+f\right.}{d}$$

$$= -\frac{\left(a(d\csc(e+fx))^{1+n}\sin^{1+2\left(-\frac{1}{2}-\frac{n}{2}\right)+n}(e+fx)\sin^2(e+fx)^{\frac{1}{2}+\frac{n}{2}}\right)\text{Subst}\left(\int \frac{\left(1-x^2\right)^{\frac{1}{2}(-1-n)}}{a^2-b^2+b^2x^2}\,dx,x,\right.}{df}$$

$$= \frac{bF_1\left(\frac{1}{2};\frac{n}{2},1;\frac{3}{2};\cos^2(e+fx),-\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)\cos(e+fx)(d\csc(e+fx))^{1+n}\sin(e+fx)\sin^2(e\cdot}{\left(a^2-b^2\right)df}$$

Mathematica [B] time = 16.9243, size = 1668, normalized size = 8.18

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Csc[e + f*x])^n/(a + b*Sin[e + f*x]),x]
```

```
[Out] -(((d*Csc[e + f*x])^n*(Cot[e + f*x]*Sqrt[Sec[e + f*x]^2])^n*Tan[e + f*x]*(a
*b*(-2 + n)*AppellF1[(1 - n)/2, -n/2, 1, (3 - n)/2, -Tan[e + f*x]^2, (-1 +
b^2/a^2)*Tan[e + f*x]^2] + (-1 + n)*((a^2 - b^2)*AppellF1[1 - n/2, (-1 - n)
/2, 1, 2 - n/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2] - a^2*H
ypergeometric2F1[1/2 - n/2, 1 - n/2, 2 - n/2, -Tan[e + f*x]^2])*Tan[e + f*x
]))/(a^2*b*f*(-2 + n)*(-1 + n)*(Sec[e + f*x]^2)^(n/2)*(a + b*Sin[e + f*x])*
(-(((Sec[e + f*x]^2)^(1 - n/2)*(Cot[e + f*x]*Sqrt[Sec[e + f*x]^2])^n*(a*b*(
-2 + n)*AppellF1[(1 - n)/2, -n/2, 1, (3 - n)/2, -Tan[e + f*x]^2, (-1 + b^2/
a^2)*Tan[e + f*x]^2] + (-1 + n)*((a^2 - b^2)*AppellF1[1 - n/2, (-1 - n)/2,
1, 2 - n/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2] - a^2*Hyper
geometric2F1[1/2 - n/2, 1 - n/2, 2 - n/2, -Tan[e + f*x]^2])*Tan[e + f*x]))/
(a^2*b*(-2 + n)*(-1 + n))) - (n*(Cot[e + f*x]*Sqrt[Sec[e + f*x]^2])^(-1 + n
)*(Sqrt[Sec[e + f*x]^2] - Csc[e + f*x]^2*Sqrt[Sec[e + f*x]^2])*Tan[e + f*x]
*(a*b*(-2 + n)*AppellF1[(1 - n)/2, -n/2, 1, (3 - n)/2, -Tan[e + f*x]^2, (-1
 + b^2/a^2)*Tan[e + f*x]^2] + (-1 + n)*((a^2 - b^2)*AppellF1[1 - n/2, (-1 -
 n)/2, 1, 2 - n/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2] - a^
2*Hypergeometric2F1[1/2 - n/2, 1 - n/2, 2 - n/2, -Tan[e + f*x]^2])*Tan[e +
f*x]))/(a^2*b*(-2 + n)*(-1 + n)*(Sec[e + f*x]^2)^(n/2)) + (n*(Cot[e + f*x]*
```

```
Sqrt[Sec[e + f*x]^2])^n*Tan[e + f*x]^2*(a*b*(-2 + n)*AppellF1[(1 - n)/2, -n
/2, 1, (3 - n)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2] + (-1 + n
)*((a^2 - b^2)*AppellF1[1 - n/2, (-1 - n)/2, 1, 2 - n/2, -Tan[e + f*x]^2, (
(-a^2 + b^2)*Tan[e + f*x]^2)/a^2] - a^2*Hypergeometric2F1[1/2 - n/2, 1 - n/
2, 2 - n/2, -Tan[e + f*x]^2])*Tan[e + f*x]))/(a^2*b*(-2 + n)*(-1 + n)*(Sec[
e + f*x]^2)^(n/2)) - ((Cot[e + f*x]*Sqrt[Sec[e + f*x]^2])^n*Tan[e + f*x]*((
-1 + n)*((a^2 - b^2)*AppellF1[1 - n/2, (-1 - n)/2, 1, 2 - n/2, -Tan[e + f*x
]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2] - a^2*Hypergeometric2F1[1/2 - n/2,
1 - n/2, 2 - n/2, -Tan[e + f*x]^2])*Sec[e + f*x]^2 + a*b*(-2 + n)*(((1 - n)
*n*AppellF1[1 + (1 - n)/2, 1 - n/2, 1, 1 + (3 - n)/2, -Tan[e + f*x]^2, (-1
+ b^2/a^2)*Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(3 - n) + (2*(-1 +
b^2/a^2)*(1 - n)*AppellF1[1 + (1 - n)/2, -n/2, 2, 1 + (3 - n)/2, -Tan[e + f
*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(3 - n))
 + (-1 + n)*Tan[e + f*x]*((a^2 - b^2)*(-(((-1 - n)*(1 - n/2)*AppellF1[2 - n
/2, 1 + (-1 - n)/2, 1, 3 - n/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]
^2)/a^2]*Sec[e + f*x]^2*Tan[e + f*x])/(2 - n/2)) + (2*(-a^2 + b^2)*(1 - n/2
)*AppellF1[2 - n/2, (-1 - n)/2, 2, 3 - n/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*
Tan[e + f*x]^2)/a^2]*Sec[e + f*x]^2*Tan[e + f*x])/(a^2*(2 - n/2))) - 2*a^2*
(1 - n/2)*Csc[e + f*x]*Sec[e + f*x]*(-Hypergeometric2F1[1/2 - n/2, 1 - n/2,
 2 - n/2, -Tan[e + f*x]^2] + (1 + Tan[e + f*x]^2)^(-1/2 + n/2)))))/(a^2*b*(
-2 + n)*(-1 + n)*(Sec[e + f*x]^2)^(n/2)))))
```

Maple [F] time = 0.493, size = 0, normalized size = 0.

$$\int \frac{\left(d \csc \left(f x + e\right)\right)^{n}}{a + b \sin \left(f x + e\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(f*x+e))^n/(a+b*sin(f*x+e)),x)
```

```
[Out] int((d*csc(f*x+e))^n/(a+b*sin(f*x+e)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d \csc \left(f x + e\right)\right)^{n}}{b \sin \left(f x + e\right) + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^n/(a+b*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((d*csc(f*x + e))^n/(b*sin(f*x + e) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(d \csc \left(f x + e\right)\right)^{n}}{b \sin \left(f x + e\right) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^n/(a+b*sin(f*x+e)),x, algorithm="fricas")

[Out] integral((d*csc(f*x + e))^n/(b*sin(f*x + e) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\csc\left(e+fx\right)\right)^n}{a+b\sin\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))**n/(a+b*sin(f*x+e)),x)

[Out] Integral((d*csc(e + f*x))**n/(a + b*sin(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\csc\left(fx+e\right)\right)^n}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^n/(a+b*sin(f*x+e)),x, algorithm="giac")

[Out] integrate((d*csc(f*x + e))^n/(b*sin(f*x + e) + a), x)
```

3.829 $\int \frac{(d\csc(e+fx))^n}{(a+b\sin(e+fx))^2}\,dx$

Optimal. Leaf size=321

$$-\frac{b^2\sin^3(e+fx)\cos(e+fx)\sin^2(e+fx)^{\frac{n-1}{2}}(d\csc(e+fx))^{n+2}F_1\left(\frac{1}{2};\frac{n-1}{2},2;\frac{3}{2};\cos^2(e+fx),-\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)}{d^2f\left(a^2-b^2\right)^2}-\frac{a^2\sin(e+f\ldots}{\ldots}$$

```
[Out] -((b^2*AppellF1[1/2, (-1 + n)/2, 2, 3/2, Cos[e + f*x]^2, -((b^2*Cos[e + f*x
]^2)/(a^2 - b^2))]*Cos[e + f*x]*(d*Csc[e + f*x])^(2 + n)*Sin[e + f*x]^3*(Si
n[e + f*x]^2)^((-1 + n)/2))/((a^2 - b^2)^2*d^2*f)) - (a^2*AppellF1[1/2, (1
+ n)/2, 2, 3/2, Cos[e + f*x]^2, -((b^2*Cos[e + f*x]^2)/(a^2 - b^2))]*Cos[e
+ f*x]*(d*Csc[e + f*x])^(2 + n)*Sin[e + f*x]*(Sin[e + f*x]^2)^((1 + n)/2))/
((a^2 - b^2)^2*d^2*f) + (2*a*b*AppellF1[1/2, n/2, 2, 3/2, Cos[e + f*x]^2, -
((b^2*Cos[e + f*x]^2)/(a^2 - b^2))]*Cos[e + f*x]*(d*Csc[e + f*x])^(2 + n)*(
Sin[e + f*x]^2)^((2 + n)/2))/((a^2 - b^2)^2*d^2*f)
```

Rubi [A] time = 0.543721, antiderivative size = 321, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3238, 3869, 2824, 3189, 429}

$$-\frac{b^2\sin^3(e+fx)\cos(e+fx)\sin^2(e+fx)^{\frac{n-1}{2}}(d\csc(e+fx))^{n+2}F_1\left(\frac{1}{2};\frac{n-1}{2},2;\frac{3}{2};\cos^2(e+fx),-\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)}{d^2f\left(a^2-b^2\right)^2}-\frac{a^2\sin(e+f\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Csc[e + f*x])^n/(a + b*Sin[e + f*x])^2,x]
```


```
[Out] -((b^2*AppellF1[1/2, (-1 + n)/2, 2, 3/2, Cos[e + f*x]^2, -((b^2*Cos[e + f*x
]^2)/(a^2 - b^2))]*Cos[e + f*x]*(d*Csc[e + f*x])^(2 + n)*Sin[e + f*x]^3*(Si
n[e + f*x]^2)^((-1 + n)/2))/((a^2 - b^2)^2*d^2*f)) - (a^2*AppellF1[1/2, (1
+ n)/2, 2, 3/2, Cos[e + f*x]^2, -((b^2*Cos[e + f*x]^2)/(a^2 - b^2))]*Cos[e
+ f*x]*(d*Csc[e + f*x])^(2 + n)*Sin[e + f*x]*(Sin[e + f*x]^2)^((1 + n)/2))/
((a^2 - b^2)^2*d^2*f) + (2*a*b*AppellF1[1/2, n/2, 2, 3/2, Cos[e + f*x]^2, -
((b^2*Cos[e + f*x]^2)/(a^2 - b^2))]*Cos[e + f*x]*(d*Csc[e + f*x])^(2 + n)*(
Sin[e + f*x]^2)^((2 + n)/2))/((a^2 - b^2)^2*d^2*f)
```


Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3869

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Dist[Sin[e + f*x]^n*(d*Csc[e + f*x])^n, Int[(b +
a*Sin[e + f*x])^m/Sin[e + f*x]^(m + n), x], x] /; FreeQ[{a, b, d, e, f, n},
 x] && NeQ[a^2 - b^2, 0] && IntegerQ[m]
```

Rule 2824

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(d*sin[e + f*x])^n/((a - b*sin[e +
```

```
f*x])^m/(a^2 - b^2*sin[e + f*x]^2)^m), x], x] /; FreeQ[{a, b, d, e, f, n},
 x] && NeQ[a^2 - b^2, 0] && ILtQ[m, -1]
```

Rule 3189

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[
(ff*d^(2*IntPart[(m - 1)/2] + 1)*(d*Sin[e + f*x])^(2*FracPart[(m - 1)/2]))/
(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2]), Subst[Int[(1 - ff^2*x^2)^((m - 1)
/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, d,
 e, f, m, p}, x] &&  !IntegerQ[m]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{(d\csc(e+fx))^n}{(a+b\sin(e+fx))^2}\,dx = \frac{\int \frac{(d\csc(e+fx))^{2+n}}{(b+a\csc(e+fx))^2}\,dx}{d^2}$$
$$= \frac{\left((d\csc(e+fx))^{2+n}\sin^{2+n}(e+fx)\right)\int \frac{\sin^{-n}(e+fx)}{(a+b\sin(e+fx))^2}\,dx}{d^2}$$
$$= \frac{\left((d\csc(e+fx))^{2+n}\sin^{2+n}(e+fx)\right)\int\left(-\frac{2ab\sin^{1-n}(e+fx)}{\left(a^2-b^2\sin^2(e+fx)\right)^2}+\frac{a^2\sin^{-n}(e+fx)}{\left(a^2-b^2\sin^2(e+fx)\right)^2}+\frac{b^2\sin^{2-n}(e}{\left(-a^2+b^2\sin^2(\right.}\right.}{d^2}$$
$$= \frac{\left(a^2(d\csc(e+fx))^{2+n}\sin^{2+n}(e+fx)\right)\int \frac{\sin^{-n}(e+fx)}{\left(a^2-b^2\sin^2(e+fx)\right)^2}\,dx}{d^2} - \frac{\left(2ab(d\csc(e+fx))^{2+n}\sin^2\right.}{}$$
$$= -\frac{\left(b^2(d\csc(e+fx))^{2+n}\sin^{2+2\left(\frac{1}{2}-\frac{n}{2}\right)+n}(e+fx)\sin^2(e+fx)^{-\frac{1}{2}+\frac{n}{2}}\right)\text{Subst}\left(\int \frac{\left(1-x^2\right)^{\frac{1-n}{2}}}{\left(-a^2+b^2-b^2x^2\right)^2}\,\right.}{d^2f}$$
$$= -\frac{b^2F_1\left(\frac{1}{2};\frac{1}{2}(-1+n),2;\frac{3}{2};\cos^2(e+fx),-\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)\cos(e+fx)(d\csc(e+fx))^{2+n}\sin^3(}{\left(a^2-b^2\right)^2d^2f}$$

Mathematica [B] time = 19.0735, size = 1938, normalized size = 6.04

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Csc[e + f*x])^n/(a + b*Sin[e + f*x])^2,x]
```

```
[Out] ((d*Csc[e + f*x])^n*(Cot[e + f*x]*Sqrt[Sec[e + f*x]^2])^n*Tan[e + f*x]*(-(a
*(a^2 + b^2)*(-2 + n)*AppellF1[(1 - n)/2, -n/2, 1, (3 - n)/2, -Tan[e + f*x]
^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]) + 2*b*(a*b*(-2 + n)*AppellF1[(1 - n
)/2, -n/2, 2, (3 - n)/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2
] + (a^2 - b^2)*(-1 + n)*AppellF1[1 - n/2, (-1 - n)/2, 2, 2 - n/2, -Tan[e +
 f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]*Tan[e + f*x])))/(a^3*(a^2 - b^2
)*f*(-2 + n)*(-1 + n)*(Sec[e + f*x]^2)^(n/2)*(a + b*Sin[e + f*x])^2*(((Sec[
```

```
e + f*x]^2)^(1 - n/2)*(Cot[e + f*x]*Sqrt[Sec[e + f*x]^2])^n*(-(a*(a^2 + b^2
)*(-2 + n)*AppellF1[(1 - n)/2, -n/2, 1, (3 - n)/2, -Tan[e + f*x]^2, ((-a^2
+ b^2)*Tan[e + f*x]^2)/a^2]) + 2*b*(a*b*(-2 + n)*AppellF1[(1 - n)/2, -n/2,
2, (3 - n)/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2] + (a^2 -
b^2)*(-1 + n)*AppellF1[1 - n/2, (-1 - n)/2, 2, 2 - n/2, -Tan[e + f*x]^2, ((
-a^2 + b^2)*Tan[e + f*x]^2)/a^2]*Tan[e + f*x])))/(a^3*(a^2 - b^2)*(-2 + n)*
(-1 + n)) + (n*(Cot[e + f*x]*Sqrt[Sec[e + f*x]^2])^(-1 + n)*(Sqrt[Sec[e + f
*x]^2] - Csc[e + f*x]^2*Sqrt[Sec[e + f*x]^2])*Tan[e + f*x]*(-(a*(a^2 + b^2)
*(-2 + n)*AppellF1[(1 - n)/2, -n/2, 1, (3 - n)/2, -Tan[e + f*x]^2, ((-a^2 +
 b^2)*Tan[e + f*x]^2)/a^2]) + 2*b*(a*b*(-2 + n)*AppellF1[(1 - n)/2, -n/2, 2
, (3 - n)/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2] + (a^2 - b
^2)*(-1 + n)*AppellF1[1 - n/2, (-1 - n)/2, 2, 2 - n/2, -Tan[e + f*x]^2, ((-
a^2 + b^2)*Tan[e + f*x]^2)/a^2]*Tan[e + f*x])))/(a^3*(a^2 - b^2)*(-2 + n)*(
-1 + n)*(Sec[e + f*x]^2)^(n/2)) - (n*(Cot[e + f*x]*Sqrt[Sec[e + f*x]^2])^n*
Tan[e + f*x]^2*(-(a*(a^2 + b^2)*(-2 + n)*AppellF1[(1 - n)/2, -n/2, 1, (3 -
n)/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]) + 2*b*(a*b*(-2 +
 n)*AppellF1[(1 - n)/2, -n/2, 2, (3 - n)/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*
Tan[e + f*x]^2)/a^2] + (a^2 - b^2)*(-1 + n)*AppellF1[1 - n/2, (-1 - n)/2, 2
, 2 - n/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]*Tan[e + f*x]
)))/(a^3*(a^2 - b^2)*(-2 + n)*(-1 + n)*(Sec[e + f*x]^2)^(n/2)) + ((Cot[e +
f*x]*Sqrt[Sec[e + f*x]^2])^n*Tan[e + f*x]*(-(a*(a^2 + b^2)*(-2 + n)*(((1 -
n)*n*AppellF1[1 + (1 - n)/2, 1 - n/2, 1, 1 + (3 - n)/2, -Tan[e + f*x]^2, ((
-a^2 + b^2)*Tan[e + f*x]^2)/a^2]*Sec[e + f*x]^2*Tan[e + f*x])/(3 - n) + (2*
(-a^2 + b^2)*(1 - n)*AppellF1[1 + (1 - n)/2, -n/2, 2, 1 + (3 - n)/2, -Tan[e
 + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]*Sec[e + f*x]^2*Tan[e + f*x])/
(a^2*(3 - n)))) + 2*b*((a^2 - b^2)*(-1 + n)*AppellF1[1 - n/2, (-1 - n)/2, 2
, 2 - n/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]*Sec[e + f*x]
^2 + a*b*(-2 + n)*(((1 - n)*n*AppellF1[1 + (1 - n)/2, 1 - n/2, 2, 1 + (3 -
n)/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]*Sec[e + f*x]^2*Ta
n[e + f*x])/(3 - n) + (4*(-a^2 + b^2)*(1 - n)*AppellF1[1 + (1 - n)/2, -n/2,
 3, 1 + (3 - n)/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]*Sec[
e + f*x]^2*Tan[e + f*x])/(a^2*(3 - n))) + (a^2 - b^2)*(-1 + n)*Tan[e + f*x]
*(-(((-1 - n)*(1 - n/2)*AppellF1[2 - n/2, 1 + (-1 - n)/2, 2, 3 - n/2, -Tan[
e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]*Sec[e + f*x]^2*Tan[e + f*x])
/(2 - n/2)) + (4*(-a^2 + b^2)*(1 - n/2)*AppellF1[2 - n/2, (-1 - n)/2, 3, 3
- n/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]*Sec[e + f*x]^2*T
an[e + f*x])/(a^2*(2 - n/2))))))/(a^3*(a^2 - b^2)*(-2 + n)*(-1 + n)*(Sec[e
+ f*x]^2)^(n/2))))
```

Maple [F] time = 0.859, size = 0, normalized size = 0.

$$\int \frac{\left(d \csc \left(f x+e\right)\right)^{n}}{\left(a+b \sin \left(f x+e\right)\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(f*x+e))^n/(a+b*sin(f*x+e))^2,x)
```

```
[Out] int((d*csc(f*x+e))^n/(a+b*sin(f*x+e))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d \csc \left(f x+e\right)\right)^{n}}{\left(b \sin \left(f x+e\right)+a\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^n/(a+b*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate((d*csc(f*x + e))^n/(b*sin(f*x + e) + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(d\csc\left(fx+e\right)\right)^{n}}{b^{2}\cos\left(fx+e\right)^{2}-2\,ab\sin\left(fx+e\right)-a^{2}-b^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^n/(a+b*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral(-(d*csc(f*x + e))^n/(b^2*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2
 - b^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(d\csc\left(e+fx\right)\right)^{n}}{\left(a+b\sin\left(e+fx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))**n/(a+b*sin(f*x+e))**2,x)
```

```
[Out] Integral((d*csc(e + f*x))**n/(a + b*sin(e + f*x))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(d\csc\left(fx+e\right)\right)^{n}}{\left(b\sin\left(fx+e\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^n/(a+b*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*csc(f*x + e))^n/(b*sin(f*x + e) + a)^2, x)
```

3.830 $\int \frac{(d\csc(e+fx))^n}{(a+b\sin(e+fx))^3}\,dx$

Optimal. Leaf size=432

$$-\frac{3ab^2\sin^4(e+fx)\cos(e+fx)\sin^2(e+fx)^{\frac{n-1}{2}}(d\csc(e+fx))^{n+3}F_1\left(\frac{1}{2};\frac{n-1}{2},3;\frac{3}{2};\cos^2(e+fx),-\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)}{d^3f\left(a^2-b^2\right)^3}+\frac{b^3\sin^3(e}{}$$

```
[Out] (-3*a*b^2*AppellF1[1/2, (-1 + n)/2, 3, 3/2, Cos[e + f*x]^2, -((b^2*Cos[e +
f*x]^2)/(a^2 - b^2))]*Cos[e + f*x]*(d*Csc[e + f*x])^(3 + n)*Sin[e + f*x]^4*
(Sin[e + f*x]^2)^((-1 + n)/2))/((a^2 - b^2)^3*d^3*f) + (b^3*AppellF1[1/2, (
-2 + n)/2, 3, 3/2, Cos[e + f*x]^2, -((b^2*Cos[e + f*x]^2)/(a^2 - b^2))]*Cos
[e + f*x]*(d*Csc[e + f*x])^(3 + n)*Sin[e + f*x]^3*(Sin[e + f*x]^2)^(n/2))/(
(a^2 - b^2)^3*d^3*f) + (3*a^2*b*AppellF1[1/2, n/2, 3, 3/2, Cos[e + f*x]^2,
-((b^2*Cos[e + f*x]^2)/(a^2 - b^2))]*Cos[e + f*x]*(d*Csc[e + f*x])^(3 + n)*
Sin[e + f*x]^3*(Sin[e + f*x]^2)^(n/2))/((a^2 - b^2)^3*d^3*f) - (a^3*AppellF
1[1/2, (1 + n)/2, 3, 3/2, Cos[e + f*x]^2, -((b^2*Cos[e + f*x]^2)/(a^2 - b^2
))]*Cos[e + f*x]*(d*Csc[e + f*x])^(3 + n)*(Sin[e + f*x]^2)^((3 + n)/2))/((a
^2 - b^2)^3*d^3*f)
```

Rubi [A] time = 0.702643, antiderivative size = 432, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3238, 3869, 2824, 3189, 429}

$$-\frac{3ab^2\sin^4(e+fx)\cos(e+fx)\sin^2(e+fx)^{\frac{n-1}{2}}(d\csc(e+fx))^{n+3}F_1\left(\frac{1}{2};\frac{n-1}{2},3;\frac{3}{2};\cos^2(e+fx),-\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)}{d^3f\left(a^2-b^2\right)^3}+\frac{b^3\sin^3(e}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Csc[e + f*x])^n/(a + b*Sin[e + f*x])^3,x]
```

```
[Out] (-3*a*b^2*AppellF1[1/2, (-1 + n)/2, 3, 3/2, Cos[e + f*x]^2, -((b^2*Cos[e +
f*x]^2)/(a^2 - b^2))]*Cos[e + f*x]*(d*Csc[e + f*x])^(3 + n)*Sin[e + f*x]^4*
(Sin[e + f*x]^2)^((-1 + n)/2))/((a^2 - b^2)^3*d^3*f) + (b^3*AppellF1[1/2, (
-2 + n)/2, 3, 3/2, Cos[e + f*x]^2, -((b^2*Cos[e + f*x]^2)/(a^2 - b^2))]*Cos
[e + f*x]*(d*Csc[e + f*x])^(3 + n)*Sin[e + f*x]^3*(Sin[e + f*x]^2)^(n/2))/(
(a^2 - b^2)^3*d^3*f) + (3*a^2*b*AppellF1[1/2, n/2, 3, 3/2, Cos[e + f*x]^2,
-((b^2*Cos[e + f*x]^2)/(a^2 - b^2))]*Cos[e + f*x]*(d*Csc[e + f*x])^(3 + n)*
Sin[e + f*x]^3*(Sin[e + f*x]^2)^(n/2))/((a^2 - b^2)^3*d^3*f) - (a^3*AppellF
1[1/2, (1 + n)/2, 3, 3/2, Cos[e + f*x]^2, -((b^2*Cos[e + f*x]^2)/(a^2 - b^2
))]*Cos[e + f*x]*(d*Csc[e + f*x])^(3 + n)*(Sin[e + f*x]^2)^((3 + n)/2))/((a
^2 - b^2)^3*d^3*f)
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3869

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Dist[Sin[e + f*x]^n*(d*Csc[e + f*x])^n, Int[(b +
a*Sin[e + f*x])^m/Sin[e + f*x]^(m + n), x], x] /; FreeQ[{a, b, d, e, f, n},
```

```
x] && NeQ[a^2 - b^2, 0] && IntegerQ[m]
```

Rule 2824

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(d*sin[e + f*x])^n/((a - b*sin[e +
 f*x])^m/(a^2 - b^2*sin[e + f*x]^2)^m), x], x] /; FreeQ[{a, b, d, e, f, n},
 x] && NeQ[a^2 - b^2, 0] && ILtQ[m, -1]
```

Rule 3189

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[
(ff*d^(2*IntPart[(m - 1)/2] + 1)*(d*Sin[e + f*x])^(2*FracPart[(m - 1)/2]))/
(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2]), Subst[Int[(1 - ff^2*x^2)^((m - 1)
/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, d,
 e, f, m, p}, x] &&  !IntegerQ[m]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{(d\csc(e+fx))^n}{(a+b\sin(e+fx))^3}\,dx = \frac{\int \frac{(d\csc(e+fx))^{3+n}}{(b+a\csc(e+fx))^3}\,dx}{d^3}$$

$$= \frac{\left((d\csc(e+fx))^{3+n}\sin^{3+n}(e+fx)\right)\int \frac{\sin^{-n}(e+fx)}{(a+b\sin(e+fx))^3}\,dx}{d^3}$$

$$= \frac{\left((d\csc(e+fx))^{3+n}\sin^{3+n}(e+fx)\right)\int \left(-\frac{3a^2b\sin^{1-n}(e+fx)}{\left(a^2-b^2\sin^2(e+fx)\right)^3}+\frac{3ab^2\sin^{2-n}(e+fx)}{\left(a^2-b^2\sin^2(e+fx)\right)^3}+\frac{a^3\sin^{-n}(e+}{\left(a^2-b^2\sin^2(e+\right.}\right.}{d^3}$$

$$= \frac{\left(a^3(d\csc(e+fx))^{3+n}\sin^{3+n}(e+fx)\right)\int \frac{\sin^{-n}(e+fx)}{\left(a^2-b^2\sin^2(e+fx)\right)^3}\,dx}{d^3} - \frac{\left(3a^2b(d\csc(e+fx))^{3+n}\sin\right.}{}$$

$$= -\frac{\left(3ab^2(d\csc(e+fx))^{3+n}\sin^{3+2\left(\frac{1}{2}-\frac{n}{2}\right)+n}(e+fx)\sin^2(e+fx)^{-\frac{1}{2}+\frac{n}{2}}\right)\operatorname{Subst}\left(\int \frac{\left(1-x^2\right)^{\frac{1-n}{2}}}{\left(a^2-b^2+b^2x^2\right)^3}\right.}{d^3f}$$

$$= -\frac{3ab^2F_1\left(\frac{1}{2};\frac{1}{2}(-1+n),3;\frac{3}{2};\cos^2(e+fx),-\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)\cos(e+fx)(d\csc(e+fx))^{3+n}\sin}{\left(a^2-b^2\right)^3d^3f}$$

Mathematica [B] time = 20.3277, size = 2496, normalized size = 5.78

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Csc[e + f*x])^n/(a + b*Sin[e + f*x])^3,x]
```

```
[Out] ((d*Csc[e + f*x])^n*(Cot[e + f*x]*Sqrt[Sec[e + f*x]^2])^n*Tan[e + f*x]*(-(a
*(a^2 + 3*b^2)*(-2 + n)*AppellF1[(1 - n)/2, -1 - n/2, 2, (3 - n)/2, -Tan[e
+ f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]) + b*(4*a*b*(-2 + n)*AppellF1[
(1 - n)/2, -1 - n/2, 3, (3 - n)/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f
*x]^2)/a^2] + (-1 + n)*((3*a^2 + b^2)*AppellF1[1 - n/2, (-1 - n)/2, 2, 2 -
n/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2] - 4*b^2*AppellF1[1
 - n/2, (-1 - n)/2, 3, 2 - n/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]
^2)/a^2])*Tan[e + f*x])))/(a^4*(a^2 - b^2)*f*(-2 + n)*(-1 + n)*(Sec[e + f*x
]^2)^(n/2)*(a + b*Sin[e + f*x])^3*(((Sec[e + f*x]^2)^(1 - n/2)*(Cot[e + f*x
]*Sqrt[Sec[e + f*x]^2])^n*(-(a*(a^2 + 3*b^2)*(-2 + n)*AppellF1[(1 - n)/2, -
1 - n/2, 2, (3 - n)/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2])
 + b*(4*a*b*(-2 + n)*AppellF1[(1 - n)/2, -1 - n/2, 3, (3 - n)/2, -Tan[e + f
*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2] + (-1 + n)*((3*a^2 + b^2)*AppellF
1[1 - n/2, (-1 - n)/2, 2, 2 - n/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f
*x]^2)/a^2] - 4*b^2*AppellF1[1 - n/2, (-1 - n)/2, 3, 2 - n/2, -Tan[e + f*x]
^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2])*Tan[e + f*x])))/(a^4*(a^2 - b^2)*(-
2 + n)*(-1 + n)) + (n*(Cot[e + f*x]*Sqrt[Sec[e + f*x]^2])^(-1 + n)*(Sqrt[Se
c[e + f*x]^2] - Csc[e + f*x]^2*Sqrt[Sec[e + f*x]^2])*Tan[e + f*x]*(-(a*(a^2
 + 3*b^2)*(-2 + n)*AppellF1[(1 - n)/2, -1 - n/2, 2, (3 - n)/2, -Tan[e + f*x
]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]) + b*(4*a*b*(-2 + n)*AppellF1[(1 -
n)/2, -1 - n/2, 3, (3 - n)/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2
)/a^2] + (-1 + n)*((3*a^2 + b^2)*AppellF1[1 - n/2, (-1 - n)/2, 2, 2 - n/2,
-Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2] - 4*b^2*AppellF1[1 - n/
2, (-1 - n)/2, 3, 2 - n/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a
^2])*Tan[e + f*x])))/(a^4*(a^2 - b^2)*(-2 + n)*(-1 + n)*(Sec[e + f*x]^2)^(n
/2)) - (n*(Cot[e + f*x]*Sqrt[Sec[e + f*x]^2])^n*Tan[e + f*x]^2*(-(a*(a^2 +
3*b^2)*(-2 + n)*AppellF1[(1 - n)/2, -1 - n/2, 2, (3 - n)/2, -Tan[e + f*x]^2
, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]) + b*(4*a*b*(-2 + n)*AppellF1[(1 - n)/
2, -1 - n/2, 3, (3 - n)/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a
^2] + (-1 + n)*((3*a^2 + b^2)*AppellF1[1 - n/2, (-1 - n)/2, 2, 2 - n/2, -Ta
n[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2] - 4*b^2*AppellF1[1 - n/2,
(-1 - n)/2, 3, 2 - n/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]
)*Tan[e + f*x])))/(a^4*(a^2 - b^2)*(-2 + n)*(-1 + n)*(Sec[e + f*x]^2)^(n/2)
) + ((Cot[e + f*x]*Sqrt[Sec[e + f*x]^2])^n*Tan[e + f*x]*(-(a*(a^2 + 3*b^2)*
(-2 + n)*((4*(-a^2 + b^2)*(1 - n)*AppellF1[1 + (1 - n)/2, -1 - n/2, 3, 1 +
(3 - n)/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]*Sec[e + f*x]
^2*Tan[e + f*x])/(a^2*(3 - n)) - (2*(1 - n)*(-1 - n/2)*AppellF1[1 + (1 - n)
/2, -n/2, 2, 1 + (3 - n)/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/
a^2]*Sec[e + f*x]^2*Tan[e + f*x])/(3 - n))) + b*((-1 + n)*((3*a^2 + b^2)*Ap
pellF1[1 - n/2, (-1 - n)/2, 2, 2 - n/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[
e + f*x]^2)/a^2] - 4*b^2*AppellF1[1 - n/2, (-1 - n)/2, 3, 2 - n/2, -Tan[e +
 f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2])*Sec[e + f*x]^2 + 4*a*b*(-2 + n
)*((6*(-a^2 + b^2)*(1 - n)*AppellF1[1 + (1 - n)/2, -1 - n/2, 4, 1 + (3 - n)
/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]*Sec[e + f*x]^2*Tan[
e + f*x])/(a^2*(3 - n)) - (2*(1 - n)*(-1 - n/2)*AppellF1[1 + (1 - n)/2, -n/
2, 3, 1 + (3 - n)/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]*Se
c[e + f*x]^2*Tan[e + f*x])/(3 - n)) + (-1 + n)*Tan[e + f*x]*((3*a^2 + b^2)*
(-(((-1 - n)*(1 - n/2)*AppellF1[2 - n/2, 1 + (-1 - n)/2, 2, 3 - n/2, -Tan[e
 + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]*Sec[e + f*x]^2*Tan[e + f*x])/
(2 - n/2)) + (4*(-a^2 + b^2)*(1 - n/2)*AppellF1[2 - n/2, (-1 - n)/2, 3, 3 -
 n/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]*Sec[e + f*x]^2*Ta
n[e + f*x])/(a^2*(2 - n/2))) - 4*b^2*(-(((-1 - n)*(1 - n/2)*AppellF1[2 - n/
2, 1 + (-1 - n)/2, 3, 3 - n/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^
2)/a^2]*Sec[e + f*x]^2*Tan[e + f*x])/(2 - n/2)) + (6*(-a^2 + b^2)*(1 - n/2)
*AppellF1[2 - n/2, (-1 - n)/2, 4, 3 - n/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*T
an[e + f*x]^2)/a^2]*Sec[e + f*x]^2*Tan[e + f*x])/(a^2*(2 - n/2)))))))/(a^4*
(a^2 - b^2)*(-2 + n)*(-1 + n)*(Sec[e + f*x]^2)^(n/2))))
```

Maple [F] time = 1.06, size = 0, normalized size = 0.

$$\int \frac{\left(d\csc\left(fx+e\right)\right)^{n}}{\left(a+b\sin\left(fx+e\right)\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(f*x+e))^n/(a+b*sin(f*x+e))^3,x)
```

```
[Out] int((d*csc(f*x+e))^n/(a+b*sin(f*x+e))^3,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^n/(a+b*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\left(d\csc\left(fx+e\right)\right)^{n}}{3\,ab^{2}\cos\left(fx+e\right)^{2}-a^{3}-3\,ab^{2}+\left(b^{3}\cos\left(fx+e\right)^{2}-3\,a^{2}b-b^{3}\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^n/(a+b*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] integral(-(d*csc(f*x + e))^n/(3*a*b^2*cos(f*x + e)^2 - a^3 - 3*a*b^2 + (b^3
*cos(f*x + e)^2 - 3*a^2*b - b^3)*sin(f*x + e)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\csc\left(e+fx\right)\right)^{n}}{\left(a+b\sin\left(e+fx\right)\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))**n/(a+b*sin(f*x+e))**3,x)
```

```
[Out] Integral((d*csc(e + f*x))**n/(a + b*sin(e + f*x))**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\csc\left(fx+e\right)\right)^{n}}{\left(b\sin\left(fx+e\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^n/(a+b*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate((d*csc(f*x + e))^n/(b*sin(f*x + e) + a)^3, x)
```

3.831 $\int \left(c(d\sin(e+fx))^p\right)^n (a+b\sin(e+fx))^m\,dx$

Optimal. Leaf size=55

$$(d\sin(e+fx))^{-np}\left(c(d\sin(e+fx))^p\right)^n \text{Unintegrable}\left((a+b\sin(e+fx))^m(d\sin(e+fx))^{np},x\right)$$

```
[Out] ((c*(d*Sin[e + f*x])^p)^n*Unintegrable[(d*Sin[e + f*x])^(n*p)*(a + b*Sin[e
+ f*x])^m, x])/(d*Sin[e + f*x])^(n*p)
```

Rubi [A] time = 0.112432, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \left(c(d\sin(e+fx))^p\right)^n (a+b\sin(e+fx))^m\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c*(d*Sin[e + f*x])^p)^n*(a + b*Sin[e + f*x])^m,x]
```

```
[Out] ((c*(d*Sin[e + f*x])^p)^n*Defer[Int][(d*Sin[e + f*x])^(n*p)*(a + b*Sin[e +
f*x])^m, x])/(d*Sin[e + f*x])^(n*p)
```

Rubi steps

$$\int \left(c(d\sin(e+fx))^p\right)^n (a+b\sin(e+fx))^m\,dx = \left((d\sin(e+fx))^{-np}\left(c(d\sin(e+fx))^p\right)^n\right)\int (d\sin(e+fx))^{np}(a+b\sin$$

Mathematica [A] time = 2.50163, size = 0, normalized size = 0.

$$\int \left(c(d\sin(e+fx))^p\right)^n (a+b\sin(e+fx))^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c*(d*Sin[e + f*x])^p)^n*(a + b*Sin[e + f*x])^m,x]
```

```
[Out] Integrate[(c*(d*Sin[e + f*x])^p)^n*(a + b*Sin[e + f*x])^m, x]
```

Maple [A] time = 0.214, size = 0, normalized size = 0.

$$\int \left(c\left(d\sin\left(fx+e\right)\right)^p\right)^n \left(a+b\sin\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*sin(f*x+e))^p)^n*(a+b*sin(f*x+e))^m,x)
```

```
[Out] int((c*(d*sin(f*x+e))^p)^n*(a+b*sin(f*x+e))^m,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(d\sin\left(fx+e\right)\right)^p c\right)^n \left(b\sin\left(fx+e\right)+a\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n*(a+b*sin(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate(((d*sin(f*x + e))^p*c)^n*(b*sin(f*x + e) + a)^m, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(\left(d\sin\left(fx+e\right)\right)^p c\right)^n \left(b\sin\left(fx+e\right)+a\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n*(a+b*sin(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral(((d*sin(f*x + e))^p*c)^n*(b*sin(f*x + e) + a)^m, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \left(c\left(d\sin\left(e+fx\right)\right)^p\right)^n \left(a+b\sin\left(e+fx\right)\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))**p)**n*(a+b*sin(f*x+e))**m,x)
```

```
[Out] Integral((c*(d*sin(e + f*x))**p)**n*(a + b*sin(e + f*x))**m, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(d\sin\left(fx+e\right)\right)^p c\right)^n \left(b\sin\left(fx+e\right)+a\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n*(a+b*sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate(((d*sin(f*x + e))^p*c)^n*(b*sin(f*x + e) + a)^m, x)
```

3.832 $\int \left(c(d\sin(e+fx))^p\right)^n (a+b\sin(e+fx))^3\,dx$

Optimal. Leaf size=323

$$\frac{b\left(3a^2(np+3)+b^2(np+2)\right)\sin^2(e+fx)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(np+2);\frac{1}{2}(np+4);\sin^2(e+fx)\right)\left(c(d\sin(e+fx))^p\right)^n}{f(np+2)(np+3)\sqrt{\cos^2(e+fx)}}\cdot$$

```
[Out] -((a*b^2*(7 + 2*n*p)*Cos[e + f*x]*Sin[e + f*x]*(c*(d*Sin[e + f*x])^p)^n)/(f
*(2 + n*p)*(3 + n*p))) + (a*(3*b^2*(1 + n*p) + a^2*(2 + n*p))*Cos[e + f*x]*
Hypergeometric2F1[1/2, (1 + n*p)/2, (3 + n*p)/2, Sin[e + f*x]^2]*Sin[e + f*
x]*(c*(d*Sin[e + f*x])^p)^n)/(f*(1 + n*p)*(2 + n*p)*Sqrt[Cos[e + f*x]^2]) +
 (b*(b^2*(2 + n*p) + 3*a^2*(3 + n*p))*Cos[e + f*x]*Hypergeometric2F1[1/2, (
2 + n*p)/2, (4 + n*p)/2, Sin[e + f*x]^2]*Sin[e + f*x]^2*(c*(d*Sin[e + f*x])
^p)^n)/(f*(2 + n*p)*(3 + n*p)*Sqrt[Cos[e + f*x]^2]) - (b^2*Cos[e + f*x]*Sin
[e + f*x]*(c*(d*Sin[e + f*x])^p)^n*(a + b*Sin[e + f*x]))/(f*(3 + n*p))
```

Rubi [A] time = 0.548291, antiderivative size = 303, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2826, 2793, 3023, 2748, 2643}

$$\frac{b\left(\frac{3a^2}{np+2}+\frac{b^2}{np+3}\right)\sin^2(e+fx)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(np+2);\frac{1}{2}(np+4);\sin^2(e+fx)\right)\left(c(d\sin(e+fx))^p\right)^n}{f\sqrt{\cos^2(e+fx)}}+\frac{a\left(\frac{a^2}{np+1}+\cdots\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c*(d*Sin[e + f*x])^p)^n*(a + b*Sin[e + f*x])^3,x]
```

```
[Out] -((a*b^2*(7 + 2*n*p)*Cos[e + f*x]*Sin[e + f*x]*(c*(d*Sin[e + f*x])^p)^n)/(f
*(2 + n*p)*(3 + n*p))) + (a*(a^2/(1 + n*p) + (3*b^2)/(2 + n*p))*Cos[e + f*x
]*Hypergeometric2F1[1/2, (1 + n*p)/2, (3 + n*p)/2, Sin[e + f*x]^2]*Sin[e +
f*x]*(c*(d*Sin[e + f*x])^p)^n)/(f*Sqrt[Cos[e + f*x]^2]) + (b*((3*a^2)/(2 +
n*p) + b^2/(3 + n*p))*Cos[e + f*x]*Hypergeometric2F1[1/2, (2 + n*p)/2, (4 +
 n*p)/2, Sin[e + f*x]^2]*Sin[e + f*x]^2*(c*(d*Sin[e + f*x])^p)^n)/(f*Sqrt[C
os[e + f*x]^2]) - (b^2*Cos[e + f*x]*Sin[e + f*x]*(c*(d*Sin[e + f*x])^p)^n*(
a + b*Sin[e + f*x]))/(f*(3 + n*p))
```

Rule 2826

```
Int[((c_.)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(p_))^(n_)*((a_.) + (b_.)*sin[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c^IntPart[n]*(c*(d*Sin[e + f*x
])^p)^FracPart[n])/(d*Sin[e + f*x])^(p*FracPart[n]), Int[(a + b*Sin[e + f*x
])^m*(d*Sin[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] &&  !IntegerQ[n]
```

Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
```

```
NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int \left(c(d\sin(e+fx))^p\right)^n (a+b\sin(e+fx))^3\,dx = \left((d\sin(e+fx))^{-np}\left(c(d\sin(e+fx))^p\right)^n\right)\int (d\sin(e+fx))^{np}(a+b\sin(e+$$

$$= -\frac{b^2\cos(e+fx)\sin(e+fx)\left(c(d\sin(e+fx))^p\right)^n(a+b\sin(e+fx))}{f(3+np)} + \frac{\left((\right.}{}$$

$$= -\frac{ab^2(7+2np)\cos(e+fx)\sin(e+fx)\left(c(d\sin(e+fx))^p\right)^n}{f(2+np)(3+np)} - \frac{b^2\cos(e+}{}$$

$$= -\frac{ab^2(7+2np)\cos(e+fx)\sin(e+fx)\left(c(d\sin(e+fx))^p\right)^n}{f(2+np)(3+np)} - \frac{b^2\cos(e+}{}$$

$$= -\frac{ab^2(7+2np)\cos(e+fx)\sin(e+fx)\left(c(d\sin(e+fx))^p\right)^n}{f(2+np)(3+np)} + \frac{a\left(\frac{a^2}{1+np}+\right.}{}$$

Mathematica [A] time = 1.07811, size = 230, normalized size = 0.71

$$\frac{\sin(e+fx)\cos(e+fx)\left(c(d\sin(e+fx))^p\right)^n\left(\frac{b\left(3a^2(np+3)+b^2(np+2)\right)\sin(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{np}{2}+1;\frac{np}{2}+2;\sin^2(e+fx)\right)}{(np+2)\sqrt{\cos^2(e+fx)}}+\frac{a(np+3)\left(a^2(np+2)+3b^2(np+1\right.}{(np+1)(}\right.}{f(np+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*(d*Sin[e + f*x])^p)^n*(a + b*Sin[e + f*x])^3,x]
```

```
[Out] (Cos[e + f*x]*Sin[e + f*x]*(c*(d*Sin[e + f*x])^p)^n*(-((a*b^2*(7 + 2*n*p))/
(2 + n*p)) + (a*(3 + n*p)*(3*b^2*(1 + n*p) + a^2*(2 + n*p))*Hypergeometric2
F1[1/2, (1 + n*p)/2, (3 + n*p)/2, Sin[e + f*x]^2])/((1 + n*p)*(2 + n*p)*Sqr
t[Cos[e + f*x]^2]) + (b*(b^2*(2 + n*p) + 3*a^2*(3 + n*p))*Hypergeometric2F1
[1/2, 1 + (n*p)/2, 2 + (n*p)/2, Sin[e + f*x]^2]*Sin[e + f*x])/((2 + n*p)*Sq
```

```
rt[Cos[e + f*x]^2]) - b^2*(a + b*Sin[e + f*x])))/(f*(3 + n*p))
```

Maple [F] time = 0.451, size = 0, normalized size = 0.

$$\int \left(c\left(d\sin\left(fx+e\right)\right)^{p}\right)^{n} \left(a+b\sin\left(fx+e\right)\right)^{3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*sin(f*x+e))^p)^n*(a+b*sin(f*x+e))^3,x)
```

```
[Out] int((c*(d*sin(f*x+e))^p)^n*(a+b*sin(f*x+e))^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)+a\right)^{3} \left(\left(d\sin\left(fx+e\right)\right)^{p} c\right)^{n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n*(a+b*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)^3*((d*sin(f*x + e))^p*c)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\left(3\,ab^{2}\cos\left(fx+e\right)^{2}-a^{3}-3\,ab^{2}+\left(b^{3}\cos\left(fx+e\right)^{2}-3\,a^{2}b-b^{3}\right)\sin\left(fx+e\right)\right)\left(\left(d\sin\left(fx+e\right)\right)^{p} c\right)^{n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n*(a+b*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] integral(-(3*a*b^2*cos(f*x + e)^2 - a^3 - 3*a*b^2 + (b^3*cos(f*x + e)^2 - 3
*a^2*b - b^3)*sin(f*x + e))*((d*sin(f*x + e))^p*c)^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))**p)**n*(a+b*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)+a\right)^{3} \left(\left(d\sin\left(fx+e\right)\right)^{p} c\right)^{n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n*(a+b*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)^3*((d*sin(f*x + e))^p*c)^n, x)
```

3.833 $\int \left(c(d\sin(e+fx))^p\right)^n (a+b\sin(e+fx))^2\,dx$

Optimal. Leaf size=231

$$\frac{\left(a^2(np+2)+b^2(np+1)\right)\sin(e+fx)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(np+1);\frac{1}{2}(np+3);\sin^2(e+fx)\right)\left(c(d\sin(e+fx))^p\right)^n}{f(np+1)(np+2)\sqrt{\cos^2(e+fx)}}+\frac{2a}{}$$

```
[Out] -((b^2*Cos[e + f*x]*Sin[e + f*x]*(c*(d*Sin[e + f*x])^p)^n)/(f*(2 + n*p))) +
 ((b^2*(1 + n*p) + a^2*(2 + n*p))*Cos[e + f*x]*Hypergeometric2F1[1/2, (1 +
n*p)/2, (3 + n*p)/2, Sin[e + f*x]^2]*Sin[e + f*x]*(c*(d*Sin[e + f*x])^p)^n)
/(f*(1 + n*p)*(2 + n*p)*Sqrt[Cos[e + f*x]^2]) + (2*a*b*Cos[e + f*x]*Hyperge
ometric2F1[1/2, (2 + n*p)/2, (4 + n*p)/2, Sin[e + f*x]^2]*Sin[e + f*x]^2*(c
*(d*Sin[e + f*x])^p)^n)/(f*(2 + n*p)*Sqrt[Cos[e + f*x]^2])
```

Rubi [A] time = 0.232943, antiderivative size = 221, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2826, 2789, 2643, 3014}

$$\frac{\left(\frac{a^2}{np+1}+\frac{b^2}{np+2}\right)\sin(e+fx)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(np+1);\frac{1}{2}(np+3);\sin^2(e+fx)\right)\left(c(d\sin(e+fx))^p\right)^n}{f\sqrt{\cos^2(e+fx)}}+\frac{2ab\sin^2(e+}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c*(d*Sin[e + f*x])^p)^n*(a + b*Sin[e + f*x])^2,x]
```

```
[Out] -((b^2*Cos[e + f*x]*Sin[e + f*x]*(c*(d*Sin[e + f*x])^p)^n)/(f*(2 + n*p))) +
 ((a^2/(1 + n*p) + b^2/(2 + n*p))*Cos[e + f*x]*Hypergeometric2F1[1/2, (1 +
n*p)/2, (3 + n*p)/2, Sin[e + f*x]^2]*Sin[e + f*x]*(c*(d*Sin[e + f*x])^p)^n)
/(f*Sqrt[Cos[e + f*x]^2]) + (2*a*b*Cos[e + f*x]*Hypergeometric2F1[1/2, (2 +
 n*p)/2, (4 + n*p)/2, Sin[e + f*x]^2]*Sin[e + f*x]^2*(c*(d*Sin[e + f*x])^p)
^n)/(f*(2 + n*p)*Sqrt[Cos[e + f*x]^2])
```

Rule 2826

```
Int[((c_.)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(p_))^(n_)*((a_.) + (b_.)*sin[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c^IntPart[n]*(c*(d*Sin[e + f*x
])^p)^FracPart[n])/(d*Sin[e + f*x])^(p*FracPart[n]), Int[(a + b*Sin[e + f*x
])^m*(d*Sin[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] &&  !IntegerQ[n]
```

Rule 2789

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)])^2, x_Symbol] :> Dist[(2*c*d)/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] +
 Int[(b*Sin[e + f*x])^m*(c^2 + d^2*Sin[e + f*x]^2), x] /; FreeQ[{b, c, d, e
, f, m}, x]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3014

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*
(m + 2)), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])^m
, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \left(c(d\sin(e+fx))^p\right)^n (a+b\sin(e+fx))^2\,dx = \left((d\sin(e+fx))^{-np}\left(c(d\sin(e+fx))^p\right)^n\right)\int (d\sin(e+fx))^{np}(a+b\sin(e+$$

$$= \left((d\sin(e+fx))^{-np}\left(c(d\sin(e+fx))^p\right)^n\right)\int (d\sin(e+fx))^{np}\left(a^2+b^2\sin^2\right.$$

$$= -\frac{b^2\cos(e+fx)\sin(e+fx)\left(c(d\sin(e+fx))^p\right)^n}{f(2+np)} + \frac{2ab\cos(e+fx)\,{}_2F_1\left(\right.}{}$$

$$= -\frac{b^2\cos(e+fx)\sin(e+fx)\left(c(d\sin(e+fx))^p\right)^n}{f(2+np)} + \frac{\left(a^2+\frac{b^2(1+np)}{2+np}\right)\cos(e}{}$$

Mathematica [A] time = 0.331043, size = 152, normalized size = 0.66

$$-\frac{\cos(e+fx)\sin^2(e+fx)^{\frac{1}{2}(-np-1)}\left(c(d\sin(e+fx))^p\right)^n\left(a\left(a\sin(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(1-np);\frac{3}{2};\cos^2(e+fx)\right)+2b\sqrt{\sin^2(e+}\right.\right.}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*(d*Sin[e + f*x])^p)^n*(a + b*Sin[e + f*x])^2,x]
```

```
[Out] -((Cos[e + f*x]*(Sin[e + f*x]^2)^((-1 - n*p)/2)*(c*(d*Sin[e + f*x])^p)^n*(b
^2*Hypergeometric2F1[1/2, (-1 - n*p)/2, 3/2, Cos[e + f*x]^2]*Sin[e + f*x] +
 a*(a*Hypergeometric2F1[1/2, (1 - n*p)/2, 3/2, Cos[e + f*x]^2]*Sin[e + f*x]
 + 2*b*Hypergeometric2F1[1/2, -(n*p)/2, 3/2, Cos[e + f*x]^2]*Sqrt[Sin[e + f
*x]^2])))/f)
```

Maple [F] time = 0.382, size = 0, normalized size = 0.

$$\int \left(c\left(d\sin\left(fx+e\right)\right)^p\right)^n\left(a+b\sin\left(fx+e\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*sin(f*x+e))^p)^n*(a+b*sin(f*x+e))^2,x)
```

```
[Out] int((c*(d*sin(f*x+e))^p)^n*(a+b*sin(f*x+e))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)+a\right)^2\left(\left(d\sin\left(fx+e\right)\right)^p c\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n*(a+b*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)^2*((d*sin(f*x + e))^p*c)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(b^2\cos\left(fx+e\right)^2-2\,ab\sin\left(fx+e\right)-a^2-b^2\right)\left(\left(d\sin\left(fx+e\right)\right)^p c\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n*(a+b*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral(-(b^2*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2)*((d*sin(f*x
 + e))^p*c)^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(c\left(d\sin\left(e+fx\right)\right)^p\right)^n\left(a+b\sin\left(e+fx\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))**p)**n*(a+b*sin(f*x+e))**2,x)
```

```
[Out] Integral((c*(d*sin(e + f*x))**p)**n*(a + b*sin(e + f*x))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\sin\left(fx+e\right)+a\right)^2\left(\left(d\sin\left(fx+e\right)\right)^p c\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n*(a+b*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)^2*((d*sin(f*x + e))^p*c)^n, x)
```

3.834 $\int \left(c(d\sin(e+fx))^p\right)^n (a+b\sin(e+fx))\,dx$

Optimal. Leaf size=163

$$\frac{a\sin(e+fx)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(np+1);\frac{1}{2}(np+3);\sin^2(e+fx)\right)\left(c(d\sin(e+fx))^p\right)^n}{f(np+1)\sqrt{\cos^2(e+fx)}}+\frac{b\sin^2(e+fx)\cos(e+fx)\,{}_2F_1$$

```
[Out] (a*Cos[e + f*x]*Hypergeometric2F1[1/2, (1 + n*p)/2, (3 + n*p)/2, Sin[e + f*
x]^2]*Sin[e + f*x]*(c*(d*Sin[e + f*x])^p)^n)/(f*(1 + n*p)*Sqrt[Cos[e + f*x]
^2]) + (b*Cos[e + f*x]*Hypergeometric2F1[1/2, (2 + n*p)/2, (4 + n*p)/2, Sin
[e + f*x]^2]*Sin[e + f*x]^2*(c*(d*Sin[e + f*x])^p)^n)/(f*(2 + n*p)*Sqrt[Cos
[e + f*x]^2])
```

Rubi [A] time = 0.114439, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2826, 2748, 2643}

$$\frac{a\sin(e+fx)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(np+1);\frac{1}{2}(np+3);\sin^2(e+fx)\right)\left(c(d\sin(e+fx))^p\right)^n}{f(np+1)\sqrt{\cos^2(e+fx)}}+\frac{b\sin^2(e+fx)\cos(e+fx)\,{}_2F_1$$

Antiderivative was successfully verified.

```
[In] Int[(c*(d*Sin[e + f*x])^p)^n*(a + b*Sin[e + f*x]),x]
```

```
[Out] (a*Cos[e + f*x]*Hypergeometric2F1[1/2, (1 + n*p)/2, (3 + n*p)/2, Sin[e + f*
x]^2]*Sin[e + f*x]*(c*(d*Sin[e + f*x])^p)^n)/(f*(1 + n*p)*Sqrt[Cos[e + f*x]
^2]) + (b*Cos[e + f*x]*Hypergeometric2F1[1/2, (2 + n*p)/2, (4 + n*p)/2, Sin
[e + f*x]^2]*Sin[e + f*x]^2*(c*(d*Sin[e + f*x])^p)^n)/(f*(2 + n*p)*Sqrt[Cos
[e + f*x]^2])
```

Rule 2826

```
Int[((c_.)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(p_))^(n_)*((a_.) + (b_.)*sin[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c^IntPart[n]*(c*(d*Sin[e + f*x
])^p)^FracPart[n])/(d*Sin[e + f*x])^(p*FracPart[n]), Int[(a + b*Sin[e + f*x
])^m*(d*Sin[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] &&  !IntegerQ[n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int \left(c(d\sin(e+fx))^p\right)^n (a+b\sin(e+fx))\,dx = \left((d\sin(e+fx))^{-np}\left(c(d\sin(e+fx))^p\right)^n\right)\int (d\sin(e+fx))^{np}(a+b\sin($$

$$= \left(a(d\sin(e+fx))^{-np}\left(c(d\sin(e+fx))^p\right)^n\right)\int (d\sin(e+fx))^{np}\,dx + \frac{(b(}{}$$

$$= \frac{a\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(1+np);\frac{1}{2}(3+np);\sin^2(e+fx)\right)\sin(e+fx)\,(c}{f(1+np)\sqrt{\cos^2(e+fx)}}$$

Mathematica [A] time = 0.248195, size = 129, normalized size = 0.79

$$\frac{\sqrt{\cos^2(e+fx)}\tan(e+fx)\left(c(d\sin(e+fx))^p\right)^n\left(a(np+2)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(np+1);\frac{1}{2}(np+3);\sin^2(e+fx)\right)+b(np+1)\sin(\right.}{f(np+1)(np+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*(d*Sin[e + f*x])^p)^n*(a + b*Sin[e + f*x]),x]
```

```
[Out] (Sqrt[Cos[e + f*x]^2]*(c*(d*Sin[e + f*x])^p)^n*(a*(2 + n*p)*Hypergeometric2
F1[1/2, (1 + n*p)/2, (3 + n*p)/2, Sin[e + f*x]^2] + b*(1 + n*p)*Hypergeomet
ric2F1[1/2, 1 + (n*p)/2, 2 + (n*p)/2, Sin[e + f*x]^2]*Sin[e + f*x])*Tan[e +
 f*x])/(f*(1 + n*p)*(2 + n*p))
```

Maple [F] time = 0.165, size = 0, normalized size = 0.

$$\int \left(c\left(d\sin\left(fx+e\right)\right)^p\right)^n \left(a+b\sin\left(fx+e\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*sin(f*x+e))^p)^n*(a+b*sin(f*x+e)),x)
```

```
[Out] int((c*(d*sin(f*x+e))^p)^n*(a+b*sin(f*x+e)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)+a\right)\left(\left(d\sin\left(fx+e\right)\right)^p c\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n*(a+b*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)*((d*sin(f*x + e))^p*c)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\sin\left(fx+e\right)+a\right)\left(\left(d\sin\left(fx+e\right)\right)^p c\right)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n*(a+b*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral((b*sin(f*x + e) + a)*((d*sin(f*x + e))^p*c)^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c\left(d\sin\left(e+fx\right)\right)^{p}\right)^{n}\left(a+b\sin\left(e+fx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))**p)**n*(a+b*sin(f*x+e)),x)
```

```
[Out] Integral((c*(d*sin(e + f*x))**p)**n*(a + b*sin(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)+a\right)\left(\left(d\sin\left(fx+e\right)\right)^{p}c\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n*(a+b*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)*((d*sin(f*x + e))^p*c)^n, x)
```

3.835 $\int \frac{(c(d\sin(e+fx))^p)^n}{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=204

$$\frac{b\cos(e+fx)\sin^2(e+fx)^{-\frac{np}{2}}\left(c(d\sin(e+fx))^p\right)^n F_1\left(\frac{1}{2};-\frac{np}{2},1;\frac{3}{2};\cos^2(e+fx),-\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)}{f\left(a^2-b^2\right)}-\frac{a\cot(e+fx)\sin^2(e+$$

```
[Out] (b*AppellF1[1/2, -(n*p)/2, 1, 3/2, Cos[e + f*x]^2, -((b^2*Cos[e + f*x]^2)/(
a^2 - b^2))]*Cos[e + f*x]*(c*(d*Sin[e + f*x])^p)^n)/((a^2 - b^2)*f*(Sin[e +
 f*x]^2)^((n*p)/2)) - (a*AppellF1[1/2, (1 - n*p)/2, 1, 3/2, Cos[e + f*x]^2,
 -((b^2*Cos[e + f*x]^2)/(a^2 - b^2))]*Cot[e + f*x]*(Sin[e + f*x]^2)^((1 - n
*p)/2)*(c*(d*Sin[e + f*x])^p)^n)/((a^2 - b^2)*f)
```

Rubi [A] time = 0.338127, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2826, 2823, 3189, 429}

$$\frac{b\cos(e+fx)\sin^2(e+fx)^{-\frac{np}{2}}\left(c(d\sin(e+fx))^p\right)^n F_1\left(\frac{1}{2};-\frac{np}{2},1;\frac{3}{2};\cos^2(e+fx),-\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)}{f\left(a^2-b^2\right)}-\frac{a\cot(e+fx)\sin^2(e+$$

Antiderivative was successfully verified.

```
[In] Int[(c*(d*Sin[e + f*x])^p)^n/(a + b*Sin[e + f*x]),x]
```

```
[Out] (b*AppellF1[1/2, -(n*p)/2, 1, 3/2, Cos[e + f*x]^2, -((b^2*Cos[e + f*x]^2)/(
a^2 - b^2))]*Cos[e + f*x]*(c*(d*Sin[e + f*x])^p)^n)/((a^2 - b^2)*f*(Sin[e +
 f*x]^2)^((n*p)/2)) - (a*AppellF1[1/2, (1 - n*p)/2, 1, 3/2, Cos[e + f*x]^2,
 -((b^2*Cos[e + f*x]^2)/(a^2 - b^2))]*Cot[e + f*x]*(Sin[e + f*x]^2)^((1 - n
*p)/2)*(c*(d*Sin[e + f*x])^p)^n)/((a^2 - b^2)*f)
```

Rule 2826

```
Int[((c_.)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(p_))^(n_)*((a_.) + (b_.)*sin[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c^IntPart[n]*(c*(d*Sin[e + f*x
])^p)^FracPart[n])/(d*Sin[e + f*x])^(p*FracPart[n]), Int[(a + b*Sin[e + f*x
])^m*(d*Sin[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] && !IntegerQ[n]
```

Rule 2823

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[a, Int[(d*Sin[e + f*x])^n/(a^2 - b^2*Sin[e + f*x]^
2), x], x] - Dist[b/d, Int[(d*Sin[e + f*x])^(n + 1)/(a^2 - b^2*Sin[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3189

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[
(ff*d^(2*IntPart[(m - 1)/2] + 1)*(d*Sin[e + f*x])^(2*FracPart[(m - 1)/2]))/
(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2]), Subst[Int[(1 - ff^2*x^2)^((m - 1)
/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, d,
 e, f, m, p}, x] && !IntegerQ[m]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\left(c(d\sin(e+fx))^p\right)^n}{a+b\sin(e+fx)}\,dx = \left((d\sin(e+fx))^{-np}\left(c(d\sin(e+fx))^p\right)^n\right)\int \frac{(d\sin(e+fx))^{np}}{a+b\sin(e+fx)}\,dx$$

$$= \left(a(d\sin(e+fx))^{-np}\left(c(d\sin(e+fx))^p\right)^n\right)\int \frac{(d\sin(e+fx))^{np}}{a^2-b^2\sin^2(e+fx)}\,dx - \frac{\left(b(d\sin(e+fx))^{-np}\right.}{}$$

$$= \frac{\left(b\sin^2(e+fx)^{-\frac{np}{2}}\left(c(d\sin(e+fx))^p\right)^n\right)\operatorname{Subst}\left(\int \frac{\left(1-x^2\right)^{\frac{np}{2}}}{a^2-b^2+b^2x^2}\,dx, x, \cos(e+fx)\right)}{f} - \frac{\left(ad(d\sin(\right.}{}$$

$$= \frac{bF_1\left(\frac{1}{2};-\frac{np}{2},1;\frac{3}{2};\cos^2(e+fx),-\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)\cos(e+fx)\sin^2(e+fx)^{-\frac{np}{2}}\left(c(d\sin(e+fx))^p\right)}{\left(a^2-b^2\right)f}$$

Mathematica [B] time = 18.1763, size = 1811, normalized size = 8.88

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*(d*Sin[e + f*x])^p)^n/(a + b*Sin[e + f*x]),x]
```

```
[Out] ((Sec[e + f*x]^2)^((n*p)/2)*(c*(d*Sin[e + f*x])^p)^n*Tan[e + f*x]*(Tan[e +
f*x]/Sqrt[Sec[e + f*x]^2])^(n*p)*((a^2 - b^2)*(1 + n*p)*AppellF1[1 + (n*p)/
2, (-1 + n*p)/2, 1, 2 + (n*p)/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x
]^2)/a^2]*Tan[e + f*x] + a*(b*(2 + n*p)*AppellF1[(1 + n*p)/2, (n*p)/2, 1, (
3 + n*p)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2] - a*(1 + n*p)*H
ypergeometric2F1[1 + (n*p)/2, (1 + n*p)/2, 2 + (n*p)/2, -Tan[e + f*x]^2]*Ta
n[e + f*x])))/(a^2*b*f*(1 + n*p)*(2 + n*p)*(a + b*Sin[e + f*x])*(((Sec[e +
f*x]^2)^(1 + (n*p)/2)*(Tan[e + f*x]/Sqrt[Sec[e + f*x]^2])^(n*p)*((a^2 - b^2
)*(1 + n*p)*AppellF1[1 + (n*p)/2, (-1 + n*p)/2, 1, 2 + (n*p)/2, -Tan[e + f*
x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]*Tan[e + f*x] + a*(b*(2 + n*p)*Appe
llF1[(1 + n*p)/2, (n*p)/2, 1, (3 + n*p)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*
Tan[e + f*x]^2] - a*(1 + n*p)*Hypergeometric2F1[1 + (n*p)/2, (1 + n*p)/2, 2
 + (n*p)/2, -Tan[e + f*x]^2]*Tan[e + f*x])))/(a^2*b*(1 + n*p)*(2 + n*p)) +
(n*p*(Sec[e + f*x]^2)^((n*p)/2)*Tan[e + f*x]^2*(Tan[e + f*x]/Sqrt[Sec[e + f
*x]^2])^(n*p)*((a^2 - b^2)*(1 + n*p)*AppellF1[1 + (n*p)/2, (-1 + n*p)/2, 1,
 2 + (n*p)/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]*Tan[e + f
*x] + a*(b*(2 + n*p)*AppellF1[(1 + n*p)/2, (n*p)/2, 1, (3 + n*p)/2, -Tan[e
+ f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2] - a*(1 + n*p)*Hypergeometric2F1[1
+ (n*p)/2, (1 + n*p)/2, 2 + (n*p)/2, -Tan[e + f*x]^2]*Tan[e + f*x])))/(a^2*
b*(1 + n*p)*(2 + n*p)) + (n*p*(Sec[e + f*x]^2)^((n*p)/2)*Tan[e + f*x]*(Tan[
e + f*x]/Sqrt[Sec[e + f*x]^2])^(-1 + n*p)*(Sqrt[Sec[e + f*x]^2] - Tan[e + f
*x]^2/Sqrt[Sec[e + f*x]^2])*((a^2 - b^2)*(1 + n*p)*AppellF1[1 + (n*p)/2, (-
1 + n*p)/2, 1, 2 + (n*p)/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/
a^2]*Tan[e + f*x] + a*(b*(2 + n*p)*AppellF1[(1 + n*p)/2, (n*p)/2, 1, (3 + n
*p)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2] - a*(1 + n*p)*Hyperg
eometric2F1[1 + (n*p)/2, (1 + n*p)/2, 2 + (n*p)/2, -Tan[e + f*x]^2]*Tan[e +
```

```
 f*x])))/(a^2*b*(1 + n*p)*(2 + n*p)) + ((Sec[e + f*x]^2)^((n*p)/2)*Tan[e +
f*x]*(Tan[e + f*x]/Sqrt[Sec[e + f*x]^2])^(n*p)*((a^2 - b^2)*(1 + n*p)*Appel
lF1[1 + (n*p)/2, (-1 + n*p)/2, 1, 2 + (n*p)/2, -Tan[e + f*x]^2, ((-a^2 + b^
2)*Tan[e + f*x]^2)/a^2]*Sec[e + f*x]^2 + (a^2 - b^2)*(1 + n*p)*Tan[e + f*x]
*((2*(-a^2 + b^2)*(1 + (n*p)/2)*AppellF1[2 + (n*p)/2, (-1 + n*p)/2, 2, 3 +
(n*p)/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]*Sec[e + f*x]^2
*Tan[e + f*x])/(a^2*(2 + (n*p)/2)) - ((1 + (n*p)/2)*(-1 + n*p)*AppellF1[2 +
 (n*p)/2, 1 + (-1 + n*p)/2, 1, 3 + (n*p)/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*
Tan[e + f*x]^2)/a^2]*Sec[e + f*x]^2*Tan[e + f*x])/(2 + (n*p)/2)) + a*(-(a*(
1 + n*p)*Hypergeometric2F1[1 + (n*p)/2, (1 + n*p)/2, 2 + (n*p)/2, -Tan[e +
f*x]^2]*Sec[e + f*x]^2) + b*(2 + n*p)*((2*(-1 + b^2/a^2)*(1 + n*p)*AppellF1
[1 + (1 + n*p)/2, (n*p)/2, 2, 1 + (3 + n*p)/2, -Tan[e + f*x]^2, (-1 + b^2/a
^2)*Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(3 + n*p) - (n*p*(1 + n*p)
*AppellF1[1 + (1 + n*p)/2, 1 + (n*p)/2, 1, 1 + (3 + n*p)/2, -Tan[e + f*x]^2
, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(3 + n*p)) -
2*a*(1 + (n*p)/2)*(1 + n*p)*Sec[e + f*x]^2*(-Hypergeometric2F1[1 + (n*p)/2,
 (1 + n*p)/2, 2 + (n*p)/2, -Tan[e + f*x]^2] + (1 + Tan[e + f*x]^2)^((-1 - n
*p)/2)))))/(a^2*b*(1 + n*p)*(2 + n*p))))
```

Maple [F] time = 0.144, size = 0, normalized size = 0.

$$\int \frac{\left(c\left(d\sin\left(fx+e\right)\right)^{p}\right)^{n}}{a+b\sin\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*sin(f*x+e))^p)^n/(a+b*sin(f*x+e)),x)
```

```
[Out] int((c*(d*sin(f*x+e))^p)^n/(a+b*sin(f*x+e)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(d\sin\left(fx+e\right)\right)^{p}c\right)^{n}}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n/(a+b*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate(((d*sin(f*x + e))^p*c)^n/(b*sin(f*x + e) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(\left(d\sin\left(fx+e\right)\right)^{p}c\right)^{n}}{b\sin\left(fx+e\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n/(a+b*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral(((d*sin(f*x + e))^p*c)^n/(b*sin(f*x + e) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c\left(d\sin\left(e+fx\right)\right)^{p}\right)^{n}}{a+b\sin\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))**p)**n/(a+b*sin(f*x+e)),x)
```

```
[Out] Integral((c*(d*sin(e + f*x))**p)**n/(a + b*sin(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(d\sin\left(fx+e\right)\right)^{p}c\right)^{n}}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n/(a+b*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate(((d*sin(f*x + e))^p*c)^n/(b*sin(f*x + e) + a), x)
```

3.836 $\int \frac{(c(d\sin(e+fx))^p)^n}{(a+b\sin(e+fx))^2}\,dx$

Optimal. Leaf size=322

$$\frac{2ab\cos(e+fx)\sin^2(e+fx)^{-\frac{np}{2}}\left(c(d\sin(e+fx))^p\right)^n F_1\left(\frac{1}{2};-\frac{np}{2},2;\frac{3}{2};\cos^2(e+fx),-\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)}{f\left(a^2-b^2\right)^2}-\frac{b^2\sin(e+fx)\cos(}{}$$

```
[Out] (2*a*b*AppellF1[1/2, -(n*p)/2, 2, 3/2, Cos[e + f*x]^2, -((b^2*Cos[e + f*x]^
2)/(a^2 - b^2))]*Cos[e + f*x]*(c*(d*Sin[e + f*x])^p)^n)/((a^2 - b^2)^2*f*(S
in[e + f*x]^2)^((n*p)/2)) - (b^2*AppellF1[1/2, (-1 - n*p)/2, 2, 3/2, Cos[e
+ f*x]^2, -((b^2*Cos[e + f*x]^2)/(a^2 - b^2))]*Cos[e + f*x]*Sin[e + f*x]*(S
in[e + f*x]^2)^((-1 - n*p)/2)*(c*(d*Sin[e + f*x])^p)^n)/((a^2 - b^2)^2*f) -
 (a^2*AppellF1[1/2, (1 - n*p)/2, 2, 3/2, Cos[e + f*x]^2, -((b^2*Cos[e + f*x
]^2)/(a^2 - b^2))]*Cot[e + f*x]*(Sin[e + f*x]^2)^((1 - n*p)/2)*(c*(d*Sin[e
+ f*x])^p)^n)/((a^2 - b^2)^2*f)
```

Rubi [A] time = 0.508049, antiderivative size = 322, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2826, 2824, 3189, 429, 16}

$$\frac{2ab\cos(e+fx)\sin^2(e+fx)^{-\frac{np}{2}}\left(c(d\sin(e+fx))^p\right)^n F_1\left(\frac{1}{2};-\frac{np}{2},2;\frac{3}{2};\cos^2(e+fx),-\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)}{f\left(a^2-b^2\right)^2}-\frac{b^2\sin(e+fx)\cos(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c*(d*Sin[e + f*x])^p)^n/(a + b*Sin[e + f*x])^2,x]
```

```
[Out] (2*a*b*AppellF1[1/2, -(n*p)/2, 2, 3/2, Cos[e + f*x]^2, -((b^2*Cos[e + f*x]^
2)/(a^2 - b^2))]*Cos[e + f*x]*(c*(d*Sin[e + f*x])^p)^n)/((a^2 - b^2)^2*f*(S
in[e + f*x]^2)^((n*p)/2)) - (b^2*AppellF1[1/2, (-1 - n*p)/2, 2, 3/2, Cos[e
+ f*x]^2, -((b^2*Cos[e + f*x]^2)/(a^2 - b^2))]*Cos[e + f*x]*Sin[e + f*x]*(S
in[e + f*x]^2)^((-1 - n*p)/2)*(c*(d*Sin[e + f*x])^p)^n)/((a^2 - b^2)^2*f) -
 (a^2*AppellF1[1/2, (1 - n*p)/2, 2, 3/2, Cos[e + f*x]^2, -((b^2*Cos[e + f*x
]^2)/(a^2 - b^2))]*Cot[e + f*x]*(Sin[e + f*x]^2)^((1 - n*p)/2)*(c*(d*Sin[e
+ f*x])^p)^n)/((a^2 - b^2)^2*f)
```

Rule 2826

```
Int[((c_.)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(p_))^(n_)*((a_.) + (b_.)*sin[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c^IntPart[n]*(c*(d*Sin[e + f*x
])^p)^FracPart[n])/(d*Sin[e + f*x])^(p*FracPart[n]), Int[(a + b*Sin[e + f*x
])^m*(d*Sin[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] &&  !IntegerQ[n]
```

Rule 2824

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(d*sin[e + f*x])^n/((a - b*sin[e +
 f*x])^m/(a^2 - b^2*sin[e + f*x]^2)^m), x], x] /; FreeQ[{a, b, d, e, f, n},
 x] && NeQ[a^2 - b^2, 0] && ILtQ[m, -1]
```

Rule 3189

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[
(ff*d^(2*IntPart[(m - 1)/2] + 1)*(d*Sin[e + f*x])^(2*FracPart[(m - 1)/2]))/
(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2]), Subst[Int[(1 - ff^2*x^2)^((m - 1)
/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, d,
 e, f, m, p}, x] && !IntegerQ[m]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\left(c(d\sin(e+fx))^p\right)^n}{(a+b\sin(e+fx))^2}\,dx = \left((d\sin(e+fx))^{-np}\left(c(d\sin(e+fx))^p\right)^n\right)\int \frac{(d\sin(e+fx))^{np}}{(a+b\sin(e+fx))^2}\,dx$$

$$= \left((d\sin(e+fx))^{-np}\left(c(d\sin(e+fx))^p\right)^n\right)\int\left(\frac{a^2(d\sin(e+fx))^{np}}{\left(a^2-b^2\sin^2(e+fx)\right)^2}-\frac{2ab\sin(e+fx)(d\sin}{\left(a^2-b^2\sin^2(e+\right.}\right.$$

$$= \left(a^2(d\sin(e+fx))^{-np}\left(c(d\sin(e+fx))^p\right)^n\right)\int\frac{(d\sin(e+fx))^{np}}{\left(a^2-b^2\sin^2(e+fx)\right)^2}\,dx-\left(2ab(d\sin(e+fx\right.$$

$$= \frac{\left(b^2(d\sin(e+fx))^{-np}\left(c(d\sin(e+fx))^p\right)^n\right)\int\frac{(d\sin(e+fx))^{2+np}}{\left(-a^2+b^2\sin^2(e+fx)\right)^2}\,dx}{d^2}-\frac{\left(2ab(d\sin(e+fx))^{-np}\right.}{}$$

$$= -\frac{a^2F_1\left(\frac{1}{2};\frac{1}{2}(1-np),2;\frac{3}{2};\cos^2(e+fx),-\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)\cot(e+fx)\sin^2(e+fx)^{\frac{1}{2}(1-np)}\left(c(d\sin\right.}{\left(a^2-b^2\right)^2f}$$

$$= \frac{2abF_1\left(\frac{1}{2};-\frac{np}{2},2;\frac{3}{2};\cos^2(e+fx),-\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)\cos(e+fx)\sin^2(e+fx)^{-\frac{np}{2}}\left(c(d\sin(e+fx)\right.}{\left(a^2-b^2\right)^2f}$$

Mathematica [B] time = 19.0039, size = 2036, normalized size = 6.32

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*(d*Sin[e + f*x])^p)^n/(a + b*Sin[e + f*x])^2,x]
```

```
[Out] -(((Sec[e + f*x]^2)^((n*p)/2)*(c*(d*Sin[e + f*x])^p)^n*Tan[e + f*x]*(Tan[e
+ f*x]/Sqrt[Sec[e + f*x]^2])^(n*p)*(-(a*(2 + n*p)*((a^2 + b^2)*AppellF1[(1
+ n*p)/2, (n*p)/2, 1, (3 + n*p)/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f
*x]^2)/a^2] - 2*b^2*AppellF1[(1 + n*p)/2, (n*p)/2, 2, (3 + n*p)/2, -Tan[e +
 f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2])) + 2*b*(a^2 - b^2)*(1 + n*p)*A
ppellF1[1 + (n*p)/2, (-1 + n*p)/2, 2, 2 + (n*p)/2, -Tan[e + f*x]^2, ((-a^2
```

```
+ b^2)*Tan[e + f*x]^2)/a^2]*Tan[e + f*x]))/(a^3*(a^2 - b^2)*f*(1 + n*p)*(2
+ n*p)*(a + b*Sin[e + f*x])^2*(-(((Sec[e + f*x]^2)^(1 + (n*p)/2)*(Tan[e + f
*x]/Sqrt[Sec[e + f*x]^2])^(n*p)*(-(a*(2 + n*p)*((a^2 + b^2)*AppellF1[(1 + n
*p)/2, (n*p)/2, 1, (3 + n*p)/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]
^2)/a^2] - 2*b^2*AppellF1[(1 + n*p)/2, (n*p)/2, 2, (3 + n*p)/2, -Tan[e + f*
x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2])) + 2*b*(a^2 - b^2)*(1 + n*p)*Appe
llF1[1 + (n*p)/2, (-1 + n*p)/2, 2, 2 + (n*p)/2, -Tan[e + f*x]^2, ((-a^2 + b
^2)*Tan[e + f*x]^2)/a^2]*Tan[e + f*x]))/(a^3*(a^2 - b^2)*(1 + n*p)*(2 + n*p
))) - (n*p*(Sec[e + f*x]^2)^((n*p)/2)*Tan[e + f*x]^2*(Tan[e + f*x]/Sqrt[Sec
[e + f*x]^2])^(n*p)*(-(a*(2 + n*p)*((a^2 + b^2)*AppellF1[(1 + n*p)/2, (n*p)
/2, 1, (3 + n*p)/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2] - 2
*b^2*AppellF1[(1 + n*p)/2, (n*p)/2, 2, (3 + n*p)/2, -Tan[e + f*x]^2, ((-a^2
 + b^2)*Tan[e + f*x]^2)/a^2])) + 2*b*(a^2 - b^2)*(1 + n*p)*AppellF1[1 + (n*
p)/2, (-1 + n*p)/2, 2, 2 + (n*p)/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e +
f*x]^2)/a^2]*Tan[e + f*x]))/(a^3*(a^2 - b^2)*(1 + n*p)*(2 + n*p)) - (n*p*(S
ec[e + f*x]^2)^((n*p)/2)*Tan[e + f*x]*(Tan[e + f*x]/Sqrt[Sec[e + f*x]^2])^(
-1 + n*p)*(-(a*(2 + n*p)*((a^2 + b^2)*AppellF1[(1 + n*p)/2, (n*p)/2, 1, (3
+ n*p)/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2] - 2*b^2*Appel
lF1[(1 + n*p)/2, (n*p)/2, 2, (3 + n*p)/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Ta
n[e + f*x]^2)/a^2])) + 2*b*(a^2 - b^2)*(1 + n*p)*AppellF1[1 + (n*p)/2, (-1
+ n*p)/2, 2, 2 + (n*p)/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^
2]*Tan[e + f*x])*(Sqrt[Sec[e + f*x]^2] - Tan[e + f*x]^2/Sqrt[Sec[e + f*x]^2
]))/(a^3*(a^2 - b^2)*(1 + n*p)*(2 + n*p)) - ((Sec[e + f*x]^2)^((n*p)/2)*Tan
[e + f*x]*(Tan[e + f*x]/Sqrt[Sec[e + f*x]^2])^(n*p)*(2*b*(a^2 - b^2)*(1 + n
*p)*AppellF1[1 + (n*p)/2, (-1 + n*p)/2, 2, 2 + (n*p)/2, -Tan[e + f*x]^2, ((
-a^2 + b^2)*Tan[e + f*x]^2)/a^2]*Sec[e + f*x]^2 + 2*b*(a^2 - b^2)*(1 + n*p)
*Tan[e + f*x]*((4*(-a^2 + b^2)*(1 + (n*p)/2)*AppellF1[2 + (n*p)/2, (-1 + n*
p)/2, 3, 3 + (n*p)/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]*S
ec[e + f*x]^2*Tan[e + f*x])/(a^2*(2 + (n*p)/2)) - ((1 + (n*p)/2)*(-1 + n*p)
*AppellF1[2 + (n*p)/2, 1 + (-1 + n*p)/2, 2, 3 + (n*p)/2, -Tan[e + f*x]^2, (
(-a^2 + b^2)*Tan[e + f*x]^2)/a^2]*Sec[e + f*x]^2*Tan[e + f*x])/(2 + (n*p)/2
)) - a*(2 + n*p)*((a^2 + b^2)*((2*(-a^2 + b^2)*(1 + n*p)*AppellF1[1 + (1 +
n*p)/2, (n*p)/2, 2, 1 + (3 + n*p)/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e +
 f*x]^2)/a^2]*Sec[e + f*x]^2*Tan[e + f*x])/(a^2*(3 + n*p)) - (n*p*(1 + n*p)
*AppellF1[1 + (1 + n*p)/2, 1 + (n*p)/2, 1, 1 + (3 + n*p)/2, -Tan[e + f*x]^2
, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]*Sec[e + f*x]^2*Tan[e + f*x])/(3 + n*p)
) - 2*b^2*((4*(-a^2 + b^2)*(1 + n*p)*AppellF1[1 + (1 + n*p)/2, (n*p)/2, 3,
1 + (3 + n*p)/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]*Sec[e
+ f*x]^2*Tan[e + f*x])/(a^2*(3 + n*p)) - (n*p*(1 + n*p)*AppellF1[1 + (1 + n
*p)/2, 1 + (n*p)/2, 2, 1 + (3 + n*p)/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[
e + f*x]^2)/a^2]*Sec[e + f*x]^2*Tan[e + f*x])/(3 + n*p)))))/(a^3*(a^2 - b^2
)*(1 + n*p)*(2 + n*p)))))
```

Maple [F] time = 0.394, size = 0, normalized size = 0.

$$\int \frac{\left(c \left(d \sin \left(f x + e\right)\right)^{p}\right)^{n}}{\left(a + b \sin \left(f x + e\right)\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*sin(f*x+e))^p)^n/(a+b*sin(f*x+e))^2,x)
```

```
[Out] int((c*(d*sin(f*x+e))^p)^n/(a+b*sin(f*x+e))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(d\sin\left(fx+e\right)\right)^{p} c\right)^{n}}{\left(b\sin\left(fx+e\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n/(a+b*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate(((d*sin(f*x + e))^p*c)^n/(b*sin(f*x + e) + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(\left(d\sin\left(fx+e\right)\right)^{p} c\right)^{n}}{b^{2}\cos\left(fx+e\right)^{2}-2\,ab\sin\left(fx+e\right)-a^{2}-b^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n/(a+b*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral(-((d*sin(f*x + e))^p*c)^n/(b^2*cos(f*x + e)^2 - 2*a*b*sin(f*x + e)
 - a^2 - b^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c\left(d\sin\left(e+fx\right)\right)^{p}\right)^{n}}{\left(a+b\sin\left(e+fx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))**p)**n/(a+b*sin(f*x+e))**2,x)
```

```
[Out] Integral((c*(d*sin(e + f*x))**p)**n/(a + b*sin(e + f*x))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(d\sin\left(fx+e\right)\right)^{p} c\right)^{n}}{\left(b\sin\left(fx+e\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n/(a+b*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate(((d*sin(f*x + e))^p*c)^n/(b*sin(f*x + e) + a)^2, x)
```

3.837 $\int \frac{\left(c(d\sin(e+fx))^p\right)^n}{(a+b\sin(e+fx))^3}\,dx$

Optimal. Leaf size=428

$$\frac{3a^2b\cos(e+fx)\sin^2(e+fx)^{-\frac{np}{2}}\left(c(d\sin(e+fx))^p\right)^n F_1\left(\frac{1}{2};-\frac{np}{2},3;\frac{3}{2};\cos^2(e+fx),-\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)}{f\left(a^2-b^2\right)^3}+\frac{b^3\cos(e+fx)\sin}{}$$

```
[Out] (3*a^2*b*AppellF1[1/2, -(n*p)/2, 3, 3/2, Cos[e + f*x]^2, -((b^2*Cos[e + f*x
]^2)/(a^2 - b^2))]*Cos[e + f*x]*(c*(d*Sin[e + f*x])^p)^n)/((a^2 - b^2)^3*f*
(Sin[e + f*x]^2)^((n*p)/2)) + (b^3*AppellF1[1/2, (-2 - n*p)/2, 3, 3/2, Cos[
e + f*x]^2, -((b^2*Cos[e + f*x]^2)/(a^2 - b^2))]*Cos[e + f*x]*(c*(d*Sin[e +
 f*x])^p)^n)/((a^2 - b^2)^3*f*(Sin[e + f*x]^2)^((n*p)/2)) - (3*a*b^2*Appell
F1[1/2, (-1 - n*p)/2, 3, 3/2, Cos[e + f*x]^2, -((b^2*Cos[e + f*x]^2)/(a^2 -
 b^2))]*Cos[e + f*x]*Sin[e + f*x]*(Sin[e + f*x]^2)^((-1 - n*p)/2)*(c*(d*Sin
[e + f*x])^p)^n)/((a^2 - b^2)^3*f) - (a^3*AppellF1[1/2, (1 - n*p)/2, 3, 3/2
, Cos[e + f*x]^2, -((b^2*Cos[e + f*x]^2)/(a^2 - b^2))]*Cot[e + f*x]*(Sin[e
+ f*x]^2)^((1 - n*p)/2)*(c*(d*Sin[e + f*x])^p)^n)/((a^2 - b^2)^3*f)
```

Rubi [A] time = 0.651891, antiderivative size = 428, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2826, 2824, 3189, 429, 16}

$$\frac{3a^2b\cos(e+fx)\sin^2(e+fx)^{-\frac{np}{2}}\left(c(d\sin(e+fx))^p\right)^n F_1\left(\frac{1}{2};-\frac{np}{2},3;\frac{3}{2};\cos^2(e+fx),-\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)}{f\left(a^2-b^2\right)^3}+\frac{b^3\cos(e+fx)\sin}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c*(d*Sin[e + f*x])^p)^n/(a + b*Sin[e + f*x])^3,x]
```


```
[Out] (3*a^2*b*AppellF1[1/2, -(n*p)/2, 3, 3/2, Cos[e + f*x]^2, -((b^2*Cos[e + f*x
]^2)/(a^2 - b^2))]*Cos[e + f*x]*(c*(d*Sin[e + f*x])^p)^n)/((a^2 - b^2)^3*f*
(Sin[e + f*x]^2)^((n*p)/2)) + (b^3*AppellF1[1/2, (-2 - n*p)/2, 3, 3/2, Cos[
e + f*x]^2, -((b^2*Cos[e + f*x]^2)/(a^2 - b^2))]*Cos[e + f*x]*(c*(d*Sin[e +
 f*x])^p)^n)/((a^2 - b^2)^3*f*(Sin[e + f*x]^2)^((n*p)/2)) - (3*a*b^2*Appell
F1[1/2, (-1 - n*p)/2, 3, 3/2, Cos[e + f*x]^2, -((b^2*Cos[e + f*x]^2)/(a^2 -
 b^2))]*Cos[e + f*x]*Sin[e + f*x]*(Sin[e + f*x]^2)^((-1 - n*p)/2)*(c*(d*Sin
[e + f*x])^p)^n)/((a^2 - b^2)^3*f) - (a^3*AppellF1[1/2, (1 - n*p)/2, 3, 3/2
, Cos[e + f*x]^2, -((b^2*Cos[e + f*x]^2)/(a^2 - b^2))]*Cot[e + f*x]*(Sin[e
+ f*x]^2)^((1 - n*p)/2)*(c*(d*Sin[e + f*x])^p)^n)/((a^2 - b^2)^3*f)
```


Rule 2826

```
Int[((c_.)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(p_))^(n_)*((a_.) + (b_.)*sin[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c^IntPart[n]*(c*(d*Sin[e + f*x
])^p)^FracPart[n])/(d*Sin[e + f*x])^(p*FracPart[n]), Int[(a + b*Sin[e + f*x
])^m*(d*Sin[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] && !IntegerQ[n]
```

Rule 2824

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(d*sin[e + f*x])^n/((a - b*sin[e +
 f*x])^m/(a^2 - b^2*sin[e + f*x]^2)^m), x], x] /; FreeQ[{a, b, d, e, f, n},
```

```
x] && NeQ[a^2 - b^2, 0] && ILtQ[m, -1]
```

Rule 3189

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[
(ff*d^(2*IntPart[(m - 1)/2] + 1)*(d*Sin[e + f*x])^(2*FracPart[(m - 1)/2]))/
(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2]), Subst[Int[(1 - ff^2*x^2)^((m - 1)
/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, d,
 e, f, m, p}, x] &&  !IntegerQ[m]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\left(c(d\sin(e+fx))^p\right)^n}{(a+b\sin(e+fx))^3}\,dx = \left((d\sin(e+fx))^{-np}\left(c(d\sin(e+fx))^p\right)^n\right)\int \frac{(d\sin(e+fx))^{np}}{(a+b\sin(e+fx))^3}\,dx$$

$$= \left((d\sin(e+fx))^{-np}\left(c(d\sin(e+fx))^p\right)^n\right)\int\left(\frac{a^3(d\sin(e+fx))^{np}}{\left(a^2-b^2\sin^2(e+fx)\right)^3}-\frac{3a^2b\sin(e+fx)(d\sin}{\left(a^2-b^2\sin^2(e+\right.}\right.$$

$$= \left(a^3(d\sin(e+fx))^{-np}\left(c(d\sin(e+fx))^p\right)^n\right)\int \frac{(d\sin(e+fx))^{np}}{\left(a^2-b^2\sin^2(e+fx)\right)^3}\,dx - \left(3a^2b(d\sin(e+f\right.$$

$$= \frac{\left(b^3(d\sin(e+fx))^{-np}\left(c(d\sin(e+fx))^p\right)^n\right)\int \frac{(d\sin(e+fx))^{3+np}}{\left(-a^2+b^2\sin^2(e+fx)\right)^3}\,dx}{d^3} + \frac{\left(3ab^2(d\sin(e+fx))^{-np}\right.}{}$$

$$= -\frac{a^3F_1\left(\frac{1}{2};\frac{1}{2}(1-np),3;\frac{3}{2};\cos^2(e+fx),-\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)\cot(e+fx)\sin^2(e+fx)^{\frac{1}{2}(1-np)}\left(c(d\sin\right.}{\left(a^2-b^2\right)^3 f}$$

$$= \frac{3a^2bF_1\left(\frac{1}{2};-\frac{np}{2},3;\frac{3}{2};\cos^2(e+fx),-\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)\cos(e+fx)\sin^2(e+fx)^{-\frac{np}{2}}\left(c(d\sin(e+fx\right.}{\left(a^2-b^2\right)^3 f}$$

Mathematica [B] time = 20.2075, size = 2660, normalized size = 6.21

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*(d*Sin[e + f*x])^p)^n/(a + b*Sin[e + f*x])^3,x]
```

```
[Out] -(((Sec[e + f*x]^2)^((n*p)/2)*(c*(d*Sin[e + f*x])^p)^n*Tan[e + f*x]*(Tan[e
+ f*x]/Sqrt[Sec[e + f*x]^2])^(n*p)*(-(a*(2 + n*p)*((a^2 + 3*b^2)*AppellF1[(
```

1 + n*p)/2, -1 + (n*p)/2, 2, (3 + n*p)/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Ta
n[e + f*x]^2)/a^2] - 4*b^2*AppellF1[(1 + n*p)/2, -1 + (n*p)/2, 3, (3 + n*p)
/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2])) + b*(3*a^2 + b^2)
*(1 + n*p)*AppellF1[1 + (n*p)/2, (-1 + n*p)/2, 2, 2 + (n*p)/2, -Tan[e + f*x
]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]*Tan[e + f*x] - 4*b^3*(1 + n*p)*Appe
llF1[1 + (n*p)/2, (-1 + n*p)/2, 3, 2 + (n*p)/2, -Tan[e + f*x]^2, ((-a^2 + b
^2)*Tan[e + f*x]^2)/a^2]*Tan[e + f*x]))/(a^4*(a^2 - b^2)*f*(1 + n*p)*(2 + n
p)(a + b*Sin[e + f*x])^3*(-(((Sec[e + f*x]^2)^(1 + (n*p)/2)*(Tan[e + f*x]
/Sqrt[Sec[e + f*x]^2])^(n*p)*(-(a*(2 + n*p)*((a^2 + 3*b^2)*AppellF1[(1 + n*
p)/2, -1 + (n*p)/2, 2, (3 + n*p)/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e +
f*x]^2)/a^2] - 4*b^2*AppellF1[(1 + n*p)/2, -1 + (n*p)/2, 3, (3 + n*p)/2, -T
an[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2])) + b*(3*a^2 + b^2)*(1 +
n*p)*AppellF1[1 + (n*p)/2, (-1 + n*p)/2, 2, 2 + (n*p)/2, -Tan[e + f*x]^2, (
(-a^2 + b^2)*Tan[e + f*x]^2)/a^2]*Tan[e + f*x] - 4*b^3*(1 + n*p)*AppellF1[1
 + (n*p)/2, (-1 + n*p)/2, 3, 2 + (n*p)/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Ta
n[e + f*x]^2)/a^2]*Tan[e + f*x]))/(a^4*(a^2 - b^2)*(1 + n*p)*(2 + n*p))) -
(n*p*(Sec[e + f*x]^2)^((n*p)/2)*Tan[e + f*x]^2*(Tan[e + f*x]/Sqrt[Sec[e + f
*x]^2])^(n*p)*(-(a*(2 + n*p)*((a^2 + 3*b^2)*AppellF1[(1 + n*p)/2, -1 + (n*p
)/2, 2, (3 + n*p)/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2] -
4*b^2*AppellF1[(1 + n*p)/2, -1 + (n*p)/2, 3, (3 + n*p)/2, -Tan[e + f*x]^2,
((-a^2 + b^2)*Tan[e + f*x]^2)/a^2])) + b*(3*a^2 + b^2)*(1 + n*p)*AppellF1[1
 + (n*p)/2, (-1 + n*p)/2, 2, 2 + (n*p)/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Ta
n[e + f*x]^2)/a^2]*Tan[e + f*x] - 4*b^3*(1 + n*p)*AppellF1[1 + (n*p)/2, (-1
 + n*p)/2, 3, 2 + (n*p)/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a
^2]*Tan[e + f*x]))/(a^4*(a^2 - b^2)*(1 + n*p)*(2 + n*p)) - (n*p*(Sec[e + f*
x]^2)^((n*p)/2)*Tan[e + f*x]*(Tan[e + f*x]/Sqrt[Sec[e + f*x]^2])^(-1 + n*p)
(-(a(2 + n*p)*((a^2 + 3*b^2)*AppellF1[(1 + n*p)/2, -1 + (n*p)/2, 2, (3 +
n*p)/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2] - 4*b^2*AppellF
1[(1 + n*p)/2, -1 + (n*p)/2, 3, (3 + n*p)/2, -Tan[e + f*x]^2, ((-a^2 + b^2)
*Tan[e + f*x]^2)/a^2])) + b*(3*a^2 + b^2)*(1 + n*p)*AppellF1[1 + (n*p)/2, (
-1 + n*p)/2, 2, 2 + (n*p)/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)
/a^2]*Tan[e + f*x] - 4*b^3*(1 + n*p)*AppellF1[1 + (n*p)/2, (-1 + n*p)/2, 3,
 2 + (n*p)/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]*Tan[e + f
x])(Sqrt[Sec[e + f*x]^2] - Tan[e + f*x]^2/Sqrt[Sec[e + f*x]^2]))/(a^4*(a^
2 - b^2)*(1 + n*p)*(2 + n*p)) - ((Sec[e + f*x]^2)^((n*p)/2)*Tan[e + f*x]*(T
an[e + f*x]/Sqrt[Sec[e + f*x]^2])^(n*p)*(b*(3*a^2 + b^2)*(1 + n*p)*AppellF1
[1 + (n*p)/2, (-1 + n*p)/2, 2, 2 + (n*p)/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*
Tan[e + f*x]^2)/a^2]*Sec[e + f*x]^2 - 4*b^3*(1 + n*p)*AppellF1[1 + (n*p)/2,
 (-1 + n*p)/2, 3, 2 + (n*p)/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^
2)/a^2]*Sec[e + f*x]^2 + b*(3*a^2 + b^2)*(1 + n*p)*Tan[e + f*x]*((4*(-a^2 +
 b^2)*(1 + (n*p)/2)*AppellF1[2 + (n*p)/2, (-1 + n*p)/2, 3, 3 + (n*p)/2, -Ta
n[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]*Sec[e + f*x]^2*Tan[e + f*x
])/(a^2*(2 + (n*p)/2)) - ((1 + (n*p)/2)*(-1 + n*p)*AppellF1[2 + (n*p)/2, 1
+ (-1 + n*p)/2, 2, 3 + (n*p)/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]
^2)/a^2]*Sec[e + f*x]^2*Tan[e + f*x])/(2 + (n*p)/2)) - 4*b^3*(1 + n*p)*Tan[
e + f*x]*((6*(-a^2 + b^2)*(1 + (n*p)/2)*AppellF1[2 + (n*p)/2, (-1 + n*p)/2,
 4, 3 + (n*p)/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]*Sec[e
+ f*x]^2*Tan[e + f*x])/(a^2*(2 + (n*p)/2)) - ((1 + (n*p)/2)*(-1 + n*p)*Appe
llF1[2 + (n*p)/2, 1 + (-1 + n*p)/2, 3, 3 + (n*p)/2, -Tan[e + f*x]^2, ((-a^2
 + b^2)*Tan[e + f*x]^2)/a^2]*Sec[e + f*x]^2*Tan[e + f*x])/(2 + (n*p)/2)) -
a*(2 + n*p)*((a^2 + 3*b^2)*((-2*(-1 + (n*p)/2)*(1 + n*p)*AppellF1[1 + (1 +
n*p)/2, (n*p)/2, 2, 1 + (3 + n*p)/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e +
 f*x]^2)/a^2]*Sec[e + f*x]^2*Tan[e + f*x])/(3 + n*p) + (4*(-a^2 + b^2)*(1 +
 n*p)*AppellF1[1 + (1 + n*p)/2, -1 + (n*p)/2, 3, 1 + (3 + n*p)/2, -Tan[e +
f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]*Sec[e + f*x]^2*Tan[e + f*x])/(a^
2*(3 + n*p))) - 4*b^2*((-2*(-1 + (n*p)/2)*(1 + n*p)*AppellF1[1 + (1 + n*p)/
2, (n*p)/2, 3, 1 + (3 + n*p)/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]
^2)/a^2]*Sec[e + f*x]^2*Tan[e + f*x])/(3 + n*p) + (6*(-a^2 + b^2)*(1 + n*p)
*AppellF1[1 + (1 + n*p)/2, -1 + (n*p)/2, 4, 1 + (3 + n*p)/2, -Tan[e + f*x]^

```
2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]*Sec[e + f*x]^2*Tan[e + f*x])/(a^2*(3
+ n*p))))))/(a^4*(a^2 - b^2)*(1 + n*p)*(2 + n*p)))))
```

Maple [F] time = 0.436, size = 0, normalized size = 0.

$$\int \frac{\left(c\left(d\sin\left(fx+e\right)\right)^{p}\right)^{n}}{\left(a+b\sin\left(fx+e\right)\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*sin(f*x+e))^p)^n/(a+b*sin(f*x+e))^3,x)
```

```
[Out] int((c*(d*sin(f*x+e))^p)^n/(a+b*sin(f*x+e))^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(d\sin\left(fx+e\right)\right)^{p}c\right)^{n}}{\left(b\sin\left(fx+e\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n/(a+b*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] integrate(((d*sin(f*x + e))^p*c)^n/(b*sin(f*x + e) + a)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\left(\left(d\sin\left(fx+e\right)\right)^{p}c\right)^{n}}{3\,ab^{2}\cos\left(fx+e\right)^{2}-a^{3}-3\,ab^{2}+\left(b^{3}\cos\left(fx+e\right)^{2}-3\,a^{2}b-b^{3}\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n/(a+b*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] integral(-((d*sin(f*x + e))^p*c)^n/(3*a*b^2*cos(f*x + e)^2 - a^3 - 3*a*b^2
+ (b^3*cos(f*x + e)^2 - 3*a^2*b - b^3)*sin(f*x + e)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c\left(d\sin\left(e+fx\right)\right)^{p}\right)^{n}}{\left(a+b\sin\left(e+fx\right)\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))**p)**n/(a+b*sin(f*x+e))**3,x)
```

```
[Out] Integral((c*(d*sin(e + f*x))**p)**n/(a + b*sin(e + f*x))**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(d \sin\left(fx+e\right)\right)^{p} c\right)^{n}}{\left(b \sin\left(fx+e\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n/(a+b*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate(((d*sin(f*x + e))^p*c)^n/(b*sin(f*x + e) + a)^3, x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
```

```
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
    ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
```

```
                print("both result and optimal complex");
             fi;
             #both result and optimal complex
             if leaf_count_result<=2*leaf_count_optimal then
                return "A";
             else
                return "B";
             end if
          else #result contains complex but optimal is not
             if debug then
                print("result contains complex but optimal is not");
             fi;
             return "C";
          end if
       else # result do not contain complex
            # this assumes optimal do not as well
            if debug then
                print("result do not contain complex, this assumes optimal do
    not as well");
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
               if debug then
                  print("leaf_count_result<=2*leaf_count_optimal");
               fi;
               return "A";
            else
               if debug then
                  print("leaf_count_result>2*leaf_count_optimal");
               fi;
               return "B";
            end if
       end if
  else #ExpnType(result) > ExpnType(optimal)
       if debug then
          print("ExpnType(result) > ExpnType(optimal)");
       fi;
       return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
```

```
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
```

```
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
```

```
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
```

```
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len
    (flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                         #since this estimate of leaf count is bit lower than
```

```
                      #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','
    hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
    sagemath

```

```
def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
```

```
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
    debug = False;

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```